KWAME NKRUMAH UNIVERSITY OF SCIENCE AND TECHNOLOGY, KUMASI

ANALYSIS OF WAITING LINES AT ELECTRICITY COMPANY OF GHANA(ECG) PAY POINT CENTRE USING QUEUING THEORY: A CASE STUDY OF ECG, DICHEMSO BRANCH.

BY

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DECLARATION

I hereby declare that this submission is my own work towards the award of the M.Sc degree and that, to the best of my knowledge, it contains no material previously published by another person nor material which had been accepted for the award of any other degree of the university, except where due acknowledgment had been made in the text.

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DEDICATION

I dedicate this thesis to my **friends** and **family**.

ABSTRACT

This thesis reviews the application of queuing theory. The M/M/N queuing model was used to analyze waiting lines at the Credit Customer Pay Point Centre(CCPC) and Prepaid Customer Pay Point Centre(PCPC) of the Electricity Company of Ghana(ECG) at Dichemso-branch. The purpose was to find the average number of customers waiting in queue and system and the average waiting time in queue and system at the pay points. Other performance measurements of the queueing system was also found. An excel spreadsheet was used to organize and analyze the data collected at the pay points. The number of customers that arrived at the CCPC was found to be higher than the number of customers that arrived at the PCPC. CCPC was found to be busier than PCPC. The average utilization factor at the CCPC and PCPC was 92.82% and 88.67% respectively. The average of arrival rate and service rate at the CCPC was found to be 0.7500 and 0.8458 respectively. The average of arrival rate and service rate at the PCPC was found to be 0.7133 and 0.7685 respectively.

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CONTENTS

LIST OF ABBREVIATION

ECG .... Electricity Company of Ghana

CCPC .... Credit Customer Pay Point Centre

PCPC .... Prepaid Customer Pay Point Centre

FCFS .... First Come First Served

LCFS .... Last Come First Served

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CHAPTER 1

INTRODUCTION

1.1 Introduction

Queuing theory is the body of knowledge which deals with the analysis of queues (or waiting lines) whereby customers arrive at a service facility and have to wait before they receive the desire service.

A queuing system consist of a server, a number of customers who demand service and a queue of customers waiting to be served. According to Banks et al. (2001), Queuing models provide the analyst with a powerful tool for designing and evaluating the performance of queuing systems. A queuing system is basically made up of arrival or inputs to the system, queuing discipline and the service facility.

Queues are a common phenomenon in our daily activities. They may, for example, take the form of patients waiting for treatment by doctors or nurses at a hospital, a ship and barges waiting for dock workers to load and unload at a harbor, customers waiting for transactions handled by a teller at a bank etc. Queues are formed as a result of demand exceeding services demanded at a facility. Queues are also formed because of limited resources.

Whenever customers arrive at a service facility, some of them have to wait before they receive the desire. It means that the customer has to wait for his/her turn, may be in a line. This situation can be frustrating when customers has to wait very long before they are served in a queue and bring dissatisfaction to them. These waiting times waste the time of customers that could have used used been used in productive activities. Long waiting can reduce revenue collected at pay points.

According to Nafees (2007),sometimes, insufficiencies in service also occur due to an undue wait in service may be because of new employee. Delays in service jobs beyond their due time result in losing future business opportunities. Queuing theory is the study of waiting in all these various situations. It uses queuing models to represent the various types of queuing systems that arise in practice. The models enable finding an appropriate balance between the cost of service and the amount of waiting.

Queuing theory may be extended to cover a wide variety of contention situations, such as how customers check-out lines form (and how they can be eliminated), how many calls a telephone switch can handle, how many computer uses can share a mainframe, and how many doors an office building should have. These are diverse applications, but their solutions all involve the same dynamics.

This study is designed to assist Electricity Company of Ghana(ECG), Dichemso branch, managers to acquire detailed information about waiting lines at the pay point center and to determine the number of pay counters that will reduce waiting times considerably.

In this chapter, the background to the study, statement of the problem, objective of the study and the significance of the study are carefully discussed. It also highlights the research methodology used, the scope, limitations and the organization of the study.

1.2 Background to the study

Queues can be refer to as items, customers, a signal in a line awaiting some kind of service. Queuing time is the amount of time a vehicle , customer or a thing spends before being attended to for some kind of service. Queues may be finite or infinite.

Queuing theory is an analytical method that models a system allowing queue, calculates it's performances and determine its' properties in other to help managers in decision making. System performance may include mean waiting time, percentile of the waiting time, utilization of the server, throughput(i.e number of customers served per unit time), average number of customers waiting etc.

Queuing models have been very useful in many practical application or field of study in the areas such as, health care centers, transportation systems, communication systems etc. Basically, a queuing model is characterized by; arrival process of customers, the behavior of customers, the service time, discipline capacity, waiting room or a lane. Queuing models can be of several types depending on its' characteristics.

MAJAKWARA (2009) ,queuing analysis provides a conceptual framework and insights of how queuing systems behave. queuing analysis seeks to understand system behavior using real life data, project from an existing system to a future system, develop an analytic model for use in designing a system and create simulations that models a system. A central challenge in designing and managing a service facility is to achieve a desired balance between operational efficiency and service quality.

ECG pay point centres at the Ashanti-East, Regional office, Dichemso, has different pay counters that delivers services to customers who uses prepaid meters and credit meters. There are two customer pay point5s centres at the branch office. These are Credit Customer Pay point Centre(CCPC) and Prepaid Customer Pay point Centre(PCPC). A single server attended to customers at both pay point centres. Both prepaid and credit customers formed a single queue to receive services at the various counters.

1.2.1 A Brief History and Organizational Structure Of ECG

Electricity Company of Ghana(ECG) is a public electricity distribution company in Ghana. It operations are under the Ministry of Energy of Ghana. The company incorporated under the companies code, 1963 in February 1997. It started as the

Electricity department on 1st April, 1947 and later became the Electricity division in 1962. Subsequently, it was converted into the Electricity Corporation of Ghana by NLC Decree 125 in 1967. ECG was to remain the entity solely responsible for electricity supply and the distribution networks nationwide for the next two decades.

The Government created the Northern Electricity Department(NED) as a subsidiary of Volta River Authority(VRA) in 1987 which took over from ECG the responsibility for the management of electric power distribution in the Northern Region of Ghana. The company (ECG) is however responsible for the distribution of electricity in the Southern part of Ghana.

The organizational structure of ECG is made up of nine-member Board of Directors who govern the company on the behalf of the Government of Ghana. The company is headed by a Managing Director. The company is generally organized into thirteen (13) departments which includes the customer care department.

Customer pay points are under the companies customer care department. These pay points can be found at all ECG offices where customers go to pay their electricity bills and also purchase their prepaid credits.

1.2.2 ECG Electricity Meters

ECG has two main types of meters used to measure the consumption of electricity by customers. These are the Credit and prepaid meters. In Ghana, credit meters was the only kind used until between 1994 and 1995 when the prepayment program started on a pilot basis in Accra, Tema and Kumasi for residential and non-residential purposes.

The credit meter is the type of meter that allows customers to use energy before they pay for the amount consumed. The meter records the amount of energy consume in digits form. These digits are recorded by an EGC worker at the end of every month. A bill is submitted to the consumer at the end of every month

for payment at any ECG pay point. Failure to pay bills within a specific period resorts to a disconnection of power supply to customers by the company.
Paid meters are the type of meters that allows customers to get power supply only when they have already paid for it. These meters accepts ECG prepaid cards that consumers use to purchase an amount of energy at any ECG pay point. With this type of meters, there are no bills supply to consumers and no disconnections.

1.3 Statement of the Problem

ECG seeks to provide quality, reliable and safe electricity services to support the socio-economic and development of Ghana. Customers has to pay for these services at ECG customer pay points. These revenue accumulated is used by management to bear their operational cost. Thought it is the responsibility of customers to pay for these services in time, customers sometimes encounter long queues at pay point centres. These queues dissatisfies customers who has to wait for a long time before services are rendered. This can cause delays in payments and also reduce revenue collected. Queuing at pay points waste the time of customers that could have been used in other productive activities for the development of the economy. Hence, it is wealth managing pay points centers in a way to reduce waiting times of customers. In this thesis, waiting lines at ECG, Dichemso branch is analyzed using queuing theories.

1.4 Objectives

1. The purpose of this study is to analyze waiting lines at ECG,Dichemso branch pay point using M/M/N queuing model.

2. Determine an average arrival and service rates from data collected.

1.5 Significance of the Study

The findings of this study is intended to assist managers of Dichemso, ECG pay point managers to reduce the waiting time of customers at the pay point centre. This will improve customer service satisfaction and encourage them to pay bills in time.

The findings of this study will help the managers to gather enough information about waiting lines at the pay point centres and be able to determine the number of pay counters appropriate to reduce waiting times.

Determining the optimal number of pay counters from the findings of this study, will minimize waiting times which will affect the development of the economy positively since customers can use their wasted times in other productive activities.

1.6 Methodology

M/M/N queuing model will be used in this thesis. The data used for the study will be obtained from the pay point centers. A stop watch will be used to record the times that customers arrive and the time that customers spend at the counter receiving services. The data will be collected in five working days. The researcher will use an hour at both CCPC and PCPC in each day of data collection. Excel spread sheet will be used in the analysis. The reading material will be obtained from the Internet and the library of KNUST.

1.7 Organization of the study

This thesis comprises of five chapters. Chapter one emphasizes on the background, problem statement, objectives, scope, limitations and research methodology. In chapter two, relevant and related literatures was reviewed. Chapter three puts forward the research methodology of the study. Chapter four deals with the data collection and analysis. The summary of the finding is also presented in chapter four. Finally, chapter five presents the conclusion and recommendations of the study.

CHAPTER 2

LITERATURE REVIEW

2.1 Overview

This chapter emphasizes on the review of relevant literatures on applications of queuing theory. The history behind queuing theory is discussed. Research papers that highlights queuing systems, the rudimentary of queuing theory and its' characteristics are also reviewed in this chapter.

2.2 Related works on applications of queuing theory

Nafees (2007), analyzed queuing systems for an empirical data of supermarket checkout service unit as an example. The model designed for this example is a multiple queues multiple-server model. The study required an empirical data which included arrival time in the queue of checkout operating unit(server), departure time and service time.

In Sharma et al. (2013), queuing theory is the mathematical study of waiting lines and it is very useful to define modern information technologies requiring innovations that are based on modeling, analyzing to deals as well as the procedure of traffic control of daily life of human like telecommunications, reservation counter, super market, big bazaar, picture cinema, hall ticket window and also to determine the sequence of computer operations, computer performance, health services, airport traffic, airline ticket sales. The paper discussed the approach of queuing theory and queuing models.

Duder and Rosenwein (2001), used queuing based 'rule-of-thumb' formulas to estimate the cost of abandonment and to determine the optimum number of operators. Whitt (2005) also used queuing analysis for staffing a call center, considering the proportion of servers present as a random variable.

Cugnasca (2007), provided useful information to build availability models for computer systems used in airspace control centers based on analytical models provided by queuing theories. The researchers used queuing models to establish availability parameters related to a data center operation and its management issues.

May (2012), analyzed data from a large open pit gold mine and applied to a multichannel queuing model representative of the loading process of the haul cycle. She stated that, one method of fleet selection involves the application of queuing theory to the haul cycle and most mining haul routes consist of four main components; loading, loading hauling, dumping and unloaded hauling to return to the loader. The outputs of the model was compared against the actual truck data to evaluate the validity of the queuing model developed.

WOENSEL and VANDAELE (2007), presented an overview of different analytic queuing models for traffic on road networks. They shown that queuing models can be used to adequately model uninterrupted traffic flows. An analytical application tool to facilitate the optimal positioning of the counting points on a highway was also presented in the paper.

Brown (2012), aimed at increasing understanding of system variables on the accuracy of simple queuing models. A queuing model was proposed that combines G/G/1 modeling techniques for rework with effective processing time techniques for machine availability and the accuracy of this model was tested under varying levels of rework, external arrival variability and machine variability. The research shown that the model performed best under exponential arrival patterns and can perform well even under high rework conditions. Generalization was made with regards to the use of this tool for allocation of jobs to specific workers and/or machines based on known rework rates with the ultimate aim of queue time minimization.

Lakshmi and Lyer (2013), reviewed the contributions and applications of queuing theory in the field of health care management problems. They proposed a system of classification of health care areas which are examined with the assistance of queuing models. Their goal was to provide sufficient information to analysis who are interested in using queuing theory to model a health care process and who want to locate the details of relevant models.

Alfares (2009), presented the modeling and solution of a real-life operator scheduling problem at a call center. Queuing and integer programming models were combined to minimize the total weekly labor cost while providing an acceptable service level for each hour of each day of the week. The models determined optimum staffing levels and employee weekly work schedules for meeting a varying workload for each hour of the week. Queuing analysis was applied to data on the number and duration of calls in order to estimate minimum hourly labor demand.

Chiang et al. (2013), presented two scheduling policies to stay away from the risk of over-provisioning and under-provision in both quality of servers and service rate. They modeled a cloud server farm as an M/M/R queuing model, such that system congestion cost and balking event loss can be estimated analytically. A cost function is developed by taking system operating cost, working mode cost, system congestion cost etc. into consideration. Simulation result showed that the optimal quality of service rate can be obtained to minimum cost.

Obamiro (2010), presented the results of a study that evaluates the effectiveness of a queuing model in identifying the ante-natal queuing system efficiency parameters. Tora Optimization System was used to analyze data collected from ante-natal care unit of a public teaching hospital in Nigeria over three-week period. The study showed that pregnant mothers spent less time in the queue and system in the first week than during the other succeeding two weeks.

