KWAME NKRUMAH UNIVERSITY OF SCIENCE AND TECHNOLOGY, KUMASI

FORECASTING MORTALITY RATE: THE PERSPECTIVE OF KNUST GUSSS

By

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Declaration

I declare that I have personally, under supervision, undertaken the study herein submitted towards the award of MPhil degree and to the best of my knowledge contains no material previously published by another person nor material accepted for the award of any other degree of the university, except where due acknowledgement made in the text.

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Dedication

This research is dedicated to my wonderful parents Mr. George and Mrs. Beatrice Boateng, my siblings Prince, Bright and Jesse and my children yet unborn and all loved ones.

Abstract

Longevity risk is something that is coming to the notice of the world. For over some time now, life expectancy is increasing rapidly and pension fund managers are on the lookout. In this research, we sought to the estimation of mortality rate, measurement of longevity and forecast of mortality rates. The Cairns-Blake-Dowd (CBD) (2006) model was used for longevity and Holt's linear trend method was used for the forecast of the mortality rates. The research models were applied on data from the Kwame Nkrumah University of Science and Technology section of the Ghana University Staff Superannuation Scheme pensioners within the period of 2005 to 2015. At the end of the research, mortality rates were estimated by the application of the force of mortality. The estimated rates were found to directly depend on the number of deaths. longevity was measured and was found to go with higher ages. That is, the chance for an eighty (80)year old to live longer than expected after retirement is higher than that of a seventy (70) year old. Future mortality rates predicted. At the end, alpha (0.6) and beta (0.4) was found to be the best forecast point with future mortality rates ranging between 0.010 and 0.017.

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Chapter 1

INTRODUCTION

1.1 Background of Study

There is time for everything in life, hence a time to work and a time to retire from service. Pension is when one retires from active service. During the work life of the individual, contributions are made to secure him/her on retirement and this is termed Pension Plan. The contribution made is a percentage of the employees' basic salary. The issue of pension funds is the length of time between the first payment and the receiving of benefits which can be affected by inflation.

Surprisingly, there was not much awareness about the historical development of age pensions. Public sector pensions can be dated back to Ancient Rome, which gave pensions to personnel of the military. Augustus formalized a pension plan for veterans in 13BC, which promised a pension for those with 20 years in service. The pension plan, sort to offer a rate of replacement between 2/3rds and 3/4 of a laborers' income and was financed by tax but only for a few years. The fall of the Roman Empire ended these arrangements.

The first official superannuation fund was instituted for officials of the customs in 1712 and a century after, around 1810 the Parliament of Britain took initial procedures in setting up a government pension scheme. Towards the end of the 19th century bigger non-governmental companies began to introduce occupational pension schemes of their own. (Frank Eich, 2009)

The term "pension plan" refers to a workplace retirement benefit. It used to be known as the defined-benefit plans which were once the norm for retirement. With deferred-benefit pension plan, you or your employer contribute into a fund while you work. The employer then provides you with income for the rest of your life when you retire. Pension payments can be tied to your income when you retire and often get indexed for inflation.

Pension is when an organized amount is regularly given to a person, normally after retirement. Usually under already stated terms. A pensioner or retiree is the person who is no longer in service and receives the retirement pension.

Retirement plan and superannuation describe a pension approved on retirement. A pension plan is called retirement plans, pension schemes and superannuation plans in United States, United Kingdom and Ireland and Australia and New Zealand respectively. A pension plan set up by an employer on behalf of the employees benefit is normally known as occupational/employer pension.(ERISA)

In contributory pension plan both the employer and the employee contributes to the plan where as in non-contributory pension plan only the employer contribute to the plan. Defined contribution and Defined benefit pension plans are the two types of pension plans.

1.1.1 Defined Benefit Pension Plan

A defined benefit (DB) pension plan is a pension whereby the employer assures a particular benefit on retirement that is already estimated by a formula developed on the earnings history, number of years in service and age of the employee rather than directly depending on the investment returns. Governmental, non-governmental and a huge number of businesses offer defined benefit plans. A defined benefit plan is 'defined' in the sense that the benefit formula is known in advance. A defined benefit pension plan defines the benefit for the employee on retirement.

The formula used is subject to the employee's earnings towards the end of service which is termed terminal or final salary. With this formula, benefits are expressed as a percentage of average earnings for the number of years in service of the worker.

Defined benefit plans are often funded solely by the contributions of the employer under the non-governmental section. For sole owner companies with few employees, higher percentage of the benefits is given to the employer. However, only employee contributions are needed under defined benefit plan in the government sector.

The cost of defined benefit plan is calculated with much efforts hence an actuary or actuarial software is of necessity. With the best of applications however, the defined benefit cost plan estimate will always be subjected to economic and financial beliefs. The mean retirement age and the length of time the employee is still alive, the interest earned by the pension plan investments and extra levies like those approved by the Pension Benefit Guaranty Corporation in America are some of the assumptions. The benefit is for sure but the contribution is uncertain even if it is calculated by a professional. (Thomson West, 2013)

1.1.2 Define Contribution Pension Plan

A defined contribution (DC) pension plan is a retirement plan in which the employer and the employee together or separately make contributions regularly into the pension fund. Separate accounts are set up for each contributor and the benefits paid are a combination of the contributed amounts and the interest on the investments of the funds. Only the employer contributions are certain but not the benefits for the future. In defined contribution, benefits for the future change frequently on the basis of the investment earnings. The defined contribution pension that is often operated are the savings and thrift pension. Under this pension plan, the worker contributes a portion of his or her salary that is predetermined into the individual accounts created, all or part of which is matched by the employer. Due to the cost of administration and the stress free of ensuring the employers legal responsibility for a defined contribution plan, no actuary is needed to calculate the lump sum equivalent like what is done for defined benefit plan. In actual fact, defined contribution pension plan is generally easy. (Cannon et al 2012). Defined contributions are classified as original, modified and hybrid defined contribution pension plans. (Yan Li et al 2014)

- Original defined contribution plan is where much details are provided to determine the annual contributions the employer made for each employee and any other relevant regulations.
- Modified defined contribution plan is also the case where the original DC plan alone is not enough for employees as more compensation expenses are necessary. In this case, there should be an initial accumulated amount considering the past service Years for the employee which is invested until retirement. The modified DC plan may be easy to accept. Here more expenditure is required for employee with longer past service years.
- Hybrid defined contribution plan is where the plan guarantees more effectively the profits. The hybrid DC plan is similar to defined benefit pension plan.in this situation, the plan manager keeps track of the contributions made into the individual accounts and guarantees a crediting pension level. The plan fund is affected when there is either an increase or decrease in the employee's salary. Here there is compensation for an employee who needs assistance.

1.2 Problem Statement

We all want a long lasted life with good health and a maintained level of normal way of life that satisfies us. Apparently we would not like to run of money when we go on retirement. In recent times due to good health care people are living longer and its having an effect on pension schemes. When one goes on retirement a portion of his pension benefit is paid as a lump sum and the remaining part is spread for a minimum of fifteen years and a maximum of twenty years. After these years if the pensioner is alive he/she will receive the monthly pension benefit continually until death. The longer the pensioner lives the more the pension scheme pays. For instance Turner 2006 reports that in the last century (100 years), life expectancy has greatly been under estimated for about 18 months for each ten (10) years.Life expectancy after retirement is also underestimated. Pensioners are living longer than expected. The reduction in mortality rate has a huge effect on the viability of pension funds since under estimating life expected after retiring from active service will lead to amounts not growing large before retirement.

1.3 Objectives

Pension funds pay monthly pension benefit to each pensioner starting from the date of retirement and continues until death. The employer and the employee who will be the future beneficiary can finance the pension plan for the employee's entire working period, however, any shortcomings of the pension funds in the future is normally the employer's obligation. It is therefore important for the employer with high confidence and precision be able to predict the complete amount that will be needed to cater for future pension duties.

Hence the main aim of this research is:

- To estimate mortality rates

- To measure longevity
- To forecast future mortality rates.

1.4 Scope of Study

The subject of pension is one that is of interest to all workers and even their dependents. The study of this research is geographically focused on the working class in government sector particularly senior members of Kwame Nkrumah University of Science and Technology (KNUST) who contribute to Ghana University Staff Superannuation Scheme (GUSSS).

1.5 Methodology

The Cairns-Blake-Dowd (2006) mortality model is a mortality model for higher ages designed for longevity (Annuties and Pensions) and long term mortality rates. The CBD model expliots relative simplicity of mortality curve at higher ages and alos good for management. The CBD model is an extrapolative model. Mortality rates have reduced which pre-supposes that future mortality rates needs to be taken noticed of.

1.6 Justification

In developing countries, populations are aging much faster than developed countries and this is a cause for great concern. Life expectancy in the Ghana has been fixed at 55 years 4 months and 59 years 6 months which show a hike in life expectancy from 50 years 3 months and 53years 8 months for males and females respectively in 1984 according to the Ghana Statistical Service report in 2003. The private pension scheme market in Ghana is yet to be explored. As reported by the Ststistical services, there is the implication that there is a reduction in mortality rate since people (retirees) are living longer than expected.

Hence as people (retirees) are ageing and living longer pension fund houses should be able to pay monthly pension benefit to retirees until death.

1.7 Organization of the study

The research has been structured into five chapters.

Chapter one describes the introduction which is made up of the general background of study, problem statement, objectives of the research, justification, scope of study, methodology and organization of research.

Chapter two will present the various literatures that are connected to the study. That is a detailed description of the study from a concerned point of view.

Chapter three has to do with the methodology. It is the backbone of the research in relation to the method used.

Chapter four is where the data collection and analysis is done. The results is also presented here.

Chapter five is where we give the summary, conclusion and recommendations on the research.

Chapter 2

LITERATURE REVIEW

2.1 Introduction

Pension is when one retires from active service and a fixed amount is regularly (monthly) paid to the retiree from the pension scheme that the person was contributing to durung the working period. During the work life of the individual, contributions are made to secure him/her on retirement and this is termed Pension Plan. The contribution made is a percentage of the employees' basic salary. The issue of pension funds is the length of time between the first payment and the receiving of benefits. The monthly pension benefit will be paid until the death of the retiree.

2.2 Introduction of pensions in Ghana and Pension Schemes at KNUST

Pension scheme in Ghana dates back to 1946 and has gone through various changes and developments from pre-independence through to post-independence. A non-contributory pension scheme which is the first scheme in the country was introduced by the government in 1946 to take care of the workers of the colonial administration office retirement benefits. The CAP 30 pension scheme which was initiated by the Pensions Ordinance Number 42 (CAP 30) and superannuation schemes for public or government workers which included certified teachers, university lecturers and Gold Coast workers were established in 1950 through to the early 1960s. However, the CAP 30 was not able to take care of all public and private workers hence the Social Security Act (No. 279) known as Provident Fund was passed in 1965 to pay lump sum benefits to the aged, invalids and survivor's benefits. Social Security and National Insurance Trust (SSNIT) in 1972 under the NRCD 127 to regulate the National Social Security Scheme which was to replace the Social Security Act, 1965 (Act 279) was created. After twenty five years of operations, it was converted to Social Security Pension Scheme which invested in long term government bonds with low interest rate. Due to the insufficient benefits that were paid, the Social Security Act, 1991 (PNDC Law 2427) was made into law to change the 1972 Provident Fund to a Defined Benefit scheme to bring improvement into retirement entitlement of workers. The current National Pension Act, 2004 (Act 766) was passed into law which was led by the Bediako Commission set up in 2004. The new ACT introduced a three-tier pension scheme to give better pension benefits to workers who payments into it and requires 18.5% over all contribution of with the employer contributing 13% and the worker paying the remaining 5.5%.

The breakdown of the contributions made is as follows:

a. First tier; is controlled by the SSNIT and is required by law for all employees in the government and non-governmental sectors. It is 13% out of the total contributions out of which 2.5% goes to the scheme of National Health Insurance.

b. Second tier; is funded fully by workers and carefully looked after by approved Trustees privately supported by Managers and Custodians of Pension Fund. It was formed to mainly to give those who make payments into it one time benefits. It is 5% out of the total contributions.

c. Third tier; is provident fund and private pension schemes which is not forced by law but out of the employers own will and supported by tax incentives for workers in both the public and private domain.

