# KWAME NKRUMAH UNIVERSITYOF SCIENCE AND TECHNOLOGY, KUMASI 

## COLLEGE OF SCIENCE

DEPARTMENT OF MATHEMATICS


MEASURING THE EFFICIENCY OF BASIC STUDENTS PERFORMANCE USING DATA ENVELOPMENT ANALYSIS (DEA)

A DISSERTATION PRESENTED TO THE DEPARTMENT OF MATHEMATICS IN PARTIAL FULFILLMENT FOR THE AWARD OF MPHIL (HONS) APPLIED MATHEMATICS DEGREE BY

## DECLARATION

I hereby declare that I have wholly undertaken this dissertation herein submitted.


I declare that I have supervised this student and that he has my permission to submit this dissertation for assessment.

Mr. CHARLES SEBIL (Supervisor)
(Sign)
(Date)

Dr. R. K Avuglah
(Head of Department) (Sign) (Date)

## DEDICATION

I dedicate this work to my lovely mother, Mary Owusu and my siblings Francis and Justice. Thank you for always being there for me. I am grateful!


## ACKNOWLEDGEMENT

My heartfelt thanks go to the Almighty God for the grace He immeasurably lavished on me throughout this thesis work. It has indeed been the doing of the Lord

Also, I want to acknowledge Mr. Charles Sebil for taking the pain to guide me through this work. You really challenged me to cover miles I never dreamt of covering by myself. God bless you for allowing yourself to be used as a vessel to reshape my thinking.

Special thanks go to the Mathematics Department of KNUST for giving me such an exposure. Through this project, I have been exposed to some of the diverse ways that what we have been learning in class is applied in real life situations.

I am also grateful to the Adansi North Education Directorate for their support in diverse ways not forgetting the Head teachers and staff at Abu Bonsra D/A JHS and Adiembra D/A JHS.

Not forgetting Mr. and Mrs Acquah-Djan, Mr. and Mrs Asiedu, Mr. J.N.Saka-Hesse, Pearl Asirifi Adjei, Eugene and Edith Queenstar Fuachie.


#### Abstract

The increasing interest in the measurement of the performance and efficiency of Basic students has led to the development of performance indicators, each of which attempts to measure the output (input) of a group of nearly homogeneous products (factors of production). The Data Envelopment Analysis (DEA) methodology is used to evaluate the performance indicators in other to obtain an overall performance measure through the comparison of a group of decision makers. This Project conducts an application of the DEA methodology in the assessment of the efficiency of the Basic students in the Adansi North District of the Ashanti Region of Ghana. The major indicators included the student attendance, teachers attendance and student continuous assessment as inputs and the students B.E.C.E performance as output. The results revealed the students who were more efficient in terms of student attendance, student continuous assessment and average input. Finally, I discuss about the existence of differences in the strength and weaknesses between the sampled students.




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## LIST OF ACRONYMS AND ABBREVIATIONS

NOTATION
DEA
DMU
CCR
DCCR
BECE
GES
COTVET
ICT
UNICEF
SRMPR
EMIS PGER

PNER
NAB
NCTE
GoG
USAID
UK-AID
MDGs
NER
GER
GPI
FGER
MGER
SDA
SBA
WAEC

## MEANING

Data Envelopment Analysis Decision Making Unit Charnes, Cooper and Rhodes
Dual Charnes, Cooper and Rhodes Basic Education Certificate Examination

Ghana Education Service
Council for Technical Vocational Education and Training Information and Communication Technology United Nations International Child Education Fund Statistics, Research, Information Management and Public Relations Education Management and Information Systems Primary Gross Enrolment Rate Primary Net Enrolment Rate National Accreditation Board National Council for Tertiary Education Government of Ghana
United States Agency for International Development United Kingdom Agency for International Development Millennium Development Goals Net Enrolment Rate Gross Enrolment Rate Gender Parity Index Female Gender Enrolment Ratio Male Gender Enrolment Ratio Seventh Day Adventist School Based Assessment West African Examination Council

## Appendix A

## Data Collected:

Table 4.3.1 shows the data of students at Abu Bonsra D/A JHS in the Fomena Circuit.

|  | INPUTS <br> DMUs |  |  | $\mathbf{U}_{\mathbf{1}}$ |
| :--- | :---: | :---: | :---: | :---: |
| $\mathbf{U}_{\mathbf{2}}$ | $\mathbf{U}_{\mathbf{3}}$ | OUTPUT (V) |  |  |
| AB01 | 61.00 | 61.00 | 79.00 | $\mathbf{9 7 . 2 0}$ |
| AB02 | 64.00 | 61.00 | 82.00 | $\mathbf{9 6 . 3 0}$ |
| AB03 | 58.00 | 61.00 | 71.00 | $\mathbf{9 3 . 5 0}$ |
| AB04 | 62.00 | 61.00 | 66.00 | $\mathbf{9 1 . 6 0}$ |
| AB05 | 61.00 | 61.00 | 70.00 | $\mathbf{9 0 . 7 0}$ |
| AB06 | 61.00 | 61.00 | 50.00 | $\mathbf{8 1 . 4 0}$ |
| AB07 | 54.00 | 61.00 | 49.00 | $\mathbf{7 8 . 7 0}$ |
| AB08 | 58.00 | 61.00 | 43.00 | $\mathbf{7 5 . 9 0}$ |
| AB09 | 59.00 | 61.00 | 54.00 | $\mathbf{7 2 . 2 0}$ |
| AB10 | 60.00 | 61.00 | 45.00 | $\mathbf{6 8 . 5 0}$ |

## Source:: Author's Construct, April 2016.

The linear programming formulated out of the data for the above academic years are as follows:

$$
\begin{gathered}
\text { DMUs for AB01: Max }(\mathrm{Z})=97.2 \mathrm{v} \\
\text { Subject to } \\
61 \mathrm{u}_{1}+61 \mathrm{u}_{2}+79 \mathrm{u}_{3} \leq 97.2 \mathrm{v}[\mathrm{AB} 01] \\
64 \mathrm{u}_{1}+61 \mathrm{u}_{2}+82 \mathrm{u}_{3} \leq 96.3 \mathrm{v}[\mathrm{AB} 02] \\
58 \mathrm{u}_{1}+61 \mathrm{u}_{2}+71 \mathrm{u}_{3} \leq 93.5 \mathrm{v}[\mathrm{AB} 03] \\
62 \mathrm{u}_{1}+61 \mathrm{u}_{2}+66 \mathrm{u}_{3} \leq 91.6 \mathrm{v} \quad[\mathrm{AB} 04] \\
61 \mathrm{u}_{1}+61 \mathrm{u}_{2}+70 \mathrm{u}_{3} \leq 90.7 \mathrm{v} \quad[\mathrm{AB} 05] \\
61 \mathrm{u}_{1}+61 \mathrm{u}_{2}+50 \mathrm{u}_{3} \leq 81.4 \mathrm{v}[\mathrm{AB} 06] \\
54 \mathrm{u}_{1}+61 \mathrm{u}_{2}+49 \mathrm{u}_{3} \leq 78.7 \mathrm{v} \quad[\mathrm{AB} 07] \\
58 \mathrm{u}_{1}+61 \mathrm{u}_{2}+43 \mathrm{u}_{3} \leq 75.9 \mathrm{v}[\mathrm{AB} 08] \\
59 \mathrm{u}_{1}+61 \mathrm{u}_{2}+54 \mathrm{u}_{3} \leq 72.2 \mathrm{v}[\mathrm{AB} 09] \\
60 \mathrm{u}_{1}+61 \mathrm{u}_{2}+45 \mathrm{u}_{3} \leq 68.5 \mathrm{v}[\mathrm{AB} 10] \\
61 \mathrm{u}_{1}+61 \mathrm{u}_{2}+79 \mathrm{u}_{3}=1
\end{gathered}
$$

$$
\mathrm{u}_{1}, \mathrm{u}_{2}, \mathrm{u}_{3}, \mathrm{v} \geq 0
$$

DMUs for AB 02 : $\operatorname{Max}(\mathrm{Z})=96.3 \mathrm{v}$

## Subject to

$61 \mathrm{u}_{1}+61 \mathrm{u}_{2}+79 \mathrm{u}_{3} \leq 97.2 \mathrm{v} \quad[\mathrm{AB} 01]$
$64 \mathrm{u}_{1}+61 \mathrm{u}_{2}+82 \mathrm{u}_{3} \leq 96.3 \mathrm{v} \quad[\mathrm{AB} 02]$
$58 \mathrm{u}_{1}+61 \mathrm{u}_{2}+71 \mathrm{u}_{3} \leq 93.5 \mathrm{v} \quad[\mathrm{AB} 03]$
$62 \mathrm{u}_{1}+61 \mathrm{u}_{2}+66 \mathrm{u}_{3} \leq 91.6 \mathrm{v} \quad[\mathrm{AB} 04]$
$61 \mathrm{u}_{1}+61 \mathrm{u}_{2}+70 \mathrm{u}_{3} \leq 90.7 \mathrm{v} \quad[\mathrm{AB} 05]$
$61 \mathrm{u}_{1}+61 \mathrm{u}_{2}+50 \mathrm{u}_{3} \leq 81.4 \mathrm{v} \quad[\mathrm{AB} 06]$
$54 \mathrm{u}_{1}+61 \mathrm{u}_{2}+49 \mathrm{u}_{3} \leq 78.7 \mathrm{v} \quad[\mathrm{AB} 07]$
$58 \mathrm{u}_{1}+61 \mathrm{u}_{2}+43 \mathrm{u}_{3} \leq 75.9 \mathrm{v} \quad[\mathrm{AB} 08]$
$59 \mathrm{u}_{1}+61 \mathrm{u}_{2}+54 \mathrm{u}_{3} \leq 72.2 \mathrm{v} \quad[\mathrm{AB} 09]$
$60 \mathrm{u}_{1}+61 \mathrm{u}_{2}+45 \mathrm{u}_{3} \leq 68.5 \mathrm{v}[\mathrm{AB} 10]$

$$
64 u_{1}+61 u_{2}+82 u_{3}=1 u_{1},
$$

$$
\mathrm{u}_{2}, \mathrm{u}_{3}, \mathrm{v} \geq 0
$$

DMUs for AB 03 : $\operatorname{Max}(\mathrm{Z})=93.5 \mathrm{v}$

Subject to
$61 u_{1}+61 u_{2}+79 u_{3} \leq 97.2 v \quad[A B 01]$
$64 \mathrm{u}_{1}+61 \mathrm{u}_{2}+82 \mathrm{u}_{3} \leq 96.3 \mathrm{v} \quad[\mathrm{AB} 02]$
$58 \mathrm{u}_{1}+61 \mathrm{u}_{2}+71 \mathrm{u}_{3} \leq 93.5 \mathrm{v} \quad[\mathrm{AB} 03]$
$62 u_{1}+61 u_{2}+66 u_{3} \leq 91.6 v \quad[A B 04]$
$61 u_{1}+61 u_{2}+70 u_{3} \leq 90.7 \mathrm{v}[A B 05]$
$61 \mathrm{u}_{1}+61 \mathrm{u}_{2}+50 \mathrm{u}_{3} \leq 81.4 \mathrm{v} \quad[\mathrm{AB} 06]$
$54 u_{1}+61 u_{2}+49 u_{3} \leq 78.7 \mathrm{v}[A B 07] 58 u_{1}+61 u_{2}+43 u_{3} \leq 75.9 \mathrm{v}[A B 08]$
$59 \mathrm{u}_{1}+61 \mathrm{u}_{2}+54 \mathrm{u}_{3} \leq 72.2 \mathrm{v} \quad[\mathrm{AB} 09]$

$$
\begin{gathered}
60 u_{1}+61 u_{2}+45 u_{3} \leq 68.5 v \quad[A B 10] \\
58 u_{1}+61 u_{2}+71 u_{3}=1 u_{1} \\
u_{2}, u_{3}, v \geq 0
\end{gathered}
$$

DMUs for AB 04 : $\operatorname{Max}(Z)=91.6 \mathrm{v}$

Subject to

$$
\begin{aligned}
61 \mathrm{u}_{1}+61 \mathrm{u}_{2}+79 \mathrm{u}_{3} & \leq 97.2 \mathrm{v}[\mathrm{AB} 01] \\
64 \mathrm{u}_{1}+61 \mathrm{u}_{2}+82 \mathrm{u}_{3} & \leq 96.3 \mathrm{v}[\mathrm{AB} 02] \\
58 \mathrm{u}_{1}+61 \mathrm{u}_{2}+71 \mathrm{u}_{3} & \leq 93.5 \mathrm{v}[\mathrm{AB} 03] \\
62 \mathrm{u}_{1}+61 \mathrm{u}_{2}+66 \mathrm{u}_{3} & \leq 91.6 \mathrm{v}[\mathrm{AB} 04] \\
61 \mathrm{u}_{1}+61 \mathrm{u}_{2}+70 \mathrm{u}_{3} & \leq 90.7 \mathrm{v}[\mathrm{AB} 05] \\
61 \mathrm{u}_{1}+61 \mathrm{u}_{2}+50 \mathrm{u}_{3} & \leq 81.4 \mathrm{v}[\mathrm{AB} 06] \\
54 \mathrm{u}_{1}+61 \mathrm{u}_{2}+49 \mathrm{u}_{3} & \leq 78.7 \mathrm{v}[\mathrm{AB} 07] \\
58 \mathrm{u}_{1}+61 \mathrm{u}_{2}+43 \mathrm{u}_{3} & \leq 75.9 \mathrm{v}[\mathrm{AB} 08] \\
59 \mathrm{u}_{1}+61 \mathrm{u}_{2}+54 \mathrm{u}_{3} & \leq 72.2 \mathrm{v}[\mathrm{AB} 09] \\
60 \mathrm{u}_{1}+61 \mathrm{u}_{2}+45 \mathrm{u}_{3} & \leq 68.5 \mathrm{v}[\mathrm{AB} 10] \\
62 \mathrm{u}_{1}+61 \mathrm{u}_{2} & +66 \mathrm{u}_{3}=1 \mathrm{u}_{1},
\end{aligned}
$$

DMUs for $A B 05$ : $\operatorname{Max}(Z)=90.7 v$

## Subject to

$$
\begin{aligned}
61 \mathrm{u}_{1}+61 \mathrm{u}_{2}+79 \mathrm{u}_{3} & \leq 97.2 \mathrm{v}[\mathrm{AB} 01] \\
64 \mathrm{u}_{1}+61 \mathrm{u}_{2}+82 \mathrm{u}_{3} & \leq 96.3 \mathrm{v}[\mathrm{AB} 02] \\
58 \mathrm{u}_{1}+61 \mathrm{u}_{2}+71 \mathrm{u}_{3} & \leq 93.5 \mathrm{v}[\mathrm{AB} 03] 62 \mathrm{u}_{1}+61 \mathrm{u}_{2}+66 \mathrm{u}_{3} \leq 91.6 \mathrm{v}[\mathrm{AB} 04] \\
61 \mathrm{u}_{1}+61 \mathrm{u}_{2}+70 \mathrm{u}_{3} & \leq 90.7 \mathrm{v}[\mathrm{AB} 05] \\
61 \mathrm{u}_{1}+61 \mathrm{u}_{2}+50 \mathrm{u}_{3} & \leq 81.4 \mathrm{v}[\mathrm{AB} 06] \\
& x i i
\end{aligned}
$$



DMUs for $A B 06$ : $\operatorname{Max}(Z)=81.4 v$

## Subject to

$$
\begin{aligned}
& 61 \mathrm{u}_{1}+61 \mathrm{u}_{2}+79 \mathrm{u}_{3} \leq 97.2 \mathrm{v}[\mathrm{AB} 01] \\
& 64 \mathrm{u}_{1}+61 \mathrm{u}_{2}+82 \mathrm{u}_{3} \leq 96.3 \mathrm{v}[\mathrm{AB} 02] \\
& 58 \mathrm{u}_{1}+61 \mathrm{u}_{2}+71 \mathrm{u}_{3} \leq 93.5 \mathrm{v}[\mathrm{AB} 03] \\
& 62 \mathrm{u}_{1}+61 \mathrm{u}_{2}+66 \mathrm{u}_{3} \leq 91.6 \mathrm{v}[\mathrm{AB} 04] \\
& 61 \mathrm{u}_{1}+61 \mathrm{u}_{2}+70 \mathrm{u}_{3} \leq 90.7 \mathrm{v}[\mathrm{AB} 05] \\
& 61 \mathrm{u}_{1}+61 \mathrm{u}_{2}+50 \mathrm{u}_{3} \leq 81.4 \mathrm{v}[\mathrm{AB} 06] \\
& 54 \mathrm{u}_{1}+61 \mathrm{u}_{2}+49 \mathrm{u}_{3} \leq 78.7 \mathrm{v} \quad[\mathrm{AB} 07] \\
& 58 \mathrm{u}_{1}+61 \mathrm{u}_{2}+43 \mathrm{u}_{3} \leq 75.9 \mathrm{v}[\mathrm{AB} 08] \\
& 59 \mathrm{u}_{1}+61 \mathrm{u}_{2}+54 \mathrm{u}_{3} \leq 72.2 \mathrm{v}[\mathrm{AB} 09] \\
& 60 \mathrm{u}_{1}+61 \mathrm{u}_{2}+45 \mathrm{u}_{3} \leq 68.5 \mathrm{v}[\mathrm{AB} 10] \\
& 61 \mathrm{u}_{1}+61 \mathrm{u}_{2}
\end{aligned}
$$

DMUs for AB 07 : $\operatorname{Max}(\mathrm{Z})=78.7 \mathrm{v}$
Subject to

$$
\begin{aligned}
& 61 u_{1}+61 u_{2}+79 u_{3} \leq 97.2 v \quad[\mathrm{AB} 01] \\
& 64 u_{1}+61 u_{2}+82 u_{3} \leq 96.3 v \quad[\mathrm{AB} 02]
\end{aligned}
$$

$$
\begin{aligned}
& 58 u_{1}+61 u_{2}+71 u_{3} \leq 93.5 v \quad[A B 03] \\
& 62 \mathrm{u}_{1}+61 \mathrm{u}_{2}+66 \mathrm{u}_{3} \leq 91.6 \mathrm{v} \quad[\mathrm{AB} 04] \\
& 61 u_{1}+61 u_{2}+70 u_{3} \leq 90.7 \mathrm{v} \quad[\mathrm{AB} 05] \\
& 61 u_{1}+61 u_{2}+50 u_{3} \leq 81.4 v \quad[A B 06] \\
& 54 \mathrm{u}_{1}+61 \mathrm{u}_{2}+49 \mathrm{u}_{3} \leq 78.7 \mathrm{v} \quad[\mathrm{AB} 07] \\
& 58 \mathrm{u}_{1}+61 \mathrm{u}_{2}+43 \mathrm{u}_{3} \leq 75.9 \mathrm{v} \quad[\mathrm{AB} 08] \\
& 59 \mathrm{u}_{1}+61 \mathrm{u}_{2}+54 \mathrm{u}_{3} \leq 72.2 \mathrm{v} \quad[\mathrm{AB} 09] \\
& 60 u_{1}+61 u_{2}+45 u_{3} \leq 68.5 v \quad[A B 10] \\
& 54 u_{1}+61 u_{2}+49 u_{3}=1 u_{1}, \\
& \mathrm{u}_{2}, \mathrm{u}_{3}, \mathrm{v} \geq 0 \\
& \text { DMUs for AB08: } \operatorname{Max}(Z)=75.9 v \\
& \text { Subject to } \\
& 61 u_{1}+61 u_{2}+79 u_{3} \leq 97.2 v \quad[A B 01] \\
& 64 u_{1}+61 u_{2}+82 u_{3} \leq 96.3 v \quad[A B 02] \\
& 58 u_{1}+61 u_{2}+71 u_{3} \leq 93.5 v \quad[A B 03] \\
& 62 u_{1}+61 u_{2}+66 u_{3} \leq 91.6 v \quad[A B 04] \\
& 61 u_{1}+61 u_{2}+70 u_{3} \leq 90.7 v \quad[A B 05] \\
& 61 u_{1}+61 u_{2}+50 u_{3} \leq 81.4 v \quad[A B 06] \\
& 54 u_{1}+61 u_{2}+49 u_{3} \leq 78.7 \mathrm{v} \quad[\mathrm{AB} 07] \\
& 58 u_{1}+61 u_{2}+43 u_{3} \leq 75.9 \mathrm{v} \quad[\mathrm{AB} 08] \\
& 59 \mathrm{u}_{1}+61 \mathrm{u}_{2}+54 \mathrm{u}_{3} \leq 72.2 \mathrm{v} \quad[\mathrm{AB} 09] \\
& 60 u_{1}+61 u_{2}+45 u_{3} \leq 68.5 v \quad[A B 10] 58 u_{1}+61 u_{2}+43 u_{3}=1 u_{1}, u_{2}, u_{3}, v \\
& \geq 0
\end{aligned}
$$

DMUs for $\mathrm{AB} 09: \operatorname{Max}(Z)=72.2 \mathrm{v}$
Subject to
$x i v$

```
\(61 u_{1}+61 u_{2}+79 u_{3} \leq 97.2 \mathrm{v} \quad[\mathrm{AB} 01]\)
\(64 u_{1}+61 u_{2}+82 u_{3} \leq 96.3 v \quad[A B 02]\)
\(58 u_{1}+61 u_{2}+71 u_{3} \leq 93.5 v \quad[A B 03]\)
\(62 u_{1}+61 u_{2}+66 u_{3} \leq 91.6 v \quad[A B 04]\)
\(61 u_{1}+61 u_{2}+70 u_{3} \leq 90.7 v \quad[A B 05]\)
\(61 u_{1}+61 u_{2}+50 u_{3} \leq 81.4 v \quad[A B 06]\)
\(54 \mathrm{u}_{1}+61 \mathrm{u}_{2}+49 \mathrm{u}_{3} \leq 78.7 \mathrm{v} \quad[\mathrm{AB} 07]\)
\(58 u_{1}+61 u_{2}+43 u_{3} \leq 75.9 \mathrm{v} \quad[\mathrm{AB} 08]\)
\(59 \mathrm{u}_{1}+61 \mathrm{u}_{2}+54 \mathrm{u}_{3} \leq 72.2 \mathrm{v} \quad[\mathrm{AB} 09]\)
\(60 u_{1}+61 u_{2}+45 u_{3} \leq 68.5 v \quad[A B 10]\)
    \(59 u_{1}+61 u_{2}+54 u_{3}=1 u_{1}\),
        \(\mathrm{u}_{2}, \mathrm{u}_{3}, \mathrm{v} \geq 0\)
```

DMUs for AB 10 : $\operatorname{Max}(\mathrm{Z})=68.5 \mathrm{v}$
$61 u_{1}+61 u_{2}+79 u_{3} \leq 97.2 v \quad[A B 01]$
$64 u_{1}+61 u_{2}+82 u_{3} \leq 96.3 v \quad[A B 02]$
$58 u_{1}+61 u_{2}+71 u_{3} \leq 93.5 v \quad[A B 03]$
$62 u_{1}+61 u_{2}+66 u_{3} \leq 91.6 v \quad[A B 04]$
$61 \mathrm{u}_{1}+61 \mathrm{u}_{2}+70 \mathrm{u}_{3} \leq 90.7 \mathrm{v} \quad[\mathrm{AB} 05]$
$61 \mathrm{u}_{1}+61 \mathrm{u}_{2}+50 \mathrm{u}_{3} \leq 81.4 \mathrm{v} \quad[\mathrm{AB} 06]$
$54 \mathrm{u}_{1}+61 \mathrm{u}_{2}+49 \mathrm{u}_{3} \leq 78.7 \mathrm{v} \quad[A B 07] 58 \mathrm{u}_{1}+61 \mathrm{u}_{2}+43 \mathrm{u}_{3} \leq 75.9 \mathrm{v}[A B 08]$
$59 \mathrm{u}_{1}+61 \mathrm{u}_{2}+54 \mathrm{u}_{3} \leq 72.2 \mathrm{v} \quad[\mathrm{AB} 09]$
$60 \mathrm{u}_{1}+61 \mathrm{u}_{2}+45 \mathrm{u}_{3} \leq 68.5 \mathrm{v}$ [AB10]
$60 u_{1}+61 u_{2}+45 u_{3}=1 u_{1}$,
$\mathrm{u}_{2}, \mathrm{u}_{3}, \mathrm{v} \geq 0$

## Appendix B

Table 4.3.1.1 shows the relative efficiency of Students Attendance $\left(\mathrm{U}_{1}\right)$ against output of Students B.E.C.E performance (V) for students at Abu Bonsra D/A JHS at Fomena Circuit.

| DMUs | $\mathrm{U}_{1}$ | $\mathrm{U}_{2}$ | $\mathrm{U}_{3}$ | V | $\mathrm{~V} / \mathrm{U}_{1}$ | $\left(\boldsymbol{\Phi}_{1}\right)$ | $\mathrm{CCR}\left(\boldsymbol{\Phi}_{\mathbf{1}}\right)$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| AB03 | 58 | 61 | 71 | 93.5 | 1.612069 | $\mathbf{1 . 0 0 0 0 0 0}$ | 1.00 |
| AB01 | 61 | 61 | 79 | 97.2 | 1.593443 | 0.988446 | 0.99 |
| AB02 | 64 | 61 | 82 | 96.3 | 1.504688 | 0.933389 | 0.93 |
| AB05 | 61 | 61 | 70 | 90.7 | 1.486885 | 0.922346 | 0.92 |
| AB04 | 62 | 61 | 66 | 91.6 | 1.477419 | 0.916474 | 0.92 |
| AB07 | 54 | 61 | 49 | 78.7 | 1.457407 | 0.904060 | 0.90 |
| AB06 | 61 | 61 | 50 | 81.4 | 1.334426 | 0.827772 | 0.83 |
| AB08 | 58 | 61 | 43 | 75.9 | 1.308621 | 0.811765 | 0.81 |
| AB09 | 59 | 61 | 54 | 72.2 | 1.223729 | 0.759105 | 0.76 |
| AB10 | 60 | 61 | 45 | 68.5 | 1.141667 | 0.708200 | 0.71 |

Source: Author's Construct, April 2016.
Key: $\operatorname{CCR}\left(\mathbf{\Phi}_{1}\right)$ is the relative efficiency for input $u_{I}$

Table 4.3.1.2 shows the relative efficiency of Teachers Attendance $\left(\mathrm{U}_{2}\right)$ against output of Students B.E.C.E performance (V) for students at Abu Bonsra D/A JHS at Fomena Circuit.

| DMUs | U1 | U2 | U3 | V | V/U2 | ( $\left.\boldsymbol{\Phi}_{2}\right)$ | CCR $\left(\boldsymbol{\Phi}_{2}\right)$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| AB01 | 61 | 61 | 79 | 97.2 | 1.593443 | $\mathbf{1 . 0 0 0 0 0 0}$ | 1.00 |
| AB02 | 64 | 61 | 82 | 96.3 | 1.578689 | $\mathbf{0 . 9 9 0 7 4 1}$ | 0.99 |
| AB03 | 58 | 61 | 71 | 93.5 | 1.532787 | $\mathbf{0 . 9 6 1 9 3 4}$ | 0.96 |
| AB04 | 62 | 61 | 66 | 91.6 | 1.501639 | $\mathbf{0 . 9 4 2 3 8 7}$ | 0.94 |
| AB05 | 61 | 61 | 70 | 90.7 | 1.486885 | $\mathbf{0 . 9 3 3 1 2 8}$ | 0.93 |


| AB06 | 61 | 61 | 50 | 81.4 | 1.334426 | $\mathbf{0 . 8 3 7 4 4 9}$ | 0.84 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| AB07 | 54 | 61 | 49 | 78.7 | 1.290164 | $\mathbf{0 . 8 0 9 6 7 1}$ | 0.81 |
| AB08 | 58 | 61 | 43 | 75.9 | 1.244262 | $\mathbf{0 . 7 8 0 8 6 4}$ | 0.78 |
| AB09 | 59 | 61 | 54 | 72.2 | 1.183607 | $\mathbf{0 . 7 4 2 7 9 8}$ | 0.74 |
| AB10 | 60 | 61 | 45 | 68.5 | 1.122951 | $\mathbf{0 . 7 0 4 7 3 3}$ | 0.70 |

Key: $\operatorname{CCR}\left(\mathbf{\Phi}_{2}\right)$ is the relative efficiency for input $u_{2}$

Table 4.3.1.3 shows the relative efficiency of Students Continuous Assessment $\left(\mathrm{U}_{3}\right)$ against output of Students B.E.C.E performance (V) for students at Abu Bonsra D/A JHS at Fomena Circuit.

| DMUs | U1 | U2 | U3 | V | V/U3 | ( $\left.\boldsymbol{\Phi}_{3}\right)$ | CCR( $\left.\boldsymbol{\Phi}_{3}\right)$ |
| :--- | :--- | :--- | :--- | :---: | :---: | :---: | :---: |
| AB08 | 58 | 61 | 43 | 75.9 | 1.765116 | $\mathbf{1 . 0 0 0 0 0 0}$ | 1.00 |
| AB06 | 61 | 61 | 50 | 81.4 | 1.628000 | $\mathbf{0 . 9 2 2 3 1 9}$ | 0.92 |
| AB07 | 54 | 61 | 49 | 78.7 | 1.606122 | $\mathbf{0 . 9 0 9 9 2 4}$ | 0.91 |
| AB10 | 60 | 61 | 45 | 68.5 | 1.522222 | $\mathbf{0 . 8 6 2 3 9 2}$ | 0.86 |
| AB04 | 62 | 61 | 66 | 91.6 | 1.387879 | $\mathbf{0 . 7 8 6 2 8 2}$ | 0.79 |
| AB09 | 59 | 61 | 54 | 72.2 | 1.337037 | $\mathbf{0 . 7 5 7 4 7 8}$ | 0.76 |
| AB03 | 58 | 61 | 71 | 93.5 | 1.316901 | $\mathbf{0 . 7 4 6 0 7 1}$ | 0.75 |
| AB05 | 61 | 61 | 70 | 90.7 | 1.295714 | $\mathbf{0 . 7 3 4 0 6 7}$ | 0.73 |
| AB01 | 61 | 61 | 79 | 97.2 | 1.23038 | $\mathbf{0 . 6 9 7 0 5 3}$ | 0.70 |
| AB02 | 64 | 61 | 82 | 96.3 | 1.17439 | $\mathbf{0 . 6 6 5 3 3 3}$ | 0.67 |

## Source: Author's Construct, April 2016.

Table 4.3.1.4 shows the relative efficiency of Average input of student $\left(\mathrm{U}^{+}\right)$against output of Students B.E.C.E performance (V) for students at Abu Bonsra D/A JHS at Fomena Circuit.

| DMUs | $\mathrm{U}_{1}$ | $\mathrm{U}_{2}$ | $\mathrm{U}_{3}$ | $\mathrm{U}^{+}$ | V | $\mathrm{V} / \mathrm{U}^{+}$ | $\boldsymbol{\Phi}_{+}$ | $\mathbf{C C R}\left(\boldsymbol{\Phi}^{+}\right)$ |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| AB03 | 58 | 61 | 71 | 63.33333 | 93.5 | 1.476316 | 1.000000 | $\mathbf{1 . 0 0}$ |
| AB01 | 61 | 61 | 79 | 67.00000 | 97.2 | 1.450746 | 0.982680 | $\mathbf{0 . 9 8}$ |
| AB04 | 62 | 61 | 66 | 63.00000 | 91.6 | 1.453968 | 0.984863 | $\mathbf{0 . 9 8}$ |
| AB07 | 54 | 61 | 49 | 54.66667 | 78.7 | 1.439634 | 0.975153 | $\mathbf{0 . 9 8}$ |
| AB05 | 61 | 61 | 70 | 64.00000 | 90.7 | 1.417188 | 0.959949 | $\mathbf{0 . 9 6}$ |


| AB06 | 61 | 61 | 50 | 57.33333 | 81.4 | 1.419767 | 0.961696 | $\mathbf{0 . 9 6}$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| AB02 | 64 | 61 | 82 | 69.00000 | 96.3 | 1.395652 | 0.945362 | $\mathbf{0 . 9 5}$ |
| AB08 | 58 | 61 | 43 | 54.00000 | 75.9 | 1.405556 | 0.952070 | $\mathbf{0 . 9 5}$ |
| AB09 | 59 | 61 | 54 | 58.00000 | 72.2 | 1.244828 | 0.843199 | $\mathbf{0 . 8 4}$ |
| AB10 | 60 | 61 | 45 | 55.33333 | 68.5 | 1.237952 | 0.838541 | $\mathbf{0 . 8 4}$ |

## Source: Author's Construct, April 2016.

Key: $\operatorname{CCR}\left(\Phi^{+}\right)$is the relative efficiency for average input $U^{+}$

## Appendix C

Table 4.3.2 shows the data of students at Adiembra D/A JHS in the Asokwa Circuit.

