LOCATION OF AN ADDITIONAL LANDFILL SITE IN FIVE SUB-METROS OF THE KUMASI METROPOLIS

By

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DECLARATION

I hereby declare that this submission is my own work towards the MSc. and that, to the best of my knowledge, it contains no material previously published by another person nor material which has been accepted for the award of any other degree of the University, except where due acknowledgment has been made in the text.

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ABSTRACT

A lot of studies and research have been done on location of landfill sites, fire stations, hospitals, ambulance and other such essential public facilities since their services have become an essential part of man's daily life. The services they provide do come in handy to the beneficiaries when the need arise.

Several methods have been used to solve these location problems. This thesis considers the problem of locating an additional landfill site (obnoxious facility) as a p-center problem under the condition that an existing facility is already located in the Kumasi Metropolis.

The Berman and Drezner (2008) was used to locate a landfill site in five sub metros put together as a network to form 56 nodes of the Kumasi Metropolis which had one existing facility.

Eight sites, namely Buokrom, Sepetimpo, Duase, Asabi, Asokore Mampong, Pakoso, Aperade and Manhyia were determined by the method.

Factor rating analysis was used to select Asokore Mampong and the farthest distance travelled to the landfill site at the new location (Asokore Mampong) was determined to be 17km. The facility would serve the residents of these sub metros effectively if they are sited at the place indicated.

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LIST OF ABBREVIATIONS

ACRONYMS	MEANING
MSPDM	Modified Shortest Path Distance Matrix
ONL	Optimal New Location
APSPDM	All Pairs Shortest Path Distance Matrix
APSP	All Pairs Shortest Path
WLPR	Weak Linear Programming Relaxation
WIP	Weak Integer Programming
SLPR	Strong Linear Programming Relaxation
MSWM	Municipal Solid Waste Management

DEDICATION

To the Glory of God

I dedicate this project to my dear wife, daughter and siblings for their prayers and support.

ACKNOWLEDGEMENT

It is with sincere gratitude that I take this opportunity to recognize those who have played a major role in bringing this work to its full realization. It has been satisfying to see all the pieces come together, often in ways much better than I expected.

It is therefore my pleasure to express my indebtedness to my supervisor, MR. KWEKU DARKWAH, Head of department of Mathematics, KNUST, who was always ready to assist me in one way or the other and also helped me in the compilation of this work, may the Almighty God richly bless him. I also express my profound gratitude to all the lecturers in the Mathematics Department who contributed in diverse ways for the successful completion of this project.

Also, to Mr. Jonathan Annan, Lecturer, School of Business, KNUST, I say may the Lord richly bless you and your family.

To Mr. and Mrs. Amissah Arthur, I say thank you for your prayers and support.

To my good friend Amo-Asante Kwadwo, I say thanks for assisting me with some vital information needed for this project.

CHAPTER 1

INTRODUCTION

1.1 BRIEF OVERVIEW OF CHAPTER 1

Facility location is the process of identifying the best location for a service commodity or production facility.

Location refers to the act of putting something in place where that thing can be identified. In a wide view, location problems deals with finding the appropriate site where one or more facilities should be placed, in order to optimize some specified criteria, which are usually related to the distance from the facilities to the demand points.

This optimization may differ depending on the particular objective function chosen. The function could be either; to minimize average travel time or cost, minimize average responds time, minimize maximum travel time or cost, and or maximize net income (Amponsah, 2007).

A facility is considered as a physical entity that provides services. Facility location problems arise in a wide set of practical applications in different fields of study: management, economy, production planning and many others (Peton, 2002), normally is classified into three categories: desirable (non-obnoxious), semi-obnoxious and obnoxious (non-desirable), (Welch et al., 1997).

We are interested in locating desirable facilities, in most location problems. Schools, hospitals, fire stations, post offices, warehouses, ambulances and production plants are all considered desirable facilities in this sense. A facility sometimes, produces a negative or undesirable effect, which may be present even though a high degree of accessibility is required to the facility. If the undesirable effect exceeds the accessibility required, then the facility is said to be obnoxious. Some examples are waste disposal sites, nuclear power stations, military installations, pollution producing industrial plants, recycling centers and airports. Although necessary for society, these facilities undesirable and often dangerous to their surroundings, (Erkut et al, 1989).

Brimberg and Juel introduced the term semi-desirable facility in 1998. They argued that the facilities cannot be classified as being purely desirable or purely obnoxious. Sometimes though a facility produces a negative or undesirable effect, this effect may be present even though a high degree of accessibility is required by the facility.

This paper aims to locate a Landfill site as an example of an obnoxious facility. Landfill sites are useful and necessary for the community, but they are a source of negative effects, such as solid and liquid waste materials that emits unpleasant smell from the landfill site makes it undesirable. Also waste which is transferred from various transfer sites to the landfill site pollutes the environment and becomes harmful to the inhabitants. The combination of these two points makes this facility obnoxious, (Gordillo et al, 2007).

A location problem is considered as a conditional p-center problem. When we are given the locations of q existing service facilities and we are to locate p additional service facilities, so as to minimize the maximum distance between the demand points, each to its nearest facility, whether existing or new. The conditional location problem is then to locate p new facilities to serve a set of demand points given that q facilities are already located. When q is equal to zero $(q = o)$, the problem is unconditional.

In conditional p-centre problems, once the new p locations are determined, a demand can be served either by one of the existing or by one of the new facilities whichever is the closest facility to the demand (Berman, 2008).

1.2 HISTORICAL BACKGROUND OF LANDFILL SITES IN GHANA

A disposal site where solid waste, such as paper, glass, and metal, is buried between layers of dirt and other materials in such a way as to reduce contamination of the surrounding land is called landfill site. Modern landfills are often lined with layers of absorbent material and sheets of plastic to keep pollutants from leaking into the soil and water.

A landfill is an engineered site where waste is being deposited. The landfill can either be a hole in the ground, or built on the surface of the ground. The purpose of a sanitary or engineered landfill is to dispose the waste in a way that keeps the effluent from the waste separated from the surrounding environment.

The history of the solid waste is at least as old as the time before people had not yet lived in the cities. Before people started to move to the cities, the waste, which was made up mostly of organic materials derived from the plants, was used as fuel, crop fertilizer, or was fed to livestock. The communities who lived on hunting and gathering simply moved away when the garbage heap became a problem. This type of waste management is still practiced by people in some rural areas of Ghana [Solid Waste Overview, 2002].

The more concentrated the populations in the cities became, the bigger the garbage heaps grew. People could not just pack up and move to another city when their heap got too big. As cities became populated, they started to spread out and became increasingly farther away from their food sources. The organic waste was no longer useful to the people, so it became "*garbage*." The old habits of throwing wastes out the door to animals or into the garden caused public health problems in the densely populated cities.

Waste management practices in Ghana are guided by the Environmental Sanitation Policy of 1999 which was revised in the year 2010. This document spells out the roles of the various stakeholders including the private sector as well as the Growth and Poverty Reduction Strategy (GPRS II) which prescribes public-private partnership in solid waste management and other legislatives documents.

Principally, there are two solid waste collection systems in Ghana - door to door collection system which takes place in the low density areas of the urban centres, and communal collection system in the high density areas. There is some amount of pre-collection in areas of poor accessibility based on the use of Manual and Motorized Tricycles introduced by Zoom lion.

1.3 BACKGROUND STUDY OF KUMASI METROPOLIS

The Kumasi Metropolitan Area is situated in the Ashanti Region and is the second largest city in Ghana next to the capital Accra. The population growth rate in Kumasi is 5.5 % per year (Ghana Statistical Service, 2005). The city is situated in central

Ghana in the forest zone about 270 km north-west of Accra. The city of Kumasi is also known as the garden city of Africa because of the many trees and green areas. Kumasi lies at an altitude of 250-350 m above sea level in the moist semi-deciduous South-East Ecological Zone. The climate is categorized as sub-equatorial, with a daily average minimum and maximum temperature in the metropolis around 21.5 °C and 30.7 °C respectively. The temperature does not vary much over the year. The average humidity is in the range of 60 % to 84 % depending on the season (KMA, 2006).

Between 1967 and 2006 the mean annual rainfall was 1350 mm (Erni, 2007). There are two rainy seasons: March to July and September to October. Many rivers are crossing the city, such as the Wiwi, Sisai, Subin, Nsuben, Oda and Owabi among others. These rivers are contaminated with waste in many places in the metropolis.

As Kumasi lies in the middle of Ghana it has been, and still is, a natural trading place. The market in Kumasi, Kejetia market, is one of the largest in Western Africa.

The number of citizens in Kumasi was 1 915 179 in year 2009, projected from data for the year 2000. The number of people living in Kumasi is increasing fast. The population growth rate was 5.4 % in 2008 (Acheamfuor, 2008). The fast raising of population in the city and in its outskirts moves the boundaries of the city. Kumasi covers a larger area each year since suburbs grow together with each other and with the city itself.

The waste generation per day is about 0.6 kg/person (Ketibuah et al., 2005). This gives an amount of solid waste produced in Kumasi at current date of approximately 1100 ton/day from industries, households and municipal areas. From this quantity, approximately 65-70 % of the waste generated in the city is being collected (Mensah et al., 2008).

A study was conducted and problems identified were;

1. Difficulties in acquiring suitable landfill sites.
2. Difficulties in conveying solid waste by road due to worsening traffic problems and the lack of alternative transport options.
3. The weak demand for composting as an option for waste treatment and disposal.

Kumasi has one landfill situated at Kaase serving the whole town. Today, all of the collected solid waste in the municipality of Kumasi is transported to the landfill site at Kaase. Kaase is under Asokwa Sub-Metro in Kumasi. The landfill in Kumasi is an engineered landfill. An engineered landfill is a waste disposal site where measurements have been taken to prevent environmental impact from the waste (The basics of landfill, 2003). It was started in 2003 and has an expected lifetime of 15 years (Mensah et al., 2003). It is supplied with vertical gas-outlets built as the waste amount is increasing and the landfill is growing. The gas that is being produced is not collected at present, but wells for a gas collection system are continuously installed as the landfill grows. When the landfill at Dompoase was constructed, the stream of the Oda River was redirected to avoid toxic and harmful substance to be flushed into the stream (Adjei-Boateng, pers. comm).

1.4 PROBLEM STATEMENT

The problem of solid waste disposal has become one of the most serious environmental problems facing many cities in Ghana. As noted by Mensah and Larbi (2005), the phenomenon is being aggregated by indiscriminate dumping, difficulties in acquiring suitable landfill sites, difficulties in conveying solid waste by road due to worsening traffic problems and the lack of alternative transport options; and the weak demand for composting as an option for waste treatment and disposal. The "UN sanitation report on Ghana" posted by the Ghanaian Journal (2010) emphasized that the wastes are sent to a few dumpsites, but majority end up in drains, streams and open places. Based on this report, a second landfill site was recommended for Kumasi Metropolis. This work therefore seeks to find the optimal site for an additional landfill site in the metropolis using the conditional p-centre model.

1.5 OBJECTIVES OF THE STUDY

1. To model the location of an additional Landfill site in the Kumasi metropolis as a conditional p-centre problem.
2. To solve the conditional p-centre problem using Berman and Drezner's algorithm.

1.6 METHODOLOGY

The objective of the study is to locate a Landfill site in Kumasi Metropolis using the conditional P-centre model.

Data on road distances between communities would be collected and used.

Flyod-Warshall algorithm will be used to find the distance matrix, $\mathrm{d}(i, j)$ for all pairs shortest path.

Materials will be obtained from the Mathematics department, KNUST library and the internet.

1.7 JUSTIFICATION OF THE STUDY

Urban solid waste issues represent major problems to the governments of developing nations, (Zerbock, 2003). Health problems, pressure on local budgets, and dirty environments are among the many issues presented by urban solid wastes.

Poor management of urban solid waste result in a chain of impairments to the Ghanaian economy; the health implications of poor waste management can be very damaging to the people's health when exposed to these unsanitary conditions.

Solid waste management is thus an integral part of basic urban service to improve environmental health. With an additional landfill site in the Kumasi metropolis, it would in turn solve waste management issues in the Kumasi Metropolis and also Ghana as a whole. It is hoped that the findings of this study would help inform Kumasi Metropolitan Assembly the right site to locate a landfill site in Kumasi.

1.8 THESIS ORGANIZATION

Chapter 1 presents the study background, significance, objectives of the study and structure of the thesis.

The second chapter deals with the literature review.

Chapter 3 presents the research methodology.

Data, analysis and discussions would be considered in Chapter 4.

Conclusion and recommendation of the study is in Chapter 5.

CHAPTER 2

LITERATURE REVIEW

2.0 INTRODUCTION

This chapter provides a brief review of the extensive literature that exists in the area of network optimization problems, as relevant to our study. In the area of network distribution and location, and discuss emergency response problems and related location-allocation models on networks.

Finally, we examine continuous location-allocation problems such as the rectilinear distance Euclidean distance, and l_p distance location-allocation problems, and also provide a brief review of the pure location counterparts of these location-allocation problems.

2.1 LOCATION ALLOCATION MODELS

Most methodologies in the literature of minimizing emergency response times utilize location-allocation models. There has been an enormous amount of research work carried out in the field of location-allocation modelling since the formulation of the first location-allocation problems by (Cooper, 1963). The simplest location-allocation problem is the Weber problem addressed by (Friedrich, 1929), which involves locating a production centre so as to minimize the aggregate weighted distance from various raw material sources. The seminal work in this area was on the p-median problem, initially formulated by (Hakimi, 1964-1965). The p-median problem model has been used to solve service facility location problems that are analogous to the location of switching centres in electrical networks. Given a graph $G(N, A)$, the p-median problem requires us to locate a set of p supply points in G, so that the total (or average) transportation cost of the supply from the nearest supply point to the demand nodes (having known demands) that are located on the nodes (N) of $G$is minimized. Hakimi showed that at least one such optimal solution locates the supply points on the nodes of the network. Hakimi also defined a set of m points X^*m in a deterministic undirected graph G as a set of absolute m-medians if for every $X_m \subseteq G$, the sum of the weight response times from X_m to the demand nodes is greater than or equal to the sum of the weighted response times from X^*m. He showed that when the utility attributes for travel cost are convex, there exists at least one m-node subset of G which is a set of absolute m-medians.

Torgeas et al. (1971) proposed a set covering model to locate emergency service facilities. One problem with this formulation is that the optimal solution may yield a very high value for the number of supply depots to be located. Also, the model does not assign any priority to the location of supply depots at pre-existing facilities.

Toward this end, Hendrick et al. (1974) provided a hierarchical model whose objective function is formulated such that facilities are assigned to pre-existing sites, provided their use does not increase the value of the objective function value of the set covering problem. An alternative model for locating emergency vehicle depots is the maximal covering location problem (Eaton et al., 1985) that has been used to locate medical rescue vehicles in the city of Austin, Texas. This model attempts to provide maximum response coverage to the network subject to a constraint on the maximum number of response vehicles available.

Handler and Mirchandani (1979) have analyzed the case of solving location-allocation problems on probabilistic networks. The formulation here considers the travel times to be stochastic. The authors have shown that for convex travel cost functions, Hakimi's result would continue to hold true, but that the optimal solution obtained by replacing the travel cost functions by their expected values and solving the resultant deterministic p-median problem would not be the same solving the stochastic case. Daskin (1983) proposed the use of a maximal expected covering location model for locating emergency response vehicles based on the idea that not all vehicles allocated to serve a particular zone in the network would actually be available during times of emergency.

2.1.1 Sequential location problems

Cavalier and Sherali (1983) have studied sequential location problems on chain tree graphs based on myopic, long-range, and discounted present worth policies. Using a Markovian assumption to describe the dynamic nature of the network, Berman (1997) developed a model for the repositioning of emergency service vehicles, in order to minimize long-run expected costs and showed that at least one set of optimal location of m units exists on the nodes of the network.

Cavalier and Sherali (1985) have analyzed static and sequential location-allocation problems on undirected networks in which the demands can occur on links having uniform probability distributions. The authors showed that except for the 1-median case, the problem is generally nonconvex.

2.1.2 P-median and p-centre problems

Several computational complexity result for the p-median and p-center problems are available in the literature, Tansel et al (1983).Hakimi et al., (1978) showed that Hakimi, (1964) method for finding the absolute 1-centre (Hakimi, 1965) can be implemented in $O(|E|\log n)$ effort, and prescribe a computational refinement which reduces the effort to $O(|E|\log n)$ for the weighted case.

Further refinements were obtained by Kariv and Hakimi (1979), resulting in an $O(|E|n\log n)$ algorithm for the weight case $O(|E|n)$ for the unweighted case, where $|E|$ is the number of arcs, and n is the number of nodes in the network, respectively.

Minieka (1981) developed an $O(n^3)$ algorithm for solving the unweighted 1-centre problem.

Kariv and Hakimi (1979) showed that the p-centre and p-median problems on a general network are NP-Hard. They also showed that the weight case can be reduced to a computationally finite one, and presented an algorithm whose complexity is $O\left(\frac{|E|^p\left(n^{2p-1}\right)\log n}{p-1!}\right)$.

