LOCATION OF AMBULANCE AT STUDENTS' RESIDENCE, KNUST.

(A VERTEX TWO-CENTRE PROBLEM)

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BY

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DECLARATION

I hereby declare that this thesis is the result of my own original research and that no part of it has been submitted to any institution or organization anywhere for the award of a degree. All inclusion for the work of others has been duly acknowledged.

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ABSTRACT

Fire outbreaks, flood and health issues are problems that confront man daily. Their occurrences always need emergency attention within the shortest possible time. One way to deal with this problem on KNUST campus is to locate two centres close to halls of residence to place ambulance on KNUST campus for students and lecturers.

The P-centre problem is a minimax problem, which minimizes the maximum distance between a demand and the nearest facility to the demand point. The P-centre model formulated by Daskin (1995) and the set covering model formulated by Daskin and Dean (1994) was used to model the problem of sitting two ambulances at two halls of residence on KNUST campus. The LINDO software application which uses branch and bound algorithm was employed to solve the problem. It was observed that, the facility should be located at Unity and University Hall. The two ambulances at these halls will serve all the halls.

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My special thanks go to my dear wife, Ellen for her support and encouragement. To my father, Rev. Kingsford Someah-Addae, my siblings, Brofoleshele and Agya Minla, I say God richly bless you.

DEDICATION

This thesis is dedicated to my father, Rev. Kingsford Someah-Addae; My siblings, Brofoleshele and Agya Minla; my wife, Ellen and my newly born son, Edenkema Benreholame Addae.

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CHAPTER ONE

1.0 INTRODUCTION

The P- centre model minimizes the maximum distance between any demand point and it nearest facility. In real life, we always encounter fire outbreaks, and health issues. These occurrences need emergency attention within the shortest possible time, otherwise the result may turn out to be disaster, with attendant destruction and mortality. The ambulance service is always the best option in these cases and hence its location is always of interest.

This chapter looks at the history of the ambulance service. It also includes the background study of the KNUST hospital, problem statement and objective of the research.

1.1 The ambulance

Skinner (1949) defines an ambulance as a vehicle for transporting sick or injured people to, from or between places of treatment for an illness or injury. The Oxford English dictionary (5th edition) also defines an ambulance as a vehicle used to bring medical care to patients outside the hospital or to transport the patient to hospital for follow-up care and further testing. The term ambulance originated from the Latin word “ambulare”, meaning “to move about”, which has reference to early medical care where patients were transported by hand carrying or wheeling (Wikipedia, 2008)

According to Barkley (1990), Queen Isabella of Spain in 1487 also used ambulances to treat the army during war. Dominique, Jean Larrey (1766-1842) who was Napoleon Bonaparte's chief physician used the "flying ambulances", the first of its kind by Napoleon's Army of the Rhine in 1793 to give early treatment on the battle field. Pearson and McLaughlin (1990) believe the first time wagons were referred to as ambulance was in 1854 during the Crimean War. They further indicated that during the American Civil War, vehicles for conveying the wounded off the battle field were called ambulance wagons. They were called field hospitals during the Franco-Prussian war of 1870 and the Serbo-Turkish war of 1876.

Nowadays, the word is mostly associated with land-based emergency care to those with acute illness or injuries. In a nutshell, the ambulance is used to provide pre-hospital care. The six main task executed by an ambulance service through emergency care are;

i. detection of an incidence
ii. reporting – calling for help to enable dispatch of an ambulance to the scene
iii. response – provide first aid
iv. on scene care
v. care in transit
vi. transfer to definitive care.

1.1.1 Types of Ambulance

Ambulances can be categorized according to whether or not they transport patients and under what conditions,

i. *Emergency ambulance*; this is the most common type of ambulance which provides care to patients with an acute illness or injury.

ii. *Patient Transport Ambulance*; they transport patients to, from or between places of medical treatment such as hospital or dialysis centre for urgent care.

iii. *Response Unit*; this is an ambulance which is used to reach acutely ill patient quickly and provide on scene care but lack the capacity to transport patient from the scene. This is usually backed up by an emergency ambulance.

iv. *Charity Ambulance*; this is provided by a charity organization for the purpose of taking sick children or adults on trips or vacations away from hospitals, hospices or home cares.

1.1.2 Historical background of civil Ambulance use

According to Barkley (1990), the first record of civil ambulance use was around 900AD by the Anglo-Saxons. During the crusades of the 11th century, the Knights of St. John set up hospitals to treat pilgrims wounded in battles, there is no clear evidence to suggest how the wounded made their way to the hospitals. The first known hospital based ambulance service in the United States was sited at a commercial Hospital (now the Cincinnati General) in Cincinnati, Ohio in 1865. In 1867, the city of London's Metropolitan Asylums Board in the United Kingdom received six horse-drawn ambulances for the purpose of conveying smallpox and fever patients from their homes to a hospital.

Skandalakis et al (2006) indicated that, Edward Dalton, former surgeon in the Union Army of the United States was charged with creating a hospital in lower New York. He started an ambulance service in 1869 to bring the patients to the hospital faster and in more comfort.

According to Barkley (1990) in June 1887, the St. John Ambulance Brigade was established to provide first aid and ambulance service at public events in London, now provides ambulance and first aid services in many countries around the world. In Queensland, Australia, Seymour Warrian established the Queensland Ambulance Transport Brigade on 12 September 1892. Bellevue Hospital in New York had the first horse-drawn ambulance in the United States in the year 1895. He continues to say that the first motor powered ambulance was brought into service in February 1899 at the Michael Reese Hospital, Chicago. In Germany, 1902, a civilian ambulance train was introduced for use during railway accidents.

In Ireland, the St. John Ambulance was set up in 1903 in the Guinness Brewery in St. James Gate in Dublin by Sir John Lumsden for workers. In 1910, the Brigade began its first public duty at the Royal Dublin Society. Later in 1916 when Ireland became independent, it became St. John Ambulance Brigade of Ireland.

Palliser Ambulance was introduced in 1905. It was named after Major Palliser of the Canadian Militia. The British Army helped the Canadians to introduce a limited number of automobile ambulances. In 1905, the Royal Army Medical Corps Commissioned a Straker-Squire motor ambulance van.

1.1.3 The Ghana National Ambulance Service

The National Ambulance Service of Ghana was established in 2004 but prior to that some regional and district hospitals had ambulances that were for their own use called the Hospital Based Ambulance Service (HBAS). The National Ambulance Service (NAS) administers ambulance service in Ghana. The subdivisions for the regions are the Regional Ambulance Service (RAS). The objective of the NAS is to provide pre-hospital care to the critically ill and victims of road, domestic and industrial accidents and to convey them in a professional manner to medical facilities. The operation of the NAS is multi-disciplinary and is managed by the Ministry of Health (MOH) and the Ghana National Fire Service (GNFS). There are five (5) ambulances allocated to the Ashanti Region and four (4) in the Kumasi Metropolis. The ones in Kumasi are located at KATH, Mamponteng, Ejisu, Konongo and Ahwia Nkwata. All except the KATH and Ahwia Nkwata are located at the Fire Stations.

1.1.4 Ambulance Service Providers

Looking at the diverse historical background, there have been diverse service providers to meet medical demands. The following are service providers.

(i) *Government Ambulance Service:* this is funded by local or national government.

(ii) *Fire or Police Linked Service*: operated by the local fire or police service

(iii) *Volunteer Ambulance Service:* Charities or non profit companies operate Ambulance services, both in an emergency and patient transport function. The Red Cross provides this charity service across the world on a volunteer basis.

(iv) *Private Ambulance Service:* they may provide only the patient transport elements of ambulance care, but in some places they are contracted to provide emergency care, or form a second tier response.

(v) *Combined Emergency Service*: These are full emergency service agencies, which may be found in places such as airports or large colleges and universities. Their key feature is that, all personnel are trained not only in ambulance care but in fire fighting and peace officer (police) functions. They may also be found in smaller towns and cities, where size or budget does not warrant separate services. This multi-functionality allows making the most of limited resource or budget, by having a single team respond to any emergency.

(vi) *Hospital Based Service*: this offers ambulance service to the community in connection to the hospitals.

(vii) Charity ambulance Service

(viii) Company Ambulance Service

1.1.5 Crew on Ambulance

Most ambulance services require at least two crew members to be on every ambulance. The following are common ambulance crew; first-responder, ambulance driver, ambulance care assistant, emergency medical technician, paramedics, emergency care practitioner, registered nurse and a doctor.

1.2 BACKGROUND OF STUDY

The main objective of the ambulance service is to take care of fire outbreaks, injuries and illness. The Kwame Nkrumah University of Science and Technology (KNUST) have these attendant problems of fire outbreaks, injuries and illness.

According, to the Fire Preventive Unit at KNUST, 255 cases of fire outbreaks were recorded from 1988 to 2008.

Table 1.1 below gives the breakdown of the type of fire outbreaks.

Table 1.1: Fire outbreaks on KNUST from 1988 to 2008

Types of outbreak	**No of cases**
Offices	63
Halls	34
Quarters	22
Bush fires	120
Vehicular fires	16
Total	**255**

Out of the 255 fire outbreaks, the halls reported 34 cases mainly due to electrical faults or negligence. This means, at least, there was one fire outbreak every semester at a hall of residence over the period. The frequency of the outbreaks could cause serious problem because the halls are the most populated, with the official total residence population for the 2007/2008 academic year being 8310 and the average hall population being 1188.

Again, from September 2006 to June 2008, a total of 2428 emergency cases were reported at the KNUST hospital. Out of this number, 462 were students resident in hall. This implies that we have at least 115 emergency cases a semester for the four semesters.

Table 1.2 is a table of cases reported at the emergency unit since its establishment in September 2006.

Table 1.2: Monthly Reported Emergency Cases at KNUST Hospital, Emergency Unit, From September 2006 To June 2008.

Year	2006		2007		2008	
Month	**Student**	**Total**	**Student**	**Total**	**Student**	**Total**
January	-	-	13	90	7	108
February	-	-	30	116	10	140
March	-	-	46	154	29	105
April	-	-	41	148	13	115
May	-	-	26	149	39	210
June	-	-	6	126	12	119
July	-	-	8	146		
August	-	-	7	89		
September	-	19	18	88		
October	-	36	34	140		
November	-	44	38	146		
December	-	97	21	99		

Some of the illnesses reported are haemorrhoids, acute appendicitis, laceration, acute asthma, bronchial asthma, asthmatic attack, gastritis, gastro-ententes, malaria, OTI, hypertensive, encephalopathy, epilepsy etc. According to the senior nursing officer, if these cases did not get to the hospital on time, they could have led to death. It is therefore necessary and profitable to locate sites on campus to place ambulance to serve the halls of residence.She further suggested that if at least two ambulances could be provided,the health care needs of students on campus could be well catered for.

1.3 OBJECTIVE OF STUDY

Being motivated by the background of the study, the objectives of the study are:

(i) To locate two centres close to halls of residence to place ambulance on KNUST campus for students.

(ii) To recommend to the University Administration, the establishment of an Emergency Ambulance Service for the University.

1.4 METHODOLOGY

Location of facilities such as ambulance can be considered a centre problem or set covering problem. The problem at hand is a weighted graph. This could be solved by the Maximum Covering Location Model or the Weighted P-Vertex Centre Model. The Weighted P-Vertex Centre Model was used in this thesis, because the implementation of the Weighted P-Vertex Centre Model provides a systematic procedure for arriving at the necessary coverage distance based on choice of facility sites (P). The implementation of the Maximum Covering Location Model allows predetermined choice of the coverage distance and the various choices, give different solutions. In using the Weighted P-Vertex Centre Model the Set Covering Model was employed as subroutine. The P-centre Model is actually an improvement on the set covering model.

Search on the internet was used to obtain related literature. The main Library at KNUST and the Department of Mathematics library were consulted in the course of the project.

1.5 THESIS ORGANIZATION

Chapter one covers the historical background of the ambulance service, cases of fire outbreaks and reported medical emergency cases at KNUST hospital. It also briefly discusses the methodology and the objective of the study. Chapter two contains the literature review. Chapter three contains the data collection, analysis and discussion. The last chapter covers the conclusion and recommendation.

1.6 CONCLUSION

This chapter looked at the research objective of placing ambulance service at the halls of residence of students of KNUST for the purpose of catering for students in times of medical emergencies. The methodology of the research was discussed. Ambulance was defined as a vehicle for transporting the sick or injured to (from) the hospital (Skinner, 1949). In the next chapter the set covering and the weighted P-centre models will be discussed. Methods of solutions will be provided.

CHAPTER TWO

2.0 REVIEW OF LITERATURE AND METHODS

2.1 INTRODUCTON

The P-centre model attempts to minimize the maximum performance of the system and thus address situations in which service inequity is more important than the average system performance. The P- centre model is also referred to as the minimax model since it minimizes the maximum distance (performance) between any demand point and it nearest facility. The P-centre model considers that a demand point is served by it nearest facility and therefore gives full coverage in the sense of set covering models. But set covering model may lead to an excessive number of facilities while full coverage in the P-centre model requires only a limited number (P) of facilities. In many location problems, the cost of a service from the customers' point of view is related to the distance between their habitation and the facilities that are being located. Usually, service is deemed adequate if the customer is within a given distance of the facility and is deemed inadequate if the distance exceeds the given distance.

The P-centre model has applications in many areas of life. These include the location of emergency services (ambulance services) and the selection of conservation and recreational sites. Also, delivery and routing problems often take on a set covering model. This chapter takes a look at the literature in the application of the P-Centre Model and the set covering model as well as some methods used in solving P- centre problems. These include weighted and unweighted P-centre problem on trees and general graphs.

2.2 REVIEW OF LITERATURE

2.2.1 Set Covering

Toregas (1970) defined the Location Set Covering Problem (LSCP) more than thirty years ago and in the pages of Geographical Analysis by Toregas and ReVelle (1973).This location problem involves finding the smallest number of facilities (and their locations) such that each demand is no farther than a pre-specified distance or time away from its closest facility. Such a problem is called a "covering" problem in that it requires that each demand be served or "covered" within some maximum time or distance standard. A demand is defined as covered if one or more facilities are located within the maximum distance or time standard of that demand. The classical LSCP requires that each demand is covered at least once. Church and ReVelle (1974) stated a related problem involves the location of a fixed number of facilities and seeks to maximize the coverage of demand. This second type of covering problem is called the Maximal Covering Location Problem (MCLP). Since the development of these two problems, there have been numerous applications and extensions.

