KWAME NKRUMAH UNIVERSITY OF SCIENCE AND TECHNOLOGY

OPTIMAL TREATMENT OF WATER USING LINEAR PROGRAMMING

A **CASE STUDY OF BEREKUM WATER TREATMENT PLANT**

By

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DECLARATION

I hereby declare that this submission is my own work towards the MSc. And that, to the best of my knowledge, it contains no material previously published by another person now material, which has been accepted for the award of any other degree of he university, except where due acknowledgement has been made in the text.

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ABSTRACT

Revised Simplex method was used to reduce treatment cost of water at Berekum water trearment plant. The 2011 data collected was divided into two to reflect the two major seasons-dry and wet season, we have in the country. Each data was modeled into objective functions and subject constraints. Matrices generated from each season were run on Matlab code. Result that were obtained showed a significant reduction in treatment cost compared to actual cost in the same year

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DEDICATION

I wish to dedicate this thesis to my dear mum,Fofoe Tawiah Kaanya; my uncle Onasis and my entire family for their immense contribution in diverse ways during the writing of this thesis

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An exercise like this couldn't have been done without the help and entire network of people, often too many to acknowledge in writing. My first gratitude goes to the almighty God for the grace and mercy upon my life.I wish to express my gratitude to Mr . K.F . Darkwah my supervisor, for his patience and direction.

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My appreciation also goes to my uncle and all my relatives for their support and prayers.Not forgetting all timely friends who came into my life and left priceless memories. May God bless you all.

CHAPTER 1

INTRODUCTION

Due to the expanding human population, competition for water is growing such that many of the worlds major aquifers are becoming depleted. This is due both for direct human consumption as well as agricultural irrigation by groundwater. Millions of pumps of all sizes are currently extracting groundwater throughout the world. Irrigation in dry areas such as northern China and India is supplied by groundwater, and is being extracted at an unsustainable rate. (Groundwater in Urban Development)

Water pollution is one of the main concerns of the world today. The governments of numerous countries have striven to find solutions to reduce this problem. Many pollutants threaten water supplies, but the most widespread, especially in developing countries, is the discharge of raw sewage into natural waters; this method of sewage disposal is the most common method in underdeveloped countries, but also is prevalent in quasi-developed countries such as China, India and Iran. Sewage, sludge, garbage, and even toxic pollutants are all dumped into the water. Even if sewage is treated, problems still arise. Treated sewage forms sludge, which may be placed in landfills, spread out on land, incinerated or dumped at sea. In addition to sewage, nonpoint source pollution such as agricultural runoff is a significant source of pollution in some parts of the world, along with urban storm water runoff and chemical wastes dumped by industries and governments . Competition for water has widely increased, and it has become more difficult to conciliate the necessities for water supply for human consumption, food production, ecosystems and other uses. Water administration is frequently involved in contradictory and complex problems.(Marine Protection Research and Sanitation Act).

However in Ghana, portable water coverage is very low in urban and rural areas. Hence it is important to minimize the cost of treating water to ensure its availability and it’s affordability.

1.1 BACKGROUND TO THE STUDY

Water is a chemical substance with the chemical formula H_2O. Its molecule contains one oxygen and two hydrogen atoms connected by covalent bonds. Water is a liquid at ambient conditions, but it often co-exists on Earth with its solid state, ice, and gaseous state (water vapor) .

Water also exists in a liquid crystal state near hydrophilic surface.

97% of the water on the Earth is salt water. However, only three percent is fresh water; slightly over two thirds of this is frozen in glaciers and polar ice caps. (Earth water distribution)

The remaining unfrozen freshwater is found mainly as groundwater, with only a small fraction present above ground or in the air.(Scientific fact on water)

Surface water is water in a river, lake or fresh water wetland. Surface water is naturally replenished by precipitation and naturally lost through discharge to the oceans, evaporation, evapotranspiration and sub-surface seepage.

Although the only natural input to any surface water system is precipitation within its watershed, the total quantity of water in that system at any given time is also dependent on many other factors. These factors include storage capacity in lakes, wetlands and artificial reservoirs, the permeability of the soil beneath these storage bodies, the runoff characteristics of the land in the watershed, the timing of the precipitation and local evaporation rates. All of these factors also affect the proportions of water loss.

Human activities can have a large and sometimes devastating impact on these factors. Humans often increase storage capacity by constructing reservoirs and decrease it by draining wetlands. Humans often increase runoff quantities and velocities by paving areas and channelizing stream flow.The total quantity of water available at any given time is an important consideration. Some human water users have an intermittent need for water. For example, many farms require large quantities of water in the spring, and no water at all in the winter. To supply such a farm with water, a surface water system may require a large storage capacity to collect water throughout the year and release it in a short period of time. Other users have a continuous need for water, such as a power plant that requires water for cooling. To supply such a power plant with water, a surface water system only needs enough storage capacity to fill in when average stream flow is below the power plant's need.

Nevertheless, over the long term the average rate of precipitation within a watershed is the upper bound for average consumption of natural surface water from that watershed.

Natural surface water can be augmented by importing surface water from another watershed through a canal or pipeline. It can also be artificially augmented from any of the other sources listed here, however in practice the quantities are negligible. Humans can also cause surface water to be "lost" (i.e. become unusable) through pollution. .(The world water)

It is estimated that 8% of worldwide water use is for household purposes. These include drinking water, bathing, cooking, sanitation, and gardening. Basic household water requirements have been estimated by Peter Gleick at around 50 liters per person per day, excluding water for gardens. Drinking water is water that is of sufficiently high quality so that it can be consumed or used without risk of immediate or long term harm. Such water is commonly called potable water. In most developed countries, the water supplied to households, commerce and industry is all of drinking water standard even though only a very small proportion is actually consumed or used in food preparation.

It is estimated that 22% of worldwide water is used in industry. Major industrial users include hydroelectric dams, thermoelectric power plants, which use water for cooling, ore and oil refineries, which use water in chemical processes, and manufacturing plants, which use water as a solvent. Water withdrawal can be very high for certain industries, but consumption is generally much lower than that of agriculture.

Water is used in renewable power generation. Hydroelectric power derives energy from the force of water flowing downhill, driving a turbine connected to a generator. This hydroelectricity is a low-cost, non-polluting, renewable energy source. Significantly, hydroelectric power can also be used for load following unlike most renewable energy sources which are intermittent. Ultimately, the energy in a hydroelectric powerplant is supplied by the sun. Heat from the sun evaporates water, which condenses as rain in higher altitudes and flows downhill. Pumped-storage hydroelectric plants also exist, which use grid electricity to pump water uphill when demand is low, and use the stored water to produce electricity when demand is high.

Hydroelectric power plants generally require the creation of a large artificial lake. Evaporation from this lake is higher than evaporation from a river due to the larger surface area exposed to the elements, resulting in much higher water consumption. The process of driving water through the turbine and tunnels or pipes also briefly removes this water from the natural environment, creating water withdrawal. The impact of this withdrawal on wildlife varies greatly depending on the design of the powerplant.

Pressurized water is used in water blasting and water jet cutters. Also, very high pressure water guns are used for precise cutting. It works very well, is relatively safe, and is not harmful to the environment. It is also used in the cooling of machinery to prevent overheating, or prevent saw blades from overheating. This is generally a very small source of water consumption relative to other uses.

Water is also used in many large scale industrial processes, such as thermoelectric power production, oil refining, fertilizer production and other chemical plant use, and natural gas extraction from shale rock. Discharge of untreated water from industrial uses is pollution. Pollution includes discharged solutes (chemical pollution) and increased water temperature (thermal pollution). Industry requires pure water for many applications and utilizes a variety of purification techniques both in water supply and discharge. Most of this pure water is generated on site, either from natural freshwater or from municipal grey water. Industrial consumption of water is generally much lower than withdrawal, due to laws requiring industrial grey water to be treated and returned to the environment. Thermoelectric powerplants using cooling towers have high consumption, nearly equal to their withdrawal, as most of the withdrawn water is evaporated as part of the cooling process. The withdrawal, however, is lower than in once-through cooling system.

