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Using simulated annealing approach for scheduling sports tournament
traveling problem (A case study of Ghana Football)

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CHAPTER ONE

INTRODUCTION

The problem of finding optimal schedules for professional sports leagues has attracted interests of many researchers in recent years. On the one hand, the scheduling of sport leagues is an economically important class of combinatorial optimization applications, since sport leagues generate considerable amount of revenue for major radio and television networks and neither the sporting event organizers nor the participating teams want to waste their investments and resources due to the poor schedules of games. On the other hand, sport scheduling poses a very challenging optimization problem with multiple objectives and constraints combining issues of feasibility and optimality.

The Traveling Tournament Problem (TTP), which is proposed by Easton, Nemhauser and Trick in year 2001, is a challenging sport scheduling problem abstracting the features of major league tournaments. The objective of the TTP is to find a double round-robin tournament schedule minimizing the total distance traveled by the teams and satisfying at the same time the TTP-specific constraints. One can say that the TTP is a combination of the well-known Traveling sport timetabling problem for which already various effective solution techniques exist. But the combination of the both optimality and feasibility-issues makes the TTP a much more difficult optimization problem than its individual underlying —sub-problemsl.

Since its introduction, the TTP has received considerable attention and numerous different approaches have been devised to tackle this hard optimization problem. The very first solving

techniques proposed for the TTP were exact-methods like constraint programming and integer programming, but their limit was quickly reached even for the smallest instances. Then one of the first successful metaheuristics approach using the Simulated Annealing technique was proposed by Anagnostopoulos *et al.* (2008) introducing basic neighborhoods, which are used by nearly all the subsequent meta-heuristics researches for the TTP. In the following years, it was further enhanced to the current state-of-the-art meta-heuristics for solving the TTP.

Based on the researches done so far, one can recognize that single-solution based metaheuristics (like Simulated Annealing and Tabu-Search) are performing particularly well for the TTP. The Iterated Local Search (ILS) is another single-solution based meta-heuristics technique, which can exhibit very powerful performance if properly optimized, and it has been successfully applied to various optimization problems. -

1.1 Background of the Study

The travelling tournament problem (TTP) represents the fundamental issues involved in the creating of schedule for sports leagues where the amount of team travel is an issue for many of these leagues. The scheduling problem includes a myriad of constraints based on thousands of games and hundreds of team's idiosyncrasies that vary in their content and importance from year to year, but at its heart are two basic requirements. The first is a feasibility issue in that the home and away matches must be sufficiently varied so as to avoid long home stands and road trips. The second is the goal of preventing excessive travel. For simplicity, we state this objective as minimize total travel distance. While each issue has been addressed by either integer programming or transportation problem sometimes both their combination is a relatively new problem for both groups.

Professional sports leagues are a major economic activity in Ghana and around the world. Teams and leagues do not want to waste their investments in players and structure they have laid, in consequence of poor schedules of games. Game scheduling is a difficult aspect with the multiple constraints and objectives involving the logistic, organization, economic and fairness as well as several decision makers such as league officials, team managers and TV executives.

Efficient schedules are of major interest for team's leagues, sponsors fans and the mass media. This issue may be further complicated due to the distances involved. In the case of the Ghana Premier League, a single trip from Accra to Tamale takes almost a full day journey, due to the flight and poor nature of our roads to cover a distance of approximately 1300 kilometers. The total distance travelled becomes an important variable to be minimized so as to reduce traveling cost and to give more time to the players for resting and training along a season that lasts for approximately four months. Another possible variable to be minimized is the maximum distance travelled by the teams in Ghana.

The managers and the officials tackled the problem of tournament scheduling in Ghana League using different techniques such as integer programming, genetic algorithms and simulated annealing. The traveling tournament problem is an inter-mural championship time tabling problem that abstracts certain characteristics of scheduling problem in Ghana's league. It combines tight feasibility issues with a difficult optimization problem. The objective is to minimize the total distance traveled by the teams, subject to the constraint that no team can play on a row more than four games at away or home. Since the total distance traveled is a major issue for every team in

Ghana taking part in the professional league, solving a traveling tournament problem may be a starting point for the solution of real timetabling applications in sports.

1.2 Historical Background of Ghana Football

It has been noted that Cape coast is the birth place of Ghana football and the credit goes to the students of the local government boys school. Inspired by a Jamaican Headmaster, Mr. Briton, the students were already sports conscious playing cricket and tennis. The zeal with which the students followed sports was fantastic. In 1903 a group of twenty two keen pupils of the Cape Coast government boy's school embarked upon a secret training course in football. They were trained mostly in the night, when the full moon was on at the Victoria Park, then a well kept place for official ceremonies. The first football used by the pioneers was gifts from friendly sailors who docked regularly at Cape Coast Port. Most of the sailors who landed ashore were keen and played games regularly with the Governor of the country. The group arranged and ordered some equipment, jerseys (red and yellow stripes) white long running shorts, and pairs of hose, football boots and caps. The happy band of soccer adventure who called themselves excelsior continued with their secret training and after three months planned a grand out-door ceremony at the Victoria Park on Boxing Day December 26 1903 Wilson(2001).

Cape Coast Victoria Park was lined and marked with the goal posts fixed; the first football pitch in Ghana was thus created. In the presence of top government officials the first two teams from the first football club of the country merged to introduce the game to the country. Although the match was played without any set rules the excited crowd cheered throughout and thoroughly enjoyed themselves as they watched 22 youngsters running around and kicking a globular object. It was a memorable occasion that was graced with the presence of the then

Governor Sir Fredrick Hodgson, himself a keen sportsman, it is significant to note that Excelsior played in boots from the day they introduced football into the country. With the warm reception received the young boys of Excelsior intensified their training and soon their popularity spread beyond Cape Coast having now mastered the rudiments of the game. Sir Fredrick arranged for them to play their first challenge match against a European side comprising sailors from a ship that had docked at Cape Coast port and some Europeans resident in Cape Coast. Excelsior lost the exciting match 2-1 but they really gave a good account of themselves. By popular request a return match was arranged later and Excelsior avenged defeat 3-1. Regular friendly fixtures were arranged for Excelsior and white civil servants in Cape Coast. Ships docking at the Cape Coast harbor supplied, at frequent intervals, sailor team who also played with the pioneer team of Ghana. This exercise enabled Excelsior to improve by leaps and bounds. The matches were officiated by the Europeans until 1905 when few Africans studied the laws of the game and began to handle matches. The game quickly captured the fancy of the youth of Cape Coast and like mushrooms clubs sprang up in the town.

At the time the game football was introduced into the country traveling was mostly on foot and so the football gospel did not travel fast. After playing for several months Excelsior moved to nearby towns and played demonstration games. This crusade proved extremely successful and within a matter of months the new game was being played with amazing zeal at Elmina, Saltpond and Winneba. Cape Coast, the original home of Ghana football dominated the local soccer scene until the middle forties. Clubs such as Evertons, Blankson XI, Energetic Sports, Swallows, Rose XI, Bolton Wanderers, Judges, Gardens, Titanics and Majestics were formed and reformed but

have now faded away. Venomous Vipers and Mysterious Dwarfs have sprung up from the babies, to uphold tradition of Cape Coast football.

The exciting stories of the new game gradually travelled through traders and fishermen to Sekondi – Takoradi. Real football lives in the Sekondi – Takoradi started with the construction of the railway line as the cream of the players was centered in the railway workshops. The formation of a powerful non-departmental team, Eleven Wise in 1919 marked the beginning of soccer development in the Sekondi – Takoradi area. Some of the early clubs were Mosquitoes, Western Wanderers, Jericho, Railway Apprentice, and GA United. In 1952 Mr. Semmer Wilson, District Commissioner of Sekondi – Takoradi, formed the District Football Association. This later developed into the Western Region Football Association. The year also saw the birth of Fanti United football club which later reformed into Hasaccas now one of the formidable clubs in the Western Region. It was nearly twenty years after the introduction of football in Cape Coast that the game reached Kumasi, the Garden City of Ashanti in 1920.

In 1926, the first Ashanti football club, Ashanti United which in the thirties developed into the present power Asante Kotoko, was formed by 13 young Ashanti boys headed by a young driver Kwasi Kuma and L.Y. Asamoah, a private electrician. The team was later on re-named mighty Atoms and in 1935 Atoms re-organized and christened Asante Kotoko by J.S.K Frimpong with the permission of Otumfuo, Asantehene. The Kumasi Jackson Park was built in 1935 and it became the central venue for all matches in Kumasi.

The game reached Accra quite early and by 1910 invincible, the first club ever to be formed in Accra was organized in James Town. Inspired by an acute sense of rivalry, boys in Ussher Town

accepted the challenge and in 1911 founded Accra Hearts of Oak which is today the oldest existing club in Ghana. By 1912 clubs like Energetics, Never Miss, Royalties, Osu

Pioneers, Africa, Wolves, had all been formed.

Football organization in the country has come a long way since those early colonial days when Excelsior was born. Soccer historians tell us that it was around 1943 that some effort was made at the control and organization of the sport from Accra. The early 40's brought onto the undefined soccer scene Mr. Richard —The Lion Heart Akweil, the man who made so much impact on the early days of organized football in Ghana. His efforts were directed at getting the Accra Football Association on solid grounds. Around 1947, Mr. Akwei, described as shrewd soccer organizer, mooted the idea of bringing together the major associations that had sprung up in the country. But the baby was not to be spared the usual teething problems. First, a serious split in the Accra Association had a corresponding effect on the national body and two national associations emerged from the crisis. Mr. Akwei became the president of the Gold Coast Football Union. The other, dubbed the Gold Coast and Ashanti Union had Mr. John Darkwa of Kumasi as chairman, with Mr. A.W. Mills of Accra as secretary. It was the projected tour of the United Kingdom by the Gold Coast national team in 1951 that brought the two factions together at an historic meeting at the Hudson Club, Kumasi on October 29, 1950, the two sides agreed on the formation of a United Gold Coast Amateur Football Association. Mr. Darkwa became the first chairman and Mr. Richard Akwei, the vice chairman. But peace was not to be given much of a chance, even after this union, soon after the 1951 United Kingdom tour, Mr. Richard Akwei was voted chairman of the Union at an election in Accra and that marked the beginning of a fresh crisis in the country's soccer administration. It was this very crisis that culminated in the so-called Reformation Era that eventually gave birth to the national soccer league. The definitive turn of

events, leading to the reformation in 1958 was natured by the Ashanti Football Association and fueled by the trenchant pen of Mr. Kofi Badu, indisputably the greatest sportswriter this country has ever produced. A sustained period a relentless battle was waged to remove Richard Akwei from the seat of Ghana football power. The culmination of these efforts came on September 8, 1957 when, at a general meeting of the Ghana Amateur Football Association held at Legion Hall, Accra and presided over by Sir Leslie McCarthy, chairman, of the Ghana Amateur Sports Council, Mr. Akwei resigned.

In appreciation of his pioneering role in the evolution of the country's soccer, Mr. Akwei was made special Life Patron of the Ghana Amateur Football Association. Thus ended one era and thus began another, to be dominated by one man —Ohene Djan! thirty-three years old, full of fresh ideas, dynamism and enthusiasm, Mr. Ohene Djan stepped into the shoes of —The Lion Heart at the unanimously elected Chairman of the Ghana Football Association (GFA).

The first attempt at organizing a national league was during the turbulent last days of Mr. Richard Akwei in 1959. It was an idea mooted by an Englishman, Mr. Ken Harrison resident Manager of a trading firm, R.E Harding & Co. This maiden contest was, however, poorly organized. Asante Kotoko and four other Kumasi based teams Cornerstone, Great Ashanti, Dynamos and Evergreens boycott it. The Richard Akwei administration reacted by suspending them. The competition thus took off with only the teams from the south. In the ensuing confrontation, the four boycotting clubs were able to get some other to join them, thus wrecking the league and flouting the authority of the GAFA. The competition was eventually left with only Accra Hearts of Oak and Sekondi Eleven Wise who were later declared respectively gold and silver medalists. This was perhaps the beginning of the traditional friendship between Hearts and Wise.