Provision of pension benefits with secured retirement income security, receive retirement and related benefits as and when due and also establish the same set of rules and regulations of good standards for the plan, organization and run, payment of retirement and all linked benefits for workers in the public and the private was the aim of the scheme.

Contributions are made at the close of each month which is taken from the employer's salary for the period. The worker makes a contribution of 5.5% to the social security scheme. A person can join the scheme with a minimum age of fifteen years and a maximum age of forty-five years. The pension scheme has a legal retirement age of sixty years but fifty five years for hazardous work like mining. However one can willingly retire from service at age of fifty-five. One has to contribute to the social security scheme for at least fifteen years which is equivalent one hundred and eighty (180) months to qualify for monthly benefit payment pension. However if a contributor contributes for less than fifteen years and retires, a lump sum money of his/her total contribution and 75% of the current government treasury bill rate is paid to the contributor as retirement benefit. In the event of death of a contributor who is still in service, a onetime benefit is paid to the beneficiaries of the deceased. The minimum pension payment is on 50% of the average per annum salary for the best three years of a contributors' life in service. In the case of working more than the minimum required contribution period, the amount of pension paid is hiked by $\frac{1}{2}$% for every 12 months worked up to eighty per cent maximum. Twenty five percent of the total pension benefit is paid as a lump sum and the remaining seventy five percent is paid as monthly pension benefit for fifteen years. If the pensioner dies within the fifteen years, the entitlement is paid to his/her beneficiaries but after the fifteen years no benefits are paid. However if the pensioner survives after the fifteen years he/she would continue to receive the monthly pension benefit until death.

2.2.1 Ghana University Staff Superannuation Scheme

Ghana University Staff Superannuation Scheme (GUSSS) is a pension scheme that was established on 1st January 1976. The universities involved in the scheme University of Ghana. University of Science and Technology, University of Cape Coast, University of Development Studies and all other government institutions. Members of GUSSS include university teachers and research fellows, university administrative, library and professional staff with status similar to that of university teachers. The pension scheme is administered by the Finance Officer under the control of a Management Board. Contributions are made to the scheme monthly at a rate of fifteen percent (15%) of the basic salary which is subject to change. A member qualifies for benefits under either compulsory retirement or voluntary retirement. Like SSNIT one has contribute for a minimum of fifteen years. If a person retires without reaching the minimum period, a lump sum benefit of the total contributions made (both employer and personal) plus interest at a rate to be determined by the Management Board. There are two types of benefit payments under this scheme;

1. Full Pension

   The full pension is calculated as $\frac{1}{40}$ $\times$ terminal salary $\times$ number of years contributed to the scheme. Where terminal salary is the basic salary $\times$ 12 months.

2. Reduced Pension plus Gratuity

   Here a gratuity which is $\frac{1}{4}$ $\times$ full pension $\times$ 20 is paid as a lump sum and the remaining $\frac{3}{4}$ is paid as a reduced pension. Under the reduced pension, monthly benefits are paid for a period of twenty years after retirement. After the twenty years if still alive the benefits will continue to be paid until death but if death occurs within the period, the remaining benefits will be paid to the beneficiaries. However if death occurs after the period no benefits are paid.

2.3 Some Thoughts on Longevity

During the last hundred years, expectancy of life on the rise from twenty-five (25) to thirty (30) years in most developed countries and this is good from human point of view. Money-connected institutions like pension houses, government pensions and life insurance companies had to face the risk of living longer than expected. Longevity risk inherent on retirement and lifetime benefits plan made pension houses and insurers of life pay more than usual as was due to the rise in life expectancy according to Lulian et al (2014). Hence, regulations had to be put in place to keep a balance and take charge of the inherent risks in such plans. The special feature of such insurance products was their long term maturity. Longevity risk implied maturities could reach 50 to 80 years and involve various risks that were to be calculated carefully. From a financial and economic point of view, many improved systems such as retirement bill and the setup of long-term insurance care were introduced to as a result of the ageing population. To control longevity risk, it was important to analyze the in sequence of longevity on economic dependence.

Longevity is a 'trend risk' whilst mortality is a 'variability risk'. Could we call mortality risk to hedge longevity risk? How can a trend risk be priced? Long-term horizons had financial impact like interest rate risk been pre-dominant. Even if equality between mortality and longevity risks certainly existed, it was not easy to obtain a major risk reduction between the two due their separate natures.

Indeed, the replica of death insurance contracts with life annuities was not without faults because it did not matter to people of similar class and however mortality gave much concern to the individuals insured with bigger share of the portfolio capital. As earlier stated, many companies were affected by longevity risk.

A research by Antolin and Blommestein (2007) underlined that improvement in the longevity of eighty years and above people had important influence on the gross domestic product and political decisions of the country. Population of the aged had macroeconomic after come which was a general reason of gradual decrease in the economy for some countries. Given that when life expectancy increased the issue lessened and consumption increased. The population of the ageing does not necessarily agree to economic ageing but on the other hand encourage the ageing economy with ideas in areas like health, automation of the home, cities plan and transport just to mention but a few. In most developed countries, redevelopment of the urban areas are occur in order to allow free flow of blood in the body of the old.

There had been an improvement in rate of mortality in the population of various developed nations over the years and this had important result on the insurance section financially, since several important amounts owed by the insurance firm were linked to mortality trends of the future. This doubt that concerns future of mortality brought to light the issue of longevity risk. Longevity risk has a major position in the management of insurance firm because of the beliefs about the gradual development of mortality for the future only allow the company to face its future roles in the right direction. Longevity risk represent a mini module for underwriting the risk module in the Solvency II framework.

The famous Lee-Carter model common and used by a lot of people at various places for the careful study of longevity. According to an article by Jindrová, Slavíček in 2012, dealt with the growth and the forth telling of life expected at pension by applying Lee-Carter model. Most mortality models were constructed in similar ways. Most especially, when fitted to historical data, one or more varying parameters which is time dependent be identified.

By estimating the future of these parameters, we could achieve a forecast for future death probabilities and other demographic quantities such as life expectancies. They are necessary in expressing longevity in risks of pension as well as been benchmarks for longevity-linked capital markets. (Jan Gogola, 2015)

2.4 Works Done in Relation to Longevity Risk

According to the model which was brought into existence by Ronald D. Lee and Lawrence Carter in 1992 which got expanded in the early 1990s and attempted to use reverse estimation to decide rates in past demography. The Social Security plan in the America, the US Census Bureau and the United Nations have all had a feel of the model. In the world today, the Lee-Carter model has become the model used by a lot of people as mortality forecasting technique.

According to Samwick 1999, in developed countries, decreased fertility, mortality, productivity rates and privatization of Chile social security as a means of financial development exhibited a global interest in social security organization. A lot of countries for over twenty-five years used this work to analyze social security input on saving. The characteristic change of social security systems was used to find out the success of the cash and carry system and the unfunded system had any connection with the higher government saving. Nations that used defined contribution system had higher savings rates were some of the observations made. However, countries operating the solely cash and carry systems had lower saving rates, thus increased the coverage rate.

As was done by Aspinal in 2000, the domino hypothesis of the start of age in connection with the immune insufficiency suggested the consequence of a cascade of events that began with the involution of the thymus. Involution was connected with a less thymic output leading to fewer naïve T cells which contributed to the peripheral T-cell pool. Homeostatic mechanisms, maintained the number of T cells in the peripheral pool within exact limits, produced the proliferation and prolong the survival of resident T cells that filled the niches left emptied by the absent naïve T cells. Falling thymic output would be matched by resident T-cell proliferation and with age those proliferating cells would reach their replicative limit. The accumulation of cells unable to be replicated, that produced a decline in immune function and likely to be affected by infection or certain cancers was as a result of the prolonged survival. Gender difference comparison in life-span and rate of death in each age group which resulted in infectious or parasitic disease suggested that the female immune system works more efficiently and effectively for longer period than the immune system in the male. The conclusion drawn was that involution of the thymus, and therefore thymic output, occurs more frequently in males than in females. Ageing thymic atrophy male female longevity.

According to Brouhns et al 2002, life annuities were priced by projected life tables because they contained future trends in mortality forecast. The tables, however did not represent future mortality also known as longevity risk. Benjamin and Soliman in 1993, McDonald in 1997 and McDonald et al. in 1998 all demonstrated mortality in adults and old age and was revealed the decreasing death probabilities annually. It is easily seen that the changes had effect on pricing and life annuities reserve as stated by the research done by Marocco and Pitacco in 1998 and also Olivieri in 2001. The expected present values calculations required a good estimate of what will happen to mortality later on to avoid under estimation of costs. Estimations were extensions of recent trends as far as they could be aware from the statistics of mortality. The purpose of the work was to describe the doubts inbuilt to mortality projection in the frame work of the log-bilinear Poisson regression model by Brouhns, Denuit and Vermunt in 2002. The matrix of the death rates of the Dutch from 1950 to 2000 for ages ranging between 60 and 98 was fitted to the model. The data was connected to the whole population in Dutch which had been given by the Centraal Bureau voor de Statistiek (CBS). To the end, a conclusion was made that the study of the changes in the amount of premium and of the matching probabilities were of prime concern for the decision concerning the state of reinsurance needed.

According to Ermanno 2002, mortality trends uncertainty from which longevity risk in living thongs arose was studied in their research. The influence longevity risk had on benefits on life annuities, sickness for the aged and long-term care contracts were analyzed financially. Lastly, special care was given to mortality guarantees and life annuities made flexible (in the aspect of total after-retirement income plan).

The research conducted by Lorenzo and Sibillo in 2002 presented a model that involved an in depth analysis of changes in the number of deaths at a particular place over a period of time and money related risks for group of policies. The influence of longevity risk in life annuity was studied and took into account the interactions with the source of financial risk. Particularly the randomness in choosing projected mortality rates was considered in the valuation of portfolios and reserves. Numerical examples illustrated the results and showed the behavior of the projection risk. The aim was to contribute its quota to the measurement of longevity risk component; in particular projection risk that was in connection to the selected projected mortality table used for portfolio valuations. The European Economic Area (EEA) system was used. In the end, projection risk was considered a framework with interest and mortality been random. The effective implications of the work came true in the possibility of the detailed study of changes in the number of deaths at a particular place over a period of time and money related risks and their effects in the risk of a portfolio globally.

Picone et al 2004, irrespective of the related importance, time preference, expected longevity, uncertainty and risk aversion on behavior and the effects had not been taken noticed of. The Health and Retirement Study (HRS) data was used in the assessment of the duty of risk and desired time, expected longevity and exposure on the request for three ways used for the detection of cancer of the breast and cervix. The ways are regular breast self-exams, mammograms, and Pap smears. It was observed that people with high life expectancy and less desired time were probable to undertake cancer screening while people with risk averse been less were likely to undergo testing.

Jin et al 2005, in their research conducted showed that as demographic transition proceeded regionally and globally, a reliable longevity insurance market development was essential. Such need in insurance was most needed in Asia, however longevity risk was not well managed everywhere. The formal and informal set of ideas that related to longevity risk and insurance changed demographic and policy landscape was reviewed by this research. The reasons behind the lack of success in longevity insurance markets and the introduction of new ideas in the markets and a general policy which might aid to get a market full of life and energy with variety of longevity insurance products was analyzed.

According to MacMinn research conducted in 2006, the state, businesses and people were faced with high longevity risk in various countries. Fertility risk has reduced and in connection with the increased longevity risk has increased the longevity problem with the cash and carry system of the general public government pension plans by reduction in tax and increase the amount of money paid for the period. Other tools will be required to manage longevity risk as mortality at higher ages had improved. Existed tools for longevity management include social security system by government, defined benefit plans by businesses via pension funds and life annuities by individual insures. The large numbers law would be sufficient for pension funds and insurers to manage in the total absence of longevity risk.

