Optimal Location of Additional Hospital Facility in Berekum Municipality

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Declaration

I hereby declare that this submission is my own work towards Master of Science degree and that, to the best of my knowledge, it contains no material previously published by another person nor material which has been accepted for the award of any other degree of the University, except where due acknowledgement has been made in the text.

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Abstract

Locating a facility (resources) in order to minimize the cost of satisfying some set of demands (customers) with respect to some constraints has been under study for a long time. Several location problems have been solved using a number of mathematical models and methods. In this thesis, a determination of optimal location of additional hospital in Berekum municipality was under study.

This thesis therefore considers the problem of locating a hospital facility as a p – centre problem under the condition that there are some existing facilities already located in the Berekum Municipality in the Brong Ahafo Region.

Berman and Drezner (2008) method was used on 19 – nodes network which had seven existing hospital facilities at Berekum, Akroforo, Namasua Abisaase, Koraso, Mpatasie, and Jininjini. An additional hospital facility using Berman and Drezner (2008) should be located at Nsapor with an objective function value of 5. The objective function value of 5 implies that the minimum distance travelled by the farthest patient to the new facility at Nsapor and any other health care centre is 5 kilometers.

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DEDICATION

To my daughter Maame Adjei-Serwaah Kwarteng.

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I am most grateful to the Almighty God for guiding and sustaining me throughout this course. His everlasting love and Grace has made me see the light of day and subsequent completion of this work.

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CHAPTER 1

INTRODUCTION

1.0 OVERVIEW

This chapter is the introductory part of the whole research work which comprises the background of the study, statement of the problem, objective of the study, method used, significance of the study, the scope of the study, assumptions, and the organization of the study.

1.1 BACKGROUND TO THE STUDY

Over the last 20 years, the number of health and its related issues has increased significantly. Disasters become a big concern in local and national government (Gilpin & Murphy, 2008).

Disasters become a major threat to people's lives and health. The big issue with disasters is that they may happen anytime and anywhere with no previous warning. Disasters have various forms, some natural sources and some manmade. Disasters cause huge challenges for people and organizations in all levels and fields, because disasters can destroy the infrastructures for cities, communities, and organizations.

Hospitals are one of the most important areas that face a big challenge during and after disasters. Many people are injured and die during disasters. According to the U.S. Geological Survey (2012), the estimated number for people who died from earthquakes worldwide from 2000-2012 is 812,600. People died or were injured because houses, buildings, and bridges were destroyed or damaged (USGS, 2012). Injured and sick people may run to or transfer to hospitals for the healthcare they need. At that time hospitals need to provide more healthcare and services for the larger number of injured people who just arrived at the hospitals. Considering the disaster situation, hospitals also may be affected by the disasters—their staff may be injured or unable to get to their hospital, or those hospitals may have a limited capacity and are unable to manage providing placement and care for the additional patients. This scenario may put hospitals in a challenging situation that requires them to be prepared for such hard times. People, government, and non-governmental organizations have realized the importance of preparing hospital professionals for disasters, protecting the health and safety of staff and patients and providing the best quality healthcare (WHO, 2002)

Even though there are a variety of management systems and guidelines for disaster and emergency situations, there is still a difference between hospitals in terms of their preparedness for disasters, which reflects how some hospitals have a different level of management system than others (FEMA, 2006). Differences are in many areas, such as the hospital's capability to perform the identified tasks, characteristics, and management styles; these differences are affected by the size and location of the hospital and its community (Corbaley, 2010).

1.2 BACKGROUND STUDY OF BEREKUM MUNICIPALITY

Berekum Municipal came into existence as a semiautonomous spatial unit by virtue of the decentralization policy adopted by the Government in 1988. Geographically, Berekum is a Municipal located at the western part of Ghana in the Brong-Ahafo Region. It lies between latitude 7'15' South and 8.00' North and longitudes 2'25' East and 2'50' West.

The Municipality shares boundaries with Wenchi Municipal and Jaman District to the Northeast and Northwest respectively, Dormaa Municipal to the South and Sunyani Municipal to the East. Berekum, the Municipal capital is 32km and 437km North West of Sunyani, its regional capital and Accra, the national capital respectively. Its total area constitutes about 0.7 percent of the entire

233, 588km² of Ghana, (1,635km²). The district's close proximity to Cote d' Ivoire is another remarkable feature which promotes economic and commercial activities between the

District and Cote d' Ivoire.

The human population of the Berekum Municipality according to 2010 population and housing census stood at 129628 with population growth rate of 3.3 percent (3.4%)

1.3 HOSPITALS

Identifying and understanding a problem is an important step toward better management. It is also very important in understanding the critical and essential tasks of the organization to efficiently and cost-effectively manage service outcomes. Hospitals are large and complex workplaces that have large numbers of employees from different technical, medical, and professional fields. Healthcare staff includes management, maintenance, and transportation staff as well as employees from other supporting departments. They all work for hospitals to ensure successful and comprehensive services of healthcare for patients. Moreover, besides the hospitals' core business function of providing medical care to patients, hospitals provide essential support services such as educating community members who assist people during disasters, injury and illness prevention, health examination, and disease notification

According to Slepski (2007), hospitals have differences in terms of their classification, capability to perform tasks, characteristics, and management styles, as each facility has various levels of medical care they are prepared to provide to patients. For example, these differences are usually affected by the size and strategy of that facility, because a big hospital can take in more patients, and the hospital's strategy can improve the hospital's achievements and services. Healthcare facilities are generally classified into three types, including community hospitals, medical centers, or specialty hospitals. Nearly all communities, especially large and urban communities, have all three types of facilities. Community hospitals provide basic short-term care for their patients, such as outpatient clinics and some small surgeries. The emergency services are at a lower level compared to most medical centers (Slepski, 2007). A medical center is larger than a community hospital and provides more advance healthcare services and treatments, such as cardiac surgeries, cancer treatments, and brain surgeries. It is open 24 hours, has advance emergency services and a full range of medical specialists, such as cardiac surgeons, neurosurgeons, and chemotherapists (American College of Surgeons: Committee on Trauma, 2010). Specialty hospitals usually provide specialized medical care, such as medical treatments for cancer patients, psychiatric therapy for mentally ill patients, inpatient counseling for drug addiction, various types of rehabilitation, or even short-term medical care (McGraw-Hill Concise Dictionary of Modern Medicine, 2002). This classification helps to serve the different care people need in the community. Because of the important roles that hospitals play for people and communities in general and during disasters in particular, they need to be prepared structurally and functionally to be able to respond effectively to people’s needs.

1.4 PROBLEM STATEMENT

Community health centers and other health facilities in the municipality only give first aid to patients and then refer patients to the Holy Family Hospital (HFH), resulting to congestion at the hospital due to the increasing number of out-patient attendants. This work therefore seeks to find the optimal site for an additional hospital in the municipality using the conditional p – center model.

1.5 OBJECTIVES OF THE STUDY

1. To locate an additional hospital in the Berekum municipality as a conditional p- center problem
2. To solve the conditional p – center problem using Berman and Drezner algorithm.

1.6 METHODOLOGY

The objective of the study is to locate additional hospital in Berekum municipality using the Conditional P – center model.

Data on road distances between communities were collected and used.

Floyd-Warshall algorithm was used to find the distance matrix, d (i, j) for all pairs shortest path.

The resource centers for the study are my personal laptop, the internet, the Sunyani Municipal library, the library of Kwame Nkrumah University of Science and Technology (Kumasi) and Sunyani Polytechnic library.

1.7 SIGNIFICANCE OF THE STUDY

The study will help determine the shortest distance a patient has to travel to the nearest health facility

The additional site will reduce travelling cost incurred by a patient.

The additional site will help reduce stress and over-dependence on the only existing district facility and thereby prolongs their lifespan.

1.8 SCOPE OF THE STUDY

Due to limited time and financial constraints, the study will be concentrated in the Berkum municipality in the Brong Ahafo region of the country.

1.9 ORGANIZATION OF THE STUDY

This thesis is structured as follows: Chapter 1 deals with the introduction and the background of the study. Chapter 2 deals with the related literature relevant to the study. Chapter 3 discusses the methodology used for the study. Chapter 4 presents the results and the discussion of the case study. Chapter 5 which is the last chapter, deals with the conclusion and the recommendations.

CHAPTER 2 LITERATURE REVIEW

2.0 OVERVIEW

This chapter reviews the literature existing in the area of quality in healthcare and facility location.

2.0 INTRODUCTION

According to Correia de Oliveira (2003), health care system is based on a national health service structure. Various governments have issued various statements over time shown that it is seeking some kind of geographical equity but this has never been clearly defined. There are wide inequalities in the distribution of hospital resources in Portugal with marked concentration in urban coastal areas and little information. The objective of the research is to develop methods to inform the allocation of resources hospitals so that this can be made more equitable in both current and capital spending. The methods used are a combination of methods already used in other countries and new methods to address two questions. First, to measure inequities in hospital care in terms of capital, finance and utilisation using capitation formulas. These formulas are constructed using: a multiplicative model to measure need for hospital care; a multilevel model to estimate unavoidable costs and to disentangle allocative inefficiencies of hospital care; and a flow demand model to predict hospital geographical utilisation and to compute cross-boundary flows. Second, to indicate how redistribution of hospital supply will best improve equity of utilisation and access, using location-allocation models that were designed to consider alternative policy objectives and account for patients' choice of hospitals.

Michael (1999), examines the significance of quality when included in the specification of a hospital cost function. Also, his research estimates a value for scale economies in order to determine if the average hospital experiences increasing returns to scale in the production of hospital care, verifying such findings in previous econometric studies. Furthermore, two functional forms are compared: the Cobb-Douglas and the translog. The results of this study demonstrate that quality has a significant impact on costs. This relationship is positive meaning increasing quality will also increase the cost of producing hospital care. The results for scale economies demonstrate that the average hospital experiences increasing returns to scale in the production of hospital care, which is consistent with previous research. Lastly, based on an *F*-test, this study is able to accept the translog as the appropriate functional form.

2.1 Quality in Healthcare

Although quality is an important factor in the production of healthcare services, it is difficult to quantify. To different people it may mean different things: how kind and caring is the nurse, how attractive is the facility, how good is the doctor in his abilities, is the bed clean, etc. Quality measures are not only difficult to obtain, but to define as well.

Moskowitz (1994), lists the common responses from surveys regarding what individuals would most desire from a healthcare provider. They include: responsiveness to urgent medical situations, referral to appropriate level of care, humanness, communication information, coordination and continuity of care, primary prevention, case finding, evaluation of present complaint, diagnosis, and proper management of the condition.

Unfortunately, the author describes the data for these measures of quality ―spotty‖ at best. Moskowitz continues to explain that rankings of quality are derived primarily from mortality rates, or death rates, which measure the probability of death. But the pervasive problem with this measure is different patients have different conditions which may or may not contribute to their likelihood of death.

Sadleir stated that it is important for hospitals to have strong and clear standards for managing the environment and health and safety issues. Hospitals have many environmental and health and safety issues that staff need to be aware of. Hospitals have a lot of workplaces where workers have high levels of potential exposure to a large variety of dangers such as biological, chemical, physical, and psychological. Hospital employees should be aware of how they can protect their own health and safety as well as protect their patients from all potential risks. Also, hospitals can extend their efforts to protect their communities and neighborhoods from all environmental and health and safety issues such as disease prevention, health examination and disease warning, disaster management, and environmental protection through the proper way of managing their wastes.

Sadleir (2010), also discussed that environmental health issues relating to hospitals can be easily divided into four parts: staff, patient, community, and environmental protection. He stated that hospitals have a major role in disaster management, as those disasters result in multiple victims. Hospitals should have a Hospital Disaster Committee that is responsible for the preparedness and planning, reviewing and testing the plan with mock drills. This committee should ensure the effectiveness of backup power and water supplies after disasters

Alshehri (2012), evaluated the hospital's role in a regional disaster response. He focused on identifying the level of the hospital's preparedness and its response to disasters and includes a comparison study between an urban and a rural hospital in New York state. His findings showed some differences between the urban and rural hospitals in terms of their capabilities and available resources and the effect of the community infrastructures on their preparedness. Both hospitals have similarities in the way participants view the hospital's role during a regional disaster response. It is noted that the urban hospital has more resources, a better geographical location, staff, medical centers, equipment, and supply management.

Fagbuyi and Upperman (2009) discussed the role of hospital managers in dealing with crisis situations. As a result of a survey conducted in 2007 by the members of the American Pediatric Surgical Association, they found that managers and frontline leaders with proper training and preparedness were almost four times better in responding to disasters than managers with no previous preparedness. Also, Fagbuyi and Upperman found that hospital managers and staff with defined responsibilities and roles were almost five times better in responding to disasters than managers who has no identified roles (Fagbuyi and Upperman, 2009).

Page (2006), mentioned the importance for employees to be familiar with the tasks they need to do during the crisis time because if they were unfamiliar with their tasks, then they definitely are going to fail in doing their jobs at the time of the disaster. He said that employees need to know their tasks, how to do them, and what to do if something was missing that they needed in order to do their work and where to go if they needed additional support or information (Page, 2006)

2.2 MATHEMATICAL THEORIES AND MODELS IN ANALYSING FACILITY

LOCATION PROBLEMS

Location problems have attracted a lot of interest by mathematical researchers for a long period of time due to its ability to solve numerous problems.

Farahani and Hakmatfar (2009) wrote in their introductory part of their book, ―Facility Location, Concepts, Models, Algorithms and case Studies" by emphasizing that the

mathematical science of facility location has attracted much attention in discrete and continuous optimisation over nearly last four decades. The authors explained facility location problems as ―to locate a set of facilities (resources) to minimise the cost of satisfying some set of demands (of customers) with respect to some set of constraints‖. Conducting a review in almost any topic in facility location, according to Schilling et al., (1992) is no small undertaking since academic outlets which publish facility location span a wide range of disciplines. Nevertheless some relevant literature has been covered in this review by the researcher.

Daskin (1995) explained covering location problems as a problem which tries to determine the number of facilities to cover all demand nodes under center problems. The model introduced under a title p-centre model (minimax problem). The objective this model was to find location of p facilities so that all demands are covered and the maximum distance between a demand node and the nearest facility is minimised.

Serray and Marianovz (1996) postulated a P-Median problem in a changing network in Barcelona. The authors formulated a p-median-like model to address the issue of locating new facilities when there is uncertainty. Several possible future scenarios with respect to demand and/or the travel times/distance parameters were presented. The authors wanted a strategy of positioning that will do, as well as possible, over the future scenarios. The paper presented a discrete location model formulation to address this P-Median problem under uncertainty. The model was applied to the location of fire stations in Barcelona.

Drezner (1995) in his research incorporated five objectives of p-median, p-center, two maximum covering and the minimum variance in order to minimize the maximum percent deviation from the optimum of each of these objectives for a casualty collection point location problem.

Amponsah et al., (2010) conducted study of location of ambulance emergency medical service in the Kumasi metropolis in Ghana. The authors modelled the problem as Non-Linear Maximum Expected Covering Location Problem(MEXCLP) implemented by Saydam and Aytug (2003).In their paper the researchers acknowledged that several approaches that have been employed in solving this very sensitive location problem such as Maximum Availability Location Problem (MALP) developed by Daskin (1982), ReVelle and Hogan (1989). Genetic Algorithms (GA) that uses random key coding was implemented. A formula for renormalization was introduced. Real route distances were used for computation and statistical deviation was introduced in the selection of optimal routes.

Arogundade et al., (2005) conducted a study of fire and emergency service facility location selection in Nigeria. The authors looked at the problem as variant of set covering problem. First, a mathematical model of facility location was introduced and solved by using optimization solver, TORA. Secondly, the balas additive algorithm of branch and bound techniques is used to solve the facility location problem. In analysis the authors discovered that both algorithms indicate the same number of fire stations in different locations. Also the results obtained by applying and implementing balas additive were more explanatory by specifying the names of the locations where the facilities are to be located and the names of the locations to be served by each of the facilities.

Other forms facility location theories, methods and models are models which include Factor Rating Method, Breakeven Analysis, Linear Programming, Centre of Gravity and Factor Analysis. The factor rating method is widely-accepted and used because a broad multiplicity of factors it objectively includes.

Huff (1966) proposed the famous ―Gravity Model‖ for estimating the market share captured by competitors. The gravity model states that existing customer locations attract business from a service in direct proportion to the existing locations and in inverse proportion to the distance between the service location and the existing customer locations.

Simpson (2012), indicated that locating a set of facilities (resources) in order to minimize the cost of satisfying some set of demands (of customers) with respect to some constraints has been under study for a long time.

His work determined the optimum location of Mobile Telecommunication masts owned by mobile service providers in the Cape Coast Metropolitan area. The general layout of sitting telecommunication mast is based on transmission and receiving conditions. The proliferation of masts across our landscapes nowadays has been in direct response to increased desire for mobile communication services by the public.

Factor Rating method was adopted to solve this problem. The goal of Factor Rating is for a mobile service provider to be able to carefully derive the maximum benefit that can be used to achieve the company’s objectives by considering all relevant factors involved in deciding to site a mast.

Oppong and Hodgson (1994) used the MCLP location/allocation model to develop a solution for improving accessibility to health facilities in rural Ghana. They found that the relocation of particular mobile facilities allowed for improved access without having to build new facilities.

Blake (2010), examines the accessibility of primary health care services to rural populations. In his work, demand aggregation was incorporated into an a Maximal Covering Location Problem (MCLP) accessibility model in order, first, to determine the unmet demand for primary health care services in Idaho, based on the allocation of Idaho residents‟‟ demand for primary health care to the state‟‟s existing primary health care facilities (PHCF). This procedure identified areas without adequate accessibility to PHCF (locations where demand for Primary Health Care Services (PHCS) is unmet). Secondly, the study used this model to determine potential locations (from a set of feasible sites) that could meet unmet demand determined by the initial models. In addition, the study explored the consequences of different demand locations and different driving time constraints on all of these models‟‟ results.

2.3 SOME APPROACHES TO FACILITY LOCATION PROBLEMS

In Malczewski and Ogryczak (1990) the location of hospitals is formulated as a multi-objective optimization problem and an interactive approach DIN AS, Dynamimic interactive network analysis system (Ogryczak et al., 1989) based on the so called reference point approach (Wierzbicki,1982) is presented. A real application is presented, considering eight sites for potential location and at least four new hospitals to be built, originating in hundred and sixty three alternative location patterns each of them generating many possible allocation schemes. The authors mention that the system can be used to support a group decision - making process making the final decision less subjective. They also observed that during the interactive process the decision – makers have gradually learned about the set of feasible alternatives and in consequence of this leaning process they have change their preference and priorities. Erkut and

Erkut, and Neuman (1992), present a mixed integer linear model for undesirable facility location. The objectives considered are total cost minimization, total opposition minimization and equity minimization. Caruso et al (1993) present a model for planning an urban solid waste management system. Incineration, composition and recycling are considered for the processing phase and sanitary landfills are considered for the disposal phase. Heuristic techniques (embedded in the reference point approximation) are used to solve the model and, as a consequence, ―approximate Pareto solutions‖ are obtained. By varying the reference point, different solutions can be obtained. The results for a case study (Lombardy region in Italy) are presented and discussed.

Hotelling (1929), introduced major concept in the field of location analysis was known as competitive location analysis. He discussed a method to locate a new facility considering already existing competition. The considered facilities were on a straight line. He proposed that the customers generally prefer visiting the closest service facility. He introduced the ―Hotelling‟s Proximity Rule‖ which can be used to determine the market share captured by each facility. He just considered the distance metric during his analysis

Chalmet et al (1986), non- interactive algorithm for Bi-criteria Integer Linear Programming modified to an interactive procedure by Ferreira et al (1994). Several equity measures are computed for each non-denominated solution presented to the decision-maker, in order to increase the information available to the decision –maker about the set of possible solutions.

Ferreira et al (1996) present a bi-criteria mixed integer linear model for the facility location where the objectives are the minimization of total cost and the minimization of environmental pollution at facility sites. The interactive approach of Ferreira et al (1994) is used to obtain and analyze non-dominated solutions.

Ballou (1998), states that exact centre of gravity approach is simple and appropriate for locating one depot in a region, since the transportation rate and the point volume are the only location factors. Given a set of points that represent source points and demand points, along with the volumes needed to be moved and the associated transportation rates, an optimal facility location could be found through minimizing total transportation cost. In principle, the total transportation cost is equal to the volume at a point multiplied by the transportation rate to ship to that point multiplied by the distance to that point. Furthermore, Ballou outlines the steps involved in the solution process in order to implement the exact centre of gravity approach properly.

2.4 REVIEW OF METHODS

2.4.1 INTRODUCTON

The P-centre model attempts to minimize the maximum performance of the system and thus address situations in which service inequity is more important than the average system performance. The P- centre model is also referred to as the minimax model since it minimizes the maximum distance (performance) between any demand point and it nearest facility. The Pcentre model considers that a demand point is served by it nearest facility and therefore gives full coverage in the sense of set covering models. But set covering model may lead to an excessive number of facilities while full coverage in the P-centre model requires only a limited number (P) of facilities. In many location problems, the cost of a service from the customers' point of view is related to the distance between their habitation and the facilities that are being located. Usually, service is deemed adequate if the customer is within a given distance of the facility and is deemed inadequate if the distance exceeds the given distance

The P-centre model has applications in many areas of life. These include the location of emergency services and the selection of conservation and recreational sites. Also, delivery and routing problems often take on a set covering model. This chapter takes a look at the literature in the application of the P-Centre Model and the set covering model as well as some methods used in solving P- centre problems.

2.4.2 SET RECOVERY

Toregas (1970) defined the Location Set Covering Problem (LSCP) more than thirty years ago and in the pages of Geographical Analysis by Toregas and ReVelle (1973).This location problem involves finding the smallest number of facilities (and their locations) such that each demand is no farther than a pre-specified distance or time away from its closest facility. Such a problem is called a ―covering‖ problem in that it requires that each demand be served or ―covered‖ within some maximum time or distance standard. A demand is defined as covered if one or more facilities are located within the maximum distance or time standard of that demand. The classical LSCP requires that each demand is covered at least once. Church and ReVelle (1974) stated a related problem involves the location of a fixed number of facilities and seeks to maximize the coverage of demand. This second type of covering problem is called the Maximal Covering (MCLP). Since the development of these two problems, there have been numerous applications and extensions.

Daskin (1983) formulated a probabilistic multiple cover model called the maximal expected coverage model. Hogan and ReVelle (1986) also formulated the simple back up covering model as a good example of a deterministic cover model that involves maximizing second-level coverage. Toregas (1970; 1971) was the first to recognise the possible need for multi-level coverage. Toregas defined the Multi-level Location Set Covering Problem (ML-LSCP) as a search for the smallest number of facility needed to cover each demand, a preset number of times, where the need for coverage might vary between demands.

Application of the set covering model includes airline crew scheduling (Desrocher et al, 1991). According to Daskin, Jones and Lowe (1990) it can also be applied to tool selection in flexible manufacturing systems.

2.4.3 CENTRE PROBLEM

The centre problem was first posed by Sylvester (1857), more than 150 years ago. The problem asks for the centre of the circle that has the smallest radius to cover all desired destinations. In the last several decades, the P-centre model and its extensions have been investigated and applied in the context of locating facilities such as EMS centres, hospitals, fire stations and other public facilities.

Garfinkel et al. (1977), examined the fundamental properties of the P-centre problem in order to locate a given number of emergency facilities along a road network. He modelled the P-centre problem using integer programming and the problem was successfully solved by using a binary search technique and a combination of exact tests and heuristics.

ReVelle and Hogan (1989), formulated a P-centre to locate facilities so as to minimize the maximum distance within which the EMS is available with (alpha) reliability. System congestion is considered and a derived server busy probability is used to constrain the service reliability that must be satisfied for all demands.

Hochbaun and Pathria (1998), considered the emergency facility location problem that must minimize the maximum distance on the network across all time periods using the Stochastic Pcentre models. The cost and distance between locations vary in each discrete time periods. The authors used k underlying networks to represent different periods and provided a polynomialtime, 3-approximation algorithm to obtain a solution for each problem.

Talmar, (2002), utilized a P-centre model to locate and dispatch three emergency rescue helicopters to serve the growing EMS demands due to accidents occurring during adventure holidays such as skiing, hiking and climbing the north and south Alpine mountain ranges. One of the model's aims is to minimize the maximum (worst) response time and the author used

effective heuristics to solve the problem.

In this work the focus is on the weighted P- centre problem formulated by Daskin (1995) on a general graph. The algorithm that is used to solve the weighted P- centre problem was also provided by Daskin (1995).

Schilling et al. (1979) generalized the maximal covering location problem (MCLP) model to locate emergency fire-fighting servers and depots in the city of Baltimore. In their model, known as FLEET (Facility Location and Equipment Emplacement Technique), two different types of servers need to be located simultaneously.

White and Case (1974), developed the MCLP model that does not require full coverage to all demand points. Instead, the model seeks the maximal coverage with a given number of facilities.

Daskin and Stern (1981), formulated a hierarchical objective location set covering problem (LSCP) for emergency medical service in order to find the minimum number of vehicles that are required to cover all demand areas while simultaneously maximizing the multiple coverage.

CHAPTER 3

METHODOLOGY

3.1 Formulation of the set covering model

Daskin and Dean (1994) formulated the set covering model as follows, Let (i) d = coverage distance

(ii) d_{ij} = edge distanc

$$
\text{(iii)} \quad a_{ij} = \begin{cases} 1, \text{ if candidate site } j \text{ can cover demands at node } i \text{ (i.e. } d\ d_{ij} \square \text{)} \\ 0, \text{ if not (i. e. } d\ d_{ij} \square \text{)} \end{cases}
$$

(iv) f_j = cost of locating a facility at candidate site j

(v) let the decision variables be

$$
x_j = \begin{cases} 1, \text{ if we locate at candidate site } j \\ 0, \text{ if not} \end{cases}
$$

3.2 Coverage Distance

A coverage distance d or (Dc) is a pre –specified distance, demand is deemed covered if the edge distance (d_{ij}) is less than or equal to Dc. The edge distance (d_{ij}) is the shortest direct distance between a demand node and a facility node. A node is the same as a vertex.

With these notations, the set covering problem is as follows;

$$
\text{Minimize } \square_j f_i x_i \qquad 3.2a
$$

Subject to

$$
\square_j a_{ij} x_j \square 1 \quad \square i \qquad 3.2b
$$

$$
x_j \square\ 0,1, \quad \square j \qquad 3.2c
$$

The objective function (3.2a) minimizes the total cost of the facilities that are selected. Constraints (3.2b) stipulate that each demand node *i*, must be covered by at least one facility represented by right hand side of constraints. Constraints (3.2c) are the integrality constraints. If all the costs are identical, then, the objective function becomes;

Minimize $\sum_{j} x_j$ 3.2d

Subject to

$\sum_{j} a_{ij} x_j \geq 1 \quad \forall i$ 3.2b

We stipulate a coverage distance, d , such that $d \geq dij$ implies demand node *i* can be covered by facility j. This affects constraints 3.2b, because the relationship between *dij* and *d* will determine whether a_{ij} is 1 or 0.

3.1 Example

To illustrate the formulation of the set covering problem, we consider the network shown in figure 3.1 below.

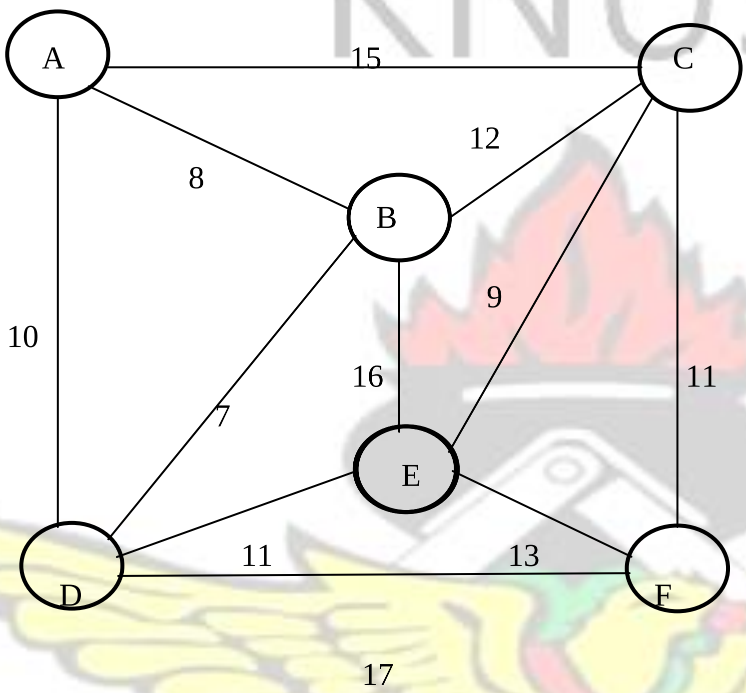

Figure 3.1: Example Network

Table 3.1 is the edge matrix of figure 3.1

Table 3.1: Table of d_{ij} values

		0	8	15	10	-	-
8	0	12	7	16	-	15	12
0	-	9	11				
		10	7	-	0	11	17
		-	16	9	11	0	13
		-	-	11	17	13	0

The coverage distance of 11 units is pre-specified to be used in this example.

Table 3.2 below shows edge distances d_{ij} such that $d_{ij} > d$ are eliminated.

Table 3.2: Table of d_{ij} values; $d > d_{ij}$: $d = 11$

From \ To	A	B	C	D	E	F
A	0	8	-	10	-	-
B	8	0	-	7	-	-
C	-	-	0	-	9	11
D	10	7	-	0	11	-
E	-	-	9	11	0	-
F	-	-	11	-	-	0

A node is assigned to itself and other nodes with non-zero entries along its row. A dash (-) means there is no direct link between the nodes

From Table 3.2 the problem is formulated as:

P3.1 Minimize $X_A + X_B + X_C + X_D + X_E + X_F$

Subject to;

(Node A assigned): $X_A + X_B + X_D \geq 1$

(Node B assigned): $X_A + X_B + X_D \geq 1$

(Node C assigned): $X_C + X_E + X_F \geq 1$

(Node D assigned): $X_A + X_B + X_D + X_E \geq 1$

(Node E assigned): $X_C + X_D + X_E \geq 1$

(Node F assigned): $X_C + X_F \geq 1$

Integrality: $X_A, X_B, X_C, X_D, X_E, X_F = 0,1$

The reduction technique was used: (i) For the column j and k, if $a_{ij} \leq a_{ik}$ for all demand nodes *i* and $a_{ij} \leq a_{ik}$ for at least one demand node *i*, then location k covers all demands covered by location *j*. Location *k* is said to dominate *j* and hence column *j* is eliminated.

(ii) For the row reduction, if $\Sigma a_{ij} = 1$ then, there is only one facility site that can cover node *i*. In such case we find location *j* such that $a_{ij} = 1$ and set $x_j = 1$. We then eliminate rows containing x_j.

From P3.1 column D dominates column A and B hence we delete column A and B. Column C dominates F hence we eliminate column F. This leads to

P3.1 Min $X_C + X_D + X_E$

S.T

$$X_D \geq 1$$

$$X_D \geq 1$$

$$X_C + X_E \geq 1$$

$$X_D + X_E \geq 1$$

$$X_C + X_D + X_E \geq 1$$

$$X_C \geq 1$$

Integrality: $X_C, X_D, X_E = 0,1$

Using the row reduction, since $\Sigma a_{ij} = 1$, holds for the first and second constraints, we set $X_D = 1$ and eliminate all the rows containing X_D. We also set $X_C = 1$ and eliminate rows containing X_C. The solution therefore is $X_C = X_D = 1$ and $X_A = X_B = X_E = X_F = 0$. The objective function equals 2; the facility will be located at X_C and X_D. From table 3.2, C covers itself, E and F. D covers itself, A, B and E.

3.3 THE MAXIMUM COVERING LOCATION MODEL

The set covering has associated problems, one of which is that the number of facilities that are needed to cover all demand nodes is likely to exceed the number that can actually be built due to budget constraints and other related issues.

Again, the set covering model treats all demands nodes identical. Under certain conditions and budgetary constraints it is appropriate to fix the number of facilities that are to be located and then maximize the number of covered demands.

Church and ReVelle (1974) formulated a Maximun Covering Location Model as follows.

Let h_i = demand at node i

P = number of facilities to locate

Decision Variables be

$$Z_i = \begin{cases} 1, & \text{if node } i \text{ is covered} \\ 0, & \text{if not} \end{cases}$$

The Maximum Covering Location Model is formulated as follows;

$$\text{Maximize } \sum_i h_i Z_i \qquad 3.3a$$

Subject to;

$$Z_i \le \sum_j a_{ij} x_j \quad \forall i \qquad 3.3b$$

$$\sum_j x_j = P \qquad 3.3c$$

$$x_j \in \{0,1\} \qquad 3.3d$$

$$Z_i \in \{0,1\} \qquad 3.3e$$

The objective function 3.3a maximizes the number of covered demands. Constraints 3.3b state that demand node *i* cannot be covered unless at least one of the facility sites that cover node *i* is selected. But, the right-hand side of constraints 3.3b which is $\sum a_{ij} x_j$ is identical to the left-hand side of constraints 3.2b. *a* gives the number of selected facilities that can cover node *i*. The constraint 3.3c stipulates that we locate not more than *P* facilities. Constraints 3.3c will be binding in the optimal solution. Constraints 3.3d and 3.3e are the integrality constraints on the decision variables. **3.4. Example**

We use the network of figure 3.2 below to illustrate the maximum covering location problem.