Aidoo (2008) located a Fire Hydrant between Independence Hall and the Administration block precisely 88m from Independence Hall without considering the demography of the students. His model was based on Absolute 1-Centre problem.

Agyapong (2009),used the Robust 1-Centre model to locate a place suitable for a student clinic which should be between Republic Hall and Independence Hall, precisely 105m from Republic Hall.

According to Daskin et al (1988) there are circumstances where the provision of a service needs more than one "covering" facility, this occurs when facilities may not always be available. For example, assume that ambulances are being located at dispatching points in order to serve demand across an urban area, and the nearest ambulance is busy, then the next closest available ambulance will need to be assigned to a call when it is received. If the closest available ambulance is farther than the service standard then that demand/call for service is not provided service within the coverage standard. To handle such issues, models have been developed that seek multiple - coverage. Two examples of multiple-coverage exist, stochastic/probabilistic and deterministic.

Daskin (1983) formulated a probabilistic multiple cover model called the maximal expected coverage model. Hogan and ReVelle (1986) also formulated the simple back up covering model as a good example of a deterministic cover model that involves maximizing second-level coverage. Toregas (1970; 1971) was the first to recognise the possible need for multi-level coverage. Toregas defined the Multi-level Location Set Covering Problem (ML-LSCP) as a search for the smallest number of facility needed to cover each demand, a preset number of times, where the need for coverage might vary between demands.

Application of the set covering model includes airline crew scheduling (Desrocher et al, 1991). According to Daskin, Jones and Lowe (1990) it can also be applied to tool selection in flexible manufacturing systems.

2.2.2 Centre Problem

The centre problem was first posed by Sylvester (1857) more than 150 years ago. The problem asks for the centre of the circle that has the smallest radius to cover all desired destinations. In the last several decades, the P-centre model and its extensions have been investigated and applied in the context of locating facilities such as EMS centres, hospitals, fire stations and other public facilities.

Garfinkel et al. (1977) examined the fundamental properties of the P-centre problem in order to locate a given number of emergency facilities along a road network. He modelled the P-centre problem using integer programming and the problem was successfully solved by using a binary search technique and a combination of exact tests and heuristics.

ReVelle and Hogan (1989) formulated a P-centre to locate facilities so as to minimize the maximum distance within which the EMS is available with (alpha) reliability. System congestion is considered and a derived server busy probability is used to constrain the service reliability that must be satisfied for all demands.

Hochbaun and Pathria (1998) considered the emergency facility location problem that must minimize the maximum distance on the network across all time periods using the Stochastic P-centre models. The cost and distance between locations vary in each discrete time periods. The authors used k underlying networks to represent different periods and provided a polynomial-time, 3-approximation algorithm to obtain a solution for each problem.

Talwar (2002) utilized a P-centre model to locate and dispatch three emergency rescue helicopters to serve the growing EMS demands due to accidents occurring during adventure holidays such as skiing, hiking and climbing the north and south Alpine mountain ranges. One of the model's aims is to minimize the maximum (worst) response time and the author used effective heuristics to solve the problem.

In this work the focus is on the weighted P- centre problem formulated by Daskin (1995) on a general graph. The algorithm that is used to solve the weighted P- centre problem was also provided by Daskin (1995).

2.3 REVIEW OF METHODS

2.3.1 Graphs

A graph G is an ordered pair of disjoint set (V,E) such that E is a subset of the set of pairs in V. Unless it is explicitly stated otherwise, we consider only finite graphs, that is V and E are always finite. The set V is the set of vertices and E the set of edges. If G is a graph, then V=V (G) is the vertex set of G and E=E (G) is the edge set. Two edges are neighbours if they have exactly one common end vertex. A tree is a graph without any cycles. A forest is defined as a disconnected set of trees. A complete graph is a graph with **n** vertices in which each vertex is connected to the others.

2.3.2 Set covering problem

The set covering problem is to find a set of facilities with minimum cost from among a finite set of candidate facilities so that every demand node is covered by at least one facility. According to Toregas (1970) , location set covering problem involves finding the smallest number of facilities and their locations so that each demand is covered by at least one facility. The location set covering problem does not specify a prior distance covering within which a demand is covered. However, the Maximal Covering Location problem finds the facilities and their locations such that each demand is not farther than a pre-specified distance or time from its closest facility. A demand is covered if one or more facilities are located within the maximum distance or time.

2.3.3 Coverage Distance

A coverage distance (Dc/d) is a pre –specified distance, demand is deemed covered if the edge distance (d_{ij}) is less than or equal to Dc. The edge distance (d_{ij}) is the shortest direct distance between a demand node and a facility node. A node is the same as a vertex.

2.3.4 Formulation of the set covering model

Daskin and Dean (1994) formulated the set covering model as follows,

Let (i) d = coverage distance

(ii) d_{ij} = edge distance

(iii) $a_{ij} = \begin{cases} 1, \text{ if candidate site j can cover demands at node i (i.e. } d_{ij} < d \\ 0, \text{ if not (i.e. } d_{ij} > d) \end{cases}$

(iv) f_j = cost of locating a facility at candidate site j

(v) let the decision variables be

$$x_j = \begin{cases} 1, \text{ if we locate at candidate site j} \\ 0, \text{ if not} \end{cases}$$

With these notations, the set covering problem is as follows;

Minimize $\sum_{j} f_j x_j$ 2.3a

Subject to

$$\sum_{j} a_{ij} x_j \geq 1 \quad \forall i \qquad 2.3b$$

$$x_j = 0,1, \forall j \qquad 2.3c$$

The objective function (2.3a) minimizes the total cost of the facilities that are selected. Constraints (2.3b) stipulate that each demand node *i*, must be covered by at least one facility represented by right hand side of constraints. Constraints (2.3c) are the integrality constraints. If all the costs are identical, then, the objective function becomes;

Minimize $\sum_{j} x_j$ 2.3d

Subject to;

$$\sum a_{ij} x_j \geq 1 \quad \forall i \qquad 2.3b$$

We stipulate a coverage distance, d, such that $d \geq d_{ij}$ implies demand node i can be covered by facility j. This affects constraints 2.3b, because the relationship between d_{ij} and d will determine whether a_{ij} is 1 or 0.

2.3.1 Example

To illustrate the formulation of the set covering problem, we consider the network shown in figure 2.1 below.

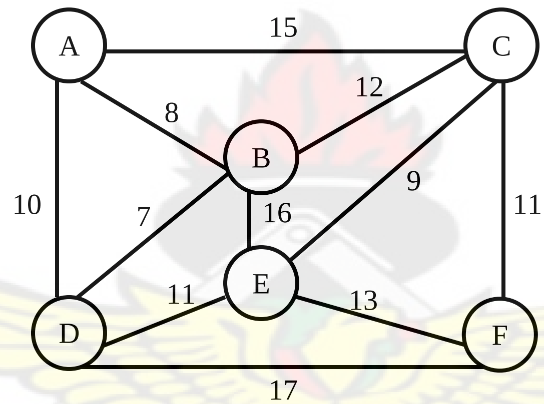

Figure 2.1: Example Network

Table 2.1 is the edge matrix of figure 2.1

Table 2.1: Table of d_{ij} values

$$d_{ij} = \begin{bmatrix} 0 & 8 & 15 & 10 & - & - \\ 306 & 0 & 12 & 7 & 16 & - \\ 15 & 12 & 0 & - & 9 & 11 \\ 10 & 7 & - & 0 & 11 & 17 \\ - & 16 & 9 & 11 & 0 & 13 \\ - & - & 11 & 17 & 13 & 0 \end{bmatrix}$$

The coverage distance of 11 units is pre-specified to be used in this example.

Table 2.2 below shows edge distances d_{ij} such that $d_{ij} > d$ are eliminated.

Table 2.2: Table of d_{ij} values; $d \geq d_{ij}$: $d = 11$

To / From	A	B	C	D	E	F
A	0	8	-	10	-	-
B	8	0	-	7	-	-
C	-	-	0	-	9	11
D	10	7	-	0	11	-
E	-	-	9	11	0	-
F	-	-	11	-	-	0

A node is assigned to itself and other nodes with non-zero entries along its row.A dash (-) means there is no direct link between the nodes

From Table 2.2 the problem is formulated as:

P2.1 Minimize $X_A + X_B + X_C + X_D + X_E + X_F$

Subject to;

(Node A assigned): $X_A + X_B + X_D \geq 1$

(Node B assigned): $X_A + X_B + X_D \geq 1$

(Node C assigned): $X_C + X_E + X_F \geq 1$

(Node D assigned): $X_A + X_B + X_D + X_E \geq 1$

(Node E assigned): $X_C + X_D + X_E \geq 1$

(Node F assigned): $X_C + X_F \geq 1$

Integrality: $X_A, X_B, X_C, X_D, X_E, X_F = 0,1$

The reduction technique was used:

(i) For the column j and k, if $a_{ij} \leq a_{ik}$ for all demand nodes i and $a_{ij} \leq a_{ik}$ for at least one demand node i, then location k covers all demands covered by location j. Location k is said to dominate j and hence column j is eliminated.

(ii) For the row reduction, if $\sum a_{ij} = 1$ then, there is only one facility site that can cover node i. In such case, we find location j such that $a_{ij} = 1$ and set $x_j = 1$. We then eliminate rows containing x_j.

From P2.1 column D dominates column A and B hence we delete column A and B. Column C dominates F hence we eliminate column F. This leads to

$$
\begin{aligned}
\text{P2.2}\quad \text{Min}\quad & X_C + X_D + X_E \\
\text{S.T}\quad & \\
& X_D \geq 1 \\
& X_D \geq 1 \\
& X_C + X_E \geq 1 \\
& X_D + X_E \geq 1 \\
& X_C + X_D + X_E \geq 1 \\
& X_C \geq 1 \\
& \text{Integrality: } X_C, X_D, X_E = 0,1
\end{aligned}
$$

Using the row reduction, since $\sum a_{ij} = 1$, holds for the first and second constraints, we set $X_D = 1$ and eliminate all the rows containing X_D. We also set $X_C = 1$ and eliminate rows containing X_C. The solution therefore is $X_C = X_D = 1$ and $X_A = X_B = X_E = X_F = 0$.The objective function equals 2; the facility will be located at X_C and X_D. From table 2.2, C covers itself, E and F. D covers itself, A, B and E.

2.4 THE MAXIMUM COVERING LOCATION MODEL

The set covering has associated problems, one of which is that the number of facilities that are needed to cover all demand nodes is likely to exceed the number that can actually be built due to budget constraints and other related issues.

Furthermore, the set covering model treats all demands nodes identical. Under certain conditions and budgetary constraints it is appropriate to fix the number of facilities that are to be located and then maximize the number of covered demands.

Church and ReVelle (1974) formulated a Maximun Covering Location Model as follows.

Let h_i = demand at node i

P = number of facilities to locate

Decision Variables be

$$Z_i = \begin{cases} 1, \text{ if node } i \text{ is covered} \\ 0, \text{ if not} \end{cases}$$

The Maximum Covering Location Model is formulated as follows;

$$\text{Maximize} \sum_i h_i Z_i \qquad 2.6a$$

Subject to;

$$Z_i \leq \sum_j a_{ij}\, x_j \;\; \forall i \qquad 2.6b$$

$$\sum_j x_j \leq P \qquad 2.6c$$

$$x_j = 0,1 \qquad 2.6d$$

$$Z_i = 0,1 \qquad 2.6e$$

The objective function 2.6a maximizes the number of covered demands. Constraints 2.6b state that demand node i cannot be covered unless at least one of the facility sites that cover node i is selected. But, the right-hand side of constraints 2.6b which is $\sum a_{ij} x_j$ is identical to the left-hand side of constraints 2.3b. $\sum_j a_{ij} x_j$ gives the number of selected facilities that can cover node i. The constraint 2.6c stipulates that we locate not more than

P facilities. Constraints 2.6c will be binding in the optimal solution. Constraints 2.6d and 2.6e are the integrality constraints on the decision variables.

2.4.1 Example

We use the network of figure 2.2 below to illustrate the maximum covering location problem.

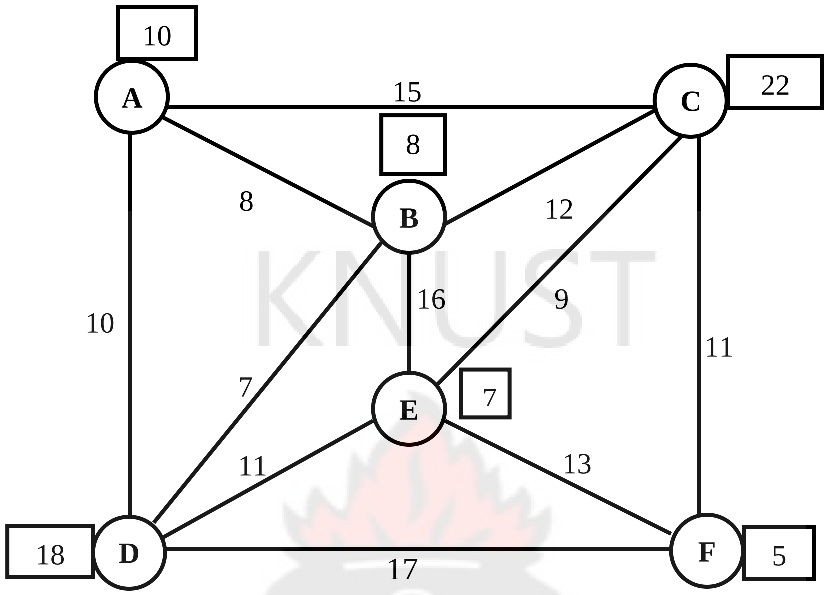

Figure 2.2: Example Network

Demand at each node is in the square box of the network. For a coverage distance of 11 units and P = 1 we have the following formulation:

P 3.1 Maximize $10\,Z_A + 8Z_B + 22\,Z_C + 18Z_D + 7Z_E + 55Z_F$

Subject to

$$X_A + X_B + X_D \geq Z_A$$

$$X_A + X_B + X_D \geq Z_B$$

$$X_C + X_E + X_F \geq Z_C$$

$$X_A + X_B + X_D + X_E \geq Z_D$$

$$X_C + X_D + X_E \geq Z_E$$

$$X_C + X_F \geq Z_F$$

(Number to Locate) $X_A + X_B\ X_C + X_D + X_E + X_F \leq 1$

Integrality $X_A, X_B, X_C, X_D, X_E, X_F = 0, 1$

$$Z_A, Z_B, Z_C, Z_D, Z_E, Z_F = 0, 1$$

2.4.2 Solution by reduction with enumeration

A reduction technique was used to solve this problem, beginning with a column reduction rule. Using the column technique on the constraints with *z* variable, we eliminate columns with subscripts A, B, since column with subscript D dominates subscript A and B. Subscript F is dominated by subscript C. We eliminate column with subscript F.