2.3 Brief History behind Queuing Theory

The history behind queuing theory goes back to primitive man. A.K(Agner Krarup)Elang published his first paper on queuing theory in 1909. Elang worked at the Copenhagan telephone exchange as an engineer. Erlangs queuing models have since been applied in the telecommunications, traffic engineering, computer networking etc.

In Acheampong (2013), Erlang developed the famous model to analyze loss probabilities of multi-channel point to point conversations. In the early 1920, he observed that a telephone system was generally characterized by either poisson input(e.i number of calls), exponential holding(service) time, and multiple channels(i.e servers) or poisson input, constant holding time and a single channel. The "application of the theory of probabilities to telephone trucking problems" was published by Modina in 1927. The uses of the queuing theory to telephone were rapidly used by many.

Thorton Fry, published a book on "probability and its engineering uses" a year after Molinas' publications. Much of the work done by Erlang was expanded in this book. The study of queues with multiple servers dates back over fifty year of the seminar paper presented by Kiefer and Wolfowitzs,MAJAKWARA (2009)

In 1951, Kendall published his work about embedded markov chains, which is the base for the calculation of queuing systems under fairly general input conditions. A naming conversion for queuing systems was also defined in his work. Lidley developed an equation allowing for results of queuing systems under fairly general input and service conditions.

In 1957, Jackson stated the investigation of networked queues thus leading to so called queuing network models. With the appearance of computers and computer networks,queuing systems and queuing networks have been identified as a powerful analysis and design tool for various applications. Queuing theory was boosted mainly by the introduction of computers and the digitization of the telecommunication infrastructures.

For engineers, the two volumes by Kleinrock(1975,1976) are perhaps the most well-known. Whole in applied mathematics, apart from the penetrating influence of Feller(1970,1971), the single server queue of Cohen(1969) is regarded as a landmark. The evolution of queuing theory still continues.(Acheampong, 2013)

2.4 Works Related to Queuing Systems

Gurumurthi and Benjaafar (2004), considered queuing systems with multiple classes of customers and heterogeneous servers where customers have the flexibility of being processed by more than one server and servers possess the capacity of processing more than one customer class. They provided a unified framework for the modeling and analysis of these systems under arbitrary customer and server flexibility and for a rich set of control policies that includes customer/server- priority schemes for server and customer selection. Several insight into the effect of system configuration and control policies was generated. In particular, they examined the relationship between flexibility, control policies and throughput under varying assumption for system parameters

Filipowicz and Kwiecien (2008), described queuing systems and queuing networks which are successfully used for performance analysis of different systems such as computer, communications, transportation networks and manufacturing. Classical Markovian systems were incorporated with exponential service times and a Poisson arrival process and queuing systems with individual service. The researchers studied oscillating queuing systems with Cox and Weibull Service time distribution as an example of Markovian systems. Some examples of queuing theory application was given. The application of closed BCMP networks in the health care area and performance evaluation of an information system was presented. An architecture called the Queuing Network-Model Human Processor was finally presented in their work.

Schwartz et al. (2006), derived stationary distributions of joint queuing length and inventory processes in explicit product form for various M/M/1-systems with inventory under continuous review and different inventory management policies and with lost sales. They considered demand to be Poisson and service times to calculate performance measures of the respective systems. In case of an infinite waiting room, the key result showed that the limiting distributions of the queue length processes are the same as in the classical M/M/1/inf system.

Tseytlin (2009), modeled the "Emergency Department(ED)- to -Internal Ward(IW) Process" as a queuing system with heterogeneous server pools, where the pools represented the ward and servers were beds. This system was analyzed under various queuing-architectures and routing policies, in search for fairness and good operational performance. This queuing system with a single centralized queue and several server pools, form an inverted-V model. A Randomized Most-Ideal routing policy(RMI) was introduced where each arriving customer joins one of the available pools, with a probability that equals the proportion of ideal servers in this pool out of the overall number of ideal servers in the system. It was realized that faster pools have lower average occupancy-rate than slower pools, but higher "flux" than slower pools. The closed-form analysis by an asymptotic analysis in the Quality and Efficiency Driven(QED) regime, where efficiency is carefully balanced against service quality was supplemented. It was emphasized that that a disadvantage of RMI is its being randomized. In order to achieve additional practical insight, by accommodating some analytically intractable features and testing various routing policies, we model the ED-to-IW process by a computer simulation.

Javanshir et al. (2011), studied a specific type of non-standard queuing systems. They explained that in such a system, delay in serving and exiting as a result of specific layout of the servers is inevitable. The purpose of their work was to present a hybrid model for non-standard queuing systems with layout constraints by using queuing theory concepts. Their model was aimed at computing major parameters like mean waiting time in queue for these systems and being able to evaluate similar non-standard queuing systems. A filling station was given as an example of non-standard queuing systems which can be analyzed by their proposed model.They concluded that, managers can evaluate and analyze their system by using their model and also achieve a better cognition of their system in order to improve it in the future.

Barak and Fallahnezhad (2012), studied two models of planning queuing systems and it's effect on the cost of each system by using two fuzzy queuing models of M/M/1 and M/E2/1. This was done regarding the fact that getting a suitable combination of the human resources and service station is one of the impatient issues in the most service and manufacturing environments. Two different fuzzy queuing models based on the cost of each model and fuzzy ranking methods were used to select optimal model due to resulted complexity. The paper resulted in a new approach for comparing different queuing models in the fuzzy environment (regarding the obtained data from the real conditions) that it can be more effective than deterministic queuing models. Also, a sensitivity analysis was carried out to help the decision maker in selecting the optimal model.

2.5 Works Related to Characteristics of Queuing Systems

According to Kumar and Sharma (2014), customer impatience has a very negative impact on a queuing system under investigation. From a business point of view, firms lose their potential customers due to customer impatience, which affects their business as a whole. They emphasized that, if the firm employ certain customer retention strategies, then there are chances that a certain fraction of impatient customers can be retain in the queuing system. A study of finite waiting space markovian single server queuing model with discouraged arrivals, reneging and retention of reneged customers was conducted. The study state solution of the model was derived iteratively. The measure of effectiveness of the queuing model was also obtained.

Wang et al. (2010), surveyed queuing systems with impatient customers in accordance with various dimensions. Firstly, impatient behaviors such as balking and reneging was introduced with its various rules proposed in literature. Also, analytic solutions, numerical solutions and simulation modeling of queues with impatient customers were investigated. They finally proposed the optimization of both from the perspective of customers and providers and some research tendencies in the field are included.

Bath and Terwiesch (2012), studied patient abandonment from a hospital emergency department queue. They found that patients are influence by what they see round them in the waiting room. Such as , waiting room census, arrivals and departures. Patients are also sensitive to being "overtaken" in the queue. Finally, patients also appear to make inferences about the severity of the patients around them and respond differently to people more sick and lee sick moving through the system. The researcher concluded that in order to reduce patient abandonment, hospitals should actively manage what patients see around and what information they have regarding the wait.

Kwok (1999), investigated the waiting time or delay of a broad class of mobile communication system-oriented queuing models. Both exact analytic and approximation algorithm techniques were employed in order to obtain the exact and approximate waiting distribution of the model. The researcher proposed different queuing models in order to describe different characteristics of mobile systems. Different server control mechanism was implemented in order to optimize channel(or server) utilization.

according to Jouini (2012),In a warehouse where items are stacked upwards, the unit on top is taken to fulfill an order, hence, customers (in this case the units) are served under LCFS. In computer systems, it is also quite common that the server works on jobs that are on the top of the stack. A further application is for broadband communications systems using asynchronous transfer mode. In a multi-access communications system, the LCFS discipline is preferred to FCFS when designing the splitting algorithm with tree structure.

Caulkins (2007), raised the question of whether some degree of randomization in queue discipline might be welfare enhancing in certain queues for which the cost of waiting is a concave function of waiting time, so that increased variability in waiting times may be good not bad for aggregate customer welfare. Such concavity may occur if the cost of waiting asymptotically approach some maximum(e.g. for patients seeking organ transplants who will not live be-young a certain threshold time) or if the customer incurs a fixed cost if there is any wait at all (e.g. for knowledge workers seeking a service or piece of information that is required to proceed with their current task,so any delay forces them to incur "set up charge" associated with switching tasks)

In medical procedures, patients face substantial risk of complication or death when treatment (for example organ transplantation) is delayed. When a queue is formed in such a situation, it is more appropriate to serve the patients according to the urgency of their requirements. When the condition of a patient deteriorates to a certain level, the treatment may become no longer required Jouini (2012).

Xuanming and Zenios (2004), developed and analyzed a queuing model to examine the role of patient choice on the high rate of organ refusals in the kidney transplant waiting system. The model used was an M/M/1 queue with homogeneous patients and exponential reneging. Patients joined the waiting system and organ transplants were reflected by the service process. Under an assumption of perfect and complete information, it was demonstrated that the queuing discipline was a potent instrument that can be used to maximize social welfare. They stated that FCFS amplifies patients desire to refuse offers of marginal quality and generates excessive organ wastage but LCFS contains the inefficiencies engendered by patient choice and achieves optimal organ utilization. Using data from the U.S. transportation system, the researchers demonstrated that the welfare improvements possible from a better control of patient choice are equivalent to 25per increase in the supply of organs.

Jouini (2012), considered a single class multi-server queuing systems working under the LCFS discipline of service. Customers waited a random length of time service to begin. A LCFS M/M/S+m queue was considered and focused to derive new results for virtual waiting time and the sojourn time in the queue.

Platz and Osterdal (2012), considered a game in which agents choose their arrival time to a service facility where they can line up only after a given point in time. They showed that in terms of equilibrium expected utility, the FIFI queue has inferior welfare properties in their model. They showed that the FIFO queuing discipline is the worst in terms of utility and welfare among all work-conserving stochastic queue discipline, while the LCFS discipline comes off best.

Sahu and Sahu (2014) focused on the bank lines system. They proposed an M/M/1:FCFS/inf/inf for queuing management system in a bank with a single channel. The model illustrated in the bank for customers on a level with service is the single-channel queuing model with poisson arrival and exponential service time (M/M/1). Data was obtained from a bank in the city. Four week average customer arrival was taken as the input data for both the queuing model and the service rate was obtained by the average service rate for customers they have given.

machining of a certain steel item may consist of cutting, turning, knurling, drilling, grinding and packaging operations. Each of which is performed by a single facility in a series (Sharma, 2006).

2.6 Performance Measures of Queuing System

According to William (2003), waiting line models consists of mathematical formulas and relations used to determine the operating characteristics of these lines. Among these features are;

- The probability that there is no item in the system.

- The average of the items in the waiting line.
- The average of the existent items in the system.
- The average time an item spends in the waiting line.
- The average time an item spends in the system.
- The probability that an item has to wait for the system.

In Kamba et al. (2012),approximations to various performance measures in queuing systems have received considerable attention because these measures have wide applicability. They proposed two methods to approximate the queuing characteristics of a GI/M/1 system. The first method was parametric in nature, using only the first three moments of the arrival distribution. Their second method treads the known path of approximating the arrival distribution, by a mixture of two exponential distributions and by matching the first three moments. Numerical examples and optimal analysis of performance measures of GI/M/1 queues were provided to illustrate the efficacy of the methods and are compared with benchmark approximations.

Gkougkouli et al. (2014), presented a new fuzzy approach to queuing modeling to eliminate the disadvantages of point estimation and relevant paradoxes when calculating the efficiency of a queue, in case of unreliable queue data available. Both poisson arrival rate and exponential service time were considered by using fuzzy estimators constructed from statistical data. The introduction of fuzzy estimators in performance measures of M/M/S queuing systems was presented to address the central estimation issue under uncertainty.

2.7 Queue Management Systems

Ramasamy and Chua (2012),the main objective of any queue management system is to achieve a better quality service to customers. In its most basic form, a queue management system will issue a queue ticket to a customer and later announce the ticket number when service is available, eliminating the need to stand in line while waiting. A couple of interviews and questionnaires were conducted to bank customers to find out their satisfaction level about the current system and acceptance of the proposed system. In this way, the queue management system helped to provide comfort as well as fairness to customers, by allowing them to maintain their position in the queue while they are seated comfortably or engaged in constructive activity. In other words, the customers still have to wait for long hours for their turn. A new way of queue management system called Queue Management System with SMS notification was introduced.

Gosha (2007), Suggested that a Queue Management System such as QueueAdmin will improve the satisfaction of a barber's shop customers as well as their barbers. The tool used in their study, QueueAdmin, is a database driven, online application to manage the different waiting list of a barbershop. In other to provide better functionality and to maximize use of all the information collected. They classified QueueAdmin under three interface, namely, the administrative interface, the employee interface and the customer interface. A multi-modal using touch screen technology as well as wireless web interface for use with cell phones and Personal Digital Assistants(PDAs) was made available instead of limiting these interfaces with standard keyboard input.

Quinn (HETS) ,Reviewed queue management strategies in context of urban traffic congestion. Although attention was focused mainly on signal control measures, both static and dynamic, non-signal technique were also mentioned. It was concluded that the formulation and use of each strategy will depend on whether the objective is to postpone, to handle or to recover from highly saturated conditions. suggestion was made that, in the context of a signal controlled urban corridor, a combination of reverse signal progression and metering would be required to form the most appropriate queue management strategy.

Silva et al. (2014), described and examined 'chronos', used Queue Management System(QMS) at a pharmacy service Out-patient Consulting waiting Rooms(OCR) and to present the results after 2 years of use.