(Water facts andTrends)

Ghana Water Company Limited (GWCL) is responsible for treatment and distribution of water in Ghana. It is divided into three main divisions and they are:

1. Water supplies

2. Water distributors

3. Administrators.

1.2 PROFILE OF GWCL

The first public water supply system in Ghana, then Gold Coast, was established in Accra just before World War I. Extensions were made to Cape Coast, Winneba and Kumasi in the1920s.

During this period, the water supply systems were managed by the Hydraulic Division of Public Works Department. With time the responsibilities of the Hydraulic Division were widened to include the planning and development of water supply systems in other parts of the country.

In 1948, the Department of Rural Water Development was established to engage in the development and management of rural water supply through the drilling of bore holes and construction of wells for rural communities.

After Ghana's independence in 1957, a Water Supply Division, with headquarters in Kumasi, was set up under the Ministry of Works and Housing with responsibilities for both urban and rural water supplies.

During the dry season of 1959, there was severe water shortage in the country. Following this crisis, an agreement was signed between the Government of Ghana and the World Health Organisation (WHO) for a study to be conducted into the water sector development of the country.

In line with the recommendations of the WHO, the Ghana Water and Sewerage Corporation (GWSC), was established in 1965 under an Act of Parliament (Act 310) as a legal public utility entity. GWSC was to be responsible for:

- water supply and sanitation in rural as well as urban areas.
- the conduct of research on water and sewerage as well as the making of engineering surveys and plans.
- the construction and operation of water and sewerage works,
- the setting of standards and prices and collection of revenues.

In the late 1970s and early 1980s, the operational efficiency of GWSC declined to very low levels mainly as a result of the deterioration in pipe connections and pumping systems. A World Bank report in 1998 states that: "The water supply systems in Ghana deteriorated rapidly during the economic crises of the 1970´s and early 1980´s when Government's ability to adequately operate and maintain essential services was severely constrained.

To reverse the decline in water supply services, interventions in the area of sector reforms and project implementation were made in 1970, 1981 and 1988. Though some gains were derived from these interventions, their general impact on service delivery was very disappointing. Due to the failure of these interventions to achieve the needed results, several efforts were made to improve efficiency within the water supply sector in Ghana especially during the era of the Economic Recovery Programme from 1983 to 1993.

During this period, loans and grants were sought from the World Bank and other donors for the initiation of rehabilitation and expansion programmes, to train personnel and to buy transport and maintenance equipment.

In addition, user fees for water supply were increased and subsidies on water tariffs were gradually removed for GWSC to achieve self-financing.

The government at that time approved a formula for annual tariff adjustments to enable the corporation generate sufficient funds to cover all annual recurrent costs as well as attain some capacity to undertake development projects.

In 1987, a "Five-Year Rehabilitation and Development Plan" for the sector was prepared which resulted in the launching of the Water Sector Restructuring Project (WSRP). The reforms were aimed at reducing unaccounted for water, introducing rationalisation through reduction of the workforce, hiring of professionals and training of the remaining staff.

Accordingly, a number of organisational reforms within the Ghanaian water sector were initiated in the early 1990s. As a first step, responsibilities for sanitation and small towns water supply were decentralized from Ghana Water and Sewerage Corporation to the District Assemblies in 1993.

In 1994 the Environmental Protection Agency (EPA) was established to ensure that water operations did not cause any harm to the environment. The Water Resources Commission (WRC) was founded in 1996 to be in charge of overall regulation and management of water resources utilization. In 1997, the Public Utilities Regulatory Commission (PURC) came into being with the purpose of setting tariffs and quality standards for the operation of public utilities.

With the passage of Act 564 of 1998, Community Water and Sanitation Agency (CWSA) was established to be responsible for management of rural water supply systems, hygiene education and provision of sanitary facilities. After the establishment of CWSA, 120 water supply systems serving small towns and rural communities were transferred to the District Assemblies and Communities to manage under the community-ownership and management scheme.

Finally, pursuant to the Statutory Corporations (Conversion to Companies) Act 461 of 1993 as amended by LI 1648, on 1st July 1999, GWSC was converted into a 100% state owned limited liability, Ghana Water Company Limited, with the responsibility for urban water supply only. (Adombire, 2007)

1.3 BEREKUM TREATMENT STATION AND DISTRIBUTION

People in Berekum pay so much for water due to high cost of its treatment by GWC. Those who are unable to pay their bills get their taps disconnected. This leaves such people with no choice but to go in for untreated water from other sources. As such there is prevalence of water borne diseases. Lives are therefore lost because this practice.

The problem at hand is for GWC to optimize (minimize) the cost of treating water with respect to:

- the cost of bags of chemical use in purifying water;
- the cost of fuel used;
- the cost of electricity used

in other to make it more affordable and accessible.

1.4 OBJECTIVES OF THE STUDY

1. Model water treatment cost as a Linear Programming problem.

2. Minimize treatment cost of water at Berekum water treatment plant.

1.5 METHODOLOGY

The problem of water treatment will be modeled as a linear programming problem. Revised Simplex method will be used to develop the Mathematics Model. The Revised Simplex method is preferred over interior point method because revised simplex methods approach the boundary of the feasible set in the limit.

Data will be collected from Berekum water treatment plant.

Software program on MATLAB will be developed using the mathematics model to run the data.

1.6 SIGNIFICANCE OF THE STUDY

Underground water is the main source of raw water to the Berekum water treatment plant. Treated water from this source serves the Berekum municipality and its environs. Irregular supply of water year round due to high cost of treatment therefore affects the livelihood of people depending on it.

Application of findings from this research will help reduce cost of water treatment. This means that there will be regular supply of water and so people will not resort to other sources of water.

Quantity of water produced would increase and so could be extended to areas that do not have access to tap water. In effect, it will:

- help reduce the amount of money that government will spend to treat people with water borne diseases.
- reduce the number of lives that are lost out of water borne related diseases
- help increase productivity in other sectors of the economy since manpower needed in those areas will not spend hours they are to use at their work places to treat themselves of water borne diseases.
- serve as a basis for more research to be made on this area.

1.7 THE SCOPE OF THE STUDY

The research aims at developing a water treatment cost model.

The model will be used to determine the optimum cost of treating water at Berekum headwork.

The work will intend to analyze the cost involved in water treatment and to distribute the same quantity (volume) of water to Berekum Municipality and its environs at a minimal cost, taking the major seasons in the year into account.

1.8 LIMITATIONS OF THE STUDY

The major limitations of this study are:

1. The research will not cover all aspects of water treatment and supply.

   Example:

   - Cost of civil structures of the treatment plant.

- Supply chain

2. The research will not determine the optimal consumption of water used to maintain the plant.

1.9 ORGANIZATION OF THE STUDY

Chapter 1 deals with the general introduction of the study. Chapter 2 gives a review of the existing theoretical and empirical literature. Chapter 3 deals with methodology. Chapter 4 deals with data collection, analysis, estimate and discussion of results. The concluding chapter, Chapter 5 summaries the findings and also provides conclusions and recommendations.

CHAPTER 2

LITERATURE REVIEW

2.0 INTRODUCTION

This chapter reviews the literature on the application of linear programming:

2.1 LINEAR PROGRAMMING

A study conducted by Jianq et al., (2004) proposed a linear programming based method to estimate arbitrary motion from two images. The proposed method always finds the global optimal solution of the linearized motion estimation energy function and thus is much more robust than the traditional motion estimation schemes. As well, the method estimates the occlusion map and motion field at the same time. To further reduce the complexity-reduced pure linear programming method they presented a two phase scheme to estimating the dense motion field. In the first step, the estimated a relative sparse motion field for the edge pixels using a non-regular sampling scheme, based on the proposed linear programming method. In the second step, they set out a detail-preserving variation variational method to upgrade the result into a dense motion field. The proposed scheme is much faster than a purely linear programming based dense motion estimation scheme. And, since they used a global optimization method linear programming in the first estimation step, they proposed two-phase scheme was also significantly more robust than a pure variation.

Biswal et Li., (1998) developed an approach to solve probabilistic linear programming problems with exponential random variables. The first step involves obtaining the probability density function (p.d.f.) of the linear combination of n independent exponential random variables. Probabilistic constraints are then transformed to the deterministic constraints using the p.d.f. The resulting non-linear deterministic model is then solved using a non-linear programming solution method.