## INPUTS

| DMUs | $\mathbf{U}_{\mathbf{1}}$ | $\mathbf{U}_{\mathbf{2}}$ | $\mathbf{U}_{\mathbf{3}}$ | OUTPUT (V) |
| :---: | :---: | :---: | :---: | :---: |
| AD01 | 62.00 | 62.00 | 70.00 | $\mathbf{9 5 . 4 0}$ |
| AD02 | 62.00 | 62.00 | 78.00 | $\mathbf{9 5 . 4 0}$ |
| AD03 | 61.00 | 62.00 | 57.00 | $\mathbf{9 1 . 7 0}$ |
| AD04 | 62.00 | 62.00 | 53.00 | $\mathbf{9 0 . 8 0}$ |
| AD05 | 63.00 | 62.00 | 52.00 | $\mathbf{8 6 . 1 0}$ |
| AD06 | 61.00 | 62.00 | 46.00 | $\mathbf{8 7 . 0 0}$ |
| AD07 | 61.00 | 62.00 | 46.00 | $\mathbf{8 5 . 2 0}$ |
| AD08 | 62.00 | 62.00 | 44.00 | $\mathbf{8 1 . 5 0}$ |
| AD09 | 63.00 | 62.00 | 41.00 | $\mathbf{8 0 . 6 0}$ |
| AD10 | 61.00 | 62.00 | 43.00 | $\mathbf{7 5 . 9 0}$ |

The linear programming formulated out of the data for the above academic years are as follows:

DMUs for AD01: $\operatorname{Max}(Z)=95.4 v$

Subject to

$$
\begin{aligned}
& 62 \mathrm{u}_{1}+62 \mathrm{u}_{2}+70 \mathrm{u}_{3} \leq 95.4 \mathrm{v}[\mathrm{AD} 01] \\
& 62 \mathrm{u}_{1}+62 \mathrm{u}_{2}+78 \mathrm{u}_{3} \leq 95.4 \mathrm{v}[\mathrm{AD} 02] \\
& 61 \mathrm{u}_{1}+62 \mathrm{u}_{2}+57 \mathrm{u}_{3} \leq 91.7 \mathrm{v}[\mathrm{AD} 03] \\
& 62 \mathrm{u}_{1}+62 \mathrm{u}_{2}+53 \mathrm{u}_{3} \leq 90.8 \mathrm{v}[\mathrm{AD} 04] \\
& 63 \mathrm{u}_{1}+62 \mathrm{u}_{2}+52 \mathrm{u}_{3} \leq 86.1 \mathrm{v}[\mathrm{AD} 05]
\end{aligned}
$$



DMUs for AD02: $\operatorname{Max}(\mathrm{Z})=95.4 \mathrm{v}$

Subject to
$62 \mathrm{u}_{1}+62 \mathrm{u}_{2}+70 \mathrm{u}_{3} \leq 95.4 \mathrm{v} \quad[\mathrm{AD} 01]$
$62 \mathrm{u}_{1}+62 \mathrm{u}_{2}+78 \mathrm{u}_{3} \leq 95.4 \mathrm{v} \quad[\mathrm{AD} 02]$
$61 u_{1}+62 u_{2}+57 u_{3} \leq 91.7 \mathrm{v} \quad[\mathrm{AD} 03]$
$62 u_{1}+62 u_{2}+53 u_{3} \leq 90.8 v \quad[A D 04]$
$63 u_{1}+62 u_{2}+52 u_{3} \leq 86.1 v \quad[A D 05]$
$61 u_{1}+62 u_{2}+46 u_{3} \leq 87.0 v \quad[A D 06]$
$61 u_{1}+62 u_{2}+46 u_{3} \leq 85.2 \mathrm{v} \quad[A D 07]$
$62 \mathrm{u}_{1}+62 \mathrm{u}_{2}+44 \mathrm{u}_{3} \leq 81.5 \mathrm{v} \quad[$ AD08]
$63 \mathrm{u}_{1}+62 \mathrm{u}_{2}+41 \mathrm{u}_{3} \leq 80.6 \mathrm{v}$ [AD09]
$61 u_{1}+62 u_{2}+43 u_{3} \leq 75.9 v \quad[A D 10]$

$$
62 u_{1}+62 u_{2}+78 u_{3}=1 u_{1},
$$

$\mathrm{u}_{2}, \mathrm{u}_{3}, \mathrm{v} \geq 0$

DMUs for AD03: Max $(Z)=91.7 \mathrm{v}$

Subject to
$62 \mathrm{u}_{1}+62 \mathrm{u}_{2}+70 \mathrm{u}_{3} \leq 95.4 \mathrm{v} \quad[\mathrm{AD} 01]$
xix

$$
\begin{aligned}
62 \mathrm{u}_{1}+62 \mathrm{u}_{2}+78 \mathrm{u}_{3} & \leq 95.4 \mathrm{v} \quad[\mathrm{AD} 02] \\
61 \mathrm{u}_{1}+62 \mathrm{u}_{2}+57 \mathrm{u}_{3} & \leq 91.7 \mathrm{v} \quad[\mathrm{AD} 03] \\
62 \mathrm{u}_{1}+62 \mathrm{u}_{2}+53 \mathrm{u}_{3} & \leq 90.8 \mathrm{v} \quad[\mathrm{AD} 04] \\
63 \mathrm{u}_{1}+62 \mathrm{u}_{2}+52 \mathrm{u}_{3} & \leq 86.1 \mathrm{v} \quad[\mathrm{AD} 05] \\
61 \mathrm{u}_{1}+62 \mathrm{u}_{2}+46 \mathrm{u}_{3} & \leq 87.0 \mathrm{v} \quad[\mathrm{AD} 06] \\
61 \mathrm{u}_{1}+62 \mathrm{u}_{2}+46 \mathrm{u}_{3} & \leq 85.2 \mathrm{v} \quad[\mathrm{AD} 07] \\
62 \mathrm{u}_{1}+62 \mathrm{u}_{2}+44 \mathrm{u}_{3} & \leq 81.5 \mathrm{v} \quad[\mathrm{AD} 08] \\
63 \mathrm{u}_{1}+62 \mathrm{u}_{2}+41 \mathrm{u}_{3} & \leq 80.6 \mathrm{v} \quad[\mathrm{AD} 09] \\
61 \mathrm{u}_{1}+62 \mathrm{u}_{2}+43 \mathrm{u}_{3} & \leq 75.9 \mathrm{v} \quad[\mathrm{AD} 10] \\
61 \mathrm{u}_{1}+62 \mathrm{u}_{2} & +57 \mathrm{u}_{3}=1 \mathrm{u}_{1}, \\
\mathrm{u}_{2}, \mathrm{u}_{3}, \mathrm{v} & \geq 0
\end{aligned}
$$

DMUs for AD04: $\operatorname{Max}(Z)=90.8 \mathrm{v}$

Subject to
$62 \mathrm{u}_{1}+62 \mathrm{u}_{2}+70 \mathrm{u}_{3} \leq 95.4 \mathrm{v} \quad[\mathrm{AD} 01]$
$62 \mathrm{u}_{1}+62 \mathrm{u}_{2}+78 \mathrm{u}_{3} \leq 95.4 \mathrm{v} \quad[A D 02] 61 \mathrm{u}_{1}+62 \mathrm{u}_{2}+57 \mathrm{u}_{3} \leq 91.7 \mathrm{v}$ [AD03]

$$
62 \mathrm{u}_{1}+62 \mathrm{u}_{2}+53 \mathrm{u}_{3} \leq 90.8 \mathrm{v} \quad[\mathrm{AD} 04]
$$

$$
63 \mathrm{u}_{1}+62 \mathrm{u}_{2}+52 \mathrm{u}_{3} \leq 86.1 \mathrm{v} \quad[\mathrm{AD} 05]
$$

$$
61 \mathrm{u}_{1}+62 \mathrm{u}_{2}+46 \mathrm{u}_{3} \leq 87.0 \mathrm{v} \quad[\mathrm{AD} 06]
$$

$$
61 u_{1}+62 u_{2}+46 u_{3} \leq 85.2 v \quad[\mathrm{AD} 07]
$$

$$
62 \mathrm{u}_{1}+62 \mathrm{u}_{2}+44 \mathrm{u}_{3} \leq 81.5 \mathrm{v} \quad[\mathrm{AD} 08]
$$

$$
63 u_{1}+62 u_{2}+41 u_{3} \leq 80.6 v \quad[A D 09]
$$

$$
61 \mathrm{u}_{1}+62 \mathrm{u}_{2}+43 \mathrm{u}_{3} \leq 75.9 \mathrm{v} \quad[\mathrm{AD} 10]
$$

$$
62 \mathrm{u}_{1}+62 \mathrm{u}_{2}+53 \mathrm{u}_{3}=1 \mathrm{u}_{1}
$$

$\mathrm{u}_{2}, \mathrm{u}_{3}, \mathrm{v} \geq 0$

DMUs for AD05: $\operatorname{Max}(\mathrm{Z})=86.1 \mathrm{v}$
Subject to
$62 \mathrm{u}_{1}+62 \mathrm{u}_{2}+70 \mathrm{u}_{3} \leq 95.4 \mathrm{v} \quad[\mathrm{AD} 01]$
$62 \mathrm{u}_{1}+62 \mathrm{u}_{2}+78 \mathrm{u}_{3} \leq 95.4 \mathrm{v} \quad[\mathrm{AD} 02]$
$61 \mathrm{u}_{1}+62 \mathrm{u}_{2}+57 \mathrm{u}_{3} \leq 91.7 \mathrm{v} \quad[\mathrm{AD} 03]$
$62 \mathrm{u}_{1}+62 \mathrm{u}_{2}+53 \mathrm{u}_{3} \leq 90.8 \mathrm{v} \quad[\mathrm{AD} 04]$
$63 \mathrm{u}_{1}+62 \mathrm{u}_{2}+52 \mathrm{u}_{3} \leq 86.1 \mathrm{v} \quad[\mathrm{AD} 05]$
$61 \mathrm{u}_{1}+62 \mathrm{u}_{2}+46 \mathrm{u}_{3} \leq 87.0 \mathrm{v} \quad[\mathrm{AD} 06]$
$61 \mathrm{u}_{1}+62 \mathrm{u}_{2}+46 \mathrm{u}_{3} \leq 85.2 \mathrm{v} \quad[\mathrm{AD} 07]$
$62 \mathrm{u}_{1}+62 \mathrm{u}_{2}+44 \mathrm{u}_{3} \leq 81.5 \mathrm{v} \quad[\mathrm{AD} 08]$
$63 \mathrm{u}_{1}+62 \mathrm{u}_{2}+41 \mathrm{u}_{3} \leq 80.6 \mathrm{v} \quad[\mathrm{AD} 09]$
$61 \mathrm{u}_{1}+62 \mathrm{u}_{2}+43 \mathrm{u}_{3} \leq 75.9 \mathrm{v} \quad[\mathrm{AD} 10]$
$63 \mathrm{u}_{1}+62 \mathrm{u}_{2}+52 \mathrm{u}_{3}=1 \mathrm{u}_{1}$,

## DMUs for AD06: $\operatorname{Max}(\mathrm{Z})=87.0 \mathrm{v}$

## Subject to

$62 \mathrm{u}_{1}+62 \mathrm{u}_{2}+70 \mathrm{u}_{3} \leq 95.4 \mathrm{v} \quad[\mathrm{AD} 01]$
$62 u_{1}+62 u_{2}+78 u_{3} \leq 95.4 v \quad[A D 02]$
$61 u_{1}+62 u_{2}+57 u_{3} \leq 91.7 v \quad[A D 03]$

$$
62 u_{1}+62 u_{2}+53 u_{3} \leq 90.8 v \quad[A D 04]
$$

$$
63 u_{1}+62 u_{2}+52 u_{3} \leq 86.1 v \quad[A D 05]
$$

$$
61 \mathrm{u}_{1}+62 \mathrm{u}_{2}+46 \mathrm{u}_{3} \leq 87.0 \mathrm{v} \quad[\mathrm{AD} 06]
$$

$$
61 \mathrm{u}_{1}+62 \mathrm{u}_{2}+46 \mathrm{u}_{3} \leq 85.2 \mathrm{v} \quad[\mathrm{AD} 07]
$$

$$
62 u_{1}+62 u_{2}+44 u_{3} \leq 81.5 v \quad[\mathrm{AD} 08]
$$



DMUs for AD07: $\operatorname{Max}(\mathrm{Z})=85.2 \mathrm{v}$

## Subject to

$62 \mathrm{u}_{1}+62 \mathrm{u}_{2}+70 \mathrm{u}_{3} \leq 95.4 \mathrm{v} \quad[\mathrm{AD} 01]$
$62 \mathrm{u}_{1}+62 \mathrm{u}_{2}+78 \mathrm{u}_{3} \leq 95.4 \mathrm{v} \quad[\mathrm{AD} 02]$
$61 \mathrm{u}_{1}+62 \mathrm{u}_{2}+57 \mathrm{u}_{3} \leq 91.7 \mathrm{v}$ [AD03]
$62 u_{1}+62 u_{2}+53 u_{3} \leq 90.8 v \quad[A D 04]$
$63 \mathrm{u}_{1}+62 \mathrm{u}_{2}+52 \mathrm{u}_{3} \leq 86.1 \mathrm{v}$ [AD05]
$61 u_{1}+62 u_{2}+46 u_{3} \leq 87.0 v \quad[A D 06]$
$61 u_{1}+62 u_{2}+46 u_{3} \leq 85.2 v \quad[A D 07]$
$62 \mathrm{u}_{1}+62 \mathrm{u}_{2}+44 \mathrm{u}_{3} \leq 81.5 \mathrm{v} \quad[\mathrm{AD} 08]$
$63 u_{1}+62 u_{2}+41 u_{3} \leq 80.6 v \quad[A D 09] 61 u_{1}+62 u_{2}+43 u_{3} \leq 75.9 v$ [AD10]

$$
61 u_{1}+62 u_{2}+46 u_{3}=1 u_{1},
$$ $\mathrm{u}_{2}, \mathrm{u}_{3}, \mathrm{v} \geq 0$

DMUs for AD08: $\operatorname{Max}(Z)=81.5 v$

Subject to

$$
\begin{aligned}
& 62 \mathrm{u}_{1}+62 \mathrm{u}_{2}+70 \mathrm{u}_{3} \leq 95.4 \mathrm{v}[\mathrm{AD} 01] \\
& 62 \mathrm{u}_{1}+62 \mathrm{u}_{2}+78 \mathrm{u}_{3} \leq 95.4 \mathrm{v}[\mathrm{AD} 02] \\
& 61 \mathrm{u}_{1}+62 \mathrm{u}_{2}+57 \mathrm{u}_{3} \leq 91.7 \mathrm{v}[\mathrm{AD} 03] \\
& 62 \mathrm{u}_{1}+62 \mathrm{u}_{2}+53 \mathrm{u}_{3} \leq 90.8 \mathrm{v}[\mathrm{AD} 04]
\end{aligned}
$$

$$
\begin{aligned}
63 \mathrm{u}_{1}+62 \mathrm{u}_{2}+52 \mathrm{u}_{3} & \leq 86.1 \mathrm{v} \quad[\mathrm{AD} 05] \\
61 \mathrm{u}_{1}+62 \mathrm{u}_{2}+46 \mathrm{u}_{3} & \leq 87.0 \mathrm{v} \quad[\mathrm{AD} 06] \\
61 \mathrm{u}_{1}+62 \mathrm{u}_{2}+46 \mathrm{u}_{3} & \leq 85.2 \mathrm{v} \quad[\mathrm{AD} 07] \\
62 \mathrm{u}_{1}+62 \mathrm{u}_{2}+44 \mathrm{u}_{3} & \leq 81.5 \mathrm{v} \quad[\mathrm{AD} 08] \\
63 \mathrm{u}_{1}+62 \mathrm{u}_{2}+41 \mathrm{u}_{3} & \leq 80.6 \mathrm{v} \quad[\mathrm{AD} 09] \\
61 \mathrm{u}_{1}+62 \mathrm{u}_{2}+43 \mathrm{u}_{3} & \leq 75.9 \mathrm{v} \quad[\mathrm{AD} 10] \\
62 \mathrm{u}_{1}+62 \mathrm{u}_{2} & +44 \mathrm{u}_{3}=1 \mathrm{u}_{1}, \\
\mathrm{u}_{2}, \mathrm{u}_{3}, \mathrm{v} & \geq 0
\end{aligned}
$$

DMUs for AD09: $\operatorname{Max}(Z)=80.6 v$

## Subject to

$62 \mathrm{u}_{1}+62 \mathrm{u}_{2}+70 \mathrm{u}_{3} \leq 95.4 \mathrm{v} \quad[\mathrm{AD} 01]$
$62 \mathrm{u}_{1}+62 \mathrm{u}_{2}+78 \mathrm{u}_{3} \leq 95.4 \mathrm{v} \quad[\mathrm{AD} 02]$
$61 u_{1}+62 u_{2}+57 u_{3} \leq 91.7 \mathrm{v}$ [AD03]
$62 u_{1}+62 u_{2}+53 u_{3} \leq 90.8 v \quad[A D 04] 63 u_{1}+62 u_{2}+52 u_{3} \leq 86.1 v$ [AD05]
$61 u_{1}+62 u_{2}+46 u_{3} \leq 87.0 v \quad[A D 06]$
$61 u_{1}+62 u_{2}+46 u_{3} \leq 85.2 v \quad[A D 07]$
$62 u_{1}+62 u_{2}+44 u_{3} \leq 81.5 v \quad[A D 08]$
$63 u_{1}+62 u_{2}+41 u_{3} \leq 80.6 v \quad[A D 09]$
$61 u_{1}+62 u_{2}+43 u_{3} \leq 75.9 v \quad[A D 10]$
$63 u_{1}+62 u_{2}+41 u_{3}=1 u_{1}$,
$\mathrm{u}_{2}, \mathrm{u}_{3}, \mathrm{v} \geq 0$

DMUs for AD10: $\operatorname{Max}(\mathrm{Z})=75.9 \mathrm{v}$

Subject to

$$
\begin{aligned}
& 62 \mathrm{u}_{1}+62 \mathrm{u}_{2}+70 \mathrm{u}_{3} \leq 95.4 \mathrm{v} \quad[\mathrm{AD} 01] \\
& 62 \mathrm{u}_{1}+62 \mathrm{u}_{2}+78 \mathrm{u}_{3} \leq 95.4 \mathrm{v} \quad[\mathrm{AD} 02] \\
& 61 u_{1}+62 u_{2}+57 u_{3} \leq 91.7 \mathrm{v} \quad[\mathrm{AD} 03] \\
& 62 u_{1}+62 u_{2}+53 u_{3} \leq 90.8 v \quad[A D 04] \\
& 63 u_{1}+62 u_{2}+52 u_{3} \leq 86.1 v \quad[A D 05] \\
& 61 u_{1}+62 u_{2}+46 u_{3} \leq 87.0 v \quad[A D 06] \\
& 61 u_{1}+62 u_{2}+46 u_{3} \leq 85.2 \mathrm{v} \quad[\mathrm{AD} 07] \\
& 62 \mathrm{u}_{1}+62 \mathrm{u}_{2}+44 \mathrm{u}_{3} \leq 81.5 \mathrm{v} \quad[\mathrm{AD} 08] \\
& 63 u_{1}+62 u_{2}+41 u_{3} \leq 80.6 v \quad[A D 09] \\
& 61 \mathrm{u}_{1}+62 \mathrm{u}_{2}+43 \mathrm{u}_{3} \leq 75.9 \mathrm{v} \quad[\mathrm{AD} 10] \\
& 61 u_{1}+62 u_{2}+43 u_{3}=1 u_{1}, \\
& \mathrm{u}_{2}, \mathrm{u}_{3}, \mathrm{v} \geq 0
\end{aligned}
$$

## Appendix D

Table 4.3.2.1 shows the relative efficiency of Students Attendance $\left(U_{1}\right)$ against output of Students B.E.C.E performance (V) for students at Adiembra D/A JHS at Asokwa Circuit.

| DMUs | U1 | U2 | U3 | V | V/U1 | $\boldsymbol{\Phi}_{1}$ | CCR( $\left.\boldsymbol{\Phi}_{\mathbf{1}}\right)$ |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| AD01 | 62 | 62 | 70 | 95.4 | 1.538710 | 1.000000 | 1.00 |
| AD02 | 62 | 62 | 78 | 95.4 | 1.538710 | 1.000000 | 1.00 |
| AD03 | 61 | 62 | 57 | 91.7 | 1.503279 | 0.976974 | 0.98 |
| AD04 | 62 | 62 | 53 | 90.8 | 1.464516 | 0.951782 | 0.95 |
| AD06 | 61 | 62 | 46 | 87.0 | 1.426230 | 0.926900 | 0.93 |
| AD07 | 61 | 62 | 46 | 85.2 | 1.396721 | 0.907722 | 0.91 |
| AD05 | 63 | 62 | 52 | 86.1 | 1.366667 | 0.909124 | 0.89 |
| AD08 | 62 | 62 | 44 | 81.5 | 1.314516 | 0.854298 | 0.85 |
| AD09 | 63 | 62 | 41 | 80.6 | 1.279365 | 0.831453 | 0.83 |
| AD10 | 61 | 62 | 43 | 75.9 | 1.244262 | 0.808640 | 0.81 |

## Source: Author's Construct, April 2016.

Key: $\operatorname{CCR}\left(\boldsymbol{\Phi}_{\mathbf{1}}\right)$ is the relative efficiency for student attendance $U_{l}$

Table 4.3.2.2 shows the relative efficiency of Teachers Attendance $\left(U_{2}\right)$ against output of Students B.E.C.E performance (V) for students at Adiembra D/A JHS at Asokwa Circuit.

| DMUs | U1 | U2 | U3 | V | V/U2 | $\boldsymbol{\Phi}_{\mathbf{2}}$ | CCR $\left(\boldsymbol{\Phi}_{\mathbf{2}}\right)$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| AD01 | 62 | 62 | 70 | 95.4 | 1.538710 | 1.000000 | 1.00 |
| AD02 | 62 | 62 | 78 | 95.4 | 1.538710 | 1.000000 | 1.00 |
| AD03 | 61 | 62 | 57 | 91.7 | 1.479032 | 0.961216 | 0.96 |
| AD04 | 62 | 62 | 53 | 90.8 | 1.464516 | 0.951782 | 0.95 |
| AD06 | 61 | 62 | 46 | 87.0 | 1.403226 | 0.911950 | 0.91 |
| AD05 | 63 | 62 | 52 | 86.1 | 1.388710 | 0.902516 | 0.90 |
| AD07 | 61 | 62 | 46 | 85.2 | 1.374194 | 0.893082 | 0.89 |
| AD08 | 62 | 62 | 44 | 81.5 | 1.314516 | 0.854298 | 0.85 |
| AD09 | 63 | 62 | 41 | 80.6 | 1.300000 | 0.844864 | 0.84 |
| AD10 | 61 | 62 | 43 | 75.9 | 1.224194 | 0.795597 | 0.80 |

Source: Author's Construct, April 2016.
Key: $\operatorname{CCR}\left(\boldsymbol{\Phi}_{2}\right)$ is the relative efficiency for teachers attendance $U_{2}$

Table 4.3.2.3 shows the relative efficiency of Students Continuous Assessment $\left(\mathrm{U}_{3}\right)$ against output of Students B.E.C.E performance (V) for students at Adiembra D/A JHS at Asokwa Circuit.

| DMUs | U1 | U2 | U3 | V | V/U3 | $\boldsymbol{\Phi}_{3}$ | CCR( $\left.\boldsymbol{\Phi}_{3}\right)$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| AD09 | 63 | 62 | 41 | 80.6 | 1.965854 | 1.000000 | 1.00 |
| AD06 | 61 | 62 | 46 | 87.0 | 1.891304 | 0.962078 | 0.96 |
| AD07 | 61 | 62 | 46 | 85.2 | 1.852174 | 0.942173 | 0.94 |
| AD08 | 62 | 62 | 44 | 81.5 | 1.852273 | 0.942223 | 0.94 |
| AD10 | 61 | 62 | 43 | 75.9 | 1.765116 | 0.897888 | 0.90 |
| AD04 | 62 | 62 | 53 | 90.8 | 1.713208 | 0.871483 | 0.87 |
| AD05 | 63 | 62 | 52 | 86.1 | 1.655769 | 0.842265 | 0.84 |
| AD03 | 61 | 62 | 57 | 91.7 | 1.608772 | 0.818358 | 0.82 |
| AD01 | 62 | 62 | 70 | 95.4 | 1.362857 | 0.693265 | 0.69 |
| AD02 | 62 | 62 | 78 | 95.4 | 1.223077 | 0.622161 | 0.62 |

Source: Author's Construct, April 2016.
Key: $\operatorname{CCR}\left(\boldsymbol{\Phi}_{3}\right)$ is the relative efficiency for teacher's attendance $U_{3}$

Table 4.3.2.4 shows the relative efficiency of Average input of student $\left(\mathrm{U}^{+}\right)$against output of Students B.E.C.E performance (V) for students at Adiembra D/A JHS at Asokwa Circuit.

| DMUs | $\mathrm{U}_{1}$ | $\mathrm{U}_{2}$ | $\mathrm{U}_{3}$ | $\mathrm{U}^{+}$ | V | $\mathrm{V} / \mathrm{U}^{+}$ | $\boldsymbol{\Phi}_{+}$ | $\mathbf{C C R}\left(\boldsymbol{\Phi}^{+}\right)$ |
| :--- | :--- | :--- | :--- | :---: | :---: | :---: | :---: | :---: |
| AD04 | 62 | 62 | 53 | 59.00000 | 90.8 | 1.538983 | 0.996506 | 1.00 |
| AD06 | 61 | 62 | 46 | 56.33333 | 87.0 | 1.544379 | 1.000000 | 1.00 |
| AD03 | 61 | 62 | 57 | 60.00000 | 91.7 | 1.528333 | 0.989610 | 0.99 |
| AD07 | 61 | 62 | 46 | 56.33333 | 85.2 | 1.512426 | 0.979310 | 0.98 |
| AD01 | 62 | 62 | 70 | 64.66667 | 95.4 | 1.475258 | 0.955244 | 0.96 |
| AD05 | 63 | 62 | 52 | 59.00000 | 86.1 | 1.459322 | 0.944925 | 0.94 |
| AD08 | 62 | 62 | 44 | 56.00000 | 81.5 | 1.455357 | 0.942358 | 0.94 |
| AD09 | 63 | 62 | 41 | 55.33333 | 80.6 | 1.456627 | 0.943180 | 0.94 |
| AD02 | 62 | 62 | 78 | 67.33333 | 95.4 | 1.416832 | 0.917412 | 0.92 |
| AD10 | 61 | 62 | 43 | 55.33333 | 75.9 | 1.371687 | 0.888180 | 0.89 |

Source: Author's Construct, April 2016. $\operatorname{Key:~} \operatorname{CCR}\left(\boldsymbol{\Phi}^{+}\right)$is the relative efficiency for average input $U^{+}$

## Appendix E

Table 4.3.3 shows the data of students at Abu Bonsra D/A JHS and Adiembra D/A JHS in the Fomena and Asokwa Circuits respectively.

