INVENTORY CONTROL MODELING FOR WATER LEVEL SCHEDULING: THE CASE STUDY OF AKOSOMBO DAM, GHANA

by

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DECLARATION

I hereby declare that this submission is my own work towards the MSc. and that, to the best of my knowledge, it contains no material previously published by another person nor material which has been accepted for the award of any other degree of the university, except where due acknowledgement has been made in the text.

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DEDICATION

I dedicate this thesis to my children.

ACKNOWLEDGEMENT

I wish to express my deepest and profound gratitude to Dr. F. T. Oduro for taking time off his heavy schedule to supervise this work.

Furthermore, I thank him for his positive and constructive criticisms which enabled me to complete this thesis.

Lastly, I am thankful to all those who in diverse ways helped to make this thesis a success.

ABSTRACT

The thesis is a construction of quantitative models to serve as some of control measures or policies for managers of Akosombo dam to determine optimal energy generation and water quantity release scheduling from the dam all times. The data used for the thesis was collected from the Transmission Systems Department, Volta River Authority, Ghana. The data were daily time series of Akosombo dam water levels from 1998-2007 and daily data of energy generated from Akosombo dam from 2000 to 2007. First order and second order autoregressive models were formulated using least-square fitting method purposely for forecasting future daily dam water levels even though series is random.

Furthermore, inventory analysis was performed on the data collected to produce optimal energy demand and optimal release quantity of water from the dam in a sustainable way and control options for the management of the dam. Finally, queue theoretic concept was used to model the occurrence of queue of water in the Akosombo dam. Probability of queue of water (traffic intensity) and probability of no queue (stockout probability) throughout a year at 68%and 95% service levels were determined.

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CHAPTER ONE

INTRODUCTION

1.1 BACKGROUND TO THE STUDY

The hydro-electric power supply of Akosombo dam falls below what is expected during seasons when there is scarcity of rainfall. The government of Ghana as well as the Volta River Authority is often criticized during such a period. The following are some of the concerns:

1. The inability to maintain the expected water level of the dam and failure to serve the consumers at the expected service levels during the dry season.
2. The inability to supplement the Akosombo dam's power supply from other sources when there is severe drought.

The insufficient hydro-electric power supply affects the growth of the economy in many areas including education, businesses, industrializations, domestic power consumptions and others.

1.1.1 The River Volta

The river Volta is very far from belonging only to Ghana. Indeed, its water springs from not less than six West African states and almost two-thirds of its 150,000 square mile (241,401.6 square kilometers) basin is outside Ghana. These countries are Burkina Faso, Togo, and Benin and to a lesser extent the Ivory Coast and Mali. But the 61,000 square miles (98,169.984 square kilometers) of the water body that lie within the boundaries of Ghana constitute the crucial part; it is there that the combined waters of the White, Black and Red Volta together with the Oti join forces to form the massive flow that with the construction of the Akosombo dam had in a matter of two years filled a lake the size of Lancashire and by the time that lake Volta had fully filled in, had doubled its area. The lake now covers an area of 3,275 square miles (5,251.266square kilometers). The main stream of the Volta which is about 1000 miles in length rises in the Kong mountains 25 miles (40.2336km) out of the Bourkina Faso town of Bobo–Dioulasso and after flowing first north–east and due south for some 320miles (514.99008km) as Black Volta, it continues due south down Ghana's western boundary for a further 200miles (321.8688km) before it passes through a narrow gorge at Bui where a second hydro-electric project is currently ongoing. From Bui, after a southwards curve, the Black Volta winds north-east and east again until it joins the White Volta (another Bourkina Faso offspring) and together they combine to flow southwards for the remaining 300 miles (482.8032km) to the sea. For its part, the White Volta starts life only a few miles across the hills that separate it from one of the watersheds of the Black Volta but combining with its sister river the Red Volta to form a movement around the capital city of Ouagadougou. It then flows across the Ghana border to join the Black Volta. One other major tributary is the Oti which though comprising only some 18percent of total catchment area of the Volta Basin contributes between 30 and 40 percent of the annual flow of water. Mason (1984)

1.1.2 The Akosombo Dam

The Akosombo dam is situated within the river Volta's only gorge at the point where the river cleaves the Akwapim-Togo range of hills. This is the first time that a river system of such a size has been artificially controlled so near its estuary and the consequent economic advantages are obvious. In spite of the variety of its sources, the lake Volta conforms to a remarkable regular time-table of rise and fall. The extent of these is always uncertain. Precise records of water levels on the lake have been kept. The river is mostly at its lowest in March each year and at its highest at the end of September or early October. Mason (1984)

Fig 1.1.2: The Akosombo Dam

Table 1.1.2: Characteristics of the Akosombo Dam

Construction began	1961
Opening Date	1965
Length	2,165.33 feet (660m)
Width (at base)	1,200.77 feet (366m)
Capacity	148*10^12 litres
Catchment area	8502 square kilometers
Maximum water level	278 feet
Minimum water level	240 feet

The development of the dam (see Table 1.4.1) was undertaken by the Ghana government and funded in part by the International Bank for Reconstruction and Development of the World Bank, the United States and the United Kingdom.

1.1.3 The Challenge

It is indeed a long cry from the day in 1966 when the Volta River Authority was obliged to 'boil' the waters of the Volta because prior to the opening of the Valco Smelter in 1967 and to supplying the mines, a single generator at Akosombo yielded more power than the nation's demand of 75MW. Five years later, when Akosombo was operating five of its six generators , the nation's consumption had more than doubled, Valco had stepped up its demand by one-third and Ghana was exporting (also for foreign currency) 25MW to the Communaute Electrique du Benin (CEB) in Togo and Dahomey (now Benin). In another five years Valco was taking its total of 400MW, CEB had doubled its demand to 50MW and Ghana's own consumption including the mines had risen to 260MW. Akosombo power had virtually reached its limit. It was at this point that the Volta River Authority (VRA) using its own financial resources as a base went out to raise the necessary loans with which to construct its 'mini-Akosombo' dam downstream at Kpong. In another five years, the VRA was again thought to be ahead of the game with enough surplus energy to enable the overworked Akosombo to enjoy some relief. If the time comes for the ongoing construction of Bui dam to fill, over a period of eighteen months, the corresponding flow of water entering Lake Volta would be reduced and the energy generation would fall. But on the completion of Bui, there would be surplus hydro-electric energy in Ghana. The Bui project on the Black Volta is planned as a five-year construction and reservoir filling project.

Recent studies into the potential of the River Oti which supplies Lake Volta with one-third of its water suggest that a dam at or near the town of Juale in the Northern Region of Ghana could be capable of generating a further 200MW. Bui was subsequently studied by Halcrow (1954), Hydro-Project of USSR (1964), Kaiser (1971) and by the Snowy Mountain Engineering Corporation (SMEG) of Australia in 1976. With minor variations, all concluded that Bui will be the second largest hydro-electric project in Ghana after Akosombo with a clay core rockfill dam of nearly the same height as Akosombo-426feet to Akosombo's 438 feet capable of generating half of Akosombo's 882MW which is about thrice the Kpong generating power.

Today, Akosombo dam generates over 1200MW due to the replacement of old dam turbines. If Ghana's plans for its macro-economy are able to include some selected industrial projects, for example a ferro-manganese plant using locally mined minerals for major agricultural investment and development, then some additional power generation will soon become necessary quite independent of the broader regional projects. The dry cycle which has enveloped West Africa since 1971 and has contributed so much to suffering in the Sahel has led to the inflow of water to the Lake Volta falling below average. Early in 1983, a policy of power cuts was introduced and the Valco Smelter as the major consumer was required to reduce its consumption in stages so that by June 1983, only a token quantity of power was being supplied to VALCO (which later came to a halt). In the extreme circumstances of continuing drought conditions, it became necessary in November 1983 for Mr. Louis Casely-Hayford, then CEO of VRA, to announce the introduction of scheduled national power cuts throughout the country. He believed that this strategy would minimize the risk of total collapse of the power system. Mason (1984)

1.1.4 Optimal Hydro-Electric Power Generation

The valuation and determination of optimal operating strategies of hydro-electric power generation facilities has long been one of the focuses of research interest. The output of hydro-electric facilities depends nonlinearly on the height of the water in the reservoir and the flow rate.

Hydro-electric generators trap potential energy by collecting water behind the dam. This water can then be released, turning turbines to generate electric power. The optimal operation of hydro-electric generation facilities depends on several factors:

- The unpredictable inflows of water that replenish the reservoir
- The power function that determines the amount of power a generator produces and is a function of the turbine head and the flow rate of the water
- The maximum flow rate that depends nonlinearly on the turbine
- The environmental regulations that often dictate maximum and minimum amount of water to be released.

1.2 STATEMENT OF THE PROBLEM

Due to the problems listed below there is a need for mathematical models to help control the water level of the Akosombo dam in a sustainable way:

1) Inflows of rain water that replenish the reservoir and energy demand are random making the control of the dam water level difficult.

2) Increasing hydro-electric energy demand domestically, commercially and industrially over-burdens the dam.

3) Frequent and severe occurrences of drought necessitate energy load-shedding

4) The environmental conditions often dictate the amount of water to be released from the dam to generate energy.

1.3 OBJECTIVES

The main objectives of this study are to construct:

1) Time series model(s) for predicting Akosombo dam water level.

2) Inventory control and queue models for achieving optimal energy generation scheduling or optimal water quantity release scheduling.

1.4 METHODOLOGY

The data for the study was collected from Transmission System Department, Volta River Authority, Ghana. The data comprises:

1) Daily time series of Akosombo dam water levels.
2) Daily time series of energy demand data from Akosombo dam

The literature review of the study was obtained from internet and books. The mathematical tools used were time series analysis, inventory control analysis and queue theoretic models. The trajectories of the water levels and the corresponding energy demand data were discussed. We constructed time series stationary AR(1) and AR(2) models for predicting future water levels of Akosombo dam. Furthermore, EOQ and Lot-size inventory models were used to determine how much energy should be generated and when to generate it. We also determine how much water should be released and when to release using the inventory models. We also determined feasible service level and stockout probability using queue theoretic and inventory models.

The computer software used for the study was MATLAB and EXEL.

1.5 JUSTIFICATION

The social and economic significances of the study shown below could improve Akosombo dam delivery to support Ghana economic growth.

1) The study should provide time series stationary models for forecasting water levels. This could help the managers of the dam to understand visual patterns of the water

levels to enable them predict the corresponding energy demand that should be generated.

2) The study should provide energy generation and water release scheduling mechanism to improve upon the performance of the dam.
3) The study should provide feasible service levels and respective stockout probabilities which may control the water levels from going below what is expected.

1.6 LIMITATION

Secondary data was used. Eight years daily monthly time series data points were analyzed. Regression was used which had its own error margins. Averages of data points were mostly used for the simulations. MATLAB output contained a lot of truncated values therefore must have given rise to truncated errors.

1.7 ORGANISATION OF STUDY

Chapter one deals with the background, statement of problem, objectives, methodology, justification, limitations and structure of study.

Chapter two reviews times series (AR(1) and AR(2)) stationary models, inventory models, and queue models.

Chapter three which deals with data analysis and modeling discusses water levels sample data and energy demand sample data of the Akosombo dam using the models in chapter two to construct time series models for forecasting water levels, inventory models to determine how much energy demand should be generated, when to generate it, or how much water should be released and when to release it. We also determine stockout probability of water in the dam using traffic intensity concept of a queue.

Chapter four presents the summary of findings the study has achieved. We also state conclusions and make recommendations to stakeholders and managers of the dam. Appendices and references then follow the chapter four.

CHAPTER TWO

TIME SERIES, INVENTORY CONTROL AND QUEUE THEORETIC MODELS

2.1 INTRODUCTION

Two problems of importance to organizations holding stock of items are (1) Deciding when to place an order for replenishment of the stock and (2) Deciding how large an order to place. That is two types of uncertainties must be considered:

(a) The quantity of items that will be demanded during a given period

(b) The time that will elapse between placing an order and the actual delivery of the item.

The major problem of inventory is how to establish optimal stock levels and this is a difficult problem because of the uncertainty of the supply and demand for the commodity.

Using mathematical models, we could construct policies to control the system. One of the objectives is the minimization of cost of inventory while satisfying demand for the stock. Another objective is to ensure that "stockout" is limited and that surplus stocks are also not carried. Stockout occurs when there is insufficient stock to meet quantity demands. On the other hand, surplus stocks result in increased storage or holding cost.

Autoregressive time series stationary process is used to predict stock level when given immediate past level(s).

Queue theoretic process also helps us to measure the probability of queue intensity of stock.

2.2 TIME SERIES ANALYSIS

A time series is a set of observation associated with time. It enhances understanding of the past and present pattern of change in aid of forecasting.

Time series is a family of random variables $\{X(t) \in T\}$ where t is time parameter. The values assumed by the process are called STATES and the set of possible values is called the STATE SPACE. If T is countably infinite {T= 0, 1, 2…} or countably finite, then the state space is discrete. In case T= $\{t: -\infty \le t \le \infty = (-\infty, \infty\}$ or T = $\{t: 0 \le t \le \infty$ } or more generally T is any finite or infinite interval, then the state space is said to be continuous.

2.2.1 Stationary Time Series

Let $\{X(t),\ \mathrm{t} \in \mathrm{T}\}$ be a time series. It is said to be stationary with respect to the mean if the mean E[X(t)] = m is a constant. Also it is said to be stationary with respect to the variance if the variance $E[(X(t)-m)^2] = \sigma^2(t) = \sigma^2$ is a constant. A weaker form of the stationary time series process is the concept of a covariance stationary process, A covariance stationary process {X(t), t$\in$T} has second moments E[$X(t)^2$]$< \infty$, a constant mean E[X(t)]=m and a covariance E[(X(t)-m)(X(s)-m)] that depends only on the time difference $|t-s|$. Sanders (1990)

2.2.1.1 Autoregressive Models

A pth-order autoregressive time series model which is a stationary process denoted AR(p) is defined as $X_t = \mathrm{a}_0 + \sum_{i=1}^{p} a_i X_{t-i} + \varepsilon_t$ where $a_1, ..., a_p$ are the parameters of the model, $X_{t-1}, ..., X_{t-p}$ are immediate past time points and ε_t is white noise. The model is based on parameters a_i where i=1….p. There is a direct correspondence between these parameters and the covariance function of the process, and this correspondence can be inverted to determine the parameters from the autocorrelation function using the Yule-Walker equations:

$\gamma_m = \sum_{k=1}^{p} a_i \gamma_{m-k} + \sigma_\varepsilon{}^2 \delta_m$ where m=0,…,p yielding p+1 equations. γ_m is the autocorrelation function of X, σ_ε is the standard deviation of the input noise process and δ_m is the kronecker delta function. Because the last part of the equation is non-zero only if m=0, the equation is usually solved by representing it as a matrix for m>0, thus getting equation:

$$\begin{bmatrix} \gamma_1 \\ \gamma_2 \\ \gamma_3 \\ . \\ . \\ . \end{bmatrix} = \begin{bmatrix} \gamma_0 & \gamma_{-1} & \gamma_{-2} & \cdots \\ \gamma_1 & \gamma_0 & \gamma_{-1} & \cdots \\ . & . & . & \cdots \\ . & . & . & \cdots \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix}$$ and for m=0, $\gamma_0 = \sum_{k=1}^{p} a_k \gamma_{-k} + \sigma_\varepsilon{}^2$ Terence (1990)

Another way of determining the model parameters is by means of least squares fitting method. In general AR(p) case, we can write:

$$\begin{aligned} X_{p+1} &= a_0 + a_1 X_p + a_2 X_{p-1} + ... + a_p X_1 + \varepsilon_{p-1} \\ X_{p+2} &= a_0 + a_1 X_{p+1} + a_2 X_p + ... + a_p X_2 + \varepsilon_{P-2} \\ . &= . \quad . \quad . \quad . \quad . \\ . &= . \quad . \quad . \quad . \quad . \\ X_N &= a_0 \quad a_1 X_{N-1} + \quad a_2 X_{N-2} + .. a_1 X_2 + \varepsilon_{p-2} \end{aligned}$$

In matrix form,

$$\begin{bmatrix} X_{p+1} \\ X_{p+2} \\ . \\ . \\ X_N \end{bmatrix} = \begin{bmatrix} 1\ X_p\ X_{p-1} ....X_1 \\ 1\ X_{p+1}\ X_p ....X_2 \\ .... \\ .... \\ 1\ X_{N-1}\ X_{N-2} ....X_{N-p} \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ . \\ . \\ a_p \end{bmatrix} + \begin{bmatrix} \varepsilon_{p+1} \\ \varepsilon_{p+2} \\ . \\ . \\ \varepsilon_N \end{bmatrix}$$ which is in the form Y= XB + E

$$\begin{bmatrix} \hat{a}_1 \\ \hat{a}_2 \\ . \\ . \\ \hat{a}_N \end{bmatrix} = \left(X^{'} X \right)^{-1} X^{'} Y$$

The residual variance is $\sigma_e^{\ 2} = \frac{1}{N-p}\sum_{t=p+1}^{N}\left(X_t - \sum_{i=1}^{p} X_{t-1}\hat{a}_i\right)^2$. Gottman (1981)

AR(1) Model

AR (1) model is first autoregressive model given by $x_t = a_0 + a_1 x_{t-1} + e_t$ where e_t is white noise with zero mean and variance σ^2. We can write each point as follows:

$x_2 = a_0 + a_1 x_1 + e_2$
$x_3 = a_0 + a_1 x_2 + e_3$
....
....
$x_N = a_0 + a_1 x_{N-1} + e_N$

In matrix form:

$$\begin{bmatrix} x_2 \\ x_3 \\ . \\ . \\ x_N \end{bmatrix} = \begin{bmatrix} 1 & \mathrm{x}_1 \\ 1 & \mathrm{x}_2 \\ \multicolumn{2}{c}{\cdots\cdots} \\ \multicolumn{2}{c}{\cdots\cdots} \\ 1 & \mathrm{x}_{\mathrm{N-1}} \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \end{bmatrix} + \begin{bmatrix} e_2 \\ . \\ . \\ . \\ e_N \end{bmatrix} \quad \text{where } X_2 = \begin{bmatrix} x_2 \\ x_3 \\ . \\ . \\ x_N \end{bmatrix}, \quad X_1 = \begin{bmatrix} x_1 \\ x_2 \\ . \\ . \\ x_{N-1} \end{bmatrix} \quad \text{and X=} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ . \\ . \\ x_N \end{bmatrix}$$

<u>Stationary condition, mean and variance of AR(1) model</u>

If $|a_1| < 1$, then the model is stationary. That is there is a covariance-stationary process for x_t.

If $|a_1| \geq 1$, then the errors accumulate instead of die out over time. If $|a_1| = 1$, then x_t exhibits a unit root and can also be considered as a random walk which is not wide sense stationary.

Assuming $|a_1| < 1$ and denoting the mean by μ, we get:

$$E(x_t) = E(a_0) + a_1 E(x_{t-1}) + E(e_t)$$

$$\Rightarrow \ \mu = \mathrm{a}_0 + a_1\mu + 0$$

Thus $\mu = \frac{a_0}{1-a_1}$

In particular, if $a_0 = 0$, then the mean is 0. The variance can be shown to equal

$$Var(x_t) = E(x_t^{\,2}) - \mu^2 = \frac{\sigma^2}{1-a_1^{\,2}}$$

The auto-covariance is given by $B_n = E\left(x_{t+n}x_t\right) - \mu^2 = \dfrac{\sigma^2}{1-a_1^{\,2}}a_1^{|n|}$

It can be seen that the auto-covariance function decays with a decay time $\tau = \dfrac{-1}{\ln(a_1)}$.

$B_n = Ka_1^{|n|}$ where K is independent of n. Note that $a_1^{|n|} = \ell^{\ln a_1 |n|}$ and match this to the exponential decay law $\ell^{-n/\tau}$. Terence (1990)

AR(2) Model

AR(2) model is 2nd order auto regressive model given by $x_t = a_0 + a_1 x_{t-1} + a_2 x_{t-2} + e_t$. We can write each point as follows:

$$x_3 = a_0 + a_1 x_2 + a_2 x_1 + e_3$$
$$x_4 = a_0 + a_1 x_3 + a_2 x_2 + e_4$$
$$x_5 = a_0 + a_1 x_4 + a_2 x_3 + e_5$$
$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$
$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$
$$x_N = a_0 + a_1 x_{N-1} + a_2 x_{N-2} + e_N$$

In matrix form:

$$\begin{bmatrix} x_3 \\ x_4 \\ x_5 \\ . \\ . \\ . \\ x_N \end{bmatrix} = \begin{bmatrix} 1 & \mathrm{x}_2 & \mathrm{x}_1 \\ 1 & \mathrm{x}_3 & \mathrm{x}_2 \\ 1 & \mathrm{x}_4 & \mathrm{x}_3 \\ \ldots\ldots\ldots\ldots & & \\ \ldots\ldots\ldots\ldots & & \\ \ldots\ldots\ldots\ldots & & \\ 1 & \mathrm{x}_{\mathrm{N-1}} & \mathrm{x}_{\mathrm{N-2}} \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \end{bmatrix} + \begin{bmatrix} e_3 \\ e_4 \\ e_5 \\ . \\ . \\ . \\ e_N \end{bmatrix} \text{ where } X_3 = \begin{bmatrix} x_3 \\ x_4 \\ x_5 \\ . \\ . \\ . \\ x_N \end{bmatrix}, \quad X_2 = \begin{bmatrix} x_2 \\ x_3 \\ x_4 \\ . \\ . \\ . \\ x_{N-1} \end{bmatrix}, \quad X_1 = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ . \\ . \\ . \\ x_{N-2} \end{bmatrix} \text{ and}$$

$$\mathrm{X} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ . \\ . \\ . \\ x_N \end{bmatrix}$$

Stationary condition of AR(2) model

The model is $x_t = a_0 + a_1 x_{t-1} + a_2 x_{t-2} + e_t$. Using the fact that autocorrelations must be less than unity and each factor in the denominator and numerator must be positive, we can derive the conditions:

$-1 < a_2 < 1$

$a_2 + a_1 < 1$

$a_2 - a_1 < 1$

For values of a_1 and a_2 outside this range, the series is not stationary. Gottman (1981)

2.3 INVENTORY CONTROL MODELS

2.3.1 Introduction

Inventory, the increasingly popular synonym for stock refers to accumulation of raw materials, semi-finished goods or finished goods held by organization for use in future. In each case, the function of the stocks is to act as a cushion between uneven flows so that the process of production and distribution can continue without interruption.

Some reasons organizations or individuals maintain stock are the difficulties in predicting sales levels, production times, demand and usage needs exactly. Thus, inventory serves as a buffer against uncertain and fluctuating usage and keeps a supply of items available in case the items are needed by the organization or its customers. No customer ever has to wait for any but the shortest time between placing his order and receiving the goods and that no machine is ever idle because materials are not available, large stocks must be held. Large stocks give security against interruption of the process.

Even though inventory serves an important role but the cost associated with it is high. Goods that are in stock represent working capital tied up and not available for re-investment in new supplies of materials or for other uses. Every dollar worth of excess inventory represents a real cost to the business of the loss of earning power of the tied-up capital.

The minimum stock level is the level below which stocks should not be normally allowed to fall. If stocks go below this level, there is the very real danger of a stockout resulting in production stoppage.

The re-order level is higher than the minimum stock level but lower than maximum stock level. The maximum level is the level above which stocks should not be allowed to rise.

The optimal stock level is the level in which inventory cost attains minimum with rise in customer satisfaction.

Holding cost is a cost associated by keeping or carrying a given level of stock. These costs depend on the size of the inventory or the stock. Too great a supply of stock results in high storage costs, excessive capital being locked up. Other holding costs are insurance, taxes, breakages, warehousing costs etc. These costs may be stated as a percentage of the inventory investment.

The ordering cost includes preparation of voucher, the processing of the order including payment, postage, telephone, transportation, invoice verification, labour cost etc.

Stockout cost is a cost associated by keeping stock below minimum level. Examples are loss of customer's goodwill, loss of period sales, reduced profit etc.

Cost of capital is a cost a firm incurs to obtain capital for investment. It is part of holding cost associated with maintaining inventory. Stafford (1969)

2.3.2 EOQ Model

2.3.2.1 Introduction

Economic Order quantity model is a model used to determine optimal order quantity at minimum cost. The assumptions of the model are as follows

(1) Average demand is fixed.

(2) Demand pattern is periodic.

(3) The cost per order is constant and does not depend on the quantity ordered

(4) The average purchase cost per unit item (m) is constant and does not depend on the quantity ordered

(5) The lead time for an order is constant

(6) The average inventory holding cost per unit time is constant.

Note that the first and second assumptions imply that the demand series should be stationary in the mean and variance.