Eight clubs selected from the Municipalities of Accra, Kumasi, Sekondi and Cape Coast play one another regularly on home and away basis, always fielding their strongest available side. The pioneer clubs were Hearts of Oak and Great Olympics (Accra); Asante Kotoko and Conerstone (Kumasi); Hasaacas and Eleven Wise (Sekondi); Mysterious Dwarfs and Venomous Vipers (Cape Coast).

1.3 Statement of the Problem

To solve a real-world sports scheduling problem it is apparent that a profound understanding of the relevant requests and requirements presented by the league is a prerequisite for developing an effective solution method. In most cases the most important goal is to minimize the number of breaks. There are various reasons why breaks should be minimized in a sports schedule: fans do not like long periods without home games, consecutive home games reduce gate receipts, and long sequence of home or away games might influence the team's current position in the tournament. Apart from minimizing the number of breaks, several other issues play a role in sports scheduling, e.g. minimizing the total traveling distance, creating a compact schedule, avoiding a team playing against all the strong teams consecutively.

An outline of the typical constraint of the sports scheduling problem is presented below. These constraints are representative of many scheduling scenarios within the area of sports scheduling. There is no strict distinction between hard and soft constraints. They are given by the instances themselves. The goal is to find a feasible solution that is the most acceptable for the sports league owner. That is, a solution that has no hard constraint violations and that minimizes the weighted sum of the soft constraints violations. The weights will also be given by the instances themselves.

1.4 Aims and Objective of the Research

The main goals set for this thesis are:

1. To model the league fixtures of the Ghana Football Association as Travelling Tournament problem.
2. Determine the optimal solution using Simulated Annealing method.

1.5 Methodology

Statement of problem for this research work was how to minimize sports travelling tournament schedules and its effect on team management. The researcher developed optimal solution to this problem by using Simulated Annealing approach. Data was collected from the Ghana Football Association and four top league clubs in the country. The top league clubs were Kumasi Asante Kotoko Football Club, Accra Hearts of Oak, Aduana Stars and Heart of Lions. The type of data collected was the travelling schedule of teams, cost per distance travel for both home and away matches. Statistical Package for Social Sciences (SPSS 12.0 Version) was used for analyzing the data collected from the respondents. Kotoko Express, Graphic Sports, Ghana Football Association Journal and the internet were used as the source of materials.

1.6 Justifications

The thesis will be useful for organizers and administrators of football clubs in country as well as other stakeholders such as sports writers, football fans and team managers or owners who contributed the development of football in the country who may require further information on

sports tournament travel problem. This will help them to know when and where to move and how they will go about their normal duties on sports and their traveling expenses they will use for the season.

1.7 Organization of the Thesis

The rest of the thesis will be organized as follows. In Chapter 2, we give a formal description of the Traveling Tournament Problem and discuss current state-of-the-art approaches. The general principles of Annealing Approach to solve the TTP will be discussed in Chapter 3. Chapter 4 comprises the discussion and experimental results of the approach. Chapter 5 deals with the conclusion and recommendations. The thesis concludes with closing remarks and the outlook on future works.



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CHAPTER TWO

LITERATURE REVIEW

2.0 Traveling Tournament Problem

Nemhauser *et al*, (2001) introduced the Traveling Tournament Problem, which is considered one of the most challenging sport scheduling problems to date. Given n teams with n even and an $n \times n$ symmetric distance matrix D , where $D(i; j)$ represents the distance between the teams of D_i and D_j . The goal in solving the traveling tournament problem is to find a valid double round robin schedule, such that the total traveling distance of all teams is minimized.

2.1 Current State-of-the-Art heuristics

According to De Werra, (1980), although the TTP is a relatively new problem, it has attracted interests of many researchers due to its practical relevance and its surprisingly high degree of difficulty, which results from the combination of two well-known problems of finding the shortest tour (optimality) and timetabling sport tournaments satisfying certain constraints (feasibility). When the TTP has been introduced for the first time, the initial approaches proposed for solving the TTP were exact methods like integer programming, constraint programming and hybrid methods. But even for small instances, it was extremely difficult for an exact algorithm to solve them in reasonable time.

The current most sophisticated exact algorithm for the TTP is proposed by Uthus (1998). It is based on branch-and-bound technique and is capable of solving the National League benchmark instances optimally up to the size of 10 teams.

One of the first successful metaheuristics for the TTP has been designed by Anagnostopoulos *et al.* (2001) using the Simulated Annealing framework. It is one of the most successful heuristics approaches for the TTP and it has produced numerous best upper bounds for most of the publicly available benchmark-sets. But the excellent solution quality comes with very long computation time, spending days of computation for larger instances. The most valuable contribution of their work was the design and definition of the neighborhoods, which have been used nearly by all following metaheuristics for the TTP. The key idea of their neighborhoods is to distinguish between hard constraints and soft constraints. The hard constraints must be satisfied all the time during the search, whereas the soft constraints can be occasionally violated. This idea stems from the observation that some constraints in the TTP are extremely difficult to repair during the search, once they are violated. After the Simulated Annealing approach, more single-solution-based metaheuristics followed.

Another very successful metaheuristic based on Tabu search was developed by Schaerf *et al.* (2000). Their algorithm uses composite neighborhoods based on the same neighborhoods of. Through further fine-tuning of the moves and careful analytical study about the effectiveness of the different composite neighborhoods, they were able to obtain very good results, which are comparable to the best results in the literature.

An interesting hybrid metaheuristic approach was introduced by Lim *et al.* (2002) which divides the search-space in two parts. The algorithm alternates between two components to improve the current solution. The first component, using a Simulated Annealing algorithm, tries to improve the solution by optimizing the timetable with a fixed team assignment, whereas the second component, which incorporates the hill-climbing technique, searches for better team assignment with a fixed timetable. So the fundamental idea in this approach is to improve the timetable, when a good team assignment has been found, and to search for a better team assignment, if the timetable looks promising.

Four years later after the first version of the Simulated Annealing approach was proposed, a population-based extension has been proposed in by Nurmi *et al.* (2010). This extension made the parallelization of the first SA algorithm possible and it produced the current upper-bounds for numerous benchmark instances running on a cluster of 60 Intel-based, dualcores, dual-processor Dell Power edge 1855 blade servers.

In recent years, other promising heuristics techniques based on Ant Colony Optimization and Hyper-Heuristic have been proposed for solving the TTP (Briskorn *et al.*, 2006). In the past, there were already 2 ACO-based attempts for the TTP, but their results were relatively poor.

The new ACO approach proposed by Uthus (2012) which incorporates some advanced extensions like forward checking and conflict-directed back jumping algorithm is able to improve greatly on the solution quality compared to the previous ACO-based attempts. His new results are competitive with those of the state-of-the-art heuristics.

The Hyper-Heuristic method proposed by Misir *et al.* (2004) also gives very promising performance. Their Hyper-Heuristic is composed of a simple selection mechanism based on a learning automaton and a novel acceptance mechanism, which they call as the Iteration Limited Threshold Accepting criterion. Despite of the simple and general nature of the Hyper-Heuristic, their method is able to produce very good solutions in relatively short amount of time.

2.2 Distance Minimization and the Travelling Tournament Problem

Although the above gives a complete formulation for the TTP, the lower bounds provided by its linear programming relaxation are very weak. To improve this formulation, Trick (2003) suggested adding the so called odd-set constraints for each week.

Trick, (2005), formulated an alternative (and much better) approach which reformulate by redefining the decision variable. We shall return to the issue of problem reformulation in Section 4, where alternative formulations for a variant of the traveling tournament problem will be explored.

Urrutia *et al.* (2007), the mirrored traveling tournament problem and the traveling tournament problem with predefined venues (Costa *et al.*, 2009) are two variants of the traveling tournament problem. The first has the additional constraint that games played in round t are exactly the same played in round $t + (n - 1)$ for $t = 1 \dots n - 1$, but with reversed venues. The second is a single round robin variant of the TTP, in which the venue of each game to be played is known beforehand.

Trick (2010), gives benchmark instances and their best lower and upper bounds for the widely studied case of the TTP with $L = 1$ and $U = 3$. The TTP and its variants have been tackled by different exact and approximate solution methods.

Easton *et al.* (2003) proposed the first integer programming approach for exactly solving the TTP, where the so-called independent lower bound later improved by Urrutia *et al.* (2007) was originally presented.

Rasmussen *et al.* (2006) developed an exact two-phase hybrid approach which generates all feasible patterns in a first phase using constraint programming and assigns teams to patterns in the second phase using integer programming.

Cheung (2008) solved to optimality the mirrored and non-mirrored benchmark TTP instances with eight teams.

Uthus *et al.* (2011) developed an iterative problem that was able to find optimal solutions to the largest benchmark instances solved to date, involving ten teams. Metaheuristics are among the most effective solution strategies for solving combinatorial optimization problems in practice and have been largely applied in the solution of the TTP and its variants. Among the main algorithmic contributions we cite the hybrid algorithms proposed by Anagnostopoulos *et al.* (2003; 2006) for the TTP, based on simulated annealing and exploring both feasible and infeasible schedules, and by Ribeiro and Urrutia (2007b) for the mirrored TTP, in which components borrowed from the GRASP and ILS metaheuristics are combined and an ejectionchain mechanism is used to generate perturbations. Numerical results for the mirrored variant have been later

improved by Van Hentenryck and Vergados (2006), extending their previous work developed for the general case.

Bhattacharyya (2009) gave the first NP-completeness proof for the variant of the TTP where no constraints exist on the number of consecutive home games or away games of a team.

Later, Thielen *et al.* (2011) have shown that the original TTP is strongly NP-complete when the upper bound on the maximal number of consecutive away games is set to 3.

Finding a good schedule is not an easy challenge, as wishes from various stakeholders (the league, clubs, fans, TV, police, etc.) are often conflicting. Indeed, over the last decade, sport scheduling has received an increased interest from researchers from fields as operations research, computer science, and mathematics.

Kendall, *et al.* (2010) give a recent overview of the research done so far in sports scheduling, and classify the contributions according to the methodology used and the application, where soccer turns out to be the most popular topic. There are quite a few papers that present a solution approach for a specific soccer league in Europe (e.g., Bartsch, Drexler & Kroger (2006) for Austria and Germany, Della Croce & Oliveri (2006) for Italy, Rasmussen (2008) for Denmark, and Roossens & Spieksma (2009) for Belgium). There are also a couple of papers that try to classify sports scheduling problems.

Bartsch *et al.* (2006) give a survey of a number of sports scheduling problems discussed in the literature and indicate what type of constraints occur.

Nurmi *et al.* (2010) provided a more elaborate classification of the various constraints involved.

These authors present a framework for a sports scheduling problem with 36 types of constraints, modeled from various professional sports leagues, including a set of artificial and real-world instances, with the best solutions found.

Nevertheless, as far as we are aware, there is only one paper that does not focus on the process of obtaining a solution, but instead exclusively focuses on the actual solutions of sports scheduling problem: the schedules. Over a decade ago, Griggs & Rosa (1996) published a short paper entitled a tour of European soccer schedules 2 or testing the popularity of GK2n". For the season 1994/1995, they examined schedules of the highest division in 25 European soccer competitions. They focused on identifying the competitions that made use of the so called canonical schedule" and found that it is used in 16 of these competitions .This paper can be seen as a follow-up of the work by Griggs & Rosa (1996): we revisit the 25 competitions they listed in 1996. These competitions still form a balanced sample of strong and weak soccer competitions in Europe. We look at the schedules for season 2008/2009 (or the 2008 schedules for countries as Norway, where the soccer season corresponds with the calendar year), and verify whether they have a number of interesting properties. Thus, our goal in this work is modest: to investigate the schedules according to which today's soccer competitions are being played. This gives insights in

the diversity of the presence of different properties, and provides an answer to the question what features are apparently considered important in European soccer schedules. Notice that this type of information is usually not explicitly available, as the properties of a schedule often result from compromises on meetings with members from the association. Further, we will compare our findings with those of Griggs & Rosa (1996) and comment on the potential of further optimizing today's schedules. We also introduce the concept of ranking-balanced, which compares the number of home games played by each team after each round, and allows to express whether a or not fair ranking can be produced after each round. In the remainder of this paper, when we discuss a competition, we mean its highest division, to which we refer as the first division. We use n for the number of teams taking part in a competition, and l for the number of matches between a pair of teams in a (stage of) a competition. Matches are grouped in so-called rounds" meaning that they are scheduled to be played on the same day or weekend. In order to draw any conclusions about popular features in a soccer schedule, it is important to consider the fixtures as they were scheduled before the start of the season. We got this information from websites as www.the-sports.org, www.rsssf.com, and www.gooolal.com. These fixtures regularly differ from the order according to which the matches are actually played.

