KWAME NKRUMAH UNIVERSITY OF SCIENCE AND TECHNOLOGY, KUMASI

OPTIMAL LOCATION OF DISTRIBUTION CENTER FOR LIQUEFIED PETROLEUM GAS - USING FLOYD?S SHORTEST ROUTE ALGORITHM
(A CASE STUDY: BRONG AHAFO REGION OF GHANA)

By

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(B.ED MATHEMATICS)

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CERTIFICATION

I herein certified that this work was carried out solely by KWARTENG, FRANCIS (PG) in the department of mathematics, institute of Distance Learning in partial fulfillment of the requirements for the award of Master of Science in Industrial Mathematics.

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DECLARATION

I, hereby declare that this submission is my own work towards the award of MSc and that, to the best of my knowledge, it contains no materials previously published by another person nor material which has been accepted for the award of any other degree of the University, except where due acknowledgement has been made in the text.

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DEDICATION

To the glory of Almighty

I dedicate this thesis to my wife, Margaret Aduhene Kwarteng, my children; Yvonne Aduhene Kwarteng, Giscard Aduhene Kwarteng and Gilbert Aduhene Kwarteng and all my family members for their moral and mutual support.

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ABSTRACT

Liquefied Petroleum Gas demand in Ghana has grown steadily since the LPG promotion program in early 1990's. The country currently has no downstream LPG Distribution Centers (depots) in any of the ten regions, apart from the primary storage facility located at the TOR. This results in long queues and delay during rush period even when there may be gas in storage. The LPG filling stations in the country get their supply from Tema with little regard for travel cost. In this thesis, a mathematical model for the determination of cost effective distribution center for Liquefied Petroleum Gas (LPG) is constructed. It involves using the Floyd's Shortest – Route Algorithm for all pairs shortest-path-network problem and distribution center site selection model. The study revealed that an amount of GH¢5580.00 and GH¢13424.00 would be spent in the cost-effective site and most expensive site respectively. It also established that locating distribution center at Techiman, the cost-effective location could save TOR an amount of GH¢6783.00 on every distribution. Among the recommendations offered was that since the model minimizes distribution cost, thereby increasing the savings, TOR and other institutions planners could adapt the model in deciding where to site a distribution center.

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LIST OF ABBREVIATIONS

ALS: Advanced life-Support unit

AOMCs: Association of Oil Marketing Companies

AT: Atebubu

BA: Banda

BE: Bechem

BK: Berekum

BLS: Basic Life Support Unit

DO: Dormaa

DR: Drobo

EMS: Emergency Medical Service

EOQ: Economic Order Quantity

FLEET: Facility Location and Equipment Emplacement Technique

GO: Goaso

JE: Jema

KE: Kenyasi

KN: Kintampo

KU: Kukuom

KW: Kwame Danso

LPG: Liquefied Petroleum Gas

LSCP: Location Set Covering Problem

MCLP: Maximal Covering Location Problem

MP: Mathematical Programming

NK: Nkoranza

NPA: National Petroleum Authority

NS: Nsawkaw

OR: Operation Research

SA: Sampa

SP: Shortest Path

SQM: Stochastic Queue Median

SU: Sunyani

TE: Techiman

TOPISI: Technique For Order Preference by Similarity to Ideal Solution

TOR: Tema Oil Refinery

WE: Wenchi

YE: Yeji

Chapter 1

INTRODUCTION

1.1 Background of Study

There is an increasing patronage of Liquefied Petroleum Gas (LPG) throughout the world. Subsequently, the supplies of LPG are growing to meet demand. In 1985, world supply was approximately 114 million tonnes and this was expected to increase to 240 million tonnes in 2005 Purvin and Gertz, (2000). Like many other developing countries, the Government of Ghana has since the early 1990s been promoting the use of Liquefied Petroleum Gas. This was necessitated largely by the unreliable electricity supply and its attendant high tariffs and the depletion of forests as a result of the usage of biomass fuel. In Ghana, LPG is primarily used for cooking in households, commercial establishments and hotels. Some industries also use it as a source of fuel for their processes. Its use is also rapidly growing in the transport sector as a substitute fuel for premium. An adequate and reliable supply of LPG therefore plays a vital role in the sustainable development of the country. LPG is supplied to the market from both the local refinery and imports. However, domestic LPG production is limited and imports account for the vast majority of the total supply to the market. The Tema Oil Refinery (TOR) is responsible for all domestic supplies of LPG in Ghana and also handles all imports of LPG and refined products. Current refinery production of LPG is about 18 000 tonnes per year. The National Petroleum Authority (NPA), the regulator of Ghana's petroleum downstream industry is to conduct an in depth nationwide study on the demand (domestic and vehicles) and supply of liquefied petroleum gas (LPG) in the country. The release further disclosed that: "PA is also liaising with the Association of oil Marketing Companies (AOMCs) to ensure that LPG companies extend equal supplies of LPG to the three Northern regions and Brong Ahafo, noted to have acute shortages of the commodity." (http://thechronicle.com.gh/?p=28218 (13/9/2013)) According to the world Liquefied Petroleum Gas Association (WLPGA), more than 9 million vehicle s in 38 countries currently operate on LP gas. Liquefied Petroleum Gas has been recognized for a long time as an environmentally attractive fuel by those close to the gas industry but in recent years environmental issues have generated discussion about the relative merits of fuels among a much wider range of experts. LP gas appears to be gaining an increased share of the market because of its availability, environmental advantages and efficiency in use. For this reason a company is looking for a cost effective Distribution Center to be sited in one of the district capitals in Brong Ahafo Region. To select a site, a variety of influential decision variables should be simultaneously assimilated in the process of decision making and this has made the subject potentially complex (Clark and McCleary, 1995; Amiri et al. 2008) and has attracted the attention of researchers. The majority of studies focused on facility location problem, the selection of the sites where new facilities are to be established is restricted to a finite set of available candidate locations. The simplest setting of such a problem is the one in which p facilities are to be selected to minimize the total (weighted) distances or costs for supplying customer demands. This is the so-called p-median problem which has attracted much attention in the literature. Daskin (1995). This setting assumes that all candidate sites are equivalent in terms of the setup cost for locating a new facility. When this is not the case, the objective function can be extended with a term for fixed facility location costs and as a result, the number of facilities to be established typically becomes an endogenous decision. The P-center model is also referred to as the minimax model since it minimizes the maximum distance between any demand point and it's nearest facility. The P-center model considers a demand point is served by its nearest facility and therefore full coverage to all demand points is always achieved. However, unlike the full coverage in the set covering models, which may lead to excessive number of facilities, the full coverage in the P-center model requires only a limited number (P) of facilities.

The center problem was first posed by Sylvester (1857) more than one hundred years ago. The problem asks for the center of a circle that has the smallest radius to cover all desired destinations. In the last several decades, the P-center model and its extensions have been investigated and applied in the context of locating facilities such as EMS centers, hospitals, fire station, and other public facilities. In order to locate a given number of emergency facilities along a road network, Garfinkel et al. (1977) examined the fundamental properties of the P-center problem.

The best way to locate a Distribution Center is to use Operations Research techniques such as network models in decision making. This will provide the decision makers the right methodology to evaluate relevant travel costs and to select least expensive location site.

1.2 Problem Statement

The use of the Liquefied Petroleum Gas (LPG) is gaining more and more recognition in the country Ghana. As the population increased with corresponding increase in the demand for LPG, it has become necessary to establish a Distribution Center (depot) in Brong Ahafo Region to reduce the haulage of LPG by numerous tanker vehicles from Tema Oil Refinery (TOR) in order to curb accidents and also free our roads from the damage caused by these heavy vehicles. The Distribution Center may also ensure reliable and equitable distribution of the products to the region which are far from the Tema Oil Refinery in Tema. The problem, therefore, prompted the researcher to locate a single service point (Distribution Center) in Brong Ahafo Region using Floyd's shortest-Route Algorithm to minimize travel distances and find the least-expensive distances (paths) between all the vertices.

1.3 The Objectives of the Study

The objective of the study is to determine the optimal location of a Distribution Center for LPG using the Floyd's Shortest - Route Algorithm in the Brong Ahafo Region of Ghana. Specifically, the study seeks:

1. To construct a mathematical model for the determination of an optimal location of the Distribution Center in the Brong Ahafo Region.
2. To minimize the travel distances of transporting Liquefied Petroleum Gas from Tema Oil Refinery to all district capitals in the Brong Ahafo Region.
3. To find the least-expensive distance between all vertices.
4. To locate a distribution center that is not too far from all the districts to ensure prompt service.
5. To recommend to stakeholders the most appropriate location for the Distribution Center in the Brong Ahafo Region.

1.4 Methodology

The problem is to locate a Distribution Center in the Brong Ahafo Region so as to minimize the travel distances and travel cost between the new facility and the existing facility using the Floyd's Shortest-Route Algorithm strategy. The data for this study will consist of both primary and secondary collected from the study area in Ghana. The data on road distances between districts were taken from Town and Country Planning Department from Brong Ahafo

Regional Office. The travel cost (Diesel used per journey) was obtained from tanker drivers and filling station owners (managers) in Brong Ahafo Region. A Matlab code was written for Floyd's shortest-route algorithm and to determine the shortest travel distance among all town pairs. The travel cost was determined. The data, that is, shortest travel distance, T_{ij} and travel cost, G_j was used as input data into the proposed location site model formulated. This was then analysed using Matlab codes.

1.5 Significance of the Study

The study is significant because of the following reasons:

1. The finding of the shortest distance and the optimal location will be the information for planning, give an effective distance for the computation and subsequently a central location for a Distribution Center.

2. It will reduce the average travel time and distances from all the existing locations to the Distribution Center.

3. It will help to conserve fuel since the algorithm provides average shortest distances for all various retail outlets in the region.

4. t will also help reduce the number of death cases when LPG are being transported to the district capitals in Brong Ahafo Region.

1.6 Organization of the Thesis

This thesis consists of five chapters: including this chapter. Chapter Two is on Literature Review which takes stock of what other people have written on the topic in terms of theories or concepts, scientific research studies and the overall goal of classifying how the present study intends to address the gap, silence or weakness in the existing literature. Chapter Three explains the methodology that is being used for the study. The findings and discussions will be presented in Chapter Four. Lastly, Chapter Five will discuss on the conclusion and recommendations.

Chapter 2

LITERATURE REVIEW

2.1 Introduction

This chapter reviews the literature existing in the area of facility location. Facility location problems date back to the 17th century when Fermat (1643) and Cavalieri, et al. (1647) simultaneously introduced the concept, although this theory is widely contested by location analyst experts. Late in 18th century, Pierre Varignon presented "The Varignon Frame" which an analog solution to the planar minisum location problem. However, it started gathering more interest when Weber (1909) presented the Planar Euclidean single facility minisum problem in 1909. The Weber problem considers the example of locating a warehouse in the best possible location such that the distance traveled between the warehouse and the customer is minimized. Weiszfeld (1937) conceptualised an iterative method to solve the minisum Euclidean problem today referred to as the Weiszfeld's procedure.

Hakimis (1964) introduced a seminal paper on locating one or more points on a network with the objective to minimize the maximum distance. Francis et al. (1983) presented a survey paper in location analysis which defined four classes of location problems and described algorithms to optimize them. They are continuous planar, discrete planar, mixed planar and discrete network problems.Daskin et al (1998) reviewed various strategic location problems where they emphasized that a good facility location decision is a critical element in the success of any supply chain. They explained median problems, centre problems, covering problems, and other dynamic location problem formulations in the context of a supply chain environment.

Facility location problems span many research fields like operation research, mathematics, statistics, urban planning designing, etc. Location analysis goes back to the influential book of the German industrial author, Weber (1909). The research was motivated by observing a warehouse operation and its inefficiencies. Weber considered the single warehouse location problem and evaluated it such that the travel distances for pickups and replenishment were reduced. Other notable work in this field was by Fermat (1643), who solved the location problem for three points constituting a triangle. Another major concept in the field of location analysis was the concept of competitive location analysis introduced by Hotelling (1929). The paper discussed a method to locate a new facility considering already existing competition. The considered facilities were on a straight line. He proposed that the customers generally prefer visiting the closest service facility. He introduced the Hotelling's Proximity Rule? which can be used to determine the market share captured by each facility. He just considered the distance metric during his analysis. The Hotelling model was extended by Drezner

(1993) who introduced the concept of varying attractiveness among competing facilities. He analyzed cost and quality factors in addition to distance metric involved.

Huff (1966) proposed the famous "Gravity Model"? for estimating the market share captured by competitors. The gravity model states that existing customer locations attract business from a service in direct proportion to the existing locations and in inverse proportion to the distance between the service location and the existing customer locations. Baumol and Wolfe (1958) have solved the location problem for minimum total delivery cost with nonlinear programming. Others have incorporated stochastic functions to account for demand and/or supply, Rosenthal, White and Young (1978); Wesolowsky (1977). Other approaches that have been employed include dynamic programming. Geoffrion, (1978); multivariate statistics using multidimensional scaling, Asami, et al. (1989) and search procedures , Kuehn et al (1963). Randhawa et al, (1995) proposed a solution approach to facility location selection problems while integrating analytical and multi-criteria decision-making models. Houshyar and White (1997) developed a mathematical model heuristics approach that assigns N machines to N equal-sized locations on a given site such that the total adjacent flow between the machines is maximized. The proposed mode based on 0-1 integer programming formulation which may produce an optimal, but infeasible solution, followed by the heuristic which begins with the 0-1 integer solution and generates a feasible solution. Chu (2002) presented a fuzzy TOPISI (technique for order preference by similarity to ideal solution) method-based approach for the plant location selection problems. The ratings and weight assigned by decision makers are first normalized rating of each alternative location for each criterion is then developed. A closeness coefficient is proposed to determine the ranking order of the alternatives.

Klose and Drexl (2005) reviewed in detail the contributions to the current state-of-the-art related to continuous location models, network location models, mixed-integer programming models and their applications to location selection decision. Yong (2006) proposed s new fuzzy TOPSIS method which deals with the selection of plant location decision-making problems in linguistic environment, where the ratings of various alternative locations under different criteria and their weights are assessed in linguistic terms represented by fuzzy numbers.

Ferreira et al (1996) present a bi-criteria mixed integer linear model for the facility location where the objectives are the minimization of total cost and the minimization of environmental pollution at facility sites. The interactive approach of Ferreira et al (1994) is used to obtain and analyze non-dominated solutions. Giannikos (1998) presents a discrete model for the location of disposal or treatment facilities and transporting hazardous waste through a network linking the population centers that produce the waste and the candidate locations for the treatment facilities method to choose the location for a waste treatment facility in a region of Finland.