Therefore $X_A = X_B = X_F = 0$.The problem reduces to

P 3.2 Maximize $10Z_A + 8Z_B + 22Z_C + 18Z_D + 7Z_E + 55Z_F$

Subject to

$$
\begin{aligned}
X_D &\geq Z_A \\
X_D &\geq Z_B \\
X_C + X_E &\geq Z_C \\
X_D + X_E &\geq Z_D \\
X_C + X_D + X_E &\geq Z_E \\
X_C &\geq Z_F
\end{aligned}
$$

No. to locate; $X_C + X_D + X_E \leq 1$

Integrality $X_C, X_D, X_E = 0,1$

$Z_A, Z_B, Z_C, Z_D, Z_E, Z_F = 0,1$

Since the row reduction technique can not be applied we use total enumeration. That is if $X_D = 1$ then $Z_A = Z_B = Z_D = Z_E = 1$, Objective function = 10 + 8 + 18 + 7 = 43.

If $X_C = 1$, then $Z_C = Z_E = Z_F = 1$, Objective function = 22 + 7 + 55 = 84

If $X_E = 1$ then $Z_C = Z_D = Z_E =1$, Objective function = 22 + 18 + 7 = 47

Since the problem is a maximization, we choose $X_C = 1$ which gives us the maximum objective function value of 84. Hence facility will be located at X_C.

If we are to locate two facilities that is P = 2, then it is either (X_D, X_E) or (X_D, X_C) or (X_C, X_E) If it is X_D, X_C then objective function equals 120. If it is X_D, X_E then objective function equals 65. If it is X_C, X_E then objective function equals 102.

Therefore the facility will be located at X_D, X_C

2.5 THE GREEDY ADDING ALGORITHM

Many algorithms have been proposed for solving the maximum covering model. The Greedy Algorithm is one of such algorithms. The Greedy Algorithm finds the best solution at each step of the algorithm without looking ahead to see how the current decision will impact on later decisions and alternatives. The greedy algorithm does not guarantee optimality. Assuming we are to locate only one facility (i.e. P = 1) we would solve the problem optimally by simply evaluating how many demands each candidate site covers (candidate site i covers $\sum_i a_{ij} h_i$ demands) and select the site that covers the most demands.

The algorithm is summarized in the flowchart of figure 2.3.

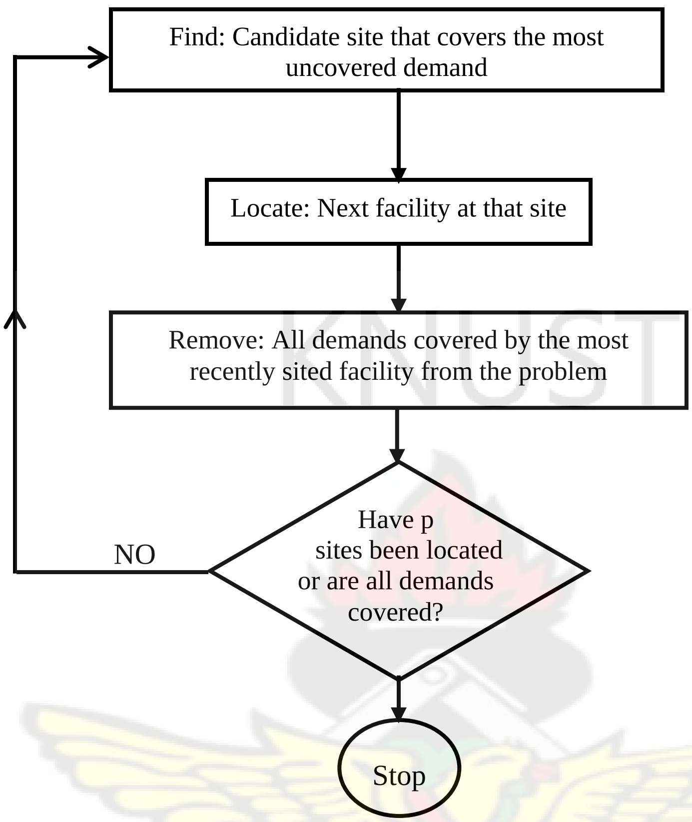

Figure 2.3: A flow chart for the greedy algorithm

2.5.1 Example

We use Figure 2.2 to illustrate the Greedy algorithm.

Using a coverage distance $D_C = 9$ the table below lists the demand nodes covered by each candidate site.

Table 2.2a Coverage by each candidate site with D_C = 9

Candidate Site	Nodes now Covered	Demand Covered
A	A, B,	18
B	A, B, D	36
C	C, E,	29
D	B, D,	26
E	C, E	29
F	F,	55

We remove node F which has the highest demand. The next removal is from node B which has demand coverage of 36. Nodes A, B and D will be removed from the problem because they are covered by B. The result is table 2.2b below.

Table 2.2b Coverage by candidate site after locating at nodes B and F

Candidate site	Nodes now covered	Demand now covered
A	-	0
B	-	0
C	C, E	29
D	-	0
E	C, E	29
F	-	0

With node C and E left, either may be selected, if either of node C or node E is selected the objective function value becomes 29.

2.6 SOLUTION BY LAGRANGIAN RELAXATION

The Lagrangian Relaxation provides us with an upper bound on the value of the objective function. Recalling the maximum covering location problem ,

P 3 Maximize $\sum_{j} h_i Z_i$ 2.6a

Subject to

$$Z_i \leq \sum_{j} a_{ij} x_j \quad \forall i \qquad 2.6b$$

$$\sum_{j} X_j \leq P \qquad 2.6c$$

$$x_j = 0, 1 \quad \forall j \qquad 2.6d$$

$$Z_i = 0, 1 \quad \forall i \qquad 2.6e$$

We first relax one or more of the constraints by using Lagrange multipliers and bring the constraints into the objective function. After relaxing the relevant constraints, 2.6b using Langrange multipliers λ_i the following problem is obtained

P4 $$\underset{\lambda}{\text{Min}} \; \underset{x\,z}{\text{M ax}} \left\{ \sum_{i} h_i z_i + \sum_{i} \lambda_i \left(\sum_{j} a_{ij} x_j - z_i \right) \right\} \qquad 2.10a$$

Subject to

$$\sum_{i} x_j \leq P \qquad 2.10b$$

$$x_j = 0, 1 \quad \forall j \qquad 2.10c$$

$$Z_i = 0, 1 \quad \forall i \qquad 2.10d$$

We try to maximize the objective function with respect to the decision variables Z_i and x_j for a given λ and minimize the resulting function with respect to the Lagrangian variables λ. For fixed λ_i find the x, z that give maximum objective of the function in the curly bracket. For the various maximum objectives corresponding to the λ_i, find the one which is minimum. Simplifying the problem further leads to

P5; $$\underset{\lambda}{\text{Min}}\ \underset{x\ z}{\text{Max}} \left\{ \sum_i (h_i - \lambda_i) Z_i + \sum_j \left(\sum_i a_{ij} \lambda_i \right) x_j \right\} \qquad 2.10e$$

Subject to

$$\sum_j x_j \le P \qquad 2.10b$$

$$x_i = 0, 1 \qquad \forall j \qquad 2.10c$$

$$Z_i = 0, 1 \qquad \forall i \qquad 2.10d$$

$$\lambda_i \ge 0 \qquad \forall i \qquad 2.10f$$

To solve Lagrangian Relaxation, we need the following:

The Lower Bound, for each iteration is given as $\sum (h_i)$ for all demand nodes satisfying $\sum a_{ij}\, x_i > 0$ and denoted by LB. The upper bound (UB) is the solution of the Lagrangian objective function 2.10e, when all demand nodes are covered. A Langragian solution is called an upper bound. The least Lagrangian objective value from previous iteration is the best upper bound, this is used for the current iteration

2.6.1 Algorithm for Lagrangian Relaxation

Step 1: Choose an appropriate coverage distance, Dc, and α^1 is usually chosen to be 2.

Step 2: Choose an initial value for the step size, $\lambda_i^{\,1} = \bar{h} + 0.5\,(h_i - \bar{h})$ where $\bar{h}$ is the average of the demand.

Step 3: For n[th] iteration, put $Z_i = 1$ if $(h_i - \lambda_i)$ is greater than zero, if not zero.

Step 4: Find x_j^n, where $x_j^n = 1$ for the largest P number of value(s) of $\sum_i a_{ij}\lambda_i$ calculated based on the j values determined.

Step 5: Calculated $LB^n = \sum h_i$ given that $(\sum_j a_{ij} x_j^n - Z_i^n) > 0$ is satisfied for all i.

Step 6: Calculate $UB^n = \sum(h_i - \lambda_i^n) Z_i^n + \sum(\sum_i a_{ij}\lambda_i^n) x_i^n$

Step 7: Determine $$t^n = \frac{\alpha^n \left(UB^n - LB\right)}{\sum_i \left[\left(\sum_j a_{ij} x_j^n\right) - Z_i^n \right]^2}$$

Where n is the n^{th} iteration

The iteration is terminated if at least one of the following is true,

1. A prescribed number of iterations are done.
2. $UB^n = LB^n$ or UB^n is close to LB^n within a given tolerance.
3. α^n become small such that there is no change in λ_i^n

Otherwise find increment in n and find λ_i^{n+1} from

$$\lambda_i^{n+1} = \text{Max}\left[0, \lambda_i^n - t^n \left(\sum_j a_{ij} x_j^n - Z_i^n \right) \right]$$

And

$$\alpha^{n+1} = 0.5\alpha^n \text{ if } (UB^n - UB^{n+1}) \geq 0$$

otherwise $\alpha^{n+1} = \alpha^n$

Step 8: Go to step 3.The solution is at $x_j^n = 1$ and the objective function value is $UB^{n.}$

Using figure 2.2, the Lagrangian method is illustrated below.

We choose Dc = 10 and $\lambda_i = \bar{h} + 0.5\,(h_i - \bar{h})$ for the first iteration where $\bar{h} = 20$

Table 2.3 gives a summary of the first Lagrangian iteration

Table 2.3 First iteration of Lagrangian Relaxation i,j = A, B, C, D, E ,F

Node	h_i	λ_i	Z_i	$(h_i - \lambda_i)Z_i$	$\sum_i a_{ij}\lambda_i$	x_j	$\left(\sum_i a_{ij}\lambda_i\right)x_j$	$\sum_j a_{ij}x_j$	$\sum_j a_{ij}x_j - Z_i$
A	10	15	0	0	48	1	48	2	2
B	8	14	0	0	48	1	48	2	2
C	22	21	1	1	34.5	0	0	0	-1
D	18	19	0	0	48	0	0	2	2
E	7	13.5	0	0	34.5	0	0	0	0
F	55	37.5	1	17.5	37.5	0	0	0	1

Upper bound ($UB^{1)}$) = 114.5

Lower bound (LB) =36.0

Best current upper bound = 114.5

Best current lower bound =36.0

$\alpha = 2$, $t^1 = 11.2142$, $\bar{h} = 20$

Table 2.4 gives the summary of the second Lagrangian iteration

Table 2.4 Second iteration of Lagrangian Relaxation

Node	h_i	λ_i	Z_i	$(h_i - \lambda_i)Z_i$	$\sum_i a_{ij}\lambda_i$	x_j	$\left(\sum_i a_{ij}\lambda_i\right)x_j$	$\sum_j a_{ij}x_j$	$\sum_j a_{ij}x_j - Z_i$
A	10	0	1	10	0	0	0	0	-1
B	8	0	1	8	0	0	0	0	-1
C	22	32.21	0	0	45.71	1	45.71	1	1
D	18	0	1	18	0	0	0	0	-1
E	7	13.5	0	0	45.71	0	0	1	1
F	55	48.71	1	6.29	48.71	1	1	1	0

Upper bound (UB^2) = 136.71

Lower bound (LB) = 84.0

Best current upper bound = 114.5

Best current lower bound = 84.0

α = 1, t^2 = 10.5429

The iteration continues to iteration six where the *LB* is close to the *UB*. That is 91 and 91.75 respectively and the facility will be located at X_A and X_F. Summary of the iterations is contained in table 2.6, x_j = 1 implies selection. Node A will cover nodes A, B and D. Node F will also cover nodes F, C and E.

Table 2.5 is the summary of the Lagrangian relaxation calculation.

Table 2.5 Summary of the Lagrangian relaxation calculation

Iteration	z_1 to z_6	x_1 to x_6	***LB***	$\boldsymbol{UB^n}$	**Best** $\boldsymbol{UB^n}$	α^n	$\boldsymbol{t^n}$
1	0,0,1,0,0,1	1,1,0,0,0	36	114.5	114.5	2	11.2142
2	1,1,0,1,0,1	0,0,1,0,0	84	136.71	114.5	1	10.543
3	0,0,1,1,1,1	1,0,0,0,0,1	91	98.46	98.46	1	1.864
4	1,0,0,1,1,1	0,0,1,0,0,1	91	94.31	94.31	1	1.105
5	1,0,0,1,1,1	1,0,0,0,0,1	91	93.86	93.86	1	1.429
6	1,1,0,1,1,1	1,0,0,0,0,1	91	91.75	91.75	1	0.75

2.7 CENTRE PROBLEM

The P-centre problem is a minimax problem, because the model minimizes the maximum distance between a demand and the nearest facility to the demand.

In set covering model, we require that all demands, h_i, be covered. Instead of using a pre-specified coverage distance and asking the model to minimize the number of facilities needed to cover all demand nodes as done in set covering, the centre problem requires the model to minimize the coverage distance such that each demand node is covered by at least one of the facilities within the pre-specified distance.

Under the P-centre problem, we have the vertex centre problem, which seeks to locate the facilities on the nodes of the network. There is also, the absolute centre problem that seeks to locate facilities at anywhere on the network, that is either at the nodes or on the links of the network.

2.8 THE ABSOLUTE CENTRE PROBLEM ON A TREE

We have the absolute centre problem on an unweighted tree in which all of the demands are equal. We also have the weighted tree in which the weights associated with each of the nodes are not equal. A tree is a network which has no loop in the connected nodes.