2.3 Minimizing Travel Distance

The minimization of travel distance becomes relevant when teams travel from one away game to the next without returning home. In this setup huge savings can be obtained when long trips are applied and teams located close together are visited on the same trip. The interest in minimizing travel distances arose from the increasing travel costs due to the oil crises in the 1970's.

This led to a request for efficient solution methods capable of finding good solutions for practical applications and a number of papers on distance minimization have appeared since 1976.

Easton, *et al* (2001) proposed the traveling tournament problem which has received most of the attention concerned with minimizing travel distances since then.

2.4 The Traveling Tournament Problem

Trick *et al.* presented the traveling tournament problem (TTP) in 2001. The problem is motivated by the problem of scheduling major league baseball and it is formulated to capture the fundamental difficulties of minimizing the travel distance for a sports league. By using the TTP as benchmark problems, it is possible to develop and compare solution methods which, afterwards, can be specialized for the various constraints present in practical applications. The TTP can be formulated as follows.

Input: n , the number of teams; D an n by n integer distance matrix; L, U integer parameters. Output: A double round robin tournament on the n teams such that the number of consecutive home games and consecutive away games are between L and U inclusive, and the total distance travelled by the teams is minimized. Furthermore, two additional requirements are mentioned. The first is a mirroring constraint requiring that the schedule is mirrored and the second is a norepeater constraint requiring that two teams cannot play two games against each other in two consecutive slots. Notice that at most one of the two requirements is relevant since the norepeater constraint is always satisfied in a mirrored schedule.

Nemhauser *et al.* (2001) present a method based on the independent lower bound (IB), which they define to be the sum of the minimum travel distances for each team when they are considered independently. The solution method generates pattern sets with as many trips as possible and a corresponding timetable minimizing the travel distance is found afterwards. In this setup, a strengthening of the IB can be used to check optimality and, as long as this bound is below the best solution, the algorithm continues.

Benoist, *et al* (2002), apply a hybrid algorithm combining Lagrange relaxation and CP. The algorithm has a hierarchical architecture consisting of three components. The main component is a CP model capturing the entire problem and capable of solving the problem by itself. However, a global constraint is introduced in order to improve the bounds during the search. This global constraint corresponds to the second component and it contains a Lagrange controller using either sub-gradient or modified gradient techniques to adjust the Lagrange multipliers for the third component consisting of a perturbed sub-problem for each team. The sub-problem for a given team i schedules all the games associated with team i such that team i 's travel distance is minimized.

Subsequently, Easton, *et al* (2003), presents another hybrid IP/CP solution method. This is a branch and price (column generation) algorithm in which the columns correspond to tours for the teams. The master problem is a linear programming problem assigning teams to tours, while the pricing problem for generating tours is a CP problem.

The next approach for the TTP was a simulated annealing algorithm by Anagnostopoulos, *et al* (1999) called TTSA. From an initial schedule found by a simple backtrack search TTSA

searches for improving solutions using five kinds of moves: Swap Homes, Swap Rounds, Swap Teams, Partial Swap Rounds and Partial Swap Teams. By applying these moves, the structure of the schedule is destroyed but for each move a corresponding ejection chain is able to restore the structure. In this way the algorithm is able to satisfy all hard constraints during the search, whereas the soft constraints may be violated. The hard constraints include the round robin constraints while the no-repeater is considered a soft constraint. The number of violated soft constraints is incorporated in the objective function to force the algorithm towards feasible solutions. TTSA randomly selects a move and it is performed with probability 1 if it leads to an improving solution and otherwise the probability depends on the resulting increase in travel distance plus the current $\|temperature\|$. The TTSA were able to improve all the current best known upper bounds for the NL instances with more than 10 teams and, in a recent paper by Hentenryck and Vergados (2005) the TTSA are further refined to handle mirrored tournaments. In Phase 1 they first use the canonical schedule to obtain a timetable with placeholders and afterwards they construct a matrix of consecutive opponents. Each entry (i, j) of the matrix gives the number of times another team meets i and j consecutively and this is used in Phase 2 when teams are assigned to placeholders. A simple heuristic assigns teams located close together to placeholders who are met consecutively by many teams.

Finally, Phase 3 uses two steps to obtain a pattern set. In Step 1 a constructive method generates an initial pattern set and afterwards Step 2 performs local search to improve the pattern set.

Ribeiro *et al.* (1999) also presents a heuristic method combining GRASP and iterated local search (ILS) which they call *GRILS-mTTP*. The GRILS-mTTP performs a number of iterations all starting with the algorithm explained above for generating an initial schedule. Afterwards, a local search is applied to obtain a locally optimal solution and then GRILS-mTTP iterates between a perturbation procedure and a local search until some re-initialization criterion is satisfied.

Henz (2004) proposes to combine large neighborhood search and CP to overcome the problem of getting away from local optima. He uses five types of moves which all relax a substantial part of the given schedule. For instance the move called Relax rounds does not only exchange two slots but it relaxes all variables associated with a number of slots. CP is then applied to obtain a new schedule given the partial schedule which has not been relaxed.

As mentioned above Ribeiro *et al.* (1999) present the instance class with constant distances and show that minimizing travel distance for these instances is equivalent to maximizing the number of breaks. They also derived upper bounds on the number of breaks for unconstrained single round robin tournaments, equilibrated single round robin tournaments, unconstrained double round robin tournaments and double round robin tournaments with a maximum of three consecutive home games and three consecutive away games. The limit on consecutive home games and away games in the last kind resembles the bounds from the benchmark TTP instances. By separating these instances into three classes $((n - 1) \bmod 3 = 0, (n - 1) \bmod 3 = 1$ and $(n - 1) \bmod 3 = 2)$ the following bounds were obtained.

$$\begin{aligned}
 & \leq 14, & \text{if } n \leq 4 \\
 & \leq 4(n^2 - n)/3 \leq 4n - 20, & \text{if } ((n-1) \bmod 3 = 0 \text{ and } n \leq 4 \\
 UB_{MTTP} & \leq 4(n^2 - 2n) & \text{if } ((n-1) \bmod 3 = 1, \text{ if } \\
 & /3, & ((n-1) \bmod 3 = 2, \\
 & \leq 4(n^2/3 - n),
 \end{aligned}$$

The corresponding mirrored constant distance TTP is solved by the GRILS-mTTP presented in and the algorithm is able to solve the instances with 4, 6, 8, 10, 12 and 16 teams to optimality by obtaining solutions which reach the upper bound stated above. The constant distance TTP was also considered by Rasmussen and Trick (2004) who used the PGBA to solve the problem. They were able to prove optimality for all the mirrored instances with 18 teams or less and all the non-mirrored instances with 16 teams or less by using the algorithm for maximizing breaks instead of minimizing breaks.

Vergados *et al.* (2006) have also used their TTSA approach and improved the best solution for mirrored instance with 20 teams and the best solutions for the non-mirrored instances with 18-24 teams.

2.5 Applications of Simulated Annealing Approach

According to Schreuder (2005), the Simulated Annealing framework is very simple but at the same time very powerful concept. If properly tuned and optimized, it can often become even a state-of-the-art algorithm. In this section we will give you a quick overview of some combinatorial problems, to which the Simulated Annealing Approach has been successfully applied. Scheduling of sports events and competitions mentioned in the literature are organized and have been studied considering two major factors, namely the pattern of home/ away games of the participating teams

and the distance these teams have to travel according to the order that is specified by the programmed schedule. Therefore, sports scheduling problems which are addressed in this paper fall into two main categories. The goal of the first category is to minimize the number of breaks, *i.e.* two consecutive home game or two consecutive away games, whereas the objective of the second one is to minimize the overall distance which teams have to travel.

De Werra (2001) has discussed the problems of the first category and application of theoretical techniques to these problems. The second class of problems is mainly the center of attention for its application in leagues.

Campbell *et al.* (2004) has studied a scheduling problem of a basketball league, in which a two-phase approach is proposed to solve the problem.

Bridge *et al.* (1999) explored a similar scheduling problem on National Basketball Association (NBA) and as a solution they proposed an Integer Programming model for the mentioned problem that was computationally infeasible to carry out, since the size of the problem was large.

Furthermore, they applied a modified version of Campbell *et al.* (2004) two-phase method. Ferland and Fleurent considered the scheduling of National Hockey League (NHL) which its teams was split into two groups of Eastern and Western Conferences so was their games and corresponding schedule.

Costa (2003), proposed the first meta-heuristic approach to minimize the travelling distance as an objective for sports scheduling problem in the form of a Tabu Search/Genetic Algorithm integration.

Additionally, Wright (1998) has presented a Simulated Annealing method in order to schedule the National Basketball league of New Zealand.

It was Easton *et al.* (1998) who introduced the Travelling Tournament Problem. In this problem which is originated from the Major League Baseball, in addition to minimizing overall travelling distance, certain constraints should be satisfied, *i.e.* feasibility constraints, making the problem more difficult to solve. Numerous approaches have been proposed to solve TTP. Among these approaches are a combination of Lagrange Relaxation (LR) and Constraint Programming (CP), a collaborative scheme by Benoist *et al.* (2010) a hybrid Integer Programming-Constraint Programming algorithm by Easton *et al.* (1987) and a Simulated Annealing algorithm by Anagnostopoulos *et al.* (2004). In the latter method, a distinction has been made between soft and hard constraints.

Furthermore, Lee *et al.* (2006) in addition to creating an IP model with no-repeat constraint offered TS for solving the problem. Lim *et al.* proposed a hybrid SA-Hill algorithm that is a combination of Simulated Annealing and Hill-Climbing methods. Also, considerable effort has been dedicated to solving the Traveling Tournament Problem quite recently and new solution methods have been developed for TTP.

In one of the most recent works in this area, Tajbakhsh *et al.* proposed a hybrid Particle Swarm Optimization (PSO) and Simulated Annealing algorithm in which the results from the PSO section of the algorithm is used as an initial solution for the SA section of the algorithm.

CHAPTER THREE

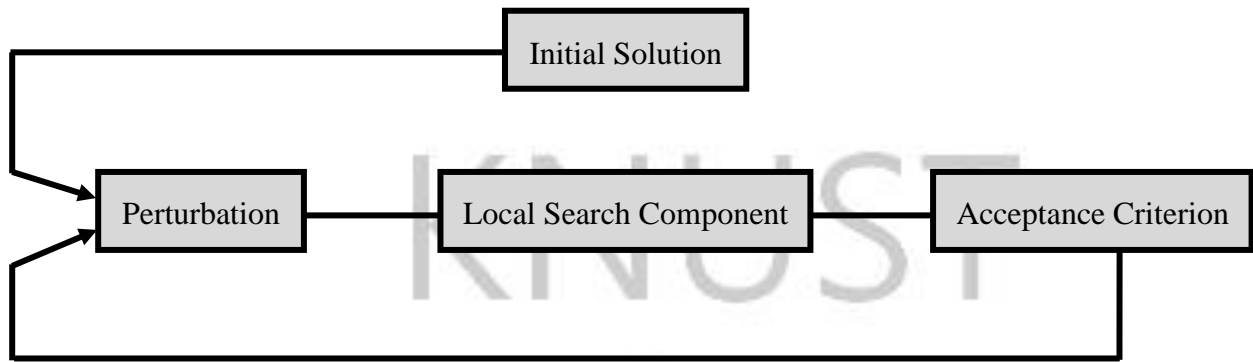
METHODOLOGY

There is an excellent overview of the Simulated Annealing Approaches where they present the main principles of the ILS in terms of the three main components. In this chapter, we will give brief descriptions about these components and discuss their impacts on the overall performance of the ILS. Simulated Annealing Approach is both conceptually and practically very simple meta heuristics framework. The basic idea behind the ILS is to use the embedded local-search component iteratively restarting it from different promising areas in the searchspace. Then how can one actually identify the promising areas for the restarts? In one extreme end, we can determine the next restarting point in completely random fashion, where we then get a simple Random-Restart scheme. But for many problems, this scheme is very unlikely to perform well because without using any information of the previous search the algorithm will most likely just stray —blindly— in the search-space. On the other extreme end, we can always restart from the —best position— found so far, but this strategy will increase the danger of getting easily stuck in local optima.

