6354

# Optimal Loan Allocation Mix Policy Of A Bank Using Markov Chain Model Analysis

BY

AHIAVE KWAME EMMANUEL B.Ed



A Thesis Submitted to the Department of Mathematics

Kwame Nkrumah University of Science and Technology

in Partial Fulfillment of the Requirement for the degree of

MASTER OF PHILOSOPHY

Department of Mathematics Faculty of Physical Science

College of Science

JUNE, 2013

## DECLARATION

This work was the result of my field research, except for references to other works carried out by other researchers which have been duly acknowledged. It has to be noted however that this work has not been submitted for the award of any other degree elsewhere apart from Kwame Nkrumah University if Science and Technology (KNUST). I am therefore responsible for the views expressed and the further accuracy of its consents.



EMMANUEL KWAME AHIAVE SIGNATURE .................. DATE 13/05/2013

(PG 3007409)

MR. F.K. DARKWAH SIGNATURE .......................... DATE 31/5/2013

(SUPERVISOR)

DR. F.K. DARKWAH SIGNATURE .......................... DATE 31/5/2013.

HEAD OF DEPARTMENT

## ABSTRACT

A Markov chain is a natural probability model for accounts receivable. If the transition matrix of the Markov chain were known, forecasts could be done for future loans for each state. The secondary data for this study was collected from a Ghana-based financial institution that operates through 149 branches nationwide . The data was collected over a span of two years from January 2010 to December 2011.

The objectives of the study are as follow:

(1) To obtain optimal loan allocation mix policy, this could be used as guiding principle on future allocation purpose.

(2) To forecast loan disbursement proportions.

This paper applies a Markov chain model to subprime loans that appear neither homogeneous nor stationary to obtain optimal loan allocation mix policy, and forecast loan disbursement proportions. Realizing its importance Markov Chain Market Share model was applied to inter temporal data of loan disbursements of the selected bank.

By applying Estimate Transition Matrix, scope for probability of loan switching among its types was calculated to suggest the probable mix of loan portfolio. From the results it was suggested that the loan proportions among various types were as follows: Housing (19.40 %), Others (29.30 %), Business (29.30 %) and Education (21.80 %). These proportions can be taken as guideline percentage within the government norms for the priority sector.

# TABLE OF CONTENTS

DECLARATION....ii

ABSTRACT....iii

TABLE OF CONTENT....iv

LIST OF TABLE AND FIGURES....vii

LIST OF ABRIVIATION....viii

DEDICATION....ix

ACKNOWLEDGEMENT....x

CHAPTER ONE

INTRODUCTION....1

1.0 BACKGROUND OF STUDY....1-7

1.1 SPECIFYING A MAKOV CHAIN....7-8

1.2 STATEMENT OF THE PROBLEM....8

1.3 OBJECTIVE OF THE STUDY....9

1.4 SIGNIFICANCE OF THE STUDY....9

CHAPTER TWO

LITERATURE REVIEW....10

2.0 INTRODUCTION....10-14

2.1 MOKOV CHAIN MODEL....14-15

2.2 SPATIAL-TEMPORAL MODEL....,15-16

2.3MULTI SITE MODEL ....,16

2.4 SINGLE SITE MODE.... ....,17

2.5 SPECTRAL METHOD....,18-19

2.6 ARTIFICIAL NEURAL NETWORK....19-23

2.7 CONCEPTUAL RAINFALL-RUNOFF MODEL....23-25

2.8 EXPLANATORY MODELS FOR FORECASTING................................25-26

2.9 MARKOV CHAIN AND LOAN................................................................28

**CHAPTER THREE**

METHODOLOGY..................................................................................38

3.1 INTRODUCTION...........................................................................38

3.2 MARKOV PROBABILITY MODEL.......................................................39

3.3 ESTIMATION OF PROBABILITY TRANSITION MATRIX..........................40-41

3.4 FORMAL DEFINITION....................................................................41

**3.5** VARIATION ..........................................................................41- 42

**3.6** MARKOV CHAINS......................................................................43-44

3.7 CALCULATING THE EXPECTED NUMBER OF STEPS TO ABSORPTION......44-45

3.8 PERIODICITY...............................................................................46

3.8.1 RECURRENCE............................................................................46

3.8.2 ERGODICITY...............................................................................47

3.8.3 STEADY-STATE ANALYSIS AND LIMITING DISTRIBUTIONS.......................48

3.8.4 STEADY-STATE ANALYSIS AND THE TIME-INHOMOGENEOUS MARKOV CHAIN.............................................................................................49

3.8.5 FINITE STATE SPACE....................................................................50

3.8.6 TIME-HOMOGENEOUS MARKOV CHAIN WITH A FINITE STATE SPACE.....51

3.8.7 REVERSIBLE MARKOV CHAIN........................................................52



**CHAPTER FOUR**

~~**4.1 DATA**~~ COLLECTION..........................................................................55

4.2 DATA PROCESSING..............................................................................56

4.3 ESTIMATING TRANSITION PROBABILITY MATRIX..............................56-58

4.4 TRANSITION PROBABILITY MATRIX..............................................58-59

4.5 STATIONARITY/HOMOGENEITY OF THE PROCESS ..............................59-63

4.6 STEADY STATE DISTRIBUTION AND THE FIRST PASSAGE TIME................64-66

4.7 FORECAST ON LOAN DISBURSEMENT PROPORTION..................................66-67

**4.8 DISCUSSION** ........................................................................67-68

**CHAPTER FIVE**

**CONCLUSION AND RECOMMENDATION**......................................................69

5.1 CONCLUSION.........................................................................................69

5.2 RECOMMENDATION...............................................................................69

REFERENCES.............................................................................................70-73

## LIST OF TABLES

Table 4.0 represents the type of loan and its code

Table 4.1: Actual Loan Disbursement and Proportion for Four Loan Types

Table 4.2: Transition Probability Values

Table 4.3: Actual and Backcast Proportion of Loan Disbursement

Table 4.4: Forecast of Monthly Loan Proportion for the Year 2012

## LIST OF FIGURES



Figure 4.1: Pictorial Representation of the Transition Probability Matrix

Figure 4.2: Housing Loan: Actual and Backcast Proportion

Figure 4.3: Business Loan – Actual and Backcast Portfolio

Figure 4.4: Education Loan – Actual and Backcast Proportion

Figure 4.5: Other Loan – Actual and Backcast Proportion

# LIST OF ABRIVIATION

| | |
|---|---|
| FAIR ISAAC CORPORATION | FICO |
| BANK OF AMERICA | BOfA |
| ACCRA STOCK EXCHANGE | ASE |
| GLOBAL CIRCULATION MODELS | GCMS |
| ELLIPTICAL CELL POISSON PROCESS MODEL | EPPM |
| GAUSSIAN DISPLACEMENTS SPATIAL-TEMPORAL MODEL | GDSTM |
| RANDOM ELLIPSE SPATIAL-TEMPORAL MODEL | RESTM |
| AUTO-REGRESSIVE MOVING AVERAGE | ARMA |
| JAPAN METEOROLOGICAL AGENCY | JMA |
| GENETIC PROGRAMMING | GP |
| EL NI NO SOUTHERN OSCILLATION | ENSO |
| SOUTHERN OSCILLATION INDEX | SOI |
| SEA SURFACE TEMPERATURE | SST |
| CONCEPTUAL RAINFALL-RUNOFF MODELS | CRRMS |
| PATTERN SEARCH | SP |
| MARKOV CHAIN MONTE CARLO | MCMC |
| BANKING AND FINANCIAL INSTITUTIONS ACT | BAFIA |
| PROFIT BEFORE TAX | PBT |
| TRANSITION PROBABILITY MATRIX. | TPM |

# DEDICATION

This work is dedicated to the glory of the Lord.

## ACKNOWLEDGEMENT

The greatest thanks go to the Almighty God for his love, grace and mercies bestowed upon me throughout my entire life and education.

I would like to express heaviest gratitude and indebtedness to my Principal supervisor Mr. F.K. Darkwah for his scholastic guidance, constant encouragement in estimable to help, valuable suggestion and great support throughout my study at KNUST. Without his continual encouragement this would have been a lonely journey. He has given me a great freedom to pursue an independence work more importantly he demonstrated this faith in my ability and encouraged me to rise to the occasion of any challenges.

Finally, My acknowledgement will be incomplete if I do not make mention of the following persons; Mr. Ben Hoenyenugah for support and encouragement and Mr. Ayimah for Proof – reading and editing dissertation.

# CHAPTER ONE

# INTRODUCTION

## 1.0 Background of the Study

In the mid-80's loan processing was mostly a manual process. Consumers filled out paper applications and this information was entered into a proprietary machine provided by the credit bureaus. The credit bureaus did not score or grade the applicant, but simply provided a list of debts and payment history to the lender. The loan officer's job was to review this information and attempt to make an objective loan decision.



In the late 80's the Fair Isaac Corporation (FICO) devised the FICO score in an effort to predict a consumer's likelihood to have a major derogatory event in the next two years. Initially this was used sparingly but over the next 10 years the FICO score became the guideline for credit decisions.



As credit decisioning became simpler, loan origination began to be automated. Whereas most lenders were typing loan contracts in the early 80's, by the early 90's most were using computers to generate loan documents. As the 90's came to a close, loan application processing became more prevalent. Instead of simply reviewing paper applications for making a loan decision, many lenders used a computer system to actually process the application as well as create the loan documents. Today as the internet becomes a more integral part of life, consumers are beginning to prefer to apply for loans electronically. Automating the process In order to improve the customer experience and speed up the process you need the right tools.

First, you need a secure method of receiving loan applications via the internet. Then you need the ability to automatically retrieve the credit report and render a timely decision to the customer. Once the decision has been rendered you need to originate the loan and produce the loan documentation. Upon acceptance of the loan terms by the consumer the loan information must be transferred to a loan servicing system.

The Kwik-Loan Solution Kwik-Loan solves this problem with a comprehensive software solution that handles all aspects of the loan process from application to servicing. The Kwik-App and Kwik Dealer modules handle the entry of consumer and dealer loan applications. Once these applications are submitted, Kwik-Decision takes over and renders an automatic decision (approved, denied or pending) to the applicant. Manual decisions on pending

applications can be rendered via the Kwik-Manage module. Once a loan has been approved the documents can be produced and transferred electronically for customer acceptance. Then they may be transferred to a 3rd party LOS system via Kwik-Connect or automatically completed within the EnCompass SAAS solution.

