KWAME NKRUMAH UNIVERSITY OF SCIENCE AND TECHNOLOGY, KUMASI

PENSION FUND ASSET ALLOCATION UNDER THE MARKOWITZ MODEL: THE CASE OF SOCIAL SECURITY AND NATIONAL INSURANCE TRUST (SSNIT)

BY

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A THESIS SUBMITTED TO THE DEPARTMENT OF MATHEMATICS, KWAME NKRUMAH UNIVERSITY OF SCIENCE AND TECHNOLOGY IN PARTIAL FUFILLMENT OF THE REQUIREMENT FOR THE DEGREE OF Msc. ACTUARIAL SCIENCE

April, 2016

DECLARATION

I hereby declare that this submission is my own work towards the award of the MSc degree and that, to the best of my knowledge, it contains no material previously published by another person nor material which had been accepted for the award of any other degree of the university, except where due acknowledgement had been made in the text.

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DEDICATION

I dedicate this work to the Almighty God for taking me through this process. Special thanks also goes to my wife Salomey Quaye for her support and encouragement.

ABSTRACT

Investors all over the world and most especially pension fund managers will all the time try to reduce risk and achieve higher returns as well. The managing of an investment portfolio requires careful selection of assets to invest in, as well as managing the proportions of funds to be channelled into a particular asset. This calls for the rational behind this study. I will investigate the various investments undertaken by Ghana's state pension fund scheme, the Social Security and National Insurance Trust (SSNIT). The data used was generated from the trust's financial statements spanning from 2004 to 2013. The methodology used here was the Morkowitz Model. This model allowed us to assign weights to various investments classes by transposing the expected returns and risk associated with them. The result showed that should the pension fund be interested in minimizing the portfolio expected risk at a given return of 18.40% from their pool of investments, then they should invest 53.65% in students' loan, 19.56% in short term investment, 19.55% in properties, 5.87% in investment available for sale, 1.37% in investment held to maturity, zero percent in treasury bills and loans & receivables On the other hand if the fund wants to maximize the portfolio expected returns at a given risk level of 3.60% (being the lowest risk for all the assets), then 28.85% of the total investment portfolio is to be channelled to the risk free asset, 26.76% to student loans, 24.19% to short term investments, 10.3% to properties, 9.22% to investment available for sale, 0.96% to loans and receivables and zero to investment held to maturity.

ACKNOWLEDGMENTS

This work wouldn't have been possible without the effort of great people who contributed to make this a success. I thank the Almighty God for the protection, strength and knowledge given me through out my studies. Thanks be to God. A very big thank you goes to my Supervisor, Mr. Derrick Asamoah Owusu who through his support, encouragement and direction has made this become a reality. I also owe a debt of gratitude to Mr. Nana K, Frimpong, a letcturer at the Department of Mathematics at the KNUST, for he has helped to refine and shape my knowledge in Actuarial Science. I finally wish to take this opportunity to offer a sincere thanks to my family, friends and loved ones for supporting me spiritually and financially throughout the programme. To my new baby girl, Ewurakua Owusua Owusu-Siaw, welcome to the world. Any shortcomings and inadequacies of this study are my sole responsibility.

CONTENTS

LIST OF ABBREVIATION

SSNIT ....Social Security and National Insurance Trust DB

....Defined Benefit

M-V .... Mean-Variance

MPT .... Modern Portfolio Theory

TB ....Treasury Bill IVP

....Investment in Properties

IVS ....Investment available for sale IVM

.... Investment Held to Maturity

LR .... Loans and Receivables STI

....Short Term Loan

SL .... Student loan

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CHAPTER 1

Introduction

1.1 Background of the Study

A lot of strategies have over the years been employed to properly allocate assets of individual and institutions such as pension funds. The most sought for by these participants is to effectively manage the assets of the funds to be able to hedge against risk by meeting expected benefit payment obligations. Due to the risk of losing their investments as a result of investing in a particular asset, many participants have become aware of the need to diversify. These strategies include the maximization of returns for a specified risk and also minimization of risk for a specified return. There is however less much study in relation to the optimal allocation of assets of a portfolio. This study analyses the various investment opportunities available to the Social Security and National Insurance Trust (SSNIT) of Ghana and recommend various combinations to be undertaken to achieve higher returns and low risk.

1.2 Pension Funds Investments

To be able to make adequate funds available to meet future obligations within the short term and long term, pension funds invest in capital markets. Funds of pension houses are usually the main background for long term domestic capital. Funds are initially routed through risk free investments. Upon maturity, some are channelled to other investment areas which turn to generate good returns than pension funds assets notwithstanding higher expected risk (Vives, 1999)

In developed countries like the United Kingdom, there has been significant increase in pension funds investment in hedge funds since 2006. The pension funds often overtake the stock market in terms of performance by significant levels. Canada, United States, Holland, etc are example of countries that have channelled their funds into hedge funds for returns. according to Stewart, 2007, despite the variability in estimations, as high as 20% of American and European pension funds are channelled to hedge funds for investment while the Japanese can go as high as 40%.

Some pension funds have also sought to invest in socially beneficial investment such as the energy industry. Other socially responsible investments are affordable housing for the poor, city car parks, etc all in the case of SSNIT. These strategies tend to increase financial returns and social amenities.

There are recently the direction of pension fund assets into new classes of assets for better returns of which infrastructure is a typical example. Infrastructure has exhibited its potential to match other long tern investment and a source of diversity in investment. In the past pension funds preferred infrastructure investment for listed institutions or through real estates portfolios. There is however the growing of large funds investing in private equities and sometimes directly. The leading countries in such investment strategy is Australia, Canada and the Dutch (Inderst, 2009). The Ontario Municipal Employees retirement System (OMERS)has invested lots of billions of Canadian Dollars into infrastructure via its subsidiary, Borealis Infrastructure which was set up in the year 1998. A United States pension house, CalPERS,as part of its investment strategy agreed to invest about USD7.2 billion in infrastructure.

Other pension funds with similar strategy is the Washington State Pension Plan, Alaska permanent Fund Corporation, etc. Similar situation can be talked about in the United Kingdom as USS, BT, RailPen, etc have announce entering into infrastructure. (Inderst, 2009)

1.3 Performance of SSNIT investment funds

The Social Security and National Insurance Trust has over the years ventured in the real estates industry. According to SSNIT annual report, 2013 there has also been investments in other areas such as banking industry, manufacturing industry, hospitality industry, students' loan, government bonds, etc.

The trust has segmented their investment assets into the following portfolio: • Investment Properties: These are long term investments and carried at market values determined periodically. Investment properties are not subject to depreciation

- Investment available for sale: This relates to investment in listed and unlisted equities
- Investment held to maturity:This relates to Government bonds and HFC mortgage bonds
- Loans and receivables: Represent advances to companies less related impairment allowance
- Student loans: Loans offered to students during their study at the university
- Short term investment: Investment in short term government securities.