They used a Retrospective cohort study. Cohort A and B consultations was made before and after QMS implementation. The main outcome measures used in the study were general data, activity record, patient consultation, average waiting time and appointment compliance. At the QMS, patients arriving at the OCR, for which they have an appointment, confirm their arrival by placing their health card in a reader in OCR, which prints out a ticket with the room number and time of consultation, arrival time and correlative number. The pharmacist checks the patients using the computer screen in the consulting room and clicks on call to notify the patients, who hear acoustic signal and see their number on a screen after attending to the patient.

They concluded that, the QMS eliminated manual system for recording work done, provide information about openings and closing times. It also eliminates FIFO queues and provides real time information on the patients in the waiting room. Finally, it improves arrival flows, reduces unscheduled patients checking in and reduces waiting times.

2.8 Summary

This chapter reviewed some related works on applications of queuing theory and a brief history behind queuing theory. Some literatures on queuing systems and its' characteristics was also reviewed. Queue management systems and the performance measures of queuing systems was finally discussed.

CHAPTER 3

METHODOLOGY

3.1 Introduction

This chapter gives a brief description of queuing systems and it's characteristics. Probability distributions that are commonly used in analysis of queuing systems are discussed. The distributions that are discussed are Poisson, exponential, uniform, gamma, Erlang, hyper exponential distributions. The Poisson process is also highlighted. $M/M/1$ and $M/M/N$ queuing models are elaborated. A brief description of $M/G/1$, $M/D/1$, $M/E_k/1$, $M/M/1/K$ queuing models are also discussed. A numerical example is given for the various queuing models that are discussed in this chapter.

3.2 Queuing Systems

A queuing system can be described as any system where by customers arrive for some services and has to wait for services for sometime and later leaves the system after being served.

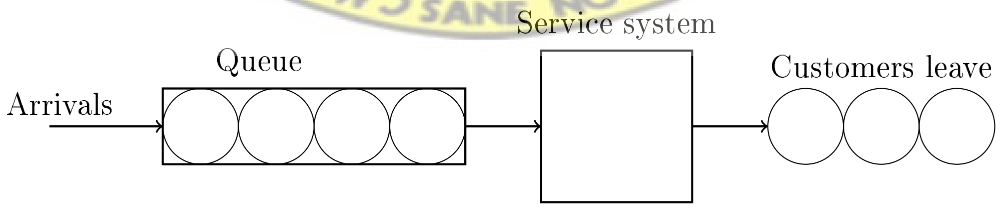

Figure 3.1: Queuing system

Examples of queuing systems are enormous, some common examples are, queuing situation at supermarket, highway toll booth, banks, machine maintenance etc. In the telecommunications, callers sometimes has to wait for switching equipments

Table 3.1: Examples of input and out put processes

Setting	Input processes	Output processes
Doctor's office	Patients	Treatment by doctors and nurses
Computer systems	Program to be run	Computer precesses jobs
Banks	Customers	Transactions handled by tellers
Machine Maintenance	Broken machines	Maintenance team fix machines

to forward their calls. In general, a queuing is made up of arrival inputs, queuing discipline and a service entity.

3.3 Characteristics of Queuing Systems

A queuing system involves customers who enter a system, wait in line(i.e queue), served and leaves the system within a time interval. Customers are served by a server when any of the servers are idle according to a rule given by a queuing discipline.

The characteristics of queuing systems can be classified under the three main parts of waiting lines systems. These are the arrival or input to the system, queue discipline and the service facility. The arrival to the system is characterized by the population size, behavior of customers and statistical distribution. The characteristics under queue discipline include whether it is limited or unlimited in length and the discipline of the customers. The design and statistical distribution of service times characterizes the service facility.

Queuing models are generally based on the above stated characteristics. Theses characteristics can be presented by Kendall's notation. That is in the form A/B/C/N, where

- A - inter-arrival distribution
- B - service time distribution
- C - number of parallel servers
- N - number of customers allowed in the system

3.3.1 Arrival characteristics

These are the size of the arrival population, behavior of arrivals and pattern of arrivals (i.e statistical distribution)

The size of the arrival population can be classified as limited or unlimited population. In a limited population arrival, there is a restriction on the number of customers who can enter the system for services. For example, in a factory with twenty(20) machines and a maintenance team where there are machine breakdowns. The maintenance team that provides the services has a limit of twenty(20) machines that they can serve whenever there is a machine breakdown. An unlimited population size arrival has infinite number of customers who can enter the system for services at any point in time (i.e open for all customers). For instance, cars arriving at a tollbooth, shoppers arriving at a supermarket, students arriving at an accountants office to pay fees. Most queuing models assume infinite arrival population.

The pattern at a system is determine by a probability distribution. Customers arrive at a service facility according to some known schedules or else they arrive randomly. When customer arrivals are independent of one another then arrival are considered randomly. In many applications, customers arrive according to a Poisson distribution (i.e exponential arrival time). Customers may arrive one after the other or in batches. An example of batch arrivals is a group of people arriving to board an elevator.

It is often assumed in queuing models that an arrival customer is a patient customer and will wait in a queue until services are provided and do not switch between lines. Unfortunately, customers are sometimes inpatient and may leave the queue before they are being served, this situation is referred to as reneging. For example, in call centers, customers will hang up when they have to wait for too long before an operator is available and they possibly try again after a while. The situation whereby customers refuse to join the waiting line because it is too long to suit their needs is also referred to as balking. In the event whereby there are multiple queues, customers may decided to move from one queue to the other if he or she thinks the other queue moves faster, this is termed as jockeying.

3.3.2 Queue Discipline

Queue discipline is the rule or priority rule used for determining order in which customers are served in a waiting line when the server completes the service of a current customer. Some of the commonest queue discipline used in queuing modeling are First-Come-First-Served(FCFS) , Last-Come-First-Served(LCFS) and priority.

FCFS discipline is the most widely used discipline in the application of queuing theory. Customers are served on first come first served basis. For example, considering a queue at a cinema ticket booth, the customers who comes first are served first one after the other.

In LCFS discipline, customers who comes in last are served first. That is, customers are being served in a reverse order in which they entered the queue.

Under priority discipline, services are provided on priority bases. It is usually used together with the other discipline. For instance, a queue at a hospital by patients usually considers FCFS discipline, but patients involved in emergency cases are given the priority to receive services without joining the queue.

3.3.3 Service capacity

The service capacity is characterized by the the design of the service facility and the service time distributions.

The design of the system indicates the number of servers and queues that a system facility has. The main types of system design are Single server with single queue, Multiple, parallel servers with single queue, Multiple, parallel facility with multiple queue and service facilities in series.

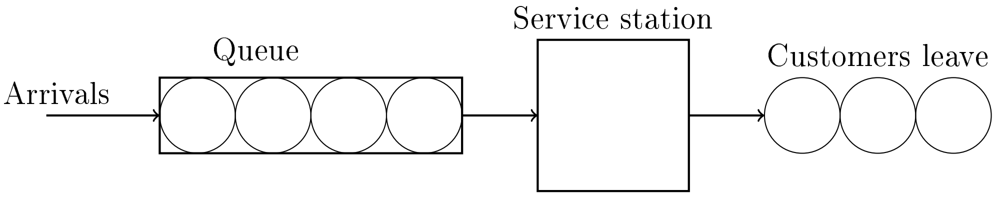

Figure 3.2: Single server, with single queue

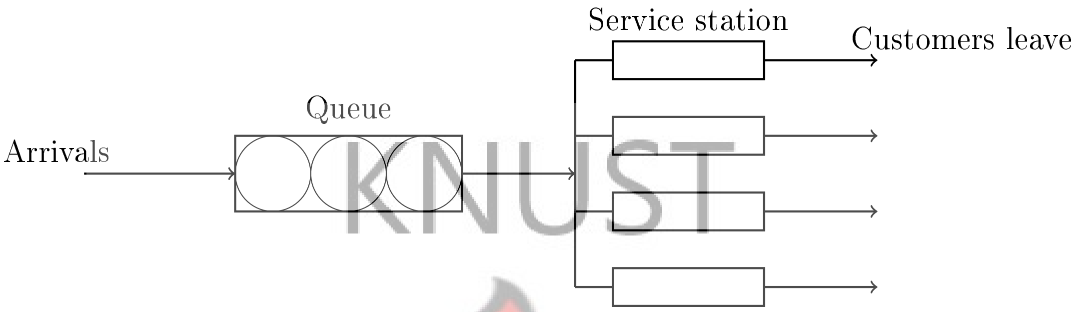

Figure 3.3: Multiple, parallel facility with single queue

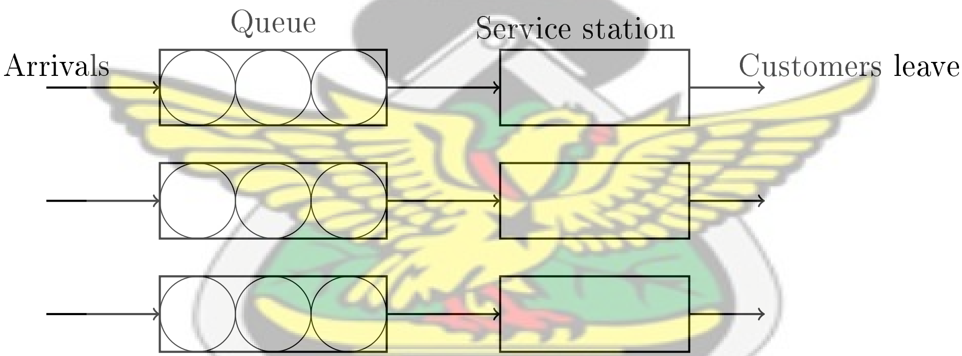

Figure 3.4: Multiple, parallel facilities with multiple queue

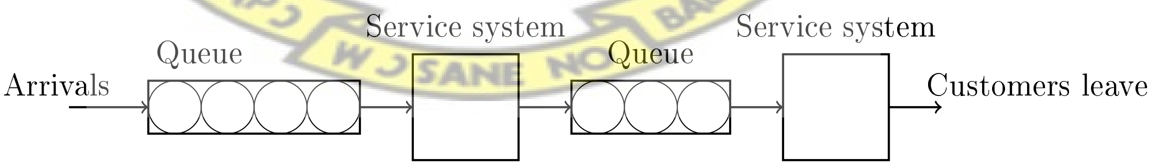

Figure 3.5: Facilities in series

The service time distributions are similar to that of arrival time distribution. Service times can be random or constant. For example, in a machined-performed service operation such as automatic car wash, service times are constant.

3.4 Probability Distributions

Some commonly used probability that are found useful for describing the distribution of random variable in queuing theory are Poisson, exponential, uniform, gamma, Erlang, hyper exponential distributions. All of these distributions stated are continuous probability distributions except the Poisson distribution.

3.4.1 Poisson distribution

This distribution is among the discrete probability distribution for counts of events that occur randomly in a given interval or space. It is the most applicable in queuing theory. Generally, if X is the number of events in a given interval and λ is the mean number of events per interval, then the probability of observing x events in a given interval or space is given by;

$$P(X = x) = \frac{\lambda^x \exp^{-\lambda}}{x!} \qquad \text{x= 0,1,2,3,...}$$

This can also be express as ; $X \sim P_0(\lambda)$. λ is the key parameter in fitting a Poisson distribution. Some properties of Poisson distribution are;

i. Occurrence of events are independently and randomly distributed in a time or space.

ii. The probability that two or more events occur simultaneously is zero.

iii. No matter how small t is, there is a positive probability that an event will occur in the interval $[s, s + t]$.

3.4.2 Exponential distribution

If a continuous variable X , has an exponential distribution, then it's probability density function is given by

$$f_x(x/\lambda) = \begin{cases} \lambda e^{-\lambda x}, & \text{for } x > 0; \\ 0, & \text{for } x \leq 0; \end{cases} \quad \text{where } \lambda > 0 \text{ is the rate of the distribution}$$

This is usually used to model the time until something happens in a process. The mean of an exponential distribution is given by $E[X] = \frac{1}{\lambda}$ with variance $var[X] = \frac{1}{\lambda^2}$.

The memoryless property of an exponential distribution

Considering that at time O, an alarm clock which will ring after a time X that is exponentially distributed with rate λ is started. Let X be the life time of the clock. Then for any $t > o$, we have,

$$\rho(X > t) = \int_t^{\infty} \lambda e^{-\lambda x} dx$$

$$\rho(X > t) = \frac{\lambda - e^{-\lambda x}}{\lambda}|_a^b$$

$$\rho(X > t) = e^{-\lambda t}$$

If we wait for sometime and return at time s to discover that the alarm has not yet gone off. implying, the event $X > s$ has been observed. If we let Y represent the remaining lifetime of the clock given that $X > s$, then,

$$\rho(Y > t|X > s) = \rho(X > s + t|X > s)$$

$$\rho(Y > t|X > s) = \frac{\rho(X > s + t)}{\rho(X > s)}$$

$$\rho(Y > t|X > s) = \frac{e^{-\lambda(s+t)}}{e^{-\lambda s}} = e^{-\lambda t}$$

This implies that the distribution of Y does not depend on s. (i.e Y after time s has the same distribution as the original lifetime X). This property of the exponential distribution is called the memoryless property. This property is exclusive to exponential distribution.