Hakimi (1964) solved the absolute 1-centre problem for a general network by prescribing an algorithm. In the special case of tree-graphs, Goldman (1972) solved the unweighted 1-centre problem, based on the repeated application of the "trichotomy theorem" that either determines the edge on which the absolute centre lies, or reduces the search to the two sub trees obtained by removing all interior points on that edge. Halfin (1974) refined Goldman's algorithm, improving it from $O(n^2)$ to $O(n)$. An algorithm of complexity $O(n^2\log n)$ is described by Kariv and Hakimi (1979) for finding the absolute p-centre of a vertex weighted tree network. These complexity results have been improved since, and several algorithms have been presented in the literature having comparable orders of complexity.

An efficient $O(n)$ “tree-trimming algorithm to find the 1-median was presented by Hua Lo-Keng et al. (1962). Matula and Kolde (1976) suggested an $O(n^3 p^2)$ algorithm for finding the p-median on a tree network. Kariv and Hakimi (1979) proposed an $O(n^2 p^2)$ algorithm for the same problem.

In the specific case of traffic incident response models, Zografos et al. (1993) used a districting model to obtain optimal locations of vehicles that minimize the total average incident responds workload per vehicle on freeways, subject to a constraint on the maximum number of available vehicles.

2.1.3 Mixed Integer Programming model

Daskin (1987) constructed a mixed-integer programming (MIP) model for the simultaneous location, dispatching, and routing of incident response vehicles. Pal and Sinha (1997) also used an MIP model to determine optimal locations for response vehicles that minimizes annual response vehicle costs, given the frequencies of incidents at potential sites in the network, and subject to a constraint on the maximum number of vehicles.

2.1.4 Dynamic Allocation models

Dynamic location problems arise in situations where current supply centers may have to be re-evaluated with time due to time-varying characteristics of the network. Orda and Rom (1990) have prescribed an algorithm that myopically determines the location sequence over a given horizon by solving 1-median problems on a network having time varying edge weights, giving a starting optimal location.

A stochastic emergency responds model presented by Jarvis (1975) employs a hypercube model of Larson (1974) in developing vehicle dispatching strategies and location plans.

Among dynamic models for emergency response, the fire –engine relocation model proposed and tested by Kolesar et al. (1974) offers an interesting approach. Upon the occurrence of a fire-related incident, the unassigned vehicles in depots are optimally repositioned to offset the loss in coverage with respect to future fire mishaps due to current dispatched vehicles. Nathanail and Zografos (1995) have developed a simulation tool for evaluating the effectiveness of freeway incident response operations. In the case where multiple incidents need to be responded to, they created a priority list of incidents based on the nature of incident and generated responds plans based on this priority and under various dispatch policies. The authors did not attempt to describe policies that can offset the effect of loss in coverage due to unavailable servers on the system. This study also noted that using multiple emergency vehicles to respond to incidents can significantly reduce response time. Most of these models assume that the closest available vehicle is dispatched to the site of the current incident. Several authors including Carter et al. (1972) have pointed out the shortcomings accompanying this assumption.

Mirchandani and Odoni (1979) noted that if there is a high probability that response units will not be available, a policy of dispatching the closest response unit may not be optimal for certain performance criteria. The hypercube queuing model proposed by Larson (1974) estimates the probability that each unit will be busy under various dispatch policies. Daskin and Haghani (1984) demonstrated that for stochastic travel times, dispatching multiple response vehicles using the shortest path from the vehicle depot to the current incident site may not be optimal with respect to minimizing the expected arrival time to the first response vehicle, and they proposed an iterative method to obtain optimal multiple routes.

Another important observation on routing was proposed by Carter et al. (1972). They noted that when service to anticipated future demands is considered, dispatching the nearest available vehicle may not be an optimal choice.

Vehicle inspection is an important tool that can be used to regulate hazardous traffic flow and to minimize the occurrence of hazmat-related incidents. While most of the focus in the literature has been on strategies to optimally respond to such incidents (Sherli et al., 1997), and on the routing and scheduling of hazmat carriers (List et al., 1991), the inspection strategy is based on the premise that it is better to prevent such an accident rather than respond to it. The main task in such a strategy is to decide on the location of the inspection station, so that a measure of risk associated with the flow of uninspected hazmat vehicles through the network is minimized.

Research on facility location is abundant. Jia, et al (2005) researched into the determination of the optimal emergency medical service (EMS) facility locations to address the needs generated by large-scale emergencies. Although facility location models were used to place any type of service in a network, from hospital to fire stations to triage areas, in their work they discussed only the problem of locating medical supplies for large-scale emergencies.

In particular, they concentrated on the problem of where to deploy local medical stocks such as the protective equipment and antidotes against a dirty bomb attack, and the problem of how to position local staging centres to receive, repackage and distribute the medical supplies from the strategic national stockpiles (SNS) for a major natural disaster or a bioterrorist attack. They first surveyed general facility location problems and identified models used to address common emergency situations, such as house fires and regular health care needs. They then analyze the characteristics of large-scale emergencies. Their general facility location model can be cast as a covering model, a P-median model or a P-centre model, each situated for different needs in a large-scale emergency. Illustrative examples were given to show how the proposed model could be used to optimize the locations of facilities for medical supplies to address large-scale emergencies in the Los Angeles area.

2.2 BRIEF OVERVIEW OF SOLID WASTE AND NATIONAL DEVELOPMENT

One of the negative effects of increased prosperity is an escalation in the quantities of wastes produced, since it is well documented that per capita waste generation is related to economic output, (Brito et al, 2003).

Waste or garbage is any material generated by human activity that is considered useless, superfluous, valueless or unwanted and is disposed of in the environment. After collection, this waste may be dumped into landfill sites or destined for composting, incineration or recycling. Solid waste generated in urban centres may contain both domestic and commercial waste, along with industrial waste, thus constituting a complex mixture of different substances, of which some are hazardous to health, (Gouveia *et al*, 2010).

Yahaya *et al* (2010) asserts that one of the greatest threats to national development of most developing countries has been the increasing generation of municipal or urban waste mainly attributed to rapid urbanization. Though there has been increased attention given by government in recent years to handle this problem in a safe and hygienic manner through municipal solid waste management programmes (MSWM), it is still getting off hands.

In its scope, municipal or urban solid waste management includes all administrative, financial, legal, planning, and engineering functions involved in the whole spectrum of solutions to problems of solid wastes thrust upon the community by its inhabitants (Tchobanaglous *et al*, 1997). The solid waste management includes various techniques such as thermal treatment, biological treatment, land filling and recycling (Kontos et al. 2005).

Out of these solid waste management techniques, land filling is the most common method used in many countries (Yesilnacar, 2005). However, in today's society, finding a site to locate undesirable facilities is becoming a significant problem, (Erkut and Moran 1991). Particularly, landfill sitting is a big issue due to the prevalent ‘not in my backyard (NIMBY)’ and ‘not in anyone’s backyard (NIABY)’ concerns from the general public (Kao and Lin 1996). Sitting of landfill is now very essential because of the imperative nature of landfills due to population expansion and the corresponding volume of garbage.

2.3 SOLID WASTE MANAGEMENT

Management of solid waste is associated with the control of generation, storage, collection, transfer and transport, processing, and disposal of solid wastes in a manner that is in accord with the best principles of public health, economics, engineering, conservation, aesthetics, and other environmental considerations. In its scope, it includes all administrative, financial, legal, planning and engineering functions involved in the whole spectrum of solutions to problems of solid wastes thrust upon the community by its inhabitants (Tchobanaglous, et al, 1997)

2.3.1 Categories of solid waste

Solid waste categorization is usually associated with its generation and collection points. People associate waste collection with the periodic collection of household waste. However, the problem is more complex. Besides residential customers, waste companies also have industrial customers, whose requirements differ from typical residential wishes. Industrial customers typically produce larger amounts of waste, which requires another pick-up system. In addition, the collection of recyclables is becoming increasingly important in a society where resources are perishable and environmental concern is growing. The following table describes the categories of wastes in relation to their sources of generation:

Table 2.1: Categories of solid waste

Type of solid waste	Typical facilities, activities, or locations where wastes are generated	Source
Spoiled food wastes, agricultural wastes, rubbish, and hazardous wastes	Field and row crops, orchards, vineyards, diaries, feedlots, farms, etc	Agricultural
Industrial process wastes, scrap materials, etc.; nonindustrial waste including food waste, rubbish, ashes, demolition and construction wastes, special wastes, and hazardous waste.	Construction, fabrication, light and heavy manufacturing, refineries, chemical plants, power plants, demolition, etc.	Industrial
Paper, cardboard, plastics, wood, food wastes, glass, metal wastes, ashes, special wastes, etc.	Stores, restaurants, markets, office buildings, hotels, auto repair shops,	Commercial a Institutional
Special waste, rubbish, general waste, paper, plastics, metals, food waste, etc.	Includes residential, commercial and Institutions	Municipal solid waste

Adapted from Hester *et al*., 2002.

2.3.1.1 Municipal solid waste

The term municipal solid waste (MSW) is normally assumed to include all of the waste generated in a community, with the exception of waste generated by municipal services, treatment plants, and industrial and agricultural processes (Tchnobanoglous, *et al.*, 2002). In the urban context the term municipal solid wastes is of special importance. The term refers to all wastes collected and controlled by the municipality and comprises of most diverse 13 categories of wastes. It comprises of wastes from several different sources such as, domestic wastes, commercial wastes, institutional wastes and building materials wastes. The sources of municipal wastes are given in table 2.2 below:

Table 1.2: Sources of municipal waste

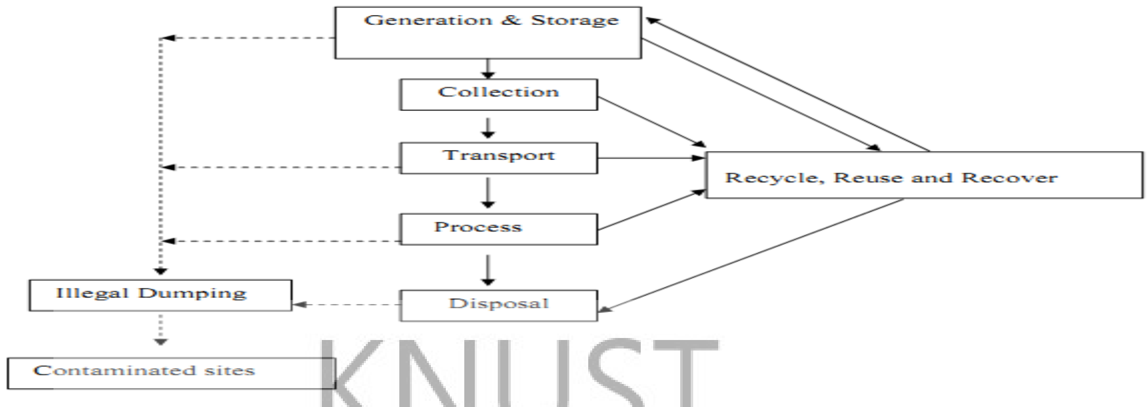
Source	Examples
Residential	Single family homes, duplexes, town houses, apartments
Commercial	Office buildings, shopping malls, warehouses, hotels, airports, restaurants
Industrial	Packaging of components, office wastes, lunchroom and restroom wastes (but not industrial process wastes)
Institutional	Schools, medical facilities, prisons

Source: Tchnobanoglous, *et al.*, 2002

2.3.2 Solid waste management stream

To implement proper waste management, various aspects have to be considered such as Waste generation (source reduction), Waste handling and sorting, storage and processing at the source (onsite storage), Collection, Sorting, processing and transformation, transfer and transport, and Disposal (The Expert Committee, 2000). Figure 2.1 below shows the interrelationship between the functional elements in solid waste management.

Figure 2.1: Solid waste management



Source: Ramachandra et al., 2007

2.3.3 Solid waste management techniques

Many approaches to waste management exist. Generally, solid waste is managed through controlled dumping, sanitary landfills, incineration, recycling or reuse.

2.3.3.1 Controlled dumping

Controlled dumping refers to the use of landfills as terminal endpoints for refuse. It is the preferred method of disposal in many towns in most developing countries because it is the most affordable and requires the least maintenance. The sanitation policies recommend controlled dumping with cover as the preferred option for all small towns and rural areas. In most communities, controlled dumping sites are located on riverbanks and in depressed areas such as in borrow pits, surface mining areas, ravines, old quarries and valleys. Generally, the standard of operation and maintenance on these sites are inadequate. There are often no mechanical equipment for spreading and compaction of waste, which means little reduction in waste volumes. Fly and rodent control are often neglected and there are serious problems with littering, (Mensah, 2005).

2.3.3.2 Sanitary landfill sites

This option is the recommended choice for most municipal waste management in and around the world solid waste disposal for the metropolitan and municipal. Land filling is also considered the most feasible option from the point of view of costs and level of environmental impact, (Mensah, 2005). Landfills are generally sited based on considerations of access to collection vehicles rather than hydrological or public health considerations, (Palczynski, 2002). Most of the landfill sites are open dumping areas. Disposal of wastes through land filling is becoming more difficult because existing landfill sites are filling up at a very fast rate. Consequently, the construction of new landfill sites has become more difficult because of land scarcity, increase of land prices and high demands, especially in urban areas, (Tchobanoglous, 1993).

2.3.3.3 Recycling and reuse

Recycling and reuse is the process by which materials otherwise destined for disposal are collected, reprocessed or remanufactured and are reused. The recycling and reuse (the use of a product more than once in its same form for the same or other purpose) sector of waste management in cities of developing countries is potentially high. Its economic assessment is a difficult task since it is practiced in an informal way, (Boyle, 2000).

2.3.4 Solid waste management practices

In Africa, waste management practices vary from country to country. For example, the Government of Comoros endorsed a National Environment Action Plan (NEAP) in

August 1994, providing the country with a comprehensive strategy for environmental management and protection as well as a proposal for investment (World Bank, 1995). Four main NEAP strategic priorities provide the operational guidelines for this proposed project:

- to collect environmental information that is required for proper environmental management planning and decision-making;
- to strengthen the institutional framework for environmental management and coordination at all levels;
- to provide training for environmental specialists, increase the environmental awareness of the public and bring about effective public participation in environmental management; and
- to preserve and restore the equilibrium of the ecological process, protect biodiversity and promote the rational use of natural resources for the benefit of present and future generations.

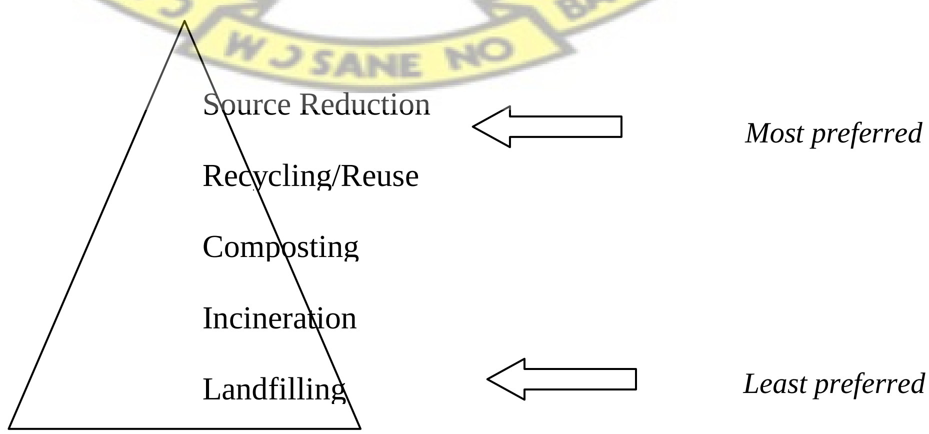
Furthermore, NEAP acknowledges that restored economic growth and successful family planning program are the only long run options for relieving pressure on the environment. In Uganda, environmental policymaking remains largely a function of the central government, but implementation of policies and legislation is passed to the districts. The decentralization process is being supported under the World Bank-financed Environment Management Project, launched in Financial Year 1996 (FY96).

In Accra, the capital of Ghana, the World Bank-facilitated regional MELISSA initiative (Managing Environment Locally in Sub-Saharan Africa), co-financed by the European Union, Sweden, and Norway, carried out its first pilot operation in FY98 to help create the basis for privatizing the management of solid waste. A number of African countries have been implementing integrated waste management (IWM). It refers to the complementary use of a variety of practices to safely and effectively handle municipal solid waste. The strategy used to develop an integrated waste management system is to identify the level or levels at which the highest values of individual and collective materials can be recovered. The most favourable is reduction, which suggests using less to begin with and reusing more, thereby saving material production, resource cost, and energy.

2.3.5 Integrated waste management system (IWMS)

As defined by Tchobanoglous *et al.* (1993), integrated waste management (IWM) is the selection and application of appropriate techniques, technologies, and management programs to achieve specific waste management objectives and goals. Understanding the inter-relationships among various waste activities makes it possible to create an IWM plan where individual components complement one another.IWM systems follow general hierarchy of waste management, which is shown in Figure 2.2 below:

Figure 1.2: Solid waste management hierarchy



Adapted from Palczynski, 2002

For each of the process elements, there is a dependence upon how effective each preceding element has been. This is of great importance when considering each element in the sequence. It can be seen that if there is no effort applied in the prime processes, then the secondary processes such as incineration or land filling must be capable of accepting and processing all of the components of the waste materials. This not only affects the quantity of municipal solid waste to be processed, it also affects its combustion characteristics, and its composition.