YU Wei-qiang 2006, considered the regular increasing of human life expectancy with deficiency that showed up in the pension fund of many countries and companies. The clear success of catastrophe bonds and mortality indexed bonds, discovered a hedging method, securitization of longevity risk. The price of the bond was calculated by the use of Markovian chain and random numeric simulation. In the end, suggestions were received to aid solve the difficulties in the conversion of longevity risk in the current capital market.

The research by Stallard 2006, showed the basics in the modeling of longevity risk that was used without proof in forecast models structured to involve past experience into the future years' experience demographically. Stochastic forecasts were needed to express the uncertainty of survival functions forecasted for a defined group of people, which included doubt about process variance, errors of the parameter and model mis specification. This work looked at the written works on the form and causes of changes in longevity in the past and the deterministic and stochastic forecasting efforts subject on the data. It was revealed that credible other sets of forecasting beliefs have been obtained from the same past data, which implied the developed method into a single well organized and easy to understand forecast model. Based on existed forecasts, it was indicated that the doubt ranges for older group of survival functions would be manageable. However, doubt ranges for younger group of people would be larger.

Also Blake et al 2007, illustrated the expected relation to functions of post male survival rates with the aid of survivor fan charts. The fan charts used a mortality model for males that provided a good fit to England and Wales mortality data. It was established that no one can avoid the Grim Reaper, hence the doubt in survivorship was great in males above age 90. It was confirmed that a ‘toxic tail’ existed for firms like annuity and providers of pension which enabled them to continue payments until death. Again uncertainty in the mortality model parameters led to an increase in the estimated widths of the fan charts.

According to Denuit et al 2007, risk associated with living longer than expected was a major problem for insurers and pension funds most especially in the sale of annuity products. The conversion of the risk could get great awareness for hedge. This work proposed to form survivor bonds which could be sold by insurers directly. The survivor bond was subject to a general mortality measure for the transparency of the product. The Lee-Carter model for forecasting mortality was used to tell the amount to pay for a risky survivor coupon bond based on the measurement.

Loisel and Serant 2007, in their work stated that, one intensity stochastic process like a jump-diffusion process or independent collective processes found in most stochastic mortality models was used to model the gradual development of stochastic survival probabilities. A new model that took age related connections into consideration was proposed. The gradual development of longevity was key to life insurance, reinsurance companies and pension schemes. As researched by Millossovich and Biffis (2006), stated that the control of similar types of business involved the careful thinking of several groups of policyholders, different in type like taken out policy, age, sex, state of health and others at the same time. The objective of the work was to propose and adjust a stochastic mortality model that took into age and period related into account. The so called stochastic logit’s Deltas model; Dahl (2004); Luciano and Vigna (2005a, b); Lin and Cox (2005); Milevsky and Promislow (2001) recently got knowledge on the various types of models.

According to Booth and Tickle 2008, the continual increase in the advancement of life expectancy limits had revealed to light the main essence of forecasting mortality. Mortality forecasting had been rechecked under belief that something would happen, estimate and gave reasons since 1980. There exist a long history on mortality modeling. After the publication of Gompertz law of mortality in 1825, there has been the proposal many more models. Two decades ago, relatively simple methods that had the involvement of a fair degree of subjective judgment were used; pollard (1987). The major developments in the methodology of mortality modeling and forecasting since 1980 were reviewed. The review updated and elaborated previous reviews of mortality forecasting. The paper sort to make aware and update the wide ranging aspect of demographic forecasting by Booth in 2006 which added more insight knowledge on methods of mortality modeling and forecasting particularly extrapolative methods for readership in actuarial. The work centered on previous methods with better data quality which was easy to access in developed countries.

According to Coughlan et al 2008, transfer of longevity risk through the capital markets was a real issue. There could be a hedge in capital markets with longevity risk instruments that made reference to the emergence of a new market which was bound to happen by pension plans and providers of annuity. The major partakers of the market were hedgers which include pension plans and providers of annuity. Investment banks, broker dealers and end investors which constitute hedge funds and many more were some of the organizations that helped one another in the making of agreements. The idea of a different approach based on the example of managed risk that did not required transfer of risk and was same with the manner other risks related to pensions were managed. A hedging rules for longevity cantered on standard based hedges approach was presented. The intended results of this hedge was maximized by careful calculation of the method and further verified by the hedge effectiveness tests.

Longevity hedges that were customized would be the choice of some hedgers who were ready to pay the extra premium which was above the standard hedge cost was addressed.

Similarly by Sherris and Wills 2008, longevity risk was still one of the led things that challenged present time financial markets and control. For over a century this risk has been the major risks faced by actuaries, life insurance and reinsurance companies. The total number of the aged around the world had brought about the necessity for new ideas in the management of longevity risk and the introduction of new markets for hedge of this risk. Financial markets and ideas inceptions which could be made to hedge longevity risk ideally were considered and also looked at what was learnt from the insurance linked assets market that could be used in risk funding the finance markets successfully.

Tuljapurkar 2008, for the past hundred years, human life expectancy had increased substantially around the world and longevity in the modern world was largely determined by the age at which adults die. The manner in which higher average ages at death for both sexes had lasted for some time now in most countries. The Lee-Carter method could be used for mortality forecasts and provide stochastic projections of average longevity. This risk is not a bother to those who made decisions concerning consumption, savings and bequests but rather on issues about records on the economy in the areas of wealth of the total group of people, medical status, financial state and the fiscal flows in governmental and non-governmental pensions. Actual models for valuation of annuity were probable not to be linear, hence values of annuity were expected to be dependent on the various parts of longevity risk.

According to Wang et al 2008, in recent years, market of reverse mortgage has expanded with speed in economy of countries with lots of industries. The start of change in a particular group of people did placed quite a number of all people living together in a house in an age frame where reverse mortgages could develop in the near future. Reverse mortgages were concerned with various risks from the providers point which could further avoid the growth of the products financially. This paper addressed the method of moving from one side to another financial risks that were connected to the products. A demonstration was made on how to make structures for reverse mortgages such as the ones used in insurance products. The advantages of developed survivor agreements and exchange for products of reverse mortgage were investigated. Premiums were calculated and examined on how longevity risk and financial investors may be transferred. In the end, there was economic benefits that were derived from the developed survivor agreements to convert longevity risk to products of reverse mortgage.

According to Fong et al 2009, in order to aid pension plan participants manage their longevity risk, compulsory annuitization was at times under defined-contribution pension schemes suggested as the way forward. The research sort to the annuity market and discussed the positives and negatives of a suggestion to authorize annuitization of the Central Provident Fund (CPF) in Singapore. The assessment of if a person who partakes in the plan might benefit high from annuity returns per premium and oppose selection costs that were low under the new annuitization authority, the pricing of the several annuity policies were evaluated. The results concluded that non-governmental annuity providers now offered better money value of annuities with worth of money values in direction with those observed in various developed states.

Kogure et al 2009, made use of a Bayesian approach in the comparison of models for forecasting mortality rates under the knowledge of Lee-Carter method. The actual normal log-bilinear formulation of the method and the recent proposed Poisson log-bilinear formulation were put into consideration. The deterministic and the stochastic trend models as well as the stationary or no trend models were compared. Markov chain Monte Carlo methods were used to select predictive distributions to calculate the marginal likelihoods. The Japan male mortality rates from 1970 to 2003 was used in the application of this model. At the end of the work, the stochastic trend model was most appropriate for the forecast of mortality rates for the normal and the Poisson formulation and used to evaluate longevity risk for the population at age 65 thereafter.

Richter et al 2009, in the research conducted showed longevity risk was a main problem for the public, individuals and providers of annuity. The risk of not organized changes to mortality patterns general bore a huge accumulated losses for insurers. The research proposed a whole new different type of life annuities with knowledge on benefits of actual mortality and detailed actuarial tools. Same adjustment to the design of the product existed in making profit related annuities. The undetermined actual cost on long term official written agreement was found in the non-governmental health insurance of Germany so the idea was not original, but was in the event of longevity risk. The longevity risk according to insurers transfer might prevent loss so the main goal was in the detailed Monte-Carlo simulation on statements that ask if and to what limit the products were of importance to policyholders contrary to a c conventional annuity product compared.

According to Richards and Currie 2009, several important obligations were aware of the direction of mortality that would happen later and so this research presented some methods in smooth fitting of the models to past mortality data recently. The influence of the models on mortality estimate and the subsequent effect of the estimation on the products of finance were demonstrated. Our research done used the Lee Carter models. The use of the data and in the family of Lee-Carter models gave way that estimation of mortality could be changed.

It was concluded that the beliefs on modeling was materially financed and hence there was a need model risk exist when looking at longevity connected debts especially under payments of amount of money at regular intervals and pensions.

Shang Qin and Qin Xue-zhi 2009, looked at the aging trend of the Chinese population and the features of interest rates market. The death intensity by Feller process which described the term structure of interest rate with Cox-Ingersoll-Ross (CIR) model was modelled by this work. Future mortality could be forecasted by sensitivity test and rationally setting of parameters in the death intensity model with data from China's life tables. The work also discussed how survival probabilities improved the impact pension present value. The results showed that longevity risk might increase the costs of products with the condition that other financial risk hedged effectively. There was the implication that life insurance should give much attention to the design of pension annuities and the strategy to hedge longevity risk.

Similarly Steur 2009, the liabilities of pension funds with defined benefit pension systems were responsive to mortality changes more than before. Life expected had increased in most of the states of the world. In the Netherlands, male life expectancy of 65-year old had increased from 78 years in 1980 to about 82 years in 2008 according to Statistics Netherlands report. However, this information was profitable to the world as a whole, which increased longevity that had a negative influence on business that provided income for the aged. Decrease in mortality rates rapidly than expected implied pensions benefits needed to be paid quite longer than expected which in effect increased pension liabilities of pension fund houses and providers of annuity. The research focused in the area of longevity risk and the chance of success of the financial market for longevity derived variables.

Stevens in 2009, investigated the effect of longevity risk engaging on the different annuities types. An instrument for life cycle with expected utility in which a person was faced with investment and longevity risk were taken into consideration. Aside idiosyncratic, systematic longevity risk was also allowed which opposed the existed literature. The expected lifetime service subjected to the type of regular payments for a period of time which was acquired was compared. However, the purchase of regular payments with deferred period and that of regular payments at the beginning of the period at retirement date were compared hence the service gained was negligible.

According to Anja et al 2010, the western world observed the average increase in the lifetime over the past century of its people. Considerable uncertainty existed in connection to the future development of mortality even though past trends showed extra changes in mortality rate. Longevity risk was the uncertainty referred to. The work reviewed the current state concerning longevity risk. The modeling of future mortality was initially discussed which included Lee Carter (J Am Statistics Association 87:659–671, 1992) and other methods. Also the essence of longevity risk for having enough portfolios of pension, products of life insurance and the investigation of the possibilities for longevity risk management were also looked at. More importantly managed longevity risk via pension and or insurance redesign was considered.

According to Meyer and Reichenstein 2010, a study was conducted to examine measures for singles and couples who are taking a decision on when to start Social Security benefits. Start date for singles or start dates for couples that would maximize the present value of benefits and minimize longevity risk where the main factors one has to consider when taking the decisions. People who are not married and were with not enough life expected were to commence benefits early while those with enough life expected could wait for some time. Benefits could begin at 70 if one wanted to minimize longevity risk. The decisions for married people revolve around the partners and their survivor benefits. Longevity risk is maximized when the low earning spouse delays benefits until 70. Lastly, this work discussed the option whereby someone can repay advance benefits and start benefits afresh which is termed as the do-over option.

Ngai and Sherris 2010, there had been underestimation of the longevity risk of people as the survival chances had improved in the developed states for some time now. The work investigated why there were no changes in the strategies of hedge effectiveness for the control of longevity risk which made use of connected and produced longevity like q-forwards for the sale of products such as deferred annuity, measured life annuity and regular payments for a period of time with guaranteed entire life benefits directly to the people. A logit independent time model for mortality rates which depended on age was the mortality model. Australian data was used to evaluate the model. Static hedging used q-forwards or longevity connected to reduce life annuities longevity risk but for annuities that were deferred small was concluded. Inflation indicated annuities with no change hedge of longevity was less effective as a result of risk associated with the general rise in the prices of goods and services. Longevity risk hedging added value to the products.