2.8.1 Absolute 1 – center on a weighted tree

The solution is computed as follows;

$$\beta_{ij} = \frac{h_i h_j \; d\,(i,\, j)}{h_i + h_j}$$

Where i and j are nodes, d (i, j) is the edge distance between nodes i and j and h_i and h_j are respectively demand weights. B_{ij} is the demand weighted distance between nodes i and j. Let F be candidate node and T be demand node. We find $B_{FT} = \max_{ij} (B_{ij})$.

We further locate a point $[h_T / (h_F + h_T), d\,(F, T)]$ from node F on the unique path from F to T or equivalently, locate a point $[h_F / (h_F + h_T)]\, d\,(F, T)$ from node T on the unique path from T to F. The computation is done as follows;

Step 1: Compute one row of β_{ij} elements

Step 2: Find the maximum element in the row that was just computed

Step 3: Compute the β_{ij} in the column in which the maximum β_{ij} element occurred in step 2.

Step 4: Find the maximum element in the column that was first computed

Step 5: Compute the elements β_{ij} in the row in which the maximum β_{ij} element occurred in step 4.

Note: We can compute all β_{ij} values and the largest β_{ij} value chosen

2.8.2 Example

Figure 2.4 is a weighted tree

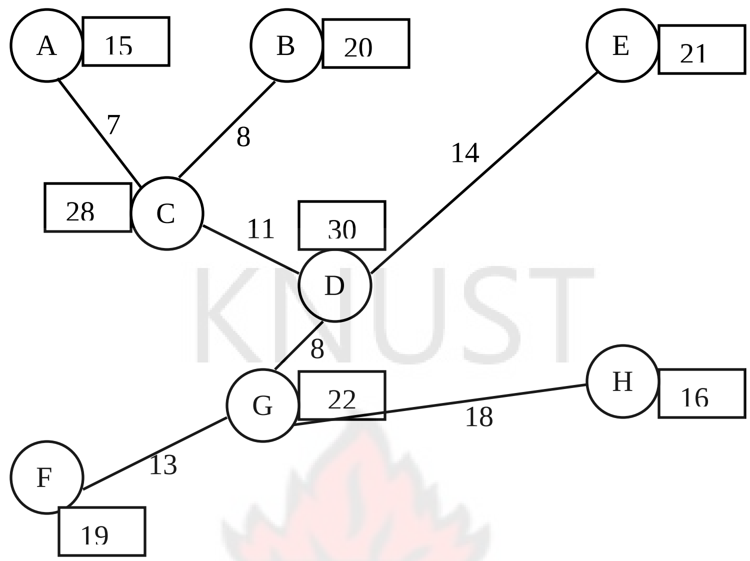

Figure 2.4: Example of tree network

$$\beta_{ij} = \frac{h_i h_j \; d\,(i, j)}{h_i + h_j}$$

$$\beta_{AB} = \frac{(15 \times 20)(15)}{35} = 128.57$$

$$\beta_{AC} = \frac{(15 \times 28)(7)}{43} = 68.37$$

When all β_{ij} are calculated we get table 2.6 below;

Table 2.6 β_{ij} values

To / From	A	B	C	D	E	F	G	H
A	0.00	128.57	68.37	180	280	326.91	231.89	340.65
B	128.57	0.00	93.33	22.8	338.05	389.74	282.86	400.00
C	68.37	93.33	0	159.31	300	362.21	234.08	376.73
D	180	228	159.31	0	175.94	244.29	101.54	271.30
E	280	338.05	300	175.94	0	349.13	236.37	508.54
F	326.91	389.74	362.21	244.29	349.13	0	132.54	269.26
G	231.89	282.86	234.08	101.54	236.37	132.54	0	166.74
H	340.65	400.00	376.73	271.30	508.54	269.26	166.74	0.00

We find $\beta_{FT} = \max_{ij} (\beta_{ij})$

$B_{FT} = \beta_{EH} = \beta_{HE} = 508.54$

Point on $\beta_{HE} = \dfrac{h_H\ d\left(H,E\right)}{h_H + h_E}$

Point on $\beta_{HE} = \dfrac{16}{16+21}(40)$

$= 17.30$

The facility will be located 17.30 units from node *H* to node *E*

2.8.3 Absolute 1 – centre on an unweighted tree

This is a tree in which all the demands are equal. Since all the demands are equal, we can normalize them so that they are all equal to one.

2.8.4 Algorithm for absolute 1–centre on an unweighted tree

Step 1

Pick any point on the tree and find the vertex that is farthest away from the point that was picked, let this be vertex V_1.

Step 2

Find the vertex that is farthest from V_1 and call it V_2

Step 3

The absolute 1-centre of the unweighted tree is at midpoint of the unique path from V_1 and V_2. The vertex 1-centre of the unweighted tree is at the vertex of the tree that is closest to the absolute 1-centre.

2.8.5 Example

Figure 2.5 is an unweighted tree network which was used to find the absolute 1-centre

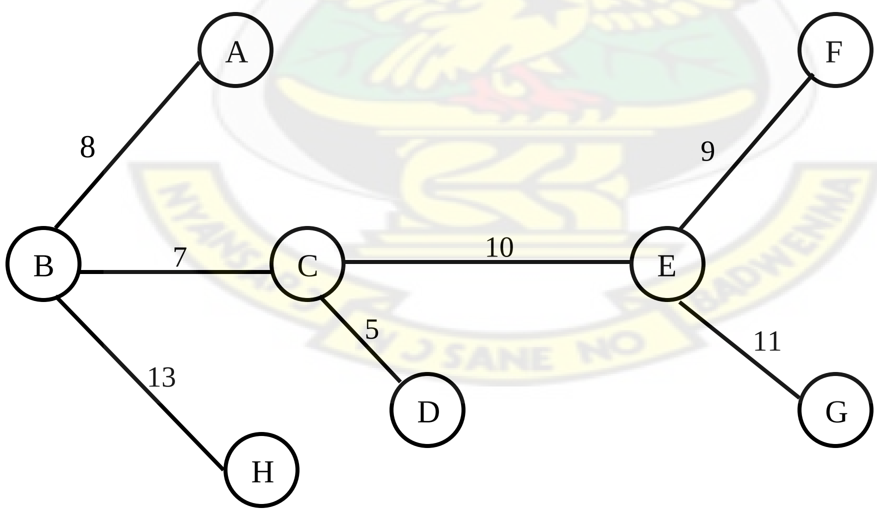

Figure 2.5: Example of an unweighted tree network

We pick A as initial point G is the farthest point from A and therefore G is V_1

H is also the farthest point from V_1 hence H is labelled V_2. The midpoint between V_1 and V_2 is 20.5. This is the absolute 1-center. But it is 0.5 from C on the CE link and 9.5 from E on the CE link. Since it is closer to C, C is the vertex 1-center and the objective function is 21.

2.8.6 Absolute 2 – centre on the unweighted tree

We modify the algorithm for the absolute 1-center as follows;

Step 1
Using the algorithm for the absolute 1-centre, find the absolute centre

Step 2
Delete from the tree the arc containing the absolute centre. This divides the tree into a forest of two disconnected subtrees.

Step 3
Use the absolute 1 –centre algorithm to find the absolute 1-centre for each subtree

2.8.7 Example

From the previous example (of section 2.11.1) we remove arc CE, which leads to Figure 2.6a and Figure 2.6b

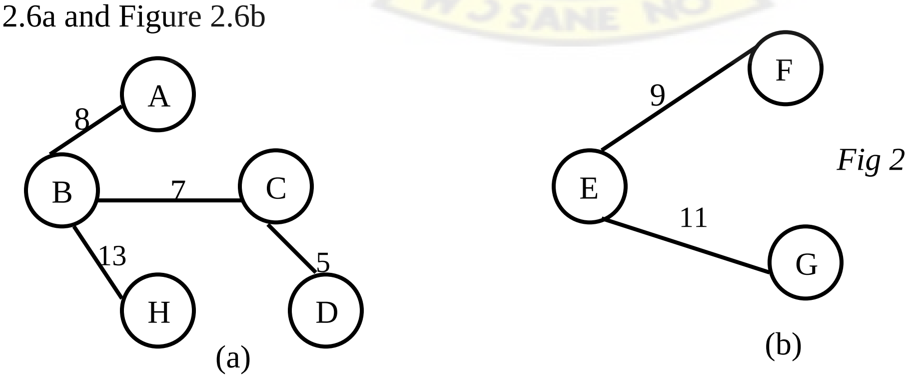

Figure 2.6: Absolute 2-center on the unweighted tree

For the Figure 2.6(a), we pick B, H is the farthest from B. H is V_1, D is also the farthest from H and D becomes V_2. The midpoint between V_1 and V_2 is 12.5. This is located 0.5 from B on the BH link which is closer to the Vertex B and therefore B is the vertex 1-center. The figure 2.6(b), will have E to be the vertex 1-center.

So therefore the facility would be located at absolute centre of 0.5 from B on BH link.

2.9 THE UNWEIGHTED VERTEX P–CENTRE PROBLEM ON A GENERAL GRAPH

The approach of solving this problem is based on searching over the range of coverage distances for the smallest coverage distance that allows all nodes to be covered.

2.9.1 Vertex P-center problem formulation

Daskin (1995) formulated the P-centre problem as follows;

Let a_{ij} = distance from demand node i to candidate facility site j

h_i = demand at node i

p = number of facilities to locate

Decision variables.

$$x_j = \begin{cases} 1, & \text{if we locate at candidate site } j. \\ 0, & \text{if not} \end{cases}$$

Y_{ij} = fraction of demand at node i that is served by a facility at node j

W = maximum distance between a demand node and the nearest facility.

The problem is formulated as follows,

$$\text{minimize } W \qquad 2.13a$$

Subject to

$$\sum_j Y_{ij} = 1 \qquad \forall i \qquad 2.13b$$

$$\sum_j x_j = P \qquad 2.13c$$

$$Y_{ij} \le x_j \qquad \forall i, j \qquad 2.13d$$

$$W \ge \sum_j a_{ij} Y_{ij} \quad \forall i \qquad 2.13e$$

$$x_j = 0, 1 \qquad \forall j \qquad 2.13f$$

$$Y_{ij} \ge 0 \qquad 2.13h$$

In some cases, the demand – weighted distance is considered and constraint 2.13e becomes $W \ge h_i \sum_j a_{ij} Y_{ij} \;\; \forall i$ 2.13e[1].

2.9.2 Algorithm for the unweighted vertex P-centre problem on a general graph

Step 1

Set D_C^H to a suitably large number, that is D_C^H = (n-1) $\max\limits_{i,j}$ (d_{ij}). Also set D_C^L = 0

Step 2

Set D_C = [(D_C^L + D_C^H)/ 2], D_C is approximated downward to the nearest integer.

Step 3

Solve a set covering problem with a coverage distance D_C where $P*$ (D_C) is an intermediate solution

Step 4

If P^* (D_C) $\leq P$, reset D_C^H to D_C, otherwise reset D_C^L to D_C + 1

Step 5

If $D_C^L \neq D_C^H$, go to step 2; otherwise stop, D_C^L is the optimal value of the objective function and the locations corresponding to the set covering solution for this coverage distance are the optimal locations for the P-centre problem.

That is, we stop when $D_C^L = D_C^H$.When the iterations are completed with $D_C^L = D_C^H$, we consider the D_C values and check for the D_C value that equals the stopping D_C^L value. The solution is found at the iteration that gives this D_C value.

Below is an example of an unweighted graph.

2.9.3 Example

Below is figure 2.7 which is an unweighted graph on which we are to solve for a vertex 2- centre.

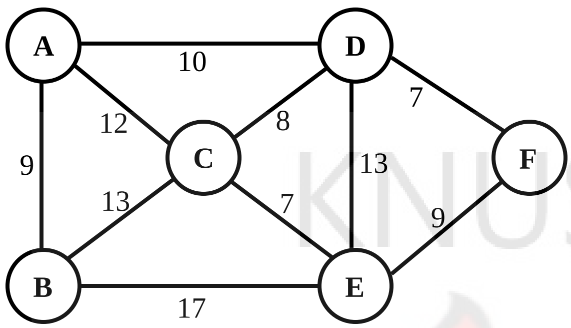

Figure 2.7: Example of P-centre on a General Graph

To solve the above unweighted problem, we choose P = 2

$$D_C^L = 0$$

$$D_C^H = (\text{n - 1}) \max_{i,j} (d_{ij})$$

$$= (6\text{-}1)\ 17 = 85$$

$$D_C = (D_C^H + D_C^L)/\ 2$$

$$= (85 + 0)/\ 2 = 42.5$$

$$\therefore\ D_C = 42$$

The set covering problem is stated below

P6.1 Minimize $X_A + X_B + X_C + X_D + X_E + X_F$

Subject to;

$$
\begin{aligned}
X_A + X_B + X_C + X_D \quad & \geq 1 \\
X_A + X_B + X_C + \quad X_E \quad & \geq 1 \\
X_A + X_B + X_C + X_D + X_E \quad & \geq 1 \\
X_A + \quad X_C + X_D + X_E + X_F \quad & \geq 1 \\
X_D + X_E + X_F \quad & \geq 1
\end{aligned}
$$

Integrality constraints; $X_A, X_B, X_C, X_D, X_E, X_F = 0, 1$

Using the column reduction technique, we have $X_A < X_C$, $X_B < X_C$, $X_F < X_E$, so columns A, B and F will be eliminated. This leads to,

P6.2 Minimize $X_C + X_D + X_E$

Subject to:

$$
\begin{aligned}
X_C + X_D \quad & \geq 1 \\
X_C + \quad X_E \quad & \geq 1 \\
X_C + X_D + X_E \quad & \geq 1 \\
X_D + X_E \quad & \geq 1
\end{aligned}
$$

Integrality constraints $X_C, X_D, X_E = 0, 1$

For the row reduction if $a_{mj} \leq a_{nj}$ then n row will be eliminated.

We have

$$\text{Min} \quad X_C + X_D + X_E$$

Subject to;

$$X_C + X_D, X_E \geq 1$$

$$X_C + \qquad X_E \geq 1$$

$$X_D + X_E \geq 1$$

The problem cannot be reduced further, so we use LINDO programme to solve problem P6.3.

The solution is $P^*(42) = 2$

Again we reset D_C = 10 and obtained $P^*(10) = 3$

Table 2.7 below is the summary of the runs using Lindo programme.