Costa et al (2008) develop two bi-criteria models for single allocation hub location problems. In both models the total cost is the first criteria to be minimized. Instead of using capacity constraints to limit the amount of flow that can be received by the hubs, a second objective function is used, trying to minimize the time to process the flow entering the hubs. In the first model, total time is considered as the second criteria and, in the second model, the maximum service time for the hubs are minimized. Non-dominated solutions are generated using an interactive decision-aid approach developed for bi-criteria integer linear programming problems. Both bi-criteria models are tested on a set of instances, analyzing the corresponding non-dominated solutions set and studying the reasonableness of the hubs flow charge for these non-dominated solutions. Ballou (1998) discusses a selected number of facility location methods for strategic planning. He further classifies the more practical methods into a number of categories in the logistics network, which include single-facility location, multi-facility location, dynamic facility location, retail and service location. Christopher and Wills (1972) comprehensively present that whether the problem of depot location is static or dynamic, Infinite Set approaches and Feasible Set approach can be identified. The infinite set approach assumes that a warehouse is flexible to be located anywhere in a certain area. The feasible set approach assumes that only a finite number of known sites are available as warehouse locations. They believe the centre of gravity method is a sort of infinite set model.

Goldengorin et al, (2000) considered the simple plant location problem. This problem often appears as a sub-problem in other combinatorial problems. Several branch and bound techniques have been developed to solve these problems. The thesis considered new approaches called branch and peg algorithms, where pegging refers to assigning values to variables outside the branching process. An exhaustive computational experiment shows that the new algorithms generate less than 60% of the number of sub-problems generated by branch and bound algorithms, and in certain cases requires less than 10% of the execution times required by branch and bound algorithms. Firstly, for each sub-problem generated in the branch and bound tree, a powerful pegging procedure is applied to reduce the size of the sub-problem. Secondly, the branching function is based on predictions made using the Beresnev function of the sub-problem at hand. They saw that branch and peg algorithms comprehensively outperform branch and bound algorithms using the same bound, taking on the average, less than 10% of the execution time of branch and bound algorithms when the transportation cost matrix is dense. The main recommendation from the results of the experiment is that branch and peg algorithms should be used to solve SPLP instances. Ballou (1998) states that exact centre of gravity approach is simple and appropriate for locating one depot in a region, since the transportation rate and the point volume are the only location factors. Given a set of points that represent source points and demand points, along with the volumes needed to be moved and the associated transportation rates, an optimal facility location could be found through minimizing total transportation cost. In principle, the total transportation cost is equal to the volume at a point multiplied by the transportation rate to ship to that point multiplied by the distance to that point. Furthermore, Ballou outlines the steps involved in the solution process in order to implement the exact centre of gravity approach properly.

2.2 Pure Location Problem

A pure location problem deals with optimally locating one or more new service facilities to serve the demand at existing customer locations. Selecting different distance measures, and by using different constraints, we obtain different variants of the multi or single facility location problem. Furthermore, we can introduce additional variants by restricting new facility locations to specific sites, and by letting demands be either stochastic or distributed over areas.

2.2.1 Location-Allocation Problems

Since 1963, when the first location-allocation model was formulated by Cooper (1963), there has been extensive research on the field. The simplest location-allocation problem is the Weber problem addressed by Friedrich (1929). This paper discussed the steps in locating a machine so as to minimize the sum of the weighted distances from all the raw materials sources. The seminal work in this area was on the p-median problem, initially formulated by Hakimi (1964). The median problem was considered on a graph and the objective function was to reduce the average or the sum of the transportation costs from the service facility to the demand locations. It was derived that one of the optimal solutions locates the service facility on one of the nodes of the network.

2.2.2 P - Central Problems

The p-center problem by Hakimi (1965) ; is the problem of minimizing the maximum distance that demand is from its closet facility given that there are pre-determined numbers of facilities. Hakimi's work established important results in location theory and sparked theoretical interest among researchers.

The vertex p-center problem restricts the set of candidate facility sites to the nodes of the network. The absolute p -center problem permits the facilities to be anywhere along the arcs. Both versions are examined in weighted and un-weighted situations. The absolute center problem can be approximated by the vertex center problem by adding nodes to the network. The objective of the p-center problem is to locate p new facilities, called centers, on a network G in order to minimize the maximum weighted distance between a node and its nearest facility. The methods developed for solving this problem are quite different from those for the p-median problem, even though the two problems are related. The first approach developed was by Hakimi (1964) who proposed an enumerative approach for p=1 to specifically locate a local center on each link, and thereby to determine the overall optimal location. A more effective method was suggested by Christofides (1975), who showed that one needs to consider only a subset of the links for an optimal location. However, this approach is unable to solve general p-center problems, Erkut et al. (1992) presented a polynomial time, binary search algorithm to solve the distance constrained p-center problem. More methods have been proposed and tested to solve the p-center problem, such as the exact algorithms due to Christofides and Viola (1971), Granfinkel et al. (1977), and various heuristics proposed by Singer (1968). Garfinkel et al. (1977) examined the fundamental properties of the P-centre problem in order to locate a given number of emergency facilities along a road network. He modelled the P-centre problem using integer programming and the problem was successfully solved by using a binary search technique and a combination of exact tests and heuristics. ReVelle and Hogan (1989) formulated a P-centre to locate facilities so as to minimize the maximum distance within which the EMS is available with (alpha) reliability. System congestion is considered and a derived server busy probability is used to constrain the service reliability that must be satisfied for all demands. Hochbaun and Pathria (1998) considered the emergency facility location problem that must minimize the maximum distance on the network across all time periods using the Stochastic P-centre models. The cost and distance between locations vary in each discrete time periods. The authors used k underlying networks to represent different periods and provided a polynomial-time, 3-approximation algorithm to obtain a solution for each problem. Talwar (2002) utilized a P-centre model to locate and dispatch three emergency rescue helicopters to serve the growing EMS demands due to accidents occurring during adventure Drezner (1984) presented heuristic and optimal algorithms for the p-center problem in the plane. The heuristic method yielded results for problems with up to n = 2000 and p = 10 whereas the optimal method solved problems with up to n = 30, p = 5 or n = 40, p = 4. Watson-Gandy [1984] suggested an algorithm that can optimally solve problems with up to about 50 demand points and 3 centers in reasonable time. Agarwal and Sharir (1998) discuss efficient approximate algorithms for geometric optimization, which includes the Euclidean p-center in d dimensions. Hale and Moberg (2004) give a broad review on location problems, which includes the Euclidean p-center problem where the Euclidean distances are and where U = i is a set of m users and V = j a set of n potential locations for facilities in the plane. In these works, Tabu search and Variable Neighborhood Search methods as well as an optimal method are used, and the efficiency of these methods for small and large problems is evaluated. It should be noted that this Euclidean problem is equivalent to the p-center problem on networks where the possible location of the facilities are on the vertices and where the minimum distances between the demand and potential supply points are given. Recent works on these two versions of the discrete problem include algorithms given by Caruso et al. (2003). The latter authors describe an efficient exact method for this p-center problem. Their algorithm finds the solution by updating, at each step, an upper or lower bound on the optimal solution. A tightlower bound to the optimal value is found in an initial phase of the algorithm, which consists of solving linear programming sub-problems.

2.2.3 P - MEDIAN

The p-median Problem (p-M) is to locate p new facilities, called medians, on the network G in order to minimize the sum of the weighted distances from each node to its nearest new facility Francis et al., (1992). If $p \geq 2$, then this problem can be viewed as a location-allocation problem (LAP). This is because the location of the new facilities will determine the allocation of their service in order to best satisfy the nodal demands. Berman and Drezner (2007), discuss the conditional p-median and p-center problems on a network. Demand nodes are served by the closest facility whether existing or new. Rather than creating a new location for an artificial facility and force the algorithm to locate a new facility there by creating an artificial demand point, the distance matrix was just modified. They suggested solving both conditional problems by defining a modified shortest distance matrix. The formulation they presented in this paper provided better results than those obtained by the best known formulation. The work presented in this thesis is based on this paper. Location problems have been widely addressed in the literature. In the first study in this vein, Weber (1929) studied the problem of determining the location of a warehouse to minimize the total distance between the warehouse and the customers (which is called a Weber problem). Hakimi (1965) found the optimum location of a "switching centre"? in a communication network and located the best place to construct a police station? in a highway system. Berman and Krass (2002) considered a generalization of the maximal cover location problem, which allows for partial customer coverage, with the degree of coverage being a non increasing step function of the distance to the nearest facility. A demand point is only considered covered if a facility is available to service the demand point within a specified distance . The p-median problem, introduced by Hakimi (1964), is to determine the location of p facilities to minimize the average (total) distance between demand points and facilities. Subsequently, ReVelle and Swain (1920) formulated the p-median problem as a linear integer program and used a branch and- bound algorithm to solve the problem. In contrast to p median models, which concentrate on optimizing the overall performance of the system, the p-centre model attempts to minimize the maximum distance between each demand point and its nearest facility, so the p-centre model is also called the minimax model. Cavalier and Sherali (1986) presented exact algorithms to solve the p-median problem on a chain graph and the 2-median problem on a tree graph, where the demand density functions are assumed to be piecewise uniform. For the incapacitated p-median problem, Chiu (1987) addressed the

1- median problem on a general network as well as on a tree network. Dynamic location considerations on networks are addressed by Sherali (1991). Recently, Francis et al. (1993) developed a median-row-column aggregation algorithm to solve large-scale rectilinear distance p-median problems. . Sherali and Rizzo (1991) solved an unbalanced, capacitated p-median problem on a chain graph with a continuum of link demands. For solving this problem, they considered two unbalanced cases, the deficit and over-capacitated cases, provided a first-order characterization of optimality for these two problems and developed an enumerative algorithm based on a partitioning of the dual space. There are still further variants that include capacity restrictions on links, probabilistic travel times on links, and maximum distance constraints. It is worthwhile to note that the p-median model has been extended and expanded in a number of ways. Since its formulation the P-median model has been enhanced and applied to a wide range of emergency facility location problems. Carbone (1974) formulated a deterministic P-median model with the objective of minimizing the distance traveled by a number of users to fixed public facilities such as medical or day-care centers. Recognizing the number of users at each demand node is uncertain, the author further extended the deterministic P-median model to a chance constrained model. The model seeks to maximize a threshold and meanwhile ensure the probability that the total travel distance below the threshold is smaller than a specified level . Calvo and Marks (1973) constructed a P-median model to locate multi-level health care facilities including central hospitals, community hospitals and local reception centers. Themodel seeks to minimize distance and user costs, and maximize demand and utilization. Later, the hierarchical

P-median model was improved by Tien et al. (1983) and Mirchandani (1987) by introducing new features and allowing various allocation schemes to overcome the deficient organization problem across hierarchies. Paluzzi (2004) discussed and tested a P-median based heuristic location model for placing emergency service facilities for the city of Carbondale. The goal of this model is to determine the optimal location for placing a new fire station by minimizing the total aggregate distance from the demand sites to the fire station. The results were compared with the results from other approaches and the comparison validated the usefulness and effectiveness of the P-median based location model. One major application of the P-median models is to dispatch EMS units such as ambulances during emergencies. Carson and Batta (1990) proposed a P-median model to find the dynamic ambulance positioning strategy for campus emergency service. The model uses scenarios to represent the demand conditions at different times. The ambulances are relocated in different scenarios in order to minimize the average response time to the service calls. Berlin et al. (1976) investigated two P-median problems to locate hospitals and ambulances. The first problem has a major attention to patient needs and seeks to minimize the average distance from the hospitals to the demand points and the average ambulance response time from ambulance bases to demand points. In the second problem, a new objective is added in order to improve the performance of the system by minimizing the average distance from ambulance bases to hospitals. Mandell (1998) developed a P-median model and used priority dispatching to optimally locate emergency units for a tiered EMS system that consists of advanced life-support (ALS) units and basic life-support (BLS) units. The model can also be used to examine other system parameters including the balance between ALS and BLS units, and different dispatch rules. Uncertainties have also been considered in many P-median models. Mirchandani (1980) examined a P-median problem to locate fire-fighting emergency units with consideration of stochastic travel characteristics and demand patterns. The author took into account the situations that a facility may not be available to serve a demand and used a Markov process to create a system in which the states were specified according to demand distribution, service and travel time, and server availability. Serra and Marianov (1999) implemented a P-median model and introduced the concept of regret and minmax objectives when locating fire station for emergency services in Barcelona. The authors explicitly addressed in their model the issue of locating facilities when there are uncertainties in demand, travel time or distance. In addition, the model uses scenarios to incorporate the variation of uncertainties and seeks to give a compromise solution by minimizing the maximum regret over the scenarios. P-median models have also been extended to solve emergency service location problems in a queuing theory context. An example is the stochastic queue median (SQM) model due to Berman et al. (1985). The SQM model seeks to optimally dispatch mobile servers such as emergency response units to demand points and locate the facilities so as to minimize average cost of response.

2.2.4 COVERING PROBLEMS

Unlike the p-median problem which seeks to minimize the total travel distance, covering models are based on the concept of acceptable proximity. The objective of covering models is to provide "coverage" to demand points. A demand point is considered as covered only if a facility is available to service the demand point within a distance limit. Covering models can be classified according to several criteria. One of such criteria is the type of objective, which allows us to distinguish between two types of formulations.The first type belongs to the Location Set Covering Problem (LSCP). The Location Set Covering Problem (LSCP) seeks to locate the minimum number of facilities that will cover all demands within a specified maximum distance Toregas et al. (1971). The problem is applied to emergency services location where a given amount of the population must be within a predefined maximum distance from a facility. The limit on maximum distance (or response time) is adopted to ensure that demands (emergency calls) are answered in timely fashion.

The second type can be classified as the Maximal Covering Location Problem (MCLP), which maximizes covered customer demand, given a limited number of facilities. The MCLP was first introduced in Church and ReVelle (1974). Church and Meadows (1979) provided a pseudo-Hakimi property for the MCLP. This property states that for any network, there exists a finite set of points that will contain at least one of the optimal solutions to the MCLP. Daskin and Stern (1981), Hogan and ReVelle (1986) developed the MCLP that contains a secondary "backup" coverage objective. Berman and Krass (2002) showed that the MCLP with a step coverage function is equivalent to the incapacitated facility location problem, Cornuejols et al (1990). They developed two IP formulations for the problem and showed an interesting result that the LP relaxations of both formulations provide the same value of the upper bound.

In a recent paper, Berman et al. (2003) investigated the MCLP with a coverage decay function whose value decreases from full coverage at the lowest pre-specified radius to no coverage at the highest pre-specified radius. Daskin (1983) provided a probabilistic formulation of the problem in which the probability of an arbitrary server being busy is specified exogenously. The objective, then, is to locate facilities so as to maximize the expected number of demand that a facility can cover. Daskin's formulation is sometimes referred to as the Maximal Expected Covering Location Problem.

Application of the set covering model includes airline crew scheduling, Desrocher et al. (1991). Covering models are the most widespread location models for formulating the emergency facility location problems. LSCP is an earlier statement of the emergency facility location problem by Toregas et al. (1971) and it aims to locate the least number of facilities that are required to cover all demand points. Since all the demand points need to be covered in LSCP, regardless of their population, remoteness, and demand quantity, the resources required for facilities could be excessive. Recognizing this problem, White and Case (1974) developed the MCLP model that does not require full coverage to all demand points. Instead, the model seeks the maximal coverage with a given number of facilities. The MCLP, and different variants of it, have been extensively used to solve various emergency service location problems. A notable example is the work of Eaton et al. (1985) that used MCLP to plan the emergency medical service in Austin, Texas. The solution gives a reduced average emergency response time even with increased calls for service.