3.4.3 Uniform Distribution

The uniform distribution has random variable X restricted to a finite interval $[a, b]$ and has a constant density over the interval. The density function $f(x)$ is ,

$$f(x) = \begin{cases} \frac{1}{b-a}, & \text{for } a \leq x \leq b; \\ 0, & otherwise; \end{cases}$$

And its distribution function is given by

$$F(x) = \rho(X \leq x) = \begin{cases} 0, & x < a; \\ \frac{(x-a)}{(b-a)}, & a \leq x < b; \\ 1, & x \geq b; \end{cases}$$

Its' mean expectation for X is defined as,

$$E(X) = \int_{-\infty}^{\infty} xf(x)dx = \frac{b+a}{2}$$

Also, using the formula for variance, we have ,

$$v(X) = E(X^2) - [E(x)]^2$$

$$v(x) = \int_a^b x^2 \frac{1}{b-a} dx - \left(\frac{b+a}{2}\right)^2 = \frac{(b-a)^2}{12}$$

3.4.4 Gamma distribution

If the random variable Y is gamma distributed with parameters α and β, then the likelihood of Y is

$$\rho(Y) = \frac{(\beta)^\alpha}{\Gamma(\alpha)} Y^{\alpha-1} e^{-\beta Y}$$

where the gamma function $\Gamma(x)$ is defined as $\Gamma(x) = \int_o^\alpha t^{x-1} e^{-t} dt$, $Y, \alpha > 0$ and

$\beta > 0$. The parameter α is called the shape parameter and β is called the inverse scale parameter. The mean of a gamma-distributed variable is given by $\frac{\alpha}{\beta}$ with a variance of $\frac{\alpha}{\beta^2}$. If $\alpha > 1$, then there is a mode of $\frac{(\alpha-1)}{\beta}$. If $\alpha = 1$, then the gamma function reduces to an exponential density function.

3.4.5 Erlang Distribution

The random variable X that has the Erlang distribution with positive shape parameter n and positive scale parameter α is expressed as $X \sim Erlang(n, \alpha)$. An Erlang random variable X with rate α and n stages has probability density function,

$$f(x) = \frac{x^{n-1}e^{\frac{-x}{\alpha}}}{\alpha^n(n-1)!} \qquad \text{x} > 0$$

Erlang distribution is a special case of the gamma distribution when the shape parameter is an integer. It can be used to model the time to complete n operations in series where each operation requires an exponential period of time to complete. It was developed by A.K. Erlang to examine the number of telephone calls which might be made at the same time to the operators of a telephone switching station.

3.4.6 Hyper-exponential Distribution

A random variable X is hyper exponentially distributed if X is with probability $\rho_i, i = 1, ..., K$ an exponential random variable X_i with mean $\frac{1}{\mu_i}$. Its' density function is given by

$$f(t) = \sum_{i=1}^{K} \rho_i \mu_i e^{-\mu_i t} \qquad \text{t} > 0$$

with mean expectation of,

$$E(x) = \sum_{i=1}^{K} \frac{\rho_i}{\mu_i}$$

3.5 Poisson Process

According to MAJAKWARA (2009),Poisson process is an extremely useful process for modeling purposes in many practical applications. For example, to model arrival processes for queuing models or demand processes for inventory systems. It is empirically found that in many circumstances, the arising stochastic processes can be well approximated by a Poisson process.

A Poisson process on the interval $[0, \infty]$ counts the number of times some primitive event has occurred during the time $[0, t]$. Considering an arrival process $[N(t), t \geq 0]$, where $N(t)$ denotes the total number of arrivals up to time t, with $N(0) = 0$, then, the following assumptions are made about the 'process' $N(t)$.

i. The distribution of $N(t+h)-N(t)$ is the same for each $h > 0$ (i.e independent of t). The distribution of $N(t)$ is the same for all intervals of length t, no matter where the interval begins.(i.e stationary increments)

ii. $N(0) = 0, N(t)$ is integer valued, right continuous and nondecreasing in t, with probability 1

iii. ρ(more than one arrival between t and $t+h$) $= o(h)$, that is, $\rho(N(h) \geq 2) = o(h)$.

3.6 M/M/1 Queuing model

The M/M/1 queuing model is used to model single server queuing systems that has inter arrival time and service times following exponential distribution.The inter arrival times has mean $\frac{1}{\lambda}$ and service times has mean $\frac{1}{\mu}$ where λ and μ are arrival and service rate respectively . Arrivals of customers follows a Poisson process. This model require that $\frac{\lambda}{\mu} < 1$. This model assumes the FCFS queue discipline and infinite population.

3.6.1 Model formulation

The exponential distribution enables us to describe the number of customers in the system at time t. We assume that the behavior of the system does not have any memory(i.e the memoryless property). The interval between events is $h = t_2 - t_1$ where h is small.

Let

$$\lambda = arrival\ rate\ and\ \mu = service\ rate$$

$$\rho_n(t) = the\ probability\ that\ there\ are\ n\ customers\ at\ time(t)$$

$$\lambda h = the\ probability\ of\ one\ arrival\ at\ a\ given\ time\ period\ (h)$$

$$\mu h = the\ probability\ of\ one\ service\ at\ a\ given\ time\ period(h)$$

$$\rho_n(t{+}h) = the\ probability\ that\ there\ are\ (n)\ customers\ in\ the\ system\ at\ time(t{+}h)$$

$$\rho_{n-1}(t) = the\ probability\ that\ there\ are\ (n-1)\ customers\ at\ time(t)$$

$$\rho_{n+1}(t) = the\ probability\ that\ there\ are\ (n+1)\ customers\ at\ time(t)$$

Assuming that arrival distribution is Poisson and service time is exponential. Then, based on the memoryless property we can write that,

$$\rho_n(t+h) = [\rho_{n-1}(t) \times \lambda h(1-\mu h)] + [\rho_{n+1}(t) \times \mu h(1-\lambda h)] + [\rho_n(t) \times (1-\mu h)(1-\lambda h)] \tag{3.1}$$

$$\rho_n(t+h) = \lambda h \rho_{n-1}(t) + (1-(\lambda+\mu)h)\rho_n(t) + \mu h \rho_{n+1}(t) + 0(h) \tag{3.2}$$

$$\rho'_n(t) = \rho_{n-1}\lambda + \rho_{n+1}(t)\mu - \rho_n(t)(\lambda+\mu) \qquad \text{n=1,2,...} \tag{3.3}$$

At steady state, where $t \to \infty$, then $\rho'_n(t) \to 0$ and $\rho_n(t) \to \rho_n$. The equilibrium probability ρ_n gives the equation

$$0 = -\lambda\rho_0 + \mu\rho_1 \tag{3.4}$$

$$0 = \lambda\rho_{n-1} - (\lambda+\mu)\rho_n + \mu\rho_{n+1} \qquad \text{n=1,2,...} \tag{3.5}$$

It follows that, the probability ρ_n also satisfy

$$\sum_{n=0}^{\infty}\rho_n = 1 \tag{3.6}$$

Equation 3.6 is called normalization equation. From equation 3.7 , solving in terms of ρ_n gives, $\rho_1 = \rho\rho_0$ where the utilization factor, $\rho = \frac{\lambda}{\mu}$. When $n = 1$, $\rho_2 = \rho^2\rho_0$

By recursively expressing all probabilities in terms of ρ_0, that is,

$$\rho_1 = \rho\rho_0$$

$$\rho_2 = \rho\rho_1 = \rho^2\rho_0$$

$$\rho_3 = \rho\rho_2 = \rho^3\rho_0$$

Generally,

$$\rho_n = \rho^n\rho_0 \qquad \text{n=0,1,2,...} \tag{3.7}$$

Substituting equation 3.7 into equation 3.6 gives,

$$\sum_{n=0}^{\infty}\rho^n\rho_0 = 1 \tag{3.8}$$

$$\rho_0\left[\frac{1}{1-\rho}\right] = 1 \tag{3.9}$$

$$\rho_0 = 1 - p \tag{3.10}$$

This implies that, $\rho_n = \rho^n(1-\rho)$, $n \geq 1$, this holds only if $\rho = \frac{\lambda}{\mu} < 1$, otherwise there will be infinite sum. We can now derive expressions for performance measures of a system from the equilibrium properties above as follows

If L_s is the expected number of people in the system, then

$$
\begin{aligned}
L_s &= \sum_{n=0}^{\infty} n\rho_n \\
&= \sum_{n=0}^{\infty} n\rho^n(1-\rho) \\
&= (1-\rho)\rho \sum_{n=0}^{\infty} (\rho^n)' \\
&= (1-\rho)\rho\left(\frac{1}{(1-\rho)}\right)' \\
&= \frac{\rho}{1-\rho}
\end{aligned}
\tag{3.11}
$$

Little's Equation

Now, Using Little's equation, $L_s = \lambda W_s$ and $L_q = \lambda W_q$ where L_q is the average number of people in the queue, W_s is the average time spent by people in the system and W_q is the average time spent by people in the queue , the following equations can be derived,

1 The probability that a person is waiting to be served

$$P_w = \frac{\lambda}{\mu}$$

2 The probability that there is no person in the system

$$\rho_0 = 1 - \rho = 1 - \frac{\lambda}{\mu}$$

3 The probability that there are n customers in the system

$$\rho_n = \left(\frac{\lambda}{\mu}\right)\rho_0$$

4 the average number of people in the queue

$$L_q = \frac{\lambda^2}{\mu(\mu - \lambda)}$$

5 the average number of people in the system

$$L_s = L_q + \frac{\lambda}{\mu}$$

6 the average time spent by people in the queue

$$W_q = \frac{L_q}{\lambda}$$

7 the average time spent by people in the system

$$W_s = W_q + \frac{1}{\mu}$$

3.6.2 Numerical example

Vehicles arrive at a toll booth at an average rate of 350 vehicle per hour. Average waiting time at the toll booth is $12s$ per vehicle. If both arrivals and departures assumes exponential distribution, Find the performance measurements of the system.

Solution:

Mean arrival rate, $\lambda = 270V/hr$ and mean service rate $\mu = \frac{3600}{12}V/hr = 300V/hr$.

1 The probability that a vehicle is waiting to be served(i.e traffic intensity)

$$P_w = \frac{270}{300} = 0.9$$

2 The probability that there is no vehicle in the system

$$\rho_0 = 1 - \rho = 1 - 0.9 = 0.1$$

3 the average number of vehicles in the queue

$$L_q = \frac{270^2}{300(300-270)} = 8.1 \ vehicles$$

4 the average number of vehicles in the system

$$L_s = 8.1 + 0.9 = 9 \ vehicles$$

5 the average time spent by vehicle in the queue

$$W_q = \frac{8.1}{270} = 0.03 \ hr$$

6 the average time spent by vehicle in the system

$$W_s = 0.03 + \frac{1}{300} = 0.0633 \ hr$$

3.7 M/M/N Queuing Model

This model is used to model queuing systems that has a single queue with more that one server(i.e N servers). Arrival of customers follows Poisson distribution with an exponentially distributed service times. The model assumes infinite population and FCFS queuing discipline.μ is the average service rate for N identical service counters in parallel and $\frac{\lambda}{c\mu} < 1$.

If λ_n and μ_n are arrival rate and service rate of n number of customers respectively, then

$$\lambda_n = \lambda$$

$$\mu_n = n\mu \qquad n < N$$

$$\mu_n = N\mu \qquad n \geq N$$

From M/M/1 model formulation ,$\rho_n = \rho^n \rho_0$ implies that,$\rho_n = \frac{\lambda^n}{\mu^n}\rho_0$. This can be rewritten as

$$\begin{aligned}
\rho_n &= \frac{\lambda^n}{\mu 2\mu 3\mu ... n\mu}\rho_0 \\
&= \left(\frac{\lambda}{\mu}\right)^n \frac{1}{n!}\rho_0 \qquad n < N \\
&= \frac{\lambda}{N!\mu^n N^{n-N}}\rho_0 \qquad n \geq N
\end{aligned}$$

From the normalization equation $\sum_{n=0}^{\infty} \rho_n = 1$ implies that,

$$\sum_{n=0}^{N-1} \frac{\rho^n}{n!}\rho_0 + \sum_{n=N}^{\infty} \frac{\rho^n}{N!N^{n-N}}\rho_0 = 1$$

$$\rho_0 \left[\sum_{n=0}^{N-1} \frac{\rho^n}{n!} + \sum_{n=N}^{\infty} \frac{\rho^N \rho^{n-N}}{N!N^{n-N}}\right] = 1$$

$$\rho_0 \left[\sum_{n=0}^{N-1} \frac{\rho^n}{n!} + \frac{\rho^N}{N!}\left\{1 + \frac{\rho}{N} + \left(\frac{\rho}{N}\right)^2 + ... + \infty\right\}\right] = 1$$

$$\rho_0 \left[\sum_{n=0}^{N-1} \frac{\rho^n}{n!} + \frac{\rho^N}{N!}\frac{1}{1-\frac{\rho}{N}}\right] = 1 \qquad \text{since, } \frac{\lambda}{N\mu} < 1$$

Therefore,

$$\rho_0 = \left[\sum_{n=0}^{N-1} \frac{\rho^n}{n!} + \frac{\rho^N}{N!}\frac{1}{1-\frac{\rho}{N}}\right]^{-1}$$

Using the expressions for ρ_n and ρ_0 we can derive L_q and then use the Littles equations to obtain L_q,W_s and W_q as follows

1. The probability that there is no person in the system

$$\rho_0 = \left[\sum_{n=0}^{N-1} \frac{\rho^n}{n!} + \frac{\rho^N}{N!} \frac{1}{1 - \frac{\rho}{N}} \right]^{-1}$$

2. the average number of people in the queue

$$L_q = \sum_{n=N}^{\infty} (n - N)\rho_n$$
$$= \frac{\rho^{N+1}\rho_0}{(N-1)!(N-\rho)^2}$$

3. the average number of people in the system

$$L_s = \rho + L_q$$

4. the average time spent by people in the queue

$$W_q = \frac{L_q}{\lambda}$$

5. the average time spent by people in the system

$$W_s = W_q + \frac{1}{\lambda}$$

3.7.1 Numerical Example

A bank has two tellers. Customers arrive and joins a queue at a rate of 0.1 customers/min. Services are delivered at the rate of 0.125 customers/min. If both arrivals and departures assumes exponential distribution, find the performance measurements of the system.