Table 2.7 Summary of runs using LINDO programme

Run	D_C^L	D_C^H	D_C	**P* (D_C)**	**Location**
1	0	85	42	2	X_A, X_D
2	0	42	21	2	X_A, X_D
3	0	21	10	3	X_B, X_E, X_F
4	11	21	16	2	X_D, X_E
5	11	16	13	2	X_D, X_E
6	11	13	12	2	X_A, X_F
7	11	12	11	2	X_A, X_E
8	11	11	Stop	Stop	

We stop at iteration 8, because $D_C^L = D_C^H = 11$. The solution is at iteration 7 where D_C^L of iteration 8 equals the D_C of iteration 7.

The optimal solution has objective function value of 11. Facilities should be located at nodes A and E. Node A covers A, D and B. Node E covers E, F and C.

2.9.4 Weighted P-vertex problem on a general graph

A demand weighted graph has demand at each node. The algorithm in section 2.21 can readily be extended to solve the weighted vertex P-centre problem.

To account for the demand ,h_i, the initial D_C^H is modified as follows;

$D_C^H = (n - 1)\, [\max_{i,j} (d_{ij})]\, [\max (h_i)]$.

Also, for the facility to cover a demand, node i, $d_{ij}\, h_i \leq D_C$

Where h_i is the demand weight at node i.

2.9.3 Algorithm for the weighted P-vertex problem on a general graph

Step 1

Set D_C^H to a suitably large number. That is $D_C^H = (\text{n-1})\, [\max_{i,j} (d_{ij})]\, [\max (h_i)]$.

Also set $D_C^L = 0$

Step 2

Set $D_C = [(D_C^L + D_C^H)/ 2]$, D_C is approximated downward to the nearest integer.

Step 3

Solve a set covering problem with a coverage distance D_C where $P^*\ (D_C)$ is an intermediate solution

Step 4

If $P^*\,(D_C) \leq P$, reset D_C^H to D_C, otherwise reset D_C^L to $D_C + 1$

Step 5

If $D_C^L \neq D_C^H$, go to step 2; otherwise stop, D_C^L is the optimal value of the objective function and the locations corresponding to the set covering solution for this coverage distance are the optimal locations for the P-centre problem.

That is, we stop when $D_C^L = D_C^H$.When the iterations are completed with $D_C^L = D_C^H$, we consider the D_C values and check for the D_C value that equals the stopping D_C^L value. The solution is found at the iteration that gives this D_C value.

2.9.4 Example

Below is Figure 2.8 which is a weighted graph on which we are to solve for a vertex 2-centre

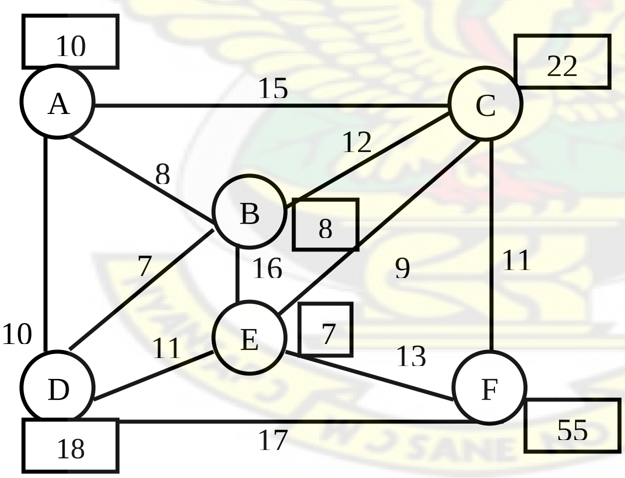

Figure 2.8: Example of vertex 2- centre on a weighted graph

$$n = 6, \qquad D_C^L = 0$$

$$D_C^H = (6\text{-}1)\ (17)\ (55) = 4675$$

$$D_C = (D_C^H + D_C^L) / 2 = 2337$$

Below is the solution by LINDO

Table 2.8 Summary of iteration using LINDO Programme

Iteration	D_C^L	D_C^H	D_C	**P* (D_C)**	**Location**
1	0	4675	2337	2	X_A, X_E
2	0	2337	1168	2	X_A, X_E
3	0	1168	584	2	X_A, X_E
4	0	584	292	2	X_B, X_E
5	0	292	146	3	X_B, X_C, X_F
6	147	292	219	3	X_A, X_E, X_F
7	220	292	255	2	X_A, X_F
8	220	255	237	3	X_A, X_E, X_F
9	238	255	246	2	X_A, X_F
10	238	246	242	2	X_A, X_E
11	238	242	240	3	X_A, X_E, X_F
12	241	242	241	3	X_A, X_E, X_F
13	242	242	Stop		

We stop at iteration 13, because we have $D_C^L = D_C^H$. The solution is at iteration 10 where D_C^L of iteration 13 equals D_C of iteration 10. The optimal value of the objective function is 242 and the facilities should be located at X_A and X_F. X_A covers X_A, X_D and X_B. X_E covers X_E, X_C and X_F.

2.10 CONCLUSION

This chapter examined the literature with respect to set covering and centre problems. Set covering problem involves finding the smallest number of facilities (and their locations) such that each demand is no farther than a pre-specified distance or time away from the closest facility. The P-centre problem minimizes the maximum distance between a demand and the nearest facility to the demand point. In the next chapter the P-centre model will be used to solve a real life problem of locating two (P=2) ambulances at the

KNUST students hall of residence. The LINDO software application which uses branch and bound algorithm was employed to solve the problem. Data of student populations in the halls and the inter-hall distances will be used.

CHAPTER THREE

3.0 DATA COLLECTION, ANALYSIS AND DISCUSSION

3.1 INTRODUCTION

In this chapter, the P-centre model would be used to locate two ambulances on KNUST students' residence. The data collected on inter-hall distances will be used to draw a network graph of the students' residence. The halls populations will be added to obtain a demand weighted network graph from which the coverage matrix (a_{ij}) is obtained and used as an input for the iterations in the solution to the problem. An instance of the model of Daskin and Dean (1994) found in section (2.3) will be formulated with the input data. The problem instance will then be iterated using the LINDO programming software which employs a branch and bound algorithm. The programme was run on Intel Pentium (iv) HP laptop of processor speed of 2.64 GHz. The results are discussed.

3.2 SOURCE OF DATA

Student populations in the hall of residence were collected from the secretaries of the various halls. The populations are for the 2007/2008 academic year. Table 3.1 is a table of student population in the hall of residence.

Table 3.1: Student Population in the Hall of Residence at KNUST

Hall	Population
Guss Hostel (A)	935
University Hall (B)	1190
Independence Hall (C)	1176
Unity Hall (D)	1925
Republic Hall (E)	1208
Queens Hall (F)	1164
Africa Hall (G)	712
Total	**8310**

Graph of KNUST student s' residence

Data of inter-hall distances was obtained from the office of the Technical Instructor at the Geomatic Engineering Department. Table 3.2 is a table of inter-hall distances.

Table 3.2: Inter-hall distances

From	To	Distance/m
Guss Hostel	University Hall	306
University Hall	Queens Hall	950
University Hall	Independence Hall	1050
Independence Hall	Unity Hall	340
Independence Hall	Republic Hall	210
Unity Hall	Republic Hall	380
Unity Hall	Africa Hall	400
Republic Hall	Queens Hall	100
Queens Hall	Africa Hall	375

Figure 3.1 is the graph of KNUST students Halls of residence and their links constructed from the data of table 3.2.

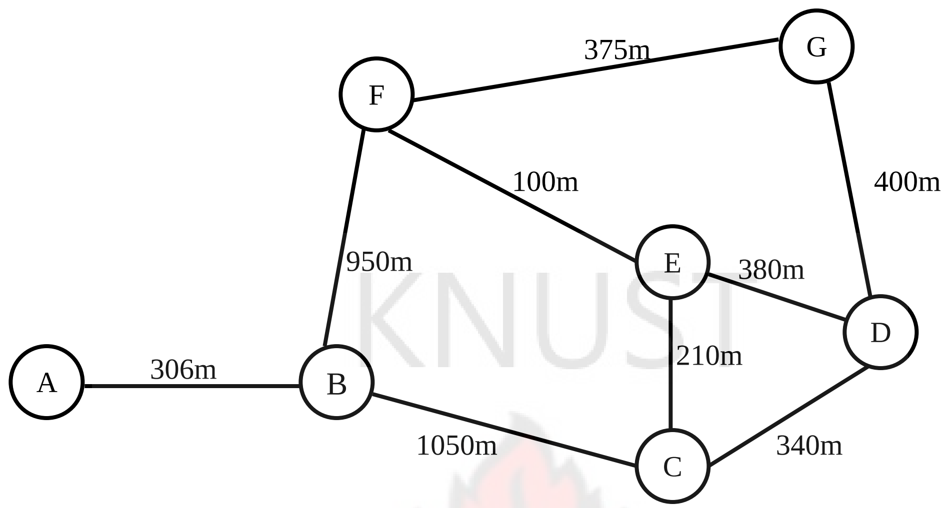

Figure 3.1: Graph of KNUST students halls of residence

We fuse the table of population with the graph and we get the graph of figure 3.2 below.

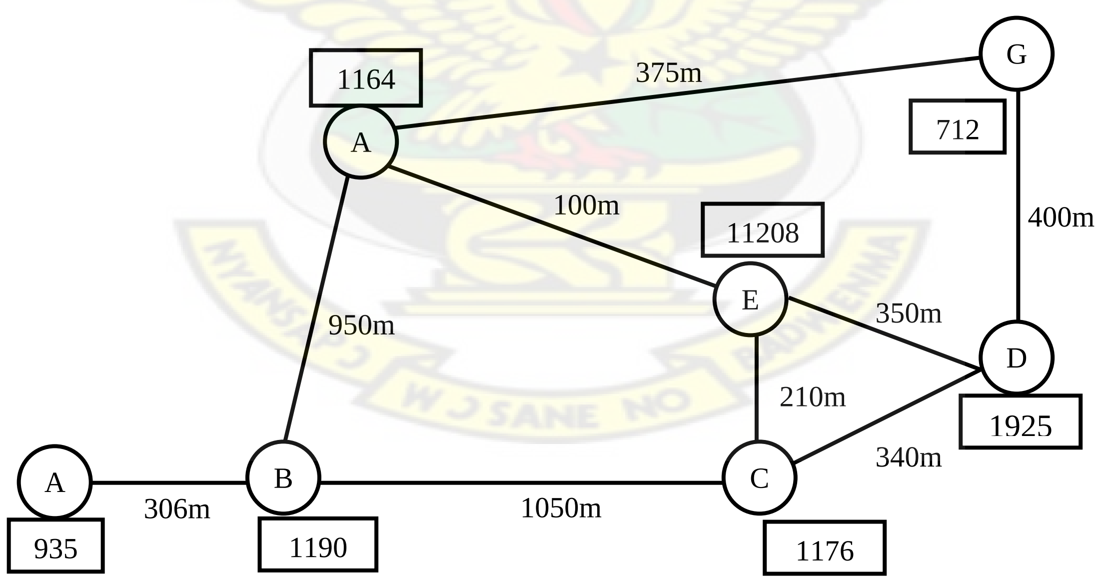

Figure 3.2: Graph of KNUST students Halls of residence with population

3.3 SOLUTION

We begin the problem by setting out the d_{ij} matrix and h_i matrix then $h_i d_{ij}$ matrix.

Table 3.3 gives the matrix of inter-hall distances obtained from table 3.2

Table 3.3: d_{ij} values

$$d_{ij} = \begin{bmatrix} 0 & 306 & - & - & - & - & - \\ 306 & 0 & 1050 & - & - & 950 & - \\ - & 1050 & 0 & 340 & 250 & - & - \\ - & - & 340 & 0 & 380 & - & 400 \\ - & - & 210 & 380 & 0 & 100 & - \\ - & 950 & - & - & 100 & 0 & 375 \\ - & - & - & 400 & - & 375 & 0 \end{bmatrix}$$

The student populations in the hall is given as a vector h_i shown below

$h_i = [935 \quad 1190 \quad 1176 \quad 1925 \quad 1208 \quad 1164 \quad 712]$

Table 3.4 gives demand weighted distances $h_i d_{ij}$

Table 3.4: Table of $h_i d_{ij}$ values

To j / From i	**A**	**B**	**C**	**D**	**E**	**F**	**G**
A	0	286110	-	-	-	-	-
B	364140	0	1,249,500	-	-	1,130,500	-
C	-	1,234600	0	399840	246960	-	-
D	-	-	654500	0	731,500	-	770,000
E	-	-	253.680	459,040	0	120,800	-
F	-	1,105,800	-	-	116,400	0	828,768
G	-	-	-	284800	-	267,000	0

Note: dash (-) means there is no direct link between a demand site *i* and a candidate node *j*.

3.3.1 Iterations

Iteration 1: Setting out of the model

Using the model and the algorithm for the weighted P-Vertex Centre,

We set $D_C^L = 0$

$D_C^H = (n\text{-}1)\ [\max_{ij}\ (d_{ij})]\ [\max_i\ (h_i)]$

$= (7\text{-}1)\ (1925)\ (1050) = 12{,}127{,}500$

$D_C = (\ D_C^L + D_C^H\ )/2 = (0+12127500)/2$

$= 6063750$

The coverage matrix is set out as Table 3.5 below

Table 3.5: Table of coverage matrix

$$a_{ij} = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 & 1 \end{bmatrix}$$

Where i = A, B, ….., G

j = A, B, ….., G

From Table 3.2 all $h_i d_{ij}$ satisfy $h_i d_{ij} \leq D_C$ and the set covering problem is expanded as follows:

Minimize $\quad X_A + X_B + X_C + X_D + X_E + X_F + X_G$

Subject to;

$$\begin{aligned} X_A + X_B & \geq 1 \\ X_A + X_B + X_C + X_F & \geq 1 \\ X_B + X_C + X_D + X_E & \geq 1 \\ X_C + X_D + X_E + X_G & \geq 1 \\ X_C + X_D + X_E + X_F & \geq 1 \\ X_B + X_E + X_F + X_G & \geq 1 \\ X_D + X_F + X_G & \geq 1 \end{aligned}$$

INTEGRALITY CONSTRAINT: $X_A, X_B, X_C, X_D, X_E, X_F, X_G = \{0, 1\}$

Using LINDO software gives the solution for the first iteration as follows.

$X_A = X_G = 1$, $X_B = X_C = X_D = X_E = X_F = 0$

Objective function = 2, therefore, P* (60637500) = 2

Iteration 2

Since P* (D_C) = P in previous iteration

We reset D_C^H to Dc, D_C^L = 0, D_C^H = 6063750, D_C = 3031875

From table 3.2, all $h_i d_{ij} \leq D_C$, hence all demand node will be covered as in the first iteration. The coverage matrix, a_{ij} ,of Table 3.5 will not change, therefore the set covering problem formulation will be the same as in iteration 1.