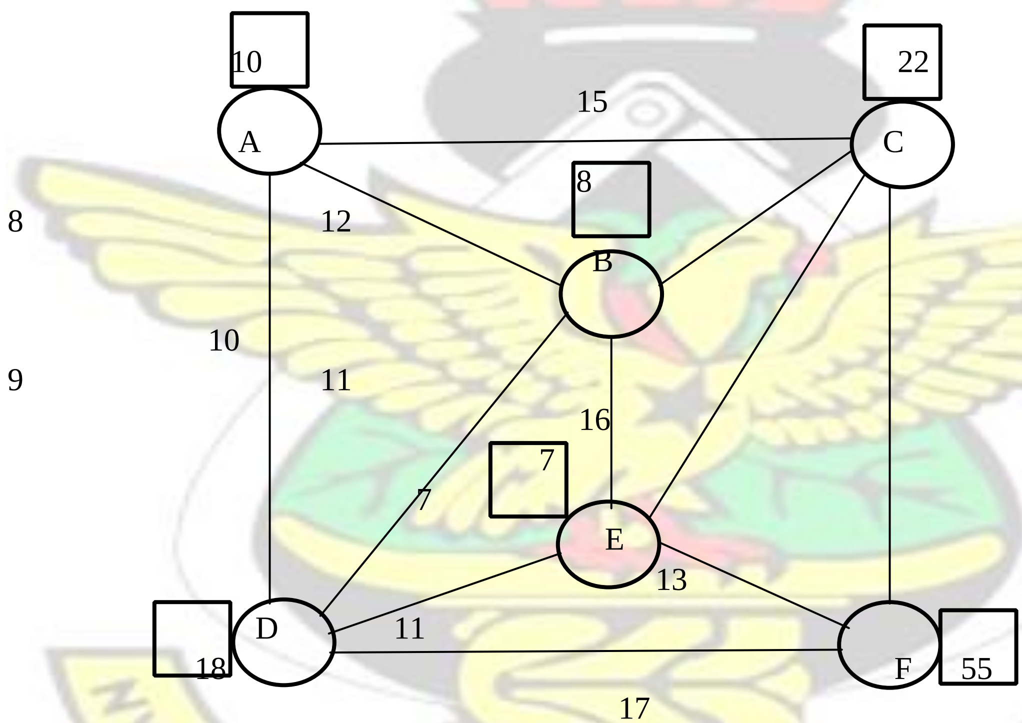

Figure 3.2: Example Network with demand at each node.

Demand at each node is in the square box of the network. For a coverage distance of 11 units and P = 1 we have the following formulation:

P 3.2 Maximize $10Z_A + 8Z_B + 22Z_C + 18Z_D + 7Z_E + 55Z_F$

Subject to

$$X_A + X_B + X_D \geq Z_A$$
$$X_A + X_B + X_D \geq Z_B$$
$$X_C + X_E + X_F \geq Z_C$$
$$X_A + X_B + X_D + X_E \geq Z_D$$
$$X_C + X_D + X_E \geq Z_E$$

3.5. Solution by reduction with enumeration

A reduction technique was used to solve this problem, beginning with a column reduction rule. Using the column technique on the constraints with *z* variable, we eliminate columns with subscripts A, B, since column with subscript D dominates subscript A and B. Subscript F is dominated by subscript C. We eliminate column with subscript F

Therefore $X_A = X_B = X_F = 0$.The problem reduces to

P 3.3 Maximize $10Z_A + 8Z_B + 22Z_C + 18Z_D + 7Z_E + 55Z_F$

Subject to

$$X_D \geq Z_A$$
$$X_D \geq Z_B$$
$$X_C + X_E \geq Z_C \qquad X_D + X_E \geq Z_D$$

$$X_C + X_D + X_E \geq Z_E$$

$$X_C \geq Z_F$$

No. to locate; $\quad X_C + X_D + X_E \leq 1$

Integrality $\quad X_C , X_D , X_E = 0,1$

$$Z_A , Z_B , Z_C , Z_D , Z_E , Z_F = 0,1$$

Since the row reduction technique cannot be applied we use total enumeration. That is if $X_D = 1$ then $Z_A = Z_B = Z_D = Z_E = 1$, Objective function = 10 + 8 + 18 + 7 = 43.

If $X_C = 1$, then $Z_C = Z_E = Z_F = 1$, Objective function = 22 + 7 + 55 = 84

If $X_E = 1$ then $Z_C = Z_D = Z_E =1$, Objective function = 22 + 18 + 7 = 47

Since the problem is a maximization, we choose $X_C = 1$ which gives us the maximum objective function value of 84. Hence facility will be located at X_C.

If we are to locate two facilities that is P = 2, then it is either (X_D, X_E) or (X_D, X_C) or (X_C, X_E)

If it is X_D, X_C then objective function equals 120. If it is X_D, X_E then objective function equals 65. If it is X_C, X_E then objective function equals 102.

Therefore the facility will be located at X_D, X_C

3.6 THE P-CENTER PROBLEM

The *p*-center problem is the problem of locating *p* (facilities) in order to minimizes the maximum response time (the time between a demand site and the nearest facility), using a given number of *p*. With the above definition and the decision variable;

W = The maximum distance between a demand node and the facility to which is assigned.

$$y_{ij} = \begin{cases} 1, \text{ if the demand node is assigned to a facility at node } j \\ 0, \text{ if not} \end{cases}$$

The p – centre problem therefore can be formulated as follows:

Minimize W ……………………………………………….. (1)

Subject to:

$$\sum_{j \in J} x_j \le p \text{……………………………..………..} \quad (2)$$

$$\sum_{j \in J} x_j = 1 \quad \forall i \in I \text{ ……………………….} \quad (3)$$

$$y_{ij} - x_j \le 0 \quad \forall i \in I, j \in J \ldots\ldots\ldots\ldots \quad (4)$$

$$W - \sum_{j \in J} h_i d_{ij} y_{ij} \ge 0 \quad \forall i \in I \ldots\ldots\ldots\ldots \quad (5)$$

$$x_j \in \{0,1\} \quad \forall j \in J \ldots\ldots\ldots\ldots \quad (6)$$

$$y_{ij} \in \{0,1\} \quad \forall i \in I, j \in J \ldots\ldots\ldots\ldots \quad (7)$$

The objective function (1) minimizes the maximum demand – weighted distance between each demand node and its closet open facility. Constraint set (2) stipulates that p facilities are to be located. Constraint set (3) requires that each demand node be assigned to exactly one facility. Constraint set (4) restricts demand node assignments only to open facilities. Constraint set (5) defines the lower bound on the maximum demand – weighted distance, which is being minimized. Constraint set (6) established the sitting decision variable as binary. Constraint set (7) requires the demand at a node to be assigned to one facility only. Constraint set (7) can be replaced by $y_{ij} \ge 0 \quad \forall i \in I, j \in J$ because constraint set (4) guarantees that $y_{ij} \le 1$. If some y_{ij} are fractional, we simply assign node i to its closet open facility (Current et al, 2001).

3.7 THE CONDITIONAL P-CENTER PROBLEM

The conditional location problem is to locate p new facilities to serve a set of demand points given that q facilities are already located. When $q = 0$, the problem is unconditional. In the conditional $p$center problems, once the new p locations are determined, a demand can be served either by one of the existing or by one of the new facilities whichever is the closest facility to the demand. Consider a network G = (N, L), where

N = the set of nodes, |N| = n

L = the set of links.

Let $d(x, y)$ be the shortest distance between any $x, y \in G$. Suppose that there is a set Q

(|Q| = q) of existing facilities. Let… $Y = (Y_1, \ldots, Y_q)$ and $X = (X_1, X_2, \ldots, X_p)$ be vectors of size q and p respectively, where Y_i is the location of existing facility i and Y_i is the location of new facility i .

Without any loss of generality we do not need to assume that $Y \in N_i$. The conditional p-center location problem is to;

$$Min\ G[x] = \max_{i=1,\ldots,n} \min\{d(X,i), d(Y,i)\}$$

Where (X, i), and (Y, i), is the shortest distance from the closest facility in respectively to the node i ,(Berman and Simchi-Levi, 1990).

3.8 BERMAN AND DREZNER'S ALGORITHM

Berman and Drezner (2008) discuss a very simple algorithm that solves the conditional p-center problem on a network. The algorithm requires one-time solution of an unconditional p-center problem using an appropriate shortest distance matrix. Rather than creating a new location for an artificial facility and force the algorithm to locate a new facility there by creating an artificial demand point, they just modify the distance matrix.

3.8.1 Algorithm

Step 1 Let D be a distance matrix with rows corresponding to demands and columns corresponding to potential locations.

Step 2 solved the conditional problem by defining a modified shortest distance matrix, from D to $\hat{D}$, where $\hat{D}_{ij} = \min\{(d_{ij}), (\min_{k \in Q}\{d_{ik}\})\}, \forall i \in A, j \in C$ (center)

Note that $\hat{D}$ is not symmetric even when D is symmetric.

The unconditional p-center problem using the appropriate $\hat{D}$ solves the conditional p-center problem. This is so since if the shortest distance from i node to the new p facilities are larger than $\min_{k \in Q}\{d_{ik}\}$, then the shortest distance to the existing q facilities is utilized. Note that the size of $\hat{D}$ is $n \times |C|$ for the conditional p-center.

Step 3 Find the optimal new location using $\hat{D}$ for the network with the objective function

$$Min\ G[x] = \max_{i=1,\dots,n} \min\{d(X,i), d(Y,i)\}$$

Illustrative example of Berman and Drezner's Algorithm.

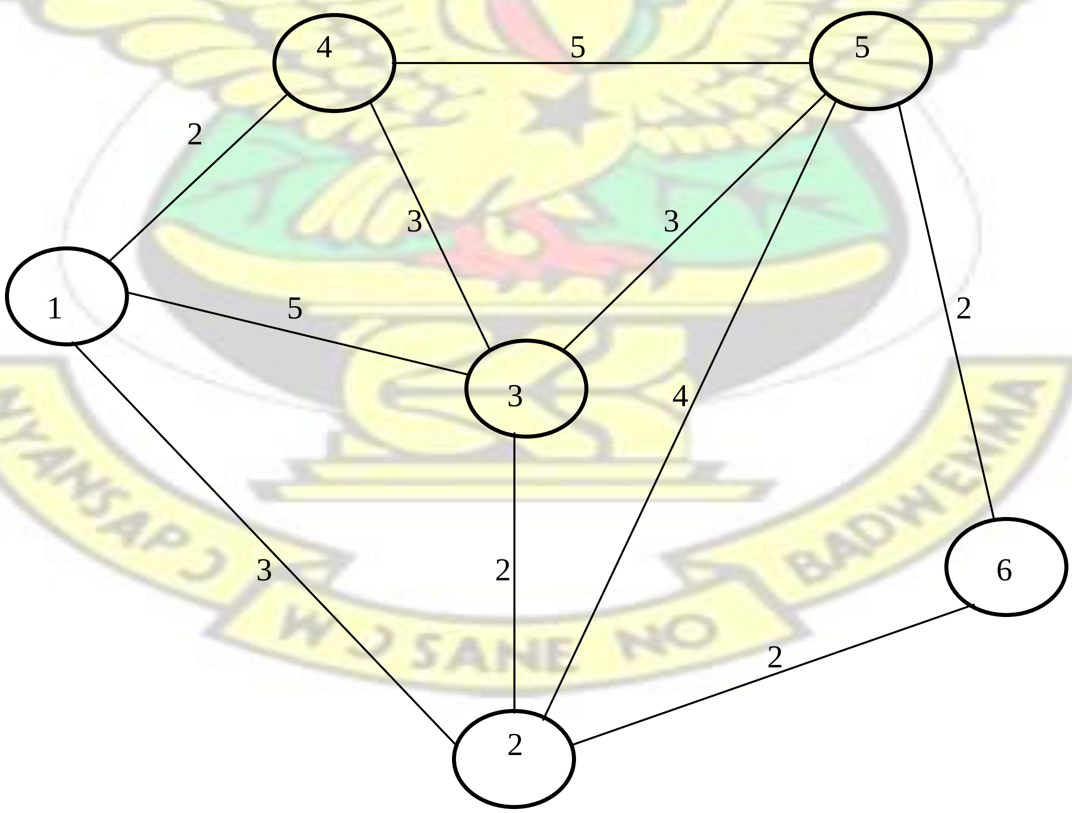

Figure 3.3: Sample network for *p*-center problem

Table 3.3 Matrix Representation of Figure 3.1

Nodes	1	2	3	4	5	6
1	-	3	5	2	∞	∞
2	3	-	2	∞	4	2
3	5	2	-	3	3	∞
4	2	∞	3	-	5	∞
5	∞	4	3	5	-	2
6	∞	2	∞	∞	2	-

Step 1: Table 3.6 below is all pair shortest path (distance matrix), D obtained by using Floyd's algorithm on figure 3.1. Suppose that nodes 1 and 5 are the existing set of facilities and an additional one facility is to be located (i.e. p = 1).

Table 3.4 All pair shortest path (distance matrix), D

Nodes	**1**	**2**	**3**	**4**	**5**	**6**
1	-	3	5	2	7	5
2	3	-	2	5	4	2
3	5	2	-	3	3	4
4	2	5	3	-	5	7
5	7	4	3	5	-	2
6	5	2	4	7	2	-

Table 3.5 All pair shortest paths distance matrix, *D*

Demand Nodes	Potential Location					
	1	**2**	**3**	**4**	**5**	**6**
1	0	3	5	2	7	5
2	3	0	2	5	4	2
3	5	2	0	3	3	4

4	2	5	3	0	5	7
5	7	4	3	5	0	2
6	5	2	4	7	2	0

Step 2: Modified shortest distance matrix is obtained using $\hat{D} = \min\{d_{ij} \min_{k \in Q}\{d_{ik}\}\}$ as shown below:

For node 1 $i = 1, j = 1$ and $Q = \{1,5\}$

$\hat{D}_{11} = \min\{d_{11}, \min\{d_{11\,15}, d\ ,\}\}$

$= \min\{0, \min\{0,7\}\}$

$= 0$

$i = 1, j = 2$

$\hat{D}_{12} = \min\{d_{12}, \min\{d_{11\,15}, d\ ,\}\}$

$= \min\{3, \min\{0,7\}\}$

$= 0$

For node 2

$i = 2, j = 1$ and $Q = \{1,5\}$

$\hat{D}_{21} = \min\{d_{21}, \min\{d\ d_{21},\ _{25},\}\}$

$= \min\{3, \min\{3,4\}\}$

$= 3$

$i = 2, j = 2$

$\hat{D}_{22} = \min\{d_{22}, \min\{d_{21}, d_{25},\}\}$

$= \min\{0, \min\{3,4\}\}$

$= 0$

Table 3.6 summarizes the results obtained above.

Table 3.6 Modified Distance Matrix, $\hat{D}$

Demand Nodes	Potential Location					
	1	**2**	**3**	**4**	**5**	**6**
1	0	0	0	0	0	0
2	3	0	2	3	3	2
3	3	2	0	3	3	3
4	2	2	2	0	2	2
5	0	0	0	0	0	0
6	2	2	2	2	2	0

Nodes 1 and 5 in Q been the nodes with existing facility are removed and presented in Table 3.10 below.

Table 3.7 Modified Distance Matrix, $\hat{D}$ with nodes 1 and 5 removed

Demand Nodes	Potential Location					
	1	**2**	**3**	**4**	**5**	**6**
2	3	0	2	3	3	2
3	3	2	0	3	3	3
4	2	2	2	0	2	2
6	2	2	2	2	2	0

Step 3: Find the optimal new location using the modified distance matrix $\hat{D}$ and objective function, $Min\ G[x(\ )] = \max_{i=1,\dots,n} \min\{d(X,i), d(Y,i)\}$

With $X = \{1,2,3,4,5,6\}$, $Y = \{1,5\}$ and $i = \{2,3,4,6\}$

For node 1 $i = 1$

$\min\{d(1,1), d(1,1), d(5,1)\}$

$= \min\{0,0,0\}$

$= 0$

$i = 2$

$\min\{d(1,2), d(1,2), d(5,2)\}$

$= \min\{3,3,3\}$

$= 3$

For node 2 i

$\square$1

min{ (2,1), (1,1), (5,1)}d d d

$\square$ min{0,0,3}

$\square$ 0

i $\square$2

min{ (2,2), (1,2), (5,2)}d d d

$\square$ min{0,3,3}

$\square$ 0

The results are then summarized and presented in Table 3.11 below.

Table 3.8 Optimal Location, $Min(g(x))$ using $\hat{D}$

Demand Node	**Potential Location**						
	1	**2**	**3**	**4**	**5**	**6**	**Max**
2	0	0	2	2	0	2	**2**
3	0	2	0	2	0	2	**2**
4	0	3	3	0	0	2	**3**
6	0	2	3	2	0	2	**3**
Minimum							**2**

The optimal new location should be at either node 2 or 3 with an objective function value of 2

CHAPTER 4

DATA COLLECTION, ANALYSIS AND RESULTS

4.1 DATA COLLECTION

Data on distances in kilometers connecting towns and villages in the Berekum municipality was collected from the Department of Feeder Roads, Berekum.

Again data on the number of health centres and where they are situated was also collected from the municipal health directorate, Berekum.

To ensure that the location decisions resulting from the model are not only profitable but also equitable and sustainable, there was a need to develop a nineteen-node network, taking into consideration the nineteen major towns. Below are the nineteen major towns and their respective assigned nodes, shown in Table 4.1

Table 4.1 Towns in Berekum Municipality and their codes

Town	Node	Town	Node
Berekum	1	Fententaa	11
Kato	2	Jinijini	12

Mpatasie	3	Domfete	13
Mpatapo	4	Jamedede	14
Kutre No.2	5	Ayimom	15
Namasua	6	Nsapor	16
Kutre No.1	7	Benkasa	17
Akroforo	8	Biadan	18
Abisaase	9	Senase	19
Koraso	10		

Having a community hospital at Berekum, clinic at Jinijini, Community based Health and Preventive Services(CHPS) compounds at Akroforo, Namasua, Abisaase, Koraso, and rehap centre at Mpatasie. These communities form the set of existing facilities, with nodes; 1, 3, 6, 8, 9,10 and 12 respectively. The data is then developed into a network of figure 4.1 below.

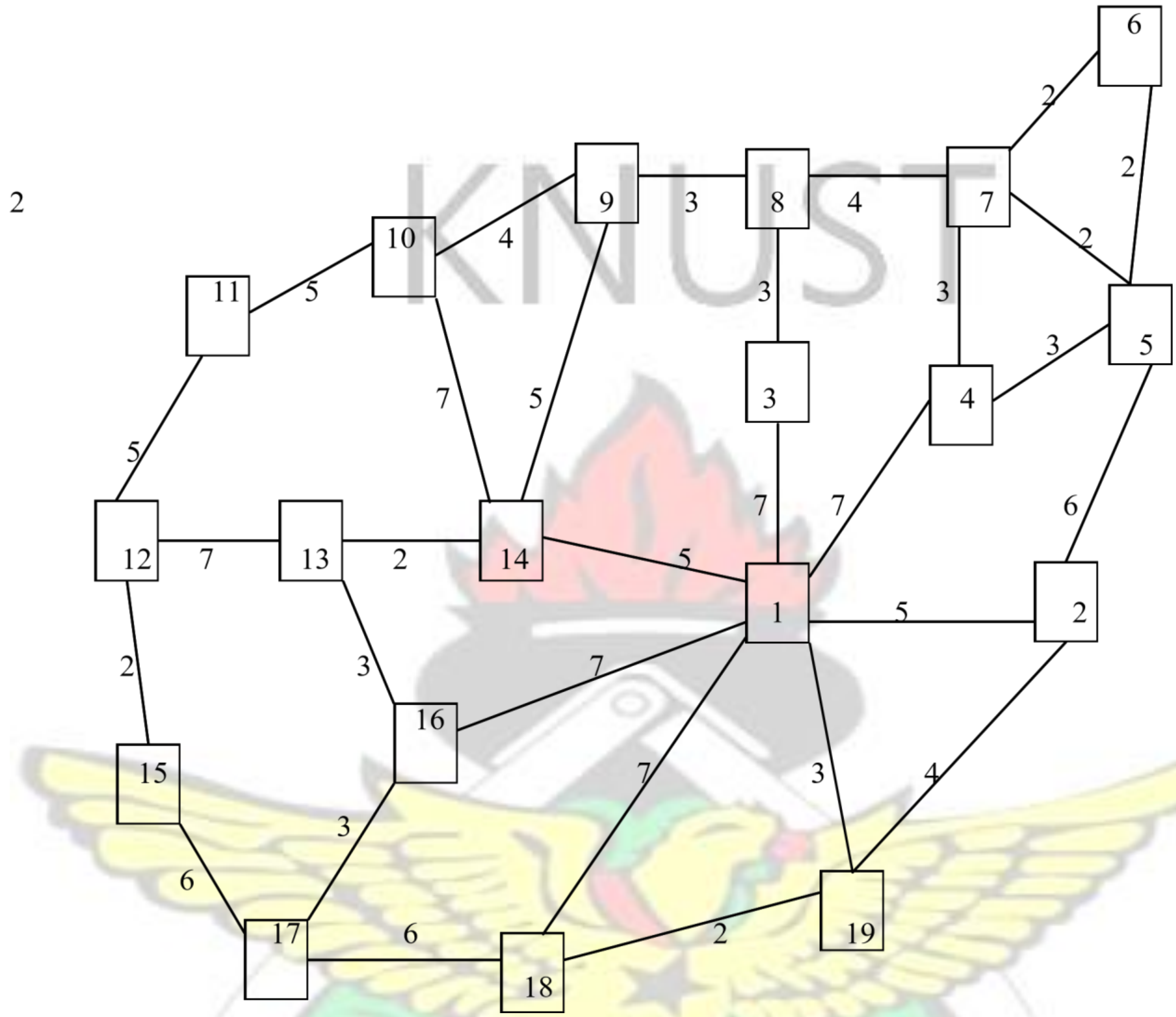

Figure 4.1 Road Network of Berekum Municipality.

Using the network in Figure 4.1 and all pair shortest path distance matrix, *D* is developed and shown in Table 4.2 below by using the Floyd-Warshall algorithm. Matlab program software was used for the coding of the Floyd-Warshall algorithm.

Table 4.2 All Pairs Shortest Path Distance Matrix *D*

Demand Nodes	Potential locations																		
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
1	0	5	7	7	10	12	10	10	10	12	17	14	7	5	16	7	10	5	3
2	5	0	12	9	6	8	8	12	15	17	22	19	12	10	18	12	12	6	4

3	7	12	0	10	9	9	7	3	6	10	15	20	13	11	22	14	17	12	10
4	7	9	10	0	3	5	3	7	10	14	19	21	14	12	23	14	17	12	10
5	10	6	9	3	0	2	2	6	9	13	18	23	16	14	24	17	18	12	10
6	12	8	9	5	2	0	2	6	9	13	18	23	16	14	25	19	20	14	12
7	10	8	7	3	2	2	0	4	7	11	16	21	14	12	23	17	20	14	12
8	10	12	3	7	6	6	4	0	3	7	12	17	10	8	19	13	16	15	13
9	10	15	6	10	9	9	7	3	0	4	9	14	7	5	16	10	13	15	13
10	12	17	10	14	13	13	11	7	4	0	5	10	9	7	12	12	15	17	15
11	17	22	15	19	18	18	16	12	9	5	0	5	12	12	7	15	13	19	20
12	14	19	20	21	23	23	21	17	14	10	5	0	7	9	2	10	8	14	16
13	7	12	13	14	16	16	14	10	7	9	12	7	0	2	9	3	6	12	10
14	5	10	11	12	14	14	12	8	5	7	12	9	2	0	11	5	8	10	8
15	16	18	22	23	24	25	23	19	16	12	7	2	9	11	0	9	6	12	14
16	7	12	14	14	17	19	17	13	10	12	15	10	3	5	9	0	3	9	10
17	10	12	17	17	18	20	20	16	13	15	13	8	6	8	6	3	0	6	8
18	5	6	12	12	12	14	14	15	15	17	19	14	12	10	12	9	6	0	2
19	3	4	10	10	10	12	12	13	13	15	20	16	10	8	14	10	8	2	0

4.2 CONDTIONAL P-CENTRE FORMULATION

We use the Berman and Drezner's algorithm (2008) to solve the problem of locating an addtioal hospital. The conditional p- center problem is formulated as:

$$Min\ G[x(\ )] = \max_{i=1,\dots,n} \min\{d(X, i), d(Y, i)\}$$

Let $d(x,y)$ be the shortest distance between any $x, y \in G.$ Suppose that there is a set Q (|Q| = q) of existing facilities. Let $Y = (Y_1, Y_2, Y_3, \dots, Y_q)$ and $X = (X_1, X_2, X_3, \dots, X_p)$ be vectors of size q and p respectively, where Y_i is the location of existing facility i and X_i is the location of new facility. Where $d(X,i)$ and $d(Y,i)$ is the shortest distance from the closest facility in X and Y respectively to the node i, (Berman and Simchi-Levi, 1990).

Considering the set of location of new facilities X={1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16,17,18,19} and the set of location of existing facilities Y,={1,3,6,8,9,10,12} .

Where i = {2,4,5,7,11,13, 14,15,16,17,18,19}

4.2.1 BERMAN AND DREZNER'S ALGORITHM

Step 1: Let D be a distance matrix with rows corresponding to demands and columns corresponding to potential locations.

Step 2: Solved the conditional problem by defining a modified shortest distance matrix,

from D to $\hat{D}$, where $\hat{D}_{ij} = \min\{(d_{ij}),(\min_{k \in Q}\{d_{ik}\})\}, \forall i \in A, j \in C$ (center) (Center).

Even though D is symmetric but $\hat{D}$ is not symmetric.

Step 3: Find the optimal new location using $\hat{D}$ for the network with the objective function

$$Min\ G[x] = \max_{i=1,\ldots,n} \min\{d(X,i), d(Y,i)\}$$

4.3 BERMAN AND DREZNER'S SOLUTION

Considering the all pair shortest path distance for the nineteen node network of the Berekum municipality in Table 4.2 above, a new shortest path distance matrix is formed.

Thus from D to $\hat{D}$.

CALCULATION OF MODIFIED SHORTEST DISTANCE MATRIX, $\hat{D}$

By defining a modified shortest distance matrix $\hat{D}$ where

$\hat{D}_{ij} = \min\{d_{ij} \min_{k \in Q}\{d_{ik}\}\} \forall i \in N, j \in C$ (Center).

For Node 1, $Q = \{1,3,6,8,9,10,12\}$ $i = 1, j = 1$

$\hat{D}_{11} = \min\{d_{11}, \min\{d_{11}, d_{13}, d_{16}, d_{18}, d_{19}, d_{1\,10}, d_{1\,12}\}\}$

$= \min\{0, \min\{0,7,12,10,10,12,14\}\}$
$= 0$

$i = 1, j = 2$

$\hat{D}_{12} = \min\{d_{12}, \min\{d_{11}, d_{13}, d_{16}, d_{18}, d_{19}, d_{1\,10}, d_{1\,12}\}\}$

$= \min\{5, \min\{0,7,12,10,10,12,14\}\}$
$= 0$

Table 4.3 below summarizes the results of $\hat{D}$ into a modified shortest distance matrix.

The results of $\hat{D}$ for the nineteen nodes is shown in Appendix A

Table 4.3 Modified Shortest Distance Matrix, $\hat{D}$

Demand nodes	Potential locations																		
	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**
1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
2	5	0	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	4
3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
4	5	5	5	0	3	5	3	5	5	5	5	5	5	5	5	5	5	5	5
5	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2
6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
7	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2
8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

9	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
10	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
11	5	5	5	5	5	5	5	5	5	5	0	5	5	5	5	5	5	5	5
12	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
13	7	7	7	7	7	7	7	7	7	7	7	7	0	2	7	3	6	7	7
14	5	5	5	5	5	5	5	5	5	5	5	5	2	0	5	5	5	5	5
15	2	2	2	2	2	2	2	2	2	2	2	2	2	2	0	2	2	2	2
16	7	7	7	7	7	7	7	7	7	7	7	7	3	5	7	0	3	7	7
17	8	8	8	8	8	8	8	8	8	8	8	8	6	8	6	3	0	6	8
18	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	0	2
19	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	2	0

Comparing road distances with existing set of nodes $Q \in \{1,3,6,8,9,10,12\}$ the minimum is always zero. The set of demand nodes of the existing facilities is removed from the modified shortest path distance matrix and this is shown in Table 4.4.

Table 4.4 below shows the summary of results $\min(g(x))$ using $\hat{D}$. Details of the results are presented in appendix B.

Table 4.4 Modified shortest path distance matrix, $\hat{D}$ with set Q removed row-wise

Demand nodes	Potential locations																		
	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**
2	5	0	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	4
4	5	5	5	0	3	5	3	5	5	5	5	5	5	5	5	5	5	5	5

5	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2
7	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2
11	5	5	5	5	5	5	5	5	5	5	0	5	5	5	5	5	5	5	5
13	7	7	7	7	7	7	7	7	7	7	7	7	0	2	7	3	6	7	7
14	5	5	5	5	5	5	5	5	5	5	5	5	2	0	5	5	5	5	5
15	2	2	2	2	2	2	2	2	2	2	2	2	2	2	0	2	2	2	2
16	7	7	7	7	7	7	7	7	7	7	7	7	3	5	7	0	3	7	7
17	8	8	8	8	8	8	8	8	8	8	8	8	6	8	6	3	0	6	8
18	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	0	2
19	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	2	0

4.3.2 FINDING THE OPTIMAL LOCATION

Using table 4.3 the optimal new location using the modified shortest distance matrix, with the objective function;

$$Min\ G[x(\ )] = \max_{i=1,\ldots,n} \min\{d(X,i), d(Y,i)\}\text{ with}$$

$X = \{1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16,17,18,19\}$, $Y = \{1,3,6,8,9,10,12\}$ and $i = \{2,4,5,7,11,13,14,15,16,17,18,19\}$

For node 1

$i = 1$

$\min\{d(1,1), d(1,1), d(3,1), d(6,1), d(8,1), d(9,1), d(10,1), d(12,1)\} = \min\{0,0,0,0,0,0,0,0\}$

$= 0$

$i = 2$

$\min\{d(1,2), d(1,2), d(3,2), d(6,2), d(8,2), d(9,2), d(10,2), d(12,2)\} = \min\{5,5,5,5,5,5,5,5\}$

$= 5$

The results are then summarized and presented in Table 4.5 below

Table 4.5 Optimal Location $\min(g(x))$ using $\hat{D}$

Demand Nodes	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	Max
2	0	0	0	5	2	0	2	0	0	0	5	0	7	5	2	7	8	5	3	**8**
4	0	5	0	0	2	0	2	0	0	0	5	0	7	5	2	7	8	5	3	**8**
5	0	5	0	3	0	0	2	0	0	0	5	0	7	5	2	7	8	5	3	**8**
7	0	5	0	3	2	0	0	0	0	0	5	0	7	5	2	7	8	5	3	**8**
11	0	5	0	5	2	0	2	0	0	0	0	0	7	5	2	7	8	5	3	**8**
13	0	5	0	5	2	0	2	0	0	0	5	0	0	2	2	3	6	5	3	**6**
14	0	5	0	5	2	0	2	0	0	0	5	0	2	0	2	5	8	5	3	**8**
15	0	5	0	5	2	0	2	0	0	0	5	0	7	5	0	7	6	5	3	**7**
16	0	5	0	5	2	0	2	0	0	0	5	0	3	5	2	0	3	5	3	**5**
17	0	5	0	5	2	0	2	0	0	0	5	0	6	5	2	3	0	5	3	**6**
18	0	5	0	5	2	0	2	0	0	0	5	0	7	5	2	7	6	0	2	**7**
19	0	4	0	5	2	0	2	0	0	0	5	0	7	5	2	7	8	2	0	**8**
Minimum																				**5**

Analysis of the results show that the optimal new location is at **node 16**

4.4 DISCUSSION

Having used Berman and Drezner's algorithm on the nineteen node-network to solve a conditional 1-centre problem as shown in table 4.4 and subsequently in table 4.5 with the existing facilities at Berekum, Mpatasie, Namasua, Akroforo, Abisaase, Koraso, and Jinijini, the optimal location of the new hospital is at **node 16 (Nsapor)** with an objective function $\min g(x) = 5$. This is to say that the minimum distance a patient has to travel to a closest health facility will not be more than 5 kilometers.

CHAPTER 5

CONCLUSION AND RECOMMENDATIONS

5.1 CONCLUSION

Matlab code was developed for Floyd-Warshall's algorithm which was used to find all pair shortest path between nodes using table 4.1 which resulted in (19×19) matrix in Table 4.2. The model formulation for the optimal location of an additional hospital facility in the Berekum Municipality is indicated on page 42. The model was solved using Berman and Drezner (2008) algorithm as shown in page 43.

Considering the objective function $\min g(x)$ from Berman and Drezner (2008), the hospital facility should be located at **node 16 (Nsapor)**.

The minimum objective function value obtained was 5 kilometers. This implies that the minimum distance travelled by the farthest patient to the new facility at Nsapor and any other health care centre is 5 kilometers.

5.2 RECOMMENDATIONS

The government, nongovernmental organization (churches) and other cooperate bodies who would like to establish additional hospital facility in the Berekum Municipality should locate it at Nsapor.

Though all road networks are equal good in the municipality but it should be improved for easy accessibility and response time of patients to the health facilities.