Reuter and Deventer (2003) proposed two linear models, the second being a subset of the first, for the simulation of flotation plants by use of linear programming. The first linear model produced the circuit structure, as well as the optimal flow rates of the valuable element between any number of flotation banks incorporating any number of recycle mills. An optimal grade for the valuable element in the concentrate was given by the second model. Operating conditions in the flotation banks and recycle mills were included as bounds in these models, permitting their possible application in expert systems. The simulated circuit structure, concentrate grade and recoveries closely resembled those of similar industrial flotation plants. The only data required by the simulation models were the feed rates of the species of an element, and their separation factors which were estimated from a multiparameter flotation model.

Greenberg et al., (1986) developed a framework for model formulation and analysis to support operations and management of large-scale linear programs from the combined capabilities of camps and analyze. Both the systems were reviewed briefly and the interface which integrates the two systems was then described. The model formulation, matrix generation, and model management capability of camps and the complementary model and solution analysis capability of analyze were presented within a unified framework. Relevant generic functions were highlighted, and an example was presented in detail to illustrate the level of integration achieved in the current prototype system. Some new results on discourse models and model management support were given in a framework designed to move toward an 'intelligent' system for linear programming modeling and analysis

Cherubini et al., (2009) described an optimization model which aims at minimizing the maximum link utilization of IP telecommunication networks under the joint use of the traditional

IGP protocols and the more sophisticated MPLS-TE technology. The survivability of the network was taken into account in the optimization process implementing the path restoration scheme.This scheme benefits of the Fast Re-Route (FRR) capability allowing service providers to offer high availability and high revenue SLAs (Service Level Agreements). The hybrid IGP/MPLS approach relies on the formulation of an innovative Linear Programming mathematical model that, while optimizing the network utilization, provides optimal user performance, efficient use of network resources, and 100% survivability in case of single link failure. The possibility of performing an optimal exploitation of the network resources throughout the joint use of the IGP and MPLS protocols provides a flexible tool for the ISP (Internet Service Provider) networks traffic engineers. The efficiency of the proposed approach was validated by a wide experimentation performed on synthetic and real networks. The obtained results showed that a small number of LSP tunnels have to be set up in order to significantly reduce the congestion level of the network while at the same time guaranteeing the survivability of the network.They applied this approach to a quadratic-cost single-commodity network design problem, comparing the newly developed algorithm with those based on both the standard continuous relaxation and the two usual variants of PR relaxation.

A linear programming model for a river basin was developed by Avdelas et al (1992) to include almost all water-related economic activity both for consumers and producers. The model was so designated that the entire basin or basin sub-division could be analyzed. The model included seven sectors, nine objective function criteria, and three river-flow levels. Economic basis for conflicts among sectors over incidence of cost allocation and level of economic activity were traced to some chosen objective. The disposal of untreated household waste water, particularly from the rural household, directly into the river was consistent with maximizing net benefits and minimizing costs. For each of the three industries analyzed separately, paper, wool and tanning, public treatment of industrial waste water was the optimal treatment process in one or more of the solutions.

To investigate how farmers could sustain an economically viable agricultural production in salt-affected areas of Oman, Naifer et al (2010), divided a sample of 112 farmers into three groups according to the soil salinity levels, low salinity, medium salinity and high salinity. Linear programming was used to maximize each type of farm's gross margin under water, land and labor constraints. The economic losses incurred by farmers due to salinity were estimated by comparing the profitability of the medium and high salinity farms to the low salinity farm's gross margin. Results showed that when salinity increased from low salinity to medium salinity level the damage was US$ 1,604 ha^{-1} and US$ 2,748 ha^{-1} if it increased from medium salinity to high salinity level. Introduction of salt-tolerant crops in the cropping systems showed that the improvement in gross margin was substantial thus attractive enough for medium salinity farmers to adopt the new crops and/or varieties to mitigate the effect of water salinity.

Frizzone et al (1997) developed a separable linear programming model, considering a set of technical factors which might influence the profit of an irrigation project. The model presented an objective function that maximized the net income and specified the range of water availability. It was assumed that yield functions in response to water application were available for different crops and described very well the water-yield relationships. The linear programming model was developed genetically, so that, the rational use of the available water resource could be included in an irrigation project. Specific equations were developed and applied in the irrigation project "Senator Nilo Coelho" (SNCP), located in Petrolina – Brazil. Based on the water-yield functions considered, cultivated land constraints, production costs and products prices, it was concluded that the model was suitable for the management of the SNCP, resulting in optimal cropping patterns.

Chung et al (2008) considered a municipal water supply system over a 15-year planning period with initial infrastructure and possibility of construction and expansion during the first and sixth year on the planning horizon. Correlated uncertainties in water demand and supply were applied on the form of the robust optimization approach of Bertsimas and Sim to design a reliable water supply system. Robust optimization aims to find a solution that remains feasible under data uncertainty. It was found that the robust optimization approach addressed parameter uncertainty without excessively affecting the system. While they applied their methodology to hypothetical conditions, extensions to real-world systems with similar structure were straightforward. Therefore, their study showed that this approach was a useful tool in water supply system design that prevented system failure at a certain level of risk.

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Matthews (2005) evaluated and optimized the utility of the nurse personnel at the Internal Medicine Outpatient Clinic of Wake Forest University Baptist Medical Center. Linear programming (LP) was employed to determine the effective combination of nurses that would allow for all weekly clinic tasks to be covered while providing the lowest possible cost to the department. A specific sensitivity analysis was performed to assess just how sensitive the outcome was to the stress of adding or deleting a nurse to or from the payroll. The nurse employee cost structure in this study consisted of five certified nurse assistants (CNA), three licensed practicing nurses (LPN), and five registered nurses (RN). The LP revealed that the outpatient clinic should staff four RNs, three LPNs, and four CNAs with 95 percent confidence of covering nurse demand on the floor.

Khan et al (2005) used Linear Programming Model to calculate the crop acreage, production and income of cotton zone. This was carried out in the three districts of the Bahawalpur. These three districts were collected by purposive sampling technique. The study was conducted on 4652 acres of the irrigated areas from the three districts. Crops included in the model were wheat, basmati rice, IRRI rice, cotton and sugar cane. The results showed that the cotton was the only crop, which gained acreage by about 10% at the expense of all other crops. Overall optimal crop acreage decreased by 1.76%, while optimal income was increased by 3.28% as compared to the existing solutions. The study reported that Bahawalpur division was more or less operating at the optimal level.

The study was conducted by Kumar and Khepar (1980) to demonstrate the usefulness of alternative levels of water use over the fixed yield approach when there is a constraint on water. In the multi-crop farm models used, a water production function for each crop was included so that one had the choice of selecting alternative levels of water use depending upon water availability. Water production functions for seven crops, viz. wheat, gram, mustard, berseem, sugarcane, paddy and cotton, based on experimental data from irrigated crops were used. The fixed yield model was modified incorporating the stepwise water production functions using a separable programming technique. The models were applied on a selected canal command area and optimal cropping patterns determined. Sensitivity analysis for land and water resources was also conducted. The water production function approach gave better possibilities of deciding upon land and water resources.

Mousavi et al (2004) presented a long-term planning model for optimizing the operation of Iranian Karoon-Dez reservoir system using an interior-point algorithm. The system is the largest multi-purpose reservoir system in Iran with hydropower generation, water supply, and environmental objectives. The focus was on resolving the dimensionality of the problem of optimization of a multi-reservoir system operation while considering hydropower generation and water supply objectives. The weighting and constraints methods of multi-objective programming were used to assess the trade-off between water supply and hydropower objectives so as to find noninferior solutions. The computational efficiency of the proposed approach was demonstrated using historical data taken from Karoon-Dez reservoir system.

Heidari (2007) formulated and solved ground water management model based on the linear systems theory using linear programming. The model maximized the total amount of pound water that could be pumped from the system subject to the physical capability of the system and institutional constraints. The results were compared with analytical and numerical solutions. Then, this model was applied to the Pawnee Valley area of south-central Kansas. The results of this application supported the previous studies about the future ground water resources of the Valley. These results provided a guide for the ground water resources management of the area over the next ten years.