Solution:

1. The probability that there is no person in the system,

$$\rho_0 = \left[\sum_{n=0}^{1} \frac{(0.8)^n}{n!} + \frac{(0.8)^2}{2!}\frac{1}{0.6}\right]^{-1}$$
$$= \left[\frac{(0.8)^0}{0!} + \frac{(0.8)^1}{1!} + \frac{(0.8)^2}{2!}\frac{1}{0.6}\right]^{-1}$$
$$= 0.429\ (approx.43per.ofthetime)$$

2. The average number of people in the queue,

$$L_q = \frac{(0.8)^3 0.429}{(1!)(2-0.8)^2} = 0.152$$

3. The average number of people in the system,

$$L_s = 0.8 + 0.152 = 0.952$$

4. Average time spent in queue,

$$W_q = \frac{0.152}{\frac{1}{10}} = 1.52\ min$$

5. Average time spent in system,

$$W_s = 1.52 + 8 = 9.52\ min$$

3.8 M/G/1 Queuing Model

In this model, arrival times are Poisson distributed(i.e inter arrival time distribution has negative exponential) and service time can have any distribution with known mean (μ) and standard deviation (σ). It has a single server. It has infinite population and assumes FCFS queuing discipline. It is used when $\mu > \lambda$ where λ is the arrival rate. The mean, $\mu = \frac{1}{E_s}$, where $E[service] = E_s$ is known. $Var[s] = \sigma^2 = \frac{1}{\mu^2}$. The probability per time unit for a transition from the state

$(N = n)$ to the state $(N = n - 1)$, that is,for departure of a customer, depends also on the time the customer in service has already spent in the server.

At steady state, where λ, E_s and σ^2 are known parameters, we use Pollaczek and Kinchin formula to find L_q and then use Littles equation to find L_s, W_q and W_s

Performance measurements:

1. $\rho = \frac{\lambda}{\mu}$ and $\rho_0 = 1 - \rho$

2. The average number of people in the queue,

$$L_q = \frac{\lambda^2\mu^2 + (\lambda E_s)^2}{2(1 - \lambda E_s)}$$

3. The average number of people in the system,

$$L_s = L_q + \lambda E_s$$

4. Average time spent in queue,

$$W_q = \frac{L_q}{\lambda}$$

5. Average time spent in system,

$$W_s = W_q + E_s$$

3.8.1 Numerical Example

An average of 15 cars per hour arrive according to a Poisson process at a work shop with single service facility for service. Assume the service time is uniformly distributed between $[2, 4]$ minute. Find the performance measurement at steady state.

Solution

$\lambda = 15\ cars/hr = 0.25\ car/min$, $E_s = \frac{4+2}{2} = 3\,\text{min}$ and $var(s) = \frac{(4-2)^2}{12} = 0.3333$

1.

$$L_q = \frac{(0.0625)(0.3333) + (0.25(3))^2}{2(1 - 0.25(3))} = 1.167$$

2.

$$L_s = 1.167 + 0.333(3) = 2.1669$$

3.

$$W_q = \frac{1.167}{0.3333} = 3.5614 \; min$$

4.

$$W_s = 3.5014 + 3 = 6.5014 \; min$$

3.9 M/D/1 Queuing Model

This model has the same assumptions and characteristics as the M/M/1 and M/G/1 queuing models except that, service times of the system is constant.(i.e not random).It is used in situations whereby service times are deterministic. The results for an $M/D/1$ model can be obtained using the $M/G/1$ model by setting the standard deviation of the service time to 0.(i.e $\sigma = 0$) At steady state the performance measurements are given by,

1.

$$L_s = \frac{\lambda}{\mu} + \frac{1}{2}\frac{(\lambda)^2}{\mu(\mu - \lambda)}$$

2.

$$L_q = \frac{1}{2}\frac{(\lambda)^2}{\mu(\mu - \lambda)}$$

3.

$$W_s = \frac{1}{\mu} + \frac{1}{2}\frac{\lambda}{\mu(\mu - \lambda)}$$

4.

$$W_q = \frac{1}{2}\frac{\lambda}{\mu(\mu - \lambda)}$$

3.9.1 Numerical Example

Arrivals at an automated car wash are Poisson distributed at a rate of 30 per hour. The time to complete a service is a constant 90 second. Determine the performance measurements of the system.

Solution:

$\lambda = 0.5$ per/min , $E(s) = \frac{1}{\mu} = 1.5$ min implies $\mu = 0.6667$ per/min.

1. The machine utilization; $\rho = \frac{\lambda}{\mu} = \frac{0.5}{0.6667} = 0.75$

$$L_q = \frac{3}{4} + \frac{1}{2}\frac{(0.75)^2}{1-0.75} = \frac{9}{8} = 1.125 \ cars$$

2.

$$W_q = \frac{9}{8}/\frac{1}{2} = 2.25 \ min$$

3.

$$W_s = 2.25 + 1.5 = 3.75 \ min$$

4.

$$L_s = 1.125 + 0.75 = 1.875 \ cars$$

3.10 M/Ek/1 Queuing Model

Arrivals follows a Poisson process , while service time follows an Erlang(K) probability distribution in this model. The system has a single server. The queue capacity of the system is infinite with FCFS queuing discipline. The k indicate the number of phases.

Let $Y_1, +Y_2+, ..., +Y_k$ be independent and identically distributed exponential random variables with parameter $k\mu$. Then the random variable

$$Y_1 + Y_2+, ..., +Y_k$$

has an Erlang distribution with parameters (μ, k), with mean $= \frac{1}{\mu}$ and variance $= \frac{1}{k\mu^2}$

Applying the POLLACZEK-KHINTCHINE equation, at steady state the performance measurements are given by,

1. $$L_q = \frac{1+k}{2k}\frac{(\lambda)^2}{\mu(\mu-\lambda)}$$

2. $$L_s = \frac{\lambda}{\mu}\frac{1+k}{2k}\frac{(\lambda)^2}{\mu(\mu-\lambda)}$$

3. $$W_q = \frac{1+k}{2k}\frac{\lambda}{\mu(\mu-\lambda)}$$

4. $$W_s = \frac{1}{\mu} + \frac{1+k}{2k}\frac{\lambda}{\mu(\mu-\lambda)}$$

3.10.1 Numerical Example

The registration of a student at a university requires three steps to be completed. The time taken to perform each step follows an exponential distribution with mean with a service time of 15 minutes and each one independent of the other. Students arrive at the registration center according to a Poisson input process with mean rate of one per student hour. Determine the average number of delayed students, L_q for this system.

Solution:

The service time distribution will be Erlang of order $k = 3$. $\lambda = \frac{1}{60}$ per minute and $\mu = \frac{1}{45}$ per minute.

$$L_q = \frac{1+3}{2*3}\frac{(\frac{1}{60})^2}{(\frac{1}{45}-\frac{1}{60})} = \frac{2}{3}\frac{135}{60} = 1.5 \; patients$$

3.11 M/M/1/K Queuing Model

This model has inter arrival and exponential service times. It has a single server with identical service time distributions. The system capacity is limited in population with FCFS queuing discipline. Its steady state probability ρ_k, is given by

$$\rho_k = a^k \rho_0$$
$$\rho_0 = \frac{1-a}{1-a^{k+1}} \qquad \text{where } a = \frac{\lambda}{\mu}$$

The mean number of customers in the system in the queue L_q, is given by

$$L_q = \frac{a}{1-a} - \frac{(k+1)a^{k+1}}{1-a^{k+1}}$$

The other performance measures can be found by using relations between L_q, L_s, W_q and W_s.

3.11.1 Numerical Example

In a small convenience store, there is room for only 6 customers. The owner serves all of these customers. On average, it takes a customer 4 minute to pay for purchase. Customers arrive at an average of 1 for 5 minutes. If a customer finds the shop full, he/she will leave immediately. What fraction of time will the owner be in the shop on his own. What is the mean number of customers in the store?

Solution: $\lambda = 0.2\ permin$, $\mu = 0.25\ permin$, and $k = 6$

1.
$$\rho_0 = \frac{1-0.8}{1-0.8^{6+1}} = 0.2531$$

2.

$$L_q = \frac{0.8}{1-0.8} - \frac{(6+1)0.8^{6+1}}{1-0.8^{6+1}} = 18576$$

All the queuing models that has been discussed so far has analytical solutions. There are some queuing models that are so complex to be solved analytically, such as $G/G/N$ queuing model. Such models are analyzed by simulations.

CHAPTER 4

COLLECTION AND ANALYSIS OF DATA

4.1 Collection of data

A primary data is used in this thesis. Observations were made at the Electricity Company of Ghana(ECG) pay point centres. The data was collected at both Prepaid Customer Pay point Centre(PCPC) and Credit Customer Pay point Centre(CCPC) of Electricity Company of Ghana(ECG) at Dichemso-Branch. The data was collected within the hour of $10:00$am to $11:00$am from Monday to Friday of $23rd$ to $27th$ February,2015.

Arrival and service times were carefully recorded during each session of the data collection. A customer was considered to have arrived when he/she joins the queue at the pay point center. The waiting time in queue ended immediately the customer gets to the counter for service. The service time was recorded from the time that the customer gets to the counter for service and the time that the customer departs. Customers were served on a first come, first served basis.

Figure 4.1 shows the data collected on Wednesday. The columns shows the number of customers, arrival times, service start times and departure times respectively.

No. of Customers	Arrival times	Service Start times	Departure times	Service times
1	10:01	10:04:30	10:05:20	00:50
2	10:03	10:05:20	10:06:10	00:50
3	10:04	10:06:10	10:07:10	01:00
4	10:05	10:07:10	10:08:40	01:30
5	10:06	10:08:40	10:09:30	00:50
6	10:08	10:09:30	10:10:40	01:10
7	10:09	10:10:40	10:12:40	02:00
8	10:11	10:13:10	10:14:00	00:50
9	10:12	10:14:00	10:15:00	01:00
10	10:14	10:15:00	10:15:50	00:50
11	10:15	10:17:20	10:19:00	01:40
12	10:18	10:19:00	10:20:10	01:10
13	10:19	10:20:10	10:21:00	00:50
14	10:19	10:21:30	10:23:10	01:40
15	10:20	10:23:10	10:24:00	00:50
16	10:22	10:25:00	10:26:00	01:00
17	10:25	10:26:00	10:27:00	01:00
18	10:27	10:27:00	10:27:50	00:50
19	10:28	10:28:00	10:29:30	01:30
20	10:29	10:30:30	10:31:40	01:10
21	10:30	10:31:40	10:33:40	02:00
22	10:31	10:33:40	10:34:40	01:00
23	10:31	10:34:40	10:35:30	00:50
24	10:32	10:35:30	10:37:10	01:40
25	10:33	10:37:10	10:38:10	01:00
26	10:35	10:38:10	10:39:10	01:00
27	10:36	10:39:10	10:41:10	02:00
28	10:38	10:41:10	10:42:40	01:30
29	10:39	10:42:40	10:43:50	01:10
30	10:40	10:43:50	10:45:00	01:10
31	10:43	10:45:00	10:46:30	01:30
32	10:46	10:46:30	10:48:10	01:40
33	10:48	10:48:10	10:49:00	00:50
34	10:49	10:49:00	10:50:00	01:00
35	10:50	10:50:00	10:52:00	02:00
36	10:52	10:52:00	10:53:30	01:30
37	10:53	10:53:30	10:54:40	01:10
38	10:56	10:56:00	10:57:40	01:40
39	10:56	10:57:40	10:58:30	00:50
40	10:57	10:58:30	10:59:40	01:10
41	10:58	10:59:40	11:00:30	00:50
42	10:59	11:00:30	11:02:00	01:30
Total				51:30

Figure 4.1: Collected data on Wednesday at PCPC

<u>Results of number of customers that arrived in the week</u>

Table 4.1 shows the number of customers that arrived at the pay point centre during the data collection period. The first column shows the dates of the data collection while the second and third columns shows the number of customers that arrived at the pay points, during the data collection period. The last row of the table shows the total number of customers that arrived at both CCPC and PCPC.

Table 4.1: Number of customer arrivals

Number of customers arrived		
Days	CCPC	PCPC
Monday	48	49
Tuesday	46	37
Wednesday	38	42
Thursday	40	35
Friday	53	51
Total	225	214

Table 4.1 shows that 225 customers arrived at the CCPC and 214 customers arrived at the PCPC. The highest number of customers that arrived at the pay point was 53 and occurred on Friday, at the CCPC. The smallest number of customers that arrived at the centre was 35 and occurred on Thursday at the PCPC. The total number of customers that arrived at all the pay point centre during the data collection period was 439. 51.3% (approx.) of the customers arrived at the CCPC while 48.7% (approx.) of the customers arrived at the PCPC.

Arrival process on Wednesday

Figure 4.2 shows the arrival process at the CCPC on Wednesday. The number of customers are represented on the horizontal axis with its corresponding probability represented on the vertical axis.