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APPENDIX A

CALCULATION OF MODIFIED SHORTEST DISTANCE MATRIX, $\hat{D}$

$\hat{D}_{ij} = \min\{(d_{ij}),(\min\{d_{ik}\})\}, \forall i \in A, j \in C$ (Centre)

$k \in Q$

For Node 1, $Q = \{1,3,6,8,9,10,12\}$

$i = 1, j = 1$

$$\hat{D}_{11} = \min\{(d_{11}, \min\{d_{11}, d_{13}, d_{16}, d_{18}, d_{19}, d_{1\,10}, d_{1\,12}\})\}$$

$$= \min\{0, \min\{0,7,12,10,10,12,14\}\}$$

$$= 0$$

$i = 1, j = 2$

$$\hat{D}_{12} = \min\{(d_{12}, \min\{d_{11}, d_{13}, d_{16}, d_{18}, d_{19}, d_{1\,10}, d_{1\,12}\})\}$$

$$= \min\{5, \min\{0,7,12,10,10,12,14\}\}$$

$$= 0$$

$i = 1, j = 3$

$$\hat{D}_{13} = \min\{(d_{13}, \min\{d_{11}, d_{13}, d_{16}, d_{18}, d_{19}, d_{1\,10}, d_{1\,12}\})\}$$

$$= \min\{7, \min\{0,7,12,10,10,12,14\}\}$$

$$= 0$$

$i = 1, j = 4$

$$\hat{D}_{14} = \min\{(d_{14}, \min\{d_{11}, d_{13}, d_{16}, d_{18}, d_{19}, d_{1\,10}, d_{1\,12}\})\}$$

$$= \min\{7, \min\{0,7,12,10,10,12,14\}\}$$

$$= 0$$

$i = 1, j = 5$

$$\hat{D}_{15} = \min\{(d_{15}, \min\{d_{11}, d_{13}, d_{16}, d_{18}, d_{19}, d_{1\,10}, d_{1\,12}\})\}$$

$$= \min\{10, \min\{0,7,12,10,10,12,14\}\}$$

$$= 0$$

$i = 1, j = 6$

$$\hat{D}_{16} = \min\{(d_{16}, \min\{d_{11}, d_{13}, d_{16}, d_{18}, d_{19}, d_{1\,10}, d_{1\,12}\})\}$$

$$= \min\{12, \min\{0,7,12,10,10,12,14\}\}$$

$$= 0$$

$i = -1, j = 7$

$$\hat{D}_{17} = \min\{(d_{17}, \min\{d_{11}, d_{13}, d_{16}, d_{18}, d_{19}, d_{1\,10}, d_{1\,12}\})\}$$
$$= \min\{10, \min\{0,7,12,10,10,12,14\}\}$$
$$= 0$$

$i = -1, j = 8$

$$\hat{D}_{18} = \min\{(d_{18}, \min\{d_{11}, d_{13}, d_{16}, d_{18}, d_{19}, d_{1\,10}, d_{1\,12}\})\}$$
$$= \min\{10, \min\{0,7,12,10,10,12,14\}\}$$
$$= 0$$

$i = -1, j = 9$

$$\hat{D}_{19} = \min\{(d_{19}, \min\{d_{11}, d_{13}, d_{16}, d_{18}, d_{19}, d_{1\,10}, d_{1\,12}\})\}$$
$$= \min\{10, \min\{0,7,12,10,10,12,14\}\}$$
$$= 0$$

$i = -1, j = 10$

$$\hat{D}_{1\,10} = \min\{(d_{1\,10}, \min\{d_{11}, d_{13}, d_{16}, d_{18}, d_{19} d_{1\,10}, d_{1\,12}\})\}$$
$$= \min\{12, \min\{0,7,12,10,10,12,14\}\}$$
$$= 0$$

$i = -1, j = 11$

$$\hat{D}_{1\,11} = \min\{(d_{1\,11}, \min\{d_{11}, d_{13}, d_{16}, d_{18}, d_{19} d_{1\,10}, d_{1\,12}\})\}$$
$$= \min\{17, \min\{0,7,12,10,10,12,14\}\}$$
$$= 0$$

$i = -1, j = 12$

$$\hat{D}_{1\,12} = \min\{(d_{1\,12}, \min\{d_{11}, d_{13}, d_{16}, d_{18}, d_{19} d_{1\,10}, d_{1\,12}\})\}$$
$$= \min\{14, \min\{0,7,12,10,10,12,14\}\}$$
$$= 0$$

$i = 1, j = 13$

$$\hat{D}_{1\,13} = \min\{(d_{1\,13}, \min\{d_{1\,1}, d_{1\,3}, d_{1\,6}, d_{1\,8}, d_{1\,9}, d_{1\,10}, d_{1\,12}\})\}$$
$$= \min\{7, \min\{0,7,12,10,10,12,14\}\}$$
$$= 0$$

$i = 1, j = 14$

$$\hat{D}_{1\,14} = \min\{(d_{1\,14}, \min\{d_{1\,1}, d_{1\,3}, d_{1\,6}, d_{1\,8}, d_{1\,9}, d_{1\,10}, d_{1\,12}\})\}$$
$$= \min\{5, \min\{0,7,12,10,10,12,14\}\}$$
$$= 0$$

$i = 1, j = 15$

$$\hat{D}_{1\,15} = \min\{(d_{1\,15}, \min\{d_{1\,1}, d_{1\,3}, d_{1\,6}, d_{1\,8}, d_{1\,9}, d_{1\,10}, d_{1\,12}\})\}$$
$$= \min\{16, \min\{0,7,12,10,10,12,14\}\}$$
$$= 0$$

$i = 1, j = 16$

$$\hat{D}_{1\,16} = \min\{(d_{1\,16}, \min\{d_{1\,1}, d_{1\,3}, d_{1\,6}, d_{1\,8}, d_{1\,9}, d_{1\,10}, d_{1\,12}\})\}$$
$$= \min\{7, \min\{0,7,12,10,10,12,14\}\}$$
$$= 0$$

$i = 1, j = 17$

$$\hat{D}_{1\,17} = \min\{(d_{1\,17}, \min\{d_{1\,1}, d_{1\,3}, d_{1\,6}, d_{1\,8}, d_{1\,9}, d_{1\,10}, d_{1\,12}\})\}$$
$$= \min\{10, \min\{0,7,12,10,10,12,14\}\}$$
$$= 0$$

$i = 1, j = 18$

$$\hat{D}_{1\,18} = \min\{(d_{1\,18}, \min\{d_{1\,1}, d_{1\,3}, d_{1\,6}, d_{1\,8}, d_{1\,9}, d_{1\,10}, d_{1\,12}\})\}$$
$$= \min\{5, \min\{0,7,12,10,10,12,14\}\}$$
$$= 0$$

$i = 1, j = 19$

$$\hat{D}_{1\,19} = \min\{(d_{1\,19}, \min\{d_{1\,1}, d_{1\,3}, d_{1\,6}, d_{1\,8}, d_{1\,9}, d_{1\,10}, d_{1\,12}\})\}$$
$$= \min\{3, \min\{0,7,12,10,10,12,14\}\}$$

$= 0$

For Node 2 $i = 2, j = 1$

$$\hat{D}_{21} = \min\{d_{21}, \min\{d_{21}, d_{23}, d_{26}, d_{28}, d_{29}, d_{2\,10}, d_{2\,12}\}\}$$
$$= \min\{5, \min\{5,12,8,12,15,17,19\}\}$$

$$= 5$$

$i = 2, j = 2$

$$\hat{D}_{22} = \min\{d_{22}, \min\{d_{21}, d_{23}, d_{26}, d_{28}, d_{29}, d_{2\,10}, d_{2\,12}\}\}$$
$$= \min\{0, \min\{5,12,8,12,15,17,19\}\}$$

$$= 0$$

$i = 2, j = 3$

$$\hat{D}_{23} = \min\{d_{23}, \min\{d_{21}, d_{23}, d_{26}, d_{28}, d_{29}, d_{2\,10}, d_{2\,12}\}\}$$

$$= \min\{12, \min\{5,12,8,12,15,17,19\}\}$$

$$= 5$$

$i = 2,\ j = 4$

$$\hat{D}_{24} = \min\{d_{24}, \min\{d_{21}, d_{23}, d_{26}, d_{28}, d_{29}, d_{2\,10}, d_{2\,12}\}\}$$

$$= \min\{9, \min\{5,12,8,12,15,17,19\}\}$$

$$= 5$$

$i = 2,\ j = 5$

$$\hat{D}_{25} = \min\{d_{25}, \min\{d_{21}, d_{23}, d_{26}, d_{28}, d_{29}, d_{2\,10}, d_{2\,12}\}\}$$

$$= \min\{6, \min\{5,12,8,12,15,17,19\}\}$$

$$= 5$$

$i = 2,\ j = 6$

$$\hat{D}_{26} = \min\{d_{26}, \min\{d_{21}, d_{23}, d_{26}, d_{28}, d_{29}, d_{2\,10}, d_{2\,12}\}\}$$

$$= \min\{8, \min\{5,12,8,12,15,17,19\}\}$$

$$= 5$$

$i = 2,\ j = 7$

$$\hat{D}_{27} = \min\{d_{27}, \min\{d_{21}, d_{23}, d_{26}, d_{28}, d_{29}, d_{2\,10}, d_{2\,12}\}\}$$

$$= \min\{8, \min\{5,12,8,12,15,17,19\}\}$$

$$= 5$$

$i = 2,\ j = 8$

$$\hat{D}_{28} = \min\{d_{28}, \min\{d_{21}, d_{23}, d_{26}, d_{28}, d_{29}, d_{2\,10}, d_{2\,12}\}\}$$

$$= \min\{12, \min\{5,12,8,12,15,17,19\}\}$$

$$= 5$$

$\hat{D}$

$i = 2, j = 9$

$\hat{D}_{29} = \min\{d_{29}, \min\{d\ d\ d\ d\ d_{21}, {}_{23}, {}_{26}, {}_{28}, {}_{29}, d_{2\ 10}, d_{2\ 12}\}\}$

$= \min\{15, \min\{5,12,8,12,15,17,19\}\}$

$= 5$

$i = 2, j = 10$

${}_{2\ 10} = \min\{(d_{2\ 10}, \min\{d_{21}, d_{23}, d_{26}, d_{28}, d_{29}\ d_{2\ 10}, d_{2\ 12}\})\}$

$= \min\{17, \min\{5,12,8,12,15,19,19\}\}$

$= 5$

$i = 2, j = 11$

$\hat{D}_{2\ 11} = \min\{(d_{2\ 11}, \min\{d\ d\ d\ d\ d_{21}, {}_{23}, {}_{26}, {}_{28}, {}_{29}\ d_{2\ 10}, d_{2\ 12}\})\}$

$= \min\{22, \min\{5,12,8,12,15,19,19\}\}$

$= 5\ i = 2,$

$j = 12$

$\hat{D}_{2\ 12} = \min\{(d_{2\ 12}, \min\{d\ d\ d\ d\ d_{21}, {}_{23}, {}_{26}, {}_{28}, {}_{29}\ d_{2\ 10}, d_{2\ 12}\})\}$

$= \min\{19, \min\{5,12,8,12,15,19,19\}\}$

$= 5$

$i = = 2, 13$

$\hat{D}_{2\ 13} = \min\{(d_{2\ 13}, \min\{d\ d\ d\ d\ d_{21}, {}_{23}, {}_{26}, {}_{28}, {}_{29}\ d_{2\ 10}, d_{2\ 12}\})\}$

$= \min\{12, \min\{5,12,8,12,15,19,19\}\}$

$= 5\ i = 2,$

$j = 14$

$\hat{D}_{2\ 14} = \min\{(d_{2\ 14}, \min\{d_{21}, d_{23}, d_{26}, d_{28}, d_{29}\ d_{2\ 10}, d_{2\ 12}\})\}$

$= \min\{10, \min\{5,12,8,12,15,19,19\}\}$

$= 5$

$i = 2, j = 15$

$\hat{D}_{2\ 15} = \min\{(d_{2\ 15}, \min\{d\ d\ d\ d\ d_{21}, {}_{23}, {}_{26}, {}_{28}, {}_{29}\ d_{2\ 10}, d_{2\ 12}\})\}$

$= \min\{18, \min\{5,12,8,12,15,19,19\}\}$

$\hat{D}$

$= 5$

$i = 2, j = 16$

$\hat{D}_{2\,16} = \min\{(d_{2\,16}, \min\{d_{21}, d_{23}, d_{26}, d_{28}, d_{29}\, d_{2\,10}, d_{2\,12}\})\}$

$= \min\{12, \min\{5,12,8,12,15,19,19\}\}$

$= 5$

$i = 2, j = 17$

$\hat{D}_{2\,17} = \min\{(d_{2\,17}, \min\{d_{21}, d_{23}, d_{26}, d_{28}, d_{29}\, d_{2\,10}, d_{2\,12}\})\}$

$= \min\{12, \min\{5,12,8,12,15,19,19\}\}$

$= 5$

$i = 2, j = 18$

$_{2\,18} = \min\{(d_{2\,18}, \min\{d_{21}, d_{23}, d_{26}, d_{28}, d_{29}\, d_{2\,10}, d_{2\,12}\})\}$

$= \min\{6, \min\{5,12,8,12,15,19,19\}\}$

$= 5$

$i = 2, j = 19$

$\hat{D}_{2\,19} = \min\{(d_{2\,19}, \min\{d_{21}, d_{23}, d_{26}, d_{28}, d_{29}\, d_{2\,10}, d_{2\,12}\})\}$

$= \min\{4, \min\{5,12,8,12,15,19,19\}\}$

$= 4$

For Node 3 i

$= 3, j = 1$

$\hat{D}_{31} = \min\{d_{31}, \min\{d_{31}, d_{33}, d_{36}, d_{38}, d_{39}, d_{3\,10}, d_{3\,12}\}\}$

$= \min\{7, \min\{7,0,9,3,6,10,20\}\}$

$= 0$

$i = 3, j = 2$

$\hat{D}_{32} = \min\{d_{32}, \min\{d_{31}, d_{33}, d_{36}, d_{38}, d_{39}, d_{3\,10}, d_{3\,12}\}\}$

$= \min\{12, \min\{7,0,9,3,6,10,20\}\}$

$= 0$

$\hat{D}$

$i = 3, j = 3$

$$\hat{D}_{33} = \min\{d_{33}, \min\{d_{31}, d_{33}, d_{36}, d_{38}, d_{39}, d_{3\,10}, d_{3\,12}\}\}$$
$$= \min\{0, \min\{7,0,9,3,6,10,20\}\}$$
$$= 0$$

$i = 3, j = 4$

$$\hat{D}_{34} = \min\{d_{34}, \min\{d_{31}, d_{33}, d_{36}, d_{38}, d_{39}, d_{3\,10}, d_{3\,12}\}\}$$
$$= \min\{10, \min\{7,0,9,3,6,10,20\}\}$$

$$= 0$$

$i = 3, j = 5$

$$\hat{D}_{35} = \min\{d_{35}, \min\{d_{31}, d_{33}, d_{36}, d_{38}, d_{39}, d_{3\,10}, d_{3\,12}\}\}$$
$$= \min\{9, \min\{7,0,9,3,6,10,20\}\}$$

$$= 0$$

$i = 3, j = 6$

$$\hat{D}_{36} = \min\{d_{36}, \min\{d_{31}, d_{33}, d_{36}, d_{38}, d_{39}, d_{3\,10}, d_{3\,12}\}\}$$
$$= \min\{9, \min\{7,0,9,3,6,10,20\}\}$$

$$= 0$$

$i = 3, j = 7$

$$\hat{D}_{37} = \min\{d_{37}, \min\{d_{31}, d_{33}, d_{36}, d_{38}, d_{39}, d_{3\,10}, d_{3\,12}\}\}$$

$$= \min\{7, \min\{7,0,9,3,6,10,20\}\}$$

$$= 0$$

$i = 3, j = 8$

$$\hat{D}_{38} = \min\{d_{38}, \min\{d_{31}, d_{33}, d_{36}, d_{38}, d_{39}, d_{3\,10}, d_{3\,12}\}\}$$

$$= \min\{3, \min\{7,0,9,3,6,10,20\}\}$$

$$= 0$$

$i = 3, j = 9$

$$\hat{D}_{39} = \min\{d_{39}, \min\{d_{31}, d_{33}, d_{36}, d_{38}, d_{39}, d_{3\,10}, d_{3\,12}\}\}$$

$$= \min\{6, \min\{7,0,9,3,6,10,20\}\}$$

$$= 0$$

$i = 3, j = 10$

$$\hat{D}_{3\,10} = \min\{(d_{3\,10}, d_{31}, d_{33}, d_{36}, d_{38}, d_{39}\, d_{3\,10}, d_{3\,12}\}\}$$

$$= \min\{10, \min\{7,0,9,3,6,10,20\}\}$$

$$= 0$$

$i = 3, j = 11$

$$\hat{D}_{3\,11} = \min\{(d_{3\,11}, d_{31}, d_{33}, d_{36}, d_{38}, d_{39}\, d_{3\,10}, d_{3\,12}\}\}$$

$$= \min\{15, \min\{7,0,9,3,6,10,20\}\}$$

$$= 0$$

$i = 3, j = 12$

$$\hat{D}_{3\,12} = \min\{(d_{3\,12}, d_{31}, d_{33}, d_{36}, d_{38}, d_{39}\, d_{3\,10}, d_{3\,12}\}\}$$

$$= \min\{20, \min\{7,0,9,3,6,10,20\}\}$$

$$= 0$$

$i = 3, j = 13$

$\hat{D}_{3\,13} = \min\{(d_{3\,13}, d_{31}, d_{33}, d_{36}, d_{38}, d_{39}, d_{3\,10}, d_{3\,12}\}\}$

$= \min\{13, \min\{7,0,9,3,6,10,20\}\}$

$= 0$

$i = 3, j = 14$

$\hat{D}_{3\,14} = \min\{(d_{3\,14}, d_{31}, d_{33}, d_{36}, d_{38}, d_{39}, d_{3\,10}, d_{3\,12}\}\}$

$= \min\{11, \min\{7,0,9,3,6,10,20\}\}$

$= 0$

$i = 3, j = 15$

$\hat{D}_{3\,15} = \min\{(d_{3\,15}, d_{31}, d_{33}, d_{36}, d_{38}, d_{39}, d_{3\,10}, d_{3\,12}\}\}$

$= \min\{22, \min\{7,0,9,3,6,10,20\}\}$

$= 0$

$i = 3, j = 16$

$\hat{D}_{3\,16} = \min\{(d_{3\,16}, d_{31}, d_{33}, d_{36}, d_{38}, d_{39}, d_{3\,10}, d_{3\,12}\}\}$

$= \min\{14, \min\{7,0,9,3,6,10,20\}\}$

$= 0$

$i = 3, j = 17$

$\hat{D}_{3\,17} = \min\{(d_{3\,17}, d_{31}, d_{33}, d_{36}, d_{38}, d_{39}, d_{3\,10}, d_{3\,12}\}\}$

$= \min\{17, \min\{7,0,9,3,6,10,20\}\}$

$= 0$

$i = 3, j = 18$

$\hat{D}_{3\,18} = \min\{(d_{3\,18}, d_{31}, d_{33}, d_{36}, d_{38}, d_{39}, d_{3\,10}, d_{3\,12}\})\}$

$= \min\{12, \min\{7,0,9,3,6,10,20\}\}$

$= 0$

$i = 3, j = 19$

$\hat{D}_{3\,19} = \min\{(d_{3\,19}, d_{31}, d_{33}, d_{36}, d_{38}, d_{39}, d_{3\,10}, d_{3\,12}\}\}$

$= \min\{10, \min\{7,0,9,3,6,10,20\}\}$

$= 0$

For Node 4 $i = 4, j = 1$

$$\hat{D}_{41} = \min\{d_{41}, \min\{d_{41}, d_{43}, d_{46}, d_{48}, d_{49}, d_{4\,10}, d_{4\,12}\}\}$$
$$= \min\{7, \min\{7,10,5,7,10,14,21\}\}$$
$$= 5$$

$i = 4, j = 2$

$$\hat{D}_{42} = \min\{d_{42}, \min\{d_{41}, d_{43}, d_{46}, d_{48}, d_{49}, d_{4\,10}, d_{4\,12}\}\}$$
$$= \min\{9, \min\{7,10,5,7,10,14,21\}\}$$
$$= 5$$

$i = 4, j = 3$

$$\hat{D}_{43} = \min\{d_{43}, \min\{d_{41}, d_{43}, d_{46}, d_{48}, d_{49}, d_{4\,10}, d_{4\,12}\}\}$$
$$= \min\{10, \min\{7,10,5,7,10,14,21\}\}$$
$$= 5$$

$i = 4, j = 4$

$$\hat{D}_{44} = \min\{d_{44}, \min\{d_{41}, d_{43}, d_{46}, d_{48}, d_{49}, d_{4\,10}, d_{4\,12}\}\}$$
$$= \min\{0, \min\{7,10,5,7,10,14,21\}\}$$
$$= 0$$

$i = 4, j = 5$

$$\hat{D}_{45} = \min\{d_{45}, \min\{d_{41}, d_{43}, d_{46}, d_{48}, d_{49}, d_{4\,10}, d_{4\,12}\}\}$$
$$= \min\{3, \min\{7,10,5,7,10,14,21\}\}$$
$$= 3$$

$i = 4, j = 6$

$$\hat{D}_{46} = \min\{d_{46}, \min\{d_{41}, d_{43}, d_{46}, d_{48}, d_{49}, d_{4\,10}, d_{4\,12}\}\}$$

$= \min\{5, \min\{7,10,5,7,10,14,21\}\}$

$= 5$

$i = 4, j = 7$

$\hat{D}_{47} = \min\{d_{47}, \min\{d_{41}, d_{43}, d_{46}, d_{48}, d_{49}, d_{4\,10}, d_{4\,12}\}\}$

$= \min\{3, \min\{7,10,5,7,10,14,21\}\}$

$= 3$

$i = 4, j = 8$

$\hat{D}_{48} = \min\{d_{48}, \min\{d_{41}, d_{43}, d_{46}, d_{48}, d_{49}, d_{4\,10}, d_{4\,12}\}\}$

$= \min\{7, \min\{7,10,5,7,10,14,21\}\}$

$= 5$

$i = 4, j = 9$

$\hat{D}_{49} = \min\{d_{49}, \min\{d_{41}, d_{43}, d_{46}, d_{48}, d_{49}, d_{4\,10}, d_{4\,12}\}\}$

$= \min\{10, \min\{7,10,5,7,10,14,21\}\}$

$= 5$

$i = 4, j = 10$

$$\hat{D}_{4\,10} = \min\{(d_{4\,10}, d_{41}, d_{43}, d_{46}, d_{48}, d_{49}\, d_{4\,10}, d_{4\,12}\}\}$$
$$= \min\{14, \min\{7,10,5,7,10,14,21\}\}$$
$$= 5$$

$i = 4, j = 11$

$$\hat{D}_{4\,11} = \min\{(d_{4\,11}, d_{41}, d_{43}, d_{46}, d_{48}, d_{49}\, d_{4\,10}, d_{4\,12}\}\}$$
$$= \min\{19, \min\{7,10,5,7,10,14,21\}\}$$
$$= 5$$

$i = 4, j = 12$

$$\hat{D}_{4\,12} = \min\{(d_{4\,12}, d_{41}, d_{43}, d_{46}, d_{48}, d_{49}\, d_{4\,10}, d_{4\,12}\}\}$$
$$= \min\{21, \min\{7,10,5,7,10,14,21\}\}$$
$$= 5$$

$i = 4, j = 13$

$$\hat{D}_{4\,13} = \min\{(d_{4\,13}, d_{41}, d_{43}, d_{46}, d_{48}, d_{49}\, d_{4\,10}, d_{4\,12}\}\}$$
$$= \min\{14, \min\{7,10,5,7,10,14,21\}\}$$
$$= 5$$

$i = 4, j = 14$

$$\hat{D}_{4\,14} = \min\{(d_{4\,14}, d_{41}, d_{43}, d_{46}, d_{48}, d_{49}\, d_{4\,10}, d_{4\,12}\}\}$$
$$= \min\{12, \min\{7,10,5,7,10,14,21\}\}$$
$$= 5$$

$i = 4, j = 15$

$$\hat{D}_{4\,15} = \min\{(d_{4\,15}, d_{41}, d_{43}, d_{46}, d_{48}, d_{49}\, d_{4\,10}, d_{4\,12}\}\}$$
$$= \min\{23, \min\{7,10,5,7,10,14,21\}\}$$

i □ j

$\hat{D}$ 10, 12}}

□ 5

i □ 4, j □16

$\hat{D}_{4\ 16}$ □ min{($d_{4\ 16}$,$d\ d\ d\ d\ d_{41}$, 43, 46, 48, 49 $d_{4\ 10}$, $d_{4\ 12}$}}

□ min{14,min{7,10,5,7,10,14,21}}

□ 5

i □ 4, j □17

$\hat{D}_{4\ 17}$ □ min{($d_{4\ 17}$,d_{41},d_{43},d_{46},d_{48},d_{49} $d_{4\ 10}$, $d_{4\ 12}$}}

□ min{17,min{7,10,5,7,10,14,21}}

□ 5

4, □18

4 18 □ min{($d_{4\ 18}$,d_{41},d_{43},d_{46},d_{48},d_{49} d_4 d_4

□ min{12,min{7,10,5,7,10,14,21}}

□ 5

i □ 4, j □19

$\hat{D}_{4\ 19}$ □ min{($d_{4\ 19}$,$d\ d\ d\ d\ d_{41}$, 43, 46, 48, 49 $d_{4\ 10}$, $d_{4\ 12}$}}

□ min{10,min{7,10,5,7,10,14,21}}

□ 5

For Node 5 i

□5, j □1

$\hat{D}_{51}$ □ min{d_{51},min{$d\ d\ d\ d\ d_{51}$, 53, 56, 58, 59 ,$d_{5\ 10}$, $d_{5\ 12}$}}

□ min{10,min{10,9,2,6,9,13,23}}

□ 2

i □ 5, j □ 2

i □ j

$\hat{D}$ $_{10},$ $_{12}\}\}$

$\hat{D}_{52}$ □ $\min\{d_{52},\min\{d\ d\ d\ d\ d_{51},\ _{53},\ _{56},\ _{58},\ _{59}\ ,d_{5\ 10},\ d_{5\ 12}\}\}$

□ $\min\{6,\min\{10,9,2,6,9,13,23\}\}$

□ 2 i □

5, j □3

$\hat{D}_{53}$ □ $\min\{d_{53},\min\{d\ d\ d\ d\ d_{51},\ _{53},\ _{56},\ _{58},\ _{59}\ ,d_{5\ 10},\ d_{5\ 12}\}\}$

□ $\min\{9,\min\{10,9,2,6,9,13,23\}\}$

□ 2

i □ 5, j □ 4

$\hat{D}_{54}$ □ $\min\{d_{54},\min\{d_{51},d_{53},d_{56},d_{58},d_{59}\ ,d_{5\ 10},\ d_{5\ 12}\}\}$

□ $\min\{3,\min\{10,9,2,6,9,13,23\}\}$

□ 2

i □ 5, j □5

$\hat{D}_{55}$ □ $\min\{d_{55},\min\{d\ d\ d\ d\ d_{51},\ _{53},\ _{56},\ _{58},\ _{59}\ ,d_{5\ 10},\ d_{5\ 12}\}\}$

□ $\min\{0,\min\{10,9,2,6,9,13,23\}\}$

□ 0

i □ 5, j □ 6

$\hat{D}_{56}$ □ $\min\{d_{56},\min\{d\ d\ d\ d\ d_{51},\ _{53},\ _{56},\ _{58},\ _{59}\ ,d_{5\ 10},\ d_{5\ 12}\}\}$

□ $\min\{2,\min\{10,9,2,6,9,13,23\}\}$

□ 2

5, □ 7

$_{57}$ □ $\min\{d_{57},\min\{d\ d\ d\ d\ d_{51},\ _{53},\ _{56},\ _{58},\ _{59}\ ,d_{5}$ d_{5}

□ $\min\{2,\min\{10,9,2,6,9,13,23\}\}$

□ 2 i □

5, j □8

$i = j$

$\hat{D}$ $\quad {}_{10}, \quad {}_{12}\}\}$

$$\hat{D}_{58} = \min\{d_{58},\min\{d_{51},d_{53},d_{56},d_{58},d_{59},d_{5\,10}, d_{5\,12}\}\}$$
$$= \min\{6,\min\{10,9,2,6,9,13,23\}\}$$
$$= 2$$

$i = 5, j = 9$

$$\hat{D}_{59} = \min\{d_{59},\min\{d_{51}, d_{53}, d_{56}, d_{58}, d_{59}, d_{5\,10}, d_{5\,12}\}\}$$
$$= \min\{9,\min\{10,9,2,6,9,13,23\}\}$$
$$= 2$$

$i = 5, j = 10$

$$\hat{D}_{5\,10} = \min\{(d_{5\,10},\min\{ d_{51}, d_{53}, d_{56}, d_{58}, d_{59}\, d_{4\,10}, d_{4\,12}\}\}$$
$$= \min\{13,\min\{10,9,2,6,9,13,23\}\}$$
$$= 2$$

$i = 5, j = 11$

$$\hat{D}_{5\,11} = \min\{(d_{5\,11},\min\{ d_{51}, d_{53},d_{56},d_{58},d_{59}\, d_{4\,10}, d_{4\,12}\}\}$$
$$= \min\{18,\min\{10,9,2,6,9,13,23\}\}$$
$$= 2$$

$i = 5, j = 12$

$$\hat{D}_{5\,12} = \min\{(d_{5\,12},\min\{ d_{51}, d_{53}, d_{56}, d_{58}, d_{59}\, d_{4\,10}, d_{4\,12}\}\}$$
$$= \min\{23,\min\{10,9,2,6,9,13,23\}\}$$
$$= 2$$

$i = 5, j = 13$

$$\hat{D}_{5\,13} = \min\{(d_{5\,13},\min\{ d_{51}, d_{53}, d_{56}, d_{58}, d_{59}\, d_{4\,10}, d_{4\,12}\}\}$$
$$= \min\{16,\min\{10,9,2,6,9,13,23\}\}$$
$$= 2$$

$i \neq j$

$\hat{D}$ $_{10},$ $_{12}\}\}$

$i = 5,\ j = 14$

$$\hat{D}_{5\,14} = \min\{(d_{5\,14}, \min\{d\ d_{51}, d_{53}, d_{56}, d_{58}, d_{59}\ d_{4\,10}, d_{4\,12}\}\}$$

$$= \min\{14, \min\{10,9,2,6,9,13,23\}\}$$

$$= 2$$

$i = 5,\ j = 15$

$\hat{D}_{5\,15} = \min\{(d_{5\,15}, \min\{d_{51}, d_{53}, d_{56}, d_{58}, d_{59}, d_{4\,10}, d_{4\,12}\}\}$

$= \min\{24,\min\{10,9,2,6,9,13,23\}\}$

$= 2$

$i = 5,\ j = 16$

$\hat{D}_{5\,16} = \min\{(d_{5\,16}, \min\{d_{51}, d_{53}, d_{56}, d_{58}, d_{59}, d_{4\,10}, d_{4\,12}\}\}$

$= \min\{17,\min\{10,9,2,6,9,13,23\}\}$

$= 2$

$i = 5,\ j = 17$

$\hat{D}_{5\,17} = \min\{(d_{5\,17}, \min\{d_{51}, d_{53}, d_{56}, d_{58}, d_{59}, d_{4\,10}, d_{4\,12}\}\}$

$= \min\{18,\min\{10,9,2,6,9,13,23\}\}$

$= 2$

$i = 5,\ j = 18$

$\hat{D}_{5\,18} = \min\{(d_{5\,18}, \min\{d_{51}, d_{53}, d_{56}, d_{58}, d_{59}, d_{4\,10}, d_{4\,12}\}\}$

$= \min\{12,\min\{10,9,2,6,9,13,23\}\}$

$= 2$

$i = 5,\ j = 19$

$\hat{D}_{5\,19} = \min\{(d_{5\,19}, \min\{d_{51}, d_{53}, d_{56}, d_{58}, d_{59}, d_{4\,10}, d_{4\,12}\}\}$

$= \min\{10,\min\{10,9,2,6,9,13,23\}\}$

$= 2$

For Node 6 $i = 6,\ j = 1$

$\hat{D}_{61} = \min\{d_{61}, \min\{d_{61}, d_{63}, d_{66}, d_{68}, d_{69}, d_{6\,10}, d_{6\,12}\}\}$

$= \min\{12,\min\{12,9,0,6,9,13,23\}\}$

$= 0$

$i = 6,\ j = 2$

$\hat{D}_{62} = \min\{d_{62}, \min\{d_{61}, d_{63}, d_{66}, d_{68}, d_{69}, d_{6\,10}, d_{6\,12}\}\}$

$= \min\{12,\min\{12,9,0,6,9,13,23\}\}$

$= 0$

$i = 6\ j = 3$

$\hat{D}_{63} = \min\{d_{63},\min\{d_{61}, d_{63}, d_{66}, d_{68}, d_{69}, d_{6\,10}, d_{6\,12}\}\}$

$= \min\{9,\min\{12,9,0,6,9,13,23\}\}$

$= 0$

$i = 6\ j = 4$

$\hat{D}_{64} = \min\{d_{64},\min\{d_{61},d_{63},d_{66},d_{68},d_{69}, d_{6\,10}, d_{6\,12}\}\}$

$= \min\{5,\min\{12,9,0,6,9,13,23\}\}$

$= 0$

$i = 6\ j = 5$

$\hat{D}_{65} = \min\{d_{65},\min\{d_{61}, d_{63}, d_{66}, d_{68}, d_{69}, d_{6\,10}, d_{6\,12}\}\}$

$= \min\{2,\min\{12,9,0,6,9,13,23\}\}$

$= 0$

$i = 6\ j = 6$

$\hat{D}_{66} = \min\{d_{66},\min\{d_{61}, d_{63}, d_{66}, d_{68}, d_{69}, d_{6\,10}, d_{6\,12}\}\}$

$= \min\{0,\min\{12,9,0,6,9,13,23\}\}$

$= 0$

$i = 6\ j = 7$

$\hat{D}_{67} = \min\{d_{67},\min\{d_{61}, d_{63}, d_{66}, d_{68}, d_{69}, d_{6\,10}, d_{6\,12}\}\}$

$= \min\{2,\min\{12,9,0,6,9,13,23\}\}$

$= 0$

$i = 6\ j = 8$

$\hat{D}_{68} = \min\{d_{68},\min\{d_{61}, d_{63}, d_{66}, d_{68}, d_{69}, d_{6\,10}, d_{6\,12}\}\}$

$= \min\{6,\min\{12,9,0,6,9,13,23\}\}$

$= 0$

$i = 6\ j = 9$

$\hat{D}_{69} = \min\{d_{69}, \min\{d_{61}, d_{63}, d_{66}, d_{68}, d_{69}, d_{6\ 10}, d_{6\ 12}\}\}$

$= \min\{9, \min\{12,9,0,6,9,13,23\}\}$

$= 0$

$i = 6\ j = 10$

$\hat{D}_{6\ 10} = \min\{(d_{6\ 10}, \min\{d_{61}, d_{63}, d_{66}, d_{68}, d_{69}\, d_{6\ 10}, d_{6\ 12}\}\}$

$= \min\{13, \min\{12,9,0,6,9,13,23\}\}$

$= 0$

$i = 6,\ j = 11$

$\hat{D}_{6\ 11} = \min\{(d_{6\ 11}, \min\{d_{61}, d_{63}, d_{66}, d_{68}, d_{69}\, d_{6\ 10}, d_{6\ 12}\}\}$

$= \min\{18, \min\{12,9,0,6,9,13,23\}\}$

$= 0$

$i = 6,\ j = 12$

$\hat{D}_{6\ 12} = \min\{(d_{6\ 12}, \min\{d_{61}, d_{63}, d_{66}, d_{68}, d_{69}\, d_{6\ 10}, d_{6\ 12}\}\}$

$= \min\{23, \min\{12,9,0,6,9,13,23\}\}$

$= 0$

$i = 6,\ j = 13$

$\hat{D}_{6\ 13} = \min\{(d_{6\ 13}, \min\{d_{61}, d_{63}, d_{66}, d_{68}, d_{69}\, d_{6\ 10}, d_{6\ 12}\}\}$

$= \min\{16, \min\{12,9,0,6,9,13,23\}\}$

$= 0$

$i = 6,\ j = 14$

$\hat{D}_{6\ 14} = \min\{(d_{6\ 14}, \min\{d_{61}, d_{63}, d_{66}, d_{68}, d_{69}\, d_{6\ 10}, d_{6\ 12}\}\}$

$= \min\{14, \min\{12,9,0,6,9,13,23\}\}$

$= 0$

$i = 6,\ j = 15$

$\hat{D}_{6\ 15} = \min\{(d_{6\ 15}, \min\{d_{61}, d_{63}, d_{66}, d_{68}, d_{69}\, d_{6\ 10}, d_{6\ 12}\}\}$