Therefore, for the second iteration

$X_A = X_G = 1$, $X_B = X_C = X_D = X_E = X_F = 0$

Objective function = 2, therefore P* (3031875) = 2

Iteration 3

Since P* (D_C) = P in previous iteration

We reset D_C^H to D_C , D_C^L = 0, D_C^H = 3031875, D_C = 15159387

This will lead to the same formulation as in iteration 1 and iteration 2 since all $h_i d_{ij}$ satisfy $h_i d_{ij} \leq D_C$

Therefore $X_A = X_G = 1$, $X_B = X_C = X_D = X_E = X_F = 0$

Objective function = 2, therefore P* (1515937) = 2

Iteration 4

Since P* (D_C) = P in previous iteration, we reset $D_C^L = 0, D_C^H = 1515937$, $D_C = 757{,}968$

The coverage matrix, a_{ij} , will change due to change in D_C

$$a_{ij} = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 1 \end{bmatrix}$$

The problem is formulated as follows

$$\text{Minimize } X_A + X_B + X_C + X_D + X_E + X_F + X_G$$

S.T

$$\begin{aligned} X_A + X_B &\geq 1 \\ X_A + X_B &\geq 1 \\ X_C + X_D + X_E &\geq 1 \\ X_C + X_D + X_E &\geq 1 \\ X_C + X_D + X_E + X_F &\geq 1 \\ X_E + X_F &\geq 1 \\ X_D + X_F + X_G &\geq 1 \end{aligned}$$

Integrality constraints: $X_A, X_B, X_C, X_D, X_E, X_F, X_G = \{0,1\}$

Using LINDO software, we have,

$X_B = X_D = X_F = 1, X_A = X_C = X_E = X_G = 0$

Objective function = 3, hence P* (757,968) = 3

P* (D_C) > P (= 2)

For the next iteration, we set $D_C^L = D_C + 1$ because P*(D_C) > P and D_C^H is set to it previous value. $D_C = \frac{D_C^L + D_C^H}{2} = \frac{757969+1515937}{2} = 1{,}136{,}953$. We check this value against $h_i d_{ij}$ to see whether the coverage matrix, a_{ij}, changes to warrant new formulation. In this case, the formulation will change. The results of the successive iterations are shown in Table 3.3.

Table 3.6 shows the total number of iterations and the solution at each iteration.

Table 3.6 Table of Results

Iteration	D_C^L	D_C^H	D_C	a_{ij} **matrix**	**P*(** D_C **)**	**Location**
1	0	12127500	6063750	Original	2	X_A, X_G
2	0	6063750	3031875	Same	2	X_A, X_G
3	0	3031875	1515937	Same	2	X_A, X_G
4	0	1515937	757,968	Changed	3	X_B, X_D, X_F
5	757969	1515937	1,136,953	Changed	3	X_A, X_D, X_G
6	1,136954	1515937	1,326,445	Changed	2	X_B, X_D
7	1,136,954	1,326,445	1,231,699	Same	2	X_B, X_D
8	1,184327	1,231,699	1,184,326	Changed	3	X_A, X_D, X_G
9	1,184327	1,231,699	1,208,013	Changed	2	X_B, X_D
10	1,184,327	1,208,013	1,196,170	Same	2	X_B, X_D
11	1,184,327	1,196,170	1,190,248	Same	2	X_B, X_D
12	1,184,327	1,190,248	1,187,287	Same	2	X_B, X_D
13	1,184,327	1,187,287	1,185,807	Same	2	X_B, X_D
14	1,184,327	1,185,807	1,185,807	Same	2	X_B, X_D
15	1,184,327	1,185,067	1184697	Same	2	X_B, X_D
16	1,184,327	1,184,697	1184512	Same	2	X_B, X_D
17	1,184,327	1,184,512	1184419	Same	2	X_B, X_D
18	1,184,327	1,184,419	1184373	Same	2	X_B, X_D
19	1,184,327	1,184,373	1184350	Same	2	X_B, X_D
20	1,184,327	1,184,350	1184338	Same	2	X_B, X_D
21	1,184,327	1,184,338	1184332	Same	2	X_B, X_D
22	1,184,327	1,184,332	1184329	Same	2	X_B, X_D
23	1,184,327	1,184,329	1184328	Same	2	X_B, X_D
24	1,184,327	1,184,328	1184327	Same	2	X_B, X_D
25	1,184,327	1,184,327	Stop	Same	2	X_B, X_D

At iteration 25, D_C^L = D_C^H hence we stop the iterations. We have reached an optimal solution since $D_C^L = D_C^H$ .The solution is at iteration 24 which has D_C =1184327= D_C^L of iteration 25.From Table 3.3, the facilities are to be located at XB and XD, that is at University Hall and Unity Hall respectively. The optimal value is 1,184,327. Appendix A shows details of the iterations. The D_C values are the objective function value and P* (D_C) values are the number of sites located at each iteration.

3.4 DISCUSSION

3.4.1 Coverage matrix, a_{ij}

Looking at the matrix a_{ij}, whenever the a_{ij} value changes the location changes. From iteration 1 to 3, the a_{ij} is unchanged and hence the locations also did not change. At iteration 4,5 and 8 the a_{ij} values changed and this affected the corresponding locations being ($X_{B,}X_{D,}X_F$),($X_{A,}X_{D,}X_G$),($X_{A,}X_D$,X_G) respectively.

3.4.2 Coverage Distance (D_C) and coverage matrix , a_{ij} ,values

The coverage distance D_C is the solution at each iteration. The relationship between the Dc and the demand weighted distance $h_i d_{ij}$ in the form $h_i d_{ij} \leq$ Dc, determine the coverage matrix. When the coverage matrix changes the formulation changes.

3.4.3 The stopping criteria

The iteration ended when the $D_C^L = D_C^H$ at iteration 25. That is $D_C^L = D_C^H$ = 1, 184,327 but the solution was at iteration 24 where D_C^L of iteration 25 equals the D_C of iteration 24. The D_C of iteration of 24 is the optimal value of the objective function which is the weighted coverage distance of 1,184,327 and the optimal locations are X_B and X_D that is University Hall and Unity Hall respectively.

3.4.4 Nodes covered

Table 3.7 below gives the weighted distances and this gives an indication of which nodes will be covered

Table 3.7: Table of $h_i d_{ij}$ values

To *j* / From *i*	**A**	**B**	**C**	**D**	**E**	**F**	**G**
A	0	286110	-	-	-	-	-
B	**364140**	**0**	**1,249,500**	-	-	**1,130,500**	-
C	-	1,234600	0	399840	246960	-	-
D	-	-	**654500**	**0**	**731,500**	-	**770,000**
E	-	-	253.680	459,040	0	120,800	-
F	-	1,105,800	-	-	116,400	0	828,768
G	-	-	-	284800	-	267,000	0

D_C = 1184328

At node X_B that is University Hall, it will cover Guss Hostel, Queens and University Hall. At node X_D that is Unity Hall, it will cover Independence Hall, Republic Hall, Africa Hall and Unity Hall. University Hall cannot cover Independence Hall since the condition is not satisfied.

3.5 CONCLUSION

For each iteration the coverage distance, Dc, is calculated from $D_C = (D_C^L + D_C^H)/2$ where D_C^L and D_C^H are respectively the current minimum and maximum coverage distances. The Dc is then used to determine the coverage matrix, a_{ij}, which is used as input for the LINDO software for each iteration.

The D_C of iteration 24 is the optimal value of the objective function which is equal to the weighted coverage distance of 1,184,327 and the optimal locations are X_B and X_D that is University Hall and Unity Hall respectively. The vertex 2-centre model has successfully been used to locate two ambulances at Unity and University Hall.

CHAPTER FOUR

4.0 CONCLUSION AND RECOMMENDATION

4.1 CONCLUSION

The results of chapter three, section 3.2.1 show that the two ambulance facilities should be located at University Hall (B) and Unity Hall (D).

The facilities if located at University Hall will cover or serve the University Hall, Guss Hostel and Queens Hall. The other at Unity Hall will serve Unity Hall, Africa Hall, Independence and Republic Hall. The facility should be located at open space near the two halls and on the side of the halls where sheds could be provided for the ambulances. This will not interfere with traffic movement in front of the halls. Rooms at the basement and the ground floor should be made available to be used as offices by the ambulance personnel. Figure 3.2 below shows the sites for the facilities on the network graph of KNUST students' halls of residence. Nodes B (University Hall) and D(Unity Hall) are the sites to place the facilities.

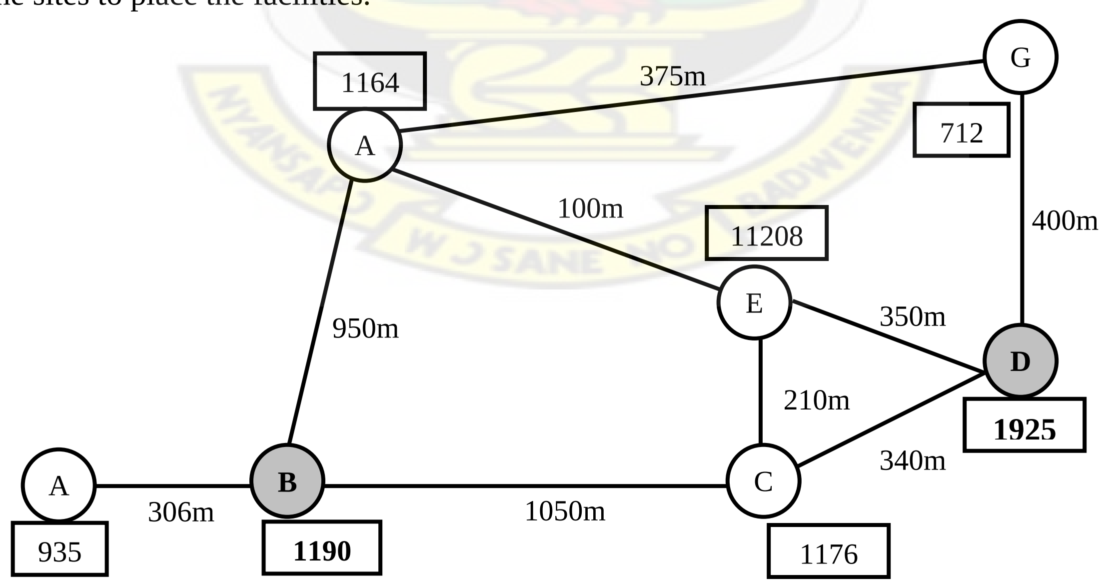

Figure 3.2: Graph of KNUST students Halls of residence with population

Due to the academic nature of the university environment, flash light and low pitch siren should be fitted on the ambulance so as not to disturb students learning.

When the Maximum Covering Location Model was applied to the network graph and the Lagrangian Relaxation was used to solve it a different solution was obtained (see appendix B). That is the facility should be located at Republic and Queens Hall. However this will not give the optimal solution since only 5 of the 7 halls will be covered. Thus the Guss Hostel and University Hall will not be covered. This is explained by the fact that the coverage distance was fixed throughout the iteration could not provide coverage for such halls from the located facilities. Thus the maximum covering model is not useful for problems that require full coverage of all demand nodes.

4.2 RECOMMENDATION

The following are being recommended

1. The University administration could purchase ambulance to be located at the University Hall and Unity Hall.
2. The problem can also be extended to include Bomso and other localities where non-residential students are mostly found.

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APPENDICES

APPENDIX A: Details of solution to the ambulance problem

I. ALGORITHM FOR THE WEIGHTED P-VERTEX PROBLEM GENERAL GRAPH

Step 1

Set D_C^H to a suitably large number. That is D_C^H = (n-1) [$\max\limits_{i,j}$ (d_{ij})] [max (h_i)].

Also set D_C^L = 0

Step 2

Set D_C = [(D_C^L + D_C^H)/ 2], D_C is approximated downward to the nearest integer.

Step 3

Solve a set covering problem with a coverage distance D_C where $P*$ (D_C) is an intermediate solution

Step 4

If P^* (D_C) $\leq P$, reset D_C^H to D_C, otherwise reset D_C^L to D_C + 1

Step 5

If D_C^L $\neq$ D_C^H, go to step 2; otherwise stop, D_C^L is the optimal value of the objective function and the locations corresponding to the set covering solution for this coverage distance are the optimal locations for the P-centre problem.

That is, we stop when $D_C^L = D_C^H$.When the iterations are completed with $D_C^L = D_C^H$, we consider the D_C values and check for the D_C value that equals the stopping D_C^L value. The solution is found at the iteration that gives this D_C value.

II. Iterations using the Vertex P-centre algorithm.

Iteration 1: Formulation of the model

We set D_C^L = 0

D_C^H = (n-1) [max$_{ij}$ (d_{ij})] [max$_i$ (h_i)]

= (7-1) (1925) (1050) = 12,127,500

D_C = (D_C^L + D_C^H)/2 = (0+12127500)/2 = 6063750

The coverage matrix is set out as below

$$a_{ij} = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 & 1 \end{bmatrix}$$

Where i = A, B, ….., G

j = A, B, ….., G

From Table 3.2 all $h_i d_{ij} \leq D_C$ and the set covering problem is expanded as follows.

Minimize $\quad X_A + X_B + X_C + X_D + X_E + X_F + X_G$

Subject to;

$$
\begin{array}{llllllll}
X_A + X_B & & & & & & & \geq 1 \\
X_A + X_B + X_C + & & & X_F & & & & \geq 1 \\
X_B + X_C + X_D + X_E & & & & & & & \geq 1 \\
X_C + X_D + X_E + & & X_G & & & & & \geq 1 \\
X_C + X_D + X_E + X_F & & & & & & & \geq 1 \\
X_B + & X_E + X_F + X_G & & & & & & \geq 1 \\
X_D + & X_F + X_G & & & & & & \geq 1
\end{array}
$$

Integrality constraint: $X_A, X_B, X_C, X_D, X_E, X_F, X_G = \{0, 1\}$

Using LINDO software gives the solution for the first iteration as follows.