Schilling et al. (1979) generalized the MCLP model to locate emergency fire-fighting servers and depots in the city of Baltimore. In their model, known as FLEET (Facility Location and Equipment Emplacement Technique), two different types of servers need to be located simultaneously. A demand point is regarded as ”covered”? only if both servers are located within a specified distance.

2.2.5 Shortest Route Problem

In 1959, Edsger Wybe Dijkstra, a Dutch computer scientist, proposed two algorithms for the solution of two fundamental graph theory problems: i) the minimum weight spanning tree problem and ii) the shortest path problem. Sniedovich (2005) comments that the original Dijkstra’s algorithm constitutes an iterative procedure for the solution of dynamic programming functional equation associated with the shortest path problem given that the arc lengths are non-negative. Dijkstra’s algorithm for the single-source (i.e. one-to-one, one-to-some and one-to-all) shortest path problem is one of the most celebrated algorithms in computer science and a very popular algorithm in operations research. There have been many refinements and modifications as well as numerical experiments (evaluations) that have brought a significant an improvement in the performance of the algorithm especially due to the use of some new data structures. As Dijkstra’s algorithm can be interpretated as a breadth-first search, a lot of unnecessary steps might be carried out before reaching the destination node. Starting from both sides might reduce the computational effort significantly Berge, et al. (1965). However, if the two-sided procedure is terminated as soon as a node has been processed from both directions, there is no guarantee that this node is actually on the shortest path from the start node i to destination node j. Vahrenkamp and Mattfeld (2007).

The A* algorithm is a more general approach than Dijkstra's algorithm for finding the shortest path between two nodes in a graph, Hart et al. (1968) for a first approach with a correction in Hart et al. (1972). As stated above, Dijkstra's algorithm performs as a breadth-first search where the next node to be looked at is the one with minimum distance to the starting node (given in Dist). The A^* algorithm introduces a heuristic to determine the order in which nodes are selected in the search process.

This heuristic is a sum of two terms. The first term is the distance to the current node k (given as Dist [k]), the second one is an estimation of the distance to the destination node j, usually denoted as h(k). In most implementations h(k) is computed as the Euclidean distance from the considered node to the destination node. If coordinates of all nodes are available, the distance can be calculated by Pythagoras' theorem. Dijkstra's algorithm can be viewed as a special case of the A* algorithm where h(k) = 0 for all nodes k. Nilsson (1980).

The Floyd-Warshall algorithm takes as input an adjacency matrix representation. This maintains two types of matrices, i) distance matrix D_t and ii) precedence matrix U_t , in each iteration and takes initial distance matrix D_0 and initial precedence matrix U_0 as input. Then proceeds for n iterations, where n is the number of nodes in the distance matrix. The nth iteration gives the optimal/final distance matrix D_t =n and the final precedence matrix U_t =n. The optimal distance matrix D_n represents the shortest distances between any two nodes in the network and the corresponding shortest paths can be traced out from the final precedence matrix U_t.

2.2.6 SITE SELECTION MODEL

The site of the plant is the critical factor into consideration because the best site will create the cost savings as well as the feasibility of the project. Furthermore, it involves a substantial capital investment and results in long-term constraints on production and distribution of goods. The complexity stems from a multitude of quantitative and qualitative factors influence site choices as well as the intrinsic difficulty of making numerous tradeoffs among those factors. Most firms take many different factors into consideration before deciding where to locate their new plant such as the quality of the labor force, political and social stability, the quality of infrastructure, the links to suppliers and demanders, unemployment, etc.

Although there are a significant number of quantitative research efforts regarding site selection in domestically oriented applications, the number of papers that investigate the site selection models are comparably few. (Francis et al 1983; Thizy et al 1985; Verter and Dincer, 1995), These studies include mixed-integer programming models, mean variance approaches that consider a degree of uncertainty and also multiple criteria techniques such as goal programming. The mixed-integer programming models include the work of Haug, Cohen and Lee Haug (1985 and 1992); Cohen and Lee (1989). In the Haug paper a multi-period mixed-integer model is developed with a single objective function based on profits. Haug (1985). The profit maximization takes into consideration numerous country specific factors. The Haug paper studies the problems that high technology firms encounter when they investigate the expansion or transfer of production to a foreign country. Optimization is carried out through the cumulative average costs. Cohen and Lee (1989) presented a mixed-integer nonlinear model which investigates decisions that involves establishing a global manufacturing and distribution network. The mean variance approach was used in the studies by Hanink (1989), Hodder and Jucker (1985) and Hodder and Dincer (1986). Hanink (1985) proposed a 0-1 quadratic model without constraints, which employs the mean variance approach in an effort to choose the optimal "portfolio"? of sites by maximizing the expected return of investment. Their mixed, 0-1 quadratic, uncapacitated model was solved by decomposition methods (Jucker and Carlson, 1976; Hodder and Jucker, 1985b). Hodder and Dincer (1986) provided a more elaborate formulation that included simultaneous location and financing decisions. Finally, Schniederjans and Hoffman (1992) and Min and Melachrinoudis (1996) used the goal programming technique to handle conflicting goals in the international site selection process. Schniedderjans and Hoffman (1992) presented a decision process based on a deterministic 0-1 goal programming model that handles the problem of the acquisition of a foreign firm.

Badri (1996) develops a Goal Programming model to make location-allocation decisions in the presence of multiple conflicting factors. In addition, Badri (1999) proposes the model for making facility site decisions on a global scale by the application of the AHP and Multi-Objective Goal-

Programming methodology. Recently, Marrewijk (2002) proposes the modern general equilibrium models of multinationals. The models deal with the decision process for selecting the location of headquarters and production plants for four different types of firms in the manufacture sector. The factors being considered are the marginal production costs, the amount of labor needed to transport one unit of manufacture, the firm and the plant levels fixed costs. In this paper, the decision support model is proposed for site selection of Distribution Plant for Liquefied Petroleum gas in Brong Ahafo Region of Ghana.

Chapter 3

METHODOLOGY

3.1 INTRODUCTION

This chapter discusses the methodology, mathematical tools and algorithm used to locate or select the site for a Distribution Center in the Brong Ahafo Region of Ghana.

3.2 MATHEMATICAL MODELLING

A Mathematical model involves creating an abstract system of equations which describes and helps reasoning about a real life system. It is a mathematical representation of a process, device or concept by means of a number of variables which are defined to represent the inputs, outputs, and internal states of the device or process, and a set of equations and inequalities describing the interaction of these variables. It is a mathematical theory or system together with its axioms. The mathematical model serves the purpose of finding an optimal solution to a planning or decision problem, answer a variety of what-if questions and establish understandings of the relationships among the input data to extrapolate past data to derive meaning.

Mathematical models include techniques such as Linear Programming (LP), Computer Simulation, Decision Theory, Regression Analysis, Economic Order Quantity (EOQ), and Break-Even Analysis. Rutherford (1994).

3.2.1 MATHEMATICAL PROGRAMMING

Mathematical Programming (MP) is the use of mathematical models; particularly optimizing models, to assist in taking decisions. It is one of a number of Operations Research (OR) techniques.

OPTIMIZATION

Optimization is the process by which the best solution is selected from among several possible solutions. Most engineering problems, including those associated with the analysis, design, construction, operation, and maintenance, involve decision making. Usually, there will be a criterion that is to be minimized or maximized while satisfying several social, economical, physical, and technological constraints. There will be a number of parameters that can be varied in the decision making process. As the number of parameters increases, it becomes necessary to use systematic procedures for solving the decision making problems.

Definition of an optimization problem

The formulation of an optimization problem involves the development of a mathematical model for the physical or engineering problem. In practice, several assumptions have to be made to develop a reasonably simple mathematical model that can predict the behavior of the system fairly accurately. The results of optimization will be different with different mathematical model of the same physical system. Hence, it is necessary to have a good mathematical model of the system, so that the results of optimization can be used to improve the performance of the system. A general optimization problem can be stated in mathematical form as Find

$$X = \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ . \\ . \\ . \\ x_n \end{Bmatrix}$$

That *maximizes* f(×)

Subject to

$g_j\ (x) \leq 0; j = 1, 2, ..., m;$

And

$h_k = 1, 2, ..., p;$

where x_i ($i = 1, 2, , n$) are the decision variables x is the vector of decision variables, $f(x)$ is the merit, or criterion, or objective function g_j is the jth inequality constraint function that is required to be less than or equal to zero, $h_k(x)$ is the kth equality constraint function that is required to be equal to zero, n is the number of decision variables, m is the number of inequality constraints, and p is the number of equality constraints.

Terminology

Decision variables

The formulation of an optimization problem begins with the identification of a set of variables that can be varied to change the performance of the system. These are called the decision or design variables, and their values are freely controlled by the decision maker. A set of numerical values, one for each decision variables, constitutes a solution (acceptable or unacceptable) to the optimization problem.

Objective function

When different solutions are obtained by changing the decision variables, a criterion is needed to judge whether one solution is better than another. This criterion, when expressed in terms of the decision variables, is called the objective, merit, or cost function. The interest of the decision maker is to select suitable values for the decision variables so as to minimize or maximize the objective function.

Inequality constraints

Candidate solution

A candidate solution is a member of a set of possible solutions to a given problem. It is simply in the set that satisfies all constraints; that is, it is in the set of feasible solutions. Algorithms for solving various types of optimization problems often narrow the set of candidate solutions down to a subset of the feasible solutions, whose points remain as candidate solutions while the other feasible solutions are excluded as candidates. The space of all candidate solutions, before any feasible points have been excluded, is called the feasible region. A feasible solution is an acceptable solution to the decision maker in terms of the constraints, but may or may not minimize the objective function.

Optimum solution

A feasible solution that minimizes the objective function is known as the optimum solution.

TYPES OF OPTIMIZATION PROBLEMS

Optimization problems are also known as mathematical programming problems. In some practical situations, the constraints may be absent, and the problem reduces to

Find

$$X = X = \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ . \\ . \\ . \\ x_n \end{Bmatrix}$$

That $maximizes\ f(X)$

The problem is known as a constrained optimization problem. Depending on the nature of functions, Optimization problems can be classified as linear and nonlinear programming problems. A linear programming problem is one in which all the functions involved, namely, $f(x), g_j(x) and h_k(x)$ are linear in terms of the design variables. A nonlinear programming problem is one in which at least one of the functions is nonlinear in terms of the decision variables. Specifically, the least-travel location uses the objective function as follows:

$$Minimize\ \sum_{j}^{N} D_j \times (\sum_{i}^{N} L_i \times (T_{ij} + G_j)),$$

$$Subjectto \sum_{j}^{N} D_j \geq 1............, ...$$

Where

ij=towns

L_i = number of days traveled ,

G_j = incidental expenses at town j,

T_{ij} = the cost of travel to town j from town i ,

D_j = 1 if distance travel in town j, 0 otherwise

3.2.2 NETWORK MODELS

One of the most prominent OR techniques is network programming (in which the problem can be modeled as a network). Network models are applicable to an enormous variety of decision problems that can be modeled as networks optimization problems and solved efficiently and effectively. Some of these decision problems are really physical problems such as transportation or flow of commodities. However, many network problems are more of an abstract representation of processes or activities such as the critical path activity network in project management. The family of network optimization problems includes the following prototype models: shortest path, assignment, critical path, max flow, transportation, and min cost flow problems. These problems are easily stated by using a network of arcs, and nodes.

3.2.3 SHORTEST ROUTE PROBLEM

The shortest route (path) problem is the problem of determining the best way to traverse a network to get from an origin to a given destination as cheaply as possible. Suppose that in a given network there are m nodes and n arcs (i.e. edges) and a cost C_{ij} associated with each arc ($itoj$) in the network. Formally, the Shortest Path (SP) problem is to find the shortest (least cost) path from the start node 1 to the finish node m and the cost C_{ij} of the path is the sum of the costs on the arcs in the path. Arsham (1998). In graph theory, the shortest route problem is the problem of finding a route (path) between two vertices (or nodes) such that the sum of the weights of its constituent edges is minimized. Danny, et al (1996). It determines the shortest route between a source and destination in a transportation network. An example is finding the quickest way to get from one location to another on a road map; in this case, the vertices represent locations and the edges represent segments of road and are weighted by the time needed to travel that segment (Hamdy, 2007). Formally, given a weighted graph (that is, a set V of vertices, a set E of edges, and a real - valued weight function $f : E \longrightarrow R$), and one element v of V^1, find a path p from v to a V' of V so that

$$\sum_{p \in P} f(p)$$

is minimal among all paths connecting v to V'

3.2.4 ALGORITHMS OF SHORTEST WAY BETWEEN TWO TOWNS

Literature presents several algorithms which find shortest way between two points from concrete graph node to all the other ones. They are: Dijkstra, Bellman-Ford, A* Search, Johnson and Floyd-Warshall algorithms. According to Cherkassky et al (1996), Dijkstra's algorithm solves the single-pair, single-source, and single-destination shortest path problems, Bellman-Ford algorithm solves the single-source problem if edge weights may be negative , A* Search algorithm solves for single pair shortest path using heuristics to try to speed up the search whilst the last two solves all pairs shortest paths. Floyd's algorithm is the simplest and fastest. Thus, Floyd-Warshall algorithm will be considered for the purpose of this study.