Isa (1990) used of Linear Programming (LP) and other mathematical procedures to evaluate watershed and perpetuity constraints on forest land use for a selected scenario in Terengganu, Peninsular Malaysia. The LP model provided a range of feasible solutions for decision making. Equations were derived for the model to show interaction of sedimentation due to road construction, timber harvesting, and other related forest management activities. Sensitivity analysis was used to test model behavior. Results indicated the constraining effects of sedimentation upon forest revenues when sedimentation was allowed to vary within the feasible region of the model (i.e., from 600,000 m^3/decade up to 1,150,000 m3/decade

Banks and Fleck (2010) applied Linear programming techniques to ground-water- flow model in order to determine optimal pumping scenarios for 14 extraction wells located downgradient of a landfill and upgradient of an estuary. The model was used to simulate flow as well as the effects of a pump-and-treat remediation system designed to capture contaminated ground water from the water-table aquifer before it reached the adjacent estuary. The objective function involved varying pumping rates and frequencies to maximize capture of ground water from the water-table aquifer. At the same time, the amount of water extracted and needing treatment was minimized. The constraints placed on the system insured that only ground water from the landfill was extracted and treated. To do this, a downward gradient from the disposal area toward the extraction wells was maintained.

A groundwater management problem in a coastal karstic aquifer in Crere, Greece subject to environmental criteria was studied by Karterakis et al (2007) using classical linear programming and heuristic optimization methodologies. A numerical simulation model of the unconfined coastal aquifer was first developed to represent the complex non-linear physical system. Then the classical linear programming optimization algorithm of the Simplex method was used to solve the groundwater management problem where the main objective was the hydraulic control of the saltwater intrusion. A piecewise linearization of the non-linear optimization problem was obtained by sequential implementation of the Simplex algorithm and a convergence to the optimal solution was achieved. The solution of the non-linear management problem was also obtained using a heuristic algorithm. A Differential Evolution (DE) algorithm that emulates some of the principles of evolution was used. A comparison of the results obtained by the two different optimization approaches was then presented.

Finally, a sensitivity analysis was employed in order to examine the influence of the active pumping wells in the evolution of the seawater intrusion front along the coastline.

Turgeon (1986) developed a parametric mixed-integer linear programming (MILP) method for selecting the sites on the river where reservoirs and hydroelectric power plants were to be built and then determining the type and size of the projected installations. The solution depended on the amount of money the utility was willing to invest, which itself was a function of what the new installations would produce. This method was used based on the fact that the branch-and-bound algorithm for selecting the sites to be developed (and consuming most of the computer time) was solved a minimum number of times. Between the points where the MILP problem was solved, LP parametric analysis was applied.

Khaled (2004) developed four models of optimal water allocation with deficit irrigation in order to determine the optimal cropping plan for a variety of scenarios. The first model (Dynamic programming model (DP)) allocated a given amount of water optimally over the different growth stages to maximize the yield per hectare for a given crop, accounting for the sensitivity of the crop growth stages to water stress. The second model (Single Crop Model) tried to find the best allocation of the available water both in time and space in order to maximize the total expected yield of a given crop. The third model (Multi crop Model) was an optimization model that determined the optimal allocation of land and water for different crops. It showed the importance of several factors in producing an optimal cropping plan. The output of the models was prepared in a readable form to the normal user by the fourth model (Irrigation Schedule Model).

Vimonsatit et al., (2003) proposed a linear programming (LP) formulation for the evaluation of the plastic limit temperature of flexibly connected steel frames exposed to fire. Within a framework of discrete models and piecewise linearized yield surfaces, the formulation was derived based on the lower-bound theorem in plastic theory, which lead to a compact matrix form of an LP problem. The plastic limit temperature was determined when the equilibrium and yield conditions were satisfied. The plastic mechanism can be checked from the dual solutions in the final simplex tableau of the primal LP solutions. Three examples were presented to investigate the effects of the partial-strength beam-to-column joints. Eigenvalue analysis of the assembled structural stiffness matrix at the predicted limit temperature was performed to check for structural instability. The advantage of the proposed method is that it is simple, computationally efficient, and its solutions provide the necessary information at the limit temperature. The method can be used as an efficient tool to a more refined but computationally expensive step-by-step historical deformation analysis.

Belotti et al., (2005) proposed to tackle large-scale instances of Maximum feasible subsystem using randomized and thermal variants of the classical relaxation method for solving systems of linear inequalities. They established lower bounds on the probability that these methods identify an optimal solution within a given number of iterations. These bounds, which are expressed as a function of a condition number of the input data, imply that with probability one these randomized methods identify an optimal solution after finitely many iterations. Computational results obtained for medium- to large-scale instances arising in the design of linear classifiers, in the planning of digital video broadcasts and in the modeling of the energy functions driving protein folding, indicate that an efficient implementation of such a method perform very well in practice.Industrial switching involves moving materials on rail cars within or between industrial complexes and connecting with other rail carriers. Planning tasks include the making up of trains with a minimum shunting effort, the feasible and timely routing through an in-plant rail network on short paths, and assigning and scheduling of locomotives under safety and network capacity aspects. A human planner must often resort to routine and simple heuristics, not least for the reason of unavailability of computer aided suggestions.

Becker (1995) explored the implications of the transformation of the system of water resources allocation to the agricultural sector in Israel from a one in which allotments were allocated to the different users without any permission to trade with water rights. A mathematical planning model was used for the entire Israeli agricultural sector, in which an 'optimal' allocation of the water resources was found and compared to the existing one. The results of the model were used in order to gain insight into the shadow price of the different water bodies in Israel (about eight). These prices could be used to grant property rights to the water users themselves in order to guarantee rational behaviour of water use, since no one could sell their rights at the source itself. From the dual prices of the primal problem they could forecast the equilibrium prices and their implications for the different users. The results showed that there was a potential budgetary benefit of 28 million dollars when capital cost was not included and 64 millions dollars when it was included

Hoesein and Limantara (2010) studied the optimization of water supply for irrigation at Jatimlerek irrigation area of 1236 ha. Jatimlerek irrigation scheme was intended to serve more than one district. The methodology consisted of optimization water supply for irrigation with Linear Programming. Results were used as the guidance in cropping pattern and allocating water supply for irrigation at the area. Linear programming model was applied by Hassan (2004) to calculate the optimal crop acreage, production and income of the irrigated Punjab. Crops included in the models were wheat, Basmati rice, IRRI rice, cotton, sugarcane, maize, potato, gram and mong / mash. The results showed that the irrigated agriculture in the Punjab was more or less operating at the optimal level. Over all cropped acreage in the Optimal solution decreased by 0.37 % as compared to the existing acreage. However, in the optimal cropping pattern some crops like cotton and pluses gained acreage by 9-10 % each, while maize and Basmati rice remained unchanged. On the other hand crops like wheat, IRRI rice, potato and sugarcane lost acreage by 4-11 %. As a result of optimum croppingpattern income, increased by 1.57 %.

Tsakiris and Spiliotis (2004) treated the Systems Analysis formulation problem of water allocation to various users as a linear programming problem with the objective of maximizing the total productivity. This was intended to solve one of the basic problems of Water Resource Management in the allocation of water resources to various users in an optimal and equitable way respecting the constraints imposed by the environment. In this work a fuzzy set representation of the unit revenue of each use together with a fuzzy representation of each set of constraints, were used to expand the capabilities of the linear programming formulation. Numerical examples were presented for illustrative purposes and useful conclusions are derived.

Konickova (2006) said a linear programming problem whose coefficients are prescribed by intervals is called strongly unbounded if each linear programming problem obtained by fixing coefficients in these intervals is unbounded. In the main result of the paper a necessary and sufficient condition for strong unboundedness of an interval linear programming problem was described. In order to have a full picture they also showed conditions for strong feasibility and strong solvability of this problem. The necessary and sufficient conditions for strong feasibility, strong solvability and strong unboundedness can be verified by checking the appropriate properties by the finite algorithms. Checking strong feasibility and checking strong solvability are NP-hard. This shows that checking strong unboundedness is NP-hard as well.

Optimal solutions of Linear Programming problems may become severely infeasible if the nominal data is slightly perturbed.