The Simulated Annealing Approach considers the embedded local-search heuristic as a kind of black box component and uses its output as a basis for determining the next starting point, trying to guide the search into the promising areas. In doing so, the nature and strength of the perturbation of the local-search's output is critical for the performance of the ILS. If the perturbation is too weak, meaning that not enough new attributes are introduced into the current search point, the algorithm risks getting stuck early in local optima. On the other hand, if the perturbation is too strong, we will lose too much information from the previous search. In worst case, it will be then not better than just restarting the search from a random starting point. Besides perturbation, there is another important aspect of the ILS, to which we should pay close attention, namely the criteria how to accept the local optima found by the embedded local search component for the next iteration. To this end there are several different strategies to consider, which we will discuss in detail later on. In summary, we can modularize the ILS framework into following three main components

- ❖ Perturbation
- ❖ Local-search component
- ❖ Acceptance criterion

Having these individual components cleanly modularized reduces the complexity of the framework and makes it easier to optimize the overall performance by fine-tuning the components independently. Of course, it should be clear that the components cannot function completely independent from each other. In order to achieve maximum performance, we also should carefully study and understand their correlations and impacts on each other, which will vary from problem to problem. The main —work-flow— of the ILS framework can be depicted as in Figure 3.1.



We start with an initial solution, which is usually generated using some random constructive procedures. Then we use the local-search component to obtain a local optimum, which is either accepted or discarded according to the chosen acceptance criterion. Then the local search component restarts with a new starting point, which is obtained by perturbing the current accepted solution. The ILS-template, formulated as a pseudo-code, is given in Algorithm 1.

Algorithm 1 Template of the ILS

```

1:  $s = \text{generate Initial Solution } (s)$ 
2: while ! Stop Condition do
3:    $s^{\wedge} = \text{perturbate } (s)$ 
4:    $s^{\wedge\wedge} = \text{local search } (s^{\wedge})$ 
5:   if acceptance criterion  $(s, s^{\wedge\wedge})$  then
6:      $s = s^{\wedge\wedge}$ 
7:   end if
8: end while
9: return  $s$ 
  
```

Initial Solution

Until now, we have somewhat neglected the question how to generate initial solutions for the ILS and what influence they have on the overall performance. Simply stated, one can either start from fully random initial solutions or may try to use greedy procedures in order to construct good-quality start solutions. But in general, there is not always a clear best choice regarding the initial solutions for the ILS. Sometimes greedy initial solutions appear to be recommendable when one has to obtain good-solutions quickly. Some experiments have shown that for certain problems the ILS performs in average better with greedy initial solutions, when short computation time is given. For much longer running-time, the meaning of the initial solution may become less relevant, since in most cases much of the initial properties will get lost during long search. Here, the user may choose the strategy, which is easiest to implement.

3.1 Perturbation

The perturbation component is a crucial component, which allows the ILS to escape from local optima. The ILS tries to modify the local optimum in a certain way, so that the local-search component can jump to another promising region in the next iteration. At this point, we introduce the notion perturbation strength, which specifies how strong the current local optimum will be modified. Obviously we should be very careful in choosing the appropriate perturbation strength for the ILS. If the perturbation is too strong, we run the risk of losing good properties found in the previous searches, which is against the concept of the ILS. On the other hand, if we are too —petty with the perturbation, the chance to successfully escape from local optima will be very low. So as you can see, one of the most important aspects in fine tuning the ILS will be the task of finding a nice balance for the perturbation strength considering the points mentioned above. For example,

there are different strategies proposed in the past to handle the perturbation strength during the search:

- ❖ **Static:** the perturbation length is fixed a priori before the search and is no longer modified during the search.
- ❖ **Dynamic:** the perturbation length is modified dynamically during the search without taking the search history into account (can be random variations of the perturbation length in a certain interval).
- ❖ **Adaptive:** the perturbation length is modified dynamically during the search exploiting the information (i.e. about the shape of the landscape) gathered during the search. Finding effective perturbation methods is a highly —problem-specific matter and depends also on the used embedded local-search heuristic. One important aspect to consider is that the perturbation shouldn't be easily undone by the local-search component; otherwise one will fall back into the local-optimum just visited. Furthermore, one should try to exploit as much problem-specific properties as possible in the perturbation component complementing possible shortcomings of the local-search component.

3.2 Local-Search Component

In many overview articles for the ILS, the local search component is described as a —black box module, for which we can use practically any existing single-solution based metaheuristics. This gives us two advantages using the Iterated Local Search framework. Since we treat the embedded local-search as a black box component, we don't have many —dependency problems between the framework and the local-search component. Therefore, if necessary we can just swap the local-search component without altering the whole framework again. As mentioned

above, we can use any existing metaheuristics as the embedded local-search component. If there already exists a well performing heuristic for the given problem, then we can quickly develop a potentially better performing ILS-version reusing the existing local-search algorithm as the embedded local search component.

At this point, we want to reemphasize the main principle of the ILS. Roughly speaking, the ILS is nothing more than a —simple walk in S^* , which can be seen as a subset of the original search-space S consisting of local optima produced by the embedded local search. So, we can think of the local-search component as a mapping function, which maps the original searchspace, S into the subset of local optima S^* :

$$S^* := \{ \text{localSearch}(s) \mid s \in S \}$$

Note also that, no explicit neighborhood is defined for the walk in S^* , but instead the components perturbation and acceptance criterion determine the next neighbor to visit. You may have already noticed that ideally S^* should be a small compact set of local optima, which contains the global optimum. In order to get a high quality mapping, one, of course, needs a powerful local-search component, which returns high quality local optima. In general, we can assume the better the embedded local-search, the better the corresponding ILS. For example in case of the TSP, the Lin-Kernighan heuristic is better than the 3-opt local-search. Researchers have shown that the ILS embedding the Lin-Kernighan heuristics gives better results than the ILS using the 3-opt local-search Rebeiro & Urrutia (1998). But high quality comes usually with a high price, namely long running-time. If the computation-time is heavily limited, it would be probably better idea to use a less powerful but faster embedded local-search in order to get useful results more quickly. As

already mentioned for the perturbation, an important aspect to consider when choosing the local-search component is the —collaboration between the local-search and the perturbation component. The rule of thumb is that local-search shouldn't systematically undo the changes made by the perturbation component.

3.3 Acceptance Criterion

Alongside the perturbation component, the acceptance criterion will also have a great influence on the effectiveness of the ILS framework. We consider the ILS as a heuristic approach, which —random-walks in the search-space S^* consisting of local optima defined by the embedded local-search component. The perturbation mechanism together with the local search component defines the transition from one local optimum $s^* \in S^*$ to the —neighboring local optimum. $s^{**} \in S^*$ and the acceptance criterion determines whether the neighbor s^{**} will be accepted or not for the next iteration. The chosen acceptance criterion has a critical influence on the balance between intensification and diversification of the search. On the one hand, we can define the acceptance criterion to accept only better local optima than the current one. We call such strategy as the Better acceptance criterion which can be defined for the minimization problem as follows:

Better(s' , s'') = 1 if $\text{Cost}(s') < \text{Cost}(s'')$
 0 otherwise

As you can intuitively see, this criterion is an extreme one, which clearly advocates strong intensification. At the opposite extreme, one can work with a strategy called RandomWalk (RW) acceptance criterion which always chooses the most recently visited local optimum, irrespective of its cost: $RW(s^{\wedge}; s^{\wedge\wedge}) = s^{\wedge\wedge}$

This criterion strongly favors diversification over intensification, because every solution in S^{\wedge} is accepted for the next step. Obviously in order to find an appropriate balance between these two extremes, we need to find a way to encourage both intensification and diversification in an adequate manner. One of the very successful acceptance criteria applied to the ILS was a simulated annealing type acceptance criterion, which we will denote as the LSMC acceptance criterion, reminiscent of the term large step Markov chains used for one of the first ILS algorithms with this type of acceptance criterion. The LSMC criterion accepts always $s^{\wedge\wedge}$, if it is better than the current local optimum s^{\wedge} . Otherwise, if $s^{\wedge\wedge}$ is worse than s^{\wedge} , a certain probability p , with which $s^{\wedge\wedge}$ will be accepted, is calculated based on the difference in qualities of s^{\wedge} and $s^{\wedge\wedge}$. The bigger the gap between s^{\wedge} and $s^{\wedge\wedge}$ is, the less the chance that $s^{\wedge\wedge}$ will be accepted.

Given s^{\wedge} and its qualitative worse neighbor $s^{\wedge\wedge}$, the acceptance probability p can be calculated as

$$p = \frac{\text{Cost}(s^{\wedge}) - \text{Cost}(s^{\wedge\wedge})}{T}$$

Where T is a parameter called temperature, which controls the balance between intensification and diversification.

3.4 Neighborhoods for the Local-Search Component

In this study neighborhoods have been defined based on the ideas introduced in chapter three. Five categories of neighbourhoods were considered, which can be three different classes regarding the magnitude of introduced changes:

- ❖ N_1 : Swap Homes (micro move)
- ❖ N_2 ; N_3 : Swap Rounds, Swap Teams (macro moves)
- ❖ N_4 ; N_5 : Partial Swap Rounds, Partial Swap Teams (generalized moves)

A rough estimation about the approximate size of the search-space, the obvious upper-bound for a TTP-instance of size n can be given as $t=1 (2n - 2) = (2n - 2)^n$, since for each of n teams, there are $(2n - 2)$ permutation possibilities to arrange the order of the matches. But this is actually too big for estimation, because it also includes all the invalid schedules violating the Double-RoundRobin constraint and the TTP-specific At-Most and No-Repeat constraints. In order to refine our estimation, let's consider a double-round-robin (DRR) schedule with 6 teams given in Table 1.

Table 3.1: Double round – robin with 6 teams

T/W	W_1	W_2	W_3	W_4	W_5	W_6	W_7	W_8	W_9	W_{10}
T_1	$+T_6$	$-T_2$	$+T_4$	$+T_3$	$-T_5$	$-T_4$	$-T_3$	$+T_5$	$+T_2$	$-T_6$
T_2	$+T_5$	$+T_1$	$-T_3$	$-T_6$	$+T_4$	$-T_3$	$+T_6$	$-T_4$	$-T_1$	$-T_5$
T_3	$-T_4$	$+T_5$	$+T_2$	$-T_1$	$+T_6$	$-T_2$	$+T_1$	$-T_6$	$-T_5$	$+T_4$
T_4	$+T_3$	$+T_6$	$-T_1$	$-T_5$	$-T_2$	$+T_1$	$+T_5$	$+T_2$	$-T_6$	$-T_3$
T_5	$-T_2$	$-T_3$	$+T_6$	$+T_4$	$+T_1$	$-T_6$	$-T_4$	$-T_1$	$+T_3$	$+T_2$
T_6	$-T_1$	$-T_4$	$-T_5$	$+T_2$	$-T_3$	$+T_5$	$-T_2$	$+T_3$	$+T_4$	$+T_1$

Based on the feasible DRR-schedule, I can think of some modification —moves that we can apply to the schedule and maintain at the same time the feasibility of the Double-Round-

Robin constraint. Firstly, we will leave the most and No Repeat constraints out. If we just look at the columns of the schedule, it's easy to recognize that swapping the columns (rounds) doesn't violate the DRR-feasibility at all. For example, look at the rounds W_3 and W_6 : below.