$= \min\{25, \min\{12,9,0,6,9,13,23\}\}$

$= 0$

$i = 6, j = 16$

$\hat{D}_{6\,16} = \min\{(d_{6\,16}, \min\{d_{61}, d_{63}, d_{66}, d_{68}, d_{69}\, d_{6\,10}, d_{6\,12}\}\}$

$= \min\{19, \min\{12,9,0,6,9,13,23\}\}$
$= 0$

$i = 6, j = 17$

$\hat{D}_{6\,17} = \min\{(d_{6\,17}, \min\{d_{61}, d_{63}, d_{66}, d_{68}, d_{69}\, d_{6\,10}, d_{6\,12}\}\}$
$= \min\{20, \min\{12,9,0,6,9,13,23\}\}$

$= 0$

$i = 6, j = 18$

$\hat{D}_{6\,18} = \min\{(d_{6\,18}, \min\{d_{61}, d_{63}, d_{66}, d_{68}, d_{69}\, d_{6\,10}, d_{6\,12}\}\}$

$= \min\{14, \min\{12,9,0,6,9,13,23\}\}$
$= 0$

$i = 6, j = 19$

$\hat{D}_{6\,19} = \min\{(d_{6\,19}, \min\{d_{61}, d_{63}, d_{66}, d_{68}, d_{69}\, d_{6\,10}, d_{6\,12}\}\}$

$= \min\{12, \min\{12,9,0,6,9,13,23\}\}$
$= 0$

For Node 7 i
$= 7, j = 1$

$\hat{D}_{71} = \min\{d_{71}, \min\{d_{71}, d_{73}, d_{76}, d_{78}, d_{79}, d_{7\,10}, d_{7\,12}\}\}$

$= \min\{10, \min\{10,7,2,4,7,11,21\}\}$
$= 2$ $i = 7,$
$j = 2$

$\hat{D}_{72} = \min\{d_{72}, \min\{d_{71}, d_{73}, d_{76}, d_{78}, d_{79}, d_{7\,10}, d_{7\,12}\}\}$

$= \min\{8, \min\{10,7,2,4,7,11,21\}\}$
$= 2$

$i = 7, j = 3$

$\hat{D}_{73} = \min\{d_{73}, \min\{d_{71}, d_{73}, d_{76}, d_{78}, d_{79}, d_{7\,10}, d_{7\,12}\}\}$

$= \min\{10, \min\{10,7,2,4,7,11,21\}\}$

$= 2$

$i = 7, j = 4$

$\hat{D}_{74} = \min\{d_{74}, \min\{d_{71}, d_{73}, d_{76}, d_{78}, d_{79}, d_{7\,10}, d_{7\,12}\}\}$

$= \min\{3, \min\{10,7,2,4,7,11,21\}\}$

$= 2$

$i = 7, j = 5$

$\hat{D}_{75} = \min\{d_{75}, \min\{d_{71}, d_{73}, d_{76}, d_{78}, d_{79}, d_{7\,10}, d_{7\,12}\}\}$

$= \min\{2, \min\{10,7,2,4,7,11,21\}\}$

$= 2$

$i = 7, j = 6$

$\hat{D}_{76} = \min\{d_{76}, \min\{d_{71}, d_{73}, d_{76}, d_{78}, d_{79}, d_{7\,10}, d_{7\,12}\}\}$

$= \min\{2, \min\{10,7,2,4,7,11,21\}\}$

$= 2$

$i = 7, j = 7$

$\hat{D}_{77} = \min\{d_{77}, \min\{d_{71}, d_{73}, d_{76}, d_{78}, d_{79}, d_{7\,10}, d_{7\,12}\}\}$

$= \min\{0, \min\{10,7,2,4,7,11,21\}\}$

$= 2$

$i = 7, j = 8$

$\hat{D}_{78} = \min\{d_{78}, \min\{d_{71}, d_{73}, d_{76}, d_{78}, d_{79}, d_{7\,10}, d_{7\,12}\}\}$

$= \min\{4, \min\{10,7,2,4,7,11,21\}\}$

$= 2$

$i = 7, j = 9$

$\hat{D}_{79} = \min\{d_{79}, \min\{d_{71}, d_{73}, d_{76}, d_{78}, d_{79}, d_{7\,10}, d_{7\,12}\}\}$

$= \min\{7, \min\{10,7,2,4,7,11,21\}\}$

$= 2$ $i =$

$7, j = 10$

$$\hat{D}_{7\,10} = \min\{(d_{7\,10}, \min\{d_{71}, d_{73}, d_{76}, d_{78}, d_{79}\, d_{7\,10}, d_{7\,12}\}\}$$
$$= \min\{11, \min\{10,7,2,4,7,11,21\}\}$$
$$= 2$$

$i = 7, j = 12$

$$\hat{D}_{7\,12} = \min\{(d_{7\,12}, \min\{d_{71}, d_{73}, d_{76}, d_{78}, d_{79}\, d_{7\,10}, d_{7\,12}\}\} =$$
$$\min\{21, \min\{10,7,2,4,7,11,21\}\}$$
$$= 2$$

$i = 7, j = 13$

$$\hat{D}_{7\,13} = \min\{(d_{7\,13}, \min\{d_{71}, d_{73}, d_{76}, d_{78}, d_{79}\, d_{7\,10}, d_{7\,12}\}\}$$
$$= \min\{14, \min\{10,7,2,4,7,11,21\}\}$$
$$= 2$$

$i = 7, j = 14$

$$\hat{D}_{7\,14} = \min\{(d_{7\,14}, \min\{d_{71}, d_{73}, d_{76}, d_{78}, d_{79}\, d_{7\,10}, d_{7\,12}\}\}$$
$$= \min\{12, \min\{10,7,2,4,7,11,21\}\}$$
$$= 2$$

$i = 7, j = 15$

$$\hat{D}_{7\,15} = \min\{(d_{7\,15}, \min\{d_{71}, d_{73}, d_{76}, d_{78}, d_{79}\, d_{7\,10}, d_{7\,12}\}\} =$$
$$\min\{23, \min\{10,7,2,4,7,11,21\}\}$$
$$= 2$$

$i = 7, j = 16$

$$\hat{D}_{7\,16} = \min\{(d_{7\,16}, \min\{d_{71}, d_{73}, d_{76}, d_{78}, d_{79}\, d_{7\,10}, d_{7\,12}\}\} =$$
$$\min\{17, \min\{10,7,2,4,7,11,21\}\}$$
$$= 2$$

$i = 7, j = 17$

$$\hat{D}_{7\,17} = \min\{(d_{7\,17}, \min\{d_{71}, d_{73}, d_{76}, d_{78}, d_{79}\, d_{7\,10}, d_{7\,12}\}\}$$
$$= \min\{20, \min\{10,7,2,4,7,11,21\}\}$$
$$= 2$$

$i = 7, j = 18$

$$\hat{D}_{7\,18} = \min\{(d_{7\,18}, \min\{d_{71}, d_{73}, d_{76}, d_{78}, d_{79}\, d_{7\,10}, d_{7\,12}\}\}$$
$$= \min\{14, \min\{10,7,2,4,7,11,21\}\}$$

$= 2$

$i = 7, j = 19$

$\hat{D}_{7\,19} = \min\{(d_{7\,19}, \min\{d_{71}, d_{73}, d_{76}, d_{78}, d_{79}, d_{7\,10}, d_{7\,12}\}\}$

$= \min\{12, \min\{10,7,2,4,7,11,21\}\}$

$= 2$

For Node 8 $i = 8, j = 1$

$\hat{D}_{81} = \min\{d_{81}, \min\{d_{81}, d_{83}, d_{86}, d_{88}, d_{89}, d_{8\,10}, d_{8\,12}\}\}$

$= \min\{10, \min\{10,3,6,0,3,7,17\}\}$

$= 0$

$i = 8, j = 2$

$\hat{D}_{82} = \min\{d_{82}, \min\{d_{81}, d_{83}, d_{86}, d_{88}, d_{89}, d_{8\,10}, d_{8\,12}\}\}$

$= \min\{10, \min\{10,3,6,0,3,7,17\}\}$

$= 0$

$i = 8, j = 3$

$\hat{D}_{83} = \min\{d_{83}, \min\{d_{81}, d_{83}, d_{86}, d_{88}, d_{89}, d_{8\,10}, d_{8\,12}\}\}$

$= \min\{3, \min\{10,3,6,0,3,7,17\}\}$

$= 0$

$i = 8, j = 4$

$\hat{D}_{84} = \min\{d_{84}, \min\{d_{81}, d_{83}, d_{86}, d_{88}, d_{89}, d_{8\,10}, d_{8\,12}\}\}$

$= \min\{7, \min\{10,3,6,0,3,7,17\}\}$

$= 0$

$i = 8, j = 5$

$\hat{D}_{85} = \min\{d_{85}, \min\{d_{81}, d_{83}, d_{86}, d_{88}, d_{89}, d_{8\,10}, d_{8\,12}\}\}$

$= \min\{6, \min\{10,3,6,0,3,7,17\}\}$

$= 0$

$i = 8, j = 6$

$$\hat{D}_{86} = \min\{d_{86}, \min\{d_{81}, d_{83}, d_{86}, d_{88}, d_{89}, d_{8\,10}, d_{8\,12}\}\}$$

$$= \min\{6, \min\{10,3,6,0,3,7,17\}\}$$

$$= 0$$

$i = 8, j = 7$

$$\hat{D}_{87} = \min\{d_{87}, \min\{d_{81}, d_{83}, d_{86}, d_{88}, d_{89}, d_{8\,10}, d_{8\,12}\}\}$$

$$= \min\{4, \min\{10,3,6,0,3,7,17\}\}$$

$$= 0$$

$i = 8, j = 8$

$$\hat{D}_{88} = \min\{d_{88}, \min\{d_{81}, d_{83}, d_{86}, d_{88}, d_{89}, d_{8\,10}, d_{8\,12}\}\}$$

$$= \min\{0, \min\{10,3,6,0,3,7,17\}\}$$

$$= 0$$

$i = 8, j = 9$

$$\hat{D}_{89} = \min\{d_{89}, \min\{d_{81}, d_{83}, d_{86}, d_{88}, d_{89}, d_{8\,10}, d_{8\,12}\}\}$$

$$= \min\{3, \min\{10,3,6,0,3,7,17\}\}$$

$$= 0$$

$i = 8, j = 10$

$$\hat{D}_{8\,10} = \min\{(d_{8\,10}, \min\{d_{81}, d_{83}, d_{86}, d_{88}, d_{89}, d_{8\,10}, d_{8\,12}\}\}$$

$$= \min\{7, \min\{10,3,6,0,3,7,17\}\}$$

$$= 0$$

$i = 8, j = 11$

$$\hat{D}_{8\,11} = \min\{(d_{8\,11}, \min\{d_{81}, d_{83}, d_{86}, d_{88}, d_{89}, d_{8\,10}, d_{8\,12}\}\}$$

$$= \min\{12, \min\{10,3,6,0,3,7,17\}\}$$

$$= 0$$

$i = 8, j = 12$

$$\hat{D}_{8\,12} = \min\{(d_{8\,12}, \min\{d_{81}, d_{83}, d_{86}, d_{88}, d_{89}, d_{8\,10}, d_{8\,12}\}\}$$
$$= \min\{17, \min\{10,3,6,0,3,7,17\}\}$$
$$= 0$$

$i = 8,\ j = 13$

$$\hat{D}_{8\,13} = \min\{(d_{8\,13}, \min\{d_{81}, d_{83}, d_{86}, d_{88}, d_{89}, d_{8\,10}, d_{8\,12}\}\} = \min\{10, \min\{10,3,6,0,3,7,17\}\}$$
$$= 0$$

$i = 8,\ j = 14$

$$\hat{D}_{8\,14} = \min\{(d_{8\,14}, \min\{d_{81}, d_{83}, d_{86}, d_{88}, d_{89}, d_{8\,10}, d_{8\,12}\}\} = \min\{8, \min\{10,3,6,0,3,7,17\}\}$$
$$= 0$$

$i = 8,\ j = 15$

$$\hat{D}_{8\,15} = \min\{(d_{8\,15}, \min\{d_{81}, d_{83}, d_{86}, d_{88}, d_{89}, d_{8\,10}, d_{8\,12}\}\} = \min\{19, \min\{10,3,6,0,3,7,17\}\}$$
$$= 0$$

$i = 8,\ j = 16$

$$\hat{D}_{8\,16} = \min\{(d_{8\,16}, \min\{d_{81}, d_{83}, d_{86}, d_{88}, d_{89}, d_{8\,10}, d_{8\,12}\}\}$$
$$= \min\{13, \min\{10,3,6,0,3,7,17\}\}$$

$$= 0$$

$i = 8,\ j = 17$

$$\hat{D}_{8\,17} = \min\{(d_{8\,17}, \min\{d_{81}, d_{83}, d_{86}, d_{88}, d_{89}, d_{8\,10}, d_{8\,12}\}\} = \min\{16, \min\{10,3,6,0,3,7,17\}\}$$
$$= 0$$

$i = 8,\ j = 18$

$$\hat{D}_{8\,18} = \min\{(d_{8\,18}, \min\{d_{81}, d_{83}, d_{86}, d_{88}, d_{89}, d_{8\,10}, d_{8\,12}\}\}$$
$$= \min\{15, \min\{10,3,6,0,3,7,17\}\}$$

$$= 0$$

$i = 8,\ j = 19$

$\hat{D}_{8\,19} = \min\{(d_{8\,19}, \min\{d_{81}, d_{83}, d_{86}, d_{88}, d_{89}, d_{8\,10}, d_{8\,12}\}\}$

$= \min\{13,\min\{10,3,6,0,3,7,17\}\}$

$= 0$

For Node 9 $i = 9, j = 1$

$\hat{D}_{91} = \min\{d_{91},\min\{d_{91}, d_{93}, d_{96}, d_{98}, d_{99}, d_{9\,10}, d_{9\,12}\}\}$

$= \min\{10,\min\{10,6,9,3,0,4,14\}\}$

$= 0$

$i = 9, j = 2$

$\hat{D}_{92} = \min\{d_{92},\min\{d_{91}, d_{93}, d_{96}, d_{98}, d_{99}, d_{9\,10}, d_{9\,12}\}\}$

$= \min\{15,\min\{10,6,9,3,0,4,14\}\}$

$= 0$

$i = 9, j = 3$

$\hat{D}_{93} = \min\{d_{93},\min\{d_{91}, d_{93}, d_{96}, d_{98}, d_{99}, d_{7\,10}, d_{9\,12}\}\}$

$= \min\{6,\min\{10,6,9,3,0,4,14\}\}$

$= 0$

$i = 9, j = 4$

$\hat{D}_{94} = \min\{d_{94},\min\{d_{91}, d_{93}, d_{96}, d_{98}, d_{99}, d_{9\,10}, d_{9\,12}\}\}$

$= \min\{10,\min\{10,6,9,3,0,4,14\}\}$

$= 0$

$i = 9, j = 5$

$\hat{D}_{95} = \min\{d_{95},\min\{d_{91}, d_{93}, d_{96}, d_{98}, d_{99}, d_{9\,10}, d_{9\,12}\}\}$

$= \min\{9,\min\{10,6,9,3,0,4,14\}\}$

$= 0$

$i = 9, j = 6$

$$\hat{D}_{96} = \min\{d_{96}, \min\{d_{91}, d_{93}, d_{96}, d_{98}, d_{99}, d_{9\,10}, d_{9\,12}\}\}$$
$$= \min\{9, \min\{10,6,9,3,0,4,14\}\}$$
$$= 0$$

$i = 9, j = 7$

$$\hat{D}_{97} = \min\{d_{97}, \min\{d_{91}, d_{93}, d_{96}, d_{98}, d_{99}, d_{9\,10}, d_{9\,12}\}\}$$
$$= \min\{10, \min\{10,6,9,3,0,4,14\}\}$$
$$= 0$$

$i = 9, j = 8$

$$\hat{D}_{98} = \min\{d_{98}, \min\{d_{91}, d_{93}, d_{96}, d_{98}, d_{99}, d_{9\,10}, d_{9\,12}\}\}$$
$$= \min\{3, \min\{10,6,9,3,0,4,14\}\}$$
$$= 0$$

$i = 9, j = 9$

$$\hat{D}_{99} = \min\{d_{99}, \min\{d_{91}, d_{93}, d_{96}, d_{98}, d_{99}, d_{9\,10}, d_{9\,12}\}\}$$
$$= \min\{0, \min\{10,6,9,3,0,4,14\}\}$$
$$= 0$$

$i = 9, j = 10$

$$\hat{D}_{9\,10} = \min\{(d_{9\,10}, \min\{d_{91}, d_{93}, d_{96}, d_{98}, d_{99}\, d_{9\,10}, d_{9\,12}\}\}$$
$$= \min\{4, \min\{10,6,9,3,0,4,14\}\}$$
$$= 0$$

$i = 9, j = 11$

$$\hat{D}_{9\,11} = \min\{(d_{9\,11}, \min\{d_{91}, d_{93}, d_{96}, d_{98}, d_{99}\, d_{9\,10}, d_{9\,12}\}\}$$
$$= \min\{9, \min\{10,6,9,3,0,4,14\}\}$$

$= 0$

$i = 9, j = 12$

$$\hat{D}_{9\,12} = \min\{(d_{9\,12}, \min\{d_{91}, d_{93}, d_{96}, d_{98}, d_{99}, d_{9\,10}, d_{9\,12}\}\}$$
$$= \min\{14, \min\{10,6,9,3,0,4,14\}\}$$

$= 0$

$i = 9, j = 13$

$$\hat{D}_{9\,13} = \min\{(d_{9\,13}, \min\{d_{91}, d_{93}, d_{96}, d_{98}, d_{99}, d_{9\,10}, d_{9\,12}\}\}$$
$$= \min\{4, \min\{10,6,9,3,0,4,14\}\}$$

$= 0$

$i = 9, j = 14$

$$\hat{D}_{9\,14} = \min\{(d_{9\,14}, \min\{d_{91}, d_{93}, d_{96}, d_{98}, d_{99}, d_{9\,10}, d_{9\,12}\}\}$$
$$= \min\{5, \min\{10,6,9,3,0,4,14\}\}$$

$= 0$

$i = 9, j = 15$

$$\hat{D}_{9\,15} = \min\{(d_{9\,15}, \min\{d_{91}, d_{93}, d_{96}, d_{98}, d_{99}, d_{9\,10}, d_{9\,12}\}\}$$
$$= \min\{16, \min\{10,6,9,3,0,4,14\}\}$$

$= 0$

$i = 9, j = 16$

$$\hat{D}_{9\,16} = \min\{(d_{9\,16}, \min\{d_{91}, d_{93}, d_{96}, d_{98}, d_{99}, d_{9\,10}, d_{9\,12}\}\}$$
$$= \min\{10, \min\{10,6,9,3,0,4,14\}\}$$

$= 0$

$i = 9, j = 17$

$\hat{D}_{9\,17} = \min\{(d_{9\,17}, \min\{d_{91}, d_{93}, d_{96}, d_{98}, d_{99}, d_{9\,10}, d_{9\,12}\}\}$

$= \min\{13, \min\{10,6,9,3,0,4,14\}\}$

$= 0$

$i = 9, j = 18$

$\hat{D}_{9\,18} = \min\{(d_{9\,18}, \min\{d_{91}, d_{93}, d_{96}, d_{98}, d_{99}, d_{9\,10}, d_{9\,12}\}\}$

$= \min\{15, \min\{10,6,9,3,0,4,14\}\}$

$= 0$

$i = 9, j = 19$

$\hat{D}_{9\,19} = \min\{(d_{9\,13}, \min\{d_{91}, d_{93}, d_{96}, d_{98}, d_{99}, d_{9\,10}, d_{9\,12}\}\}$

$= \min\{13, \min\{10,6,9,3,0,4,14\}\}$

$= 0$

For Node 10

$i = 10, j = 1$

$\hat{D}_{10\,1} = \min\{(d_{10\,1}, \min\{d_{10\,1}, d_{10\,3}, d_{10\,6}, d_{10\,8}, d_{10\,9}, d_{10\,10}, d_{10\,12}\}\}$

$= \min\{12, \min\{12,10,13,7,4,0,10\}\}$

$= 0$

$i = 10, j = 2$

$\hat{D}_{10\,2} = \min\{(d_{10\,2}, \min\{d_{10\,1}, d_{10\,3}, d_{10\,6}, d_{10\,8}, d_{10\,9}, d_{10\,10}, d_{10\,12}\}\}$

$= \min\{17, \min\{12,10,13,7,4,0,10\}\}$

$= 0$

$i = 10, j = 2$

$\hat{D}_{10\,3} = \min\{(d_{10\,3}, \min\{d_{10\,1}, d_{10\,3}, d_{10\,6}, d_{10\,8}, d_{10\,9}, d_{10\,10}, d_{10\,12}\}\} =$
$\min\{10, \min\{12,10,13,7,4,0,10\}\}$

$= 0$

$i = 10, j = 4$

$\hat{D}_{10\,4} = \min\{(d_{10\,4}, \min\{d_{10\,1}, d_{10\,3}, d_{10\,6}, d_{10\,8}, d_{10\,9}, d_{10\,10}, d_{10\,12}\}\}$

$= \min\{14, \min\{12,10,13,7,4,0,10\}\}$

$= 0$

$i = 10, j = 5$

$\hat{D}_{10\,5} = \min\{(d_{10\,5}, \min\{d_{10\,1}, d_{10\,3}, d_{10\,6}, d_{10\,8}, d_{10\,9}, d_{10\,10}, d_{10\,12}\}\} = \min\{13, \min\{12,10,13,7,4,0,10\}\}$

$= 0$

$i = 10, j = 6$

$\hat{D}_{10\,6} = \min\{(d_{10\,6}, \min\{d_{10\,1}, d_{10\,3}, d_{10\,6}, d_{10\,8}, d_{10\,9}, d_{10\,10}, d_{10\,12}\}\} = \min\{13, \min\{12,10,13,7,4,0,10\}\}$

$= 0$

$i = 10, j = 7$

$\hat{D}_{10\,7} = \min\{(d_{10\,7}, \min\{d_{10\,1}, d_{10\,3}, d_{10\,6}, d_{10\,8}, d_{10\,9}, d_{10\,10}, d_{10\,12}\}\} = \min\{11, \min\{12,10,13,7,4,0,10\}\}$

$= 0$ $i = 10, j = 8$

$\hat{D}_{10\,8} = \min\{(d_{10\,8}, \min\{d_{10\,1}, d_{10\,3}, d_{10\,6}, d_{10\,8}, d_{10\,9}, d_{10\,10}, d_{10\,12}\}\}$

$= \min\{7, \min\{12,10,13,7,4,0,10\}\}$

$= 0$

$i = 10, j = 9$

$\hat{D}_{10\,9} = \min\{(d_{10\,9}, \min\{d_{10\,1}, d_{10\,3}, d_{10\,6}, d_{10\,8}, d_{10\,9}, d_{10\,10}, d_{10\,12}\}\}$

$= \min\{4, \min\{12,10,13,7,4,0,10\}\}$

$= 0$

$i = 10, j = 11$

$$\hat{D}_{10\ 11} = \min\{(d_{10\ 11}, \min\{d_{10\ 1}, d_{10\ 3}, d_{10\ 6}, d_{10\ 8}, d_{10\ 9}, d_{10\ 10}, d_{10\ 12}\}\}$$

$$= \min\{5, \min\{12,10,13,7,4,0,10\}\}$$
$$= 0$$

$i = 10,\ j = 12$

$$\hat{D}_{10\ 12} = \min\{(d_{10\ 12}, \min\{d_{10\ 1}, d_{10\ 3}, d_{10\ 6}, d_{10\ 8}, d_{10\ 9}, d_{10\ 10}, d_{10\ 12}\}\}$$

$$= \min\{10, \min\{12,10,13,7,4,0,10\}\}$$
$$= 0$$

$i = 10,\ j = 13$

$$\hat{D}_{10\ 13} = \min\{(d_{10\ 13}, \min\{d_{10\ 1}, d_{10\ 3}, d_{10\ 6}, d_{10\ 8}, d_{10\ 9}, d_{10\ 10}, d_{10\ 12}\}\}$$

$$= \min\{9, \min\{12,10,13,7,4,0,10\}\}$$
$$= 0$$

$i = 10,\ j = 14$

$$\hat{D}_{10\ 14} = \min\{(d_{10\ 14}, \min\{d_{10\ 1}, d_{10\ 3}, d_{10\ 6}, d_{10\ 8}, d_{10\ 9}, d_{10\ 10}, d_{10\ 12}\}\} = \min\{7, \min\{12,10,13,7,4,0,10\}\}$$
$$= 0$$

$i = 10,\ j = 15$

$$\hat{D}_{10\ 15} = \min\{(d_{10\ 15}, \min\{d_{10\ 1}, d_{10\ 3}, d_{10\ 6}, d_{10\ 8}, d_{10\ 9}, d_{10\ 10}, d_{10\ 12}\}\}$$
$$= \min\{12, \min\{12,10,13,7,4,0,10\}\}$$

$$= 0$$

$i = 10,\ j = 16$

$$\hat{D}_{10\ 16} = \min\{(d_{10\ 16}, \min\{d_{10\ 1}, d_{10\ 3}, d_{10\ 6}, d_{10\ 8}, d_{10\ 9}, d_{10\ 10}, d_{10\ 12}\}\}$$
$$= \min\{12, \min\{12,10,13,7,4,0,10\}\}$$

$$= 0$$

$i = 10, j = 17$

$\hat{D}_{10\,17} = \min\{(d_{10\,17}, \min\{d_{10\,1}, d_{10\,3}, d_{10\,6}, d_{10\,8}, d_{10\,9}, d_{10\,10}, d_{10\,12}\}\}$

$= \min\{15, \min\{12,10,13,7,4,0,10\}\}$

$= 0$

$i = 10, j = 18$

$\hat{D}_{10\,18} = \min\{(d_{10\,18}, \min\{d_{10\,1}, d_{10\,3}, d_{10\,6}, d_{10\,8}, d_{10\,9}, d_{10\,10}, d_{10\,12}\}\}$

$= \min\{17, \min\{12,10,13,7,4,0,10\}\}$

$= 0$

$i = 10, j = 19$

$\hat{D}_{10\,19} = \min\{(d_{10\,19}, \min\{d_{10\,1}, d_{10\,3}, d_{10\,6}, d_{10\,8}, d_{10\,9}, d_{10\,10}, d_{10\,12}\}\}$

$= \min\{15, \min\{12,10,13,7,4,0,10\}\}$

$= 0$

For Node 11

$i = 11, j = 1$

$\hat{D}_{11\,1} = \min\{(d_{11\,1}, \min\{d_{11\,1}, d_{11\,3}, d_{11\,6}, d_{11\,8}, d_{11\,9}, d_{11\,10}, d_{11\,12}\}\}$

$= \min\{17, \min\{17,15,18,12,9,5,5\}\}$

$= 5$

$i = 11, j = 2$

$\hat{D}_{11\,2} = \min\{(d_{11\,2}, \min\{d_{11\,1}, d_{11\,3}, d_{11\,6}, d_{11\,8}, d_{11\,9}, d_{11\,10}, d_{11\,12}\}\}$

$= \min\{19, \min\{17,15,18,12,9,5,5\}\}$

$= 5$

$i = 11, j = 3$

$$\hat{D}_{11\,3} = \min\{(d_{11\,3}, \min\{d_{11\,1}, d_{11\,3}, d_{11\,6}, d_{11\,8}, d_{11\,9}, d_{11\,10}, d_{11\,12}\}\}$$
$$= \min\{15, \min\{17,15,18,12,9,5,5\}\}$$

$$= 5$$

$i = 11, j = 4$

$$\hat{D}_{11\,4} = \min\{(d_{11\,4}, \min\{d_{11\,1}, d_{11\,3}, d_{11\,6}, d_{11\,8}, d_{11\,9}, d_{11\,10}, d_{11\,12}\}\}$$
$$= \min\{19, \min\{17,15,18,12,9,5,5\}\}$$

$$= 5$$

$i = 11, j = 5$

$$\hat{D}_{11\,5} = \min\{(d_{11\,5}, \min\{d_{11\,1}, d_{11\,3}, d_{11\,6}, d_{11\,8}, d_{11\,9}, d_{11\,10}, d_{11\,12}\}\} = \min\{18, \min\{17,15,18,12,9,5,5\}\}$$
$$= 5$$

$i = 11, j = 6$

$$\hat{D}_{11\,6} = \min\{(d_{11\,6}, \min\{d_{11\,1}, d_{11\,3}, d_{11\,6}, d_{11\,8}, d_{11\,9}, d_{11\,10}, d_{11\,12}\}\} = \min\{18, \min\{17,15,18,12,9,5,5\}\}$$
$$= 5$$

$i = 11, j = 7$

$$\hat{D}_{11\,7} = \min\{(d_{11\,7}, \min\{d_{11\,1}, d_{11\,3}, d_{11\,6}, d_{11\,8}, d_{11\,9}, d_{11\,10}, d_{11\,12}\}\} = \min\{16, \min\{17,15,18,12,9,5,5\}\}$$
$$= 5$$

$i = 11, j = 8$

$$\hat{D}_{11\,8} = \min\{(d_{11\,8}, \min\{d_{11\,1}, d_{11\,3}, d_{11\,6}, d_{11\,8}, d_{11\,9}, d_{11\,10}, d_{11\,12}\}\} = \min\{12, \min\{17,15,18,12,9,5,5\}\}$$
$$= 5$$

$i = 11, j = 9$

$$\hat{D}_{11\,9} = \min\{(d_{11\,9}, \min\{d_{11\,1}, d_{11\,3}, d_{11\,6}, d_{11\,8}, d_{11\,9}, d_{11\,10}, d_{11\,12}\}\}$$
$$= \min\{9, \min\{17,15,18,12,9,5,5\}\}$$

$$= 5$$

$i = 11, j = 10$

$$\hat{D}_{11\,10} = \min\{(d_{11\,10}, \min\{d_{11\,1}, d_{11\,3}, d_{11\,6}, d_{11\,8}, d_{11\,9}, d_{11\,10}, d_{11\,12}\}\}$$
$$= \min\{5, \min\{17,15,18,12,9,5,5\}\}$$

$$= 5$$

$i = 11, j = 11$

$$\hat{D}_{11\,11} = \min\{(d_{11\,11}, \min\{d_{11\,1}, d_{11\,3}, d_{11\,6}, d_{11\,8}, d_{11\,9}, d_{11\,10}, d_{11\,12}\}\}$$
$$= \min\{0, \min\{17,15,18,12,9,5,5\}\}$$

$$= 0$$

$i = 11,\ j = 12$

$$\hat{D}_{11\,12} = \min\{(d_{11\,12}, \min\{d_{11\,1}, d_{11\,3}, d_{11\,6}, d_{11\,8}, d_{11\,9}, d_{11\,10}, d_{11\,12}\}\} =$$
$$\min\{5, \min\{17,15,18,12,9,5,5\}\}$$
$$= 5$$

$i = 11,\ j = 13$

$$\hat{D}_{11\,13} = \min\{(d_{11\,13}, \min\{d_{11\,1}, d_{11\,3}, d_{11\,6}, d_{11\,8}, d_{11\,9}, d_{11\,10}, d_{11\,12}\}\} =$$
$$\min\{13, \min\{17,15,18,12,9,5,5\}\}$$
$$= 5$$

$i = 11,\ j = 14$

$$\hat{D}_{11\,14} = \min\{(d_{11\,14}, \min\{d_{11\,1}, d_{11\,3}, d_{11\,6}, d_{11\,8}, d_{11\,9}, d_{11\,10}, d_{11\,12}\}\} =$$
$$\min\{12, \min\{17,15,18,12,9,5,5\}\}$$
$$= 5$$

$i = 11,\ j = 15$

$$\hat{D}_{11\,15} = \min\{(d_{11\,15}, \min\{d_{11\,1}, d_{11\,3}, d_{11\,6}, d_{11\,8}, d_{11\,9}, d_{11\,10}, d_{11\,12}\}\} =$$
$$\min\{7, \min\{17,15,18,12,9,5,5\}\}$$
$$= 5$$

$i = 11,\ j = 16$

$$\hat{D}_{11\,16} = \min\{(d_{11\,16}, \min\{d_{11\,1}, d_{11\,3}, d_{11\,6}, d_{11\,8}, d_{11\,9}, d_{11\,10}, d_{11\,12}\}\}$$
$$= \min\{15, \min\{17,15,18,12,9,5,5\}\}$$

$$= 5$$

$i = 11,\ j = 17$

$$\hat{D}_{11\,17} = \min\{(d_{11\,17}, \min\{d_{11\,1}, d_{11\,3}, d_{11\,6}, d_{11\,8}, d_{11\,9}, d_{11\,10}, d_{11\,12}\}\}$$
$$= \min\{13, \min\{17,15,18,12,9,5,5\}\}$$

i j

$\hat{D} =$

$=$

$=$

$= 5$

$i = 11, j = 18$

$$\hat{D}_{11\,18} = \min\{(d_{11\,18}, \min\{d_{11\,1}, d_{11\,3}, d_{11\,6}, d_{11\,8}, d_{11\,9}, d_{11\,10}, d_{11\,12}\}\}$$
$$= \min\{19, \min\{17,15,18,12,9,5,5\}\}$$

$= 5$

$= 11, = 19$

$$_{11\,19} \quad \min\{(d_{11\,19}, \min\{d_{11\,1}, d_{11\,3}, d_{11\,6}, d_{11\,8}, d_{11\,9}, d_{11\,10}, d_{11\,12}\}\}$$
$$\min\{20, \min\{17,15,18,12,9,5,5\}\}$$

5

For Node 12

$i = 12, j = 1$

$$\hat{D}_{12\,1} = \min\{(d_{12\,1}, \min\{d_{12\,1}, d_{12\,3}, d_{12\,6}, d_{12\,8}, d_{12\,9}, d_{12\,10}, d_{12\,12}\}\} =$$
$$\min\{14, \min\{14,20,23,17,14,10,0\}\}$$
$$= 0$$

$i = 12, j = 2$

$$\hat{D}_{12\,2} = \min\{(d_{12\,2}, \min\{d_{12\,1}, d_{12\,3}, d_{12\,6}, d_{12\,8}, d_{12\,9}, d_{12\,10}, d_{12\,12}\}\} =$$
$$\min\{19, \min\{14,20,23,17,14,10,0\}\}$$
$$= 0$$