In 2013, the trust invested (gross) a total of GHS618.91 million representing 18.23% over 2012 position of GHS523.49 million. Out of this investments, the nominal returns gained on their investment portfolio was 29.48% as opposed to 29.50% in 2012 indicating as 43.80% increment.

1.4 Defined Benefit

Defined Benefit Plan is a pension plan where an employer or a sponsor agrees to provide stated benefits to an employee on his or her retirement. Payment can be done on monthly basis as per the agreement and benefits determined in advance through a formula based on the past earnings of service and age instead of the individual's return on investment.

The use of defined benefit plans have been used by institutions, both public and private, to reward employees in lieu of increased wage.

The formula for determining benefit under the defined benefit plan is known at the beginning of the contract. Defined contribution plan on the other hand, though the formula for contribution is known in advance, has benefit payment not known in advance.

The applicable formula used depend on the final salary of the employee. The benefit derived is as a proportion of the average salary of the worker for a specified years of service at the end of his or her career.

There is the exclusivity of funding of defined benefit plans by the contribution of the employer within the private sector. Defined benefits, in the public sector, on most occasion necessitate the contribution of the worker over the period. There is the possibility of the plan facing benefits or deficit challenges as a result of higher pension obligations are vice versa. The contributions to the fund are usually made by both the employer and the employee or can be done separately. Investment risk, under the defined benefit plan are usually taken care of by the employer and can benefit from surpluses.

1.5 Statement of Problem

There is little information relating to the level of risk of pension fund companies and what proportion to invest in assets to be able to spread the risks for some expected returns. Due the lack of the understanding of the levels of risk associated with investment opportunities, financial losses may be incurred. There is therefore a difficult task faced by pension fund managers in allocating resources efficiently. There is the zeal by every investor to increase returns with minimum risk on their investment. In decision making, risk plays a very important role. It is therefore important to quantify the level of risk associated with our investment.

The problem confronting investors is that they are unable to quantify the risk that will also lead to higher returns on their investment. In an ideal world, enjoying higher returns will lead to higher risk. The question here is, to be able to maximize returns on investment, how much risk is the investor prepared to take? Decision making are affected negatively when investors are unable to quantify risk. This same problem is real to the nation's pension sector. From our point of view if SSNIT, our case study, is able to identify how sensitive their investment portfolio is, they will to be able to spread risk by allocating appropriate proportion of their fund into the numerous investment opportunities available to them.

1.6 Objectives of the Study

The objective of the study is to:

1. Quantify the risk of investment of the various investment opportunities available to SSNIT.
2. The proportion of funds which is to be invested in the various investment opportunities to gain optimal returns with tolerable investment risk or minimize investment risk.

1.7 Significance of the Study

Asset allocation remains investors' most important decision. This study will provide a framework for determining the relatively importance of active management and asset allocation in portfolio performance. As more people do right kind of investment by allocation resources in viable market, the economy grows thereby reducing inflation rate which makes people have value for their money. The study will also be used as academic reference book for further studies. This study can help SSNIT diversify funds and reduce risk by spreading their risk. Quantifying risk of SSNIT is very important to decision making. This will help not only SSNIT but other investors and analysts for national development.

1.8 Methodology

Secondary data was used to undertake the study. This data comprises of the various investments done by the Social Security and National Investment Trust over the period 2003 to 2013. SSNIT has classified their investments into the following six categories: Investment in properties, Investment to maturity Investment available for sale, Student loan, Loans and receivables and Short term Investment Microsoft excel programming add-in and excel solver was used to run the figures obtained for the Markowitz model.

1.9 Limitation of the Study

The study was conducted within a very short time period with relatively little data availability. This prevented the researcher from adding on other factors that is considered significant to the study.

1.10 Organization of the Study

As already noted the first chapter briefly gives a background study of defined benefit. It goes further to state the problem, objective, significance, methodology, limitations and structure of the study. A brief background on the method used is also provided. Chapter two provides the theoretical basis for this research. It will also introduce the reader. Chapter three talks about the methodology used in the research. Chapter four details the primary data collected for the research, findings and the analysis of these data. Finally, the fifth chapter presents the summary, conclusion and recommendation from the researcher.

CHAPTER 2

LITERATURE REVIEW

2.1 Introduction

Markowitz model indicates that with continuous addition to investment portfolio, the risk associated with such assets, as determined by the standard deviation or variance of total return, reduces. The Markowitz framework of portfolio optimization indicates that as assets are further complemented to the portfolio of investments, the risk associated with that specific portfolio declines constantly. The associated risk of the portfolio is measured by the standard deviation. The weighted average is used to represent the expected returns of the portfolio.

2.2 Markowitz Portfolio Selection

Harry Markowitz (1952, 1959) designed a portfolio-selection technique, of which was named the modern portfolio theory (MPT). Before Markowitz's model development, asset-selection frameworks concentrated on gains spawned by opportunities under investment. Portfolios were built on those assets that were identified to have had the best opportunities for benefit with a lower risk. With this assertion, there were conclusion by investors that stocks of railroad provided good risk-reward traits of which portfolios were constructed from.

The main focus of the Markowitz framework was on return of which it raised risk to a standard level of importance, there the concept of portfolio risk was developed. Markowitz became the pioneer to carefully and thoroughly indicate how the variance of a portfolio can be minimize through the influence of diversification, noting that risk was an essential cause and variance a standard way to quantify risk. His ideas were based on the fact that investors emphasize their identification of portfolios through overall risk-reward traits other than the collation of portfolios from investment which individually portraits attractive risk-reward factors.

Markowitz frame work indicated that investors should not expect additional returns devoid of growing the portfolio's risk. The Efficient Frontier under Markowitz framework is the set of all portfolios where expected returns, based on some risk level, attains a maximum

Various assumptions are considered on the behavior of investors and financial market under the Markowitz model;

1. Investors can estimate a probability distribution indicating likely returns during a period.

2. Investors are exposed to one-period utility functions where utility can be maximized within the context of diminishing marginal utility of wealth.

3. Investors uses the variability about the values of expected return to measure risk.

4. The means and variance of the returns for a period are of essence to the investors.

5. Expected return and risk as used by investors are measured by the first two moments of the probability distribution of returns-expected value and variance.

6. Desire returns and circumvent risk.

7. Financial markets are frictionless.

2.3 Measurement of Return and Risk

We assume, in this section, that investors measure returns by calculating the expected value by applying probability distribution of expected gains for a portfolio.

The fluctuating nature around the anticipated value of the probability distribution of gains is assumed to be quantified by risk. The variance and standard deviation has been the most recognized measure of variability. By computing the weighted average return of the investment within the portfolio, the returns are achieved.

In a single security, the sum of squared deviation about the mean represents the variance. The standard deviation becomes the square root of the variance. The weighted average covariance of returns on distinct investments is equivalent to variance of portfolio combination of investments

The average value of sample period determines the mean returns for each distinct investment. The variance is calculated as the average of squared deviation around the average of the sample. The covariance becomes the average figure of the cross product of deviations.