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T/W	W ₁	W ₂	W ₆	W ₄	W ₅	W ₃	W ₇	W ₈	W ₉	W ₁₀
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T_1	$+T_6$	$-T_2$	$-T_4$	$+T_3$	$-T_5$	$+T_4$	$-T_3$	$+T_5$	$+T_2$	$-T_6$
T_2	$+T_5$	$+T_1$	$-T_3$	$-T_6$	$+T_4$	$-T_3$	$+T_6$	$-T_4$	$-T_1$	$-T_5$

T/W	W_1	W_2	W_3	W_4	W_5	W_6	W_7	W_8	W_9	W_{10}
T_1	$+T_6$	$-T_2$	$+T_4$	$+T_3$	$-T_5$	$-T_4$	$-T_3$	$+T_5$	$+T_2$	$-T_6$
T_2	$+T_5$	$+T_1$	$-T_3$	$-T_6$	$+T_4$	$-T_3$	$+T_6$	$-T_4$	$-T_1$	$-T_5$
T_3	$-T_4$	$+T_5$	$+T_2$	$-T_1$	$+T_6$	$-T_2$	$+T_1$	$-T_6$	$-T_5$	$+T_4$
T_4	$+T_3$	$+T_6$	$-T_1$	$-T_5$	$-T_2$	$+T_1$	$+T_5$	$+T_2$	$-T_6$	$-T_3$
T_5	$-T_2$	$-T_3$	$+T_6$	$+T_4$	$+T_1$	$-T_6$	$-T_4$	$-T_1$	$+T_3$	$+T_2$
T_6	$-T_1$	$-T_4$	$-T_5$	$+T_2$	$-T_3$	$+T_5$	$-T_2$	$+T_3$	$+T_4$	$+T_1$



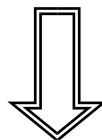
Swapping



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As indicated, the modified schedule is still a valid DRR-schedule. This means, given a certain DRR-schedule, we have already $(2n - 2)$ possibilities to permutate the order of the rounds producing different valid DRR-schedules. In addition, I can

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also recognize that the swapping of two teams doesn't violate the DRR feasibility either. Considering, the schedule given in Table 1, we can imagine this time the teams $T_1, T_2 \dots T_n$ as kind of placeholders, to which arbitrary teams can be assigned. Then obviously there are n different team-assignments to consider.

Again, we can consider one relevant example below:

T/W	W_1	W_2	W_3	W_4	W_5	W_6	W_7	W_8	W_9	W_{10}
T_1	$+T_6$	$-T_2$	$+T_4$	$+T_3$	$-T_5$	$-T_4$	$-T_3$	$+T_5$	$+T_2$	$-T_6$
T_2	$+T_5$	$+T_1$	$-T_3$	$-T_6$	$+T_4$	$-T_3$	$+T_6$	$-T_4$	$-T_1$	$-T_5$
T_3	$-T_4$	$+T_5$	$+T_2$	$-T_1$	$+T_6$	$-T_2$	$+T_1$	$-T_6$	$-T_5$	$+T_4$
T_4	$+T_3$	$+T_6$	$-T_1$	$-T_5$	$-T_2$	$+T_1$	$+T_5$	$+T_2$	$-T_6$	$-T_3$
T_5	$-T_2$	$-T_3$	$+T_6$	$+T_4$	$+T_1$	$-T_6$	$-T_4$	$-T_1$	$+T_3$	$+T_2$
T_6	$-T_1$	$-T_4$	$-T_5$	$+T_2$	$-T_3$	$+T_5$	$-T_2$	$+T_3$	$+T_4$	$+T_1$

New team assignment, where the teams T_2 and T_4 are swapped:

T_1	T_1
T_2	T_4
T_3	T_3
T_4	T_2
T_5	T_5
T_6	T_6

It is now evident that some possibilities to produce different DRR-schedules from a given DRR-schedule. Thus, even if the search-space contains only valid DRR-schedules, there would still have to deal with at least $(2n - 2)$ possible solution candidates.

In sum, the search-space would consist of total $(2n - 2)^n$ possible candidates, if we allow that all the three constraints can be violated. If we include all the invalid schedules in the searchspace, then the large size of the search-space will make it very difficult to find even valid solutions satisfying all the three constraints, optimizing the travel-distance. Thus, our neighborhoods consist of only valid DRR-schedules, whereas the other two constraints can be violated.

3.5 Swap-Homes Neighborhood

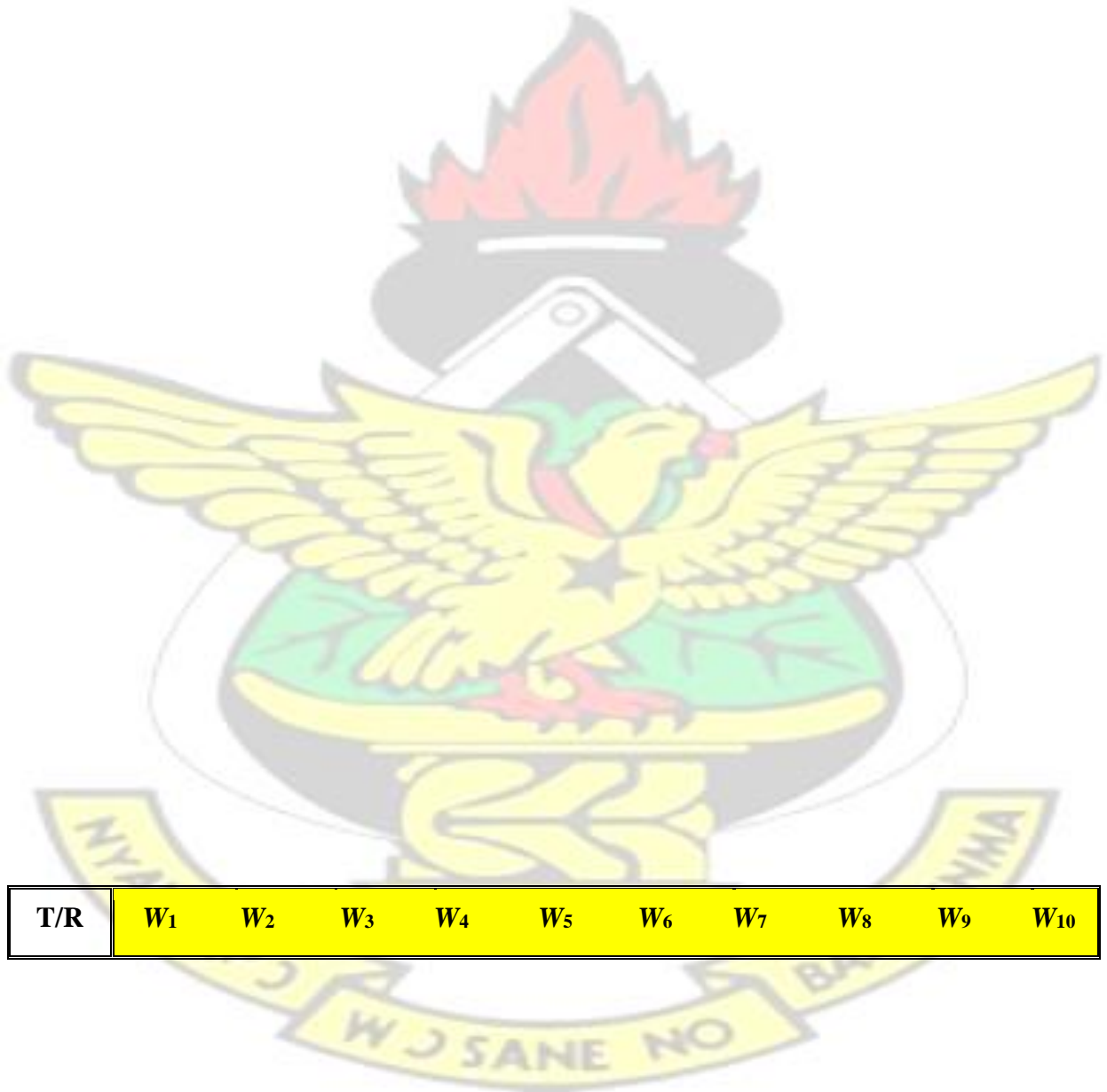
From the five neighborhoods mentioned earlier, the Swap-Homes neighborhood offers the —smallest local-move, meaning that the number of changes caused by this move is minimal.

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Given a valid DRR-schedule, this move swaps the home/away states of the teams T_i and T_j , where $i \neq j$. Let's say that team T_i plays T_j in round W_1 at home and team T_j plays T_i in round R_k at home, where $i \neq j$ and $k \neq 1$. Then after swapping the home/away states of T_i and T_j , team T_i plays T_j in round W_k at home and team T_j plays T_i in round W_1 at home. IT can be inferred that there is $O(n^2)$ possible neighbors in this neighborhood. It is important to note that —movestrength is always 4, since only 4 games are affected by this move. The procedure for applying the Swap-Homes move is depicted in Table 2.

Table 3.2: Swapping the home/away teams

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T/R	W ₁	W ₂	W ₃	W ₄	W ₅	W ₆	W ₇	W ₈	W ₉	W ₁₀
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T_1	$+T_6$	$+T_4$	$+T_2$	$+T_3$	$-T_5$	$-T_4$	$-T_3$	$+T_5$	$-T_2$	$-T_6$
T_2	$+T_5$	$-T_6$	$-T_1$	$-T_5$	$+T_4$	$+T_3$	$+T_6$	$-T_4$	$+T_1$	$-T_3$
T_3	$-T_4$	$+T_5$	$+T_4$	$-T_1$	$+T_6$	$-T_2$	$+T_1$	$-T_6$	$-T_5$	$+T_2$
T_4	$+T_3$	$-T_1$	$-T_3$	$-T_6$	$-T_2$	$+T_1$	$+T_5$	$+T_2$	$+T_6$	$-T_5$
T_5	$-T_2$	$-T_3$	$+T_6$	$+T_2$	$+T_1$	$-T_6$	$-T_4$	$-T_1$	$+T_3$	$+T_4$
T_6	$-T_1$	$+T_2$	$-T_5$	$+T_4$	$-T_3$	$+T_5$	$-T_2$	$+T_3$	$-T_4$	$+T_1$
T/R	W₁	W₂	W₃	W₄	W₅	W₆	W₇	W₈	W₉	W₁₀
T_1	$+T_6$	$-T_2$	$+T_4$	$+T_3$	$-T_5$	$-T_4$	$-T_3$	$+T_5$	$+T_2$	$-T_6$
T_2	$+T_5$	$+T_1$	$-T_3$	$-T_6$	$+T_4$	$-T_3$	$+T_6$	$-T_4$	$-T_1$	$-T_5$
T_3	$-T_4$	$+T_5$	$+T_2$	$-T_1$	$+T_6$	$-T_2$	$+T_1$	$-T_6$	$-T_5$	$+T_4$
T_4	$+T_3$	$+T_6$	$-T_1$	$-T_5$	$-T_2$	$+T_1$	$+T_5$	$+T_2$	$-T_6$	$-T_3$
T_5	$-T_2$	$-T_3$	$+T_6$	$+T_4$	$+T_1$	$-T_6$	$-T_4$	$-T_1$	$+T_3$	$+T_2$
T_6	$-T_1$	$-T_4$	$-T_5$	$+T_2$	$-T_3$	$+T_5$	$-T_2$	$+T_3$	$+T_4$	$+T_1$

Swapping the home/away roles of teams T_2 and T_4



KNUST

3.6 Swap-Rounds Neighborhood



T/W	W_1	W_2	W_3	W_4	W_5	W_6	W_7	W_8	W_9	W_{10}
T_1	$+T_6$	$-T_4$	$+T_2$	$+T_3$	$-$	$-T_2$	$-T_3$	$+T_5$	$-T_2$	$-T_6$
					T_{405}					