$i = 12, j = 3$

$i \quad j$

$\hat{D}$ =

$$\hat{D}_{12\,3} = \min\{(d_{12\,3}, \min\{d_{12\,1}, d_{12\,3}, d_{12\,6}, d_{12\,8}, d_{12\,9}, d_{12\,10}, d_{12\,12}\}\}$$
$$= \min\{20, \min\{14,20,23,17,14,10,0\}\}$$
$$= 0$$

$i = 12, j = 4$

$$\hat{D}_{12\,4} = \min\{(d_{12\,4}, \min\{d_{12\,1}, d_{12\,3}, d_{12\,6}, d_{12\,8}, d_{12\,9}, d_{12\,10}, d_{12\,12}\}\}$$
$$= \min\{21, \min\{14,20,23,17,14,10,0\}\}$$
$$= 0$$

$i = 12, j = 5$

$$\hat{D}_{12\,5} = \min\{(d_{12\,5}, \min\{d_{12\,1}, d_{12\,3}, d_{12\,6}, d_{12\,8}, d_{12\,9}, d_{12\,10}, d_{12\,12}\}\}$$
$$= \min\{23, \min\{14,20,23,17,14,10,0\}\}$$
$$= 0$$

$i = 12, j = 6$

$$\hat{D}_{12\,6} = \min\{(d_{12\,6}, \min\{d_{12\,1}, d_{12\,3}, d_{12\,6}, d_{12\,8}, d_{12\,9}, d_{12\,10}, d_{12\,12}\}\}$$
$$= \min\{23, \min\{14,20,23,17,14,10,0\}\}$$
$$= 0$$

$= 12, = 7$

$$\hat{D}_{12\,7} = \min\{(d_{12\,7}, \min\{d_{12\,1}, d_{12\,3}, d_{12\,6}, d_{12\,8}, d_{12\,9}, d_{12\,10}, d_{12\,12}\}\}$$
$$= \min\{21, \min\{14,20,23,17,14,10,0\}\}$$
$$= 0$$

$i = 12, j = 8$

$i \quad j$

$\hat{D}$ =

=

=

$\hat{D}_{12\,8} = \min\{(d_{12\,8}, \min\{d_{12\,1}, d_{12\,3}, d_{12\,6}, d_{12\,8}, d_{12\,9}, d_{12\,10}, d_{12\,12}\}\} =$
$\min\{17, \min\{14,20,23,17,14,10,0\}\}$
$= 0$

$i = 12, j = 9$

$\hat{D}_{12\,9} = \min\{(d_{12\,9}, \min\{d_{12\,1}, d_{12\,3}, d_{12\,6}, d_{12\,8}, d_{12\,9}, d_{12\,10}, d_{12\,12}\}\} =$
$\min\{14, \min\{14,20,23,17,14,10,0\}\}$
$= 0$

$i = 12, j = 10$

$\hat{D}_{12\,10} = \min\{(d_{12\,10}, \min\{d_{12\,1}, d_{12\,3}, d_{12\,6}, d_{12\,8}, d_{12\,9}, d_{12\,10}, d_{12\,12}\}\}$
$= \min\{10, \min\{14,20,23,17,14,10,0\}\}$

$= 0$

$i = 12, j = 11$

$\hat{D}_{12\,11} = \min\{(d_{12\,1}, \min\{d_{12\,11}, d_{12\,3}, d_{12\,6}, d_{12\,8}, d_{12\,9}, d_{12\,10}, d_{12\,12}\}\}$
$= \min\{5, \min\{14,20,23,17,14,10,0\}\}$

$= 0$

$i = 12, j = 12$

$\hat{D}_{12\,12} = \min\{(d_{12\,12}, \min\{d_{12\,1}, d_{12\,3}, d_{12\,6}, d_{12\,8}, d_{12\,9}, d_{12\,10}, d_{12\,12}\}\}$
$= \min\{0, \min\{14,20,23,17,14,10,0\}\}$

$= 0$

i j

$\hat{D} =$

$i = 12, j = 13$

$$\hat{D}_{12\,13} = \min\{(d_{12\,13}, \min\{d_{12\,1}, d_{12\,3}, d_{12\,6}, d_{12\,8}, d_{12\,9}, d_{12\,10}, d_{12\,12}\}\}$$
$$= \min\{7, \min\{14,20,23,17,14,10,0\}\}$$
$$= 0$$

$= 12, = 14$

i j

$$\hat{D}_{12\,14} = \min\{(d_{12\,14}, \min\{d_{12\,1}, d_{12\,3}, d_{12\,6}, d_{12\,8}, d_{12\,9}, d_{12\,10}, d_{12\,12}\}\} = \min\{9, \min\{14,20,23,17,14,10,0\}\}$$

$$= 0$$

$i = 12, j = 15$

$$\hat{D}_{12\,15} = \min\{(d_{12\,15}, \min\{d_{12\,1}, d_{12\,3}, d_{12\,6}, d_{12\,8}, d_{12\,9}, d_{12\,10}, d_{12\,12}\}\} = \min\{2, \min\{14,20,23,17,14,10,0\}\}$$

$$= 0$$

$i = 12, j = 16$

$$\hat{D}_{12\,16} = \min\{(d_{12\,16}, \min\{d_{12\,1}, d_{12\,3}, d_{12\,6}, d_{12\,8}, d_{12\,9}, d_{12\,10}, d_{12\,12}\}\} = \min\{10, \min\{14,20,23,17,14,10,0\}\}$$

$$= 0$$

$i = 12, j = 17$

$$\hat{D}_{12\,17} = \min\{(d_{12\,1}, \min\{d_{12\,1}, d_{12\,3}, d_{12\,6}, d_{12\,8}, d_{12\,9}, d_{12\,10}, d_{12\,12}\}\}$$

$$= \min\{14, \min\{14,20,23,17,14,10,0\}\}$$

$$= 0$$

$i = 12, j = 18$

$$\hat{D}_{12\,18} = \min\{(d_{12\,18}, \min\{d_{12\,1}, d_{12\,3}, d_{12\,6}, d_{12\,8}, d_{12\,9}, d_{12\,10}, d_{12\,12}\}\}$$

$$= \min\{14, \min\{14,20,23,17,14,10,0\}\}$$

$$= 0$$

$i = 12, j = 19$

$\hat{D}_{12\,19} = \min\{(d_{12\,19}, \min\{d_{12\,1}, d_{12\,3}, d_{12\,6}, d_{12\,8}, d_{12\,9}, d_{12\,10}, d_{12\,12}\}\}$

$= \min\{14, \min\{14,20,23,17,14,10,0\}\}$

$= 0$

$i = 13,\ j = 1$

$\hat{D}_{12\,19} = \min\{(d_{12\,19}, \min\{d_{12\,1}, d_{12\,3}, d_{12\,6}, d_{12\,8}, d_{12\,9}, d_{12\,10}, d_{12\,12}\}\}$

$= \min\{14, \min\{14,20,23,17,14,10,0\}\}$

$= 0$

For Node 13

$i = 13,\ j = 1$

$\hat{D}_{13\,1} = \min\{(d_{13\,1}, \min\{(d_{13\,1} d_{13\,3}, d_{13\,6}, d_{13\,8}, d_{13\,9}, d_{13\,10}, d_{13\,12}\}\}$

$= \min\{7, \min\{7,13,16,10,7,9,7\}\}$

$= 7$

$i = 13,\ j = 2$

$\hat{D}_{13\,2} = \min\{(d_{13\,2}, \min\{(d_{13\,1}, d_{13\,3}, d_{13\,6}, d_{13\,8}, d_{13\,9}, d_{13\,10}, d_{13\,12}\}\}$

$= \min\{12, \min\{7,13,16,10,7,9,7\}\}$

$= 7$

$i = 13,\ j = 3$

$\hat{D}_{13\,3} = \min\{(d_{13\,3}, \min\{(d_{13\,1}, d_{13\,3}, d_{13\,6}, d_{13\,8}, d_{13\,9}, d_{13\,10}, d_{13\,12}\}\}$

$= \min\{13, \min\{7,13,16,10,7,9,7\}\}$

$= 7$

$i = 13,\ j = 4$

$\hat{D}_{13\,4} = \min\{(d_{13\,4}, \min\{(d_{13\,1}, d_{13\,3}, d_{13\,6}, d_{13\,8}, d_{13\,9}, d_{13\,10}, d_{13\,12}\}\}$

$= \min\{14, \min\{7,13,16,10,7,9,7\}\}$

$= 7$

$i = 13,\ j = 5$

$\hat{D}_{13\,5} = \min\{(d_{13\,5}, \min\{(d_{13\,1}, d_{13\,3}, d_{13\,6}, d_{13\,8}, d_{13\,9}, d_{13\,10}, d_{13\,12}\}\}$

$= \min\{16, \min\{7,13,16,10,7,9,7\}\}$

$= 7$

$i = 13,\ j = 6$

$$\hat{D}_{13\,6} = \min\{(d_{13\,6},\min\{(d_{13\,1},d_{13\,3},\ d_{13\,6},\ d_{13\,8},\ d_{13\,9},\ d_{13\,10},\ d_{13\,12}\}\}$$

$$= \min\{16,\min\{7,13,16,10,7,9,7\}\}$$

$$= 7$$

$i = 13,\ j = 7$

$$\hat{D}_{13\,7} = \min\{(d_{13\,7},\min\{(d_{13\,1},d_{13\,3},\ d_{13\,6},\ d_{13\,8},\ d_{13\,9},\ d_{13\,10},\ d_{13\,12}\}\}$$

$$= \min\{14,\min\{7,13,16,10,7,9,7\}\}$$

$= 7$ i
$= 13,\ j = 8$

$$\hat{D}_{13\,8} = \min\{(d_{13\,8},\min\{(d_{13\,1},d_{13\,3},\ d_{13\,6},\ d_{13\,8},\ d_{13\,9},\ d_{13\,10},\ d_{13\,12}\}\}$$

$$= \min\{10,\min\{7,13,16,10,7,9,7\}\}$$

$= 7$ i
$= 13,\ j = 9$

$$\hat{D}_{13\,9} = \min\{(d_{13\,9},\min\{(d_{13\,1},d_{13\,3},\ d_{13\,6},\ d_{13\,8},\ d_{13\,9},\ d_{13\,10},\ d_{13\,12}\}\}$$

$$= \min\{7,\min\{7,13,16,10,7,9,7\}\}$$

$$= 7$$

$i = 13,\ j = 10$

$$\hat{D}_{13\,10} = \min\{(d_{13\,10},\min\{(d_{13\,1},d_{13\,3},\ d_{13\,6},\ d_{13\,8},\ d_{13\,9},\ d_{13\,10},\ d_{13\,12}\}\}$$

$$= \min\{9,\min\{7,13,16,10,7,9,7\}\}$$

$$= 7$$

$i = 13,\ j = 11$

$$\hat{D}_{13\,11} = \min\{(d_{13\,11},\min\{(d_{13\,1},d_{13\,3},\ d_{13\,6},\ d_{13\,8},\ d_{13\,9},\ d_{13\,10},\ d_{13\,12}\}\}$$

$$= \min\{12,\min\{7,13,16,10,7,9,7\}\}$$

$= 7$ $i = 13,$
$j = 12$

$$\hat{D}_{13\,12} = \min\{(d_{13\,12},\min\{(d_{13\,1},d_{13\,3},\ d_{13\,6},\ d_{13\,8},\ d_{13\,9},\ d_{13\,10},\ d_{13\,12}\}\}$$

$$= \min\{7,\min\{7,13,16,10,7,9,7\}\}$$

$= 7$ $i = 13,$
$j = 13$

$$\hat{D}_{13\,13} = \min\{(d_{13\,13},\min\{(d_{13\,1},d_{13\,3},\ d_{13\,6},\ d_{13\,8},\ d_{13\,9},\ d_{13\,10},\ d_{13\,12}\}\}$$

$= \min\{0,\min\{7,13,16,10,7,9,7\}\}$

$= 0 \quad i = 13,$

$j = 14$

$\hat{D}_{13\ 14} = \min\{(d_{13\ 14},\min\{(d_{13\ 1},d_{13\ 3},\ d_{13\ 6},\ d_{13\ 8}\ ,\ d_{13\ 9},\ d_{13\ 10},\ d_{13\ 12}\}\}$

$= \min\{2,\min\{7,13,16,10,7,9,7\}\}$

$= 2$

$i = 13,\ j = 15$

$\hat{D}_{13\ 15} = \min\{(d_{13\ 15},\min\{(d_{13\ 1},d_{13\ 3},\ d_{13\ 6},\ d_{13\ 8}\ ,\ d_{13\ 9},\ d_{13\ 10},\ d_{13\ 12}\}\}$

$= \min\{9,\min\{7,13,16,10,7,9,7\}\}$

$= 7$

$i = 13,\ j = 16$

$\hat{D}_{13\ 16} = \min\{(d_{13\ 16},\min\{(d_{13\ 1},d_{13\ 3},\ d_{13\ 6},\ d_{13\ 8}\ ,\ d_{13\ 9},\ d_{13\ 10},\ d_{13\ 12}\}\}$

$= \min\{3,\min\{7,13,16,10,7,9,7\}\}$

$= 3$

$i = 13,\ j = 17$

$\hat{D}_{13\ 17} = \min\{(d_{13\ 17},\min\{(d_{13\ 1},d_{13\ 3},\ d_{13\ 6},\ d_{13\ 8}\ ,\ d_{13\ 9},\ d_{13\ 10},\ d_{13\ 12}\}\}$

$= \min\{6,\min\{7,13,16,10,7,9,7\}\}$

$= 6$

$i = 13,\ j = 18$

$\hat{D}_{13\ 18} = \min\{(d_{13\ 18},\min\{(d_{13\ 1},d_{13\ 3},\ d_{13\ 6},\ d_{13\ 8}\ ,\ d_{13\ 9},\ d_{13\ 10},\ d_{13\ 12}\}\}$

$= \min\{12,\min\{7,13,16,10,7,9,7\}\}$

$= 7$

$i = 13,\ j = 19$

$\hat{D}_{13\ 19} = \min\{(d_{13\ 19},\min\{(d_{13\ 1},d_{13\ 3},\ d_{13\ 6},\ d_{13\ 8}\ ,\ d_{13\ 9},\ d_{13\ 10},\ d_{13\ 12}\}\}$

$= \min\{10,\min\{7,13,16,10,7,9,7\}\}$

$= 7$

For Node 14 $i = 14, j = 1$

$$\hat{D}_{14\,1} = \min\{(d_{14\,1}, \min\{(d_{14\,1}, d_{14\,3}, d_{14\,6}, d_{14\,8}, d_{14\,9}, d_{14\,10}, d_{14\,12}\}\}$$
$$= \min\{5, \min\{5,11,14,8,5,7,9\}\}$$
$$= 5$$

$i = 14, j = 2$

$$\hat{D}_{14\,2} = \min\{(d_{14\,2}, \min\{(d_{14\,1}, d_{14\,3}, d_{14\,6}, d_{14\,8}, d_{14\,9}, d_{14\,10}, d_{14\,12}\}\}$$
$$= \min\{10, \min\{5,11,14,8,5,7,9\}\}$$
$$= 5$$

$i = 14, j = 3$

$$\hat{D}_{14\,3} = \min\{(d_{14\,3}, \min\{(d_{14\,1}, d_{14\,3}, d_{14\,6}, d_{14\,8}, d_{14\,9}, d_{14\,10}, d_{14\,12}\}\}$$
$$= \min\{11, \min\{5,11,14,8,5,7,9\}\}$$
$$= 5$$

$i = 14, j = 4$

$$\hat{D}_{14\,4} = \min\{(d_{14\,4}, \min\{(d_{14\,1}, d_{14\,3}, d_{14\,6}, d_{14\,8}, d_{14\,9}, d_{14\,10}, d_{14\,12}\}\}$$
$$= \min\{12, \min\{5,11,14,8,5,7,9\}\}$$
$$= 5$$

$i = 14, j = 5$

$$\hat{D}_{14\,5} = \min\{(d_{14\,5}, \min\{(d_{14\,1}, d_{14\,3}, d_{14\,6}, d_{14\,8}, d_{14\,9}, d_{14\,10}, d_{14\,12}\}\}$$
$$= \min\{14, \min\{5,11,14,8,5,7,9\}\}$$
$$= 5$$

$i = 14, j = 6$

$$\hat{D}_{14\,6} = \min\{(d_{14\,6}, \min\{(d_{14\,1}, d_{14\,3}, d_{14\,6}, d_{14\,8}, d_{14\,9}, d_{14\,10}, d_{14\,12}\}\}$$
$$= \min\{14, \min\{5,11,14,8,5,7,9\}\}$$
$$= 5$$

$i = 14,\ j = 7$

$$\hat{D}_{14\,7} = \min\{(d_{14\,7}, \min\{(d_{14\,1}, d_{14\,3}, d_{14\,6}, d_{14\,8}, d_{14\,9}, d_{14\,10}, d_{14\,12}\}\}$$
$$= \min\{12, \min\{5,11,14,8,5,7,9\}\}$$
$$= 5$$

$i = 14,\ j = 8$

$$\hat{D}_{14\,8} = \min\{(d_{14\,8}, \min\{(d_{14\,1}, d_{14\,3}, d_{14\,6}, d_{14\,8}, d_{14\,9}, d_{14\,10}, d_{14\,12}\}\}$$
$$= \min\{8, \min\{5,11,14,8,5,7,9\}\}$$
$$= 5$$

$i = 14,\ j = 9$

$$\hat{D}_{14\,9} = \min\{(d_{14\,9}, \min\{(d_{14\,1}, d_{14\,3}, d_{14\,6}, d_{14\,8}, d_{14\,9}, d_{14\,10}, d_{14\,12}\}\}$$
$$= \min\{5, \min\{5,11,14,8,5,7,9\}\}$$
$$= 5$$

$i = 14,\ j = 10$

$$\hat{D}_{14\,10} = \min\{(d_{14\,10}, \min\{(d_{14\,1}, d_{14\,3}, d_{14\,6}, d_{14\,8}, d_{14\,9}, d_{14\,10}, d_{14\,12}\}\}$$
$$= \min\{7, \min\{5,11,14,8,5,7,9\}\}$$
$$= 5$$

$i = 14,\ j = 11$

$$\hat{D}_{14\,11} = \min\{(d_{14\,11}, \min\{(d_{14\,1}, d_{14\,3}, d_{14\,6}, d_{14\,8}, d_{14\,9}, d_{14\,10}, d_{14\,12}\}\}$$
$$= \min\{12, \min\{5,11,14,8,5,7,9\}\}$$
$$= 5$$

$i = 14,\ j = 12$

$$\hat{D}_{14\,12} = \min\{(d_{14\,12}, \min\{(d_{14\,1}, d_{14\,3}, d_{14\,6}, d_{14\,8}, d_{14\,9}, d_{14\,10}, d_{14\,12}\}\}$$
$$= \min\{9, \min\{5,11,14,8,5,7,9\}\}$$

$$= 5$$

$i = 14,\ j = 13$

$$\hat{D}_{14\,13} = \min\{(d_{14\,13}, \min\{(d_{14\,1}, d_{14\,3}, d_{14\,6}, d_{14\,8}, d_{14\,9}, d_{14\,10}, d_{14\,12}\}\}$$

$= \min\{2, \min\{5,11,14,8,5,7,9\}\}$
$= 2$

$i = 14, j = 14$

$i = 14$, $j = 14$

$$\hat{D}_{14\,14} = \min\{(d_{14\,14}, \min\{(d_{14\,1}, d_{14\,3}, d_{14\,6}, d_{14\,8}, d_{14\,9}, d_{14\,10}, d_{14\,12}\}\}$$
$$= \min\{0, \min\{5,11,14,8,5,7,9\}\}$$
$$= 0$$

$i = 14$, $j = 15$

$$\hat{D}_{14\,15} = \min\{(d_{14\,15}, \min\{(d_{14\,1}, d_{14\,3}, d_{14\,6}, d_{14\,8}, d_{14\,9}, d_{14\,10}, d_{14\,12}\}\}$$
$$= \min\{11, \min\{5,11,14,8,5,7,9\}\}$$
$$= 5$$

$i = 14$, $j = 16$

$$\hat{D}_{14\,16} = \min\{(d_{14\,16}, \min\{(d_{14\,1}, d_{14\,3}, d_{14\,6}, d_{14\,8}, d_{14\,9}, d_{14\,10}, d_{14\,12}\}\}$$
$$= \min\{5, \min\{5,11,14,8,5,7,9\}\}$$
$$= 5$$

$i = 14$, $j = 17$

$$\hat{D}_{14\,17} = \min\{(d_{14\,17}, \min\{(d_{14\,1}, d_{14\,3}, d_{14\,6}, d_{14\,8}, d_{14\,9}, d_{14\,10}, d_{14\,12}\}\}$$
$$= \min\{8, \min\{5,11,14,8,5,7,9\}\}$$
$$= 5$$

$i = 14$, $j = 18$

$$\hat{D}_{14\,18} = \min\{(d_{14\,18}, \min\{(d_{14\,1}, d_{14\,3}, d_{14\,6}, d_{14\,8}, d_{14\,9}, d_{14\,10}, d_{14\,12}\}\}$$
$$= \min\{10, \min\{5,11,14,8,5,7,9\}\}$$
$$= 5$$

$i = 14$, $j = 19$

$$\hat{D}_{14\,19} = \min\{(d_{14\,19}, \min\{(d_{14\,1}, d_{14\,3}, d_{14\,6}, d_{14\,8}, d_{14\,9}, d_{14\,10}, d_{14\,12}\}\}$$
$$= \min\{8, \min\{5,11,14,8,5,7,9\}\}$$
$$= 5$$

For Node 15

$i = 15$, $j = 1$

$\hat{D}$ = $\min\{(d$

=

=

i

$\hat{D}$

$$\hat{D}_{15\,1} = \min\{(d_{15\,1}, \min\{(d_{15\,1}, d_{15\,3}, d_{15\,6}, d_{15\,8}, d_{15\,9}, d_{15\,10}, d_{15\,12}\}\}$$
$$= \min\{16, \min\{16,22,25,19,16,12,2\}\}$$
$$= 2$$

$i = 15,\ j = 2$

$$\hat{D}_{15\,2} = \min\{(d_{15\,2}, \min\{(d_{15\,1}, d_{15\,3}, d_{15\,6}, d_{15\,8}, d_{15\,9}, d_{15\,10}, d_{15\,12}\}\}$$
$$= \min\{18, \min\{16,22,25,19,16,12,2\}\}$$
$$= 2$$

$i = 15,\ j = 3$

$$\hat{D}_{15\,3} = \min\{(d_{15\,3}, \min\{(d_{15\,1}, d_{15\,3}, d_{15\,6}, d_{15\,8}, d_{15\,9}, d_{15\,10}, d_{15\,12}\}\}$$
$$= \min\{22, \min\{16,22,25,19,16,12,2\}\}$$
$$= 2$$

$i = 15,\ j = 4$

$$\hat{D}_{15\,4} = \min\{(d_{15\,4}, \min\{(d_{15\,1}, d_{15\,3}, d_{15\,6}, d_{15\,8}, d_{15\,9}, d_{15\,10}, d_{15\,12}\}\}$$
$$= \min\{23, \min\{16,22,25,19,16,12,2\}\}$$
$$= 2$$

$i = 15,\ j = 5$

$$\hat{D}_{15\,5} = \min\{(d_{15\,5}, \min\{(d_{15\,1}, d_{15\,3}, d_{15\,6}, d_{15\,8}, d_{15\,9}, d_{15\,10}, d_{15\,12}\}\}$$
$$= \min\{24, \min\{16,22,25,19,16,12,2\}\}$$
$$= 2$$

i j

$i = 15,\ j = 6$

$\hat{D}_{15\ 6} = \min\{(d_{15\ 6}, \min\{(d_{15\ 1}, d_{15\ 3}, d_{15\ 6}, d_{15\ 8}, d_{15\ 9}, d_{15\ 10}, d_{15\ 12}\}\}$

$= \min\{25, \min\{16,22,25,19,16,12,2\}\}$
$= 2$

$i = 15,\ j = 7$

$\hat{D}_{15\ 7} = \min\{(d_{15\ 7}, \min\{(d_{15\ 1}, d_{15\ 3}, d_{15\ 6}, d_{15\ 8}, d_{15\ 9}, d_{15\ 10}, d_{15\ 12}\}\}$

$= \min\{23, \min\{16,22,25,19,16,12,2\}\}$
$= 2$

$i = 15,\ j = 8$

$\hat{D}_{15\ 8} = \min\{(d_{15\ 8}, \min\{(d_{15\ 1}, d_{15\ 3}, d_{15\ 6}, d_{15\ 8}, d_{15\ 9}, d_{15\ 10}, d_{15\ 12}\}\}$

$= \min\{19, \min\{16,22,25,19,16,12,2\}\}$
$= 2$

$i = 15,\ j = 9$

$\hat{D}_{15\ 9} = \min\{(d_{15\ 9}, \min\{(d_{15\ 1}, d_{15\ 3}, d_{15\ 6}, d_{15\ 8}, d_{15\ 9}, d_{15\ 10}, d_{15\ 12}\}\}$
$= \min\{16, \min\{16,22,25,19,16,12,2\}\}$

$= 2$

$= 15,\ = 10$
$_{15\ 10}$ $_{15\ 10}, \min\{(d_{15\ 1}, d_{15\ 3}, d_{15\ 6}, d_{15\ 8}, d_{15\ 9}, d_{15\ 10}, d_{15\ 12}\}\}$
$\min\{12, \min\{16,22,25,19,16,12,2\}\}$

2

$= 15,\ j = 11$

$_{15\ 11} = \min\{(d_{15\ 11}, \min\{(d_{15\ 1}, d_{15\ 3}, d_{15\ 6}, d_{15\ 8}, d_{15\ 9}, d_{15\ 10}, d_{15\ 12}\}\}$

$= \min\{7, \min\{16,22,25,19,16,12,2\}\}$
$= 2$

i j

$\hat{D}$ $=$ min{(d

$=$

$=$

i

$\hat{D}$

$i = 15,\ j = 12$

$$\hat{D}_{15\ 12} = \min\{(d_{15\ 12}, \min\{(d_{15\ 1}, d_{15\ 3}, d_{15\ 6}, d_{15\ 8}, d_{15\ 9}, d_{15\ 10}, d_{15\ 12}\}\}$$

$$= \min\{2, \min\{16,22,25,19,16,12,2\}\}$$

$$= 2$$

$i = 15,\ j = 13$

$$\hat{D}_{15\ 13} = \min\{(d_{15\ 13}, \min\{(d_{15\ 1}, d_{15\ 3}, d_{15\ 6}, d_{15\ 8}, d_{15\ 9}, d_{15\ 10}, d_{15\ 12}\}\}$$

$$= \min\{9, \min\{16,22,25,19,16,12,2\}\}$$

$= 2$ $i = 15$,

$j = 14$

$$\hat{D}_{15\ 14} = \min\{(d_{15\ 14}, \min\{(d_{15\ 1}, d_{15\ 3}, d_{15\ 6}, d_{15\ 8}, d_{15\ 9}, d_{15\ 10}, d_{15\ 12}\}\}$$

$$= \min\{11, \min\{16,22,25,19,16,12,2\}\}$$

$$= 2$$

$i = 15,\ j = 15$

$$\hat{D}_{15\ 15} = \min\{(d_{15\ 15}, \min\{(d_{15\ 1}, d_{15\ 3}, d_{15\ 6}, d_{15\ 8}, d_{15\ 9}, d_{15\ 10}, d_{15\ 12}\}\}$$

$$= \min\{0, \min\{16,22,25,19,16,12,2\}\}$$

$$= 0$$

$i = 15,\ j = 16$

$$\hat{D}_{15\ 16} = \min\{(d_{15\ 16}, \min\{(d_{15\ 1}, d_{15\ 3}, d_{15\ 6}, d_{15\ 8}, d_{15\ 9}, d_{15\ 10}, d_{15\ 12}\}\}$$

$$= \min\{9, \min\{16,22,25,19,16,12,2\}\}$$

$$= 2$$

$i = 15,\ j = 17$

$$\hat{D}_{15\ 17} = \min\{(d_{15\ 17}, \min\{(d_{15\ 1}, d_{15\ 3}, d_{15\ 6}, d_{15\ 8}, d_{15\ 9}, d_{15\ 10}, d_{15\ 12}\}\}$$

i j

□ min{6,min{16,22,25,19,16,12,2}}

□ 2

□15, □18

i *j*

$\hat{D}$ $=$ $\min\{(d$

$=$

$=$

i

$\hat{D}$

$_{15\,18}$ $_{15\,18}, \min\{(d_{15\,1}, d_{15\,3}, d_{15\,6}, d_{15\,8}, d_{15\,9}, d_{15\,10}, d_{15\,12}\}\}$

$\min\{12, \min\{16,22,25,19,16,12,2\}\}$

2

$= 15, j = 19$

$_{15\,19} = \min\{(d_{15\,19}, \min\{(d_{15\,1}, d_{15\,3}, d_{15\,6}, d_{15\,8}, d_{15\,9}, d_{15\,10}, d_{15\,12}\}\}$

$= \min\{14, \min\{16,22,25,19,16,12,2\}\}$

$= 2$

For Node 16 i

$= 16, j = 1$

$\hat{D}_{16\,1} = \min\{(d_{16\,1}, \min\{(d_{16\,1}, d_{16\,3}, d_{16\,6}, d_{16\,8}, d_{16\,9}, d_{16\,10}, d_{16\,12}\}\}$

$= \min\{7, \min\{7,14,19,13,10,12,10\}\}$

$= 7$

$i = 16, j = 2$

$\hat{D}_{16\,2} = \min\{(d_{16\,2}, \min\{(d_{16\,1}, d_{16\,3}, d_{16\,6}, d_{16\,8}, d_{16\,9}, d_{16\,10}, d_{16\,12}\}\}$

$= \min\{12, \min\{7,14,19,13,10,12,10\}\}$

$= 7$

$i = 16, j = 3$

$\hat{D}_{16\,3} = \min\{(d_{16\,3}, \min\{(d_{16\,1}, d_{16\,3}, d_{16\,6}, d_{16\,8}, d_{16\,9}, d_{16\,10}, d_{16\,12}\}\}$

i j

$\hat{D}$ = min{(d

=

=

i

$\hat{D}$

= min{14,min{7,14,19,13,10,12,10}}

= 7

$i = 16, j = 4$

$$\hat{D}_{16\,4} = \min\{(d_{16\,4}, \min\{(d_{16\,1}, d_{16\,3}, d_{16\,6}, d_{16\,8}, d_{16\,9}, d_{16\,10}, d_{16\,12}\}\}$$

$= \min\{14, \min\{7,14,19,13,10,12,10\}\}$

$= 7$

$i = 16, j = 5$

$$\hat{D}_{16\,5} = \min\{(d_{16\,5}, \min\{(d_{16\,1}, d_{16\,3}, d_{16\,6}, d_{16\,8}, d_{16\,9}, d_{16\,10}, d_{16\,12}\}\}$$

$= \min\{17, \min\{7,14,19,13,10,12,10\}\}$

$= 7$

=16, = 6

16 6 16 6,min{($d_{16\,1}$,$d_{16\,3}$, $d_{16\,6}$, $d_{16\,8}$, $d_{16\,9}$, $d_{16\,10}$, $d_{16\,12}$}}

min{19,min{7,14,19,13,10,12,10}}

7

=16, j = 7

16 7 = min{($d_{16\,7}$,min{($d_{16\,1}$,$d_{16\,3}$, $d_{16\,6}$, $d_{16\,8}$, $d_{16\,9}$, $d_{16\,10}$, $d_{16\,12}$}}

= min{17,min{7,14,19,13,10,12,10}}

= 7

$i = 16, j = 8$

$$\hat{D}_{16\,8} = \min\{(d_{16\,8}, \min\{(d_{16\,1}, d_{16\,3}, d_{16\,6}, d_{16\,8}, d_{16\,9}, d_{16\,10}, d_{16\,12}\}\}$$

$= \min\{13, \min\{7,14,19,13,10,12,10\}\}$

$= 7$

$\hat{D} = \min\{(d$

$=$

$=$

i

$\hat{D}$

$i = 16, j = 9$

$\hat{D}_{16\,9} = \min\{(d_{16\,9}, \min\{(d_{16\,1}, d_{16\,3}, d_{16\,6}, d_{16\,8}, d_{16\,9}, d_{16\,10}, d_{16\,12}\}\}$

$= \min\{10, \min\{7,14,19,13,10,12,10\}\}$

$= 7$ $i = 16,$

$j = 10$

$\hat{D}_{16\,10} = \min\{(d_{16\,10}, \min\{(d_{16\,1}, d_{16\,3}, d_{16\,6}, d_{16\,8}, d_{16\,9}, d_{16\,10}, d_{16\,12}\}\}$

$= \min\{12, \min\{7,14,19,13,10,12,10\}\}$

$= 7$

$i = 16, j = 11$

$\hat{D}_{16\,11} = \min\{(d_{16\,11}, \min\{(d_{16\,1}, d_{16\,3}, d_{16\,6}, d_{16\,8}, d_{16\,9}, d_{16\,10}, d_{16\,12}\}\}$

$= \min\{15, \min\{7,14,19,13,10,12,10\}\}$

$= 7$

$i = 16, j = 13$

$\hat{D}_{16\,13} = \min\{(d_{16\,13}, \min\{(d_{16\,1}, d_{16\,3}, d_{16\,6}, d_{16\,8}, d_{16\,9}, d_{16\,10}, d_{16\,12}\}\}$

$= \min\{3, \min\{7,14,19,13,10,12,10\}\}$

$= 3$

$i = 16, j = 14$

$\hat{D}_{16\,14} = \min\{(d_{16\,14}, \min\{(d_{16\,1}, d_{16\,3}, d_{16\,6}, d_{16\,8}, d_{16\,9}, d_{16\,10}, d_{16\,12}\}\}$

$= \min\{5, \min\{7,14,19,13,10,12,10\}\}$

$= 5$

i j

$i = 16, j = 15$

$\hat{D}_{16\,15} = \min\{(d_{16\,15},\min\{\{d_{16\,1},d_{16\,3}, d_{16\,6}, d_{16\,8}, d_{16\,9}, d_{16\,10}, d_{16\,12}\}\}$

$= \min\{9,\min\{7,14,19,13,10,12,10\}\}$

$= 7$

$i = 16, j = 16$

$\hat{D}_{16\,16} = \min\{(d_{16\,16},\min\{\{d_{16\,1},d_{16\,3}, d_{16\,6}, d_{16\,8}, d_{16\,9}, d_{16\,10}, d_{16\,12}\}\}$