The symbols used are as follows**:**

- unit cost of procuring and holding goods C_T

Total cost of procuring and holding inventory T_c

- cost of placing an order (cost per order) C_0

- size of quantity ordered q

- sales per period of time D

- unit cost of the inventory item m

- holding cost rate I

Suppose that our requirements are considered over a fairly long time period designated T. Assuming that we start with a full optimum batch size q in stock. Demand continuous at constant rate till q is used up in time period t. Then there should be instant replenishment of size q as shown in the figure 2.3.2. Stafford (1969)

2.3.2.2 Inventory Level Trajectory

The equation of the inventory level trajectory for the EOQ model is a periodic piecewise linear function of the form q(t)= mt +q . From figure 2.3.2, the slope (m) of the trajectory of each cycle is given by m=$\frac{-q}{T}$ where q is the maximum inventory level

$$q(t) = \frac{-q}{T}t + q$$

$$= q\left(1 - \frac{t}{T}\right) \quad 0<t<T \text{ (domain for } 1^{st} \text{ cycle)}$$

Generally, q(t)= $\left(1-\frac{t-kT}{T}\right)$ for kT<t<(k+1)T where k$\in z^{+}$

Average inventory = $\frac{1}{T}\int_{0}^{T} q(t)dt$

$$=\frac{q}{T}\int_{0}^{T}\left(1-\frac{t}{T}\right)dt$$

$$=\frac{q}{T}\left(t-\frac{t^{2}}{2T}\right)_{0}^{T}$$

$$=\frac{q}{T}\left(T-\frac{T^{2}}{2T}\right)$$

$$=\frac{q}{T}\left(T-\frac{T}{2}\right)=\frac{q}{2}$$

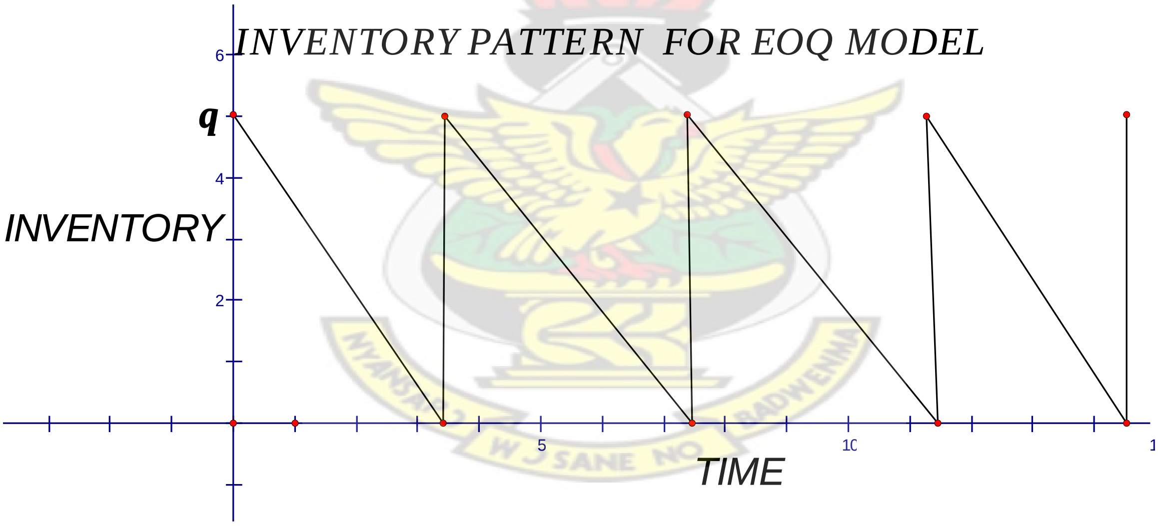

Fig. 2.3.2: The trajectory of EOQ model

2.3.2.3 Development of the EOQ Model

We develop below the EOQ model using total inventory cost model.

Preparing the model for assembly

The first expression that we must derive is one for the cost of purchasing one unit of whatever goods we are considering. The total cost of purchasing will consist of ordering cost plus total purchase price. The total purchase price being quantity ordered multiplied by unit price. The expression required will be:

Cost of procuring batch = C_0 + mq

Cost per unit purchased = (C_0 + mq)/q

The next expression required is that for the cost of holding an item in stock for one time period; that is for one month or whatever period is appropriate, this last item has been given as a percentage of unit cost of purchasing C_0/q +m . If this percentage is 100I, then the equivalent decimal fraction will be p .The cost of holding one stock unit for one time period is:

$$I\left(\frac{C_0}{q}+m\right)$$

The average length of time for which goods remain in stock is given by

$$q/2D$$

Therefore the total cost of ordering and holding one stock unit is given by

$$C_T=\left(\frac{C_0}{q}+m\right)+\ \mathrm{I}\left(\frac{\mathrm{C}_0}{\mathrm{q}}+m\right)\left(\frac{q}{2D}\right)$$

Method 1

The requirement now is to find the batch size that will minimize the cost per unit item (C_T)

$$C_T = \left(\frac{C_0}{q} + m\right) + \text{ I}\left(\frac{\text{C}_0}{\text{q}} + m\right)\left(\frac{q}{2D}\right)$$

$$= \frac{C_0}{q} + m + \frac{IC_0}{2D} + \left(\frac{\text{Im}}{2D}\right)q$$

To find the minimum cost, we must differentiate C_T with respect to q and then put the derivative equal to zero to find the stationary point:

$$\frac{dC_T}{dq} = \frac{\text{Im}}{2D} - C_0 q^{-2}$$

At minimum point, $\frac{dC_0}{dq} = 0$

$$\frac{\text{Im}}{2D} = \frac{C_0}{q^2}$$

Therefore, $Q_m = \sqrt{\frac{2DC_0}{\text{Im}}}$

The symbol Q_m is called optimal order quantity or optimum batch size. It is the quantity to order to minimize total inventory cost. It is used to develop cost-effective inventory management decisions. The second derivative is $d^2 C_T / dq^2 = \text{ C}_0 / q^3$

Because the value of the second derivative is positive, Q_m is the minimum cost solution.

Stafford (1969)

Method 2

The holding cost can be calculated using the average inventory. It is obtained by multiplying the average inventory by the cost of carrying one unit in inventory for the stated period. The period selected for the model is up to you; it could be 1 week, 1 month, 1 year or more. However, because the holding cost for many industries and businesses is expressed as an annual percentage, most inventory models are developed on an annual cost basis.

C_h =Im= (annual holding cost rate) (unit cost of inventory item)

Therefore the general equation for the annual holding cost for the average inventory of (q/2) units is as follows:

Annual holding cost = (average inventory)(annual holding cost per unit)

$$= \frac{q}{2} C_h$$

Also, Annual ordering cost = (number of orders per year)(cost per order)

$$= \left(\frac{D}{q}\right) C_0$$

Where D is the annual demand for the product. Therefore, the total annual cost denoted as T_c is given by :

T_c = (Annual holding cost) + (Annual ordering cost)

$$= \left(\frac{q}{2}\right) C_h + \left(\frac{D}{q}\right) C_0$$

To find the minimum total cost we must differentiate T_c with respect to q and then put the derivative to zero to find the stationary point.

$$\frac{dT_c}{dq} = \frac{C_h}{2} - \frac{D}{q^2} C_0 = 0$$

$$C_h q^2 = 2DC_0$$

$$q^2 = \frac{2DC_0}{C_h}$$

Therefore, $Q_m = \sqrt{\frac{2DC_0}{C_h}}$

Taylor (2006)

<u>Interpretation of EOQ model</u>

The formula is:

$$Q_m = \sqrt{\frac{2DC_0}{C_h}}$$

The optimal order quantity (Q_m) becomes larger as C_0, the ordering cost increases. It again increases as sales (D) increases. It declines as m, the unit cost increases.

2.3.2.4 Cost- Effective Inventory Decision for the EOQ Model

The recommended inventory decision is to order Q_m units whenever the inventory reaches the reorder point of r units.

Safety stock units = r-d

The anticipated annual cost are as follows:

(1) Holding cost, normal inventory = $\left(\frac{q}{2}\right)C_h$

(2) Holding cost, safety stock = $(r-d)C_h$

(3) Ordering cost = $\left(\frac{D}{q}\right)C_0$

At optimal:

Holding cost, normal inventory = $\left(\frac{Q_m}{2}\right)C_h$

$$\left(\frac{Q_m}{2}\right)C_h < \left(\frac{q}{2}\right)C_h$$

Ordering cost = $\left(\frac{D}{Q_m}\right)C_0$

$$\left(\frac{D}{Q_m}\right)C_0 < \left(\frac{D}{q}\right)C_0$$

NB:

(1) $\left(\frac{Q_m}{2}\right)C_h = \left(\frac{D}{Q_m}\right)C_0$

(2) When demand is uncertain and can be expressed in probabilistic terms, a larger total cost occurs in the form of larger holding costs because more inventory must be maintained to limit the number of stockouts

(3) Service level is the percentage of all order cycles that do not experience a stockout.

2.3.3 Probabilistic Inventory Models

Probabilistic Inventory Models are types of models in which the demand for the item fluctuates and can be described in probabilistic terms. In probabilistic demand, the number of units demanded varies considerably from time to time.

2.3.3.1 Order-quantity, Reorder Point Model with Probabilistic Demand

These involve inventory models with multi-period order quantity, reorder point inventory models with probabilistic demand. Here, the inventory system operates continuously with many repeating periods or cycles; inventory can be carried from one period to the next. Whenever the inventory position reaches the reorder point, an order for Q_m units is placed.

Because demand is probabilistic, the time the reorder point will be reached, the time between orders and the time the order of Q_m units will arrive in inventory cannot be determined in advance. With probabilistic demand, occasional shortages may occur. Demand data indicate that demand during a lead time can be described by a normal probability distribution with mean d and standard deviation s.

<u>The how-much-to-order decision</u>

(a) The expected annual demand if the delivery time(lead time) is in days

=(mean demand during period)(365 days per year)/(lead time)

(b) The expected annual demand if the lead time is in week(s)

= (mean demand during period)(52 weeks per year)/(lead time)

(c) The expected annual demand if the lead time is in months

= (mean demand during period)(12 months per year)/(lead time)

We then apply the EOQ model as an approximation of the best order quantity with D representing the expected annual demand.

We expect $Q_m = \sqrt{\dfrac{2DC_0}{C_h}}$ units per order to be a good approximation of the optimal order quantity. Even if annual demand is less than D units or greater than D units, an order quantity of Q_m units should be a relatively low-cost order size. We have established the Q_m unit order quantity by ignoring the fact that demand is probabilistic. We can then anticipate placing approximately $\dfrac{D}{Q_m}$ number of orders per year with an average of approximately T working days between orders. Taylor (2006)

The reorder point

The lead-time demand probability distribution used is the normal probability distribution. The curve is symmetrical about the line X=d

X= normal variate of demand during lead time period with mean d and variance s^2. The probability density function is given by:

$$F(x) = \frac{1}{s\sqrt{2\pi}} \ell^{-(X-d)^2/2s^2}$$

F(x) is maximum when X=d . Approximately 68.26% of the distribution lies within one standard deviation of the mean. Approximately 95.44% lies within 2 standard deviations of the mean. Approximately 99.74% of the distribution lies within 3 standard deviation of the mean.

The random variable z = (X-d)/s has standard normal distribution N (0,1)

z = standard score

X = d + sz

The reorder point r can be found by using this distribution. It is inventory position at which a new order should be placed.

r = d + zs where d = mean of lead-time demand distribution

s = standard deviation of lead-time demand distribution

z = number of standard deviations necessary to obtain the acceptable stockouts probability

The when-to-order decision

We now want to establish when-to-order decision rule or reorder point (r). With a mean lead-time of d units, we must first suggest a d-unit reorder point.

Lead time

This is delivery period for a new order.

Cycle time (T)

The period between orders is referred to as the cycle time.

$$T = (\text{length of time of operation per period}) \times \frac{\textit{optimal}\ \text{order quantity}}{\text{demand per period of time}}$$

When to experience stockout

When the demand during the lead-time period exceeds the reorder point, then stockout or shortage occurs.

Probability of stockout

$$\text{Probability of stockout} = \frac{\textit{Number}\ \text{of stockouts per period}}{\text{number of orders made during that period}}$$

We could now calculate cost per stockout and add to total cost equation. It is well noted that attempting to avoid stockouts completely will require high reorder point, high inventory and an associated high holding cost. Taylor (2006)

2.3.4 Economic Order Quantity for Lot-Size Model

2.3.4.1 Introduction

Lot-size is the number of units in an order.

In this model, we determine how much to order and when to order depending on how much is in stock. Daily inventory buildup during the arrival phase is p-d where

p = daily arrival rate of inventory goods in the inventory system

d = daily demand rate

The assumptions of the model are as follows:

(1) Average demand is fixed

(2) Demand pattern is periodic

(3) The average cost per unit item is constant

(4) The average holding cost per unit item is constant

(5) The average cost per order is constant

(6) p>d during the production run

Note that the 1st and 2nd assumptions imply that the demand series should be stationary in the mean and variance.

The Lot-size model is designed for production situations in which once supply begins, demand begins. During the supply run, demand would be reducing the inventory while supply would be adding to inventory. We assumed that the supply rate exceeds the demand rate during the supply run. Therefore the excess supply would cause a gradual inventory build-up during the supply period. When the supply run is completed, the continuing demand will cause the inventory to gradually decline until a new supply run is started. Taylor (2006)

2.3.4.2 Inventory Level Trajectory Of Lot-Size Model

The equation of the inventory level trajectory for the lot-size model is also a periodic piecewise linear function involving the form q(t)=m(t). The slope(m) of the trajectory of each cycle is given by m=$\left(\frac{p}{T}-\frac{d}{T}\right)$ = $\frac{1}{T}(p-d)$ where p-d is the inventory build-up function as shown in figure 2.3.4. The equation of the trajectory is q(t) = $\frac{1}{T}(p-d)t$.

Maximum inventory (height of inventory at time t) is the inventory at the end of arrivals run = (p-d) t where t = number of days for arrivals run

If we schedule Q lot-size at time t, then Q = pt

This implies t = $\dfrac{Q}{p}$

Maximum inventory = (p-d) $\dfrac{Q}{p}$

Average inventory (area of triangle per cycle)

$$= \frac{1}{2}\int_0^T q(t)dt \; = \frac{1}{2}\int_0^T (\frac{1}{T}(p-d)\frac{Q}{P})dt$$

$$= \frac{1}{2}\left(\frac{1}{T}(p-d)\frac{Q}{p}\right)t)^T{}_0$$

$$= \frac{1}{2}\left(\frac{1}{T}(p-d)\frac{Q}{p}\right)T$$

$$= \frac{1}{2}(p-d)\frac{Q}{p}$$

$$= \frac{1}{2}\left(1-\frac{d}{p}\right)Q$$

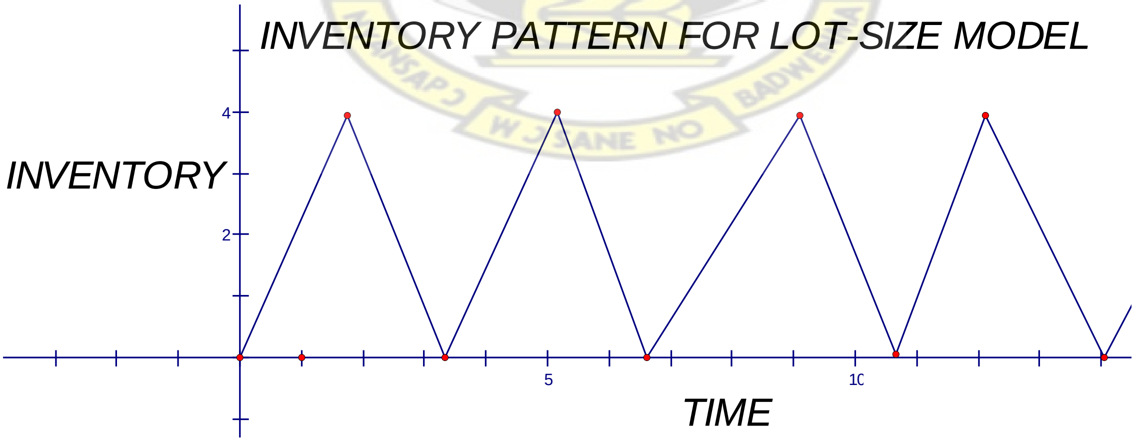

Fig 2.3.4: trajectory of Lot-Size model

2.3.4.3 Development of the Optimal Order Quantity for Lot-Size Model

We develop below the Lot-size model through the construction of the total inventory cost model.

Let:

Annual holding cost = (Average inventory) (annual holding cost per unit)

$$= \frac{1}{2}\left(1-\frac{d}{p}\right)QC_h \ldots(1)$$

Annual ordering cost = (Number of orders per year) (cost per order)

Annual ordering cost = $(\frac{D}{Q})C_0$ … (2) where D represents annual total demand.

Thus, total annual cost (T_c) model is given by

$$T_c = (1) + (2) = \frac{1}{2}\left(1-\frac{d}{p}\right)QC_h + \left(\frac{\mathrm{D}}{\mathrm{Q}}\right)C_0$$

P = total annual production of the inventory

Therefore, $\frac{d}{p} = \frac{D}{L} / \frac{P}{L} = \frac{D}{P}$

When production run ceases, demand continues and inventory declines. This situation gives rise to stockout most especially when the inventory system is probabilistic.

In the Akosombo Dam, it is assumed that p>d . Annually, D> P since energy demand continues during dry season when no water flows into the dam.

$$T_c = \frac{1}{2}\left(1-\frac{P}{D}\right)QC_h + \left(\frac{\mathrm{D}}{\mathrm{Q}}\right)C_0$$

At minimum cost, $\frac{dT_c}{dQ} = 0 \Rightarrow \frac{1}{2}\left(1-\frac{P}{D}\right)C_h - \frac{D}{Q^2}C_0 = 0$

It implies that $\frac{1}{2}\left(1-\frac{P}{D}\right)C_hQ^2 = 2DC_0$

$$Q^2 = \frac{2DC_0}{\left(1-\frac{P}{D}\right)C_h}$$

Therefore, $$Q_m = \sqrt{\frac{2DC_0}{\left(1-\frac{P}{D}\right)C_h}}$$

Number of orders to be placed annually (N) = $\frac{D}{Q_m}$

Average working days between orders per year (T) = $\frac{LQ_m}{D}$

Stockout and service level

Stockout: Stockout occurs when there is insufficient stock to satisfy customers demand.

Service level = 1 - P (stockout)

Taylor (2006), Anderson (2004)

2.3.4.4 Effective Inventory Cost Decision for Lot-Size Model

(1) Holding cost, normal inventory = $\frac{1}{2}\left(1-\frac{P}{D}\right)QC_h$

(2) Minimum holding cost $=\frac{1}{2}\left(1-\frac{P}{D}\right)Q_mC_h$

i.e $$\frac{1}{2}\left(1-\frac{P}{D}\right)Q_mC_h < \frac{1}{2}\left(1-\frac{P}{D}\right)QC_h$$

(3) Ordering cost = $\left(\frac{D}{Q}\right)C_0$

(4) Minimum ordering cost = $\left(\frac{D}{Q_m}\right)C_0$

(5) $\left(\frac{D}{Q_m}\right)C_0 < \left(\frac{D}{Q}\right)C_0$

(6) $\frac{1}{2}\left(1-\frac{P}{D}\right)Q_m C_h = \left(\frac{D}{Q_m}\right)C_h$

2.3.5 Periodic Review Inventory System

2.3.5.1 Introduction

An alternative to the continuous review system is the periodic review inventory system. With a periodic review, the inventory may be checked and orders placed on a weekly, bi-weekly, tri-weekly, monthly or some other periodic basis.

2.3.5.2 Replenishment Level (M)

It is inventory level at which the order quantity should be demanded at the review period. If the normal probability distribution is used then:

M= d + zs where

d = mean demand

z = number of standard deviations necessary to obtain the acceptable stockout probability

s = standard deviation of the distribution

2.3.5.3 How-Much-To-Order Decision

The how-much-to-order decision at any review period is determined using the model

Let:

q = M-X where

q represents the order quantity at review period

M = replenishment level

X = the inventory on hand at review period which varies since demand is probabilistic

Taylor (2006), Anderson (2004)

2.4 QUEUE THEORETIC MODEL:

2.4.1 Introduction

Queue situations have been part of most people's lives for many years. We queue at supermarkets, banks, we queue for buses and sit in cars queueing. There are many situations similar to these everyday queues; example is rain water in a reservoir (dam) waiting to be released to generate electricity etc. Mathematical models are devised to minimize queue. On contrary to Akosombo dam situation, we device models that can keep queue in the dam to sustain the dam at optimal level.

Individuals or materials arrive at the end of a queue, wait in the queue, receive the service and then leave the system. Schematically, the situation is:

Arrivals $\rightarrow$ queue $\rightarrow$ service $\rightarrow$ exit from system

The simplest situation consists of a single queue and a single service point. An alternative system often seen in banks and elsewhere is where there are several service points each with its own queue. A further variant would be to have a single queue in which the queue members have to go to any service point which happens to be vacant.

The queue discipline may allow priorities or may be on a 1^{st} come, 1^{st} served basis. The arrival pattern may vary; people or whatever queue units are concerned may arrive at regular intervals or may arrive randomly.

Le the average number of arrivals in the given time be p, the average time between arrivals will be $\frac{1}{p}$ of the given time. Similarly, let the average number of services completed in the given time be d, the average time taken for each service is $\frac{1}{d}$ of the given time. Stafford (1969)

2.4.2 Birth-Death Process

A useful class of Markov processes when analyzing queue systems are birth-death processes. The states represent current size and the transitions are limited to birth and death.

2.4.2.1 States Transitions

When birth occurs, the process goes from state i to i+1. The transition intensity from state i to i+1 is designated $\lambda_i \geq 0$ for $i \geq 0$. When death occurs, the process goes from state i to i-1. The transition intensity from i to i-1 is designated $\mu_i \geq 0$ for $i \geq 1$

2.4.2.2 The M/M/1 Queue System

The M/M/1 is a single server queue with infinite buffer size. Arrivals are random and departure times are also random. Even though arrivals are random, we can estimate the average number of arrivals that may be expected in the chosen time unit. The average rate of arrivals is designated as p and the average rate of departure is designated as d. That is:

$$\lambda_i = p \text{ and } \mu_i = d \text{ for all i}$$

The differential equations for the probability that the system is in state k at time t are:

$$P_0^{'} = \mu_1 P_1(t) - \lambda_0 P_0(t)$$
$$P_k^{'}(t) = \lambda_{k-1} P_{k-1}(t) + \mu_{k+1} P_{k+1}(t) - (\lambda_k + \mu_k) P_k(t)$$

2.4.2.3 The M/M/1/K Queue

The M/M/1/K queue is a single server queue with a finite buffer of size K.

$$\lambda_i = p \text{ for } 0 \leq i \leq K$$
$$\mu_i = d \text{ for } 1 \leq i \leq K$$

The differential equations for the probability that the system is in state k at time t are:

$P_0^{'} = \mu_1 P_1(t) - \lambda_0 P_0(t)$

$P_k^{'}(t) = \lambda_{k-1} P_{k-1}(t) + \mu_{k+1} P_{k+1}(t) - (\lambda_k + \mu_k) P_k(t)$

For $k \leq K$, $\mathrm{P_k}^{'}(t) = 0$

2.4.2.4 Equilibrium Solution

A queue is said to be in equilibrium if the limit $\lim_{t\to\infty} P_k(t)$ exists.

Assume $P_k = \lim_{t\to\infty} P_k(t)$[*probability* of finding birth-death system in state k]. This is

equilibrium probabilities of finding k customers in the system. In equilibrium, $P_k^{'} = 0$ is zero.

Using M/M/1 queue or M/M/1/K for example, the steady-state (equilibrium) equations are:

$0 = \mu_1 P_1(t) - \lambda_0 P_0(t)$

$\Rightarrow \mu_1 P_1(t) = \lambda_0 P_0(t)$ ...... (1), also,

$(\lambda_k + \mu_k) P_k(t) = \lambda_{k-1} P_{k-1} + \mu_{k+1} P_{k+1}(t)$ ... (2)

This can be reduced if $\lambda_k = p$ and $\mu_k = d$ for all k (the homogeneous case) to

$pP_k(t) = dP_{k+1}$ for $k \geq 0$

Conservation of flow is as follows:

Input flow= output flow

Flow rate into state k= $\lambda_{k-1} P_{k-1} + \mu_{k+1} P_{k+1}$

Flow rate out of the state k= $(\lambda_k + \mu_k) P_k$

In equilibrium,

$\lambda_{k-1} P_{k-1} + \mu_{k+1} P_{k+1} = (\lambda_k + \mu_k) P_k$

$\Rightarrow \sum \mathrm{P_k} = 1$

From (1) above, $P_1 = \frac{\lambda_0}{\mu} P_0(t)$

If k=1, from (2) above,

$\lambda_0 P_0 + \mu_2 P_2 = (\lambda_1 + \mu_1) P_1$ but $P_1 = \frac{\lambda_0}{\mu_1} P_0$

$\Rightarrow \lambda_0 P_0 + \mu_2 P_2 = (\lambda_1 + \mu_1) \frac{\lambda_0}{\mu_1} P_0$

Therefore, $P_2 = \frac{\lambda_0 \lambda_1}{\mu_1 \mu_2} P_0 \ldots$ (3)

Generalizing, we have: $P_k = \frac{\lambda_0 \lambda_1 \ldots \lambda_{k-1}}{\mu_1 \mu_2 \ldots \mu_k} P_0$

$P_k = P_0 \prod_{i=0}^{k-1} \frac{\lambda_i}{\mu_{i+1}}$ k=0,1,2,... $\sum_k P_k = 1$

$$P_0 = \frac{1}{1 + \sum_{k=1}^{\infty} \prod_{i=0}^{k-1} \frac{\lambda_i}{\mu_{i+1}}}$$

Define:

$$S_1 = \sum_{k=1}^{\infty} \prod_{i=0}^{k-1} \frac{\lambda_i}{\mu_{i+1}}, \qquad S_2 = \sum_{k=1}^{\infty} \frac{1}{\lambda_k \prod_{i=0}^{k-1} \frac{\lambda_i}{\mu_{i+1}}}$$

All states:

(a) ergodic iff $\begin{matrix} S_1 < \infty \\ S_2 = \infty \end{matrix}$ $\rightarrow$ equilibrium probabilities

(b) recurrent null iff $\begin{matrix} S_1 = \infty \\ S_2 = \infty \end{matrix}$

(c) transient iff $\begin{matrix} S_1 = \infty \\ S_2 < \infty \end{matrix}$

Steady-state probabilities are said to exist if and only if condition for ergodicity is satisfied.