2.4 Efficient Portfolio

A n-number of combination of assets can be used to estimate the efficient portfolios. Two risky assets, for instance, will be examined under the efficient asset allocation. The understanding of the characteristics of portfolio with two risky assets is assumed to pave way to understand more, portfolio with n-number of risky assets

- Two risky-asset Portfolio

There is the need to appreciate how the uncertainties of investments interrelate. We can realized that one of the major element of portfolio risk is the magnitude at which the two assets associate positively or negatively. The extent of correlation between the benefits of investments determines the extent at which variance of return is decreased by two risky asset portfolio. The Standard deviation of the portfolio will be the weighted average of the individual investment standard deviations,which is for exceptional case for perfectly positive correlated investment. With this, there will be no advantage from diversification. Irrespective of the percentage of each asset, A and B, the standard deviation and the mean are merely weighted averages. Figure 3.1

Indicates a perfect positive correlation for a combination of opportunity set. The positive correlation shows a straight line moving sections of the assets. presented in this scenario, portfolios are deemed to be adequate with the option among portfolios lying only on preference to risk. Positive correlation is achieved when diversification is consideration.

2.5 The Concept of Markowitz Efficient Frontier

Various potential combination of assets can be accepted by the space of the risk return. The space represents the gathering of all potential portfolios. The efficient frontier is represented by the line along the upper edge of the point. There is lower risk for levels of return for the combination of assets moving on the efficient frontier. Again, the greatest of all portfolios channeling out the best returns are the combination of assets moving on the efficient frontier line. The efficient frontier represents the intersection of the combination of assets which has the lowest variance or standard deviation and same asset with highest returns. Figure 2.1 Indicates to possible investors, the set of opportunity in investment for which various combination of risk and returns given by portfolios by asset A and B in different proportions. The risk return combination of all assets is depicted by the curve moving through asset A and B. The portfolio which sits to the northwest of Figure 2.1 is desirous by potential investors. They represents portfolios of high expected returns, thus towards the north of figure 2.1, and low risk, towards the west of figure 2.1.

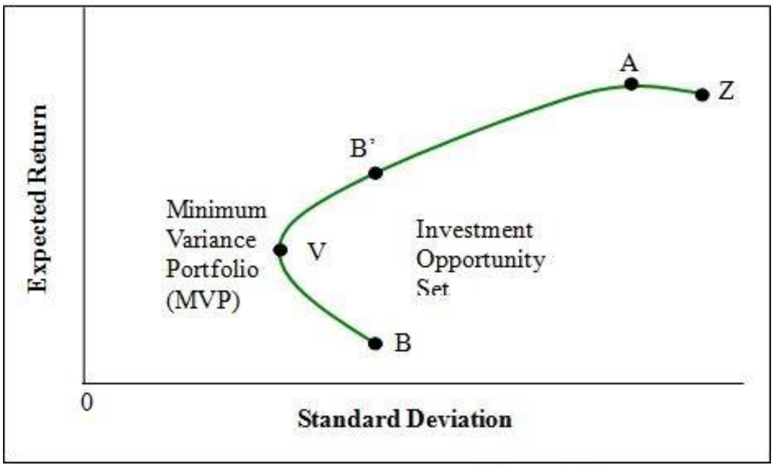

Figure 2.1: Investment opportunity set for asset A and asset B

The possible opportunity set which represents the combination of all possible assets is depicted within the area BVAZ. From figure 2.1 Investors can forgo the combination of portfolio that is below the minimum-variance, point V,. These are deemed inefficient. Again, assets which sits on the frontier, VA, is not a portfolio to be desired by investors. Thus these assets do not meet the requirement of maximizing expected returns to achieve some level of risk, the standard deviation, or reducing risk given some level of returns. This has been depicted in figure 2.1 when the portfolios are compared as indicated by the points B and B'. B' is on most occasions, better than B, as investors will normally go in for more expected returns than less returns for a specified level of risk. With similar scenario, investors wi;; go in for B than for V due to the fact that B has higher returns and at the same time lower associated risk. The mean-variance portfolio is represented by the portfolio at point V as there isn't any portfolio with a lower standard deviation. All possible efficient portfolios is represented by the curve VA, indicating the set of assets which provides the highest possible returns for each portfolio risk level.

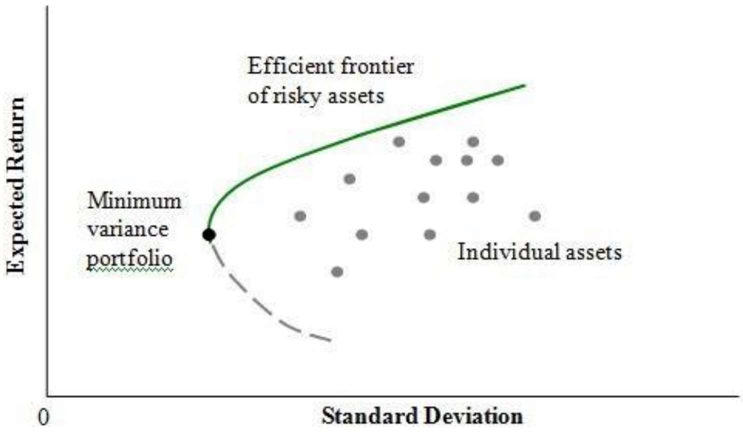

Figure 2.2: The efficient frontier of risky assets and individual assets

Combinations of investments lying on the negatively sloping part of the curve is subordinated to the combinations that lies above it on the increasing part of the curve as it commands higher expected returns and same associated risk. Making the best choice from the upward sloping part of the curve is not so noticeable. This is due to the fact that the part with the higher expected returns comes with corresponding higher risk. The investor’s eagerness to trade off risk in relation to expected returns will determine the best of choice.

• Short Selling

A lot of possible setbacks can prevent an investor from selecting combination of portfolios from the efficient frontier curve. Such setback is short sales which is like the normal market transacting being regulated. It is the situation where borrowed assets are sold with the speculation that prices of the asset will decreased. In situations were the asset price declines, there is the purchase of equivalent asset at that new lower price and returned to the original lender.

We will currently relax the assumption of no short selling, where investors can sell the lowest-return asset B (we will assume here that, $E(r_a \geq E(r_b))$) and $\sigma_a \geq \sigma_b$. Where the quantity of short sales is unobstructed, and by regular short selling of B and investing in A, infinite returns could be generated by the investor.

Figure 2.3 depicts efficient frontier with constraint portfolios. The upper bound of portfolio with the highest return will now not be A but infinity. This has been indicated by the arrow place on top of the efficient frontier. On another hand, the investor can short sell security A which has the highest return and divert the funds to the security with lowest yield, B. With this the investor achieves a return which is less than the asset with the lowest return. Without restrictions concerning the amount of short selling, there could be the realization infinitely negative returns, which will clear the lower bound of asset B on the efficient frontier. Thus short selling will usually increase choice of various investments from the minimum-variance portfolio toward +/- infinity.