3.2.5 FLOYD'S SHORTEST PATH ALGORITHM

Floyd's algorithm is a more generalized algorithm compared to Dijkstra's because it determines the shortest route between any two nodes in the network. The algorithm represents an n - node network as a square matrix with n rows and n columns. Entry (i, j) of a matrix gives the distance d_{ij} from node i to node j , which is finite if i is linked directly to j , and infinite otherwise. The Floyd's algorithm is based on a simple intuitive logic. It states that if the travel to a node from its preceding node can be made shorter by traveling via another node, which is linked to the preceding node, it is always advisable to travel via the extra node so that the travel distance is minimum. This can be stated mathematically as follows:

Given three nodes i, j and k as shown in the Figure 3.1, with the connecting distances shown on three arcs, it is shorter to reach j from i passing through k if

$$d_{ik} + d_{kj} < d_{ij}$$

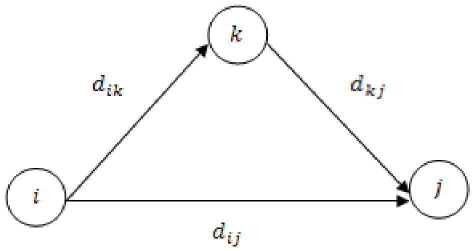

Fig. 3.1: A Directed Graph Showing Nodes i, j, and k

In such a case, it is optimal to replace the direct route (i, j) by the sum of routes (i, k) and (k, j). A systematic method to exhaust all routes joining every node set I, k, j is to first form a matrix D_0. This matrix is formed by the distances (costs) between all the possible pairs of nodes in the network. According to Hamdy (2007), a triple operation exchange is then applied to the chosen nodes using the following steps: Step 0: Defined the starting distance $n * n$ matrix Do and node sequence matrix So as given in Figure 3.2. The diagonal elements are marked with (o) to indicate that they are blocked. Set step number k equal to 1 $ie.k = 1$.

$$D_0$$

$$\begin{array}{c|cccccc|}
 & 1 & 2 & \cdots & j & \cdots & n \\
\hline
1 & 0 & d_{12} & \cdots & d_{1j} & \cdots & d_{1n} \\
2 & d_{21} & 0 & \cdots & d_{2j} & \cdots & d_{2n} \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
i & d_{i1} & d_{i2} & \cdots & d_{ij} & \cdots & d_{in} \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
n & d_{n1} & d_{n2} & \cdots & d_{nj} & \cdots & 0 \\
\hline
\end{array}$$

$$S_0$$

$$\begin{array}{c|cccccc|}
 & 1 & 2 & \cdots & j & \cdots & n \\
\hline
1 & 0 & 2 & \cdots & j & \cdots & n \\
2 & 1 & 0 & \cdots & j & \cdots & n \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
i & 1 & 2 & \cdots & j & \cdots & n \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
n & 1 & 2 & \cdots & j & \cdots & 0 \\
\hline
\end{array}$$

Fig. 3.2: Starting distance matrix and node sequence matrix

General step k: Define row k and column k as pivot row and pivot column. As explained earlier, row k and column k intersect at a diagonal element. Apply the triple operation to each element d_{ij} in D_{k-1} (i.e. on all elements in D_{k-1}, which are not on the diagonal and not on the selected row and column),

for all i and j . If the condition $d_{ik} + d_{kj<d_{ij},(1\neq k,j\neq k and i\neq j)}$, is satisfied, make the following changes: (a) Create new matrix D_k by replacing d_{ij} in D_{k-1} with $d_{ik} + d_{kj}$.

(b) Create new matrix S_k by replacing S_{ij} in S_{k-1} with k . Set $k = k+1$. If $k = n+1$, stop; else repeat step k.

Step k of the algorithm can be explained by representing D_{k-1} as shown in figure 3.3. The intersection of row k and the column k defines the current pivot row and column. Row i represents any of the rows$1, 2, ..., k-1,$, and row p represents any of the rows $k+1, k+2, ..., n$. Similarly, column j represents any of the columns 1,2,..., k-1 , and column q represents any of the columns $k+1, k+2, ..., n.$. The triple operation can be applied as follows: If the sum of the elements on the pivot row and the pivot column (shown by squares) is smaller than the associated intersection element (shown by a circle), then it is optimal to replace the intersection distances (or the values in the circles) by the sum of the pivot distances (values in the squares).

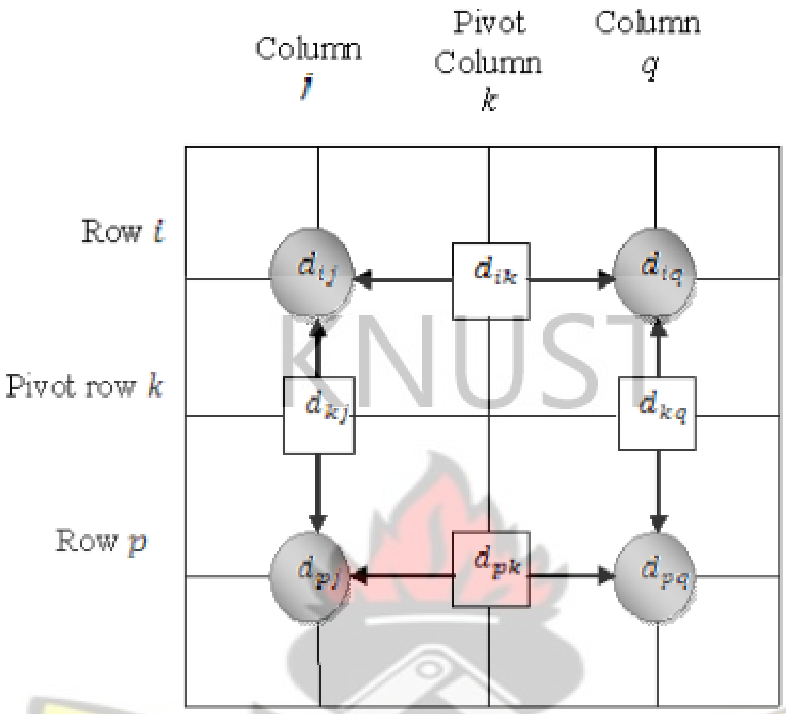

Fig. 3.3: Schematic Diagram showing the Pivot Manipulation in a Floyds Matrix

After n steps the shortest route between any two nodes i and j can be determined as the entry d_{ij} in the matrices D_n and S_n using the following rules:

1. From D_n , d_{ij} gives the shortest distance between nodes i and j.

2. From S_n, determine the intermediate node $k = S_{ij}$ that yields the route $i \to k \to j$. If $S_{ik} = k$ and $S_{ik} = j$, stop; all the intermediate nodes of the route have been found. Otherwise, repeat the procedure between nodes i and k , and between nodes k and j. The if conditions of the algorithm are as follows (they are given in a programmer friendly way to help in its coding and understanding):

If $(i \neq k, jkandi \neq j)$

$d_{ik} \neq \infty, andd_{ik} \neq \infty,$

if$d_{ij} = \infty,$

$ord_{ij} = \infty, d_{ik} + d_{kj} < d_{ij}$

then make the following change: Create D_k by replacing d_{ij} in D_{k-1} with $d_{ik} + d_{kj}$

These conditions can be shown in the form of a flow chart as follows:

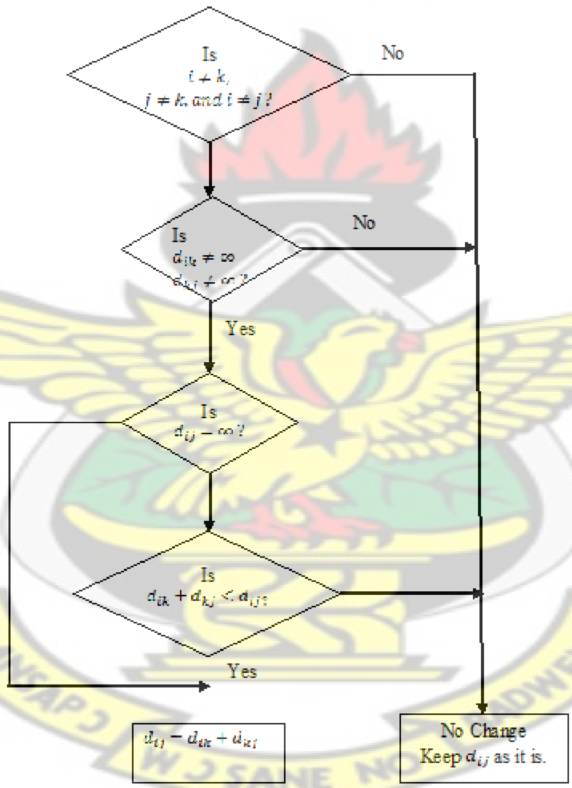

Figure 3.4: Flow Chart Showing the Floyd's Algorithm

Pseudocode:

```
Procedure Floyd-Warshall (G)

for i  = 1 to n do   initializing the source matrix

for  j  = 1 to n do

d[i, j, 0] = w(i, j)

s[i, j] = 0      ** s[i, j] keeps the break point vertex

for k  = 1 to n do  number of iterations

for i  = 1 to n do

for j  = 1 to n do

 if $i\ne j \& k \ne j \& i\ne k;$

if d[i, k, k  1]+d[k, j, k  1] < d[i, j, k  1] then
```

```
d[i, j, k] = d[i, k, k  1]+d[k, j, k  1];

s[i, j] = k;

else

d[i, j, k] = d[i, j, k  1]

end

end

end

end

end
```

This algorithm gives the shortest route between any two nodes (towns). This leads to a final updated version of Adjacency matrix with respective shortest route distances (cost) as the matrix elements. The computation complexity is given by n^3 (Dequan, 2009)

The Floyd's algorithm correct distance labels in a systematic way until they represent the shortest path distance. It is more robust and involves lesser computational overhead in large networks. Moreover, practical experience also indicates that Floyd's algorithm is faster than Dijkstra's algorithm in MATLAB simulation (Shang and Ruml, 2004).

Example 1

(Shortest Route Problem using Floyd's Algorithm)

For the network in Figure 3.5, find the shortest routes between every two nodes. The distances (in miles) are given on the arcs. Arc (3, 5) is directional, so that no traffic is allowed from node 5 to node 3. All the other arcs allow two-way traffic.

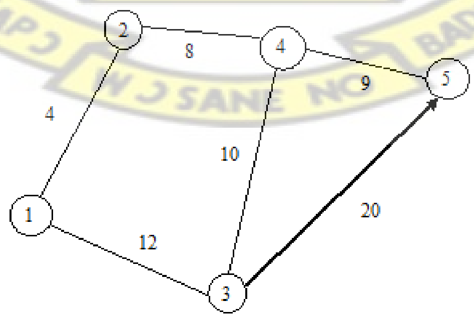

Figure 3.5: Network for example 1

We let the numbers in column 1 and row 1 of each network be the origin and destination nodes respectively.

Iteration 0. The matrices D_o and S_o give the initial representation of the network. D_o is symmetrical, except that $d_{53} = \infty$ because no traffic is allowed from node 5 to node 3.

D_0

Node	1	2	3	4	5
1	0	4	12	8	8
2	4	0	8	8	8
3	12	8	o	10	20
4	8	8	10	0	9
5	8	8	8	9	0

S_0

Node	1	2	3	4	5
1	0	2	3	4	5
2	1	0	3	4	5
3	1	2	0	4	5
4	1	2	3	0	5
5	1	2	3	4	0

Iteration 1. Set $k = 1$. The pivot row and column are shown by the lightly shaded second row and second column in the D_o matrix. The darker cells, d_{23} and d_{32} , are the only ones that can be improved by the triple operation. Thus, D_1 and S_1 are obtained from D_o and S_o in the following manner:

1.Replace d_{23} and d_{32} with $d_{21} + d_{13} = 4 + 12 = 16$ and set $S_{23} = 1$.

2. Replace d_{32} with $d_{31} + d_{12} = 12 + 4 = 16$ and set $S_{32} = 1$. Iteration

D_1

	1	2	3	4	5
1	0	4	12	8	8
2	4	0	16	8	8
3	12	16	0	10	20
4	8	8	10	0	9
5	8	8	8	9	0

S_1

	1	2	3	4	5
1	0	2	3	4	5
2	1	0	1	4	5
3	1	1	0	4	5
4	1	2	3	0	5
5	1	2	3	4	0

2. Set k = 2 , as shown by the lightly shaded row and column in D_1 . The triple operation is applied to the darker cells in D_1 and S_1 . The resulting changes are shown in both D_2 and S_2

D_2

	1	2	3	4	5
1	0	4	12	12	8
2	4	0	16	8	8
3	12	16	0	10	20
4	12	8	10	0	9
5	8	8	8	9	0

S_2

	1	2	3	4	5
1	0	2	3	2	5
2	1	0	1	4	5
3	1	1	0	4	5
4	2	2	3	0	5
5	1	2	3	4	0

Figure 3.8:Showing and Matrices

Iteration 3. Set , as shown by the shaded row and column in D_2 . The new matrices are given by D_3 and S_3

D_3

	1	2	3	4	5
1	0	4	12	12	32
2	4	0	16	8	17
3	12	16	0	10	20
4	12	8	10	0	9
5	8	8	8	9	0

S_3

	1	2	3	4	5
1	0	2	3	2	3
2	1	0	1	4	3
3	1	1	0	4	5
4	2	2	3	0	5
5	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]

Iteration 4. Set k = 4 , as shown by the shaded row and column in D_3 . The new matrices are given by D_4 and S_4 .

Iteration 5. Set k = 5, as shown by the shaded row and column in D_4 .

	1	2	3	4	5
1	0	4	12	12	**21**
2	4	0	**16**	8	**17**
3	12	**16**	0	10	**19**
4	12	8	10	0	9
5	**21**	**17**	**19**	9	0

S_4

	1	2	3	4	5
1	0	2	3	2	**4**
2	1	0	**4**	4	**4**
3	1	**4**	0	4	**4**
4	2	2	3	0	5
5	**4**	**4**	**4**	4	0

Figure 3.10: Showing and Matrices

No further improvements are possible in this iteration.

The final matrices D_4 and S_4 contain all the information needed to determine the shortest route between any two nodes in the network. For example, from D_4 , the shortest distance ($leastcost$) from node 1 to node 5 is $d_{15} = 12$ miles.

To determine the associated route, remember that a segment (i, j) represents a direct link only if s_{ij}. Otherwise, i and j are linked through at least one other intermediate node. Because $s_{15} = 4 \neq 5$, the route is initially given as $1 \to 4 \to 5$.

Now, because $s_{14} = 2 \neq 4$, the segment $(1, 4)$ is not a direct link, and $1 \to 4$ is replaced with $1 \to 2 \to 4$, and the route $1 \to 4 \to 5$ now becomes $1 \to 2 \to 4 \to 5$. Next, because $s_{12} = 2$, $s_{24} = 4$, and $s_{45} = 5$, no further dissecting is needed, and $1 \to 2 \to 4 \to 5$ defines the shortest route.

3.3 Solving the Least-cost site Problem

The solution methodology used to optimize the results for this problem involves simple arithmetic. Given distance in kilometers between all the district capitals to locate a distribution center that is not too far from all the districts, the total travel cost is the sum of the products of the number of litres of diesel used between each town and the price per litre from that town to a candidate town. The additional cost is the incidental expenses for the days to off load the consignment from town i to town j . We can best illustrate the methodology by example.