2.4 COST OF SOLID WASTE

Solid wastes have the potential to pollute all the vital components of living environment (i.e., air, land and water) at local and at global levels. The problem is compounded by trends in consumption and production patterns and by continuing urbanization of the world. As identified by Ramachandra *et al*. (2007), the problem is more acute in developing nations than in developed nations, as the economic growth as well as urbanization is more rapid. The cost of solid waste is more than just the transport and landfill gate fee. True cost of solid waste, especially municipal waste includes the cost of raw material, the loss of product, the loss of production costs in raw materials and processing (including water and energy), labour costs, treatment costs, collection costs, and disposal costs.

CHAPTER 3
METHODOLOGY

3.0 NETWORK LOCATION MODELS

Network location problems are concerned with finding the right locations to place one or more facilities in a network of demand points. One of the most famous problems for dealing with locating facilities is the p-centre problem. In this problem, the maximum distance of customers (demand points) from the facilities is an important factor. This problem is known as the minimax location problem. N demand points are located in space and they are in fixed positions in static facility location.

3.1 TYPES OF FACILITIES

3.1.1 Non- Obnoxious Facilities

Most services are provided by desirable or non-obnoxious facilities. There are facilities that bring comfort to customers and are pleasant in the neighbourhood. They may include supermarkets, warehouses, shops, garages, banks etc. As the customer needs access to the facility providing service, it is beneficial if these facilities are sited close to the customers who need their services.

3.1.2 Semi- Obnoxious Facilities

Sometimes a facility that requires a high degree of accessibility provides a negative or undesirable effect. For example, a football stadium provides entertainment and so requires a large amount of access to enable supporters to attain a game. On the other hand, on a match day, local non-football fans will have to be content with the noise and traffic generated.

The generation of noise and traffic will be unpleasant for locals who are not attending the match and who will therefore describe the facility as undesirable. The combination of the two makes this facility semi-obnoxious.

Another example is a hospital with an ambulance. Here access is needed for the treatment of the local population particularly on emergency days. On the other hand the siren of the ambulance may be too noisy to others who might not need its service at the moment in time.

3.1.3 Obnoxious Facilities

An obnoxious facility is one which is useful but has undesirable effect on the inhabitants and users in an area. Examples include landfill sites, equipment which emits pollutants such as noise and radiation or warehouses that contain flammable materials. Other obnoxious facilities include machines that are potential sources of hazards to nearby machines and workers. Further obnoxious facilities are the nuclear power stations, military installations. Although necessary for society, these facilities are undesirable.

3.2 LOCATION MODELS

We discuss two approaches to sitting a single facility on a road network.

1. Methods that do not make use of the road links;

a. Location Break Even Analysis

b. Centre of gravity

c. Factor rating method

2. Methods that make use of road links and which includes Median problems and Centre Problems.

3.2.1 The Location Break-Even Analysis

The location break-even analysis is the use of cost-volume analysis to make an economic comparison of location alternative. By identifying fixed and variable cost and graphing them for each location, we can determine which location provides the lowest cost. Location break-even analysis can be done mathematically or graphically. The graphical approach has the advantage of providing the range of volume over which each location is preferable.

The location break-even analysis method employs three steps, these are:

- Determine the fixed and variable cost for doing business at each location.
- Plot the cost for each location, with cost on the vertical axis of the graph and volume on the horizontal axis.
- Select the location that has the lowest total cost for the expected volume of business.

The location break-even analysis is determined by the equation;

$Y = ax + b$, where;

a = Variable cost

b = fixed cost

X= Volume of business, *Y*= Cost of business

3.2.2 Centre of Gravity Method

The centre of gravity method is a mathematical technique used for finding the location of a distribution centre that will minimize distribution cost. For instance in the location of a market, the method takes into account the volume of goods shipped to those markets and shipping cost in finding the best location for the distribution centre. The first step in the centre of gravity method is to place the locations on a coordinate system. The coordinates of each location must be carefully noted. The origin of the coordinate system is arbitrary, just as long as the relative distances are correctly represented. This can be done easily by placing a grid over an ordinary map of the location in question. The centre of gravity is determined by equations (1) and (2) below;

$$C_x = \frac{\sum d_{ix} W_i}{\sum W_i} \quad \text{......................................(1)}$$

$$C_y = \frac{\sum d_{iy} W_i}{\sum W_i} \quad \text{......................................(2)}$$

Where,

$C_x = X$ -coordinate of the centre of gravity

$C_y = Y$ -coordinate of the centre of gravity

$d_{ix} = X$ -coordinate of location i

$d_{iy} = Y$ -coordinate of location i

$W_i =$ Volume of goods to or from location i

The centre of gravity is then determined by equation (1) and (2) above. Once x and y -coordinates have been obtained, we place that new location on the previously described map. If that particular location does not fall directly on a city, simply locate nearest city and place the new distribution there. In the case where there is more than one city that can be used as possible location, the factor rating method can be used to select one. This method could be implemented when locating a Library Complex on a network of towns or cities.

3.2.3 The Factor Rating Method

The factor rating method is a method used to find a suitable location for a facility considering a number of factors. The factors include: labour cost (wages, unionization, and productivity), labour availability, proximity to raw materials and supplier, proximity to markets, state and local government fiscal policies, environmental regulations, utilities, site cost, transportation, and quality of life issues within the community, foreign exchange and quality of government. When using the factor rating method, the following six steps must be followed strictly.

These are:

- Develop a list of relevant factors.
- Assign a weight to each factor to reflect its relative importance in management's objective.
- Develop a scale for each factor (for example, 1 to 10 or 1 to 100)
- Have management or related people score each relevant factor, using the scale developed above.
- Multiply the score by the weight assigned to each factor and total the score for each location.

- Make a recommendation based on the maximum point score; considering the result of quantitative approaches as well.

(Amponsah, 2007)

Table 3.1 illustrates an example for the factor rating analysis for which a company must decide among three sites for the construction of a new satellite clinic. The firm selected seven factors listed below as a basis for evaluation and have assigned rating weights on each factor.

Table 2.1: Relative scores on factors for a Satellite Clinic

Factor	Factor Name	Rating Weight	Ratio of Rate	Location A	Location B	Location C
1	Land Acquisition	5	0.25	25	20	20
2	Power-source available and cost	3	0.15	12	10.5	15
3	Workforce attitude and cost	4	0.2	6	12	14
4	Population size	2	0.1	1	8	6
5	Community desirability	2	0.1	9	6	8
6	Equipment suppliers in area	3	0.15	7.5	9	13.5
7	Economic activities	1	0.05	4.5	3	3
Total				65	68.5	79.5

Clearly from their respective aggregate score shown in table 3.1, location C would be recommended since it has the highest aggregate.

3.3 FACILITY NODES AND NETWORK POPULATION CENTRES

Location simply refers to the strategy of putting a facility in place where it can be identified as serving inhabitants staying at population centres around the facility. The points of placement of the facilities are called facility nodes and the population centres are called demand nodes. These nodes are linked by paths or streets which are called edges. A node may simultaneously serve as facility node and demand node. A graph is defined as $G(V,E)$ consisting of a finite set of vertices (V) and a finite set of edges (E) such that $V \times V \rightarrow E$. Example if $V_i, V_j \in V$ then $(V_i, V_j) \in E$ if there is an edge between V_i and V_j which implies there exist an edge distance $e(V_i, V_j) \neq 0$. A network is a physical implementation of a graph. Example:

- A network of roads: The vertices are towns and the edges are road links.
- An electrical network: The vertices are junctions of resistors, inductors and capacitors and edges are wire links to junctions.

3.3.1 Median Problem

The median problem is to find the location of p facilities on a network so that the total cost is minimized. The cost of serving demands at node i is given by the product of the demand at node i and the distance (d_{ij}) between demand node i and the nearest jth facility to node i. This problem may be formulated using the following notation:

Inputs

$h_i =$ demand at node i

$d_{ij} =$ distance between demand node i and candidate site j

$P =$ number of facilities to locate

Decision variables

$$X_i = \begin{cases} 1, \text{if we locate at candidate site } j \\ 0, \text{if not} \end{cases}$$

$$Y_{ij} = \begin{cases} 1, \text{demands at node } i \text{ are served by a facility at node } j \\ 0, \text{if not} \end{cases}$$

With this notation the median problem may be formulated as follows:

Minimize $\sum_i \sum_j h_i d_{ij} Y_{ij}$ ....(1)

Subject to $\sum Y_{ij} = 1 \ \forall \ i$...(2)

$\sum_j X_j = P$...(3)

$Y_{ij} - X_i \leq 0 \ \forall \ i, j$..(4)

$X_j = 0, 1 \ \forall \ j$...(5)

$Y_{ij} = 0, 1 \ \forall \ i, j$..(6)

The objective function (1) minimizes the total demand-weighted distance between each demand node and the nearest facility. Constraint (2) requires each demand node i to be assigned to exactly one facility j. Constraint (3) states that exactly P facilities are to be located. Constraints (4) link the location variables, X_j and the allocation variables Y_{ij}. They state that demands at node i can only be assigned to a facility at location $j(Y_{ij} = 1)$ if a facility is located at node $i(X_j = 1)$. Constraints (5) and (6) are the standard integrality conditions. The median formulation given above assumes that facilities are located on the nodes of the network (Hamiki, 1995).

3.3.2 Centre Problem

The centre problem is defined as the location of a number of facilities such that all the nodes are covered. The centre problem requires the model to minimize the coverage distance such that each demand node is covered by one of the facilities to be sited within the endogenously determined coverage distance. The centre problem is a minimax problem.

The 1-center problem is a classical optimization problem that looks at the location of a single facility such that all the demand nodes are covered. Under the 1-center problem, we have the vertex centre problem, which seeks to locate the facilities on the nodes of a network. There is also the absolute centre problem that seeks to locate facilities at anywhere on the network.

3.3.3 Vertex P-Centre Problem formulation

Let $a_{ij} = 1$ distance from demand node i to candidate facility site j

$h_i =$ Demand at node i

$P =$ Number of facilities to locate

Decision variables,

$$x_j = \begin{cases} 1, if\ we\ locate\ at\ candidate\ site\ j \\ 0, if\ not \end{cases}$$

$Y_{ij} =$ Fraction of demand at node i that is served by a facility at node j

W = maximum distance between a demand node and the nearest facility.

The problem is formulated as follows,

Minimize W $\quad$ 2.13a

Subject to

$$\sum_j Y_{ij} = 1 \qquad \forall\, i \qquad 2.13b$$

$$\sum_j X_j = P \qquad 2.13c$$

$$Y_{ij} \le X_j \qquad \forall\, i, j \qquad 2.13d$$

$$W \ge \sum_j a_{ij} Y_{ij} \qquad \forall\, i \qquad 2.13e$$

$$x_j = 0,1 \qquad \forall\, j \qquad 2.13f$$

$$Y_{ij} \ge 0 \qquad 2.13g$$

In some cases, the demand-weighted distance is considered and constraint 2.13e becomes

$$W \ge h_i \sum_j a_{ij} Y_{ij} \qquad \forall\, i \qquad 2.13e$$

3.4 THE MAXIMUM COVERING LOCATION MODEL

The set covering has associated problems, one of which is that the number of facilities that are needed to cover all demand nodes is likely to exceed the number that can actually be built due to budget constraints and other related issues.

Moreover, the set covering model treats all demands nodes identical. Under certain conditions and budgetary constraints it is ideal to fix the number of facilities to be located and then maximize the number of covered demands. Church and ReVelle (1974) formulated a Maximum Covering Location Model as follows.

Let $h_i =$ demand at node i

$P =$ Number of facilities to locate

Decision Variables be

$$Z_i = \begin{cases} 1, \textit{if node } i \textit{ is covered} \\ 0, \textit{if not} \end{cases}$$

The Maximum Covering Location Model is formulated as follows;

Maximize $\sum_i h_i Z_i$ 2.6a

Subject to;

$Z_i \leq \sum_j a_{ij} x_j \quad \forall i$ 2.6b

$\sum_j x_j \leq P$ 2.6c

$x_j = 0,1$ 2.6d

$Z_i = 0,1$ 2.6e

The objective function 2.6a maximizes the number of covered demands. Constraint 2.6b state that demand node i cannot be covered unless at least one of the facility sites that cover node i is selected. But, the right-hand side of constraints 2.6b which is $\sum a_{ij} x_j$ is identical to the left-hand side of constraints 2.3b. $\sum a_{ij}x_j$ gives the number of selected facilities that can cover node i .The constraint 2.6c indicates that we locate not more than P facilities. Constraints 2.6c will be binding in the optimal solution. Constraints 2.6d and 2.6e are the integrality constraints on the decision variables.

3.5 THE UNCAPACITATED FACILITY LOCATION PROBLEM

The uncapacitated facility location problem, UFL problem, is used to model many applications. Some applications are: bank account allocation, clustering analysis, lock-box location, location of offshore drilling platforms, economic lot sizing, machine scheduling and inventory management, portfolio management and the design of communication networks. For example, the bank account location problem arises from the fact that the clearing time for a cheque depends on a city i where it is cashed and the city j where the paying bank is located. A company that pays bills by cheque to clients in several locations finds it useful to open accounts in several strategically located banks. It pays the bill to the client in city i from a bank in city j that maximizes the clearing time.

The UFL problem consists of:

- a set $J = \{1, 2, 3, ..., n\}$ of potential sites in locating facilities,
- a set $I = \{1, 2, 3, ..., m\}$ of clients whose demands need to be served by the facilities,
- a profit c_{ij} for each facility $j \in J$ and client $i \in I$. The profit is made by satisfying the demand of client i from a facility j

- a fixed nonnegative cost f_j for each facility $j \in J$ to be used in setting up facility j

The uncapacitated facility location problem is to select a given number S of the facilities $S \subseteq J$. And to assign each client to exactly one facility such that the difference between the profits from S and the fixed costs is maximized. The integer linear programming (ILP) formulation is given by:

Maximize $$Z = \sum_{i \in I} \sum_{j \in J} c_{ij} y_{ij} - \sum_{j \in J} f_j x_j \qquad \text{.......................(1)}$$

Subject to: $$\sum_{i \in I} y_{ij} = 1 \qquad \forall i \in I \qquad \text{....................(2)}$$

$$y_{ij} \le x_j \qquad \forall i \in I, \forall j \in J \qquad \text{...................(3)}$$

$$x_j \in \{0,1\} \qquad \forall j \in J \qquad \text{......................(4)}$$

$$y_{ij} \in \{0,1\} \qquad \forall i \in I, j \in J \qquad \text{.....................(5)}$$

Represent each potential site $j \in J$ by x_j and a facility is opened at site $j \in J$ if $x_j = 1$ otherwise, $x_j = 0$ .Represent the satisfaction of demand of client $i \in I$ from facility $j \in J$ by y_{ij}. Demand of client i is served by the facility at site j if $y_{ij} = 1$ otherwise, $y_{ij} = 0$.

Equation (3) may be relaxed to obtain $\sum_{i \in I} y_{ij} \le m x_j \quad \forall j \in J$ ....(6)

This is the result of summing the m constraints $y_{ij} \le x_j$. In such case the optimization problem is called weak integer programming (WIP).

When linear programming relaxation method is used to solve the ILP, the problem is called strong linear programming relaxation (SLPR). Also, when linear programming relaxation method is used to solve the WIP, the problem is known as weak linear programming relaxation (WLPR). In such cases $x_j \in \{0,1\}$ is replaced by $0 \le x_j \le 1$ and $y_{ij} \in \{0,1\}$ is replaced by $y_{ij} \ge 0$.

SLPR is formulated as

(SLPR) Maximize $$Z = \sum_{i \in I}\sum_{j \in J} c_{ij} y_{ij} - \sum_{j \in J} f_j x_j \qquad \text{.......................(7)}$$

Subject to: $$\sum_{i \in I} y_{ij} = 1 \qquad \forall i \in I \qquad \text{.....................(8)}$$

$$y_{ij} \le x_j \qquad \forall i \in I, \forall j \in J \qquad \text{.....................(9)}$$

$$x_j \le 1 \qquad \forall j \in J \quad \text{.......................(10)}$$

$$x_j \ge 0 \quad \forall j \in J \quad \text{.....................(11)}$$

$$y_{ij} \ge 0 \quad \forall i \in I, j \in J \quad \text{.....................(12)}$$

The dual formulation to the SLPR is obtained by using arguments for the simplex of LP method. The coefficients of the objective function of the dual are obtained from equations $(8),(9)$ and (10) . They are $m(from\, I)$ and $n(from\, J)$. Introducing new variables u_i and t_j we have the objective function;

$$\text{Minimize} \quad W = \sum_{i \in I} u_i + \sum_{j \in J} t_j$$

The corresponding equations (8) and (9) at the RHS are f_j and c_{ij}. We the constraints

$$t_j - \sum_{i \in I} w_{ij} \geq -f_j \qquad \forall j \in J$$

$$u_i + w_{ij} \geq c_{ij} \qquad \forall i \in I, \forall j \in J$$

$$u_i \text{ free} \qquad \forall i \in I$$

$$w_{ij} \geq 0 \qquad \forall i \in I, \forall j \in J$$

$$t_j \geq 0$$

The dual of SLPR is, therefore, given as;

$$\text{(Dual SLPR)} \quad \text{Min} \quad W = \sum_{i \in I} u_i + \sum_{j \in J} t_j \qquad \text{.....................(13)}$$

$$\text{Subject to} \quad t_j - \sum_{i \in I} w_{ij} \geq -f_j \qquad \forall j \in J \qquad \text{.....................(14)}$$

$$u_i + w_{ij} \geq c_{ij} \qquad \forall i \in I, \forall j \in J \qquad \text{.....................(15)}$$

$$u_i \text{ free} \qquad \forall i \in I \qquad \text{.....................(16)}$$

$$w_{ij} \geq 0 \qquad \forall i \in I, \forall j \in J \qquad \text{.....................(17)}$$

$$t_j \geq 0 \qquad \forall j \in J \qquad \text{....................(18)}$$

Rather than solving the relaxed integer programming problem we reduce the dual problem to condensed form and solve by heuristic method. For instance, if all the variables u_i have been given fixed values then from equations (15) and (17)

$u_i + w_{ij} \geq c_{ij}$ and $w_{ij} \geq 0$.