About Compass Technologies Founded in 1999, and headquartered in Buford, GA, Compass Technologies is an established leader in providing a total loan software solution to the loan industry.

Jack Marinel Submitted on June 28, 2010

I presume the intent of the above opinion is to suggest that "assumable" loans would be a viable financing alternative in today's real estate market.Yet, as a California real estate licensee since 1978 I have somewhat of a different recall of the events that took place. See Wellenkamp v. Bank of America; Invalidation of Automatically enforceable Due-on-sale Clauses at http://www.jstor.org/pss/3480063

Many lenders had "due-on-sale clauses" in their trust deeds for years leading up to the landmark decision by California Supreme Court in 1978.



In 1975 Cynthia Wellenkamp hired attorney Fred Crane to represent her as a defendant in a case where Bank of America was the plaintiff. BofA attempted to enforce the due-on-sale clause found in the trust deed signed by the prior owners Birdie, Fred and Dorothy Mans. Ms. Wellkenkamp had not assumed the existing loan, but merely took title to an owner occupied home, "subject to" the $19,100 loan from BofA. The case went all the way to the California Supreme Court.

In 1978, the court ruled BofA could not justify calling the note due and payable. One reason was that BofA had more security for their loan than they did when they made the loan to the original borrower and Ms. Wellenkamp had kept the payments current on the loan.



This ruling opened the flood gates in California for buyers to take title "subject-to" existing loans. No formal loan assumption was required on loans from state chartered institutions. Yet, the prior owner/borrower was still held ultimately responsibility for the promissory note they had signed and responsible for any deficiencies.

The non-enforceability of the "due-on-sale clause" was overturned in 1982 when a federal law entitled the "Garn/St. Germain" bill originated in the US Senate and was passed by

Congress. It put the enforceability back into the deeds of trust in California. This had a serious dampening effect on the market.

FHA and VA loans allowed non-qualifiers to take loans "subject to" until the end of the 1980's. VA loans could be assumed by another Veteran who substituted their "VA eligibility" which had the effect of releasing the original VA borrower from liability on the loan. A simple "subject to" purchase did not release liability.

Today the "due-on-sale clause" is alive and well. Yet many buyers are taking title to properties subject to existing liens. Why aren't lenders automatically enforcing their right to call the loan due and payable? It is simple. To do so they might have to initiate a foreclosure action. If the new owner on title is current on their payments and the lender is receiving a higher yield than today's historically low rates; why would a lender pursue such a course?

Ghana's economic well-being and recovery program were closely tied to significant levels of foreign loans and assistance, especially from the World Bank and the International Monetary Fund. Altogether, between 1982 and 1990 foreign and multilateral donors disbursed a total of approximately US$3.5 billion in official development assistance; at the same time, the country's external debt reached US$3.5 billion. By 1991, the largest bilateral donors were Germany, the United States, Japan, and Canada, which together provided Ghana with US$656 million in development assistance. The largest multilateral donors in 1991 included the European Community, the IMF, and the International Development Association, which furnished almost US$435 million to Ghana.

In addition, the government obtained five IMF programs amounting to approximately US$1.6 billion: three standby loans, simultaneous Extended Fund Facility and Structural Adjustment

Facility loans, and an Enhanced Structural Adjustment Facility loan in 1988. The government signed more than twenty policy-based program loans with the World Bank. The World Bank also sponsored six consultative group meetings; the first, held in November 1983, resulted in pledges of US$190 million. Between 1984 and 1991, almost US$3.5 billion more was raised at five additional meetings.

In 1991 Ghana successfully raised the country's first syndicated loan in almost twenty years in the amount of US$75 million. The loan's collateral was a proportion of the country's cocoa crop. Special arrangements were made to ensure that a specific amount of the crop was purchased using letters of credit. Then in March 1992, the IMF announced that following the expiration of Ghana's third and final arrangement under the Enhanced Structural Adjustment Facility, Ghana needed no further IMF financing. Even so, the Ghanaian government asked the IMF to monitor progress on the country's economic program and to continue policy dialogue.

In early 1994, Ghana accepted the obligations of Article VIII of the IMF's Articles of Agreement. According to the IMF, Ghana will no longer impose restrictions on payments and transfers for current international transactions or engage in discriminatory currency arrangements or multiple currency practice without IMF approval. Ghana's decision undoubtedly will enhance its image with foreign investors and bankers.

Over the past one decade the banking industry in Ghana had gone through many structural changes in terms of increase in branch network, provision of wide range of banking services and acceleration of credit activities in different ways. The financial crisis in 1997-98 has created a tremendous pressure in the banking sector which was sorted out by means of consolidation process carried out by the Ghanaian Central Bank.

The Central Bank envisaged the merger schemes to combat the crisis and termed some of the merged banks as anchor banks, to accelerate the economic growth. The survival of any banking sector normally depends on their ability to improve their efficiency and effectiveness in their product offerings. There are three main banking objectives:

(1) It has to ensure that its business should run as usual by ensuring that its debts do not exceed its liability.

(2) The bank must maintain its liquidity; i.e. the bank should be able to meet withdrawals at any given point of time.



(3) The bank has to generate profits for the stockholders (profitability). Thus, the bank should maintain an appropriate fund's portfolio for their survival and growth. The variance of the actual loan sanctions and its allocation over a period of 24 months in the retail bank selected for the study revealed two important findings. Firstly, the loan allocation policy adopted by the bank management is suspectingly based on non documented hybrid model. Secondly, the switching of loan allocation from one type to another is also possible. These two findings suggested that there should be a systematic method of loan portfolio management, in order to maximize the interest income of the bank. The current study attempts to devise a loan allocation policy to different type of loans using Markov Chain Market Share Model. Kosubud and Stokes (1980) suggested that Markov Chain application in the business situation application is rich in terms of economics and policy implications. In this study an attempt has been made to estimate the transition matrix using inter-temporal data on loan disbursements. This provides the probability of loan switching among its state. Simulation process was also carried out to calculate the expected income of interest from all loan types using the actual loan disbursement data and Markov proportions to evaluate the superiority of Markov Chain approach.

Most of the study of probability has dealt with independent trials processes. These processes are the basis of classical probability theory and much of statistics. Modern probability theory studies chance processes for which the knowledge of previous outcomes influences predictions for future experiments. In principle, when we observe a sequence of chance experiments, all of the past outcomes could influence our predictions for the next experiment. For example, this should be the case in predicting a student's grades on a sequence of exams in a course. But to allow this much generality would make it very difficult to prove general results.



In 1907, A. A. Markov began the study of an important new type of chance process. In this process, the outcome of a given experiment can affect the outcome of the next experiment; this type of process is called a Markov chain.

## 1.1 Specifying a Markov Chain

We describe a Markov chain as follows: We have a set of *states*, $S = \{s_1, s_2, \ldots, s_r\}$.

The process starts in one of these states and moves successively from one state to another. Each move is called a *step*. If the chain is currently in state $s_i$, then it moves to state $s_j$ at the next step with a probability denoted by $p_{ij}$ , and this probability does not depend upon which states the chain was in before the current state.

The probabilities $p_{ij}$ are called *transition probabilities*. The process can remain in the state it is in, and this occurs with probability $p_{ii}$. An initial probability distribution, defined on *S*, specifies the starting state. Usually this is done by specifying a particular state as the starting state.

The transition matrix is called the "migration matrix" in Gupton et al. (1997) for Credit Metrics. The estimation of the continuous time Markov chain transition probabilities is introduced in Fleming (1978) and more recently in Monteiro et al. (2006). While the issue of correlation between issuers discussed in that research may be applicable to a portfolio of mortgages, a mortgage is a simple accounts receivable discrete-time Markov chain model with no arbitrage or hedging opportunities that require more complicated model features. The statistical problem of interest is to estimate the transition matrix using a sample of observed monthly loan movements between delinquency states. The estimation is complicated by the frequent observation in studies that the Markov chain is neither homogeneous nor stationary. Betancourt (2006) concluded repayment for Freddie Mac data on prime mortgages was neither homogeneous nor stationary, and estimated transition matrices produce poor forecasts. This paper proposes two innovative estimation methods for the transition matrix based on two observations that improve forecasting precision.

R. A. Howard provides us with a picturesque description of a Markov chain as a frog jumping on a set of lily pads. The frog starts on one of the pads and then jumps from lily pad to lily pad with the appropriate transition probabilities.

### 1.2 Statement of the Problem

The survival of any banking sector normally depends on their ability to improve their efficiency and effectiveness in their product offerings Berger, A.N., and G.F.Udel, (1996). Luckett, D.G. (1984) explains that there are three main banking objectives.: (1) It has to ensure that its business should run as usual by ensuring that its debts do not exceed its liability. (2) The bank must maintain its liquidity; i.e the bank should be able to meet withdrawals at any given point of time. Finally, the bank has to generate profits for the stockholders (profitability)[3]. Thus, the bank should maintain an appropriate funds portfolio for their survival and growth. The variance of the actual loan sanctions and its allocation over a period of 24 months in the retail bank selected for the study revealed two important findings. Firstly, the loan allocation policy adopted by the bank management is suspectingly based on non documented hybrid model. Secondly, the switching of loan allocation from one type to another is also the bank loan portfolio of the selected bank is composed of three main strategic business possible. These two findings suggested that there should be a systematic method of loan portfolio management, in order to maximize the interest income of the bank. The current study attempts to devise a loan allocation policy to different type of loans using Markov Chain Market Share Model operations namely Retail Banking, Business Banking and Corporate Banking with an individual share of 39%, 28% and 33% respectively as at January 2002. The Business Banking caters for small and medium-sized companies (paid up capital up to GH¢1.0 million) and generally concentrates on business loan and trade financing related to their business. The Corporate Banking, serves to the top-tier Ghanaian conglomerates or corporate sector of listed or about to be listed in the Accra Stock Exchange (ASE), in the form of loans such as revolving credits, huge capital expenditure loan, bridging loan, multi-million project undertakings by way of either term loan, overdraft or floating rate loan, and other package of trade finance. The retail banking emphasizes on individual customer loans like housing, small business (to sole-proprietors, partnership or small size companies with paid-up capital up to GH¢ 250,000) Education and miscellaneous loans such as staff housing, trust receipt, and personal overdraft facilities. The latter miscellaneous loans are classified as "other loan". In the current study the focus was made only on retail banking unit. The reason for selecting the retail banking was on two folds.