3.3.1 Solving the Least-cost Site Selection Model

Tema Oil Refinery in Tema, wants to locate a site in one of the district capitals that would be the shortest route and cost effective distribution center for the company. The company considers the distance between various district capitals, the drivers' duty expenses incurred in performance of duties, which include the costs of diesel, meals, and incidentals. The least Travel-cost site, location minimizes total costs and is derived as follows:

$$Minimize \sum_{j}^{N} D_j \times (\sum_{i}^{N} L_i \times (T_{ij} + G_j)),$$

Subject to

$$\sum_{j}^{N} D_j \geq 1............,$$

Where

ij=towns

L_i = number of days traveled

G_j = incidental expenses at town

$T_i j$ = the cost of travel to town j from town i,

$D_j = 1$ if distance travel in town 0 otherwise

The constraint ensures that at least one candidate city is found. In many cases only one city i will provide the least cost among all N cities so $D_i = 1$

and $D_k = 0$ for k $\neq$ i. The constraint will be satisfied because $\sum_j^N D_j = 1$

for these cases. However, we need to provide for the possibility that there may be a case where two cities provide the least cost, and for this case the constraint will be satisfied because $\sum_j^N D_j > 1$

Example 2

Consider the situation in which a Company distributing LPG from a distribution center to all the four towns. The expense data varies by town (Table 3.2). To compute the total cost for distributing in Wiawso, we calculate travel costs from all points of origin:

Table 3.1: Expenses Data

Origin Town	Number of Days Travel	Incidental expenses(IE) Per Day (GH)
Wiawso	1	55
Bibiani	1	58
Dwinase	1	65
Bekwai	1	48

Table 3.2: Travel Costs

Route	Round Trip (km)	Cost per km (GH)	Total Cost (GH)
Wiawso - Bibiani	41	0.6492	26.6
Wiawso - Bekwai	13	0.6492	8.4
Wiawso - Dwinase	5	0.6492	3.2
Bibiani - Bekwai	28	0.6492	18.2
Bibiani - Dwinase	48	0.6492	31.2

Total travel Cost from distribution center to all points of origin are: $(Costperkm)\times$
$(Wiawso - Bibiani) = (41) \times (GH0.6492) = GH26.6$

$(Costperkm) \times (Wiawso - Bekwai) = (13) \times (GH0.6492) = GH8.4$

$(Costperkm) \times (Wiawso - Bibiani) = (5) \times (GH0.6492) = GH3.2$

$(Costperkm) \times (Bibiani - Bekwai) = (28) \times (GH0.6492) = GH18.2$

$(Costperkm) \times (Wiawso - Bibiani) = (48) \times (GH0.6492) = GH31.2$

3.3.2 How The Cost of Travel, T_{ij} , is Determined

The travel cost T_{ij} is based on the quantity of diesel used between town i and j (called a town pair) is based on one-way travel. Since no truck (tanker) station services every other station, a one-way trip is often composed of multiple legs. For example, a trip from Wiawso to Bibiani usually means a transfer in either Bekwai or Dwinase through Bekwai and then non-stop to Bibiani. The cheapest route is determined by solving a shortest route (path) problem where we have four nodes (Wiawso, Dwenasi, Bekwai and

Bibiani) and the following five arcs or town pairs: Wiawso - Bekwai, Wiawso - Dwenase, Dwenase - Bekwai, Bekwai - Bibiani, Wiawso - Bibiani. The cost for each arc, c_{ij} , is the amount of diesel used to off load the consignment from town i to town j Hence the travel cost from Wiawso to Bekwai and back, $T_{Wiawso,Bekwai}$, would be the (total cost of the least cost path from Wiawso to Bekwai)$\times$ 2. Any shortest-path algorithm can be used for this problem. But Floyd's algorithm is more robust and involves lesser computational overhead in large networks (Ravi, 2004).

Example 3

(Numerical example of the Least-cost Travel Site model) A gas company wants to locate distribution center in one of the four towns. Determine the least travel-cost location. Data are provided as follows: Duration, $L_i = 1$ day

Table 3.4: Towns and their incidental expenses

Town	L_i= days	G_jIncidental expenses(IE)
Wiawso	1	55
Bibiani	1	58
Dwinase	1	65
Bekwai	1	48

Table 3.5: Round-trip costs (T_{ij})

	Wiawso	Bibiani	Dwinase	Bekwai
Wiawso	0	106	6	34
Bibiani	106	0	112	72
Dwinase	6	112	0	40
Bekwai	34	72	40	0

Solution

The towns in column 1 of table 3.5 are the origin towns and the towns in columns 2 through 5 are the location for distribution center. Therefore, if Bibiani is the location for distribution center, the travel cost from Wiawso to Bibiani and back,T_{12} , is 106. Therefore if $D_1 = 1$ (Wiawso is the location for distribution center) and D_2 through $D_5 = 0$ we have

Table 3.6: Cost for location in Wiawso

i	L_i	T_{i1}	G_jIE	$L_i \times (T_{i1+G_1})$
1	1	0	0	0
2	1	106	55	161
3	1	6	55	61
4	1	34	55	89

The total cost for Wiawso is 0 + 161 + 61 + 89 = 311. The travel distance and per diem from Wiawso is zero since the distribution center is located at this station. Also if Therefore if $D_2 = 1$ (Bibiani is the location for distribution center) and $D_{1,}D_3$ through $D_5 = 0$ we have

Table 3.7: Cost for location in Bibiani

i	L_i	T_{i1}	G_jIE	$L_i \times (T_{i1+G_1})$
1	1	106	58	164
2	1	0	0	0
3	1	112	58	170
4	1	72	58	130

The total cost for Bibiani is 164 + 0 + 170 + 130 = 464. Repeating this process for the other three towns, we get these total costs:

Table 3.8: Total costs for locating distribution center in various towns

j	Distribution Center Location	Total Cost (Gh)
1	Wiawso	311
2	Bibiani	464
3	Dwinase	353
4	Bekwai	*290

* Least-Cost distribution Center

By inspection, it could be observed from Table 3.8 that, the least-cost for Distribution Center is Bekwai. It quickly becomes obvious that while manually calculating all of the costs is a straight-forward, albeit time-consuming task, a MATLAB Programme is therefore developed to speed up the calculations.

Chapter 4

DATA ANALYSIS AND MODELLING

4.1 GENERAL OVERVIEW

This chapter analyses the secondary data used as input to the proposed site selection model of the study and discusses results obtained from the analysis. The data analysis was implemented using the two developed MATLAB programmes (leasttravelcost.m and leastcost-site.m). Specifically the Floyd's algorithm was used to determine the least travel costs as input parameters and the other programme was used to solve the site selection model.

4.2 DATA COLLECTION

The Government Agency that was contacted for the secondary data and other important information for this thesis was the Regional Town and Country

Planning Department (Sunyani) where the map of the Brong Ahafo Region was obtained. The State Transport Cooperation (STC) in Sunyani, where distances between District capitals were obtained, Hills Gas Tankers for quantity of diesel used per kilometer and the price per litre of diesel was obtained from Total Filling Station in Techiman. The Floyd's Algorithm was coded using MATLAB and used to minimize the travel costs across the nineteen (19) district capital towns in Brong Ahafo Region of Ghana. The data from Hills Gas was about incidental expenses for each trip.

Table 4.1: Summary of Data and their Source

Data	Source
Distances between District Capitals in the Region	State Transport Corporation (Sunyani)
Incidental Expenses, Quantity of fuel used	Hills Gas Company
Names of District Capitals in the Region	Town and Country Planning Department (Sunyani)

4.3 DATA ANALYSIS

4.3.1 MODEL INPUT DATA

The travel costs (distances and diesel used) for each town were calculated and also analyzed using the Floyd's Algorithm code. The actual code is included in Appendix A. The summary of both data are given below as the input data. The incidental expenses is written against each town. Duration of travel (days), $L_i = 1$.

Table 4.2: Summary of incidental expenses

$TownIndex$	(Town)	IE (GH)	Town Index	(Town)	IE (GH)
1	Sunyani, SU	40	11	Banda, BA	45
2	Techiman, TE	40	12	Drobo, DR	45
3	Nkoranza, NK	40	13	Berekum, BK	40
4	Atebubu, AT	50	14	Dormaa, DO	45
5	Kwame Danso, KW	50	15	Kukuom, KU	50
6	Yeji, YE	50	16	Goaso, GO	45
7	Kintampo, KN	45	17	Kenyasi, KE	45
8	Jema, JE	45	18	Bechem, BE	40
9	Wenchi, WE	40	19	Sampa, SA	45
10	Nsawkaw, NS	45			

Table 4.2 depicts the incidental expenses at each capital town per day. The output of the Floyd's Algorithm is presented in Table 4.3. These results represent the travel costs of round-trip per person in Ghana Cedis (GH). The entries below the main diagonal symmetrically equal those above it.

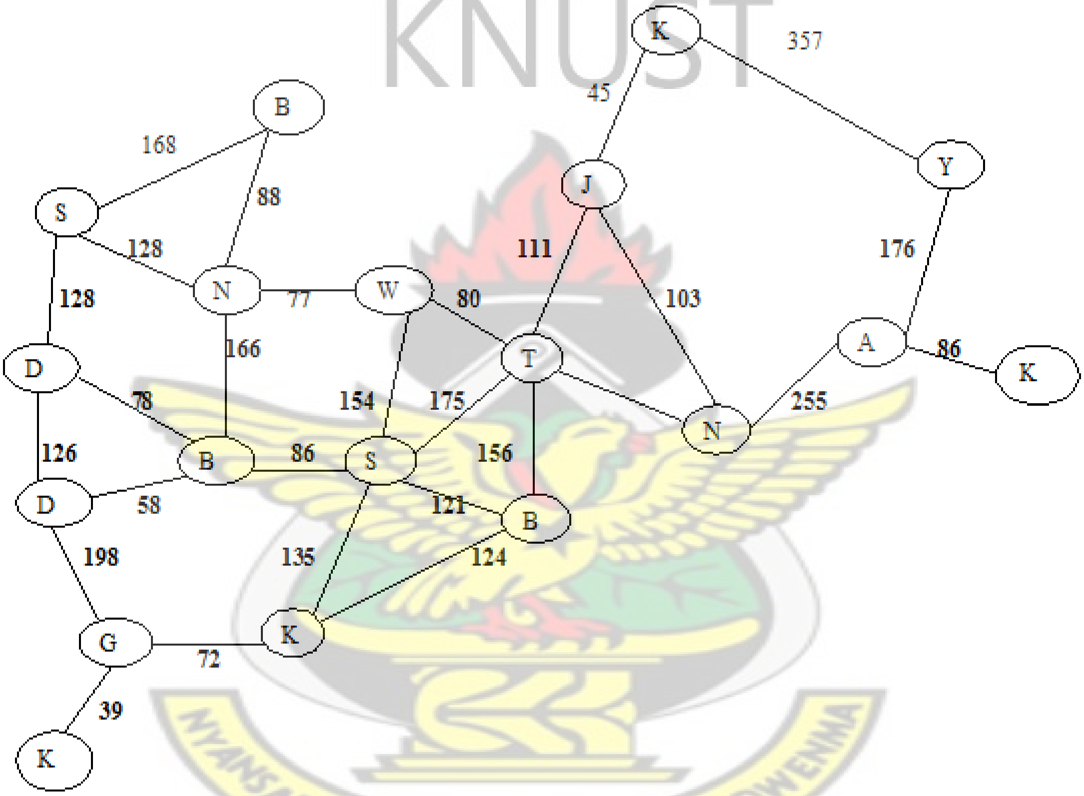

Figure 4.1: Network of Nineteen District Capitals in Brong Ahafo Region

Table 4.3a: Floyd Matrix of the ROUND - TRIP TRAVEL COST ( ) IN (B/A)

	SU	TE	NK	AT	KW	YE	KN	JE	WE
SU	0	175	249	504	590	680	331	286	154
TE	175	0	74	329	415	505	156	111	80
NK	249	74	0	255	341	431	148	103	154
AT	504	329	255	0	86	176	403	358	409
KW	590	415	341	86	0	262	489	444	495
YE	680	505	431	176	262	0	357	402	585
KN	331	156	148	403	489	357	0	45	236
JE	286	111	103	358	444	402	45	0	191
WE	154	80	154	409	595	585	236	191	0
NS	231	157	231	486	572	662	313	268	77
BA	319	245	319	574	660	750	401	356	165
DR	164	339	413	668	754	844	495	450	318
BK	86	261	335	590	676	766	417	372	240
DO	144	319	393	648	734	824	475	430	298
KU	246	421	495	750	836	926	577	532	400
GO	207	382	456	711	797	887	538	493	361
KE	135	310	384	639	725	815	466	421	289
BE	121	296	370	625	711	801	452	407	275
SA	292	285	359	614	700	790	441	396	205
TOTAL	4914	4860	5510	8825	10287	11463	6740	6065	4932

Table 4.3b: Floyd Matrix of the ROUND - TRIP TRAVEL COST ( ) IN (B/A)

	NS	BA	DR	BK	DO	KU	GO	KE	BE	SA
SU	231	319	164	86	144	246	207	135	121	359
TE	157	245	339	261	319	391	352	280	156	285
NK	231	319	413	333	393	465	426	354	230	359
AT	496	574	668	590	648	720	681	609	495	614
KW	572	660	754	676	734	806	767	695	571	700
YE	662	750	844	766	824	896	857	785	661	790
KN	313	401	495	417	475	547	508	436	312	441
JE	268	356	450	372	430	502	463	391	267	396
WE	77	165	318	240	298	400	361	289	236	205
NS	0	88	244	166	224	461	422	366	313	128
BA	88	0	296	254	312	549	510	454	401	168
DR	244	296	0	78	126	363	324	299	285	372
BK	166	254	78	0	58	295	256	221	207	294
DO	224	312	126	58	0	237	198	270	265	352
KU	461	549	363	295	237	0	39	111	235	589
GO	422	510	324	256	198	39	0	72	196	550
KE	366	454	299	221	270	111	72	0	124	494
BE	352	440	285	207	265	235	196	124	0	480
SA	128	168	128	206	254	491	452	427	413	0
TOTAL	5448	6896	6588	5484	6209	7754	7091	6318	5478	7576

Table 4.4a: CODE (TOWN) SEQUENCE (S_i)

	SU	TE	NK	AT	KW	YE	KN	JE	WE
SU	0	2	2	3	4	4	8	2	9
TE	1	0	3	3	4	4	8	8	9
NK	2	2	0	4	4	4	8	8	2
AT	3	3	3	0	5	6	8	3	3
KW	4	4	4	4	0	4	8	4	4
YE	4	4	4	4	4	0	7	7	4
KN	8	8	8	8	8	6	0	8	8
JE	2	2	3	3	4	7	7	0	2
WE	1	2	2	3	4	4	8	2	0
NS	9	9	9	9	9	9	9	9	9
BA	10	10	10	10	10	10	10	10	10
DR	13	13	13	13	13	13	13	13	13
BK	1	1	2	3	4	4	8	2	1
DO	13	13	13	13	13	13	13	13	13
KU	17	17	17	17	17	17	17	17	17
GO	17	17	17	17	17	17	17	17	17
KE	1	1	2	3	4	4	8	2	1
BE	1	1	2	3	4	4	8	2	1
SA	13	10	10	10	10	10	10	10	10

Table 4.4b: CODE (TOWN) SEQUENCE (S_i)

	BA	DR	BK	DO	KU	GO	KE	BE	SA
SU	10	13	13	13	17	17	17	18	10
TE	10	13	1	13	18	18	18	18	10
NK	10	13	1	13	18	18	18	2	10
AT	10	13	1	13	18	18	18	3	10
KW	10	13	1	13	18	18	18	4	10
YE	10	13	1	13	18	18	18	4	10
KN	10	13	1	13	18	18	18	8	10
JE	10	13	1	13	18	18	18	2	10
WE	10	13	1	13	17	17	1	2	10
NS	11	13	13	13	16	14	9	9	19
BA	0	19	10	13	16	14	10	10	19
DR	13	0	13	14	16	14	13	13	13
BK	10	12	0	14	16	14	1	1	10
DO	13	12	13	0	16	16	16	13	13
KU	16	16	16	16	0	16	16	17	16
GO	14	14	14	14	15	0	17	17	14
KE	10	13	1	16	16	16	0	18	10
BE	10	13	1	13	17	17	17	0	10
SA	11	12	12	12	16	14	13	13	0