T_2	$+T_5$	$+T_3$	$-T_1$	$-T_5$	$+T_4$	$+T_1$	$+T_6$	$-T_4$	$+T_1$	$-T_3$
T_3	$-T_4$	$-T_2$	$+T_4$	$-T_1$	$+T_6$	$+T_5$	$+T_1$	$-T_6$	$-T_5$	$+T_2$
T_4	$+T_3$	$+T_1$	$-T_3$	$-T_6$	$-T_2$	$+T_6$	$+T_5$	$+T_2$	$+T_6$	$-T_5$

The Swap-Rounds move simply swaps two rounds W_k and W_l in the given configuration, where $k \neq l$. Swapping two rounds is considered to be a —macro— move, meaning that the changes introduced by this move are of quite disruptive nature. The number of affected games by this move is obviously $2 * n$ and there are $O(n^2)$ possible neighbors in this neighborhood. Obviously, after swapping two rounds, the new resulting schedule is still a valid DRR-schedule, thus no further repair action is required. The procedure for applying the Swap-Rounds move is depicted in **Table 3.3: Swapping round 2 and 6**

T/W	W_1	W_2	W_3	W_4	W_5	W_6	W_7	W_8	W_9	W_{10}
T_1	$+T_6$	$-T_2$	$+T_4$	$+T_3$	$-T_5$	$-T_4$	$-T_3$	$+T_5$	$+T_2$	$-T_6$
T_2	$+T_5$	$+T_1$	$-T_3$	$-T_6$	$+T_4$	$-T_3$	$+T_6$	$-T_4$	$-T_1$	$-T_5$
T_3	$-T_4$	$+T_5$	$+T_2$	$-T_1$	$+T_6$	$-T_2$	$+T_1$	$-T_6$	$-T_5$	$+T_4$
T_4	$+T_3$	$+T_6$	$-T_1$	$-T_5$	$-T_2$	$+T_1$	$+T_5$	$+T_2$	$-T_6$	$-T_3$
T_5	$-T_2$	$-T_3$	$+T_6$	$+T_4$	$+T_1$	$-T_6$	$-T_4$	$-T_1$	$+T_3$	$+T_2$
T_6	$-T_1$	$-T_4$	$-T_5$	$+T_2$	$-T_3$	$+T_5$	$-T_2$	$+T_3$	$+T_4$	$+T_1$

Swapping the round R_2 and R_6



3.7 Swap-Teams Neighborhood

Similar to the Swap-Rounds move, swapping two teams is a —macroll move, which introduces up to $4 * (2n - 4)$ changes in the given schedule, and is the most disruptive move of the five neighborhoods. Given two teams T_i and T_j , the Swap-Teams move swaps the games of T_i and T_j at every round, except when they play against each other. Obviously, the number of affected rounds is $2n - 4$ and at each round, the number of changed games is always 4. This move is also similar to the Swap-Rounds move as it does not violate the DRR-feasibility after the application either. The procedure for applying the Swap-Teams move is shown in Table 4. **Table 3.4:** Swapping the teams T_2 and T_4

T/W	W_1	W_2	W_3	W_4	W_5	W_6	W_7	W_8	W_9	W_{10}
T_1	$+T_6$	$-T_2$	$+T_4$	$+T_3$					$+T_2$	$-T_6$
T_2	$+T_5$	$+T_1$	$-T_3$	$-T_6$	$-T_5$	$-T_3$	$+T_6$	$+T_5$	$-T_1$	$-T_5$
T_3	$-T_4$	$+T_5$	$+T_2$	$-T_1$	$+T_4$	$-T_2$	$+T_1$	$-T_4$	$-T_5$	$+T_4$
T_4	$+T_3$	$+T_6$	$-T_1$	$-T_5$	$+T_6$	$+T_1$	$+T_5$	$-T_6$	$-T_6$	$-T_3$
T_5	$-T_2$	$-T_3$	$+T_6$	$+T_4$	$+T_1$	$-T_6$	$-T_4$	$-T_1$	$+T_3$	$+T_2$
T_6	$-T_1$	$-T_4$	$-T_5$	$+T_2$	$-T_3$	$+T_5$	$-T_2$	$+T_3$	$+T_4$	$+T_1$

Swapping the teams T_2 and T_4



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3.8 Swap-Partial-Rounds Neighborhood

Despite the neighborhoods been straight-forward and relatively simple to understand, however the local-moves given by these neighborhoods are not sufficient for an effective search resulting only in limited search-space. The Swap-Partial-Rounds neighborhood. Like the SwapRounds move, two parameters W_i and W_j are required, which specify the rounds that are being swapped. In addition, we also need a team T_k as the third parameter, from which the games at rounds R_i and R_j should be swapped. Obviously, swapping just the two games will violate the DRR-constraint of the schedule, but there is a deterministic way to define a sequence of —repairing movements| in order to restore the DRR-feasibility after the swapping. The procedure for applying the Swap-Partial-Rounds move is depicted in Table 3.5.

Table 3.5: Swap - Partial Round move

T/W	W ₁	W ₂	W ₃	W ₄	W ₅	W ₆	W ₇	W ₈	W ₉
<i>T</i> ₁	$+T_6$ 2	$-T$ $+T_1$	$+T_4$	$+T_3$	$-T_5$	$-T_4$	$-T_3$	$+T_5$	$+T$ 2
<i>T</i> ₂		$+T_5$	$-T_3$	$-T_6$	$+T_4$	$-T_3$	$+T_6$	$-T_4$	$-T_1$ $-T_5$
<i>T</i> ₃	$+T_5$		$+T_2$	$-T_1$	$+T_6$	$-T_2$	$+T_1$	$-T_6$	
<i>T</i> ₄	$-T_4$								
<i>T</i> ₅	$+T_3$	$+T_6$	$-T_1$	$-T_5$	$-T_2$	$+T_1$	$+T_5$	$+T_2$	$-T_6$
<i>T</i> ₆	$-T_2$	$-T_3$	$+T_6$	$+T_4$	$+T_1$	$-T_6$	$-T_4$	$-T_1$	$+T_3$
<i>T</i> ₆	$-T_1$	$-T_4$	$-T_5$	$+T_2$	$-T_3$	$+T_5$	$-T_2$	$+T_3$	$+T_4$

Partial-swapping the weeks *W*₂ and



*W*₉ for the team *T*₂

T/W	W ₁	W ₂	W ₃	W ₄	W ₅	W ₆	W ₇	W ₈	W ₉	W ₁₀
<i>T</i> ₁	$+T_6$	$+T_4$	$+T_2$	$+T_3$	$-T_5$	$-T_4$	$-T_3$	$+T_5$	$-T_2$	$-T_6$
<i>T</i> ₂	$+T_5$	$-T_6$	$-T_1$	$-T_5$	$+T_4$	$+T_3$	$+T_6$	$-T_4$	$+T_1$	$-T_3$
<i>T</i> ₃	$-T_4$	$+T_5$	$+T_4$	$-T_1$	$+T_6$	$-T_2$	$+T_1$	$-T_6$	$-T_5$	$+T_2$
<i>T</i> ₄	$+T_3$	$-T_1$	$-T_3$	$-T_6$	$-T_2$	$+T_1$	$+T_5$	$+T_2$	$+T_6$	$-T_5$
<i>T</i> ₅	$-T_2$	$-T_3$	$+T_6$	$+T_2$	$+T_1$ 43	$-T_6$	$-T_4$	$-T_1$	$+T_3$	

T_6	$-T_1$	$+T_2$	$-T_5$	$+T_4$	$-T_3$	$+T_5$	$-T_2$	$+T_3$	$-T_4$	$+T_4$	$+T_1$
-------	--------	--------	--------	--------	--------	--------	--------	--------	--------	--------	--------

As shown in Table 3.5 the Swap-Partial-Rounds move does not swap the —whole columns, however only the parts which are necessary to maintain the DRR-validity. As already indicated in Table 3.5 the parts that are needed to be swapped can be determined in a deterministic fashion. For example, taking a closer look at the above example, Team T_2 plays against team T_1 and T_6 at rounds W_2 and W_9 , which therefore should be swapped. It is important to note that the home/away states of the games are irrelevant in this case, so we concentrate only on teams. After swapping the relevant games of T_2 , one can see that the games of T_1 and T_6 also should be swapped at rounds W_2 and W_9 , since they are affected by the first swap. Swapping games of T_1 and T_6 further affects the team T_4 and gives us a total set of teams $\{T_1, T_2, T_4, T_6\}$, whose games must be swapped at rounds W_2 and W_9 in order to repair the violation of the DRR-constraint.

Swap-Partial-Teams Neighborhood

Similarly, the Swap-Partial-Rounds neighborhood, the Swap-Partial-Teams neighborhood is the further generalization of the Swap-Teams neighborhood. Given two teams T_i, T_j and round W_k , the Swap-Partial-Teams move swaps the games of T_i and T_j at the round W_k and repairs the schedule afterwards to make it a valid DRR-schedule again. In addition, there is an important precondition that T_i does not play against T_j at the round W_k . Similar to the Swap-Partial-Rounds move, the repair-chain can be determined in a deterministic way. The move-strength varies from case to case and is given by the actual length of the repair-chain. In the extreme case, the

repairchain can have the length $(2n - 4)$, in which case the move equals to the respective Swap-Teams move. The procedure for applying the Swap-Partial-Teams move is depicted in Table 6.

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Table 3.6: Partial Swap teams T_2 and T_4 at W_9

T/W	W_1	W_2	W_3	W_4	W_5	W_6	W_7	W_8	W_9	W_{10}
T_1	$+T_6$	$-T_2$	$+T_4$	$+T_3$	$-T_5$	$-T_4$	$-T_3$	$+T_5$	$+T_2$	$-T_6$
T_2	$+T_5$	$+T_1$	$-T_3$	$-T_6$	$+T_4$	$-T_3$	$+T_6$	$-T_4$	$-T_1$	$-T_5$
T_3	$-T_4$	$+T_5$	$+T_2$	$-T_1$	$+T_6$	$-T_2$	$+T_1$	$-T_6$	$-T_5$	$+T_4$
T_4	$+T_3$	$+T_6$	$-T_1$	$-T_5$	$-T_2$	$+T_1$	$+T_5$	$+T_2$	$-T_6$	$-T_3$
T_5	$-T_2$	$-T_3$	$+T_6$	$+T_4$	$+T_1$	$-T_6$	$-T_4$	$-T_1$	$+T_3$	$+T_2$
T_6	$-T_1$	$-T_4$	$-T_5$	$+T_2$	$-T_3$	$+T_5$	$-T_2$	$+T_3$	$+T_4$	$+T_1$

Partial swapping the teams T_2 and T_4 at the round R_9



T/W	W_1	W_2	W_3	W_4	W_5	W_6	W_7	W_8	W_9	W_{10}
T_1	$+T_6$	$-T_2$	$+T_4$	$+T_3$	$-T_5$	$-T_4$	$-T_3$	$+T_5$	$+T_2$	$-T_6$
T_2	$+T_5$	$+T_1$	$-T_3$	$-T_6$	$+T_4$	$-T_3$	$+T_6$	$-T_4$	$-T_1$	$-T_5$

T_3	$-T_4$	$+T_5$	$+T_2$	$-T_1$	$+T_6$	$-T_2$	$+T_1$	$-T_6$	$-T_5$	$+T_4$
T_4	$+T_3$	$+T_6$	$-T_1$	$-T_5$	$-T_2$	$+T_1$	$+T_5$	$+T_4$	$-T_6$	$-T_3$
T_5	$-T_2$	$-T_3$	$+T_6$	$+T_4$	$+T_1$	$-T_6$	$-T_4$	$-T_1$	$+T_3$	$+T_4$
T_6	$-T_1$	$-T_4$	$-T_5$	$+T_2$	$-T_3$	$+T_5$	$-T_2$	$+T_3$	$+T_4$	$+T_1$

3.9 Connectivity of the neighborhoods

Having defined the neighborhoods for our embedded local-search component, it becomes to explore the connectivity of our neighborhoods. Indeed, this is crucial as there is the need to test, if it is theoretically possible to reach every valid solution in the search-space with the local-moves given by our neighborhoods. Let's denote our neighborhoods with following abbreviations:

N_1 : Swap-Homes Neighborhood

N_2 : Swap-Rounds Neighborhood

N_3 : Swap-Teams Neighborhood

N_4 : Swap-Partial-Rounds Neighborhood

N_5 : Swap-Partial-Teams Neighborhood

The experiment designed as follows:

- ❖ Generate a random DRR-schedule s .
- ❖ Generate another random DRR-schedule s'
- ❖ Test if we can reach the schedule s' starting from the schedule s using only local transformations given by the neighborhoods $[N_1; : : ; N_5]$.