(1) The retail loan portfolio is usually greater than the other portfolios.

(2) Retail loan products are generally popular in branch banking networks.



## 1.3 Objectives of the study

The objectives of the study are as follow:

(1) To obtain optimal loan allocation mix policy, this could be used as guiding principle on future allocation purpose.

(2) To forecast loan disbursement proportions

## 1.4 Significance of the study

Any company in the business of making loans will require a forecast of future cash flows on their loans. The demands placed on the forecast are particularly heavy for companies who securitize their loans. In essence, a group of loans is set aside in a trust. The cash own from these loans (principal and interest) is used to support bonds sold to investors. The proper valuation of these bonds will depend critically on the amount and timing of losses and prepayments over the life of the loans. Securitized loans form an unchanging pool. These pools are small enough to expect noticeable variation in loss and prepayment rates even within a stable environment.

# CHAPTER TWO

# LITERATURE REVIEW

## 2.0 Introduction

Markov chains are related to Brownian motion and the ergodic hypothesis, two topics in physics which were important in the early years of the twentieth century, but Markov appears to have pursued this out of a mathematical motivation, namely the extension of the law of large numbers to dependent events.



Markov chain models have been used in advanced baseball analysis since 1960, although their use is still rare. Each half-inning of a baseball game fits the Markov chain state when the number of runners and outs are considered. During any at-bat, there are 24 possible combinations of number of outs and position of the runners. Markov chain models can be used to evaluate runs created for both individual players as well as a team. Markov chain models have been used to analyze statistics for game situations such as bunting and base stealing and differences when playing on grass vs. astroturf.

Several applications have emerged that involve generating sequences of synthetic daily data that represent time series of climatic variable aggrieved to, e.g., a monthly time scale – a procedure referred to as " stochastic disaggregation" or "temporal downscaling" – for use with hydrological or biological_simulation models. These include: (a) predicting crop production impacts of climate change scenarios; Mearns et al. (1996), Semenov and Barrow (1997), Mavromatis and Jones (1998), (b) predicting crop yield based on seasonal climate forecasts; Hansen and Indeje (2004), (c) analyzing crop yields variability using long-tem monthly meteorological records where the original daily observations have been lost or are otherwise unavailable Boer et al. (2004), and (d) interpolating between station, e.g., to create gridded daily meteorological time series data sets; Kittelet et'al. (2004).

The need to preserved key statistical properties of the historic daily time series justified the use of a stochastic model in each of these applications. Crops respond not to climatic averages, but to the dynamic nonlinear interactions between daily sequences of whether, soil water, and nutrient balance. The statistical properties of rainfall are particularly important because of its influence on processes, such as solute leaching, soil erosion and crop water stress response, which depend on soil water balance dynamics. Any biases in variability of daily weather can seriously distort crop models prediction.



Osborn and Hulme, (1997), study spatial averaging or interpolation of daily weather data among stations that tends to distort day-to-day variability, biasing simulated crop response.

Hansen and Jones (2000), also study a particular problem for predicting crop response to the soil water balance is the tendency for spatial averaging to increase the frequency of days with rain and reduce the means intensity of rainfall events. This distortion can result in ether under-prediction due to reduced dry spell duration, deWit and van Keulen (1987), Carbone (1993), Mearns et all., (1996), were also of the same view.

Mearan et al. (1995), Mavromatis and Jones (1999), opines that the same challenge arises when using the output of physically based global circulation models (GCMs) to predict crop response to either climate change scenarios or predicted seasonal climate variations. Although GCMs operate on sub-daily time steps, the spatial averaging the occurs within grid cells distorts the variability of daily whether sequences, generally resulting in too many rainfall events, with too little rain per event, with too little rain per event, suggested in the views of Goddard et al. (2001). Therefore, their predictions are typically aggregated into monthly or seasonal (ie., 3 months) anomalies.

Wilks (1992), Katz (1996), Mearns et al. (1997), suggested two general approaches that are used with stochastic weather generators to disaggregate monthsly climatic means into daily realizations. Thes most common is to adjust the input parameters of the stochastic model to match taget means or other statistics. Understanding the statistical properties of a stochastic weather generato allows one to manipulate its input parameters to reproduce a wide range of statistical properties of interest, such as means, variances and the relative influence of the number of stoms (i.e., frequency) and the type of storm (i.e., the intensity distribution ) on total rainfall,. This approach has been applied to climate change impact studies and disaggregation of seasonal climate forecasts.

Multivariate techniques have been underlined as suitable and powerful tool to find hydrologically homogenous region or to classify meteorological data such as rainfall. Principle component analysis, factor analysis and different cluster techniques have been used to classify daily rainfall patterns and their relationship to atmospheric circulation.

Romero et al. (1999), classify rainfall into spatio-temporal pattern in Iran, Singh (1999), classify flood and drought years and Stahl and Demuth (1999), classify streams flow drought. They used cluster analysis for regionalization involves grouping of various observations and variables into clusters, so that each cluster is composed of observations or variables with similar characteristics such as geographical, physical, statistical or stochastic behavior.

Mosely (1981), used hierarchical cluster analysis on rivers in New Zealand and Tasker (182), compared methods of difining homogenous regions including cluster analysis with a complete linkage algorithm. Acerman (1985), and Acerman Sinclair (1986), concluded that clustering has some intrinsic worth to explain the observed variation in data. Gottschalk (1985), applied cluster and principal component analysis to the territory of Sweden and found that cluster analysis is an appropriate method to used on a national scale with heterogeneous hydrogological regimes.

Nathan and McMahon (1990), performed hierarchical cluster analysis for the prediction low flow of rain characteristics in southeaster, Australia. They found that Ward method with a similarity measure based on the squared Euclidean distance is the best method for cluster analysis.

Cox and Isham (1994), presented an interesting classification of rainfall models in three types: empirical statistical models, dynamic models and intermediate stochastic models. The idea behind this classification is the amount of physical realism incorporated into the models structure. In the empirical case, there is no attempt to incorporate physical modeling of the atmosphere but to the empirical stochastic models to the available data. While the second type of models are pure physically based models, the third group is a combination of both method by which certan physical process of rainfall structure as for example, rain cells, rain bands and cell clusters, are described with a stochastic approach.

Andrade et al (1998), Miranda and Andrade (1999), and Miranda et al, (2004), used concepts of graph theory to analyze spatial patterns in time correlation function among rain events, using recorded data from a set of stations in Northeast Brazil. In previous contributions they investigated properties of rain events in this region with concept of statistical scale invariance within the data, which can be expressed in terms of temporal and spatial Hurst exponents. The method they used herein is similar to that proposed for the analysis of brain activity signals by Eguiluz et al ( 2005). Within this approach, non-local spatial dependence is estimated by evaluating Pearson coefficient between time series of pairs of stations.

Le Cam (1961), in his fundamental work, propane models for spatial-temporal precipitation based on stochastic point processes, this approach developed rapidly in the 1980's through a series of papers by Waymire et al. (1984), such models are based on a hierarchical structure in which rainfall field occur in a temporal Poisson process, rain bands (storms) occur within each field in a spatial Poisson process (the rate of which may reflect orography and seasonality), and rain cells occur in each storm, clustering space and time. Typically the cells, storms and field move: in the simplest models, all components have a common velocity. They assume stochastic stationarity in both time and space. Thus, in fitting the models, they treat each month separately, and use data for a relatively homogenous 6 spatial region.

Reodriguez-Iturbe et al. (1987, 1988), generalize that the spatial – temporal models that they developed were spatial analogues of models that they used successfully to represent the temporal process of rainfall at a single rain gauge, investigation in Cox and Isham (1988). The multi-site models similarly generalize the models the modes of Cox and Isham (199). All of these models have the desirable feature that they preserve the structure of the single-site models in their marginal properties.

## 2.1 Markov Chain Model

Liu et al, (2009), and Markov chain has been widely applied in the disciplines of natural science, engineering, economics and management. This approach has also been widely used in drought forecasting, Lohani and Loganathan, (1997), Lohani et al, (1998).

Paulo and Pereira (2007) stated that the Markov chain modeling approach is useful in understanding the stochastic characteristics of droughts and rainfall through the analysis of probabilities for each severity class, times for reaching the non drought class from any drought severity state, and residence times in each drought class. They found that the approach can be satisfactorily be used as a predictive tool for forecasting transitions among drought severity classes up to 3 months ahead.



Lohani and Loganathan (1997) and Lohani et all. (1998) developed an early warning system for drought management using the PDSI and the Markov chain, in two climatic areas of Virginia (U.S.A). The same approach was also adopted for developing a meteorological droght/rainfall forecasting model by Liu et al. (2009) in Laohahe catchment in northern China. In their study, spatio-temporal distributions of PDSI were analyzed and forecasted by Markov chain.



Steinemann (2003) adopted six classes of severity, from wet to dry conditions, similar to those PDSI, and used the Markov chain to characterize probabilities for drought class and duration in a class. The results obtained were used to propose triggers for early-activating of the drought preparedness plans at the basin scale.

Liu et al. (2009) demonstrated two advantages of the Markov chain technique for forecasting drought and rainfall conditions. They were (1) the predictive performance increased greatly as the severity of drought increased, and (2) the predictive performance was always satisfactory for drought state transitions, and the prediction performance as acceptable for the successive and smooth states.

## 2.2 Spatial-temporal Models

Northrop (1996), generalized this model in the case where cells are elliptical rather than circular (it is referred to as the elliptical cell Poisson process model (EPPM). EPPM is likely to be more realistic, especially in the cases where banding is apparent in the radar images. These cells are also identifiable by the elliptical contours of their spatial autocorrelation plots. This model requires two extra parameter, the eccentricity and orientation of cells, which are both assumed to be common to all cells.

Northrop (1996), have investigated a modified version of EPPM model, the temporal clustering of cells is achieved using a Bartlett-Lewis structure as above. Additionally, spatial clustering is incorporated using a Neyman-Scott-type mechanism in which the displacements of the cell origins from the storm centre follow a bivariate distribution in a space. A range of storm shapes (e.g. bands large masses) can be produced by variation of the parameters of the spatial clustering distribution. An important modification to the model of Cox and Isham (188) is to have the storm centre moving with the same velocity as the cells so that cells are born within the existing structure of the storm. Two spatial clustering distributions are considered.

