KWAME NKRUMAH UNIVERSITY OF SCIENCE AND

TECHNOLOGY, KUMASI

MODELING OF PRODUCTION PLAN AND SCHEDULING OF MANUFACTURING PROCESS FOR A BEVERAGE COMPANY IN GHANA

By

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A THESIS SUBMITTED TO THE DEPARTMENT OF MATHEMATICS, KWAME NKRUMAH UNIVERSITY OF SCIENCE AND TECHNOLOGY IN PARTIAL FUFILLMENT OF THE REQUIREMENT FOR THE DEGREE OF MSC INDUSTRIAL MATHEMATICS

November, 2015

Declaration

I hereby declare that this submission is my own work towards the award of the MSc degree and that, to the best of my knowledge, it contains no material previously published by another person nor material which had been accepted for the award of any other degree of the university, except where due acknowledgement had been made in the text.

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Dedication

I dedicate this work to God Almighty and my dad Mr. Boahen.

Abstract

Production scheduling is a decision-making process that is used in manufacturing and service industries to achieve e ciency and minimize production cost. The goal with this research is to optimize the production and Scheduling of products of a beverage company by developing a model that displays the optimal production plan for a given planning horizon. The model is designed in a way that it can be implemented on any kind of Beverage Company: The model can also be implemented on di erent kind of production to make a production plan. A Mixed Linear Programming Model to support decision making is presented, in order to nd the optimal production plan, by maximizing the production and reducing cost. Data from Promasidor Ghana Limited was applied to the model. Results from the model were analyzed and sensitivity analysis was carried out. I found out that the mixed linear programming model is an e cient approach to solve a production planning and Scheduling problems.

Acknowledgements

My deepest, heartfelt thanks, rst of all goes to the Almighty God for granting me guidance and knowledge in coming out with this important study.

I sincerely thank my supervisor Prof. S. K. Amponsah for his help, guidance and insightful suggestions. I also thank Dr. Dominic Otoo , Mr. Charles Sebil and all the lecturers who in one way or the other taught me in the course of my study. I am very grateful for all their inputs in various ways and at various times. I also thank my boss Ms. Lucy Diawuo, the assistant registrar of Institute of Distant Learning (IDL) and all of my friends and colleagues for giving me reasons to persevere and work hard, to laugh, and most of all support through the good times and bad.

Most importantly I thank God for life and strength to go through this course successfully. I am sincerely grateful.

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CHAPTER 1

INTRODUCTION

1.1 BACKGROUND OF THE STUDY

The challenge of improving production plan, manufacturing processes and scheduling has inspired many di erent researchers. For manufacturers, the task of meeting the ever rising demand, customer expectations and lowering production costs in an environment of more goods, more complexity, more alternatives and competition is placing great stress on the e ectiveness of planning of activities in the manufacturing process. Most companies have already adopted solutions with varying degrees of planning and scheduling skills. However, production managers admit that these same systems are becoming an old fashion, lacking the swiftness, exibility and sensitivity to manage the increasing complex production atmosphere.

Due to government believe in private sector being the engine of economic growth, beverage companies have grown rapidly over the past twenty(20) years in Ghana and competition among companies is so high that planning and scheduling of resources must be pro cient for any of these companies to be pro table and able to survive competition. In modern manufacturing scheduling problems, it is of importance to e ciently utilize diverse resources. Treating set up times separately from processing times allows operations to be performed simultaneously and hence improve resource utilization. This is particularly important in modern production management systems, such as, Just-In-Time (JIT), and Optimized Production Technology (OPT). The advantages of reducing set up times include; reduced cost, increased production speed, increased output, reduced lead times, faster charge over, increased pro tability and customer satisfaction. Planning and scheduling of products, raw materials and labour are paramount to the pro tability of any beverage company in Ghana and any part of the world.

Production planning is one of the most important activities in a production factory. Production planning represents the heart beat of any manufacturing process. According to Cai et al.(2011), production plan considers resource capacities, time periods, supply and demand over a long planning horizon at a high level. Its decision then forms the input to more detailed, shorter-term functions such as scheduling and control at the lower level, which usually have more accurate estimates of supply, demand, and capacity levels. Hence, interaction between production planning and production scheduling/control is inevitable, not only because the scheduling/control decisions are constrained by the planning decisions, but also because disruptions occurring in the execution/control stage (usually after schedule generation) may a ect the optimality and/or feasibility of both the plan and the schedule.

Mitchell (1939) discussed the role of production planning department, including routing, dispatching (issuing shop orders) and scheduling. Stevenson (2009) considers that in the decision making hierarchy, scheduling decisions are the nal step in the transformation process before actual output occurs.

Manufacturing facilities are complex, dynamic, stochastic systems. From the beginning of organized manufacturing, workers, supervisors, engineers, and managers have developed many clever and practical methods for controlling production activities. Many manufacturing organizations generate and update production schedules, which are plans that state when certain controllable activities (e.g., processing of jobs by resources) should take place. Production schedules coordinate activities to increase productivity and minimize operating costs. A production schedule can identify resource con icts, control the release of jobs to the shop, ensure that required raw materials are ordered in time, determine whether delivery promises can be met, and identify time periods available for preventive maintenance (Herrmann, 2007).

1.1.1 PROFILE OF THE STUDY AREA

In this research, Promasidor Ghana Limited, a beverage manufacturing company is used as a case study. Promasidor was founded in 1979 by Robert Rose, who left the United Kingdom in 1957 for Zimbabwe to pursue his African dream. As Chairman of Allied Lyons Africa for over 20 years, he travelled extensively across Africa and gained a unique and thorough knowledge of the food industry throughout the continent. In particular he noticed a lack of availability of the one highly nutritious product that the developed world takes for granted - milk.

He realised that with technology in the manufacture of milk powders advancing rapidly, there was an exciting opportunity to provide milk powder in small portions that could be packaged in exible sachets. It was found that removing the animal fat from the milk and replacing it with vegetable fat allowed for a longer shelf life. This meant that for the rst time, milk powder could be distributed across the vast African continent, providing access to a ordable milk to everyone in Africa.

Based on the extensive knowledge and expertise gained in manufacturing, packing and distributing products across Africa, the Group has expanded the range of Promasidor products. Their products are now purchased daily in their millions and have developed into strong brands across the African continent, with highly noticeable brand identities.

However their market places are becoming more competitive as new products become available to consumers. They must therefore continue to o er high, consistent quality, value-for-money products and strive to satisfy the high standards that our consumers have come to expect from their products.

Beverage industries have grown quickly over the years in Ghana and competition among manufacturer is so high that planning and scheduling of resources must be e cient for any of these industries to be pro table and able to survive competition. However, in today's manufacturing scheduling problems, it is of signi cance to e ciently utilize various resources. Treating set up times separately from processing times allows operations to be performed simultaneously and hence improve resource utilization (Okoli et al., 2012).

The bene ts of reducing set up times include; reduced expenses, increased production speed, increased output, reduced lead times, faster charge over, increased competitiveness, increased pro tability and customer satisfaction. Planning and scheduling of products, machines, raw materials and labour are principal to the pro tability of any plastic industry in Ghana and the world at large. Production planning is one of the most important activities in a production factory. Production planning represents the beating heart of any manufacturing process. According to Veeke and Lodewijks (2005), production planning usually ful ls its functions by determining the required capacities and materials for these orders in quantity and time.

Mitchell (1939) discussed the role of production planning department, including routing, dispatching (issuing shop orders) and scheduling. According to Stevenson (2009), in the decision making hierarchy, scheduling decisions are the nal step in the transformation process before actual output occurs. Wight (1984) puts the two key problems in the production scheduling as priorities and capacity . In other words, what should be done rst? Who should do it? The author observes that in manufacturing rms, there are multiple types of scheduling, including the detailed scheduling of shop order that shows when each operation must start and complete. A lot of researchers have done extensive work in developing e cient solution strategies, they include Grossman et al (2002), Maravelias and Sung (2008).

1.1.2 PRODUCT INFORMATION

In Ghana, Promasidor commenced operations in 1999 and through dint of hard work, their range of products have become rm favorites among consumers. Cowbell, is their leading brand and has grown to include a number of well-received line extensions: Cowbell Co ee milk, Chocomalt, Sweet milk Strawberry, Mocha, Coconut, brand extensions. Miksi- a creamy, tasti alternative - is another o t's popular milk powder brand as well as Loya milk powder, a fulll cream brand. The Promasidor group also markets Onga seasoning powders, which are available in a wide range of delicious avors ideally suited to Ghanaian tastes.

The regular additions to the Cowbell product range are testament to the Group's commitment to the provision of innovative and value-added milk products. It is this dedication to service and consistency that has contributed to the continual success of Cowbell as a brand and Promasidor as a whole.

1.2 PROBLEM STATEMENT

Scheduling and planning play a very important role in manufacturing. In industries, time, labour, maintenance and manufacturing cost should be planned in order to minimize cost and maximize total turnover. However, most manufacturing companies in sub-Saharan Africa do not employ optimization models in their planning and scheduling where Ghana is not an exception. It is of this reason why this research is undertaken to address some of the problems. The research seeks to address some of the problems faced by Promasidor as outlined below;

The company has no mathematical model which explains time spent in each sector of manufacturing

Inaccurate estimation of supply

Improper resource allocation

Manufacturing companies engaged with the production of multi-items are invariably confronted with the problem of producing just enough of each item to satisfy demand, but at same time to maximize pro t in terms of production costs, inventory costs, man-power limitations, production time and demand pro le for the products. Usually the identi cation and manner of treatments of the associated production and sales constraints determine the extent of scheduling optimality. This becomes very necessary when output targets for both production and sales have been established as strict key performance indicators for the respective departments. Predominant production scheduling problems encountered belong to at least, one of the following:

(i) Companies would always want to satisfy demand instantly, avoiding long term delivery schedules. The practice of turning away customers to make them come back later for supplies, causes losses of sales to competitors who may be located not far from work sites.

(ii) When sales team have identi ed customers preferences to combination of speci c products, shortage of one product would eventually a ect sales pattern. This implies that production scheduling would need to be optimized in such a way to ensure su ciency of all product variants in the right proportions on stock.

(iii) There is the need to establish a referenced benchmark to measure and analyse performance at end of the month. Knowledge of required optimal scheduling output will facilitate inventory control.

1.3 OBJECTIVES OF THE STUDY

The main objectives of this study are:

(i) To construct a mathematical model which provides optimal scheduling solution for production output under normal operational environment?

(ii) To link the proposed model solution to a case study area.

(iii) To interpret the outcomes of the applied model in a case study, using sensitivity analysis for various changes in the model due to impact from ignored constraint.

1.4 METHODOLOGY

Over the years, various methods have been used to solve facility planning and scheduling problems. In this thesis we shall employ Mixed Integer Linear programming model for production planning for the following context:

Multiple items with independent demand

Multiple shared resources

Linear costs

Machine idle cost

Mathematical Laboratory (Matlab) and TORA will be used for all the coding for the work.

1.5 SIGNIFICANCE OF THE STUDY

In this age of modernity, manufacturers need to make optimal use of the scarce resource available to their disposal. This is even more important in Africa where interest rate to companies are just not friendly at all to compete with the foreign companies who gets access to loans at a very low rate. It is therefore necessary for industry players to employ operation researchers to plan their work schedule in order to save time, man hours, labour, energy, space, money etc. It is of these reasons why this research work is very relevant to the company under study, the government and the consuming public at large, since cutting down on high cost of manufacturing translate to lower unit price of the commodity.

1.6 LIMITATIONS OF THE STUDY

Africa being a developing continent has its own problems with regards to accessing information from one source or the other for research purposes. Ghana, one of the developing countries in Africa is not an exception to this problem. In our quest to obtain to obtain information for our research work, we encountered some challenges some of which are categorized below.