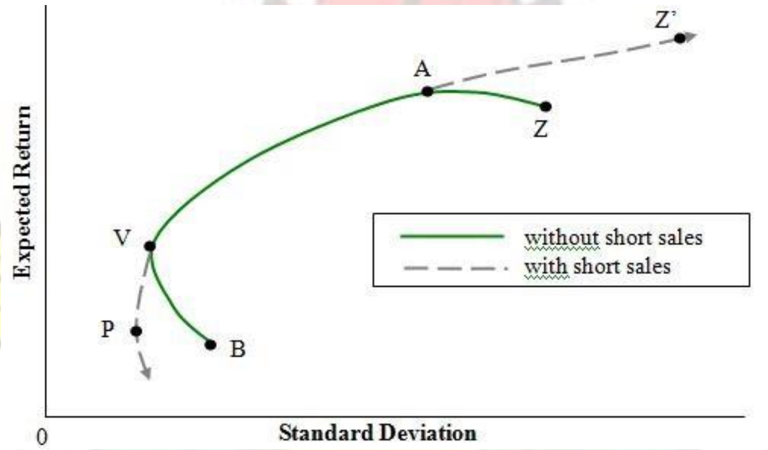

Figure 2.3: The efficient frontier of unrestricted/restricted portfolio

Relaxing the assumption of no short selling in this development of the efficient frontier involves a modification of the analysis of the efficient frontier of constraint (not allowed short sales).

2.6 National Pensions Act, 2008

The Ghanaian pension funds are subject to investment controls as specified by the national pension act, 2008, act 766. These controls are meant to ensure maximum portfolio diversification as pension fund trustees are restricted from investing all of the funds into one risky asset class. The investment controls as captured by the National Pension Act, 2008, act 766 are defined below:

Investment of Pension Fund

Section 175. A trustee or pension fund manager shall invest pension contributions received under this Act in order to obtain safe and fair returns on the amount invested.

Permitted Investments

Section 176. Subject to guidelines that the Board may issue, pension fund and assets shall be invested in any of the following:

a bonds, bills and other securities issued or guaranteed by the Bank of Ghana or the Government of Ghana;

b bonds, debenture, redeemable preference shares and other debt instruments issued by corporate entities and listed on a stock exchange registered under the Securities Industry Act, 1993, (P.N.D.C.L. 333)

c ordinary shares of limited companies listed on a Stock Exchange and registered under the Securities Investment Act 1993 (P.N.D.C.L. 333) with good records, declared and paid dividends in the preceding five years;

d bank deposits and bank securities;

e investment certificates of closed-end investment fund or hybrid investment funds listed on a Stock Exchange registered under the Investments and Securities Industry Act, 1993 (P.N.D.C.L. 333) with a good record of earning;

f units sold by open-end investment funds or specialist open-end investment funds listed on the stock exchange recognised by the Board;

g bonds and other debt securities issued by listed companies;

h real estate investment; and
i other forms of investment, that the Board may determine.

External investments

Section 177.

1. A trustee or pension fund manager may invest pension fund assets in units of investment funds except that investment fund may only be invested in the categories of investments specified in section 176 of this Act.

2. The Board may recommend to the President through the Minister responsible for Finance for approval, the investment of pension fund assets outside the country except that any amount to be invested outside, shall not exceed a percentage of total funds available for investment as determined by the Board.

3. External investments shall be subject to the Bank of Ghana foreign exchange rules

2.6.1 Restriction on Investments

Section 178. A trustee or pension fund manager shall not invest pension fund assets in the shares or any other securities issued by

a. the trustee or pension fund manager or custodian, or

b. a shareholder of the trustee or pension fund manager or custodian.

A privately-managed pension fund shall not

a. hold more than 10% of any class of security issued by a single issuer;

b. have more than 10% of its total assets in the securities issued by a single issuer other than that permitted for government and other public securities;

c. make short sales; or

d. borrow for investment purposes.

2.7 Criticisms of Restrictions

Even though the National Pension Act 2008, act 766 are applied to protect the potential investor, there exist arguments not in favor of these restrictions. Due to the rigidness of the restrictions, investment outcomes can lead to suboptimal portfolios by restricting the allocation of various investment opportunities (ChanLau, 2004:21). Thus the restrictions will not be in the good interest of pension funds if the act leads negative outcome.

Davis and Hu (2009) acknowledge that moral hazard can be reduced by restraining exposure to risky assets when quantitative restrictions are placed on assets. This leads to suboptimal portfolios which are normally below the efficient frontier. There is a direction to risk and liquidity towards individual assets instead of the portfolio in general. Pink's (1989:308) study looked at the effect of a 10% constrain on foreign assets on the performance of the Canadian mutual funds. his study compares the performance of funds with a 10% restriction against other funds which are not subject to 10% restrictions. The study showed that the restriction lead to suboptimal performance.

Pfau (2009:15) conducted a study on pension funds in Pakistan and find that investment constraints can reduce returns and at the same time increase risk. Investors with risk aversion coefficient of 5 or less are pushed to take lower expected returns with higher risk while highly conservative investors are able to find portfolios with lower volatility but at the expense of lower returns than would have been achieved without the constraints.

Act 2008, act 766 seems not to be in conformity to Modern Portfolio Theory which reduces overall portfolio risk by investing in complementary assets instead focuses on the risk of individual assets. To apply act 2008, act 766, we have to limit certain assets to m%, meaning additional constraints.

$$\pi_i \leq m, \text{for assets } i \tag{2.1}$$

2.8 Empirical Studies om Markowitz Portfolio Theory

Studies undertaken on asset allocation using the Markowitz framework in the world has been numerous. A summary of findings on existing studies conducted on optimization of assets will be provided in this section. the following literature review considers the theoretical foundation of the study.

Driessen and Leaven (2005) used the mean-variance model to establish that benefits are high for countries with significant country risk. Their studies also identified that the greatest benefits of international diversification of assets are mostly geared towards developing countries. The study used monthly data for the period running between 1985 and 2002 for 52 developed and developing countries. This established that the addition of international assets increases the sharp ratio. The sharp ratio on the other hand reduced to about an average of 18% from 21% after the imposition of short sale constraints. Offshore assets recorded a sharp ratio of 10% indicating that there exist benefits to international diversification regardless of allowance for short selling. For majority of the countries, diversification benefits decreased due to increased correlation between local and global indices. The downward movement of variance of local position was also a factor of the decrease in international benefit.

Mensah et al. (2013) applied the classical Markowitz optimization model to investigate the allocation of assets on the Ghana Stock Exchange. They used stock price data spanning between 2007 and 2010. Their studies showed that to obtain

profitable portfolio, investors should invest 90% of their wealth in non-financial assets with 10% going into financial assets. They further indicated that a risk aversion investor interested in minimizing risk, will have to invest
80% of his funds in non-financial assets and with financial assets, 20%. Also if the investor is to equally apportion his funds between financial and non-financial assets, his returns will not be as much as the minimum and the optimum risk portfolio. They concluded that, there exist a reward for risk on the Ghana Stock Exchange meanwhile the Markowitz optimization strategy does not exceed the buy and hold strategy of the market index.