It implies $w_{ij} = (c_{ij} - u_i)^+ \quad \forall i \in I, \forall j \in J$ ....(19)

Where, for any expression $b, b^+ = \max(b, 0)$.

So far as the $u_i's$ have been fixed and we are minimizing, we must assign the t_j minimum values such that from equation (14) and (18)

$t_j - \sum_{i \in I} w_{ij} \geq -f_j \quad \forall j \in J$ ....$(20a)$ and $t_j \geq 0 \ \forall j \in J$ are satisfied.

Meaning that $t_j \geq \left(\sum_{i \in I} w_{ij} - f_j \right)^+ \quad \forall j \in J$ ....$(20b)$

Since $w_{ij} = (c_{ij} - u_i)^+$, we substitute w_{ij} into $(20b)$, we get

$t_j \geq \left(\sum_{i \in I} (c_{ij} - u_i)^+ - f_j \right)^+ \quad \forall j \in J$

We have the first condensed form;

$$(CDI)\ W = \min_{u_1, \ldots, u_m} \left\{ \sum_{i \in I} u_i + \sum_{i \in I} \left(\sum_{i \in I} (c_{ij} - u_i)^+ - f_j \right)^+ \right\} \quad \text{.....................(21)}$$

Suppose an u_k in equation (21) is increased by a small number $\varepsilon > 0$, then the increased effect in $\sum_{i \in I} u_i$ in the first term is cancelled by the effect of $\sum_{i \in I} u_i$ in the bracket and the objective is not changed. But the increase of ε in u_k decreases the second term which is equal to t_j by the same amount. Under such cases there will always exist an optimal solution to the dual SLPR with

$t_j \leq 0 \ \forall j \in J$ ....(22)

This information is used to change u_i until t_j is at zero. We formulate equation (21) by splitting the two terms and add the constraint $u_i \leq \max_{j \in J}\{c_{ij}\}$ we get;

$$(CD2) \ \text{Min} \ W = \sum_{i \in I} u_i \text{ ....(23)}$$

Subject to $$\sum_{i \in I} (c_{ij} - u_i)^+ - f_j \leq 0 \quad \forall j \in J \text{ ....(24)}$$

$$u_i \leq \max_{j \in J}\{c_{ij}\} \quad \forall i \in I \text{ ....(25)}$$

3.6 THE CONDITIONAL P-CENTRE PROBLEM

The conditional location problem is to locate *p* new facilities to serve a set of demand points given that *q* facilities are already located. When *q* = 0, the problem is unconditional. In the conditional *p*-centre problems, once the new *p* locations are determined, a demand can be served either by one of the existing or by one of the new facilities whichever is the closest facility to the demand.

Consider a network G = (N, L)

Where;

N = the set of nodes, |N| = n

L = the set of links.

Let $d(x,y)$ be the shortest distance between any $x, y \in G$. Suppose that there is a set $Q(|Q| = q)$ of existing facilities. Let $Y = (Y_{1,\ldots\ldots\ldots\ldots}, Y_q)$ and $X = (X_1, X_2, \ldots\ldots, X_p)$ be vectors of size q and p respectively, where Y_i is the location of existing facility i and X_i is the location of new facility i. Without any loss of generality we do not need to assume that $Y_i \in N$. The conditional p-centre location problem is to;

$$Min\left[g(x) = \max_{i=1,\ldots,n} \min\{d(X,i), d(Y,i)\}\right]$$

Where $d(X,i)$ and $d(Y,i)$, is the shortest distance from the closest facility in X and Y respectively to the node *i*, (Berman and Simchi-Levi, 1990).

3.7 BERMAN AND SIMCHI-LEVI ALGORITHM

Berman and Simchi-Levi (1990) suggest to solve the conditional p-centre problem on a network by an algorithm that requires one-time solution of an unconditional (p + 1)-centre problem.

- Let be a distance matrix with rows corresponding to demands and columns corresponding to potential locations. For the p-centre problem the columns of *D* correspond to the set of local centres C. The idea is to create a new potential location representing all existing facilities. If a demand point is utilizing the services of an existing facility, it will use the services of the closest existing facility. Therefore, the distance between a demand point and the new location is the minimum distance calculated for all existing facilities.

- To force the creation of a facility at the new location, a new demand point is created with a distance of zero to the new potential location and a large distance to all other potential locations. The new distance matrix $\hat{\hat{D}}$ is constructed by adding a new location a_0 (a new column) to D so that the columns represent the Q existing locations and a new demand point v_0 with an arbitrary positive weight. For each demand point (node) $i, d(i, a_0) = \min_{k \in Q}\{d_{ik}\}$ and $d\{v_0, a_0\} = 0$. For each potential location (node) j, $d(v_0, j) = M$ (M is a large number). Again the nodes in Q and potential locations Q are removed.
- Find the optimal new location using $\hat{\hat{D}}$ for the network with the objective function

$$Min\left[\, g(x) = \max_{i=1,\ldots,n} \min\{d(X,i), d(Y,i)\}\right]$$

To illustrate the approach, we consider the network of figure 3.1; the numbers next to the links are lengths. Suppose that existing set of facilities are nodes *2* and *3*, and only one facility is to be located.

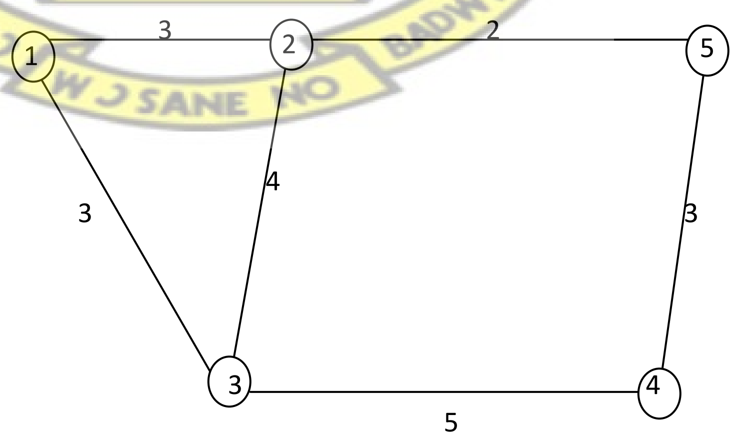


Figure 2.1: Sample network for p-centre problem

Step 1 By using either the Floyd's algorithm or the Dijkstra's algorithm, we obtain the shortest paths matrix (distance matrix) D, for the above network as shown in table 3.1. In table 3.1, column 1 and row 1 represents the demand nodes and potential location respectively, and each other row represents the interconnected distances.

Table 3.2: All pairs shortest path distance matrix, D.

Demand nodes	Potential location				
	1	2	3	4	5
1	0	3	3	8	5
2	3	0	4	5	2
3	3	4	0	5	6
4	8	5	5	0	3
5	5	2	6	3	0

Step 2 Determine a modified shortest distance matrix, $\hat{\hat{D}}$ by: adding a new location a_0 (a new column) to D and adding a new demand point v_0 (a new row) with an arbitrary positive weight to the rows. For each demand point (node) $i, d(i, a_0) = \min_{k \in Q}\{d_{ik}\}$ and $d(v_0, a_0)$. For each potential location (node) j, $d(v_0, j) = M$ (M is a large number), and this is shown in Table 3.2.

Table 3.2a: The Modified Distance Matrix, $\hat{\hat{D}}$

Demand Nodes	Potential Location					
	1	2	3	4	5	a_0
1	0	3	3	8	5	3
2	3	0	4	5	2	0
3	3	4	0	5	6	0
4	8	5	5	0	3	5
5	5	2	6	3	0	2
v_0	M	M	M	M	M	0

The nodes in Q representing the existing nodes are removed, and this is shown intable3.2b.

Table 3.2b: The modified Distance Matrix, $\hat{\hat{D}}$ with nodes 2 and 3 removed.

Demand Nodes	**Potential Location**			
	1	**4**	**5**	a_0
1	0	8	5	3
4	8	0	3	5
5	5	3	0	2
v_0	M	M	M	0

Step 3 finds the optimal new location using the distance matrix, $\hat{\hat{D}}$ and the objective function,

$$Minimize\left[g(x)=\max_{i=1,\ldots,n}\min\{d(X,i),d(Y,i)\}\right]$$

Taking the distance matrix, $\hat{\hat{D}}$

$$Minimize\left[g(x) = \max_{i=1,\ldots,n} \min\{d(X,i), d(Y,i)\}\right]$$

$$X = \{1, 4, 5, a_0\} \qquad\qquad Y = \{2, 3\}$$

At $X = 1$

$i = 1,\ d(1,1),\ d(2,1),\ d(3,1)$

$0,\quad 3,\quad 3$

$\min = 0$

$i = 2,\ d(1,2),\ d(2,2),\ d(3,2)$

$3,\quad 0,\quad 4$

$\min = 0$

$i = 3,\ d(1,3),\ d(2,3),\ d(3,3)$

$3,\quad 4,\quad 0$

$\min = 0$

$i = 4,\ d(1,4),\ d(2,4),\ d(3,4)$

$8,\quad 5,\quad 5$

$\min = 5$

$i = 5,\ d(1,5),\ d(2,5),\ d(3,5)$

$$5, \quad 2, \quad 6$$

$$\min = 2$$

At $X = 4$

$$i = 1,\ d(4,1),\ d(2,1),\ d(3,1)$$

$$8, \quad 3, \quad 3$$

$$\min = 3$$

$$i = 2,\ d(4,2),\ d(2,2),\ d(3,2)$$

$$5, \quad 0, \quad 4$$

$$\min = 0$$

$$i = 3,\ d(4,3),\ d(2,3),\ d(3,3)$$

$$5, \quad 4, \quad 0$$

$$\min = 0$$

$$i = 4,\ d(4,4),\ d(2,4),\ d(3,4)$$

$$0, \quad 5, \quad 5$$

$$\min = 0$$

$$i = 5,\ d(4,5),\ d(2,5),\ d(3,5)$$

$$3_{,} \quad 2_{,} \quad 6$$

$$\min = 2$$

Therefore $X = 1$, the maximum $= 5$ at node 4. The results are summarized and shown below in table 3.3; with column 5 representing the maximum distance between demand nodes and rows representing the minimum interconnected distances.

Table 3.3 Optimal Location $Min(g(x))$, using $\hat{\hat{D}}$

Demand nodes	1	4	5	Maximum
1	0	5	2	**5**
4	3	0	2	**3**
5	3	3	0	**3**
Minimum				**3**

3.8 BERMAN AND DREZNER'S ALGORITHM

Berman and Drezner (2008) discuss a very simple algorithm which solves the conditional p-centre problem on a network. The algorithm requires one-time solution of an unconditional p-centre problem using an appropriate shortest distance matrix. Rather than creating a new location for an artificial facility and force the algorithm to locate a new facility there by creating an artificial demand point, they just modify the distance matrix.

Step 1 Let D be a distance matrix with rows corresponding to demands and columns corresponding to potential locations.

Step 2 Solve the conditional problem by defining a modified shortest distance matrix, from D to $\hat{D}$.

$$\hat{D} = \min\left\{ d_{ij} \min_{k \in Q} \{d_{ik}\} \right\} \forall i \in N,\ j \in C(centre)$$
,

note that $\hat{D}$ is not symmetric even when D is symmetric.

The unconditional p-centre problem using the appropriate $\hat{D}$ solves the conditional p-centre problem. This is so since if the shortest distance form node i to the new p facilities are larger than $\min_{k \in Q} \{d_{ik}\}$,then the shortest distance from the existing q facilities is utilized. Notice the size of $\hat{D}$ is $n \times |C|$ for the conditional p centre.

Step 3 Find the optimal new location using $\hat{D}$ for the network with the objective function

$$Minimize\left[g(x) = \max_{i=1,\dots,n} \min\{d(X,i), d(Y,i)\} \right]$$

To demonstrate the algorithm, a 5-node network depicted in figure 3.1 is considered where the numbers next to the links are lengths. The 1-centre problem is solved. Suppose that the existing set of facilities remain are Q = {2, 3} and p = 1, the new facility to be located.

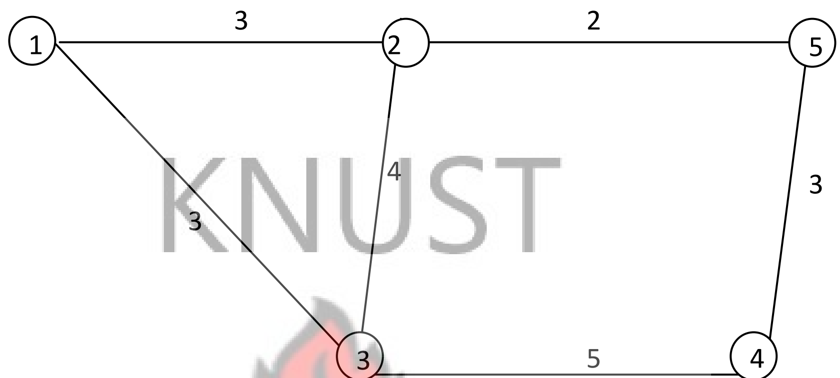


Figure 3.2: Sample network for p centre problem.

Step 1 Using either Floyd's algorithm or Dijkstra's algorithm we obtain shortest paths matrix (Distance matrix) D, for the above network. With column 1 and row 1 represents demand nodes and potential location respectively and each other row represents the interconnected distances.

Table 3.4: All pairs shortest path distance matrix, D.

Demand nodes	Potential location				
	1	2	3	4	5
1	0	3	3	8	5
2	3	0	4	5	2
3	3	4	0	5	6
4	8	5	5	0	3
5	5	2	6	3	0

Step 2 Determine a modified shortest distance matrix by:

$$\hat{D} = \min\left\{ d_{ij} \min_{k \in Q} \{d_{ik}\} \right\} \forall i \in N, j \in C(centre)$$

For node 1, $i = 1, \; j = 1$

$$\hat{D}_{11} = \min\left\{ d_{11} \min_{\{2,3\} \in Q} \{d_{12}, d_{13}\} \right\}$$

$$\hat{D}_{11} = \min\left\{ 0 \min_{\{2,3\} \in Q} \{3,3\} \right\} = 0$$

$i = 1, j = 2$

$$\hat{D}_{12} = \min\left\{ d_{12} \min_{\{2,3\} \in Q} \{d_{12}, d_{13}\} \right\}$$

$$\hat{D}_{12} = \min\left\{ 3 \min_{\{2,3\} \in Q} \{3,3\} \right\} = 3$$

$i = 1, j = 3$

$$\hat{D}_{13} = \min\left\{ d_{13} \min_{\{2,3\} \in Q} \{d_{12}, d_{13}\} \right\}$$

$$\hat{D}_{13} = \min\left\{ 3 \min_{\{2,3\} \in Q} \{3,3\} \right\} = 3$$

$i = 1, j = 4$

$$\hat{D}_{14} = \min\left\{d_{14} \min_{\{2,3\}\in Q} \{d_{12}, d_{13}\}\right\}$$

$$\hat{D}_{14} = \min\left\{8 \min_{\{2,3\}\in Q} \{3,3\}\right\} = 3$$

$$i = 1, j = 5$$

$$\hat{D}_{15} = \min\left\{d_{15} \min_{\{2,3\}\in Q} \{d_{12}, d_{13}\}\right\}$$

$$\hat{D}_{15} = \min\left\{5 \min_{\{2,3\}\in Q} \{3,3\}\right\} = 3$$

The results are then summarized and shown below in the table 3.3; with column 1 and row 1 representing demand nodes and potential locations respectively and the other rows representing interconnected distances.

Table 3.4a: All pairs shortest path distance matrix, D.

Demand nodes	Potential location				
	1	2	3	4	5
1	0	3	3	3	3
2	0	0	0	0	0
3	0	0	0	0	0
4	5	5	5	0	3
5	2	2	2	2	0

Step 2b The existing facility nodes in $Q = \{2,3\}$ are removed and the modified shortest path distance matrix, $\hat{D}$ is shown in Table 3.4b below.

Table 3.4b: Modified Shortest Path Distance Matrix, $\hat{D}$ with existing facility nodes removed.