That is there exist some k_0 such that for all k$\geq k_0$ $\frac{\lambda_k}{\mu_k}<1$. The necessary and sufficient

condition for ergodicity is that p<d in M/M/1 system.

$\lambda_k = p$ k=0,1,2...

$\mu_k = d$ k=0,1,2,...

$$P_k = P_0\prod_{i=0}^{k-1}\frac{\lambda_i}{\mu_{i+1}} = P_0\prod_{i=0}^{k-1}\frac{p}{d} = P_0(\frac{p}{d})^k \quad k \geq 0$$

To be ergodic (and hence $P_k > 0$). It implies, $S_1 < \infty$ and $S_2 = \infty$

$$\Rightarrow \ S_1 = \sum\frac{P_k}{P_0} = \sum_{k=0}^{\infty}\left(\frac{p}{d}\right)^k < \infty \quad \text{converges if and if } \frac{p}{d}<1$$

$$S_2 = \sum_{k=0}^{\infty}\frac{1}{p\left(\frac{P_k}{P_0}\right)} = \sum\frac{1}{p}\left(\frac{d}{p}\right)^k = \infty \qquad S_2 \text{ is satisfied if } \frac{\mathrm{p}}{\mathrm{d}}\leq 1$$

$$\Rightarrow \ P_0 = \frac{1}{1+\sum_{k=1}^{\infty}\left(\frac{p}{d}\right)^k} = \frac{1}{1+\frac{p/d}{1-p/d}} = 1-p/d$$

P_0 = The probability that there is no queue in the system at a given time (no one in the queue and no one being dealt with at the service point)

Let $\rho = \frac{p}{d}$, for stability, 0<ρ<1 ; ρ measures traffic intensity

$$P_k = P_0\left(\frac{p}{d}\right)^k = (1-\rho)\rho^k$$

P_k =The probability that there are k states in the system

The average number of k-states in the queue (including the occasions when no one is queue) is equal to:

$$\frac{\rho^2}{1-\rho} = \frac{p^2}{d(d-p)}$$

The expected number of k in the queue when there is a queue is given by:

$$\frac{1}{1-\rho} = \frac{d}{d-p}$$

The average waiting time: $\frac{\rho}{d(1-\rho)} = \frac{p}{d(d-p)}$

The average time spent in the system: $T = \frac{p}{d(d-p)} + \frac{1}{p} = \frac{1}{d-p}$ (this is waiting time plus service time)

Lui (1999)

Traffic intensity and its significance (ρ)

It is the ratio of average number of arrivals to the average number of departure. That is $\rho = p/d$ .It determines the probability of queueing in the system. Unless the service rate is well below the arrival rate, the service will not be adequate and the queue will be long. That is long queue occurs if and only if ρ approaches 1 .For optimality, $0 < \rho < 1$

2.4.2.5 The M/M/C Queue

The M/M/C queue is a multi-server queue with C servers and an infinite buffer. This differs from M/M/1 queue only in the service time which becomes:

$\mu_k = kd$ for k ≤ C

$\mu_k = Cd$ for k ≥ C

$\lambda_k = p$ for all k=0,1,2,...

Conditions for ergodicity is $\frac{p}{Cd} < 1$ for k ≤ C

Traffic intensity $(\rho) = \frac{p}{Cd}$ is that the service rate for a single channel must be multiplied by the number of channels(C) available. The differential equations for the probability that the system is in state k at time t are:

$$P_0^{'}(t) = \mu_1 P_1(t) - \lambda_0 P_0(t) ...(1)$$

$$P_k^{'}(t) = \lambda_{k-1} P_{k-1}(t) + (k+1)\mu_{k+1} P_{k+1}(t) - (\lambda_k + \mu_k) P_k(t) ...(2)$$

For $k \le C-1$

$$P_k^{'}(t) = \lambda_{k-1} P_{k-1}(t) + C\mu_{k+1} P_{k+1}(t) - (\lambda_k + \mu_k) P_k(t) \text{ for k} \ge \text{C} ...(3)$$

<u>Steady-state probabilities</u>

At equilibrium, $P_0^{'}(t) = 0$, $\mathrm{P_k}^{'}(t) = 0$

$$0 = \mu_1 P_1 - \lambda_0 P_0$$

$$\Rightarrow \mu_1 P_1 = \lambda_0 P_0$$

Therefore, $P_1 = \frac{\lambda_0}{\mu_1} P_0 ...*$

$$0 = \lambda_{k-1} P_{k-1} + (k+1)\mu_{k+1} P_{k+1} - (\lambda_k + \mu_k) P_k \text{ for k} \le \text{C-1}$$

If k=1, $0 = \lambda_0 P_0 + 2\mu_2 P_2 - (\lambda_1 + \mu_1) P_1$

$$0 = \lambda_0 P_0 + 2\mu_2 P_2 - (\lambda_1 + \mu_1)\frac{\lambda_0}{\mu_1} P_0$$

$$0 = \lambda_0 P_0 + 2\mu_2 P_2 - \frac{\lambda_0 \lambda_1}{\mu_1} P_0 - \lambda_0 P_0$$

$$\Rightarrow 2\mu_2 P_2 = \frac{\lambda_0 \lambda_1}{\mu_1} P_0$$

Therefore, $P_2 = \frac{\lambda_0 \lambda_1}{2\mu_1 \mu_2} P_0 ...**$

If k=2, $0 = \lambda_1 P_1 + 3\mu_3 P_3 - (\lambda_2 + \mu_2) P_2$

$$0=\lambda_1\left(\frac{\lambda_0}{\mu_1}P_0\right)+3\mu_3P_3-(\lambda_2+\mu_2)\frac{\lambda_0\lambda_1}{2\mu_1\mu_2}P_0$$

Therefore, $P_3=\dfrac{\lambda_0\lambda_1\lambda_2}{3.2\mu_1\mu_2\mu_3}P_0+\dfrac{\lambda_0\lambda_1}{3.2\mu_1\mu_3}P_0-\dfrac{\lambda_0\lambda_1}{3\mu_1\mu_3}P_0$

…………………………………………………………..

Generally, $P_k=\dfrac{\lambda_0\lambda_1\lambda_2.....\lambda_{k-1}}{k!\mu_1...\mu_k}P_0-\dfrac{\mu_2\lambda_0...\lambda_{k-1}}{k!\mu_1...\mu_k}P_0$

$$P_k=P_0\prod_{i=0}^{k-1}\frac{p}{(i+1)d}=P_0\left(\frac{p}{d}\right)^k\frac{1}{k!}$$

For $k\geq C$: $0=\lambda_{k-1}P_{k-1}+C\mu_{k+1}P_{k+1}-(\lambda_k+\mu_k)P_k$

For k=1, $0=\lambda_0P_0+C\mu_2P_2-(\lambda_1+\mu_1)P_1$

$$0=\lambda_0P_0+C\mu_2P_2-(\lambda_1+\mu_1)\frac{\lambda_0}{\mu_1}P_0$$

$$0=\lambda_0P_0+C\mu_2P_2-\frac{\lambda_0\lambda_1}{\mu_1}P_0\text{ - }\lambda_0P_0$$

$P_2=\dfrac{\lambda_0\lambda_1}{C\mu_1\mu_2}P_0$ ….***

For k=2, $0=\lambda_1P_1+C\mu_3P_3-(\lambda_2+\mu_2)P_2$

$$0=\lambda_1\left(\frac{\lambda_0P_0}{\mu_1}\right)+C\mu_3P_3-(\lambda_2+\mu_2)\left(\frac{\lambda_0\lambda_1}{C\mu_1\mu_2}P_0\right)$$

$$0=\frac{\lambda_0\lambda_1}{\mu_1}P_0+C\mu_3P_3-\frac{\lambda_0\lambda_1\lambda_2}{C\mu_1\mu_2}P_0-\frac{\lambda_0\lambda_1}{C\mu_1}P_0\ \Rightarrow$$

$P_3=\dfrac{\lambda_0\lambda_1\lambda_2}{C^2\mu_1\mu_2\mu_3}P_0+\dfrac{\lambda_0\lambda_1}{C^2\mu_1\mu_3}P_0-\dfrac{\lambda_0\lambda_1}{C\mu_1\mu_3}P_0$

……………………………………………….

Generally,

$$P_k = \frac{\lambda_0 \ldots \lambda_{k-1}}{C^{k-1}\mu_{i+1}} P_0 + \frac{\lambda_0 \ldots \lambda_{k-1}}{C^{k-1}\mu_{i+1}} P_0 - \frac{\lambda_0 \ldots \lambda_{k-1}}{C\mu_{i+1}} P_0$$

$$P_k = P_0 \prod_{i=0}^{k-1} \frac{p}{C^{k-1}Cd} + P_0 \prod_{i=0}^{k-1} \frac{p}{C^{k-1}Cd} - P_0 \prod \frac{p}{C^2 d}$$

$$P_k = P_0 \prod_{i=0}^{C-1} \frac{p}{(i+1)d} \prod_{i=0}^{k-1} \frac{p}{Cd}$$

$$P_k = P_0 \left(\frac{p}{d}\right)^k \frac{1}{C!C^{k-C}}$$

$\Rightarrow$ $P_k = \begin{matrix} P_0 (C\rho)^k & k \le C \\ P_0 \frac{\rho^k C^C}{C!} & \end{matrix}$ where $\rho = \frac{p}{Cd} < 1 \rightarrow$ expected fraction busy servers

$$P_0 = [1 + \sum_{k=1}^{C-1} \frac{(C\rho)^k}{k!} + \sum_{k=C}^{\infty} \frac{(C\rho)^k}{C!} (\frac{1}{C^{k-C}})]^{-1} = [\,[\sum_{k=0}^{C-1} \frac{(C\rho)^k}{k!} + \left(\frac{(C\rho)^k}{C!}\right)\left(\frac{1}{1-\rho}\right)]^{-1}$$

Lui (1999)

CHAPTER THREE

DATA ANALYSIS AND MODELING

3.0 INTRODUCTION

In this chapter, we aim at analyzing and discussing water levels sample data and energy demand sample data of Akosombo dam to achieve our objectives. We should be able to construct time series models for forecasting Akosombo dam water levels, inventory models to determine how much energy to be generated or water to be released and when to generate or release it from the Akosombo dam respectively.

We also determine stockout probabilities at 68% and 95% service levels for the dam inventory using inventory and queue models. At the end of the analysis, we are able to confirm which service level is feasible.

To sustain the Akosombo dam, managers of the dam should be able to allow reasonable number of stockouts and feasible service levels every year.

Let $X_{n+1} = (y + i + w_n) - q$ where X_n is the content of the dam at the release period,

y represents minimum water level. The minimum water level in the Aksombo dam is 240feet.

i represents quantity of water waiting in line in the reservoir to be released.

w_n represents Poisson arrivals at nth period, n=0, 1, 2, 3, …….

q represents size of water released from the dam at the review period

Note that if i = 0 and w_n =0 , then $X_{n+1} = y$. It implies that if possible no water should be released from the dam.

3.1 WATER LEVEL DATA DESCRIPTION

The water level data used for this study (see appendix A) was collected from the Transmission Systems Department, Volta River Authority, Ghana.

The data comprises the following:

(1) Daily time series data of Akosombo dam water levels from January, 1998-December, 2007

(2) Cost per order and cost per unit energy as at year 2008

The Akosombo dam water levels were measured in feet while the hydro-electric energy was also measured in Giga-watt hour.

Figure 3.1 displays the trajectory of the water level data from January 1998-December, 2007

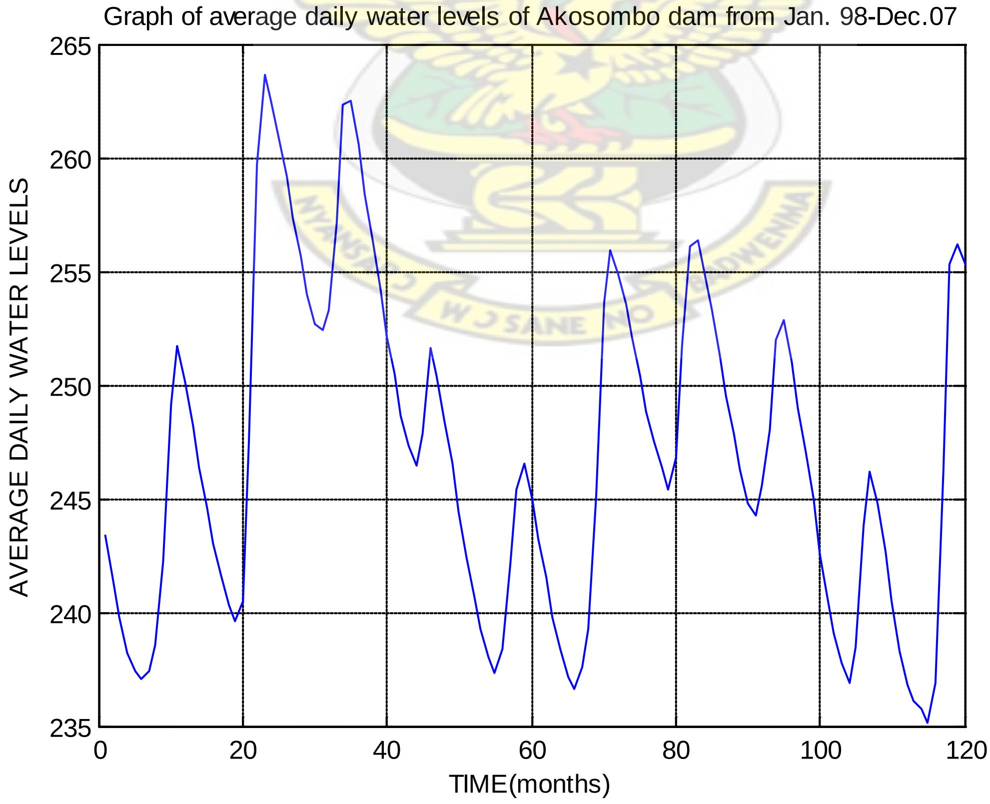

Fig 3.1: Trajectory of water level for Akosombo Dam

The visual pattern of the water levels as shown in figure 3.1 is indicative of stationarity. There were periods especially between 2^{nd} -9^{th} months, 52^{nd}-57^{th} months, 63^{rd}-69^{th} months, 102^{nd}-105^{th} and 109^{th}-116^{th} months as shown in the figure 3.1 that the water levels were 240feet or below 240feet indicating stockout period. Limiting stockout is one of the main objectives of our case study. Between periods of 50th-65th months, 96th-115th months etc, the water levels observed to be below 250feet. These unfavourable dam water levels might have caused by severe drought or untimely water release scheduling mechanism of the Akosombo dam. These might have also resulted in energy load-shedding.

3.2 ENERGY DEMAND DATA DESCRIPTION

The energy demand data used for the data (see appendix B) was also collected from the Transmission System Department, Volta River Authority. The data is daily time series energy demand generation from January, 2000-December, 2007.

The figure 3.2 displays the trajectory of the energy demand data from January, 2000-December, 2007

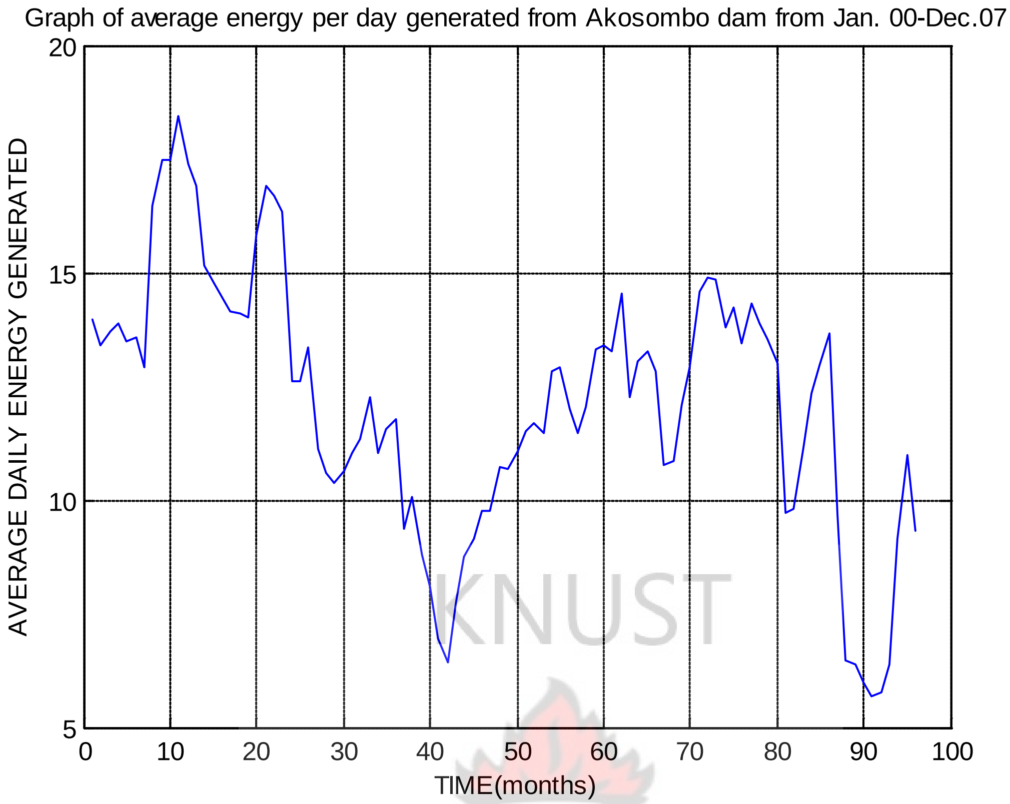

Fig 3.2: Trajectory of energy demand data for Akosombo dam

The visual pattern of energy demand data of the Akosombo dam as shown in figure 3.2 is of similar pattern as that of the water levels shown in figure 3.1. The pattern is periodic and stationary in the mean. It is observed that at low water level, low quantity of energy were generated. This shows linear relation among the energy demands and the corresponding water levels that generate them. It is also therefore a clear indication that the optimal hydro-electric generations often depend on the environmental conditions that dictate the amount of water to be released to generate energy. The figure again confirms the fact that energy generation from Akosombo dam depends on the height of water in the reservoir. It is then of major importance to control the water levels sustainably.

3.3 FITTING OF AR (1) MODEL TO WATER LEVEL DATA

The AR(1) model is in the form

$$x_t = a_0 + a_1 x_{t-1} + \varepsilon_t$$

where x_{t-1} is the immediate past time point to x_t, x_t is the point yet to be forecasted and ε_t is the white noise which is a set of errors with zero mean and constant variance. Gottman (1981)

3.3.1 Ordinary Least Squares Estimate of the Model

With reference to the average monthly water levels displayed in APPENDIX C(i), we should be able to construct AR(1) model to forecast the monthly water level at any time.

Data vector X_2 [See APPENDIX C(i)] refers to the 119 by 1 dimensional vector of predictable average daily water levels on monthly basis of Akosombo dam from January 1998-December 2007.

Data vector X_1 [See APPENDIX C(i)] refers to the 119 by 1 dimensional vector of corresponding immediate past average daily water levels on monthly basis to X_2 of Akosombo dam from January 1998-December 2007.

We regress X_2 on X_1 and the general regression model is $X_2 = a_0 + a_1 X_1 + \varepsilon_2$ where ε_2 represent all unexplained variations in X_2 caused by important but omitted variables such as vapourization etc.

NB: All the MATLAB outputs of the computations of the results below are displayed in APPENDIX C(i)

The parameters a_0 and a_1 are unknown and must be estimated using the sample data in APPENDIX C(i).

To estimate the model parameters, first calculate $S_{X_1X_1} = \sum_{i=1}^{119}(X_{1i})^2 - \frac{\left(\sum_{i=1}^{119} X_{1i}\right)^2}{119}$ which is the corrected sum of squares of X_1.

That is $S_{X_1X_1}$ = (sum of squares of the elements in X_1)-(product of mean of X_1 and sum of X_1)

= 6157.5

We again calculate $S_{X_1X_2} = \sum_{i=1}^{119} X_{1i}X_{2i} - \frac{\sum_{i=1}^{119} X_{1i}\sum_{i=1}^{119} X_{2i}}{119}$ which is the corrected sum of cross-products of X_1 and X_2.

$$S_{X_1X_2} = \left(\begin{array}{l}\textit{sum} \text{ of the product of the corresponding} \\ \text{elements in } \mathrm{X}_1 \text{ and } \mathrm{X}_2\end{array}\right) \text{-} \left(\frac{\textit{product} \text{ of sum of } \mathrm{X}_1 \text{ and sum of } \mathrm{X}_2}{119}\right)$$

= 5703.6

The slope a_1 is the change in the mean of the distribution X_2 produced by a unit change in X_1.

$$\hat{a}_1 = \frac{S_{X_1X_2}}{S_{X_1X_1}}$$

$\hat{a}_1$ = 0.9263

$\hat{a}_1$ is the unbiased estimator of a_1.

$\hat{a}_0$ = (mean of X_2)- $\hat{a}_1$ (mean of X_1)

$\hat{a}_0$ = 18.3004

The intercept $\hat{a}_0$ is the mean of the distribution of the response X_2 when X_1=0. If the range of X_1 does not include zero, then $\hat{a}_0$ has no practical interpretation. Montgomery (1982)

The fitted model is $\hat{X}_2 = \hat{a}_0 + \hat{a}_1 X_1$. The error ε_2 = (X_2 - $\hat{X}_2$). The mean of the errors is 1.8391x10^-14

The error indicates the residuals which is the observed values (X_2) minus the fitted values ($\hat{X}_2$).

Table 3.3: Some errors of fitted AR(1) model

Error type	AR(1)
mean error (ME)	1.839x10^-14
MSE	3.2474
MAE	1.0332
absolute maximum error	1.352
absolute minimum error	0.0337

Therefore, the 1st order fit of AR (1) model for Akosombo dam water level is:

$\hat{x}_t = 18.3004 + 0.9263x_{t-1}$

The model reasonably fits the data due to its negligible mean absolute error, minimum absolute error and maximum absolute error shown in table 3.3.

3.3.1.1 Stationarity Test

If $|a_1| < 1$, then the AR(1) fitted model is stationary. GOTTMAN (1981)

From the analysis, $|0.9263| < 1$. Hence model is stationary.

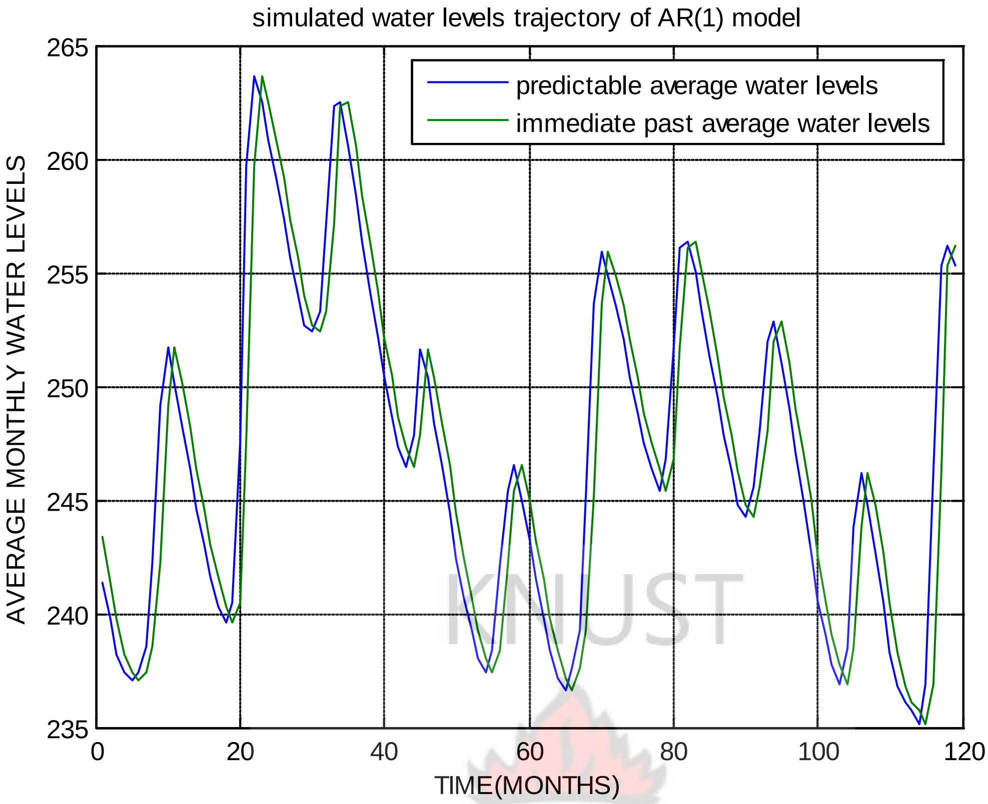

Fig 3.3: Simulated water levels trajectory of AR(1) model

3.4 FITTING OF AR(2) MODEL TO WATER LEVEL DATA

The linear least squares AR(2) fitting of Akosombo dam water levels gives rise to another constructive time series stationary model for forecasting Akosombo dam water levels called the AR(2) fitted model to Akosombo dam water levels. The sample data used are average daily water levels on monthly basis of Akosombo dam from January 1998-December 2007 [See APPENDIX C(ii)].

The AR(2) model is in the form $x_t = a_0 + a_1 x_{t-1} + a_2 x_{t-2} + \varepsilon_t$ where x_{t-1} and x_{t-2} are 2nd and 1st immediate past time points to x_t respectively and x_t is time point yet to be forecasted.

3.4.1 Ordinary Least Squares Estimate Of The Model

The AR(2) model is a multiple regression model of two independent variables. The general regression model is $X_3 = a_0 + a_1 X_2 + a_2 X_1 + \varepsilon_3$.

Data vector X_3 [See APPENDIX C(ii)] is 118 by 1 dimensional vector of water levels yet to be predicted.

Data vector X_2 [See APPENDIX C(ii)] is 118 by 1 dimensional vector of 2nd immediate past water levels to X_3.