- The right o ce to go for the required information
- O cials not willing to give out information
- Deliberate attempt to frustrate the researcher by given you successive postponement to come for data
- Lack of nancial support

1.7 ORGAINSATION OF THE STUDY

The study consists of ve chapters. Chapter one considered the background, statement of the problem and the objectives of the study. The justi cation, methodology, scope and limitations of the study were also put forward. In chapter two, we shall put forward adequate and relevant literature on production and transportation problem. Chapter three presents the research methodology of the study. Chapter four will focus on data collection and analysis. Chapter ve, which is the last chapter of the study presents the summary, conclusions and recommendations of the study.

1.8 SUMMARY

In this chapter, we looked at production and the factors that are needed for production. We also looked at an overview of the problems facing the manufacturing industries in Ghana and how to solve some of these problems. The next chapter presents literature review on production management and transportation problem.

CHAPTER 2

LITERATURE REVIEW

2.1 INTRODUCTION

In this chapter, the various researches done in the area of study are discussed. It is categorized under historical aspect of production planning and scheduling, optimal production planning, optimal production scheduling and the various mathematical models applied in solving the problems to obtain optimality. The topic of production planning and scheduling has been widely researched over the years. Once using a mathematical approach to solving manufacturing scheduling problems, diverse techniques such as Constraint Programming (CP), Mixed Integer Linear Programming (MILP), Mixed Integer Non-Linear Programming (MINLP), or even a hybrid of CP and MILP formulations can be used, depending on the type of problem and solve discrete manufacturing scheduling problems more e ectively. Generally, MILP models are used when optimization is the main goal, and CP models are used when feasibility is the main concern.

2.2 HISTORY OF PRODUCTION PLANNING AND SCHEDULING

In this section, studies by various researchers on the history of production planning and scheduling are reviewed. Herman (2006) in this book titled Hand book for Production Scheduling describes the history of production scheduling in manufacturing facilities over the last 100 years. The author discussed the ways that production scheduling has been done is critical to analyzing existing production scheduling systems and nding ways to improve them. The tools used to support decision-making in real-world production scheduling are not only mentioned but also the changes in the production scheduling systems. This story goes from the rst charts developed by Henry Gantt to advanced scheduling systems that rely on sophisticated algorithms. The book helps production schedulers, engineers, and researchers understand the true nature of production scheduling in dynamic manufacturing systems and to encourage them to consider how production scheduling systems can be improved even more. It does not only review the range of concepts and an approach used to improve production scheduling but also demonstrates their timeless importance.

The challenge of improving production scheduling has inspired many di erent approaches as stated by Herman in this paper published in 2007, the Legacy of Taylor, Gantt, and Johnson: How to Improve Production Scheduling. This paper examines the key contributions of three individuals who improved production scheduling: Frederick Taylor, who de ned the key planning functions and created a planning o ce; Henry Gantt, who provided useful charts to improve scheduling decision-making, and S.M. Johnson, who initiated the mathematical analysis of production scheduling problems. The paper presents an integrative strategy to improve production scheduling that synthesizes these complementary approaches. Finally, the paper discusses the soundness of this approach and its implications on research, education, and practice. Another article published in 2007 by Hermann, a History of Decision-Making Tools for Production Scheduling, elaborates how important production scheduling is in decision-making process that has embraced technology as computers and information systems became cheaper and easier to use. The history of production scheduling is not one of replacing human decision-makers with algorithms, however, this paper provides a historical perspective on the decision support tools that have been developed to improve production scheduling.

In the next session, the various models used by various researchers are discussed.

2.3 MATHEMATICAL MODELS USED IN PRODUCTION PLANNING AND SCHEDULING

In this session, some of the various mathematical models already used by previous researchers in this eld are analysed.

Ashayeri (1996) presented a production and maintenance planning model for the process industry, developed a model to simultaneously plan preventive maintenance and production in a process industry environment, where maintenance planning is extremely important. The model schedules production jobs and preventive maintenance jobs, while minimizing costs associated with production, backorders, corrective maintenance and preventive maintenance. The formulation of the model is exible, so that it can be adapted to several production situations. The performance of the model is discussed and alternate solution procedures are suggested.

Lippman et al. (1967) studied a model that minimizes the sum of production, employment smoothing, and inventory costs subject to a schedule of known demand requirements over a nite time horizon. The three instrumental variables are work force producing at regular-time, work force producing on overtime, and the total work force. Overtime is limited to be not more than a xed multiple of regular time. The idle portion of the work force and the levels of inventory are resultant variables. The authors postulated the following shape characteristics for the cost functions production costs are convex-like, smoothing costs are V-shaped, and holding costs are increasing, both the production and holding cost functions need not be stationary. In this paper, they provided upper and lower bounds on the cumulative regular-time plus overtime work force for any sequence of demand requirements. They also gave the form of an optimal policy when demands are monotone (either increasing or decreasing). Finally, they derived the asymptotic behavior of optimal policies when demands are monotone and the planning horizon becomes arbitrarily long. All of these results, which convey information about the numerical values of optimal policies, given speci c demands and an initial level of inventory, depend only on the shape characteristics of the cost functions.

Ramezanian (2010) considered a ow shop scheduling problem with bypass consideration for minimizing the sum of earliness and tardiness costs. They proposed a new mathematical modelling to formulate this problem. There are several constraints which are involved in their modelling such as the due date of jobs, the job ready times, the earliness and the tardiness cost of jobs, and so on. They applied adapted genetic algorithm based on bypass consideration to solve the problem. The basic parameters of this meta-heuristic are brie y discussed in this paper. Also a computational experiment is conducted to evaluate the performance of the implemented methods. The implemented algorithm could be used to solve large scale ow shop scheduling problem with bypass e ectively.

eda (2008) proposed a similar mathematical model for permutation ow shop scheduling and job shop scheduling problems. The rst problem is based on a mixed integer programming model. As the problem is NP-complete, this model can only be used for smaller instances where an optimal solution can be computed. For large instances, another model is proposed which is suitable for solving the problem by stochastic heuristic methods. For the job shop scheduling problem, a mathematical model and its main representation schemes are presented.

Pochet (2000) presented a lecture on mixed integer programming models and formulations for a speci c problem class, namely deterministic production planning problems. The objective is to present the classical optimization approaches used, and the known models, for dealing with such management problems. The rst production planning models in the general context of manufacturing planning and control systems are described and explained which sense most optimization solution approaches are based on the decomposition of the problem into single-item sub-problems. A study in detail of the reformulations for the core or simplest sub problem in production planning, the single-item uncapacitated lot-sizing problem, and some of its variants. Such reformulations are either obtained by adding variables to obtain so called extended reformulations or by adding constraints to the initial formulation. This typically allows one to obtain a linear description of the convex hull of the feasible solutions of the sub problem. Such tight reformulations for the sub problems play an important role in solving the original planning problem to optimality. A review of two important classes of extensions for the production planning models, capacitated models and multi-stage or multi-level models is done. For each, the classical modelling approaches used is described.

Mixed Integer Programming (MIP) has been used for optimizing production schedules of mines since the 1960s and is recognized as having signi cant potential for optimizing production scheduling problems for both surface and underground mining. The major problem in long-term production scheduling for underground ore bodies generally relate to the large number of variables needed to formulate a MIP model, which makes it too complex to solve. As the number of variables in the model increase, solution times are known to increase at an exponential rate. In many instances the more extensive use of MIP models has been limited due to excessive solution times. Nehring et al, (2010) reviewed in their paper, production schedule optimization studies for underground mining operations. It also presents a classical MIP model for optimized production scheduling of a sublevel stoping operation and proposes a new model formulation to signi cantly reduce solution times without altering results while maintaining all constraints. A case study is summarized investigating solution times as ve stopes are added incrementally to an initial ten stope operation, working up to a fty stope operation. It shows substantial improvement in the solution time required when using the new formulation technique. This increased e ciency in the solution time of the MIP model allows it to solve much larger underground mine scheduling problems within a reasonable time frame with the potential to substantially increase the Net Present Value (NPV) of these projects. Finally, results from the two models are also compared to that of a manually generated schedule which showed the clear advantages of mathematical programming in obtaining optimal solutions.

Lindholm et al. (2013) formulated an optimization model for the production scheduling problem at continuous production sites. The production scheduling activity produced a monthly schedule that accounted for orders and forecasted all products. The plan should be updated every day, with feedback on the actual production the previous day. The actual daily production may be lower than the planned production due to disturbances, e.g. disruptions in the supply of a utility. The work is performed in collaboration with Perstorp, a world-leading company within several sectors of the specialty chemicals market. Together with Perstorp, a list of speci cations for the production scheduling has been formulated. These are formulated mathematically in a mixed-integer linear program that is solved in receding horizon fashion. The formulation of the model aims to be general, such that it may be used for any process industrial site. Determination of the optimum production schedules over the life of a mine is a critical mechanism in open pit mine planning procedures (Gholamnejad & Moosavi, 2012). In this paper, a long-term production scheduling is used to maximize the net present value of the project under technical, nancial, and environmental constraints. Mathematical programming models are well suited for optimizing long-term production schedules of open pit mines. The two approaches to solving long-term production problems are: deterministic- and uncertainty- based approaches. Deterministic-based models are unable to deal with grade and geological uncertainties, which are two important sources of risk in mining industries. This may lead to discrepancies between actual production obtained by these algorithms and planning expectations. In this paper, a new binary integer programming model was developed for long-term production scheduling that incorporates geological uncertainty within the orebody. Then, traditional and uncertainty-based models are applied to an iron ore deposit. Results showed that the uncertainty-based approach yields more practical schedules than traditional approaches in terms of production targets

Xue et al.(2000) in their research titled An intelligent optimal production scheduling approach using constraint-based search and agent-based collaboration introduced an intelligent approach for identifying the optimal production schedule to satisfy product and manufacturing constraints. In this approach, product constraints are modeled using a feature-based product representation scheme. Manufacturing constraints are described as available resources including facilities and persons. Manufacturing requirements for producing the products, including tasks and sequential constraints for conducting these tasks, are represented as part of the product feature descriptions. The optimal production process and its timing parameter values are identi ed using constraint-based search and agent-based collaboration. The intelligent optimal production scheduling system was implemented using Smalltalk, an object oriented programming language.

Kopanos et al.(2009) used a model their work on Optimal Production Scheduling and Lot-Sizing in Dairy Plants: The Yogurt Production Line that addresses the lot-sizing and production scheduling problem in a multiproduct yogurt production line of a real-life dairy plant. A new mixed discrete/continuous-time mixed-integer linear programming model, based on the de nition of families of products, is proposed. The problem under question is mainly focused on the packaging stage, whereas timing and capacity constraints are imposed with respect to the pasteurization/homogenization and fermentation stage. Packaging units operate in parallel and share common resources. Sequence-dependent times and costs are explicitly taken into account and optimized by the proposed framework. Several scenarios for a large-scale dairy plant have been solved to optimality using the proposed model. Production bottlenecks are revealed, and several retro t design options are proposed to enhance the production capacity and exibility of the plant.