INPUTS

| DMUs | $\mathbf{U}_{\mathbf{1}}$ | $\mathbf{U}_{\mathbf{2}}$ | $\mathbf{U}_{\mathbf{3}}$ | OUTPUT (V) |
| :---: | :---: | :---: | :---: | :---: |
| AB01 | 61.00 | 61.00 | 79.00 | $\mathbf{9 7 . 2 0}$ |
| AB02 | 64.00 | 61.00 | 82.00 | $\mathbf{9 6 . 3 0}$ |
| AB03 | 58.00 | 61.00 | 71.00 | $\mathbf{9 3 . 5 0}$ |
| AB04 | 62.00 | 61.00 | 66.00 | $\mathbf{9 1 . 6 0}$ |
| AB05 | 61.00 | 61.00 | 70.00 | $\mathbf{9 0 . 7 0}$ |
| AB06 | 61.00 | 61.00 | 50.00 | $\mathbf{8 1 . 4 0}$ |
| AB07 | 54.00 | 61.00 | 49.00 | $\mathbf{7 8 . 7 0}$ |
| AB08 | 58.00 | 61.00 | 43.00 | $\mathbf{7 5 . 9 0}$ |
| AB09 | 59.00 | 61.00 | 54.00 | $\mathbf{7 2 . 2 0}$ |
| AB10 | 60.00 | 61.00 | 45.00 | $\mathbf{6 8 . 5 0}$ |
| AD01 | 62.00 | 62.00 | 70.00 | $\mathbf{9 5 . 4 0}$ |
| AD02 | 62.00 | 62.00 | 78.00 | $\mathbf{9 5 . 4 0}$ |
| AD03 | 61.00 | 62.00 | 57.00 | $\mathbf{9 1 . 7 0}$ |
| AD04 | 62.00 | 62.00 | 53.00 | $\mathbf{9 0 . 8 0}$ |
| AD05 | 63.00 | 62.00 | 52.00 | $\mathbf{8 6 . 1 0}$ |
| AD06 | 61.00 | 62.00 | 46.00 | $\mathbf{8 7 . 0 0}$ |
|  |  | $x x v i$ |  |  |


| AD07 | 61.00 | 62.00 | 46.00 | $\mathbf{8 5 . 2 0}$ |
| :--- | :--- | :--- | :--- | :--- |
| AD08 | 62.00 | 62.00 | 44.00 | $\mathbf{8 1 . 5 0}$ |
| AD09 | 63.00 | 62.00 | 41.00 | $\mathbf{8 0 . 6 0}$ |
| AD10 | 61.00 | 62.00 | 43.00 | $\mathbf{7 5 . 9 0}$ |

## Source: Author's Construct, April 2016.

The linear programming formulated out of the data for the above academic years are as follows:

$$
\begin{gathered}
\text { DMUs for AB01: Max }(\mathrm{Z})=97.2 \mathrm{v} \\
\text { Subject to } \\
61 \mathrm{u}_{1}+61 \mathrm{u}_{2}+79 \mathrm{u}_{3} \leq 97.2 \mathrm{v} \quad[\mathrm{AB} 01] \\
64 \mathrm{u}_{1}+61 \mathrm{u}_{2}+82 \mathrm{u}_{3} \leq 96.3 \mathrm{v} \quad[\mathrm{AB} 02] \\
58 \mathrm{u}_{1}+61 \mathrm{u}_{2}+71 \mathrm{u}_{3} \leq 93.5 \mathrm{v} \quad[\mathrm{AB} 03] \\
62 \mathrm{u}_{1}+61 \mathrm{u}_{2}+66 \mathrm{u}_{3} \leq 91.6 \mathrm{v} \quad[\mathrm{AB} 04] \\
61 \mathrm{u}_{1}+61 \mathrm{u}_{2}+70 \mathrm{u}_{3} \leq 90.7 \mathrm{v} \quad[\mathrm{AB} 05] \\
\hline 61 \mathrm{u}_{1}+61 \mathrm{u}_{2}+50 \mathrm{u}_{3} \leq 81.4 \mathrm{v} \quad[\mathrm{AB} 06] \\
54 \mathrm{u}_{1}+61 \mathrm{u}_{2}+49 \mathrm{u}_{3} \leq 78.7 \mathrm{v} \quad[\mathrm{AB} 07] \\
58 \mathrm{u}_{1}+61 \mathrm{u}_{2}+43 \mathrm{u}_{3} \leq 75.9 \mathrm{v} \quad[\mathrm{AB} 08] \\
59 \mathrm{u}_{1}+61 \mathrm{u}_{2}+54 \mathrm{u}_{3} \leq 72.2 \mathrm{v} \quad[\mathrm{AB} 09] \\
\hline 60 \mathrm{u}_{1}+61 \mathrm{u}_{2}+45 \mathrm{u}_{3} \leq 68.5 \mathrm{v} \quad[\mathrm{AB} 10] \\
62 \mathrm{u}_{1}+62 \mathrm{u}_{2}+70 \mathrm{u}_{3} \leq 95.4 \mathrm{v} \quad[\mathrm{AD} 01] \\
62 \mathrm{u}_{1}+62 \mathrm{u}_{2}+78 \mathrm{u}_{3} \leq 95.4 \mathrm{v} \quad[\mathrm{AD} 02] \\
x x v i i
\end{gathered}
$$

$$
\begin{aligned}
& 61 \mathrm{u}_{1}+62 \mathrm{u}_{2}+57 \mathrm{u}_{3} \leq 91.7 \mathrm{v} \text { [AD03] } \\
& 62 \mathrm{u}_{1}+62 \mathrm{u}_{2}+53 \mathrm{u}_{3} \leq 90.8 \mathrm{v} \text { [AD04] } \\
& 63 \mathrm{u}_{1}+62 \mathrm{u}_{2}+52 \mathrm{u}_{3} \leq 86.1 \mathrm{v}[\mathrm{AD} 05] \\
& 61 \mathrm{u}_{1}+62 \mathrm{u}_{2}+46 \mathrm{u}_{3} \leq 87.0 \mathrm{v} \text { [AD06] } \\
& 61 \mathrm{u}_{1}+62 \mathrm{u}_{2}+46 \mathrm{u}_{3} \leq 85.2 \mathrm{v}[\mathrm{AD} 07] \\
& 62 \mathrm{u}_{1}+62 \mathrm{u}_{2}+44 \mathrm{u}_{3} \leq 81.5 \mathrm{v}[\mathrm{AD} 08] \\
& 63 \mathrm{u}_{1}+62 \mathrm{u}_{2}+41 \mathrm{u}_{3} \leq 80.6 \mathrm{v} \text { [AD09] } \\
& 61 \mathrm{u}_{1}+62 \mathrm{u}_{2}+43 \mathrm{u}_{3} \leq 75.9 \mathrm{v}[\mathrm{AD} 10]
\end{aligned}
$$

$$
61 u_{1}+61 u_{2}+79 u_{3}=1 u_{1},
$$

$$
\mathrm{u}_{2}, \mathrm{u}_{3}, \mathrm{v} \geq 0
$$

Max
DMUs for AB 02 : $(Z)=96.3 v$

Subject to

$61 \mathrm{u}_{1}+61 \mathrm{u}_{2}+79 \mathrm{u}_{3} \leq 97.2 \mathrm{v} \quad[\mathrm{AB} 01]$
$64 \mathrm{u}_{1}+61 \mathrm{u}_{2}+82 \mathrm{u}_{3} \leq 96.3 \mathrm{v} \quad[\mathrm{AB} 02]$
$58 \mathrm{u}_{1}+61 \mathrm{u}_{2}+71 \mathrm{u}_{3} \leq 93.5 \mathrm{v} \quad[\mathrm{AB} 03]$
$62 \mathrm{u}_{1}+61 \mathrm{u}_{2}+66 \mathrm{u}_{3} \leq 91.6 \mathrm{v} \quad[\mathrm{AB} 04]$
$61 \mathrm{u}_{1}+61 \mathrm{u}_{2}+70 \mathrm{u}_{3} \leq 90.7 \mathrm{v} \quad[\mathrm{AB} 05]$
$61 \mathrm{u}_{1}+61 \mathrm{u}_{2}+50 \mathrm{u}_{3} \leq 81.4 \mathrm{v} \quad[\mathrm{AB} 06]$
$54 \mathrm{u}_{1}+61 \mathrm{u}_{2}+49 \mathrm{u}_{3} \leq 78.7 \mathrm{v} \quad[\mathrm{AB} 07]$
$58 \mathrm{u}_{1}+61 \mathrm{u}_{2}+43 \mathrm{u}_{3} \leq 75.9 \mathrm{v} \quad[\mathrm{AB} 08]$
$59 \mathrm{u}_{1}+61 \mathrm{u}_{2}+54 \mathrm{u}_{3} \leq 72.2 \mathrm{v} \quad[\mathrm{AB} 09]$
$60 \mathrm{u}_{1}+61 \mathrm{u}_{2}+45 \mathrm{u}_{3} \leq 68.5 \mathrm{v} \quad[\mathrm{AB} 10]$
$62 \mathrm{u}_{1}+62 \mathrm{u}_{2}+70 \mathrm{u}_{3} \leq 95.4 \mathrm{v}$ [AD01]
$62 u_{1}+62 u_{2}+78 u_{3} \leq 95.4 v \quad[A D 02]$
$61 \mathrm{u}_{1}+62 \mathrm{u}_{2}+57 \mathrm{u}_{3} \leq 91.7 \mathrm{v} \quad[\mathrm{AD} 03]$
$62 u_{1}+62 u_{2}+53 u_{3} \leq 90.8 v \quad[A D 04]$
$63 u_{1}+62 u_{2}+52 u_{3} \leq 86.1 v \quad[A D 05]$
$61 u_{1}+62 u_{2}+46 u_{3} \leq 87.0 v \quad[A D 06]$
$61 \mathrm{u}_{1}+62 \mathrm{u}_{2}+46 \mathrm{u}_{3} \leq 85.2 \mathrm{v}$ [AD07]
$62 u_{1}+62 u_{2}+44 u_{3} \leq 81.5 v \quad[A D 08]$
$63 u_{1}+62 u_{2}+41 u_{3} \leq 80.6 v$ [AD09]
$61 \mathrm{u}_{1}+62 \mathrm{u}_{2}+43 \mathrm{u}_{3} \leq 75.9 \mathrm{v} \quad[\mathrm{AD} 10]$
$64 u_{1}+61 u_{2}+82 u_{3}=1$
$\mathrm{u}_{1}, \mathrm{u}_{2}, \mathrm{u}_{3}, \mathrm{v} \geq 0$
xxix

## Max

DMUs for AB03: $\quad(Z)=93.5 v$


> Subject to
> $61 u_{1}+61 u_{2}+79 u_{3} \leq 97.2 v \quad[A B 01]$
> $64 u_{1}+61 u_{2}+82 u_{3} \leq 96.3 v[A B 02]$
> $58 u_{1}+61 u_{2}+71 u_{3} \leq 93.5 v \quad[A B 03]$
> $62 \mathrm{u}_{1}+61 \mathrm{u}_{2}+66 \mathrm{u}_{3} \leq 91.6 \mathrm{v} \quad[\mathrm{AB} 04]$
> $61 u_{1}+61 u_{2}+70 u_{3} \leq 90.7 \mathrm{v} \quad[\mathrm{AB} 05]$
> $61 \mathrm{u}_{1}+61 \mathrm{u}_{2}+50 \mathrm{u}_{3} \leq 81.4 \mathrm{v} \quad[\mathrm{AB} 06]$
> $54 \mathrm{u}_{1}+61 \mathrm{u}_{2}+49 \mathrm{u}_{3} \leq 78.7 \mathrm{v} \quad[\mathrm{AB} 07]$
> $58 \mathrm{u}_{1}+61 \mathrm{u}_{2}+43 \mathrm{u}_{3} \leq 75.9 \mathrm{v} \quad[\mathrm{AB} 08]$
> $59 \mathrm{u}_{1}+61 \mathrm{u}_{2}+54 \mathrm{u}_{3} \leq 72.2 \mathrm{v} \quad[\mathrm{AB} 09]$
> $60 \mathrm{u}_{1}+61 \mathrm{u}_{2}+45 \mathrm{u}_{3} \leq 68.5 \mathrm{v}-[\mathrm{AB} 10]$
> $62 \mathrm{u}_{1}+62 \mathrm{u}_{2}+70 \mathrm{u}_{3} \leq 95.4 \mathrm{v} \quad[\mathrm{AD} 01]$
> $62 u_{1}+62 u_{2}+78 u_{3} \leq 95.4 v \quad[A D 02]$
> $61 u_{1}+62 u_{2}+57 u_{3} \leq 91.7 \mathrm{v}$ [AD03]
> $62 u_{1}+62 u_{2}+53 u_{3} \leq 90.8 v \quad[A D 04]$
> $63 u_{1}+62 u_{2}+52 u_{3} \leq 86.1 v \quad[A D 05]$
> $61 u_{1}+62 u_{2}+46 u_{3} \leq 87.0 v \quad[A D 06]$
> $61 u_{1}+62 u_{2}+46 u_{3} \leq 85.2 v \quad[A D 07]$
> $62 \mathrm{u}_{1}+62 \mathrm{u}_{2}+44 \mathrm{u}_{3} \leq 81.5 \mathrm{v} \quad[\mathrm{AD} 08]$
> $63 u_{1}+62 u_{2}+41 u_{3} \leq 80.6 v$ [AD09]
> $61 u_{1}+62 u_{2}+43 u_{3} \leq 75.9 v \quad[A D 10]$
> $58 u_{1}+61 u_{2}+71 u_{3}=1$
$\mathrm{u}_{1}, \mathrm{u}_{2}, \mathrm{u}_{3}, \mathrm{v} \geq 0$

Max


$$
\begin{aligned}
& \text { DMUs for } \mathrm{AB} 04: \quad(\mathrm{Z})=91.6 \mathrm{v} \\
& \text { Subject to } \\
& 61 u_{1}+61 u_{2}+79 u_{3} \leq 97.2 v \quad[A B 01] \\
& 64 u_{1}+61 \mathrm{u}_{2}+82 \mathrm{u}_{3} \leq 96.3 \mathrm{v} \quad[\mathrm{AB} 02] \\
& 58 u_{1}+61 u_{2}+71 u_{3} \leq 93.5 v \quad[A B 03] \\
& 62 u_{1}+61 u_{2}+66 u_{3} \leq 91.6 v \quad[A B 04] \\
& 61 \mathrm{u}_{1}+61 \mathrm{u}_{2}+70 \mathrm{u}_{3} \leq 90.7 \mathrm{v} \quad[\mathrm{AB} 05] \\
& 61 \mathrm{u}_{1}+61 \mathrm{u}_{2}+50 \mathrm{u}_{3} \leq 81.4 \mathrm{v} \quad[\mathrm{AB} 06] \\
& 54 \mathrm{u}_{1}+61 \mathrm{u}_{2}+49 \mathrm{u}_{3} \leq 78.7 \mathrm{v} \quad[\mathrm{AB} 07] \\
& 58 u_{1}+61 u_{2}+43 u_{3} \leq 75.9 v \quad[A B 08] \\
& 59 \mathrm{u}_{1}+61 \mathrm{u}_{2}+54 \mathrm{u}_{3} \leq 72.2 \mathrm{v} \quad[\mathrm{AB} 09] \\
& 60 u_{1}+61 u_{2}+45 u_{3} \leq 68.5 v \quad[A B 10] \\
& 62 \mathrm{u}_{1}+62 \mathrm{u}_{2}+70 \mathrm{u}_{3} \leq 95.4 \mathrm{v} \quad[\mathrm{AD} 01] \\
& 62 \mathrm{u}_{1}+62 \mathrm{u}_{2}+78 \mathrm{u}_{3} \leq 95.4 \mathrm{v} \quad[\mathrm{AD} 02] \\
& 61 \mathrm{u}_{1}+62 \mathrm{u}_{2}+57 \mathrm{u}_{3} \leq 91.7 \mathrm{v} \quad[\mathrm{AD} 03] \\
& 62 \mathrm{u}_{1}+62 \mathrm{u}_{2}+53 \mathrm{u}_{3} \leq 90.8 \mathrm{v} \quad[\mathrm{AD} 04] \\
& 63 u_{1}+62 u_{2}+52 u_{3} \leq 86.1 v \text { [AD05] } \\
& 61 u_{1}+62 u_{2}+46 u_{3} \leq 87.0 v \quad[A D 06] \\
& 61 u_{1}+62 u_{2}+46 u_{3} \leq 85.2 v \quad[A D 07] \\
& 62 u_{1}+62 u_{2}+44 u_{3} \leq 81.5 v \quad[A D 08] \\
& 63 \mathrm{u}_{1}+62 \mathrm{u}_{2}+41 \mathrm{u}_{3} \leq 80.6 \mathrm{v} \quad[\mathrm{AD} 09] \\
& 61 \mathrm{u}_{1}+62 \mathrm{u}_{2}+43 \mathrm{u}_{3} \leq 75.9 \mathrm{v} \quad[\mathrm{AD} 10] \\
& 62 u_{1}+61 u_{2}+66 u_{3}=1
\end{aligned}
$$

xxxi

Max
$\mathrm{u}_{1}, \mathrm{u}_{2}, \mathrm{u}_{3}, \mathrm{v} \geq 0$


$$
\begin{aligned}
& 61 \mathrm{u}_{1}+61 \mathrm{u}_{2}+79 \mathrm{u}_{3} \leq 97.2 \mathrm{v}[\mathrm{AB} 01] \\
& 64 \mathrm{u}_{1}+61 \mathrm{u}_{2}+82 \mathrm{u}_{3} \leq 96.3 \mathrm{v}[\mathrm{AB} 02] \\
& 58 \mathrm{u}_{1}+61 \mathrm{u}_{2}+71 \mathrm{u}_{3} \leq 93.5 \mathrm{v}[\mathrm{AB} 03] \\
& 62 \mathrm{u}_{1}+61 \mathrm{u}_{2}+66 \mathrm{u}_{3} \leq 91.6 \mathrm{v}[\mathrm{AB} 04]
\end{aligned}
$$

$$
61 \mathrm{u}_{1}+61 \mathrm{u}_{2}+70 \mathrm{u}_{3} \leq 90.7 \mathrm{v} \quad[\mathrm{AB} 05]
$$

$$
61 \mathrm{u}_{1}+61 \mathrm{u}_{2}+50 \mathrm{u}_{3} \leq 81.4 \mathrm{v} \quad[\mathrm{AB} 06]
$$

$$
54 \mathrm{u}_{1}+61 \mathrm{u}_{2}+49 \mathrm{u}_{3} \leq 78.7 \mathrm{v} \quad[\mathrm{AB} 07]
$$

$$
58 \mathrm{u}_{1}+61 \mathrm{u}_{2}+43 \mathrm{u}_{3} \leq 75.9 \mathrm{v} \quad[\mathrm{AB} 08]
$$

$$
59 \mathrm{u}_{1}+61 \mathrm{u}_{2}+54 \mathrm{u}_{3} \leq 72.2 \mathrm{v} \quad[\mathrm{AB} 09]
$$

$$
60 \mathrm{u}_{1}+61 \mathrm{u}_{2}+45 \mathrm{u}_{3} \leq 68.5 \mathrm{v} \quad[\mathrm{AB} 10]
$$

$$
62 \mathrm{u}_{1}+62 \mathrm{u}_{2}+70 \mathrm{u}_{3} \leq 95.4 \mathrm{v} \quad[\mathrm{AD} 01]
$$

$$
62 u_{1}+62 u_{2}+78 u_{3} \leq 95.4 v \quad[A D 02]
$$

$$
61 \mathrm{u}_{1}+62 \mathrm{u}_{2}+57 \mathrm{u}_{3} \leq 91.7 \mathrm{v} \quad[\mathrm{AD} 03]
$$

$$
62 \mathrm{u}_{1}+62 \mathrm{u}_{2}+53 \mathrm{u}_{3} \leq 90.8 \mathrm{v} \quad[\mathrm{AD} 04]
$$

$$
63 u_{1}+62 u_{2}+52 u_{3} \leq 86.1 v \quad[A D 05]
$$

$$
61 \mathrm{u}_{1}+62 \mathrm{u}_{2}+46 \mathrm{u}_{3} \leq 87.0 \mathrm{v} \quad[\mathrm{AD} 06]
$$

$$
61 u_{1}+62 u_{2}+46 u_{3} \leq 85.2 \mathrm{v} \quad[\mathrm{AD} 07]
$$

$$
62 \mathrm{u}_{1}+62 \mathrm{u}_{2}+44 \mathrm{u}_{3} \leq 81.5 \mathrm{v} \quad[\mathrm{AD} 08]
$$

$$
63 u_{1}+62 u_{2}+41 u_{3} \leq 80.6 v \quad[A D 09]
$$

$$
61 \mathrm{u}_{1}+62 \mathrm{u}_{2}+43 \mathrm{u}_{3} \leq 75.9 \mathrm{v} \quad[\mathrm{AD} 10]
$$



Max
$61 u_{1}+62 u_{2}+43 u_{3} \leq 75.9 v \quad[A D 10]$
$61 u_{1}+61 u_{2}+50 u_{3}=1$


## Max

to

$61 u_{1}+61 u_{2}+79 u_{3} \quad v \quad[A B 01]$
$64 u_{1}+61 u_{2}+82 u_{3} \leq 96.3 v \quad[A B 02]$
$58 u_{1}+61 u_{2}+71 u_{3} \leq 93.5 v \quad[A B 03]$
$62 u_{1}+61 u_{2}+66 u_{3} \leq 91.6 v \quad[A B 04]$
$61 u_{1}+61 u_{2}+70 u_{3} \leq 90.7 \mathrm{v} \quad[A B 05]$
$61 \mathrm{u}_{1}+61 \mathrm{u}_{2}+50 \mathrm{u}_{3} \leq 81.4 \mathrm{v} \quad[\mathrm{AB} 06]$
$54 \mathrm{u}_{1}+61 \mathrm{u}_{2}+49 \mathrm{u}_{3} \leq 78.7 \mathrm{v} \quad[\mathrm{AB} 07]$
$58 \mathrm{u}_{1}+61 \mathrm{u}_{2}+43 \mathrm{u}_{3} \leq 75.9 \mathrm{v} \quad[\mathrm{AB} 08]$
$59 \mathrm{u}_{1}+61 \mathrm{u}_{2}+54 \mathrm{u}_{3} \leq 72.2 \mathrm{v}[\mathrm{AB} 09]$
$60 u_{1}+61 u_{2}+45 u_{3} \leq 68.5 v \quad[A B 10]$
$62 \mathrm{u}_{1}+62 \mathrm{u}_{2}+70 \mathrm{u}_{3} \leq 95.4 \mathrm{v} \quad[\mathrm{AD} 01]$
$62 \mathrm{u}_{1}+62 \mathrm{u}_{2}+78 \mathrm{u}_{3} \leq 95.4 \mathrm{v} \quad[\mathrm{AD} 02]$
$61 u_{1}+62 u_{2}+57 u_{3} \leq 91.7 \mathrm{v} \quad[A D 03]$
$62 \mathrm{u}_{1}+62 \mathrm{u}_{2}+53 \mathrm{u}_{3} \leq 90.8 \mathrm{v}$ [AD04]
$63 \mathrm{u}_{1}+62 \mathrm{u}_{2}+52 \mathrm{u}_{3} \leq 86.1 \mathrm{v}$ [AD05]
$61 \mathrm{u}_{1}+62 \mathrm{u}_{2}+46 \mathrm{u}_{3} \leq 87.0 \mathrm{v} \quad[\mathrm{AD} 06]$
$61 u_{1}+62 u_{2}+46 u_{3} \leq 85.2 v \quad[A D 07]$
$u_{1}, u_{2}, u_{3}, v \geq 0$

## Max

to


DMUs for AB08: $\quad(\mathrm{Z})=75.9 \mathrm{v}$
Subject
$61 u_{1}+61 u_{2}+79 u_{3} \quad v \quad[A B 01]$
$64 u_{1}+61 u_{2}+82 u_{3} \leq 96.3 v \quad[A B 02]$
$58 u_{1}+61 u_{2}+71 u_{3} \leq 93.5 v \quad[A B 03]$
$62 u_{1}+61 u_{2}+66 u_{3} \leq 91.6 v \quad[A B 04]$
$61 u_{1}+61 u_{2}+70 u_{3} \leq 90.7 v[A B 05]$
$61 u_{1}+61 u_{2}+50 u_{3} \leq 81.4 v \quad[A B 06]$
$54 \mathrm{u}_{1}+61 \mathrm{u}_{2}+49 \mathrm{u}_{3} \leq 78.7 \mathrm{v} \quad[\mathrm{AB} 07]$
$58 \mathrm{u}_{1}+61 \mathrm{u}_{2}+43 \mathrm{u}_{3} \leq 75.9 \mathrm{v} \quad[\mathrm{AB} 08]$
$59 \mathrm{u}_{1}+61 \mathrm{u}_{2}+54 \mathrm{u}_{3} \leq 72.2 \mathrm{v} \quad[\mathrm{AB} 09]$
$60 \mathrm{u}_{1}+61 \mathrm{u}_{2}+45 \mathrm{u}_{3} \leq 68.5 \mathrm{v} \quad[\mathrm{AB} 10]$
$62 \mathrm{u}_{1}+62 \mathrm{u}_{2}+70 \mathrm{u}_{3} \leq 95.4 \mathrm{v} \quad[\mathrm{AD} 01]$
$62 u_{1}+62 u_{2}+78 u_{3} \leq 95.4 v \quad[A D 02]$
$61 \mathrm{u}_{1}+62 \mathrm{u}_{2}+57 \mathrm{u}_{3} \leq 91.7 \mathrm{v} \quad[\mathrm{AD} 03]$
$\mathrm{u}_{1}, \mathrm{u}_{2}, \mathrm{u}_{3}, \mathrm{v} \geq 0$

## Max

to


DMUs for $\mathrm{AB} 09: \quad(\mathrm{Z})=72.2 \mathrm{v}$

## Subject

$61 u_{1}+61 u_{2}+79 u_{3} \quad v[A B 01]$
$64 u_{1}+61 u_{2}+82 u_{3} \leq 96.3 v \quad[A B 02]$
$58 u_{1}+61 u_{2}+71 u_{3} \leq 93.5 v \quad[A B 03]$
$62 u_{1}+61 u_{2}+66 u_{3} \leq 91.6 v \quad[A B 04]$
$61 u_{1}+61 u_{2}+70 u_{3} \leq 90.7 \mathrm{v} \quad[\mathrm{AB} 05]$
$61 \mathrm{u}_{1}+61 \mathrm{u}_{2}+50 \mathrm{u}_{3} \leq 81.4 \mathrm{v} \quad[\mathrm{AB} 06]$
$54 \mathrm{u}_{1}+61 \mathrm{u}_{2}+49 \mathrm{u}_{3} \leq 78.7 \mathrm{v} \quad[\mathrm{AB} 07]$
$58 \mathrm{u}_{1}+61 \mathrm{u}_{2}+43 \mathrm{u}_{3} \leq 75.9 \mathrm{v} \quad[\mathrm{AB} 08]$
$u_{1}, u_{2}, u_{3}, v \geq 0$

## Max

to

$u_{1}, u_{2}, u_{3}, v \geq 0$
xxxviii

## Max

to

$$
\begin{aligned}
& \begin{aligned}
& \leq 97.2 \\
61 \mathrm{u}_{1}+61 \mathrm{u}_{2}+70 \mathrm{u}_{3} & \leq 90.7 \mathrm{v} \quad[\mathrm{AB} 05]
\end{aligned} \\
& 61 u_{1}+61 u_{2}+50 u_{3} \leq 81.4 v \quad[A B 06] \\
& 54 \mathrm{u}_{1}+61 \mathrm{u}_{2}+49 \mathrm{u}_{3} \leq 78.7 \mathrm{v} \quad[\mathrm{AB} 07] \\
& 58 \mathrm{u}_{1}+61 \mathrm{u}_{2}+43 \mathrm{u}_{3} \leq 75.9 \mathrm{v} \quad[\mathrm{AB} 08] \\
& 59 \mathrm{u}_{1}+61 \mathrm{u}_{2}+54 \mathrm{u}_{3} \leq 72.2 \mathrm{v} \quad[\mathrm{AB} 09] \\
& 60 \mathrm{u}_{1}+61 \mathrm{u}_{2}+45 \mathrm{u}_{3} \leq 68.5 \mathrm{v} \quad[\mathrm{AB} 10] \\
& 62 \mathrm{u}_{1}+62 \mathrm{u}_{2}+70 \mathrm{u}_{3} \leq 95.4 \mathrm{v} \quad[\mathrm{AD} 01] \\
& 62 \mathrm{u}_{1}+62 \mathrm{u}_{2}+78 \mathrm{u}_{3} \leq 95.4 \mathrm{v} \quad[\mathrm{AD} 02] \\
& 61 \mathrm{u}_{1}+62 \mathrm{u}_{2}+57 \mathrm{u}_{3} \leq 91.7 \mathrm{v} \quad[A D 03] \\
& 62 u_{1}+62 u_{2}+53 u_{3} \leq 90.8 v \quad[A D 04] \\
& 63 \mathrm{u}_{1}+62 \mathrm{u}_{2}+52 \mathrm{u}_{3} \leq 86.1 \mathrm{v}[\mathrm{AD} 05] \\
& 61 u_{1}+62 u_{2}+46 u_{3} \leq 87.0 v \quad[A D 06] \\
& 61 \mathrm{u}_{1}+62 \mathrm{u}_{2}+46 \mathrm{u}_{3} \leq 85.2 \mathrm{v} \text { [AD07] } \\
& 62 u_{1}+62 u_{2}+44 u_{3} \leq 81.5 v \quad[A D 08] \\
& 63 u_{1}+62 u_{2}+41 u_{3} \leq 80.6 v \quad[A D 09] \\
& 61 u_{1}+62 u_{2}+43 u_{3} \leq 75.9 \mathrm{v} \quad[\mathrm{AD} 10] \\
& 60 u_{1}+61 u_{2}+45 u_{3}=1
\end{aligned}
$$