1. A bivariate Gaussian (normal) distribution. They refer to the resulting model as the Gaussian displacements spatial-temporal model (GDSTM):
2. A uniform distribution over a random ellipse. This gives rise to the random ellipse spatial-temporal model (RESTM)

## 2.3 Multi-site Models

Kakou (1997), suggested that multi-site models are reasonably parsimonious in their parameterization, requiring a single extra parameter, the cell duration scalar, for each new site that is included in the study were considered. The cross-correlation function of the rainfall intensity at a pair of were derived and has the implied functional form of the probability of a cell hitting two sites. It turns out that, for individual storms, this probability decays approximately exponentially with inter-site distance for sites which are well-separated and which are not aligned along the direction of the storm's movement; for sites which are closer together, the dependence is no longer exponential.



## 2.4 Single-site Model

The models described in the preceding sections were generalizations of models that have been successfully to model the temporal evolution of rainfall at a single site. A first step towards improving the performance of these models involves studying ways in which the single-site models can be improved.

Rodriguez- Iturbe et al. (1987), is of the view that one of the most obvious ways in which the basic single-site models can be extended is by allowing the different types of storm to occur so as to randomize the cell duration parameter between storms in this approach; storms have a common structure but occur at different timescales. The main advantage of such models, in practical terms, lies in their ability to reproduce well the observed probability of no rainfall at various levels of aggregation. They have investigated an alternative to the randomization of the cell duration parameter for single-site models, instead allowing for different types of storm using an inverse relationship between the duration of an event and its intensity (the motivation being that intense convective event tend to be shorter-lived than shallower stratiform systems.

Cowpetwait (1994), adopted an explicit functional form for the dependence between cell depth and cell duration, it is possible to overcome the problems of over-parameterization typically associated with attempts to model different cell types explicitly. Their work is based on the Neyman-Scott and Bartlett-Lewspoint process models, Rodriguez-Iturbe et al. (1987), which are modified to allow rain cells with stochastically dependent duration and intensity, Kakou (1997)



## 2.5 Spectral method

The method of moments suffers from a number of disadvantages. In particular, the choice of features to incorporate into the fitting procedure is subjective, and the parameter values obtained can be quite sensitive to the features used in the fitting hence model comparison can be difficult.

Brillinger and Rosenblatt (1967), makes inefficient use of available data, as only a few summary statistics are used in the fitting. In an attempt to overcome some of these difficulties, a spectral method has been developed. This method uses the sample Fourier coefficients rather thatn the original data, and makes use of the fact that, for large samples, small collections of the Fourier coefficients have a joint distribution which is approximately multivariate normal. This enables them to write down approximate likelihood functions for the mode parameters in terms of small subsets of the sample fourier coefficients.

McCullagh and Nelder (1989), combined all approximate likelihood functions, an objective function is defined which can be interpreted as a log quasi-likelihood, Chandler (1997). This ten provides a basis for objectives model comparison procedures using standard statistical techniques such as likelihood ratio tests. The method has been developed for use in fitting single-site and spatial-temporal models. The reliance on second-order properties is a potential disadvantage in distinguishing between models whose main difference is in their wet\dry interval properties. More details may be found in Chandler (1996b, 1997).

Chandler (1997), describe spectral method so far as been used extensively in the fitting of single-site models, and some preliminary work on the fitting of spatial-temporal models has also been done. The main area of interest has been in the area of model comparison, as it is here that the apparent objectivity of the method is particularly useful. In the single-site case, numerous different models have been fitted to data from the HYREX raingauge network. Rogorous procedures for model comparison, such as likelihood ratio tests, are available which allow for the different numbers of parameters in the models. We conclude that the clustering models to the data is better than does the Poisson model; also that storms tend to be asymmetric with more intense activity towards the beginning of a storm than at the end.

## 2.6 Artificial Neural Network

French et al, (1992), used Neural networks to estimate accurate information on rainfall as ~~essential~~ for the planning and management of water resources. Nevertheless, rainfall is one of the most complex and difficult elements of the hydrology cycle to understand and to model due to the complexity of the atmospheric processes that generate rainfall and the tremendous range of variation over a wide range of scales both in space and time.

Gwangeseob and Ana, (2001). DESCRIBED Neural networks as been an accurate rainfall forecasting tool which is one of the greatest challenges in operational hydrology, despite many advances in weather forecasting in recent decades. Neural networks have widely applied to model many of nonlinear hydrologic processes such as rainfall-runoff. Hsu et al. (1950), Shamseldin (1997), stream flow, Zealand et al, (1999), stream flow, Zealand et al, (1999), Campolo and Soldat (1999), Abrahart and See, (2000), groundwater management, Rogers and Dwla, (1994), water quality simulation, Maier and Dandy (1996), Maier and Dandy (1999), and rainfall forecasting.

French et al. (1992), employed a neural network to forecast two-dimensional rainfall, in advance. Their ANN model used present rainfall data, generated by a mathematical rainfall simulation model, as an input data. This work is, however, limited in a number of aspects. For example, there is a trade-off between the interactions and the training time which could not be easily balanced.

The number of hidden layers and hidden nodes seem insufficient, in comparison with the number of input and output nodes, to reserve the higher order relationship needed for adequately abstracting the process. Still, it has been considered as the first contribution to ANN's application and established a new trend in understanding and evaluating the roles of ANN in investigating complex geophysical processes.

Toth et al (2000), compared short-time rainfall prediction models for real-time flood forecasting. Different structures of auto-regressive moving average (ARMA) models, artificial neural networks and nearest – neighbors approaches were applied for forecasting storm rainfall occurring in the Sieve River basin, Italy, in the period 1992-1996 with lead times varying form 1 to 6 h. The ANN adaptive calibration application proved to be stable for lead times longer than 3 hours, but inadequate for reproducing low rainfall events.

Koizumi (1999), employed an ANN model using radar, satellite and weather-station data together with numerical products generated by the Japan Meteorological Agency (JMA)

Asian Spectral Model and the model was trained using 1 year data. It was found that the ANN skills were better than the persistence forecast (after 3 h ), the linear regression forecasting, and the numerical model precipitation prediction. As the ANN model was trained with only 1 year data, the results were limited.

Luk et al. (2000), studied and indicated that ANN is a good approach and has a high potential to forecast rainfall. The ANN is capable to model without prescribing hydrological processes, catching the complex nonlinear relation of input and output, and solving without the use of differential equations sited in Hsu et al. (1995), French et al. (1992). In addition, ANN could learn and generalize from examples to produce a meaningful solution even when the input data contain errors or is incomplete.



Luk et al. (2000), an artificial neural network ANN which is a mathematical model used for data processing inspired by the bioelectrical networks in the brain comprised of neurons and synapses. In an ANN, simple processing elements referred to as neurons are used to create networks that are capable of learning to model complex system. For example introduction to the structure and design of Artificial Neural Networks the reader is referred to Hagan et al. (1996).



Karunanithi et al., (1994) has done a number of studies into the application of ANN in the field of rainfall-runoff modeling and flood forecasting sited in the work carried out by Lorrai and Sechi, (1995); Campolo et al., (1999).

Hsu et al. (1995) compared ANN models with traditional black box models, concluding that an ANN model is capable of giving superior performance over a linear ARMAX (autoregressive moving average with exogenous input) time series approach, which observed time series of flow rate and rainfall are used as input.

Smith et al. (2004), has an alternative to the ANN, genetic programming (GP) strategy introduced, and ANN can be considered for use in forecasting the error between the outputs of physical rainfall runoff model and the observed runoff rates. A feed forward neural network has been used for this purpose and was found to provide similar accuracy to GP. A advantage of GP is that it is easier to use than an ANN approach in that it uses a function in the forecasting stage rather than a complicated network of neurons.



Gwangseob and Ana, (2001), developed an Artificial Networks (ANN), which perform nonlinear mapping between inputs and outputs, has lately provided alternative approaches to forecast rainfall. ANN were first developed in the 1940s (Mc Culloch and Pitts, 1943) and the development has experienced a renaissance with Hopfield's effort Hopfield, (1982) in 5 iterative auto-associable neural networks.

Abraham et al. (2001) used as artificial neural network with scaled conjugate gradient algorithm (ANN-CGA) and evolving fuzzy neural network (EfuNN) for predicting the rainfall time series. In the study, monthly rainfall was used as input data for training model. The authors analyzed 87 years of rainfall data in Kerala, a state in the southern part of the Indian Peninsula. The empirical results showed the neuro-fuzzy systems were efficient in terms of having better performance time and lower error rates compared to the pure neural network approach. In some cases, the deviation of the predicted rainfall from the actual rainfall was due to a delay in the actual commencement of monsoon, El Ni no Southern Oscillation (ENSO)

Manusthiparom et al. (2003), has another study of ANN that relates to El-Ni no Southern Oscillation was done and the authors investigated the correlations between El-Nin o Southern Oscillation indices, namely, Southern Oscillation Index (SOI), and sea surface temperature (SST), with monthly rainfall in Chiang Mai, Thailand, and found that the correlations were significant. For that reason, SOI, SST and historical rainfall were used as input data for standard back-propagation algorithm ANN to forecast rainfall one year ahead. The study suggested that it might be better to adopt various related climatic variables such as wind speed, cloudiness, surface temperature and air pressure as the additional predictors.



Toth et a. (2000) compared short-time rainfall prediction models for real-time flood forecasting. Different structures of auto-regressive moving average (ARMA) models, artificial neural networks, and nearest-neighbors approaches were applied for forecasting storm rainfall occurring in the Sieve River basin, Italy, in the period 1992-1996 with lead times varying form 1 to 6h. The ANN adaptive calibration application proved to be stable for lead times longer than 3 h, but inadequate for reproducing low rainfall.



Koizumi (1999), has another application which employed an ANN model using radar, satellite, and weather-station data together with numerical products generated by the Japan Meteorological Agency (JMA) Asian Spectral Model for 1-year training data. Koizumi found that ANN skills were better than persistence forecast (after 3 h), the linear regression forecasts, and numerical model precipitation prediction. As the ANN used only 1 year data for training, the results were limited. The author believed that the performance of the neural network would be improved when more training data became available. It is still unclear to what extent predictor contributed to the forecast and to what extend recent observations might improve the forecast.