$= \min\{0,\min\{7,14,19,13,10,12,10\}\}$

$= 0$

$i = 16, j = 17$

$\hat{D}_{16\,17} = \min\{(d_{16\,17},\min\{\{d_{16\,1},d_{16\,3}, d_{16\,6}, d_{16\,8}, d_{16\,9}, d_{16\,10}, d_{16\,12}\}\}$

$= \min\{3,\min\{7,14,19,13,10,12,10\}\}$

$= 3$

$i = 16, j = 18$

$\hat{D}_{16\,18} = \min\{(d_{16\,18},\min\{\{d_{16\,1},d_{16\,3}, d_{16\,6}, d_{16\,8}, d_{16\,9}, d_{16\,10}, d_{16\,12}\}\}$

$= \min\{9,\min\{7,14,19,13,10,12,10\}\}$

$= 7$

$i = 16, j = 19$

$\hat{D}_{16\,19} = \min\{(d_{16\,19},\min\{\{d_{16\,1},d_{16\,3}, d_{16\,6}, d_{16\,8}, d_{16\,9}, d_{16\,10}, d_{16\,12}\}\}$

$= \min\{10,\min\{7,14,19,13,10,12,10\}\}$

$= 7$

For Node 17

$i = 17, j = 1$

$\hat{D}$ $=$ $\min\{(d$

$=$

$=$

i

$\hat{D}$

$\hat{D}_{17\,1} = \min\{(d_{17\,1}, \min\{d_{17\,1}, d_{17\,3}, d_{17\,6}, d_{17\,8}, d_{17\,9}, d_{17\,10}, d_{17\,12}\}\}$ $=$
$\min\{10, \min\{10,17,20,16,13,15,8\}\}$

$= 8$

$i = 17,\ j = 2$

$\hat{D}_{17\,2} = \min\{(d_{17\,2}, \min\{d_{17\,1}, d_{17\,3}, d_{17\,6}, d_{17\,8}, d_{17\,9}, d_{17\,10}, d_{17\,12}\}\}$

$= \min\{12, \min\{10,17,20,16,13,15,8\}\}$

$= 8$

$i = 17,\ j = 3$

$\hat{D}_{17\,3} = \min\{(d_{17\,3}, \min\{d_{17\,1}, d_{17\,3}, d_{17\,6}, d_{17\,8}, d_{17\,9}, d_{17\,10}, d_{17\,12}\}\}$

i j

$= \min\{17,\min\{10,17,20,16,13,15,8\}\}$

$= 8$

$i = 17, j = 4$

$\hat{D}_{17\,4} = \min\{(d_{17\,4}, \min\{d_{17\,1}, d_{17\,3}, d_{17\,6}, d_{17\,8}, d_{17\,9}, d_{17\,10}, d_{17\,12}\}\} =$
$\min\{17,\min\{10,17,20,16,13,15,8\}\}$

$= 8$

$i = 17, j = 5$

$\hat{D}_{17\,5} = \min\{(d_{17\,5}, \min\{d_{17\,1}, d_{17\,3}, d_{17\,6}, d_{17\,8}, d_{17\,9}, d_{17\,10}, d_{17\,12}\}\} =$
$\min\{18,\min\{10,17,20,16,13,15,8\}\}$

$= 8$

$i = 17, j = 6$

$\hat{D}_{17\,6} = \min\{(d_{17\,6}, \min\{d_{17\,1}, d_{17\,3}, d_{17\,6}, d_{17\,8}, d_{17\,9}, d_{17\,10}, d_{17\,12}\}\}$

$= \min\{20,\min\{10,17,20,16,13,15,8\}\}$

$= 8$

$i = 17, j = 7$

$\hat{D}_{17\,7} = \min\{(d_{17\,7}, \min\{d_{17\,1}, d_{17\,3}, d_{17\,6}, d_{17\,8}, d_{17\,9}, d_{17\,10}, d_{17\,12}\}\} =$
$\min\{20,\min\{10,17,20,16,13,15,8\}\}$

$= 8$

$i = 17, j = 8$

$\hat{D}$

i j

$\hat{D}$ $\square$ $_8$ $_{12}\}\}$

$\square$

$\square$

$$\hat{D}_{17\ 8} \square \min\{(d_{17\ 8}, \min\{d_{17\ 1}, d_{17\ 3}, d_{17\ 6}, d_{17\ 8}, d_{17\ 9}, d_{17\ 10}, d_{17\ 12}\}\}$$

$\square$ min{16,min{10,17,20,16,13,15,8}}

$\square 8$

$i \square 17, j \square 9$

$$_{17}9 \square \min\{(d_{17\ 9}, \min\{d_{17\ 1}, d_{17\ 3}, d_{17\ 6}, d_{17\ 8}, d_{17\ 9}, d_{17\ 10}, d_{17\ 12}\}\}$$

$\square$ min{13,min{10,17,20,16,13,15,8}}

$\square 8$

$\square 17, \square 10$

$$_{17}10 \quad \min\{(d_{17\ 10}, \min\{d_{17\ 1}, d_{17\ 3}, d_{17\ 6}, d_{17\ 8}, d_{17\ 9}, d_{17\ 10}, d_{17\ 12}\}\}$$

min{15,min{10,17,20,16,13,15,8}}

$\square 8$

$i \square 17, j \square 11$

$$\hat{D}_{17\ 11} \square \min\{(d_{17\ 11}, \min\{d_{17\ 1}, d_{17\ 3}, d_{17\ 6}, d_{17\ 8}, d_{17\ 9}, d_{17\ 10}, d_{17\ 12}\}\} \quad \square$$

min{13,min{10,17,20,16,13,15,8}}

$\square 8$

$i \square 17, j \square 12$

$$\hat{D}_{17\ 12} \square \min\{(d_{17\ 12}, \min\{d_{17\ 1}, d_{17\ 3}, d_{17\ 6}, d_{17\ 8}, d_{17\ 9}, d_{17\ 10}, d_{17\ 12}\}\} \quad \square$$

min{8,min{10,17,20,16,13,15,8}}

$\square 8$

$\hat{D}_{18}$ $\square$ $_6, d_{18}$ $, d_{18\ 9}, d_{18\ 10}, d_{18}$

$\square$

$\square$

i j

$\hat{D}$

$i = 17, j = 13$

$\hat{D}_{17\,13} = \min\{(d_{17\,13}, \min\{d_{17\,1}, d_{17\,3}, d_{17\,6}, d_{17\,8}, d_{17\,9}, d_{17\,10}, d_{17\,12}\}\} = \min\{6, \min\{10,17,20,16,13,15,8\}\}$

$= 6$

$i = 17, j = 14$

$\hat{D}_{17\,14} = \min\{(d_{17\,14}, \min\{d_{17\,1}, d_{17\,3}, d_{17\,6}, d_{17\,8}, d_{17\,9}, d_{17\,10}, d_{17\,12}\}\}$

$= \min\{8, \min\{10,17,20,16,13,15,8\}\}$

$= 8$

$i = 17, j = 15$

$\hat{D}_{17\,15} = \min\{(d_{17\,15}, \min\{d_{17\,1}, d_{17\,3}, d_{17\,6}, d_{17\,8}, d_{17\,9}, d_{17\,10}, d_{17\,12}\}\}$

$= \min\{6, \min\{10,17,20,16,13,15,8\}\}$

$= 6$

$i = 17, j = 16$

$_{17\,16}$ $\min\{(d_{17\,16}, \min\{d_{17\,1}, d_{17\,3}, d_{17\,6}, d_{17\,8}, d_{17\,9}, d_{17\,10}, d_{17\,12}\}\}$

$\min\{3, \min\{10,17,20,16,13,15,8\}\}$

3

$= 17, = 17$

$_{17\,17}$ $\min\{(d_{17\,17}, \min\{d_{17\,1}, d_{17\,3}, d_{17\,6}, d_{17}$ $, d_{17\,9}, d_{17\,10}, d_{17}$

$\hat{D}$

i j

$\hat{D}$ $=$ ${}_{8}$ ${}_{12}\}\}$

$=$

$=$ $\min\{0,\min\{10,17,20,16,13,15,8\}\}$

0

$i = 17, j = 18$

$\hat{D}_{17\,18} = \min\{(d_{17\,18}, \min\{d_{17\,1}, d_{17\,3}, d_{17\,6}, d_{17\,8}, d_{17\,9}, d_{17\,10}, d_{17\,12}\}\} =$
$\min\{6,\min\{10,17,20,16,13,15,8\}\}$
$= 6$

$i = 17, j = 19$

$\hat{D}_{17\,19} = \min\{(d_{17\,19}, \min\{d_{17\,1}, d_{17\,3}, d_{17\,6}, d_{17\,8}, d_{17\,9}, d_{17\,10}, d_{17\,12}\}\} =$
$\min\{8,\min\{10,17,20,16,13,15,8\}\}$
$= 8$

For Node 18 i
$= 18, j = 1$

$\hat{D}_{18\,1} = \min\{(d_{18\,1}, \min\{d_{18\,1}, d_{18\,3}, d_{18\,6}, d_{18\,8}, d_{18\,9}, d_{18\,10}, d_{18\,12}\}\} =$
$\min\{5,\min\{5,12,14,15,15,17,14\}\}$
$= 5$

$i = 18, j = 2$

$\hat{D}_{18\,2} = \min\{(d_{18\,2}, \min\{d_{18\,1}, d_{18\,3}, d_{18\,6}, d_{18\,8}, d_{18\,9}, d_{18\,10}, d_{18\,12}\}\}$
$= \min\{6,\min\{5,12,14,15,15,17,14\}\}$

$\hat{D}_{18}$ $=$ ${}_{6}, d_{18}$ $, d_{18\,9}, d_{18\,10}, d_{18}$

$=$

$=$

i j

$\hat{D}$ □

□

□ 5 i

□18, j □3

$\hat{D}_{18\ 3}$ □ $\min\{(d_{18\ 3}, \min\{d_{18\ 1}, d_{18\ 3}, d_{18\ 6}, d_{18\ 8}, d_{18\ 9}, d_{18\ 10}, d_{18\ 12}\}\}$

□ $\min\{12, \min\{5,12,14,15,15,17,14\}\}$

□ 5

i □18, j □ 4

4 $\min\{(d_{18\ 4}, \min\{d_{18\ 1}, d_{18\ 3}, d_{18}$ 8 12$\}\}$

$\min\{12, \min\{5,12,14,15,15,17,14\}\}$

5

$\hat{D}$ □

□

□

$i = 18, j$

$\hat{D}_{18} = \min\{(d_{18}$

5

$5\ _5, \min\{d_{18\ 1}, d_{18\ 3}, d_{18\ 6}, d_{18\ 8}, d_{18\ 9}, d_{18\ 10}, d_{18\ 12}\}\}$

$\min\{12, \min\{5,12,14,15,15,17,14\}\}$

5

$i = 18, j = 6$

$\hat{D}_{18\ 6} = \min\{(d_{18\ 6}, \min\{d_{18\ 1}, d_{18\ 3}, d_{18\ 6}, d_{18\ 8}, d_{18\ 9}, d_{18\ 10}, d_{18\ 12}\}\} =$
$\min\{14, \min\{5,12,14,15,15,17,14\}\}$
$= 5$

$i = 18, j = 7$

$\hat{D}_{18\ 7} = \min\{(d_{18\ 7}, \min\{d_{18\ 1}, d_{18\ 3}, d_{18\ 6}, d_{18\ 8}, d_{18\ 9}, d_{18\ 10}, d_{18\ 12}\}\} =$
$\min\{14, \min\{5,12,14,15,15,17,14\}\}$
$= 5$

$i = 18, j = 8$

$\hat{D}_{18\ 8} = \min\{(d_{18\ 8}, \min\{d_{18\ 1}, d_{18\ 3}, d_{18\ 6}, d_{18\ 8}, d_{18\ 9}, d_{18\ 10}, d_{18\ 12}\}\} =$
$\min\{15, \min\{5,12,14,15,15,17,14\}\}$
$= 5$

$i = 18, j = 9$

$\hat{D}_{18} = \quad _6, d_{18} \quad , d_{18\ 9}, d_{18\ 10}, d_{18}$

i 18, j

$\hat{D}_{18}$ $\min\{(d_{18}$ $_{8}$ $_{12}\}\}$

$\hat{D}_{18\ 9} \min\{(d_{18\ 9}, \min\{d_{18\ 1}, d_{18\ 3}, d_{18\ 6}, d_{18\ 8}, d_{18\ 9}, d_{18\ 10}, d_{18\ 12}\}\}$

$\min\{15, \min\{5,12,14,15,15,17,14\}\}$

5

i 18, j 10

$\hat{D}_{18\ 10} \min\{(d_{18\ 10}, \min\{d_{18\ 1}, d_{18\ 3}, d_{18\ 6}, d_{18\ 8}, d_{18\ 9}, d_{18\ 10}, d_{18\ 12}\}\}$

$\min\{17, \min\{5,12,14,15,15,17,14\}\}$

5

i 18, j 11

11 $\min\{(d_{18\ 11}, \min\{d_{18\ 1}, d_{18\ 3}, d_{18}$ $_{8}$ $_{12}\}\}$

$\min\{19, \min\{5,12,14,15,15,17,14\}\}$

5

12

12 $_{12}, \min\{d_{18\ 1}, d_{18\ 3}, d_{18\ 6}, d_{18}$ $, d_{18}$ $9, d_{18}$ $10, d_{18}$

$\min\{14, \min\{5,12,14,15,15,17,14\}\}$

5

i 18, j 13

$\hat{D}$

$i = 18, j$

$\hat{D}_{18} = \min\{(d_{18} \quad {}_{8} \quad {}_{12}\}\}$

$=$

$=$

$\hat{D}_{18\,13} = \min\{(d_{18\,13}, \min\{d_{18\,1}, d_{18\,3}, d_{18\,6}, d_{18\,8}, d_{18\,9}, d_{18\,10}, d_{18\,12}\}\} =$
$\min\{12, \min\{5,12,14,15,15,17,14\}\}$

$= 5$

$i = 18, j = 14$

$\hat{D}_{18\,14} = \min\{(d_{18\,14}, \min\{d_{18\,1}, d_{18\,3}, d_{18\,6}, d_{18\,8}, d_{18\,9}, d_{18\,10}, d_{18\,12}\}\} =$
$\min\{10, \min\{5,12,14,15,15,17,14\}\}$

$= 5$

$i = 18, j = 15$

$\hat{D}_{18\,15} = \min\{(d_{18\,15}, \min\{d_{18\,1}, d_{18\,3}, d_{18\,6}, d_{18\,8}, d_{18\,9}, d_{18\,10}, d_{18\,12}\}\} =$
$\min\{12, \min\{5,12,14,15,15,17,14\}\}$

$= 5$

$i = 18, j = 16$

$\hat{D}_{18\,16} = \min\{(d_{18\,16}, \min\{d_{18\,1}, d_{18\,3}, d_{18\,6}, d_{18\,8}, d_{18\,9}, d_{18\,10}, d_{18\,12}\}\}$

$= \min\{9, \min\{5,12,14,15,15,17,14\}\}$

$= 5$

$i = 18, j = 17$

$\hat{D}_{18\,17} = \min\{(d_{18\,17}, \min\{d_{18\,1}, d_{18\,3}, d_{18\,6}, d_{18\,8}, d_{18\,9}, d_{18\,10}, d_{18\,12}\}\}$

$\hat{D}_{18} = \quad {}_{6}, d_{18} \quad , d_{18\,9}, d_{18\,10}, d_{18}$

$=$

$=$

$i = 18, j$

$\hat{D}_{18} = \min\{(d_{18} \quad 8 \quad 12\}\}$

$= \min\{6, \min\{5,12,14,15,15,17,14\}\}$

$= 5$

$i = 18, j = 18$

$18 \quad \min\{(d_{18\,18}, \min\{d_{18\,1}, d_{18\,3}, d_{18} \quad 8 \quad 12\}\}$

$\min\{0, \min\{5,12,14,15,15,17,14\}\}$

0

$= 19$

$19 \quad {}_{19}, \min\{d_{18\,1}, d_{18\,3}, d_{18\,6}, d_{18}, d_{18} \quad 9, d_{18} \quad 10, d_{18}$

$\min\{2, \min\{5,12,14,15,15,17,14\}\}$

2

For Node 19

$i = 19, j = 1$

$\hat{D}_{19\,1} = \min\{(d_{19\,1}, \min\{d_{19\,1}, d_{19\,3}, d_{19\,6}, d_{19\,8}, d_{19\,9}, d_{19\,10}, d_{19\,12}\}\} =$
$\min\{3, \min\{3,10,12,13,13,15,16\}\}$

$= 3$

$i = 19, j = 2$

$\hat{D} =$

i □18, j

$\hat{D}_{18}$ □ min{(d_{18} $_{8}$ $_{12}$}}

□

□

$\hat{D}_{19\,2}$ □ $\min\{(d_{19\,2}, \min\{d_{19\,1}, d_{19\,3}, d_{19\,6}, d_{19\,8}, d_{19\,9}, d_{19\,10}, d_{19\,12}\}\}$ □
min{4,min{3,10,12,13,13,15,16}}

□ 3 i

□19, j □3

$\hat{D}_{19\,3}$ □ $\min\{(d_{19\,3}, \min\{d_{19\,1}, d_{19\,3}, d_{19\,6}, d_{19\,8}, d_{19\,9}, d_{19\,10}, d_{19\,12}\}\}$ □
min{10,min{3,10,12,13,13,15,16}}

□ 3

i □19, j □ 4

$\hat{D}_{19\,4}$ □ $\min\{(d_{19\,4}, \min\{d_{19\,1}, d_{19\,3}, d_{19\,6}, d_{19\,8}, d_{19\,9}, d_{19\,10}, d_{19\,12}\}\}$

□ min{10,min{3,10,12,13,13,15,16}}

□ 3

i □19, j □5

$\hat{D}_{19\,5}$ □ $\min\{(d_{19\,5}, \min\{d_{19\,1}, d_{19\,3}, d_{19\,6}, d_{19\,8}, d_{19\,9}, d_{19\,10}, d_{19\,12}\}\}$

□ min{10,min{3,10,12,13,13,15,16}}

□ 3

i □19, j □ 6

$_{19}$ 6 $\min\{(d_{19\,6}, \min\{d_{19\,1}, d_{19\,3}, d_{19\,6}, d_{19\,8}, d_{19\,9}, d_{19\,10}, d_{19\,12}\}\}$
min{12,min{3,10,12,13,13,15,16}}

$\hat{D}_{18}$ □ $_{6}, d_{18}$ $, d_{18\,9}, d_{18\,10}, d_{18}$

□

□

i 18, j

$\hat{D}_{18}$ min{(d_{18} $_{8}$ $_{12}$}}

3

$\hat{D}$

$i \quad j$

$\hat{D} = \quad {}_{8} \quad {}_{12}\}\}$

$i = 19,\ j = 7$

$\hat{D}_{19\,7} = \min\{(d_{19\,7}, \min\{d_{19\,1}, d_{19\,3}, d_{19\,6}, d_{19\,\,}, d_{19\,\,}\quad {}_{9}, d_{19} \quad {}_{10}, d_{19}$

$\min\{12, \min\{3,10,12,13,13,15,16\}\}$

$= 3$

$i = 19,\ j = 8$

$\hat{D}_{19\,1} = \min\{(d_{19\,1}, \min\{d_{19\,1}, d_{19\,3}, d_{19\,6}, d_{19\,8}, d_{19\,9}, d_{19\,10}, d_{19\,12}\}\} =$
$\min\{13, \min\{3,10,12,13,13,15,16\}\}$
$= 3$

$i = 19,\ j = 9$

$\hat{D}_{19\,9} = \min\{(d_{19\,9}, \min\{d_{19\,1}, d_{19\,3}, d_{19\,6}, d_{19\,8}, d_{19\,9}, d_{19\,10}, d_{19\,12}\}\} =$
$\min\{3, \min\{13,10,12,13,13,15,16\}\}$
$= 3$

$i = 19,\ j = 10$

$\hat{D}_{19\,10} = \min\{(d_{19\,10}, \min\{d_{19\,1}, d_{19\,3}, d_{19\,6}, d_{19\,8}, d_{19\,9}, d_{19\,10}, d_{19\,12}\}\} =$
$\min\{15, \min\{3,10,12,13,13,15,16\}\}$
$= 3$

$i = 19,\ j = 11$

$\hat{D}_{19\,11} = \min\{(d_{19\,11}, \min\{d_{19\,1}, d_{19\,3}, d_{19\,6}, d_{19\,8}, d_{19\,9}, d_{19\,10}, d_{19\,12}\}\}$

$\hat{D} =$

i j

$\hat{D}$ □ $_{8}$ $_{12}\}\}$

□

□

□ $\min\{20,\min\{3,10,12,13,13,15,16\}\}$

□ 3

i □19, j □12

$\hat{D}_{19\ 12}$ □ $\min\{(d_{19\ 12},\min\{d_{19\ 1},d_{19\ 3},d_{19\ 6},d_{19\ 8},d_{19\ 9},d_{19\ 10},d_{19\ 12}\}\}$

□ $\min\{16,\min\{3,10,12,13,13,15,16\}\}$

□ 3

i □19, j □13

$_{19}$ 13 $\min\{(d_{19\ 13},\min\{d_{19\ 1},d_{19\ 3},d_{19\ 6},d_{19\ 8},d_{19\ 9},d_{19\ 10},d_{19\ 12}\}\}$

$\min\{10,\min\{3,10,12,13,13,15,16\}\}$

3

□19, □14

$_{19}$ 4 $\min\{(d_{19\ 14},\min\{d_{19\ 1},d_{19\ 3},d_{19\ 6},d_{19}$ $,d_{19}$ $_{9},d_{19}$ $_{10},d_{19}$

$\min\{8,\min\{3,10,12,13,13,15,16\}\}$

3

i □19, j □15

$\hat{D}_{19\ 15}$ □ $\min\{(d_{19\ 15},\min\{d_{19\ 1},d_{19\ 3},d_{19\ 6},d_{19\ 8},d_{19\ 9},d_{19\ 10},d_{19\ 12}\}\}$ □

$\min\{14,\min\{3,10,12,13,13,15,16\}\}$

□ 3

i □19, j □16

$\hat{D}_{19\ 16}$ □ $\min\{(d_{19\ 16},\min\{d_{19\ 1},d_{19\ 3},d_{19\ 6},d_{19\ 8},d_{19\ 9},d_{19\ 10},d_{19\ 12}\}\}$ □

$\min\{10,\min\{3,10,12,13,13,15,16\}\}$

$= 3$

$i = 19,\ j = 17$

$$\hat{D}_{19\,17} = \min\{(d_{19\,17}, \min\{d_{19\,1}, d_{19\,3}, d_{19\,6}, d_{19\,8}, d_{19\,9}, d_{19\,10}, d_{19\,12}\}\} = \min\{8, \min\{3,10,12,13,13,15,16\}\}$$

$= 3$

$i = 19,\ j = 18$

$$\hat{D}_{19\,18} = \min\{(d_{19\,18}, \min\{d_{19\,1}, d_{19\,3}, d_{19\,6}, d_{19\,8}, d_{19\,9}, d_{19\,10}, d_{19\,12}\}\} = \min\{2, \min\{3,10,12,13,13,15,16\}\}$$

$= 2$

$i = 19,\ j = 19$

$$\hat{D}_{19\,19} = \min\{(d_{19\,19}, \min\{d_{19\,1}, d_{19\,3}, d_{19\,6}, d_{19\,8}, d_{19\,9}, d_{19\,10}, d_{19\,12}\}\}$$

$= \min\{0, \min\{3,10,12,13,13,15,16\}\}$

$= 0$

i j

$\hat{D}$ □ $_8$ $_{12}\}\}$

□

□

APPENDIX B

FINDING THE OPTIMAL LOCATION

$Min\ G\ x[\ (\ )$ □ $\max\min_{i□1,\ldots,n}\{\ (d\ X\ i\ d\ Y\ i,\ ),\ (\ ,\ )\}]$ with

X □$\{1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16,17,18,19\}$ and Y □$\{1,3,6,8,9,10,12\}$

i □$\{2,4,5,7,11,13,14,15,16,17,18,19\}$

For node 1

i □1

$\min\{\ (1,1),\ (1,1),\ (3,1),\ (6,1),\ (8,1),\ (9,1),d\ \ d\ \ d\ \ d\ \ d\ \ d$

$d(10,1),\ (12,1)\}d\ \min\{0,0,0,0,0,0,0,0\}$

□ 0

i □ 2

min{ (1,2), (1,2), (3,2), (6,2), (8,2), (9,2),*d* *d* *d* *d* *d*

d *d*(10,2), (12,2)}*d* min{5,5,5,5,5,5,5,5}

□ 5

i □3

min{ (1,3), (1,3), (3,3), (6,3), (8,3), (9,3),*d* *d* *d* *d* *d*

d *d*(10,3), (12,3)}*d* min{0,0,0,0,0,0,0,0}

□ 0

i □ 4

min{ (1,4), (1,4), (3,4), (6,4), (8,4), (9,4),*d* *d* *d* *d*

d *dd*(10,4), (12,4)}*d*

min{5,5,5,5,5,5,5,5}

□ 5

i □ 5

min{ (1,5), (1,5), (3,5), (6,5), (8,5), (9,5),*d* *d* *d* *d*

d *dd*(10,5), (12,5)}*d*

min{2,2,2,2,2,2,2,2}

□ 2

i □ 6

min{ (1,6), (1,6), (3,6), (6,6), (8,6), (9,6),*d* *d* *d* *d* *d*

d *d*(10,6), (12,6)}*d* min{0,0,0,0,0,0,0,0}

□ 0

i □ 7

min{ (1,7), (1,7), (3,7), (6,7), (8,7), (9,7),*d* *d* *d* *d*

d *dd*(10,7), (12,7)}*d*

min{2,2,2,2,2,2,2,2}

□ 2

i □8

min{ (1,8), (1,8), (3,8), (6,8), (8,8), (9,8),*d* *d* *d* *d* *d*

d *d*(10,8), (12,8)}*d* min{0,0,0,0,0,0,0,0}

□ 0

i

$i = 9$

$d = \min\{ d(1,9), d(1,9), d(3,9), d(6,9), d(8,9), d(9,9), d(10,9), d(12,9)\} = \min\{0,0,0,0,0,0,0,0\}$

$= 0$

$i = 10$

$d = \min\{ d(1,10), d(1,10), d(3,10), d(6,10), d(8,10), d(9,10), d(10,10), d(12,10)\} = \min\{0,0,0,0,0,0,0,0\}$

$= 0$

$i = 11$

$d = \min\{ d(1,11), d(1,11), d(3,11), d(6,11), d(8,11), d(9,11), d(10,11), d(12,11)\} = \min\{5,5,5,5,5,5,5,5\}$

$= 5$

$i = 12$

$d = \min\{ d(1,12), d(1,12), d(3,12), d(6,12), d(8,12), d(9,12), d(10,12), d(12,12)\} = \min\{0,0,0,0,0,0,0,0\}$

$= 0$

$i = 13$

$d = \min\{ d(1,13), d(1,13), d(3,13), d(6,13), d(8,13), d(9,13), d(10,13), d(12,13)\} = \min\{7,7,7,7,7,7,7,7\}$

$= 7$

$i = 14$

$d = \min\{ d(1,14), d(1,14), d(3,14), d(6,14), d(8,14), d(9,14), d(10,14), d(12,14)\} = \min\{5,5,5,5,5,5,5,5\}$

$= 5$

i 15

min{ (1,15), (1,15), (3,15), (6,15), (8,15), (*d* *d* *d*
d *dd* 9,15), (10,15), (12,15)}*d* *d*
min{2,2,2,2,2,2,2,2}

 2

i 16

min{ (1,16), (1,16), (3,16), (6,16), (8,16), (*d* *d* *d*
d *dd* 9,16), (10,16), (12,16)}*d* *d*
min{7,7,7,7,7,7,7,7}

 7

i 17

min{ (1,17), (1,17), (3,17), (6,17), (8,17), (*d* *d* *d*
d *dd* 9,17), (10,17), (12,17)}*d* *d*
min{8,8,8,8,8,8,8,8}

8

18

min{ (2,18), (1,18), (3,18), (6,18), (8,18), (*d* *d* *d*
d *dd* 9,18), (10,18), (12,18)}*d* *d*
min{5,5,5,5,5,5,5,5}

 5

i 19

min{ (1,19), (1,19), (3,19), (6,19), (8,19), (*d* *d* *d*
d *dd* 9,19), (10,19), (12,19)}*d* *d*
min{3,3,3,3,3,3,3,3,3}

 3

For node 2

i 1

min{ (2,1), (1,1), (3,1), (6,1), (8,1), (9,1),*d* *d* *d* *d* *d*
d *d*(10,1), (12,1)}*d* min{0,0,0,0,0,0,0,0}

 0

i 2

i

min{ (2,2), (1,2), (3,2), (6,2), (8,2), (9,2),*d* *d* *d* *d* *d*
d *d*(10,2), (12,2)}*d* min{0,5,5,5,5,5,5,5}

□ 0

i □ 3
min{ (2,3), (1,3), (3,3), (6,3), (8,3), (9,3),*d* *d* *d* *d*
d *dd*(10,3), (12,3)}*d*

min{0,0,0,0,0,0,0,0}

□ 0

i □ 4
min{ (2,4), (1,4), (3,4), (6,4), (8,4), (9,4),*d* *d* *d* *d*
d *dd*(10,4), (12,4)}*d*

min{5,5,5,5,5,5,5,5}

□ 5

i □ 5
min{ (2,5), (1,5), (3,5), (6,5), (8,5), (9,5),*d* *d* *d* *d*
d *dd*(10,5), (12,5)}*d*

min{2,2,2,2,2,2,2,2}

□ 2

i □ 6
min{ (2,6), (1,6), (3,6), (6,6), (8,6), (9,6),*d* *d* *d* *d*
d *dd*(10,6), (12,6)}*d*

min{0,0,0,0,0,0,0,0}

□ 0

i

d *d* *d* *d* *d* *d*

= 7

min{ (2,7), (1,7), (3,7), (6,7), (8,7), (9,7),*d*(10,7), (12,7)}*d*

min{2,2,2,2,2,2,2,2}

= 2

i =8

min{ (2,8), (1,8), (3,8), (6,8), (8,8), (9,8),*d* *d* *d* *d*
d *dd*(10,8), (12,8)}*d*

min{0,0,0,0,0,0,0,0}

= 0

i = 9

min{ (2,9), (1,9), (3,9), (6,9), (8,9), (9,9),*d* *d* *d* *d*
d *dd*(10,9), (12,9)}*d*

min{0,0,0,0,0,0,0,0}

= 0 *i*

=10

min{ (2,10), (1,10), (3,10), (6,10), (8,10),
(*d* *d* *d* *d* *d* *d* 9,10), (10,10), (12,10)}*dd*
min{0,0,0,0,0,0,0,0}

= 0

i =11

min{ (2,11), (1,11), (3,11), (6,11), (8,11), (*d* *d* *d* *d*
dd 9,11), (10,11), (12,11)}*d* *d*
min{5,5,5,5,5,5,5,5}

= 5

i =12

i

d *d* *d* *d* *d* *d*

min{ (2,12), (1,12), (3,12), (6,12), (8,12),
(*d* *d* *d* *d* *d* *d* 9,12), (10,12), (12,12)}*dd*
min{0,0,0,0,0,0,0,0}

□ 0

i □13
min{ (2,13), (1,13), (3,13), (6,13), (8,13),
(*d* *d* *d* *d* *d* *d* 9,13), (10,13), (12,13)}*d d*
min{7,7,7,7,7,7,7,7}

□ 7

i □14
min{ (2,14), (1,14), (3,14), (6,14), (8,14),
(*d* *d* *d* *d* *d* *d* 9,14), (10,14), (12,14)}*dd*
min{5,5,5,5,5,5,5,5}

□ 5

□15 min{ (2,15), (1,15), (3,15), (6,15), (8,15), (9,15), (10,15), (12,15)}*d*

d

min{2,2,2,2,2,2,2,2}

□ 2

i □16
min{ (2,16), (1,16), (3,16), (6,16), (8,16),
(*d* *d* *d* *d* *d* *d* 9,16), (10,16), (12,16)}*dd*
min{7,7,7,7,7,7,7,7}

□ 7

i □17
min{ (2,17), (1,17), (3,17), (6,17), (8,17),
(*d* *d* *d* *d* *d* *d* 9,17), (10,17),
(12,17)}*d* *d*

i

d d d d d d

min{8,8,8,8,8,8,8,8}

□8

i □18

min{ (2,18), (1,18), (3,18), (6,18), (8,18),

(d d d d d d 9,18), (10,18), (12,18)}d d

min{5,5,5,5,5,5,5,5}

□ 5 i

□19

min{ (2,19), (1,19), (3,19), (6,19), (8,19),

(d d d d d d 9,19), (10,19), (12,19)}dd

min{3,3,3,3,3,3,3,3,3}

□ 3

For node 3

i □1

min{ (3,1), (1,1), (3,1), (6,1), (8,1), (9,1),d d d d d

d d(10,1), (12,1)}d min{0,0,0,0,0,0,0,0}

□ 0

i □ 2

min{ (3,2), (1,2), (3,2), (6,2), (8,2), (9,2),d d d d d

d d(10,2), (12,2)}d min{5,5,5,5,5,5,5,5}

□ 5

i □3

min{ (3,3), (1,3), (3,3), (6,3), (8,3), (9,3),d d d d d

d d(10,3), (12,3)}d min{0,0,0,0,0,0,0,0}

□ 0

□ 4

i

d d d d d d

min{ (3,4), (1,4), (3,4), (6,4), (8,4), (9,4),d(10,4), (12,4)}d

min{5,5,5,5,5,5,5,5}

 5

i 5

min{ (3,5), (1,5), (3,5), (6,5), (8,5), (9,5),d d d d
d dd(10,5), (12,5)}d

min{2,2,2,2,2,2,2,2}

 2

i 6

min{ (3,6), (1,6), (3,6), (6,6), (8,6), (9,6),d d d d d

d d(10,6), (12,6)}d min{0,0,0,0,0,0,0,0}

 0

i 7

min{ (3,7), (1,7), (3,7), (6,7), (8,7), (9,7),d d d d
d dd(10,7), (12,7)}d

min{2,2,2,2,2,2,2,2}

 2

i 8

min{ (3,8), (1,8), (3,8), (6,8), (8,8), (9,8),d d d d d

d d(10,8), (12,8)}d min{0,0,0,0,0,0,0,0}

 0

i 9

min{ (3,9), (1,9), (3,9), (6,9), (8,9), (9,9),d d d d d

d d(10,9), (12,9)}d min{0,0,0,0,0,0,0,0}

 0

i

d *d* *d* *d* *d* *d*

i □10

min{ (3,10), (1,10), (3,10), (6,10), (8,10), (*d d* *d* *d* *d*

d 9,10), (10,10), (12,10)}*d* *d* min{0,0,0,0,0,0,0,0}

□ 0

i □11

min{ (3,11), (1,11), (3,11), (6,11), (8,11), (*d* *d* *d* *d*

dd 9,11), (10,11), (12,11)}*d* *d*

min{5,5,5,5,5,5,5,5}

□ 5

i □12

min{ (3,12), (1,12), (3,12), (6,12), (8,12), (*d d* *d* *d* *d*

d 9,12), (10,12), (12,12)}*d* *d* min{0,0,0,0,0,0,0,0}

□ 0

i □13

min{ (3,13), (1,13), (3,13), (6,13), (8,13),

(*d* *d* *d* *d* *d* *d* 9,13), (10,13), (12,13)}*d d*

min{7,7,7,7,7,7,7,7}

□ 7

i □14

min{ (3,14), (1,14), (3,14), (6,14), (8,14),

(*d* *d* *d* *d* *d* *d* 9,14), (10,14), (12,14)}*dd*

min{5,5,5,5,5,5,5,5}

□ 5

i □15

min{ (3,15), (1,15), (3,15), (6,15), (8,15),

(*d* *d* *d* *d* *d* *d* 9,15), (10,15), (12,15)}*dd*

min{2,2,2,2,2,2,2,2}

□ 2

i □16

min{ (3,16), (1,16), (3,16), (6,16), (8,16),

(*d* *d* *d* *d* *d* *d* 9,16), (10,16), (12,16)}*dd*

min{7,7,7,7,7,7,7,7}

□ 7

i □17

min{ (3,17), (1,17), (3,17), (6,17), (8,17),

(*d* *d* *d* *d* *d* *d* 9,17), (10,17), (12,17)}*dd*

min{8,8,8,8,8,8,8,8}

□8

i □18

min{ (3,18), (1,18), (3,18), (6,18), (8,18),

(*d* *d* *d* *d* *d* *d* 9,18), (10,18), (12,18)}*d d*

min{5,5,5,5,5,5,5,5}

 5

i 19

min{ (3,19), (1,19), (3,19), (6,19), (8,19),
(d d d d d d 9,19), (10,19), (12,19)}dd
min{3,3,3,3,3,3,3,3,3}