DMUs for AD01: $\quad(\mathrm{Z})=95.4 \mathrm{v}$

## Subject

$u_{1}, u_{2}, u_{3}, v \geq 0$
xxxix

## Max

to


## Max

to


## Subject

$$
\begin{aligned}
& 61 \mathrm{u}_{1}+61 \mathrm{u}_{2}+79 \mathrm{u}_{3} \\
& 64 \mathrm{u}_{1}+61 \mathrm{u}_{2}+82 \mathrm{u}_{3} \leq 96.3 \mathrm{v} \\
& \left.58 \mathrm{v}_{1}+61 \mathrm{u}_{2}+71 \mathrm{u}_{3} \leq 9301\right] \\
& 62 \mathrm{u}_{1}+61 \mathrm{u}_{2}+66 \mathrm{u}_{3} \leq 91.6 \mathrm{v}[\mathrm{vB} 03] \\
& \left.61 \mathrm{u}_{1}+61 \mathrm{u}_{2}+70 \mathrm{u}_{3} \leq 90.7 \mathrm{v} 04\right] \\
& \left.61 \mathrm{u}_{1}+61 \mathrm{u}_{2}+50 \mathrm{u}_{3} \leq 81.4 \mathrm{v} 05\right]
\end{aligned}
$$

$$
54 \mathrm{u}_{1}+61 \mathrm{u}_{2}+49 \mathrm{u}_{3} \leq 78.7 \mathrm{v} \quad[\mathrm{AB} 07]
$$

$$
58 \mathrm{u}_{1}+61 \mathrm{u}_{2}+43 \mathrm{u}_{3} \leq 75.9 \mathrm{v} \quad[\mathrm{AB} 08]
$$

$$
59 \mathrm{u}_{1}+61 \mathrm{u}_{2}+54 \mathrm{u}_{3} \leq 72.2 \mathrm{v} \quad[\mathrm{AB} 09]
$$

$$
60 \mathrm{u}_{1}+61 \mathrm{u}_{2}+45 \mathrm{u}_{3} \leq 68.5 \mathrm{v} \quad[\mathrm{AB} 10]
$$

$$
62 \mathrm{u}_{1}+62 \mathrm{u}_{2}+70 \mathrm{u}_{3} \leq 95.4 \mathrm{v} \quad[\mathrm{AD} 01]
$$

$$
62 \mathrm{u}_{1}+62 \mathrm{u}_{2}+78 \mathrm{u}_{3} \leq 95.4 \mathrm{v} \quad[\mathrm{AD} 02]
$$

$$
61 \mathrm{u}_{1}+62 \mathrm{u}_{2}+57 \mathrm{u}_{3} \leq 91.7 \mathrm{v} \quad[\mathrm{AD} 03]
$$

$$
62 \mathrm{u}_{1}+62 \mathrm{u}_{2}+53 \mathrm{u}_{3} \leq 90.8 \mathrm{v} \quad[\mathrm{AD} 04]
$$

$$
63 u_{1}+62 u_{2}+52 u_{3} \leq 86.1 v \quad[A D 05]
$$

$$
61 \mathrm{u}_{1}+62 \mathrm{u}_{2}+46 \mathrm{u}_{3} \leq 87.0 \mathrm{v} \quad[\mathrm{AD} 06]
$$

$u_{1}, u_{2}, u_{3}, v \geq 0$

## Max

to


DMUs for AD03: $\quad(Z)=91.7 \mathrm{v}$

Subject
$61 u_{1}+61 u_{2}+79 u_{3} \quad v[A B 01]$
$64 \mathrm{u}_{1}+61 \mathrm{u}_{2}+82 \mathrm{u}_{3} \leq 96.3 \mathrm{v} \quad[\mathrm{AB} 02]$
$58 u_{1}+61 u_{2}+71 u_{3} \leq 93.5 v \quad[A B 03]$
$62 u_{1}+61 u_{2}+66 u_{3} \leq 91.6 v \quad[A B 04]$
$61 u_{1}+61 u_{2}+70 u_{3} \leq 90.7 \mathrm{v}[\mathrm{AB} 05]$
$61 u_{1}+61 u_{2}+50 u_{3} \leq 81.4 v \quad[A B 06]$
$54 \mathrm{u}_{1}+61 \mathrm{u}_{2}+49 \mathrm{u}_{3} \leq 78.7 \mathrm{v} \quad[\mathrm{AB} 07]$
$58 \mathrm{u}_{1}+61 \mathrm{u}_{2}+43 \mathrm{u}_{3} \leq 75.9 \mathrm{v} \quad[\mathrm{AB} 08]$
$59 \mathrm{u}_{1}+61 \mathrm{u}_{2}+54 \mathrm{u}_{3} \leq 72.2 \mathrm{v} \quad[\mathrm{AB} 09]$
$60 u_{1}+61 u_{2}+45 u_{3} \leq 68.5 v \quad[A B 10]$
$u_{1}, u_{2}, u_{3}, v \geq 0$

## Max

to


$$
\text { DMUs for AD04: } \quad(Z)=90.8 v
$$

## Subject

$$
61 \mathrm{u}_{1}+61 \mathrm{u}_{2}+79 \mathrm{u}_{3} \quad \mathrm{v}[\mathrm{AB} 01]
$$

$$
64 \mathrm{u}_{1}+61 \mathrm{u}_{2}+82 \mathrm{u}_{3} \leq 96.3 \mathrm{v} \quad[\mathrm{AB} 02]
$$

$$
58 \mathrm{u}_{1}+61 \mathrm{u}_{2}+71 \mathrm{u}_{3} \leq 93.5 \mathrm{v} \quad[\mathrm{AB} 03]
$$

$$
62 \mathrm{u}_{1}+61 \mathrm{u}_{2}+66 \mathrm{u}_{3} \leq 91.6 \mathrm{v} \quad[\mathrm{AB} 04]
$$

$$
\mathrm{u}_{1}, \mathrm{u}_{2}, \mathrm{u}_{3}, \mathrm{v} \geq 0
$$

## Max

to

$$
\begin{aligned}
& \begin{aligned}
& \leq 97.2 \\
& 61 u_{1}+61 u_{2}+70 u_{3} \leq 90.7 v \quad[A B 05]
\end{aligned} \\
& 61 u_{1}+61 u_{2}+50 u_{3} \leq 81.4 v \quad[A B 06] \\
& 54 \mathrm{u}_{1}+61 \mathrm{u}_{2}+49 \mathrm{u}_{3} \leq 78.7 \mathrm{v} \quad[\mathrm{AB} 07] \\
& 58 \mathrm{u}_{1}+61 \mathrm{u}_{2}+43 \mathrm{u}_{3} \leq 75.9 \mathrm{v} \quad[\mathrm{AB} 08] \\
& 59 \mathrm{u}_{1}+61 \mathrm{u}_{2}+54 \mathrm{u}_{3} \leq 72.2 \mathrm{v} \quad[\mathrm{AB} 09] \\
& 60 u_{1}+61 u_{2}+45 u_{3} \leq 68.5 v \quad[A B 10] \\
& 62 \mathrm{u}_{1}+62 \mathrm{u}_{2}+70 \mathrm{u}_{3} \leq 95.4 \mathrm{v} \quad[\mathrm{AD} 01] \\
& 62 \mathrm{u}_{1}+62 \mathrm{u}_{2}+78 \mathrm{u}_{3} \leq 95.4 \mathrm{v} \quad[\mathrm{AD} 02] \\
& 61 \mathrm{u}_{1}+62 \mathrm{u}_{2}+57 \mathrm{u}_{3} \leq 91.7 \mathrm{v} \quad[\mathrm{AD} 03] \\
& 62 u_{1}+62 u_{2}+53 u_{3} \leq 90.8 v \quad[A D 04] \\
& 63 \mathrm{u}_{1}+62 \mathrm{u}_{2}+52 \mathrm{u}_{3} \leq 86.1 \mathrm{v}[\mathrm{AD} 05] \\
& 61 \mathrm{u}_{1}+62 \mathrm{u}_{2}+46 \mathrm{u}_{3} \leq 87.0 \mathrm{v} \quad \text { [AD06] } \\
& 61 \mathrm{u}_{1}+62 \mathrm{u}_{2}+46 \mathrm{u}_{3} \leq 85.2 \mathrm{v} \text { [AD07] } \\
& 62 u_{1}+62 u_{2}+44 u_{3} \leq 81.5 v \quad[A D 08] \\
& 63 u_{1}+62 u_{2}+41 u_{3} \leq 80.6 v \quad[A D 09] \\
& 61 \mathrm{u}_{1}+62 \mathrm{u}_{2}+43 \mathrm{u}_{3} \leq 75.9 \mathrm{v} \quad[\mathrm{AD} 10] \\
& 62 u_{1}+62 u_{2}+53 u_{3}=1
\end{aligned}
$$

$u_{1}, u_{2}, u_{3}, v \geq 0$

## Max

to


| $61 u_{1}+61 u_{2}+79 u_{3}$ |  | [AB01] |
| :---: | :---: | :---: |
| $64 \mathrm{u}_{1}+61 \mathrm{u}_{2}+82 \mathrm{u}_{3} \leq$ | 96.3v | [AB02] |
| $58 \mathrm{u}_{1}+61 \mathrm{u}_{2}+71 \mathrm{u}_{3} \leq$ | 93.5v | [AB03] |
| $62 u_{1}+61 u_{2}+66 u_{3}$ | 91.6v | [AB04] |
| $61 \mathrm{u}_{1}+61 \mathrm{u}_{2}+70 \mathrm{u}_{3} \leq$ | 90.7 v | [AB05] |
| $61 \mathrm{u}_{1}+61 \mathrm{u}_{2}+50 \mathrm{u}_{3} \leq$ | 81.4v | [AB06] |
| $54 \mathrm{u}_{1}+61 \mathrm{u}_{2}+49 \mathrm{u}_{3} \leq$ | 78.7v | [AB07] |
| $58 \mathrm{u}_{1}+61 \mathrm{u}_{2}+43 \mathrm{u}_{3}$ | . | 8] |
| $59 \mathrm{u}_{1}+61 \mathrm{u}_{2}+54 \mathrm{u}_{3}$ | , | $9]$ |
| $60 u_{1}+61 u_{2}+45 u_{3}$ | 68.5 v | [AB10] |
| $62 \mathrm{u}_{1}+62 \mathrm{u}_{2}+70 \mathrm{u}_{3} \leq$ | 95.4v | [AD01] |
| $62 u_{1}+62 u_{2}+78 u_{3} \leq$ | 95.4v | [AD02] |
| $61 u_{1}+62 u_{2}+57 u_{3} \leq$ | 91.7 v | [AD03] |
| $62 u_{1}+62 u_{2}+53 u_{3} \leq$ | 90.8 v | [AD04] |
| $63 u_{1}+62 u_{2}+52 u_{3} \leq$ |  | [AD05] |
| $61 \mathrm{u}_{1}+62 \mathrm{u}_{2}+46 \mathrm{u}_{3} \leq$ | 87.0v | AD06] |
| $1 u_{1}+62 u_{2}+46 u_{3} \leq$ | 85.2 v | [AD07] |
| WE $\mathrm{u}_{1}, \mathrm{u}_{2}, \mathrm{u}_{3}$ | $\geq 0$ |  |

## Max

to


$$
\text { DMUs for AD06: } \quad(\mathrm{Z})=87.0 \mathrm{v}
$$

Subject

$$
61 u_{1}+61 u_{2}+79 u_{3} \quad v[A B 01]
$$

$$
64 \mathrm{u}_{1}+61 \mathrm{u}_{2}+82 \mathrm{u}_{3} \leq 96.3 \mathrm{v} \quad[\mathrm{AB} 02]
$$

$$
58 \mathrm{u}_{1}+61 \mathrm{u}_{2}+71 \mathrm{u}_{3} \leq 93.5 \mathrm{v} \quad[\mathrm{AB} 03]
$$

$$
62 u_{1}+61 u_{2}+66 u_{3} \leq 91.6 v \quad[\mathrm{AB} 04]
$$

$$
61 \mathrm{u}_{1}+61 \mathrm{u}_{2}+70 \mathrm{u}_{3} \leq 90.7 \mathrm{v} \quad[\mathrm{AB} 05]
$$

$$
61 \mathrm{u}_{1}+61 \mathrm{u}_{2}+50 \mathrm{u}_{3} \leq 81.4 \mathrm{v} \quad[\mathrm{AB} 06]
$$

$$
54 \mathrm{u}_{1}+61 \mathrm{u}_{2}+49 \mathrm{u}_{3} \leq 78.7 \mathrm{v} \quad[\mathrm{AB} 07]
$$

$$
58 \mathrm{u}_{1}+61 \mathrm{u}_{2}+43 \mathrm{u}_{3} \leq 75.9 \mathrm{v} \quad[\mathrm{AB} 08]
$$

$$
59 \mathrm{u}_{1}+61 \mathrm{u}_{2}+54 \mathrm{u}_{3} \leq 72.2 \mathrm{v} \quad[\mathrm{AB} 09]
$$

$$
60 \mathrm{u}_{1}+61 \mathrm{u}_{2}+45 \mathrm{u}_{3} \leq 68.5 \mathrm{v} \quad[\mathrm{AB} 10]
$$

$$
62 \mathrm{u}_{1}+62 \mathrm{u}_{2}+70 \mathrm{u}_{3} \leq 95.4 \mathrm{v} \quad[\mathrm{AD} 01]
$$

$$
u_{1}, u_{2}, u_{3}, v \geq 0
$$

## Max

to

|  | $\leq 97.2$ |
| ---: | :--- |
| $62 u_{1}+62 u_{2}+78 u_{3}$ | $\leq 95.4 v \quad[A D 02]$ |
| $61 u_{1}+62 u_{2}+57 u_{3}$ | $\leq 91.7 v \quad[A D 03]$ |
| $62 u_{1}+62 u_{2}+53 u_{3}$ | $\leq 90.8 v \quad[A D 04]$ |
| $63 u_{1}+62 u_{2}+52 u_{3}$ | $\leq 86.1 v \quad[A D 05]$ |
| $61 u_{1}+62 u_{2}+46 u_{3}$ | $\leq 87.0 v \quad[A D 06]$ |
| $61 u_{1}+62 u_{2}+46 u_{3}$ | $\leq 85.2 v \quad[A D 07]$ |
| $62 u_{1}+62 u_{2}+44 u_{3}$ | $\leq 81.5 v \quad[A D 08]$ |
| $63 u_{1}+62 u_{2}+41 u_{3}$ | $\leq 80.6 v \quad[A D 09]$ |
| $61 u_{1}+62 u_{2}+43 u_{3}$ | $\leq 75.9 v \quad[A D 10]$ |
| $61 u_{1}+62 u_{2}$ | $+46 u_{3}=1$ |

DMUs for AD07: $\quad(\mathrm{Z})=85.2 \mathrm{v}$
Subject
$61 u_{1}+61 u_{2}+79 u_{3} \quad v \quad[A B 01]$
$64 u_{1}+61 u_{2}+82 u_{3} \leq 96.3 v \quad[A B 02]$
$58 u_{1}+61 u_{2}+71 u_{3} \leq 93.5 v \quad[A B 03]$
$62 u_{1}+61 u_{2}+66 u_{3} \leq 91.6 v \quad[A B 04]$
$61 \mathrm{u}_{1}+61 \mathrm{u}_{2}+70 \mathrm{u}_{3} \leq 90.7 \mathrm{v} \quad[\mathrm{AB} 05]$
$\mathrm{u}_{1}, \mathrm{u}_{2}, \mathrm{u}_{3}, \mathrm{v} \geq 0$

## Max

to

$$
\begin{aligned}
& \begin{aligned}
& \leq 97.2 \\
61 u_{1}+61 u_{2}+50 u_{3} & \leq 81.4 \mathrm{v} \quad[\mathrm{AB} 06] \\
54 \mathrm{u}_{1}+61 \mathrm{u}_{2}+49 \mathrm{u}_{3} & \leq 78.7 \mathrm{v} \quad[\mathrm{AB} 07]
\end{aligned} \\
& 58 u_{1}+61 u_{2}+43 u_{3} \leq 75.9 \mathrm{v} \quad[\mathrm{AB} 08] \\
& 59 \mathrm{u}_{1}+61 \mathrm{u}_{2}+54 \mathrm{u}_{3} \leq 72.2 \mathrm{v} \quad[\mathrm{AB} 09] \\
& 60 u_{1}+61 u_{2}+45 u_{3} \leq 68.5 v \quad[A B 10] \\
& 62 \mathrm{u}_{1}+62 \mathrm{u}_{2}+70 \mathrm{u}_{3} \leq 95.4 \mathrm{v} \quad[\mathrm{AD} 01] \\
& 62 u_{1}+62 u_{2}+78 u_{3} \leq 95.4 v \quad[A D 02] \\
& 61 u_{1}+62 u_{2}+57 u_{3} \leq 91.7 v \quad[A D 03] \\
& 62 \mathrm{u}_{1}+62 \mathrm{u}_{2}+53 \mathrm{u}_{3} \leq 90.8 \mathrm{v} \quad[\mathrm{AD} 04] \\
& 63 \mathrm{u}_{1}+62 \mathrm{u}_{2}+52 \mathrm{u}_{3} \leq 86.1 \mathrm{v} \text { [AD05] } \\
& 61 u_{1}+62 u_{2}+46 u_{3} \leq 87.0 \mathrm{v} \quad[A D 06] \\
& 61 u_{1}+62 u_{2}+46 u_{3} \leq 85.2 \mathrm{v} \quad[\mathrm{AD} 07] \\
& 62 \mathrm{u}_{1}+62 \mathrm{u}_{2}+44 \mathrm{u}_{3} \leq 81.5 \mathrm{v} \quad[\mathrm{AD} 08] \\
& 63 u_{1}+62 u_{2}+41 u_{3} \leq 80.6 v \quad[A D 09] \\
& 61 u_{1}+62 u_{2}+43 u_{3} \leq 75.9 v \quad[A D 10] \\
& 61 u_{1}+62 u_{2}+46 u_{3}=1
\end{aligned}
$$

DMUs for AD08: $\quad(\mathrm{Z})=81.5 \mathrm{v}$
$\mathrm{u}_{1}, \mathrm{u}_{2}, \mathrm{u}_{3}, \mathrm{v} \geq 0$
xlviii

## Max

to

$64 \mathrm{u}_{1}+61 \mathrm{u}_{2}+82 \mathrm{u}_{3} \leq 96.3 \mathrm{v} \quad[\mathrm{AB} 02]$
$58 \mathrm{u}_{1}+61 \mathrm{u}_{2}+71 \mathrm{u}_{3} \leq 93.5 \mathrm{v} \quad[\mathrm{AB} 03]$
$62 u_{1}+61 u_{2}+66 u_{3} \leq 91.6 v \quad[A B 04]$
$61 u_{1}+61 u_{2}+70 u_{3} \leq 90.7 \mathrm{v} \quad[\mathrm{AB} 05]$
$61 \mathrm{u}_{1}+61 \mathrm{u}_{2}+50 \mathrm{u}_{3} \leq 81.4 \mathrm{v} \quad[\mathrm{AB} 06]$
$54 \mathrm{u}_{1}+61 \mathrm{u}_{2}+49 \mathrm{u}_{3} \leq 78.7 \mathrm{v} \quad[\mathrm{AB} 07]$
$58 \mathrm{u}_{1}+61 \mathrm{u}_{2}+43 \mathrm{u}_{3} \leq 75.9 \mathrm{v} \quad[\mathrm{AB} 08]$
$59 \mathrm{u}_{1}+61 \mathrm{u}_{2}+54 \mathrm{u}_{3} \leq 72.2 \mathrm{v} \quad[\mathrm{AB} 09]$
$60 \mathrm{u}_{1}+61 \mathrm{u}_{2}+45 \mathrm{u}_{3} \leq 68.5 \mathrm{v} \quad[\mathrm{AB} 10]$
$62 \mathrm{u}_{1}+62 \mathrm{u}_{2}+70 \mathrm{u}_{3} \leq 95.4 \mathrm{v} \quad[\mathrm{AD} 01]$
$62 \mathrm{u}_{1}+62 \mathrm{u}_{2}+78 \mathrm{u}_{3} \leq 95.4 \mathrm{v} \quad[\mathrm{AD} 02]$
$61 u_{1}+62 u_{2}+57 u_{3} \leq 91.7 v \quad[A D 03]$
$62 \mathrm{u}_{1}+62 \mathrm{u}_{2}+53 \mathrm{u}_{3} \leq 90.8 \mathrm{v}[\mathrm{AD} 04]$
$63 \mathrm{u}_{1}+62 \mathrm{u}_{2}+52 \mathrm{u}_{3} \leq 86.1 \mathrm{v}$ [AD05]
$61 u_{1}+62 u_{2}+46 u_{3} \leq 87.0 v \quad[A D 06]$
$61 u_{1}+62 u_{2}+46 u_{3} \leq 85.2 v \quad[A D 07]$
$62 \mathrm{u}_{1}+62 \mathrm{u}_{2}+44 \mathrm{u}_{3} \leq 81.5 \mathrm{v}$ [AD08]
$63 u_{1}+62 u_{2}+41 u_{3} \leq 80.6 v$ [AD09]
$\mathrm{u}_{1}, \mathrm{u}_{2}, \mathrm{u}_{3}, \mathrm{v} \geq 0$

## Max

to


DMUs for AD09: $\quad(\mathrm{Z})=80.6 \mathrm{v}$
Subject
$61 u_{1}+61 u_{2}+79 u_{3}$
v [AB01]
$64 u_{1}+61 u_{2}+82 u_{3} \leq 96.3 v \quad[A B 02]$
$58 \mathrm{u}_{1}+61 \mathrm{u}_{2}+71 \mathrm{u}_{3} \leq 93.5 \mathrm{v} \quad[\mathrm{AB} 03]$
$62 u_{1}+61 u_{2}+66 u_{3} \leq 91.6 v \quad[A B 04]$
$61 u_{1}+61 u_{2}+70 u_{3} \leq 90.7 \mathrm{v} \quad[\mathrm{AB} 05]$
$61 \mathrm{u}_{1}+61 \mathrm{u}_{2}+50 \mathrm{u}_{3} \leq 81.4 \mathrm{v} \quad[\mathrm{AB} 06]$
$54 \mathrm{u}_{1}+61 \mathrm{u}_{2}+49 \mathrm{u}_{3} \leq 78.7 \mathrm{v} \quad[\mathrm{AB} 07]$
$58 u_{1}+61 u_{2}+43 u_{3} \leq 75.9 \mathrm{v} \quad[\mathrm{AB} 08]$
$59 \mathrm{u}_{1}+61 \mathrm{u}_{2}+54 \mathrm{u}_{3} \leq 72.2 \mathrm{v} \quad[\mathrm{AB} 09]$
$60 \mathrm{u}_{1}+61 \mathrm{u}_{2}+45 \mathrm{u}_{3} \leq 68.5 \mathrm{v} \quad[\mathrm{AB} 10]$
$62 \mathrm{u}_{1}+62 \mathrm{u}_{2}+70 \mathrm{u}_{3} \leq 95.4 \mathrm{v} \quad[\mathrm{AD} 01]$
$62 \mathrm{u}_{1}+62 \mathrm{u}_{2}+78 \mathrm{u}_{3} \leq 95.4 \mathrm{v} \quad$ [AD02]
$61 \mathrm{u}_{1}+62 \mathrm{u}_{2}+57 \mathrm{u}_{3} \leq 91.7 \mathrm{v} \quad[\mathrm{AD} 03]$
$\mathrm{u}_{1}, \mathrm{u}_{2}, \mathrm{u}_{3}, \mathrm{v} \geq 0$

## Max

to


$$
63 u_{1}+62 u_{2}+41 u_{3}=1
$$

DMUs for AD10: $\quad(\mathrm{Z})=75.9 \mathrm{v}$

## Subject

$61 u_{1}+61 u_{2}+79 u_{3} \quad v \quad[A B 01]$
$64 \mathrm{u}_{1}+61 \mathrm{u}_{2}+82 \mathrm{u}_{3} \leq 96.3 \mathrm{v} \quad[\mathrm{AB} 02]$
$58 u_{1}+61 u_{2}+71 u_{3} \leq 93.5 v \quad[A B 03]$
$62 u_{1}+61 u_{2}+66 u_{3} \leq 91.6 v \quad[A B 04]$
$61 \mathrm{u}_{1}+61 \mathrm{u}_{2}+70 \mathrm{u}_{3} \leq 90.7 \mathrm{v} \quad[\mathrm{AB} 05]$
$61 \mathrm{u}_{1}+61 \mathrm{u}_{2}+50 \mathrm{u}_{3} \leq 81.4 \mathrm{v} \quad[\mathrm{AB} 06]$
$54 u_{1}+61 u_{2}+49 u_{3} \leq 78.7 v \quad[A B 07]$
$u_{1}, u_{2}, u_{3}, v \geq 0$

## Max

to

$61 \mathrm{u}_{1}+62 \mathrm{u}_{2}+57 \mathrm{u}_{3} \leq 91.7 \mathrm{v} \quad[\mathrm{AD} 03]$
$62 u_{1}+62 u_{2}+53 u_{3} \leq 90.8 v \quad[A D 04]$
$63 \mathrm{u}_{1}+62 \mathrm{u}_{2}+52 \mathrm{u}_{3} \leq 86.1 \mathrm{v}$ [AD05]
$61 u_{1}+62 u_{2}+46 u_{3} \leq 87.0 \mathrm{v} \quad[\mathrm{AD} 06]$
$61 \mathrm{u}_{1}+62 \mathrm{u}_{2}+46 \mathrm{u}_{3} \leq 85.2 \mathrm{v} \quad[\mathrm{AD} 07]$
$62 \mathrm{u}_{1}+62 \mathrm{u}_{2}+44 \mathrm{u}_{3} \leq 81.5 \mathrm{v} \quad[\mathrm{AD} 08]$
$63 u_{1}+62 u_{2}+41 u_{3} \leq 80.6 \mathrm{v}$ [AD09]
$61 \mathrm{u}_{1}+62 \mathrm{u}_{2}+43 \mathrm{u}_{3} \leq 75.9 \mathrm{v} \quad$ [AD10]
$61 u_{1}+62 u_{2}+43 u_{3}=1$
$u_{1}, u_{2}, u_{3}, v \geq 0$

## Appendix F

Table 4.3.3.1 shows the relative efficiency of Students Attendance $\left(\mathrm{U}_{1}\right)$ against output of Students B.E.C.E performance (V) for students at Abu Bonsra D/A JHS and Adiembra D/A JHS at Fomena and Asokwa Circuits.

| DMUs | U1 | U2 | U3 | V | V/U1 | ( $\mathbf{1}$ | CCR( $\left.\boldsymbol{\Phi}_{\mathbf{1}}\right)$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| AB03 | 58 | 61 | 71 | 93.5 | 1.612069 | 1.000000 | 1.00 |
| AB01 | 61 | 61 | 79 | 97.2 | 1.593443 | 0.988446 | 0.99 |
| AD01 | 62 | 62 | 70 | 95.4 | 1.538710 | 0.954494 | 0.95 |
| AD02 | 62 | 62 | 78 | 95.4 | 1.538710 | 0.954494 | 0.95 |
| AB02 | 64 | 61 | 82 | 96.3 | 1.504688 | 0.933389 | 0.93 |
| AD03 | 61 | 62 | 57 | 91.7 | 1.503279 | 0.932515 | 0.93 |
| AB05 | 61 | 61 | 70 | 90.7 | 1.486885 | 0.922346 | 0.92 |
| AB04 | 62 | 61 | 66 | 91.6 | 1.477419 | 0.916474 | 0.92 |
| AD04 | 62 | 62 | 53 | 90.8 | 1.464516 | 0.908470 | 0.91 |
| AB07 | 54 | 61 | 49 | 78.7 | 1.457407 | 0.904060 | 0.90 |
| AD06 | 61 | 62 | 46 | 87.0 | 1.426230 | 0.884720 | 0.88 |
| AD07 | 61 | 62 | 46 | 85.2 | 1.396721 | 0.866415 | 0.87 |
| AD05 | 63 | 62 | 52 | 86.1 | 1.366667 | 0.847772 | 0.85 |
| AB06 | 61 | 61 | 50 | 81.4 | 1.334426 | 0.827772 | 0.83 |
| AD08 | 62 | 62 | 44 | 81.5 | 1.314516 | 0.815422 | 0.82 |
| AB08 | 58 | 61 | 43 | 75.9 | 1.308621 | 0.811765 | 0.81 |
| AD09 | 63 | 62 | 41 | 80.6 | 1.279365 | 0.793617 | 0.79 |
| AD10 | 61 | 62 | 43 | 75.9 | 1.244262 | 0.771842 | 0.77 |
| AB09 | 59 | 61 | 54 | 72.2 | 1.223729 | 0.759105 | 0.76 |
| AB010 | 60 | 61 | 45 | 68.5 | 1.141667 | 0.708200 | 0.71 |

Key: $\mathbf{C C R}\left(\boldsymbol{\Phi}_{1}\right)$ is the relative efficiency for input $u_{1}$ Source:

## Author's Construct, April 2016.

Table 4.3.3.2 shows the relative efficiency of Teachers Attendance $\left(U_{2}\right)$ against output of Students B.E.C.E performance (V) for students at Abu Bonsra D/A JHS and Adiembra D/A JHS at Fomena and Asokwa Circuits.

| DMUs | U1 | U2 | U3 | V | V/U2 | $\left(\boldsymbol{\Phi}_{2}\right)$ | CCR $\left(\boldsymbol{\Phi}_{2}\right)$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| AB01 | 61 | 61 | 79 | 97.2 | 1.593443 | 1.000000 | 1.00 |
| AB02 | 64 | 61 | 82 | 96.3 | 1.578689 | 0.990741 | 0.99 |
| AD01 | 62 | 62 | 70 | 95.4 | 1.538710 | 0.965651 | 0.97 |
| AD02 | 62 | 62 | 78 | 95.4 | 1.538710 | 0.965651 | 0.97 |
| AB03 | 58 | 61 | 71 | 93.5 | 1.532787 | 0.961934 | 0.96 |
| AB04 | 62 | 61 | 66 | 91.6 | 1.501639 | 0.942386 | 0.94 |
| AB05 | 61 | 61 | 70 | 90.7 | 1.486885 | 0.933127 | 0.93 |
| AD03 | 61 | 62 | 57 | 91.7 | 1.479032 | 0.928199 | 0.93 |
| AD04 | 62 | 62 | 53 | 90.8 | 1.464516 | 0.919089 | 0.92 |
| AD06 | 61 | 62 | 46 | 87.0 | 1.403226 | 0.880625 | 0.88 |
| AD05 | 63 | 62 | 52 | 86.1 | 1.388710 | 0.871515 | 0.87 |
| AD07 | 61 | 62 | 46 | 85.2 | 1.374194 | 0.862405 | 0.86 |
| AB06 | 61 | 61 | 50 | 81.4 | 1.334426 | 0.837448 | 0.84 |
| AD08 | 62 | 62 | 44 | 81.5 | 1.314516 | 0.824953 | 0.83 |
| AD09 | 63 | 62 | 41 | 80.6 | 1.300000 | 0.815843 | 0.82 |
| AB07 | 54 | 61 | 49 | 78.7 | 1.290164 | 0.809671 | 0.81 |
| AB08 | 58 | 61 | 43 | 75.9 | 1.244262 | 0.780864 | 0.78 |
| AD10 | 61 | 62 | 43 | 75.9 | 1.224194 | 0.768269 | 0.77 |
| AB09 | 59 | 61 | 54 | 72.2 | 1.183607 | 0.742798 | 0.74 |
| AB10 | 60 | 61 | 45 | 68.5 | 1.122951 | 0.704732 | 0.70 |

Key: $\operatorname{CCR}\left(\mathbf{\Phi}_{2}\right)$ is the relative efficiency for input $u_{2}$ Source:
Author's Construct, April 2016.