In considering a manufacturing system in which a single consumable good is fabricated in a process that consists of stages in an uncertain environment. On each stage, there are a number of workstations that are assumed to have di erent operating parameters that are subject to failure, repair, and preventive maintenance which generate discrete jumps in the value of the state. A JustIn-Time manufacturing discipline is assumed for the workstations with running costs that include penalties for shortfall and surplus production. The formulation presented here for the optimal production scheduling for the manufacturing system requires extensions to the results of the LQGP problem with State Dependent Poisson Processes (SDPP) by the inclusion of coe cients for the dynamics and the costs that are parameterized by the value of the state. The cost functional used is fully quadratic which an enhancement for the LQGP problem (Westman et al., 2000)

2.4 OPTIMAL PRODUCTION PLANNING

This session concentrates on studies done mostly on optimal production planning. Production planning is meant to arrive at the framework of manufacturing operations during the period planned (Chandra Mouli et al., 2006). The aim in production planning is to determine the production capacity in terms of high level decisions such as production levels and product inventories for given marketing forecasts, and demands over a long time horizon ranging from several months up to a year.

Zied et al. (2009) in their paper on an Optimal Production/Maintenance Planning Under Stochastic Random Remand, Service Level and Failure Rate dealt with the combination between production and maintenance plan for a manufacturing system satisfying a random demand. A jointly optimization is made in order to establish an optimal production planning and scheduling maintenance strategy showing the machine degradation. The key of this study is to consider the in uence of the production rate on the degradation degree. A constrained stochastic production-maintenance planning problem under hypotheses of inventory and failure rate variables, an optimal production plan and maintenance scheduling which minimizes the average total holding, production and maintenance costs was simultaneously proved. A numerical example is studied in order to apply the developed approach.

Another paper addressed the problem of planning the usage of actuators optimally in an economic perspective. The objective is to maximize the pro t of operating a given plant during 24 hours of operation. Models of two business objectives are formulated in terms of system states and the monetary value of these objectives is established. Based on these and the cost of using the di erent actuators a pro t function has been formulated. The optimization of the pro t is formulated as an optimal control problem where the constraints include the dynamics of the plant as well as a requirement to reference tracking. A power plant is considered in this paper, where the fuel system consists of three di erent fuels; coal, gas, and oil (Kragelund et al., 2009).

An Optimum Production Planning (OPP) Model for a robot-served machine in the make-to-order (MTO) industry is proposed to minimize the production cost under deterministic order quantity and deadline constraints (Lan et al., 2007). In this paper, the operational cost of the machine and the part handling robot, as well as the xed costs for both equipments and the product holding cost are considered simultaneously into the objective of the model. This study not only implements the Lagrange Method to resolve the production planning problem, but also provides a veri ed cost-related property of the Lagrange Multiplier for budget and/or cost forecasting under the deterministic market. Through the forecasted future demand, the step-by-step algorithm to reach the optimal production plan for the probabilistic market is then constructed. In addition, the versatility and adaptability of this study are exempli ed through numerical simulation. This paper surely contributes the applicable solution to control a robot-served machine under certain market, as well as to plan the productivity of the machine and the robot in the forecasted future.

Singh et al. (2013) discussed a practical oil production planning optimization problem. In this research, for oil wells with insu cient reservoir pressure, gas is usually injected to arti cially lift oil, a practice commonly referred to as Enhanced Oil Recovery (EOR). The total gas that can be used for oil extraction is constrained by daily availability limits. The oil extracted from each well is known to be a nonlinear function of the gas injected into the well and varies between wells. The problem is to identify the optimal amount of gas that needs to be injected into each well to maximize the amount of oil extracted subject to the constraint on the total daily gas availability. The problem has long been of practical interest to all major oil exploration companies as it has the potential to derive large nancial bene t. In this paper, an infeasibility driven evolutionary algorithm is used to solve a fty-six (56) well reservoir problem which demonstrates its e ciency in solving constrained optimization problems. Furthermore, a multi-objective formulation of the problem is posed and solved using a number of algorithms, which eliminates the need for solving the (single objective) problem on a regular basis. Lastly, a modi ed single objective formulation of the problem is also proposed, which aims to maximize the pro t instead of the quantity of oil. It is shown that even with a lesser amount of oil extracted, more economic bene ts can be achieved through the modi ed formulation.

Mishra (2012) in his work on Optimal Production Planning When Final Demand is Stochastic and Inter-Related formulated the Input-Output Analytic framework, production (X) is related to nal demand (C) through the B [while B= INV(I-A), where A is the technical coe cients matrix and INV(.) means inverted (.)], such that X=BC. Generally, the elements of A and C are considered to be non-stochastic and un-correlated with each other within A and C. While non-stochasticity of the elements

of A may be (more or less) justi able, it cannot be empirically justi ed for the elements of C. Due to complementarity and substitutability, the elements of C may be correlated. This consideration introduces stochasticity and correlatedness into the elements of X. In this paper we address the problem of obtaining optimal X when the elements of C are
(stochastic and) correlated.

2.5 OPTIMAL PRODUCTION SCHEDULING

Jian et al.(2004) worked on Optimum Integrated Cast Plan for SteelmakingContinuous Casting Production Scheduling Using Improved Genetic Algorithm.

In this paper, a planning method of cast for steelmaking continuous casting production scheduling in integrated production process is studied. The integrated cast plan model is established. A modi ed genetic algorithm with adaptive operator is proposed to solve the optimum integrated cast plan problem. Simulations have been carried out with practical data in steel and iron plant and the results show that the model and the solving method are very e ective.

Mitsumori (1972) in his work on Optimum Production Scheduling of Multicommodity in Flow Line discusses how the applications of computer control in production factories have been extended from the control of mass and energy to the production control. The control at this level comprises the formulation and alteration of work schedules in accordance with the progress of work, and it may be called the control of information. This paper deals with the ow line which is the most fundamental production line in the factory and proposes and veri es the optimal scheduling method. In the case of producing multicommodities by a ow line, the optimum schedule is the one which satis es the demand for the respective products and minimizes the changeover loss. The branch-and-bound method is used to obtain an optimum schedule. The geometrical characteristics of the region in which the feasible schedules exist are used for calculating the lower bound of the objective function for the subset of feasible schedules.

Xue et al.(2004) deliberated on their piece, Optimum Cast Plan for SteelmakingContinuous Casting Production Scheduling. A planning method of cast for steelmaking continuous casting production scheduling in CIMS is studied. The cast plan model is established. An adaptive operator genetic algorithm is proposed to solve the optimum cast plan problem. The computation with practical data shows that the model and the solving method are very e ective.

Agyepong-Mensah (2011) conducted a study in Ernest Chemists Limited
(ECL), the study presented a production scheduling solution for a manufacturing rm, all in an attempt to cut down manufacturing cost and increase e ciency. The creation of an optimum production schedule requires the modelling of the scheduling problem as a balanced transportation problem. An important result upon the implementation of the model is the allocation of the optimum level of production necessary to meet a given demand at a minimum cost. The main objective of the study is to develop a quantitative model by which ECL and for that matter, manufacturing rms can meet their demand at a minimum cost. To achieve this objective the study adopted the quantitative approach in this research, by using a quantitative method to model the production problems of ECL as a balanced transportation problem, which can be solved using the simplex pivot method that makes it easy to nd the Initial Basic Feasible Solution (IBFS). A balanced transportation problem is where total supply equals total demand. To nd the basic feasible solution for the balanced transportation problem, the researcher used the Vogel's Approximation Method (VAM), and then improved the IBFS to obtain optimality by using the Modi ed Distribution Method (MODI).

After collecting the necessary data for the study, with an interview guide, the researcher came out with the optimum production schedule for ECL by using the Quantitative Manager for windows statistical software. The research revealed that, the company incurred a regular production cost of GHS 6,095,844.00 and an overtime cost of GHS 3,371,832.00, giving a total production cost of GHS 9,467,676.00 for producing 695,311cartons of the Big Joe pain reliever for the year, which were not all demanded within the period under review, without the optimum production model. With the model, the company required 596,695 cartons, at the cost of GHS 7,808,011.00, to meet its demand for the year instead. The researcher, therefore, recommends the usage of the proposed model to the management of Ernest Chemists Limited, to determine the optimum level of production to meet a given demand at a minimum cost.

Pongcharoen et al. (2002) published in their journal, Determining Optimum Genetic Algorithm Parameters for Scheduling the Manufacturing and Assembly of Complex Products. In this journal, a Genetic Algorithm-based Scheduling Tool (GAST) has been developed for the scheduling of complex products with multiple resource constraints and deep product structure. This includes a repair process that identi es and corrects infeasible schedules. The algorithm takes account of the requirement to minimise the penalties due to both the early supply of components and assemblies and the late delivery of nal products, whilst simultaneously considering capacity utilisation. The research has used manufacturing data obtained from a capital goods company. The Genetic Algorithm scheduling method produces signi cantly better delivery performance and resource utilisation than the Company plans. Genetic Algorithm programs include a number of parameters including the probabilities of crossover and mutation, the population size and the number of generations. A factorial experiment has been performed to identify appropriate values for these factors that produce the best results within a given execution time. The overall objective is to use the most e cient Genetic Algorithm parameters that achieve minimum total costs and minimum spread, to solve a very large scheduling problem that is computationally expensive. The results are compared to the corresponding plans produced by the collaborating company using simulation. It is demonstrated that in the case considered, the Genetic Algorithm scheduling method achieves on time delivery and a 63% reduction in costs.

Amponsah et al. (2011) conducted a study in Accra, Ghana, that presented a production scheduling problem for a beverage rm based in Accra, all in an attempt to cut down manufacturing cost and increase e ciency. The creation of an optimum production schedule requires the modelling of the scheduling problem as a balanced transportation problem. An important result upon the implementation of the model is the allocation of the optimum level of production necessary to meet a given demand at a minimum cost.

In 1973, Gruhl examined a quasi-optimal technique ('quasi' in that the technique discards unreasonable optimums), realized by a dynamically evolving mixed integer program. It is used to develop regional electric power maintenance and production schedules for a two to ve year planning horizon per unit production, recovery and product grade. Once the relationship between the production rate and cost is established, dynamic-programming techniques can de ne the optimum production schedule (i.e. the production schedule that maximizes the present worth of the operation) for the life of the deposit. In general, the optimum production rate is not constant for the life of the deposit, but declines gradually as the deposit is being exhausted (Roman, 1971).

Sivasubramanian et al. (2004) published a journal on Optimum Production Schedule and Pro t Maximisation Using the Concept: Theory of Constraints. They elaborated that the Theory of Constraints (TOC) is an example of a management philosophy built upon a limited number of assumptions and designed to provide a process of continuous ongoing improvement. The assumption forming one foundation of TOC is that a system's outputs are determined by its constraints. The assumptions forming another foundation are new de nitions for throughput, inventory and operating expense. These de nitions are designed to support the goal of the organisation, which, according to Goldratt, is to make money. TOC, previously referred to as Optimized Production Technology (OPT), is a production control methodology that maximises pro ts in a plant with a demonstrated bottleneck. The process used by TOC to determine product mix that will maximise pro tability is a very simple series of steps. In this article, a case study from an industry is considered to demonstrate how the application of concepts of TOC will maximise pro t for an organisation. This paper further explained how TOC plays a vital role in increasing performance through a limited number of assumptions designed to provide a continuous process of improvement, as emphasised in Total Quality Management (TQM). This paper also shows the steps involved in solving a typical TOC problem along with optimisation of resources for increased demand conditions.

Houghton and Portougal (2001) presented an analytic framework for processing planning in industries where xed batch sizes are common. The overall optimum processing plan is shown to be located on an envelope between the optimum JIT plan and the optimum level plan. These concepts provide the framework for understanding the overall optimum plan, and the framework leads to an e cient heuristic. The approach is practical, illustrated by a case study from the food industry, which shows the place of overall optimum planning within the company's planning system and its implications for company performance. In this studies, a mixed integer linear programming model approached is used. It considers both production planning and scheduling to obtain optimality.

CHAPTER 3

METHODOLOGY

In this chapter, mathematical programming is discussed. The various types of mathematical programming are stated with much emphasis on Mixed Integer programming problem. Formulation of the MILP and methods of solving MILP were also discussed.