Coulibaly (2000) stated that ninety percent of ANN models applied in the field of hydrology used the back propagation algorithm. This algorithm involves minimizing the global error by using the steepest descent or gradient approach. The network weights and biases are adjusted by moving a small step in the direction of the negative gradient of the error function during each iteration. The Advantage of this algorithm lies in its simplicity.

In the study, ANN model was applied for each 75 rain gauge stations in Bangkok, to forecast rainfall from 1 to 6 h ahead as forecast point.



### 2.7 Conceptual rainfall-runoff Models

Franchini and Galeati (1997), Conceptual rainfall-runoff models (CRRMs) have become a basic tool for flood forecasting and for catchment basin management. These models permit calculation of the runoff generated by precipitation events by similating the physical process that affect the movement of water over the through the soil. The accuracy of these calculations depends both on the structure of the model and on how the relevant parameters are defined. CRRMs generally have a large number of parameters which, because of the conceptual nature, cannot be measured directly and are therefore estimated on the basis of a calibration process which involves adjusting their values so that the simulated discharges fit the corresponding observed discharges as closely as possible. Measurement of the deviation between the two series represents the objective function. Therefore, the purpose of the calibration is ultimately to find the values of the parameter so the CRRM which reduce this deviation to a minimum or, in other words those values which minimize the objective function.

Duan et al. (1992;1993; 1994), have presented a global optimization method called SCEUA applied to the Sacramento model, Brazil and Hudlow (1981), Solomatine (1995), with reference to the TANK model, Sugawura et al. (1983), has evaluated applicability of a scheme based on the combined used of clustering, random sampling and random covering techniques. A possible alternative to these approaches is offered by the use of a genetic algorithm (GA), Holland (1975), Goldberg (1989), which is being used increasingly in the field of industrial design, Davis (1991) and in hydrology-hydraulic engineering. The GA applications described in the literature focus upon two topics, in particular; the calibration of hydraulic and hydrological models an water resources management optimization problems.

Wang (1991), and Franchini (1996), also suggest a two step procedure which associates the GA with local-search optimization techniques for a subsequent "fine-tuning' process. In water resource management the application address the optimization of aquifer monitoring systems, Cienawsky et al. (1995), Wagner (1995), and their utilization, McKinney and Lin (1994) the containment and recovery of polluted aquifers.

Rogers and Dowla, (1994) promogated the management of reservoir systems sited in Ritzel et al, (1994) and Esat and Hall (1994). The problem are addressed in complex single and multiple objective contexts and produce results which appear very promising.

Whitley and Hanson (1989), also suggested combined ways with other Artificial Intelligence methods (Neural networks) as in Rogers and Dowla (1994). However, in all these applications the term "GA" indicates an algorithm that can be formulated in very many ways, David (1991), Michalewics (1992). It is interesting therefore to judge how the different GA structures affect the ability to find the region encompassing the optimum solution in the specific field of CRRM calibration, while considering that another different algorithm will perform the subsequent "fine-tuning" process.

Hendrickson et al. (1988), analyze the characteristics of sequential use of two algorithms, the first based on the Pattern Search (SP) method, Hooke and Jeeves (1961), which is a direct search method, and the second (fine-tuning) based on the Newton method. This sequences, is shown to be a fairy good tool, primarily for the PS characteristics that make it less susceptible to irregularity of the response surface, thus less easily trapped on a local minima, and therefore, more efficient in the early stage of optimization.

Ibbirt and Donnel (1971), said; Conceptual rainfall-runoff models usually consist of a number of parameters. Most of the parameters have to be calibrated by examining the estimated and the measured discharge series. The use of function optimization method for calibrating rainfall-runoff models has been studied by Johnson and Pilgrim (1976), Jupta and Sorooshian (1985), Hendrickson et al. (1988). They found that the standard optimization methods can be easily fooled into declaring convergence far short of the true optima because of high dimensionality and irregularities contained in the objective function response such as multiple optima, unsmoothness, discontinuity, elongated ridges, flat plateaus and so on.



### 2.8 Explanatory models for forecasting

The use of explanatory models in business forecasting does not have such a long history as the use of time series methods.

Pardoe (2006) Linear regression modeling is now widely used where a variable to be forecast is modeled as linear combination of potential input variables: An interesting application of regression model to forecasting is given by Byron & Ashenfelter (1995) who use a simple regression model to predict the quality of a Grange wine using simple weather variables. However, it is far more common for regression modeling to be used to explain historical variation than for it to be used for forecasting purposes.

Hyndman & Fan( 2009). In some domains, the use of nonparametric additive models for forecasting is growing here the model is often of the form

$$\sum_{j=1}^{J} fj\,(xj;t) + et\,;$$

where *fj* is a smooth nonlinear function to be estimated non parametrically.

Hanssens et al. (2001) In advertising, there is a well-developed culture of using distributed lag regression models such as $yt = \sum \alpha \lambda\, jxt\text{-}j + et$ ;

where *xt* denotes advertising expenditure in month *t*, $0 < \alpha < 1$ and $\alpha > 0$.



**2.8.4 Markov Chain and Loan**

Ait-Sahalia, (1999), "Do Interest Rates Really Follow Continuous-Time Markov Diffusions?" Examines whether interest rates follow a diffusion process (continuous time Markov process), given that only discrete-time interest rates are available. Based on the extended period 1857 to 1995, this work finds that neither short-term interest rates nor long-term interest rates follow Markov processes, but the slope of the yield curve is a univariate Markov process and a diffusion process.

Wai-Ki CHING, Li-Min LI, Tang LI, Shu-Qin ZHANG,( 2007), In this paper, they propose a new multivariate Markov chain model for modeling multiple categorical data sequences. They then test the proposed model with synthetic data and apply it to practical sales demand data.

Stefan Waner (1995-2004)

A **Markov chain**, named for **Andrey Markov,** is a mathematical system that undergoes transitions from one state to another (from a finite or countable number of possible states) in a chain like manner. It is a **random process** endowed with the **Markov property**: the next state depends only on the current state and not on the past. Markov chains have many applications as **statistical model** of real-world processes.

Formally, a Markov chain is a discrete (discrete-time) random process with the Markov property. Often, the term "Markov chain" is used to mean a Markov process which has a discrete (finite or countable) state-space. Usually a Markov chain would be defined for a discrete set of times (i.e. a discrete-time Markov chain) although some authors use the same terminology where "time" can take continuous values. Also see continuous-time Markov process. The use of the term in Markov chain Monte Carlo methodology covers cases where the process is in discrete-time (discrete algorithm steps) with a continuous state space. The following concentrates on the discrete-time discrete-state-space case.



A "discrete-time" random process means a system which is in a certain state at each "step", with the state changing randomly between steps. The steps are often thought of as time, but they can equally well refer to physical distance or any other discrete measurement; formally, the steps are just the integers or natural numbers, and the random process is a mapping of these to states. The Markov property states that the conditional probability distribution for the system at the next step (and in fact at all future steps) *given* its current state depends only on the current state of the system, and not additionally on the state of the system at previous steps.



Since the system changes randomly, it is generally impossible to predict the exact state of the system in the future. However, the statistical properties of the system's future can be predicted. In many applications it is these statistical properties that are important.

The changes of state of the system are called transitions, and the probabilities associated with various state-changes are called transition probabilities. The set of all states and transition probabilities completely characterizes a Markov chain. By convention, we assume all possible states and transitions have been included in the definition of the processes, so there is always a next-state and the process goes on forever.

A famous Markov chain is the so-called "drunkard's walk", a **random walk** on the number line where, at each step, the position may change by +1 or −1 with equal probability. From any position there are two possible transitions, to the next or previous integer. The transition probabilities depend only on the current position, not on the way the position was reached. For example, the transition probabilities from 5 to 4 and 5 to 6 are both 0.5, and all other transition probabilities from 5 are 0. These probabilities are independent of whether the system was previously in 4 or 6.



Another example is the dietary habits of a creature who eats only grapes, cheese or lettuce, and whose dietary habits conform to the following rules:

- It eats exactly once a day.
- If it ate cheese yesterday, it will not today.
- It will eat lettuce or grapes with equal probability.
- If it ate grapes yesterday, it will eat grapes today with probability 1/10, cheese with probability 4/10 and lettuce with probability 5/10.
- If it ate lettuce yesterday, it will not eat lettuce again today but will eat grapes with probability 4/10 or cheese with probability 6/10.

This creature's eating habits can be modeled with a Markov chain since its choice depends solely on what it ate yesterday, not what it ate two days ago or even farther in the past. One statistical property that could be calculated is the expected percentage, over a long period, of the days on which the creature will eat grapes.

A series of independent events—for example, a series of coin flips—does satisfy the formal definition of a Markov chain. However, the theory is usually applied only when the probability distribution of the next step depends non-trivially on the current state.

Thyagarajan and Saiful Maznan Bin Mohamed,(2005). They found out that variance analysis of actual loan sanctions with the non-documented method of loan allocation of the selected retail bank, over a period of 24 months, revealed that there is a scope to improve their income earnings. Realizing its importance Markov Chain Market Share model was applied to inter temporal data of loan disbursements of the selected bank. By applying Estimate Transition Matrix, scope for probability of loan switching among its types was calculated to suggest the probable mix of loan portfolio. From the results it was suggested that the loan proportions among various types were as follows: Housing (32.0 %), Others (28.1 %), Business (20.0 %) and Education (19.7 %). These proportions can be taken as guideline percentage within the government norms for the priority sector. Simulation studies were also done to calculate the expected income of interest using Markov proportions and compared with the actual interest earnings to prove the superiority of the model.

Howard (1966) showed that dynamic programming based on the Markov process has application in a wide variety of situations, including maintenance and repair, financial portfolio balancing, inventory and production control, equipment replacement, and directed marketing.

If the decision involves not only selection among alternatives but also determination of timing, a static decision model is inadequate.

Ahn and Kim (1998) formulated the action-timing problem with Bayesian updating and derived decision rules based on the observation or information. They used sequential Bayesian revision for the action-timing problem and demonstrated its value using simulation. Their work provides a decision rule based on the information or observation at each stage, rather than on the revised belief. They consider only two alternatives: "accept" the current observation or "reject" in favor of another observation.

There have been many studies about the value of improving forecast accuracy.

Murphy and Ehrendorfer (1987) explored the relationship between the quality and value of imperfect forecasts. They used the Brier score as a measure of forecast accuracy, but they found that a scalar measure such as the Brier score cannot completely and unambiguously characterize the quality of the imperfect forecasts. Their research showed the relationship between forecast accuracy and forecast value represented by a multi-valued function—an accuracy/value envelope.