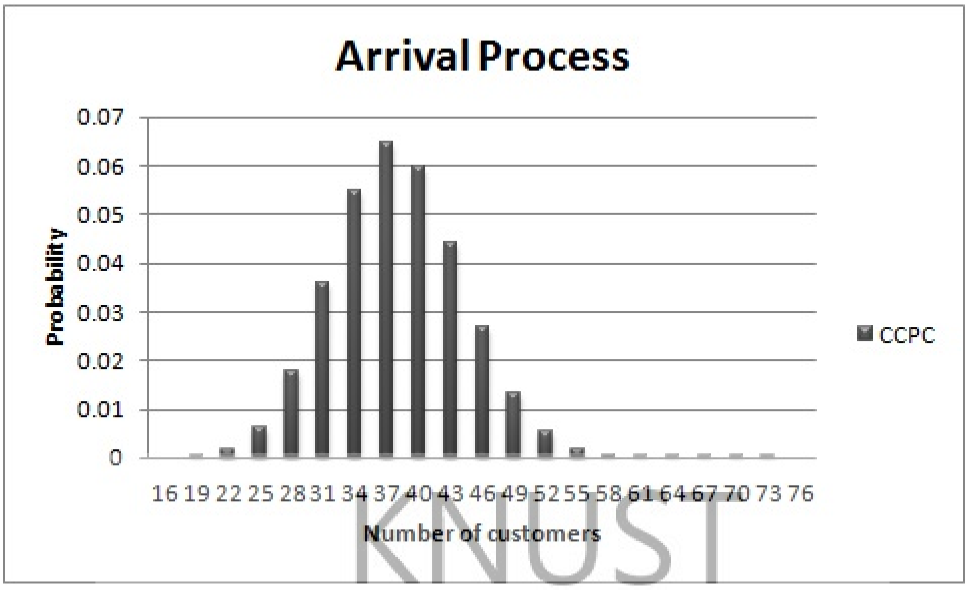

Figure 4.2: Arrival process at CCPC

Figure 4.2 shows that the mean arrival number of customers (λ) is 38.

Figure 4.3 shows the arrival process at the PCPC on Wednesday. The number of customers are represented on the horizontal axis with its corresponding probability represented on the vertical axis.

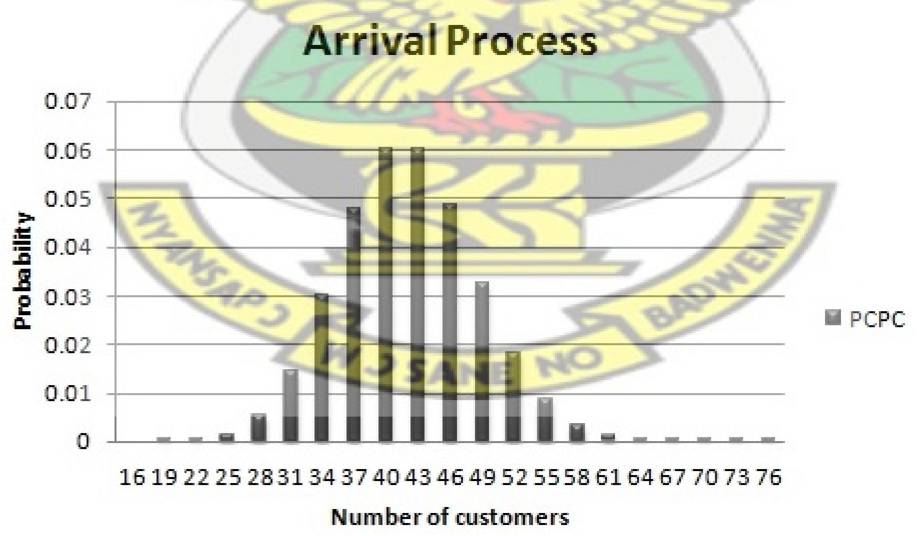

Figure 4.3: Arrival process at PCPC

Figure 4.3 shows that the mean arrival number of customers (λ) is 42.

4.2 Analysis of data

4.2.1 Arrival rates

The number of customers that arrived per unit of time is referred to as arrival rate. Table 4.2 shows the arrival count process in an interval of $10min$ at the PCPC on Wednesday. The counting starts from $10:00am$ and ends at $11:00am$. The first column shows the 10 minutes time intervals from $10:00am - 11:00am$.The second column shows the number of customers that arrived within the time interval. The last column shows the arrival rates.

Table 4.2: Arrival counts on Wednesday at PCPC

Time intervals	Number of customers	Arrival Rates(cust/min)
10:00-10:10<	7	0.7
10:10-10:20<	7	0.7
10:20-10:30<	6	0.6
10:30-10:40<	9	0.9
10:40-10:50<	5	0.5
10:50-10:59<	8	0.8
10:00-11:00<	42 customers	4.2

From table 4.2, the highest number of customers arrived within the time interval $10:30am - 10:40am$ with 9 customers. The lowest number of customers arrived within the time interval $10:40am - 10:50am$ with 5 customers.

Calculation of mean arrival rate

The mean arrival rate(λ) is given by,

$$\lambda = \frac{4.2\ cust/min}{6} = 0.7\ cust/min$$

Results of arrival rates in the week

Table 4.3 shows the arrival rates for the week starting from Monday to Friday.

The first column shows the dates while the second and the last columns shows the arrival rates at both CCPC and PCPC. The last row shows the average arrival rates for the entire week at both CCPC and PCPC.

Table 4.3: Arrival rates for the week

Arrival rates		
Dates	CCPC(cust/min)	PCPC(cust/min)
Monday	0.8000	0.8167
Tuesday	0.7667	0.6167
Wednesday	0.6333	0.7000
Thursday	0.6667	0.6250
Friday	0.8833	0.8361
Average	0.7500	0.7133

Table 4.3 shows that the highest arrival rate is 0.8833 $cust/min$ which occurred on Friday at the CCPC. The lowest of the arrival rate is 0.6167 $cust/min$ which occurred on Tuesday at the PCPC. Table 4.3 shows that the arrival rate at CCPC is higher than arrival rate at the PCPC for the entire week of the data collection.

4.2.2 Service Rates

The service rate is the number of customers that are served per unit of time. The service rate(μ) of the customers is given by,

$$\mu = \frac{1}{E(s)}$$

. Where $E(s)$ is the expectation of the service times. The $E(s)$ is also given by

$$E(s) = \frac{Total\ service\ time}{Total\ number\ of\ customers}$$

From the data of previous table 4.1 and choosing Wednesday,

$$Total\ service\ time = 51 \min 30sec$$

$$Total\ number\ of\ customers = 42\ customers$$

$$E(s) = \frac{51.5}{42} = 1.2262\ min/cust.$$

Hence the service rate is

$$\mu = \frac{1}{1.2262} = 0.8155\ cust/min$$

Results of service rates in the week

Table 4.4, shows the dates of the data collection in the first column and the service rates of both CCPC and PCPC are presented in the second and third columns respectively.

Table 4.4: Service rates for the week

Service rates for the week		
Days	CCPC(cust/min)	PCPC(cust/min)
Monday	0.8495	0.8522
Tuesday	0.8214	0.6549
Wednesday	0.7600	0.8155
Thursday	0.8163	0.6250
Friday	0.9815	0.8947
Average	0.8458	0.7685

The summary of service rates shows that the highest service rate during the data collection period is 0.9815 $cust/min$ and occurred at the CCPC on the Friday and the smallest of the service rate is 0.6250 $cust/min$ and occurred at the PCPC on Thursday.

4.2.3 Utilization Factors

The utilization factor(ρ) is the probability that the system is busy when the system is in equilibrium.

This is given by, $\rho = \frac{\lambda}{\mu}$. The Utilization factor on Wednesday at the prepaid customer pay point center is,

$$\rho = \frac{0.7000\ cust/min}{0.8155\ cust/min} = 0.8583$$

.

<u>Results of utilization factor in the week</u>

Table 4.5, shows the utilization factors in the week. The first column shows the days of the data collection. The second and third column shows the utilization factors at both CCPC and PCPC respectively.

Table 4.5: Utilization factors in the week

Utilization Factors		
Days	CCPC	PCPC
Monday	0.9417	0.9583
Tuesday	0.9333	0.9417
Wednesday	0.8333	0.8583
Thursday	0.8167	0.9250
Friday	0.9000	0.9500

Table 4.5 shows that the busiest day of the servers at both CCPC and PCPC was on Monday with 94.17% and 95.83% at both CCPC and PCPC respectively. The least of the utilization factors is 81.67% and occurred on the Thursday at the CCPC. On average, the servers at both pay points were less busy on the Wednesday with an average utilization factor of 84.58%.

4.2.4 Performance measurements

The performance measure is the specific representation of a pay point capacity, process or outcome deemed to be relevant to the assessment of performance. The Performance measurements are

1. The probability that there is no person in the system ρ_0.

2. The average number of people in the queue(L_q).

3. The average number of people in the system(L_s).

4. The average time spent by people in the queue(W_q).

5. The average time spent by people in the system(W_s).

Calculation of Performance Measurements

The performance measurements on Wednesday at the prepaid meter pay point centre are,

- The probability that there is no person in the system

$$\rho_n = \left(\frac{\lambda}{\mu}\right)\rho_0$$

$$\rho_0 = 1 - 0.8583 = 0.1417$$

- the average number of people in the queue

$$L_q = \frac{\lambda^2}{\mu(\mu - \lambda)}$$

$$L_q = \frac{0.7^2}{0.8155(0.8155 - 0.7)} = 5.2008$$

- the average number of people in the system

$$L_s = L_q + \frac{\lambda}{\mu}$$

$$L_s = 5.2008 + 0.8583 = 6.0591$$

- the average time spent by people in the queue

$$W_q = \frac{L_q}{\lambda}$$

$$W_q = \frac{5.2008}{0.7} = 0.1238$$

- the average time spent by people in the system

$$W_s = W_q + \frac{1}{\mu}$$

$$W_s = 0.1238 + \frac{1}{0.8155} = 0.1443$$

Results of Performance Measurements at CCPC in the week

Table 4.6 shows the performance measurements at the CCPC for the entire week. The first column shows the days that the data was collected. The second column shows the probability that the system is idle(P_0). The third and forth columns shows the average number of customers in queue(L_q) and system(L_s) respectively. The fifth and sixth columns shows the average waiting time in queue(W_q) and system(W_s) respectively.

Table 4.6: Performance measurements at the CCPC

	Performance measures				
Days	P_0	L_q	L_s	W_q	W_s
Monday	0.0583	15.2053	16.1469	19.0080	20.1840
Tuesday	0.0667	13.0667	14.0001	17.0460	18.2580
Wednesday	0.1667	4.1667	5.0000	6.5760	7.8960
Thursday	0.1833	3.6377	4.4543	5.4540	6.6840
Friday	0.1000	8.1000	9.0000	9.1680	10.1880

From table 4.6, The highest probability that the system is idle is 18.33% and occurred on Thursday. The least probability that the system is idle is 5.83% and occurred on Monday. The highest number of customers in queue and system are 15.2($approx$) customers and 16.1($approx$) customers respectively.The least number of customers in queue and system are 4.17($approx$) customers and 5.00($approx$) customers respectively. The longest waiting time in queue and system are 19min($approx$) and 20min($approx$) respectively and occurred on Monday. The shortest waiting time in queue and system are 6min($approx$) and 7min($approx$) and occurred on Thursday.

Results of Performance Measurements at PCPC in the week

Table 4.7 shows the performance measurements at the PCPC for the entire week. The first column shows the days that the data was collected. The second column shows the probability that the system is idle(P_0). The third and forth columns shows the average number of customers in queue(L_q) and system(L_s) respectively. The fifth and sixth columns shows the average waiting time in queue(W_q) and system(W_s) respectively.

Table 4.7: Results of performance measurements at the PCPC

	Performance measures				
Days	P_0	L_q	L_s	W_q	W_s
Monday	0.0417	22.0420	23.0000	26.9880	28.1640
Tuesday	0.0583	15.2053	16.1469	19.0080	20.1840
Wednesday	0.1417	5.2008	6.0591	7.4280	8.6580
Thursday	0.0750	11.4085	12.3335	19.5600	21.1440
Friday	0.0500	18.0501	19.0001	21.2340	22.3560

From table 4.7, The highest probability that the system is idle is 14.17% and occurred on Wednesday. The least probability that the system is ideal is 4.17% and occurred on Monday. The highest number of customers in queue and system are 22.04($approx$) customers and 23.00($approx$) customers respectively.The least number of customers in queue and system are 5.20($approx$) customers and 6.06($approx$) customers respectively. The longest waiting time in queue and system are 26.98min($approx$) and 28.16min($approx$) respectively and occurred on Monday. The shortest waiting time in queue and system are 7.43min($approx$) and 8.66min($approx$) and occurred on Wednesday.

4.2.5 Comparison of performance measurements of CCPC and PCPC for different number of servers using average of arrival and service rates

An average of the arrival and service rates at both credit and service meter pay point centers are computed by finding the sum of various rates and dividing by five. The average of arrival rate at the credit meter pay point center during the five days data collection period is given by,

$$\lambda_c = \frac{48 + 46 + 38 + 40 + 53}{5} = 45$$

Table 4.8, shows average of arrival and service rate for the entire data collected at both CCPC and PCPC.The first column shows the pay points. The second and third columns shows average of arrival rate and average of service rate respectively.

Table 4.8: Summary of Average of arrival and service rates

Pay point	Average of Arrival rate	Average of service rate
CCPC	0.7500	0.8458
PCPC	0.7133	0.7685

Table 4.8 shows that both arrival and service rates at the CCPC are higher than that of the PCPC.