Table 4.3.3.3 shows the relative efficiency of student continuous assessment $\left(\mathrm{U}_{3}\right)$ against output of Students B.E.C.E performance (V) for students at Abu Bonsra D/A JHS and Adiembra D/A JHS at Fomena and Asokwa Circuits.

| DMUs | U1 | U2 | U3 | V | V/U3 | $\left(\boldsymbol{\Phi}_{3}\right)$ | CCR $\left(\boldsymbol{\Phi}_{3}\right)$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| AD09 | 63 | 62 | 41 | 80.6 | 1.965854 | 1.000000 | 1.00 |
| AD06 | 61 | 62 | 46 | 87.0 | 1.891304 | 0.962078 | 0.96 |
| AD08 | 62 | 62 | 44 | 81.5 | 1.852273 | 0.942223 | 0.94 |
| AD07 | 61 | 62 | 46 | 85.2 | 1.852174 | 0.942173 | 0.94 |
| AD10 | 61 | 62 | 43 | 75.9 | 1.765116 | 0.897888 | 0.90 |
| AB08 | 58 | 61 | 43 | 75.9 | 1.765116 | 0.897888 | 0.90 |
| AD04 | 62 | 62 | 53 | 90.8 | 1.713208 | 0.871483 | 0.87 |
| AD05 | 63 | 62 | 52 | 86.1 | 1.655769 | 0.842265 | 0.84 |
| AB06 | 61 | 61 | 50 | 81.4 | 1.628000 | 0.828139 | 0.83 |
| AD03 | 61 | 62 | 57 | 91.7 | 1.608772 | 0.818358 | 0.82 |
| AB07 | 54 | 61 | 49 | 78.7 | 1.606122 | 0.817010 | 0.82 |
| AB10 | 60 | 61 | 45 | 68.5 | 1.522222 | 0.774331 | 0.77 |
| AB04 | 62 | 61 | 66 | 91.6 | 1.387879 | 0.705993 | 0.71 |
| AD01 | 62 | 62 | 70 | 95.4 | 1.362857 | 0.693265 | 0.69 |
| AB09 | 59 | 61 | 54 | 72.2 | 1.337037 | 0.680130 | 0.68 |
| AB03 | 58 | 61 | 71 | 93.5 | 1.316901 | 0.669888 | 0.67 |
| AB05 | 61 | 61 | 70 | 90.7 | 1.295714 | 0.659110 | 0.66 |
| AB01 | 61 | 61 | 79 | 97.2 | 1.230380 | 0.625876 | 0.63 |
| AD02 | 62 | 62 | 78 | 95.4 | 1.223077 | 0.622161 | 0.62 |
| AB02 | 64 | 61 | 82 | 96.3 | 1.174390 | 0.597394 | 0.60 |

Key: $\operatorname{CCR}\left(\boldsymbol{\Phi}_{3}\right)$ is the relative efficiency for input $u_{3}$ Source:
Author's Construct, April 2016.

Table 4.3.3.4 shows the relative efficiency of average input $\left(\mathrm{U}^{+}\right)$against output of Students B.E.C.E performance (V) for students at Abu Bonsra D/A JHS and Adiembra D/A JHS at Fomena and Asokwa Circuits.

| DMUs | $\mathrm{U}_{1}$ | $\mathrm{U}_{2}$ |  | $\mathbf{U}^{+}$ |  | V/U ${ }^{+}$ | $\boldsymbol{\Phi}_{+}$ | $\operatorname{CCR}\left(\Phi^{+}\right)$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| AD06 | 61 | 62 | 46 | 56.33333 |  | 1.544379 | 1.000000 | 1.00 |
| AD04 | 62 | 62 | 53 | 59.00000 | 90.8 | 1.538983 | 0.996506 | 1.00 |
| AD03 | 61 | 62 | 57 | 60.00000 | 91.7 | 1.528333 | 0.989610 | 0.99 |
| AD07 | 61 | 62 | 46 | 56.33333 | 85.2 | 1.512426 | 0.979310 | 0.98 |
| AB03 | 58 | 61 | 71 | 63.33333 | 93.5 | 1.476316 | 0.955929 | 0.96 |
| AD01 | 62 | 62 | 70 | 64.66667 | 95.4 | 1.475258 | 0.955244 | 0.96 |
| AD05 | 63 | 62 | 52 | 59.00000 | 86.1 | 1.459322 | 0.944925 | 0.94 |
| AD09 | 63 | 62 | 41 | 55.33333 | 80.6 | 1.456627 | 0.943180 | 0.94 |
| AD08 | 62 | 62 | 44 | 56.00000 | 81.5 | 1.455357 | 0.942358 | 0.94 |
| AB04 | 62 | 61 | 66 | 63.00000 | 91.6 | 1.453968 | 0.941458 | 0.94 |
| AB01 | 61 |  | 79 | 67.00000 | 97.2 | 1.450746 | 0.939372 | 0.94 |
| AB07 | 54 | 61 | 9 | 54.66667 | 78.7 | 1.439634 | 0.932177 | 0.93 |
| AB06 | 61 | 61 | 50 | 57.33333 | 81.4 | 1.419767 | 0.919313 | 0.92 |
| AB05 | 61 | 61 | 0 | 64.00000 | 90.7 | 1.417188 | 0.917642 | 0.92 |
| AD02 | 62 | 62 | 78 | 67.33333 | 95.4 | 1.416832 | 0.917412 | 0.92 |
| AB08 | 58 | 61 | 43 | 54.00000 | 75.9 | 1.405556 | 0.910111 | 0.91 |
| AB02 | 64 | 61 |  | 69.00000 | 96.3 | 1.395652 | 0.903698 | 0.90 |
| AD10 |  | 62 | 43 | 55.33333 | 75.9 | 1.371687 | 0.888180 | 0.89 |
| AB09 |  | 61 | 54 | 58.00000 | 72.2 | 1.244828 | 0.806038 | 0.81 |
| AB010 |  | 61 | 45 | 55.33333 | 68.5 | 1.237952 | 0.801586 | 0.80 |

Key: $\operatorname{CCR}\left(\Phi^{+}\right)$is the relative efficiency for average input $u^{+}$Source:

## Author's Construct, April 2016.

## CHAPTER ONE

### 1.0 INTRODUCTION

The continual decline in the performance of pupils at the Basic Education Certificate Examination (BECE) poses a significant threat to education in Ghana (EMIS Report (2012). This issue has become a national burden which stakeholders have strived in finding control measures. There are many factors which contribute to such performance and that is what this project seek to address and to make relevant control measures available to stakeholders. The Ministry of Education is constitutionally mandated to provide significant and quality education at all levels, to all Ghanaians for human and national development and to create a non-static education sector that can prepare and equip all citizens with relevant education and skills to produce a high standard of Human capital. (www.moe.gov.gh/2013 Annual performance report of civil service pp 62)

According to en.Wikipedia.org, the core duty of the Ministry of Education is for the administration and the coordination of policy formulation and direction regarding Education. According to www.moe.gov.gh, the Ministry is also mandated to formulate policies and seeks to set out action plans in making these policies a reality. These policies should be able to provide quality, equitable and accessible education to all citizens with emphasis on Science, Information, Communication and Technology for self -actualization and peaceful coexistence, as well as needed skills for the labor market and material development. The Ministry of Education has these multiple agencies (GES, Ghana Library Board, NACVET etc.) that assist in handling the concrete implementations of policies. All these are also done in cooperation with the Local authorities (10 regional and 138 district offices).

Ghanaian education was mainly informal before the arrival of European settlers. It was these

European settlers who built a formal education system (en.Wikipedia.org). The education Act 1987, followed by the constitution of 1992 brought about new insights and expositions to educational policies in the country.

The 2008 Education Act (Act 778) which is current in operation has its purpose to produce well balanced individuals with the requisite knowledge, skills, values, aptitudes, and attitudes to become functional and productive citizens for the total development and democratic advancement of the nation. The system of education under this Act is organized in three progressive levels known as Basic education, Second cycle education and Tertiary education. The Basic Education lasts 11 years (with 2 years in Kindergarten, 6 years in Primary School and 3 years in Junior High School) which ends on the Basic Education Certificate Examination (Ghana Education ServicelSchool Management Committee Resource Handbook pp 4, 5). With the above, the following subjects are considered to the award of the certificate: English

Language, Ghanaian Language and Culture, Social Studies, Integrated Science, Mathematics, Information and Communication Technology (ICT), French (optional), Basic Design and Technology, Religious and Moral Education (en.Wikipedia.org). Once the certificate is obtained, the pupil then proceeds to pursue secondary education.

In the Statistics, Research, Information Management and Public Relations (SRMPR) 2013 report on evaluation of workshop on research and education policy in secondary schools, pages 2-3, (submitted to International Institute for Educational Panning/student paper) the Hon Deputy Minister of Education, Hon Alex Kyeremeh commented on how education has enabled many families to change their social status and move up the social ladder within generations. He commended the Education Management and Information System (EMIS) that provided the Ministry with information on the progress in ensuring access to primary education data for all

Ghanaians. The Minister touched on some figures based on the EMIS data that there has been an increase in the Primary Gross Enrolment Rate (PGER) from 96.5\% in the 2011/2012 academic year to $105 \%$ in the 2012/2013 academic year and increase in the Primary Net Enrolment Rate (PNER) from $81.7 \%$ in the $2011 / 2012$ academic year to $84.1 \%$ in the 2012/2013 academic year.

According to the 2013 Education Sector Performance Report page 96, the total expenditure on education in 2012 was GHS 5.7 billion representing a $60 \%$ increase from GHS 3.6 billion in 2011(Submitted to All India Institute of speech and hearing/student paper).

The Millennium Development Goal report on October 2013 page 17 submitted to Leads Metropolitan University (student paper), the Goal 2 of the Millennium Development Goals (MDGs) which is to achieve universal primary education has a target to ensure that by 2015, children everywhere, boys and girls alike, will be able to complete a full course of primary schooling. This target is to be measured by three indicators which are Net Enrolment Ratio (NER) in primary education, Proportion of pupils starting grade 1 who reach last grade of primary education and literacy rate of 15-24 years old, women and men (submitted to university of London External system/student paper).

Inference from above, Government is using all resources possible within her capacity to promote education among the citizens of Ghana.

### 1.1 Background of the study

Education in Ghana especially at the basic level, has witnessed a downward trend in academic output. This has led to a decline in the Basic Education Certificate Examination (BECE) performance over the years. For about a decade now, almost $50 \%$ of all pupils who write the BECE do not get admission into the Senior High Schools or Technical and Vocational Schools. (submitted to Kwame Nkrumah University of Science and Technology/student paper) According to the Education Sector Performance Report in August 2013 page 15, the number of schools which were captured in the EMIS schools census reports that in 2008/2009 academic year, there were 11,827 public schools. 12481 in 2009/2010 academic year, 13263 in 2010/2011 academic year, 13505 in 2011/2012 academic year, 13305 in 2012/2013 academic year for the Kindergarten division (www.moe.gov.gh). There were 13510 public schools in 2008/2009 academic year, 13835 in 2009/2010 academic year, 14431 in 2010/2011 academic year, 14360 in 2011/2012, 14112 in the 2012/2013 academic year for the primary division. In the Junior High School division, there were 7656 public schools in the 2008/2009 academic year, 7969 in 2009/2010 academic year, 8462 in 2010/2011 academic year, 8336 in the 2011/2012 academic year and 8818 in the 2012/2013 academic year.

The Education Sector Performance Report in August 2013 page 37 also attested that, the Gross Enrolment Ratios (GERs) and Gender Parity Index (GPI) indicates that in 2008/2009 academic year, the Female Gender Enrolment Ratio (FGER) was 92.2 and the Male Gender Enrolment Ratio (MGER) was 93.5 making the Gender Parity Index (GPI) to be 0.99 for the Kindergarten division. FGER was 96.5 and MGER was 98.1 with a GPI 0.98 in 2009/2010 academic year. In 2010/2011 academic year, FGER was 97.5, MGER was 99.2 and GPI was 0.98. In 2011/2012, FGER was 98.4, MGER was 100.4 and GPI was 0.98 . In 2012/2013, FGER was
98.4, MGER was 112.4 and GPI of 1.03. For the Primary division, FGER was 92.79 , MGER was 96.97 and GPI of 0.96 in the 2008/2009 academic year. In 2009/2010 academic year, FGER was 93.03, MGER was 96.7 and GPI of 0.96. In 2010/2011 academic year, FGER was 94.7, MGER was 98 and GPI of 0.97 . In 2011/2012 academic year, FGER was 94.9, MGER was 97.9 and GPI of 0.97. In 2012/2013 academic year, FGER was 104.5, MGER was 105.5 and GPI of 0.99 . For the Junior High School division, it was recorded that in the 2008/2009 academic year, FGER was 76.93, MGER was 84.05 and GPI of 0.92. In 2009/2010 academic year, FGER was 76.3, MGER was 82.5 with GPI of 0.92 . In 2010/2011 academic year, FGER was 76.4, MGER was 82.6 and GPI of 0.93 . In 2011/2012 academic year, FGER was 78.1, MGER was 83 with GPI of 0.94 and in 2012/2013 academic year, FGER was 79.2, MGER was 85.2 and GPI of 0.93 .

There are at times, where teachers and other stakeholders do their best but pupils fail to perform well in the BECE. A lot of factors contribute in the pupils not performing which includes their attendance to school and continuous assessment with teaches attendance inclusive (Duckor Brent Catellano, Katherine E. Tellez, Kip Wihardini, Diah Wilson, Mark.'Examining the internal structure evidence for the performance assessment for carlifonia teachers".

Students who get the opportunity to be active in school from kindergarten to their Junior High School always do not up with the same results as students who were not active in school. There is always the need to measure the efficiency of a pupil's performance in the Basic Education Certificate Examination to be able to know where the Ghana Education Service can improve in terms of teaching delivery and how we can encourage students to perform better.

### 1.2 Profile of the study

According to www. Adansinorth.ghanadistricts.gov.gh/internet source, Adansi North is one of the 216 districts in Ghana situated in the Ashanti Region. The Adansi North District covers a total land area of 828 square kilometers with most of its communities located in the hinterland, approximately 15 towns. The number of communities along the major roads is far less than those in the hinterland. This district is predominantly farming area with the cultivation of cash crops, palm trees and citrus. Adansi North District contains six major national forest reserves and Obuasi Gold mine, one of Anglogold Ashanti's largest Gold mines. The district has 195 schools and 4 major hospitals. A greater number of the basic schools in the district are public and are situated in the hinterland.

The seven schools sampled for this project are all public schools. Though there are sixty nine (69) JHS in the district, I sampled these seven (7) schools being a school from each circuit in the District Education Directorate because they have their primary and kindergarten attached to the Junior High School for easy collection of data and their school standards are nearly homogenous.

### 1.3 Problem statement

The Adansi North District has witnessed a decline in the performance of the BECE results. This could be there is no technical way of measuring efficiency of the students. This research aids in providing a technical way of measuring the technical efficiency of students BECE performance.

### 1.4 Objectives of the study I.

## Main objective

To measure the efficiency of Basic students performance in the Adansi North District.

## II. Specific objectives

- To determine the extent to which teachers and students attendance affect the BECE results
- To determine the extent to which continuous assessment of pupils affect the BECE Results
- To make recommendations based on findings on how Basic students performance can be improved in the Adansi North District.


### 1.5 Methodology

The study employed the use of both secondary and primary data. The latter was collected from the Head teachers of the various schools. The secondary data were collected from the Adansi North Education Directorate to enable me check the validity of all primary data received from the head teachers at the various schools. The data received was modeled using Data Envelopment Analysis (DEA). The study covered a period of 8 months from August 2014 to April 2016. Challenges related to the project include the inability of getting students records when the student started their primary education other than where they attended their Junior High School. It was also difficult retrieving attendance of students when they have completed their Basic Education. Since no funds were given to me to embark the project, most of all monies spent on transportation, printing, calls etc. has to come from my own pocket.

### 1.6 Significance of the study

The performance of students at the end of their Basic Education does not only get a secondary education for them but channels the course of a profession one may end up with. When students at the Basic Education get to build a high attitude towards education, it helps to build a strong foundation for the human capacity of the Nation.

In the Statistics, Research, Information Management and Public Relations (SRMPR) 2013 report page 3, the Deputy Minister of Education, indicated that the role of research in education should not be considered only as a mere academic exercise but one that should meet the needs of policy makers and policy implementers.

In the Statistics, Research, Information Management and Public Relations (SRMPR) 2013 submitted to International Institute for Educational Planning/ student paper report on evaluation of workshop on research and education policy in secondary schools, page 8, Professor Nicholas Nsowah- Nuamah (Rector of Kumasi Polytechnic) said that, Research into education is extremely critical for the development of human capital of a nation. According to Professor Nicholas Nsowah-Nuamah, research findings can only be used as input to education and other development of policies when researchers and policy-makers cooperate closely to understand specific needs, ensure the relevance of topics that can improve a community, through dissemination and implementation of research recommendations. (vi.Unctad.org/internet source)

In the Education Sector Performance Report 2013 page 41, the distribution of National BECE results by subject with 'above average' by scores 1 to 3 , 'average' being scores 4 to 6 and 'below average' being scores 7 to 9 , it was recorded that $23 \%$ recorded above average, $54 \%$ recorded average and $23 \%$ below average for English Language. For Mathematics, 23\% recorded above average, $54 \%$ recorded average and $23 \%$ below average. For Science, 25\%
recorded above average, $54 \%$ average and $22 \%$ below average. For Social studies, $23 \%$ recorded above average, 54\% average and $23 \%$ below average in 2012.

The findings of the thesis will help stakeholders to deploy quality monitoring approaches to increase students' performance and means of encouraging students to increase attendance and for better Basic Education Certificate Examination results (www.moe.gov.gh).

### 1.7 Limitations of the study

The study considered only inputs and outputs that are paramount to the efficiency of students (icmai.in/internet source) in the Adansi North District. The study was restricted to students in the two schools (Abu Bonsra D/A JHS and Adiembra D/A JHS in the Fomena and Asokwa Circuits respectively). Among the seven circuits, the study covered two.

### 1.8 Organization of the study

Chapter one introduces the project as it gives an overview about what the project is about, how Education in Ghana started, the Ministry mandated to oversee education in Ghana and students' academic performance. Chapter two reviews works that deployed DEA on its data analysis. Chapter three indicates the methodology used in the study. Chapter four indicates a vivid description of how the data collected was modeled using the Data Envelopment Analysis and the results presented. Chapter five summarizes the major findings from the thesis with conclusions and recommendations given.

## KNUST

## CHAPTER TWO <br> LITERATURE REVIEW

Since the introduction of the methodology Data Envelopment Analysis (DEA) (submitted to University of Portsmouth) in 1978 by Charnes, Cooper and Rhodes, it has been extensively applied to the efficiency measurement of many organizations (lib.dr.iastate.edu/internet source).

Reichmann and Sommorsguter- Reichmann (2006) employed DEA to measure the technical efficiency of 118 University Libraries. In this paper, the performance of Australian libraries were compared in efficiency to other libraries from different countries. The efficiency differences between library groups, particularly between small and large libraries and between

Germanspeaking countries (Austria, Germany and Switzerland) and English-speaking Countries (the United States, Australia, and Canada). In analyzing the differences in the performance between libraries from German and English-speaking Countries, it is motivated by the assumption that, due to the different environmental conditions (e.g. more strongly developed market forces, continuous performance assessments etc) libraries from English-speaking Countries (are forced to) perform better than libraries from German-speaking Countries. This depicts that a constant performance assessment always makes an individual or organization perform extra than previous. This paper also revealed that the average efficiency for small University library groups was 68.43\% and that of larger University library groups was $65.93 \%$.

Casu and Molyneux (2003) (www.eurojournals.com/internetsource) employed Data Envelopment Analysis to investigate the efficiency of European banking systems. In this paper, investigation was done to know whether the productive efficiency of European banking systems has improved since the creation of Single Internal Market (SIM). European banking markets was examined between 1993 and 1997 using the non-parametric DEA approach (fil.wharton.upenn.edu/internet source). According to Barbara Casu 'A comparative study of efficiency in European banking"; applied Economics, 11/20/2003/Publication, the geographical coverage of the study was France, Germany, Italy, Spain and the United Kingdom. The choice of these countries was based as much on their relative economic weight inside the European Union as on the size of their respective banking sectors. A sample of 750 banks from these five Countries (the largest 150 banks by asset size in each respective country) which was drawn from the London-based International Bank Credit Analysis LTD's 'Bank scope' database (www.londonmet.ac.uk/internet source). In this study, two outputs were used according to www.ukessays.com (total loans and other earning
assets). Two inputs were also used which include total costs (interest expenses, non-interest expenses, personnel expenses) and total deposits (total customers and short term funding). The DEA Results outputs showed relatively low average efficiency levels. Nevertheless, it was possible to detect a slight improvement in the average efficiency scores over the period of analysis for almost the banking systems in the sample with the exception of Italy. However, the results showed that, the efficiency gab among countries grew even wider over the period 1993 1997(fil.Wharton.upenn.edu/internet source). It was noted that the geographical location of these banking systems had much influence on the average banking efficiency scores in every year (www.eurojournals.com/internet source). The disparity contributing to the difference in the efficiencies was due to country-specific aspects of the banking technology
(www.mafhoum.com/internet source) which constitutes the different banking regulations and the different managerial strategies implemented to face up to the new challenges brought about by information technology, financial innovation and greater competition within the European banking market (fil.Wharton.upenn.edu/internet source). In my opinion, total loans and other earning assets used as outputs are key for any organization. Managerial competence also contributes a higher extent for the efficiency of these banking systems and should have been considered as an input. This would have influenced the efficiency levels of these sample banking systems.

Adu (2012) deployed Data Envelopment Analysis in measuring the relative efficiency of four S.D.A Senior High Schools in Ghana (ir.knust.edu.gh). These sample schools included S.D.A Senior High School at Bekwai-Ashanti, S.D.A Senior High School at Kenyase, S.D.A Senior High School at Bantama and S.D.A Senior High School at Tamale. The inputs used for this study included, trained to non-trained teacher ratio, contact hours per day and Library facilities. The outputs considered also was the passes obtained by the individual schools (i.e. 8 passes, 7 passes,

6 passes, 5 passes and below) in the West African Examination Council results in the year 2011. The study was based on the assumption that, all data and weights are positive ("General discussion "Data Envelopment Analysis 2007 /Publication), efficiency score must lie between zero and unity (www.inf.fih.org/internet source) and the same weight for the target school applied to all schools. At the end of the study, three schools were found efficient (i.e. S.D.A Senior High School at Bantama, Kenyase and Bekwai) with efficiency score 1.00 while S.D.A Senior High School at Tamale showed inefficient with efficiency score of 0.22 .

Using S.D.A Senior High School at Bekwai as target Decision Making Unit (DMU) to assist S.D.A Senior High School at Tamale to improve upon her inefficiency, the reduction of contact hours per day as an input has a larger effect on the efficiency of S.D.A Senior High School at Tamale than does trained and non-trained teachers ratio and hence contact hours should be utilized efficiently by management of the school. Also, there was an indication that, using S.D.A Senior High School at Bantama as a target DMU for the improvement of S.D.A Senior High School at Tamale, the reduction in contact hours a day has a larger effect on the efficiency of Tamale S.D.A Senior High School than Library facilities. These results proved same when used S.D.A Senior High School at Kenyase as a target DMU for the improvement of Tamale S.D.A Senior High Schools inefficiency. General recommendations were made by Adu to the Adventist Education Unit. Notably among them was the unit recruiting permanent teaching staffs instead of part-time teaching staff that only prefer wasting teaching hours to commute from their main schools to the school and regular roll calls for students. This will help improve the contact hours per day. The study deployed only one academic year which to my understanding cannot be a basis for judgment. I suggest a five year be used if there will be the need to embark on such a project. This will really give substantial results for judgment. How long the school under study has been existed after the establishment should
have been considered as an input since facilities in schools are mostly supported by old students whose contributions have a direct correlation with the years the school has been in existence.

Nevena Stancheva and Vyara Angelova employed DEA to measure the efficiency of University Libraries in the 2002 and 2003 period (www.inforum.cz/internet source). They estimated the following six inputs: number of Staff, Printed edition expenses, Expenses on electronic databases and software, Building space, Wages and Technical equipment (www.bookpump.com/internet source). Their outputs were Registered readers, Customers served and Books borrowed. Five University Libraries in Varna, Bulgaria were used as the Decision Making Units (Advances in intelligent systems and computing,2014/ Publication). These decision making units included libraries in Naval Academy, Medical Academy, Technical University, University of Economics and Free University (www.bookpump.com/internet source). Their results indicated that, the Libraries in the Technical University, in the University of Economics and in the Free University form the efficiency frontier for the two periods. The Libraries in the Medical Academy and in the Naval Academy work less efficiently during the period. The Library in Naval Academy efficiency was $75.66 \%$ in year 2002 and $75.26 \%$ in year 2003. The efficiency of the Library in the Medical Academy decrease from $70.41 \%$ in year 2002 to $64.9 \%$ in year 2003 (www.inforum.cz/internet source). Recommendations were made for the Libraries in Naval Academy and in the Medical Academy in other to improve their efficiencies.

Cidalia Leal Paco and Juan Manuel Cepeda Perez deployed DEA to evaluate the impact of ICT on productivity in the hotel sector (Submitted to University of Surrey/student paper) in Potugal from 2008 to 2011. 184 hotel establishments were used in the study. Twelve (12) inputs were considered which include Number of workers, Number of computers, Number of computers adapted to
workers with special needs, Number of workers who use computers at least once a week, Number of computers with internet connection, Number of workers who use computers with internet connection at least once a week, Total number of training courses in 2007 and 2010, Total number of ICT training courses in 2007 and 2010, Total number of workers who attended the ICT courses in 2007 and 2010, Total number of workers with ICT skills and Total number of workers with ICT skills who are with Graduate Studies. Two outputs were also considered which include Total number of stays in the establishment in 2007 and 2010 and Number of stays in the establishment in 2007 and 2010 resulting from internet bookings. Their results indicated that the number of efficient hotels in the two periods is approximately the same. These were 16 hotels which remained on the frontier between 2008 and 2011. 33 efficient hotels in 2008 and 131 inefficient hotels in 2008 while 35 hotels were efficient and 149 hotels inefficient in 2011(Leal Paco, Cidalia maria and Cepeda Perez, Juan Manuel, "Assessing the impact of Information and Communication Technologies on the Portuguese hotel sector: an exploratory analysis with Data Envelopment Analysis").

Emilio Martin employed DEA to measure the performance of the Zaragoza University Departments. 52 departments within the University of Zaragoza (Spain) were used in 1999 (www.bookpump.com/internet source). Three inputs were considered for the study which include Human resources, Financial resources, Material resources. The output indicators used include Teaching indicators (credits registered* experiment coefficient, PhD credits offered), Research indicators (PhD thesis read during the last year, Researching annual incomes, Compute of department research activity). His results concluded that 36 departments operating comparatively efficient with 16 being inefficient which were near to the aim value.


## CHAPTER THREE

## METHODOLOGY

### 3.0 Introduction

This section will describe the methodology employed in this study.

### 3.1 Introduction To Data Envelopment Analysis (DEA)

Data Envelopment Analysis (DEA) is a relatively new "data oriented" approach for evaluating the performance of a set of peer entities called Decision Making Units (DMUs) which convert multiple inputs into multiple outputs (www.enbis.org/internet source), multiple inputs to a single output, a single input to a multiple output or a single input to single output. The definition of a DMU is generic and flexible. Nowadays, we have seen a great variety of how the DEA is applied in evaluating the performances of many different kinds of entities engaged in many different activities. Data Envelopment Analysis applications have used DMUs of various forms to evaluate the performance of entities. These entities might be hospitals, US Air Force wings, universities, cities, courts, business firms, and others, including the performance of countries, regions, etc. Because the Data Envelopment Analysis requires very few assumptions, it has also opened up possibilities for use in cases which have been resistant to other approaches because of the complex (often unknown) nature of the relations between the multiple inputs and outputs involved.