The towns in column one (1) of Table 4.3 are the origin towns and the towns in columns two (2) through Twenty (20) are the potential location sites. For instance, if Wenchi, WE, is the selected site, the travel cost from Sunyani, SU to Wenchi and back, $T_{1,9}$ is GH154.00 for fuel used from Sunyani. Similarly, the travel costs from Techiman, Nkoranza, Atebubu, Kwame Danso, Yeji, Kintampo, Jema, , Nsawkwaw, Banda, Drobo, Berekum, Dormaa, Kukuom, Goaso, Kenyasi, Bechem, and Sampa to Wenchi and back are GH80.00, GH154.00, GH409.00, GH495.00, GH585.00, GH236.00, GH191.00, GH77.00, GH165.00, GH318.00, GH240.00, GH298.00, GH400.00, GH361.00, GH289.00, GH275.00, and GH205.00 respectively. The values in row Twenty One (21) represent the collective travel costs to the respective potential location towns. For example, if the site is located at Banda the total travel cost is GH6896.00. Table 4.4 represents the associated routes connecting the town-pairs. The travel cost from Atebubu with index 4 to Jema, indexed 8 and back, $T_{4,8}$ is GH358.00, (Table 4.3). The associated route, $S_{4,8}$ from Table 4.4 is 3. Since the segment $S_{4,8} = 3 \neq 8$, it means Atebubu and Jema are linked through at least one intermediate town, possibly Nkoranza, indexed 3. Considering Wenchi to Bechem, $S_{9,18}$, we could deduce from Table 4.4 that $S_{9,18} = 2 \neq 18$ .This implies that from Wenchi to Bechem is not a direct link therefore Wenchi$\rightarrow$ Bechem is replaced with Wenchi $\rightarrow$ Techiman $\rightarrow$ Bechem and the route Atebubu $\rightarrow$ Jema now becomes Atebubu $\rightarrow$ Nkoranza $\rightarrow$ Jema. That is, $S_{4,8} = S_{4,3} \rightarrow S_{3,8}$. The associated travel costs of the route are GH255.00 + GH103.00 = GH358.00 (Table 4.3). Moreover, the route Berekum $\rightarrow$ Jema, $S_{13,8} = 2 \neq 8$ means that the towns are linked through at least one other intermediate town. These towns are Sunyani and

Techiman indexed 1 and 2 respectively. Thus the route becomes Berekum $\rightarrow$ Sunyani $\rightarrow$ Techiman $\rightarrow$ Jema (Table 4.4).

4.3.2 MODELLING

To solve for the least selection site, an algebraic representation model of the problem was developed to derive the minimum total costs for each town as follows: Minimize

$$\sum_{j}^{N} D_j \times (\sum_{i}^{N} L_i \times (T_{ij} + G_j)),$$

$$Subjectto \sum_{j}^{N} D_j \geq 1............,$$

Where

ij=towns

L_i = number of days traveled ,

G_j = incidental expenses at town

T_{ij} = the cost of travel to town from town ,

D_j = 1 if distance travel in town 0 otherwise

The constraint ensures that at least one candidate town is found. In many cases only one town i will provide the least travel cost among all N towns so $D_i = 1$ and $D_k = 0$ for $k \neq i$. The constraint will be satisfied because $\sum_j^N D_j = 1$ for these cases. However, we need to provide for the possibility that there may be a case where two or more towns will provide the least cost, and for this case the constraint will be satisfied because $\sum_j^N D_j > 1$. The model was coded using MATLAB and makes its calculations based on input from the user. The code is included in Appendix A. The output was displayed for analysis in the form of text files. The data in Tables 4.1 and 4.2 represent T_{ij} is the input data and for the purpose of this study, G_i incidental expenses and L_i number of days travel.

4.3.3 Results from the Model Solution

Table 4.5 below shows the summary results of the site selection model for the Distribution Center of LPG. The study revealed that Yeji, YE is the most expensive site at GH12363.00 and it is GH6783.00 more than the least travel cost alternative, Techiman, at GH5580.00. This represents a 54.87 % (GH6783.00) savings on this Distribution center. If the Oil Company could save 54.87 % on distribution LPG to various district, taken Techiman as Distribution Center, it might realize potential GH6783.00 savings based on the estimated Yeji distribution cost. Even if a conservative estimate of 10% annual savings is used, it is possible that Tema Oil Refinery could realize an annual GH1236.30 saving on distribution of LPG relating to Brong Ahafo Region of Ghana.

Table 4.5: Total Costs for Distribution in Various District Capital Towns

i	**Distribution Center**	**Total Costs**	
1	Sunyani, SU	GH¢5634.00	
2	Techiman, TE	GH¢5580.00	Least-cost Distribution Center
3	Nkoranza, NK	GH¢6230.00	
4	Atebubu, AT	GH¢9725.00	
5	Kwame Danso, KW	GH¢11187.00	
6	Yeji, YE	GH¢12363.00	
7	Kintampo, KN	GH¢7550.00	
8	Jema, JE	GH¢6875.00	
9	Wenchi, WE	GH¢5652.00	
10	Nsawkwaw, NS	GH¢6258.00	
11	Banda, BA	GH¢7706.00	
12	Drobo, DR	GH¢7398.00	
13	Berekum, BK	GH¢6204.00	
14	Dormaa, DO	GH¢7019.00	
15	Kukuom, KU	GH¢8654.00	
16	Goaso, GO	GH¢7901.00	
17	Kenyasi, KE	GH¢7128.00	
18	Bechem, BE	GH¢6198.00	
19	Sampa, SA	GH¢8386.00	

The true strength of this methodology is that the study shows the exact location and the least aggregate travel cost of GH4860.00 (Table 4.3).

TABLE 4.6: SUMMARY OF ROUTES TO COST-EFFECTIVE SITE

From	To	Round-trip cost (GH¢)	Route
1 SU	2 TE	175	1 – 2
3 NK	2 TE	74	3 – 2
4 AT	2 TE	329	4 – 3 – 2
5 KW	2 TE	415	5 – 4 – 3 – 2
6 YE	2 TE	505	6 – 4 – 3 – 2
7 KN	2 TE	156	7 – 8 – 2
8 JE	2 TE	111	8 – 2
9 WE	2 TE	80	9 – 2
10 NS	2 TE	157	10 – 9 – 2
11 BA	2 TE	245	11 – 10 – 9 – 2
12 DR	2 TE	339	12 – 13 – 1 – 2
13 BK	2 TE	261	13 – 1 – 2
14 DO	2 TE	319	14 – 13 – 1 – 2
15 KU	2 TE	421	15 – 16 – 17 – 1 – 2
16 GO	2 TE	382	16 – 17 – 1 – 2
17 KE	2 TE	310	17 – 1 – 2
18 BE	2 TE	296	18 – 2
19 SA	2 TE	285	19 – 10 – 9 – 2

Table 4.6 above is an excerpt of table 4.4 showing all the shortest routes from all the origin towns to the estimated cost-effective Distribution Center with their round-trip costs. The numbers preceding the codes of towns in columns one (1) and two (2) are the respective index of the towns. These are used in column four (4) to represent the intermediate nodes (towns) in each

route to the Distribution Center. Thus we can deduce that the route from Yeji to Techiman is 6 - 4 - 3 - 2 which denotes Yeji → Atebubu → Nkoranza → Techiman

4.4 SENSITIVITY ANALYSIS

While this model is a useful tool to aid decision making in site selection for Distribution Center, there remain several types of uncertainty associated with this method of analysis. One of such uncertainty is the effect of change of values due to inflation or otherwise.

The study has revealed that Techiman is the least cost town with the cost of GH5580.00. From Table 4.7 with a 10% increase in fuel and incidental expenses in the respective towns, Techiman remains the least cost town at GH5954 .00; an increase of GH374.00 (6.70%). Yeji remains the most expensive Distribution Center at GH13424.00; an increase of GH1061.00 representing 8.58%. At 15% increase of fuel and incidental expenses, Yeji is still the most expensive site at GH1,3999.00 with an increase of GH1,636.00 (13.23%) and Techiman remains the least cost town with GH6199.00; an increase of GH619.00 representing 11.09%. None of the respective town recorded a change in their positions. The analysis shows that the model is certain and can stand the test of time even with an appreciable percentage change of values.

Table 4.7: Total Costs for Distribution in Various District Capital Towns at 10%, and 15% increase of the Fuel and Incidental expenses

J	Distribution Center	Total Cost (10% increase)	Total Cost (15% increase)
1	SU	6268	6562
2	TE	5954	6199
3	NK	6665	6944
4	AT	10516	10969
5	KW	12131	12652
6	YE	13424	13999
7	KN	9145	10136
8	JE	7450	7697
9	WE	6240	6462
10	NS	6921	7183
11	BA	8477	8803
12	DR	8224	9360
13	BK	6809	7779
14	DO	7806	8813
15	KU	9587	10246
16	GO	8766	9373
17	KE	7918	8383
18	BE	6687	7204
19	SA	8396	10022

Least-cost Distribution Center

4.5 FINDINGS

The study was an attempt to develop methodology which could be used in the different institutions to select and optimize the most cost effective Distribution Center among many possible choices. The results generated from the data gathered aim at providing insights into the institution under study and do not necessarily translate to being representative of the entire population. The district capitals were chosen from the twenty seven district capital towns in Brong Ahafo Region of Ghana. The data was collected and analyzed using MATLAB code for Floyd's algorithm. Another MATLAB code was used to solve the model. From Table 4.5, the most expensive Distribution Center is Yeji with total cost of GH12363.00. About 92.72% (GH11463.00) constituted travel cost (Table 4.3) and 7.28% of the total expenses is estimated to be the incidental expenses. The most cost-effective Distribution Center selected is Techiman with GH4860.00 and GH5580.00 as travel and total cost respectively (Tables 4.3 and 4.5). Thus the model suggested that the subsequent Distribution Center could be located at Techiman. However, the model is flexible enough to allow institutions planners to choose a preferred destination.

Chapter 5

CONCLUSION AND RECOMMENDATIONS

5.1 INTRODUCTION

This chapter presents the summary and conclusions drawn from the study and makes some recommendations to help the TOR and other institutions to live within their budget allocated for building a distribution center or warehouses.

5.2 SUMMARY

The researcher sought to develop site selection methodology using Floyd's Shortest-route Algorithm to minimize the travel costs of distribution LPG. The purpose specifically was to determine the least expensive travel cost and this involved mathematical model for the determination of an optimal location of the Distribution Center in the Brong Ahafo Region.

The Tema Oil Refinery was used as case study. In all, the nineteen district capitals were used in the study. Data was collected from the Regional Town and Country Planning Department (Sunyani), The State Transport Cooperation (STC) in Sunyani, Hills Gas Tankers, Floyd's shortest-route algorithm was used as the mathematical tool to solve the travel cost problem and MATLAB code was used. The following were findings of the study:

On the issue of least expensive travel cost, the analysis revealed that Techiman, is the least expensive site with aggregate cost of GH5580.00. The total travel cost needed to distribute LPG from the cost-effective site was found to be GH4860.00. The model revealed that Yeji, is GH6783.00 more than the least cost alternative, Techiman. This represents a 54.87% savings on traveling cost alone. About 87.10% constitutes transportation cost and 12.90% of the total expenses are estimated to be the incidental expenses.

5.3 CONCLUSION

Based on the results from the study it was concluded that the most expensive Distribution Center (site) is Yeji and the least expensive alternative Distribution Center, that is, most cost-effective site is Techiman, in Brong Ahafo Region. The model developed reveals that if TOR adapts to the model they can make a saving of GH6783.00 on every distribution. Hence we can conclude that the shortest-route algorithm we used to develop the model can have a dramatic increase in the saving margin of the institutions in terms of distribution, should they adapt to it.

5.4 RECOMMENDATIONS

From the conclusion, we realized that using network models in decision making to select distribution center or site helps to minimize the distribution cost, thereby increase their savings. It is therefore recommended that TOR and other institutions should adapt this model in their selection of cost-effective Distribution Center or Warehouse site.

It is also recommended that institutions be educated to consult mathematicians to use Floyd's shortest-route algorithm to find an appropriate mathematical model to help them in decision making and planning events more efficiently.

Moreover, immediate funding from agencies for continued refinement of the distribution site selection model programme is recommended. Lastly, it is recommended that the study should be replicated for the Techiman Municipal using the towns for the most potential cost - effective Distribution Center or sites and create window-based user interface for the programme to help naive users of MATLAB.

REFERENCES

Agarwal ,P.K. and Sharir, M., (1998). Efficient algorithm for geometric optimization. ACM Comut. Survey. Vol 30 pp 428-458.

Arsham H,. Distribution-routes Stability Analysis of the Transportation Problem, Optimization, Vol. 43, No. 1, 47-72, 1998.

Aris, Rutherford [1978] (1994). Mathematical Modelling Techniques, New York : Dover. ISBN 0-486-68131-9

Ballou, Ronald H. (1998), "Business Logistics Management - Planning, Organizing, and Controlling the Supply Chain - Fourth Edition", Prentice Hall

Badri, M., A. (1996) Approach to Global Facility Location-Allocation Problem. International Journal of Information and Management Science. Vol.7, No.3, pp.1-20.

Baumol, W. and Wolfe, P. (1958). A Warehouse Location Problem. Operations Research.6(2), 252-263.

Berge, C., and A. Ghouila-Houri. 1965. Programming, Games, and Transportation Networks. New York: Wiley

Berman, O. and Krass, D. " The generalized maximal covering Location problem," Computer & Operations Research, vol. 29, no. 6, pp. 563-581, 2002.

Berman, O. and Larson, R.C. (1985). Optimal 2-facility network districting in the presence of queuing. Transportation Science, 19, pp. 261-277.

Berlin, G., ReVelle, C. and Elzinga, J. (1976). Determining ambulance-hospital locations for on-scene and hospital services. Environment and Planning A, 8, pp. 553-561.

Calvo, A. and Marks, H. (1973). Location of health care facilities: An analytical approach. Socio-Economic Planning Sciences, 7, pp. 407-422.

Carbone, R. (1974). Public facility location under stochastic demand, INFOR, 12.

Carson, Y. and Batta, R. (1990). Locating an ambulance on the Amherst campus of the State University of New York at Buffalo. Interfaces, 20, pp.43-49.

Caruso C., Colomi A., and Aloi. L.,(2003). Dominant, an algorithm for the p-center problem. Eur. J. Operation Research Vol 149 pp 53-64,

Christopher, Martin and Wills, Gordon (1972), "Marketing Logistics and Distribution Planning", George Allen & Unwin Ltd, Uk

Cavalieri, T. M. (1647) Exercitio sexta. In CAVALIELI, "Exercitationes geometricae." Bologna, 1647.