CHAPTER 3

METHODOLOGY

3.1 Introduction

Portfolio is a collection of various securities, that is diversification. This is concerned with the construction of investment portfolios.The starting point of a portfolio theory optimization is the work of Harry markowitz on a mean variance criterion to judge investment strategies in security markets. The model underlying the mean variance approach is a one period model, this means that decision on investment strategies are only made at the beginning of the period, the consequences of this decision will then be observed at the end of the period and there is no action in between. This gives a static model. This model is a bit simplistic and has some disadvantages and thus leads to continuous time model.

Mean Variance Approach Inputs

Markowitz (1959) developed a mathematical framework to determine the optimal portfolio which can produce a maximum returns on portfolio while reducing risk concurrently. This mathematical framework as developed by Henry Markowitz has always been denoted as the Mean-variance (M-V)model. This has helped develop the foundation of Modern Portfolio theory. The portfolio selection problems faced by pension funds and other investors is solved by the M-V model. The M-V model applies the returns based on the mean as the expected return on portfolio and applies the variance of the portfolio as the risk measure. The following are the various objective functions that are to be formulated in the mathematical model.

- Maximize the expected return for a specific risk

- Minimize risk for a specified return
- Minimize and maximize risk irrespective of the expected return and risk respectively.
- Maximize expected returns while minimizing risk
- Minimize risk below a certain threshold
- Maximize expected returns above a specifies threshold. etc.

The efficient frontier is the main output The main output is the efficient frontier, that is the set of portfolios with expected returns greater than any other with the same level or lesser risk than any other with the same or greater returns.

3.2 Mean Variance Approach

Description of the one period modes of the mean variance approach consider a market where d different securities with prices $P_1,P_2,...,P_d > 0$ at the initial time t=0 are traded. The security prices at the final time t=T are not feasible. Therefore they are modeled as non-negative random variable on some probability space (Ω,F,P). The return of the securities is given by

$$R_i(T) = \frac{P_i(T) - P_i(0)}{P_i(0)}, i = 1, ..., d \tag{3.1}$$

Assuming we know their means and variance and covariance

$$E(R_i(T)) = \mu, i = 1,...,d \tag{3.2}$$

$$Cov(R_i(T),(R_j(T))) = \sigma_{ij}, ij = 1,...,d \tag{3.3}$$

Assuming that each security is perfectly divisible, that is we can hold $\varphi_i \epsilon R$ of security i. i=1,...,d. A negative position (ie ϕ(unit of assets) < 0, for some i)in a security corresponding to a short position. To avoid the possibility of negative final wealth we do not allow such negative position in the one period certain we require $\phi_i \geq 0$, i=r,...,d

An investor with an initial wealth X>0 is assumed to hold $\phi_i \geq 0$ shares of security i, i=1,...,d with

$$\sum_{i=1}^{d} \phi_i P_i = X \qquad (3.4)$$

Then the portfolio vector $\pi = (\pi_1, \pi_2, ..., \pi_d)$ is denoted as

$$\pi_1 = \frac{\varphi_i P_i}{X} \qquad (3.5)$$

$$R^{\pi} = \sum_{i=1}^{d} \pi_1 R_i(T) \qquad (3.6)$$

This is called the corresponding portfolio return. The component of the portfolio vector represent the fraction of wealth which are invested in the corresponding assets or securities. The mean and variance of the portfolio return are given by:

$$E(R_\pi) = \sum_{i=1}^{d} \pi_i \mu_i \qquad (3.7)$$

$$Var(R_\pi) = \sum_{i=1}^{d} \sum_{j=1}^{d} \pi_i \sigma_{ij} \pi_i \qquad (3.8)$$

Where σ_{ij} is the covariance of the securities.

3.3 Selecting a Portfolio

When choosing a portfolio an investor had the aim of obtaining a return as large as possible. If the only criterion to judge this is the mean of the portfolio return then this will typically lead to investing the whole sum in the security in the largest mean return.

However, this could be very risky security or asset and thus the return can have big fluctuations. To accommodate this fact we introduce the idea of minimizing risk of such a stock or asset.

3.4 Markowitz Idea on Portfolio Selection

Markowitz looked for a balance between risk(portfolio variance) and return(portfolio mean), he considered the problem of requiring a lower bound of the portfolio return(minimum return) and then choosing from the corresponding set the portfolio vector with minimal variance, alternatively he considered a problem of setting up an upper bound for the portfolio vector with highest possible mean returns from the remaining set.

3.5 Problem Formulation in the Mean Variance Approach

We consider the task of maximizing the mean of the portfolio return $E(R^{\pi})$ under a given upper bound C_1 for the variance $Var(R^{\pi})$ (R^n −n dimensional Euclidean space)

$$maxE(R^{\pi}), \pi \epsilon R^d \tag{3.9}$$

subject to $\pi_i \geq 0$ for i=1,...,d

$$\sum_{i=1}^{d} \pi_i = 1 \tag{3.10}$$

$$Var(R)^{\pi} < C_1 \tag{3.11}$$

In words, under all possible portfolios$\pi \epsilon R^d$ we consider only those who satisfies the constraints(which form the feasible region). In particular those who have a variance that is below C_1(maximum variance the investor is willing to take), then among those portfolios determine the one with the largest expected return.

The second formulation of the mean variance approach leads to the task of minimizing the variance of the portfolio return Var(R^{π}) given a lower bound on the expected portfolio return, $E(R^{\pi}) \geq C_2$(minimum returns the investor is willing to take)

$$minVar(R^{\pi}), \pi \epsilon R^d \tag{3.12}$$

subject to $\pi \geq 0$ for i=1,...,d

d

$$\sum_{i=1} \pi_i = 1 \qquad (3.13)$$

In words, under all possible portfolios, $\pi \epsilon R^d$ consider only those that satisfies the constraints in particular those who yield at least an expected return C_2 then among those portfolio determine the one with the smallest variance.

3.6 Solution Method

The first problem is a linear optimization problem with additional quadratic constraints, for such problem there exist no special standard algorithms, how ever we can use general non-linear optimization method but this leads to inefficient algorithms.

The second problem is a quadratic problem with a positive semi definate objective matrix namely the matrix of variance(σ). This problem can be solved in a very efficient way by standard quadratic programming algorithms.

The feasible region is not empty at least one π satisfying the constraints. If we have $C_2 \leq max\mu_i$ where $1 \leq i \leq d$ if the matrix σ is a positive definate and the feasible region is non empty then the problem possess a unique solution.

3.7 Comments on the Mean Variance Approach

The mean variance approach in one period is easy to understand and the setting can be implemented in a straight forward manner. This explains its popularity in practice, however there is a trading only at time t=0, no reactions to current price changes are possible and the risk of the investment is only modelled via the variance of its returns. In general the security price model is quite simplistic and it is purely a static model, this absence of dynamics with respect to time in both the modelling of the price evolution and of the possible trading activities has to be seen as the major reason for the development of continuous time models.