Tzuling and Tzeng 2010, proposed a stochastic mortality model which was additive continuous-time that revised the Ballotta and Haberman (B&H) model in 2006. The B&H model structure suggested the hazard rate for future time was appropriate to the stochastic part. However, a high hazard rate at the base might not cause a major rise as suggested by the exponential portion of B&H (2006).The uncertainty of the future hazard rate might not be affected with age increase. The additive form which was the sum of an already stated and stochastic components were the issue that could be solved. In the doubting of the model of the hazard rate in the near future would not in any way be affected by a rise in age or the hazard rate the base. Further extension of the model was by calculating longevity risks reserves for pure endowments and several familiar products with regular payments for a period of time in UK were demonstrated. The results obtained and those of the B&H model 2006 were compared.

Similarly, Cairns 2011 in his work tackled hedging strategies development reaction towards pension plan that used longevity associated hedge tools. Difficulty due to the lack of laid down steps for the valuation of related mortality gains and losses and the subsequent needed particular set of conditions were as a result of advancement in this area. The use of the function alongside the Taylor expansion to estimate values of longevity was suggested. Thus made the development and implementation of discrete time hedge strategies easy by using q-forwards as hedge equipment. It was realized that the approximations were correct and the time independent hedge strategy was productive at reduced risk.

According to Gallie and Sherris 2011 issued insurers products of longevity risk assessment was very critical in mortality modeling and longevity risk. In the occurrence of death consideration, trends of mortality rates were not left out. Longevity was usually predicted by the use of the Lee-Carter model with one stochastic time series improvements for a period or use an age parametric model with a variable time series. A multi variable time series for the model of Heligman-Pollard function was assessed by the Vector Error Correction Models (VECM) which included stochastic trends in the long term. The model was used particularly to deaths resulted from diseases connected the movement of blood in the body in the United States for a 50-year period from 1950 to 2000 and it shown to be ahead of the Lee-Carter model and the stochastic Heligman-Pollard model.

Njenga and Sherris 2011, looked at the modeling of longevity risk and the sudden changes in mortality which had gotten noticed of by the aged in populations around the globe and it's awaited in puts financially. Longevity risk was involved with one improvement factor with impacts on the age was evaluated by the Lee-Carter original model. Financial models for pricing and management risks have gained attention alongside several factor models. This research investigated the tendency which included popular trends through combination and factors driving the volatility of mortality for developed countries which include Norway, England, Japan, Australia and USA. In the end, the results demonstrated the essence for multiple factors for mortality modeling within all the countries. Trends by the countries were stochastic. Ordinary trends and existing connections were observed for ages that benefit under mortality rates under Vector Autoregressive (VAR) model and captured long term relations with Vector Error Correction Model (VECM).

Piscopc 2011, in his work said the theory of defined contributions in 1995 started with the cash and carry pension system in Italy. Contributions paid throughout the working life of the employee were gradually put together for a period of time within the growth rate of the economy that was believed to happen until retirement. Life annuity was obtained from the conversion of the accumulated amount which took into account predicted expected life. Longevity risk due to changes in the deaths counts from values likely to occur and providers or workers of pension effects that depended on when it happened. The research analyzed the current general system of pensions and the influence of longevity risk on coefficients complete change which converted the calculated balances into pension benefits. Numerical results showed coefficients for the use of two separate mortality models; Lee Carter model and the demographic functional model. The work dealt with longevity risk on pension providers and workers effects.

Riemer-Hommel and Trauth 2011, longevity risk borne products of pension, long term care and regular payments of money into a pension fund. The quest the products had increased very fast and hence led to increase in the involvement on effective management of longevity risks, difficult life expectancy long term forecast. This work looked at some possible approaches to improve longevity risks management. The methods included finite reinsurance and capital market solutions.

Amlan 2012, in his work discussed what longevity risk was, the importance of it, methods used in longevity risk management by people in the West and lessons that could be learnt by countries around the Asian continent from the experiences from people in the West. A major risk that has affected plans on pensions, insurers and the general public together in developed and developing countries around the world were the uncertainty and the increased longevity. In this work a further discussion was done on longevity modeling and the merits as well as the setbacks of these models. Capital market and the general public had made approaches to tackle longevity risk in the Western countries, but the availability of solutions remained limited.

Vesa 2012, in order to manage and quantify risk and uncertainties that had connection to the pricing of annuities studied and came out with some tools for long term. A Monte-Carlo simulation model was made, calculated and officially started to enable one to do proper examination of pension and life insurance products. The S&P 500 annual total was modeled by a non-correlated and Normal distributed process. The US government 5 year bond annual total is modeled as a process of ARMA (1,1) followed by log transforming returns. The model used was the famous Lee and Carter (1992) where the Bayesian Monte Carlo Markov Chain (MCMC) method in the inference concerned with time index was used. The data used in this work was the Finnish mortality data. Also the data for males and females were done differently. In conclusion a manageable model for the dependence structure that allowed processes of the number of deaths in a particular place and the economy to be either non-correlated or there exist a connection between them. The model was also enabled to analyze financing longevity risk under insurance of pensions and the subsequent management cases of risk.

According to Barrieu et al 2012, did an investigation into the current gradual growth in longevity risk modeling and seek the major management of the problems for financial and insurance industries together. The work also discussed the main meanings for the development of longevity risk, giving a global perception on the real problems of longevity connected to products of pension and insurance that came about at the same time with life expected increase from 1960. In their work they also framed present and future developments that were expected to take action through the whole industry modeled to efficiently asses and manage inherited risks. At the same time the introduction of longevity increased the essence for capital markets to control and arrange risk using Insurance-Linked-Securities (ILS) as the medium.

Aro2013, in her work presented mathematical models for management of longevity risk. The development of methods for the cash flows of longevity linked liabilities hedge on the market financially was the main aim of the work. Modeling the law of a multiple variable stochastic process that gave a description on mortality and returns on asset was obtained. The stochastic model that was reached was then applied to the study of systematic and non-systematic risks in collective pensions and investigate the investment of longevity linked liabilities by investors.

Brockett et al 2013, a transaction in which there was a change of ownership of life insurance contract by the owner of the policy to another party was termed life settlement. We took a look at the general view of the market of life settlement exhibited its likelihood to be affected by longevity risk and the category of longevity linked assets. The situation on deciding how much to charge and the person to invest was duly updated with facts on the expected life insured as well as other vital information medically from an underwriter was discussed. An illustration on the use of several mortality tables which included the extended new Lee–Carter model that allowed mortality and longevity rise over time. An analysis which used separate developed standardized tables and medical facts illustrated the strong nature and many different uses of the method.

According to Hanewald et al 2013, a person's after retirement management strategy of longevity that allowed for longevity risk product ideas and loadings were analyzed. A complete market in independent state and multiple period of investment procedures were used to calculate private longevity insurance investments with separate stages of longevity risk. (GSA) plans replaced related inflation on regular interval payments products that guaranteed loadings on regular payments within a particular period of time on life products. Co-insurance group of strategies with withdrawals and group self annuitization dominated life annuities or deferred annuities.

According to the research by Lukasz et al 2013, an idea on how to get some facts with greater interest on household from likely accumulated net cash flow informed by the people in the house were accepted variant. The format of financial planning implied changes in the accumulated surplus. To illustrate this concept, there was a proposal display of household planning model financially which took longevity and untimely death risks into consideration the beneficiary as well as. A system to recognize and described methods of greater interest presented.

Maurer et al 2013, in the research sort the Variable Investment Linked Deferred Annuities (VILDAs) influence on consumption and allocation under lifecycle which gave chance to longevity risk. Insurers determine premiums in order to reduce the probability that paid out benefits do not exceed reserves in the self-insurance method. Similarly, there is an approach in which the producer prevents determined longevity risk by rectifying benefits in reaction to mortality. Most people in a particular hose at a specific time would like to partake in longevity risk that could be determined apart from situations where insurers could hedge the price a bit lower.

According to Ouburg 2013, the research was done to investigate mortality models for one or more populations using either frequentist or Bayesian inference. A Bayesian for a group of people at a particular place death count model for mortality data from Dutch was developed and generalized to a Bayesian model of two for a group of people in a specific location. This Bayesian for a group of people model of two allowed modeling of masculine and feminine death rate together. The used of Bayesian inference had multiple pros in modeling of mortality. The contribution of this work was to explain the technicalities of Monte Carlo Markov Chain (MCMC) sampling for Bayesian of one or more particular group of people models of death counts and the application of the models to the number of deaths data from Dutch. The existence of Markov chain theory; following Ross, 2007, the Metropolis-Hastings set of rules and Gibbs sampler produced Markov chains as the result. These set of rules would be used to describe the backward distribution within Bayesian death count models. England and Wales population data was used in this research to develop a framework for data reliability asses and anomalies identification. It was concluded that, large irregularities observed could be connected to uneven pattern of birth cohorts.

According to Post and Hanewald 2013, the fact of people knowing longevity risk which was survival expectations based drawn out the Survey of Health, Ageing and Retirement in Europe put in simple terms as SHARE and matched longevity risk information from the database of human death count which was provided. The relationship between the dispersion forecast of estimated survival and longevity risk that indicated to some extent that people were informed about longevity risk was realized. Survival estimates was primarily explained by those in disagreement of the chances of survival and not by knowing of the doubts. Averagely much is not there to save if people had problems with longevity risk even though the theory suggested otherwise.

According to the research by Tzuling and Tasi 2013, length of time and the outline of the prices of death life insurance and annuity products with appropriate change and similar shift instantly in the force of the number of deaths, survival probability for a year and death probability for a year denoted as μ, p and q respectively and subsequently obtained with great importance free closed form formulae. Many duration and convexity matching techniques used to tell the weights of more than two products of insurance were suggested. The Value at Risk values and the effective surplus hedge at time zero with the techniques matching were evaluated. The duration and convexity to instant appropriate change in μ and q can openly hedge the number of deaths and longevity risks.

According to Boyer et al 2014, the person who would gain in the future and the person who gave employment together could be told to financially plan throughout the beneficiary's career, often the employer would bear any shortfalls that arose. Therefore it was necessary for the person who gave the employment to predict highly the total amount that would be needed for future pension duties. Application of forecasted mortality model to the data of the pension plan of Royal Canadian mounted police and its important value to a pension fund with actuarial liability was illustrated. Compounded longevity risk was related to not having idea on how the actual mortality model was like.

Fung et al 2014, development of longevity market required adequate and structured financial instruments. A swap index based longevity were examined in this research under a stochastic model on mortality which was easy to deal with. The model was checked by the use of Australian mortality data and formulas for longevity derivative prices were obtained. The work presented many features on hedge which was shown by swap based longevity on separate assumed price of longevity risk at the market. In the end, the findings showed effects on the best possible use of longevity hedge tools with payoff structures under linear and nonlinear scenarios.

Nan Zhu and Daniel Bauer2014, observed the natural hedge method for insurers of life in order to manage internal longevity risk exposed by modifying the insurance. The non-parametric mortality forecast model prevents the idea that almost every mortality rates were determined by similar factors was considered. At the end of the work, the result showed that higher changes in mortality rates might influence the natural hedge performance. The findings based on a parametric one factor model implied that longevity risk could be hedged. The results for the non-parametric mortality model was far better.

Wong et al 2014, in their research investigated the change in time agreement with the average–variance hedge of longevity risk with protection on the level of mortality or national mortality. Hamilton Jacobi Bellman (BJH) equation was used to solve the problem of hedging in which the liabilities of insurance followed a Poisson stochastic process. Closed form hedge policy showed the need of co-integration in longevity hedge.