3.1 MATHEMATICAL PROGRAMMING

Mathematical programming is one of the most widely used operation Research technique. A major feature of mathematical programming is that its models involve optimization, which helps management to come out with optimal policies in their quest to improve e ciency in their operations.

Mathematical programming therefore, is used to nd the best or optimal solution to a problem that requires a decision or set of decisions about how best to use a set of limited resources to achieve a state goal of objectives.

The application of mathematical programming has been so successful that their use has passed out of operational research departments to become an accepted routine planning tool.

Steps involved in Mathematical Programming

Conversion of stated problem into a mathematical model that abstracts all the essential elements of the problem.

Exploration of di erent solutions of the problem.

Finding out the most suitable or optimum solution.

Types of mathematical programming.

Linear programming

Quadratic programming

Dynamic programming

3.2 LINEAR PROGRAMMING

Linear programming is a widely used mathematical modeling technique to determine the optimum allocation of scarce resources among competing demands. Resources typically include raw materials, manpower, machinery, time, money and space.
The technique is very powerful and found especially useful because of its application to many di erent types of real business problems in areas like nance, production, sales and distribution, personnel, marketing and many more areas of management.
As its name implies, the linear programming model consists of linear objectives and linear constraints, which means that the variables in a model have a proportionate relationship. The linear programing technique can be said to have a linear objective function that is optimized (either minimized or maximized) subject to linear equality or inequality constraints and sign restrictions on the variables. Since its introduction in the late 1930s it has found practical application in almost all facets of businesses.
A Linear Programming model seeks to maximize or minimize a linear function, subject to a set of linear constraints. The linear model consists of the following components:

A set of decision variables

An objective function

A set of constraints

3.2.1 DEFINITIONS

Decision Variables (X_j): In mathematical programming models, the unknown quantities are assigned symbols, which are known as variables. Thus the decision variables are symbols that represent quantities of a certain good or a production material that is either minimized or maximized in an LP model.

Objective function: A decision variable is a quantity that the decision-maker controls. For example, the number of nurses to employ during the morning shift in an emergency room may be a decision variable in an optimization model for labor scheduling. Or a linear mathematical relationship describing an objective of the rm, in terms of decision variables - this function is to be maximized or minimized.

Constraints: In mathematics, a constraint is a condition of an optimization problem that the solution must satisfy. There are several types of constraintsprimarily equality constraints, inequality constraints, and integer constraints. The set of candidate solutions that satisfy all constraints is called the feasible set.

3.2.2 ASSUMPTIONS OF THE LINEAR PROGRAMMING MODEL

The assumptions of the linear programming model are;

The parameter values are known with certainty.

The objective function and constraints exhibit constant returns to scale.

The continuity assumption: Variables can take any value within a given feasible range.

The additivity assumption: There are no interactions between the decision variables.

3.2.3 STEPS FOR DEVELOPING AN ALGEBRAIC LP MODEL

The steps include;

1. What decisions need to be made? De ne each decision variable.

2. What is the goal of the problem? Write down the objective function as a function of the decision variables.

3. What resources are in short supply and/or what requirements must be met?

4. Formulate the constraints as functions of the decision variables.

3.2.4 GENERAL FORM OF LP MODELS

The general linear programming model can be stated as:

Minimize (Maximize) $\sum C_j X_j$

Subject to $A_{ij} X_j \leq B_i$

Where

the j^{th} row may be " = " or " ≤ " or " ≥ " C_j are known as cost coefficients.

X_j are decision Variables.

A_{ij} are called structural coefficients.

B_i is known as the resource value.

3.2.5 LP TERMINOLOGY

Restrictive line: A straight line corresponding to a constraint of the model.

Vertex or extreme point: A point at which two restrictive lines are intersecting.

Solution: Each combination of the decision variables' values.

Feasible solution: A solution satisfying all the constraints.

Non feasible solution: A solution not satisfying at least one of the constraints.

Extreme point feasible solution: Is one of the vertexes of the feasible region.

Neighboring feasible solutions: They are connected with an edge (boundary) of the feasible region.

Feasible region: The (curved) region of feasible solutions formed by the restrictive lines.

Basic solution (extreme point solution): A solution corresponding to a vertex (has non-zero variables equal to the number of the constraints)

Non basic solution: A solution that is not on a vertex of the feasible region and can be feasible or non-feasible

Basic feasible solution: A basic solution that corresponds to a vertex of the feasible region and e ectively all its variables are non-negative

Slack value: Any excess of an available resource (constraint symbol > or <). Surplus

value: Any "surpassing" of a requirement (constraint symbol < or >).

Auxiliary variables: Variables that correspond to slack values and surplus values.

Basic (non basic) variable: A non-zero (zero) variable in a solution that contains decision and auxiliary variables

Mandatory or active constraint: When the resource is completely consumed or there is no surplus (zero slack or surplus respectively)

Non mandatory constraint: When a resource is not exhausted (<) or a requirement is being surpassed (>) (non-zero slack or surplus respectively)

Optimal solution: The feasible solution of an extreme point that gives to the objective function the optimal value (maximum or minimum). The optimal solution can be only one, but there are cases with unlimited optimal solutions, no optimal solution, or the value of the objective function tends to in nity. In every case the amount of feasible solutions of the extreme point is nite. When a feasible solution of an extreme point is better than all its neighboring then this solution is the optimal.

Optimal value: The value of the objective function that corresponds to the optimal solution.

Infeasibility: Occurs when a model has no feasible point.

Unboundness: Occurs when the objective can become in nitely large (max) or in nitely small (min).

Alternate solution: Occurs when more than one point optimizes the objective function

3.2.6 METHODS OF SOLVING LINEAR PROGRAMMING PROBLEM

There are two main types of solving linear programming problems. These are;

Simplex Method

Graphical method

A linear programming problem involving only two decision variables can be solved using a graphical approach however if the decision variables are more than two and the constraints are two or more, then the appropriate method to use is the Simplex Method.

3.3 SIMPLEX METHOD

The simplex method is a general mathematical solution technique for solving linear programming problems. In the simplex method, the model is put into the form of a table, and then a number of mathematical steps are performed on the table. These mathematical steps in e ect replicate the process in graphical analysis of moving from one extreme point on the solution boundary to another. However, unlike the graphical method, in which we could simply search through all the solution points to nd the best one, the simplex method moves from one better solution to another until the best one is found, and then it stops.

3.3.1 SIMPLEX ALGORITHM

The simplex method demonstrated in the previous section consists of the following steps.

1. Transform the model constraint inequalities into equations.
2. Set up the initial tableau for the basic feasible solution at the origin and compute the z_j and $c_j - z_j$ row values.
3. Determine the pivot column (entering nonbasic solution variable) by selecting the column with the highest positive value in the $c_j - z_j$ row.
4. Determine the pivot row (leaving basic solution variable) by dividing the quantity column values by the pivot column values and selecting the row with the minimum nonnegative quotient.

5. Compute the new pivot row values using the formula new tableau pivot row values = $\frac{\text{old tableau pivot row values}}{\text{pivot number}}$

6. Compute all other row values using the formula new tableau row values = old tableau row values - (corresponding coe cients in pivot column× corresponding new tableau pivot row values)

7. Compute the new z_j and $c_j - z_j$ row

8. Determine whether or not the new solution is optimal by checking the $c_j - z_j$ row. If all $c_j - z_j$ row values are zero or negative, the solution is optimal. If a positive value exists, return to step 3 and repeat the simplex steps.

3.4 ILLUSTRATION

At the Beaver Creek Pottery Company Native American artisans produce bowls (x_1) and mugs (x_2) from labor and clay. The linear programming model is formulated as;

Maximize subject to

$$Z = 40x_1 + 50x_2$$

$$x_1 + 2x_2 \leq 40$$

$$4x_1 + 3x_2 \leq 120$$

$$x_1, x_2 \geq 0$$

We convert this model into standard form by adding slack variables to each constraint as follows

Maximize $Z = 40x_1 + 50x_2 + 0s_1 + 0s_2$ subject to

$$x_1 + 2x_2 + s_1 = 40$$

$$4x_1 + 3x_2 + s_2 = 120$$

$$x_1, x_2, s_1, s_2 \geq 0$$

The slack variables, s_1 and s_2, represent the amount of unused labor and clay, respectively.

The initial simplex tableau for this model, with the various column and row headings, is shown in table below.

c_j	Basic variables	Quantity	x_1	x_2	s_1	s_2
	z_j					
	$c_j - z_j$					

The following list summarizes the steps of the simplex method tableau.

1. First, transform all inequalities to equations by adding slack variables.
2. Develop a simplex tableau with the number of columns equaling the number of variables plus three, and the number of rows equaling the number of constraints plus four.
3. Set up table headings that list the model decision variables and slack variables.
4. Insert the initial basic feasible solution, which are the slack variables and their quantity values.
5. Assign c_j values for the model variables in the top row and the basic feasible solution variables on the left side.
6. Insert the model constraint coe cients into the body of the table.

Following the above steps we have;

c_j	Quantity	40	50	0	0

Basic variables			x_1	x_2	s_1	s_2
0	s_1	40	1	2	1	0
0	s_2	120	4	3	0	1
	z_j	0	0	0	0	0
	$c_j - z_j$		40	50	0	0

We select variable x_2 as the entering basic variable because it has the greatest net increase in pro t per unit, and select s_1 row as the pivot row.

	Basic variables		40	50	0	0
c_j		Quantity	x_1	x_2	s_1	s_2
0	s_1	40	1	2	1	0
0	s_2	120	4	3	0	1
	z_j	0	0	0	0	0
	$c_j - z_j$		40	50	0	0

The same process is repeated until an optimal solution is gotten. Below is the tableau of the process.

	Basic variables		40	50	0	0
c_j		Quantity	x_1	x_2	s_1	s_2
50	x_2	20	1/2	1	1/2	0
0	s_2	60	5/2	0	-3/2	1
	z_j	1,000	25	50	25	0
	$c_j - z_j$		15	0	-25	0

c_j		Quantity	40	50	0	0

Basic variables		x_1	x_2	s_1	s_2
50 x_2	20	1/2	1	1/2	0
0 s_2	60	5/2	0	-3/2	1
z_j	1,000	25	50	25	0
$c_j - z_j$		15	0	-25	0

c_j	Basic variables	Quantity	x_1	x_2	s_1	s_2
			40	50	0	0
50	x_2	8	0	1	4/5	-1/5
40	x_2	24	1	0	-3/5	2/5
	z_j	1,360	40	50	16	6
	$c_j - z_j$		0	0	-16	-6

The optimal solution is

$x_1 = 24$ bowls $x_2 = 8$

mugs

$Z = \$1{,}360$ pro t TYPES OF LINEAR PROGRAMMING MODEL

Linear programming has the following types:

Pure integer programming Model

Binary

Mixed Integer Programming Model

PURE INTEGER PROGRAMMING MODEL

A linear programming problem in which all the decision variables must have integer values is called a pure integer programming problem. Integerprogramming model is one of the most important models in Management science. In certain LP problems fractional solutions are not realistic, one cannot produce 1 and half cars neither can one produce half a bottle of coke. Sometimes, all of the decision variables must have the value of either 0 or 1. Such problems are then called zero-one or binary programming problems. E.g. on and o of a switch.

MIXED INTEGER PROGRAMMING PROBLEM

A problem in which only some of the decision variables must have integer values is called a mixed-integer programming problem. The model is therefore mixed . When the objective function and constraints are all linear in form, then it is a Mixed Integer Linear Program (MILP).