Demand Nodes	Potential Location		
	1	4	5
1	0	3	3
4	5	0	3
5	2	2	0

Step 3 Find the optimal new location using $\hat{D}$ for the network with the objective function

$$Minimize\left[g(x) = \max_{i=1,\ldots,n} \min\{d(X,i), d(Y,i)\}\right]$$

$$X = \{1,4,5\} \qquad Y = \{2,3\}$$

At $X = 1$

$i = 1,\ d(1,1),\ d(2,1),\ d(3,1)$

$0,\ 3,\ 3$

$\min = 0$

$i = 2,\ d(1,2),\ d(2,2),\ d(3,2)$

$3,\ 0,\ 4$

$\min = 0$

$i = 3,\ d(1,3),\ d(2,3),\ d(3,3)$

$$3, \quad 4, \quad 0$$

$$\min = 0$$

$$i = 4, \; d(1,4), \; d(2,4), \; d(3,4)$$

$$8, \quad 5, \quad 5$$

$$\min = 5$$

$$i = 5, \; d(1,5), \; d(2,5), \; d(3,5)$$

$$5, \quad 2, \quad 6$$

$$\min = 2$$

Therefore at $X = 1$ the maximum = 5, at node 4.

The results are then summarized and shown in table 3.5 with column 5 representing the maximum distance between demand nodes and rows representing the minimum interconnected distances.

Table 3.5 Optimal Location $Min(g(x))$, using $\hat{\hat{D}}$

Demand nodes	**1**	**4**	**5**	**Maximum**
1	0	5	2	**5**
4	3	0	2	**3**
5	3	3	0	**3**
Minimum				**3**

From table 3.5 it is easy to verify that the optimal location is node 5 with an objective function value of 3.

CHAPTER 4

DATA COLLECTION, ANALYSIS AND RESULTS

4.0 INTRODUCTION

In this chapter a Berman and Drezner's algorithm (2008) for solving conditional p-centre on a network would be used to locate a landfill site in five sub metros put together. They are Asokwa, Manhyia, Oforikrom, Asawase and Nhyiaeso sub-metros of the Kumasi Metropolitan Assembly. The map of these sub metros will be used to draw a cyclic network with the edges being the distances between the communities. The Flyod-Warshall all-pairs shortest paths algorithm would be applied to the network to create a distance matrix and the Berman and Drezner's algorithm would be followed through to solve the problem.

4.1 DATA COLLECTION AND ANALYSIS

A map of the Kumasi Metropolis with the individual sub-metros and their respective towns was obtained from the Planning Department of the Kumasi Metropolitan Assembly. Figure A1.0 in Appendix 1.0 depict the sub-metro areas of Kumasi.

The map was prepared in 2008. The communities or settlements of the five sub-metros were identified and the ArcGIS software was used to calculate the distances between the communities.

Table 4.1: Towns under the five Sub-metros

TOWN	NODE	TOWN	NODE	TOWN	NODE	TOWN	NODE
Buokrom	BK(22)	Manhyia	MH(34)	Twumduase	TD(45)	Ahodwo	AV(17)
Duase	DU(24)	Ashtown	AS(33)	Kotei	KO(47)	Adeibeba	AZ(18)
Pakoso	PS(27)	Nsenie	NE(38)	Deduako	DK(46)	Apramang	AX(13)
Asabi	AI(25)	Oduom	OM(37)	Gyinyase	GY(52)	Nhyiaeso	NA(6)
Sepetimpon	SP(23)	Kentinkrono	KT(39)	Kyirapatre	KP(56)	Adiembra	DB(10)
Yennyawoso	YW(9)	KNUST	KN(48)	Apirabo	AJ(36)	Atasomanso	AL(11)
Dichemso	DM(21)	Ayigya	AY(49)	Dompoase	DP(20)	Anyinam	AQ(14)
Akrom	AK(29)	Bomso	BM(50)	Atonsu	AT(55)	Santase	SN(7)
Aboabo	AA(31)	Oforikrom	OF(51)	Ahinsan	AH(54)	Fankyenebra	FA(8)
Adukrom	AO(30)	Anwomaso	AN(40)	Asokwa	AE(53)	Ridge	RG(4)
Asokore Mampong	AM(26)	Boadi	BD(41)	Kaase	KA(19)	South Suntreso	SS(2)
Aparade	AR(28)	Kokoben	KB(42)	Asago	GO(15)	South Patase	SH(3)
Asawase	AW(35)	Apeadu	AU(43)	Sokoban	SK(16)	North Patase	PA(1)
Odumase	OD(32)	Emena	EM(44)	Daban	DA(12)	Danyame	DY(5)

A network was formed out of the map. The fifty-six (56) nodes in the network are the communities or settlements of the five sub-metros. The edges are the various roads linking the communities. These are access roads which link the towns or communities. Figure 4.0 below shows the network. The key to the network is displayed in Table A1.0 in Appendix 1.

Kaase is chosen as the existing landfill sites in Kumasi used in this thesis. It is the existing facility, thus node 19 in the table above or KA on the network below.

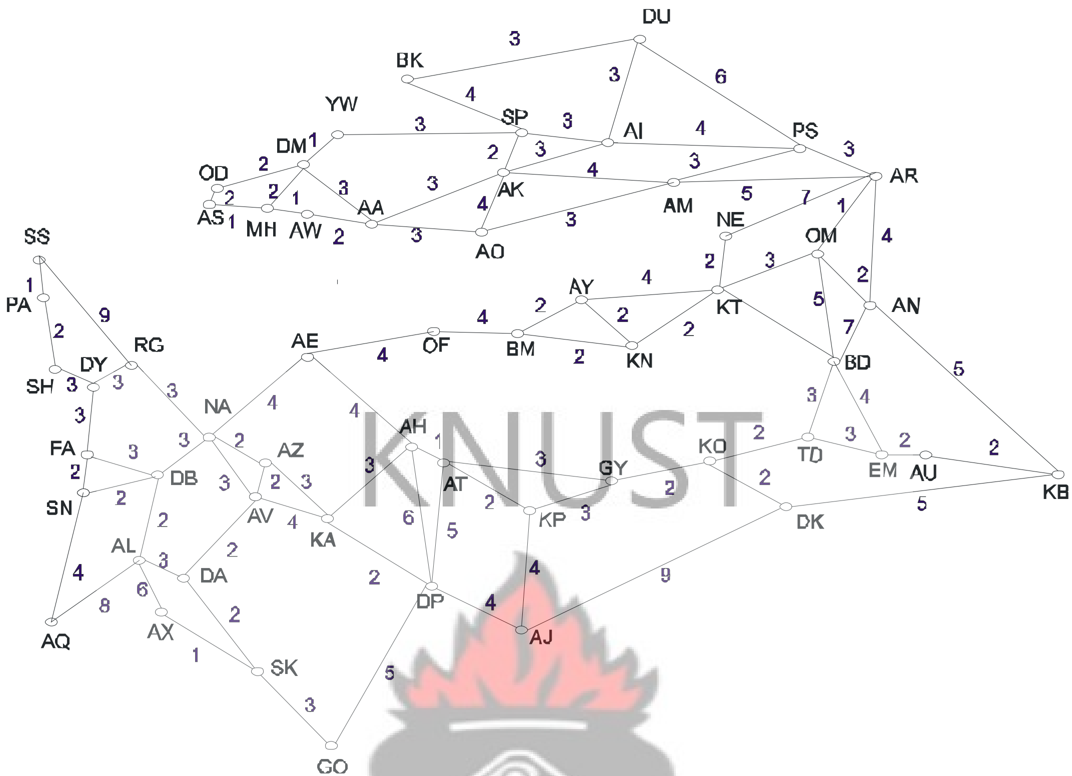


Figure 4.1: Network of communities in the five sub-metros.

The nodes of the network were put in a matrix form. Towns with direct link are indicated with their respective distance. Towns with no direct link are indicated with a dash. It is a 56 by 56 square matrix. This is shown in Table 4.3 in Appendix 2.0.

Table 4.2: Summary of Matrix of Network in Fig 4.2 Indicating Towns and their Pair of Distances.

	1	2	3	4	5	6	.	.	.	27	28	29	30	.	.	.	51	52	53	54	55	56
1	0	1	2	-	-	-	.	.	.	-	-	-	-	.	.	.	-	-	-	-	-	-
2	1	0	-	9	-	-	.	.	.	-	-	-	-	.	.	.	-	-	-	-	-	-
3	2	-	0	-	3	-	.	.	.	-	-	-	-	.	.	.	-	-	-	-	-	-
4	-	9	-	0	3	3	.	.	.	-	-	-	-	.	.	.	-	-	-	-	-	-
5	-	-	3	3	0	-	.	.	.	-	-	-	-	.	.	.	-	-	-	-	-	-
6	-	-	-	3	-	0	.	.	.	-	-	-	-	.	.	.	-	-	4	-	-	-
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27	-	-	-	-	-	-	.	.	.	0	3	-	-	.	.	.	-	-	-	-	-	-
28	-	-	-	-	-	-	.	.	.	3	0	-	-	.	.	.	-	-	-	-	-	-
29	-	-	-	-	-	-	.	.	.	-	-	0	4	.	.	.	-	-	-	-	-	-
30	-	-	-	-	-	-	.	.	.	-	-	4	0	.	.	.	-	-	-	-	-	-
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51	-	-	-	-	-	-	.	.	.	-	-	-	-	.	.	.	0	-	4	-	-	-
52	-	-	-	-	-	-	.	.	.	-	-	-	-	.	.	.	-	0	-	-	3	3
53	-	-	-	-	-	4	.	.	.	-	-	-	-	.	.	.	4	-	0	4	-	-
54	-	-	-	-	-	-	.	.	.	-	-	-	-	.	.	.	-	-	4	0	1	-
55	-	-	-	-	-	-	.	.	.	-	-	-	-	.	.	.	-	3	-	1	0	2
56	-	-	-	-	-	-	.	.	.	-	-	-	-	.	.	.	-	3	-	-	2	0

The Flyod-Warshall algorithm was applied to the matrix in Table 4.2 in to obtain the distance matrix between each pair of node as displayed in Table 4.3.

The matrix shows the length of the shortest path between respective nodes. Column one and row one represents the demand nodes and potential location respectively; the other rows also represent the interconnecting road distances.

Table 4.3: Summary of All Pairs Shortest Path Distance Matrix *D* between Pair of Nodes

	1	2	3	4	5	6	.	.	.	27	28	29	30	.	.	.	51	52	53	54	55	56
1	0	1	2	8	5	11	.	.	.	40	39	42	47	.	.	.	15	19	23	19	15	19
2	1	0	3	9	6	12	.	.	.	41	40	43	48	.	.	.	16	20	24	20	16	20
3	2	3	0	6	3	9	.	.	.	38	37	40	45	.	.	.	13	17	21	13	17	21
4	8	9	6	0	3	3	.	.	.	32	31	34	39	.	.	.	7	11	15	7	11	15
5	5	6	3	3	0	6	.	.	.	35	34	37	42	.	.	.	10	14	18	14	10	18
6	11	12	9	3	6	0	.	.	.	29	28	31	36	.	.	.	12	4	8	12	5	5
.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
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27	36	35	32	26	29	23	.	.	.	9	8	11	13	.	.	.	19	20	11	16	19	20
28	30	32	29	23	26	20	.	.	.	0	4	3	16	.	.	.	16	17	8	12	13	17
29	40	41	38	32	35	29	.	.	.	4	0	3	8	.	.	.	25	26	17	21	22	25
30	39	40	37	31	34	28	.	.	.	3	3	0	8	.	.	.	24	25	16	20	21	24
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51	11	11	11	11	11	8	.	.	.	17	16	19	24	.	.	.	4	8	4	8	9	11
52	7	7	7	7	7	7	.	.	.	22	21	24	29	.	.	.	8	4	14	12	8	14
53	7	7	7	7	7	4	.	.	.	25	24	27	32	.	.	.	0	4	8	4	10	10
54	3	3	3	3	3	3	.	.	.	26	25	28	33	.	.	.	4	0	8	12	8	14
55	4	4	4	4	4	4	.	.	.	25	24	27	32	.	.	.	5	1	9	13	3	15
56	6	6	6	6	6	6	.	.	.	25	24	27	32	.	.	.	7	3	7	11	15	17

4.2 FORMULATION OF PROBLEM INSTANCE

At this point, we use the Berman and Drezner's algorithm (2008) to solve the problem.

We begin by formulating the conditional p-centre problem as

$$Min\left[g(x)=\left\{\max_{i=1,\dots,n}\ \min\{d(X,i),d(Y,i)\}\right\}\right]$$

$$T_{ij} = \min$$

Let $Y=\{1,2,3,4,\ldots\ldots\ldots\ldots,56\}$ in table 4.3 and $d(X,i)$, $d(Y,i)$ are entries in table 4.3

4.2.1 THE BERMAN AND DREZNER'S ALGORITHM

Steps:

1. Let D be a distance matrix with rows corresponding to demands and columns corresponding to potential locations.
2. Solve the conditional problem by defining a modified shortest distance matrix, from D to $\hat{D}$

$$\hat{D} = \min\left\{\left(d_{ij}\right), \left(\min_{k \in Q}\{d_{ik}\}\right)\right\} \forall i \in N,\ j \in C(center)$$

Note that $\hat{D}$ is not symmetric even when D is symmetric.

3. Find the optimal new location using $\hat{D}$ for the network with the objective function

$$Min\left[\ g(x) = \max_{i=1,\ldots\ldots,n} \min\{d(X,i), d(Y,i)\}\right]$$

a) Defining a modified shortest distance matrix, $\hat{D} = \min\{d(X,i), d(Y,i)\}$,

where $X = \{1,2,3,4,5,6,\ldots\ldots\ldots,56\}$ is the set of facilities of new locations and $Y = \{19\}$, is the location of existing facility. $d(X,i)$ and $d(Y,i)$, is the shortest distance from the closest facility in X and Y respectively to the node i.

Table 4.4a: Summary of Modified Shortest Distance Matrix, $\hat{D}$.

	1	2	3	.	.	19	20	.	.	27	28	29	30	.	.	.	51	52	53	54	55	56
1	0	1	2	.	.	16	16	.	.	16	16	16	16	.	.	.	16	16	15	16	16	16
2	1	0	3	.	.	17	17	.	.	17	17	17	17	.	.	.	17	17	16	17	17	17
3	2	3	0	.	.	14	14	.	.	14	14	14	14	.	.	.	17	21	13	17	18	20
.	.	.	.	.	.	.	.	.	.	8	8	8	8	.	.	.	11	15	7	11	12	14
.	.	.	.	.	.	.	.	.	.	11	11	11	11	.	.	.	14	18	10	14	15	17
19	0	0	0	.	.	0	0	.	.	0	0	0	0	.	.	.	0	0	0	0	0	0
20	2	2	2	.	.	2	2	.	.	2	2	2	2	.	.	.	2	2	2	2	2	2
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27	23	23	23	.	.	23	23	.	.	4	3	7	6	.	.	.	15	16	19	20	16	20
28	20	20	20	.	.	20	20	.	.	7	5	9	8	.	.	.	12	13	16	17	13	17
29	29	29	29	.	.	29	29	.	.	3	4	0	4	.	.	.	21	22	25	26	22	26
30	28	28	28	.	.	28	28	.	.	7	3	4	0	.	.	.	20	21	24	25	21	25
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51	11	11	11	.	.	11	11	.	.	11	11	11	11	.	.	.	4	0	11	8	8	11
52	7	7	7	.	.	7	7	.	.	7	7	7	7	.	.	.	7	7	0	4	4	7
53	7	7	7	.	.	7	7	.	.	7	7	7	7	.	.	.	7	4	7	4	4	7
54	3	3	3	.	.	3	3	.	.	3	3	3	3	.	.	.	3	3	3	0	0	3
55	4	4	4	.	.	4	4	.	.	4	4	4	4	.	.	.	6	6	3	1	1	4
56	6	6	6	.	.	6	6	.	.	6	6	6	6	.	.	.	6	6	3	6	3	6

b) Defining a modified shortest distance matrix, $\hat{D}$ with the existing node (19) removed.

$$g(x) = \max_{i=1,\ldots,n} \min\{d(X,i), d(Y,i)\}$$

Table 4.4b: Summary of the modified distance matrix with the existing facility removed.

	1	2	3	4	5	6	.	.	.	27	28	29	30	.	.	.	51	52	53	54	55	56
1	0	1	2	8	5	11	.	.	.	16	16	16	16	.	.	.	15	16	15	16	16	16
2	1	0	3	9	6	12	.	.	.	17	17	17	17	.	.	.	16	17	17	17	17	17
3	2	3	0	6	3	9	.	.	.	14	14	14	14	.	.	.	13	14	13	14	14	14
4	8	8	6	0	3	3	.	.	.	8	8	8	8	.	.	.	7	8	8	8	8	8
5	5	6	3	3	0	6	.	.	.	11	11	11	11	.	.	.	10	11	11	11	11	11
6	5	5	5	3	5	0	.	.	.	5	5	5	5	.	.	.	4	5	5	5	5	5
.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
27	23	23	23	23	23	23	.	.	.	6	9	13	13	.	.	.	19	20	19	20	19	19
28	20	20	20	20	20	20	.	.	.	8	11	16	15	.	.	.	16	17	16	17	16	16
29	29	29	29	29	29	29	.	.	.	4	3	8	7	.	.	.	25	26	25	26	25	25
30	28	28	28	28	28	28	.	.	.	0	3	8	7	.	.	.	24	25	24	25	24	24
.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
51	11	11	11	11	11	8	.	.	.	11	11	11	11	.	.	.	4	8	9	8	9	11
52	7	7	7	7	7	7	.	.	.	7	7	7	7	.	.	.	7	4	3	4	3	3
53	7	7	7	7	7	4	.	.	.	7	7	7	7	.	.	.	0	4	5	4	5	7
54	3	3	3	3	3	3	.	.	.	3	3	3	3	.	.	.	3	0	1	0	1	3
55	4	4	4	4	4	4	.	.	.	4	4	4	4	.	.	.	4	1	0	1	0	2
56	6	6	6	6	6	6	.	.	.	6	6	6	6	.	.	.	6	3	2	3	2	0

c) Defining the optimal new location using $\hat{D}$ for the network with the objective function

$$Min\left[g(x) = \max_{i=1,\ldots\ldots,n} \min\{d(X,i), d(Y,i)\} \right]$$

A summary of the results is presented in Table 4.5 below. With column 55 representing the maximum distance between demand nodes and rows represent the minimum interconnected distances.