Yoshito (2004) considered the problem of finite dimensional approximation of the dual problem in abstract linear programming approach to control system design. A constraint qualification that guarantees the existence of a sequence of finite dimensional dual problems that computes the true optimal value. The result is based on the averaging integration by a probability measures. A matrix is sought that solves a given dual pair of systems of linear algebraic equations. Necessary and sufficient conditions for the existence of solutions to this problem were obtained, and the form of the solutions was found. The form of the solution with the minimal Euclidean norm was indicated. Conditions for this solution to be a rank one matrix were examined. On the basis of these results, an analysis was performed for the following two problems: modifying the coefficient matrix for a dual pair of linear programs (which can be improper) to ensure the existence of given solutions for these programs, and modifying the coefficient matrix for a dual pair of improper linear programs to minimize its Euclidean norm. Necessary and sufficient conditions for the solvability of the first problem were given, and the form of its solutions was described. For the second problem, a method for the reduction to a nonlinear constrained minimization problem was indicated, necessary conditions for the existence of solutions were found, and the form of solutions was described.

CHAPTER 3

3.0 INTRODUCTION

. This chapter reviews the methodology used for developing water treatment cost model. The first phase of this chapter talks about some procedures, the linear programming model, theoretical method used in solving it and software for solving linear programming.

3.1.0 LINEAR PROGRAMMING

Linear programming is a mathematical technique that deals with the optimization (maximizing or minimizing) of a linear function known as objective function subject to a set of linear equations or inequalities known as constraints. It is a mathematical technique which involves the allocation of scarce resources in an optimum manner, on the basis of a given criterion of optimality. The technique used here is linear because the decision variables in any given situation generate straight line when graphed. It is also programming because it involves the movement from one feasible solution to another until the best possible solution is attained. A variable or decision variables usually represent things that can be adjusted or controlled. An objective function can be defined as a mathematical expression that combines the variables to express your goal and the constraints are expressions that combine variables to express limits on the possible solutions.

Linear programs can be expressed in the form:

$$\begin{aligned} &\text{maximize} && c^\top x \\ &\text{subject to} && Ax \le b \end{aligned}$$

where **x** represents the vector of variables (to be determined), **c** and **b** are vectors of (known) coefficients and *A* is a (known) matrix of coefficients. The expression to be maximized or minimized is called the objective function ($\mathbf{c}^T\mathbf{x}$ in this case). The equations $A\mathbf{x} \le \mathbf{b}$ are the constraints which specify a convex polytope over which the objective function is to be optimized. (In this context, two vectors are comparable when every entry in one is less-than or equal-to the corresponding entry in the other. Otherwise, they are incomparable.)

Linear programming can be applied to various fields of study. It is used most extensively in business and economics, but can also be utilized for some engineering problems. Industries that use linear programming models include transportation, energy, telecommunications, and manufacturing. It has proved useful in modeling diverse types of problems in planning, routing, scheduling, assignment, and design.

3.1.1 STANDARD FORM

The Standard form is the usual and most intuitive form of describing a linear programming problem. It consists of the following four parts:

- A **linear function**
- **Problem constraints**
- **Non-negative variables**
- **Non-negative right hand side constants**

Given an m - vector, ***b*** = $(b_1,...,b_m)^T$, an n - vector, ***c*** = $(c_1,...,c_n)^T$, and an m

$$A = \begin{pmatrix} a_{11} & a_{12} & . & . & . & a_{1n} \\ a_{21} & a_{22} & . & . & . & a_{2n} \\ . & . & . & & & . \\ . & . & & . & & . \\ . & . & & & . & . \\ a_{m1} & a_{m2} & . & . & . & a_{mn} \end{pmatrix}$$

of real numbers.

3.1.2 THE STANDARD MAXIMUM PROBLEM

Find an n - vector $\boldsymbol{x} = (x_1, ..., x_n)^T$, to maximize

C = $c^T x = c_1 x_1 + ... + c_n x_n$

subject to the constraints

$$a_{11}x_1 + a_{12}x_2 + ... + a_{1n}x_n \le b_1$$

$$a_{21}x_1 + a_{22}x_2 + ... + a_{2n}x_n \le b_2$$

.

.

.

$$a_{m1}x_1 + a_{m2}x_2 + ... + a_{mn}x_n \le b_m$$

And

$x_1 \geq 0, x_2 \geq 0, \ldots, x_n \geq 0$.

3.1.3 THE STANDARD MINIMUM PROBLEM

Find an m – vector y = $(y_1, \ldots, y_m)^T$, to minimize

Z = $y^T b = y_1 b_1 + \ldots + y_m b_m$

subject to the constraints

$$y_1 a_{11} + y_2 a_{12} + \ldots + y_m a_{1m} \geq c_1$$

$$y_1 a_{12} + y_2 a_{22} + \ldots + y_m a_{m2} \geq c_2$$

.

.

.

$$y_1 a_{1n} + y_2 a_{2n} + \ldots + y_m a_{mn} \geq c_n$$

and

$$y_1 \geq 0, y_2 \geq 0, \ldots, y_m \geq 0 .$$

3.2. 0 METHODS OF SOLVING LINEAR PROGRAMMING

Basically, there are several methods of solving a linear programming problem. These are

i. The graphical(Geometrical) Method

ii. The simplex (Algebraic) Method

iii. Revised simplex method

iv Interior point methods

3.2.1 THE GRAPHICAL METHOD

This method of solving Linear Programming Problem is applicable to problems involving only two decision variables. The following steps can be followed in solving Linear Programming Problem

STEP 1 Locate and identify or define the decisions variables in accordance with problem given.

STEP 2 Formulate the problem in a standard Linear Programming model.

STEP 3 Consider each of the inequality as an equation and plot each equation on the graph as each will geometrically represents a straight line.

STEP 4 Mark the appropriate region. If the inequality constraint corresponding to that line is less than or equal to, then the region below the line lying in the first quadrant (due to the non negativity of the decision variables) is shaded. For the inequality constraint corresponding with greater than or equal to, the region above the line in the first quadrant is shaded.

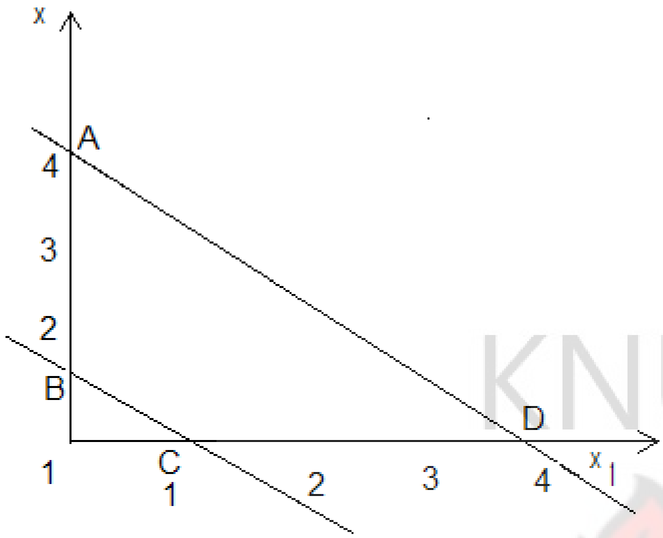
STEP 5 The points lying in common region will satisfy all the constraints simultaneously. The common region thus obtained is called the feasible region.

STEP 6 To obtain the optimum solution theoretically, a line of equal profits or line of equal cost is drawn to represent the objective function after assigning a value say zero for the objective function so as to for a straight line passing through the origin. Stretch the objective function line till the extreme points of the feasible region.

STEP 7 Draw the necessary conclusion

EXAMPLE

Minimize Z =3



Hence the minimum value Z is 3.

3.2.3 A UNIQUE OPTIMAL SOLUTION

This is where the solution to the problem occurs at one and only one extreme point of the feasible region. That is, the combination that gives the highest contribution or profit or the minimum cost or time depending on the problem at hand

Example: Max Z = 6

Now using the Gauss Jordan elimination method. Let

Table 3.2.4.2 Iteration three

		4	3	0	0	
						RHS
		1	0			
		0	1			
		4	3		0	
		0	0		0	

3.2.5 UNBOUNDED SOLUTION

This is a situation where the feasible region is not enclosed by constraints. In such situation, there may or may not be an optimal solution. However, in all cases if the feasible region is unbounded, then there exists no maximum solution but rather a minimum solution. To illustrate unbounded solution, let us consider a numerical example.