3.5 MIXED INTEGER LINEAR PROBLEM MODEL

GENERAL FORM

The general linear programming model can be stated as:

Minimize (Maximize) $\text{X } C_jX_j$

Subject to $A_{ij}X_j \leq B_i$

. $X_j \geq 0,\quad X_j \in \text{Z}$ for some j

Where

the j^{th} row may be " = " or " ≤ " or " ≥ " C_j are known as cost coe cients.

X_j are decision Variables.

$X_j{}^0s$ are integers

A_{ij} are called structural coe cients.

B_i is known as the resource value.

3.5.1 METHODS OF SOLVING MILP

Whereas the simplex method is e ective for solving linear programs, there is no single technique for solving integer programs. Instead, a number of procedures have been developed, and the performance of any particular technique appears to be highly problem-dependent. Methods to date can be classi ed broadly as following one of three approaches:

(i) Enumeration techniques, including the branch-and-bound procedure.

(ii) Cutting-plane techniques.

(iii) Group-theoretic techniques.

3.5.2 BRANCH AND BOUND

The branch and bound method is a solution approach that can be applied to a number of di erent types of problems. The branch and bound approach is based on the principle that the total set of feasible solutions can be partitioned into smaller subsets of solutions. These smaller subsets can then be evaluated systematically until the best solution is found. When the branch and bound approach is applied to a mixed integer programming problem, it is used in conjunction with the normal noninteger solution approach. The IP problem is rst solved as an LP problem by relaxing the integrality conditions. If the resultant solution (the continuous optimum) is an integer, the problem is solved; otherwise, a tree search is performed.

THE BRANCH AND BOUND ALORITHM

The steps of the branch and bound method for determining an optimal integer solution for a maximization model (with ≤ constraints) can be summarized as follows.

1. Find the optimal solution to the linear programming model with the integer restrictions relaxed.
2. At node 1 let the relaxed solution be the upper bound and the roundeddown integer solution be the lower bound.
3. Select the variable with the greatest fractional part for branching. Create two new constraints for this variable re ecting the partitioned integer values. The result will be a new ≤ constraint and a new ≥ constraint.
4. Create two new nodes, one for the ≤ constraint and one for the ≥ constraint.
5. Solve the relaxed linear programming model with the new constraint added at each of these nodes.
6. The relaxed solution is the upper bound at each node, and the existing maximum integer solution (at any node) is the lower bound.
7. If the process produces a feasible integer solution with the greatest upper bound value of any ending node, the optimal integer solution has been reached. If a feasible integer solution does not emerge, branch from the node with the greatest upper bound.
8. Return to step 3. For a minimization model, relaxed solutions are rounded up, and upper and lower bounds are reversed.

ILLUSTRATION

The owner of a machine shop is planning to expand by purchasing some new machines presses and lathes. The owner has estimated that each press purchased will increase pro t by \$100 per day and each lathe will increase pro t by \$150 daily. The number of machines the owner can purchase is limited by the cost of the machines and the available oor space in the shop. The machine purchase prices and space requirements are as follows.

	Required	
Machine	Floor Space(ft^2)	Purchase Price

Press	15	$8,000
Lathe	30	4,000

The owner has a budget of GHc 40,000 for purchasing machines and 200 square feet of available oor space. The owner wants to know how many of each type of machine to purchase to maximize the daily increase in pro t. The model below is formulated from the problem above;

maximize $Z = 100x_1 + 150x_2$ subject to

$$8{,}000x_1 + 4{,}000x_2 \leq 40{,}000$$

$$15x_1 + 30x_2 \leq 200 \quad x_1, x_2 \geq 0 \text{ and integer}$$

where x_1 = number of presses x_2= number of lathes

We solve the problem using the branch and bound method by rst solving the problem as a regular linear programming model without integer restrictions (i.e., the integer restrictions are relaxed). Thus using the Simplex method to solve the problem.

The linear programming model for the problem and the optimal relaxed solution is maximize $Z = 100x_1 + 150x_2$ subject to

$$8{,}000x_1 + 4{,}000x_2 \leq 40{,}000$$

$$15x_1 + 30x_2 \leq 200$$

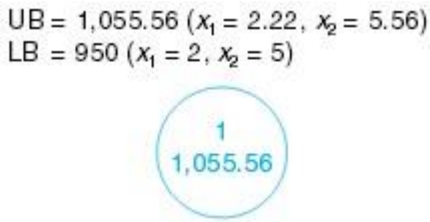
$$x_1, x_2 \geq 0$$

and

$$x_1 = 2.22,\ x_2 = 5.56$$

$$Z = 1{,}055.56$$

The branch and bound method employs a diagram consisting of nodes and branches as a framework for the solution process.



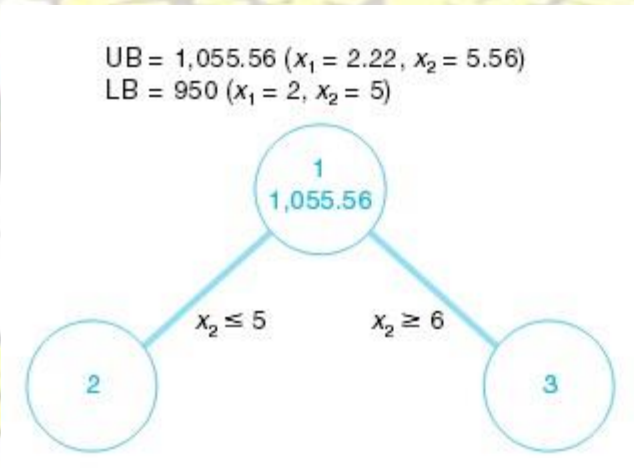
This node has two designated bounds: an upper bound (UB) of 1,055.56 and a lower bound (LB) of 950. The lower bound is the Z value for the rounded down solution, x_1 = 2 and x_2 = 5; the upper bound is the Z value for the relaxed solution, x_1 = 2.22 and x_2 = 5.56. The optimal integer solution will be between these two bounds.

X_2 = 5.56 is the greatest fractional part; thus, x_2 will be the variable that we will branch on. The following constraints are developed and added to the constraints of the main equation

$$x_2 \leq 5 \; x_2$$

$$\geq 6$$

Below is the three diagram.



First, the solution at node 2 is found by solving the following model with the constraint $x_2 \leq 5$ added.

maximize Z = $100x_1 + 150x_2$ subject to

$$8{,}000x_1 + 4{,}000x_2 \leq 40{,}000$$

$$15x_1 + 30x_2 \le 200$$

$$x_2 \le 5 \quad x_1, x_2 \ge 0$$

The optimal solution for this model with integer restrictions relaxed (using the Simplex method) is

$$x_1 = 2.5 \quad x_2$$

$$= 5$$

$$Z = 1{,}000$$

The solution at node 3 is found by solving the model with $x_2 \ge 6$ added.

maximize Z = $100x_1 + 150x_2$ subject to

$$8{,}000x_1 + 4{,}000x_2 \le 40{,}000$$

$$15x_1 + 30x_2 \le 200$$

$$x_2 \ge 6 \quad x_1, x_2 \ge 0$$

The optimal solution for this model with integer restrictions relaxed is

$$x_1 = 1.33 \quad x_2$$

$$= 6$$

$$Z = 1{,}033.33$$

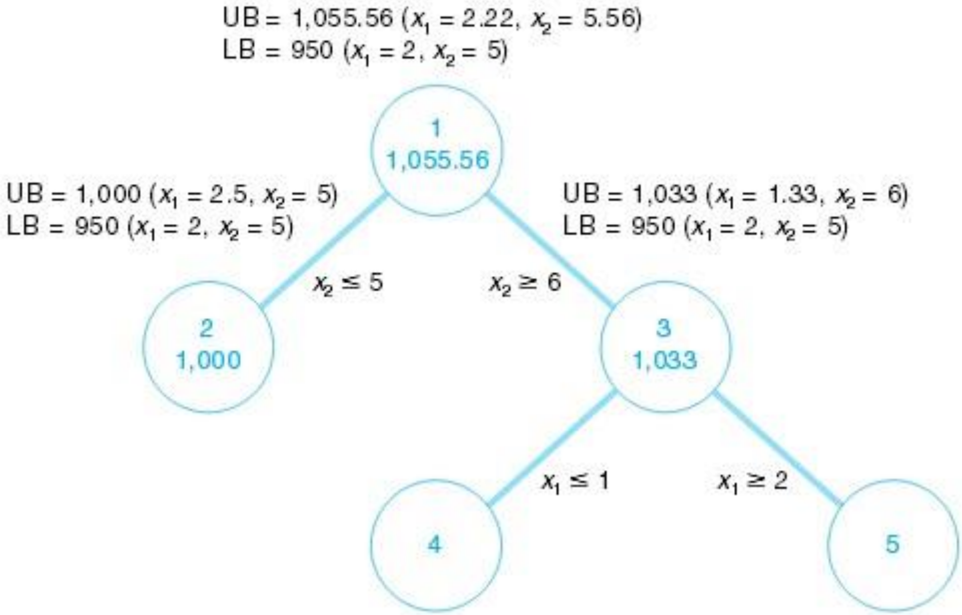
Since neither of the decision variables at node 2 and 3 are all integers, an optimal solution is not found yet. We branch on from node 3 since its giving us the highest value of Z.

At node 3, $x_1 = 1.33$ has the largest fractional part, it's the decision variable we will branch on. Two new constraints are formed,

$$x_1 \le 1 \quad x_1$$

$$\ge 2$$

Below is a tree diagram representing the solution procedure;



Now we solve the model at node 4 with $x_1 \le 1$ constraint added to constraints of the model.

maximize subject to $\quad Z = 100x_1 + 150x_2$

$$8{,}000x_1 + 4{,}000x_2 \le 40{,}000$$

$$15x_1 + 30x_2 \le 200$$

$$x_2 \ge 6 \quad x_1 \le 1$$

$$x_1, x_2 \ge 0$$

The optimal solution for this model with integer restrictions relaxed is

$$x_1 = 1 \; x_2$$

$$= 6.17$$

$$Z = 1{,}025$$

Next, we solve the model at node 5 with the constraint $x_2 \ge 2$ added to the model.

maximize $\quad Z = 100x_1 + 150x_2$

subject to

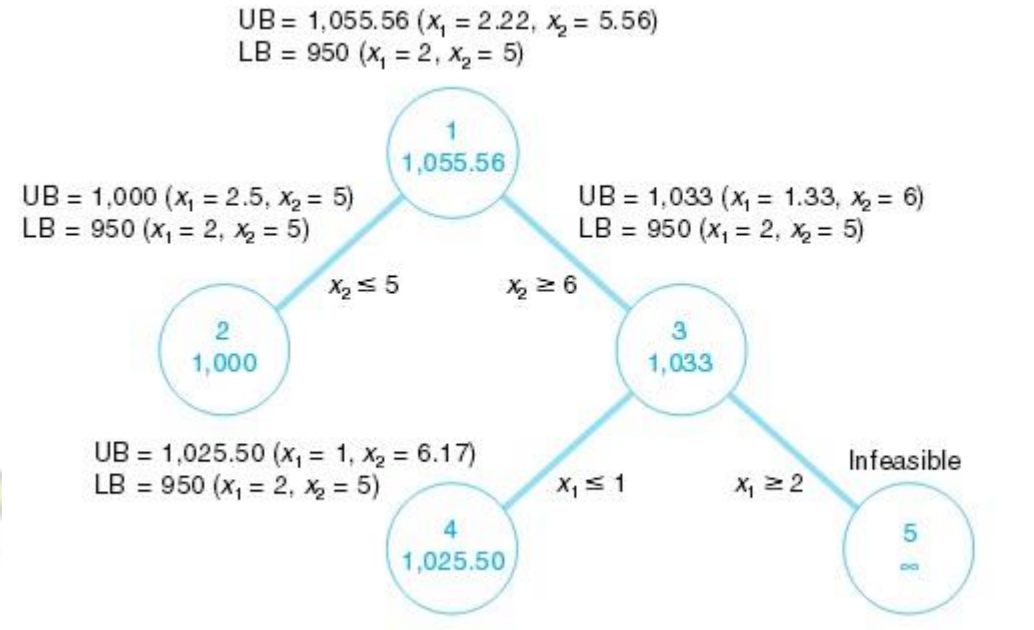
$$8{,}000x_1 + 4{,}000x_2 \le 40{,}000$$

$$15x_1 + 30x_2 \leq 200$$

$$x_2 \geq 6 \quad x_1 \geq 2$$

$$x_1, x_2 \geq 0$$

There is no feasible solution for this model. Therefore, no solution exists at node 5.