$X_A = X_G = 1$, $X_B = X_C = X_D = X_E = X_F = 0$

Objective function = 2, therefore P* (60637500) = 2

Iteration 2

Since P* (D_C) = P in previous iteration

We reset D_C^H to Dc

$D_C^L = 0$, $D_C^H = 6063750$, $D_C = 3031875$

From table 3.2, all $h_i d_{ij} \leq D_C$, hence all demand node will be covered as in the first iteration. Also the a_{ij} matrix will not change, therefore the set covering problem formulation will be the same as in iteration 1, therefore, for the second iteration

$X_A = X_G = 1$, $X_B = X_C = X_D = X_E = X_F = 0$

Objective function = 2, hence P* (3031875) = 2

Iteration 3

Since P* (D_C) = P in previous iteration

We, reset D_C^H to D_C

D_C^L = 0, D_C^H = 3031875, D_C = 15159387

This will lead to the same formulation as in iteration 1 and iteration 2 since $h_i d_{ij} \leq D_C$

Therefore $X_A = X_G = 1$, $X_B = X_C = X_D = X_E = X_F = 0$

Objective function = 2, hence P* (1515937) = 2

Iteration 4

Since P* (D_C) = P in previous iteration, we reset D_C^L = 0,

D_C^H = 1515937, D_C = 757,968

The a_{ij} matrix will change due to change in D_C

$$a_{ij} = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 1 \end{bmatrix}$$

The problem is formulated as follows

Minimize $X_A + X_B + X_C + X_D + X_E + X_F + X_G$

S.T

$$
\begin{array}{lr}
X_A + X_B & \geq 1 \\
X_A + X_B + & \geq 1 \\
X_C + X_D + X_E & \geq 1 \\
X_C + X_D + X_E & \geq 1 \\
X_C + X_D + X_E + X_F & \geq 1 \\
X_E + X_F & \geq 1 \\
X_D + \quad X_F + X_G & \geq 1
\end{array}
$$

Integrality constraints: $X_A, X_B, X_C, X_D, X_E, X_F, X_G = \{0,1\}$

Using LINDO software, we have,

$X_B = X_D = X_F = 1$, $X_A = X_C = X_E = X_G = 0$

Objective function = 3, hence P* (757,968) = 3

P* (D_C) > P (= 2)

For the next iteration, we set $D_C^L = D_C + 1$ became P*(D_C) > P and D_C^H is set to it previous value. $D_C = \frac{D_C^L + D_C^H}{2} = \frac{757969 + 1515937}{2} = 1{,}136{,}953$. We check this value against $h_i d_{ij}$ to see whether the a_{ij}'s change to warrant new formulation. In this case, the formulation will change.

Iteration 5

Since P* (D_C) = P

We set D_C^L = D_C + 1= 757,968 + 1 = 757969

D_C^H = 1515937, D_C = (757,969 + 1515, 937)/ 2= 1,136, 953

$$a_{ij} = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 & 1 \end{bmatrix}$$

The problem with D_C = 1,136,953 is thus formulated as follows;

Minimize $X_A + X_B + X_C + X_D + X_E + X_F + X_G$

Subject to;

$$\begin{aligned}
X_A + X_B & \geq 1 \\
X_A + X_B + X_C + X_F & \geq 1 \\
X_B + X_C + X_D + X_E & \geq 1 \\
X_C + X_D + X_E + X_G & \geq 1 \\
X_C + X_D + X_E + X_F & \geq 1 \\
X_B + X_E + X_F + X_G & \geq 1 \\
X_D + X_F + X_G & \geq 1
\end{aligned}$$

Integrality constraints, $X_A, X_B, X_C, X_D, X_E, X_F, X_G = \{0, 1\}$

Using LINDO software the solution is

$X_A = X_D = X_G = 1, X_B = X_C = X_E = X_F = 0$

Objective function = 3

Iteration 6

Since P* (1,136,956) = 2

We set $D_C^L = D_C + 1 = 1{,}136{,}954$

$D_C^H = 1{,}326{,}445$, $D_C = (1{,}136{,}954 + 1{,}326{,}445)/\ 2 = 1{,}231{,}699$

This will have the a_{ij} matrix as follows

$$a_{ij} = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 & 1 \end{bmatrix}$$

With $D_C = 1{,}231{,}699$ we formulate the problem as follows

Minimize $\quad X_A + X_B + X_C + X_D + X_E + X_F + X_G$

S.T

$$\begin{aligned}
X_A + X_B \quad & \geq 1 \\
X_A + X_B + X_C + X_F \quad & \geq 1 \\
X_B + X_C + X_D + X_E \quad & \geq 1 \\
X_C + X_D + X_E + X_G \quad & \geq 1 \\
X_C + X_D + X_E + X_F \quad & \geq 1 \\
X_B + X_E + X_F + X_G \quad & \geq 1 \\
X_D + X_F + X_G \quad & \geq 1
\end{aligned}$$

Integrality constraints, $X_A, X_B, X_C, X_D, X_E, X_F, X_G = \{0, 1\}$.

Using LINDO software we have

$X_B = X_D = 1$, $X_A = X_C = X_E = X_F = X_G = 0$

Objective function = 2, P* (1,231,699) = 2

Iteration 8

Since P* (1,231,699) = 2 we set $D_C^H = D_C$ = 1,231,699

D_C^L = 1,136,954, D_C = (1,136,954 + 1,231,699)/2= 1,184,326

The a_{ij} matrix is set as follows

$$a_{ij} = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 & 1 \end{bmatrix}$$

Minimize $X_A + X_B + X_C + X_D + X_E + X_F + X_G$

Subject to;

$$\begin{aligned} X_A + X_B &\geq 1 \\ X_A + X_B + X_C + X_F &\geq 1 \\ X_B + X_C + X_D + X_E &\geq 1 \\ X_C + X_D + X_E + X_G &\geq 1 \\ X_C + X_D + X_E + X_F &\geq 1 \\ X_B + X_E + X_F + X_G &\geq 1 \\ X_D + X_F + X_G &\geq 1 \end{aligned}$$

Integrality constraints, $X_A, X_B, X_C, X_D, X_E, X_F, X_G = \{0, 1\}$.

Using LINDO software we have

$X_A = X_D = X_G = 1, X_B = X_C = X_E = X_F = 0$

Objective function = 3, P* (1,184,326) = 3

Iteration 9

Since P* (1,184,326) = 3, we reset $D_C^L = D_C + 1$= 1,184,326 + 1 = 1,184,327

D_C^H = 1,231,699, D_C = (1,184,327 + 1,231,699)/ 2= 1,208,013

This will have the same a_{ij} matrix in iteration six and therefore will have the same formulation in iteration six. The solution is as follows:

$X_B = X_D = 1$, $X_A = X_C = X_E = X_F = X_G = 0$

Objective function = 2, P* (1,208,013) = 2.

Iteration 10 – 24

At this point, the formulation remains the same and has the same solution as in iteration 9.That is, $X_B = X_D = 1$, $X_A = X_C = X_E = X_F = X_G = 0$

Objective function = 2

Iteration 25

It has the same formulation but

D_C^L = 1184327 , $D^H{}_C$ D_C^H = 1184327

Since $D_C^L = D_C^H$ we stop the iteration. We have reach an optimal solution since D_C^L = D_C^H. The facility is thus located at X_B and X_D. That is at University Hall and Unity Hall. The optimal value is 1184327.

Table 3.3 Table of Results

Iteration	D_C^L	D_C^H	D_C	**P*(** D_C **)**	**Location**
1	0	12127500	6063750	2	X_A, X_G
2	0	6063750	3031875	2	X_A, X_G
3	0	3031875	1515937	2	X_A, X_G
4	0	1515937	757,968	3	X_B, X_D, X_F
5	757969	1515937	1,136,953	3	X_A, X_D, X_G
6	1,136954	1515937	1,326,445	2	X_B, X_D
7	1,136,954	1,326,445	1,231,699	2	X_B, X_D
8	1,184327	1,231,699	1,184,326	3	X_A, X_D, X_G
9	1,184327	1,231,699	1,208,013	2	X_B, X_D
10	1,184,327	1,208,013	1,196,170	2	X_B, X_D
11	1,184,327	1,196,170	1,190,248	2	X_B, X_D
12	1,184,327	1,190,248	1,187,287	2	X_B, X_D
13	1,184,327	1,187,287	1,185,807	2	X_B, X_D
14	1,184,327	1,185,807	1,185,807	2	X_B, X_D
15	1,184,327	1,185,067	1184697	2	X_B, X_D
16	1,184,327	1,184,697	1184512	2	X_B, X_D
17	1,184,327	1,184,512	1184419	2	X_B, X_D
18	1,184,327	1,184,419	1184373	2	X_B, X_D
19	1,184,327	1,184,373	1184350	2	X_B, X_D
20	1,184,327	1,184,350	1184338	2	X_B, X_D
21	1,184,327	1,184,338	1184332	2	X_B, X_D
22	1,184,327	1,184,332	1184329	2	X_B, X_D
23	1,184,327	1,184,329	1184328	2	X_B, X_D
24	1,184,327	1,184,328	1184327	2	X_B, X_D
25	1,184,327	1,184,327		2	X_B, X_D

APPENDIX B: Solution using Lagrangian Relaxation

Iteration 1

$\bar{h}$ = 1188 $\lambda_i = \bar{h} + 0.5\left(h_i - \bar{h}\right)$

λ_1^1 = 1188 + 0.5 (935 – 1188) = 1061.5

λ_2^1 = 1188 + 0.5 (1190 – 1188) = 1189

λ_3^1 = 1188 + 0.5 (1176 – 1188) = 1182

λ_4^1 = 1188 + 0.5 (1925 – 1188) = 1556.5

λ_5^1 = 1188 + 0.5 (1208 – 1188) = 1198

λ_6^1 = 1188 + 0.5 (1164 – 1188) = 1176

λ_7^1 = 1188 + 0.5 (712 – 1188) = 950

D_C = 457

Iteration 1 table: Summary of iteration

Node	h_i	λ_i	Z_i	$(h_i - \lambda_i)Z_i$	$\sum_i a_{ij}\lambda_i$	x_j	$\left(\sum_i a_{ij}\lambda_i\right)x_j$	$\left(\sum_j a_{ij}x_j\right)$	$\sum_j a_{ij}x_j - Z_i$
A	935	1061.5	0	0	2250.5	0	0	0	0
B	1190	1189	1	1	2250.5	0	0	0	-1
C	1176	1182	0	0	3935.5	0	0	2	2
D	1925	1556.5	1	368.5	4885.5	1	4885.5	1	0
E	1208	1198	1	10	5111.5	1	5111.5	1	0
F	1164	1176	0	0	3324	0	0	1	1
G	712	950	0	0	3682.5	0	0	1	1
				$\sum(h_i - \lambda_i)Z_i$ **= 379.5**			$\sum\left(\sum_i a_{ij}\lambda_i\right)x_j$ **= 9997**		

UB^1 = 379.5 + 9997 = 10376.5

LB^1 = 1176 + 1925 + 1208 + 1164 + 712 = 6185

α^1 = 2

$t^1 = \dfrac{2\,(10376.5\;\;6185)}{7} = 1197.5714$

Iteration 2

λ_1^2 = max [0, 1061.5 – 1197.5714 (0)] = 1061.5

λ_2^2 = max [0, 1189 – 1197.5714 (-1)] = 2386.5714

λ_3^2 = max [0, 1182 – 1197.5714 (2)] = 0

λ_4^2 = max [0, 1556.5 – 1197.5714 (0)] = 1556.5

λ_5^2 = max [0, 1198 – 1197.5714 (0)] = 1198

λ_6^2 = max [0, 1176 – 1197.5714 (1)]

λ_7^2 = max [0, 950 – 1197.5714 (1)] = 0

Table 2: Summary of iteration

Node	h_i	λ_i	Z_i	$(h_i - \lambda_i)Z_i$	$\sum_i a_{ij}\lambda_i$	x_j	$(\sum_i a_{ij}\lambda_i)x_j$	$(\sum_j a_{ij}x_j)$	$\sum_j a_{ij}x_j - Z_i$
A	935	1061.5	0	0	3448.07	1	3448.07	1	1
B	1190	2386.57	0	0	3448.07	1	3448.07	1	1
C	1176	0	1	1176	2754.5	0	0	0	-1
D	1925	1556.5	1	368.5	2754.5	0	0	0	-1
E	1208	1198	1	10	2754.5	0	0	0	-1
F	1164	0	1	1164	1198	0	0	0	-1
G	712	0	1	712	1556.5	0	0	0	-1
				$\sum(h_i - \lambda_i)Z_i$ **= 3430.5**			$(\sum_i a_{oj}\lambda_i)x_j$ **= 6896.14**		

UB^2 = 3420.5 + 6896.14 = 10326.64

LB^2 = 935 + 1190 = 2125

α^2 = 2 (because UB has decreased)

$t^2 = \dfrac{2(10326.64 - 2125)}{7} = 2343.3257$

Iteration 3

λ_1^3 = max [0, 1061.5 – 2343.3257 (1)] = 0

λ_2^3 = max [0, 2386.5714 – 2343.3257 (1)] = 43.2457

λ_3^3 = max [0, 0 – 2343.3257 (-1)] = 2343.3257

λ_4^3 = max [0, 1556.5 – 2343.3257 (-1)] = 3899.8257

λ_5^3 = max [0, 1198 – 2343.3257 (-1)] = 3541.3257

λ_6^3 = max [0, 0 – 2343.3257 (-1) = 2343.3257

λ_7^3 = max [0, 0 – 2343.3257 (-1)] = 2343.3257

Table 3: Summary of iteration

Node	h_i	λ_i	Z_i	$(h_i - \lambda_i)Z_i$	$\sum_i a_{ij}\lambda_i$	x_j	$(\sum_i a_{ij}\lambda_i)x_j$	$(\sum_j a_{ij}x_j)$	$\sum_j a_{ij}x_j - Z_i$
A	935	0	1	013.5	43.25	0	0	0	-1
B	1190	43.25	1	1146.75	43.25	0	0	0	-1
C	1176	2343.33	0	0	9784.49	0	0	2	2
D	1925	3899.83	0	0	12127.82	1	121.27.82	1	1
E	1208	3541.33	0	0	12127.82	1	12127.82	1	1
F	1164	2343.33	0	0	8227.99	0	0	1	1
G	712	2343.33	0	0	8586.49	0	0	1	1
				$\sum(h_i - \lambda_i)Z_i$ = 2081.75			$\sum(\sum_i a_{ij}\lambda_i)x_j$ 24255.64		

UB^3 = 2081.75 + 24255.64 = 26337.39

LB^3 = 6185

α^3 = 1 (because *UB* has increased)