Table 4.6: Summary of Optimal Location, $Min\, g(x)$ using $\hat{D}$

	1	**2**	**3**	**4**	**5**	**6**	**.**	**.**	**.**	**27**	**28**	**29**	**30**	**.**	**.**	**.**	**51**	**52**	**53**	**54**	**55**	**56**	**MAX**
1	0	1	2	8	5	5	.	.	.	23	28	31	36	.	.	.	7	3	4	3	4	4	36
2	1	0	3	8	6	5	.	.	.	23	28	31	36	.	.	.	7	3	4	3	4	4	36
3	2	3	0	6	3	5	.	.	.	23	28	31	36	.	.	.	7	3	4	3	4	4	36
4	8	9	6	0	3	3	.	.	.	23	28	31	36	.	.	.	7	3	4	3	4	4	36
5	5	6	3	3	0	5	.	.	.	23	28	31	36	.	.	.	7	3	4	3	4	4	36
6	11	12	9	3	6	0	.	.	.	23	28	31	36	.	.	.	7	3	4	3	4	4	36
.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
27	16	17	14	8	11	5	.	.	.	0	6	9	13	.	.	.	7	3	4	3	4	4	17
28	16	17	14	8	11	5	.	.	.	3	8	11	16	.	.	.	7	3	4	3	4	4	17
29	16	17	14	8	11	5	.	.	.	7	4	3	8	.	.	.	7	3	4	3	4	4	17
30	16	17	14	8	11	5	.	.	.	6	0	\3	8	.	.	.	7	3	4	3	4	4	19
.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
51	16	17	14	8	11	5	.	.	.	11	11	11	11	.	.	.	0	11	4	8	9	11	32
52	16	17	14	8	11	5	.	.	.	7	7	7	7	.	.	.	7	0	7	4	3	3	33
53	16	17	14	8	11	5	.	.	.	7	7	7	7	.	.	.	4	7	0	4	5	7	32
54	16	17	14	8	11	5	.	.	.	3	3	3	3	.	.	.	3	3	3	0	1	3	33
55	16	17	14	8	11	5	.	.	.	4	4	4	4	.	.	.	4	3	4	1	0	2	32
56	16	\17	14	8	11	5	.	.	.	6	6	6	6	.	.	.	6	3	6	3	2	0	32
MINIMUM																							17

From the results above in Table 4.5, by using the modified shortest distance matrix$\widehat{D}$, it is easy to verify that the optimal new location can be either at node 22(BK), 23(SP), 24(DU), 25(AI), 26(AM), 27(PS), 28(AR) or 34(MH) with an objective function value of 17km.

4.3 FACTOR RATING METHOD

Based on the Berman and Drezner's algorithm, eight different locations were found to be optimal, thus Buokrom (BK) or node 22, Sepetimpo (SP) or node 23, Duase (DU) or node 24, Asabi (AI) or node 25, Asokore Mampong (AM) or node26, Pakoso (PS) or node 27, Aperade (AR) or node 28 and Manhyia (MH) or node 34. To decide among the eight communities, the factor rating method is used. Considering the location of a landfill site, five relevant factors listed below is noted as shown below in Table4.6 with the respective rating weight attached to each factor.

Table 4.7: Relevant Factors and rating weight

Factor	Factor Name	Rating Weight
1	Land Acquisition	8
2	Community Desirability	4
3	Population Size	6
4	Accessibility of roads to the site	6
5	Proximity of raw materials and suppliers	5

Table 4.7 below summarizes the results opinion leaders and other people score each factor, ranging from one to hundred for all two locations.

Table 4.8 Location Rate on a 1 to 100 basis

Factor	Buokrom	Sepetimpo	Duase	Asabi	Asokore Mampong	Pakoso	Aperade	Manhyia
1	80	70	80	75	100	85	80	50
2	70	75	75	80	85	80	70	50
3	80	70	70	70	80	70	75	90
4	80	70	75	70	80	75	75	80
5	75	50	60	60	70	70	75	80

The ratio of the rating weight is multiplied by the rating scores of the communities for each particular factor. The results are then shown in Table 4.8.

Table 4.9 Rating Scores of locations

Factor	Rating Weight	Ratio Of Rate	Buokrom	Sepetimpo	Duase	Asabi	Asokore Mampong	Pakoso	Aparade	Manhyia
1	8	0.28	22.4	19.6	22.4	21	28	23.8	22.4	14
2	4	0.14	9.8	10.5	10.5	11.2	11.9	11.2	9.8	7
3	6	0.21	16.8	14.7	14.7	14.7	16.8	14.7	14.7	18.9
4	6	0.21	16.8	14.7	15.75	14.7	16.8	15.75	14.7	16.8
5	5	0.17	12.75	8.5	10.2	10.2	11.9	11.9	12.75	13.6
Total			78.55	68	73.55	71.8	85.4	77.35	74.35	70.3

Clearly from their respective aggregate scores, Asokore Mampong (AM) or node 26 would be recommended since it has the highest aggregate of 85.4.

4.4 DISCUSSION

With the algorithm demonstrated above in session 4.2, considering the 56-node network depicted in figure 4.0, and solving the conditional 1-center problem with $Q = \{KA\, or\, 19\}$ and $p = 1$. It is easy to verify that D and $\hat{D}$, is the distance matrix shown in Table 4.3 and Table 4.4. The optimal new location using the modified distance matrix $\hat{D}$, thus by using the Berman and Drezner's algorithm the new landfill site can arbitrarily be located at AM (Asokore Manpong) or node 26, Buokrom(BK) or node 22, Sepetimpo(SP) or node 23, Duase(DU) or node 24, Asabi(AI) or node 25, Pakoso(PS) or node 27, Aperade(AR) or node 28 and Manhyia(MH) or node 34, an objective function value of $Ming(x) = 17km$.

Also considering the factor rating method, it is clear that Asokore Mampong has the highest aggregate score of 85.4; hence node 26 or Asokore Mampong (AM) is recommended out of the eight communities for the location of a new landfill site in the Asawase Sub metro.

CHAPTER 5

CONCLUSION AND RECOMMENDATION

5.1 CONCLUSION

The objective of the study was to use the conditional p-centre problem to locate an additional landfill site in the Kumasi Metropolis.

Considering the objective function $Min[g(x)]$ as showed in page 62 and using the method of Berman and Drezner (2008), the facility could be located at any of these eight nodes; Asokore Manpong (AM) or node 26, Buokrom (BK) or node 22, Sepetimpo (SP) or node 23, Duase (DU) or node 24, Asabi (AI) or node 25, Pakoso (PS) or node 27, Aperade (AR) or node 28 and Manhyia (MH) or node 34. By identifying relevant factors for the location of a landfill, determination of rating weights and analysis of scores, the best alternative found among the eight communities is Asokore Mampong (AM) or node 26, which has the highest aggregate of 85.4 as compared with the other seven communities earlier mentioned in using the factor rating method.

The objective function value obtained is 17 kilometres. This implies that the coverage distance travelled by the farthest truck carrying refuse to the new landfill site at Asokore Mampong is 17 kilometres.

The facility at Asokore Mampong will serve the entire five sub metros because the model is a conditional 1-center problem. Meaning only one facility is to be sited although there is an existing facility already sited.

Figure 5.0 below shows the site for the facility on the network of towns and settlements in the five sub metro.

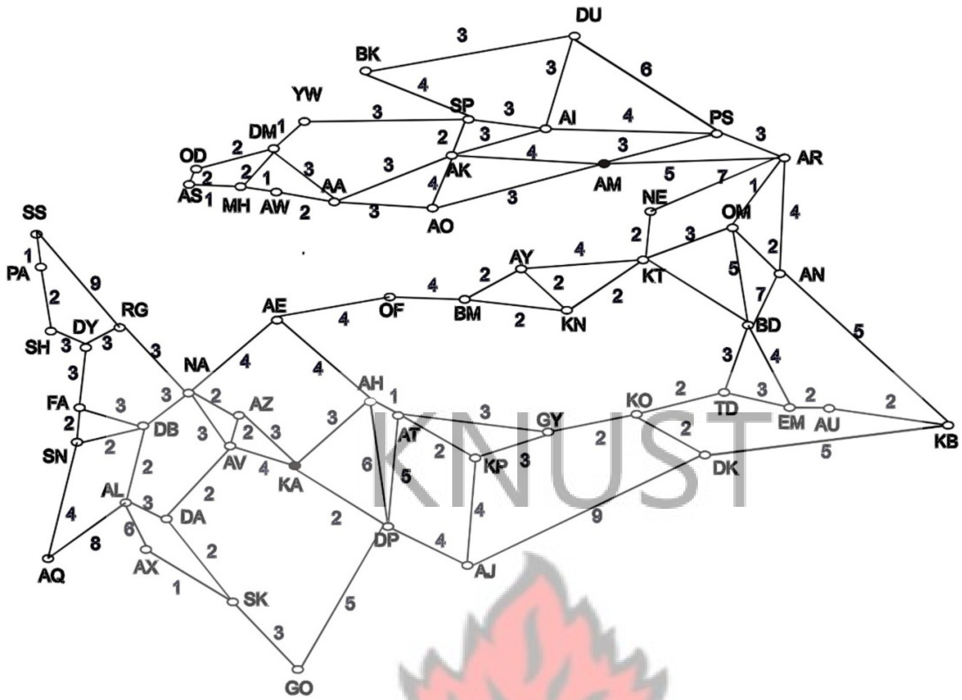

Figure 3 5.0 Network of towns and settlements in the five sub metros indicating the location of the existing and new facility.

5.2 RECOMMENDATIONS

The following recommendations are made.

1. The government or waste management companies who would like to site a landfill in the Kumasi Metropolis are advised to locate it at Asokore Mampong.
2. The facility should be located at an open space in the town with good road networks linking the town to other towns.
3. The facility should be managed in such a way that the health hazards of the surrounding communities would be minimized.
4. Researchers who would like to do further work could consider Z. Drenzer (2008) Algorithm that requires solving $0(\log n)$ unconditional p-centre problems.

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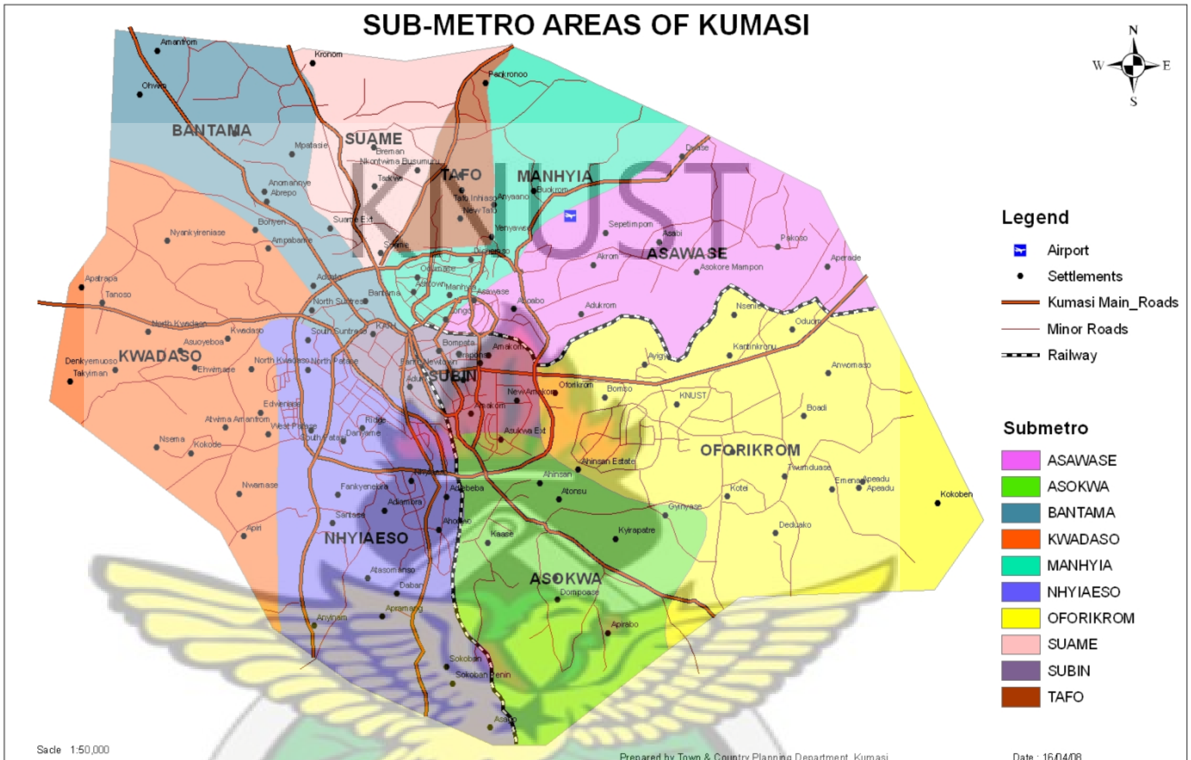
APPENDICES

APPENDIX 1.0



APPENDIX 2.0

Table A2.0 Matrix of Network in Fig 4.0 Indicating Towns and their Pair of Distances

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
1	0	1	2	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
2	1	0	-	9	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
3	2	-	0	-	3	-	-	-	-	-	-	-	-	-	-	-	-	-	-
4	-	9	-	0	3	3	-	-	-	-	-	-	-	-	-	-	-	-	-
5	-	-	3	3	0	-	-	3	-	-	-	-	-	-	-	-	-	-	-
6	-	-	-	3	-	0	-	-	-	3	-	-	-	-	-	-	3	2	-
7	-	-	-	-	-	-	0	2	-	2	-	-	-	4	-	-	-	-	-
8	-	-	-	-	3	-	2	0	-	3	-	-	-	-	-	-	-	-	-
9	-	-	-	-	-	-	-	-	0	-	-	-	-	-	-	-	-	-	-
10	-	-	-	-	-	3	2	3	-	0	2	-	-	-	-	-	-	-	-
11	-	-	-	-	-	-	-	-	-	2	0	3	6	8	-	-	-	-	-
12	-	-	-	-	-	-	-	-	-	-	3	0	-	-	-	2	2	-	-
13	-	-	-	-	-	-	-	-	-	-	6	-	0	-	-	1	-	-	-
14	-	-	-	-	-	-	4	-	-	-	8	-	-	0	-	-	-	-	-
15	-	-	-	-	-	-	-	-	-	-	-	-	-	-	0	3	-	-	-
16	-	-	-	-	-	-	-	-	-	-	-	2	1	-	3	0	-	-	-
17	-	-	-	-	-	3	-	-	-	-	-	2	-	-	-	-	0	2	4
18	-	-	-	-	-	2	-	-	-	-	-	-	-	-	-	-	2	0	3
19	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	4	3	0
20	-	-	-	-	-	-	-	-	-	-	-	-	-	-	5	-	-	-	2
21	-	-	-	-	-	-	-	-	1	-	-	-	-	-	-	-	-	-	-
22	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
23	-	-	-	-	-	-	-	-	3	-	-	-	-	-	-	-	-	-	-
24	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
25	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
26	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
27	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
28	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
29	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
30	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
31	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
32	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
33	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-

	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38
1	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
2	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
3	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
4	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
5	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
6	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
7	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
8	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
9	-	1	-	3	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
10	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
11	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
12	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
13	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
14	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
15	5	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
16	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
17	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
18	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
19	2	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
20	0	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	4	-	-
21	-	0	-	-	-	-	-	-	-	-	-	3	2	-	2	-	-	-	-
22	-	-	0	4	3	-	-	-	-	-	-	-	-	-	-	-	-	-	-
23	-	-	4	0	-	3	-	-	-	2	-	-	-	-	-	-	-	-	-
24	-	-	3	-	0	3	-	6	-	-	-	-	-	-	-	-	-	-	-
25	-	-	-	3	3	0	-	4	-	3	-	-	-	-	-	-	-	-	-
26	-	-	-	-	-	-	0	3	5	4	3	-	-	-	-	-	-	-	-
27	-	-	-	-	6	4	3	0	3	-	-	-	-	-	-	-	-	-	-
28	-	-	-	-	-	-	5	3	0	-	-	-	-	-	-	-	-	1	7
29	-	-	-	2	-	3	4	-	-	0	4	3	-	-	-	-	-	-	-
30	-	-	-	-	-	-	3	-	-	4	0	3	-	-	-	-	-	-	-
31	-	3	-	-	-	-	-	-	-	3	3	0	-	-	-	2	-	-	-
32	-	2	-	-	-	-	-	-	-	-	-	-	0	2	-	-	-	-	-
33	-	-	-	-	-	-	-	-	-	-	-	-	2	0	1	-	-	-	-