Data vector X_1 [See APPENDIX C(ii)] is 118 by 1 dimensional vector of 1st immediate past water levels to X_3.

The regression coefficients are a_0, a_1 and a_2.

NB: All the MATLAB computations of the results below are displayed in APPENDIX C(ii).

X^* is the regression matrix of X_2 and X_1 shown by MATLAB results in APPENDIX C(ii).

To obtain the regression coefficients, divide X^* by the response vector X_3. Let b be the 3 by 1 vector of the regression coefficients, that is b = $\begin{pmatrix} \hat{a}_0 \\ \hat{a}_1 \\ \hat{a}_2 \end{pmatrix}$ then from MATLAB (See appendix C(ii))

b = $X^* \backslash X_3$

$$= \begin{pmatrix} 34.0416 \\ 1.6039 \\ -0.7416 \end{pmatrix}$$

$\hat{X}_3$ is the least squares fitted model.

$\hat{X}_3 = X^* \mathrm{b}$

To validate the model, we find the mean absolute value, maximum and minimum absolute values of the deviation of the data from the model.

Therefore, the linear least-square fitted AR(2) model for Akosombo dam water levels is

$$\hat{x}_t = 34.0416 + 1.6039 x_{t-1} - 0.7416 x_{t-2}$$

Montgomery (1982), Gottman (1981), Neuman (2006)

Table 3.4: Some errors of fitted AR(2) model shown by MATLAB output in appendix C(ii)

Error type	AR(2)
ME	-1.1321x10^-13
MSE	7.9561
MAE	1.0661
absolute maximum error	1.845
absolute minimum error	0.0660

3.4.1.1 Stationarity Test

$-1 < a_2 < 1$

$a_2 + a_1 < 1$ GOTTMAN (1981)

$a_2 - a_1 < 1$

From the analysis,

-1<-0.7416<1, (-0.7416 + 1.6039)<1, (-0.7416-1.6039)<1

Hence, the AR (2) fitted model above is stationary.

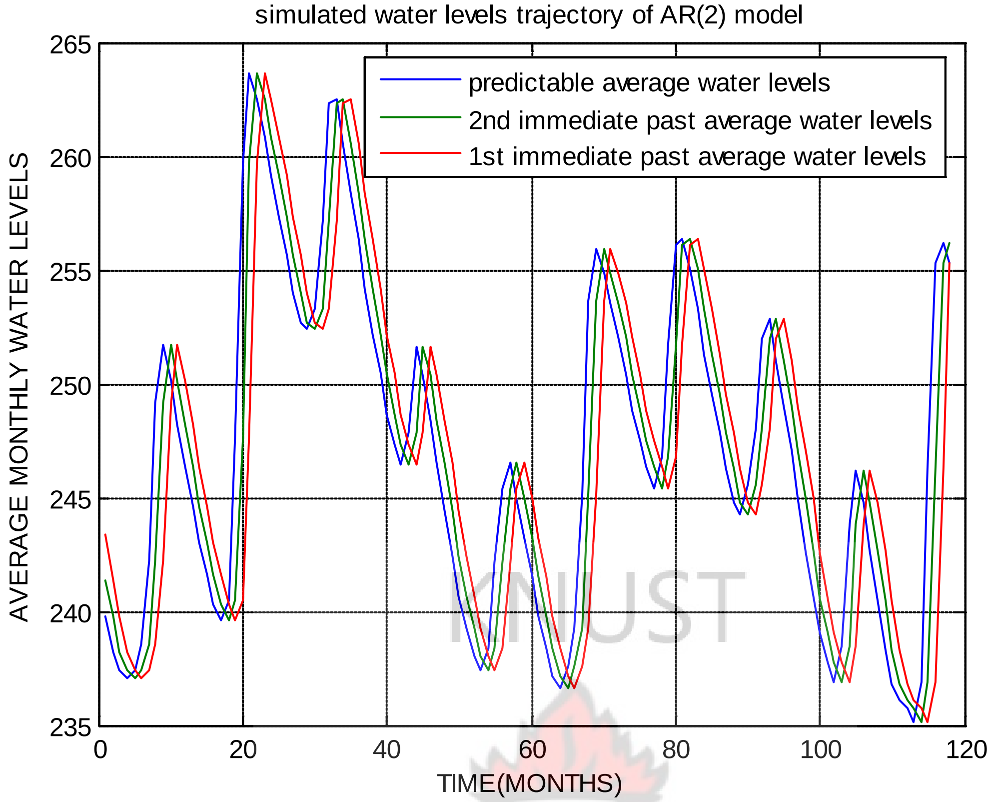

Fig 3.4: Simulated water levels trajectory of AR(2) model

3.5 (EOQ) AND LOT-SIZE MODELS ANALYSIS

3.5.1 Introduction

Our aim is to determine the optimal quantity of energy that should be generated from Akosombo dam and when to generate it at minimum cost. Again, average quantity of water that should be scheduled for release and period for release should also be determined. The associated number of cycles per year could then be determined. As visualized in figure 3.1 and figure 3.2, the energy output depends on the height of the water level. Data used for this modeling are the average monthly energy demand derived using regression analysis from the corresponding average monthly water levels from January 2000-December 2007 (See APPENDIX D).

3.5.2 Regression of Energy demand on Water level

Let the data vector E represent average monthly energy generation data from January, 2000-December, 2007 (See APPENDIX D)

Let the data vector X represent average monthly water level data from January 2000-December 2007 (See APPENDIX D)

We regress E on X to construct regression model relating energy generated to water level and also to find the expected change in E per unit change in X. The linear regression model is $E = \beta_0 + \beta_1 X + \varepsilon_E$ where ε_E represents all unexplained variations in E caused by important but omitted variables.

NB: All the MATLAB outputs computations of the results of the regression are displayed in APPENDIX D

3.5.2.1 Least Squares Estimation of β_0 And β_1

To estimate model parameters, first calculate the corrected sum of squares of X denoted by S_{XX} .

$$S_{XX} = \sum_{i=1}^{96} X_i^{\,2} - \frac{\left(\sum_{i=1}^{96} X_i\right)^2}{96}$$

S_{XX} = (sum of squares of the elements in X)-(product of mean of X and sum of X)

= 4745.8

Also, we calculate S_{XE} which is the corrected sum of cross-products of X and E.

$$S_{XE} = \sum_{i=1}^{96} X_i E_i - \frac{\sum_{i=1}^{96} X_i \sum_{i=1}^{96} E_i}{96}$$

$$S_{XE} = \left(\begin{array}{l} \textit{sum} \text{ of the product of} \\ \text{corresponding elements in X and E} \end{array}\right) - \left(\frac{\textit{product} \text{ of sum of X and sum of E}}{96}\right)$$

= 1169.2

$\hat{\beta}_1$ and $\hat{\beta}_0$ are unbiased estimators of β_1 and β_0 respectively where $\hat{\beta}_1$ is the expected change in E per unit change in X.

$$\hat{\beta}_1 = \frac{S_{XE}}{S_{XX}}$$

$\hat{\beta}_1$ = 0.2464

$\hat{\beta}_0$ =(mean of E)- $\hat{\beta}_1$ (mean of X)

$\hat{\beta}_0$ =-48.8348

Therefore, fitted model relating energy to water level is $\hat{E}$ =-48.8348 + 0.2464X

The error ε_E represents all unexplained variations in E.

ε_E =E- $\hat{E}$ computed and suppressed in APPENDIX D. The mean of the errors is -1.9429x10^-15. Montgomery (1982)

Hypothesis testing on the slope (β_1)

Suppose that we wish to test hypothesis on the slope, the appropriate hypothesis are

$H_0 : \beta_1 = 0$

$H_1 : \beta_1 \neq 0$ where H_0 is the null hypothesis and H_1 is the alternative hypothesis.

Confidence interval is given by $\hat{\beta}_1 \pm t_{\alpha/2} S_\beta$ where S_β is the standard error of $\hat{\beta}_1$. For 95% confidence interval, α = 0.05 and $\alpha/2$ = 0.025

Then using student's t-distribution table, $t_{\alpha/2}$ = 0.025 = 1.96

The error sum of squares $SS_E = \sum_{i=1}^{96}\left(E - \hat{E}\right)^2$ = 487.1935

$s_e^2 = MS_E = \frac{SS_E}{(96-2)}$ = 5.1829 where s_e is the standard error of the regression

s_e = 2.2766

$S_\beta = \frac{s_e}{(sqrt(S_{XX})}$ = 0.0330

The standard error 0.0330 shows that the fitted model $\hat{E}$ = -48.8348 + 0.2464X indicates a strong relation between the energy demand and the water level that generate it.

$t_{cal} = \frac{\hat{\beta}_1 - 0}{S_\beta}$ = (0.2464-0)/0.0330 = 7.4550

Since $\left|t_{cal}\right|$ = 7.4550>1.96, we reject H_0 and accept H_1.

Therefore, 95% confidence interval is given by

$\hat{\beta}_1 \pm t_{0.025} S_{\beta}$ = 0.2464 ± 1.96 (0.0330)

= [0.1816, 0.3111]

Montgomery (1982)

3.5.3 Determination of Optimal Order Energy Demand Ordered Per Cycle, Number of Orders(N) And Cycle Period (T) Using EOQ and Lot-size Models

Let Q_m be the optimal energy demand per order. This means that in accordance with the EOQ and Lot-size models, the optimal energy to be generated within cycle period T is Q_m.

Let the mean of E be represented by μ and is given by μ = 12.1210GWh from MATLAB output (see appendix D).

s is the standard deviation of E which measures the spread of the distribution of E values about the least squares line. Hence we expect most of the observed values to lie within 2s of their respective least squares predicted values $\hat{E}$.

s = 2.8566

Let D represent the expected annual energy generated. Working days per year is 365days.

Therefore D = 365 μ

D =

4.4242e+003GWh

Let m be the cost per unit energy received from VRA sub-station in Kumasi.

m = \$69000

C_0 is the cost per order. This cost is fixed regardless of the order quantity.

C_0 =\$109090

C_h is the annual holding cost per unit

$C_h = \hat{\beta}_1$ m

$C_h = \$1.6999e^{+004}$

(i) using the EOQ model:

$$Q_m = \sqrt{\frac{2DC_0}{C_h}}$$

Q_m =

238.2920GWh

Let N represent number of orders to be placed annually and T represent average number of working days between orders per year. Then in accordance with the EOQ model,

N = D/ Q_m

N =

18.5662

T=365/N

T =

19.6594days

Therefore, 238.2920GWh should be generated almost every three weeks at minimum cost conditions according to the EOQ model. The cycle should be repeated almost 19 times a year. We verify below that the total holding cost is equal to the total ordering cost as is expected in the EOQ model.

Let h be the total holding cost per year

h= (Q_m /2)* C_h

h =

$2.0254e+006

Let O be the total ordering cost per year

O= (D/Q_m)*C_0

O =

$ 2.0254e+006

This shows that at optimal, the total annual holding cost = total annual ordering cost. Taylor (2006), Anderson (2004)

3.5.3.1 Service Levels And Stockout Probabilities

Using the normal probability distribution, we let the normal variate be M. Therefore, M = μ + zs where μ = mean of distribution

s = standard deviation of the distribution

z = number of standard deviation necessary to obtain the acceptable stockout probability

When z = 0, then the normal curve is symmetrical at M = μ

z =1 implies that approximately 68% of the distribution lies within one standard deviation of the mean. When z=2, then approximately 95% of the distribution lies within 2 standard deviation of the mean.

Taylor (2006)

From the energy demand data analysis in section 3.5.2.1, μ = 12.1210 and s = 2.8566.

Therefore M = 12.1210 + 2.8566z. The least squares model relating energy to water level was determined to be $\hat{E}$ = -48.8348 + 0.2464X.

If z=0, M =12.1210. It implies that

P(M>12.1210) = P($\frac{M-12.1210}{2.8566}$)>$\left(\frac{12.1210-12.1210}{2.8566}\right)$

$= P\ (z>0)$

$= 1 - P(z \leq 0)$, from normal distribution table, $P(z \leq 0) = 0.5$

$= 1 - 0.5 = 0.5$

This shows that from EOQ results in section 3.5.3, $Q_m = 238.2920$GWh should be ordered per cycle at 50% service level.

(ii) using Lot-size model,

$$Q_m = \sqrt{\frac{2DC_0}{(1-0.5)C_h}}$$

$$= \frac{238.2920}{\sqrt{(1-0.5))}} = 336.99578\text{GWh}$$

$$N = \frac{D}{Q_m} = 4424.2/336.99578$$

$$= 13.12835$$

T = 365/N = 27.80days

According to Lot-size model, 336.99578GWh should be generated every 28days at 50% service level.

If z=1, M = 12.1210 + 2.8566(1) = 14.9776GWh

Using the energy-water level model, if $\hat{E} = 14.9776$, the corresponding average water level that could generate it should be $X = \left(\frac{14.9776 + 48.8348}{0.2464}\right) = 258.98$feet

$$P(M>14.9776) = P\left(\frac{M - 12.1210}{2.8566}\right) > \left(\frac{14.9776 - 12.1210}{2.8566}\right)$$

$= P(z>1)$

$= 1 - P(z \leq 1)$ (from normal distribution table, $P(z \leq 1) = 0.8413$)

$= 1 - 0.8413$

$= 0.1587$

By symmetry, $P(z>1) + P(z<-1) = 0.3174$

Therefore, when $z = \pm 1$, area within one standard deviation of the mean is 1-0.3174 = 0.6826.

It proves that approximately 68% of the energy demand distribution lies within one standard deviation of the mean.

If z =2 , M = 12.1210 + 2.8566(2) = 17.8342GWh

If $\hat{E}$ = 17.8342, the corresponding average height of water that could generate it should be

$$X = \left(\frac{17.8343 + 48.8348}{0.2464}\right) = 270.57\text{feet}$$

$$P(M>17.8342) = P\left(\frac{M - 12.1210}{2.8566}\right) > \left(\frac{17.8342 - 12.1210}{2.8566}\right)$$

$$= P(z>2)$$

$$= 1 - P(z \leq 2) \quad \text{(from normal distribution table, } P(z \leq 2) = 0.9772\text{)}$$

$$= 1 - 0.9772$$

$$= 0.0228$$

By symmetry, $P(z>2) + P(z<-2) = 0.0228 + 0.0228$

$$= 0.045$$

Therefore when $z = \pm 2$, area within 2 standard deviation of the mean is 1-0.045 = 0.9544. It means that approximately 95% of the energy demand distribution lies within 2 standard deviation of the mean.

Table 3.5.3.1: categories of service and corresponding energy demand

z	service level	energy demand	normal water level
1	68%	14.9776GWh	258.98feet
2	95%	17.8342	270.57feet

Table 3.5.3.1 shows that at any particular day that VRA should generate energy at 68% service level, the average of 14.9776GWh should be generated at about 259feet height of water. Similarly, at 95% service level, average of 17.8342GWh should be generated at 270.57feet of water level. This clearly shows that should VRA generate average of 12.1210GWh daily throughout a year at sustainable level of water as shown at section 3.5.3, then the service level for the energy demand should be below 68% annually.

Service level = 1-P(stockout)

Taylor (2006)

From queue theoretic model, P(stockout) = 1- ρ where ρ is the traffic intensity. Lui (1999)

This implies that service level = 1-(1- ρ) = ρ

<u>Determination of optimal order energy demand ordered per cycle at 68% and 95% using EOQ and Lot-size models</u>

(a) <u>68% service level</u>

Let the traffic intensity (ρ) = 0.68, then P(stockout) = 1-0.68 = 0.32

Stockout percent (32%) means that no queue of water in the dam at 32% annually and therefore VRA could not serve customers at 32% throughout a year.

From table 3.5.3.1, average daily demand at 68% should be 14.9776GWh. Then the expected annual energy demand (D) =365x14.9776 = 5466.824GWh

(i) using EOQ model:

$$Q_m = \sqrt{\frac{2DC_0}{C_h}} = \sqrt{\frac{2(5466.824)(109090)}{16999}} = 264.888\text{GWh}$$

$$N = \frac{D}{Q_m} = \frac{5466.824}{264.888} = 20.638$$

$$T = \frac{365}{N}$$

T = 17.686days

(ii) using Lot-size model:

$$Q_m = \sqrt{\frac{2DC_0}{(1-\rho)C_h}}$$

$$= \frac{264.888}{\sqrt{(1-0.68)}}$$

= 468.261GWh

$$N = \frac{D}{Q_m} = \frac{5466.824}{468.261}$$

= 11.6747

$$T = \frac{365}{N} = 31.264\text{days}$$

Therefore, at 68% service level, the optimal energy demand that should be scheduled to be generated at sustainable water level should be either 264.888GWh within 18days or 468.261GWh within 31days respectively. According to EOQ and Lot-size models, the cycle generation of energy should be one of the best methods to minimize total inventory cost. The cycle should be repeated almost 21 times annually for the EOQ and 12 times a year for the Lot-size.

(b) <u>95% service level</u>

Let the traffic intensity (ρ) = 0.95

Therefore, P(stockout)=1-0.95

= 0.05

Stockout percent (5%) only means that VRA could not serve their customers at 5% throughout a year. This service could not sustain the dam as water level could be below minimum level (240feet) about 2/3 of a year.

From table 3.5.3.1, average daily energy demand at 95% service level should be 17.8342GWh. Therefore, the expected annual energy demand (D) = 365x17.8342= 6509.483GWh

(i) using EOQ model:

$$Q_m = \sqrt{\frac{2DC_0}{C_h}}$$

$$Q_m = \sqrt{\frac{2(6509.483)(109090)}{16999}}$$

$$= 289.0474\text{GWh}$$

$$N = \frac{D}{Q_m} = \frac{6509.483}{289.047}$$

$$= 22.5205$$

$$T = \frac{365}{N} = 16.207\text{days}$$

(ii) using Lot-size model:

$$Q_m = \frac{289.0474}{\sqrt{1-0.95}}$$

$$Q_m = 1292.66\text{GWh}$$

$$N = \frac{D}{Q_m} = \frac{6509.483}{1292.66}$$

$$= 5.03573$$

$$T = \frac{365}{5.03573} = 72.48\text{days}$$

Therefore at 95% service level, either 289.0474GWh or 1292.66GWh should be scheduled to be generated within 16days or 72days respectively. The cycle should be repeated almost 23 times a year according to the EOQ model or 5 times a year according to the Lot-size model respectively.

<u>Determination of optimal release quantity of water released per cycle using periodic review, EOQ and lot-size models</u>

(a) <u>68 percent service level</u>

Let q be the order quantity at review period. Then from periodic review inventory, the model is q = replenishment level – review level

From table 3.5.3.1, replenishment level at 68% service level is 258.98feet of water. The expected review level for our analysis should be the average water level. The average water level is the water level that generates the mean energy 12.1210GWh. Using

$\hat{E} = -48.8348 + 0.2464X$, when $\hat{E}$ = 12.1210GWh, X = 247.385552feet of water level.

Therefore,

q = 258.98-247.38

= 11.6feet Taylor (2006)

Therefore, average monthly quantity of 11.6feet should be released to generate energy.

Let p be the average monthly quantity of water that queue in the dam assuming 240feet is zero. Then from data X_1 (see appendix D), p = 7.9108feet of water. Data X_1 is a vector of quantity of water that queue in the dam from January 2000-December 2007. From queue theory, service level (traffic intensity) is given by $\rho = \frac{p}{q}$

$$= \frac{7.9108}{11.6}$$

$$= 0.682$$

P(stockout) = 1-0.682

$$= 0.318$$

The expected annual release quantity (D) that should be scheduled for release is given by

D = 12x11.6

= 139.2feet of water

(i) using EOQ model:

$$Q_m = \sqrt{\frac{2DC_0}{C_h}} = \sqrt{\frac{2(139.2)(109090)}{16999}}$$

$$= 42.268\text{feet}$$

$$N = \frac{D}{Q_m}$$

$$= \frac{139.2}{42.268}$$

$$= 3.2933$$

$$T = \frac{12}{3.2933}$$

= 3.64376

= 3.64376x30days

T = 109.31days

According to EOQ model, average daily of water that should be released from the dam is 0.3867feet. If 7.9108feet of water queues in the dam monthly, then on daily basis 0.2637feet queues in the dam. Assuming water queues for 109days, then 28.7433feet should queue in the dam. The average water level should then be 240 + 28.7433 =268.7433feet. If release scheduling begins on the 109th day through another 109days, then average water level after release should be 268.7433-42.268 = 226.4753feet. In this case stockout should be 240feet-226.4753 = 13.5247feet. 240feet is assumed to be the minimum water level. Taylor (2006)

Therefore, stockout probability = $\frac{13.5247}{42.268}$

$= 0.31997$

The above explanation is shown in figure 3.5.3

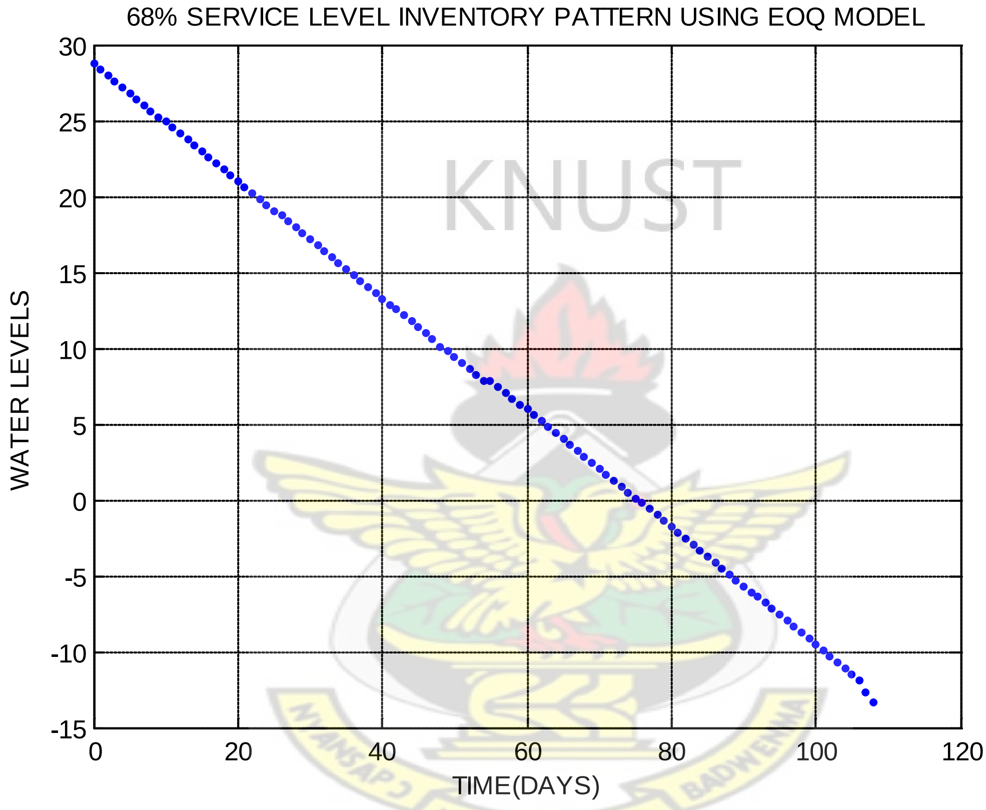

Fig 3.5.3: Trajectory of optimal release scheduling pattern at 68% service using EOQ model

Note that the origin ‘0’ as shown on figure 3.5.3 indicates 240feet water level.

Stockout probability = $\frac{13.5247}{42.268}$

$= 0.31997$

It has been shown according to EOQ model that VRA could release 42.268feet of water to generate energy at 68% service level every 109days. There could be no queue of water at 32% annually and therefore VRA could not serve customers energy 32% throughout the year.

(ii) using lot-size model:

$$Q_m = \frac{42.268}{\sqrt{1-0.68}}$$

Q_m = 74.720feet

$$N = \frac{D}{Q_m}$$

$$= \frac{139.2}{74.720}$$

= 1.8630

$$T = \frac{12}{N} = 6.4412$$

T = 6.4412x30days

T = 193.24days

According to Lot-size model, 74.720feet of water should be scheduled for release within 193days at 68% service level. Taylor (2006)

CHAPTER FOUR

SUMMARY OF FINDINGS, CONCLUSION AND RECOMMENDATIONS

4.0 INTRODUCTION

This chapter talks about various findings our research has so far achieved. We should also state our conclusions and recommendations to the stakeholders and managers of Akosombo dam.

4.1 SUMMARY OF FINDINGS

4.1.1 The Trajectories of Water Levels Data and Energy Demand Data of Akosombo Dam

The trajectories of water levels and energy demand data of Akosombo dam are both periodic and stationary in the mean. They are linearly related. The quantity of energy generated from the dam at any period depends on the height of the water in the dam. The higher the water level, the higher the energy generation and vice-versa. The optimal hydro-electric generations often depend on the environmental conditions that dictate the amount of water to be released to generate energy.

4.1.2 AR(1) and AR(2) Models for Akosombo Dam

AR(1) and AR(2) models were constructed for forecasting Akosombo dam water levels. Both models are useful and efficient because of negligible errors.

AR(1) model: $\hat{x}_t = 18.3004 + 0.9263x_{t-1}$

AR(2) model: $\hat{x}_t = 34.0416 + 1.6039x_{t-1} - 0.7416x_{t-2}$

4.1.3 EOQ, Lot-size and Periodic Review Inventory Models Analysis

The fitted model relating energy to water level was found to be $\hat{E} = -48.8348 + 0.2464X$. According to EOQ and Lot-size models, the following generation of energy from the Akosombo dam is feasible and could sustain the dam:

(1) Average of 238.2920GWh of energy should be generated every three weeks or 336.9957GWh should be generated every 28days at 50% service level respectively.