Mjelde et al. (1993) used a structure called a "forecast matrix" to represent various scenarios for climate forecast quality. A stochastic dynamic programming model was used to obtain the expected value of the various scenarios. They attempted to quantify forecast quality through two measures: entropy and variance of the forecast. They showed that the entire structure of the forecast format interacts to determine the economic value of that system.

Considine et al. (2004) examined the value of hurricane forecasts to oil and gas producers rather than the general population. Unlike the general population, the producers of crude oil and natural gas in the Gulf of Mexico respond to the threat of hurricanes by evacuating offshore drilling rigs and temporarily ceasing production. The researchers estimated the value of existing as well as more accurate hurricane forecasting information to show the value of improving forecast accuracy

They used a probabilistic cost-loss model to estimate the incremental value of hurricane forecast information for oil and gas leases in that area over the past two decades. Their research showed that forecast value dramatically increases with improvements in accuracy. They simulated a 50% improvement in 48-hr forecast accuracy, which they assumed would double the strike probability given a strike forecast, to 0.60. They used a critical threshold of the forecast of weather conditions important to the rig operator at the drilling location, such as wind speed and wave height, to distinguish a strike forecast from a no-hit forecast.

Regnier and Harr (2006) deal with the decision to prepare for an oncoming hurricane using a discrete Markov model of hurricane travel that is derived from historical tropical cyclone tracks and combined with the dynamic decision model to estimate the additional value that can be extracted from existing forecasts by anticipating updated forecasts. They used variable hurricane preparation cost, which is defined as a fraction of the maximum loss, increasing linearly or exponentially after a critical lead-time. They used a discrete Markov model for multi-period decision making with respect to a sequence of more than two forecasts with improving accuracy for a single event. Simulation was used to compare the expense in different cases.

Czajkowski (2007) developed a dynamic model of hurricane evacuation behavior in which a household's evacuation decision is framed as an optimal stopping problem where every potential evacuation time prior to the actual hurricane landfall presents the household with the choice either to evacuate or to wait one more period for a revised hurricane forecast. Czajkowski used a Markov Chain to represent the revision of hurricane status and used a state variable named "risk index" for the transition matrix. Since the risk index primarily reflects the mean of forecasted intensity of the hurricane, it contains little information about the uncertainty of the forecast.

Regnier (2008) viewed the hurricane evacuation problem from the perspective of public officials with the authority to order hurricane evacuation. She used a stochastic model of storm motion derived from historic tracks to show the relationship between lead-time and track uncertainty for Atlantic hurricanes, using a discrete Markov model. She showed that being able to tolerate no more than a 10% probability of failing to evacuate before a striking hurricane (a false negative) implies that at least 76% of evacuations will be false alarms. She also showed that reducing decision lead-times from 72 to 48 hours for major population centers could save an average of hundreds of millions of dollars for the region surrounding each target in evacuation costs annually, assuming 460 miles of coastline evacuated.

None of the many contributors to the dynamic action-timing decision problem has considered the optimal investment decision based on the improvement of track and intensity forecasts and of evacuation speed. By modeling these factors, we will derive the optimal investment policy.

Bastos, (2009). With the advent of the new Basel Capital Accord, banking organizations are invited to estimate credit risk capital requirements using an internal ratings based approach. In order to be compliant with this approach, institutions must estimate the expected loss-given-default, the fraction of the credit exposure that is lost if the borrower defaults. This study evaluates the ability of a parametric fractional response regression and a nonparametric regression tree model to forecast bank loan credit losses. The out-of-sample predictive ability of these models is evaluated at several recovery horizons after the default event. The out-of-time predictive ability is also estimated for a recovery horizon of one year. The performance of the models is benchmarked against recovery estimates given by historical averages. The results suggest that regression trees are an interesting alternative to parametric models in modeling and forecasting loss-given-default.

J. Scott Armstrong and John U. Farley, (1969) Ehrenberg's sweeping criticism of Markov brand switching models, highlights many shortcomings of these models for aggregate analysis of consumer behavior. While it has been pointed out that some of his criticisms are not entirely correct, one of Ehrenberg's themes is unquestionably valid. The models tend to break down empirically due to violations of important Markovian stability assumptions. A situation in which the assumptions of the model appear less restrictive is short-run forecasting of store choice behavior of individual families.



Hyun-cheol and Paul Choi(2010). In their paper, Hyun-cheol and Paul Choi reviewed the area of upstream information sharing in supply chain management for the current challenges and future research opportunities for modeling research. Information sharing in supply chain management has become a very much studied area in operations management field. Although downstream information sharing has been widely studied over a decade or so, upstream information sharing has not been studied widely. Therefore, we reviewed the current mathematical modeling or analytical literature in upstream information sharing to identify the relevant issues and potential research ideas.



Terwiesch et al,( 2006). Forecast sharing in a supply chain using linear price contracts often leads to inefficiencies as the buyer has an incentive to inflate demand forecasts to ensure sufficient supply. Recent research in supply chain contracting has focused on one-shot relationships, and has identified various contracts that align incentives in the supply chain and induce the buyer to reveal forecast information truthfully. In this paper, Justin, Morris, Teck and Christian investigate the effect of having an infinitely repeated supplier relationship in achieving supply chain coordination. They analyze a capacity game with forecast sharing under information asymmetry. They establish conditions under which a buyer operating with a linear price contract reveals demand information truthfully with his supplier, who in turn allocates system-optimal capacity, both assuming that their business relationship is long-term. They show that in a repeated forecast sharing game coordination can be achieved when the industry is stable, both parties value their long-term relationship, and over-forecasting is easy to detect.



Swanson et al, (2004). In this paper Michael, Philip and Norman discuss the current state-of-the-art in estimating, evaluating, and selecting among non-linear forecasting models for economic and financial time series. review theoretical and empirical issues, including predictive density, interval and point evaluation and model selection, loss functions, data-mining, and aggregation. In addition, they argue that although the evidence in favor of constructing forecasts using non-linear models is rather sparse, there is reason to be optimistic. However, much remains to be done. Finally, we outline a variety of topics for future research, and discuss a number of areas which have received considerable attention in the recent literature, but where many questions remain.



Timmermann et al, (2004). This paper provides a general framework for forecasting time series subject to discrete structural breaks. Hashem ,Davide and Allan propose a Bayesian estimation and prediction procedure that allows for the possibility of new breaks over the forecast horizon, taking account of the size and duration of past breaks (if any) by means of a hierarchical Markov chain method. Predictions are formed as weighted averages of scenarios with different numbers of breaks by integrating over the hyper parameters from the meta distributions characterizing the stochastic break point process. In an application to US

Treasury bill rates, we end that the proposed Bayesian regime averaging procedure leads to better out-of-sample forecasts than alternative methods that ignore breaks, particularly at long horizons.

Nikolaos Demiris, (2004). This thesis is concerned with statistical methodology for the analysis of stochastic SIR (Susceptible→Infective→Removed) epidemic models. They adopt the Bayesian paradigm and they developed suitably tailored Markov chain Monte Carlo (MCMC) algorithms. The focus is on methods that are easy to generalize in order to accommodate epidemic models with complex population structures. Additionally, the models are general enough to be applicable to a wide range of infectious diseases. They introduce the stochastic epidemic models of interest and the MCMC methods they shall use and they review existing methods of statistical inference for epidemic models. They develop algorithms that utilise multiple precision arithmetic to overcome the well-known numerical problems in the calculation of the final size distribution for the generalised stochastic epidemic. Consequently, They used these exact results to evaluate the precision of asymptotic theorems previously derived in the literature. They also use the exact final size probabilities to obtain the posterior distribution of the threshold parameter $R_0$. They proceed to develop methods of statistical inference for an epidemic model with two levels of mixing. This model assumes that the population is partitioned into subpopulations and permits infection on both local (within-group) and global (population-wide) scales. They adopt two different data augmentation algorithms. The first method introduces an appropriate latent variable, the final severity, for which they have asymptotic information in the event of an outbreak among a population with a large number of groups. In the last part of this thesis we use a random graph representation of the epidemic process and they impute more detailed information about the infection spread.

**Jouchi Nakajima and Yuki Teranishi,(2009).** To investigate the banking sector integration across euro area countries in terms of loan interest rate stickiness, we estimate structural loan rate curves for 12 euro area countries using time-varying regressions with stochastic volatility. Our results show that the loan rates are sticky to a policy interest rate in all countries for all loan maturities, the degree of stickiness differs across the countries, and the degree of difference is more prominent for longer loan maturities. For short-term loans, the loan rate stickiness decreases and for intermediate- and long-term loans the loan rate stickiness converge to average levels during the sample periods. Banking integration in the euro area is not yet complete, but the degree of heterogeneity in the loan rate stickiness decreases.

# CHAPTER THREE

# METHODOLOGY

## 3.1 Introdution

**3.2 Markov Probability Model:** The probability of switching a loan disbursement from loan type $i$ to loan type $j$ is a conditional probability and can be represented by the transition matrix $P=\left[p_{ij}\right]$ such that $\sum_{j=1}^{m} p_{ij}=1$. Indices $i$ refer to the number of loan type. For example $p_{2,1}$ represents the probability of a change in loan disbursement from business to housing in the next period of time. While $p_{i,i}$ represents the probability of no change in loan disbursement for loan type $i$. The stochastic model used to explain the loan disbursement behavior is a Markov Chain with finite number of states $\{E\}$ Markov process $\{X_t\}$ with discrete time $t$ such that $p_{i,j}$ in general represents the probability of the process moving from state $i$ at time $t$- 1 to state $j$ at time $t$. In this study we assume that the loan disbursement for type $i$ in the next period $t$ (month) is only determined by the loan disbursement at the preceding period $t$-1. In other words, the "history" of loan disbursement before time t-1 does not influence the future loan disbursement. This is known as a first order time dependency. In statistical notation it is represented by $P\left(X_t=j \mid X_0, X_1, \ldots X_{t-1}=i\right)=P\left(X_t=j \mid X_{t-1}=i\right)$

Furthermore, it is also assumed that the underlying variable that are responsible for the generation of loan disbursement do not change overtime, such that the transition probability has a stationary property i.e. $P\left(X_t=i \mid X_{t-1}=i\right)=p_{ij}(t+1)=p_{ij}$ for all $t$.