 3

For node 4

i 1

min{ (4,1), (1,1), (3,1), (6,1), (8,1), (9,1),d d d d d
d d(10,1), (12,1)}d min{0,0,0,0,0,0,0,0}

 0

i 2

min{ (4,2), (1,2), (3,2), (6,2), (8,2), (9,2),d d d d d
d d(10,2), (12,2)}d min{5,5,5,5,5,5,5,5}

 5

i 3

min{ (4,3), (1,3), (3,3), (6,3), (8,3), (9,3),d d d d
d dd(10,3), (12,3)}d
min{0,0,0,0,0,0,0,0}

 0

i 4

min{ (4,4), (1,4), (3,4), (6,4), (8,4), (9,4),d d d d
d dd(10,4), (12,4)}d
min{0,5,5,5,5,5,5,5}

 0

i 5

min{ (4,5), (1,5), (3,5), (6,5), (8,5), (9,5),d d d d
d dd(10,5), (12,5)}d
min{2,2,2,2,2,2,2,2}

 2

i

d

i □ 6

min{ (4,6), (1,6), (3,6), (6,6), (8,6), (9,6),*d* *d* *d* *d*

d *dd*(10,6), (12,6)}*d*

min{0,0,0,0,0,0,0,0}

□ 0

i □ 7

min{ (4,7), (1,7), (3,7), (6,7), (8,7), (9,7),*d* *d* *d* *d*

d *dd*(10,7), (12,7)}*d*

min{2,2,2,2,2,2,2,2}

□ 2

i □8

min{ (4,8), (1,8), (3,8), (6,8), (8,8), (9,8),*d* *d* *d* *d*

d *dd*(10,8), (12,8)}*d*

min{0,0,0,0,0,0,0,0}

□ 0

□ 9

min{ (4,9), (1,9), (3,9), (6,9), (8,9), (9,9),*d* *d* *d* *d*

d *d*(10,9), (12,9)}*d*

min{0,0,0,0,0,0,0,0}

□ 0

i □10

min{ (4,10), (1,10), (3,10), (6,10), (8,10),

(*d* *d* *d* *d* *d* *d* 9,10), (10,10), (12,10)}*dd*

min{0,0,0,0,0,0,0,0}

□ 0

i □11

□

$$\min\{d(4,11), d(1,11), d(3,11), d(6,11), d(8,11), d(9,11), d(10,11), d(12,11)\} = \min\{5,5,5,5,5,5,5,5\}$$

$= 5$

$i = 12$

$$\min\{d(4,12), d(1,12), d(3,12), d(6,12), d(8,12), d(9,12), d(10,12), d(12,12)\} = \min\{0,0,0,0,0,0,0,0\}$$

$= 0$

$i = 13$

$$\min\{d(4,13), d(1,13), d(3,13), d(6,13), d(8,13), d(9,13), d(10,13), d(12,13)\} = \min\{7,7,7,7,7,7,7,7\}$$

$= 7$

$i = 14$

$$\min\{d(4,14), d(1,14), d(3,14), d(6,14), d(8,14), d(9,14), d(10,14), d(12,14)\} = \min\{5,5,5,5,5,5,5,5\}$$

$= 5$

$i = 15$

$$\min\{d(4,15), d(1,15), d(3,15), d(6,15), d(8,15), d(9,15), d(10,15), d(12,15)\} = \min\{2,2,2,2,2,2,2,2\}$$

$= 2$

$i = 16$

$$\min\{d(4,16), d(1,16), d(3,16), d(6,16), d(8,16), d(9,16), d(10,16), d(12,16)\} = \min\{7,7,7,7,7,7,7,7\}$$

$= 7$

i

$\square 17$

min{ (4,17), (1,17), (3,17), (6,17), (8,17),
(d d d d d d 9,17), (10,17),
(12,17)}d d

min{8,8,8,8,8,8,8,8}

$\square 8$

i $\square 18$

min{ (4,18), (1,18), (3,18), (6,18), (8,18),
(d d d d d d 9,18), (10,18), (12,18)}d d
min{5,5,5,5,5,5,5,5}

$\square$ 5

i $\square 19$

min{ (4,19), (1,19), (3,19), (6,19), (8,19),
(d d d d d d 9,19), (10,19), (12,19)}dd
min{3,3,3,3,3,3,3,3}

$\square$ 3

For node 5

i $\square 1$

min{ (5,1), (1,1), (3,1), (6,1), (8,1), (9,1),d d d d d

d d(10,1), (12,1)}d min{0,0,0,0,0,0,0,0}

$\square$ 0

i $\square 2$

min{ (5,2), (1,2), (3,2), (6,2), (8,2), (9,2),d d d d d

d d(10,2), (12,2)}d min{5,5,5,5,5,5,5,5}

$\square$ 5

i $\square$ 3

$\square$

i

min{ (5,3), (1,3), (3,3), (6,3), (8,3), (9,3),d d d d
d dd(10,3), (12,3)}d

min{0,0,0,0,0,0,0,0}

 0

i 4
min{ (5,4), (1,4), (3,4), (6,4), (8,4), (9,4),d d d d
d dd(10,4), (12,4)}d

min{3,5,5,5,5,5,5,5}

 3

i 5
min{ (5,5), (1,5), (3,5), (6,5), (8,5), (9,5),d d d d
d dd(10,5), (12,5)}d

min{0,2,2,2,2,2,2,2}
0
 6
min{ (5,6), (1,6), (3,6), (6,6), (8,6), (9,6),d d d d
d d(10,6), (12,6)}d

min{0,0,0,0,0,0,0,0}

 0

i 7
min{ (5,7), (1,7), (3,7), (6,7), (8,7), (9,7),d d d d
d dd(10,7), (12,7)}d
min{2,2,2,2,2,2,2,2}

 2

i 8
min{ (5,8), (1,8), (3,8), (6,8), (8,8), (9,8),d d d d
d dd(10,8), (12,8)}d

i

d

min{0,0,0,0,0,0,0,0}

 0

i 9

min{ (5,9), (1,9), (3,9), (6,9), (8,9), (9,9),*d* *d* *d* *d*
d *dd*(10,9), (12,9)}*d*

min{0,0,0,0,0,0,0,0}

 0 *i*

10

min{ (5,10), (1,10), (3,10), (6,10), (8,10),
(*d* *d* *d* *d* *d* *d* 9,10), (10,10), (12,10)}*d d*
min{0,0,0,0,0,0,0,0}

 0

i 11

min{ (5,11), (1,11), (3,11), (6,11), (8,11), (*d* *d* *d* *d*
dd 9,11), (10,11), (12,11)}*d* *d*
min{5,5,5,5,5,5,5,5}

 5

i 12

min{ (5,12), (1,12), (3,12), (6,12), (8,12),
(*d* *d* *d* *d* *d* *d* 9,12), (10,12), (12,12)}*d d*
min{0,0,0,0,0,0,0,0}

 0

i 13

$i = 13$

$d = \min\{d(5,13), d(1,13), d(3,13), d(6,13), d(8,13), d(9,13), d(10,13), d(12,13)\}$
$= \min\{7,7,7,7,7,7,7,7\}$

$= 7$

$i = 14$

$d = \min\{d(5,14), d(1,14), d(3,14), d(6,14), d(8,14), d(9,14), d(10,14), d(12,14)\}$
$= \min\{5,5,5,5,5,5,5,5\}$

$= 5$

$i = 15$

$d = \min\{d(5,15), d(1,15), d(3,15), d(6,15), d(8,15), d(9,15), d(10,15), d(12,15)\}$
$= \min\{2,2,2,2,2,2,2,2\}$

$= 2$

$i = 16$

$d = \min\{d(5,16), d(1,16), d(3,16), d(6,16), d(8,16), d(9,16), d(10,16), d(12,16)\}$
$= \min\{7,7,7,7,7,7,7,7\}$

$= 7$

$i = 17$

$d = \min\{d(5,17), d(1,17), d(3,17), d(6,17), d(8,17), d(9,17), d(10,17), d(12,17)\}$
$= \min\{8,8,8,8,8,8,8,8\}$

$= 8$

$i = 18$

$d = \min\{d(5,18), d(1,18), d(3,18), d(6,18), d(8,18), d(9,18), d(10,18), d(12,18)\}$
$= \min\{5,5,5,5,5,5,5,5\}$

$= 5$

i

d

i 19

min{ (5,19), (1,19), (3,19), (6,19), (8,19),

(*d* *d* *d* *d* *d* *d* 9,19), (10,19), (12,19)}*d* *d*

min{3,3,3,3,3,3,3,3,3}

 3

For node 6

i 1

min{ (6,1), (1,1), (3,1), (6,1), (8,1), (9,1),*d* *d* *d* *d* *d*

d *d*(10,1), (12,1)}*d* min{0,0,0,0,0,0,0,0}

 0

i 2

min{ (6,2), (1,2), (3,2), (6,2), (8,2), (9,2),*d* *d* *d* *d* *d*

d *d*(10,2), (12,2)}*d* min{5,5,5,5,5,5,5,5}

 5

d

i 3

min{ (6,3), (1,3), (3,3), (6,3), (8,3), (9,3),d d d d d

d (10,3), (12,3)}d min{0,0,0,0,0,0,0,0}

 0

i 4

min{ (6,4), (1,4), (3,4), (6,4), (8,4), (9,4),d d d d

d dd(10,4), (12,4)}d

min{5,5,5,5,5,5,5,5}

 5

i 5

min{ (6,5), (1,5), (3,5), (6,5), (8,5), (9,5),d d d d

d dd(10,5), (12,5)}d

min{2,2,2,2,2,2,2,2}

 2

i 6

min{ (6,6), (1,6), (3,6), (6,6), (8,6), (9,6),d d d d d

d d(10,6), (12,6)}d min{0,0,0,0,0,0,0,0}

 0

i 7

min{ (6,7), (1,7), (3,7), (6,7), (8,7), (9,7),d d d d

d dd(10,7), (12,7)}d

min{2,2,2,2,2,2,2,2}

 2

i 8

min{ (6,8), (1,8), (3,8), (6,8), (8,8), (9,8),d d d d d

d d(10,8), (12,8)}d min{0,0,0,0,0,0,0,0}

i

 0

i 9

min{ (6,9), (1,9), (3,9), (6,9), (8,9), (9,9),*d* *d* *d* *d* *d*

d *d*(10,9), (12,9)}*d* min{0,0,0,0,0,0,0,0}

 0

i 10

min{ (6,10), (1,10), (3,10), (6,10), (8,10), (*d d* *d* *d* *d*

d 9,10), (10,10), (12,10)}*d* *d* min{0,0,0,0,0,0,0,0}

 0

11

min{ (6,11), (1,11), (3,11), (6,11), (8,11), (*d* *d* *d* *d*

dd 9,11), (10,11), (12,11)}*d* *d*

min{5,5,5,5,5,5,5,5}

 5

i 12

min{ (6,12), (1,12), (3,12), (6,12), (8,12), (*d d* *d* *d* *d*

d 9,12), (10,12), (12,12)}*d* *d* min{0,0,0,0,0,0,0,0}

 0

i 13

min{ (6,13), (1,13), (3,13), (6,13), (8,13),

(*d* *d* *d* *d* *d* *d* 9,13), (10,13), (12,13)}*d d*

min{7,7,7,7,7,7,7,7}

 7

i 14

min{ (6,14), (1,14), (3,14), (6,14), (8,14),

(*d* *d* *d* *d* *d* *d* 9,14), (10,14), (12,14)}*dd*

min{5,5,5,5,5,5,5,5}

 5

i 15

i

min{ (6,15), (1,15), (3,15), (6,15), (8,15),
(*d* *d* *d* *d* *d* *d* 9,15), (10,15), (12,15)}*d d*
min{2,2,2,2,2,2,2,2}

□ 2

i □16
min{ (6,16), (1,16), (3,16), (6,16), (8,16),
(*d* *d* *d* *d* *d* *d* 9,16), (10,16), (12,16)}*d d*
min{7,7,7,7,7,7,7,7}

□ 7

i □17
min{ (6,17), (1,17), (3,17), (6,17), (8,17),
(*d* *d* *d* *d* *d* *d* 9,17), (10,17),
(12,17)}*d* *d*
min{8,8,8,8,8,8,8,8}

□8

i □18
min{ (6,18), (1,18), (3,18), (6,18), (8,18),
(*d* *d* *d* *d* *d* *d* 9,18), (10,18), (12,18)}*d d*
min{5,5,5,5,5,5,5,5}

□ 5

□19
min{ (6,19), (1,19), (3,19), (6,19), (8,19),
(*d* *d* *d* *d* *d* *d* 9,19), (10,19), (12,19)}*d d*
min{3,3,3,3,3,3,3,3,3}

□ 3
For node 7
i □1
min{ (7,1), (1,1), (3,1), (6,1), (8,1), (9,1),*d* *d* *d* *d* *d*
d *d*(10,1), (12,1)}*d* min{0,0,0,0,0,0,0,0}

i
= 0

i = 2
min{ (7,2), (1,2), (3,2), (6,2), (8,2), (9,2),*d* *d* *d* *d* *d*
d *d*(10,2), (12,2)}*d* min{5,5,5,5,5,5,5,5}

= 5

i = 3
min{ (7,3), (1,3), (3,3), (6,3), (8,3), (9,3),*d* *d* *d* *d*
d *dd*(10,3), (12,3)}*d*

min{0,0,0,0,0,0,0,0}

= 0

i = 4
min{ (7,4), (1,4), (3,4), (6,4), (8,4), (9,4),*d* *d* *d* *d*
d *dd*(10,4), (12,4)}*d*

min{3,5,5,5,5,5,5,5}

= 3

i = 5
min{ (7,5), (1,5), (3,5), (6,5), (8,5), (9,5),*d* *d* *d* *d*
d *dd*(10,5), (12,5)}*d*

min{2,2,2,2,2,2,2,2}

= 2

i = 6
min{ (7,6), (1,6), (3,6), (6,6), (8,6), (9,6),*d* *d* *d* *d*
d *dd*(10,6), (12,6)}*d*

min{0,0,0,0,0,0,0,0}

= 0

i = 7
min{ (7,7), (1,7), (3,7), (6,7), (8,7), (9,7),*d* *d* *d* *d*
d *dd*(10,7), (12,7)}*d*

i

min{0,2,2,2,2,2,2,2,2}

□ 0

□8

min{ (7,8), (1,8), (3,8), (6,8), (8,8), (9,8),*d* *d* *d* *d*

d *dd*(10,8), (12,8)}*d*

min{0,0,0,0,0,0,0,0,0}

□ 0

i □ 9

min{ (7,9), (1,9), (3,9), (6,9), (8,9), (9,9),*d* *d* *d* *d*

d *dd*(10,9), (12,9)}*d*

min{0,0,0,0,0,0,0,0,0}

□ 0 *i*

□10

min{ (7,10), (1,10), (3,10), (6,10), (8,10),

(*d* *d* *d* *d* *d* *d* 9,10), (10,10), (12,10)}*dd*

min{0,0,0,0,0,0,0,0,0}

□ 0

i □11

min{ (7,11), (1,11), (3,11), (6,11), (8,11), (*d* *d* *d* *d*

dd 9,11), (10,11), (12,11)}*d* *d*

min{5,5,5,5,5,5,5,5,5}

□ 5

i □12

min{ (7,12), (1,12), (3,12), (6,12), (8,12),

(*d* *d* *d* *d* *d* *d* 9,12), (10,12), (12,12)}*dd*

min{0,0,0,0,0,0,0,0,0}

□ 0

i

i = 13
min{ (7,13), (1,13), (3,13), (6,13), (8,13),
(*d* *d* *d* *d* *d* *d* 9,13), (10,13), (12,13)}*d d*
min{7,7,7,7,7,7,7,7}

= 7

i = 14
min{ (7,14), (1,14), (3,14), (6,14), (8,14),
(*d* *d* *d* *d* *d* *d* 9,14), (10,14), (12,14)}*dd*
min{5,5,5,5,5,5,5,5}

= 5

i = 15
min{ (7,15), (1,15), (3,15), (6,15), (8,15),
(*d* *d* *d* *d* *d* *d* 9,15), (10,15), (12,15)}*d d*
min{2,2,2,2,2,2,2,2}

= 2

= 16
min{ (7,16), (1,16), (3,16), (6,16), (8,16),
(*d* *d* *d* *d* *d* *d* 9,16), (10,16), (12,16)}*dd*
min{7,7,7,7,7,7,7,7}

= 7

i = 17
min{ (7,17), (1,17), (3,17), (6,17), (8,17),
(*d* *d* *d* *d* *d* *d* 9,17), (10,17),
(12,17)}*d* *d*
min{8,8,8,8,8,8,8,8}

=8

i = 18
min{ (7,18), (1,18), (3,18), (6,18), (8,18),
(*d* *d* *d* *d* *d* *d* 9,18), (10,18), (12,18)}*d d*
min{5,5,5,5,5,5,5,5}

= 5

i

i □19

min{ (7,19), (1,19), (3,19), (6,19), (8,19),

(*d* *d* *d* *d* *d* *d* 9,19), (10,19), (12,19)}*dd*

min{3,3,3,3,3,3,3,3,3}

□ 3

For node 8

i □1

min{ (8,1), (1,1), (3,1), (6,1), (8,1), (9,1),*d* *d* *d* *d* *d*

d *d*(10,1), (12,1)}*d* min{0,0,0,0,0,0,0,0}

□ 0

i □ 2

min{ (8,2), (1,2), (3,2), (6,2), (8,2), (9,2),*d* *d* *d* *d* *d*

d *d*(10,2), (12,2)}*d* min{5,5,5,5,5,5,5,5}

□ 5

i □3

min{ (8,3), (1,3), (3,3), (6,3), (8,3), (9,3),*d* *d* *d* *d* *d*

d *d*(10,3), (12,3)}*d* min{0,0,0,0,0,0,0,0}

□ 0

i □ 4

min{ (8,4), (1,4), (3,4), (6,4), (8,4), (9,4),*d* *d* *d* *d*

d *dd*(10,4), (12,4)}*d*

min{5,5,5,5,5,5,5,5}

□ 5

□ 5

min{ (8,5), (1,5), (3,5), (6,5), (8,5), (9,5),*d* *d* *d* *d*

d *dd*(10,5), (12,5)}*d*

min{2,2,2,2,2,2,2,2}

i
$= 2$

$i = 6$
$\min\{d(8,6), d(1,6), d(3,6), d(6,6), d(8,6), d(9,6), d(10,6), d(12,6)\} = \min\{0,0,0,0,0,0,0,0\}$

$= 0$

$i = 7$
$\min\{d(8,7), d(1,7), d(3,7), d(6,7), d(8,7), d(9,7), d(10,7), d(12,7)\}$

$\min\{2,2,2,2,2,2,2,2\}$

$= 2$

$i = 8$
$\min\{d(8,8), d(1,8), d(3,8), d(6,8), d(8,8), d(9,8), d(10,8), d(12,8)\} = \min\{0,0,0,0,0,0,0,0\}$

$= 0$
$i = 9$
$\min\{d(8,9), d(1,9), d(3,9), d(6,9), d(8,9), d(9,9), d(10,9), d(12,9)\} = \min\{0,0,0,0,0,0,0,0\}$

$= 0$

$i = 10$
$\min\{d(8,10), d(1,10), d(3,10), d(6,10), d(8,10), d(9,10), d(10,10), d(12,10)\} = \min\{0,0,0,0,0,0,0,0\}$

$= 0$

$i = 11$
$\min\{d(8,11), d(1,11), d(3,11), d(6,11), d(8,11), d(9,11), d(10,11), d(12,11)\}$
$\min\{5,5,5,5,5,5,5,5\}$

$= 5$

$i = 12$

i

$\min\{d(8,12), d(1,12), d(3,12), d(6,12), d(8,12), d(9,12), d(10,12), d(12,12)\} = \min\{0,0,0,0,0,0,0,0\}$

$= 0$

$i = 13$

$\min\{d(8,13), d(1,13), d(3,13), d(6,13), d(8,13), d(9,13), d(10,13), d(12,13)\} = \min\{7,7,7,7,7,7,7,7\}$

$= 7$

$i = 14$

$\min\{d(8,14), d(1,14), d(3,14), d(6,14), d(8,14), d(9,14), d(10,14), d(12,14)\} = \min\{5,5,5,5,5,5,5,5\}$

$= 5$

$i = 15$

$\min\{d(8,15), d(1,15), d(3,15), d(6,15), d(8,15), d(9,15), d(10,15), d(12,15)\} = \min\{2,2,2,2,2,2,2,2\}$

$= 2$

$i = 16$

$\min\{d(8,16), d(1,16), d(3,16), d(6,16), d(8,16), d(9,16), d(10,16), d(12,16)\} = \min\{7,7,7,7,7,7,7,7\}$

$= 7$

$i = 17$

$\min\{d(8,17), d(1,17), d(3,17), d(6,17), d(8,17), d(9,17), d(10,17), d(12,17)\} = \min\{8,8,8,8,8,8,8,8\}$

$= 8$

i

i □18

min{ (8,18), (1,18), (3,18), (6,18), (8,18),

(d d d d d d 9,18), (10,18), (12,18)}d d

min{5,5,5,5,5,5,5,5}

□ 5

i □19

min{ (8,19), (1,19), (3,19), (6,19), (8,19),

(d d d d d d 9,19), (10,19), (12,19)}$d$$d$

min{3,3,3,3,3,3,3,3,3}

□ 3

For node 9

i □1

min{ (9,1), (1,1), (3,1), (6,1), (8,1), (9,1),d d d d d

d d(10,1), (12,1)}d min{0,0,0,0,0,0,0,0}

□ 0

i 2

min{ (9,2), (1,2), (3,2), (6,2), (8,2), (9,2),*d* *d* *d* *d* *d*
d *d*(10,2), (12,2)}*d* min{5,5,5,5,5,5,5,5}

 5

i 3

min{ (9,3), (1,3), (3,3), (6,3), (8,3), (9,3),*d* *d* *d* *d* *d*
d *d*(10,3), (12,3)}*d* min{0,0,0,0,0,0,0,0}

 0

i 4

min{ (9,4), (1,4), (3,4), (6,4), (8,4), (9,4),*d* *d* *d* *d*
d *dd*(10,4), (12,4)}*d*
min{5,5,5,5,5,5,5,5}

 5

i 5

min{ (9,5), (1,5), (3,5), (6,5), (8,5), (9,5),*d* *d* *d* *d*
d *dd*(10,5), (12,5)}*d*
min{2,2,2,2,2,2,2,2}

 2

i 6

min{ (9,6), (1,6), (3,6), (6,6), (8,6), (9,6),*d* *d* *d* *d* *d*
d *d*(10,6), (12,6)}*d* min{0,0,0,0,0,0,0,0}

 0

i 7

min{ (9,7), (1,7), (3,7), (6,7), (8,7), (9,7),*d* *d* *d* *d*
d *dd*(10,7), (12,7)}*d*
min{2,2,2,2,2,2,2,2}

 2

i 8

min{ (9,8), (1,8), (3,8), (6,8), (8,8), (9,8),*d* *d* *d* *d* *d*
d *d*(10,8), (12,8)}*d* min{0,0,0,0,0,0,0,0}

 0

i 9
min{ (9,9), (1,9), (3,9), (6,9), (8,9), (9,9),*d d d d d*
d d(10,9), (12,9)}*d* min{0,0,0,0,0,0,0,0}
 0

i 10
min{ (9,10), (1,10), (3,10), (6,10), (8,10), (*d d d d d*
d 9,10), (10,10), (12,10)}*d d* min{0,0,0,0,0,0,0,0}
 0

i 11
min{ (9,11), (1,11), (3,11), (6,11), (8,11), (*d d d d*
dd 9,11), (10,11), (12,11)}*d d*
min{5,5,5,5,5,5,5,5}
 5

i 12
min{ (9,12), (1,12), (3,12), (6,12), (8,12), (*d d d d d*
d 9,12), (10,12), (12,12)}*d d* min{0,0,0,0,0,0,0,0}
 0

i 13
min{ (9,13), (1,13), (3,13), (6,13), (8,13),
(*d d d d d d* 9,13), (10,13), (12,13)}*d d*
min{7,7,7,7,7,7,7,7}
 7

i 14
min{ (9,14), (1,14), (3,14), (6,14), (8,14),
(*d d d d d d* 9,14), (10,14), (12,14)}*dd*
min{5,5,5,5,5,5,5,5}
 5

i

i 15

min{ (9,15), (1,15), (3,15), (6,15), (8,15),
(d d d d d d 9,15), (10,15), (12,15)}d d
min{2,2,2,2,2,2,2,2}

 2

i 16

min{ (9,16), (1,16), (3,16), (6,16), (8,16),
(d d d d d d 9,16), (10,16), (12,16)}dd
min{7,7,7,7,7,7,7,7}

 7 i

17

min{ (9,17), (1,17), (3,17), (6,17), (8,17),
(d d d d d d 9,17), (10,17), (12,17)}dd
min{8,8,8,8,8,8,8,8}

8

18

min{ (9,18), (1,18), (3,18), (6,18), (8,18),
(d d d d d d 9,18), (10,18), (12,18)}d d
min{5,5,5,5,5,5,5,5}

 5

i 19

min{ (9,19), (1,19), (3,19), (6,19), (8,19),
(d d d d d d 9,19), (10,19), (12,19)}dd
min{3,3,3,3,3,3,3,3,3}

 3

For node

10 i 1

min{ (10,1), (1,1), (3,1), (6,1), (8,1), (9,1)d d d d d

d , (10,1), (12,1)}d d min{0,0,0,0,0,0,0,0}

 0

i □ 2

min{ (10,2), (1,2), (3,2), (6,2), (8,2), (9,2)*d* *d* *d* *d* *d*

d , (10,2), (12,2)}*d* *d* min{5,5,5,5,5,5,5,5}

□ 5

i □3

min{ (10,3), (1,3), (3,3), (6,3), (8,3), (9,3)*d* *d* *d* *d* *d*

d , (10,3), (12,3)}*d* *d* min{0,0,0,0,0,0,0,0}

□ 0

i □ 4

min{ (10,4), (1,4), (3,4), (6,4), (8,4), (9,4)*dd* *dd* *d*

d, (10,4), (12,4)}*d* *d*

min{5,5,5,5,5,5,5,5}

□ 5

i □ 5

min{ (10,5), (1,5), (3,5), (6,5), (8,5), (9,5)*d* *dd d* *d*

d, (10,5), (12,5)}*d* *d*

min{2,2,2,2,2,2,2,2}

□ 2

i □6

min{ (1,6), (1,6), (3,6), (6,6), (8,6), (9,6),*d* *d* *d* *d* *d*

d *d*(10,6), (12,6)}*d* min{0,0,0,0,0,0,0,0}

□ 0

i

□ 7

min{ (10,7), (1,7), (3,7), (6,7), (8,7), (9,7)*dd* *dd* *d*
d, (10,7), (12,7)}*d* *d*

min{2,2,2,2,2,2,2,2}

□ 2

i □8

min{ (10,8), (1,8), (3,8), (6,8), (8,8), (9,8)*d* *d* *d* *d* *d*
d , (10,8), (12,8)}*d* *d* min{0,0,0,0,0,0,0,0}

□ 0

i □ 9

min{ (10,9), (1,9), (3,9), (6,9), (8,9), (9,9)*d* *d* *d* *d* *d*
d , (10,9), (12,9)}*d* *d* min{0,0,0,0,0,0,0,0}

□ 0

i □10

min{ (10,10), (1,10), (3,10), (6,10), (8,10),*d* *d* *d* *d* *d*
d(9,10), (10,10), (12,10)}*d* *d* min{0,0,0,0,0,0,0,0}

□ 0

i □11

min{ (10,11), (1,11), (3,11), (6,11), (8,11),*d* *d* *d* *d*
dd(9,11), (10,11), (12,11)}*d* *d*
min{5,5,5,5,5,5,5,5}

□ 5

i □12

min{ (10,12), (1,12), (3,12), (6,12), (8,12),*d* *d* *d* *d* *d*
d(9,12), (10,12), (12,12)}*d* *d* min{0,0,0,0,0,0,0,0}

□ 0

i

i 13

min{ (10,13), (1,13), (3,13), (6,13), (8,13),d d d
d dd(9,13), (10,13), (12,13)}d d
min{7,7,7,7,7,7,7,7}

 7

i 14

min{ (10,14), (1,14), (3,14), (6,14), (8,14),d d d
d dd(9,14), (10,14), (12,14)}d d
min{5,5,5,5,5,5,5,5}

 5

15

min{ (10,15), (1,15), (3,15), (6,15), (8,15),d d d
d dd(9,15), (10,15), (12,15)}d d
min{2,2,2,2,2,2,2,2}

 2

i 16

min{ (10,16), (1,16), (3,16), (6,16), (8,16),d d d
d dd(9,16), (10,16), (12,16)}d d
min{7,7,7,7,7,7,7,7}

 7

i 17

min{ (10,17), (1,17), (3,17), (6,17), (8,17),d d d
d dd(9,17), (10,17), (12,17)}d d
min{8,8,8,8,8,8,8,8}

8

i 18

i

min{ (10,18), (1,18), (3,18), (6,18), (8,18),d d d
d dd(9,18), (10,18), (12,18)}d d
min{5,5,5,5,5,5,5,5}

 5

i 19
min{ (10,19), (1,19), (3,19), (6,19), (8,19),d d d
d dd(9,19), (10,19), (12,19)}d d
min{3,3,3,3,3,3,3,3,3}

 3

For node11 i
1

min{ (11,1), (1,1), (3,1), (6,1), (8,1), (9,1)d dd d d
d, (10,1), (12,1)}d d _

min{0,0,0,0,0,0,0,0}

 0

i 2
min{ (11,2), (1,2), (3,2), (6,2), (8,2), (9,2)d d d d d
d , (10,2), (12,2)}d d min{5,5,5,5,5,5,5,5}

 5

 3
min{ (11,3), (1,3), (3,3), (6,3), (8,3), (9,3)d dd d d
d, (10,3), (12,3)}d d

min{0,0,0,0,0,0,0,0}

 0

i 4

i

$\min\{ (11,4), (1,4), (3,4), (6,4), (8,4), (9,4)dd \quad dd \quad d$
$d, (10,4), (12,4)\}d \quad d$
$\min\{5,5,5,5,5,5,5,5\}$

□ 5

i □ 5
$\min\{ (11,5), (1,5), (3,5), (6,5), (8,5), (9,5)d \quad dd\ d \quad d$
$d, (10,5), (12,5)\}d \quad d$
$\min\{2,2,2,2,2,2,2,2\}$

□ 2

i □ 6
$\min\{ (11,6), (1,6), (3,6), (6,6), (8,6), (9,6)d \quad ddd \quad d$
$d, (10,6), (12,6)\}d \quad d$
$\min\{0,0,0,0,0,0,0,0\}$

□ 0

i □ 7
$\min\{ (11,7), (1,7), (3,7), (6,7), (8,7), (9,7)dd \quad dd \quad d$
$d, (10,7), (12,7)\}d \quad d$
$\min\{2,2,2,2,2,2,2,2\}$

□ 2

i □8
$\min\{ (11,8), (1,8), (3,8), (6,8), (8,8), (9,8)d \quad dd\ d \quad d$
$d, (10,8), (12,8)\}d \quad d$

$\min\{0,0,0,0,0,0,0,0\}$
□ 0

i □ 9
$\min\{ (11,9), (1,9), (3,9), (6,9), (8,9), (9,9)d \quad dd\ d \quad d$
$d, (10,9), (12,9)\}d \quad d$

i

min{0,0,0,0,0,0,0,0}

□ 0 i

□10

min{ 11,10), (1,10), (3,10), (6,10), (8,10), (d d d d

dd 9,10), (10,10), (12,10)}d d

min{0,0,0,0,0,0,0,0}

□ 0

□11

min{ 11,11), (1,11), (3,11), (6,11), (8,11), (d d d d

dd 9,11), (10,11), (12,11)}d d

min{0,5,5,5,5,5,5,5}

□ 0

i □12

min{ (11,12), (1,12), (3,12), (6,12), (8,12),d d d

d dd(9,12), (10,12), (12,12)}d d

min{0,0,0,0,0,0,0,0}

□ 0

i □13

min{ (11,13), (1,13), (3,13), (6,13), (8,13),d d d

d dd(9,13), (10,13), (12,13)}d d

min{7,7,7,7,7,7,7,7}

□ 7

i □14

min{ (11,14), (1,14), (3,14), (6,14), (8,14),d d d

d dd(9,14), (10,14), (12,14)}d d

min{5,5,5,5,5,5,5,5}

□ 5

i □15

i

min{ (11,15), (1,15), (3,15), (6,15), (8,15),d d d
d dd(9,15), (10,15), (12,15)}d d
min{2,2,2,2,2,2,2,2}