(2) At 68% service level, the optimal energy demand that should be scheduled to be generated should either be 264.888GWh every 18days or 468.261GWh every 31days.

According to periodic review inventory, EOQ and Lot-size models, the following release of water from the dam is feasible and sustainable:

(1) According to EOQ model, average of 42.268feet of water should be released from the dam every 109.31days at 68% service level.

(2) According to Lot-size model, 74.720feet of water should be scheduled for release every 193days at 68% service level.

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4.2 CONCLUSION

The AR(1) model $\hat{x}_t = 18.3004 + 0.9263x_{t-1}$ and AR(2) model

$\hat{x}_t = 34.0416 + 1.6039x_{t-1} - 0.7416x_{t-2}$ have been constructed for forecasting Akosombo dam water levels at any period.

VRA could generate 238.2920GWh every 3 weeks or 336.995GWh every 28days at 50% service level. The authority could also generate 264.888GWh every 18days or 468.261GWh every 31days respectively at 68% service level. VRA again could release 42.268feet of water every 109.31days or 74.720feet of water every 193days respectively at 68% service level. Service at 95% is not feasible.

4.3 RECOMMENDATIONS

Based on the findings so far arrived at, in order to ensure proper running of the Akosombo dam, the following recommendations are made:

(1) The stakeholders and managers of the dam should use the time series models $\hat{x}_t = 18.3004 + 0.9263x_{t-1}$ or $\hat{x}_t = 34.0416 + 1.6039x_{t-1} - 0.7416x_{t-2}$ for forecasting Akosombo dam water levels.

(2) Inventory and queue models should be used for Akosombo dam energy generation and water level scheduling.

(3) The release scheduling process should be in rainy season.

(4) The water level during release scheduling period should be above 250feet to reduce the probability of water level going below 240feet

(5) In this research, we used discrete markov process (discrete queue and inventory markov chains) simulation technique. There should be further research study using

continuous markov process simulation techniques such as stochastic optimal control models and AR(X) models.

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4.5 APPENDIX A

1998

DATE	JAN.	FEB.	MARCH	APRIL	MAY	JUNE
1	244.55	242.24	240.60	238.85	237.74	236.98
2	244.50	242.16	240.54	238.80	237.72	236.95
3	244.42	242.08	240.50	238.74	237.70	236.93
4	244.34	242.02	240.46	238.68	237.67	236.93
5	244.25	241.96	240.40	238.62	237.65	236.93
6	244.17	241.88	240.36	238.58	237.65	236.93
7	244.09	241.82	240.32	238.54	237.65	236.93
8	244.00	241.77	240.26	238.50	237.63	236.93
9	243.92	241.70	240.20	238.47	237.60	236.93
10	243.84	241.64	240.15	238.42	237.57	236.96
11	243.77	241.59	240.08	238.38	237.55	236.98
12	243.68	241.52	240.00	238.32	237.52	236.98
13	243.60	241.46	239.92	238.28	237.49	236.98
14	243.52	241.40	239.85	238.25	237.46	237.00
15	243.43	241.35	239.77	238.22	237.42	237.00
16	243.34	241.30	239.72	238.20	237.38	237.03
17	243.26	241.25	239.67	238.15	237.36	237.05
18	243.20	241.20	239.63	238.10	237.34	237.05
19	243.13	241.15	239.58	238.06	237.34	237.05
20	243.07	241.08	239.53	238.03	237.34	237.03
21	242.98	241.00	239.48	238.00	237.32	237.03
22	242.92	240.95	239.42	237.98	237.29	237.06
23	242.86	240.90	239.38	237.96	237.26	237.10
24	242.80	240.86	239.30	237.92	237.24	237.15
25	242.72	240.82	239.24	237.89	237.22	237.15
26	242.64	240.78	239.16	237.86	237.18	237.18
27	242.55	240.72	239.10	237.83	237.15	237.20
28	242.48	240.66	239.04	237.81	237.12	237.20
29	242.42		238.98	237.78	237.10	237.25
30	242.35		238.93	237.76	237.06	237.25
31	242.30		238.90		237.02	
TOTAL	7545.08	6759.26	7432.47	7146.96	7359.74	7111.14
MEAN	243.39	241.40	239.76	238.23	237.41	237.04

1998

DATE	JULY	AUG.	SEPT.	OCT.	NOV.	DEC.
1	237.28	237.85	239.98	245.45	251.80	251.00
2	237.30	237.85	240.10	245.75	251.90	250.95
3	237.30	237.85	240.25	246.00	251.95	250.90
4	237.30	237.90	240.35	246.25	251.95	250.84
5	237.32	237.98	240.50	246.55	251.98	250.78
6	237.32	238.02	240.60	246.90	251.98	250.72
7	237.30	238.04	240.75	247.10	251.98	250.67
8	237.30	238.04	240.90	247.30	252.00	250.62
9	237.32	238.04	241.10	247.60	252.00	250.56
10	237.35	238.08	241.20	247.90	252.00	250.52
11	237.35	238.08	241.30	248.25	251.97	250.48
12	237.37	238.08	241.50	248.55	251.95	250.42
13	237.37	238.12	241.60	248.80	251.92	250.36
14	237.37	238.15	241.80	249.10	251.90	250.30
15	237.37	238.20	242.00	249.42	251.87	250.25
16	237.32	238.25	242.15	249.60	251.84	250.20
17	237.32	238.32	242.30	249.80	251.80	250.14
18	237.32	238.40	242.45	250.05	251.75	250.08
19	237.32	238.50	242.60	250.30	251.70	250.02
20	237.30	238.62	242.80	250.50	251.65	249.95
21	237.30	238.70	243.00	250.60	251.58	249.89
22	237.35	238.85	243.30	250.78	251.53	249.82
23	237.40	239.00	243.50	250.95	251.47	249.76
24	237.45	239.08	243.65	251.10	251.40	249.68
25	237.50	239.20	243.90	251.25	251.34	249.58
26	237.55	239.28	244.15	251.40	251.28	249.50
27	237.60	239.38	244.30	251.50	251.22	249.44
28	237.68	239.48	244.50	251.60	251.16	249.35
29	237.74	239.55	244.85	251.65	251.10	249.28
30	237.80	239.76	245.25	251.70	251.05	249.22
31	237.85	239.88		251.74		249.17
TOTAL	7359.75	7394.53	7266.63	7725.44	7551.02	7754.45
MEAN	237.41	238.53	242.22	249.21	251.70	250.14

1999

DATE	JAN.	FEB.	MARCH	APRIL	MAY	JUNE
1	249.12	247.13	245.54	243.78	242.38	240.83
2	249.06	247.07	245.48	243.72	242.32	240.80
3	249.00	247.00	245.42	243.68	242.26	240.77
4	248.94	246.95	245.35	243.62	242.20	240.73
5	248.86	246.90	245.30	243.56	242.15	240.70
6	248.82	246.85	245.24	243.50	242.08	240.67
7	248.77	246.80	245.16	243.46	242.02	240.63
8	248.72	246.74	245.08	243.42	241.95	240.58
9	248.67	246.67	245.00	243.37	241.90	240.54
10	248.62	246.59	244.93	243.32	241.85	240.50
11	248.56	246.51	244.88	243.26	241.80	240.45
12	248.50	246.44	244.82	243.20	241.74	240.42
13	248.42	246.37	244.74	243.13	241.68	240.38
14	248.32	246.30	244.68	243.08	241.63	240.35
15	248.27	246.24	244.62	243.04	241.60	240.32
16	248.20	246.20	244.57	243.00	241.56	240.28
17	248.14	246.16	244.52	242.96	241.52	240.23
18	248.08	246.12	244.46	242.93	241.48	240.20
19	248.02	246.08	244.40	242.88	241.45	240.15
20	247.95	246.04	244.35	242.83	241.41	240.10
21	247.87	246.00	244.30	242.80	241.37	240.05
22	247.80	245.95	244.25	242.75	241.32	240.00
23	247.72	245.90	244.20	242.70	241.26	239.95
24	247.65	245.85	244.15	242.66	241.22	239.90
25	247.60	245.78	244.10	242.63	241.16	239.86
26	247.52	245.72	244.06	242.58	241.12	239.82
27	247.45	245.65	244.04	242.52	241.06	239.78
28	247.38	245.58	243.98	242.48	241.00	239.74
29	247.30		243.90	242.46	240.97	239.70
30	247.25		243.87	242.43	240.92	239.67
31	247.20		243.82		240.88	
TOTAL	7693.78	6897.59	7583.21	7291.81	7489.26	7208.1
MEAN	248.19	246.34	244.62	243.06	241.59	240.27

1999

DATE	JULY	AUGUST	SEPT.	OCT.	NOV.	DEC.
1	239.63	239.64	242.55	254.10	263.60	263.20
2	239.60	239.66	242.85	254.55	263.68	263.16
3	239.60	239.70	243.18	255.00	263.73	263.12
4	239.60	239.73	243.40	255.40	263.78	263.07
5	239.58	239.73	243.70	256.00	263.82	263.04
6	239.58	239.73	244.00	256.45	263.85	263.02
7	239.58	239.73	244.30	256.72	263.88	262.98
8	239.58	239.75	244.58	257.20	263.90	262.93
9	239.60	239.80	244.90	257.62	263.90	262.90
10	239.60	239.85	245.20	258.10	263.90	262.85
11	239.60	239.95	245.64	258.45	263.88	262.80
12	239.60	240.05	246.00	258.75	263.88	262.75
13	239.60	240.10	246.30	259.10	263.85	262.70
14	239.60	240.10	246.70	259.45	263.82	262.65
15	239.60	240.12	247.12	259.85	263.80	262.59
16	239.62	240.12	247.45	260.12	263.77	262.53
17	239.62	240.15	247.75	260.50	263.74	262.46
18	239.62	240.20	248.10	260.80	263.70	262.40
19	239.60	240.30	248.64	261.10	263.67	262.35
20	239.58	240.45	249.10	261.46	263.63	262.30
21	239.54	240.60	249.72	261.70	263.60	262.25
22	239.54	240.70	250.05	261.90	263.56	262.20
23	239.54	240.90	250.50	262.20	263.53	262.14
24	239.50	241.00	251.05	262.45	263.50	262.08
25	239.52	241.12	251.45	262.75	263.45	262.02
26	239.52	241.40	251.90	262.95	263.40	261.98
27	239.52	241.50	252.40	263.10	263.35	261.93
28	239.52	241.70	252.80	263.20	263.32	261.88
29	239.54	241.85	253.30	263.32	263.28	261.82
30	239.56	242.10	253.70	263.45	263.24	261.76
31	239.60	242.30		263.50		261.69
TOTAL	7426.95	7454.03	7428.33	8051.24	7910.01	8137.50
MEAN	239.58	240.45	247.61	259.72	263.67	262.5

2000

DATE	JAN.	FEB.	MARCH	APRIL	MAY	JUNE
1	261.62	260.03	258.26	256.42	254.88	253.15
2	261.56	260.00	258.22	256.35	254.84	253.10
3	261.52	259.96	258.16	256.30	254.80	253.06
4	261.48	259.90	258.10	256.26	254.75	253.02
5	261.44	259.84	258.05	256.22	254.70	252.98
6	261.40	259.80	258.00	256.18	254.63	252.95
7	261.35	259.75	257.90	256.14	254.54	252.93
8	261.30	259.70	257.87	256.07	254.46	252.90
9	261.24	259.64	257.83	256.02	254.41	252.85
10	261.18	259.57	257.80	255.98	254.36	252.80
11	261.10	259.50	257.75	255.93	254.32	252.76
12	261.03	259.42	257.70	255.87	254.26	252.73
13	260.98	259.35	257.63	255.84	254.20	252.70
14	260.92	259.27	257.55	255.78	254.13	252.65
15	260.86	259.20	257.47	255.74	254.07	252.60
16	260.82	259.14	257.38	255.70	254.01	252.56
17	260.78	259.08	257.28	255.64	253.95	252.53
18	260.72	259.00	257.16	255.60	253.90	252.50
19	260.66	258.94	257.08	255.56	253.84	252.50
20	260.58	258.88	257.00	255.52	253.78	252.47
21	260.54	258.80	256.95	255.45	253.72	252.45
22	260.50	258.73	256.90	255.40	253.66	252.42
23	260.45	258.66	256.83	255.36	253.60	252.40
24	260.40	258.58	256.78	255.30	253.55	252.40
25	260.36	258.52	256.74	255.24	253.50	252.37
26	260.30	258.46	256.70	255.18	253.45	252.37
27	260.25	258.40	256.67	255.10	253.40	252.33
28	260.18	258.38	256.64	255.02	253.36	252.30
29	260.14	258.30	256.58	254.96	253.32	252.30
30	260.10		256.53	254.92	253.26	252.26
31	260.06		256.48		253.20	
TOTAL	8085.82	7516.8	7978.06	7671.05	7874.85	7579.34
MEAN	260.83	259.20	257.36	255.70	254.03	252.64

2000

DATE	JULY	AUGUST	SEPT.	OCT.	NOV.	DEC.
1	252.24	252.36	255.05	260.20	263.13	261.51
2	252.24	252.36	255.12	260.55	263.10	261.50
3	252.24	252.36	255.20	260.70	263.06	261.44
4	252.26	252.38	255.25	260.90	263.03	261.38
5	252.26	252.40	255.30	261.00	263.01	261.36
6	252.26	252.40	255.36	261.20	262.98	261.28
7	252.30	252.43	255.50	261.40	262.96	261.24
8	252.34	252.47	255.70	261.50	262.93	261.16
9	252.37	252.50	255.80	261.70	262.90	261.10
10	252.40	252.60	255.95	261.90	262.86	261.03
11	252.44	252.70	256.20	262.05	262.80	261.00
12	252.47	252.85	256.40	262.20	262.70	260.93
13	252.50	252.95	256.54	262.32	262.67	260.86
14	252.54	253.10	256.70	262.45	262.62	260.80
15	252.58	253.20	256.80	262.60	262.58	260.73
16	252.60	253.30	256.96	262.70	262.54	260.65
17	252.63	253.40	257.20	262.75	262.48	260.55
18	252.67	253.43	257.50	262.80	262.42	260.50
19	252.67	253.50	257.70	262.90	262.38	260.42
20	252.67	253.60	257.90	262.92	262.34	260.34
21	252.62	253.65	258.20	262.94	262.28	260.26
22	252.59	253.73	258.45	263.00	262.22	260.18
23	252.57	253.85	258.60	263.10	262.14	260.10
24	252.54	253.95	258.80	263.15	262.07	259.02
25	252.51	254.15	259.05	263.20	262.00	259.94
26	252.48	254.25	259.30	263.25	261.94	259.88
27	252.45	254.38	259.44	263.25	261.88	259.83
28	252.42	254.50	259.68	263.25	261.80	259.74
29	252.40	254.65	259.85	263.23	261.72	259.66
30	252.38	254.80	260.00	263.20	261.60	259.58
31	252.36	254.90		263.17		259.50
TOTAL	7826	7853.10	7715.50	8131.48	7875.14	8078.49
MEAN	252.45	253.33	257.18	262.31	262.50	260.60

2001

DATE	JAN.	FEB.	MARCH	APRIL	MAY	JUNE
1	259.46	257.29	255.27	253.08	251.28	249.56
2	259.40	257.22	255.19	253.00	251.23	249.50
3	259.32	257.15	255.10	252.92	251.18	249.44
4	259.27	257.07	255.01	252.86	251.11	249.39
5	259.20	257.02	254.92	252.78	251.04	249.32
6	259.13	256.96	254.86	252.72	251.00	249.26
7	259.04	256.88	254.80	252.67	250.95	249.20
8	258.98	256.80	254.74	252.62	250.90	249.13
9	258.90	256.74	254.68	252.56	250.84	249.07
10	258.82	256.68	254.60	252.50	250.78	249.00
11	258.74	256.62	254.52	252.42	250.73	248.94
12	258.67	256.56	254.45	252.34	250.65	248.89
13	258.60	256.50	254.36	252.28	250.61	248.82
14	258.52	256.45	254.28	252.22	250.56	248.74
15	258.45	256.38	254.23	252.16	250.50	248.66
16	258.38	256.31	254.17	252.12	250.45	248.58
17	258.30	256.24	254.10	252.08	250.40	248.50
18	258.22	256.15	254.02	252.00	250.33	248.43
19	258.14	256.07	253.95	251.93	250.28	248.36
20	258.06	255.99	253.87	251.87	250.23	248.28
21	258.00	255.91	253.80	251.80	250.18	248.21
22	257.95	255.83	253.72	251.75	250.12	248.15
23	257.88	255.75	253.66	251.70	250.06	248.10
24	257.82	255.67	253.58	251.65	250.00	248.05
25	257.76	255.59	253.50	251.58	249.95	248.05
26	257.69	255.51	253.42	251.53	249.89	248.05
27	257.61	255.43	253.36	251.48	249.83	248.00
28	257.57	255.35	253.30	251.42	249.78	247.95
29	257.50		253.24	251.38	249.72	247.92
30	257.43		253.20	251.34	249.67	247.88
31	257.36		253.13		249.60	
TOTAL	8010.17	7178.12	7879.03	7564.76	7763.87	7459.43
MEAN	258.39	256.36	254.16	252.16	250.45	248.65

2001

DATE	JULY	AUGUST	SEPT.	OCT.	NOV.	DEC.
1	247.84	246.78	246.30	250.55	251.44	249.31
2	247.80	246.75	246.33	250.65	251.40	249.25
3	247.77	246.72	246.36	250.85	251.35	249.20
4	247.72	246.68	246.40	251.00	251.29	249.13
5	247.70	246.65	246.45	251.12	251.20	249.07
6	247.65	246.62	246.48	251.28	251.14	249.01
7	247.60	246.60	246.52	251.45	251.05	248.96
8	247.56	246.56	246.57	251.55	250.96	248.91
9	247.52	246.53	246.64	251.70	250.86	248.86
10	247.47	246.50	246.80	251.78	250.79	248.80
11	247.43	246.47	247.00	251.82	250.71	248.74
12	247.40	246.44	247.10	251.88	250.64	248.69
13	247.40	246.42	247.26	251.88	250.56	248.62
14	247.37	246.40	247.40	251.90	250.48	248.55
15	247.33	246.40	247.60	251.90	250.39	248.48
16	247.30	246.40	247.78	251.90	250.32	248.42
17	247.28	246.42	247.90	251.90	250.25	248.36
18	247.24	246.44	248.10	251.94	250.19	248.30
19	247.20	246.44	248.25	251.94	250.12	248.23
20	247.17	246.42	248.40	251.90	250.05	248.16
21	247.13	246.42	248.50	251.86	249.98	248.08
22	247.10	246.40	248.65	251.80	249.91	248.02
23	247.06	246.40	248.85	251.75	249.85	247.95
24	247.03	246.38	249.10	251.72	249.79	247.89
25	247.00	246.36	249.20	251.69	249.72	247.84
26	246.96	246.34	249.40	251.64	249.66	247.77
27	246.92	246.34	249.60	251.60	249.59	247.71
28	246.90	246.32	249.85	251.57	249.52	247.64
29	246.87	246.30	250.10	251.54	249.45	247.56
30	246.83	246.30	250.30	251.50	249.38	247.50
31	246.80	246.30		251.47		247.45
TOTAL	7666.35	7640.50	7435.19	7799.19	7512.04	7700.46
MEAN	247.30	246.47	247.84	251.58	250.40	248.40

2002

DATE	JAN.	FEB.	MARCH	APRIL	MAY	JUNE
1	247.38	245.39	243.38	241.50	239.88	238.64
2	247.33	245.32	243.30	241.44	239.84	238.60
3	247.28	245.25	243.25	241.39	239.80	238.55
4	247.22	245.18	243.20	241.32	239.75	238.50
5	247.16	245.10	243.13	241.25	239.70	238.45
6	247.12	245.02	243.05	241.18	239.66	238.40
7	247.07	244.96	242.98	241.11	239.61	238.35
8	247.02	244.88	242.90	241.05	239.56	238.30
9	246.98	244.81	242.85	240.99	239.52	238.25
10	246.93	244.73	242.80	240.93	239.48	238.20
11	246.87	244.66	242.75	240.87	239.44	238.16
12	246.82	244.58	242.70	240.82	239.41	238.12
13	246.76	244.44	242.65	240.76	239.38	238.07
14	246.71	244.36	242.60	240.71	239.35	238.03
15	246.64	244.36	242.53	240.66	239.33	238.00
16	246.57	244.27	242.47	240.61	239.30	237.96
17	246.49	244.18	242.40	240.56	239.26	237.91
18	246.41	244.09	242.33	240.51	239.23	237.86
19	246.34	244.02	242.26	240.46	239.19	237.82
20	246.27	243.94	242.20	240.41	239.16	237.79
21	246.20	243.89	242.15	240.37	239.12	237.77
22	246.12	243.82	242.08	240.33	239.08	237.74
23	246.05	243.76	242.02	240.28	239.04	237.70
24	245.98	243.69	241.97	240.23	239.00	237.67
25	245.91	243.62	241.90	240.18	238.95	237.65
26	245.84	243.56	241.83	240.13	238.90	237.65
27	245.77	243.51	241.75	240.08	238.88	237.63
28	245.70	243.46	241.68	240.03	238.82	237.60
29	245.63		241.64	239.98	238.78	237.60
30	245.55		241.60	239.93	238.74	237.58
31	245.47		241.56		238.69	
TOTAL	7641.59	6842.85	7515.91	7220.07	7417.85	7140.55
MEAN	246.50	244.39	242.45	240.67	239.28	238.02

2002

DATE	JULY	AUGUST	SEPT.	OCT.	NOV.	DEC.
1	237.55	237.60	239.82	244.30	246.70	245.84
2	237.52	237.64	239.94	244.30	246.72	245.79
3	237.50	237.66	240.08	244.35	246.72	245.72
4	237.48	237.68	240.18	244.35	246.74	245.64
5	237.46	237.68	240.32	244.40	246.76	245.59
6	237.46	237.70	240.50	244.45	246.76	245.54
7	237.46	237.72	240.75	244.50	246.78	245.47
8	237.44	237.76	241.00	244.55	246.78	245.40
9	237.42	237.80	241.22	244.62	246.78	245.32
10	237.40	237.84	241.40	244.72	246.80	245.27
11	237.40	237.90	241.60	244.82	246.80	245.23
12	237.40	237.95	241.78	244.90	246.78	245.18
13	237.40	238.00	241.85	245.02	246.75	245.13
14	237.40	238.06	241.95	245.12	246.72	245.07
15	237.38	238.12	242.02	245.24	246.70	245.02
16	237.35	238.20	242.32	245.37	246.67	244.96
17	237.33	238.30	242.44	245.46	246.65	244.92
18	237.31	238.38	242.62	245.58	246.57	244.90
19	237.30	238.50	242.92	245.66	246.52	244.85
20	237.30	238.62	243.10	245.76	246.48	244.80
21	237.30	238.72	243.23	245.88	246.42	244.72
22	237.32	238.85	243.33	246.00	246.36	244.65
23	237.34	238.96	243.45	246.10	246.30	244.58
24	237.38	239.05	243.57	246.18	246.25	244.50
25	237.42	239.15	243.70	246.30	246.19	244.45
26	237.45	239.20	243.82	246.40	246.13	244.38
27	237.47	239.34	243.92	246.48	246.08	244.30
28	237.50	239.45	244.00	246.55	246.00	244.22
29	237.53	239.55	244.15	246.60	245.94	244.14
30	237.56	239.62	244.25	246.63	245.90	244.08
31	237.58	239.70		246.67		244.03
TOTAL	7360.11	7390.7	7265.23	7607.26	7395.75	7593.68
MEAN	237.40	238.41	242.17	245.40	246.53	244.96

2003

DATE	JAN.	FEB.	MARCH	APRIL	MAY	JUNE
1	243.98	242.32	240.66	238.94	237.84	236.52
2	243.92	242.27	240.60	238.88	237.80	236.50
3	243.88	242.20	240.55	238.82	237.75	236.50
4	243.83	242.15	240.50	238.77	237.70	236.50
5	243.78	242.09	240.46	238.72	237.65	236.48
6	243.74	242.04	240.42	238.66	237.60	236.48
7	243.70	242.00	240.37	238.62	237.54	236.46
8	243.64	241.95	240.31	238.58	237.49	236.44
9	243.58	241.89	240.25	238.53	237.43	236.44
10	243.52	241.84	240.18	238.47	237.38	236.44
11	243.48	241.76	240.10	238.42	237.34	236.42
12	243.43	241.66	240.02	238.38	237.30	236.42
13	243.37	241.58	239.95	238.34	237.25	236.44
14	243.32	241.52	239.88	238.30	237.20	236.46
15	243.26	241.43	239.83	238.28	237.14	236.46
16	243.20	241.43	239.77	238.28	237.08	236.48
17	243.15	241.40	239.72	238.28	237.04	236.50
18	243.08	241.37	239.66	238.26	237.00	236.54
19	243.00	241.32	239.60	238.22	236.97	236.58
20	242.95	241.26	239.55	238.22	236.94	236.60
21	242.88	241.18	239.50	238.22	236.90	236.63
22	242.82	241.10	239.44	238.20	236.87	236.65
23	242.76	241.03	239.40	238.17	236.83	236.70
24	242.70	240.98	239.35	238.14	236.80	236.76
25	242.65	240.92	239.27	238.10	236.77	236.80
26	242.60	240.85	239.25	238.04	236.73	236.86
27	242.56	240.79	239.20	237.98	236.70	236.90
28	242.53	240.73	239.14	237.94	236.66	236.94
29	242.49		239.08	237.90	236.62	236.98
30	242.44		239.04	237.88	236.59	237.04
31	242.38		239.00		236.55	
TOTAL	7538.62	6763.1	7434.07	7150.54	7351.46	7097.92
MEAN	243.18	241.54	239.81	238.35	237.14	236.60