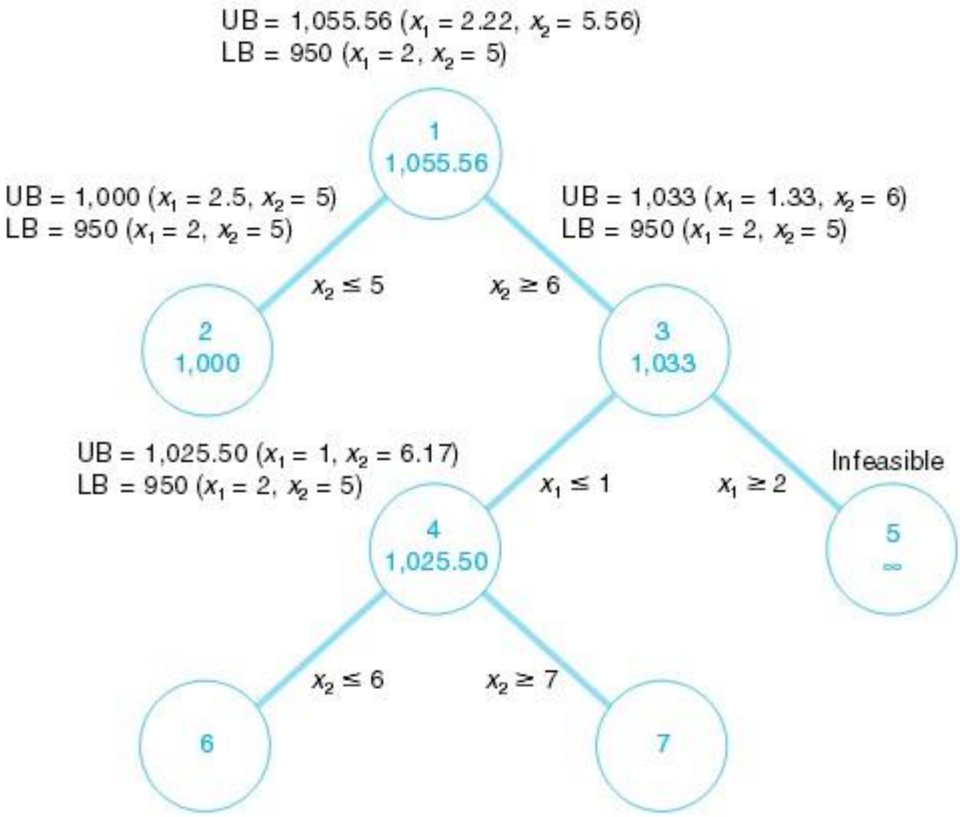


At node 4 $x_2 = 6.17$ is having the greatest fractional part, so it's the decision variable on which we will branch. Two new constraints are formed

$$x_2 \leq 6 \quad x_2$$

$$\geq 7$$

Below is a tree diagram representing the solution procedure;



Now we solve the model at node 6 with $x_1 \leq 6$ constraints added to constraints of the model.

maximize subject to $\quad Z = 100x_1 + 150x_2$

$$8,000x_1 + 4,000x_2 \leq 40,000$$

$$15x_1 + 30x_2 \leq 200$$

$$x_2 \geq 6 \; x_1 \leq 1 \; x_2$$

$$\leq 6 \; x_1, x_2 \geq 0$$

The optimal solution for this model with integer restrictions relaxed is

$$x_1 = 1 \; x_2$$

$$= 6$$

$$Z = 1,000$$

Next, we solve the model at node 7 with the constraint $x_2 \geq 7$ added to the model.

maximize $\quad Z = 100x_1 + 150x_2$ subject to

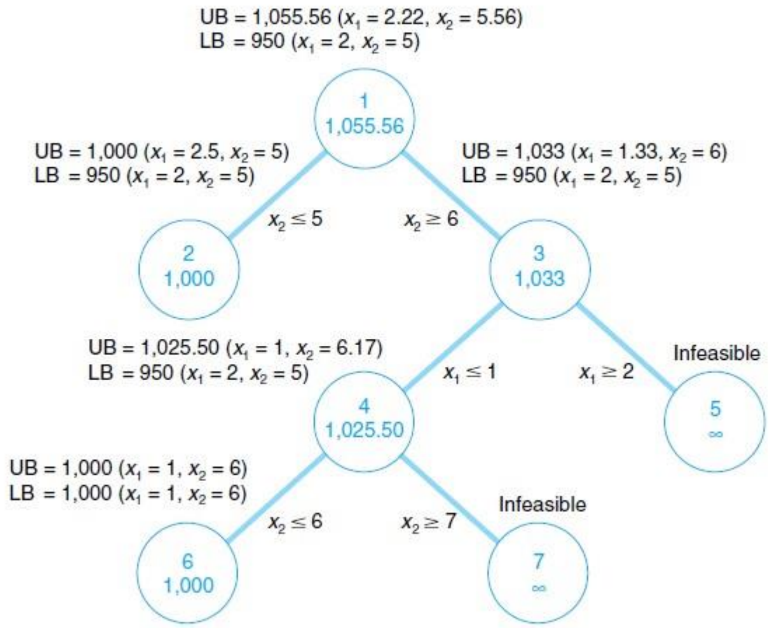
$$8{,}000x_1 + 4{,}000x_2 \leq 40{,}000$$

$$15x_1 + 30x_2 \leq 200$$

$$x_2 \geq 6 \; x_1 \leq 1 \; x_2$$

$$\geq 7 \; x_1, x_2 \geq 0$$

There is no feasible solution for this model. Therefore, no solution exists at node 7.



The optimal solution is therefore at node 6.

3.6 SENSITIVITY ANALYSIS

Sensitivity analysis is the study of how the uncertainty in the output of a mathematical model or system (numerical or otherwise) can be apportioned to di erent sources of uncertainty in its inputs or its technique used to determine how di erent values of an independent variable will impact a particular dependent variable under a given set of assumptions. This technique is used within speci c boundaries that will depend on one or more input variables. Sensitivity analysis is a way to predict the outcome of a decision if a situation turns out to be di erent compared to the key prediction(s).

Sensitivity analysis can be useful for a range of purposes, including

Testing the robustness of the results of a model or system in the presence of uncertainty.

Increased understanding of the relationships between input and output variables in a system or model.

Uncertainty reduction: identifying model inputs that cause signi cant uncertainty in the output and should therefore be the focus of attention if the robustness is to be increased (perhaps by further research).

Sensitivity analysis using the simplex method. While this is not as e cient or quick as using the computer, close examination of the simplex method for performing sensitivity analysis can provide a more thorough understanding.

3.6.1 ILLUSTRATION

Lets consider the problem below.

maximize $Z = 160x_1 + 200x_2$ subject to

$$2x_1 + 4x_2 \leq 40$$

$$18x_1 + 18x_2 \leq 216$$

200	x_2	8	0	1	1/2	-1/18	0
160	x_1	4	1	0	-1/2	1/9	0
0	s_3	48	0	0	6	-2	1
	z_j	2240	160	200	20	20/3	0
	$c_j - z_j$		0	0	-20	-20/3	0
	Basic		160	200	0	0	0
c_j	variables	Quantity	x_1	x_2	s_1	s_2	s_3

$$24x_1 + 24x_2 \leq 240$$

$x_1, x_2 \geq 0$ The optimal simplex tableau is given below.

Table 3.1: Optimal simplex tableau

SENSITIVITY ANALYSIS OF DECISION VARIABLES

Here, sensitivity analysis is performed to determine the range over which c_j can be changed without altering the optimal solution. The coe cients in the objective function will be represented symbolically as c_j (the same notation used in the simplex tableau). Thus,

	Basic variables		$160 + \Delta$	200	0	0	0
c_j		Quantity	x_1	x_2	s_1	s_2	s_3
200	x_2	8	0	1	1/2	-1/18	0
$160 + \Delta$	x_1	4	1	0	-1/2	1/9	0
	s_3	48	0	0	6	-2	1
	z_j	$2240 + \Delta$	$160 + \Delta$	200	$20 - \Delta/2$	$20/3 + \Delta/9$	0
	$c_j - z_j$		0	0	$-20 + \Delta/2$	$-20/3 - \Delta/9$	0

$$c_1 = 160$$

$$c_2 = 200$$

First, lets consider a Δ change for c_1. This will change the c_1 value from $c_1 = 160$ to $c_1 = 160 + \Delta$. Because c1 is a decision variable the simplex tableau changes with respect to the change in c_1. The transformed simplex tableau is represented below.

The solution shown in Table above will remain optimal as long as the $c_j - z_j$ row values remain negative. Thus, for the solution to remain optimal

$$-20 + \Delta/2 \leq 0$$

and

$$-20/3 - \Delta/9 \le 0$$

Both of these inequalities must be solved for Δ.

$$-20 + \Delta/2 \le 0$$

$$\Delta/2 \le 20$$

$$\Delta \le 40$$

and

$$-20/3 - \Delta/9 \le 0$$

$$-\Delta/9 \le 20/3$$

$$-\Delta \le 60$$

$$\Delta \ge -60$$

Thus $\Delta \le 40$ and $\Delta \ge -60$. But $c_1 = 160 + \Delta$; therefore, $\Delta = c_1 - 160$.

Substituting $\Delta = c_1 - 160$ into these inequalities yields

$$\Delta \le 40$$

$$c_1 - 160 \le 40 \; c_1$$

$$\le 200$$

and

$$\Delta \ge -60 \; c_1 -$$

$$160 \ge -60 \; c_1 \ge$$

$$100$$

Therefore, the range of values of c_1 over which the solution basis will remain optimal is

$$100 \le c_1 \le 200$$

Secondly, lets consider a Δ change in c_2 so that $c_2 = 200 + \Delta$. The e ect of this change in the nal simplex tableau is shown in Table below.

	Basic variables		160	200 + Δ	0	0	0
c_j		Quantity	x_1	x_2	s_1	s_2	s_3
200 + Δ	x_2	8	0	1	1/2	-1/18	0
160	x_1	4	1	0	-1/2	1/9	0
	s_3	48	0	0	6	-2	1
	z_j	$2240 + 8\Delta$	160	$200 + \Delta$	$20 + \Delta/2$	$20/3 - \Delta/18$	0
	$c_j - z_j$		0	0	$-20 - \Delta/2$	$-20/3 + \Delta/18$	0

The solution shown in table above will remain optimal as long as the $c_j - z_j$ row values remain negative. Thus, for the solution to remain optimal

$$-20 - \Delta/2 \leq 0$$

and

$$-20/3 + \Delta/18 \leq 0$$

Solving these inequalities for Δ gives

$$-20 - \Delta/2 \leq 0$$

$$-\Delta/2 \leq 20$$

$$\Delta \geq -40$$

and

$$-20/3 + \Delta/18 \leq 0$$

$$-\Delta/18 \leq 20/3$$

$$\Delta \leq 120$$

Thus $\Delta \geq -40$ and $\Delta \leq 120$. But $c_2 = 200 + \Delta$; therefore, $\Delta = c_2 - 200$.