The approach to test the reachability between two random DRR-schedules is simple. Just devise a simple heuristic, which tries to minimize the hamming-distance between the starting schedule s and the target schedule s' by applying only the moves from $[N_1; : : : ; N_5]$. That is, the objective function is the hamming-distance function, which calculates the number of differences between s and s' .

Summary

In this chapter a detailed description of the Simulated Annealing framework was given. The Simulated Annealing framework consists of three main components, namely embedded local search, perturbation component and acceptance criterion. In general, each of these components can be optimized individually, but if we want to achieve maximum performance out of Simulated Annealing, we should also try to gain deeper understanding how they influence each other and fine-tune them together —globally|. We have presented some selected successful applications of the Simulated Annealing to various combinatorial optimization problems. Most notably, the ILS framework seems to be well suited both for the TSP and the scheduling problems. This gives us an extra motivation and hope in choosing the Simulated Annealing framework for solving the TTP.

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CHAPTER FOUR

DISCUSSIONS AND EXPERIMENTAL RESULTS

The Traveling Tournament Problem, which is considered one of the most challenging sport scheduling problems to date, was originally introduced by Easton *et al.* (2001). Given n teams with n even and an $n \times n$ symmetric distance matrix D , where $D(i; j)$ represents the distance between the teams of D_i and D_j . The goal in solving the traveling tournament problem is to find a valid double round robin schedule, such that the total traveling distance of all teams is minimized. A schedule is valid for the traveling tournament problem, if it satisfies the following constraints:

- ❖ *Double Round-Robin constraint*: Each team plays with each other team exactly two times, once in its own city and once in its opponent's city

- ❖ *At Most constraint*: Each team must play no more than u and no less than l consecutive games in or away from the home city. u is a number prescribed by the association.
- ❖ *No Repeat constraint*: It is not allowed that two teams are playing each other in two consecutive rounds.

We call a schedule feasible, if it satisfies all the constraints above, otherwise infeasible. Note that, if u is set to $n - 1$, then finding the schedule with the shortest traveling distance for one team T_i is equivalent to solving the Traveling Salesman Problem. It is somewhat misleading to name the second constraint as the *At Most* constraint, since we have both lower- and upper bounds for the number of consecutive home- and away-games, but for most of the benchmark instances available in the literature the parameter l is set to 1, so that many researchers adopted the notion *At Most* constraint, because they consider only the upper bound. It is obvious that a double round-robin schedule for n teams (n even) consists of at least $2n - 2$ rounds and for the regular TTP we shall only consider double round-robin schedules with this minimum number of rounds. In this work, a schedule is represented by an $n \times (2n - 2)$ matrix S of integer numbers where the teams are assigned unique positive integer numbers from $[1..n]$. The entry $S_{i,j}$ of the schedule matrix (see Table 4.1) represents the game, which is played by the team T_i in round R_j . The game entry $S_{i,j}$ is a positive integer permutation number representing the away Team T , if the team T_i plays at home against the away team in round W_j in home city i . On the other hand if the game takes place in the opponent's home city, $S_{i,j}$ will be represented with a negative integer number $-T$.

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The day of the national premier league where 16 (x) is an even number of teams. The national league is probably the well- researched area of the TTP and a virtually for all the researchers studying area in the country. The computation of the results from the league fixtures and their travelling distance over the years and more and more teams are added where the sub-division teams are added by the recent ones. The three sub-division teams are the teams in the Premier league.

P_1 teams with size from 6 to 16,

P_2 Teams with size from 6 to 14 and

P_3 Teams with sizes from 6- 16

Parameter Setting

Despite of the relatively simple ILS components; there are actually some parameters to define for the TTIL sports. The embedded local search component is a simple Hill Climbing heuristic without any parameters to tune. The parameters for the perturbation components define the interval in

which the perturbation strength varies. At this point one should notice that TTIL sports tries to achieve diversification mainly by occasionally accepting the worse solutions and not so much by very high perturbation. The tool for escaping local optima is LSMC acceptance criterion, whose parameters have critical impacts on the performance and I have spent considerable effort to determining the best values for them.

In summary,

K_{\min} : the minimum perturbation strength

K_{\max} : The maximum perturbation strength

T_{\max} : The maximum (initial) Temperature for LSMC acceptance criterion λ :
number of iterations before the next worse solution acceptance rate is checked.

δ : if the number of accepted worse solution is smaller than δ : then the temperature is reset to T_{\max} .

δ : The rate with which the temperature is lowered. ω : The rate with which the penalty weight of the soft constraints is increased or decreased. λ : The upper- bound for the lower number of violation in the country is best infeasible solution.

Extensive try and error experiment. I would have determined the reasonable values for most of the parameters which are stable across all the leagues, except for the T_{\max} . T_{\max} is surely one of the most influential parameters for TTIL. Sports because if it is too high or too low, the balance diversification and intensification is easily distorted. Again, it is believed that the best value for

T_{\max} actually depend on the individual league and its sizes. As it has been already pointed out that it is reasonable to take the average distance between two teams' sites into consideration when experimenting with initial temperature parameter for simulated- annealing type acceptance criteria. The average distance calculated for each of our league is given in the table below.

Leagues	Teams _{6n}	Teams _{8n}	Teams _{10n}	Teams _{12n}	Teams _{14n}	Teams _{16n}
P ₁	649	623	624	790	1094	1194
P ₂	35	38	45	51	57	61
P ₃	4198	4167	4130	3910	3954	

Average Distance between Two Team Sites

Considering the average distance given above I have experiment with the following temperature conditions for T_{\max} (10, 20, 30, 50, 200, 400, 600, 800, 1000, 2000).

As expected the best T_{\max} value from the league to league, probably depends on the individual team sizes and the average distance between two teams sites. It is also noticeable that T_{\max} does not have critical impacts on the solution quantity for smaller league because they are already solve optimally by the embedded local search. The formal parameter setting, which I have determined based on the various experiment with T_{\max} and extensively try and error experiment for other parameters are given in Table 4.2 they are used for all of our computational Leagues and the Teams. As you can see, all values for K_{\max} are rather small values, so it seems to be best when the

perturbation does not get too disruptive. As expected, the perimeter ω should be set to a moderate value so that the penalty weight does not fluctuate too much once it is settled.

At the, cooling speed δ seems to be too low, but since I update the temperature at each iteration. This is actually a reasonable value. The parameter λ controls how often the league team's solution is checked and it shows the value 200 for λ is stable for most of the leagues. The parameter α does not actually appear to have significant impacts as long as it is set to a reasonably small value. As far as soft restarting mechanism is concerned. I just restart the algorithm when during the last 15 temperature reset no improvement could be achieved. In along so, the initial solution for the next round is obtained by applying five (5) random moves selected from the neighborhood (N_1, \dots, N_5) to the current best solution.

Table 4.2: Final parameter setting for each league (N denotes the term size)

Zones	K_{min}	K_{max}	ω	δ	λ	α	β	T_{max}
P ₁ L ₆	2	3	1.1	0.999	500	3	3	200
P ₁ L ₈	2	4	1.1	0.999	500	3	3	300
P ₁ L ₁₀	2	5	1.1	0.999	500	3	5	400
P ₁ L ₁₂	2	6	1.1	0.999	500	3	6	500
P ₁ L ₁₄	2	7	1.1	0.999	500	3	6	500
P ₁ L ₁₆	2	8	1.1	0.9995	1000	3	7	550

P ₂ L ₆	2	3	1.1	0.999	500	3	3	10
P ₂ L ₈	2	4	1.1	0.999	500	3	3	20
P ₂ L ₁₀	2	5	1.1	0.999	500	3	5	30
P ₂ L ₁₂	2	6	1.1	0.999	500	3	6	30
P ₂ L ₁₄	2	7	1.1	0.999	500	3	6	30
P ₂ L ₁₆	2	8	1.1	0.9995	1000	3	7	30
P ₃ L ₆	2	3	1.1	0.999	500	3	3	400
P ₃ L ₈	2	4	1.1	0.999	500	3	3	400
P ₃ L ₁₀	2	5	1.1	0.999	500	3	5	500
P ₃ L ₁₂	2	6	1.1	0.999	500	3	6	500
P ₃ L ₁₄	2	7	1.1	0.999	500	3	6	1000
P ₃ L ₁₆	2	8	1.1	0.999	500	3	6	100

RESULTS FOR THE NATIONAL LEAGUE

As already named, the national league is well-studied for which numerous computational results from different methods are reported. Among different heuristics approaches in the studied the researcher picked 5 leading method (including the current state of the art met heuristics) to compute the results with. Simulated Annealing (TTSA)

Composite-neighborhood Tabu search (CNTS)

Hybridization of simulated Annealing and Hill-climbing (SAHC)

Ant colony optimization (AFC-TTP)

Learning Hyper-Heuristic (LHH)

Two separate evaluations with different timeout setting for the national premier league were conducted for the first experiment. The researcher tested the TTLL sports 6 times for each National league and adjusted the time out corresponding to the average time-value report in doing so, he also tried to take the machine difference into account, but of course, the difference can be only roughly approximated for the second experiment he run the TTIL spots 6 times for each National league and this he set the time outs corresponding to the average time-value.

DISTANCE

League/Teams	Min	Average	Max	Std. Dev
P L ₆	23916	23916.0	23916	0
P L ₈	39721	39721.0	39721	0
P L ₁₀	59583	59632.6	59727	60.592
P L ₁₂	113360	114391.7	115289	708.349
P L ₁₄	197230	199182.4	201638	1436.473
P L ₁₆	281644	286178.0	289547	2190.686

BEST TIME

League/Teams	Min	Average	Max	Std. Dev	Timeout
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P L ₆	0	0.4	2	0.6663	10
P L ₈	4	60	198	56.896	300
P L ₁₀	489	2652	4305	1167.318	4700
P L ₁₂	410	3088	4688	1140.322	4700
P L ₁₄	1873	3739.5	4688	874.921	4700
P L ₁₆	2225	3686.8	4673	751.776	4700

Table 4.3: Results of the TTP on the national premier league with short timeout

DISTANCE

Teams	Min	Avg	Max	Std - dev.
PL ₁₀	59583	59655.8	59910	119.33
PL ₁₂	112960	118820.8	115586	798.296
PL ₁₄	114802	117185.1	201132	1688.066
PL ₁₆	277088	280868.3	283951	1943.884

LONG TIME

TEAMS	MIN	AVG	MAX	STD	TIMEOUT
PL ₁₀	30	1605.7	3816	1277.917	4600
PL ₁₂	374	4979.3	6937	1940.038	7000
PL ₁₄	5178	12577	18881	4721.297	19000
PL ₁₄	10140	24433.2	283951	8216.410	32700

Table 4.4: Results of TTP on the national league with long time out.