2003

DATE	JULY	AUGUST	SEPT.	OCT.	NOV.	DEC.
1	237.10	238.43	241.20	250.00	255.70	255.57
2	237.18	238.50	241.33	250.40	255.75	255.52
3	237.24	238.55	241.50	250.75	255.80	255.46
4	237.28	238.58	241.70	251.05	255.85	255.41
5	237.28	238.60	241.95	251.35	255.88	255.36
6	237.28	238.55	242.25	251.67	255.90	255.29
7	237.28	238.53	242.70	252.00	255.93	255.25
8	237.28	238.53	243.00	252.20	255.95	255.21
9	237.30	238.53	243.25	252.44	255.95	255.18
10	237.32	238.55	243.45	252.75	255.97	255.13
11	237.34	238.60	243.70	252.95	255.97	255.07
12	237.36	238.70	244.00	253.05	256.00	255.02
13	237.36	238.76	244.30	253.30	256.02	254.96
14	237.36	238.80	244.60	253.55	256.04	254.90
15	237.38	238.85	244.85	253.80	256.04	254.85
16	237.38	238.93	245.10	254.00	256.04	254.80
17	237.40	239.00	245.40	254.30	256.04	254.76
18	237.45	239.08	245.66	254.53	256.02	254.73
19	237.50	239.20	246.00	254.70	256.00	254.70
20	237.55	239.30	246.40	254.80	255.97	254.67
21	237.62	239.42	246.70	254.92	255.95	254.65
22	237.68	239.60	247.01	254.96	255.92	254.62
23	237.74	239.80	247.50	255.10	255.90	254.57
24	237.82	239.95	247.80	255.22	255.87	254.52
25	237.90	240.15	248.14	255.34	255.83	254.46
26	238.00	240.40	248.50	255.45	255.80	254.42
27	238.06	240.60	248.80	255.50	255.75	254.39
28	238.16	240.70	249.00	255.54	255.70	254.37
29	238.24	240.80	249.30	255.60	255.65	254.34
30	238.30	240.95	249.56	255.63	255.61	254.30
31	238.38	241.10		255.67		254.25
TOTAL	7364.52	7418.04	7354.65	7862.52	7676.80	7900.73
MEAN	237.57	239.29	245.16	253.63	255.89	254.86

2004

DATE	JAN.	FEB.	MARCH	APRIL	MAY	JUNE
1	254.20	252.87	251.20	249.48	248.06	246.89
2	254.15	252.82	251.15	249.44	248.03	246.87
3	254.12	252.78	251.10	249.40	248.00	246.84
4	254.08	252.74	251.05	249.35	247.95	246.80
5	254.03	252.70	251.02	249.30	247.89	246.77
6	254.00	252.65	250.98	249.25	247.83	246.75
7	253.95	252.60	250.94	249.21	247.78	246.72
8	253.90	252.54	250.90	249.18	247.74	246.70
9	253.86	252.48	250.85	249.15	247.70	246.67
10	253.82	252.42	250.80	249.10	247.66	246.64
11	253.78	252.36	250.73	249.05	247.61	246.60
12	253.75	252.30	250.66	249.00	247.57	246.55
13	253.70	252.23	250.60	248.94	247.53	246.50
14	253.65	252.16	250.56	248.88	247.48	246.44
15	253.58	252.10	250.50	248.82	247.45	246.39
16	253.54	252.04	250.44	248.77	247.42	246.35
17	253.50	251.98	250.37	248.71	247.40	246.30
18	253.45	251.92	250.30	248.66	247.37	246.25
19	253.40	251.85	250.23	248.60	247.34	246.20
20	253.35	251.78	250.17	248.57	247.32	246.15
21	253.30	251.72	250.10	248.52	247.28	246.10
22	253.27	251.66	250.05	248.46	247.24	246.05
23	253.24	251.60	250.00	248.41	247.21	246.00
24	253.20	251.55	249.93	248.37	247.18	245.94
25	253.16	251.50	249.86	248.32	247.14	245.88
26	253.13	251.43	249.79	248.26	247.10	245.83
27	253.10	251.38	249.73	248.22	247.06	245.77
28	253.06	251.31	249.67	248.18	247.02	245.72
29	253.02	251.25	249.62	248.15	246.99	245.67
30	252.98		249.57	248.10	246.95	245.62
31	252.93		249.52		246.92	
TOTAL	7860.20	7310.72	7762.39	7463.87	7671.22	7389.96
MEAN	253.55	252.09	250.40	248.80	247.46	246.33

2004

DATE	JULY	AUGUST	SEPT.	OCT.	NOV.	DEC.
1	245.56	245.58	248.80	254.75	256.73	255.83
2	245.51	245.66	248.95	254.95	256.75	255.79
3	245.46	245.72	249.10	255.10	256.75	255.74
4	245.42	245.80	249.30	255.20	256.73	255.68
5	245.38	245.88	249.50	255.28	256.70	255.62
6	245.34	246.00	249.70	255.40	256.70	255.58
7	245.32	246.10	249.90	255.55	256.68	255.52
8	245.30	246.16	250.06	255.65	256.65	255.46
9	245.28	246.22	250.30	255.75	256.62	255.40
10	245.28	246.26	250.55	255.85	256.60	255.34
11	245.28	246.30	250.80	255.97	256.57	255.28
12	245.30	246.35	251.10	256.10	256.53	255.22
13	245.32	246.45	251.30	256.18	256.50	255.16
14	245.30	246.60	251.55	256.28	256.47	255.10
15	245.35	246.75	251.80	256.36	256.43	255.05
16	245.37	246.83	252.00	256.43	256.38	254.98
17	245.37	246.90	252.20	256.48	256.34	254.92
18	245.39	246.97	252.40	256.52	256.29	254.85
19	245.39	247.03	252.58	256.54	256.25	254.80
20	245.40	247.08	252.80	256.57	256.20	254.75
21	245.40	247.14	252.90	256.59	256.17	254.70
22	245.40	247.18	253.05	256.60	256.14	254.64
23	245.40	247.28	253.25	256.60	256.10	254.59
24	245.40	247.40	253.50	256.60	256.07	254.55
25	245.40	247.55	253.70	256.62	256.04	254.50
26	245.40	247.72	253.90	256.64	256.00	254.44
27	245.40	247.85	254.10	256.66	255.97	254.40
28	245.42	248.00	254.25	256.68	255.94	254.34
29	245.45	248.20	254.40	256.68	255.90	254.28
30	245.48	248.40	254.55	256.70	255.87	254.26
31	245.52	248.65		256.70		254.24
TOTAL	7606.99	7652.01	7552.29	7939.98	7691.07	7905.01
MEAN	245.39	246.84	251.74	256.13	256.37	255.05

2005

DATE	JAN.	FEB.	MARCH	APRIL	MAY	JUNE
1	254.10	252.23	250.35	248.63	247.00	245.28
2	254.05	252.15	250.30	248.56	246.97	245.22
3	254.00	252.07	250.22	248.50	246.92	245.18
4	253.95	251.99	250.17	248.45	246.87	245.15
5	253.90	251.93	250.12	248.40	246.84	245.12
6	253.86	251.85	250.07	248.35	246.80	245.10
7	253.82	251.78	250.03	248.28	246.74	245.06
8	253.78	251.70	249.98	248.21	246.70	245.02
9	253.72	251.63	249.94	248.16	246.67	244.98
10	253.66	251.55	249.88	248.10	246.63	244.95
11	253.59	251.48	249.82	248.04	246.58	244.92
12	253.52	251.40	249.74	247.98	246.52	244.90
13	253.44	251.32	249.66	247.91	246.46	244.87
14	253.37	251.26	249.60	247.85	246.39	244.84
15	253.32	251.20	249.56	247.80	246.32	244.80
16	253.27	251.14	249.50	247.75	246.26	244.75
17	253.20	251.07	249.44	247.70	246.20	244.70
18	253.13	251.00	249.40	247.66	246.14	244.65
19	253.07	250.92	249.35	247.61	246.07	244.60
20	253.00	250.85	249.31	247.57	246.00	244.56
21	252.95	250.79	249.27	247.52	245.94	244.53
22	252.90	250.72	249.23	247.47	245.90	244.50
23	252.83	250.66	249.18	247.42	245.85	244.48
24	252.77	250.60	249.14	247.38	245.80	244.46
25	252.70	250.53	247.10	247.35	245.73	244.43
26	252.64	250.48	249.04	247.31	245.66	244.43
27	252.56	250.44	249.98	247.25	245.60	244.43
28	252.48	250.40	248.92	247.18	245.53	244.40
29	252.42		248.85	247.11	245.46	244.40
30	252.36		248.77	247.04	245.39	244.40
31	252.30		248.70		245.33	
TOTAL	7850.66	7035.14	7735.62	7434.54	7633.27	7343.11
MEAN	253.25	251.26	249.54	247.82	246.23	244.77

2005

DATE	JULY	AUGUST	SEPT.	OCT.	NOV.	DEC.
1	244.40	244.80	246.28	250.12	253.40	251.97
2	244.40	244.88	246.38	250.20	253.40	251.90
3	244.40	244.98	246.50	250.30	253.42	251.82
4	244.40	245.10	246.60	250.40	253.42	251.76
5	244.37	245.20	246.70	250.50	253.40	251.70
6	244.35	245.25	246.85	250.62	253.38	251.62
7	244.35	245.30	246.95	250.70	253.35	251.54
8	244.32	245.34	247.05	250.85	253.32	251.48
9	244.48	245.38	247.20	251.00	253.28	251.43
10	244.23	245.43	247.35	251.20	253.23	251.37
11	244.20	245.46	247.50	251.35	253.18	251.32
12	244.15	245.48	247.60	251.50	253.13	251.27
13	244.12	245.50	247.70	251.65	253.10	251.22
14	244.09	245.54	247.85	251.80	253.05	251.17
15	244.07	245.57	248.00	251.94	253.00	251.12
16	244.03	245.60	248.10	252.10	252.95	251.06
17	244.00	245.65	248.20	252.25	252.90	251.00
18	244.00	245.68	248.35	252.38	252.84	250.93
19	244.00	245.72	248.50	252.50	252.78	250.87
20	244.00	245.74	248.60	252.60	252.72	250.80
21	244.02	245.77	248.70	252.74	252.66	250.74
22	244.05	245.80	248.80	252.86	252.58	250.70
23	244.10	245.80	248.92	252.92	252.50	250.63
24	244.15	245.83	249.05	253.00	252.44	250.56
25	244.18	245.85	249.20	253.10	252.36	250.50
26	244.23	245.85	249.35	253.18	252.30	250.45
27	244.30	245.88	249.50	253.24	252.25	250.40
28	244.40	245.92	249.70	253.30	252.20	250.33
29	244.46	246.00	249.85	253.35	252.13	250.25
30	244.58	246.10	250.00	253.38	252.05	250.17
31	244.68	246.20		253.40		250.10
TOTAL	7571.29	7612.60	7441.33	7810.43	7586.72	7782.18
MEAN	244.24	245.57	248.04	251.95	252.89	251.04

2006

DATE	JAN.	FEB.	MARCH	APRIL	MAY	JUNE
1	250.03	249.92	246.03	243.75	241.49	239.73
2	249.97	247.86	245.96	243.67	241.44	239.68
3	249.90	247.78	245.88	243.59	241.36	239.64
4	249.82	247.72	245.80	243.51	241.31	239.60
5	249.74	247.65	245.73	243.43	241.25	239.56
6	249.67	247.59	245.65	243.35	241.17	239.51
7	249.60	247.52	245.58	243.27	241.10	239.46
8	249.54	247.44	245.52	243.19	241.04	239.41
9	249.48	247.37	245.45	243.10	240.96	239.36
10	249.42	247.31	245.37	243.02	240.89	239.32
11	249.36	247.25	245.30	242.94	240.82	239.27
12	249.30	247.19	245.23	242.85	240.74	239.23
13	249.22	247.12	245.16	242.76	240.66	239.18
14	249.15	247.05	245.08	242.67	240.60	239.13
15	249.07	247.00	245.00	242.58	240.54	239.08
16	249.00	246.95	244.93	242.50	240.48	239.04
17	248.92	246.89	244.86	242.42	240.43	238.00
18	248.84	246.83	244.80	242.34	240.37	238.96
19	248.78	246.78	244.73	242.27	240.32	238.92
20	248.72	246.73	244.67	242.22	240.27	238.88
21	248.65	246.67	244.59	242.15	240.22	238.85
22	248.59	246.59	244.50	242.07	240.17	238.82
23	248.52	246.52	244.47	241.98	240.12	238.78
24	248.44	246.44	244.36	241.90	240.07	238.75
25	248.37	246.36	244.30	241.83	240.02	238.72
26	248.30	246.28	244.22	241.77	239.98	238.69
27	248.22	246.20	244.14	241.70	239.94	238.65
28	248.16	246.11	244.06	241.64	239.90	238.57
29	248.10		243.98	241.58	239.87	238.57
30	248.04		243.90	241.53	239.82	238.53
31	247.97		243.82		239.78	
TOTAL	7718.89	6917.12	7595.07	7277.58	7457.13	7172.89
MEAN	249.00	247.04	244.94	242.59	240.55	239.10

2006

DATE	JULY	AUGUST	SEPT.	OCT.	NOV.	DEC.
1	238.49	236.99	236.73	241.10	246.10	245.65
2	238.45	236.96	236.73	241.30	246.20	245.61
3	238.42	236.93	236.75	241.50	246.30	245.57
4	238.38	236.90	236.79	241.70	246.30	245.53
5	238.34	236.90	236.84	241.90	246.30	245.49
6	238.30	236.90	236.90	242.00	246.30	245.44
7	238.25	236.92	237.00	242.15	246.30	245.39
8	238.21	236.94	237.08	242.30	246.35	245.34
9	238.17	236.94	237.20	242.55	246.38	245.29
10	238.13	236.94	237.40	242.70	246.40	245.25
11	238.08	236.94	237.55	242.90	246.42	245.20
12	238.04	236.94	237.65	243.15	246.42	245.08
13	237.98	236.92	237.85	243.30	246.40	245.08
14	237.92	236.90	238.00	243.50	246.30	245.00
15	237.87	236.90	238.15	243.80	246.32	244.92
16	237.82	236.90	238.30	243.95	246.27	244.84
17	237.77	236.87	238.50	244.05	246.23	244.77
18	237.72	236.84	238.70	244.20	246.26	244.70
19	237.66	236.82	238.90	244.40	246.20	244.63
20	237.60	236.80	239.10	244.70	246.13	244.56
21	237.55	236.80	239.40	244.90	246.10	244.49
22	237.50	236.80	239.55	245.00	246.05	244.42
23	237.45	236.80	239.70	245.20	246.00	244.35
24	237.40	236.80	239.85	245.40	245.95	244.28
25	237.34	236.78	240.00	245.50	245.90	244.21
26	237.29	236.75	240.20	245.60	245.86	244.15
27	237.24	236.75	240.40	245.70	245.82	244.09
28	237.18	236.75	240.60	245.80	245.77	244.02
29	237.13	236.75	240.75	246.00	245.73	243.95
30	237.08	236.75	240.90	246.00	245.69	243.89
31	237.03	236.73		246.00		243.82
TOTAL	7371.80	7342.61	7153.47	7558.25	7384.75	7589.01
MEAN	237.80	236.86	238.45	243.81	246.16	244.81

2007

DATE	JAN.	FEB.	MARCH	APRIL	MAY	JUNE
1	243.75	241.47	239.35	237.32	236.30	235.86
2	243.68	241.40	239.27	237.27	236.27	235.84
3	243.61	241.33	239.20	237.22	236.24	235.82
4	243.55	241.27	239.12	237.17	236.22	235.80
5	243.49	241.20	239.05	237.13	236.19	235.80
6	243.42	241.13	238.97	237.09	236.17	235.78
7	243.36	241.05	238.89	237.05	236.15	235.78
8	243.30	240.97	238.82	237.02	236.13	235.78
9	243.23	240.90	238.75	236.99	236.10	235.78
10	243.16	240.83	238.68	236.96	236.08	235.78
11	243.09	240.76	238.61	236.93	236.06	235.80
12	243.00	240.69	238.55	236.90	236.06	235.80
13	242.93	240.62	238.48	236.87	236.06	235.80
14	242.85	240.55	238.41	236.83	236.04	235.78
15	242.78	240.47	238.33	236.80	236.04	235.78
16	242.70	240.39	238.26	236.77	236.04	235.78
17	242.62	240.30	238.20	236.74	236.04	235.76
18	242.55	240.22	238.14	236.71	236.04	235.74
19	242.47	240.14	238.08	236.68	236.04	235.72
20	242.39	240.06	238.02	236.65	236.04	235.70
21	242.31	239.99	237.96	236.62	236.04	235.68
22	242.24	239.90	237.90	236.60	236.04	235.66
23	242.17	239.82	237.84	236.58	236.02	235.64
24	242.09	239.74	237.76	236.55	236.00	235.62
25	242.01	239.66	237.72	236.51	235.98	235.60
26	241.93	239.59	237.66	236.47	235.96	235.58
27	241.85	239.51	237.60	236.43	235.94	235.56
28	241.77	239.49	237.54	236.39	235.94	235.54
29	241.70		237.48	236.36	235.92	235.52
30	241.62		237.43	236.33	235.90	235.50
31	241.54		237.37		235.88	
TOTAL	7523.16	6733.44	7387.30	7104	7317.86	7071.60
MEAN	242.68	240.48	238.30	236.80	236.06	235.72

2007

DATE	JULY	AUGUST	SEPT.	OCT.	NOV.	DEC.
1	235.48	235.28	239.85	252.80	256.40	255.71
2	235.46	235.36	240.25	253.10	256.40	255.67
3	235.44	235.46	240.55	253.40	256.40	255.65
4	235.42	235.56	240.95	253.75	256.45	255.65
5	235.39	235.61	241.40	254.05	256.50	255.65
6	235.36	235.61	241.80	254.20	256.50	255.63
7	235.33	235.68	242.30	254.55	256.40	255.61
8	235.30	235.73	242.85	254.70	256.40	255.58
9	235.27	235.80	243.25	254.90	256.40	255.56
10	235.24	235.88	243.80	255.10	256.40	255.53
11	235.21	236.00	244.30	255.30	256.38	255.50
12	235.18	236.10	244.80	255.40	256.38	255.46
13	235.16	236.18	245.40	255.50	256.38	255.41
14	235.13	236.24	246.10	255.60	256.34	255.35
15	235.10	236.34	246.50	255.70	256.30	255.30
16	235.07	236.48	246.95	255.80	256.25	255.25
17	235.04	236.60	247.45	255.85	256.25	255.22
18	235.02	236.75	247.85	255.85	256.22	255.19
19	235.00	237.00	248.30	256.00	256.16	255.15
20	234.98	237.20	248.75	256.00	256.10	255.10
21	234.96	237.40	249.20	256.05	256.05	255.07
22	234.96	237.60	249.20	256.10	256.00	255.05
23	234.96	237.78	249.62	256.10	255.96	255.02
24	234.96	238.00	250.00	256.10	255.93	255.00
25	234.96	238.20	250.45	256.15	255.90	254.97
26	234.98	238.38	251.00	256.20	255.88	254.93
27	235.01	238.54	251.30	256.25	255.85	254.90
28	235.05	238.80	251.70	256.35	255.82	254.87
29	235.10	239.04	252.20	256.35	255.80	254.84
30	235.15	239.27	252.50	256.35	255.75	254.79
31	235.20	239.50		256.40		254.74
TOTAL	7289.96	7343.59	7387.20	7915.85	7686	7913.37
MEAN	235.16	236.89	246.24	255.35	256.20	255.27

4.6 APPENDIX B

2000

DATE	JAN.	FEB.	MARCH	APRIL	MAY	JUNE
1	13.42	13.70	13.72	13.61	14.21	11.35
2	13.03	13.18	14.23	12.40	14.17	14.03
3	14.35	13.28	13.92	14.44	14.26	14.12
4	14.74	13.80	12.58	14.19	14.22	13.81
5	14.67	12.71	11.16	13.59	14.43	13.75
6	13.98	13.47	14.05	14.21	13.31	13.77
7	14.27	13.85	14.18	14.25	12.90	13.70
8	13.72	13.72	13.81	13.34	14.37	13.61
9	13.47	14.05	13.45	13.08	14.39	14.14
10	13.64	13.33	14.23	14.10	13.95	13.56
11	14.37	14.56	13.19	14.01	14.11	12.59
12	14.68	12.74	13.24	14.36	14.26	13.96
13	14.63	12.98	14.30	14.35	12.91	13.64
14	14.87	13.62	13.44	14.28	13.86	14.00
15	14.14	13.08	13.84	12.64	13.87	13.98
16	13.49	12.48	13.97	13.06	13.89	13.63
17	14.39	12.73	13.74	14.61	13.17	13.96
18	14.45	13.53	12.61	13.98	14.00	12.08
19	14.25	12.99	12.44	14.52	13.91	13.97
20	14.28	13.15	13.16	14.48	13.95	13.99
21	13.50	13.65	13.56	13.37	12.25	13.81
22	13.27	13.25	14.64	13.00	13.68	13.72
23	12.18	14.06	14.61	12.85	13.83	13.57
24	13.92	14.09	14.60	14.18	13.77	12.65
25	14.12	13.37	14.28	14.44	13.99	12.20
26	14.11	12.99	13.21	14.47	13.86	13.95
27	13.77	12.40	13.25	13.92	13.75	14.16
28	14.34	13.81	14.06	13.91	11.39	13.95
29	13.26	14.10	14.45	14.19	11.26	13.96
30	12.87		14.69	14.37	11.03	13.70
31	14.15		14.20		11.16	
TOTAL	432.33	388.67	424.81	416.20	418.11	407.31
MEAN	13.95	13.38	13.70	13.87	13.49	13.58

2000

DATE	JULY	AUGUST	SEPT.	OCT.	NOV.	DEC.
1	13.14	17.28	17.91	16.00	18.18	18.45
2	11.86	15.52	16.91	17.06	18.56	18.03
3	13.98	16.98	16.46	17.55	18.56	17.82
4	12.43	16.69	18.17	17.63	17.77	17.96
5	11.09	15.10	18.18	17.35	17.56	18.13
6	11.31	15.86	18.06	17.30	18.12	18.66
7	11.01	16.13	17.92	16.79	17.96	18.58
8	11.25	17.07	18.10	17.22	18.75	18.87
9	10.95	17.09	16.66	18.10	18.53	14.91
10	11.00	17.20	16.65	18.18	18.81	14.89
11	11.02	17.57	17.77	17.76	18.38	15.05
12	10.98	15.13	17.19	17.65	18.01	14.72
13	10.87	15.16	17.94	17.37	19.09	15.25
14	10.96	15.73	18.14	16.69	18.52	17.26
15	11.86	15.48	18.02	17.68	18.60	18.59
16	12.22	15.70	17.34	17.59	19.17	18.57
17	13.19	16.06	16.42	17.50	19.05	16.81
18	13.51	16.09	17.90	16.05	17.87	18.76
19	13.81	14.75	17.96	16.10	17.07	16.58
20	13.72	14.46	17.57	16.02	19.03	18.90
21	13.39	15.84	17.66	16.23	18.52	18.38
22	11.98	17.12	18.05	16.11	17.53	18.29
23	13.72	17.36	16.71	18.65	18.69	16.93
24	14.76	17.37	16.20	18.93	19.28	17.76
25	13.67	17.60	17.30	19.03	17.78	17.35
26	13.00	16.40	17.41	18.47	18.31	18.30
27	14.66	16.58	17.67	18.81	19.24	18.22
28	15.81	17.53	17.62	18.50	18.83	18.41
29	16.85	17.91	17.96	16.25	18.86	16.18
30	16.49	18.19	16.79	18.38	18.74	15.09
31	15.98	18.08		18.73		15.01
TOTAL	400.42	511.01	524.64	541.68	553.37	538.71
MEAN	12.92	16.48	17.49	17.47	18.45	17.38

2001

DATE	JAN.	FEB.	MARCH	APRIL	MAY	JUNE
1	14.75	15.20	15.53	13.72	14.66	14.32
2	18.16	14.93	15.15	14.21	14.73	14.05
3	18.87	14.98	15.60	13.99	14.61	12.67
4	18.80	14.42	15.05	14.62	14.59	14.46
5	18.48	14.60	14.90	14.73	13.19	14.14
6	17.05	15.33	14.97	14.38	14.16	14.23
7	16.92	15.31	14.56	14.90	14.29	14.44
8	18.65	15.26	14.94	13.80	14.56	14.37
9	18.35	15.11	14.93	14.86	14.36	14.11
10	18.20	15.47	14.86	14.96	14.46	14.06
11	17.61	15.01	13.25	14.88	14.28	14.51
12	17.62	15.36	14.80	14.83	14.73	14.04
13	17.20	15.58	15.16	14.88	13.11	14.08
14	17.20	14.94	15.19	14.67	14.37	14.02
15	18.46	15.19	15.02	12.84	14.17	13.47
16	18.38	15.06	14.87	13.30	13.80	14.07
17	18.35	15.53	14.95	14.39	14.72	14.45
18	18.26	15.26	13.94	14.70	14.44	14.05
19	18.50	15.34	14.76	14.26	12.37	13.97
20	17.09	15.21	14.43	14.60	12.81	14.40
21	16.60	15.20	14.55	14.82	14.43	14.41
22	15.12	15.06	14.57	14.12	14.62	14.20
23	14.91	15.03	14.56	14.81	14.36	14.32
24	15.15	15.43	14.45	14.91	14.18	14.35
25	15.12	15.48	13.91	14.69	14.36	13.24
26	15.51	15.00	14.12	14.59	14.41	13.54
27	15.16	14.89	14.79	14.17	13.93	14.09
28	14.66	14.90	14.72	15.06	14.28	14.66
29	14.72		14.82	14.29	14.01	14.45
30	15.20		14.99	14.90	13.65	14.25
31	15.03		14.63		13.89	
TOTAL	524.08	424.08	456.97	433.88	438.53	423.42
MEAN	16.91	15.15	14.74	14.46	14.15	14.11