Further more the complexity of discrete time multi period models increases every rapidly with increasing numbers of period and even in times of fast computers cannot be solved real time.

3.8 Two Asset Modelling

$$R_p = w_a R_a + w_b R_b \tag{3.14}$$

The variance of the portfolio is given by;

$$Var(R_p) = w_a^2 \sigma_a^2 \sigma_a^2 + (1 - w_a)^2 \sigma_b^2 + 2w_a(1 - w_a)\sigma_a \sigma_b \rho_{ab} \tag{3.15}$$

For minimum variance

$$\frac{dVar(R_p)}{dw_a} = 0 \tag{3.16}$$

This implies

$$W_a(\sigma_a^2 + \sigma_b^2 - 2\rho_{ab}\sigma_a\sigma_b\rho_{ab})$$

$$w_a = \frac{\sigma_b^2 - \rho_{ab}\sigma_a\sigma_b}{\sigma_a^2 + \sigma_b^2 - 2\rho_{ab}\sigma_a\sigma_b} \tag{3.17}$$

The weight at which we have minimum risk, $B_m = 1$, where B is the risk with respect to the matrix,

$$B_{im}\frac{cov(R_i, R_m)}{\sigma_i\sigma_m} \tag{3.18}$$

but,

$$= cov(R_m, R_m)$$

$$= \sigma_{m2}$$

Hence

$$B_{im} = \frac{\sigma_m^2}{\sigma_m^2} = 1 \quad (3.19)$$

3.8.1 The Impact of the Correlation Coefficient on the Portfolio

$$Var(R_p) = w_a^2\sigma_a^2 + w_b^2\sigma_b^2 + 2\rho_{ab}\sigma_a\sigma_b w_a w_b \quad (3.20)$$

ρ_{ab} must be as to reduce $Var(R_p)$. The variance which measures the risk of the portfolio significantly. It must approach 1 as much as possible.

$\rho_{ab} = 1$ (Perfect Correlation)

$$w_a = \frac{\sigma_b^2 - \sigma_a\sigma_b}{\sigma_a^2 + \sigma_b^2 - 2\sigma_a\sigma_b}$$

$$w_a = \frac{\sigma_b}{\sigma_a - \sigma_b}, (w_b = 1 - w_a) \quad (3.21)$$

$\rho_{ab} = -1$

$$w_a = \frac{\sigma_b}{\sigma_a + \sigma_b}, (w_b = 1 - w_a) \quad (3.22)$$

$\rho = 0$ (Independent)

$$w_a = \frac{\sigma_b^2}{\sigma_a^2 + \sigma_b^2} \quad (3.23)$$

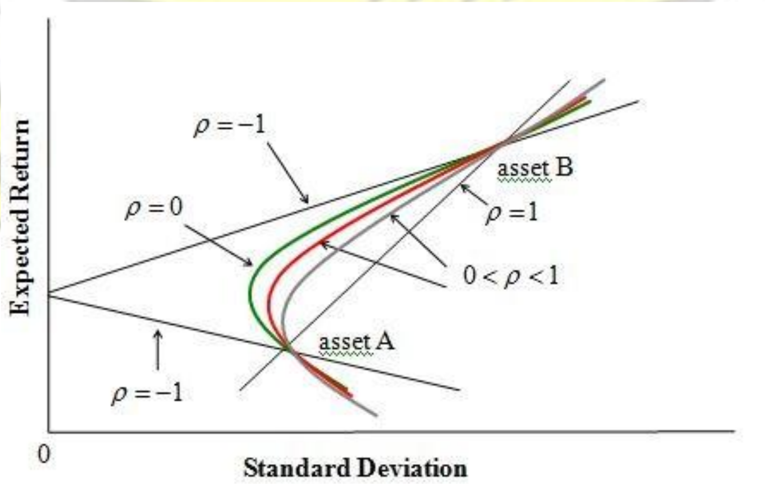

Figure 3.1: Investment opportunity sets for asset A and asset B with various correlation coefficients

Perfect positive correlation is the only case in which there is no benefit from diversification. With any correlation coefficient less than 1.0($\rho < 1$), there will be a diversification effect, the portfolio standard deviation is less than the weighted average of the standard deviations of the component securities. Therefore, there are benefits to diversification whenever asset returns are less than perfectly correlated. Our analysis has ranged from very attractive diversification benefits ($\rho_{ab} < 0$) to no benefits at all$\rho_{ab} = 1.0$. For ρ_{ab} within this range, the benefits will be somewhere in between. Negative correlation between a pair of assets is also possible. Where negative correlation is present, there will be even greater diversification benefits. Again, let us start with an extreme. With perfect negative correlation, we substitute $\rho_{ab} = -1$ in equation(3.21) and simplify it in the same way as with positive perfect correlation. Here, too, we can complete the square, this time, however, with different results.

With perfect negative correlation, the benefits from diversification stretch to the limit. An investor can reduce portfolio risk simply by holding instruments which are not perfectly correlated. In other words, investors can reduce their exposure to individual asset risk by holding a diversified portfolio of assets. Diversification will allow for the same portfolio return with reduced risk.

3.9 Allocation of Asset of Pension Plan

The risk and expected return on portfolio constructed from n different securities can be described in terms of their weight

$$w_i = \frac{X_i S_i(0)}{X(0)}, i = 1, ..., n \qquad (3.24)$$

Where X_i is the number of shares of type *i* in the portfolio, $S_i(0)$ is the initial price of security *i*, and X(0) is the initial amount invested in the portfolio.

we have a one row matrix.$w = [w_1, w_2, ..., w_n]$ with

$$Uw^T = 1 \qquad (3.25)$$

Where

$$u=[1,1,1,...,1]$$

is a one row matrix with all n entries equal to 1, w^T is a one column matrix, the transpose of w, the usual matrix multiplication rules apply. The attainable set consist of all portfolios with weights satisfying equation(3.25) and that portfolio is called the attainable portfolios. Suppose that the returns on the securities are $R_1,R_2,...,R_n$, the expected return $\mu_i = E(R_i)$ for i=1,...,n, will also be a one row matrix $m = [\mu_1,\mu_2,\mu_3,...,\mu_n]$.

The covariance between returns denoted by $C_{ij} = cov(R_iR_j)$ with

$$C_{ij} = \begin{pmatrix} C_{11} & C_{12} & . & . & . & C_{1n} \\ C_{21} & C_{22} & . & . & . & C_{2n} \\ C_{n1} & C_{n2} & . & . & . & C_{nn} \end{pmatrix}$$

The covariance matrix is symmetric and positive definate. C_{ii} is equal to the variance of returns(Var(R_i)) The expected return $\mu_p = E(R_p)$ and the variance $\sigma_p{}^2 = Var(R_p)$ of the return. $R_P = w_aR_a + w_bR_b +,...,+ w_nR_n$ on a portfolio with weight w=$[w_a,w_b,...,w_n]$ are given by

$$\{\mu_p = mw^T, \sigma_p^2 = wCw^T\} \qquad (3.26)$$

NB: Assuming C is an inverse or invertible matrix.