Cavalier, T., and Sherali, H. D., "Network Location Problems with Continuous Link Demands: p-Medians on a Chain and 2- Medians on a Tree", European Journal of, Operational Research, Vol. 23, [1986], pp. 246-255.

Dequan, Z. Guolin, W. "Optimization of Floyd Algorithm for shortest path problem", JOURNAL OF XUCHANG UNIVERSITY, vol. 2, no. 28, pp 10-13, 2009

Christofides, N. and Vioda, P., (1971). "The Optimum Location of Multi-Centers on a graph", Operations Research Quarterly, Vol 22,pp 145-154

Christofides, N. (1975). Graph Thoery : An algarithmic Approach, Academy Press, New York. Christopher, Martin and Wills, Gordon (1972), "Marketing Logistics and Distribution Planning", George Allen & Unwin Ltd, Uk

Chu T, (2002), selecting plant location via a fuzzy topsis approach, International Journal in Advanced Manufacturing Technology, Vol 20,pp 859-864.

Cohen, M. A. and Lee, H. L. (1989), "Resource deployment analysis of global manufacturing and distribution networks". Journal of and Manufacturing Operation

Cooper, L. (1963) "Location-alocation problems" Operation Research, 11: 331-343.

Costa, M. G., M. E. Captivo and J. Climaco., (2008) "Capacitated Single Allocation Hub Location Problem - A Bicriteria Approach", Computers & Operations Research, 35 (11): 3671-3695 .

Chen, Danny Z (1996). "Developing algorithms and software for geometric path planning problems". ACM Computing Surveys 28 (4es): 18.

Cherkassky et al (1996), Shortest paths algorithms: theory and experimental evaluation. Mathematical Programming: Series A and B, 73, pp. 129-174.

Daskin, M. (1983). The maximal expected covering location model: Formulation, properties and heuristic solution. Transportation Science, 17(1), pp. 48-70.

Daskin, M. S. and Owen, S. H. (1998) "Two new location of covering problems: the partial covering p-center problem and the partial set covering." Geographical Analysis, 1: 50-100.

Dijksta, E. W. (1959). A Note on Two Problems in Connection with Graphs, Numeriche Mathematik, 1, 269-271.

Drezner. Z.,(1984). The p-center problem-heuristic and optimal algorithm. Operations. Research, Vol pp 741-748.

Drezner, Z. (1993) "recent result in competitive facility location." Studies in Locational Analysis, 4: 97-103.

Drezner, Z. H.W. Hamacher(Eds), Facility Location: Applications and Theory, Springer, New York, 2004.

Eaton, D.J., Daskin, M.S., Simmons, D., Bulloch, B. and Jansma, G. (1985). Determining emergency medical deployment in Austin, Texas. Interfaces, 15(1), pp. 96-108.

Erkut, E. and S.Neuman, (1992) "A Multiobjective Model for location undesirable facilities", Annals of operations Research, 40:209-227.

Farahani R.Z. and Asgari, N., (2007).Combination of MCAM and covering techniques in a hierarchical model for facility location: A case study, European Journal of Operational Research, Vol 76, pp 1839-1858.

Ferreira, C., J.Climaco and J. Paixo, (1994). "The location-covering problem: a bicriterion interactive approach", investigacin Operativa, 4:119-139

Ferreira, C., B. S. Santos, M. E. Captivo, J. Climaco and C. C Sliva, (1996). "Multiobjective location of unwelcome or central facilities involving environmental aspects - a prototype of a decision support system". JORBEL (Beligium Journal of Operations Researche, Stastistics and Computer Science) , 36:159-72

Fermat, P. D. (1643) Methodus de maximis et minimis. In Tannery, H. (Ed), "Fermat-Oeuvres I.", Paris, 1896, supplement, Paris.

Francis, R. L. ,McGinnis, L. F. and White, J. A. (1983) "Locational analysis," European Journal of Operational Research, 12: 220-252.

Francis, R. L., andCabot, A. V., (1972). "Properties of a Multifacility

Location Problem Involving Euclidean Distances", Naval Research Logistics Quarterly, Vol. 19, pp. 335-353.

Friedrich, F. (1929) "Alfred Weber's theory of the location of industries." Chicago University Press, Chicago, Illinois.

Garfinkel, R. S., Neebe, A. W. and Rao, M. R. (1977). The m-center problem: Minimax facility location. Management Science, 23, pp. 1133-1142.

Geoffrion, A. (1978). A Guide to Computer Assisted Methods for Distribution System Planning. Sloan Management Review. 24(5), 535-544.

Giannikos, I., (1998). " Amultiobjective programming model for locating treatment sites and routng hazardous wastes", European Journal of operation research, 104;333-342

Goldengorin, B., Ghosh, D. and Sierksma, G. (2000), Equivalent instances of the simple plant location problem, SOM Research Report-00A54, University of Groningen, The Netherlands. . Hakimis, S. L. (1964) "Optimum location of switching centres and absolute centre and medians of a graph." Operational Research, 12: 450-459.

Hale S. H., and Moberg, C. R. Location science reserch: A review. Ann. Oper. Res., 123:21-35, 2004.

Hakimi, S. L., (1965).Optimum Distribution of switching centers in a communication Network and some Related Graph Theoretic Problems", Operation Research, Vol13, pp 462-475.

Hart, P. E., N. J. Nilsson, and B. Raphael. 1972. "Correction to 'A Formal Basis for the Heuristic Determination of Minimum Cost Paths"'. SIGART Newsletter 37:28-29.

Hogan, K. and ReVelle, C. (1986). Concepts and applications of backup coverage. Management Science, 32, pp.1434-1444.

Hotelling, H. (1929) "Stability in competition." Economic Journal, 39: 41-57.

Houcbaum, D. S. and Pathria, A., (1998). Location centers in a dynamically changing network and related problems. Locating science, Vol 6, pp 243 - 256.

Huff, D.L. 1966. A programmed solution for approximating an optimum retail location. Land Economics 42, 293-303

Houshyar, A. White B. " A simulation Study: "Comparison of solution procedures to a Facility Location Problem," journal of Computers & industrial Engineering, Vol 32, No 1, pp 77-87, 1997. (http://thechronicle.com.gh/?p=28218

A. Klose, A. Drexl, Facility location models for distribution system design, European Journal of Operational Research 162 (2005) 4-29.

Kuehn, A. and Hamburger, M. (1963). A Heuristic Program for Locating Warehouses. Management Science. 9(4), 643-666.

Mandell, M.B. (1998). A P-median approach to locating basic life support and advanced life support units. Presented at the CORS/INFORMS National Meeting, Montreal, April, 1998.

Marrewijk, C. V. (2002) International Trade & The World Economy. Oxford University Press.

Mirchandani, P.B. (1987). Generalized hierarchical facility location. Transportation Science, 21, pp. 123-125.

M.S. Daskin, Network and Discrete Location: Models, Algorithms, and Applications, Wiley, New York, 1995. Nilsson, N. J. 1980. Principles of

Artificial Intelligence. Palo Alto: Tioga Publishing Company.

Paluzzi, M. (2004). Testing a heuristic P-median location allocation model for siting emergency service facilities. Paper Presented at the Annual Meeting of Association of American Geographers, Philadelphia, PA.

Purvin, & Gertz. (2000). Continued LPG demand growth changes historical trade patterns. Oil & Gas Journal. [Online] Available: www.purvingertz.com

Randhawa S.U and West, (1995). An integrated approach to facility location problem, computers and industrial engineering, Vol 29, pp 261-265.

Rardin, R. L. (2003). Optimization in Operations Research, Pearson Education, Delhi, India.

Revelle, C. and Hogan, K. (1986). The maximum reliability location problem and a-reliable p-center problem. Derivatives of the probabilistic location set covering problem. Annals of Operation Research, 18, pp. 155 - 174.

ReVelle, C. S. Eiselt, H. A. Location analysis: A synthesis and survey, European Journal of Operational Research 165 (2005) 1-19.

ReVelle, C. S. Eiselt, H. A. Daskin, M. S. A bibliography for some fundamental problem categories in discrete location science, European Journal of Operational Research 184 (2008) 817-848.

Saaty, T. (1996). Multi-criteria Decision Making: The Analytic Hierarchy Process. Pittsburgh, RWS Publications.

Schilling, D., Elzinga, D., Cohon, J., Church, R. and ReVelle, C. (1979). The TEAM/FLEET Models for simultaneous facility and equipment siting. Transportation Science, 13, pp. 163-175.

Serra., D and Marianov, V. (1999). The P-median problem in a changing network: The case of Barcelona. Location Science, 4, 1999.

Shang Y. and Ruml W. Improved mds-based localization. In Proceedings of IEEE InfoCom'04, 2004. Pp 1-12

Sherali, H. D., "Capacitated. Balanced, Sequential Location-Allocation Problems on Chains and Trees", Mathematical Programming, Vol. 49, [1991], pp. 381-396.

Sylvester, J.J. (1857). A question in the geometry of situation. Quarterly Journal of Pure and Applied Mathematics, 1, pp. 79.

Shortest Path Problem.http://www.answers.com/topic/shortest-path-problem (August. 24, 2013; 8: 40 AM)

Taha Hamdy A. (2007). Operations Research, An Introduction (8th Ed.), Pearson Prentice Hall

Tansel, B., Francis, R. and Lowe, T. (1989). Location on Networks: A Survey. Management Science. 29, 482-492.

Tien, J.M., El-Tell, K. and Simons, G. (1983). Improved formulations to the hierarchical health facility location-allocation problem. IEEE Transactions in Systems Management and Cybernetics, SMC-13, pp. 1128-1132.

Toregas, C., Swain, R., ReVelle, C. and Bergman, L. (1971). The location of emergency service facility. Operations Research, 19, pp. 1363-1373.

Vahrenkamp, R., and D. C. Mattfeld. 2007. Logistiknetzwerke: Modelle fur Standortwahl und Tourenplanung. Wiesbaden: Gabler. www.fearme.com/misc/alg/node88.ht

Weber, A. (1909) "Ueber den standort der industrien. Erster Teil. Reine Teorie der standorte. Mit einem mathematischen Anhang von G. PICK."Verlag. Tubingen, Germany.

Weizfeld, E. (1937) "Sur le point pour la somme des distances de n point donnes est minimum.” Tohoku Mathematical journal, 43: 355-386.

Wesolowsky, G. (1977). Probabilistic Weights in the One-dimensional Facility Location Problem. Management Science. 24(2), 224-229

White, J. and Case, K. (1974). On covering problems and the central facility location problem. Geographical Analysis, 281, 1974.

Yong D.(2006). Plant Location selection based on Fuzzy Topsis, International Journal of Advanced Manufacturing Technology, Vol 28, pp 839-844.

appendix Appendix A: MATLAB Code for Floyd’s Shortest Path Algorithm and Least Expensive Model Leasttravelcost.m

```
PART 1: This Code determines the least travel cost matrix and node(Town)sequenc
%This a code for
%Floyd's shortest path algorithm
 %The input parmeters are your starting travel cost matrix Do.
 %It serves you with your final least travel cost matrix and town sequence matr

 n=input('Enter the number of towns :  ');
   Towns=cell(1,n);
   for i=1:n;  %This allows entry of the names of the towns
     Towns(1,i)=input('Enter the names of the towns one by one: ');
          end
             Townsinv=Towns';
              D=zeros(n);
                for i=1:n;
                for j=1:n;
```

```
if i~=j;
 disp(['provide travel distance from ' Townsinv(i,1) 'to' Town
    D_o(i,j)=input('travel distance =' );
          end
      end
   end
   h=D_o;
    for i=1:n;          %code for initializing the source matri
           j=1:n;
           s(i,j)=j;
   end
    for k=1:n;   %number of iterations
        for i=1:n;
            for j=1:n;
              if i~=j & k~=j & i~=k;
                   if h(i,j)>h(i,k)+h(k,j);
                       h(i,j)=h(i,k)+h(k,j);
                       s(i,j)=k;
                   else
                       h(i,j)=h(i,j);
                       s(i,j)=s(i,j);
                   end
                end
            end
        end
```

```
end
for i=1:n;
    s(i,i)=0;
end
 D=h;
disp(['The number of iterations is '  ]);
disp(k);
disp('The initial travel distance matrix across towns');
disp(D_o);
disp('The least travel distance matrix');
disp(D);
disp('The town sequence matrix');
disp(s);
```

```
PART 2: CODE FOR COST-EFFECTIVE DISTRIBUTION CENTER
n=input('put the size of matrix here: ');
d=input('the matrix here: ');
v=d;
Towns=cell(1,n);L=zeros(n,1);G=L; G=zeros(n);T=G;T_cost=G;
T=v; Total_costmatrix=L;
for i=1:n;
    Towns(1,i)=input('Enter the names of the towns one by
```

```
end
for i=1:n;
    disp(['Provide info on' Towns(1,i)]);
   L(i,1)= input('Number of days travel = ');
    G(1,i)=input('incidental expenses = ');
    G(:,i)=G(1,i);
    G(i,i)=0;
    T_cost=L(i,1)*(T(:,i)+G(:,i));
    disp(['Cost shcedule for ' Towns(1,i)]);
    j=1:n;
    disp([j' T_cost]);Total_costmatrix(i,1)=sum(T_cost);
    disp(['Total Cost']);
    disp([sum(T_cost)]);
end
[value,dim]=min(Total_costmatrix);
disp(['Least Cost Town ' ]);
disp([Towns(1,dim)]);
disp(['Least Cost' ]);
disp([value ]);
```

Appendix B:

INPUT DATA OF FLOYD'S ALGORITHM CODE

```
Enter the number of towns :  19
Enter the names of the towns one by one: {'SU'}
Enter the names of the towns one by one: {'TE'}
Enter the names of the towns one by one: {'NK'}
Enter the names of the towns one by one: {'AT'}
Enter the names of the towns one by one: {'KW'}
Enter the names of the towns one by one: {'YE'}
Enter the names of the towns one by one: {'KN'}
Enter the names of the towns one by one: {'JE'}
Enter the names of the towns one by one: {'WE'}
Enter the names of the towns one by one: {'NS'}
Enter the names of the towns one by one: {'BA'}
Enter the names of the towns one by one: {'DR'}
Enter the names of the towns one by one: {'BK'}
 Enter the names of the towns one by one: {'DO'}
 Enter the names of the towns one by one: {'KU'}
 Enter the names of the towns one by one: {'GO'}
 Enter the names of the towns one by one: {'KE'}
 Enter the names of the towns one by one: {'BE'}
 Enter the names of the towns one by one: {'SA'}
 [1x25 char]    'SU'    'to'    'TE'

 travel cost =175
 [1x25 char]    'SU'    'to'    'NK'
```

```
travel cost =inf
[1x25 char]    'SU'    'to'    'AT'

travel cost =inf
[1x25 char]    'SU'    'to'    'KW'

travel cost =inf
[1x25 char]    'SU'    'to'    'YE'

travel cost =inf
[1x25 char]    'SU'    'to'    'KN'

travel cost =inf
[1x25 char]    'SU'    'to'    'JE'

travel cost =inf
[1x25 char]    'SU'    'to'    'WE'

travel cost =154
[1x25 char]    'SU'    'to'    'NS'

travel cost =inf
[1x25 char]    'SU'    'to'    'BA'
```

```
travel cost =inf

[1x25 char]    'SU'    'to'    'DR'


travel cost =inf

[1x25 char]    'SU'    'to'    'BK'


travel cost =86

[1x25 char]    'SU'    'to'    'DO'


travel cost =inf

[1x25 char]    'SU'    'to'    'KU'


travel cost =inf

[1x25 char]    'SU'    'to'    'GO'


travel cost =inf

[1x25 char]    'SU'    'to'    'KE'


travel cost =135

[1x25 char]    'SU'    'to'    'BE'
```