The efficiency evaluation of systems and production activities has been practiced, and studied, for a long time. In the field of microeconomics, the concept of production functions provides an
inputoutput relationship. From the output orientation, a production function shows the maximum output rate (measuring efficiency: 'case analysis of highway maintenance patras'"; journal of SocioEconomics, 199121/ Publication) that can be achieved from a specified usage rate of inputs. From the input orientation, the production function shows the minimum input usage rate that can produce a specified output rate. Efficiency can then be measured relative to the frontier defined by the production function (Zerafat et al (2015)). It has been recognized that the efficiency evaluation is a very complex problem.

The usual measure of efficiency (www.ner.org.za/internet source) ie:
output
Efficiency ___ is often inadequate for the evaluation of the efficiency of systems and input
production activities with multiple inputs and outputs related to different resources, activities and environmental factors (submitted to Management Development Institute/student paper). Two basic approaches used for calculating efficiency are the parametric and nonparametric approaches (measuring efficiency:"case analysis of highway maintenance patrase", journal of SocioEconomics, 199121/Publication). The parametric approach requires that the shape of the frontier be guessed beforehand by specifying a particular function relating output to input (www.escience.upm.edu.my/internet source) while the nonparametric approach does not require any assumption on the functional form (www.inderscience.com/internet source). DEA is a nonparametric approach for measuring the efficiency of decision making units (DMUs). (Xiaoming,XU, the Feng, and Zhu Li-yun. "A slacks-based measure of super-efficiency in DEA with interval data", 2013 international conference on Management Science and Engineering, $20^{\text {th }}$ Annual Conference proceedings, 2013/Publication)

### 3.2 About Data Envelopment Analysis (DEA)

In the 1970's Charnes, Cooper, and Rhodes in the 1970's (submitted to Associatie K.U.

Leuven/student paper) developed a new nonparametric research tool called Data Envelopment Analysis (DEA) (bondynamics.com/internet source) to measure efficiency by combining several measures of input and output into a single measure of efficiency (Mohhidin Othman. '"Total factor productivity efficiency changes in a Malaysian hotel chain" International journal of Revenue Management 2010/Publication). Some key characteristics of DEA are summarized as follows (submitted to Istanbul Biigi University/student paper).

- DEA is used to measure the efficiency of homogenous units called decision making units (DMUs), which consume the same type of inputs and produce the same type of outputs
- DEA is a nonparametric approach; hence, there is no restriction on the functional form that relates inputs to outputs.
- DEA is a fractional mathematical programming technique. However, it can be converted into a linear programming model and solved by a standard LP solver.
- DEA generalizes the concept of the single-input, single-output technical efficiency measure of Farrell [2] to the multiple-input and multiple-output to a virtual input. Specially, efficiency is defined as a ratio of a weighted sum of outputs to a weighted sum of inputs.
- DEA is an approach focused on frontiers instead of central tendencies. It evaluates the efficiency of each DMU relative to similar DMUs. Thus, it provides an efficiency frontier or envelope for all considered DMUs rather than fitting a regression plane through the center of the data.
- DEA determines the relative efficiency of one DMU at a time over all other DMUs by finding the most favorable weights from the viewpoint of that, "target", DMU.
- Alternatives for making each inefficient DMU efficient can be seen by projecting them onto the efficient frontier.
- DEA offers the following three possible orientations to efficiency analysis"
(1) Input-Oriented. With this orientation, we try to minimize the input usage to produce given output levels for each DMU
(2) Output-Oriented. With this orientation, we try to produce the highest possible output levels for a given input usage for each DMU.
(3) Base-Oriented. With this orientation, we try to produce the optimal usage of the inputs and the optimal production of the outputs simultaneously. Both input and output can be controlled in a Base-Oriented model.


Figure 2.2 illustrates the projection for each orientation for the case of one input and one output.


### 3.3 DIFINITIONS OF KEY TERMS USEFUL IN DATA ENVELOPMENT ANALYSIS (DEA)

BARNES ET AL (BCC): The BCC model is used in Frontier Analysis when a variable return to scale relationship is assumed between inputs and outputs (www.banxia.com/internet source)

CHARNES, COOPER AND RHODES (CCR): The CCR model is used in frontier analysis when a constant return to scale relationship is assumed between inputs and outputs (Liping Ding. 'Evaluation of the capability of personal software process based on data envelopment analysis" Lecture notes in computer science, 2006/Publgication).

Decision Making Units: DMU's is the name used to describe the units being analyzed in the project (USir.Salford.ac.uk/internet source).

Efficiency frontier: It is the envelope that describes the best performance among several Decision Making Units (DMUs). It consists of the units in the data set which became most efficient in transforming their inputs into outputs. The units that are used for the frontier are those classified as being $100 \%$ efficient (USir.Salford.ac.uk/internet source)..

Efficiency score: DEA result in each unit is allocated a score between 0-1 (100\%). A unit with a score of $100 \%$ is relatively efficient, a unit with a score of less than $100 \%$ is relatively inefficient (USir.Salford.ac.uk/internet source).

Environmental factors: This is neither an economic resource nor product but rather an attribute of the environment where the units operates (USir.Salford.ac.uk/internet source). It may be measured directly or surrogate measures.

Inputs: This is any resource used by a unit in realizing its outputs. (USir.Salford.ac.uk/internet source).

Output: These are the products and services derived after processing the input. (www.banxia.com/internet source).

Output maximization: It is the DEA mode adopted when the analysis tries to maximize the output produced for a fixed amount of input (www.banxia.com/internet source).

Scale efficiency: A unit is scale efficient if its size of operation is optimal. If its size of operation is either increased or decreased its efficiency will drop (USir.Salford.ac.uk/internet source). It is calculated by dividing aggregate efficiency by technical efficiency (submitted to University of Central Florida/student paper).

### 3.4 How the DEA Works

The analysis compares the relative efficiency of organizational "units" such as bank branches, hospitals, vehicles, shops and other instances where units perform similar tasks. These units utilize similar resources, referred to as inputs, to generate similar outputs. For example, in measuring the efficiency of basic students performance, a student has inputs of continuous assessment, class attendance and teacher attendance to school and output of Basic Education Certificate Examination. However, there can be considerable differences in how each student unit combines inputs into outputs. In addition, there may also be differences in potential among each students caused by the educational level of their parents, distance from their houses to the school etc.

Frontier Analyst generates efficiency scores for all units being analyzed. It shows how much inefficient units need to reduce their inputs or increase their outputs in order to become efficient. Frontier Analyst therefore not only helps managers answer the question "How well are the units doing?" but also "How much could they improve?" It suggests performance targets, such as, unit A should be able to produce $15 \%$ more output with their staffing level or unit B should be able to reduce costs by $25 \%$ and still produce the same level of outputs. It also identifies the units which are performing best and their operating practices can then be examined to establish a guide to "best practice" for others to emulate.

Frontier Analyst presents the results of an efficiency study very effectively, using high powered graphics, so that you can see and understand the information that the analysis provides more clearly. It offers various ways of visualizing the results and shows in detail which units are performing the best and why they are doing so. It graphically displays performance information relating to an inefficient unit and shows the difference between its performance and the "best practice" units to which it has been compared.

Many organizations such as banks, hospitals, airlines, government departments and local authorities are using this analysis. It is used by managers in these organizations to perform a number of tasks including:

- Resource allocation: reallocating from the inefficient to the efficient
- Identification of "best practice"
- Identification of "poor practice"
- Target Setting
- Monitoring efficiency changes over time
- Rewards for good performance
- Planning site locations


### 3.5 Data Envelopment Analysis Models

Examples of DEA models are the CCR, BCC, Norman and Stoker models etc. The model to be used in this study is the CCR model. An introduction of this model is stated below.

### 3.6 The CCR Model

### 3.6.1 Introduction

In DEA modeling (CCR model), we assume that there are number ( n ) DMUs, each of which has ' $m$ ' inputs and ' $r$ ' outputs of common types. All inputs and outputs are assumed to be nonnegative, but at least one input and one output are positive (www.iytte.com). The following notations were used throughout the study.

Indices:
$\mathrm{i}=1,2 \ldots \mathrm{n}, \mathrm{j}=1,2 \ldots \mathrm{~m}, \mathrm{k}=1,2 \ldots \mathrm{r}$, (submitted to World Maritime

University/student paper) Notation:
$\mathrm{DMU}_{\mathrm{i}}$ is the $\mathrm{i}^{\text {th }} \mathrm{DMU}$
$\mathrm{DMU}_{\mathrm{o}}$ is the target DMU,(M-hikari.com/internet source) $\mathrm{X}_{\mathrm{ji}}$ is the
amount of input j consumed by $\mathrm{DMU}_{\mathrm{i}}, \mathrm{x}_{\mathrm{i}}=\left(\mathrm{x}_{\mathrm{j}}\right)_{\mathrm{m} \times 1}$ is the column vector
of inputs consumed by $D M U_{i}, x_{o}=\left(x_{j o}\right)_{m \times 1}$ is the column vector of inputs
consumed by the target DMU,
$\mathrm{X}=\left(\mathrm{x}_{\mathrm{j}}\right)_{\mathrm{m} \times \mathrm{n}}$ is the matrix of inputs,(Lertworasirikul, $S$ "Fuzzy data envelopment analysis (DEA): a possibility approach; "Fuzzy sets and systems, 20031016/Publication) $\mathrm{y}_{\mathrm{ki}}$ is the amount of output k produced by $\mathrm{DMU}_{\mathrm{i}}$, (Asian journal on Quality, volume 12, Issue 1 (2012-08-06) /Publication) $\mathrm{y}_{\mathrm{i}}=\left(\mathrm{y}_{\mathrm{ki}}\right)_{\mathrm{r} \times 1}$ is the column vector of outputs
produced by $\mathrm{DMU}_{\mathrm{i}}, \mathrm{y}_{\mathrm{o}}=\left(\mathrm{y}_{\mathrm{ko}}\right)_{\mathrm{r} \times 1}$ is the column vector of outputs produced
by the target DMU,
$\mathrm{Y}=\left(\mathrm{y}_{\mathrm{k}}\right)_{\mathrm{r} \times \mathrm{n}}$ is the matrix of outputs,(www.iytte.com)
$\mathrm{U}_{\mathrm{j}}$ is the weight of input j , (Publication)
$\mathrm{U}=\left(\mathrm{u}_{\mathrm{j}}\right)_{\mathrm{m} \times 1}$ is the column vector of input weights,
$\mathrm{V}=\left(\mathrm{V}_{\mathrm{k}}\right)_{\mathrm{r} \times 1}$ is the column vector of output weights, (www.iytte.com)
$\lambda=\left(\lambda_{i}\right)_{n \times 1}$ is the matrix of outputs, $\lambda \in \mathrm{R}^{\mathrm{n}}$ is the column vector of a linear combination of n DMUs (www.m-hikari.com/internet source)
$\theta$ is the objective value (efficiency) of the Charnes-Cooper-Rhodes (CCR) model.

### 3.6.2 Input - Oriented CCR Model

In the CCR model, the multiple -inputs and multiple -outputs of each DMU are aggregated into a single virtual input and a single virtual output, respectively. The input-oriented CCR model for target DMUo can be expressed by the following fractional programming model (India studies in Business and Economics, 2014/Publication):
$\operatorname{Max} \Theta=\mathrm{v}_{1} \mathrm{y}_{10}+\mathrm{v}_{2} \mathrm{y}_{20}+\ldots+\mathrm{v}_{\mathrm{r}} \mathrm{y}_{\mathrm{r} 0}$

```
            \mp@subsup{u}{1}{}\mp@subsup{\textrm{X}}{10}{}+\mp@subsup{\textrm{U}}{2}{}\mp@subsup{\textrm{X}}{20}{}+\ldots+...+\mp@subsup{u}{m}{}\mp@subsup{\textrm{X}}{\textrm{m}0}{}
s.t v1y10+v2y20+\ldots+
```



```
\(\mathrm{u}_{\mathrm{m}} \geq 0\)
\(\mathrm{v} 1, \mathrm{v} 2, \ldots, \mathrm{vr} \geq 0\)
```

Let $\Theta^{*}, u^{*}$ and $v^{*}$ be the optimal objective value (efficiency value) (www.m-hikari.com/internet source), the optimal input weights and the optimal output weights (Asian journal on Quality, volume 12, Issue 1 (2012-08-06)/Publication), respectively.

The objective of this model is to determine the input weights and output weights that maximize the ratio of a virtual output to a virtual input for DMUo. The constraints restrict the ratio of the virtual outputs to the virtual inputs for every DMU to be less than or equal to one (1). This implies that the maximum efficiency, $\theta^{*}$, is at most one (1) (India studies in Business and Economics, 2014/Publication). In the input-oriented CCR model, a DMU is inefficient if it is possible to reduce any input without increasing any other inputs and achieve the same level of output (www.iytte.com).

Under the assumption that all outputs and inputs have non-zero worth (www.ner.org.za/internet source), DMUo in the above model will be efficient if $\theta^{*}$ is equal to 1 . If $\theta^{*}<1$, it is possible to produce the given output $\left(\mathrm{y}_{10}, \mathrm{y}_{20}, \ldots, \mathrm{y}_{\mathrm{r} 0}\right)$ using a smaller vector of inputs which may be obtained as a linear combination of the input vectors of other DMUs ("Least squares formalism", Measurement uncertainties in science and Technology, 2005/ Publication). The efficiencies of all DMUs are obtained by solving model (3.1) n times, once for each DMU as the target DMU:

Charnes and Cooper developed a transformation from a linear fractional programming problem to an equivalent linear programming problem. By using His transformation; the fractional CCR model (3.6) can be transformed into the following linear programming model (India studies in Business and Economics, 2014/Publication):

Max $\theta=\mathrm{v}_{1} \mathrm{y}_{10}+\mathrm{v}_{2} \mathrm{y}_{20}+\ldots+\mathrm{v}_{\mathrm{r}} \mathrm{y}_{\mathrm{r} 0}$
s.t $\mathrm{u}_{1 \mathrm{X}_{10}}+\mathrm{u}_{2} \mathrm{X}_{20}+\ldots+\mathrm{u}_{\mathrm{m}} \mathrm{X}_{\mathrm{m} 0}=1$
$v_{1} y_{1 i}+v_{2} y_{2 i}+\ldots+v_{r y r i} \leq u_{1} x_{1 i}+u_{2} x_{2 i}+\ldots+u_{m x m i}, i=1, \ldots, n$
$u 1, u_{2}, \ldots, u_{m} \geq 0$
$\mathrm{v}_{1}, \mathrm{v}_{2}, \ldots, \mathrm{v}_{\mathrm{r}} \geq 0$

The above linear CCR model and its dual can be written in the following vector-matrix form (www.management.ase.ro/internet source): (CCR) $\max \mathrm{v}^{\mathrm{T}} \mathrm{y}_{0}$
s.t $\mathbf{u}^{\mathrm{T}} \mathrm{X}_{0}=1$
$-u^{T} X+V^{T} y \leq 0$
$u \geq 0 \mathrm{v}$
$\geq 0$

According to Lertworasirikul,S.: ' 'Fuzzy data envelopment analysis (DEA): a possibility approach
"Fuzzy sets and systems, 20031016 "/Publication)
(DCCR) $\quad \min \theta$
s.t $\theta x_{0}-X \lambda \geq 0$
$\mathrm{Y} \lambda \geq \mathrm{Y}_{0}$
$\lambda \geq 0$

Note that the Dual Charnes, Cooper and Rhodes (DCCR) model has a feasible solution, for $\theta=1, \lambda_{i}=0$ for $\mathrm{i} \neq 0$, and $\lambda_{0}=1$.

Therefore, the value $\theta^{*}$ of the DCCR model depicting its optimality do not exceed the constraint (submitted to Birla Institute of Technology and Science Pilani/student paper)
$Y \lambda \geq y_{0}$ forces $\lambda$ to be a nonzero vector. This along with $\theta x_{0}-X \lambda \geq 0$ implies that $\theta^{*}>0$.
Therefore, $0<\theta^{*} \leq 1$. Thus, the DCCR model has an optimal solution.
From the strong duality theorem of linear programming, the CCR model also has an optimal solution and the optimal objective values of the CCR and DCCR models are equal (www.management.ase.ro/internet source).

### 3.6.3 Interpretation of the CCR Model

The target DMU $\left(\mathrm{DMU}_{0}\right)$ is being compared with a linear combination of other DMUs. The objective of the CCR model is to find a vector of weights such that the efficiency of DMU, relative to other DMUs is maximized, provided that no other DMUs or linear combination of other DMUs could achieve the same output levels with smaller amount of any input (Submitted to Technowledge Turkey/Student paper).

### 3.6.4 Interpretation of the DCCR model

DMUo is efficient if no linear combination of other DMUs can produce the same or higher output levels using less of all inputs (Submitted to Technowledge Turkey/Student paper). $\theta$ indicates a possible proportional reduction in inputs ( $\mathrm{x}_{0}$ ). Reduction in inputs x 0 can be viewed as a radial movement from $\left(\mathrm{x}_{0}, \mathrm{y}_{0}\right)$ toward the production frontier.
$\theta^{*}=1$ implies that no linear combination of other DMUs has $\mathrm{X} \lambda<\mathrm{x}_{0}$ and $\mathrm{Y} \lambda<\mathrm{y}_{0}$.

Otherwise, we can further reduce $\theta^{*}$ while $\mathrm{X} \lambda<\theta^{*} \mathrm{x}_{0}$ still holds. Thus $\theta^{*}$ is not an optimal solution because we can find $\theta<\theta^{*}$ that satisfies all the constraints.

On the other hand, $\Theta^{*}<1$ indicates that the resulting linear combination of DMUs acts as a benchmark for DMU0. $\theta$ *can also be interpreted as the largest ratio of $x_{0}$ to $X^{\lambda}$ which outputs are at least equalized, i.e. $\mathrm{Y}^{\lambda}<\mathrm{y}_{0}$.

### 3.6.5 Determination of Efficiency

To which DMUs are efficient, we introduce the definition of Pareto-Koopmans efficiency (www.m-hiari.com/internet source) as follows:

Definition of Pareto-Koopmans Efficiency (Lertworasirikul,S. : '"Fuzzy data envelopment analysis (DEA): a possibility approach "Fuzzy sets and systems, 20031016 "/Publication): A DMU is fully efficient if and only if it is impossible to improve any input or output without worsening some other inputs or outputs.

From the above definition, the $\mathrm{DMU}_{0}$ with $\Theta^{*}=1$ may not be Pareto-Koopmans Efficient if it is possible to make additional improvement (lower input or higher output) without worsening any other input or output. Therefore, we introduce a vector of input excesses ( $\mathrm{s}^{-}$) and output shortfalls ( $\mathrm{s}^{+}$) as follows:
$\mathrm{s}^{-}=\Theta \mathrm{x}_{0}-\mathrm{X} \lambda$ ands $^{+}=\mathrm{Y} \lambda-\mathrm{y}_{0}$

Where $\mathrm{s}^{-} \geq 0, \mathrm{~s}^{+} \geq 0$ are defined as slack vectors for any feasible solution $(\theta, \lambda)$ of the DCCR model (3.4)

Based on the slack vectors, a DMU is Pareto-Koopmans efficient if it satisfies the following two conditions:
(1) $\theta^{*}=1$
(2) $\mathrm{s}^{-}=0$ and $\mathrm{s}^{+}=0$

The first condition is referred to as a weak efficiency, technical efficiency of "Farrell efficiency" after M. J. Farrell (1957) ("The CCR model and production correspondence", Data Envelopment Analysis, 2002/Publication).

For the CCR model, the Pareto-Koopmans efficiency is called the CCR efficiency (isl.yonsei.ac.kr/internet source).

We summarize the CCR efficiency conditions for a DMU as follows.

1. If $\theta^{*}<1$, then the DMU is CCR efficient.
2. If $\Theta^{*}=1$, (India studies in Business and Economics, 2014/Publication) and there I nonzero slacks, i.e. $s^{* *} \neq 0$, or $\mathrm{s}^{+*} \neq 0$, then the DMU is CCR -inefficient. From the complementary slackness conditions of linear programming ("The CCR model and production correspondence", Data Envelopment Analysis, 2002/Publication) the elements of the vectors $u^{*}$ and $\mathrm{v}^{*}$ corresponding to the positive slacks must be zero.
(etd.adm.unipi.it/internet source) Thus, the DMU with $\Theta^{*}=1$ is CCR-inefficient if there is not at least one optimal $\mathbf{u}^{*}$ and $\mathrm{v}^{*}$ such that $\mathrm{u}^{*}>0$ and $\mathrm{v}^{*}>0$ (Submitted to University of Manchester/student paper).
3. If $\Theta^{*}=1$ with zero slack, then the DMU is CCR-efficient. From the strong theorem of complementarily, there exist optimal $\mathrm{u}^{*}$ and $\mathrm{v}^{*}$ such that $\mathrm{u}^{*}>0$ and $\mathrm{v}^{*}>0$ (The CCR model and production correspondence", Data Envelopment Analysis, 2002/Publication).
3.7 Strengths of Data Envelopment Analysis
(Omani,Omar Khalifeh. "Measuring Efficiency levels of the Jordannian and U.A.E Banking industries: An application of Data Envelopment Analysis/ Publication). Data Envelopment Analysis (DEA) can be a powerful tool when used wisely. Some of the characteristics that make it powerful according to a thesis submitted to university of Petroleum and Energy studies /student paper are:
4. DEA can handle multiple inputs and output methods.
5. It does not require an assumption of a functional function relating inputs to be outputs.
6. DMU's are directly compared against a peer or a combination of peers.
7. Inputs and outputs can have very different units.
8. capable of being used with any input-output measurement
9. no need to explicitly specify a mathematical form for the production function
10. proven to be useful in uncovering relationships that remain hidden for other methodologies

### 3.8 LIMITATIONS OF THE DATA ENVELOPMENT ANALYSIS (DEA)

There are some limitations to this technique as submitted to Higher Education Commission, Pakistan/student paper. Some of these include the following as submitted to Universiti Teknologi, MARA/student paper:

1. Since DEA is an extreme point technique, a little error in measurement can course significant problems.
2. DEA is good at estimating relative efficiencies of a DMU but it converges very slowly at absolute efficiency.
3. Since DEA is a non-paramagnetic technique, statistical hypothesis test are different.
4. Since a standard formulation of DEA creates a separate linear programming problem for each DMU, large problem can be computationally intensive.

### 3.9 ASSUMPTIONS ABOUT DATA ENVELOPMENT ANALYSIS (DEA)

Like any other mathematical tool, DEA has its own set of assumptions. The assumptions that apply to be the formulation and evaluation of DEA problems are:

1. When the same set of input and output coefficients is applied to all other service units being compared, no service unit will exceed $100 \%$ or 0.1 efficiency (submitted to Techknowledge Turkey/student paper).
2. Linearity of the objective function: It is impossible to use these techniques if the objective function is not linear.
3. Non negative inputs, that is, inputs cannot be negative, e.g. we cannot have negative staff.
4. Given the same production schedule, different producers should be able to produce the same output at the same level of efficiency (www.iriti.org/internet source).

### 3.10 ASSESSMENT AT THE BASIC LEVEL

### 3.10.1 SCHOOL BASED ASSESSMENT (SBA)

Assessment at the Basic level is partly school based. It is expressly stated in the syllabus that "the form of assessment in schools will follow the requirements of school based assessment...". The SBA is to consist of 12 assessments per year. Guidelines for the SBA are provided in the Teacher's Handbook on School Based Assessment. The scores of each student are to be recorded in his/ her Cumulative record book.

The scores are submitted to the West African Examination Council (WAEC) at the end of the Junior High School as continuous assessment score.

The School Based Assessment also creates room for each District to sit for a common examination set by neutral examiners other than their teachers at the end of each term to enable students have a feel of the external BECE set by WAEC.

### 3.10.2 PUBLIC EXAMINATION

The public exam is called the Basic Education Certificate Examination (BECE). It is conducted by the West African Examination Council (WAEC) Ghana. It is a high state exit point examination taken at the end of the JHS. It is based on the national curriculum. There are two papers for each subject; multiple choice, paper 1 and theory, paper 2.

### 3.10.3 GRADING

The candidate is graded on his/ her continuous assessment and external examination, BECE scores contributed on the 30:70 ratio respectively. The 9-point stanine grading system is used.

## CHAPTER FOUR

## DATA COLLECTION AND ANALYSIS

### 4.0 Introduction

The chapter exposes us to the data used, how Data Envelopment Analysis (DEA) was applied in calculating the efficiency score of the Basic Students Performance.

The process begins with comprehending the data and how it was analyzed. After the inputs and outputs have been defined from the data source, the CCR model is employed in calculating the technical efficiency of the Basic Students performance.

### 4.1 Description Of Data

Records for students at their respective schools within a Circuit in the Adansi North District Education Directorate were used. There are several reasons why these records were used for the current research. First, the school records give a least deviation of actual records of students as compared to when these records are taken as secondary data from the district Education Directorate. Secondly, some specific questions about students could be asked from teachers at specific schools. Thirdly, it gives the researcher the opportunity to discuss peculiar issues which might influence the performance of each student under study. Also, the data are readily accessible from the individual schools as compared to the District Education Directorate. It is also easy to obtain and transform the data into a format suitable for spreadsheet programs (e.g., Lotus 1-2-3 or Microsoft Excel) and statistical programs (e.g. SPSS).

### 4.2 Description of Data Sets

The current study deployed the use of the following variables.

## Output variable(s)

Basic Education Certificate Examination (B.E.C.E) results at the end of the Basic School education (V)

## Input variable(s)

Students Attendance ( $\mathrm{U}_{1}$ )
Teachers Attendance $\left(\mathrm{U}_{2}\right)$
Students Continuous Assessment ( $\mathrm{U}_{3}$ )
The Decision Making Units (DMUs) chosen for the research work are students from Abu Bonsra D/A JHS (AB01 - AB10) from Fomena Circuit and Adiembra D/A JHS (AD01-AD10) from Asokwa Circuit.

### 4.3 Modeling the Efficiency of Data Sets

Using the data sets deployed earlier, the Data Envelopment Analysis for the Basic Students Performance can be formulated as follows:
$\left(L P_{0}\right) \max \theta=u_{1} y_{1 o}+u_{2} y_{2 o}+\ldots .+u_{s} y_{s o}$ Subject to $v_{1} x_{1 o}$
$+v_{2} x_{2 o}+\ldots . .+v_{m} x_{m o}=1 u_{1} y_{l j}+u 2 y_{2 j}+\ldots . .+u s y_{s j} \leq v_{1 x_{l j}}+v_{2} x_{2 j}$
$+\ldots . .+v_{m} x_{m j}$

$$
(j=1 \ldots \ldots . . . n)
$$

$v 1, v_{2}, \ldots v_{n} \geq 0, u_{1}, u_{2}, \ldots u_{n} \geq 0$
$y_{r}=$ amount of output $r \quad u_{r}=$ weight
assigned to output $r \quad x_{i}=$ amount of
output $i$ $v_{i}=$ weight assigned to output $i$ $\qquad$

### 4.4 OBSERVATIONS

### 4.4.1 Observations from students at Abu Bonsra D/A JHS at Fomena Circuit (Appendix B)

The technical efficiency of the Students in Abu Bonsra D/A JHS at the Fomena Circuit used in this project as DMUs varies significantly with values raging from 0.71 to 1.00 on the input of Students Attendance $\left(\mathrm{U}_{1}\right)$ against the output of students B.E.C.E. performance $(\mathrm{V})$. The technical efficiency ranges from 0.70 to 1.00 on the input of Teachers Attendance $\left(\mathrm{U}_{2}\right)$ against the output of students B.E.C.E. performance $(\mathrm{V})$.The technical efficiency ranges from 0.67 to 1.00 on the input of Students Continuous Assessment $\left(\mathrm{U}_{3}\right)$ against the output of students B.E.C.E. performance (V).The technical efficiency ranges from 0.84 to 1.00 on the input of Students Continuous Assessment $\left(\mathrm{U}^{+}\right)$against the output of students B.E.C.E. performance (V).

Among the ten students, $(\mathrm{AB} 01, \mathrm{AB} 02, \mathrm{AB} 03, \mathrm{AB} 04, \mathrm{AB} 05, \mathrm{AB} 06, \mathrm{AB} 07, \mathrm{AB} 08, \mathrm{AB} 09$, AB10), AB03 obtained 1.00 efficiency on the input of Students Attendance $\left(\mathrm{U}_{1}\right)$ against the output of students B.E.C.E. performance (V). This student is the best performer in terms of Attendance.

Among the ten students, ( $\mathrm{AB} 01, \mathrm{AB} 02, \mathrm{AB} 03, \mathrm{AB} 04, \mathrm{AB} 05, \mathrm{AB} 06, \mathrm{AB} 07, \mathrm{AB} 08, \mathrm{AB} 09$, AB10), AB01 obtained 1.00 efficiency on the input of Teachers Attendance $\left(\mathrm{U}_{2}\right)$ against the output of students B.E.C.E. performance (V). This student is the best performer in terms of Attendance.

Among the ten students, (AB01, $\mathrm{AB} 02, \mathrm{AB} 03, \mathrm{AB} 04, \mathrm{AB} 05, \mathrm{AB} 06, \mathrm{AB} 07, \mathrm{AB} 08, \mathrm{AB} 09$, AB10), AB08 obtained 1.00 efficiency on the input of Students Continuous Assessment $\left(\mathrm{U}_{3}\right)$ against the output of students B.E.C.E. performance $(\mathrm{V})$. This student is the best performer in terms of Attendance.
o
Among the ten students, (AB01, AB02, AB03, AB04, AB05, AB06, AB07, AB08, AB09, AB10), AB03 obtained 1.00 efficiency on the Average input ( $\mathrm{U}^{+}$) against the output of students B.E.C.E. performance (V). This student is the best performer in terms of Attendance and Continuous assessment (holding teachers attendance to class constant for all the ten students).