Furthermore, the probability relations $\sum_{j=1}^{m} p_{ij}=1$ and $0 \leq p_{i,j} \leq 1$ must also be satisfied

**3.3 Estimation of Probability Transition Matrix:** The estimation of the probability transition matrix plays a major and crucial role in the study of a Markov process. If a process that follows a known probability distribution, the estimation can be made with less difficult. For micro economic data that traces the movement from any given state to another states, the estimation procedure follows that of a multinomial distribution, that is $p_{ij} = \frac{n_{ij}}{n_i}$ where $n_{ij}$ is the number of time the process moves from state $i$ to state $j$ and $n_i$ is the number of time the process is in state $i$. However for the macro economic data the estimation procedure is quite tedious. Among the several techniques considered, Bayesian estimation is the best, but among the non Bayesian, they proposed the following ranking: maximum likelihood (MLE), weighted least squares, unweighted restricted least square, minimum absolute deviation and the unrestricted least square estimator. In this study, however, the estimation of the transition matrix is made by using the unweighted ordinary least square techniques.

Following Lee, et al (1965), the first order conditional probability can be rewritten as

$$P(x_t = j) = \sum_{t=1}^{m} P(x_t = j \mid x_{t-1} = i) = \sum_{t=1}^{m} (x_{t-1} = i \mid x_t = j) P(x_{t-1} = i)$$

Or

$q_j(t) = \sum_{t=1}^{m} q_i(t-1) p_{i,j}$ , where $q_j(.)$ and $p(.)$ represent the unconditional probability. If $q_{j(t)}$ is replaced by the observed proportion $y_{j(t)}$, then the sample observation may be assumed to be generated by the following stochastic relation.

$$y_j(t) = \sum_{i=1}^{m} y_i(t-1) p_{i,j} + u_j(t)$$

Or

$Y_j = X_j P_j + U_j$ Where $Y_j, X_j$, and $P_j$ are defined as follows. $Y_j$ is a vector of proportion for loan type $j$. $X$ with $(t-1)$ components $X_j$ is a matrix of proportion with dimension of $(t-1)$ by $m$. $P_j$ is a probability vector $\left(P_{i,j}, i = 1,2,3,...,m\right)$. $U_j$ is a vector of random error. Similarly, for all $i$ and $j$ the possible movements of the process are described in the following equation. $Y = XP + U$,

Where $Y' = \left[Y_1', Y_2', ..., Y_m'\right], P' = \left[P_1', P_2', ..., P_m'\right], U' = \left[U'_1, U'_2, ..., U'_m\right]$

and $X$ is a block diagonal matrix with $X_1 = X_2 = ...X_m$. Thus the above equation is used to estimate $P$ by the ordinary least square (OLS) technique subject to the non negativity and equality constraints; i.e.

$\min\left[U'U = (Y - XP)'(Y - XP)\right]$ such that $GP = IP \geq 0$. Where $G = \left[I_1, I_2, ..., I_m\right]$ and $I_j$ is the identity matrix. This optimization problem can be solved by the quadratic programming routine provided that $(X'X)$ is non-singular. Under this formulation however, the error terms are not uncorrelated, thus $P$ (the estimated $P$) is an unbiased but consistent estimator of $P$.



### 3.4 Formal definition

A Markov chain is a sequence of **random variables** $X_1, X_2, X_3, ...$ with the Markov property, namely that, given the present state, the future and past states are independent. Formally,

$$\Pr\left(X_{n+1} = x / X_1 = x_1, X_2 = x_2, ..., X_n = x_n\right) = \Pr\left(X_{n+1} x / X_n = x_n\right).$$

The possible values of $X_i$ form a countable set $S$ called the **state space** of the chain.

Markov chains are often described by a directed graph, where the edges are labeled by the probabilities of going from one state to the other states.

### 3.5 Variations

- Continuous-time Markov processes have a continuous index.
- **Time-homogeneous Markov chains** (or **stationary Markov chains**) are processes where

$$\Pr(X_{n+1} = x | X_n = y) = \Pr(X_n = x | X_{n-1} = y)$$

for all $n$. The probability of the transition is independent of $n$.

- A **Markov chain of order** $m$ (or a Markov chain with memory $m$) where $m$ is finite, is a process satisfying

$$\begin{aligned} &\Pr(X_n = x_n | X_{n-1} = x_{n-1}, X_{n-2} = x_{n-2}, \dots, X_1 = x_1) \\ = &\Pr(X_n = x_n | X_{n-1} = x_{n-1}, X_{n-2} = x_{n-2}, \dots, X_{n-m} = x_{n-m}) \text{ for } n > m \end{aligned}$$

In other words, the future state depends on the past $m$ states. It is possible to construct a chain $(Y_n)$ from $(X_n)$ ~~which has~~ the 'classical' Markov property as follows:

Let $Y_n = (X_n, X_{n-1}, \dots, X_{n-m+1})$, the ordered $m$-tuple of $X$ values. Then $Y_n$ is a Markov chain with state space $S^m$ and has the classical Markov property.

- An additive Markov chain of order $m$ where $m$ is finite, is where

$$\Pr(X_n = x_n | X_{n-1} = x_{n-1}, X_{n-2} = x_{n-2}, \dots, X_1 = x_1) = \sum_{r=1}^{m} f(x_n, x_{n-r}, r)$$

for $n > m$.

Example 1

If a Markov system is in state $i$, there is a fixed probability, $p_{ij}$, of it going into state $j$ the next time step, and is called a transition probability.

A Markov system can be illustrated by means of a state transition diagram, which is a diagram showing all the states and transition probabilities.

The matrix P whose *ijth* entry is $p_{ij}$ is called the transition matrix associated with the system. The entries in each row add up to 1. Thus, for instance, a 2 x 2 transition matrix P would be set up as in the following figure.

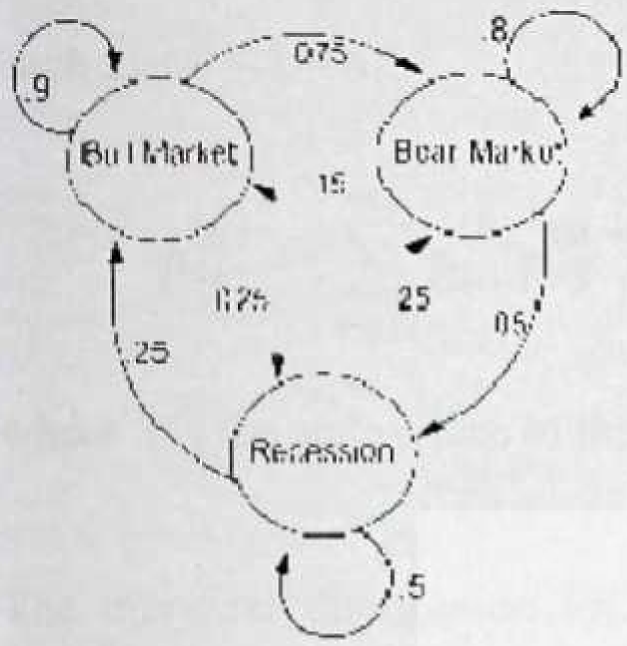
$$\begin{array}{ccc} & \text{To} & \\ & \begin{array}{cc} ① & ② \end{array} & \\ \text{From} \begin{array}{c} ① \\ ② \end{array} & \begin{bmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{bmatrix} & \begin{array}{l} \text{arrows originating in State 1} \\ \text{arrows originating in State 2} \end{array} \end{array}$$

$$\begin{array}{cc} & \text{To} \\ & \begin{array}{ccc} ① & ② & ③ \end{array} \\ \text{From} \begin{array}{c} ① \\ ② \\ ③ \end{array} & \begin{bmatrix} 0.2 & 0.8 & 0 \\ 0.4 & 0 & 0.6 \\ 0.5 & 0.35 & 0.15 \end{bmatrix} \end{array}$$

Distribution Vectors and Powers of the Transition Matrix

**Example 2**

A simple example is shown in the figure 3.1 below, using a directed graph to picture the state transitions. The states represent whether the economy is in a bull market, a bear market, or a recession, during a given week. According to the figure, a bull week is followed by another bull week 90% of the time, a bear market 7.5% of the time, and a recession the other 2.5%. From this figure it is possible to calculate, for example, the long-term fraction of time during which the economy is in a recession, or on average how long it will take to go from a recession to a bull market.




FIGUER 3.1Transition Diagram



A finite state machine can be used as a representation of a Markov chain. Assuming a sequence of independent and identically distributed input signals (for example, symbols from a binary alphabet chosen by coin tosses), if the machine is in state $y$ at time $n$, then the probability that it moves to state $x$ at time $n + 1$ depends only on the current state.

### 3.6 Markov chains

The probability of going from state $i$ to state $j$ in $n$ time steps is

$$p_{ij}^{(n)} = \Pr(X_n = j \mid X_0 = i)$$

and the single-step transition is

$$p_{ij} = \Pr(X_1 = j \mid X_0 = i).$$

For a time-homogeneous Markov chain:

$$p_{ij}^{(n)} = \Pr(X_{k+n} = j \mid X_k = i)_{\text{and}}$$
$$p_{ij} = \Pr(X_{k+1} = j \mid X_k = i).$$

The $n$-step transition probabilities satisfy the Chapman-Kolmogorov equation, that for any $k$ such that $0 < k < n$,

$$p_{ij}^{(n)} = \sum_{r \in S} p_{ir}^{(k)} p_{rj}^{(n-k)}$$

where $S$ is the state space of the Markov chain.

The marginal distribution $\Pr(X_n = x)$ is the distribution over states at time $n$. The initial distribution is $\Pr(X_0 = x)$. The evolution of the process through one time step is described by

$$\Pr(X_n = j) = \sum_{r \in S} p_{rj} \Pr(X_{n-1} = r) = \sum_{r \in S} p_{rj}^{(n)} \Pr(X_0 = r).$$

**Example 3**

Trees in a forest are assumed in this simple model to fall into four age groups: $b(k)$ denotes the number of baby trees in the forest (age group 0-15 years) at a given time period $k$; similarly $y(k)$,$m(k)$ and $o(k)$ denote the number of young trees (16-30 years of age), middle-aged trees (age 31-45), and old trees (older than 45 years of age), respectively. The length of one time period is 15 years.