EXAMPLE

Max $Z = 4$

Table 3.2.5.0 Iteration one

		4	3	0	0	
						RHS
0		1	-6	1	0	5
0		3	0	0	1	11
		0	0	0	0	
		4	3	0	0	

Table 3.2.5.1 Iteration two

		4	3	0	0	
						RHS
0		0	-6	1		
4		1	0	0		
		4	0	0		
		0	3	0		

Unbounded ness occurs in this solution, because there is an entering variable in the second iteration but there is no leaving variable in the same iteration.

3.2.6 NO SOLUTION

There may also be a situation where there is no solution to the problem at hand. In such case, there will be no feasible region ±hence; the bounded area will be empty.

3.3.0 SIMPLEX METHOD

It is a systematic way of examining the vertices of the feasible region to determine the optimal value of the objective function. Simplex usually starts at the corner that represents doing nothing. It moves to the neighbouring corner that best improves the solution. It does this over and over again, making the greatest possible improvement each time. When no more improvement can be made, the most attractive corner corresponding to the optimal solution has been found.

3.3.1 THE STANDARD MAXIMUM FORM FOR A LINEAR PROGRAM

A standard maximum problem is a linear program in which the objective is to maximize an objective function of the form:

$$Z = C_1X_1 + C_2X_2 + \ldots + C_nX_n$$

subject to:

$$a_{11}x_1 + a_{12}x_2 + \ldots + a_{1n}x_n \le b_1$$

$$a_{21}x_1 + a_{22}x_2 + \ldots + a_{2n}x_n \le b_2$$

$$a_{m1}x_1 + a_{m2}x_2 + \ldots + a_{mn}x_n \le b_m$$

where $x_1, x_2, \ldots, x_n \ge 0$

and $b_j \ge 0$ for $j = 1, 2, \ldots, m$

3.3.2 THE SIMPLEX TABLEAU

To set up the simplex tableau for a given objective function and its constraints, add none negative slack variable s_i to the constraints. This is to convert the constraints into equations. The constraints therefore become:

0	s_2	a_{21}	a_{22}	…	a_{2n}	0	1	…	0	b_2
. . .	. . .	. . .	. . .		. . .	. . .	. . .		. . .	. . .
0	s_m	a_{m1}	a_{m2}	…	a_{mn}	0	0	…	1	b_m
	z_j	0	0	…	0	0	0	…	0	0
	$c_j - z_j$	c_1	c_2	…	c_n	0	0	0	0	

C_B is the objective function coefficients for each of the basic variables.

Z_j is the increase the value of the objective function that will result if one unit of the variable corresponding to the j^{th} column of the matrix formed from the coefficients of the variables in the constraints is brought into the basis (thus if the variable is made a basic variable with a value of one)

$C_j - Z_j$ called the Net Evaluation Row, is the net change in the value of the objective function if one unit of the variable corresponding to the j^{th} column of the matrix (formed from the coefficient of the variables in the constraints), is brought into solution.

From the $C_j - Z_j$ row we locate the column that contains the largest positive number and this becomes the Pivot Column. In each row we now divide the value in the RHS by the positive entry in the pivot column (ignoring all zero or negative entries) and the smallest one of these ratios gives the pivot row. The number at the intersection of the pivot column and the pivot row is called the PIVOT.

We then divide the entries of that row in the matrix by the pivot and use row operation to reduce all other entries in the pivot column, apart from the pivot, to zero.

3.3.3 THE STOPPING CRITERION

The optimal solution to the linear program problem is reached when all the entries in the net evaluation row, that is, $C_j - Z_j$, are all negative or zero.

3.3.4 MINIMIZING THE OBJECTIVE FUNCTION

Standard form of LP problem consists of a maximizing objective function. Simplex method is described based on the standard form of LP problems. If the problem is a minimization type, the objective function is multiplied through by – 1 so that the problem becomes maximization one.

Min F = - Max F

3.3.5 CONSTRAINTS OF THE $\geq$ TYPE

The LP problem with 'greater-than-equal-to' ($\geq$) constraint is transformed to its standard form by subtracting a non negative surplus variable from it:

$$a_i x \geq b_i$$

is equivalent to

$$a_i x - s_i = b_i \text{ and } s_i \geq 0 .$$

3.3.6 CONSTRAINTS WITH NEGATIVE RIGHT HAND SIDE CONSTANTS

Multiply both side of the constraint by – 1 and add either an artificial variable or a surplus and artificial variable as required. Assuming we have the constraint:

$$-2x_1 + 7x_2 \leq -10$$

Multiplying both sides by a negative gives:

$$2x_1 - 7x_2 \geq 10 .$$

To convert the new constraint into equality, we add both a surplus and artificial variable as follow:

$$2x_1 - 7x_2 - 1s_1 + 1A_1 = 10$$

where s_1 and A_1 are surplus and artificial variables respectively.

3.3.7 EQUALITY CONSTRAINT

Situation where any of the constraints is of the linear programming is of the form:

$$a_1 x_1 + a_2 x_2 + ... + a_n x_n = b ,$$

The single constraint is replaced with the following two constraints:

$$a_1x_1 + a_2x_2 + ... + a_nx_n \leq b \text{ and } a_1x_1 + a_2x_2 + ... + a_nx_n = b.$$

The usual procedure is then applied.

3.3.8 UNCONSTRAINED VARIABLES

If some variable x_j is unrestricted in sign, replace it everywhere in the formulation by $x'_j - x''_j$, where $x'_j \geq 0$ and $x''_j \geq 0$.

EXAMPLE

Max $Z = 100$

Table 3.3.8 0 iteration one

		100	-100	200	-200	0	0	-M	
									RHS
0		5	-5	7	-7	1	0	0	30
-M		5	-5	2	-2	0	-1	1	5
		-5M	5M	-2M	2M	0	M	-M	-5M
		100+5M	100+5M	200+2M	-200-2M	0	-M	0	

Table 3.3.8 1 iteration two

		100	-100	200	-200	0	0	-M	
									RHS
-100		-1	1	0	0	0.08	0.28	-0.28	1
200		0	0	1	-1	0.20	0.20	-0.20	5
		100	-100	200	-200	32	12	-12	900
		0	0	0	0	-32	-12	-M+12	

However, to determine the optimal solution to the original problem, these variable must be reconnected to their original.

Thus the solution to the original problem does indeed have one variable with negative value (i. e

STEP 5: If there exist no negative entry appearing on the RIGHT HAND SIDE column of the initial tableau, proceed to obtain the optimum basic feasible solution

STEP 6: If there exist a negative entry on the Right Hand Side column of the initial tableau,

i. identify the most negative at the Right Hand Side , this row is the pivot row

ii. Select the most negative entry in the pivoting row to the left of the Right Hand Side. This entry is the pivot element

iii. Reduce the pivot element to 1 and the other entries on the pivot column to 0 using elementary row operation

STEP 7: Repeat step 6 as long as there is a negative entry on the Right Hand Side column. When no negative entry exists on the Right Hand Side column, except in the last row, we proceed to find the optimal solution.

MIXED CONSTRAINTS

Min Z = 5

5		1	0		0			1
		5	3		M			3M +12
		0	0		0			

3.4.0 PRIMAL DUAL METHODS

It is one of the three main categories of the interior point methods. The primal dual algorithm operates simultaneously on the primal and the dual linear programming.

3.4.1 THE PRIMAL PROBLEM

Given the linear programming problem in the standard form:

(P) minimize $c^T x$

subject to $Ax = b$, $x \geq 0$

where $c \in R^n$, $A \in R^{m \times n}$ and $b \in R^m$ are given data, and $x \in R^n$ is the decision variable.

The dual (D) to the primal (P) can be written as:

(D) maximize $b^T y$

subject to $A^T y + s = c$, $s \geq 0$

with variables $y \in R^m$ and $s \in R^n$

The centering parameter (σ)

sIt balances the movement towards the central path against the movement toward optimal solutions. If $\sigma = 1$, then the updates move towards the center of the feasible region. If $\sigma = 0$, then the update step is in the direction of the optimal solution.