Appendix C:

THE OUTPUT DATA OF FLOYD'S ALGORITHM CODE FOR SENSITIVITY ANALYSIS - 10% INCREASE OF INPUT

The initial travel cost matrix across towns

0	175	Inf	Inf	Inf	Inf	Inf	Inf	154	Inf	Inf	Inf	86	Inf	Inf	Inf	135	121	Inf
175	0	74	Inf	Inf	Inf	Inf	111	80	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	156	Inf
Inf	74	0	255	Inf	Inf	Inf	103	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf
Inf	Inf	255	0	86	176	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf
Inf	Inf	Inf	86	0	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf
Inf	Inf	Inf	176	Inf	0	357	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf
Inf	Inf	Inf	Inf	Inf	357	0	45	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf
Inf	111	103	Inf	Inf	Inf	45	0	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf
154	80	Inf	Inf	Inf	Inf	Inf	Inf	0	77	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf
Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	77	0	88	Inf	166	Inf	Inf	Inf	Inf	Inf	128
Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	88	0	Inf	Inf	Inf	Inf	Inf	Inf	Inf	168
Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	0	78	126	Inf	Inf	Inf	Inf	Inf
86	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	166	Inf	78	0	58	Inf	Inf	Inf	Inf	Inf
Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	126	58	0	Inf	198	Inf	Inf	Inf
Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	0	39	Inf	Inf	Inf
Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	198	39	0	72	Inf	Inf
135	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	72	0	124	Inf
121	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	124	0	Inf
Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	128	168	128	Inf	Inf	Inf	Inf	Inf	Inf	0

The least travel cost matrix

0	175	249	504	590	680	331	286	154	231	319	164	86	144	246	207	135	121	359
175	0	74	329	415	505	156	111	80	157	245	339	261	319	391	352	280	156	285
249	74	0	255	341	431	148	103	154	231	319	413	335	393	465	426	354	230	359
504	329	255	0	86	176	403	358	409	486	574	668	590	648	720	681	609	485	614
590	415	341	86	0	262	489	444	495	572	660	754	676	734	806	767	695	571	700
680	505	431	176	262	0	357	402	585	662	750	844	766	824	896	857	785	661	790
331	156	148	403	489	357	0	45	236	313	401	495	417	475	547	508	436	312	441
286	111	103	358	444	402	45	0	191	268	356	450	372	430	502	463	391	267	396
154	80	154	409	495	585	236	191	0	77	165	318	240	298	400	361	289	236	205
231	157	231	486	572	662	313	268	77	0	88	244	166	224	461	422	366	313	128
319	245	319	574	660	750	401	356	165	88	0	296	254	312	549	510	454	401	168
164	339	413	668	754	844	495	450	318	244	332	0	78	126	363	324	299	285	372
86	261	335	590	676	766	417	372	240	166	254	78	0	58	295	256	221	207	294
144	319	393	648	734	824	475	430	298	224	312	126	58	0	237	198	270	265	352
246	421	495	750	836	926	577	532	400	461	549	363	295	237	0	39	111	235	589
207	382	456	711	797	887	538	493	361	422	510	324	256	198	39	0	72	196	550

The town sequence matrix

0	2	2	3	4	4	8	2	9	9	10	13	13	13	17	17	17	18	10
1	0	3	3	4	4	8	8	9	9	10	13	1	13	18	18	18	18	10
2	2	0	4	4	4	8	8	2	9	10	13	2	13	18	18	18	2	10
3	3	3	0	5	6	8	3	3	9	10	13	3	13	18	18	18	3	10
4	4	4	4	0	4	8	4	4	9	10	13	4	13	18	18	18	4	10
4	4	4	4	4	0	7	7	4	9	10	13	4	13	18	18	18	4	10
8	8	8	8	8	6	0	8	8	9	10	13	8	13	18	18	18	8	10
2	2	3	3	4	7	7	0	2	9	10	13	2	13	18	18	18	2	10
1	2	2	3	4	4	8	2	0	10	10	13	1	13	17	17	1	2	10
9	9	9	9	9	9	9	9	9	0	11	13	13	13	16	14	9	9	19
10	10	10	10	10	10	10	10	10	10	0	19	10	13	16	14	10	10	19
13	13	13	13	13	13	13	13	13	13	13	0	13	14	16	14	13	13	13
1	1	2	3	4	4	8	2	1	10	10	12	0	14	16	14	1	1	10
13	13	13	13	13	13	13	13	13	13	13	12	13	0	16	16	16	13	13
17	17	17	17	17	17	17	17	17	16	16	16	16	16	0	16	16	17	16
17	17	17	17	17	17	17	17	17	14	14	14	14	14	15	0	17	17	14
1	1	2	3	4	4	8	2	1	9	10	13	1	16	16	16	0	18	10
1	1	2	3	4	4	8	2	1	9	10	13	1	13	17	17	17	0	10
13	10	10	10	10	10	10	10	10	10	11	12	12	12	16	14	13	13	0

The initial travel distance matrix across towns

0	193	Inf	Inf	Inf	Inf	Inf	Inf	169	Inf	Inf	Inf	95	Inf	Inf	Inf	149	133	Inf
193	0	81	Inf	Inf	Inf	Inf	122	88	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	172	Inf
Inf	81	0	281	Inf	Inf	Inf	113	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf
Inf	Inf	281	0	95	194	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf
Inf	Inf	Inf	95	0	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf
Inf	Inf	Inf	194	Inf	0	393	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf
Inf	Inf	Inf	Inf	Inf	393	0	113	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf
Inf	122	113	Inf	Inf	Inf	113	0	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf
169	88	Inf	Inf	Inf	Inf	Inf	Inf	0	85	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf
Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	85	0	97	Inf	183	Inf	Inf	Inf	Inf	Inf	141
Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	97	0	Inf	Inf	Inf	Inf	Inf	Inf	Inf	185
Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	0	86	139	Inf	Inf	Inf	Inf	141
95	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	183	Inf	86	0	64	Inf	Inf	Inf	Inf	Inf
Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	139	64	0	Inf	218	Inf	Inf	Inf
Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	0	43	Inf	Inf	Inf
Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	218	43	0	79	Inf	Inf
149	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	79	0	136	Inf
133	172	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	136	0	Inf
Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	141	185	141	Inf	Inf	Inf	Inf	Inf	Inf	0

The least travel distance matrix

0	193	274	555	650	749	428	315	169	254	351	181	95	159	271	228	149	133	322
193	0	81	362	457	556	235	122	88	173	270	374	288	352	430	387	308	172	314
274	81	0	281	376	475	226	113	169	254	351	455	369	433	511	468	389	253	395
555	362	281	0	95	194	507	394	450	535	632	736	650	714	792	749	670	534	676
650	457	376	95	0	289	602	489	545	630	727	831	745	809	887	844	765	629	771
749	556	475	194	289	0	393	506	644	729	826	930	844	908	986	943	864	728	870
428	235	226	507	602	393	0	113	323	408	505	609	523	587	665	622	543	407	549
315	122	113	394	489	506	113	0	210	295	392	496	410	474	552	509	430	294	436
169	88	169	450	545	644	323	210	0	85	182	350	264	328	440	397	318	260	226
254	173	254	535	630	729	408	295	85	0	97	269	183	247	508	465	403	345	141
351	270	351	632	727	826	505	392	182	97	0	326	280	344	605	562	500	442	185
181	374	455	736	831	930	609	496	350	269	326	0	86	139	400	357	330	314	141
95	288	369	650	745	844	523	410	264	183	280	86	0	64	325	282	244	228	227
159	352	433	714	809	908	587	474	328	247	344	139	64	0	261	218	297	292	280
271	430	511	792	887	986	665	552	440	508	605	400	325	261	0	43	122	258	541
228	387	468	749	844	943	622	509	397	465	562	357	282	218	43	0	79	215	498

The town sequence matrix

0	2	2	3	4	4	8	2	9	9	10	13	13	13	17	17	17	18	13
1	0	3	3	4	4	8	8	9	9	10	13	1	13	18	18	18	18	10
2	2	0	4	4	4	8	8	2	9	10	13	2	13	18	18	18	2	10
3	3	3	0	5	6	8	3	3	9	10	13	3	13	18	18	18	3	10
4	4	4	4	0	4	8	4	4	9	10	13	4	13	18	18	18	4	10
4	4	4	4	4	0	7	7	4	9	10	13	4	13	18	18	18	4	10
8	8	8	8	8	6	0	8	8	9	10	13	8	13	18	18	18	8	10
2	2	3	3	4	7	7	0	2	9	10	13	2	13	18	18	18	2	10
1	2	2	3	4	4	8	2	0	10	10	13	1	13	17	17	1	2	10
9	9	9	9	9	9	9	9	9	0	11	13	13	13	16	14	9	9	19
10	10	10	10	10	10	10	10	10	10	0	19	10	13	16	14	10	10	19
13	13	13	13	13	13	13	13	13	13	19	0	13	14	16	14	13	13	19
1	1	2	3	4	4	8	2	1	10	10	12	0	14	16	14	1	1	12
13	13	13	13	13	13	13	13	13	13	13	12	13	0	16	16	16	13	12
17	18	18	18	18	18	18	18	17	16	16	16	16	16	0	16	16	17	16
17	18	18	18	18	18	18	18	17	14	14	14	14	14	15	0	17	17	14
1	18	18	18	18	18	18	18	1	9	10	13	1	16	16	16	0	18	13
1	2	2	3	4	4	8	2	2	9	10	13	1	13	17	17	17	0	13
13	10	10	10	10	10	10	10	10	10	11	12	12	12	16	14	13	13	0

THE OUTPUT DATA OF FLOYD'SALGORITHM CODE FOR SENSITIVITY ANALYSIS – 15% INCREASE OF INPUT

The initial travel distance matrix across towns

0	201	Inf	Inf	Inf	Inf	Inf	Inf	177	Inf	Inf	Inf	99	Inf	Inf	Inf	155	139	Inf
201	0	85	Inf	Inf	Inf	Inf	128	92	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	179	Inf
Inf	85	0	293	Inf	Inf	Inf	118	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf
Inf	Inf	293	0	99	202	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf
Inf	Inf	Inf	99	0	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf
Inf	Inf	Inf	202	Inf	0	411	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf
Inf	Inf	Inf	Inf	Inf	411	0	52	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf
Inf	128	118	Inf	Inf	Inf	52	0	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf
177	92	Inf	Inf	Inf	Inf	Inf	Inf	0	89	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf
Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	89	0	101	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf
Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	101	0	Inf	Inf	Inf	Inf	Inf	Inf	Inf	193
Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	0	90	145	Inf	Inf	Inf	Inf	147
99	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	191	Inf	90	0	74	Inf	Inf	Inf	Inf	Inf
Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	145	74	0	Inf	228	Inf	Inf	Inf
Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	0	45	Inf	Inf	Inf
Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	228	45	0	83	Inf	Inf
155	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	83	0	156	Inf
139	179	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	156	0	Inf
Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	Inf	147	193	Inf	Inf	Inf	Inf	Inf	Inf	Inf	0

The least travel distance matrix

0 201 286 579 678 781 381 329 177 266 367 189 99 173 283 238 155 139 336

201 0 85 378 477 580 180 128 92 181 282 390 300 374 463 418 335
179 475

286 85 0 293 392 495 170 118 177 266 367 75 385 459 548 503 420
264 560

579 378 293 0 99 202 463 411 470 559 660 768 678 752 841 796 713
557 853

678 477 392 99 0 301 562 510 569 658 759 867 777 851 940 895 812
656 952

781 580 495 202 301 0 411 463 672 761 862 970 880 954 1043 998 915
759 1055

381 180 170 463 562 411 0 52 272 361 462 570 480 554 643 598 515
359 655

329 128 118 411 510 463 52 0 220 309 410 518 428 502 591 546 463
307 603

177 92 177 470 569 672 272 220 0 89 190 366 276 350 460 415 332
271 383

266 181 266 559 658 761 361 309 89 0 101 455 365 439 549 504 421
360 294

367 282 367 660 759 862 462 410 190 101 0 556 466 540 650 605 522
461 193

189 390 475 768 867 970 570 518 366 281 340 0 90 145 418 373 344
328 147

99 300 385 678 777 880 480 428 276 191 292 90 0 74 347 302 254
238 237

173 374 459 752 851 954 554 502 350 265 366 145 74 0 273 228 311
312 292

283 463 548 841 940 1043 643 591 460 538 639 418 347 273 0 45 128
284 565

238 418 503 796 895 998 598 546 415 493 594 373 302 228 45 0 83
239 520

The town sequence matrix

0	2	2	3	4	4	8	2	9	9	10	13	13	13	17	17	17	18	13
1	0	3	3	4	4	8	8	9	9	10	13	1	13	18	18	18	18	11
2	2	0	4	4	4	8	8	2	9	10	13	2	13	18	18	18	2	11
3	3	3	0	5	6	8	3	3	9	10	13	3	13	18	18	18	3	11
4	4	4	4	0	4	8	4	4	9	10	13	4	13	18	18	18	4	11
4	4	4	4	4	0	7	7	4	9	10	13	4	13	18	18	18	4	11
8	8	8	8	8	6	0	8	8	9	10	13	8	13	18	18	18	8	11
2	2	3	3	4	7	7	0	2	9	10	13	2	13	18	18	18	2	11
1	2	2	3	4	4	8	2	0	10	10	13	1	13	17	17	1	2	11
9	9	9	9	9	9	9	9	9	0	11	13	9	13	17	17	9	9	11
10	10	10	10	10	10	10	10	10	10	0	13	10	13	17	17	10	10	19
13	13	13	13	13	13	13	13	13	13	19	0	13	14	16	14	13	13	19
1	1	2	3	4	4	8	2	1	10	10	12	0	14	16	14	1	1	12
13	13	13	13	13	13	13	13	13	13	13	12	13	0	16	16	16	13	12
17	18	18	18	18	18	18	18	17	16	16	16	16	16	0	16	16	17	16
17	18	18	18	18	18	18	18	17	14	14	14	14	14	15	0	17	17	14
1	18	18	18	18	18	18	18	1	9	10	13	1	16	16	16	0	18	13
1	2	2	3	4	4	8	2	2	9	10	13	1	13	17	17	17	0	13
10	10	10	10	10	10	10	10	10	10	11	13	10	13	17	17	10	10	0