2001

DATE	JULY	AUGUST	SEPT.	OCT.	NOV.	DEC.
1	12.94	13.69	16.33	17.35	14.98	13.50
2	14.15	14.12	16.77	17.61	15.98	12.80
3	14.30	14.21	16.90	18.27	17.90	14.54
4	14.32	14.16	16.93	18.05	16.11	13.77
5	14.42	13.66	16.99	18.13	17.85	13.47
6	14.31	14.13	16.77	17.25	17.85	13.23
7	13.28	14.13	16.98	16.37	17.90	13.58
8	13.54	14.13	17.09	18.07	18.10	13.51
9	14.34	14.18	15.01	18.20	18.17	12.98
10	14.36	14.21	17.01	18.15	17.54	12.49
11	14.37	15.32	17.79	17.61	17.51	12.06
12	14.36	15.22	16.96	18.17	16.58	11.91
13	14.42	14.25	17.28	17.31	17.87	11.87
14	13.88	15.54	17.56	17.05	17.90	11.62
15	13.92	17.39	16.24	18.49	17.55	11.75
16	14.12	17.45	16.52	18.37	17.98	11.91
17	14.32	17.29	17.00	18.16	17.38	11.64
18	14.34	16.79	17.34	18.43	16.10	12.49
19	14.00	15.99	17.39	18.41	15.89	12.59
20	13.95	17.21	17.59	16.06	15.07	12.90
21	13.82	17.41	17.84	14.78	15.03	12.88
22	13.38	16.85	16.53	14.95	15.10	12.90
23	13.93	17.39	16.09	14.98	14.59	11.77
24	14.01	17.30	17.01	14.90	14.85	12.51
25	14.19	16.56	17.27	14.77	14.55	11.59
26	13.92	16.03	17.52	14.59	14.57	11.89
27	14.22	16.85	17.24	14.36	13.93	12.37
28	13.39	17.60	17.29	14.66	14.10	12.66
29	14.35	17.41	15.88	14.64	15.50	12.30
30	14.06	17.38	16.43	14.54	15.21	12.28
31	13.88	17.13		14.91		12.43
TOTAL	434.79	489.98	507.55	517.59	489.64	390.19
MEAN	14.03	15.81	16.92	16.70	16.32	12.59

2002

DATE	JAN.	FEB.	MARCH	APRIL	MAY	JUNE
1	11.65	13.75	13.33	9.92	9.11	11.00
2	12.62	11.88	13.52	11.69	10.03	10.79
3	12.54	17.78	10.18	11.52	9.97	9.98
4	13.41	12.98	11.18	10.57	9.34	10.20
5	12.34	12.98	10.85	11.12	9.49	10.34
6	11.49	13.18	10.78	10.20	10.61	10.74
7	12.37	12.77	12.60	10.58	11.19	10.64
8	12.48	12.56	10.40	10.49	10.44	9.76
9	13.16	13.54	10.31	10.67	10.56	8.33
10	12.56	12.46	9.77	10.90	11.22	8.36
11	12.38	12.48	9.65	10.28	10.61	9.06
12	12.26	13.55	10.50	10.98	9.45	10.07
13	11.89	14.81	11.01	10.78	11.74	10.30
14	11.61	14.41	11.59	8.47	12.12	9.50
15	12.11	14.17	13.54	10.43	11.87	11.32
16	12.13	13.14	11.26	9.84	9.67	10.57
17	12.05	13.62	11.80	10.39	11.51	12.63
18	12.66	14.06	11.60	10.75	8.34	11.61
19	13.03	12.81	10.92	9.83	9.69	11.07
20	13.07	13.20	11.00	9.29	10.84	10.39
21	13.49	14.58	12.78	11.55	9.65	11.35
22	13.12	12.19	12.74	11.96	11.85	11.66
23	13.87	12.31	10.97	11.17	12.15	10.00
24	13.18	12.50	9.27	10.48	11.20	11.34
25	12.58	12.72	10.81	11.16	10.21	12.38
26	11.72	13.29	10.70	10.01	9.43	11.68
27	12.91	13.35	10.62	9.13	9.20	10.99
28	12.66	12.93	10.20	10.48	9.93	11.45
29	13.23		10.43	11.27	9.29	10.69
30	13.25		10.18	11.54	9.88	10.57
31	13.79		9.72		11.48	
TOTAL	391.61	374.00	344.21	317.45	322.07	318.77
MEAN	12.63	13.36	11.10	10.58	10.39	10.63

2002

DATE	JULY	AUGUST	SEPT.	OCT.	NOV.	DEC.
1	11.09	10.68	10.95	11.88	10.99	11.97
2	11.69	10.71	13.29	12.56	9.50	11.56
3	11.28	11.02	14.89	11.39	10.65	11.96
4	11.56	11.54	13.18	11.78	11.46	12.58
5	9.99	10.59	13.95	11.78	11.23	12.16
6	8.75	10.59	12.59	11.37	11.49	13.62
7	11.33	11.37	12.08	10.73	11.36	12.97
8	11.42	11.43	11.38	10.66	11.83	12.17
9	11.36	10.39	12.98	10.99	12.11	11.97
10	11.32	10.30	11.69	11.98	11.20	11.69
11	11.37	11.05	11.99	12.24	12.35	11.28
12	11.13	10.83	14.31	11.49	11.58	11.01
13	9.98	10.92	12.40	10.68	12.06	10.78
14	10.95	10.92	11.16	11.93	12.65	11.71
15	10.92	11.07	9.84	11.10	11.73	11.62
16	11.26	11.53	12.10	10.76	10.65	11.37
17	10.93	11.39	12.08	10.89	11.73	11.93
18	11.02	11.29	11.12	10.20	10.65	11.50
19	10.97	11.64	11.14	10.97	10.78	11.99
20	11.73	11.51	11.69	9.55	11.64	11.86
21	10.96	10.78	11.91	11.46	11.51	12.26
22	11.43	11.19	11.06	10.56	11.82	12.41
23	11.41	11.10	12.84	10.26	11.90	11.81
24	10.78	12.03	13.05	10.75	11.93	11.49
25	10.64	10.61	12.68	10.28	12.00	10.95
26	10.69	11.13	12.44	9.90	11.96	10.48
27	11.41	12.78	12.60	9.50	11.87	11.58
28	10.08	12.48	12.45	9.87	11.91	11.45
29	10.93	13.95	11.05	11.50	11.72	11.71
30	11.79	12.57	12.25	11.30	12.16	11.74
31	12.51	12.18		11.35		11.31
TOTAL	342.68	351.57	367.14	341.66	346.42	364.87
MEAN	11.05	11.34	12.24	11.02	11.55	11.77

2003

DATE	JAN.	FEB.	MARCH	APRIL	MAY	JUNE
1	8.78	9.76	8.40	10.31	6.94	5.22
2	9.30	7.52	6.98	9.49	8.54	5.40
3	9.37	8.93	8.62	8.85	8.42	5.64
4	9.36	8.78	8.83	8.22	8.66	6.73
5	9.00	11.33	9.00	7.34	8.06	5.98
6	9.67	10.98	9.23	7.36	6.69	5.38
7	11.29	11.46	10.45	8.82	7.66	5.19
8	11.97	10.92	8.93	10.25	6.87	5.48
9	9.93	10.39	7.54	10.33	7.75	5.01
10	9.09	8.10	9.18	8.72	6.33	5.10
11	10.60	8.92	9.22	7.38	6.03	5.21
12	8.79	8.70	9.14	6.25	7.15	5.99
13	10.54	10.01	8.68	6.25	6.76	7.71
14	10.38	10.40	9.04	7.43	6.65	6.99
15	10.18	8.87	8.52	7.64	7.85	6.89
16	9.56	11.45	7.45	8.43	7.01	8.67
17	10.06	11.33	9.19	8.35	7.33	8.54
18	10.34	10.95	9.33	6.68	5.24	8.19
19	8.79	9.75	8.34	6.84	4.78	8.71
20	9.17	9.84	8.41	6.18	6.29	8.58
21	9.12	10.79	8.82	6.64	7.41	7.77
22	9.03	10.73	9.79	8.27	9.14	6.15
23	9.11	10.25	7.23	8.55	9.08	5.58
24	8.32	10.27	10.10	8.57	7.44	5.88
25	8.99	11.91	10.43	10.22	4.80	6.19
26	8.42	11.19	8.72	8.31	4.65	6.06
27	8.68	9.45	8.93	6.65	5.94	6.60
28	7.87	9.22	9.32	8.16	5.74	5.79
29	8.34		8.37	8.47	6.97	5.71
30	7.76		7.37	8.28	7.10	6.24
31	8.94		9.26		6.05	
TOTAL	290.73	282.20	272.82	243.24	215.33	192.58
MEAN	9.38	10.08	8.80	8.11	6.95	6.42

2003

DATE	JULY	AUGUST	SEPT.	OCT.	NOV.	DEC.
1	6.59	8.07	8.69	9.84	8.63	13.33
2	5.97	6.73	9.13	9.75	7.98	13.18
3	6.38	6.63	9.49	10.48	10.26	13.03
4	7.90	9.51	9.22	9.14	9.13	10.27
5	7.19	11.21	8.83	8.26	9.43	12.23
6	6.28	10.90	8.03	9.93	9.38	12.10
7	6.42	10.65	7.24	10.85	10.11	11.28
8	6.39	10.73	9.00	10.43	8.42	12.96
9	8.36	9.81	9.93	10.01	8.46	13.20
10	8.35	9.50	9.57	10.29	8.80	12.85
11	9.43	8.83	9.94	9.44	9.30	12.81
12	7.47	8.61	9.64	8.39	10.00	12.59
13	6.60	8.92	9.29	10.28	9.61	10.22
14	9.26	8.37	7.69	9.45	9.54	9.96
15	9.15	8.93	9.44	9.52	8.63	11.41
16	8.87	7.92	9.95	9.36	8.12	11.54
17	9.93	7.17	10.10	8.89	9.62	11.00
18	8.39	8.98	9.88	8.81	9.80	9.78
19	7.28	9.02	10.07	7.31	10.24	10.17
20	6.47	8.65	9.55	10.86	9.65	9.74
21	7.42	8.49	7.94	12.34	9.88	9.29
22	7.56	8.29	9.09	10.87	9.60	10.95
23	7.74	7.93	10.55	9.23	7.99	9.79
24	8.33	7.02	9.74	9.63	9.05	9.51
25	8.42	8.66	8.57	8.64	8.97	8.01
26	8.06	8.92	9.41	9.10	11.36	7.64
27	6.64	8.79	8.17	10.89	13.24	9.04
28	8.20	9.12	7.54	9.87	13.32	8.34
29	9.29	9.07	9.39	9.92	12.85	8.59
30	8.17	8.81	9.45	9.99	11.95	8.22
31	7.94	7.12		10.07		9.02
TOTAL	240.45	271.36	274.53	301.84	293.32	332.05
MEAN	7.76	8.75	9.15	9.74	9.78	10.71

2004

DATE	JAN.	FEB.	MARCH	APRIL	MAY	JUNE
1	7.59	10.40	10.98	11.17	10.49	9.97
2	8.57	10.41	11.74	11.11	10.81	10.43
3	8.65	12.45	11.58	11.73	12.27	12.64
4	8.23	11.14	11.48	11.31	12.87	13.12
5	9.76	10.69	11.37	12.51	13.17	13.00
6	9.33	11.61	10.43	11.95	11.85	12.12
7	9.98	10.43	9.72	12.38	11.52	12.82
8	10.39	9.80	11.25	12.46	11.84	13.12
9	11.01	11.28	10.69	11.96	11.37	14.01
10	10.01	11.38	12.39	11.74	12.68	13.24
11	9.35	11.37	12.21	10.15	11.62	13.66
12	9.59	11.13	12,25	10.49	11.63	13.39
13	12.02	12.16	12.28	11.21	12.09	12.29
14	11.35	10.69	10.68	11.87	11.98	12.67
15	9.84	9.55	12.36	11.35	11.19	12.78
16	12.02	10.27	12.31	12.19	10.69	13.14
17	10.72	11.53	12.55	11.49	12.60	12.85
18	10.35	11.14	12.67	11.46	12.78	13.18
19	11.42	11.69	12.02	12.52	12.48	12.90
20	11.81	11.19	10.26	12.63	11.83	11.95
21	10.77	10.87	10.20	12.83	11.71	13.14
22	10.64	10.84	11.61	12.20	10.47	13.18
23	11.97	12.35	11.20	11.93	10.26	13.93
24	10.82	12.60	11.69	11.31	11.01	13.73
25	11.87	12.68	11.89	11.24	10.99	13.25
26	12.31	11.86	11.14	11.85	10.45	12.74
27	11.97	10.82	11.99	9.46	11.71	11.93
28	12.37	8.76	11.66	12.24	11.16	12.74
29	12.40	9.72	11.46	11.20	10.15	13.44
30	12.62		11.18	12.63	9.65	13.38
31	11.46		11.92		10.37	
TOTAL	331.19	320.81	357.16	350.57	355.69	384.74
MEAN	10.68	11.06	11.52	11.69	11.47	12.82

2004

DATE	JULY	AUGUST	SEPT.	OCT.	NOV.	DEC.
1	12.43	10.64	11.68	8.49	12.72	14.00
2	13.39	12.63	11.84	9.77	13.50	13.56
3	12.98	12.28	11.65	11.28	14.14	13.66
4	12.14	12.73	10.22	12.24	13.94	13.38
5	13.50	12.38	10.34	12.57	13.62	12.85
6	13.55	12.20	11.67	12.65	12.32	13.50
7	13.56	11.35	12.05	12.51	12.22	13.23
8	13.81	10.79	12.27	13.43	13.24	13.89
9	13.75	12.23	12.27	12.19	14.29	13.83
10	12.37	12.43	12.67	11.55	13.91	13.68
11	11.22	12.27	11.83	13.06	13.60	14.10
12	12.86	12.84	10.35	14.01	14.19	12.61
13	13.37	13.13	12.11	13.79	12.95	13.52
14	11.66	11.69	12.79	13.79	12.78	14.00
15	14.38	10.87	12.82	13.19	13.18	13.62
16	13.92	11.92	13.45	12.39	13.71	13.82
17	13.26	12.06	13.48	12.11	12.96	13.81
18	11.55	11.89	11.99	12.91	13.67	13.39
19	13.12	12.30	11.33	13.85	12.99	13.20
20	13.27	12.68	12.09	13.88	12.54	13.04
21	13.46	11.53	11.80	13.52	12.03	14.09
22	13.15	10.95	12.05	13.39	13.98	13.31
23	13.25	12.31	10.28	12.45	14.19	13.59
24	12.51	13.19	9.37	10.97	13.27	12.96
25	11.77	12.49	9.78	12.37	13.07	12.20
26	12.63	12.73	10.89	12.48	13.08	12.47
27	13.01	12.66	12.26	11.43	13.49	13.47
28	13.10	12.32	10.93	9.27	12.49	13.05
29	12.49	9.58	8.79	8.89	13.27	13.60
30	12.75	11.31	8.41	9.38	13.72	13.22
31	11.92	11.67		10.24		12.85
TOTAL	400.13	372.05	343.44	374.00	399.06	415.50
MEAN	12.91	12.00	11.45	12.06	13.30	13.40

2005

DATE	JAN.	FEB.	MARCH	APRIL	MAY	JUNE
1	11.89	15.33	14.85	13.70	11.28	13.30
2	11.48	15.76	14.99	12.92	12.36	12.83
3	12.46	15.57	13.12	11.68	13.78	12.56
4	12.95	14.92	13.45	13.07	14.12	12.94
5	13.30	14.24	11.41	12.44	12.47	12.63
6	13.06	12.95	11.54	13.42	13.10	12.78
7	13.59	13.63	12.73	13.54	12.70	13.63
8	11.93	14.13	14.23	12.83	11.69	13.66
9	10.79	14.34	12.16	12.68	13.22	13.36
10	11.55	15.10	15.09	10.78	13.55	13.30
11	12.92	15.53	15.48	13.31	14.16	12.21
12	13.10	14.64	14.44	15.72	13.97	11.20
13	12.09	13.15	11.30	13.98	13.67	12.33
14	13.69	15.34	15.33	13.21	13.79	13.39
15	13.30	15.20	13.68	12.48	12.29	13.47
16	11.72	15.80	12.76	11.96	12.92	13.05
17	13.76	15.80	9.86	11.93	13.93	12.76
18	14.30	15.47	9.95	12.66	12.57	12.34
19	13.62	15.01	9.71	12.69	13.25	11.73
20	13.00	13.11	10.26	14.14	13.80	12.99
21	13.80	15.44	12.10	14.35	14.06	13.42
22	13.57	15.73	12.82	13.81	13.67	12.72
23	12.22	15.96	11.82	12.14	13.31	13.07
24	14.08	16.01	9.76	11.36	14.59	14.46
25	15.27	13.02	10.04	13.36	14.36	11.35
26	15.20	10.86	9.63	13.79	12.17	12.04
27	15.40	9.99	9.75	13.38	13.78	12.85
28	15.50	14.79	9.73	13.94	13.61	12.71
29	13.11		12.16	13.57	13.01	13.21
30	13.31		12.39	12.26	13.48	12.43
31	15.20		13.32		12.44	
TOTAL	411.16	406.82	379.86	391.10	411.08	384.72
MEAN	13.26	14.53	12.25	13.04	13.26	12.82

2005

DATE	JULY	AUGUST	SEPT.	OCT.	NOV.	DEC.
1	11.63	10.25	10.97	13.20	13.30	15.30
2	10.93	10.70	10.74	11.56	13.25	14.27
3	10.77	11.37	10.49	13.11	13.36	14.54
4	12.30	11.34	10.54	13.13	13.10	14.71
5	12.25	11.06	10.79	12.70	13.31	16.20
6	11.97	10.99	12.33	12.45	12.65	14.72
7	12.20	10.84	11.22	12.84	14.13	13.40
8	12.30	11.85	11.43	9.68	14.95	15.66
9	13.19	10.86	13.54	11.49	14.55	15.98
10	9.74	11.88	11.89	11.58	14.25	14.92
11	10.32	11.26	10.81	11.89	15.38	14.16
12	12.10	11.01	12.46	11.87	14.33	13.94
13	10.16	10.62	12.50	13.40	13.61	15.40
14	10.71	10.23	12.98	12.96	14.67	14.65
15	10.98	10.46	12.70	12.20	14.68	15.83
16	10.42	10.71	12.64	11.23	14.10	16.09
17	9.68	10.94	11.18	13.43	15.03	15.88
18	9.96	10.67	11.22	14.29	15.29	14.58
19	10.38	11.69	13.06	13.43	14.58	15.14
20	10.21	10.18	13.03	13.09	14.76	15.96
21	10.30	10.06	12.93	14.46	14.77	15.69
22	10.94	11.02	13.20	13.24	15.95	15.12
23	9.96	11.15	12.48	12.57	16.14	15.29
24	9.57	10.80	12.40	13.41	16.07	14.34
25	10.00	10.48	10.96	14.81	16.00	13.81
26	10.31	11.36	12.30	14.27	14.61	13.23
27	10.21	11.05	12.67	14.15	13.71	14.25
28	10.82	10.02	12.12	14.24	15.13	15.28
29	10.37	10.17	13.34	14.13	16.04	14.74
30	9.60	11.00	13.36	12.20	16.20	14.97
31	9.25	10.88		13.51		14.16
TOTAL	333.54	336.90	362.28	400.52	437.90	462.21
MEAN	10.76	10.87	12.08	12.92	14.60	14.91

2006

DATE	JAN.	FEB.	MARCH	APRIL	MAY	JUNE
1	13.63	13.88	13.91	14.09	13.58	14.92
2	13.89	13.22	14.14	13.32	14.13	15.34
3	14.86	13.49	14.53	12.94	15.53	13.39
4	15.86	13.03	13.39	14.26	15.57	12.99
5	16.03	10.83	12.45	13.20	15.68	13.76
6	16.39	14.53	13.38	12.52	14.57	15.15
7	13.19	14.40	14.46	13.01	14.46	14.93
8	12.21	14.33	14.07	12.68	15.12	13.95
9	16.10	13.63	14.95	12.67	16.22	14.24
10	16.07	14.08	15.78	12.78	13.85	12.71
11	17.13	13.10	14.09	13.35	14.71	12.55
12	16.22	12.38	13.59	14.29	14.48	14.20
13	16.82	13.56	15.82	15.51	12.32	14.38
14	16.56	14.33	15.50	14.62	12.58	14.77
15	14.41	13.92	15.97	14.64	15.32	14.56
16	15.74	14.08	13.62	13.71	13.68	13.65
17	14.91	13.80	13.01	14.87	14.99	12.92
18	14.76	14.28	13.73	15.95	14.59	12.51
19	14.72	12.53	13.40	15.99	13.56	13.67
20	14.95	13.67	13.59	14.79	12.71	13.54
21	13.56	14.29	14.19	13.02	11.82	13.49
22	15.12	14.10	13.16	12.28	14.58	14.99
23	16.46	14.51	13.49	12.54	14.63	13.15
24	16.29	14.72	14.09	12.23	16.22	12.81
25	13.98	13.70	14.32	11.27	14.67	12.29
26	14.41	14.18	14.00	11.90	14.12	13.56
27	14.06	14.78	15.64	12.06	13.92	14.51
28	13.01	15.29	14.84	12.89	12.24	14.51
29	11.35		15.87	12.73	14.63	13.83
30	13.23		14.39	13.61	14.90	15.00
31	13.67		14.07		15.12	
TOTAL	459.59	386.64	441.44	403.72	444.50	416.27
MEAN	14.83	13.81	14.24	13.46	14.34	13.88

2006

DATE	JULY	AUGUST	SEPT.	OCT.	NOV.	DEC.
1	13.52	13.92	11.86	11.22	11.05	11.49
2	12.17	13.23	11.01	11.24	11.22	9.71
3	12.70	13.59	9.40	10.95	11.03	9.33
4	13.56	12.99	10.20	11.93	10.10	9.88
5	14.69	14.29	10.52	10.75	9.61	11.16
6	13.75	11.76	10.42	10.51	10.52	11.20
7	14.22	13.02	9.82	9.48	10.47	11.61
8	13.09	12.95	9.95	8.75	10.37	10.67
9	12.31	13.36	9.09	10.32	11.95	9.47
10	13.24	12.86	8.87	10.25	11.22	12.10
11	13.41	13.24	9.17	8.75	11.10	12.73
12	13.64	12.54	9.16	9.66	10.59	12.88
13	13.59	11.90	9.10	9.59	11.33	13.17
14	15.04	12.62	9.50	9.02	12.12	13.73
15	13.84	12.79	9.61	9.17	13.37	14.51
16	12.80	12.85	9.43	9.89	12.78	11.67
17	13.71	15.33	8.72	10.17	12.37	11.08
18	14.39	15.01	8.86	9.28	11.08	13.49
19	13.43	12.72	8.93	9.15	10.31	13.63
20	13.19	11.75	8.77	9.39	12.10	13.61
21	15.55	12.66	9.14	9.27	11.84	14.00
22	14.09	13.49	9.31	8.82	11.31	13.93
23	13.82	13.31	9.76	9.82	11.91	14.66
24	14.93	13.49	8.88	10.53	12.10	13.49
25	12.98	13.97	9.41	9.73	10.11	11.64
26	12.96	13.27	11.30	9.85	8.86	11.42
27	12.71	12.46	10.75	10.16	11.13	13.49
28	13.05	11.92	9.93	8.61	11.46	13.70
29	14.49	12.07	10.34	8.64	10.54	13.89
30	11.80	12.23	10.37	9.79	11.40	13.30
31	13.31	11.73		9.76		12.43
TOTAL	419.98	403.32	291.58	304.45	335.35	383.07
MEAN	13.55	13.01	9.72	9.82	11.18	12.36

2007

DATE	JAN.	FEB.	MARCH	APRIL	MAY	JUNE
1	11.54	11.10	13.51	6.04	6.96	6.08
2	12.83	11.90	13.09	5.39	8.46	6.41
3	11.83	10.81	12.79	5.81	6.58	5.73
4	12.10	11.88	11.96	6.22	6.32	6.50
5	12.27	14.10	12.47	6.55	6.22	6.49
6	11.37	14.03	11.49	5.85	5.72	6.24
7	10.90	14.65	10.36	4.93	6.41	5.61
8	13.15	14.32	10.29	3.91	6.35	5.88
9	13.37	14.15	10.58	4.32	6.32	6.08
10	15.58	12.15	9.84	5.95	6.27	4.74
11	15.90	11.54	10.13	6.03	6.46	5.83
12	13.97	13.48	11.73	6.28	5.87	6.34
13	13.91	15.61	11.38	8.35	6.21	6.24
14	10.34	13.80	11.10	7.19	6.61	5.68
15	12.59	14.32	9.79	6.81	6.63	6.37
16	13.06	14.60	8.41	7.47	6.54	4.70
17	13.37	16.15	6.72	6.13	6.54	3.99
18	12.81	13.97	6.78	5.78	6.53	5.87
19	13.06	16.39	8.67	5.74	6.58	5.97
20	12.02	15.79	8.47	6.17	6.02	6.09
21	11.96	14.53	8.87	5.01	6.48	6.02
22	12.75	14.53	9.14	4.68	6.68	6.39
23	14.32	13.75	8.83	7.96	6.65	6.42
24	13.60	12.14	8.31	8.69	6.56	6.28
25	13.56	11.26	6.82	8.39	6.39	6.38
26	13.58	13.16	7.26	7.72	4.89	6.49
27	13.32	14.26	7.49	8.76	5.85	6.22
28	13.66	14.14	7.85	5.92	5.85	6.36
29	13.88		8.14	6.87	6.30	6.46
30	12.68		9.38	9.07	6.49	5.30
31	12.63		7.93		6.40	
TOTAL	401.91	382.51	299.58	193.99	198.14	179.16
MEAN	12.96	13.66	9.66	6.47	6.39	5.97