Also Li and O'Hare 2015, proposed that mortality projection in the future was of basic importance to insurance companies, pension providers and government welfare systems as mortality forecasts accuracy were needed to determine the right amount of insurance premiums, pension benefits and contribution rates. Therefore, there was an increasing need to better understand mortality in order to increase the future mortality forecasts accuracy. Various approaches had been made to project future mortality rates of different types of models (Booth and Tickle, 2008). Majority of the models tend to identify patterns in age, time or cohort dimensions in the mortality data and extract these patterns made projections on future mortality rates as stated in Lee Carter, 1992; Cairns et al., 2006. The primary interest of this paper was to study the question of whether local or global information was more appropriate to be used and if should be preferred in the mortality forecasting process. Two pairs of mortality models Li et al 2015a and Li et al 2015b were used for comparison. The work was done using male mortality data of Great Britain from 1950 to 2009 for ages 50-89. At the end of the study, it was realized that, in the forecasting process, local information seemed to have greater predictive power and so it should be given more weight when doing future mortality projection.

Luciano et al 2015, research provided procedures for natural hedging in life insurance and annuity business that was either written on one or more generations with longevity and interest presence together. Hedge delta and gamma against a group of people with longevity risk based was achieved from the closed-form solutions. The UK data that was used for survivor and bond dynamics revealed that hedge was effective even when restoring balance was not happening. The correlation between mortality of different generations and the hedge of longevity risk of one particular group of people with products on other group of people were exploited.

Chapter 3

METHODOLOGY

3.1 Overview

The importance and issue of longevity and the risk associated with it has gained international and worldwide recognition. The aggregate of longevity risk is its determined and unchangeable nature; Milevsky et al. 2006. In this research, we will use 'the two factor stochastic' mortality model of Cairns-Blake-Dowd (2006) model as the instrument to estimate mortality rates, measure longevity risk and use Holt's linear method to forecast future mortality rates.

3.2 History of the Model

The Cairns-Blake-Dowd model is a mortality stochastic model structured by three Professors named Andrew Cairns of Maxwell Institute and Heriot Watt University, David Blake and Kevin Dowd of the Pensions Institute at Cass Business School. The CBD model and Cairns et al will be used more often and interchangeably. The CBD model is one of the extrapolative projection models that are commonly used for mortality forecast. The model would only be credible if trends of the past continue. Mortality rates have been decreasing but the decrease have been variable. The uncertainty of future mortality rates has to be put in consideration when forecasting.

The CBD model

1. Has the structure for higher ages mortality modeling
2. Generates a simple mortality curve at higher ages

3. Has the form for modeling pensions and annuities longevity risk for long term mortality risk

4. is a stochastic model

The CBD model is applied to risks like Communication, Longevity, Survivor, Mortality and the Projection of annuity prices.

Advantages of the CBD model:

1. mortality log rates are in line at ages above 40

2. Uses two time length parameter impact to take advantage of mortality rates and difference in rates at high age

3. Robust

4. Effects on age is simple

5. Easy adjustment to parameter uncertainty

Disadvantages of the CBD model:

1. It is not good for small data

2. It is not good for ages below forty (40)

Most stochastic mortality models are fitted to already existed data on one or more time based varying indexes (k_t) are noticed. By estimating the indexes, we can obtain a prediction on future death probabilities and other changes in the number of deaths in a place over a period of time like expectancies of life. They are good for measuring pension risks on longevity and for creating capital markets that are longevity related.

Cairns et al. (2006) 'two factor stochastic mortality model' is formulated as

$$\log\left(\frac{q_{x,t}}{1-q_{x,t}}\right) = k_t^1 + k_t^2(x-\bar{x}) \tag{3.1}$$

$$\text{logit } q_{x,t} = k_t^1 + k_t^2(x-\bar{x}) \tag{3.2}$$

Where:

$q_{x,t}$: Probability that a person age x will die at time t.

$barx$: Average of the age range under consideration and

k_t^1, k_t^2 : Are the period mortality indexes(Mean and Variance of mortality rate within a period).

Notation

The death rate for period x at time t is denoted $m(t, x)$. Hence

$$m(t,x) = \frac{\text{number of deaths for the period } x \text{ at time } t}{\text{number of people exposed to death for period } x \text{ at time } t} \tag{3.3}$$

Relationship between m(t,x) and q(t,x)

The function of the death rate at time t for individuals at age x which is termed force of mortality and denoted $\mu(t, x)$.
The death rate and the mortality rate are quite near in value to one another. Some relations are made for the formalization of the relationship;

- The force of mortality remains unchanged over each year and age. Expressed mathematically as for any given t and x, for all $0 \leq s, u < 1$, then $\mu(t + s, x + u) = \mu(t; s)$;
- The size of the population is assumed to remain unchanged over age and time.

The assumptions then imply that

1. If $m(t;x) = \mu(t;x)$;

2. Then $q(t;x) = 1 - \exp[-\mu(t;x)] = 1 - \exp[-m(t;x)]$

The overall principle of extrapolative mortality modeling is to extract one or more mortality indexes from historical data; project the index (es) forward to obtain a mortality forecast. In the CBD model, k t is a bivariate time series vector and it is normally modeled by a bivariate vector random walk.

3.3 Implication of the CBD Mortality Indexes

- ▷ As older ages are been approached, payouts by annuity insurers and sponsors of pensions are positively related to mortality productivity. The total productivity of mortality is higher than expected when financial duties are huge. (k_t^1 is lower than expected).

- ▷ Mortality productivity at old age is higher than at younger age and as such annuity providers and pension sponsors problem worsens in a fixed total mortality productivity. (k_t^2 is lower than expected).

- ▷ Payouts are negatively connected to productivity of mortality at younger ages for life insurers selling term insurance products therefore total mortality productivity is lower than expected. (k_t^1 is higher than expected)

- ▷ Mortality productivity at younger age is less at higher age and as such life insurers' problem worsens in a fixed total mortality productivity. (k_t^2 is lower than expected)

3.4 Method of Analysis

- A historic data of pensioners under the GUSSS pension scheme at KNUST from 2005 to 2015 is used.

- The needed information from the data are the number of death for a period and the number exposed to death.
- Maximum likelihood Estimate (MLE) of $Z(t) = (k_t^1, k_t^2)$ for $t = 2005, ..., 2015$ and age 70 to 95.

3.5 Logit

The logit is a link function or a transformation of a parameter. The logit function is the opposite of the sigmoidal logistic function or logistic transform. Given the function parameter with probability p,the term (p/1-p) is called the odds. The natural logarithm of the odds is known as the logit function.

$$\text{Logit } (p) = \log\left(\frac{p}{1-p}\right) \tag{3.4}$$

3.6 Maximum Likelihood Estimation

A model is made up of its parameters and as soon as the data is put together, one has better evaluation for the fit of goodness through the parameter estimation method. The estimation of the parameters are done by the Least Squares Estimation (LSE) and Maximum Likelihood Estimation (MLE).

Let $X_1, X_2, X_3, X_4, ..., X_n$ be a random sample of size n from a population which is $N(\mu, \sigma^2)$ Where

$$\mu = \bar{X} = \frac{1}{n}\sum_{i=1}^{n} X_i \qquad \text{and} \tag{3.5}$$

$$\sigma^2 = \frac{1}{n}\sum_{i=1}^{n} (X_i - \mu) \tag{3.6}$$

3.7 Forecasting

The main aim of forecasting is to predict the future using the time series data at hand. A good forecast should be timely, accurate, reliable and easy to understand.

Time series is a collection of past data at regular intervals of change to be forecasted. In this research, we are going to use the double exponential smoothing to forecast.

3.8 Holt Linear Trend Method

Holt extension of the single exponential smoothing is used for forecasting data with either or both seasonality and trend. Hence the double exponential smoothing simply put as the Holt linear trend method. The method is made up of a forecast equation and two smoothing equations for the level and the trend as described below:

$$\text{Forecast equation:} \quad \hat{F}_{t+h} = \hat{y}_{t+h/t} = s_t + h\mu_t \tag{3.7}$$

$$\text{Level equation:} \quad s_t = \alpha y_t + (1-\alpha)(s_{t-1} + \mu_{t-1}) \tag{3.8}$$

$$\text{Trend equation:} \quad \mu_t = \beta(s_t - s_{t-1}) + (1-\beta)\mu_{t-1} \tag{3.9}$$

Where estimate of the level of series at time t is denoted s_t an estimate of trend also known as slope of series at time t is u_t. The smoothing parameter α for level where $0 \leq \alpha \leq 1$ and β is the smoothing parameter for trend where $0 \leq \beta \leq 1$. The time to be forecasted is represented as h. The h-step forecast is equal to the last estimate of level added to h multiplied by the last trend value estimated. The forecasts are therefore linear function of h. In this research, the mortality rates will be forecasted quarterly for five years giving us a 20-step ahead forecast in the R-code software.

Chapter 4

DATA ANALYSIS AND DISCUSSIONS

4.1 Introduction

This chapter contains the analysis of the data collected. Discussions are made on the key findings of the study in a theoretical manner on longevity risk. Here we look at how the various parameters are estimated and have a discussion on them.

4.2 Data Source

A secondary data was used for this research. The data was collected from the Ghana University Staff Superannuation Scheme (GUSSS) Kwame Nkrumah University of Science and Technology (KNUST) chapter. The historical data collected was for pensioners who are still alive and those who have died from age 70 to 95 within the period of 2005 to 2015.

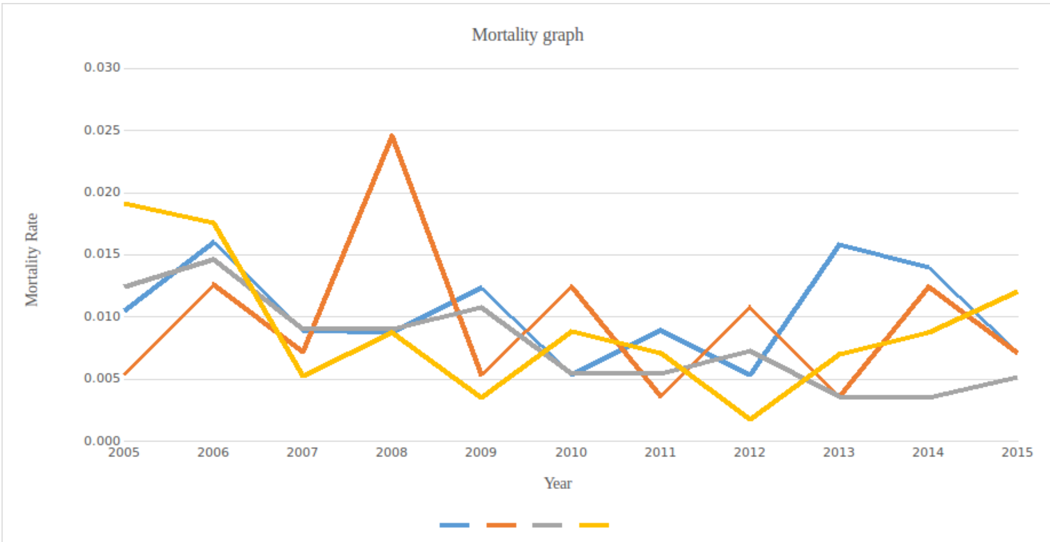

Figure 4.1: Mortality Rate from 2005 to 2015

The above graph is the mortality for each quarter in the various years from 2005 to 2015. The blue colour represent the first quater, the orange represent the second quater, the grey represent the third quater and the yellow represent the last quater of a year. The mortality increases when the number of deaths within the quarter is high and vice versa. The mortality rate is estimated using the constant force of mortality. There is a huge rise in mortality in the year 2008 during the second quarter and the least mortality is realized in the year 2012 in the last quarter.