<u>**Results of Performance Measurements for different number of servers**</u>

Using the average of arrival and service rates, the performance measurements of the system can be calculated for different number of servers as in the diagram below.

Table 4.9: Performance measurement

Perf. Measurements	m/m/1		m/m/2		m/m/3	
	CCPC	PCPC	CCPC	PCPC	CCPC	PCPC
ρ	0.8867	0.9282	0.4433	0.4641	0.2956	0.3094
L_q	6.9394	12.0023	0.2169	0.2548	0.0283	0.0339
L_s	7.8261	12.9305	1.1036	1.1830	0.9150	0.9621
W_q	0.1542	0.2804	0.0048	0.0060	0.0006	0.0008
W_s	0.1739	0.3021	0.0245	0.0276	0.0203	0.0225
P_0	0.1133	0.0718	0.3857	0.3660	0.4090	0.3918

Table 4.9 shows that an increase in the number of servers from one to two servers indicates that the servers will be 44.33% and 46.41% busy at both credit and prepaid pay point centers respectively.

At the CCPC, the number of customers waiting in queue decreased from 6.9394 customers to 0.2169 customers. The number of customers waiting in the system decreased from 7.8261 customers to 1.1036 customers. The average waiting time in the queue decreased from 9.2520min to 0.2880min and the average time in the system also decreased from 10.4340min to 1.4700min.

At the PCPP, the average number of customers waiting in queue decreased from 12.0023 customers to 0.2548 customers and the average number waiting in the system decreased from 12.9305 customers to 1.1830 customers. The average waiting time in the queue and the system decreased from 16.8240min to 0.3600min and 18.1260 to 1.6560 respectively.

Finally, Increasing the number of servers from two to three also shows a decrease in the performance measurements at all the pay points.

4.2.6 Discussions of results

The analysis shows that the number of customers that arrived at the CCPC was higher than the number of customers that arrived at the PCPC during the data collection period. The arrival rate of customers at the CCPC on Friday was the highest of the arrival rates with an arrival rate of 0.8833$cust/min$. The least of the arrival rate was estimated to be $0.6167 cust/min$ and occured at the PCPC on Tuesday. The highest service rate was also found to be $0.9815 cust/min$ and occurred on Friday at CCPC. The least service rate was estimated to be $0.6250 cust/min$ and occurred on Thursday at PCPC. The largest of the average number of customers waiting in the system was estimated to be 23 customers and occurred on Monday at the PCPC and this corresponded to a largest average waiting time(i.e in the system) of $28.1640 min$. The least of the average number of customers waiting in the system was estimated to be 3.6377 customers and occurred on Thursday at the CCPC and this corresponded to the least average waiting time(i.e in the system) of $6.6840 min$. The average of the arrival and service rate at the CCPC for the entire week was higher than that of the PCPC. Using the average of arrival and service rates, the performance measurements at both CCPC and PCPC decreased with the an increase in the number of servers.

CHAPTER 5

CONCLUSION AND RECOMMENDATIONS

5.0.7 Conclusion

This thesis reviews the application of queuing models. The $M/M/N$ queuing model is used to analyze the waiting lines operations at the Electricity Company of Ghana pay-point centre at Dichemso-Branch. The data that was used in the analysis was collected at both credit and prepaid customer pay point centres.

The study reveals that the number of customers that waited in queue and system at the Prepaid customer pay points(PCPC) was higher than the number of customers that waited in queue and system at the Credit Customer Pay point Centre(CCPC).

The number of customers that arrived at the CCPC was higher than that of PCPC with 51.3% of customers arriving at the CCPC against 48.7% of customers arriving at the PCPC. It was observed that customers spent more time for receiving services at the PCPC counter than the time that customers spent at the CCPC counters. This was mainly due to the slow computer network system at the PCPC.

Thought the average of arrival rate and service rate at the CCPC was higher than that of the PCPC, the PCPC was busier than the CCPC with utilization factor of 92.82% against 88.67%. Hence the performance measurements at the PCPC was higher than that of CCPC. This implies that there will be an incresing number of customers waiting in line if the number of customer arrivals at the PCPC increases to the number of customer arrivals at the CCPC.

Using the average of arrival and service rate, the outcome of the analysis shows a decrease in the performance measurements when the number of servers are increased from one server to two or three.

5.0.8 Recommendations

In order to improve the operations within the waiting line at the pay point centres based on the results of the analysis, the following recommendations are made,

1. The efficiency of the computer network system at both pay points centres must be enhanced to reduce the service time.

2. The number of servers at the PCPC must be increased from one to two during peak hours of the day and months to reduce the service time.

3. The managers of Electricity Company of Ghana must increase the number of pay points vendors in the Ashanti-East district to reduce the number of customers that come to pay their bills and purchase credits at the Dichemso-Branch, which is the Ashanti-East Regional office.

   Any or a combination of the recommendations above can be used to reduce waiting times at the pay point centre.

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APPENDIX A: COLLECTED DATA FOR THE STUDY

No. of Customers	Arrival times	Service Start times	Departure times	Service times
1	10:00	10:05:00	10:06:40	01:40
2	10:01	10:06:40	10:07:30	00:50
3	10:02	10:07:30	10:08:30	01:00
4	10:04	10:08:30	10:09:20	00:50
5	10:05	10:09:20	10:11:20	02:00
6	10:07	10:11:20	10:12:10	00:50
7	10:08	10:12:10	10:13:20	01:10
8	10:09	10:13:20	10:14:30	01:10
9	10:10	10:14:30	10:15:30	01:00
10	10:11	10:15:30	10:16:20	00:50
11	10:12	10:16:20	10:17:50	01:30
12	10:14	10:17:50	10:18:40	00:50
13	10:14	10:18:40	10:19:50	01:10
14	10:15	10:19:50	10:20:40	00:50
15	10:16	10:20:40	10:21:50	01:10
16	10:17	10:21:50	10:23:30	01:40
17	10:18	10:23:30	10:24:30	01:00
18	10:19	10:24:30	10:25:40	01:10
19	10:20	10:25:40	10:27:10	01:30
20	10:21	10:27:10	10:28:00	00:50
21	10:23	10:28:00	10:28:50	00:50
22	10:24	10:28:50	10:31:30	02:40
23	10:25	10:31:30	10:32:30	01:00
24	10:25	10:32:30	10:33:20	00:50
25	10:27	10:33:20	10:34:30	01:10
26	10:28	10:34:30	10:36:10	01:40
27	10:29	10:36:10	10:37:00	00:50
28	10:31	10:37:00	10:38:00	01:00
29	10:32	10:38:00	10:39:30	01:30
30	10:35	10:39:30	10:40:30	01:00
31	10:36	10:40:30	10:42:00	01:30
32	10:37	10:42:00	10:42:50	00:50
33	10:39	10:42:50	10:43:50	01:00
34	10:39	10:43:50	10:45:20	01:30
35	10:40	10:45:20	10:47:00	01:40
36	10:41	10:47:00	10:49:00	02:00
37	10:42	10:49:00	10:50:00	01:00
38	10:42	10:50:00	10:50:50	00:50
39	10:43	10:50:50	10:51:50	01:00
40	10:44	10:51:50	10:53:00	01:10
41	10:45	10:53:00	10:55:00	02:00
42	10:47	10:55:00	10:55:50	00:50
43	10:49	10:55:50	10:57:00	01:10
44	10:51	10:57:00	10:58:40	01:40
45	10:52	10:58:40	10:59:40	01:00
46	10:55	10:59:40	11:01:10	01:30
47	10:56	11:01:10	11:02:00	00:50
48	10:58	11:02:00	11:03:30	01:30
Total				56:30

Figure 5.1: Data collected on Monday at CCPC

No. of Customers	Arrival times	Service Start times	Departure times	Service times
1	10:00	10:05:00	10:05:20	00:50
2	10:01	10:05:50	10:06:10	00:50
3	10:03	10:06:40	10:07:10	01:30
4	10:04	10:08:10	10:08:40	01:00
5	10:05	10:09:10	10:10:20	01:10
6	10:06	10:10:20	10:11:10	00:50
7	10:08	10:11:10	10:12:00	00:50
8	10:09	10:12:00	10:12:30	00:30
9	10:10	10:12:30	10:13:30	01:00
10	10:10	10:13:30	10:15:30	02:00
11	10:12	10:15:30	10:17:00	01:30
12	10:13	10:17:00	10:17:50	00:50
13	10:16	10:17:50	10:19:30	01:40
14	10:18	10:19:30	10:21:10	01:40
15	10:19	10:21:10	10:22:10	01:00
16	10:21	10:21:10	10:22:20	01:10
17	10:24	10:24:00	10:25:00	01:00
18	10:25	10:25:00	10:26:30	01:30
19	10:25	10:26:30	10:28:10	01:40
20	10:26	10:28:10	10:29:00	00:50
21	10:27	10:29:00	10:30:10	01:10
22	10:28	10:30:10	10:31:00	00:50
23	10:29	10:31:00	10:33:30	02:30
24	10:30	10:33:30	10:34:30	01:00
25	10:31	10:34:30	10:35:20	00:50
26	10:33	10:35:20	10:36:10	00:50
27	10:34	10:36:10	10:37:20	01:10
28	10:35	10:37:20	10:39:00	01:40
29	10:36	10:39:00	10:40:00	01:00
30	10:37	10:40:00	10:42:00	02:00
31	10:38	10:42:00	10:42:50	00:50
32	10:39	10:42:50	10:43:50	01:00
33	10:39	10:43:50	10:45:00	01:10
34	10:40	10:45:00	10:46:30	01:30
35	10:42	10:46:30	10:48:00	01:30
36	10:43	10:48:00	10:48:50	00:50
37	10:44	10:48:50	10:49:50	01:00
38	10:45	10:49:50	10:51:50	02:00
39	10:46	10:51:50	10:52:40	00:50
40	10:47	10:52:40	10:54:20	01:40
41	10:48	10:54:20	10:55:10	00:50
42	10:49	10:55:10	10:56:20	01:10
43	10:50	10:56:20	10:57:20	01:00
44	10:52	10:57:20	10:58:50	01:30
45	10:53	10:58:50	11:00:20	01:30
46	10:54	11:00:20	11:01:10	00:50
47	10:56	11:01:10	11:02:10	01:00
48	10:57	11:02:10	11:03:00	00:50
49	10:59	11:03:00	11:04:10	01:10
Total				57:30

Figure 5.2: Data collected on Monday at PCPC

No. of Customers	Arrival times	Service Start times	Departure times	Service times
1	10:01	10:05:30	10:06:30	01:00
2	10:02	10:06:30	10:07:30	01:00
3	10:03	10:07:30	10:09:00	01:30
4	10:05	10:09:00	10:10:00	01:00
5	10:06	10:10:00	10:10:50	00:50
6	10:07	10:10:50	10:12:20	01:30
7	10:09	10:12:20	10:13:20	01:00
8	10:09	10:13:20	10:14:20	01:00
9	10:11	10:14:20	10:16:20	02:00
10	10:12	10:16:20	10:17:20	01:00
11	10:13	10:17:20	10:18:20	01:00
12	10:14	10:18:20	10:19:50	01:30
13	10:15	10:19:50	10:20:50	00:50
14	10:15	10:20:50	10:21:50	01:00
15	10:16	10:21:50	10:24:20	02:30
16	10:17	10:24:20	10:25:20	01:00
17	10:18	10:25:20	10:26:20	01:00
18	10:20	10:26:20	10:27:50	01:30
19	10:21	10:27:50	10:28:40	00:50
20	10:22	10:28:40	10:29:40	01:00
21	10:23	10:29:40	10:31:10	01:30
22	10:26	10:31:10	10:32:10	01:00
23	10:27	10:32:10	10:34:10	02:00
24	10:28	10:34:10	10:35:40	01:30
25	10:29	10:35:40	10:36:40	01:00
26	10:31	10:36:40	10:37:40	01:00
27	10:32	10:37:40	10:38:40	01:00
28	10:33	10:38:40	10:40:10	01:30
29	10:35	10:40:10	10:41:40	01:30
30	10:36	10:41:40	10:42:40	01:00
31	10:37	10:42:40	10:44:40	02:00
32	10:39	10:44:40	10:45:40	01:00
33	10:40	10:45:40	10:46:30	00:50
34	10:42	10:46:30	10:47:30	01:00
35	10:42	10:47:30	10:48:30	01:00
36	10:43	10:48:30	10:49:20	01:30
37	10:44	10:49:20	10:50:50	01:30
38	10:46	10:50:50	10:51:50	01:00
39	10:47	10:51:50	10:53:50	02:00
40	10:49	10:53:50	10:55:20	01:30
41	10:50	10:55:20	10:56:50	01:30
42	10:51	10:56:50	10:57:50	01:00
43	10:52	10:57:50	10:58:30	00:50
44	10:54	10:58:30	10:59:30	01:00
45	10:56	10:59:30	11:00:20	00:50
46	10:58	11:00:20	11:01:30	01:10
Total				56:00