2007

DATE	JULY	AUGUST	SEPT.	OCT.	NOV.	DEC.
1	4.73	5.84	5.33	7.26	9.14	8.56
2	5.70	5.51	6.11	7.81	9.30	7.48
3	6.22	5.78	6.48	8.58	7.86	9.16
4	6.47	4.62	6.32	8.84	7.70	9.39
5	6.13	4.87	6.09	9.44	9.55	9.00
6	6.16	5.41	6.01	8.63	10.84	8.82
7	4.82	6.03	6.23	8.40	10.37	7.51
8	4.94	6.12	6.03	9.00	9.75	7.89
9	5.67	6.14	5.74	8.84	9.50	7.70
10	6.13	5.96	6.30	8.94	13.54	10.99
11	6.05	5.88	6.27	7.52	8.15	10.11
12	6.04	5.80	6.05	7.78	11.21	9.33
13	6.50	5.64	6.36	8.22	12.05	9.39
14	5.03	6.27	6.58	8.09	11.97	9.52
15	5.87	6.03	6.24	10.02	13.11	9.80
16	5.83	5.98	6.28	10.66	13.80	7.96
17	6.12	6.08	6.22	10.45	12.88	9.35
18	6.36	5.79	6.47	10.17	12.43	9.68
19	5.75	5.73	6.61	8.86	13.13	9.54
20	5.91	8.10	5.99	7.67	13.09	9.64
21	5.78	5.65	7.44	7.84	14.35	9.37
22	5.43	6.53	6.19	9.88	13.46	8.56
23	6.18	5.70	6.25	10.92	10.64	7.39
24	6.00	5.81	6.60	10.65	7.68	9.71
25	5.85	5.57	6.72	11.92	9.41	9.79
26	5.66	5.48	6.92	12.64	10.48	10.69
27	5.32	5.85	7.03	8.05	11.61	12.36
28	4.34	5.94	7.06	7.76	11.91	10.46
29	4.40	5.73	6.67	9.22	9.94	10.82
30	5.11	5.70	7.02	10.15	10.62	9.75
31	5.47	5.24		9.66		9.61
TOTAL	175.97	178.78	191.61	283.85	329.47	289.33
MEAN	5.68	5.77	6.39	9.16	10.98	9.33

4.7 APPENDIX C(i)

>> AVERAGE DAILY WATER LEVELS PER MONTH OF AKOSOMBO DAM FROM JANUARY, 1998-DECEMBER, 2007

>> X=[243.39, 241.40, 239.76, 238.23, 237.41, 237.04, 237.41, 238.53, 242.22, 249.21 251.7, 250.14, 248.19, 246.34, 244.62, 243.06, 241.59, 240.27, 239.58, 240.45, 247.61 259.72, 263.67, 262.50, 260.83, 259.20, 257.36, 255.70, 254.03, 252.64, 252.45, 253.33 257.18, 262.31, 262.50, 260.60, 258.39, 256.36, 254.16, 252.16, 250.45, 248.65, 247.30 246.47, 247.84, 251.58, 250.40, 248.40, 246.50, 244.39, 242.45, 240.67, 239.28, 238.02 237.40, 238.41, 242.17, 245.40, 246.53, 244.96, 243.18, 241.54, 239.81, 238.35, 237.14 236.60, 237.57, 239.29, 245.16, 253.63, 255.89, 254.86, 253.55, 252.09, 250.40, 248.80 247.46, 246.33, 245.39, 246.84, 251.74, 256.13, 256.37, 255.05, 253.25, 251.26, 249.54 247.82, 246.23, 244.77, 244.24, 245.57, 248.04, 251.95, 252.89, 251.04, 249.00, 247.04 244.94, 242.59, 240.55, 239.10, 237.80, 236.86, 238.45, 243.81, 246.16, 244.81, 242.68 240.48, 238.30, 236.80, 236.06, 235.72, 235.16, 236.89, 246.24, 255.35, 256.20, 255.27]';

X_2 is 119 by 1 dimensional vector of predictable average daily water levels of Akosombo dam from 1998-2007

>> X_2=[241.40, 239.76, 238.23, 237.41, 237.04, 237.41, 238.53, 242.22, 249.21, 251.70 250.14, 248.19, 246.34, 244.62, 243.06, 241.59, 240.27, 239.58, 240.45, 247.61, 259.72 263.67, 262.50, 260.83, 259.20, 257.36, 255.70, 254.03, 252.64, 252.45, 253.33, 257.18 262.31, 262.50, 260.60, 258.39, 256.36, 254.16, 252.16, 250.45, 248.65, 247.30, 246.47 247.84, 251.58, 250.40, 248.40, 246.50, 244.39, 242.45, 240.67, 239.28, 238.02, 237.40 238.41, 242.17, 245.40, 246.53, 244.96, 243.18, 241.54, 239.81, 238.35, 237.14, 236.60 237.57, 239.29, 245.16, 253.63, 255.89, 254.86, 253.55, 252.09, 250.40, 248.80, 247.46 246.33, 245.39, 246.84, 251.74, 256.13, 256.37, 255.05, 253.25, 251.26, 249.54, 247.82

246.23, 244.77, 244.24, 245.57, 248.04, 251.95, 252.89, 251.04, 249.00, 247.04, 244.94
242.59, 240.55, 239.10, 237.80, 236.86, 238.45, 243.81, 246.16, 244.81, 242.68, 240.48
238.30, 236.80, 236.06, 235.72, 235.16, 236.89, 246.24, 255.35, 256.20, 255.27]';

X_1 is 119 by 1 dimensional vector of corresponding immediate past average daily water levels of Akosombo dam from 1998-2007

>> X_1 =[243.39, 241.40, 239.76, 238.23, 237.41, 237.04, 237.41, 238.53, 242.22, 249.21
251.70, 250.14, 248.19, 246.34, 244.62, 243.06, 241.59, 240.27, 239.58, 240.45, 247.61
259.72, 263.67, 262.50, 260.83, 259.20, 257.36, 255.70, 254.03, 252.64, 252.45, 253.33
257.18, 262.31, 262.50, 260.60, 258.39, 256.36, 254.16, 252.16, 250.45, 248.65, 247.30
246.47, 247.84, 251.58, 250.40, 248.40, 246.50, 244.39, 242.45, 240.67, 239.28, 238.02
237.40, 238.41, 242.17, 245.40, 246.53, 244.96, 243.18, 241.54, 239.81, 238.35, 237.14
236.60, 237.57, 239.29, 245.16, 253.63, 255.89, 254.86, 253.55, 252.09, 250.40, 248.80
247.46, 246.33, 245.39, 246.84, 251.74, 256.13, 256.37, 255.05, 253.25, 251.26, 249.54
247.82, 246.23, 244.77, 244.24, 245.57, 248.04, 251.95, 252.89, 251.04, 249.00, 247.04
244.94, 242.59, 240.55, 239.10, 237.80, 236.86, 238.45, 243.81, 246.16, 244.81, 242.68
240.48, 238.30, 236.80, 236.06, 235.72, 235.16, 236.89, 246.24, 255.35, 256.20]';

>> $S_{X_1X_1}$ =sum(X_1.^2)-mean(X_1)*sum(X_1)= 6157.5

>> $S_{X_1X_2}$ =sum(X_1.* X_2)-(sum(X_1)*sum(X_2))/119 = 5703.6

>> $\hat{a}_1 = \frac{S_{X_1X_2}}{S_{X_1X_1}}$

$\hat{a}_1$ = 0.9263

$\hat{a}_1$ is the unbiased estimator of a_1.

>> $\hat{a}_0$ =mean(X_2)- $\hat{a}_1$ *mean(X_1)

$\hat{a}_0 =$

18.3004

>> The fitted model is $\hat{X}_2 = \hat{a}_0 + \hat{a}_1 X_1$

>> $\hat{X}_2 = \hat{a}_0 + \hat{a}_1 * X_1;$

>> meanerror=mean($X_2 - \hat{X}_2$)

meanerror =

1.8391e-014

>> MAE=mean (abs($X_2 - \hat{X}_2$)) = 1.0332

>> maxerror=max (abs($X_2 - \hat{X}_2$))

maxerror = 1.352

minerror = min(abs($X_2 - \hat{X}_2$))

= 0.0337

SS_E = 930.8664

MSE = $\frac{SS_E}{(119-2)}$

= 7.9561

4.8 APPENDIX C(ii)

>> X represents average daily water levels on monthly basis from January, 1998-December, 2007

>> X=[243.39, 241.40, 239.76, 238.23, 237.41, 237.04, 237.41, 238.53, 242.22, 249.21
251.70, 250.14, 248.19, 246.34, 244.62, 243.06, 241.59, 240.27, 239.58, 240.45, 247.61
259.72, 263.67, 262.50, 260.83, 259.20, 257.36, 255.70, 254.03, 252.64, 252.45, 253.33
257.18, 262.31, 262.50, 260.60, 258.39, 256.36, 254.16, 252.16, 250.45, 248.65, 247.30
246.47, 247.84, 251.58, 250.40, 248.40, 246.50, 244.39, 242.45, 240.67, 239.28, 238.02
237.40, 238.41, 242.17, 245.40, 246.53, 244.96, 243.18, 241.54, 239.81, 238.35, 237.14
236.60, 237.57, 239.29, 245.16, 253.63, 255.89, 254.86, 253.55, 252.09, 250.40, 248.80
247.46, 246.33, 245.39, 246.84, 251.74, 256.13, 256.37, 255.05, 253.25, 251.26, 249.54
247.82, 246.23, 244.77, 244.24, 245.57, 248.04, 251.95, 252.89, 251.04, 249.00, 247.04
244.94, 242.59, 240.55, 239.10, 237.80, 236.86, 238.45, 243.81, 246.16, 244.81, 242.68
240.48, 238.30, 236.80, 236.06, 235.72, 235.16, 236.89, 246.24, 255.35, 256.20, 255.27]';

X_3 is 118 by 1 dimensional vector of predictable average daily water levels from January 1998-December 2007

>> X_3 =[239.76, 238.23, 237.41, 237.04, 237.41, 238.53, 242.22, 249.21, 251.70, 250.14
248.19, 246.34, 244.62, 243.06, 241.59, 240.27, 239.58, 240.45, 247.61, 259.72, 263.67
262.50, 260.83, 259.20, 257.36, 255.70, 254.03, 252.64, 252.45, 253.33, 257.18, 262.31
262.50, 260.60, 258.39, 256.36, 254.16, 252.16, 250.45, 248.65, 247.30, 246.47, 247.84
251.58, 250.40, 248.40, 246.50, 244.39, 242.45, 240.67, 239.28, 238.02, 237.40, 238.41
242.17, 245.40, 246.53, 244.96, 243.18, 241.54, 239.81, 238.35, 237.14, 236.60, 237.57
239.29, 245.16, 253.63, 255.89, 254.86, 253.55, 252.09, 250.40, 248.80, 247.46, 246.33
245.39, 246.84, 251.74, 256.13, 256.37, 255.05, 253.25, 251.26, 249.54, 247.82, 246.23
244.77, 244.24, 245.57, 248.04, 251.95, 252.89, 251.04, 249.00, 247.04, 244.94, 242.59

240.55, 239.10, 237.80, 236.86, 238.45, 243.81, 246.16, 244.81, 242.68, 240.48, 238.30 236.80, 236.06, 235.72, 235.16, 236.89, 246.24, 255.35, 256.20, 255.27]';

X_2 is 118 by 1 dimensional vector of 2nd immediate past water levels

>> X_2=[241.40, 239.76, 238.23, 237.41, 237.04, 237.41, 238.53, 242.22, 249.21, 251.70 250.14, 248.19, 246.34, 244.62, 243.06, 241.59, 240.27, 239.58, 240.45, 247.61, 259.72 263.67, 262.50, 260.83, 259.20, 257.36, 255.70, 254.03, 252.64, 252.45, 253.33, 257.18 262.31, 262.50, 260.60, 258.39, 256.36, 254.16, 252.16, 250.45, 248.65, 247.30, 246.47 247.84, 251.58, 250.40, 248.40, 246.50, 244.39, 242.45, 240.67, 239.28, 238.02, 237.40 238.41, 242.17, 245.40, 246.53, 244.96, 243.18, 241.54, 239.81, 238.35, 237.14, 236.60 237.57, 239.29, 245.16, 253.63, 255.89, 254.86, 253.55, 252.09, 250.40, 248.80, 247.46 246.33, 245.39, 246.84, 251.74, 256.13, 256.37, 255.05, 253.25, 251.26, 249.54, 247.82 246.23, 244.77, 244.24, 245.57, 248.04, 251.95, 252.89, 251.04, 249.00, 247.04, 244.94 242.59, 240.55, 239.10, 237.80, 236.86, 238.45, 243.81, 246.16, 244.81, 242.68, 240.48 238.30, 236.80, 236.06, 235.72, 235.16, 236.89, 246.24, 255.35, 256.20]';

X_1 is also 118 by 1 dimensional vector of 1st immediate past water levels

>> X_1=[243.39, 241.40, 239.76, 238.23, 237.41, 237.04, 237.41, 238.53, 242.22, 249.21 251.70, 250.14, 248.19, 246.34, 244.62, 243.06, 241.59, 240.27, 239.58, 240.45, 247.61 259.72, 263.67, 262.50, 260.83, 259.20, 257.36, 255.70, 254.03, 252.64, 252.45, 253.33 257.18, 262.31, 262.50, 260.60, 258.39, 256.36, 254.16, 252.16, 250.45, 248.65, 247.30 246.47, 247.84, 251.58, 250.40, 248.40, 246.50, 244.39, 242.45, 240.67, 239.28, 238.02 237.40, 238.41, 242.17, 245.40, 246.53, 244.96, 243.18, 241.54, 239.81, 238.35, 237.14 236.60, 237.57, 239.29, 245.16, 253.63, 255.89, 254.86, 253.55, 252.09, 250.40, 248.80 247.46, 246.33, 245.39, 246.84, 251.74, 256.13, 256.37, 255.05, 253.25, 251.26, 249.54 247.82, 246.23, 244.77, 244.24, 245.57, 248.04, 251.95, 252.89, 251.04, 249.00, 247.04

244.94, 242.59, 240.55, 239.10, 237.80, 236.86, 238.45, 243.81, 246.16, 244.81, 242.68

240.48, 238.30, 236.80, 236.06, 235.72, 235.16, 236.89, 246.24, 255.35]';

>> X^*=[ones(size(X_2)) X_2 X_1]

X^* =

```
    1.0000  241.4000  243.3900
    1.0000  239.7600  241.4000
    1.0000  238.2300  239.7600
    1.0000  237.4100  238.2300
    1.0000  237.0400  237.4100
    1.0000  237.4100  237.0400
    1.0000  238.5300  237.4100
    1.0000  242.2200  238.5300
    1.0000  249.2100  242.2200
    1.0000  251.7000  249.2100
    1.0000  250.1400  251.7000
    1.0000  248.1900  250.1400
    1.0000  246.3400  248.1900
    1.0000  244.6200  246.3400
    1.0000  243.0600  244.6200
    1.0000  241.5900  243.0600
    1.0000  240.2700  241.5900
    1.0000  239.5800  240.2700
    1.0000  240.4500  239.5800
    1.0000  247.6100  240.4500
    1.0000  259.7200  247.6100
    1.0000  263.6700  259.7200
    1.0000  262.5000  263.6700
    1.0000  260.8300  262.5000
    1.0000  259.2000  260.8300
    1.0000  257.3600  259.2000
    1.0000  255.7000  257.3600
    1.0000  254.0300  255.7000
    1.0000  252.6400  254.0300
    1.0000  252.4500  252.6400
    1.0000  253.3300  252.4500
    1.0000  257.1800  253.3300
    1.0000  262.3100  257.1800
    1.0000  262.5000  262.3100
    1.0000  260.6000  262.5000
    1.0000  258.3900  260.6000
    1.0000  256.3600  258.3900
    1.0000  254.1600  256.3600
    1.0000  252.1600  254.1600
    1.0000  250.4500  252.1600
    1.0000  248.6500  250.4500
    1.0000  247.3000  248.6500
```

1.0000	246.4700	247.3000
1.0000	247.8400	246.4700
1.0000	251.5800	247.8400
1.0000	250.4000	251.5800
1.0000	248.4000	250.4000
1.0000	246.5000	248.4000
1.0000	244.3900	246.5000
1.0000	242.4500	244.3900
1.0000	240.6700	242.4500
1.0000	239.2800	240.6700
1.0000	238.0200	239.2800
1.0000	237.4000	238.0200
1.0000	238.4100	237.4000
1.0000	242.1700	238.4100
1.0000	245.4000	242.1700
1.0000	246.5300	245.4000
1.0000	244.9600	246.5300
1.0000	243.1800	244.9600
1.0000	241.5400	243.1800
1.0000	239.8100	241.5400
1.0000	238.3500	239.8100
1.0000	237.1400	238.3500
1.0000	236.6000	237.1400
1.0000	237.5700	236.6000
1.0000	239.2900	237.5700
1.0000	245.1600	239.2900
1.0000	253.6300	245.1600
1.0000	255.8900	253.6300
1.0000	254.8600	255.8900
1.0000	253.5500	254.8600
1.0000	252.0900	253.5500
1.0000	250.4000	252.0900
1.0000	248.8000	250.4000
1.0000	247.4600	248.8000
1.0000	246.3300	247.4600
1.0000	245.3900	246.3300
1.0000	246.8400	245.3900
1.0000	251.7400	246.8400
1.0000	256.1300	251.7400
1.0000	256.3700	256.1300
1.0000	255.0500	256.3700
1.0000	253.2500	255.0500
1.0000	251.2600	253.2500
1.0000	249.5400	251.2600
1.0000	247.8200	249.5400
1.0000	246.2300	247.8200
1.0000	244.7700	246.2300
1.0000	244.2400	244.7700
1.0000	245.5700	244.2400
1.0000	248.0400	245.5700

1.0000	251.9500	248.0400
1.0000	252.8900	251.9500
1.0000	251.0400	252.8900
1.0000	249.0000	251.0400
1.0000	247.0400	249.0000
1.0000	244.9400	247.0400
1.0000	242.5900	244.9400
1.0000	240.5500	242.5900
1.0000	239.1000	240.5500
1.0000	237.8000	239.1000
1.0000	236.8600	237.8000
1.0000	238.4500	236.8600
1.0000	243.8100	238.4500
1.0000	246.1600	243.8100
1.0000	244.8100	246.1600
1.0000	242.6800	244.8100
1.0000	240.4800	242.6800
1.0000	238.3000	240.4800
1.0000	236.8000	238.3000
1.0000	236.0600	236.8000
1.0000	235.7200	236.0600
1.0000	235.1600	235.7200
1.0000	236.8900	235.1600
1.0000	246.2400	236.8900
1.0000	255.3500	246.2400
1.0000	256.2000	255.3500

>> b= $X^* \backslash X_3$

b =

34.0416

1.6039

-0.7416

>> $\hat{X}_3 = X^*$*b;

>> ME=mean($X_3 - \hat{X}_3$)

ME =

-1.1321e-013

>> MAE=mean(abs($X_3 - \hat{X}_3$))

MAE =

1.3661

>> SIGMA=std($X_3 - \hat{X}_3$)

SIGMA =

1.8937

>> maxerror=max(abs($X_3 - \hat{X}_3$))

maxerror =

1.845

>> minerror=min(abs($X_3 - \hat{X}_3$))

minerror = 0.0660

4.9 APPENDIX D

```
>> E=[13.95, 13.38, 13.70, 13.87, 13.49, 13.58, 12.92, 16.48, 17.49, 17.47, 18.45, 17.38
16.91, 15.15, 14.74, 14.46, 14.15, 14.11, 14.03, 15.81, 16.92, 16.70, 16.32, 12.59, 12.63
13.36, 11.10, 10.58, 10.39, 10.63, 11.05, 11.34, 12.24, 11.02, 11.55, 11.77, 9.38, 10.08 8.80,
8.11, 6.95, 6.42, 7.76, 8.75, 9.15, 9.74, 9.78, 10.71, 10.68, 11.06, 11.52, 11.69, 11.47, 12.82,
12.91, 12.00, 11.45, 12.06, 13.30, 13.40, 13.26, 14.53, 12.25, 13.04, 13.26 12.82, 10.76,
10.87, 12.08, 12.92, 14.60, 14.91, 14.83, 13.81, 14.24, 13.46, 14.34, 13.88 13.55, 13.01, 9.72,
9.82, 11.18, 12.36, 12.96, 13.66, 9.66, 6.47, 6.39, 5.97, 5.68, 5.77, 6.39, 9.16, 10.98, 9.33]';
>> X=[260.83, 259.20, 257.36, 255.70, 254.03, 252.64, 252.45, 253.33, 257.18, 262.31
262.50, 260.60, 258.39, 256.36, 254.16, 252.16, 250.45, 248.65, 247.30, 246.47, 247.84
251.58, 250.40, 248.40, 246.50, 244.39, 242.45, 240.67, 239.28, 238.02, 237.40, 238.41
242.17, 245.40, 246.53, 244.96, 243.18, 241.54, 239.81, 238.35, 237.14, 236.60, 237.57
239.29, 245.16, 253.63, 255.89, 254.86, 253.55, 252.09, 250.40, 248.80, 247.46, 246.33
245.39, 246.84, 251.74, 256.13, 256.37, 255.05, 253.25, 251.26, 249.54, 247.82, 246.23
244.77, 244.24, 245.57, 248.04, 251.95, 252.89, 251.04, 249.00, 247.04, 244.94, 242.59
240.55, 239.10, 237.80, 236.86, 238.45, 243.81, 246.16, 244.81, 242.68, 240.48, 238.30
236.80, 236.06, 235.72, 235.16, 236.89, 246.24, 255.35, 256.20, 255.27]';
```

>> S_{XX} =sum (X.^2)-mean(X)*sum(X)

S_{XX} =

4.7458e+003

>> S_{XE} =sum (X.*E)-(sum(X)*sum (E))/96

S_{XE} =

1.1692e+003

>> $\hat{\beta}_1 = \frac{S_{XE}}{S_{XX}}$

$\hat{\beta}_1 = 0.2464$

>> $\hat{\beta}_0$ =mean (E)- $\hat{\beta}_1$ *mean(X)

$\hat{\beta}_0$ =-48.8348

>> The fitted model relating energy to water level is $\hat{E}$ =-48.8348 + 0.2464X

>> $\hat{E} = \hat{\beta}_0 + \hat{\beta}_1$ *X;

>> ε_E =E- $\hat{E}$;

>> meanerror=mean(E- $\hat{E}$)

meanerror $= -1.9429e^{-015}$

>> μ represents the mean of E

>> μ =mean(E)

μ =12.1210GWh

>> s=std(E)

s = 2.8566

>> D represents the expected annual energy generated

>> working days per year is 365

>> D= μ *365

D =

4.4242e+003GWh

>> m is the cost per unit energy at Volta River Authority. This was received from VRA substation in Kumasi

>> m is 6.9cent per 1kwh

>> m is 0.069US dollar per 1kwh. 1GWh is the same as 10^6kwh

>> Therefore, m is 0.069*10^6dollars per 1GWh. That is:

>> m=0.069*10^6

m = $69000

>> C_0 is the cost per order. This cost is fixed regardless of the order quantity.

>> C_0 =$109090

>> $C_h = \hat{\beta}_1$ *m

C_h = 1.6999e^{+004}$

>> SS_E = sum((E-e).^2) = 487.1935

>> Q_m =sqrt(2*D*C_0 / C_h)

Q_m =

238.2920GWh

>> N represents number of orders to be placed annually and T represents average working days between orders per year

>> N=D/Q_m

N =

18.5662

>> T=365/N

T =

19.6594days

>> Let h be the total holding cost per year

>> h= (Q_m/2)*C_h

h =

$2.0254e+006

>> O is the total ordering cost per year

>> O= (D/ Q_m)* C_0

O =

$ 2.0254e+006

>> X_1 =[20.83 19.2 17.36 15.70 14.03 12.64 12.45 13.33 17.18 22.31 22.50 20.60 18.39 16.36 14.16 12.16 10.45 8.65 7.30 6.47 7.84 11.53 10.40 8.40 6.50 4.39 2.45 0.67 0 0 0 0 2.17 5.40 6.53 4.96 3.18 1.54 0 0 0 0 0 0 5.16 13.63 15.89 14.86 13.55 12.09 10.40 8.80 7.46 6.33 5.39 6.84 11.74 16.13 16.37 15.05 13.25 11.26 9.54 7.82 6.23 4.77 4.24 5.57 8.04 11.95 12.89 11.04 9.00 7.04 4.94 2.59 0.55 0 0 0 0 3.81 6.16 4.81 2.68 0.48 0 0 0 0 0 0 6.24 15.35 16.20 15.27]';

>> S=size(X_1)

S =

96 1

>> p=mean(X_1)

p =

7.9108

4.10 APPENDIX E

% matlab simple code for drawing graphs, figures of Akosombo dam water level patterns

% graphs of service levels inventory pattern for the Lot- size model

% m is the number of data points

% trajectory of Akosombo dam water levels

% trajectory of Akosombo dam energy demand data

```
for i=1:m
    a=input('enter values of [t,X]:');
    A=[A;a];
end
t=A(:,1);
X=A(:,2);
plot(t,X,'.')
title('Graph of average daily water levels on monthly basis of Akosombo dam
from Jan. 1998-Dec. 2007')
xlabel('TIME(MONTHS)')
ylabel('AVERAGE DAILY WATER LEVELS')
grid
```