Substituting $\Delta = c_2 - 200$ into these inequalities yields

$$\Delta \geq -40 \quad c_2 -$$

$$200 \geq -40 \quad c_2 \geq$$

$$160$$

and

$$\Delta \leq 120 \; c_2 -$$

$$200 \leq 120 \; c_2 \leq$$

$$320$$

Therefore, the range of values of c_2 over which the solution basis will remain optimal is

$$160 \leq c_2 \leq 320$$

The ranges for both objective function coe cients are as follows.

$$100 \leq c_1 \leq 200$$

$$160 \leq c_2 \leq 320$$

However, these ranges re ect a possible change in either c_1 or c_2, not simultaneous changes in both c_1 and c_2.

SENSITIVITY ANALYSIS OF CONSTRAINTS

To demonstrate the e ect of a change in the quantity values of the model constraints, we will again use the example we use previously

maximize $Z = 160x_1 + 200x_2$ subject to

$$2x_1 + 4x_2 \leq 40$$

$$18x_1 + 18x_2 \leq 216$$

$$24x_1 + 24x_2 \leq 240$$

$$x_1, x_2 \geq 0$$

The purpose of sensitivity analysis of constraints is to determine the range for q_i over which the optimal variable mix will remain the same and the shadow price will remain the same.

As in the case of the c_j values, the range for q_i can be determined directly from the optimal simplex tableau. As an example, consider a Δ increase in the number of labor hours. The model constraints become

$$2x_1 + 4x_2 \leq 40 + 1\Delta$$

$$18x_1 + 18x_2 \leq 216 + 0\Delta$$

$$24x_1 + 24x_2 \leq 240 + 0\Delta$$

The changes in the quantity column (constraints) are represented as the coe cients in the s_1 column of the simplex tableau below

	Basic variables		160	200	0	0	0
c_j		Quantity	x_1	x_2	s_1	s_2	s_3
0	s_1	$40 + 1\Delta$	2	4	1	0	0
0	s_2	$216 + 0\Delta$	18	18	0	1	0
0	s_3	$240 + 0\Delta$	24	12	0	0	1
	z_j	0	0	0	0	0	0
	$c_j - z_j$		160	200	0	0	0

This process will carry through each subsequent tableau, so the s1 column values will duplicate the changes in the quantity column in the nal tableau.

	Basic variables		160	200	0	0	0
c_j		Quantity	x_1	x_2	s_1	s_2	s_3
200	x_2	$8 + \Delta/2$	0	1	1/2	-1/18	0

160	x_1	$4 - \Delta/2$	1	0	-1/2	1/9	0
0	s_3	$48 + 6\Delta$	0	0	6	-2	1
	z_j	$2240 + 20\Delta$	160	200	20	20/3	0
	$c_j - z_j$		0	0	-20	-20/3	0

Recall that a requirement of the simplex method is that the quantity values not be negative. If any q_i value becomes negative, the current solution will no longer be feasible and a new variable will enter the solution. Thus, the inequalities

$$8 + \Delta/2 \geq 0 \ 4$$

$$- \Delta/2 \geq 0$$

$$48 + 6\Delta \geq 0$$

are solved for Δ

$$8 + \Delta/2 \geq 0$$

$$\Delta/2 \geq -8$$

$$\Delta \geq -16$$

$$4 - \Delta/2 \geq 0$$

$$-\Delta/2 \geq -4$$

$$\Delta \leq 8$$

and

$$48 + 6\Delta \geq 0$$

$$6\Delta \geq -48$$

$$\Delta \geq -8$$

Thus $\Delta \geq -16$, $\Delta \leq 8$ and $\Delta \geq -8$. But $q_1 = 40 + \Delta$; therefore, $\Delta = q_1 - 40$.

Substituting $\Delta = q_1 - 40$ into these inequalities yields

$$\Delta \geq -16 \ q_1 -$$

$$40 \ \geq -16 \ \ q_1 \ \geq$$

$$24$$

$$\Delta \leq 8$$

$$q_1 - 40 \leq 8\ q_1$$

$$\leq 48$$

$$\Delta \geq -8$$

$$q_1 - 40 \geq -8\ q_1 \geq$$

$$32$$

Summarizing these inequalities, we have

$$24 \leq 32 \leq q_1 \leq 48$$

The value of 24 can be eliminated, since q_1 must be greater than 32; therefore the optimal range is

$$32 \leq q_1 \leq 48$$

As long as q_1 remains in this range, the present basic solution variables will remain the same and feasible. However, the quantity values of those basic variables may change. In other words, although the variables in the basis remain the same, their values can change.

To determine the range for q_2 the s_2 column values are used to develop the Δ inequalities

$$8 - \Delta/18 \geq 0$$

$$4 + \Delta/9 \geq 0$$

$$48 - 2\Delta \geq 0$$

The inequalities are solved as follows.

$$8 - \Delta/18 \geq 0$$

$$-\Delta/18 \geq -8$$

$$\Delta \leq 144$$

$$4 + \Delta/9 \geq 0$$

$$\Delta/9 \geq -4$$

$$\Delta \geq -36$$

and

$$48 - 2\Delta \geq 0$$

$$-2\Delta \geq -48$$

$$\Delta \leq 24$$

Thus $\Delta \leq 144$, $\Delta \geq -36$ and $\Delta \leq 24$. But $q_2 = 216 + \Delta$; therefore, $\Delta = q_2 - 216$.

Substituting $\Delta = q_2 - 216$ into these inequalities yields

$\Delta \leq 144$ $q_2 -$
$216 \leq 144$ $q_2 \leq$
360

$\Delta \geq -36$ $q_2 -$
$216 \geq -36$ $q_2 \geq$
180

$\Delta \leq 24$ $q_2 -$
$216 \leq 24$ $q_2 \leq 240$ Summarizing these inequalities, we have

$$180 \leq q_2 \leq 240 \leq 360$$

The value of 240 can be discarded, since its smaller than 320; therefore the optimal range is

$$180 \leq q_2 \leq 360$$

As long as q_2 remains in this range, the present basic solution variables will remain the same and feasible. However, the quantity values of those basic variables may change. In other words, although the variables in the basis remain the same, their values can change.

CHAPTER 4

DATA COLLECTION AND ANALYSIS

This chapter presents model formulation and analysis of the model.

4.1 PLANNING AND SCHEDULING MODEL

The production planning model is developed for addressing medium range time horizon decisions. The objective of the production planning model is to minimize the production costs. Production costs are inventory costs and set up costs of end products, intermediate products, inventory cost of by-products and recovered raw materials and cost of fresh materials. Scheduling decisions determine start time and completion time of a job on each machine. The production plan imposes constraints on the scheduling model. We now provide the formulation of the production model/scheduling model for a milk production caompany.

Decision variables

$X(i,j)$ produced quantity of product i at shift j.

1. j = number of shifts
2. i = number of independent products.

Y_i = 1 if product i is produced at shift j. 0 otherwise.

Parameters

$C_{i,j}$ = cost of producing milk product type i at shift j

$S_{i,j}$ = Selling price of milk product type i SP =

Total amount of bulk milk(kg)

D_i = The customer's demand of milk product type i

CP_j = The capacity of the packaging machine at shift j

T_j = total available machine time at shift j

$T_{i,j}$ = required production time of product i at shift j

P_i = The size of milk product type i FC =
Fixed Cost

Objective Function

$$\sum_i X_{i,j} Y_{i,j} S_i - \sum_i X_{i,j} Y_{i,j} C_{i,j} - FC \qquad (4.1)$$

subject to

$$\sum P_i X_{i,j} \le CP_j \qquad (4.2)$$

$$\sum X_{i,j} \ge Y_{i,j} D_i \qquad (4.3)$$

$$\sum X_{i,j} T_{i,j} \le Y_{i,j} T_j \qquad (4.4)$$

$$X_{i,j} \ge 0 \quad \text{and} \quad X_{i,j} \in \text{integer} \qquad (4.5)$$

The objective function (4.1) seeks to maximize the net pro t.

Constraint (4.2) states the capacity of the packaging machine per shift. Constraint (4.3) states that the number of milk produced per day can supply the customer's demand for each milk product type.

Constraint (4.4) states the time of the packaging machine per shift.

Constraint (6) is speci cation of the decision variables.

4.2 CASE STUDY

Promasidor Ghana Limited produces 3 milk products consisting of plain powdered milk imported from Indonesia. The plain bulk milk weighs 3600kg . Each day consist of 3 shifts that are 8 hours each. The capacity of the packaging machine at each shift is 750kg.

The table below represents summary of the production details.

	weight	Production time	Demand	cost per shift	Selling price
Product 1	200g	8s	5000pcs	[0.25 0.25 0.27]	0.35
Product 2	300g	11s	2250pcs	[0.38 0.38 0.40]	0.46
Product 3	500g	18s	1000pcs	[0.80 0.80 0.84]	0.90

Table 4.1: summary of production options

Fixed cost = 7,600GHc per month

4.2.1 MODEL

The model below is based on daily operations of the company.

Maximize $\sum_{i=1}^{3}\sum_{j=1}^{3}(C_{i,j}-S_i)X_{i,j}$

Subject to

$$8X_{1,1} + 11X_{1,2} + 18X_{1,3} \leq 80*60*60$$
$$8X_{2,1} + 11X_{2,2} + 18X_{2,3} \leq 80*60*60 \quad \text{time constraint.}$$
$$8X_{3,1} + 11X_{3,2} + 18X_{3,3} \leq 80*60*60$$

$$0.2X_{1,1} + 0.3X_{1,2} + 0.5X_{1,3} \leq 750$$
$$0.2X_{2,1} + 0.3X_{2,2} + 0.5X_{2,3} \leq 750 \quad \text{production capacity constraint.}$$
$$0.2X_{3,1} + 0.3X_{3,2} + 0.5X_{3,3} \leq 750$$

$$\left.\begin{array}{llll} X_{1,1}+ & X_{2,1}+ & X_{3,1} & \geq 5000 \\ X_{1,2}+ & X_{2,2}+ & X_{3,2} & \geq 2250 \\ X_{1,3}+ & X_{2,3}+ & X_{3,3} & \geq 1000 \end{array}\right\}$$

$X_{1,1}, ..., X_{3,3} \geq 0 \qquad X_{i,j} \;\epsilon\; \mathbb{Z}$ demand constraint.

4.2.2 MODEL ANALYSIS AND RESULT DISCUSSION

TORA is used to solve the model above. The decision variables are being transformed so that the TORA software could be used.