The duality Gap (μ)

It is the difference between the primal and dual objective functions. Theoretically, these two quantities are equal and so give a result of zero (0) at optimality. In practice however, the algorithm drives the result down to a small amount. This is given by the equation:

$$\mu \equiv \frac{1}{n}(x^T s) = c^T x - b^T y$$

While $\mu \geq \varepsilon$, Newton's method is applied until $\mu \leq \varepsilon$ when the algorithm terminates. ε is a positive fixed number.

The general standard minimum problem and the dual standard maximum problem may be together illustrated as:

Table 3.4.1 shows standard minimum and dual maximum constraints

	x_1	x_2	$\cdots$	x_n	
y_1	a_{11}	a_{12}	$\cdots$	a_{1n}	$\geq b_1$

y_2	a_{21}	a_{22}	$\cdots$	a_{2n}	$\geq b_2$
. . .	. . .	. . .		. . .	. . .
y_n	a_{m1}	a_{m2}	$\cdots$	a_{mn}	$\geq b_m$
	$\leq c_1$	$\leq c_2$	$\cdots$	$\leq c_n$	

3.4.2 THE PRIMAL-DUAL ALGORITHM

Initialization

1. Choose $\beta, \gamma \in (0,1)$ and $(\varepsilon_P, \varepsilon_D, \varepsilon_G) > 0.$

Choose (x^0, y^0, s^0) such that $(x^0, s^0) > 0$ and $\left\| X^0 s^0 - \mu_0 e \right\| \leq \beta \mu_0$

where $\mu_0 = \dfrac{(x^0)^T s^0}{n}$.

2 Set $k = 0$

3. Set $r_p^k = b - Ax^k$, $r_D^k = c - Ak^T y^k - s^k$, $\mu_k = \dfrac{(x^k)^T s^T}{n}$

4. Check the termination. If $\|r_P^k\| \le \varepsilon_P$, $\|r_D^k\| \le \varepsilon_D$, $(x^k)^T s^k \le \varepsilon_G$, then terminate.

5. Compute the direction by solving the system

$$\begin{bmatrix} A & 0 & 0 \\ 0 & A^T & I \\ S^k & 0 & A^k \end{bmatrix}\begin{bmatrix} d_x \\ d_y \\ d_s \end{bmatrix} = \begin{bmatrix} r_P^k \\ r_D^k \\ -X^k s^k + \gamma\mu_k e \end{bmatrix}$$

6. Compute the step size

$\alpha = \max\{\alpha' : \|X(\alpha)s(\alpha) - \mu(\alpha)e\| \le \beta(\alpha), \forall\, \alpha \in [0,\alpha']\}$, where

$x(\alpha) = x^k + \alpha d_x$, $s(\alpha) = s^k + \alpha d_s$ and $\mu(\alpha) = \dfrac{x^T(\alpha)s(\alpha)}{n}$.

7. Update $x^{k+1} = x^k + \alpha_k d_x$, $y^{k+1} = y^k + \alpha d_y$, $s^{k+1} = s^k d_s$

8. Set $k = k+1$, and go to step

EXAMPLE:

PROBLEM PI(PRIMAL)

Min

PROBLEM DI (DUAL)

Max 15

		1	-2	0	0	0	
							RHS
0			0	1	0		13
0			0	0	1		1
-2			1	0	0		2
	Dual vs		-2	0	0		-4
	r. cost		0	0	0		

The optimal solution is (

X

Together with the relation

Where I is the identity matrix that appeared in the solution of a given problem.

Go to step 1

EXAMPLE:

Max Z = 3

Y =

Solution after one iteration :

CHAPTER 4

DATA ANALYSIS AND RESULTS

4.1 DATA COLLECTION

The data was collected from Berekum water treatment plant. They consist of cost and quantity of chemical, electricity and fuel that were used to treat water from January to December, 2011.

The data was categorized into two aspects to reflect the two major seasons in the year. These are the dry and wet seasons. The dry season is from November to March and the wet season is from April to October.

Dry season

Table 4.1 TABLE SHOWING COST OF CHEMICAL, ELECTRICITY AND FUEL

Month	Chemical(GH¢)	Electricity	Fuel
November	608.04	21128.12	128.59
December	614.16	21732.21	133.09
January	621.64	8784.44	135.00
February	590.94	6890.66	140.65
March	624.36	8174.64	134.23

Table 4.2 TABLE SHOWING QUANTITY OF CHEMICAL, ELECTRICITY AND FUEL

Month	Chemical(Bags)	Electricity(kw/h)	Fuel(Litre)
November	651	41493	110
December	654	42360	120
January	662	38524	125
February	603	30298	142
March	659	36748	130

Wet season

4.3 TABLE SHOWING COST OF CHEMICAL, ELECTRICITY AND FUEL

Month	Chemical (GH¢)	Electricity(GH¢)	Fuel(GH¢)
April	628.44	9285.79	138.61
May	602.69	9484.61	138.61
June	520.22	8641.56	145.22
July	528.48	7455.91	150.20
August	618.96	13848.69	213.21
Sept	599.88	18040.27	135.23
October	612.84	21186.09	128.59

Table 4.4 TABLE SHOWING QUANTITY OF CHEMICAL ELECTRICITY AND FUEL

Month	Chemical(Bag)	Electricity(kw/h)	Fuel(Litre)
April	661	41407	126
May	666	42595	126
June	608	35828	149

July	612	32753	167
August	674	40362	151
September	647	39556	138
October	671	41605	145

4.2 CALCULATION OF PARAMETERS

The objective function is formulated based on the following factors: total monthly chemical cost, total monthly electricity cost and total monthly fuel cost (include transport and other lubricants).

Cost = Price x Quantity

Hence Total Treatment Cost =

4.2.1 CONSTRAINTS COEFFICIENT

The seasonal average cost, average quantities and usage ratio of chemical, electricity and fuel are presented in tables 4.2.1,4.2.2,4.2.2,4.2.3 and 4.2.4 The seasonal average cost and quantity were obtained by using the general formula below for both seasons

$$\text{Seasonal average cost/quantity} = \frac{\text{Total monthly cost/quantity}}{\text{Number of months}}$$

The usage ratios were calculated by using the formula:

$$\text{Usage Ratio} = \frac{\text{Seasonal cost}}{\text{Seasonal quantity}}$$

4.2.1TABLE SHOWING USAGE RATIO OF CHEMICAL, ELECTRICITY AND FUEL FOR DRY SEASON

	Chemical	Electricity	Fuel
Average Cost	611.82	13342.01	134.31
Average Quantity	645.80	37884.60	125.40
Ratio/Unit	0.94	0.35	1.07

4.2.2 TABLE SHOWING COST/QUANTITY AND THE USAGE RATIO OF FUEL FOR DRY SEASON

Chemical House		Pump House		Transportation	
Cost(GH¢)	Quantity	Cost(GH¢)	Quantity	Cost(GH¢)	Quantity
123.35	14.32	200.81	16.71	301	84.23
8.61		9.69		3.57	

4.2.3TABLE SHOWING USAGE RATIO OF CHEMICAL, ELECTRICITY AND FUEL FOR WET SEASON

	Chemical	Electricity	Fuel
Average Cost	587.358	12563.274	149.952
Average Quantity	648.428	39158.00	143.142
Ratio/Unit	0.905	0.3208	1.0475

4.2.4 TABLE SHOWING THE COST/QUANTITY AND THE USAGE RATIO OF FUEL FOR WET SEASON

Chemical House		Pump House		Transportation	
Cost(GH¢)	Quantity	Cost(GH¢)	Quantity	Cost(GH¢)	Quantity
332.00	18.30	417.00	20.04	502.08	26.65
18.14		20.80		18.83	

4.2.2 MODEL FOR DRY SEASON

Minimize Total Cost =0.92

The matrices were put in the Matlab program code and ran on AMD Anthlon™ 64 $\times$ 2 Dual-Core processor TK-57, 32-bit operating system, 1.90GHz speed, Windows Vista Toshiba laptop computer.