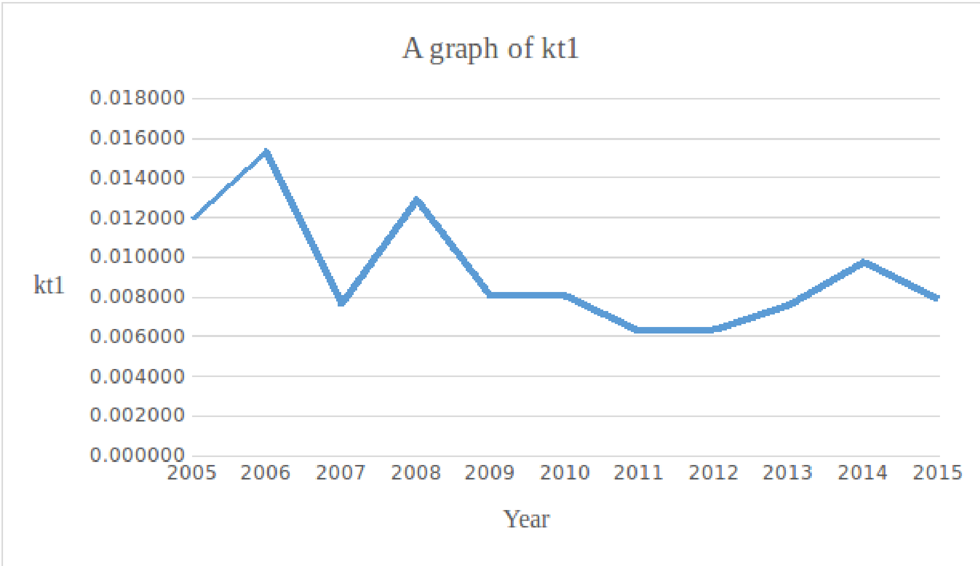

Figure 4.2: is the estimates of k_t^1 from 2005 to 2015

The above graphs are the graphical representation of the mortality indexes.

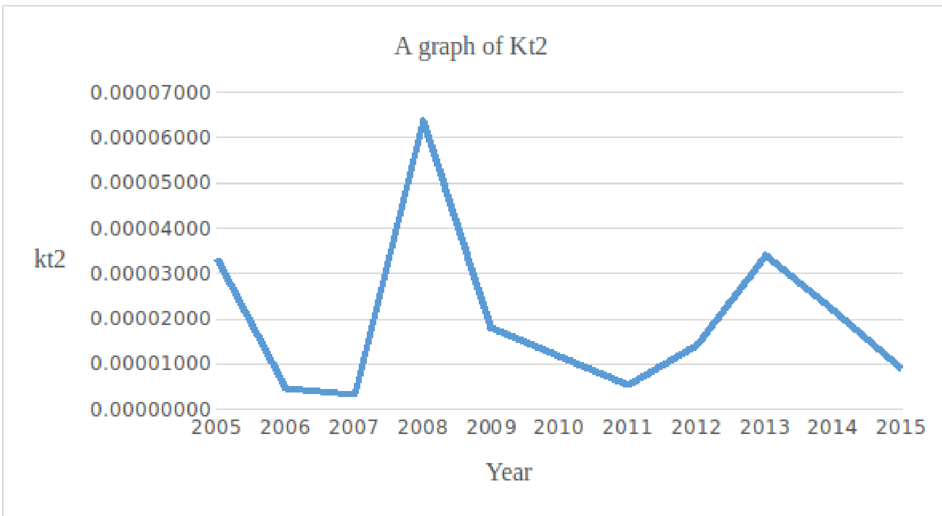

Figure 4.3: is the estimates of k_t^2 from 2005 to 2015

They are computed by calculating the mean and the variance respectively from the quaterly mortality rates of a year.

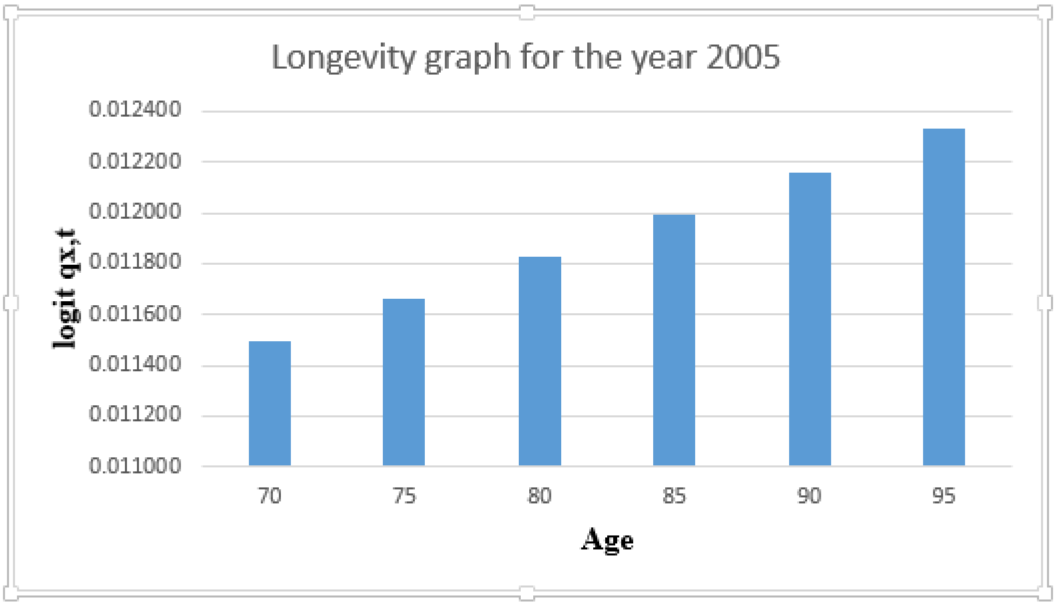

Figure 4.4.1: Longevity for year 2005

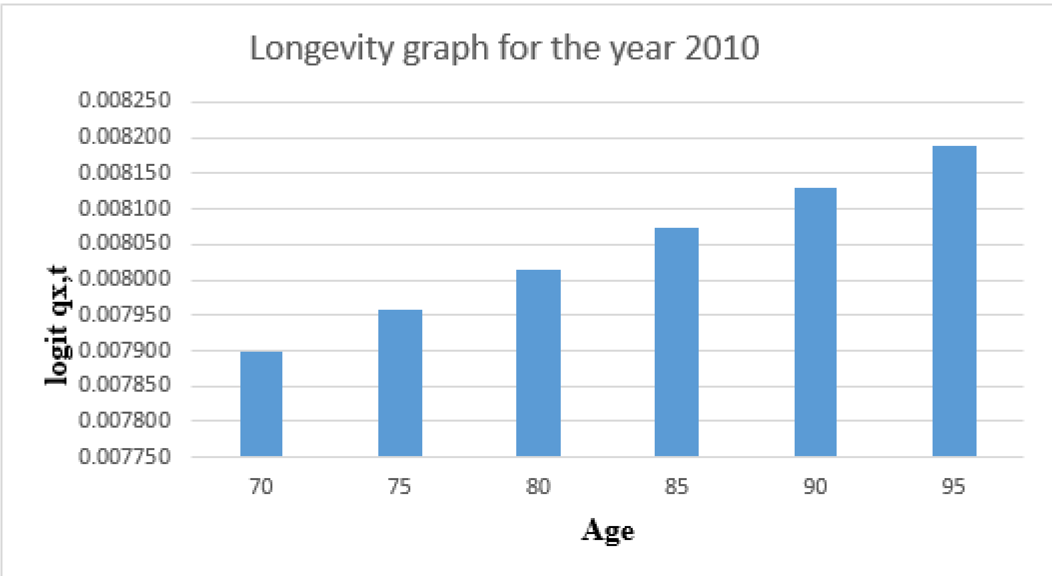

Figure 4.4.2: Longevity for year 2010

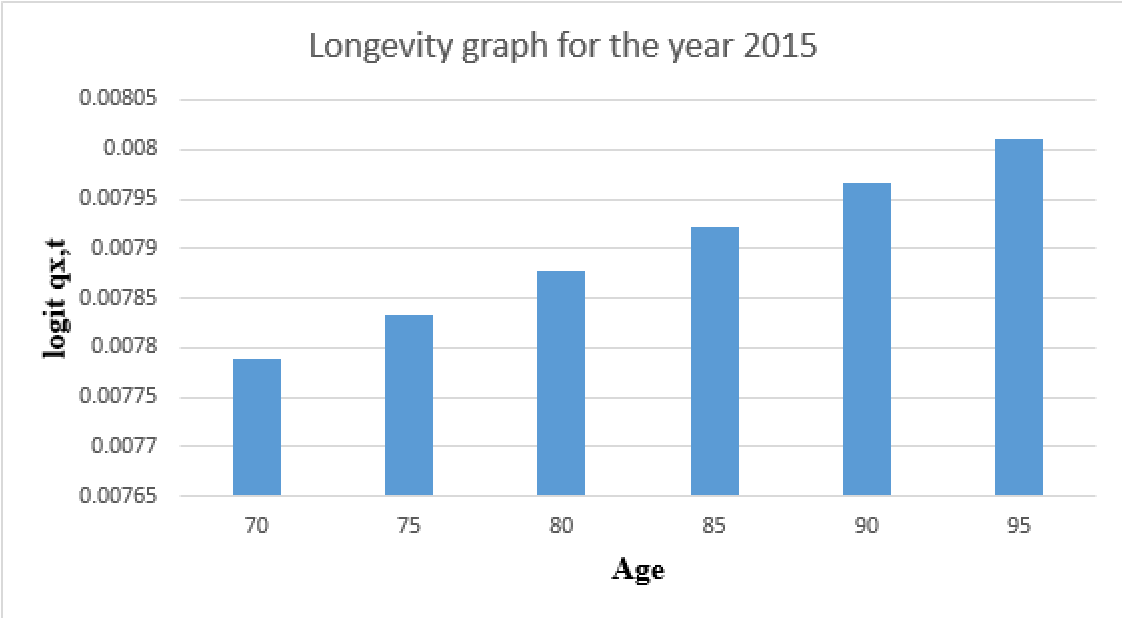

Figure 4.4.3: Longevity for year 2015

The representation of longevity against age for the KNUST GUSSS pensioners for 2005, 2010 and 2015. From all the above graphs, we realized that there is an increasing order of longevity risk. As the ages are increasing the longevity also increases. Also the increase is linear. From the three graphs, it is realised that the probablity that an older person (85 years) will live longer than expected after retirement is higher than that of a person with a lower age (70 years).

Below are the forecast of the mortality rate for each quarter for the next five years where the smoothing parameters alpha and beta are 0.1 step down or up at a 95% prediction level. The tables below are the forecast points for the various smoothing parameters.

Table 4.5: Forecast of future mortality rates

Alpha = 0, Beta = 1				
Year/Quarter	Point Forecast	Lo 95	Hi 95	Hi 95 - Lo 95
2016 Q1	-0.2223860000	-0.4807337000	0.0359617400	0.5166954400
2016 Q2	-0.2275610000	-0.8052441000	0.3501221000	1.1553662000
2016 Q3	-0.2327360000	-1.1993847000	0.7339127200	1.9332974200
2016 Q4	-0.2379110000	-1.6529398000	1.1771178300	2.8300576300
2017 Q1	-0.2430860000	-2.1590441000	1.6728721000	3.8319162000
2017 Q2	-0.2482610000	-2.7127413000	2.2162193400	4.9289606400
2017 Q3	-0.2534360000	-3.3102476000	2.8033756500	6.1136232500
2017 Q4	-0.2586110000	-3.9485547000	3.4313327500	7.3798874500

Alpha = 0.1, Beta = 0.9				
Year/Quarter	Point Forecast	Lo 95	Hi 95	Hi 95 - Lo 95
2016 Q1	0.0065222416	-0.0095945120	0.0226390000	0.0322335120
2016 Q2	0.0060171520	-0.0285869860	0.0406212900	0.0692082760
2016 Q3	0.0055120624	-0.0513551900	0.0623793200	0.1137345100
2016 Q4	0.0050006973	-0.0773936250	0.0874075700	0.1648011950
2017 Q1	0.0045018832	-0.1063410840	0.1153448500	0.2216859340
2017 Q2	0.0039967936	-0.1379313340	0.1459249200	0.2838562540
2017 Q3	0.0034917040	-0.1719589820	0.1789423900	0.3509013720
2017 Q4	0.0029866143	-0.2082597050	0.2143293000	0.4225890050