Figure 5.3: Data collected on Tuesday at CCPC

No. of Customers	Arrival times	Service Start times	Departure times	Service times
1	10:01	10:06:30	10:08:00	01:30
2	10:03	10:08:00	10:09:30	01:30
3	10:04	10:09:30	10:10:30	01:00
4	10:06	10:10:30	10:11:30	01:00
5	10:07	10:11:30	10:12:30	01:00
6	10:08	10:12:30	10:13:30	01:00
7	10:10	10:13:30	10:15:00	01:30
8	10:11	10:15:00	10:17:00	02:00
9	10:13	10:17:00	10:18:00	01:00
10	10:15	10:18:00	10:19:00	01:00
11	10:17	10:19:00	10:20:30	01:30
12	10:18	10:20:30	10:22:00	01:30
13	10:20	10:22:00	10:24:30	02:30
14	10:21	10:24:30	10:25:30	02:00
15	10:23	10:25:30	10:26:30	01:00
16	10:25	10:26:30	10:28:00	01:30
17	10:26	10:28:00	10:29:00	01:00
18	10:27	10:29:00	10:31:00	02:00
19	10:29	10:31:00	10:32:30	01:30
20	10:30	10:32:30	10:33:30	01:00
21	10:32	10:33:30	10:34:30	01:00
22	10:34	10:34:30	10:37:40	03:00
23	10:35	10:37:30	10:38:30	01:00
24	10:37	10:38:30	10:39:30	01:00
25	10:38	10:39:30	10:41:00	01:30
26	10:40	10:41:00	10:43:00	02:00
27	10:41	10:43:00	10:45:30	02:30
28	10:43	10:45:30	10:47:00	01:30
29	10:45	10:47:00	10:49:00	02:00
30	10:46	10:49:00	10:50:00	01:00
31	10:48	10:50:00	10:51:30	01:30
32	10:49	10:51:30	10:54:30	03:00
33	10:51	10:54:30	10:57:00	02:30
34	10:52	10:57:00	10:58:00	01:00
35	10:54	10:58:00	10:59:30	01:30
36	10:57	10:59:30	11:01:00	01:30
37	10:58	11:01:00	11:02:00	01:00
Total				56:30

Figure 5.4: Data collected on Tuesday at PCPC

No. of Customers	Arrival times	Service Start times	Departure times	Service times
1	10:02	10:07:00	10:08:30	01:30
2	10:03	10:08:30	10:09:30	01:00
3	10:05	10:09:30	10:10:30	01:00
4	10:06	10:10:30	10:12:30	02:00
5	10:09	10:12:30	10:13:20	00:50
6	10:10	10:13:20	10:14:30	01:10
7	10:10	10:14:30	10:16:10	01:40
8	10:12	10:16:10	10:17:00	00:50
9	10:15	10:17:00	10:18:40	01:40
10	10:16	10:18:40	10:19:30	00:50
11	10:18	10:19:30	10:21:00	01:30
12	10:21	10:21:00	10:24:00	03:00
13	10:22	10:24:00	10:25:00	01:00
14	10:23	10:25:00	10:25:50	00:50
15	10:24	10:25:50	10:27:00	01:10
16	10:26	10:27:00	10:28:40	01:40
17	10:28	10:28:40	10:29:30	00:50
18	10:30	10:30:00	10:31:10	01:10
19	10:30	10:31:10	10:32:00	00:50
20	10:31	10:32:00	10:33:00	01:00
21	10:32	10:33:00	10:33:50	00:50
22	10:33	10:33:50	10:35:20	01:30
23	10:35	10:35:20	10:36:20	01:00
24	10:36	10:36:20	10:38:20	02:00
25	10:37	10:38:20	10:39:20	01:00
26	10:37	10:39:20	10:40:30	01:10
27	10:39	10:40:30	10:41:40	01:10
28	10:42	10:42:00	10:43:30	01:30
29	10:43	10:43:30	10:45:10	01:40
30	10:45	10:45:10	10:47:10	02:00
31	10:47	10:47:10	10:50:10	03:00
32	10:49	10:50:10	10:51:40	01:30
33	10:50	10:51:40	10:52:30	00:50
34	10:51	10:52:30	10:53:30	01:00
35	10:53	10:53:30	10:55:10	01:40
36	10:54	10:55:10	10:56:40	01:30
37	10:57	10:57:00	10:58:00	01:00
38	10:58	10:58:00	10:59:10	01:00
Total				50:00

Figure 5.5: Data collected on Wednesday at CCPC

No. of Customers	Arrival times	Service Start times	Departure times	Service times
1	10:01	10:04:30	10:05:20	00:50
2	10:03	10:05:20	10:06:10	00:50
3	10:04	10:06:10	10:07:10	01:00
4	10:05	10:07:10	10:08:40	01:30
5	10:06	10:08:40	10:09:30	00:50
6	10:08	10:09:30	10:10:40	01:10
7	10:09	10:10:40	10:12:40	02:00
8	10:11	10:13:10	10:14:00	00:50
9	10:12	10:14:00	10:15:00	01:00
10	10:14	10:15:00	10:15:50	00:50
11	10:15	10:17:20	10:19:00	01:40
12	10:18	10:19:00	10:20:10	01:10
13	10:19	10:20:10	10:21:00	00:50
14	10:19	10:21:30	10:23:10	01:40
15	10:20	10:23:10	10:24:00	00:50
16	10:22	10:25:00	10:26:00	01:00
17	10:25	10:26:00	10:27:00	01:00
18	10:27	10:27:00	10:27:50	00:50
19	10:28	10:28:00	10:29:30	01:30
20	10:29	10:30:30	10:31:40	01:10
21	10:30	10:31:40	10:33:40	02:00
22	10:31	10:33:40	10:34:40	01:00
23	10:31	10:34:40	10:35:30	00:50
24	10:32	10:35:30	10:37:10	01:40
25	10:33	10:37:10	10:38:10	01:00
26	10:35	10:38:10	10:39:10	01:00
27	10:36	10:39:10	10:41:10	02:00
28	10:38	10:41:10	10:42:40	01:30
29	10:39	10:42:40	10:43:50	01:10
30	10:40	10:43:50	10:45:00	01:10
31	10:43	10:45:00	10:46:30	01:30
32	10:46	10:46:30	10:48:10	01:40
33	10:48	10:48:10	10:49:00	00:50
34	10:49	10:49:00	10:50:00	01:00
35	10:50	10:50:00	10:52:00	02:00
36	10:52	10:52:00	10:53:30	01:30
37	10:53	10:53:30	10:54:40	01:10
38	10:56	10:56:00	10:57:40	01:40
39	10:56	10:57:40	10:58:30	00:50
40	10:57	10:58:30	10:59:40	01:10
41	10:58	10:59:40	11:00:30	00:50
42	10:59	11:00:30	11:02:00	01:30
Total				51:30

Figure 5.6: Collected data on Wednesday at PCPC

No. of Customers	Arrival times	Service Start times	Departure times	Service times
1	10:01	10:05:30	10:06:30	01:00
2	10:02	10:06:30	10:07:30	01:00
3	10:04	10:07:30	10:09:00	01:30
4	10:04	10:09:00	10:10:00	01:00
5	10:05	10:10:00	10:11:00	01:00
6	10:07	10:11:00	10:12:00	01:00
7	10:09	10:12:00	10:13:00	01:00
8	10:10	10:13:00	10:14:00	01:00
9	10:12	10:14:00	10:15:30	01:30
10	10:13	10:15:30	10:16:30	01:00
11	10:14	10:16:30	10:18:00	01:30
12	10:16	10:18:00	10:19:00	01:00
13	10:17	10:19:00	10:20:00	01:00
14	10:19	10:20:00	10:21:30	01:30
15	10:21	10:21:30	10:22:30	01:00
16	10:23	10:23:00	10:25:00	02:00
17	10:24	10:25:00	10:26:00	01:00
18	10:27	10:27:00	10:28:30	01:30
19	10:29	10:29:00	10:30:00	01:00
20	10:29	10:30:00	10:32:00	02:00
21	10:30	10:32:00	10:33:30	01:30
22	10:31	10:33:30	10:34:30	01:00
23	10:33	10:34:30	10:35:30	01:00
24	10:34	10:35:30	10:37:00	01:30
25	10:35	10:37:00	10:38:00	01:00
26	10:37	10:38:00	10:39:00	01:00
27	10:38	10:39:00	10:40:30	01:30
28	10:41	10:41:00	10:42:00	01:00
29	10:42	10:42:00	10:44:00	02:00
30	10:43	10:44:00	10:45:30	01:30
31	10:44	10:45:30	10:46:30	01:00
32	10:45	10:46:30	10:47:30	01:00
33	10:46	10:47:30	10:49:00	01:30
34	10:48	10:49:00	10:50:00	01:00
35	10:52	10:52:00	10:53:30	01:30
36	10:53	10:53:30	10:54:30	01:00
37	10:54	10:54:30	10:55:30	01:00
38	10:55	10:55:30	10:56:30	01:40
39	10:57	10:57:00	10:59:00	00:50
40	10:58	10:59:00	10:59:30	00:30
Total				49:00

Figure 5.7: Data collected on Thursday at CCPC

No. of Customers	Arrival times	Service Start times	Departure times	Service times
1	10:02	10:05:30	10:05:20	01:00
2	10:04	10:06:30	10:08:30	02:00
3	10:05	10:08:30	10:09:30	01:00
4	10:07	10:09:30	10:11:00	01:30
5	10:09	10:11:00	10:12:00	01:00
6	10:10	10:12:00	10:14:30	02:30
7	10:12	10:14:30	10:16:00	01:30
8	10:15	10:16:00	10:17:30	01:30
9	10:16	10:17:30	10:18:30	01:00
10	10:18	10:18:30	10:20:30	02:00
11	10:19	10:20:30	10:21:30	01:00
12	10:21	10:21:30	10:22:30	01:00
13	10:22	10:22:30	10:24:00	01:30
14	10:24	10:24:00	10:25:30	01:30
15	10:25	10:25:30	10:28:30	03:00
16	10:26	10:28:30	10:30:30	02:00
17	10:28	10:30:30	10:31:30	01:00
18	10:30	10:31:30	10:32:30	01:00
19	10:31	10:32:30	10:36:30	04:00
20	10:33	10:36:30	10:39:00	02:30
21	10:35	10:39:00	10:42:00	03:00
22	10:38	10:42:00	10:44:30	02:30
23	10:40	10:44:30	10:46:00	01:30
24	10:41	10:46:00	10:47:00	01:00
25	10:44	10:47:00	10:48:00	01:00
26	10:46	10:48:00	10:49:00	01:00
27	10:47	10:49:00	10:50:30	01:30
28	10:48	10:50:30	10:52:30	02:00
29	10:50	10:52:30	10:55:00	02:30
30	10:51	10:55:00	10:56:00	01:00
31	10:53	10:56:00	10:57:00	01:00
32	10:54	10:57:00	10:58:30	01:30
33	10:57	10:58:30	10:59:30	01:00
34	10:58	10:59:30	11:00:30	01:00
35	10:59	11:00:30	11:01:00	00:30
Total				51:30

Figure 5.8: Data collected on Thursday at PCPC

No. of Customers	Arrival times	Service Start times	Departure times	Service times
1	10:00	10:02:40	10:03:20	00:40
2	10:01	10:03:20	10:04:20	01:00
3	10:01	10:04:20	10:05:20	01:00
4	10:02	10:05:20	10:06:00	00:40
5	10:03	10:06:00	10:06:40	00:40
6	10:04	10:06:40	10:07:20	00:40
7	10:05	10:07:20	10:08:50	01:30
8	10:05	10:08:50	10:09:30	00:40
9	10:06	10:09:30	10:10:10	00:40
10	10:07	10:10:10	10:11:10	01:00
11	10:08	10:11:10	10:12:10	01:00
12	10:10	10:12:10	10:12:50	00:40
13	10:11	10:12:50	10:14:20	01:30
14	10:11	10:14:20	10:15:00	00:40
15	10:13	10:15:00	10:15:40	00:40
16	10:13	10:15:40	10:16:40	01:00
17	10:14	10:16:40	10:17:20	00:40
18	10:16	10:17:20	10:18:20	01:00
19	10:17	10:18:20	10:19:00	00:40
20	10:18	10:19:00	10:20:00	01:00
21	10:20	10:20:00	10:20:40	00:40
22	10:20	10:20:40	10:22:10	01:30
23	10:22	10:22:10	10:22:50	00:40
24	10:23	10:23:00	10:24:00	01:00
25	10:25	10:25:00	10:26:30	01:30
26	10:25	10:26:30	10:27:10	00:40
27	10:26	10:27:10	10:28:00	00:50
28	10:28	10:28:00	10:29:00	01:00
29	10:30	10:30:00	10:32:00	02:00
30	10:30	10:32:00	10:33:30	01:30
31	10:31	10:33:30	10:34:10	00:40
32	10:32	10:34:10	10:35:00	00:50
33	10:33	10:35:00	10:35:40	00:40
34	10:34	10:35:40	10:36:20	00:40
35	10:35	10:36:20	10:37:50	01:30
36	10:36	10:37:50	10:38:30	00:40
37	10:38	10:38:30	10:39:30	01:00
38	10:40	10:40:00	10:41:30	01:30
39	10:41	10:41:30	10:42:10	00:40
40	10:42	10:42:10	10:42:50	00:40
41	10:43	10:43:00	10:44:00	01:00
42	10:44	10:45:00	10:46:00	01:00
43	10:45	10:46:00	10:46:40	00:40
44	10:46	10:46:40	10:47:40	01:00
45	10:47	10:47:40	10:48:40	01:00
46	10:48	10:48:40	10:49:40	01:00
47	10:50	10:50:00	10:52:00	02:00
48	10:51	10:52:00	10:53:30	01:30
49	10:53	10:53:30	10:55:30	02:00
50	10:54	10:55:30	10:56:10	00:50
51	10:55	10:56:10	10:57:10	01:00
52	10:57	10:57:10	10:58:10	01:00
53	10:58	10:58:10	11:00:10	02:00
Total				54:00

Figure 5.9: Data collected on Friday at CCPC