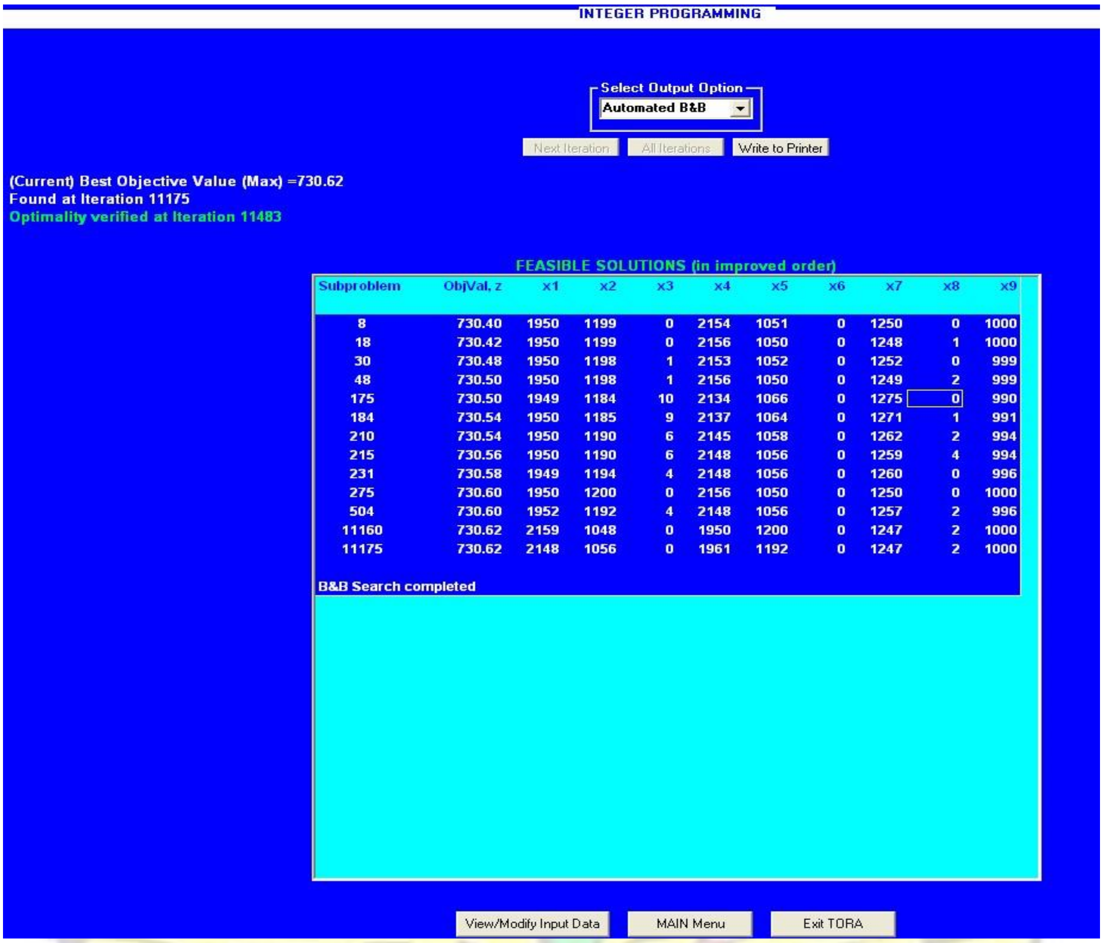
$X_{1,1} = x1$, $X_{1,2} = x2$, $X_{1,3} = x3$

$X_{2,1} = x4$, $X_{2,2} = x5$, $X_{2,3} = x6$

$X_{3,1} = x7$, $X_{3,2} = x8$, $X_{3,3} = x9$.

	x1	x2	x3	x4	x5	x6	x7	x8	x9	Enter <, >, or =	R.H.S.
Var. Name											
Maximize	0.10	0.08	0.06	0.10	0.08	0.06	0.08	0.06	0.04		
Constr 1	0.20	0.30	0.50	0.00	0.00	0.00	0.00	0.00	0.00	<=	750.00
Constr 2	0.00	0.00	0.00	0.20	0.30	0.50	0.00	0.00	0.00	<=	750.00
Constr 3	0.00	0.00	0.00	0.00	0.00	0.00	0.20	0.30	0.50	<=	750.00
Constr 4	1.00	0.00	0.00	1.00	0.00	0.00	1.00	0.00	0.00	>=	5000.00
Constr 5	0.00	1.00	0.00	0.00	1.00	0.00	0.00	1.00	0.00	>=	2250.00
Constr 6	0.00	0.00	1.00	0.00	0.00	1.00	0.00	0.00	1.00	>=	1000.00
Constr 7	8.00	11.00	18.00	0.00	0.00	0.00	0.00	0.00	0.00	<=	28800.00
Constr 8	0.00	0.00	0.00	8.00	11.00	18.00	0.00	0.00	0.00	<=	28800.00
Constr 9	0.00	0.00	0.00	0.00	0.00	0.00	8.00	11.00	18.00	<=	28800.00
Lower Bound	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00		
Upper Bound	infinity	infinity	infinity	infinity	infinity	infinity	infinity	infinity	infinity		
Unrestr'd (y/n)?	n	n	n	n	n	n	n	n	n		

Figure 4.1: Typical TORA window showing the problem formulated



Subproblem	ObjVal, z	x1	x2	x3	x4	x5	x6	x7	x8	x9
8	730.40	1950	1199	0	2154	1051	0	1250	0	1000
18	730.42	1950	1199	0	2156	1050	0	1248	1	1000
30	730.48	1950	1198	1	2153	1052	0	1252	0	999
48	730.50	1950	1198	1	2156	1050	0	1249	2	999
175	730.50	1949	1184	10	2134	1066	0	1275	0	990
184	730.54	1950	1185	9	2137	1064	0	1271	1	991
210	730.54	1950	1190	6	2145	1058	0	1262	2	994
215	730.56	1950	1190	6	2148	1056	0	1259	4	994
231	730.58	1949	1194	4	2148	1056	0	1260	0	996
275	730.60	1950	1200	0	2156	1050	0	1250	0	1000
504	730.60	1952	1192	4	2148	1056	0	1257	2	996
11160	730.62	2159	1048	0	1950	1200	0	1247	2	1000
11175	730.62	2148	1056	0	1961	1192	0	1247	2	1000

Figure 4.2: Feasible Solutions of Branch and Bound Method

From the solution produced by the TORA Software package

$x1 = X_{1,1} = 2148$ $x2 = X_{1,2} = 1056$ $x3 = X_{1,3} = 0$

$x4 = X_{2,1} = 1961$ $x5 = X_{2,2} = 1192$ $x6 = X_{2,3} = 0$

$x7 = X_{3,1} = 1247$ $x8 = X_{3,2} = 2$ $x9 = X_{3,3} = 1000$

Best max Objective value = 730.62

The company operates 6 days a week. Hence net pro t is 6*4*730.627,600. At the end of the month the net pro t is 9,934.88GHc when the proposed model is used in the scheduling and production process.

To operate at optimum level during shift 1 production only 2148 pcs of product 1 and 1056 pcs of product 2 has to be produced. Product 3 must not be produced at shift 1 at all. Product 3 is not produced due to production capacity constraint and products 1 and 2 have the most pro t per pack.

At shift 2, only 1961 pcs of product 1 and 1192 pcs of product 2 has to be produced. Product 3 must not be produced at shift 1 at all. Product 3 is not produced due to production capacity constraint and products 1 and 2 have the most pro t per pack.

At shift 3, 1247 pcs of product 1, 2 pcs of product 2 and 1000 pcs of product 3 has to be produced.

4.3 SENSITIVITY ANALYSIS

Sensitivity Analysis

Variable	Current Obj Coeff	Min Obj Coeff	Max Obj Coeff	Reduced Cost
x1:	0.10	0.09	0.10	0.00
x2:	0.08	0.08	0.09	0.00
x3:	0.06	-infinity	0.07	0.01
x4:	0.10	0.10	0.10	0.00
x5:	0.08	0.08	0.08	0.00
x6:	0.06	-infinity	0.07	0.01
x7:	0.08	0.08	0.08	0.00
x8:	0.06	-infinity	0.06	0.00
x9:	0.04	0.04	0.20	0.00
Constraint	**Current RHS**	**Min RHS**	**Max RHS**	**Dual Price**
1 (<)	750.00	746.25	776.25	0.00
2 (<)	750.00	746.25	infinity	0.00
3 (<)	750.00	678.75	770.00	0.40
4 (>)	5000.00	-infinity	5356.25	0.00
5 (>)	2250.00	1200.00	2400.00	-0.06
6 (>)	1000.00	600.00	1142.50	-0.16
7 (<)	28800.00	27750.00	28950.00	0.01
8 (<)	28800.00	25950.00	28950.00	0.01
9 (<)	28800.00	28000.00	infinity	0.00

Table 4.2: Sensistivity analysus window from TORA Software

At shift 1, the decision variables x_1, x_2, x_3 has [0.10 0.09 .07] as maximum objective function coe cients respectively. Thus holding all other things constant, a 0.01 increase in x_2 will not a ection the solution whilst 0.01 increase in x_3 will reduce the contribution of the x_3 component by 0.01. The minimum objective coe cient of x_1 is 0.09 and this change will not have any e ect on the solution. The minimum objective function coe cient of x_3 is -in nity because x_3 is not produced at shift 3 at all. Hence this change in objective function coe cient cannot a ect the solution. Shift 1 and 2 has the same production plan therefore what ever happens at shift 1 just repeats itself at shift 2.

At shift 3, the decision variables x_7, x_8, x_9 has [0.08 0.06 .20] as maximum objective function coe cients respectively. A n upward adjust in any of the decision variables here does not a ect the optimality of the solution. x_8 also has a minimum of in nity since only 2 pcs are produced. Hence a change in that direction does a ect the solution.

Constraints 1 and 2 have a RHS range of [746.25 776.25] and [746.25 in nity]. A change in the RHS of any of these two constraints does not a ect the dual price. Thus there is no change in the dual price. Constraint 3 has a RHS range of [678.25 770.00]. A RHS change in this range results in 0.40 change in the dual price.

Constraints 4 and 5 have a RHS range of [$-\infty$ 5356.25] and [1200 2400]. A change in the RHS of any of these two constraints will a ect the dual price 0.0 and -0.05 repectively. Thus there is no change in the dual price in the case of constraint 4. Constraint 6 has a RHS range of [600 1142.50]. A RHS change in this range results in -0.15 change in the dual price.

CHAPTER 5

CONCLUSIONS AND RECOMMENDATIONS

In this thesis, we investigated production planning and scheduling model formulation and its application to a beverage company. We argued that these problems need clear-cut models that capture the relevant aspects of production, and e cient algorithms to nd their close-to-optimal solutions in a reasonable time. The branch and bound Algorithm was adopted to solve the model which yields highly accurate results.

5.1 SUMMARY OF FINDINGS

At the end of our simulations we found out the more pcs of product 1 can be produced given the necessary constraints. Thus to maximize pro t, 263 more pcs of product 1 should be produce. This falls in line with the demand of product 1. Product 1 has the highest demand among all products and has the highest pro t margin per pack.

The least amount of products happens at shift 3. This is true because shift 3 has the highest cost of operation/production. Only 2 pcs of Product 2 and 1247 pcs of product 2 are produced at shift 3. This few amounts are only produced at shift 3 due to production capacity constraints at shifts 1 and 2.

Given the low demand of product 3, more resources should be channeled into the production of products 1 and 2 since the return the most pro t.

5.2 CONCLUSIONS

At the end of this thesis we have come up with the following conclusions;

A generalized scheduling and production model was developed. This model can be adopted in most beverage industries as its captures the essential elements of a typical production and scheduling problem. The model is MILP one that can be solved with the legendary branch and bound algorithm.

We report implementation of the production planning and scheduling models in a real life case of a beverage company in Ghana. The results of the models indicate substantial savings over the actual company performance. Sensitivity analysis on the results is provided evaluate various production plans and schedules.

5.3 RECOMMENDATIONS

We recommend that beverage companies in Ghana adopt the generalized model for scheduling and production planning to help boost their pro t margins.

In line with the ndings of this research and ndings necessary attention must be paid to the production of products at each shift. Since the cost of production at some shifts are higher than others. The results from the production and Scheduling model must be adhered to strictly.

We also recommend Promasidor Ghana Limited to increase the production capacity of shifts 1 and 2 and discard shift 3 all together since only few products are produced and then cost of production is also the highest.

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Appendix A