Alpha = 0.2, Beta = 0.8				
Year/Quarter	Point Forecast	Lo 95	Hi 95	Hi 95 - Lo 95
2016 Q1	0.0073975030	-0.0060540240	0.0208490300	0.0269030540
2016 Q2	0.0073538060	-0.0203445820	0.0350521900	0.0553967720
2016 Q3	0.0073101090	-0.0373035560	0.0519237700	0.0892273260
2016 Q4	0.0072664130	-0.0566247970	0.0711576200	0.1277824170
2017 Q1	0.0072227160	-0.0780646280	0.0925100600	0.1705746880
2017 Q2	0.0071790190	-0.1014373690	0.1157954100	0.2172327790
2017 Q3	0.0071353230	-0.1265974750	0.1408681200	0.2674655950
2017 Q4	0.0070916260	-0.1534274150	0.1676106700	0.3210380850

Alpha = 0.3, Beta = 0.7				
Year/Quarter	Point Forecast	Lo 95	Hi 95	Hi 95 - Lo 95
2016 Q1	0.0076859800	-0.0053367590	0.0207087200	0.0260454790
2016 Q2	0.0080090600	-0.0176757960	0.0336939200	0.0513697160
2016 Q3	0.0083332141	-0.0321222800	0.0487865600	0.0809088400
2016 Q4	0.0086552220	-0.0484966520	0.0658071000	0.1143037520
2017 Q1	0.0089783030	-0.0666209120	0.0845775200	0.1511984320
2017 Q2	0.0093013830	-0.0863514990	0.1049542700	0.1913057690
2017 Q3	0.0096244640	-0.1075729520	0.1268218800	0.2343948320
2017 Q4	0.0099475450	-0.1301906810	0.1500857700	0.2802764510

Alpha = 0.4, Beta = 0.6				
Year/Quarter	Point Forecast	Lo 95	Hi 95	Hi 95 - Lo 95
2016 Q1	0.0084705270	-0.0042191610	0.0211602200	0.0253793810
2016 Q2	0.0091510800	-0.0147917760	0.0330939400	0.0478857160
2016 Q3	0.0098316320	-0.0269465780	0.0466098400	0.0735564180
2016 Q4	0.0105121850	-0.0406258420	0.0616502100	0.1022760520
2017 Q1	0.0111927370	-0.0557145390	0.0781000100	0.1338145490
2017 Q2	0.0118732900	-0.0721090580	0.0958556400	0.1679646980
2017 Q3	0.0125538420	-0.0897222110	0.1148299000	0.2045521110
2017 Q4	0.0132343950	-0.1084808190	0.1349496100	0.2434304290

Alpha = 0.5, Beta = 0.5				
Year/Quarter	Point Forecast	Lo 95	Hi 95	Hi 95 - Lo 95
2016 Q1	0.0093835690	-0.0031318950	0.0218990300	0.0250309250
2016 Q2	0.0102795200	-0.0122830540	0.0328420900	0.0451251440
2016 Q3	0.0111754710	-0.0225234470	0.0118743900	0.0343978370
2016 Q4	0.0120714230	-0.0339133290	0.0580561700	0.0919694990
2017 Q1	0.0129673740	-0.0463986840	0.0723334300	0.1187321140
2017 Q2	0.0138633260	-0.0599142470	0.0876409000	0.1475551470
2017 Q3	0.0147592770	-0.0743996800	0.1039182300	0.1783179100
2017 Q4	0.0156552290	-0.0898019460	0.1211124000	0.2109143460

Alpha = 0.7, Beta = 0.3				
Year/Quarter	Point Forecast	Lo 95	Hi 95	Hi 95 - Lo 95
2016 Q1	0.0108214900	-0.0017208290	0.0233638100	0.0250846390
2016 Q2	0.0116780900	-0.0088928450	0.0322490200	0.0411418650
2016 Q3	0.0125346900	-0.0162033780	0.0412727500	0.0574761280
2016 Q4	0.0133912800	-0.0239418740	0.0507244440	0.0746663180
2017 Q1	0.0142478800	-0.0321756470	0.0606714100	0.0928470570
2017 Q2	0.0151044800	-0.0409163220	0.0711252800	0.1120416020
2017 Q3	0.0159610800	-0.0501572930	0.0820794500	0.1322367430
2017 Q4	0.0168176800	-0.0598855580	0.0935209100	0.1534064680

Alpha = 0.8, Beta = 0.2				
Year/Quarter	Point Forecast	Lo 95	Hi 95	Hi 95 - Lo 95
2016 Q1	0.0113509500	-0.0012798630	0.0239817700	0.0252616330
2016 Q2	0.0120230200	-0.0077069470	0.0317529900	0.0394599370
2016 Q3	0.1269509000	-0.0137995390	0.0391897100	0.0529892490
2016 Q4	0.0133671500	-0.0199552320	0.0466895300	0.0666447620
2017 Q1	0.0140392200	-0.0263003770	0.0543788100	0.0806791870
2017 Q2	0.0147112800	-0.0328852820	0.0623078500	0.0951931320
2017 Q3	0.0153833350	-0.0397310290	0.0704977300	0.1102287590
2017 Q4	0.0160554100	-0.0468455460	0.0789563700	0.1258019160

Alpha = 0.9, Beta = 0.1				
Year/Quarter	Point Forecast	Lo 95	Hi 95	Hi 95 - Lo 95
2016 Q1	0.0117003700	-0.0011238760	0.0245246200	0.0256484960
2016 Q2	0.0120255900	-0.0070390240	0.0310902000	0.0381292240
2016 Q3	0.0123508100	-0.0121498810	0.0368515000	0.0490013810
2016 Q4	0.0126760300	-0.0169588070	0.0123108600	0.0292696670
2017 Q1	0.0130012500	-0.0216479610	0.0476504600	0.0692984210
2017 Q2	0.0133264700	-0.0263044000	0.0529573300	0.0792617300
2017 Q3	0.0136516900	-0.0309759580	0.0582793300	0.0892552880
2017 Q4	0.0139769100	-0.0356911920	0.0636450000	0.0993361920

Alpha = 1, Beta = 0				
Year/Quarter	Point Forecast	Lo 95	Hi 95	Hi 95 - Lo 95
2016 Q1	0.0069040000	-0.0089686790	0.0227766791	0.0317453581
2016 Q2	0.0071290000	-0.0207183580	0.0241766358	0.0448949938
2016 Q3	-0.0034460000	-0.0309382870	0.0240462867	0.0549845737
2016 Q4	-0.0086210000	-0.0403663580	0.0231224358	0.0634887938
2017 Q1	-0.0137960000	0.0492883900	0.0216963895	-0.0275920005
2017 Q2	-0.0189710000	-0.0578509650	0.0199089648	0.0777599298
2017 Q3	-0.0241460000	-0.0661411620	0.0178491617	0.0839903237
2017 Q4	-0.0293210000	-0.0742157160	0.0155737162	0.0897894322

Alpha = 1, Beta = 1				
Year/Quarter	Point Forecast	Lo 95	Hi 95	Hi 95 - Lo 95
2016 Q1	0.0189900000	-0.0014268760	0.0394068800	0.0408337560
2016 Q2	0.0259010000	-0.0386628320	0.0904648300	0.1291276620
2016 Q3	0.0328120000	-0.0712940510	0.1369180500	0.2082121010
2016 Q4	0.0397230000	-0.1060826620	0.1855286600	0.2916113220
2017 Q1	0.0466340000	-0.1438019460	0.2370699500	0.3808718960
2017 Q2	0.0535450000	-0.1845546490	0.2916446500	0.4761992990
2017 Q3	0.0604560000	-0.2282822360	0.3491942400	0.5774764760
2017 Q4	0.0673670000	-0.2748822150	0.4096162100	0.6844984250

Alpha = 0.6, Beta = 0.4				
Year/Quarter	Point Forecast	Lo 95	Hi 95	Hi 95 - Lo 95
2016 Q1	0.0101766600	-0.0023128170	0.0226661400	0.0249789570
2016 Q2	0.0111183400	-0.0103693690	0.0326060600	0.0429754290
2016 Q3	0.0120600300	-0.0190385270	0.0431585800	0.0621971070
2016 Q4	0.0130017100	-0.0284964550	0.0544998700	0.0829963250
2017 Q1	0.0139433900	-0.0387497810	0.0666365600	0.1053863410
2017 Q2	0.0148850700	-0.0497713600	0.0795415000	0.1293128600
2017 Q3	0.0158267500	-0.0615273990	0.0931809100	0.1547083090
2017 Q4	0.0167684400	-0.0739846290	0.1075215000	0.1815061290
2018 Q1	0.0177101200	-0.0871122610	0.1225325000	0.2096447610
2018 Q2	0.0186518000	-0.1008823710	0.1381859700	0.2390683410
2018 Q3	0.0195934800	-0.1152697660	0.1544567300	0.2697264960
2018 Q4	0.0205351600	-0.1302517030	0.1713220300	0.3015737330
2019 Q1	0.0214768400	-0.1458075980	0.1887612900	0.3345688880
2019 Q2	0.0224185300	-0.1619188743	0.2067558000	0.3686746743
2019 Q3	0.0233602100	-0.1785680630	0.2252884800	0.4038565430
2019 Q4	0.0243018900	-0.1957399110	0.2443436900	0.4400836010
2020 Q1	0.0252435700	-0.2134198890	0.2639070300	0.4773269190
2020 Q2	0.0261852500	-0.2315946960	0.2839652000	0.5155598960
2020 Q3	0.0271269400	-0.2502520020	0.3045058700	0.5547578720
2020 Q4	0.0280686200	-0.2693803370	0.3255175700	0.5948979070

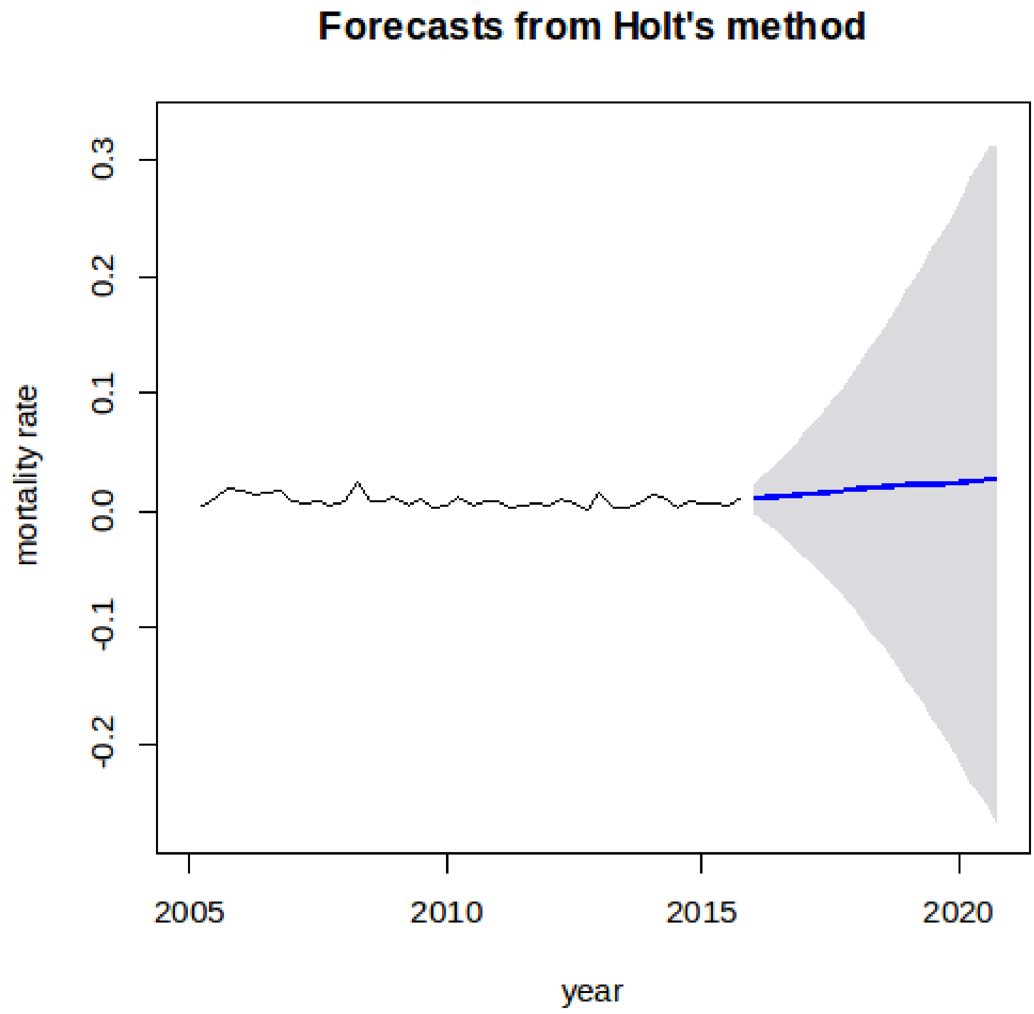

Figure 4.5: This is the graphical representation of the forecast of mortality rate

4.3 Discussions

From figure 4.1, we noticed that, mortality rates were going up and down depending on the number of deaths. The graph is in colours blue, orange, grey and yellow which represent quarter 1, 2, 3 and 4 respectively. We realized that there is a high rise in mortality rate of 0.024517 which is around 2.5% in 2008 during the second quarter because the number of deaths was 14 out of the 29 total number of deaths for that year. The 14 deaths in that year is also the highest for the period in which the data was collected. The least mortality rate of 0.001768 which was about 0.18% was observed in the last quarter of the year 2012 with 1 person dying out of the 14 people who died in that year. This is the least death for the period the information was collected. All the other mortality rate are within the range of 0.18% and 2.5%.