The result of both evaluations given in table 4.3 and 4.4, where all time values are given in seconds. The result from the first evaluation Table 4.3) are compared with the results of table 4.4). The results obtained from the second experiment with longer timeout (Table 4.4) are compared with the results from the longer experiment, since I always get optimal solutions for these small Teams. It should be also noted that the comparison are made under the roughly same experiment regarding the computation – time and the number of runs per team. However the results obtained from much longer time test- runs carried of 20 runs per team. The results of the individual comparisons given in Table 4.5, Table 4.6, Table 4.7, Table 4.8 and Table 4.9. The evaluation and comparisons show very promising and favorable results for TTP. TTP is always able to solve the small teams Pl₆ and Pl₈ to optimality in very short time. The comparison with learning Hyper-Huristic shows that TTP is capable of producing very good solutions in short amount of time for all national leagues and

TTP exhibits through better average solution qualities and more stability thus the LHH. However LHH produced better minimum results than TTP for teams Pl₁₂, Pl₁₄ and Pl₁₆.

LHH

Teams	Min	Average	Std- Dev	Avg (Time)
Pl ₆	23916	23916	0	300
Pl ₈	39721	39801	172	1800
Pl ₁₀	59583	60046	335	3600
Pl ₁₂	112873	115828	1313	3600
Pl ₁₄	196058	201256	2779	3600
Pl ₁₆	279330	288113	4267	3600

TTIL

Teams	Min	Average	Std- Dev	Avg (Time)	Teams
Pl ₆	23916	2391.0	0	10	0
Pl ₈	39721	39721.0	0	300	-0.2
Pl ₁₀	59583	59632.6	60.0	4700	-.07
Pl ₁₂	113360	114391.7	708.3	4700	-1.2
Pl ₁₄	197230	199182.4	1436.5	4700	-1.0
Pl ₁₆	281644	286178.0	2199.7	4700	-0.7

Table 4.5: Comparison of TTIL spot with LHH on National Premier League.

AFC - TTP

Teams	Min	Average	Std- Dev	Avg (Time)
Pl ₁₀	59634	59928.3	155.47	4969.51
Pl ₁₂	112521	114437.4	895.7	7660.07
Pl ₁₄	196849	198950.5	1294.43	20870.07
Pl ₁₆	278456	285529.6	3398.57	35931.27

TTIL Sport

Teams	Min	Average	Std- Dev	Avg (Time)	(Avg /diff)
Pl ₁₀	59583	59655.8	119.3	4600	-0.5
Pl ₁₂	112960	113820.8	798.3	7000	-0.5
Pl ₁₄	194802	197185.1	1688.1	19000	-0.9
Pl ₁₆	277088	280868.3	1943.9	32700	-1.6

Table 4.6: Comparison of TTIL sport with AFC – TTP on national league.**TTAS**

Teams	Min	Average	Std- Dev	Avg (Time)
Pl ₁₀	59583	59605.96	53.36	40268.62

Pl ₁₂	112800	113853.00	467.91	68505.26
Pl ₁₄	190368	192931.86	1188.08	23357.35
Pl ₁₆	267194	275015.88	2488.02	192086.55

TTIL Spot

Teams	Min	Average	Std.- Dev.	Avg. (Time)	Avg. Diff.
Pl ₁₀	59583	59655.8	119.3	4600	0.08
Pl ₁₂	112960	113820.8	798.3	7000	-0.02
Pl ₁₄	194802	197185.1	1688.1	19000	-2.2
Pl ₁₆	194802	280868.3	1943.9	32700	-2.1

Table 4.7: Comparison of TTIL sport with TTSA on National league.

The results compared for teams Pl₁₀, Pl₁₂, Pl₁₄. The TTIL sport outperforms AFC-TTP both in average performance than teams of minimum solution. Pl₁₂, finds slightly better minimum solution compared with TTSA which is the current best met heuristics for the TTP. I claimed that, the results are at least comparable, where the biggest gap in average solution is 2.2% for Pl₁₂. The results for team Pl₁₆ comparable with other methods, the result for the Pl₁₆ were worse than originally anticipated.

First I suspected that TTIL spots could have problems with the large size of 16 teams.

but it seems to be rather in terms specific problem because TTIL sport produces very good results for the teams in the National F.A Cup with the same team size of it in the future would investigate thoroughly of I can obtain better result for the National league with 16 teams and the F.A cup with 16 teams.

CNTS

Teams	Min	Average	Std Dev	Avg (Time)
Pl ₁₀	59878	60424.2	823.9	7056.7
Pl ₁₂	113729	114880.6	948.2	10577.3
Pl ₁₄	194807	187284.2	2698.5	29635.5
Pl ₁₆	275296	279465.8	3242.4	57022.4

TTIL Sport

Teams	Min	Average	Std Dev	Avg (Time)	Avg. Dif.
Pl ₁₀	59583	59655.8	119.3	4600	-1.3
Pl ₁₂	112960	113820.8	798.3	7000	-0.9
Pl ₁₄	194802	197185.1	1688.1	19000	-0.05
Pl ₁₆	277088	280868.3	1943.9	32700	0.5

Table 4.8: Comparison of TTIL with CNTS on the Natural premier League

SAHC

Teams	Min	Average	Std Dev	Avg (Time)
Pl ₁₀	59821	60375.0	552.72	61619.6
Pl ₁₂	115089	116792.3	1069.59	82322.0
Pl ₁₄	196363	197769.9	731.52	96822.4
Pl ₁₆	274673	278477.9	1885.55	111935.2

TTIL

Teams	Min	Average	Std Dev	Avg (Time)	(Avg /diff)
Pl ₁₀	59583	59655.8	119.3	4600	-1.2
Pl ₁₂	115089	116792.8	798.3	7000	-2.5
Pl ₁₄	194802	197185.1	1688.1	19000	-0.3
Pl ₁₆	277088	280868.3	1943.9	32700	-0.8

Table 4.9: Comparison of TTIL spot with SAHC on the national Premier league.

The result computed favorably with other approaches with longer computation-time in general TTIL Spots shown slightly better average performance than CNTS and SAHC for the teams Pl₁₀, Pl₁₂ and Pl₁₄ even when SAHC has much longer computation – time for each teams. TTIL spot also find better solution than CNTS and SAHC for the team in the national premier league in Ghana.

Best Results for the National Premier League In Ghana

The researcher presented the global best results for the Ghana Premier league which was obtained during the research and comprise with the best results in the study. It would be noted that the best result for teams Pl_{12} and Pl_{14} presented before but the various individual experiment I have conducted in the course of this work. They obtained without the soft restarting mechanism and the parameter settings, with which I have to work with than are given in Table 4.8

Teams	K_{min}	K_{max}	ω	δ	N	δ	α	T_{max}	Time
Pl_{12}	2	6	1.1	0.999	500	3	5	600	2700
Pl_{14}	2	7	1.1	0.999	500	5	3	800	32204

Table 4.8: parameter settings for best National league results.

The best results slightly better than those obtained in 2007 except for the largest Pl_{16} teams and are slightly worse than the best results for Pl_{12} , Pl_{14} , Pl_{16} teams. Obviously, the current performing for the TTP is the Simulated Annealing approach TTSA and its extensions. But as you can see, the best results are not far away from their best results. All in all, it is believed that TTIL spots produces very competitive results and shows great potential for future studies.

Results for the Top Club League

As is it was done for the national league, I adjust the computation-time of my experiment to be comparable with those reported and a test for algorithm 10 times for each team. The results are reported in table 4.6.1 and compared with the results of LHH in table 4.9. As you can see the results, TTIL spots is able to solve the small team PL_6 and PL_8 always to the optimality and LHH is again out performed by TTIL spots. Additionally, I give the difference between the experiments best value and the current best results for the top team club league fixtures. The current best results for the top league fixtures are

DISTANCE

Teams	Min	Aveg	Max	Std-Dev
PL_6	2173	2173	2173	0
PL_8	3040	3040	3040	0
PL_{10}	5300	5312	5312	7
PL_{12}	782	1921	1921	658
PL_{14}	9906	10180	10180	158

BEST TIME

Team	Min	Avg	Max	Std-Dev	Timeout
PL_6	0	0.002	0.017	0.009	1

PL ₈	0.05	1.29	4.77	2.45	30
PL ₁₀	3.10	38.56	68.40	32.69	30
PL ₁₂	2.50	41.91	77.43	37.48	470
PL ₁₄	8.97	31.86	50.08	20.60	470

Table 4.10: Results of TTIL spot on the Top League Clubs.

Teams	TTIL Spot	Best	Dif (%)
PL ₆	2172.75	2172.75	0
PL ₈	3010.15	3040.15	0
PL ₁₀	5300.12	5272.15	0.26
PL ₁₂	7821.50	7281.27	3.58
PL ₁₄	9906.47	9527.20	1.95

Table 4.11: Comparison with Best Results of the Top League Clubs Fixtures **CONCLUSION**

In this work, the researcher has proposed a novel approach based on the iterated local search framework for solving the challenging Travelling Tournament Problem. First, he developed a basic ILS approach, TTILS basis to assess the applicability of the ILS principle to the TTP. The initial results of TTILS basis were promising and based on the insights gained by analyzing TTILS basic. He further optimized and extended TTILS basic which led to the final version

TTIL Spot. The proposed TTIL spot's incorporated perturbation mechanism, which uses random moves from higher-order neighborhood to perturbation solution and varies its strength well.

TTIL spot's embeds simple Hill-Climbing as the local-search component where I have experimented with the different neighborhood. In order to compute difficult local optima, TTIL spot's uses simulated annealing type acceptance criterion with non-monotonic cooling schedule.

TTIL spot's incorporates the additional extensions and other mechanism and implemented the proposed algorithm. TTIL spot's and conducted extensive computational experiment on selected teams in the country premier leagues. The results of the experiment are compared with others leagues in the experiment are compound with other leagues in the country.

TTIL spot's is able to solve the smaller teams to optimality in only few seconds and then most of the other compared approaches, being only second to the current best-performing simulated annealing approach TTSA.

CHAPTER FIVE

CONCLUSION AND RECOMMENDATION

Sport league scheduling has received considerable attention in recent years, since these applications involve significant revenues for television networks and generate challenging combinatorial optimization problems. In the following paragraphs, I first summarize the most important conclusions drawn from previous chapters of the thesis; we then indicate possible extensions and future research directions related to this thesis; I conclude with some general remarks.

As an abstract contribution, this thesis shows that, contrary to common belief, Local Search is, in fact, an effective method for tackling sport scheduling problems. This is demonstrated by our approach to two very important problems in Sport Scheduling: the Break Minimization problem and the TTP.

For the Break Minimization Problem, I propose a simulated annealing scheme, based on a simple connected neighborhood and a systematic scheme for cooling the temperature and deciding

termination. The resulting algorithm is conceptually simple and easy to implement; yet, it always finds optimal solutions on the instances used in evaluating the state-of-the-art algorithm of regardless of its starting points. More importantly, BMSA exhibits excellent performance and significantly outperforms earlier approaches on instances with more than 14 teams. In the case of the Traveling Tournament Problem proposed in my simulated annealing algorithm, TTSA, is able (with suitable enhancements) to match or significantly improve the best known solutions for most instances of the TTP, both in its original form, and in a number of variants, including different distance metrics and mirroring constraints. The gains are higher for larger instances. The key to these results is the design of a sophisticated neighborhood that is very well adapted to the problem's particular structure.

I have demonstrated that it is possible to reduce the number of pair clashes without a statistical difference to the distance that has to be travelled by the football fans. This provides the police with the ability to reduce their costs for these two days, which might have included paying overtime. I hope that I would be able to discuss these results with the football authorities and the police in order for them to validate our work and to provide us with potential future research directions. I already recognise that some pair clashes might provide the police with more problems than others and it might be worth prioritising certain clashes so that these can be removed, rather than removing less high profile fixtures. As a longer term research aim, I would like to include in our model details about public transport as some routes might be more different than other routes, even if they are shorter. I also plan to run my algorithms for every future season, as well as for previous seasons. Executing the algorithm is not the main issue. Data collection provides the real challenge due to the distance data that has to be collected.

There are a variety of open issues that need to be addressed. On the theoretical side, it would be interesting to determine if the neighborhood is connected. On the practical side, it would be interesting to explore other meta-heuristics that may improve the efficiency of the algorithm. This is a challenging task however, since it seems critical to consider a large neighborhood to obtain high-quality solutions. However, preliminary experimental results with fast cooling schedules are encouraging.

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