### 4.4.2 Observations from students at Adiembra D/A JHS at Asokwa Circuit (Appendix D)

2. The efficiency of the Students in Adiembra D/A JHS at the Asokwa Circuit used in this project as DMUs varies significantly with values raging from 0.81 to 1.00 on the input of Students Attendance $\left(\mathrm{U}_{1}\right)$ against the output of students B.E.C.E. performance $(\mathrm{V})$. The technical efficiency ranges from 0.80 to 1.00 on the input of Teachers Attendance $\left(\mathrm{U}_{2}\right)$ against the output of students B.E.C.E. performance (V). The technical efficiency ranges from 0.62 to 1.00 on the input of Students Continuous Assessment $\left(\mathrm{U}_{3}\right)$ against the output of students B.E.C.E. performance (V). The technical efficiency ranges from 0.89 to 1.00 on the input of Students Continuous Assessment $\left(\mathrm{U}^{+}\right)$against the output of students B.E.C.E. performance (V) (etdUwaterloo.ca/internet source).

* Among the ten students, (AD01, AD02, AD03, AD04, AD05, AD06, AD07, AD08, AD09, AD10), AD01 and AD02 obtained 1.00 efficiency on the input of Students Attendance ( $\mathrm{U}_{1}$ ) against the output of students B.E.C.E. performance (V). These students are the best performers in terms of Attendance.

Among the ten students, (AD01, AD02, AD03, AD04, AD05, AD06, AD07, AD08, AD09, AD10), AD01 and AD02 obtained 1.00 efficiency on the input of Teachers Attendance

2
$\left(\mathrm{U}_{2}\right)$ against the output of students B.E.C.E. performance $(\mathrm{V})$. These students are the best performers in terms of Attendance.

Among the ten students, (AD01, AD02, AD03, AD04, AD05, AD06, AD07, AD08, AD09, AD10), AD09 obtained 1.00 efficiency on the input of Students Continuous Assessment $\left(\mathrm{U}_{3}\right)$ against the output of students B.E.C.E. performance $(\mathrm{V})$. This student is the best performer in terms of Attendance.

Among the ten students, (AD01, AD02, AD03, AD04, AD05, AD06, AD07, AD08, AD09, AD10), AD04 and AD06 obtained 1.00 efficiency on the Average input $\left(\mathrm{U}^{+}\right)$against the output of students B.E.C.E. performance (V). These students are the best performers in terms of Attendance and Continuous assessment (holding teachers attendance to class constant for all the ten students).

### 4.4.3 Observations from students at Abu Bonsra D/A JHS and Adiembra D/A JHS at Fomena and Asokwa Circuits respectively. (Appendix E)

2. The efficiency of the Students in Abu Bonsra D/A JHS at the Fomena Circuit and Adiembra D/A JHS at the Asokwa Circuit used in this project as DMUs varies significantly with values raging from 0.71 to 1.00 on the input of Students Attendance $\left(\mathrm{U}_{1}\right)$ against the output of students B.E.C.E. performance(V). The technical efficiency ranges from 0.70 to 1.00 on the input of Teachers Attendance $\left(\mathrm{U}_{2}\right)$ against the output of students B.E.C.E. performance (V). The technical efficiency ranges from 0.60 to 1.00 on the input of Students Continuous Assessment $\left(\mathrm{U}_{3}\right)$ against the output of students B.E.C.E. performance $(\mathrm{V})$. The technical efficiency ranges from 0.80 to 1.00 on the average input $\left(\mathrm{U}^{+}\right)$against the output of students B.E.C.E. performance (V) (etdUwaterloo.ca/internet source).

Among the twenty students, $(\mathrm{AB} 01, \mathrm{AB} 02, \mathrm{AB} 03, \mathrm{AB} 04, \mathrm{AB} 05, \mathrm{AB} 06, \mathrm{AB} 07, \mathrm{AB} 08$,
.
AB09, AB10, AD01, AD02, AD03, AD04, AD05, AD06, AD07, AD08, AD09, AD10), AB03 obtained 1.00 efficiency on the input of Students Attendance $\left(\mathrm{U}_{1}\right)$ against the output of students B.E.C.E. performance (V). This student is the best performer in terms of Attendance.

Among the twenty students, (AB01, $\mathrm{AB} 02, \mathrm{AB} 03, \mathrm{AB} 04, \mathrm{AB} 05, \mathrm{AB} 06, \mathrm{AB} 07, \mathrm{AB} 08$, AB09, AB10, AD01, AD02, AD03, AD04, AD05, AD06, AD07, AD08, AD09, AD10), AB01 obtained 1.00 efficiency on the input of Teachers Attendance $\left(\mathrm{U}_{2}\right)$ against the output of students B.E.C.E. performance (V). This student is the best performer in terms of Attendance.

Among the twenty students, $(\mathrm{AB} 01, \mathrm{AB} 02, \mathrm{AB} 03, \mathrm{AB} 04, \mathrm{AB} 05, \mathrm{AB} 06, \mathrm{AB} 07, \mathrm{AB} 08$, $\mathrm{AB} 09, \mathrm{AB} 10, \mathrm{AD} 01, \mathrm{AD} 02, \mathrm{AD} 03, \mathrm{AD} 04, \mathrm{AD} 05, \mathrm{AD} 06, \mathrm{AD} 07, \mathrm{AD} 08, \mathrm{AD} 09, \mathrm{AD} 10)$, AD09 obtained 1.00 efficiency on the input of Students Continuous Assessment $\left(\mathrm{U}_{3}\right)$ against the output of students B.E.C.E. performance (V). This student is the best performer in terms of Attendance.
( Among the twenty students, (AB01, AB02, AB03, AB04, AB05, AB06, AB07, AB08, $\mathrm{AB} 09, \mathrm{AB} 10, \mathrm{AD} 01, \mathrm{AD} 02, \mathrm{AD} 03, \mathrm{AD} 04, \mathrm{AD} 05, \mathrm{AD} 06, \mathrm{AD} 07, \mathrm{AD} 08, \mathrm{AD} 09, \mathrm{AD} 10)$, AD06 and AD04 obtained 1.00 efficiency on the Average input $\left(\mathrm{U}^{+}\right)$against the output of students B.E.C.E. performance (V). These students are the best performers in terms of Attendance and Continuous assessment (holding teachers attendance to class constant for all the ten students).
$\infty$


### 4.5 IMPROVEMENTS



The fundamental assumption in the computation of relative efficiency is that, if a student PQ is capable of producing $\mathrm{Y}(\mathrm{PQ})$ units of output with the usage of $\mathrm{X}(\mathrm{AB})$ of inputs, then other students should produce likewise to be efficient. Performance targets can be set for inefficient students to assist them reach relative efficiency of $100 \%$ (1.00) in comparison with the most efficient student.

### 4.5.1 Students at Abu Bonsra D/A JHS in the Fomena Circuit

4.5.1.1 Improvement for students at Abu Bonsra D/A JHS on student attendance.

The student AB03 has operated in an environment similar to that of the other students. Using his/her achievement as a benchmark become significant. (Mishra, RohitaKumar. Input, " Benchmarking scheme for retail stores efficiency" International journal of marketing studies, 2009/Publication). Input target for inefficient students will be the amount of input that will enable the student to have the same ratio of efficiency as AB03 in terms of Student Attendance.

Using the input from Students Attendance $\left(\mathrm{U}_{1}\right)$ and the corresponding efficiencies, Input Target $=$ Actual input $\times$ Relative Efficiency

For AB01:
Input Target $=61 \times 0.99=60.39$
This means that if ABO 1 operates using $60.39 \sim 60$ as input with the same output then it will be considered as efficient as AB 03 student.
i.e. Slack $=$ actual input - target input

$$
=61-60=1
$$

Input Slack can also be expressed as a percentage (IJMS, Editor, "International journal of Marketing studies, Vol 1, No. 2, November 2009, all in one pdf file".

Input Slack percentage $=\frac{\text { Input slack }}{\text { Actual input }} \times 100 \%$

$$
=\frac{1}{61} \times 100 \%
$$

$$
=1.64 \%
$$

Thus if student AB01 has to be efficient as AB03 student, it should produce the same output using 1.64 percent less input.
Output Target $=$ Actual output $\times$ Relative Efficiency
Output Target $=97.2 \times 0.99=96.23$
This means that if ABO1 operates using 96.23 ~ 96 as output with the same input (depicting a higher B.E.C.E overall results), then it will be considered as efficient as AB03 student.

Output Slack $=$ actual output - target output

$$
=97.2-96=1.2
$$

Output Slack can also be expressed as a percentage.
Output Slack percentage $=\frac{\text { output slack }}{\text { Actual output }} \times 100 \%$

$$
=\frac{1.2}{97.2} \times 100 \%
$$

$$
=1.23 \%
$$

Thus if student AB 01 has to be efficient as AB 03 student, it should produce the same input using 1.23 percent more output.

For AB02:
Input Target $=64 \times 0.93=59.52$
This means that if ABO 2 student operates using $59.52 \sim 60$ as input with the same output then it will be considered as efficient as AB03 student.
Input Slack $=$ actual input - target input

$$
=64-60=4
$$

Input Slack can also be expressed as a percentage (Mishra, RohitaKumar. Input, "
Benchmarking scheme for retail stores efficiency" International journal of marketing studies,2009/Publication).
Input Slack percentage $=\frac{\text { Input slack }}{\text { Actual input }} \times 100 \%$

$$
\begin{aligned}
& =\frac{4}{64} \times 100 \% \\
& =6.25 \%
\end{aligned}
$$

Thus if student AB 02 has to be efficient as AB 03 student, it should produce the same output using 6.25 percent less input.

Output Target $=$ Actual output $\times$ Relative Efficiency
Output Target $=96.3 \times 0.93=89.559$
This means that if ABO 2 operates using 89.559 ~ 90 as output with the same input (depicting a higher B.E.C.E overall results), then it will be considered as efficient as AB03
student. i.e. output Slack $=$ actual output $-\operatorname{target}$ output

$$
=96.3-90=6.3
$$

Output Slack can also be expressed as a percentage.
Output Slack percentage $=\frac{\text { output slack }}{\text { Actual output }} \times 100 \%$

$$
=\frac{6.3}{96.3} \times 100 \%
$$

$$
=6.54 \%
$$

Thus if student AB 02 has to be efficient as AB 03 student, it should produce the same input using 6.54 percent more output.

For AB05:
Input Target $=61 \times 0.92=56.12$
This means that if ABO5 student operates using 56.12 ~ 56 as input with the same output then it will be considered as efficient as AB03 student. i.e. input Slack $=$ actual input - target input

$$
=61-56=5
$$

Input Slack can also be expressed as a percentage.

$$
\begin{aligned}
\text { Input Slack percentage } & =\frac{\text { Input slack }}{\text { Actual input }} \times 100 \% \\
& =\frac{5}{61} \times 100 \% \\
=8.20 & \%
\end{aligned}
$$

Thus if student AB05 has to be efficient as AB03 student, it should produce the same output using 8.20 percent less input.

Output Target $=$ Actual output $\times$ Relative Efficiency
Output Target $=90.7 \times 0.92=83.44$
This means that if ABO5 student operates using $83.44 \sim 83$ as output with the same input (depicting a lower B.E.C.E results), then it will be considered as efficient as AB 03 student.
i.e. output Slack $=$ actual output - target output

$$
=90.7-83=7.7
$$

Output Slack can also be expressed as a percentage.
Output Slack percentage $=\frac{\text { output slack }}{\text { Actual output }} \times 100 \%$

$$
\begin{aligned}
& \quad=\frac{7.7}{90.7} \times 100 \% \\
& =8.49 \%
\end{aligned}
$$

Thus if student AB 05 has to be efficient as AB 03 student, it should produce the same input using 8.49 percent more output.

For AB04:
Input Target $=62 \times 0.92=57.04$
This means that if ABO4 student operates using $57.04 \sim 57$ as input with the same output then it will be considered as efficient as AB03 student.
i.e. input Slack $=$ actual input - target input

$$
=62-57=5
$$

Input Slack can also be expressed as a percentage.
Input Slack percentage $=\frac{\text { Input slack }}{\text { Actual input }} \times 100 \%$

$$
=\frac{5}{62} \times 100 \%
$$

$$
=8.07 \%
$$

Thus if student AB04 has to be efficient as AB03 student, it should produce the same output using 8.07 percent less input.

Output Target $=$ Actual output $\times$ Relative Efficiency

Output Target $=91.6 \times 0.92=84.27$
This means that if ABO4 student operates using $84.27 \sim 84$ as output with the same input (depicting a lower B.E.C.E results), then it will be considered as efficient as AB 03 student.
i.e. output Slack $=$ actual output - target output

$$
=91.6-84=7.6
$$

Output Slack can also be expressed as a percentage.
Output Slack percentage $=\frac{\text { output slack }}{\text { Actual output }} \times 100 \%$

$$
\begin{aligned}
& \quad=\frac{7.6}{91.6} \times 100 \% \\
& =8.30 \%
\end{aligned}
$$

Thus if student AB 04 has to be efficient as AB 03 student, it should produce the same input using 8.30 percent more output.

For AB07:
Input Target $=54 \times 0.90=48.60$
This means that if ABO7 operates using $48.60 \sim 49$ as input with the same output then it will be considered as efficient as AB 03 student.
i.e. input Slack $=$ actual input - target input

$$
=54-49=5
$$

Input Slack can also be expressed as a percentage.
Input Slack percentage $=\frac{\text { Input slack }}{\text { Actual input }} \times 100 \%$
$\square=\frac{5}{54} \times 100 \%$

$$
=9.26 \%
$$

Thus if student AB07 has to be efficient as AB03 student, it should produce the same output using 9.26 percent less input.

Output Target $=$ Actual output $\times$ Relative Efficiency
Output Target $=78.7 \times 0.90=70.83$
This means that if ABO7 operates using 70.83 ~ 71 as output with the same input (depicting a lower B.E.C.E results), then it will be considered as efficient as AB 03 student.
i.e. output Slack $=$ actual output - target output

$$
=78.7-71=7.7
$$

Output Slack can also be expressed as a percentage.
Output Slack percentage $=\frac{\text { output slack }}{\text { Actual output }} \times 100 \%$

$$
=\frac{7.7}{78.7} \times 100 \%
$$

$$
=9.78 \%
$$

Thus if student AB 07 has to be efficient as AB 03 student, it should produce the same input using 9.78 percent more output.

## For AB06:

Input Target $=61 \times 0.83=50.63$
This means that if ABO6 operates using 50.63 ~ 51 as input with the same output then it will be considered as efficient as AB03 student.
i.e. input Slack $=$ actual input - target input

$$
=61-51=10
$$

Input Slack can also be expressed as a percentage.
Input Slack percentage $=\frac{\text { Input slack }}{\text { Actual input }} \times 100 \%$

$$
\begin{aligned}
& =\frac{10}{61} \times 100 \% \\
& =16.39 \%
\end{aligned}
$$

Thus if student AB 06 has to be efficient as AB 03 student, it should produce the same output using 16.39 percent less input.

Output Target $=$ Actual output $\times$ Relative Efficiency
Output Target $=81.4 \times 0.83=67.56$
This means that if ABO6 operates using 67.56~68 as output with the same input (depicting a lower B.E.C.E results), then it will be considered as efficient as AB 03 student.
i.e. output Slack $=$ actual output - target output

$$
=81.4-68=13.4
$$

Output Slack can also be expressed as a percentage.
Output Slack percentage $=\frac{\text { output slack }}{\text { Actual output }} \times 100 \%$

$$
=16.46 \%
$$

Thus if student AB 06 has to be efficient as AB 03 student, it should produce the same input using 16.46 percent more output.

For AB08:
Input Target $=58 \times 0.81=46.98$
This means that if ABO8 student operates using $46.98 \sim 47$ as input with the same output then it will be considered as efficient as AB03 student.
i.e. input Slack $=$ actual input - target input

$$
=58-47=11
$$

Input Slack can also be expressed as a percentage.
Input Slack percentage $=\frac{\text { Input slack }}{\text { Actual input }} \times 100 \%$

$$
\begin{aligned}
& =\frac{11}{58} \times 100 \% \\
& =18.97 \%
\end{aligned}
$$

Thus if student AB 08 has to be efficient as AB 03 student, it should produce the same output using 18.97 percent less input.

Output Target $=$ Actual output $\times$ Relative Efficiency
Output Target $=75.9 \times 0.81=61.48$
This means that if ABO8 operates using 61.48 ~61 as output with the same input (depicting a lower B.E.C.E results), then it will be considered as efficient as AB 03 student.
i.e. output Slack $=$ actual output - target output

$$
=75.9-61=14.9
$$

Output Slack can also be expressed as a percentage.

Output Slack percentage $=\frac{\text { output slack }}{\text { Actual output }} \times 100 \%$

$$
\begin{aligned}
& =\frac{14.9}{75.9} \times 100 \% \\
& =19.63 \%
\end{aligned}
$$

Thus if student AB 08 has to be efficient as AB 03 student, it should produce the same input using 19.63 percent more output.

## For AB09:

Input Target $=59 \times 0.76=44.84$
This means that if ABO9 student operates using $44.84 \sim 45$ as input with the same output then it will be considered as efficient as AB 03 student.
i.e. input Slack $=$ actual input - target input

$$
=59-45=14
$$

Input Slack can also be expressed as a percentage.

$$
\begin{aligned}
\text { Input Slack percentage } & =\frac{\text { Input slack }}{\text { Actual input } \times 100 \%} \\
& =\frac{14}{59} \times 100 \% \\
& =23.73 \%
\end{aligned}
$$

Thus if student AB09 has to be efficient as AB03 student, it should produce the same output using 23.73 percent less input.

Output Target $=$ Actual output $\times$ Relative Efficiency
Output Target $=72.2 \times 0.76=54.87$
This means that if ABO 9 student operates using $54.87 \sim 55$ as output with the same input (depicting a lower B.E.C.E results), then it will be considered as efficient as AB 03 student.
i.e. output Slack $=$ actual output - target output

$$
=72.2-55=17.2
$$

Output Slack can also be expressed as a percentage.
Output Slack percentage $=\frac{\text { output slack }}{\text { Actual output }} \times 100 \%$

$$
=\frac{17.2}{72.2} \times 100 \%
$$

$$
=23.82 \%
$$

Thus if student AB 09 has to be efficient as AB03 student, it should produce the same input using 23.82 percent more output.

For AB10:
Input Target $=60 \times 0.71=42.60$
This means that if ABO10 student operates using 42.60~43 as input with the same output then it will be considered as efficient as AB 03 student.
i.e. input Slack $=$ actual input - target input

$$
=60-43=23
$$

Input Slack can also be expressed as a percentage.

$$
\begin{aligned}
\text { Input Slack percentage } & =\frac{\text { Input slack }}{\text { Actual input }} \times 100 \% \\
& =\frac{23}{60} \times 100 \%
\end{aligned}
$$

$$
=38.33 \%
$$

Thus if student AB 010 has to be efficient as AB 03 student, it should produce the same output using 38.33 percent less input.

Output Target $=$ Actual output $\times$ Relative Efficiency
Output Target $=68.5 \times 0.71=48.64$
This means that if ABO10 operates using $48.64 \sim 49$ as output with the same input (depicting a lower B.E.C.E results), then it will be considered as efficient as AB 03 student.
i.e. output Slack $=$ actual output - target output

$$
=68.5-49=19.5
$$

Output Slack can also be expressed as a percentage.
Output Slack percentage $=\frac{\text { output slack }}{=\text { Actual output }} \times 100 \%$

$$
=\frac{19.5}{68.5} \times 100 \%
$$

$$
=28.47 \%
$$

Thus if student AB 010 has to be efficient as AB 03 student, it should produce the same input using 28.47 percent more output.
4.5.1.2 Improvement for students at Abu Bonsra D/A JHS on student continuous assessment.

The student AB 08 has operated in an environment similar to that of the other students. Using his/her achievement as a benchmark is significant. Input target for inefficient students will be the amount of input that will enable the student to have the same ratio of efficiency as AB08 in terms of Student Continuous Assessment.

Using the input from Students Continuous Assessment $\left(\mathrm{U}_{3}\right)$ and the corresponding efficiencies, Input Target $=$ Actual input $\times$ Relative Efficiency

For AB06:
Input Target $=50 \times 0.92=46$
This means that if ABO6 operates using $40.00 \sim 40$ as input with the same output then it will be considered as efficient as AB 08 student.
i.e. input Slack $=$ actual input - target input

$$
=50-46=4
$$

Input Slack can also be expressed as a percentage.
Input Slack percentage $=\frac{\text { Input slack }}{\text { Actual input }} \times 100 \%$
$=\frac{4}{50} \times 100 \%$
$=8.00 \%$
Thus if student AB 06 has to be efficient as AB 08 student, it should produce the same output using 8.00 percent less input.

Output Target $=$ Actual output $\times$ Relative Efficiency
Output Target $=81.4 \times 0.92=74.89$
This means that if ABO6 operates using 74.89~75 as output with the same input (depicting a lower B.E.C.E overall results), then it will be considered as efficient as AB08 student.
i.e. output Slack $=$ actual output - target output

$$
=81.4-75=6.4
$$

Output Slack can also be expressed as a percentage.
Output Slack percentage $=\frac{\text { output slack }}{\text { Actual output }} \times 100 \%$

$$
=\frac{6.4}{81.4} \times 100 \%
$$

$$
=7.86 \%
$$

Thus if student AB06 has to be efficient as AB08 student, it should produce the same input using 7.86 percent less output.

## For AB07:

Input Target $=49 \times 0.91=44.59$
This means that if ABO 7 operates using $44.59 \sim 45$ as input with the same output then it will be considered as efficient as AB08 student.
i.e. input Slack $=$ actual input - target input

$$
=49-45=4
$$

Input Slack can also be expressed as a percentage.
Input Slack percentage $=\frac{\text { Input slack }}{\text { Actual input }} \times 100 \%$

$$
\begin{aligned}
& \quad=\frac{4}{49} \times 100 \% \\
& =8.16 \%
\end{aligned}
$$

Thus if student AB07 has to be efficient as AB08 student, it should produce the same output using 8.16 percent less input.

Output Target $=$ Actual output $\times$ Relative Efficiency
Output Target $=78.7 \times 0.91=71.62$
This means that if ABO7 operates using 71.62 ~ 72 as output with the same input (depicting a lower B.E.C.E overall results), then it will be considered as efficient as AB 08 student.
i.e. output Slack $=$ actual output - target output

$$
=78.7-72=6.7
$$

Output Slack can also be expressed as a percentage.
Output Slack percentage $=\frac{\text { output slack }}{\text { Actual output }} \times 100 \%$

$$
\begin{aligned}
& =\frac{6.7}{78.7} \times 100 \% \\
& =8.51 \%
\end{aligned}
$$

Thus if student AB 07 has to be efficient as AB 08 student, it should produce the same input using 7.86 percent less output.

For AB10:
Input Target $=45 \times 0.86=38.7$
This means that if AB 10 operates using $38.7 \sim 39$ as input with the same output then it will be considered as efficient as AB 08 student.
i.e. input Slack $=$ actual input - target input

$$
=45-39=6
$$

## Input Slack can also be expressed as a percentage.

Input Slack percentage $=\frac{\text { Input slack }}{\text { Actual input }} \times 100 \%$

$$
=\frac{6}{45} \times 100 \%
$$

$$
=13.33 \%
$$

Thus if student AB 10 has to be efficient as AB 08 student, it should produce the same output using 13.33 percent less input.

Output Target $=$ Actual output $\times$ Relative Efficiency
Output Target $=68.5 \times 0.86=58.91$
This means that if AB 10 operates using $58.91 \sim 59$ as output with the same input (depicting a lower B.E.C.E overall results), then it will be considered as efficient as AB 08 student.
i.e. output Slack $=$ actual output - target output

$$
=68.5-59=9.5
$$

Output Slack can also be expressed as a percentage.
Output Slack percentage $=\frac{\text { output slack }}{\text { Actual output }} \times 100 \%$

$$
\begin{aligned}
& =\frac{9.5}{68.5} \times 100 \% \\
& =13.87 \%
\end{aligned}
$$

Thus if student AB 10 has to be efficient as AB 08 student, it should produce the same input using 13.87 percent less output.

For AB04:
Input Target $=66 \times 0.79=52.14$
This means that if AB04 operates using $52.14 \sim 52$ as input with the same output then it will be considered as efficient as AB08 student.
i.e. input Slack $=$ actual input $-\operatorname{target}$ input

$$
=66-52=14
$$

Input Slack can also be expressed as a percentage.
Input Slack percentage $=\frac{\text { Input slack }}{\text { Actual input }} \times 100 \%$

$$
=\frac{14}{66} \times 100 \%
$$

$$
=21.21 \%
$$

Thus if student AB04 has to be efficient as AB08 student, it should produce the same output using 21.21 percent less input.

Output Target $=$ Actual output $\times$ Relative Efficiency
Output Target $=91.6 \times 0.79=72.36$
This means that if AB 10 operates using $72.36 \sim 72$ as output with the same input (depicting a lower B.E.C.E overall results), then it will be considered as efficient as AB 08 student.
i.e. output Slack $=$ actual output $-\operatorname{target}$ output

$$
=91.6-72=19.6
$$

Output Slack can also be expressed as a percentage.
Output Slack percentage $=\frac{\text { output slack }}{\text { Actual output }} \times 100 \%$

$$
=\frac{19.6}{91.6} \times 100 \%
$$

$$
=21.40 \%
$$

Thus if student AB04 has to be efficient as AB08 student, it should produce the same input using 13.87 percent less output.

For AB09:
Input Target $=54 \times 0.76=41.04$
This means that if AB09 operates using 41.04~41 as input with the same output then it will be considered as efficient as AB08 student.
i.e. input Slack $=$ actual input - target input

$$
=54-41=13
$$

Input Slack can also be expressed as a percentage.
Input Slack percentage $=\frac{\text { Input slack }}{\text { Actual input }} \times 100 \%$

$$
=\frac{13}{54} \times 100 \%
$$

$$
=24.07 \%
$$

Thus if student AB09 has to be efficient as AB08 student, it should produce the same output using 24.07 percent less input.

Output Target $=$ Actual output $\times$ Relative Efficiency
Output Target $=72.2 \times 0.76=54.87$
This means that if AB 10 operates using $54.87 \sim 55$ as output with the same input (depicting a lower B.E.C.E overall results), then it will be considered as efficient as AB 08 student.
i.e. output Slack $=$ actual output - target output

$$
=72.2-55=17.2
$$

Output Slack can also be expressed as a percentage.
Output Slack percentage $=\frac{\text { output slack }}{=\text { Actual output }} \times 100 \%$

$$
\begin{aligned}
& =\frac{17.2}{72.2} \times 100 \% \\
& =23.82 \%
\end{aligned}
$$

Thus if student AB 09 has to be efficient as AB 08 student, it should produce the same input using 23.82 percent less output.

For AB03:
Input Target $=71 \times 0.75=53.25$
This means that if AB03 operates using 53.25 ~ 53 as input with the same output then it will be considered as efficient as AB08 student.
i.e. input Slack $=$ actual input - target input

$$
=71-53=18
$$

Input Slack can also be expressed as a percentage.

$$
\begin{aligned}
\text { Input Slack percentage } & =\frac{\text { Input slack }}{\text { Actual input } \times 100 \%} \\
& =\frac{18}{71} \times 100 \% \\
& =25.35 \%
\end{aligned}
$$

Thus if student AB 03 has to be efficient as AB 08 student, it should produce the same output using 25.35 percent less input.

Output Target $=$ Actual output $\times$ Relative Efficiency
Output Target $=93.5 \times 0.75=70.13$
This means that if AB03 operates using 70.13 ~ 70 as output with the same input (depicting a lower B.E.C.E overall results), then it will be considered as efficient as AB 08 student.
i.e. output Slack $=$ actual output - target output

$$
=93.5-70=23.5
$$

Output Slack can also be expressed as a percentage.
Output Slack percentage $=\frac{\text { output slack }}{\text { Actual output }} \times 100 \%$

$$
=\frac{23.5}{93.5} \times 100 \%
$$

$$
=25.13 \%
$$

Thus if student AB 03 has to be efficient as AB 08 student, it should produce the same input using 25.13 percent less output.

For AB05:
Input Target $=70 \times 0.73=51.10$

This means that if AB 05 operates using $51.10 \sim 51$ as input with the same output then it will be considered as efficient as AB08 student.
i.e. input Slack $=$ actual input - target input

$$
=70-51=19
$$

Input Slack can also be expressed as a percentage.
Input Slack percentage $=\frac{\text { Input slack }}{\text { Actual input }} \times 100 \%$

$$
\begin{aligned}
& \quad=\frac{19}{70} \times 100 \% \\
& =27.14 \%
\end{aligned}
$$

Thus if student AB05 has to be efficient as AB08 student, it should produce the same output using 27.14 percent less input.

Output Target $=$ Actual output $\times$ Relative Efficiency
Output Target $=90.7 \times 0.73=66.21$
This means that if AB05 operates using $66.21 \sim 66$ as output with the same input (depicting a lower B.E.C.E overall results), then it will be considered as efficient as AB 08 student.
i.e. output Slack $=$ actual output -


Output Slack can also be expressed as a percentage.
Output Slack percentage $=\frac{\text { output slack }}{\text { Actual output }} \times 100 \%$

$$
=\frac{24.7}{90.7} \times 100 \%
$$

$$
=27.23 \%
$$

Thus if student AB 05 has to be efficient as AB 08 student, it should produce the same input using 27.23 percent less output.

## For AB01:

Input Target $=79 \times 0.70=55.30$
This means that if AB 01 operates using $55.30 \sim 55$ as input with the same output then it will be considered as efficient as AB 08 student.
i.e. input Slack $=$ actual input - target input

$$
=79-55=24
$$

Input Slack can also be expressed as a percentage.
Input Slack percentage $=\frac{\text { Input slack }}{\text { Actual input }} \times 100 \%$

$$
=\frac{24}{79} \times 100 \%
$$

$$
=30.38 \%
$$

Thus if student AB 01 has to be efficient as AB 08 student, it should produce the same output using 30.38 percent less input.