	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56
1	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
2	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
3	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
4	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
5	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
6	-	-	-	-	-	-	-	-	-	-	-	-	-	-	4	-	-	-
7	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
8	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
9	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
10	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
11	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
12	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
13	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
14	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
15	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
16	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
17	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
18	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
19	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	3	-	-
20	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	6	5	-
21	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
22	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
23	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
24	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
25	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
26	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
27	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
28	-	4	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
29	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
30	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
31	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
32	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
33	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
34	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
35	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
36	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
37	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
38	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
39	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
40	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
41	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
42	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
43	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
44	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
45	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
46	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
47	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
48	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
49	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
50	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
51	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
52	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
53	-	-	-	-	-	4	-	-	-	-	-	-	-	-	-	-	-	-	-
54	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	3
55	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
56	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-

	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38
34	-	2	-	-	-	-	-	-	-	-	-	-	-	1	0	1	-	-	-
35	-	-	-	-	-	-	-	-	-	-	-	2	-	-	1	0	-	-	-
36	4	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	0	-	-
37	-	-	-	-	-	-	-	-	1	-	-	-	-	-	-	-	-	0	-
38	-	-	-	-	-	-	-	-	7	-	-	-	-	-	-	-	-	-	0
39	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	3	2
40	-	-	-	-	-	-	-	-	4	-	-	-	-	-	-	-	-	2	-
41	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	5	-
42	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
43	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
44	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
45	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
46	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	9	-	-
47	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
48	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
49	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
50	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
51	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
52	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
53	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
54	6	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
55	5	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
56	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	4	-	-

	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56
34	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
35	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
36	-	-	-	-	-	-	-	9	-	-	-	-	-	-	-	-	-	4
37	3	2	5	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
38	2	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
39	0	-	3	-	-	-	-	-	-	2	4	-	-	-	-	-	-	-
40	-	0	7	5	-	-	-	-	-	-	-	-	-	-	-	-	-	-
41	3	7	0	-	-	4	3	-	-	-	-	-	-	-	-	-	-	-
42	-	5	-	0	2	-	-	5	-	-	-	-	-	-	-	-	-	-
43	-	-	-	2	0	2	-	-	-	-	-	-	-	-	-	-	-	-
44	-	-	4	-	2	0	3	-	-	-	-	-	-	-	-	-	-	-
45	-	-	3	-	-	3	0	-	2	-	-	-	-	-	-	-	-	-
46	-	-	-	5	-	-	-	0	2	-	-	-	-	-	-	-	-	-
47	-	-	-	-	-	-	2	2	0	-	-	-	-	2	-	-	-	-
48	2	-	-	-	-	-	-	-	-	0	2	2	-	-	-	-	-	-
49	4	-	-	-	-	-	-	-	-	2	0	2	-	-	-	-	-	-
50	-	-	-	-	-	-	-	-	-	2	2	0	4	-	-	-	-	-
51	-	-	-	-	-	-	-	-	-	-	-	4	0	-	4	-	-	-
52	-	-	-	-	-	-	-	-	2	-	-	-	-	0	-	-	3	3
53	-	-	-	-	-	-	-	-	-	-	-	-	4	-	0	4	-	-
54	-	-	-	-	-	-	-	-	-	-	-	-	-	-	4	0	1	-
55	-	-	-	-	-	-	-	-	-	-	-	-	-	3	-	1	0	2
56	-	-	-	-	-	-	-	-	-	-	-	-	-	3	-	-	2	0

APPENDIX 3.0

Table A3.0 Shortest Distance Matrix between Pair of Nodes

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
1	0	1	2	8	5	11	10	8	44	11	13	16	19	14	21	18	14	13	16
2	1	0	3	9	6	12	11	9	45	12	14	17	20	15	22	19	15	14	17
3	2	3	0	6	3	9	8	6	42	9	11	14	17	12	19	16	12	11	14
4	8	9	6	0	3	3	8	6	36	6	8	8	11	12	13	10	6	5	8
5	5	6	3	3	0	6	5	3	39	6	8	11	14	9	16	13	9	8	11
6	11	12	9	3	6	0	5	6	33	3	5	5	8	9	10	7	3	2	5
7	10	11	8	8	5	5	0	2	38	2	4	7	10	4	12	9	8	7	10
8	8	9	6	6	3	6	2	0	39	3	5	8	11	6	13	10	9	8	11
9	44	45	42	36	39	33	38	39	0	36	38	38	41	42	39	40	36	35	33
10	11	12	9	6	6	3	2	3	36	0	2	5	8	6	10	7	6	5	8
11	13	14	11	8	8	5	4	5	38	2	0	3	6	8	8	5	5	7	9
12	16	17	14	8	11	5	7	8	38	5	3	0	3	11	5	2	2	4	6
13	19	20	17	11	14	8	10	11	41	8	6	3	0	14	4	1	5	7	9
14	14	15	12	12	9	9	4	6	42	6	8	11	14	0	16	13	12	11	14
15	21	22	19	13	16	10	12	13	39	10	8	5	4	16	0	3	7	9	7
16	18	19	16	10	13	7	9	10	40	7	5	2	1	13	3	0	4	6	8
17	14	15	12	6	9	3	8	9	36	6	5	2	5	12	7	4	0	2	4
18	13	14	11	5	8	2	7	8	35	5	7	4	7	11	9	6	2	0	3
19	16	17	14	8	11	5	10	11	33	8	9	6	9	14	7	8	4	3	0
20	18	19	16	10	13	7	12	13	34	10	11	8	9	29	28	31	36	35	34
21	45	46	43	37	40	34	39	40	1	37	39	39	42	43	40	41	37	36	34
22	43	44	41	35	38	32	37	38	7	35	37	40	41	38	39	35	34	32	33
23	41	42	39	33	36	30	35	36	3	33	35	35	38	39	36	37	33	32	30
24	40	41	38	32	35	29	34	35	9	32	34	34	37	38	35	36	32	31	29
25	38	39	36	30	33	27	32	33	6	30	32	32	35	36	33	34	30	29	27
26	36	37	34	28	31	25	30	31	9	28	30	30	33	34	31	32	28	27	25
27	34	35	32	26	29	23	28	29	10	26	28	28	31	32	29	30	26	25	23
28	30	32	29	23	26	20	25	26	13	23	25	25	28	29	26	27	23	22	20

	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38
1	18	45	43	41	40	38	36	40	39	42	47	46	45	44	22	30	29	27	32
2	19	46	44	42	41	39	37	41	40	43	48	47	46	45	23	31	30	28	33
3	16	43	41	39	38	36	34	38	37	40	45	44	43	42	20	28	27	25	30
4	10	37	35	33	32	30	28	32	31	34	39	38	37	36	14	22	21	19	24
5	13	40	38	36	35	33	31	35	34	37	42	41	40	39	17	25	24	22	27
6	7	34	32	30	29	27	25	29	28	31	36	35	34	33	11	19	18	16	21
7	12	39	37	35	34	32	30	34	33	36	41	40	39	38	16	24	23	21	26
8	13	40	38	36	35	33	31	35	34	37	42	41	40	39	17	25	24	22	27
9	34	1	7	3	9	6	9	5	7	4	3	4	3	4	33	14	19	17	16
10	10	37	35	33	32	30	28	32	31	34	39	38	37	36	14	22	21	19	24
11	11	39	37	35	34	32	30	34	33	36	41	40	39	38	15	24	23	21	26
12	8	39	37	35	34	32	30	34	33	36	41	40	39	38	12	24	23	21	26
13	9	42	40	38	37	35	33	37	36	39	44	43	42	41	13	27	26	24	29
14	16	43	41	39	38	36	34	36	37	40	45	44	43	42	20	28	27	25	30
15	5	40	38	36	35	33	31	35	34	37	42	41	40	39	9	25	25	23	27
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17	6	37	35	33	32	30	28	32	31	34	39	38	37	36	10	22	21	19	24
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21	35	0	8	4	10	7	9	6	6	3	2	3	2	3	34	15	20	18	17
22	8	0	4	3	6	10	6	6	10	9	10	11	10	11	32	13	18	16	15
23	31	4	4	0	6	3	6	2	6	5	6	7	6	7	30	11	16	14	13
24	30	10	3	6	0	3	9	6	10	9	12	13	12	11	29	10	15	13	13
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26	26	9	10	6	9	7	0	4	3	6	11	10	9	8	25	6	11	9	8
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7	24	26	26	24	21	21	19	19	19	17	13	17	9	13	19	17	9	17
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27	9	11	13	13	12	16	14	9	11	11	15	16	19	20	11	16	19	20
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54	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3
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31	32	3	9	5	9	6	6	3	3	0	5	4	3	2	31	12	17	15	14
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35	33	3	11	7	11	8	8	5	5	2	4	2	1	0	33	14	19	17	16
36	4	4	4	4	4	4	4	29	28	31	36	35	34	33	0	19	19	17	19
37	19	15	13	11	10	8	6	10	9	12	17	16	15	14	19	0	5	3	2
38	19	19	18	16	15	13	11	15	14	17	22	21	20	19	19	5	0	2	7
39	17	17	16	14	13	11	9	13	12	15	20	19	18	17	17	3	2	0	5
40	21	17	15	13	12	10	8	12	11	14	19	18	17	16	19	2	7	5	0
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42	16	16	16	16	16	15	13	17	16	19	24	23	22	21	14	7	12	10	5
43	16	16	16	16	16	16	15	19	18	21	26	25	24	23	16	9	11	9	7
44	14	14	14	14	14	14	14	19	18	21	26	25	24	23	14	9	9	7	9
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48	19	20	18	16	15	13	11	15	14	17	22	21	20	19	19	5	4	2	7
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55	4	4	4	4	4	4	4	25	24	27	32	31	30	29	6	15	15	13	17
56	6	6	6	6	6	6	6	25	24	27	32	31	30	29	4	15	15	13	17

	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56
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36	19	17	19	14	14	16	14	11	9	9	19	21	21	25	26	29	30	21
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38	5	12	11	9	8	12	10	4	6	6	10	12	14	16	6	10	12	16
39	3	10	9	7	6	10	8	2	4	4	8	10	12	14	4	8	12	14
40	7	5	7	9	10	10	12	1	9	9	13	14	17	18	9	13	14	18
41	0	8	6	4	3	7	5	5	7	7	11	7	15	11	14	7	11	15
42	8	0	2	4	7	5	12	14	14	18	9	17	13	9	14	17	13	18
43	6	2	0	2	5	7	7	11	13	13	17	9	17	13	17	13	11	17
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46	7	5	7	7	4	0	2	12	14	14	16	4	12	8	12	8	4	14
47	5	7	7	5	2	2	0	10	12	12	14	2	10	6	10	6	2	14
48	5	12	11	9	8	12	10	0	2	2	6	12	10	14	10	14	6	12
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51	11	18	17	15	14	16	14	6	6	4	0	12	4	8	4	8	9	11
52	7	9	9	7	4	4	2	12	14	14	12	0	8	4	14	12	8	14
53	15	17	17	15	12	12	10	10	10	8	4	8	0	4	8	4	10	10
54	11	13	13	11	8	8	6	14	14	12	8	4	4	0	8	12	8	14
55	10	12	12	10	7	7	5	15	15	13	9	3	5	1	9	13	3	15
56	10	12	12	10	7	7	5	15	17	15	11	3	7	3	7	11	15	17

APPENDIX 4.0

Table A4.0 Modified Shortest Distance Matrix between Pair of Nodes

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
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2	1	0	3	9	6	12	11	9	17	12	14	17	17	15	17	17	15	14	17	17
3	2	3	0	6	3	9	8	6	14	9	11	14	14	12	14	14	12	11	14	14
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5	5	6	3	3	0	6	5	3	11	6	8	11	11	9	11	11	9	8	11	11
6	5	5	5	3	5	0	5	5	5	3	5	5	5	5	5	5	3	2	5	5
7	10	10	8	8	5	5	0	2	10	2	4	7	10	4	10	9	8	7	10	10
8	8	9	6	6	3	6	2	0	11	3	5	8	11	6	11	10	9	8	11	11
9	33	33	33	33	33	33	33	33	0	33	33	33	33	33	33	33	33	33	33	1
10	8	8	8	6	6	3	2	3	8	0	2	5	8	6	8	1	5	7	8	8
11	9	9	9	9	9	8	9	9	9	8	6	3	0	9	4	1	5	7	9	9
12	6	6	6	6	6	5	6	6	6	5	3	0	3	6	5	2	2	4	6	6
13	9	9	9	9	9	8	9	9	14	8	6	3	0	9	4	1	5	7	9	9
14	14	14	12	12	9	9	4	6	8	6	8	11	14	0	14	13	12	11	14	14
15	7	7	7	7	7	7	7	7	7	7	7	5	4	7	0	3	7	7	7	5
16	8	8	8	8	8	7	8	8	8	7	5	2	1	8	3	0	4	6	8	8
17	4	4	4	4	4	3	4	4	4	4	4	2	4	4	4	4	0	2	4	4
18	3	3	3	3	3	2	3	3	3	3	3	3	3	3	3	3	2	0	3	3
19	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
20	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	0
21	34	34	34	34	34	34	34	34	34	1	34	34	34	34	34	34	34	34	34	34
22	32	32	32	32	32	32	32	32	32	7	32	32	32	32	32	32	32	32	32	32
23	30	30	30	30	30	30	30	30	30	3	30	30	30	30	30	30	30	30	30	30
24	29	29	29	29	29	29	29	29	29	9	29	29	29	29	29	29	29	29	29	29
25	27	27	27	27	27	27	27	27	27	6	27	27	27	27	27	27	27	27	27	27
26	25	25	25	25	25	25	25	25	25	9	25	25	25	25	25	25	25	25	25	25
27	23	23	23	23	23	23	23	23	23	10	23	23	23	23	23	23	23	23	23	23
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4	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8
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7	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10
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13	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9
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15	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7
16	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8
17	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4
18	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3
19	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
20	2	0	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2
21	34	34	0	8	4	10	7	9	6	6	3	2	3	2	3	34	15	20
22	32	32	8	0	4	3	6	10	6	10	9	10	11	10	11	32	13	18
23	30	30	4	4	0	6	3	6	2	6	5	6	7	6	7	30	11	16
24	29	29	10	3	6	0	3	9	6	10	9	12	13	12	11	29	10	15
25	27	27	7	6	3	3	0	7	3	7	6	9	10	9	8	27	8	13
26	25	25	9	10	6	9	7	0	4	3	6	11	10	9	8	25	6	11
27	23	23	11	9	7	6	4	3	7	6	9	13	13	12	11	23	4	9
28	20	20	14	12	10	9	7	5	9	8	11	16	15	14	13	20	1	6

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2	17	17	17	17	17	17	17	17	17	17	17	17	17	16	17	17	17	17
3	14	14	14	14	14	14	14	14	14	14	14	15	11	13	14	14	14	14
4	8	8	8	8	8	8	8	8	8	8	8	15	11	7	8	8	8	8
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6	5	5	5	5	5	5	5	5	5	5	5	5	5	4	5	5	5	5
7	10	10	10	10	10	10	10	10	10	10	10	15	11	9	10	10	10	10
8	11	11	11	11	11	11	11	11	11	11	17	15	11	10	11	11	11	11
9	17	16	19	21	23	23	22	26	24	19	21	21	25	26	29	30	25	30
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11	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9
12	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6
13	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9
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15	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7
16	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8
17	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4
18	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3
19	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
20	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2
21	18	17	20	22	24	24	23	27	25	20	22	22	26	27	30	31	26	31
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23	14	13	16	18	20	20	19	23	21	16	18	18	22	23	26	27	26	27
24	13	13	15	17	19	19	18	22	20	15	17	17	21	22	25	26	25	26
25	11	10	13	15	17	17	16	20	18	13	15	15	19	20	23	24	20	24
26	9	8	11	13	15	15	14	18	16	11	13	13	17	18	21	22	18	22
27	7	6	9	11	13	13	12	16	14	9	11	11	15	16	19	20	16	20
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33	35	35	35	35	35	35	35	35	35	4	35	35	35	35	35	35	35	35	35	35
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35	33	33	33	33	33	33	33	33	33	4	33	33	33	33	33	33	33	33	33	33
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46	11	11	11	11	11	11	11	11	11	11	11	11	11	11	11	11	11	11	11	11
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48	17	17	17	17	17	17	17	17	17	17	17	17	17	17	17	17	17	16	17	17
49	17	17	17	17	17	17	17	17	17	17	17	17	17	17	17	17	17	16	17	17
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54	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3
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31	3	9	3	9	5	9	6	6	3	3	0	5	4	3	2	31	12	17
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34	2	10	2	10	6	12	9	9	6	6	3	3	1	0	1	34	15	20
35	3	11	3	11	7	11	8	8	5	5	2	4	2	1	0	33	14	19
36	6	4	6	6	6	6	6	6	6	6	6	6	6	6	6	0	6	6
37	19	19	15	13	11	10	8	6	10	10	9	12	17	16	15	14	19	0
38	19	19	19	18	16	15	13	11	15	15	14	17	19	19	19	19	19	5
39	17	17	17	16	14	13	11	9	13	13	12	15	17	17	17	17	17	3
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41	14	14	14	14	14	14	13	11	15	14	14	14	14	14	14	14	14	5
42	16	16	16	16	16	16	15	13	17	16	16	16	16	16	16	16	14	7
43	16	16	16	16	16	16	16	15	19	16	16	16	16	16	16	16	16	9
44	14	14	14	14	14	14	14	14	19	14	14	14	14	14	14	14	14	9
45	11	11	11	11	11	11	11	11	18	11	11	11	11	11	11	11	11	8
46	11	11	11	11	11	11	11	11	22	11	11	11	11	11	11	11	9	11
47	9	9	9	9	9	9	9	9	20	9	9	9	9	9	9	9	9	9
48	17	17	17	17	16	15	13	11	15	15	14	17	17	17	17	17	17	5
49	17	17	17	17	17	17	15	13	17	17	16	17	17	17	17	17	17	7
50	15	15	15	15	15	15	15	13	17	15	15	15	15	15	15	15	15	7
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52	7	7	7	7	7	7	7	7	22	7	7	7	7	7	7	7	7	7
53	7	7	7	7	7	7	7	7	25	7	7	7	7	7	7	7	7	7
54	3	3	3	3	3	3	3	3	26	3	3	3	3	3	3	3	3	3
55	4	4	4	4	4	4	4	4	25	4	4	4	4	4	4	4	4	4
56	6	6	6	6	6	6	6	6	25	6	6	6	6	6	6	6	4	6