The portfolio with the smallest variance in the attainable set has weight

$$w = \frac{\mu C^{-1}}{\mu C^{-1}\mu^T} \tag{3.27}$$

The portfolio with the smallest variance among attainable portfolio with expected return μ has weight, w given as;

$$w = \frac{\mu C^{-1}\begin{pmatrix} 1 & \mu C^{-1}m^T \\ \mu & mC^{-1}M^T \end{pmatrix} + mC^{-1}\begin{pmatrix} \mu C^{-1}\mu^T & 1 \\ mC^{-1}\mu^T & \mu_p \end{pmatrix}}{\begin{pmatrix} \mu C^{-1}\mu^T & \mu C^{-1}m^T \\ mC^{-1}\mu^T & mC^{-1}m^T \end{pmatrix}} \tag{3.28}$$

Provided that the denominator is non-zero, the weights depend linearly on μ_p. w is the weight computed using excel solver given a particular μ_p

CHAPTER 4

ANALYSIS

4.1 Introduction

In this chapter the application of the mean-variance approach of pension fund asset allocation is looked at. The problem of allocation of pension assets by a pension fund manager is solved using the mean variance approach for optimal portfolio selection. Secondary data was used to undertake the study. This data comprises of the various investments done by the Social Security and National Investment Trust over the period 2003 to 2013. SSNIT has classified their investments into the following six categories and we have the following letters for simplicity: Investment in properties (IVP), Investment to maturity (IVM), Investment available for sale (IVS), Loans and receivables (LR), Short term Investment (STI), Student Loan (SL)

We also introduced a new asset, the risk free asset, which is represented by the 1 year Treasury Bill (TB). Returns from this risk free asset was use to establish the correlation between it and the other risky assets. Matlab and Microsoft excel programming solver were used to run the figures obtained for the Markowitz model.

4.2 Analysis of Data

Portfolio of assets with their respective returns were obtained from the financial statement of Social Security and National Insurance Trust. The various asset allocation of the pension asset is solved in this chapter.

4.3 Discriptive Analysis

The descriptive statistics shows the various returns and risks associated with the different investments by SSNIT. The Mean represent returns from each portfolio and

the Standard Deviation representing the associated risk with each of the expected returns. Table 4.1 indicates that in the coming years , investment in student loan will record the highest expected returns of 20.1% while that in short term investment is expected to record the lowest of 14.2 %. Despite the significant positive returns expected from student loan, its associated risk will also be low at 13.1% compared to the other investments (with the exception of that of short term loan and the risk free asset with risk levels of 11% and 3.6% respectively). Thus even with higher expected returns, investment in student loan is expected to record lower risk. The 1-year treasury bill indicates low returns of 16.5% (compared to student loan, Property investment,Investment to maturity and Investment available for sale). This can be attributed to the lower associated
risk.

Table 4.1: Descriptive Analysis of Data

	Mean	Standard Deviation
TB	0.1646	0.0361
IVP	0.1840	0.1733
IVS	0.1680	0.2236
IVM	0.1730	0.2083
LR	0.1480	0.1452
SL	0.2010	0.1303
STI	0.1430	0.1110

4.4 Correlation Matrix

The Markowitz model studies the pair-wise correlation among investment assets. This study will also consider the correlation between the various investment assets by SSNIT and the risk free asset. Table 4.2. indicates a strong correlation of 70.1$ between Student loan and the 1 year treasury bill followed by investment held to maturity which recorded 22% correlation. On the other hand, Investment available for sale, Short term investment and investment in properties has negative correlation with the risk free asset. Investment available for sale recorded the highest negative correlation of 24.7% followed by short term investment at 19.9% and investment in properties at 13%.

Table 4.2 also depicts the correlation between the other investment assets. From the table, there is a good relationship between investment available for sale and investment in properties at 15.4% while investment held to maturity recorded a weak positive correlation with the same asset. Other assets correlation with property investments was negative. Also with the exception of student loans which recorded a positive correlation with investment available for sale (though weak at 5.1%), all other assets records negative correlation with IVS. Short term investment from table 4.2, records a good correlation with investment held to maturity at 25.7% with student loan recording a correlation of 3.1%. However loans and receivables posts a negagtive correlation of 43.2%.

With the correlation between the investment assets and the advantage of diversification, I present the allocation of investment between the various investment opportunities available to SSNIT in the next section.

Table 4.2: Correlation Matrix of the Pension Fund Investment

	TB	IVP	IVS	IVM	LR	SL	STI
TB	1						
IVP	-0.130	1					
IVS	-0.247	0.154	1				
IVM	0.220	0.093	-0.141	1			
LR	0.046	-0.152	-0.108	-0.432	1		
SL	0.701	-0.160	0.051	0.031	0.340	1	

STI	-0.199	-0.035	-0.670	0.257	-0.233	-0.498	1

4.5 Allocation of various Pension Fund Asset

In this section, we look at the various options available to SSNIT in terms of which asset is to be invested in, given the various constraints. The principle of markowitz portfolio optimization is concerned on benefits generated from portfolio diversification. Portfolio allocation rests on market risk and return from the various assets as well as the correlation between these assets. The portfolio optimization technique which concerns the highest trade-off among various assets interest will be applied. The main aim of portfolio management is maximizing returns and risk minimization. The rational for asset optimization is to allocate weights to assets that ensures a minimum level of risk for good levels of expected returns.

The aim is to recognize the combination of the Various investment opportunities available to SSNIT that brings a minimal risk with various amount of returns expected. To achieve this, we apply two different constraints. The first is that we want achieve the minimum risk (Standard Deviation) of 3.60% and its impact on the weights. Thus a risk level equal to or less that 3.60%. Secondly, what happens to the weights if we want to maximize the expected return at 18.40%. In all, the weights applied to the various assets will not exceed 1 or 100%. The study also looks at a situation when the Pension fund expects equal expected returns

Equal Expected Returns

Table 4.3 below indicates the various possible solutions for the optimization when the various constraints are applied. The table shows that should the pension fund be interested in enjoying equal weights [expected returns] from the numerous investment assets, then the total portfolio return will be 16.88% with a risk level of 4.59%.

Maximizing Expected Returns

Again if the pension fund is interested in maximizing return at a risk level of 3.60% as indicated in table 4.1, then the following weights is to be applied to the various investments:

- 10.03% of its total investment portfolio into investment properties;
- 26.76% into investment into student loans; and
- 9.22% into investment available for sale
- 0.96% in Loans and receivables.
- 24.19% in short term investment.
- 28.85% in investment in the risk free rate, the one year Treasury bill.
- none into investment held to maturity.

With these weights applied to the available investments, it can be realized that the total portfolio returns would increase to 17.12% which is an improvement compared to when equal weights are applied (expected return of 16.88%). The associated total portfolio risk of 3.60% (ie the standard deviation)is also an improvement over that with equal weights. In this case the pension fund enjoys higher returns with lower associated risk if the above weights are applied.