Output Target $=$ Actual output $\times$ Relative Efficiency
i.e. output Slack $=$ actual output -

Output Target $=97.2 \times 0.70=68.04$
This means that if AB01 operates using 68.04~68 as output with the same input (depicting a lower B.E.C.E overall results), then it will be considered as efficient as AB08 student.
target output

$$
=97.2-68=29.2
$$

Output Slack can also be expressed as a percentage.
Output Slack percentage $=\frac{\text { output slack }}{\text { Actual output }} \times 100 \%$

$$
\begin{aligned}
& =\frac{29.2}{97.2} \times 100 \% \\
& =30.04 \%
\end{aligned}
$$

Thus if student AB 01 has to be efficient as AB 08 student, it should produce the same input using 30.04 percent less output.

For AB02:
Input Target $=82 \times 0.67=30.04$
This means that if AB02 operates using 30.02 ~ 30 as input with the same output then it will be considered as efficient as AB 08 student.
i.e. input Slack $=$ actual input - target input

$$
=82-30=52
$$

Input Slack can also be expressed as a percentage.
Input Slack percentage $=\frac{\text { Input slack }}{\text { Actual input }} \times 100 \%$

$$
=\frac{52}{82} \times 100 \%
$$

$$
=63.41 \%
$$

Thus if student AB 02 has to be efficient as AB 08 student, it should produce the same output using 63.41 percent less input.
i.e. output Slack $=$ actual output -

Output Target $=$ Actual output $\times$ Relative Efficiency
Output Target $=96.3 \times 0.67=64.52$
This means that if AB 02 operates using $64.52 \sim 65$ as output with the same input (depicting a lower B.E.C.E overall results), then it will be considered as efficient as AB 08 student.

## target output

$$
=96.3-65=31.3
$$

Output Slack can also be expressed as a percentage.
Output Slack percentage $=\frac{\text { output slack }}{\text { Actual output }} \times 100 \%$

$$
=\frac{31.3}{96.3} \times 100 \%
$$

$$
=32.50 \%
$$

Thus if student AB02 has to be efficient as AB08 student, it should produce the same input using 32.50 percent less output.

### 4.5.2 Students at Adiembra D/A JHS at the Asokwa Circuit

4.5.2.1 Improvement for students at Adiembra D/A JHS on student attendance.

The student AD01 and AD02 have operated in an environment similar to that of the other students in Adiembra D/A JHS at the Asokwa Circuit. Using their performance as a benchmark is significant. Input target for inefficient students will be the amount of input that will enable the student to have the same ratio of efficiency as AD01 and AD02 in terms of Student Attendance ( $\mathrm{U}_{1}$ )
i.e. output Slack $=$ actual output -

Using the input from Students Attendance $\left(\mathrm{U}_{1}\right)$ and the corresponding efficiencies, Input
Target $=$ Actual input $\times$ Relative Efficiency

For AD03:


Input Target $=61 \times 0.98=59.78$
This means that if AD03 operates using $59.78 \sim 60$ as input with the same output then it will be considered as efficient as AD 01 and AD 02 students.
i.e. input Slack $=$ actual input - target input

$$
=61-60=1
$$

Input Slack can also be expressed as a percentage.



Thus if student AD 03 has to be efficient as AD 01 and AD 02 student, it should produce the same output using 1.64 percent less input.

Output Target $=$ Actual output $\times$ Relative Efficiency
Output Target $=91.7 \times 0.98=89.87$
This means that if AD03 operates using $89.87 \sim 90$ as output with the same input (depicting a lower B.E.C.E overall results), then it will be considered as efficient as AD01 and AD02 students.
i.e. output Slack $=$ actual output - target output

$$
=91.7-90=1.7
$$

Output Slack can also be expressed as a percentage.
Output Slack percentage $=\frac{\text { output slack }}{\text { Actual output }} \times 100 \%$

$$
=1.85 \%-\frac{1.7}{91.7} \times 100 \%
$$

Thus if student AD03 has to be efficient as AD01 and AD02 students, it should produce the same input using 1.85 percent less output.

## For AD04:

Input Target $=62 \times 0.95=58.9$

- target input

Input Slack can also be expressed as a percentage.

$$
\begin{array}{r}
\times 100 \% \\
\quad \begin{array}{r}
\text { Input slack } \\
\text { Actual input }
\end{array} \\
\times 100 \%
\end{array}
$$

This means that if AD04 operates using $58.9 \sim 59$ as input with the same output then it will be considered as efficient as AD01 and AD02 students.
i.e. input Slack = actual input

$$
=6259=3
$$

Input Slack percentage $=$

$$
\begin{aligned}
& =\frac{3}{62} \\
& =4.84 \%
\end{aligned}
$$

Thus if student AD04 has to be efficient as AD01 and AD02 students, it should produce the same output using 4.84 percent less input.

Output Target $=$ Actual output $\times$ Relative Efficiency
Output Target $=90.8 \times 0.95=86.26$
This means that if AD04 operates using $86.26 \sim 86$ as output with the same input (depicting a lower B.E.C.E overall results), then it will be considered as efficient as AD02 and AD01 students.
i.e. output Slack $=$ actual output - target output

$$
=90.8-86=4.8
$$

Output Slack can also be expressed as a percentage.
Output Slack percentage $=\frac{\text { output slack }}{\text { Actual output }} \times 100 \%$

$$
=\frac{4.8}{90.8} \times 100 \%
$$

$$
=5.29 \%
$$

- target input

Input Slack can also be expressed as a percentage.

Input Slack percentage $=\frac{\text { Input slack }}{\text { Actual input }} \times \quad 100 \%$

$$
\times 100 \%
$$

Thus if student AD04 has to be efficient as AD01 and AD02 students, it should produce the same input using 5.29 percent less output.

For AD06:
Input Target $=61 \times 0.93=56.73$
This means that if AD06 operates using $56.73 \sim 57$ as input with the same output then it will be considered as efficient as AD01 and AD02 students.
i.e. input Slack $=$ actual input

$$
\begin{aligned}
=6157 & =4 \\
& =\frac{4}{61}
\end{aligned}
$$

$$
=6.56 \%
$$

Thus if student AD06 has to be efficient as AD01 and AD02 students, it should produce the same output using 6.56 percent less input.

Output Target $=$ Actual output $\times$ Relative Efficiency
Output Target $=87 \times 0.93=80.91$
This means that if AD06 operates using 80.91~81 as output with the same input (depicting a lower B.E.C.E overall results), then it will be considered as efficient as AD01 and AD02 students.
i.e. output Slack $=$ actual output - target output

$$
\begin{aligned}
=87-81= & \\
& - \text { target input }
\end{aligned}
$$

Input Slack can also be expressed as a percentage.

$$
\times 100 \% \quad \frac{\text { Input slack }}{\text { Actual input }}
$$

$$
\times 100 \%
$$

Output Slack can also be expressed as a percentage.
Output Slack percentage $=\frac{\text { output slack }}{\text { Actual output }} \times 100 \%$

$$
\begin{aligned}
& =\frac{6}{87} \times 100 \% \\
& =6.90 \%
\end{aligned}
$$

Thus if student AD06 has to be efficient as AD01 and AD02 students, it should produce the same input using 6.90 percent less output.

For AD07:
Input Target $=61 \times 0.91=55.51$
This means that if AD07 operates using $55.51 \sim 56$ as input with the same output then it will be considered as efficient as AD01 and AD02 students.
i.e. input Slack = actual input

$$
=6156=5
$$



Input Slack can also be expressed as a percentage.

Input Slack percentage $=\frac{\text { Input slack }}{\text { Actual input }} \times \quad 100 \%$ $\times 100 \%$

$$
\begin{aligned}
& =\frac{5}{61} \\
& =8.20 \%
\end{aligned}
$$

Thus if student AD07 has to be efficient as AD01 and AD02 students, it should produce the same output using 8.20 percent less input.

Output Target $=$ Actual output $\times$ Relative Efficiency
Output Target $=85.2 \times 0.91=77.53$
This means that if AD07 operates using 77.53 ~ 78 as output with the same input (depicting a lower B.E.C.E overall results), then it will be considered as efficient as AB08 student.
i.e. output Slack $=$ actual output - target output

$$
=85.2-78=7.2
$$

Output Slack can also be expressed as a percentage.
Output Slack percentage $=\frac{\text { output slack }}{\text { Actual output }} \times 100 \%$

$$
\begin{aligned}
& =\frac{7.2}{85.2} \times 100 \% \\
& =8.45 \%
\end{aligned}
$$

Thus if student AD 07 has to be efficient as AD 01 and AD 02 students, it should produce the same input using 8.45 percent less output.

For AD05:
Input Target $=63 \times 0.89=56.07$
This means that if AD05 operates using $56.07 \sim 56$ as input with the same output then it will be considered as efficient as AD 01 and AD 02 students.
i.e. input Slack $=$ actual input - target input

$$
=63-56=7
$$

Input Slack can also be expressed as a percentage.

$$
\begin{array}{r}
=\frac{7}{63} \\
=11.11 \%
\end{array}
$$

Thus if student AD05 has to be efficient as AD01 and AD02 students, it should produce the same output using 11.11 percent less input.

## Output Target $=$ Actual output $\times$ Relative Efficiency

Output Target $=86.1 \times 0.89=76.63$
This means that if AD05 operates using 76.63 ~ 77 as output with the same input (depicting a lower B.E.C.E overall results), then it will be considered as efficient as AD01 and AD02 students.
i.e. output Slack $=$ actual output - target output

$$
=86.1-77=9.1
$$

Output Slack can also be expressed as a percentage.

$$
\begin{aligned}
\text { Output Slack percentage } & =\frac{\frac{\text { Output slack }}{\text { Actual output }} \times 100 \%}{} \begin{aligned}
& =\frac{9.1}{86.1} \times 100 \% \\
& =10.57 \%
\end{aligned}
\end{aligned}
$$

Thus if student AD 05 has to be efficient as AD 01 and AD 02 students, it should produce the same input using 10.57 percent less output.

For AD08:
Input Target $=62 \times 0.85=52.7$
This means that if AD08 operates using $52.7 \sim 53$ as input with the same output then it will be considered as efficient as AD01 and AD02 students.
i.e. input Slack $=$ actual input - target input

$$
=62-53=9
$$

Input Slack can also be expressed as a percentage.

Input Slack percentage $=\frac{\text { Input slack }}{\text { Actual input }} \times \quad 100 \%$ $\times 100 \%$

Input Slack percentage $=\frac{\text { Input slack }}{\text { Actual input }} \times 100 \%$

$$
\begin{aligned}
& \quad=\frac{9}{62} \times 100 \% \\
& =14.52 \%
\end{aligned}
$$

Thus if student AD08 has to be efficient as AD01 and AD02 students, it should produce the same output using 14.52 percent less input.

Output Target $=$ Actual output $\times$ Relative Efficiency
Output Target $=81.5 \times 0.85=69.28$
This means that if AD08 operates using $69.28 \sim 69$ as output with the same input (depicting a lower B.E.C.E overall results), then it will be considered as efficient as AD01 and AD02 students.
i.e . output Slack $=$ actual output - target output

$$
=81.5-69=12.5
$$

Output Slack can also be expressed as a percentage.
Output Slack percentage $=\frac{\text { Output slack }}{\text { Actual output }} \times 100 \%$

$$
\begin{aligned}
& \quad=\frac{12.5}{81.5} \times 100 \% \\
& =15.34 \%
\end{aligned}
$$

Thus if student AD08 has to be efficient as AD01 and AD02 students, it should produce the same input using 15.34 percent less output.

## For AD09:

Input Target $=63 \times 0.83=52.29$
This means that if AD09 operates using 52.29~52 as input with the same output then it will be considered as efficient as AD01 and AD02 students.
i.e. input Slack $=$ actual input - target input

$$
=63-52=11
$$

Input Slack can also be expressed as a percentage.


$$
\begin{aligned}
& \text { Input Slack percentage }= \\
& \frac{\text { Input slack }}{\text { Actual input }} \times 100 \% \\
&=\frac{11}{63} \times 100 \% \\
&=17.46 \%
\end{aligned}
$$

Thus if student AD09 has to be efficient as AD01 and AD02 students, it should produce the same output using 17.46 percent less input.

Output Target $=$ Actual output $\times$ Relative Efficiency
Output Target $=80.6 \times 0.83=66.90$
This means that if AD09 operates using 66.90~67 as output with the same input (depicting a lower B.E.C.E overall results), then it will be considered as efficient as AD01 and AD02 students.
i.e. output Slack $=$ actual output - target output

$$
=80.6-67=13.6
$$

Output Slack can also be expressed as a percentage.
Output Slack percentage $=\frac{\text { output slack }}{\text { Actual output }} \times 100 \%$

$$
\begin{aligned}
& \quad=\frac{13.6}{80.6} \times 100 \% \\
& =16.87 \%
\end{aligned}
$$

Thus if student AD09 has to be efficient as AD01 and AD02 students, it should produce the same input using 16.87 percent less output.

For AD10:
Input Target $=61 \times 0.81=49.41$
This means that if AD10 operates using $49.41 \sim 49$ as input with the same output then it will be considered as efficient as AD 01 and AD 02 students.
i.e. input Slack $=$ actual input - target input

$$
=61-49=12
$$

Input Slack can also be expressed as a percentage.

$$
\begin{aligned}
\text { Input Slack percentage } & =\frac{\text { Input slack }}{\text { Actual input } \times 100 \%} \\
& =\frac{12}{61} \times 100 \% \\
& =19.67 \%
\end{aligned}
$$

Thus if student AD10 has to be efficient as AD 01 and AD 02 students, it should produce the same output using 19.67 percent less input.

Output Target $=$ Actual output $\times$ Relative Efficiency
Output Target $=75.9 \times 0.81=61.48$
This means that if AD 10 operates using $61.48 \sim 61$ as output with the same input (depicting a lower B.E.C.E overall results), then it will be considered as efficient as AB08 student.
i.e. output Slack $=$ actual output - target output

$$
=75.9-61=14.9
$$

Output Slack can also be expressed as a percentage.
Output Slack percentage $=\frac{\text { output slack }}{\text { Actual output }} \times 100 \%$

$$
\begin{aligned}
& =\frac{14.9}{75.9} \times 100 \% \\
& =19.63 \%
\end{aligned}
$$

Thus if student AD10 has to be efficient as AD01 and AD02 students, it should produce the same input using 19.63 percent less output.
4.5.2.2 Improvement for students at Adiembra D/A JHS on student continuous assessment.

The student AD09 has operated in an environment similar to that of the other students at Adiembra D/A JHS in Asokwa Circuit. Using his/her achievement as a benchmark is significant. Input target for inefficient students will be the amount of input that will enable the student to have the same ratio of efficiency as AD09 in terms of Student Continuous Assessment $\left(\mathrm{U}_{3}\right)$.

Using the input from Students Continuous Assessment $\left(\mathrm{U}_{3}\right)$ and the corresponding efficiencies,

Input Target $=$ Actual input $\times$ Relative Efficiency

For AD06:
Input Target $=46 \times 0.96=44.16$
This means that if AD06 operates using 44.16 ~ 44 as input with the same output then it will be considered as efficient as AD 09 student.
i.e. input Slack $=$ actual input - target input

$$
=46-44=2
$$

Input Slack can also be expressed as a percentage.
Input Slack percentage $=\frac{\text { Input slack }}{\text { Actual input }} \times 100 \%$

$$
\begin{aligned}
& =\frac{2}{46} \times 100 \% \\
& =4.35 \%
\end{aligned}
$$

Thus if student AD06 has to be efficient as AD09 student, it should produce the same output using 4.35 percent less input.

Output Target $=$ Actual output $\times$ Relative Efficiency
Output Target $=87 \times 0.96=83.52$
This means that if AD06 operates using $83.52 \sim 84$ as output with the same input (depicting a lower B.E.C.E overall results), then it will be considered as efficient as AD09 student.
i.e. output Slack $=$ actual output - target output

$$
=87-84=3
$$

Output Slack can also be expressed as a percentage.
Output Slack percentage $=\frac{\text { output slack }}{\text { Actual output }} \times 100 \%$

$$
\begin{aligned}
& =\frac{3}{87} \times 100 \% \\
& =3.45 \%
\end{aligned}
$$

Thus if student AD06 has to be efficient as AD09 student, it should produce the same input using 3.45 percent less output.

For AD07:
Input Target $=46 \times 0.94=43.24$
This means that if AD07 operates using 43.24~43 as input with the same output then it will be considered as efficient as AD09 student.
i.e. input Slack $=$ actual input - target input

$$
=46-43=3
$$

Input Slack can also be expressed as a percentage.

$$
\begin{aligned}
& \text { Input Slack percentage }=\frac{\text { Input slack }}{\text { Actual input } \times 100 \%} \\
&=\frac{3}{46} \times 100 \% \\
&=6.52 \%
\end{aligned}
$$

Thus if student AD07 has to be efficient as AD09 student, it should produce the same output using 6.52 percent less input.

Output Target $=$ Actual output $\times$ Relative Efficiency
Output Target $=85.2 \times 0.94=80.09$
This means that if AD07 operates using 80.09~80 as output with the same input (depicting a lower B.E.C.E overall results), then it will be considered as efficient as AD09 student.
i.e. output Slack $=$ actual output - target output

$$
=85.2-80=5.2
$$

Output Slack can also be expressed as a percentage.

$$
\text { Output Slack percentage }=\frac{\text { output slack }}{\text { Actual output }} \times 100 \%
$$

$$
\begin{aligned}
& \quad \frac{-5.2}{85.2} \times 100 \% \\
& =6.10 \%
\end{aligned}
$$

Thus if student AD 07 has to be efficient as AD 09 student, it should produce the same input using 6.10 percent less output.

For AD08:
Input Target $=44 \times 0.94=41.36$
This means that if AD08 operates using 41.36~41 as input with the same output then it will be considered as efficient as AD09 student.
i.e. input Slack $=$ actual input - target input

$$
=44-41=3
$$

Input Slack can also be expressed as a percentage.

$$
\begin{aligned}
& \text { Input Slack percentage }=\frac{\text { Input slack }}{\text { Actual input } \times 100 \%} \\
&=\frac{3}{44} \times 100 \% \\
&=6.82 \%
\end{aligned}
$$

Thus if student AD08 has to be efficient as AD09 student, it should produce the same output using 6.82 percent less input.

Output Target $=$ Actual output $\times$ Relative Efficiency
Output Target $=81.5 \times 0.94=76.61$
This means that if AD08 operates using 76.61~77 as output with the same input (depicting a lower B.E.C.E overall results), then it will be considered as efficient as AB 08 student.
i.e. output Slack $=$ actual output - target output

$$
=81.5-77=4.5
$$

Output Slack can also be expressed as a percentage.
Output Slack percentage $=\frac{\text { Output slack }}{\text { Actual output }} \times 100 \%$

$$
=\frac{4.5}{81.5} \times 100 \%
$$

$$
=5.52 \%
$$

Thus if student AD 08 has to be efficient as AD 09 student, it should produce the same input using 5.52 percent less output.

For AD10:

Input Target $=43 \times 0.90=38.70$
This means that if AD10 operates using $38.7 \sim 39$ as input with the same output then it will be considered as efficient as AD09 student.
i.e. input Slack $=$ actual input - target input

$$
=43-39=4
$$

Input Slack can also be expressed as a percentage.
Input Slack percentage $=\frac{\text { Input slack }}{\text { Actual input }} \times 100 \%$

$$
\begin{aligned}
& =\frac{6}{43} \times 100 \% \\
& =9.30 \%
\end{aligned}
$$

Thus if student AD10 has to be efficient as AD09 student, it should produce the same output using 9.30 percent less input.

Output Target $=$ Actual output $\times$ Relative Efficiency
Output Target $=75.9 \times 0.90=68.31$
This means that if AD10 operates using $68.31 \sim 68$ as output with the same input (depicting a lower B.E.C.E overall results), then it will be considered as efficient as AD09 student.
i.e. output Slack $=$ actual output - target output

$$
=75.9-68=7.9
$$

Output Slack can also be expressed as a percentage.
Output Slack percentage $=\frac{\text { output slack }}{\text { Actual output }} \times 100 \%$

$$
\begin{aligned}
& =\frac{7.9}{75.9} \times 100 \% \\
& =10.41 \%
\end{aligned}
$$

Thus if student AD10 has to be efficient as AD09 student, it should produce the same input using 10.41 percent less output.

## For AD04:

Input Target $=53 \times 0.87=46.11$

This means that if AD04 operates using 46.11~46 as input with the same output then it will be considered as efficient as AD09 student.
i.e. input Slack $=$ actual input - target input

$$
=53-46=7
$$

Input Slack can also be expressed as a percentage.
Input Slack percentage $=\frac{\text { Input slack }}{\text { Actual input }} \times 100 \%$

$$
=\frac{7}{53} \times 100 \%
$$

$$
=13.21 \%
$$

Thus if student AD04 has to be efficient as AD09 student, it should produce the same output using 13.21 percent less input.

Output Target $=$ Actual output $\times$ Relative Efficiency
Output Target $=90.8 \times 0.87=78.99$
This means that if AD04 operates using 78.99~79 as output with the same input (depicting a lower B.E.C.E overall results), then it will be considered as efficient as AD09 student.
i.e. output Slack $=$ actual output - target output

$$
=90.8-79=11.8
$$

Output Slack can also be expressed as a percentage.
Output Slack percentage $=\frac{\text { output slack }}{\text { Actual output }} \times 100 \%$

$$
=\frac{11.8}{90.8} \times 100 \%
$$

$$
=13.00 \%
$$

Thus if student AD 04 has to be efficient as AD 09 student, it should produce the same input using 13.00 percent less output.

## For AD05:

Input Target $=52 \times 0.84=43.68$
This means that if AD05 operates using $43.68 \sim 44$ as input with the same output then it will be considered as efficient as AD09 student.
i.e. input Slack $=$ actual input - target input

$$
=52-44=8
$$

Input Slack can also be expressed as a percentage.
Input Slack percentage $=\frac{\text { Input slack }}{\text { Actual input }} \times 100 \%$

$$
=\frac{8}{52} \times 100 \%
$$

$$
=15.38 \%
$$

Thus if student AD05 has to be efficient as AD09 student, it should produce the same output using 15.38 percent less input.

## Output Target $=$ Actual output $\times$ Relative Efficiency

Output Target $=86.1 \times 0.84=72.32$
This means that if AD05 operates using 72.32~72 as output with the same input (depicting a lower B.E.C.E overall results), then it will be considered as efficient as AD09 student.
i.e. output Slack $=$ actual output - target output

$$
=86.1-72=14.1
$$

Output Slack can also be expressed as a percentage.
Output Slack percentage $=\frac{\text { Output slack }}{\text { Actual output }} \times 100 \%$

$$
\begin{aligned}
& \quad=\frac{14.1}{86.1} \times 100 \% \\
& =16.38 \%
\end{aligned}
$$

Thus if student AD 05 has to be efficient as AD 09 student, it should produce the same input using 16.38 percent less output.

For AD03:
Input Target $=57 \times 0.82=46.74$
This means that if AD03 operates using 46.74~47 as input with the same output then it will be considered as efficient as AD 09 student.
i.e. input Slack $=$ actual input - target input

$$
=57-47=10
$$

Input Slack can also be expressed as a percentage.
Input Slack percentage $=\frac{\text { Input slack }}{\text { Actual input }} \times 100 \%$

$$
=\frac{10}{57} \times 100 \%
$$

$$
=17.54 \%
$$

Thus if student AD03 has to be efficient as AD09 student, it should produce the same output using 17.54 percent less input.

## Output Target $=$ Actual output $\times$ Relative Efficiency

Output Target $=91.7 \times 0.82=75.19$
This means that if AD03 operates using 75.19~75 as output with the same input (depicting a lower B.E.C.E overall results), then it will be considered as efficient as AD09 student.
i.e. output Slack $=$ actual output - target output

$$
=91.7-75=16.7
$$

Output Slack can also be expressed as a percentage.
Output Slack percentage $=\frac{\text { output slack }}{\text { Actual output }} \times 100 \%$

$$
\begin{aligned}
& =\frac{16.7}{91.7} \times 100 \% \\
& =18.21 \%
\end{aligned}
$$

Thus if student AD 03 has to be efficient as AD 09 student, it should produce the same input using 18.21 percent less output.

For AD01:
Input Target $=70 \times 0.69=48.3$
This means that if AD01 operates using 48.3~48 as input with the same output then it will be considered as efficient as AD09 student.
i.e. input Slack $=$ actual input - target input

$$
=70-48=22
$$

Input Slack can also be expressed as a percentage.

Input Slack percentage $=\frac{\text { Input slack }}{\text { Actual input }} \times 100 \%$

$$
=\frac{22}{70} \times 100 \%
$$

$$
=31.43 \%
$$

Thus if student AD01 has to be efficient as AD09 student, it should produce the same output using 31.43 percent less input.

Output Target $=$ Actual output $\times$ Relative Efficiency
Output Target $=95.4 \times 0.69=65.83$
This means that if AD01 operates using $65.83 \sim 66$ as output with the same input (depicting a lower B.E.C.E overall results), then it will be considered as efficient as AD09 student.
i.e. output Slack $=$ actual output - target output

$$
=95.4-66=29.4
$$

Output Slack can also be expressed as a percentage.
Output Slack percentage $=\frac{\text { output slack }}{\text { Actual output }} \times 100 \%$

$$
=\frac{29.4}{95.4} \times 100 \%
$$

Thus if student AD 01 has to be efficient as AD 09 student, it should produce the same input using 30.82 percent less output.

## For AD02:

Input Target $=78 \times 0.62=48.36$
This means that if AD02 operates using $48.36 \sim 48$ as input with the same output then it will be considered as efficient as AD09 student.
i.e. input Slack $=$ actual input - target input

$$
=78-48=30
$$

Input Slack can also be expressed as a percentage.
Input Slack percentage $=\frac{\text { Input slack }}{\text { Actual input }} \times 100 \%$

$$
\begin{aligned}
& =\frac{30}{78} \times 100 \% \\
& =38.46 \%
\end{aligned}
$$

Thus if student AD02 has to be efficient as AD09 student, it should produce the same output using 38.46 percent less input.

Output Target $=$ Actual output $\times$ Relative Efficiency
Output Target $=95.4 \times 0.62=59.15$
This means that if AD02 operates using 59.15 ~ 59 as output with the same input (depicting a lower B.E.C.E overall results), then it will be considered as efficient as AD09 student.
i.e. output Slack $=$ actual output $-\operatorname{target}$ output

$$
=95.4-59=36.4
$$

Output Slack can also be expressed as a percentage.
Output Slack percentage $=\frac{\text { output slack }}{\text { Actual output }} \times 100 \%$
$\square=\frac{36.4}{95.4} \times 100 \%$
Thus if student AD 02 has to be efficient as AD 09 student, it should produce the same input using 38.16 percent less output.

## CHAPTER FIVE

## CONCLUSION AND RECOMMENDATION

### 5.0 CONCLUSION

Data envelopment analysis seems to be a useful tool for small data sets estimation (www.bookpump.com/internet source).

The method identifies best practices for the purpose of benchmarking (www.inforum.cz/internet) source. The analysis provides the precise corrective figure for every output and input in order to improve the efficiency of an inefficient Student in Abu Bonsra D/A JHS and Adiembra D/A JHS at Fomena and Asokwa Circuits respectively.

In my project, I deployed Data Envelopment Analysis.

The chapter summarizes the major findings and conclusions that can be drawn from them. The necessary recommendations have been made after the findings.

- From the analysis done using DEA, two students from Adiembra D/A JHS was efficient in terms of average input ( $u+$ ) as against the output of student B.E.C.E results (v) whiles eight (8) students from Adiembra D/A JHS in the Asokwa Circuit and all the ten (10) students from Abu Bonsra D/A JHS at the Fomena Circuit were inefficient because they could have performed better in the BECE results considering their attendance to school and continuous assessment.
- What contributed to the effectiveness of the two students performance was their highest attendance to school within the three year span and though had a minimal continuous assessment at the end of the three years, they were able to deliver during their B.E.C.E examination.
- There is a positive correlation between student attendance and their B.E.C.E performance. From the analysis, almost all who had a higher attendance to school within the three years had their efficiencies higher than those who recorded lower attendance. Also, students who recorded high attendance also performed better as compared to those who had low attendance to school.
- Students at Adiembra D/A JHS had higher efficiencies as compared to students at Abu Bonsra D/A JHS when their teacher's attendance is taken into consideration. This depicts that teacher attendance to school contributes greatly to the performance of the students during their B.E.C.E and also how efficient their students will be.
- From the analysis, it is clear that Student Continuous Assessment have a significant impact on the B.E.C.E results. A student with a high continuous assessment is likely to perform better than a student with a low continuous assessment. However, the extent of the difference in continuous assessment does not necessarily depict same difference during the B.E.C.E
- From the analysis, it is shown that students with a lower Continuous assessment had higher efficiencies than those with higher continuous assessment. This is because those students who recorded lower continuous assessment ended up having B.E.C.E results which does not depict their strength. There could be an external factor contributing to this output which might be some weeks before the examination or during the examination.
- There is no significant correlation between student attendance and their continuous assessment. This is because, other external factors may come to play such as parental control, extra classes etc.


### 5.1 RECOMMENDATIONS

- Ghana Education Service must see to it that, School Based Assessment (SBA) are supervised accordingly by head teachers. This will inform them of students who need much attention in other to bring weak students to a higher level in terms of performance.
- Ghana Education must see to it that, registers are marked by class teachers at the appropriate time each day an supervised by head teachers. This will inform teachers of students who are not regular to school for the appropriate action to be taken.
- Innovative programs like Inspirational Campaign which seek to increase student performance through inspiration (by giving inspirational talks to the students), also, programs geared at reducing teenage pregnancy and encouraging regular and punctual attendance among students and teachers.
- There can be award schemes instituted by Ghana Education Service to encourage students to give their maximum best in school which will definitely be seen in their B.E.C.E results
- Students who have been attaining higher continuous assessment should be encouraged not to be complacent and relax but to learn and give their best during the B.E.C.E results.
- At the terminal point, punctuality and regularity by students will not only earn them an average performance in the B.E.C.E but will help them to do well in their career to earn a living and improve national development


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