From figures 4.4.1, 4.4.2 and 4.4.3, we observed that longevity increases with higher ages. As one is ageing with higher age the longevity risk increases. There is a linear increase in longevity. Longevity is positively related to age.

From the forecast tables (table 4.5), we have the forecast points for the various values of alpha and beta. In order to get the best alpha and beta point for the best forecast point, we find the interval between the low and high prediction level. After finding the interval, we realized that alpha being 0.6 while beta is 0.4 gives the smallest interval for each quarter for the various alpha and beta points. The smaller the interval the higher the accuracy of the forecast.

Chapter 5

SUMMARY, CONCLUSION AND RECOMMENDATIONS

5.1 Summary

Retirement from active service is one thing that every person will experience one day if death does not put its icy hands on you. After retirement one looks forward to staying longer, hence longevity. Longevity is where people live longer and gets older as the time goes by and the risk associated with it is termed longevity risk. Longevity is however connected to mortality rate. Hence, in this research we estimated mortality rate using the death rate which is obtained from the number of people exposed to death and the number of pensioners within each quarter.

After getting the mortality rate, we further used the logit of the mortality rate to investigate longevity in the Cairns Blake Dowd 2006 model. The model use two mortality parameters k_t^1 and k_t^2 because they fit the qualities of been able to represent an age varying pattern of mortality improvement, new data invariant and interpretable which is important to model based mortality indexes. The CBD mortality indexes also satisfy the properties of been objective and transparent which should be fulfilled by mortality indexes in general.

The forecast of future mortality rate was done by the use of Holt Linear method also known as the double exponential smoothing. Smoothing parameters alpha and beta of lying between o and 1 that is $0 \leq \alpha \leq 1$ and $0 \leq \beta \leq 1$ were used within a prediction level of 95 percentage as alpha is increasing, beta is decreasing and vice versa. The increase or decrease is by an interval of 0.1. The distance or interval between the high and low prediction level in this case 95% is estimated by finding the difference between the two in order to find the best forecast point. For example, the forecasted mortality rate for the third quarter of 2017 is about 0.007135 with a 95% prediction interval of (-0.126597, 0.140868) for alpha of 0.2 and beta of 0.8.

5.2 Conclusion

In this research, we look at the estimation of mortality rate, measurement of longevity risk and the forecast of future mortality rates. Mortality rates were estimated for each quarter from the data collected from GUSSS KNUST. Conclusion drawn from the research was that, mortality rates was dependent on the number of deaths per period.

Longevity risk was estimated using the Cairns Blake Dowd (CBD) 2006 model in this work. A major conclusion that was observed was longevity goes with ages most especially older ages, hence as one gets older, longevity risk also increases. Hence, as people are ageing, the chance of them been alive for a little while is high. The study also includes the prediction of future mortality rate.

The forecast of the mortality rates were done for twenty periods (quarters) which is for 5 years from 2016 to 2020. At the end of the forecast for the various alpha's and their corresponding beta's, we realized that alpha=0.6 and beta=0.4 gave as the least interval, hence there is a high accuracy in the forecast because the lower the interval, the higher the accuracy of the forecast.Therefore we conclude that alpha and beta of 0.6, 0.4 respectively is a good point choice for the forecast of mortality rate. From the forecast table it was concluded that future mortality rates are within 0.010 and 0.017, as such mortality rate is very low.

However, the analysis carried out in this work could be extended to include a wider range of models with different characteristics such as the Generalized Cairns Blake Dowd (GCBD) model. There are still challenges ahead that could be considered. The existing models on mortality and longevity risk need further refinement and the need to get better understanding of them for proper and effective management.

5.3 Recommendation

People now believe that there is more years after pension. People tend to take proper care of themselves, eat healthy, exercise and go for regular checkups as they grow older in order not to die early. It is also believed that life is sweeter after retirement where the pressure at that time is quite low. Though our pension system is not the best, at retirement a pensioner receives a monthly income to cater for his or her needs until death. After realizing that people now live longer than expected from this research, the following recommendations are made for pension houses and all other institutions that are related to pensions:

A recommendation is therefore made to the pension funds to put measures in place so they are not found wanting when most of their pensioners are still alive living longer and receiving monthly benefit.

It is also recommended to pension houses to put their funds into profitable and long term investments with less risks so they do not run out of funds and be able to continually pay the retirees their monthly benefits for as long as they will live.

A recommendation is also made that once future mortalities have been predicted, pension houses as well as insurance firms that do pensions can based on it and prepare adequately for the future of their pensioners.

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Appendix A

Age: 70			
Year	K_t^1	K_t^2	logit $q_{x,t}$
2005	0.011912	0.00003336	0.011495
2006	0.015324	0.00000456	0.015267
2007	0.007632	0.00000325	0.007591
2008	0.012872	0.00006349	0.012078
2009	0.008043	0.00001804	0.007818
2010	0.008044	0.00001163	0.007899
2011	0.006288	0.00000523	0.006223
2012	0.006321	0.00001429	0.006142
2013	0.007543	0.00003401	0.007118
2014	0.009724	0.00002207	0.009448
2015	0.007899	0.00000889	0.007788

Age: 75			
Year	K_t^1	K_t^2	logit $q_{x,t}$
2005	0.011912	0.00003336	0.011662
2006	0.015324	0.00000456	0.015290
2007	0.007632	0.00000325	0.007608
2008	0.012872	0.00006349	0.012396
2009	0.008043	0.00001804	0.007908
2010	0.008044	0.00001163	0.007957
2011	0.006288	0.00000523	0.006249
2012	0.006321	0.00001429	0.006214
2013	0.007543	0.00003401	0.007288
2014	0.009724	0.00002207	0.009558
2015	0.007899	0.00000889	0.007832

Age: 80			
Year	K_t^1	K_t^2	logit $q_{x,t}$
2005	0.011912	0.00003336	0.011829
2006	0.015324	0.00000456	0.015313
2007	0.007632	0.00000325	0.007624
2008	0.012872	0.00006349	0.012713
2009	0.008043	0.00001804	0.007998
2010	0.008044	0.00001163	0.008015
2011	0.006288	0.00000523	0.006275
2012	0.006321	0.00001429	0.006285
2013	0.007543	0.00003401	0.007458
2014	0.009724	0.00002207	0.009669
2015	0.007899	0.00000889	0.007877

Age: 85			
Year	K_t^1	K_t^2	logit $q_{x,t}$
2005	0.011912	0.00003336	0.011995
2006	0.015324	0.00000456	0.015335
2007	0.007632	0.00000325	0.007640
2008	0.012872	0.00006349	0.013031
2009	0.008043	0.00001804	0.008088
2010	0.008044	0.00001163	0.008073
2011	0.006288	0.00000523	0.006301
2012	0.006321	0.00001429	0.006357
2013	0.007543	0.00003401	0.007628
2014	0.009724	0.00002207	0.009779
2015	0.007899	0.00000889	0.007921

Age: 90			
Year	K_t^1	K_t^2	logit $q_{x,t}$
2005	0.011912	0.00003336	0.012162
2006	0.015324	0.00000456	0.015358
2007	0.007632	0.00000325	0.007656
2008	0.012872	0.00006349	0.013348
2009	0.008043	0.00001804	0.008178
2010	0.008044	0.00001163	0.008131
2011	0.006288	0.00000523	0.006327
2012	0.006321	0.00001429	0.006428
2013	0.007543	0.00003401	0.007798
2014	0.009724	0.00002207	0.009890
2015	0.007899	0.00000889	0.007966

Age: 95			
Year	K_t^1	K_t^2	logit $q_{x,t}$
2005	0.011912	0.00003336	0.012329
2006	0.015324	0.00000456	0.015381
2007	0.007632	0.00000325	0.007673
2008	0.012872	0.00006349	0.013666
2009	0.008043	0.00001804	0.008269
2010	0.008044	0.00001163	0.008189
2011	0.006288	0.00000523	0.006353
2012	0.006321	0.00001429	0.006500
2013	0.007543	0.00003401	0.007968
2014	0.009724	0.00002207	0.010000
2015	0.007899	0.00000889	0.008010

Table 5.1: Mortality rate and indexes

Year	Period/ Quarter	No. of Pensioners	Exposure to death	No. of death	Death rate	Mortality Rate	K_t^1	K_t^2
			568		0	0	-	-
2005	1	1	569	6	0.0105	0.0105	0.0119	0.000034
	2	0	563	3	0.0053	0.0053		
	3	1	561	7	0.0125	0.0124		
	4	16	570	11	0.0193	0.0191		
2006	1	0	559	9	0.0161	0.0160	0.0153	0.000005
	2	0	550	7	0.0127	0.0127		
	3	1	544	8	0.0147	0.0146		
	4	27	563	10	0.0178	0.0176		
2007	1	4	557	5	0.0089	0.0089	0.0076	0.000003
	2	1	553	4	0.0072	0.0072		
	3	2	551	5	0.0091	0.0090		
	4	26	572	3	0.0053	0.0052		
2008	1	0	569	5	0.0088	0.0087	0.0129	0.00006
	2	0	564	14	0.0248	0.0245		
	3	0	550	5	0.0091	0.0090		
	4	24	569	5	0.0088	0.0087		
2009	1	0	564	7	0.0124	0.0123	0.0080	0.00002
	2	0	557	3	0.0053	0.0054		
	3	0	554	6	0.0108	0.0108		
	4	16	564	2	0.0035	0.0035		
2010	1	0	562	3	0.0053	0.0053	0.0080	0.00001
	2	0	559	7	0.0125	0.0124		
	3	0	552	3	0.0054	0.0054		
	4	14	563	5	0.0089	0.0088		
2011	1	0	558	5	0.0090	0.0089	0.0063	0.000005
	2	0	553	2	0.0036	0.0036		
	3	0	551	3	0.0054	0.0054		
	4	13	561	4	0.0071	0.0071		
2012	1	0	557	3	0.0054	0.0054	0.0063	0.00001
	2	0	554	6	0.0108	0.0108		
	3	0	548	4	0.0073	0.0073		
	4	21	565	1	0.0017	0.0018		
2013	1	0	564	9	0.0160	0.0158	0.0075	0.00003
	2	0	555	2	0.0036	0.0036		
	3	0	553	2	0.0036	0.0036		
	4	21	572	4	0.0070	0.0070		
2014	1	0	568	8	0.0141	0.0140	0.0097	0.00002
	2	0	560	7	0.0125	0.0124		
	3	17	570	2	0.0035	0.0035		
	4	0	568	5	0.0088	0.0088		
2015	1	0	563	4	0.0071	0.0071	0.0079	0.000009
	2	0	559	4	0.0072	0.0071		
	3	24	579	3	0.0052	0.0052		
	4	0	576	7	0.0122	0.0121		