Minimizing Expected Risk

Lastly, where the pension fund is interested in minimizing the portfolio expected risk at a given return of 18.40% from their pool of investments, and then they should undertake the following investments

- invest 53.65% in students' loan,
- 19.56% in short term investment,
- 19.55% in properties, 5.87% in investment available for sale,

- 1.37% in investment held to maturity, • 0% in treasury bills and loans & receivables these investments will result in total portfolio risk of 6.50% which is higher than all the other scenarios. This is not surprising because, higher returns come with higher risk.

Table 4.3: Optimal Allocation of Pension Assets

Portfolio			
	Equal WT	Max Return	Min Std. Dev.
Constrained Variable	None	at SD≤	at Mean=
Value of Constraints	N/A	3.60%	18.40%
	PTW	PTW	PTW
TB	14.29	28.85	0%
IVP	14.29	10.03	19.55%
IVS	14.29	9.22	5.87%
IVM	14.29	0.00%	1.37%
LR	14.29	0.96	0.00%
SL	14.29	26.76	53.65%
STI	14.29	24.19	19.56%
$P_{(w)}$	100	100	100
μ	16.88	17.12	18.40%
σ	4.59	3.60	6.5%

4.6 Efficient Frontier

The efficient frontier is the set of portfolios, when optimality is taken into consideration, which provides the highest expected return against a specified risk or that with lowest risk against a specified returns. All assets which lie below the efficient frontier line are considered not to be optimal. thus the returns generated are not adequate as against the associated risks.

Figure 4.1 and 4.2 below shows the efficient frontier of the expected returns and its associated risk. Inside the table is the individual assets (indicated by the dotted points).

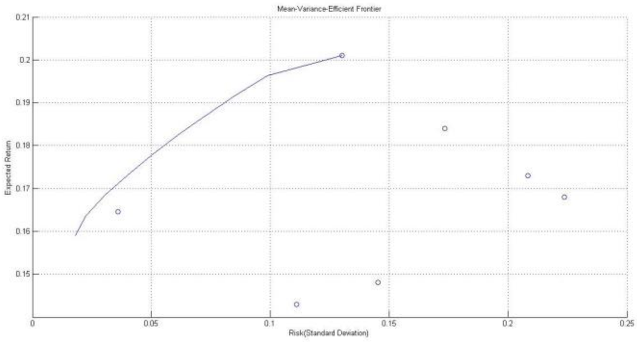

Figure 4.1: Efficient Frontier

Figure 4.1 Indicates to SSNIT, the set of investment opportunities for which various combination of risk and returns are given by the six investment assets of SSNIT. The risk-return combination of all assets is depicted by the efficient frontier curve. The portfolio which sits to the northwest of the efficient frontier should be desirous by SSNIT. They represents portfolios of high expected returns with associated low risk.

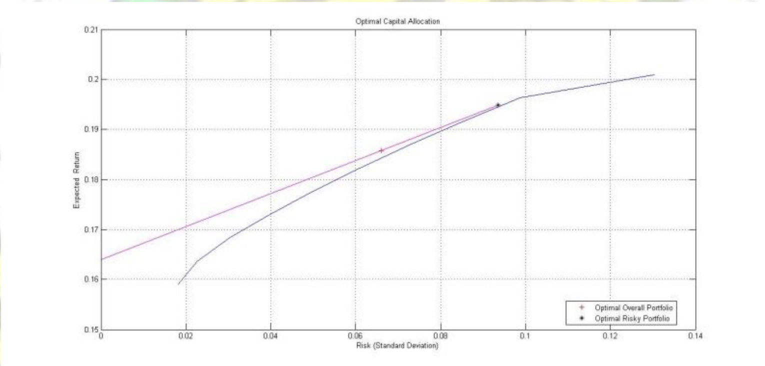

Figure 4.2: The Optimal Efficient Portfolio

Figure 4.2 indicates the optimal portfolio efficient frontier. This is the combination of the efficient frontier curve and the Capital Market line (CML). The point where the CML touches the expected return axis represent the risk free asset. It is the 1 year Treasury bill with zero risk, thus no standard deviation as shown in figure 4.2. The point of tangency between the CML and the efficient frontier represents the market folio.Combination of assets on the CML is superior to combination of assets on the efficient frontier.

CHAPTER 5

CONCLUSION and RECOMMENDATIONS

5.1 Introduction

This chapter is the final stage of the research process; it includes the summary, conclusion to the finding, the limitation of the study and recommendations future research.

5.2 Conclusion

We applied Markowitz model to the various investment portfolios of the Social Security and National Insurance Trust. Our work was to establish an optimal way of allocating their investment portfolios looking at the risks associated with it. The study quantified the various risks associated with the numerous investments opportunities available to SSNIT. It was realized that investment available for sale records the highest risk at 22.36% followed by Investment held to maturity at 20.83%. Investment in properties has a risk level of 17.33% with Loans & receivables and students' loan recording 14.52% and 13.03% respectively. Short term investment also has a risk level of
11.10%. The risk-free asset, 1 year Treasury Bill, records the lowest risk at 3.06%.

The result showed that should the pension fund be interested in minimizing the portfolio expected risk at a given return of 18.40% from their pool of investments, then they should invest 53.65% in students' loan, 19.56% in short term investment, 19.55% in properties, 5.87% in investment available for sale,
1.37% in investment held to maturity, zero percent in treasury bills and loans & receivables.

On the other hand if the fund wants to maximize the portfolio expected returns at a given risk level of 3.60% (being the lowest risk for all the assets), then 28.85% of the total investment portfolio is to be channelled to the risk free asset, 26.76% to student loans, 24.19% to short term investments, 10.3% to properties, 9.22% to investment available for sale, 0.96% to loans and receivables and zero to investment held to maturity.

5.3 Recommendations

The study demonstrate the application of Markowitz model to the allocation of various investment portfolios undertaken by the Social Security and National Insurance Trust. This will guide the trust to effectively allocate their resources to the various investment opportunities available to them.

As shown and estimated, the risk of the various investment opportunities of SSNIT were quantified showing positive trend towards the Treasury Bills. From our study based on the individual risk assessment, we recommend that SSNIT's investment should be geared towards the Treasury Bills as it records the lowest risk. Other assets such as the Short term investment and Students' loan should also be considered as they record individual risk below 15%.

Looking at the three scenarios on allocation of assets as presented above, we concluded on the following: scenario 1 (ie embarking on equal allocation of resources) give expected portfolio return of 18.88% but with a portfolio risk of 4.59%, scenarios 2 (ie Maximizing returns at a risk level of 3.60%) gives expected portfolio returns of 17.12% and scenario 3 (ie minimizing risk for a return of

18.40%) gives portfolio risk of 6.5%. We thus strongly recommend SSNIT to adapt to scenario 2 by maximizing their expected portfolio returns to 17.12% at a portfolio risk of 3.60%. The 3.60% is the lowest portfolio risk as compared to the other scenarios.

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