4.4 MATRICES FORMULATION

Using A, B and C for the matrices of left-hand side inequalities, right-hand side constants and cost functions respectively, then:

Dry Season

A=

4.5 RESULTS

Results of final test run for TOTAL WATER TREATMENT COST IN DRY SEASON after five iterations

x_b =

6360	147650	280

Optimal value Z = 54869.70

Results of final test run for TOTAL WATER TREATMENT COST IN WET SEASON after five iterations.

x_b =

8460	218660	80

Optimal value Z = 802

4.6 DISCUSSION

During the dry season, it will take (

CHAPTER 5

CONCLUSION AND RECOMMENDATION

5.1 Conclusion

The treatment cost of water at Berekum treatment plant was modeled as a Linear Programming problems to reflect the two major seasons we have in Ghana-dry and wet. The solution of the model using Revised simplex algorithm showed that water treatment cost of GH¢70440.77 and GH¢93104.10 for dry and wet seasons respectively could be optimized at GH¢54869.70 in the dry season and GH¢ 80235.00 in the wet season to treat the same quantity of water. Cost of treating water can therefore be reduced with respect to the factors that influence the treatment cost.

5.2 Recommendations.

Based on the study, the following recommendations are made.

1. A Scientific approach should be adopted to assess the minimum cost of water treatment.

2. Ghana water company limited needs to have a well structure data and Resource personnel in the area of mathematical programs, to train workers on the use of the scientific method.

3. The research was limited to Chemical, Electricity and Fuel being used by company.

4 The model will be used to determent the optimum cost of treating water at Berekum headworks. This intends to analyze the cost involved in water treatment and to distribute the same quantity of water to Berekum municipality.

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Appendix 1

TABLE SHOWING ALLOCATION OF CHEMICAL, ELECTRICITY AND FUEL FOR DRY SEASON

Dry Season	Chemical House	Pump House	Transportation
Chemical Cost	5178.00	0	0
Electricity Cost	0	45581.00	0
Fuel Cost	85.00	101.00	825.00
Total	5263.00	45682.00	825.00

Appendix 2

TABLE SHOWING ALLOCATION OF CHEMICAL, ELECTRICITY AND FUEL FOR WET SEASON

	Chemical House	Pump House	Transportation
Chemical Cost	4389	0	0
Electricity Cost	0	47671.08	0
Fuel Cost	101.00	136.00	986.00
Total	4490.00	47807.00	986.00

APPENDIX 3

Matlab code for algorithm

```
function revised()
a=[  ; ;];
c=[  ];
b=[; ; ];
   n=length(c);
   m=length(b);
   j=max(abs(c));
   if nargin<4
      minimize=0;
      inq=-ones(m,1);
   elseif nargin<5
      minimize=0;
   end
   if ~isequal(size(a),[m,n])||m~=length(inq)
      fprintf('\nError: Dimension mismatch!\n');
   else
      if minimize==1
         c=-c;
      end
      count=n;nbv=1:n;bv=zeros(1,m);av=zeros(1,m);
      for i=1:m
         if b(i)<0
            a(i,:)=-a(i,:);
            b(i)=-b(i);
         end
         if inq(i)<0
            count=count+1;
            c(count)=0;
            a(i,count)=1;
            bv(i)=count;
         elseif inq(i)==0
            count=count+1;
            c(count)=-10*j;
            a(i,count)=1;
            bv(i)=count;
            av(i)=count;
         else
            count=count+1;
            c(count)=0;
            a(i,count)=-1;
            nbv=[nbv count];
            count=count+1;
            c(count)=-10*j;
```

```
        a(i,count)=1;
        av(i)=count;
        bv(i)=count;
    end
end
A=[-c;a]
B_inv=eye(m+1,m+1);
B_inv(1,2:m+1)=c(bv);
x_b=B_inv*[1; b]
fprintf('\n............The initial tablaue................\n')
fprintf('\t z');disp(bv);
fprintf('---------------------------------------------------\n')
disp([B_inv x_b])
flag=0;count=0;of_curr=0;
while(flag~=1)
    [s,t]=min(B_inv(1,:)*A(:,nbv));
    y=B_inv*A(:,nbv(t));count=count+1;
    if(any(y(2:m+1)>0))
        fprintf('\n............The ith tablaue................\n',count)
        fprintf('\t z');disp(bv);
        fprintf('---------------------------------------------------\n')
        disp([B_inv x_b y])
        if count>1 && of_curr==x_b(1)
            flag=1;
            if minimize==1
                x_b(1)=-x_b(1);
            end
            fprintf('\nThe given problem has degeneracy!\n');
            fprintf('\nThe current objective function value=%d.\n',x_b(1));
            fprintf('\nThe current solution is:\n');
            for i=1:n
                found=0;
                for j=1:m
                    if bv(j)==i
                        fprintf('x%u = %d\n',i,x_b(1+j));found=1;
                    end
                end
                if found==0
                    fprintf('x%u = %d\n',i,0);
                end
            end
        else
            of_curr=x_b(1);
            if(s>=0)
                flag=1;
                for i=1:length(av)
```

```
                for j=1:m
                    if av(i)==bv(j)
                        fprintf('\nThe given LPP is infeasible!\n');
                        return
                    end
                end
            end
            if minimize==1
                x_b(1)=-x_b(1);
            end
            fprintf('\nReqiured optimization has been achieved!\n');
            fprintf('\nThe optimum objective function value=%d.\n',x_b(1));
            fprintf('\nThe optimum solution is:\n');
            for i=1:n;
                found=0;
                for j=1:m;
                    if bv(j)==i;
                        fprintf('x%u = %d\n',i,x_b(1+j));found=1;
                    end
                end
                if found==0;
                    fprintf('x%u = %d\n',i,0);
                end
            end
            if (s==0 && any(y(2:m+1)>0));
                fprintf('\nThe given problem has alternate optima!\n');
            end
        else
            u=10*j;
            for i=2:m+1;
                if y(i)>0;
                    if (x_b(i)/y(i))<u;
                        u=(x_b(i)/y(i));
                    end
                end
            end
        end
v=i-1;
        temp=bv(v);bv(v)=nbv(t);
            nbv(t)=temp;
            E=eye(m+1,m+1);
            E(:,1+v)=-y/y(1+v);
            E(1+v,1+v)=1/y(1+v);
            B_inv=E*B_inv;
            x_b=B_inv*[1; b]
    end
```

```
        fprintf('\nThe given problem has unbounded solution\n')
        return
           end
    end
end
```

APPENDIX 4

Result of programme run for dry season

A =

-0.9200	-0.3300	-1.0500	0	0	0
0.9400	0	8.6100	1.0000	0	0
0	0.3500	9.6900	0	1.0000	0
0	0	3.5700	0	0	1.0000

x_b =

1

8768

150365

1000

............The initial tablaue................

z 4 5 6

1	0	0	0	1
0	1	0	0	8768

0	0	1	0	150365
0	0	0	1	1000

............The ith tablaue................

z 4 5 6

1.0e+005 *

0.0000	0	0	0	0.0000	-0.0000
0	0.0000	0	0	0.0877	0.0001
0	0	0.0000	0	1.5036	0.0001
0	0	0	0.0000	0.0100	0.0000

x_b =

1.0e+005 *

0.0030

0.0636

1.4765

0.0028

The given problem has unbounded solution

>>

APPENDIX 5

Result of programme run for dry season

A =

-0.9200	-0.3300	-1.0500	0	0	0
0.9000	0	18.1400	1.0000	0	0
0	0.3200	20.8000	0	1.0000	0
0	0	18.8300	0	0	1.0000

x_b =

1

9907

217321

1800

.............The initial tablaue................

z 4 5 6

1	0	0	0	1
0	1	0	0	9907

0	0	1	0	217321
0	0	0	1	1800

............The ith tablaue................

z 4 5 6

1.0e+005 *

0.0000	0	0	0	0.0000	-0.0000
0	0.0000	0	0	0.0991	0.0002
0	0	0.0000	0	2.1732	0.0002
0	0	0	0.0000	0.0180	0.0002

x_b =

1.0e+005 *

0.0010

0.0817

2.1533

0.0010

The given problem has unbounded solution