□ 2

i □16
min{ (11,16), (1,16), (3,16), (6,16), (8,16),d d d
d dd(9,16), (10,16), (12,16)}d d
min{7,7,7,7,7,7,7,7}

□ 7

i □17
min{ (11,17), (1,17), (3,17), (6,17), (8,17),d d d
d dd(9,17), (10,17), (12,17)}d d
min{8,8,8,8,8,8,8,8}

□8

i □18
min{ (11,18), (1,18), (3,18), (6,18), (8,18),d d d
d dd(9,18), (10,18), (12,18)}d d
min{5,5,5,5,5,5,5,5}

□ 5

□19

min{ (11,19), (1,19), (3,19), (6,19), (8,19),*d d d d d*

(9,19), (10,19), (12,19)}*d d* min{3,3,3,3,3,3,3,3,3}

□ 3

For node 12

i □1

min{ (12,1), (1,1), (3,1), (6,1), (8,1), (9,1)*d d d d d*

d , (10,1), (12,1)}*d d* min{0,0,0,0,0,0,0,0}

□ 0

i □ 2

min{ (12,2), (1,2), (3,2), (6,2), (8,2), (9,2)*d d d d d*

d , (10,2), (12,2)}*d d* min{5,5,5,5,5,5,5,5}

□ 5

i □3

min{ (12,3), (1,3), (3,3), (6,3), (8,3), (9,3)*d d d d d*

d , (10,3), (12,3)}*d d* min{0,0,0,0,0,0,0,0}

□ 0

i □ 4

min{ (12,4), (1,4), (3,4), (6,4), (8,4), (9,4)*dd dd d*

d, (10,4), (12,4)}*d d*

min{5,5,5,5,5,5,5,5}

□ 5

i □ 5

min{ (12,5), (1,5), (3,5), (6,5), (8,5), (9,5)*d dd d d*

d, (10,5), (12,5)}*d d*

min{2,2,2,2,2,2,2,2}

□ 2

i □6

min{ (12,6), (1,6), (3,6), (6,6), (8,6), (9,6)*d d d d d*

d , (10,6), (12,6)}*d d* min{0,0,0,0,0,0,0,0}

□ 0

i

d

i □ 7

min{ (12,7), (1,7), (3,7), (6,7), (8,7), (9,7)*dd* *dd* *d*

d, (10,7), (12,7)}*d* *d*

min{2,2,2,2,2,2,2,2}

□ 2

i □8

min{ (12,8), (1,8), (3,8), (6,8), (8,8), (9,8)*d* *d* *d* *d* *d*

d , (10,8), (12,8)}*d* *d* min{0,0,0,0,0,0,0,0}

□ 0

i □ 9

min{ (12,9), (1,9), (3,9), (6,9), (8,9), (9,9)*d* *d* *d* *d* *d*

d , (10,9), (12,9)}*d* *d* min{0,0,0,0,0,0,0,0}

□ 0

i □10

min{ (12,10), (1,10), (3,10), (6,10), (8,10),*d* *d* *d* *d* *d*

d(9,10), (10,10), (12,10)}*d* *d* min{0,0,0,0,0,0,0,0}

□ 0

i □11

min{ (12,11), (1,11), (3,11), (6,11), (8,11),*d* *d* *d* *d*

dd(9,11), (10,11), (12,11)}*d* *d*

min{5,5,5,5,5,5,5,5}

□ 5

i □12

min{ (12,12), (1,12), (3,12), (6,12), (8,12),*d* *d* *d* *d* *d*

d(9,12), (10,12), (12,12)}*d* *d* min{0,0,0,0,0,0,0,0}

□ 0

i □13

min{ (12,13), (1,13), (3,13), (6,13), (8,13),*d* *d* *d*
d *dd*(9,13), (10,13), (12,13)}*d* *d*
min{7,7,7,7,7,7,7,7}

 7

i 14
min{ (12,14), (1,14), (3,14), (6,14), (8,14),*d* *d* *d*
d *dd*(9,14), (10,14), (12,14)}*d* *d*
min{5,5,5,5,5,5,5,5}

 5

i 15
min{ (12,15), (1,15), (3,15), (6,15), (8,15),*d* *d* *d*
d *dd*(9,15), (10,15), (12,15)}*d* *d*
min{2,2,2,2,2,2,2,2}

 2

16
min{ (12,16), (1,16), (3,16), (6,16), (8,16),*d* *d* *d* *d* *d*
(9,16), (10,16), (12,16)}*d* *d* min{7,7,7,7,7,7,7,7}

 7

i 17
min{ (12,17), (1,17), (3,17), (6,17), (8,17),*d* *d* *d*
d *dd*(9,17), (10,17), (12,17)}*d* *d*
min{8,8,8,8,8,8,8,8}

8

i 18
min{ (12,18), (1,18), (3,18), (6,18), (8,18),*d* *d* *d*
d *dd*(9,18), (10,18), (12,18)}*d* *d*
min{5,5,5,5,5,5,5,5}

 5

i 19
min{ (12,19), (1,19), (3,19), (6,19), (8,19),*d* *d* *d*
d *dd*(9,19), (10,19), (12,19)}*d* *d*
min{3,3,3,3,3,3,3,3,3}

i

d

$= 3$

For node13 i

$=1$

min{ (13,1), (1,1), (3,1), (6,1), (8,1), (9,1)d dd d d
d, (10,1), (12,1)}d d _

min{0,0,0,0,0,0,0,0}

$= 0$

$i = 2$

min{ (13,2), (1,2), (3,2), (6,2), (8,2), (9,2)d d d d d
d , (10,2), (12,2)}d d min{5,5,5,5,5,5,5,5}

$= 5$

$i = 3$

min{ (13,3), (1,3), (3,3), (6,3), (8,3), (9,3)d dd d d
d, (10,3), (12,3)}d d

min{0,0,0,0,0,0,0,0}

$= 0$

$i = 4$

min{ (13,4), (1,4), (3,4), (6,4), (8,4), (9,4)dd dd d
d, (10,4), (12,4)}d d

min{5,5,5,5,5,5,5,5}

$= 5$

$i = 5$

min{ (13,5), (1,5), (3,5), (6,5), (8,5), (9,5)d dd d d
d, (10,5), (12,5)}d d

min{2,2,2,2,2,2,2,2}

$= 2$

$i = 6$

min{ (13,6), (1,6), (3,6), (6,6), (8,6), (9,6)*dd* *ddd* *d* ,
(10,6), (12,6)}*d* *d*

min{0,0,0,0,0,0,0,0}

□ 0

i □ 7
min{ (13,7), (1,7), (3,7), (6,7), (8,7), (9,7)*dd* *dd* *d*
d, (10,7), (12,7)}*d* *d*
min{2,2,2,2,2,2,2,2}

□ 2

i □8
min{ (13,8), (1,8), (3,8), (6,8), (8,8), (9,8)*d* *dd* *d* *d*
d, (10,8), (12,8)}*d* *d*

min{0,0,0,0,0,0,0,0}

□ 0

i □ 9
min{ (13,9), (1,9), (3,9), (6,9), (8,9), (9,9)*d* *dd* *d* *d*
d, (10,9), (12,9)}*d* *d*

min{0,0,0,0,0,0,0,0}

□ 0

i □10
min{ 13,10), (1,10), (3,10), (6,10), (8,10), (*d* *d* *d*
d *dd* 9,10), (10,10), (12,10)}*d* *d*
min{0,0,0,0,0,0,0,0}

□ 0

i □11
min{ 13,11), (1,11), (3,11), (6,11), (8,11), (*d* *d* *d* *d*
dd 9,11), (10,11), (12,11)}*d* *d*
min{5,5,5,5,5,5,5,5}

□ 5

i □12

i

d

min{ (13,12), (1,12), (3,12), (6,12), (8,12),*d* *d* *d*
d *dd*(9,12), (10,12), (12,12)}*d* *d*
min{0,0,0,0,0,0,0,0}

 0

13
min{ (13,13), (1,13), (3,13), (6,13), (8,13),*d* *d* *d* *d* *d*
(9,13), (10,13), (12,13)}*d* *d* min{0,7,7,7,7,7,7,7}

 0

i 14
min{ (13,14), (1,14), (3,14), (6,14), (8,14),*d* *d* *d*
d *dd*(9,14), (10,14), (12,14)}*d* *d*
min{2,5,5,5,5,5,5,5}

 2

i 15
min{ (13,15), (1,15), (3,15), (6,15), (8,15),*d* *d* *d*
d *dd*(9,15), (10,15), (12,15)}*d* *d*
min{2,2,2,2,2,2,2,2}

 2 *i*
16
min{ (13,16), (1,16), (3,16), (6,16), (8,16),*d* *d* *d*
d *dd*(9,16), (10,16), (12,16)}*d* *d*
min{3,7,7,7,7,7,7,7}

 3

i 17
min{ (13,17), (1,17), (3,17), (6,17), (8,17),*d* *d* *d*
d *dd*(9,17), (10,17), (12,17)}*d* *d*
min{6,8,8,8,8,8,8,8}

 6

i 18

min{ (13,18), (1,18), (3,18), (6,18), (8,18),*d* *d* *d*
d *dd*(9,18), (10,18), (12,18)}*d* *d*
min{5,5,5,5,5,5,5,5}

□ 5

i □19
min{ (13,19), (1,19), (3,19), (6,19), (8,19),*d* *d* *d*
d *dd*(9,19), (10,19), (12,19)}*d* *d*
min{3,3,3,3,3,3,3,3,3}

□ 3

For node14 *i*
□1

min{ (14,1), (1,1), (3,1), (6,1), (8,1), (9,1)*d* *dd* *d* *d*
d, (10,1), (12,1)}*d* *d* _

min{0,0,0,0,0,0,0,0}

□ 0

i □ 2
min{ (14,2), (1,2), (3,2), (6,2), (8,2), (9,2)*d* *d* *d* *d* *d*
d , (10,2), (12,2)}*d* *d* min{5,5,5,5,5,5,5,5}

□ 5

i □ 3
min{ (14,3), (1,3), (3,3), (6,3), (8,3), (9,3)*d* *dd* *d* *d*
d, (10,3), (12,3)}*d* *d*
min{0,0,0,0,0,0,0,0}

□ 0

i □ 4
min{ (14,4), (1,4), (3,4), (6,4), (8,4), (9,4)*dd* *dd* *d*
d, (10,4), (12,4)}*d* *d*
min{5,5,5,5,5,5,5,5}

□ 5

i □ 5
min{ (14,5), (1,5), (3,5), (6,5), (8,5), (9,5)*d* *dd* *d* *d*
d, (10,5), (12,5)}*d* *d*

i

d

min{2,2,2,2,2,2,2,2}

 2

i 6

min{ (14,6), (1,6), (3,6), (6,6), (8,6), (9,6)*dd* *ddd* *d* ,
(10,6), (12,6)}*d* *d*

min{0,0,0,0,0,0,0,0}

 0

i 7

min{ (14,7), (1,7), (3,7), (6,7), (8,7), (9,7)*dd* *dd* *d*
d, (10,7), (12,7)}*d* *d*

min{2,2,2,2,2,2,2,2}

 2

i 8

min{ (14,8), (1,8), (3,8), (6,8), (8,8), (9,8)*d* *dd* *d* *d*
d, (10,8), (12,8)}*d* *d*

min{0,0,0,0,0,0,0,0}

 0

i 9

min{ (14,9), (1,9), (3,9), (6,9), (8,9), (9,9)*d* *dd* *d* *d*
d, (10,9), (12,9)}*d* *d*

min{0,0,0,0,0,0,0,0}

 0

i

□10
min{ 14,10), (1,10), (3,10), (6,10), (8,10), (*dd* *d* *d* *d*
d 9,10), (10,10), (12,10)}*d* *d* min{0,0,0,0,0,0,0,0}

□ 0

i □11
min{ 14,11), (1,11), (3,11), (6,11), (8,11), (*d* *d* *d* *d*
dd 9,11), (10,11), (12,11)}*d* *d*
min{5,5,5,5,5,5,5,5}

□ 5 *i*
□12
min{ (14,12), (1,12), (3,12), (6,12), (8,12),*d* *d* *d*
d *dd*(9,12), (10,12), (12,12)}*d* *d*
min{0,0,0,0,0,0,0,0}

□ 0

i □13
min{ (14,13), (1,13), (3,13), (6,13), (8,13),*d* *d* *d*
d *dd*(9,13), (10,13), (12,13)}*d* *d*
min{2,7,7,7,7,7,7,7}

□ 2

i □14
min{ (14,14), (1,14), (3,14), (6,14), (8,14),*d* *d* *d*
d *dd*(9,14), (10,14), (12,14)}*d* *d*
min{0,5,5,5,5,5,5,5}

□ 0

i □15
min{ (14,15), (1,15), (3,15), (6,15), (8,15),*d* *d* *d*
d *dd*(9,15), (10,15), (12,15)}*d* *d*
min{2,2,2,2,2,2,2,2}

□ 2

i

i 16

min{ (14,16), (1,16), (3,16), (6,16), (8,16),d d d
d dd(9,16), (10,16), (12,16)}d d
min{5,7,7,7,7,7,7,7}

 5

i 17

min{ (14,17), (1,17), (3,17), (6,17), (8,17),d d d
d dd(9,17), (10,17), (12,17)}d d
min{8,8,8,8,8,8,8,8}

8

18

min{ (14,18), (1,18), (3,18), (6,18), (8,18),d d d d d
(9,18), (10,18), (12,18)}d d min{5,5,5,5,5,5,5,5}

 5

i 19

min{ (14,19), (1,19), (3,19), (6,19), (8,19),d d d
d dd(9,19), (10,19), (12,19)}d d
min{3,3,3,3,3,3,3,3,3}

 3

For node15 i

1

min{ (15,1), (1,1), (3,1), (6,1), (8,1), (9,1)d dd d d
d, (10,1), (12,1)}d d _

min{0,0,0,0,0,0,0,0}

 0

i 2

i

d

min{ (15,2), (1,2), (3,2), (6,2), (8,2), (9,2)*d* *d* *d* *d* *d*
d , (10,2), (12,2)}*d* *d* min{5,5,5,5,5,5,5,5}

 5

i 3
min{ (15,3), (1,3), (3,3), (6,3), (8,3), (9,3)*d* *dd* *d* *d*
d, (10,3), (12,3)}*d* *d*

min{0,0,0,0,0,0,0,0}

 0

i 4
min{ (15,4), (1,4), (3,4), (6,4), (8,4), (9,4)*dd* *dd* *d*
d, (10,4), (12,4)}*d* *d*

min{5,5,5,5,5,5,5,5}

 5

i 5
min{ (15,5), (1,5), (3,5), (6,5), (8,5), (9,5)*d* *dd* *d* *d*
d, (10,5), (12,5)}*d* *d*

min{2,2,2,2,2,2,2,2}

 2

i 6
min{ (15,6), (1,6), (3,6), (6,6), (8,6), (9,6)*dd* *ddd* *d* ,
(10,6), (12,6)}*d* *d*

min{0,0,0,0,0,0,0,0}

 0

 7

i

min{ (15,7), (1,7), (3,7), (6,7), (8,7), (9,7)*dd* *dd* *d*
d, (10,7), (12,7)}*d* *d*
min{2,2,2,2,2,2,2,2}

 2

i 8
min{ (15,8), (1,8), (3,8), (6,8), (8,8), (9,8)*d* *dd* *d* *d*
d, (10,8), (12,8)}*d* *d*

min{0,0,0,0,0,0,0,0}

 0

i 9
min{ (15,9), (1,9), (3,9), (6,9), (8,9), (9,9)*d* *dd* *d* *d*
d, (10,9), (12,9)}*d* *d*

min{0,0,0,0,0,0,0,0}

 0

i 10
min{ 15,10), (1,10), (3,10), (6,10), (8,10), (*d* *d* *d*
d *dd* 9,10), (10,10), (12,10)}*d* *d*
min{0,0,0,0,0,0,0,0}

 0

i 11
min{ 15,11), (1,11), (3,11), (6,11), (8,11), (*d* *d* *d* *d*
dd 9,11), (10,11), (12,11)}*d* *d*
min{5,5,5,5,5,5,5,5}

 5

i 12
min{ (15,12), (1,12), (3,12), (6,12), (8,12),*d* *d* *d*
d *dd*(9,12), (10,12), (12,12)}*d* *d*

i

d

min{0,0,0,0,0,0,0,0}

□ 0

i □13

min{ (15,13), (1,13), (3,13), (6,13), (8,13),d d d
d dd(9,13), (10,13), (12,13)}d d
min{7,7,7,7,7,7,7,7}

□ 7

i □14

min{ (15,14), (1,14), (3,14), (6,14), (8,14),d d d
d dd(9,14), (10,14), (12,14)}d d
min{5,5,5,5,5,5,5,5}

□ 5

□15

min{ (15,15), (1,15), (3,15), (6,15), (8,15),d d d d d
(9,15), (10,15), (12,15)}d d min{0,2,2,2,2,2,2,2}

□ 0 i

□16

min{ (15,16), (1,16), (3,16), (6,16), (8,16),d d d
d dd(9,16), (10,16), (12,16)}d d
min{7,7,7,7,7,7,7,7}

□ 7

i □17

min{ (15,17), (1,17), (3,17), (6,17), (8,17),d d d
d dd(9,17), (10,17), (12,17)}d d
min{6,8,8,8,8,8,8,8}

□ 6

i □18

i

min{ (15,18), (1,18), (3,18), (6,18), (8,18),d d d
d dd(9,18), (10,18), (12,18)}d d
min{5,5,5,5,5,5,5,5,5}

 5

i 19

min{ (15,19), (1,19), (3,19), (6,19), (8,19),d d d
d dd(9,19), (10,19), (12,19)}d d
min{3,3,3,3,3,3,3,3,3}

 3

For node16 i

1

min{ (16,1), (1,1), (3,1), (6,1), (8,1), (9,1)d dd d d
d, (10,1), (12,1)}d d _

min{0,0,0,0,0,0,0,0,0}

 0

i 2

min{ (16,2), (1,2), (3,2), (6,2), (8,2), (9,2)d d d d d
d , (10,2), (12,2)}d d min{5,5,5,5,5,5,5,5,5}

 5

i 3

min{ (16,3), (1,3), (3,3), (6,3), (8,3), (9,3)d dd d d
d, (10,3), (12,3)}d d

min{0,0,0,0,0,0,0,0,0}

 0

 4

min{ (16,4), (1,4), (3,4), (6,4), (8,4), (9,4)dd dd d
d, (10,4), (12,4)}d d

i

d

min{5,5,5,5,5,5,5,5}

 5

i 5

min{ (16,5), (1,5), (3,5), (6,5), (8,5), (9,5)*d* *dd d* *d*
d, (10,5), (12,5)}*d* *d*

min{2,2,2,2,2,2,2,2}

 2

i 6

min{ (16,6), (1,6), (3,6), (6,6), (8,6), (9,6)*dd* *ddd* *d* ,
(10,6), (12,6)}*d* *d*

min{0,0,0,0,0,0,0,0}

 0

i 7

min{ (16,7), (1,7), (3,7), (6,7), (8,7), (9,7)*dd* *dd* *d*
d, (10,7), (12,7)}*d* *d*

min{2,2,2,2,2,2,2,2}

 2

i 8

min{ (16,8), (1,8), (3,8), (6,8), (8,8), (9,8)*d* *dd d* *d*
d, (10,8), (12,8)}*d* *d*

min{0,0,0,0,0,0,0,0}

 0

i 9

min{ (16,9), (1,9), (3,9), (6,9), (8,9), (9,9)*d* *ddd* *d*
d, (10,9), (12,9)}*d* *d*

min{0,0,0,0,0,0,0,0}

i

 0

i 10

min{ 16,10), (1,10), (3,10), (6,10), (8,10), (d d d
d dd 9,10), (10,10), (12,10)}d d
min{0,0,0,0,0,0,0,0}

 0

i 11

min{ 16,11), (1,11), (3,11), (6,11), (8,11), (d d d d
dd 9,11), (10,11), (12,11)}d d
min{5,5,5,5,5,5,5,5}

 5

12

min{ (16,12), (1,12), (3,12), (6,12), (8,12),d d d d d
(9,12), (10,12), (12,12)}d d min{0,0,0,0,0,0,0,0}

 0

i 13

min{ (16,13), (1,13), (3,13), (6,13), (8,13),d d d
d dd(9,13), (10,13), (12,13)}d d
min{3,7,7,7,7,7,7,7}

 3

i 14

min{ (16,14), (1,14), (3,14), (6,14), (8,14),d d d
d dd(9,14), (10,14), (12,14)}d d
min{5,5,5,5,5,5,5,5}

 5

i 15

i

d

min{ (16,15), (1,15), (3,15), (6,15), (8,15),d d d
d dd(9,15), (10,15), (12,15)}d d
min{2,2,2,2,2,2,2,2}

 2

i 16
min{ (16,16), (1,16), (3,16), (6,16), (8,16),d d d
d dd(9,16), (10,16), (12,16)}d d
min{0,7,7,7,7,7,7,7}

 0

i 17
min{ (16,17), (1,17), (3,17), (6,17), (8,17),d d d
d dd(9,17), (10,17), (12,17)}d d
min{3,8,8,8,8,8,8,8}

 3

i 18
min{ (16,18), (1,18), (3,18), (6,18), (8,18),d d d
d dd(9,18), (10,18), (12,18)}d d
min{5,5,5,5,5,5,5,5}

 5

i 19
min{ (16,19), (1,19), (3,19), (6,19), (8,19),d d d
d dd(9,19), (10,19), (12,19)}d d
min{3,3,3,3,3,3,3,3,3}

 3

For node17

$i = 1$

min{ (17,1), (1,1), (3,1), (6,1), (8,1), (9,1)*d* *dd* *d* *d*
d, (10,1), (12,1)}*d* *d* _

min{0,0,0,0,0,0,0,0}

= 0

$i = 2$

min{ (17,2), (1,2), (3,2), (6,2), (8,2), (9,2)*d* *d* *d* *d* *d*
d , (10,2), (12,2)}*d* *d* min{5,5,5,5,5,5,5,5}

= 5

$i = 3$

min{ (17,3), (1,3), (3,3), (6,3), (8,3), (9,3)*d* *dd* *d* *d*
d, (10,3), (12,3)}*d* *d*
min{0,0,0,0,0,0,0,0}

= 0

$i = 4$

min{ (17,4), (1,4), (3,4), (6,4), (8,4), (9,4)*dd* *dd* *d*
d, (10,4), (12,4)}*d* *d*
min{5,5,5,5,5,5,5,5}

= 5

$i = 5$

min{ (17,5), (1,5), (3,5), (6,5), (8,5), (9,5)*d* *dd* *d* *d*
d, (10,5), (12,5)}*d* *d*
min{2,2,2,2,2,2,2,2}

= 2

$i = 6$

min{ (17,6), (1,6), (3,6), (6,6), (8,6), (9,6)*dd* *ddd* *d* ,
(10,6), (12,6)}*d* *d*
min{0,0,0,0,0,0,0,0}

= 0

$i = 7$

min{ (17,7), (1,7), (3,7), (6,7), (8,7), (9,7)*dd* *dd* *d*
d, (10,7), (12,7)}*d* *d*
min{2,2,2,2,2,2,2,2}

□ 2

i □8
min{ (17,8), (1,8), (3,8), (6,8), (8,8), (9,8)*d* *dd* *d* *d*
d, (10,8), (12,8)}*d* *d*
min{0,0,0,0,0,0,0,0}

□ 0

i □ 9
min{ (17,9), (1,9), (3,9), (6,9), (8,9), (9,9)*d* *ddd* *d*
d, (10,9), (12,9)}*d* *d*

min{0,0,0,0,0,0,0,0}

□ 0

i □10
min{ 17,10), (1,10), (3,10), (6,10), (8,10), (*d* *d* *d*
d *dd* 9,10), (10,10), (12,10)}*d* *d*
min{0,0,0,0,0,0,0,0}

□ 0

i □11
min{ 17,11), (1,11), (3,11), (6,11), (8,11), (*d* *d* *d* *d*
dd 9,11), (10,11), (12,11)}*d* *d*
min{5,5,5,5,5,5,5,5}

□ 5

i □12
min{ (17,12), (1,12), (3,12), (6,12), (8,12),*d* *d* *d*
d *dd*(9,12), (10,12), (12,12)}*d* *d*
min{0,0,0,0,0,0,0,0}

□ 0

i □13

i

min{ (17,13), (1,13), (3,13), (6,13), (8,13),*d* *d* *d*
d *dd*(9,13), (10,13), (12,13)}*d* *d*
min{6,7,7,7,7,7,7,7}

□ 6

i □14
min{ (17,14), (1,14), (3,14), (6,14), (8,14),*d* *d* *d*
d *dd*(9,14), (10,14), (12,14)}*d* *d*
min{5,5,5,5,5,5,5,5}

□ 5

i □15
min{ (17,15), (1,15), (3,15), (6,15), (8,15),*d* *d* *d*
d *dd*(9,15), (10,15), (12,15)}*d* *d*
min{2,2,2,2,2,2,2,2}

□ 2 *i*
□16
min{ (17,16), (1,16), (3,16), (6,16), (8,16),*d* *d* *d*
d *dd*(9,16), (10,16), (12,16)}*d* *d*
min{3,7,7,7,7,7,7,7}

□ 3

□17
min{ (17,17), (1,17), (3,17), (6,17), (8,17),*d* *d* *d*
d *dd*(9,17), (10,17), (12,17)}*d* *d*
min{0,8,8,8,8,8,8,8}

□ 0

i □18
min{ (17,18), (1,18), (3,18), (6,18), (8,18),*d* *d* *d*
d *dd*(9,18), (10,18), (12,18)}*d* *d*
min{5,5,5,5,5,5,5,5}

□ 5

i □19

min{ (17,19), (1,19), (3,19), (6,19), (8,19),*d* *d* *d*
d *dd*(9,19), (10,19), (12,19)}*d* *d*
min{3,3,3,3,3,3,3,3,3}

 3

For node18 *i*
1

min{ (18,1), (1,1), (3,1), (6,1), (8,1), (9,1)*d* *dd* *d* *d*
d, (10,1), (12,1)}*d* *d* _

min{0,0,0,0,0,0,0,0}

 0

i 2
min{ (18,2), (1,2), (3,2), (6,2), (8,2), (9,2)*d* *d* *d* *d* *d*
d , (10,2), (12,2)}*d* *d* min{5,5,5,5,5,5,5,5}

 5

i 3
min{ (18,3), (1,3), (3,3), (6,3), (8,3), (9,3)*d* *dd* *d* *d*
d, (10,3), (12,3)}*d* *d*

min{0,0,0,0,0,0,0,0}

 0

i 4
min{ (18,4), (1,4), (3,4), (6,4), (8,4), (9,4)*dd* *dd* *d*
d, (10,4), (12,4)}*d* *d*

min{5,5,5,5,5,5,5,5}

 5

i 5
min{ (18,5), (1,5), (3,5), (6,5), (8,5), (9,5)*d* *dd* *d* *d*
d, (10,5), (12,5)}*d* *d*

min{2,2,2,2,2,2,2,2}

 2

i

i □ 6
min{ (18,6), (1,6), (3,6), (6,6), (8,6), (9,6)dd ddd d ,
(10,6), (12,6)}d d

min{0,0,0,0,0,0,0,0}

□ 0

i □ 7
min{ (18,7), (1,7), (3,7), (6,7), (8,7), (9,7)dd dd d
d, (10,7), (12,7)}d d
min{2,2,2,2,2,2,2,2}

□ 2

i □8
min{ (18,8), (1,8), (3,8), (6,8), (8,8), (9,8)d dd d d
d, (10,8), (12,8)}d d

min{0,0,0,0,0,0,0,0}

□ 0

i □ 9
min{ (18,9), (1,9), (3,9), (6,9), (8,9), (9,9)d dd d d
d, (10,9), (12,9)}d d

min{0,0,0,0,0,0,0,0}

□ 0

i □10
min{ 18,10), (1,10), (3,10), (6,10), (8,10), (d d d
d dd 9,10), (10,10), (12,10)}d d
min{0,0,0,0,0,0,0,0}

□ 0

i □11
min{ 18,11), (1,11), (3,11), (6,11), (8,11), (d d d d
dd 9,11), (10,11), (12,11)}d d

min{5,5,5,5,5,5,5,5}

□ 5 *i*

□12

min{ (18,12), (1,12), (3,12), (6,12), (8,12),*d* *d* *d*

d *dd*(9,12), (10,12), (12,12)}*d* *d*

min{0,0,0,0,0,0,0,0}

□ 0

i □13

min{ (18,13), (1,13), (3,13), (6,13), (8,13),*d* *d* *d*

d *dd*(9,13), (10,13), (12,13)}*d* *d*

min{7,7,7,7,7,7,7,7}

□ 7

□14

min{ (18,14), (1,14), (3,14), (6,14), (8,14),*d* *d* *d*

d *dd*(9,14), (10,14), (12,14)}*d* *d*

min{5,5,5,5,5,5,5,5}

□ 5

i □15

min{ (18,15), (1,15), (3,15), (6,15), (8,15),*d* *d* *d*

d *dd*(9,15), (10,15), (12,15)}*d* *d*

min{2,2,2,2,2,2,2,2}

□ 2

i □16

min{ (18,16), (1,16), (3,16), (6,16), (8,16),*d* *d* *d*

d *dd*(9,16), (10,16), (12,16)}*d* *d*

min{7,7,7,7,7,7,7,7}

□ 7

i □17

min{ (18,17), (1,17), (3,17), (6,17), (8,17),*d* *d* *d*

d *dd*(9,17), (10,17), (12,17)}*d* *d*

min{6,8,8,8,8,8,8,8}

□ 6

i

i 18

min{ (18,18), (1,18), (3,18), (6,18), (8,18),d d d d dd(9,18), (10,18), (12,18)}d d
min{0,5,5,5,5,5,5,5}

 0

i 19

min{ (18,19), (1,19), (3,19), (6,19), (8,19),d d d d dd(9,19), (10,19), (12,19)}d d
min{2,3,3,3,3,3,3,3,3}

 2

For node19 i

1

min{ (19,1), (1,1), (3,1), (6,1), (8,1), (9,1)d dd d d d, (10,1), (12,1)}d d _

min{0,0,0,0,0,0,0,0}

 0

i 2

min{ (19,2), (1,2), (3,2), (6,2), (8,2), (9,2)d d d d d d , (10,2), (12,2)}d d min{4,5,5,5,5,5,5,5}

 4

i

□ 3

min{ (19,3), (1,3), (3,3), (6,3), (8,3), (9,3)*d* *dd d* *d*
d, (10,3), (12,3)}*d* *d*

min{0,0,0,0,0,0,0,0}

□ 0

i □ 4

min{ (19,4), (1,4), (3,4), (6,4), (8,4), (9,4)*dd* *dd* *d*
d, (10,4), (12,4)}*d* *d*
min{5,5,5,5,5,5,5,5}

□ 5

i □ 5

min{ (19,5), (1,5), (3,5), (6,5), (8,5), (9,5)*d* *dd d* *d*
d, (10,5), (12,5)}*d* *d*

min{2,2,2,2,2,2,2,2}

□ 2

i □ 6

min{ (19,6), (1,6), (3,6), (6,6), (8,6), (9,6)*dd* *ddd* *d* ,
(10,6), (12,6)}*d* *d*
min{0,0,0,0,0,0,0,0}

□ 0

i □ 7

min{ (19,7), (1,7), (3,7), (6,7), (8,7), (9,7)*dd* *dd* *d*
d, (10,7), (12,7)}*d* *d*
min{2,2,2,2,2,2,2,2}

□ 2

i □8

i

min{ (19,8), (1,8), (3,8), (6,8), (8,8), (9,8)*d* *dd* *d* *d*
d, (10,8), (12,8)}*d* *d*

min{0,0,0,0,0,0,0,0,0}

 0

i 9
min{ (19,9), (1,9), (3,9), (6,9), (8,9), (9,9)*d* *dd* *d* *d*
d, (10,9), (12,9)}*d* *d*
min{0,0,0,0,0,0,0,0,0}

 0

i 10
min{ 19,10), (1,10), (3,10), (6,10), (8,10), (*d* *d* *d*
d *dd* 9,10), (10,10), (12,10)}*d* *d*
min{0,0,0,0,0,0,0,0,0}

 0

11
min{ 19,11), (1,11), (3,11), (6,11), (8,11), (*d* *d* *d* *d*
dd 9,11), (10,11), (12,11)}*d* *d*
min{5,5,5,5,5,5,5,5,5}

 5

i 12
min{ (19,12), (1,12), (3,12), (6,12), (8,12),*d* *d* *d*
d *dd*(9,12), (10,12), (12,12)}*d* *d*
min{0,0,0,0,0,0,0,0,0}

 0

i 13
min{ (19,13), (1,13), (3,13), (6,13), (8,13),*d* *d* *d*
d *dd*(9,13), (10,13), (12,13)}*d* *d*
min{7,7,7,7,7,7,7,7,7}

i

$= 7$

$i = 14$

$\min\{d(19,14), d(1,14), d(3,14), d(6,14), d(8,14), d(9,14), d(10,14), d(12,14)\}$

$\min\{5,5,5,5,5,5,5,5\}$

$= 5$

$i = 15$

$\min\{d(19,15), d(1,15), d(3,15), d(6,15), d(8,15), d(9,15), d(10,15), d(12,15)\}$

$\min\{2,2,2,2,2,2,2,2\}$

$= 2$

$i = 16$

$\min\{d(19,16), d(1,16), d(3,16), d(6,16), d(8,16), d(9,16), d(10,16), d(12,16)\}$

$\min\{7,7,7,7,7,7,7,7\}$

$= 7$

$i = 17$

$\min\{d(19,17), d(1,17), d(3,17), d(6,17), d(8,17), d(9,17), d(10,17), d(12,17)\}$

$\min\{8,8,8,8,8,8,8,8\}$

$= 8$

$i = 18$

$\min\{d(19,18), d(1,18), d(3,18), d(6,18), d(8,18), d(9,18), d(10,18), d(12,18)\}$

$\min\{2,5,5,5,5,5,5,5\}$

$= 2$

$= 19$

$\min\{d(19,19), d(1,19), d(3,19), d(6,19), d(8,19), d(9,19), d(10,19), d(12,19)\}$

$\min\{0,3,3,3,3,3,3,3,3\}$

$= 0$

i

i