	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56
29	13	12	15	17	19	19	18	22	20	15	17	17	21	22	25	26	22	26
30	12	11	14	16	18	18	17	21	19	14	16	16	20	21	24	25	21	25
31	15	14	17	19	21	21	20	24	22	17	19	19	23	24	27	28	24	28
32	20	19	22	24	26	26	25	29	27	22	24	24	28	29	32	33	29	33
33	19	18	21	23	25	25	24	28	26	21	23	23	27	28	31	32	28	32
34	18	17	20	22	24	24	23	27	25	20	22	22	26	27	30	31	27	31
35	17	16	19	21	23	23	22	26	24	19	21	21	25	26	29	30	26	30
36	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6
37	5	3	2	5	7	9	9	8	12	10	5	7	7	11	12	15	16	16
38	0	2	7	5	12	11	9	8	12	10	4	6	6	10	12	14	16	16
39	2	0	5	3	10	9	7	6	10	8	2	4	4	8	10	12	14	12
40	7	5	0	7	5	7	9	10	10	12	1	9	9	13	14	17	18	14
41	5	3	7	0	8	6	4	3	7	5	5	7	7	11	7	14	11	14
42	12	10	5	8	0	2	4	7	5	7	12	14	14	16	9	16	13	16
43	11	9	7	6	2	0	2	5	7	7	11	13	13	16	9	16	13	16
44	9	7	9	4	4	2	0	3	7	5	9	11	11	14	7	14	11	14
45	8	6	10	3	7	5	3	0	4	2	8	10	10	11	4	11	8	11
46	11	10	10	7	5	7	7	4	0	2	11	11	11	11	4	11	8	11
47	9	8	9	5	7	7	5	2	2	0	9	9	9	9	2	9	6	9
48	4	2	7	5	12	11	9	8	12	10	0	2	2	6	12	10	14	12
49	6	4	9	7	14	13	11	10	14	12	2	0	2	6	14	10	14	14
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52	7	7	7	7	7	7	7	4	4	2	7	7	7	7	0	7	4	7
53	7	7	7	7	7	7	7	7	7	7	7	7	7	4	7	0	4	7
54	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	0	3
55	4	4	4	4	4	4	4	4	4	4	4	6	6	6	3	4	1	4
56	6	6	6	6	6	6	6	6	6	5	6	6	6	6	3	6	3	6

APPENDIX 4.1

Table A4.1 Modified Shortest Distance Matrix, $\hat{D}$ with set Q removed

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
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2	1	0	3	9	6	12	11	9	17	12	14	17	17	15	17	17	15	14
3	2	3	0	6	3	9	8	6	14	9	11	14	14	12	14	14	12	11
4	8	8	6	0	3	3	8	6	8	6	8	8	8	8	8	8	6	5
5	5	6	3	3	0	6	5	3	11	6	8	11	11	9	11	11	9	8
6	5	5	5	3	5	0	5	5	5	3	5	5	5	5	5	5	3	2
7	10	10	8	8	5	5	0	2	10	2	4	7	10	4	10	9	8	7
8	8	9	6	6	3	6	2	0	11	3	5	8	11	6	11	10	9	8
9	33	33	33	33	33	33	33	33	0	33	33	33	33	33	33	33	33	33
10	8	8	8	6	6	3	2	3	8	0	2	5	8	6	8	7	6	5
11	9	9	9	8	8	5	4	5	9	2	0	3	6	8	8	5	5	7
12	6	6	6	6	6	5	6	6	6	5	3	0	3	6	5	2	2	4
13	9	9	9	9	9	8	9	9	9	8	6	3	0	9	4	1	5	7
14	14	14	12	12	9	9	4	6	14	6	8	11	14	0	14	13	12	11
15	7	7	7	7	7	7	7	7	7	7	7	5	4	7	0	3	7	7
16	8	8	8	8	8	7	8	8	8	7	5	2	1	8	3	0	4	6
17	4	4	4	4	4	3	4	4	4	4	4	2	4	4	4	4	0	2
18	3	3	3	3	3	2	3	3	3	3	3	3	3	3	3	3	2	0
20	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2
21	34	34	34	34	34	34	34	34	1	34	34	34	34	34	34	34	34	34
22	32	32	32	32	32	32	32	32	7	32	32	32	32	32	32	32	32	32
23	30	30	30	30	30	30	30	30	3	30	30	30	30	30	30	30	30	30
24	29	29	29	29	29	29	29	29	9	29	29	29	29	29	29	29	29	29
25	27	27	27	27	27	27	27	27	6	27	27	27	27	27	27	27	27	27
26	25	25	25	25	25	25	25	25	9	25	25	25	25	25	25	25	25	25
27	23	23	23	23	23	23	23	23	10	23	23	23	23	23	23	23	23	23
28	20	20	20	20	20	20	20	20	13	20	20	20	20	20	20	20	20	20

	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38
1	16	16	16	16	16	16	16	16	16	16	16	16	16	16	16	16	16	16	16
2	17	17	17	17	17	17	17	17	17	17	17	17	17	17	17	17	17	17	17
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6	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5
7	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10
8	11	11	11	11	11	11	11	11	11	11	11	11	11	11	11	11	11	11	11
9	33	1	7	3	9	6	9	10	7	4	3	4	3	4	33	14	19	17	16
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16	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8
17	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4
18	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3
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21	34	0	8	4	10	7	9	11	6	3	2	3	2	3	34	15	20	18	17
22	32	8	0	4	3	6	10	9	10	9	10	11	10	11	32	13	18	16	15
23	30	4	4	0	6	3	6	7	6	5	6	7	6	7	30	11	16	14	13
24	29	10	3	6	0	3	9	6	10	9	12	13	12	11	29	10	15	13	13
25	27	7	6	3	3	0	7	4	7	6	9	10	9	8	27	8	13	11	10
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27	23	11	9	7	6	4	3	0	6	9	13	13	12	11	23	4	9	7	6
28	20	14	12	10	9	7	5	3	8	11	16	15	14	13	20	1	6	4	3

	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56
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2	17	17	17	17	17	17	17	17	17	17	17	17	16	17	17	17	17	17
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8	11	11	11	11	11	11	11	11	11	11	11	11	10	11	11	11	11	11
9	19	21	23	23	22	26	24	19	21	21	25	26	29	30	29	30	29	29
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16	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8
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21	20	22	24	24	23	27	25	20	22	22	26	27	30	31	30	31	30	30
22	18	20	27	22	22	25	23	18	20	20	24	25	28	29	28	29	28	28
23	16	18	20	20	19	23	21	16	18	18	22	23	26	27	26	27	26	26
24	15	17	19	19	18	22	20	15	17	17	21	22	25	26	25	26	25	25
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26	11	13	15	15	14	18	16	11	13	13	17	18	21	22	21	22	21	21
27	9	11	13	13	12	16	14	9	11	11	15	16	19	20	19	20	19	19
28	6	8	10	10	9	13	11	6	8	8	12	13	16	17	16	17	16	16

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31	31	31	31	31	31	31	31	31	4	31	31	31	31	31	31	31	31	31
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33	35	35	35	35	35	35	35	35	4	35	35	35	35	35	35	35	35	35
34	34	34	34	34	34	34	34	34	3	34	34	34	34	34	34	34	34	34
35	33	33	33	33	33	33	33	33	4	33	33	33	33	33	33	33	33	33
36	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6
37	19	19	19	19	19	19	19	19	14	19	19	19	19	19	19	19	19	19
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39	17	17	17	17	17	16	17	17	17	17	17	17	17	17	17	17	17	17
40	21	21	21	21	21	21	21	21	16	21	21	21	21	21	21	21	21	21
41	14	14	14	14	14	14	14	14	14	14	14	14	14	14	14	14	14	14
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46	7	5	7	7	4	0	2	11	11	11	11	4	11	8	7	8	7	7
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55	4	4	4	4	4	4	4	4	4	4	4	3	4	1	0	1	0	2
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APPENDIX 4.2

Table A4.2 Optimal Location, $Min\, g(x)$ using $\hat{D}$

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16	16	17	14	8	11	5	9	10	33	7	5	2	1	13	3	0	4	3
17	14	15	12	6	9	3	8	9	33	6	5	2	5	12	7	4	0	2
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44	2	24	22	20	19	17	15	13	18	21	26	25	24	23	6	9	9	7	9
45	2	23	21	19	18	16	14	12	17	20	25	24	23	22	6	8	8	6	10
46	2	27	25	23	22	20	18	16	21	24	29	28	27	26	6	12	12	10	10
47	2	25	23	21	20	18	16	14	19	22	27	26	25	24	6	10	10	8	12
48	2	20	18	16	15	13	11	9	14	17	22	21	20	19	6	5	4	2	7
49	2	22	20	18	17	15	13	11	16	19	24	23	22	21	6	7	6	4	9
50	2	22	20	18	17	15	13	11	16	19	24	23	22	21	6	7	6	4	9
51	2	26	24	22	21	19	17	15	20	23	28	27	26	25	6	11	10	8	13
52	2	27	25	23	22	20	18	16	21	24	29	28	27	26	6	12	12	10	14
53	2	30	28	26	25	23	21	19	24	27	32	31	30	29	6	15	14	12	17
54	2	31	29	27	26	24	22	20	25	28	33	32	31	30	6	16	16	14	18
55	2	30	28	26	25	23	21	19	24	27	32	31	30	29	6	15	15	13	17
56	2	30	28	26	25	23	21	19	24	27	32	31	30	29	4	15	15	13	17

	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	Max_Dbar
29	14	16	16	14	11	11	9	15	17	15	11	7	7	3	4	3	4	4	17
30	14	16	16	14	11	11	9	14	16	15	11	7	7	3	4	3	4	4	19
31	14	16	16	14	11	11	9	17	17	15	11	7	7	3	4	3	4	4	19
32	14	16	16	14	11	11	9	17	17	15	11	7	7	3	4	3	4	4	19
33	14	16	16	14	11	11	9	17	17	15	11	7	7	3	4	3	4	4	19
34	14	16	16	14	11	11	9	17	17	15	11	7	7	3	4	3	4	4	36
35	14	16	16	14	11	11	9	17	17	15	11	7	7	3	4	3	4	4	17
36	14	16	16	14	11	19	9	17	17	15	11	7	7	3	4	3	4	4	22
37	5	7	9	9	8	11	9	5	7	7	11	7	7	3	4	3	4	4	20
38	5	12	11	9	8	11	9	4	6	6	10	7	7	3	4	3	4	4	19
39	3	10	9	7	6	10	8	2	4	4	8	7	7	3	4	3	4	4	22
40	7	5	7	9	10	10	9	1	9	9	11	7	7	3	4	3	4	4	24
41	0	8	6	4	3	7	5	5	7	7	11	7	7	3	4	3	4	4	26
42	8	0	2	4	7	5	7	12	14	14	11	7	7	3	4	3	4	4	26
43	6	2	0	2	5	7	7	11	13	13	11	7	7	3	4	3	4	4	25
44	4	4	2	0	3	7	5	9	11	11	11	7	7	3	4	3	4	4	29
45	3	7	5	3	0	4	2	8	10	10	11	4	7	3	4	3	4	4	27
46	7	5	7	7	4	0	2	12	14	14	11	4	7	3	4	3	4	4	22
47	5	7	7	5	2	2	0	10	12	12	11	2	7	3	4	3	4	4	24
48	5	12	11	9	8	11	9	0	2	2	6	7	7	3	4	3	4	4	24
49	7	14	13	11	10	11	9	2	0	2	6	7	7	3	4	3	4	4	28
50	7	14	13	11	10	11	9	2	2	0	4	7	7	3	4	3	4	4	29
51	11	16	16	14	11	11	9	6	6	4	0	7	4	3	4	3	4	4	32
52	7	9	9	7	4	4	2	12	14	14	11	0	7	3	3	3	3	3	33
53	14	16	16	14	11	11	9	10	10	8	4	7	0	3	4	4	4	4	32
54	11	13	13	11	8	8	6	14	14	12	8	4	4	0	1	0	1	3	33
55	10	12	12	10	7	7	5	15	15	13	9	3	5	1	0	1	0	2	32
KP	10	12	12	10	7	7	5	15	17	15	11	3	7	3	2	3	2	0	32
Min_Max																			17

APPENDIX 5.0

Mat lab coding for the function $Min\left[g(x) = \max\limits_{i=1,.....,n} \min\{d(X,i), d(Y,i)\} \right]$

```
% function [D hatDDbarmax_Dbarminimum_max_Dbar] = berman (A, ina, inb)

function [D hatDDbarmax_Dbarminimum_max_Dbar] = berman (A, ina)

D = A;

[n m] = size (D);

for i = 1: n

for j = 1: m

% D (i, j) = min (D (i,j), min (D (i,ina),D (i,inb)));

D (i, j) = min (D (i, j), D (i,ina));

end

end

hatD = D;

% deleting corresponding rows and column of the initial facility

% hatD ([inainb], :) = []; hatD (:, [inainb]) = [] ;

hatD (ina, :) = []; hatD (:, ina) = [];

%initial facility is Y and the remaining nodes are contained in X

% Y = [inainb];

Y = ina;

X = 1: n; X (:, Y) = [];

Dbar = zeros (n-2, m);

for k = 1: length(X)

kk = X (k);
```

```
for i = 1: n

%Dbar is the optimal location

% Dbar (k,i) = min ([A (kk,i), A (ina,i), A (inb,i)]);

Dbar (k,i) = min ([A (kk,i), A (ina,i)]);

end

end

Dbar (:, Y) = [];

%maximum of the optimal location

max_Dbar = max (Dbar');

%minimum of the maximum optimal location

minimum_max_Dbar = min (max_Dbar);
```

APPENDIX 6.0

Taking the optimal new location using the modified shortest distance matrix, $\hat{D}$ for the network with the objective function:

$$Min\left[g(x) = \max_{i=1,.....,n} \min\{d(X,i), d(Y,i)\} \right]$$

Where $Y = \{19\}$ and $X = \{1, 2,3,4,5,6,............,56\}$

For node 1

$i = 1$

$\min\{d(1,1), d(19,1)\}$

$\min\{0,16\}$

$\min\{0\}$

$i = 2$

$\min\{d(1,2), d(19,2)\}$

$\min\{1,17\}$

$\min\{1\}$

$i = 3$

$\min\{d(1,3), d(19,3)\}$

$\min\{2,14\}$

$\min\{2\}$

$i = 4$

$\min\{d(1,4), d(19,4)\}$

$i = 5$

$\min\{d(1,5), d(19,5)\}$

$\min\{5,11\}$

$\min\{5\}$

$i = 6$

$\min\{d(1,6), d(19,6)\}$

$\min\{5,5\}$

$\min\{5\}$

$i = 7$

$\min\{d(1,7), d(19,7)\}$

$\min\{10,10\}$

$\min\{10\}$

$i = 8$

$\min\{d(1,8), d(19,8)\}$

$\min\{8,8\}$

$\min\{8\}$

$i = 9$

$\min\{d(1,9), d(19,9)\}$

$\min\{33, 33\}$

$\min\{33\}$

$i = 19$

$\min\{d(1,19), d(19,19)\}$

$\min\{0,0\}$

$\min\{0\}$

$i = 20$

$\min\{d(1,20), d(19,20)\}$

$\min\{2,2\}$

$\min\{2\}$

$Maximum\ at\ node\ 32 = 36$

$\min\{8,11\}$

$\min\{8\}$

$i = 32$

$\min\{d(1,32), d(19,32)\}$

$\min\{36,36\}$

$\min\{36\}$

$i = 56$

$\min\{d(1,56), d(19,56)\}$

$\min\{6,6\}$

$\min\{6\}$