$t^3 = \dfrac{1(26337.39 - 6185)}{10}$ = 2015.239

Iteration 4

λ_1^4 = max [0, 0 – 2015.239 (-1)] = 2015.239

λ_2^4 = max [0, 43.2457 – 2015.239 (-1)] = 2058.489

λ_3^4 = max [0, 0 – 2343.33 – 2015.239 (2)] = 0

λ_4^4 = max [0, 3899.83 – 2015.239 (1)] = 1884.591

λ_5^4 = max [0, 3541.33 – 2015.239 (1) = 1526.091

λ_6^4 = max [0, 2343.33 – 2015.239 (1)] = 328.091

λ_7^4 = max [0, 2343.33 – 2015.239 (1)] = 328.091

Table 4: Summary of iteration

Node	h_i	λ_i	Z_i	$(h_i - \lambda_i)Z_i$	$\sum_i a_{ij}\lambda_i$	x_j	$\left(\sum_i a_{ij}\lambda_i\right)x_j$	$\left(\sum_j a_{ij}x_j\right)$	$\sum_j a_{ij}x_j - Z_i$
A	935	2015.24	0	0	4073.73	1	4073.73	1	1
B	1190	2058.49	0	0	4073.73	1	4073.73	1	1
C	1176	0	1	1176	3410.68	0	0	0	-1
D	1925	1884.59	1	40.41	3738.77	0	0	0	-1
E	1208	1526.09	0	0	3738.77	0	0	0	0
F	1164	328.09	1	835.91	2182.27	0	0	0	-1
G	712	328.09	1	383.91	2540.77	0	0	0	-1
				$\sum(h_i - \lambda_i)Z_i$ $= 2436.23$			$\sum\left(\sum_i a_{ij}\lambda_i\right)x_j$ $= 8147.46$		

UB^4 = 2436.23 + 8147.46 = 10583.69

LB^4 = 2125

α^4 = 1

$$t^4 = \frac{1(10583.69 - 2125)}{6} = 1409.782$$

Iteration 5

λ_1^5 = max [0, 2015.24 – 1409.782 (1)] = 605.458

λ_2^5 = max [0, 2058.49 – 1409.782 (1)] = 648.708

λ_3^5 = max [0, 0 – 1409.782 (-1)] = 1409.782

λ_4^5 = max [0, 1884.59 – 1409.782 (-1)] = 4784.808

λ_5^5 = max [0, 1526.091 – 1409.782 (0) = 1526.091

λ_6^5 = max [0, 328.09 – 1409.782 (-1)] = 1737.872

λ_7^5 = max [0, 328.09 – 1409.782 (-1)] = 1737.872

Table 5: Summary of iteration

Node	h_i	λ_i	Z_i	$(h_i - \lambda_i)Z_i$	$\sum_i a_{ij}\lambda_i$	x_j	$(\sum_i a_{ij}\lambda_i)x_j$	$(\sum_j a_{ij}x_j)$	$\sum_j a_{ij}x_j - Z_i$
A	935	605.46	1	329.54	1254.17	0	0	0	-1
B	1190	648.71	1	541.29	1254.17	0	0	0	-1
C	1176	1409.78	0	0	3410.68	0	0	2	2
D	1925	474.81	1	1450.19	5148.55	1	5148.55	1	0
E	1208	1526.09	0	0	5148.55	1	5148.55	1	1
F	1164	1737.87	0	0	5001.83	0	0	1	1
G	712	1737.87	0	0	39.50.55	0	0	1	1
				$\sum(h_i - \lambda_i)Z_i$ $= 2321.02$			$\sum(\sum_i a_{ij}\lambda_i)x_j$ $= 10297.1$		

UB^5 = 2321.02 + 10297.1 = 12618.12

LB^5 = 6185

$\alpha^5 = \frac{1}{2}$

$$t^5 = \frac{1}{2}\frac{(12618.12 - 6185)}{9} = 357.396$$

Iteration 6

λ_1^6 = max [0, 605.46 – 375.396 (-1)] = 962.856

λ_2^6 = max [0, 648.718 – 357.396 (-1)] = 1006.106

λ_3^6 = max [0, – 1409.78 – 357.396 (2)] = 694.988

λ_4^6 = max [0, 474.818 – 357.396 (0)] = 474.818

λ_5^6 = max [0, 1526.09 – 357.396 (1) = 1168.694

λ_6^6 = max [0, 1737.87 – 357.396 (1)] = 1380.474

λ_7^6 = max [0, 1737.87 – 357.396 (1)] = 1380.474

Table 6: Summary of iteration

Node	h_i	λ_i	Z_i	$(h_i - \lambda_i)Z_i$	$\sum_i a_{ij}\lambda_i$	x_j	$(\sum_i a_{ij}\lambda_i)x_j$	$(\sum_j a_{ij}x_j)$	$\sum_j a_{ij}x_j - Z_i$
A	935	926.86	0	0	1968.97	0	0	0	0
B	1190	1006.11	1	183.89	1968.97	0	0	0	-1
C	1176	694.99	1	481.01	2338.49	0	0	1	0
D	1925	474.81	1	1450.19	3719.14	0	0	1	0
E	1208	1168.69	1	39.31	3719.14	1	3719.14	1	0
F	1164	1380.47	0	0	3929.63	1	3929.63	1	1
G	712	1380.47	0	0	3235.75	0	0	1	1
				$\sum(h_i - \lambda_i)Z_i$ $= 2154.4$			$\sum(\sum_i a_{ij}\lambda_i)x_j$ $= 7648.77$		

$UB^6 = 2154.4 + 7648.77 = 9803.17$

$LB^6 = 6185$

$\alpha^6 = \frac{1}{2}$

$$t^6 = \frac{1}{2}\frac{(9803.17 - 6185)}{3} = 603.028$$

Iteration 7

λ_1^6 = max [0, 962.86 – 603.208 (0)] = 962.86

λ_2^6 = max [0, 1006.11 – 603.028 (-1)] = 1609.138

λ_3^6 = max [0, 694.99 – 603.396 (2)] = 694.988

λ_4^6 = max [0, 474.81 – 603.028 (0)] = 474.81

λ_5^6 = max [0, 1168.69 – 603.028 (0) = 1168.69

λ_6^6 = max [0, 1380.47 – 603.028 (1)] = 777.442

λ_7^6 = max [0, 1380.47 – 603.028 (1)] = 777.442

Table 7: Summary of iteration

Node	h_i	λ_i	Z_i	$(h_i - \lambda_i)Z_i$	$\sum_i a_{ij}\lambda_i$	x_j	$(\sum_i a_{ij}\lambda_i)x_j$	$(\sum_j a_{ij}x_j)$	$\sum_j a_{ij}x_j - Z_i$
A	935	926.86	0	0	2572	0	0	0	0
B	1190	1609.14	0	0	2572	0	0	0	0
C	1176	694.99	1	481.01	2338.49	0	0	2	1
D	1925	474.81	1	1450.19	3115.93	1	3115.93	1	0
E	1208	1168.69	1	39.31	3115.93	1	3115.93	1	0
F	1164	777.44	1	386.56	2723.57	0	0	1	0
G	712	777.44	0	0	2029.69	0	0	1	1
				$\sum(h_i - \lambda_i)Z_i$ = 2357.07			$\sum(\sum_i a_{ij}\lambda_i)x$ = 6231.86		

UB^7 = 2357.07 + 6231.86 = 8688.93

LB^7 = 6185

$\alpha^7 = \frac{1}{2}$

$$t^7 = \frac{1}{2}\frac{(8588.93 - 6185)}{2} = 600.9825$$

Iteration 8

λ_1^8 = max [0, 962.86 – 600.9825 (0)] = 962.86

λ_2^8 = max [0, 1609.14 – 600.9825 (0)] = 1609.14

λ_3^8 = max [0, – 694.99 – 600.9825 (1)] = 94.0075

λ_4^8 = max [0, 474.81 – 600.9825 (0)] = 474.81

λ_5^8 = max [0, 1168.69 – 600.9825 (0) = 1168.69

λ_6^8 = max [0, 777.44 – 600.9825 (0)] = 777.44

λ_7^8 = max [0, 777.44 – 600.9825 (1)] = 176.4575

Table 8: Summary of iteration

Node	h_i	λ_i	Z_i	$(h_i - \lambda_i)Z_i$	$\sum_i a_{ij}\lambda_i$	x_j	$(\sum_i a_{ij}\lambda_i)x_j$	$(\sum_j a_{ij}x_j)$	$\sum_j a_{ij}x_j - Z_i$
A	935	926.86	0	0	2572	1	2572	1	1
B	1190	1609.14	0	0	2572	1	2572	1	1
C	1176	94.00	1	1082	1737.5	0	0	0	-1
D	1925	474.81	1	1450.19	1913.96	0	0	0	-1
E	1208	1168.69	1	39.31	2514.94	0	0	0	-1
F	1164	777.44	1	386.56	2122.59	0	0	0	-1
G	712	176.46	1	535.54	1428.71	0	0	0	-1
				$\sum(h_i - \lambda_i)Z_i$ $= 3493.6$			$\sum(\sum_i a_{ij}\lambda_i)x_j$ $= 5144$		

$UB^8 = 3493.6 + 5144 = 8637.6$

$LB^8 = 2125$

$\alpha^8 = 1/4$

$$t^8 = \frac{1}{4}\frac{(8637.6 - 6185)}{7} = 232.5929$$

Iteration 9

λ_1^9 = max [0, 962.86 – 232.5929 (1)] = 730.2671

λ_2^9 = max [0, 1609.14 – 232.5929 (1)] = 1376.5471

λ_3^9 = max [0, – 94.00 – 232.5929 (-1)] = 326.5929

λ_4^9 = max [0, 474.81 – 232.5929 (-1)] = 707.4029

λ_5^9 = max [0, 1168.69 – 232.5929 (-1) = 1401.2829

λ_6^9 = max [0, 777.44 – 232.5929 (-1)] = 1010.0329

λ_7^9 = max [0, 176.46 – 232.5929 (-1)] = 409.0529

Table 9: Summary of iteration

Node	h_i	λ_i	Z_i	$(h_i - \lambda_i)Z_i$	$\sum_i a_{ij}\lambda_i$	x_j	$(\sum_i a_{ij}\lambda_i)x_j$	$(\sum_j a_{ij}x_j)$	$\sum_j a_{ij}x_j - Z_i$
A	935	730.27	1	204.73	2106.82	0	0	0	-1
B	1190	1376.35	0	0	2106.82	0	0	0	0
C	1176	326.59	1	849.41	2435.27	0	0	2	1
D	1925	707.40	1	1217.60	2844.32	1	2844.32	1	0
E	1208	1401.28	0	0	3445.3	1	3445.3	1	1
F	1164	1010.03	1	153.97	2820.36	0	0	1	0
G	712	409.05	1	302.95	2126.48	0	0	1	0
				$\sum(h_i - \lambda_i)Z_i$ $= 2728.66$			$\sum(\sum_i a_{ij}\lambda_i)x_j$ $= 6289.62$		

UB^9 = 2728.66 + 6289.62 = 9018.28

LB^9 = 6185

$\alpha^9 = 1/8$

$$t^9 = \frac{1}{8}\frac{(9018.28 - 6185)}{3} = 118.053$$

Iteration 10

λ_1^{10} = max [0, 730.27 – 118.053 (-1)] = 848.323

λ_2^{10} = max [0, 1376.55 – 118.053 (0)] = 1376.55

λ_3^{10} = max [0, – 326.59 – 118.053 (1)] = 208.537

λ_4^{10} = max [0, 707.40 – 118.053 (0)] = 707.40

λ_5^{10} = max [0, 1401.28 – 118.053 (1) = 1283.227

λ_6^{10} = max [0, 1010.03 – 118.053 (0)] = 1010.03

λ_7^{10} = max [0, 407.05 – 118.053 (0)] = 409.05

Table 10: Summary of iteration

Node	h_i	λ_i	Z_i	$(h_i - \lambda_i)Z_i$	$\sum_i a_{ij}\lambda_i$	x_j	$(\sum_i a_{ij}\lambda_i)x_j$	$(\sum_j a_{ij}x_j)$	$\sum_j a_{ij}x_j - Z_i$
A	935	848.32	1	86.68	2224.87	0	0	0	-1
B	1190	1376.55	0	0	2224.87	0	0	0	0
C	1176	208.54	1	967.46	2199.17	0	0	1	0
D	1925	707.40	1	1217.60	2608.22	0	0	1	0
E	1208	1283.23	0	0	3209.2	1	3209.2	1	1
F	1164	1010.03	1	153.97	2702.31	1	2702.31	1	0
G	712	409.05	1	302.95	2126.48	0	0	1	0
				$\sum(h_i - \lambda_i)Z_i$ $= 2728.66$			$\sum(\sum_i a_{ij}\lambda_i)x_j$ $= 5911.51$		

UB^{10} = 2728.66 + 5911.51 = 8640.17

LB^{10} = 6185

$\alpha^{10} = \frac{1}{8}$

$$t^{10} = \frac{1}{8}\frac{(8640.17 - 6185)}{2} = 153.4481$$

Table 11: Summary of iteration

Node	h_i	λ_i	Z_i	$(h_i - \lambda_i)Z_i$	$\sum_i a_{ij}\lambda_i$	x_j	$(\sum_i a_{ij}\lambda_i)x_j$	$(\sum_j a_{ij}x_j)$	$\sum_j a_{ij}x_j - Z_i$
A	935	1001.77	0	0	2378.32	0	0	0	0
B	1190	1376.55	0	0	2378.32	0	0	0	0
C	1176	208.54	1	967.46	2045.72	0	0	1	0
D	1925	707.40	1	1217.60	2454.77	0	0	1	0
E	1208	1129.78	1	78.22	3055.75	1	3055.75	1	0
F	1164	1010.03	1	153.97	2847.21	1	2847.21	1	0
G	712	409.05	1	302.95	2126.48	0	0	1	0
				$\sum(h_i - \lambda_i)Z_i$ = 2720.2			$\sum(\sum_i a_{ij}\lambda_i)x_j$ = 5902.96		

UB^{11} = 2728.66 + 6289.62 = 9018.28

LB^{11} = 6185

$\alpha^{11} = \frac{1}{8}$

$$t^{11} = \frac{1}{8}\frac{(8623.16 - 6185)}{0} = 0$$

We stop, since α^n is very small and the changes in λ_i would not help in the solution because changes in λ_i are very small.

Therefore, the facility will be located at E and F.