How does the age distribution change from one time period to the next? The model makes the following three assumptions:

- A certain percentage of trees in each age group dies.
- Surviving trees enter into the next age group; old trees remain old.
- Lost trees are replaced by baby trees.

Note that the total tree population does not change over time.

### 3.7 Calculating the Expected Number of Steps to Absorption

The number of times, starting in state i, you expect to visit state j before absorption is the ij$^{th}$ entry of Q.

To obtain information about the time to absorption in an absorbing Markov system, we first calculate the fundamental matrix Q.

The total number of steps expected before absorption equals the total number of visits you expect to make to all the non-absorbing states. This is the sum of all the entries in the i$^{th}$ row of Q.

The product QT gives the probabilities of winding up in the different absorbing states. For instance, if the i$^{th}$ row of QT is [x y z t], then starting in state i, there is a probability x of winding up in the first absorbing state, a probability y of winding up in the second absorbing state, and so on.

A state $j$ is said to be **accessible** from a state $i$ (written $i \to j$) if a system started in state $i$ has a non-zero probability of transitioning into state $j$ at some point. Formally, state $j$ is accessible from state $i$ if there exists an integer $n \geq 0$ such that

$$\Pr(X_n = j | X_0 = i) = p_{ij}^{(n)} > 0.$$

Allowing $n$ to be zero means that every state is defined to be accessible from itself.

A state $i$ is said to **communicate** with state $j$ (written $i \leftrightarrow j$) if both $i \to j$ and $j \to i$. A set of states $C$ is a **communicating class** if every pair of states in $C$ communicates with each other, and no state in $C$ communicates with any state not in $C$. It can be shown that communication in this sense is an **equivalence relation** and thus that communicating classes are the **equivalence classes** of this relation. A communicating class is **closed** if the probability of leaving the class is zero, namely that if $i$ is in $C$ but $j$ is not, then $j$ is not accessible from $i$.

That said, communicating classes need not be commutative, in that classes achieving greater periodic frequencies that encompass 100% of the phases of smaller periodic frequencies, may still be communicating classes provided a form of either diminished, downgraded, or multiplexed cooperation exists within the higher frequency class.

Finally, a Markov chain is said to be **irreducible** if its state space is a single communicating class; in other words, if it is possible to get to any state from any state.

### 3.8 Periodicity

A state $i$ has **period** $k$ if any return to state $i$ must occur in multiples of $k$ time steps. Formally, the period of a state is defined as

$$k = \gcd\{n : \Pr(X_n = i | X_0 = i) > 0\}$$

(where "gcd" is the greatest common divisor). Note that even though a state has period $k$, it may not be possible to reach the state in $k$ steps. For example, suppose it is possible to return to the state in {6, 8, 10, 12, ...} time steps; $k$ would be 2, even though 2 does not appear in this list.

If $k = 1$, then the state is said to be **aperiodic**: returns to state $i$ can occur at irregular times. Otherwise ($k > 1$), the state is said to be **periodic with period $k$**.

It can be shown that every state in a communicating class must have overlapping periods with all equivalent-or-larger occurring sample(s).

It can be also shown that every state of a bipartite graph has an even **period**.

### 3.8.1 Recurrence

A state $i$ is said to be **transient** if, given that we start in state $i$, there is a non-zero probability that we will never return to $i$. Formally, let the random variable $T_i$ be the first return time to state $i$ (the "hitting time"):

$$T_i = \inf\{n \geq 1 : X_n = i | X_0 = i\}.$$



Then, state $i$ is transient if and only if:

$$\Pr(T_i = \infty) > 0.$$

If a state $i$ is not transient (it has finite hitting time with probability 1), then it is said to be **recurrent** or **persistent**. Although the hitting time is finite, it need not have a finite expectation. Let $M_i$ be the expected return time,

$$M_i = E[T_i].$$

Then, state $i$ is **positive recurrent** if $M_i$ is finite; otherwise, state $i$ is **null recurrent** (the terms **non-null persistent** and **null persistent** are also used, respectively).

It can be shown that a state is recurrent if and only if

$$\sum_{n=0}^{\infty} p_{ii}^{(n)} = \infty.$$

A state $i$ is called **absorbing** if it is impossible to leave this state. Therefore, the state $i$ is absorbing if and only if

$$p_{ii} = 1 \text{ and } p_{ij} = 0 \text{ for } i \neq j.$$

### 3.8.2 Ergodicity

A state $i$ is said to be ergodic if it is aperiodic and positive recurrent. If all states in an irreducible Markov chain are ergodic, then the chain is said to be ergodic.

It can be shown that a finite state irreducible Markov chain is ergodic if it has an aperiodic state. A model has the ergodic property if there's a finite number $N$ such that any state can be reached from any other state in exactly $N$ steps. In case of a fully-connected transition matrix where all transitions have a non-zero probability, this condition is fulfilled with $N$=1. A model with just one out-going transition per state cannot be ergodic.

### 3.8.3 Steady-state analysis and limiting distributions

If the Markov chain is a time-homogeneous Markov chain, so that the process is described by a single, time-independent matrix $p_{ij}$, then the vector $\pi$ is called a **stationary distribution** (or invariant measure) if its entries $\pi_j$ are non-negative and sum to 1 and if it satisfies

$$\pi_j = \sum_{i \in S} \pi_i p_{ij}.$$

An irreducible chain has a stationary distribution if and only if all of its states are positive recurrent. In that case, $\pi$ is unique and is related to the expected return time:

$$\pi_j = \frac{1}{M_j}.$$

Further, if the chain is both irreducible and aperiodic, then for any $i$ and $j$,

$$\lim_{n\to\infty} p_{ij}^{(n)} = \frac{1}{M_j}.$$

Note that there is no assumption on the starting distribution; the chain converges to the stationary distribution regardless of where it begins. Such $\pi$ is called the **equilibrium distribution** of the chain.

If a chain has more than one closed communicating class, its stationary distributions will not be unique (consider any closed communicating class in the chain; each one will have its own unique stationary distribution. Any of these will extend to a stationary distribution for the overall chain, where the probability outside the class is set to zero). However, if a state $j$ is aperiodic, then

$$\lim_{n\to\infty} p_{jj}^{(n)} = \frac{1}{M_j}$$

and for any other state $i$, let $f_{ij}$ be the probability that the chain ever visits state $j$ if it starts at $i$,

$$\lim_{n\to\infty} p_{ij}^{(n)} = \frac{f_{ij}}{M_j}.$$

If a state $i$ is periodic with period $k > 1$ then the limit

$$\lim_{n\to\infty} p_{ii}^{(n)}$$

does not exist, although the limit

$$\lim_{n\to\infty} p_{ii}^{(kn+r)}$$

does exist for every integer $r$.

### 3.8.4 Steady-state analysis and the time-inhomogeneous Markov chain

A Markov chain need not necessarily be time-homogeneous to have an equilibrium distribution. If there is a probability distribution over states $\boldsymbol{\pi}$ such that

$$\pi_j = \sum_{i \in S} \pi_i \Pr(X_{n+1} = j \mid X_n = i)$$

for every state $j$ and every time $n$ then $\boldsymbol{\pi}$ is an equilibrium distribution of the Markov chain. Such can occur in Markov chain Monte Carlo (MCMC) methods in situations where a number of different transition matrices are used, because each is efficient for a particular kind of mixing, but each matrix respects a shared equilibrium distribution.

### 3.8.5 Finite state space

If the state space is finite, the transition probability distribution can be represented by a matrix, called the **transition matrix**, with the $(i, j)$th element of **P** equal to

$$p_{ij} = \Pr(X_{n+1} = j \mid X_n = i).$$

Since each row of **P** sums to one and all elements are non-negative, **P** is a right stochastic matrix.

### 3.8.6 Time-homogeneous Markov chain with a finite state space

If the Markov chain is time-homogeneous, then the transition matrix **P** is the same after each step, so the $k$-step transition probability can be computed as the $k$-th power of the transition matrix, $\mathbf{P}^k$.

The stationary distribution $\boldsymbol{\pi}$ is a (row) vector, whose entries are non-negative and sum to 1, that satisfies the equation

$$\pi = \pi \mathbf{P}.$$



In other words, the stationary distribution $\boldsymbol{\pi}$ is a normalized (meaning that the sum of its entries is 1) left eigenvector of the transition matrix associated with the eigenvalue 1.

Alternatively, $\boldsymbol{\pi}$ can be viewed as a fixed point of the linear (hence continuous) transformation on the unit **simplex** associated to the matrix **P**. As any continuous transformation in the unit simplex has a fixed point, a stationary distribution always exists, but is not guaranteed to be unique, in general. However, if the Markov chain is irreducible and aperiodic, then there is a unique stationary distribution $\pi$. Additionally, in this case $\mathbf{P}^k$ converges to a rank-one matrix in which each row is the stationary distribution $\boldsymbol{\pi}$, that is,



$$\lim_{k\to\infty} P^k = P$$

where **1** is the column vector with all entries equal to 1. This is stated by the Perron-Frobenius theorem. If, by whatever means, $\lim_{k\to\infty}\mathbf{P}^k$ is found, then the stationary distribution of the Markov chain in question can be easily determined for any starting distribution, as will be explained below.

For some stochastic matrices **P**, the limit $\lim_{k\to\infty} \mathbf{P}^k$ does not exist, as shown by this example:

$$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}.$$

Because there are a number of different special cases to consider, the process of finding this limit if it exists can be a lengthy task. However, there are many techniques that can assist in finding this limit. Let **P** be an $n\times n$ matrix, and define $\mathbf{Q}=\lim_{k\to\infty} \mathbf{P}^k$.

It is always true that



$$\mathbf{QP} = \mathbf{Q}.$$

Subtracting **Q** from both sides and factoring then yields

$$\mathbf{Q}(\mathbf{P} - \mathbf{I}_n) = \mathbf{0}_{n,n}$$

where $\mathbf{I}_n$ is the identity matrix of size $n$, and $\mathbf{0}_{n,n}$ is the zero matrix of size $n\times n$. Multiplying together stochastic matrices always yields another stochastic matrix, so **Q** must be a stochastic matrix. It is sometimes sufficient to use the matrix equation above and the fact that **Q** is a stochastic matrix to solve for **Q**.



Here is one method for doing so: first, define the function $f(\mathbf{A})$ to return the matrix **A** with its right-most column replaced with all 1's. If $[f(\mathbf{P} - \mathbf{I}_n)]^{-1}$ exists then

$$\mathbf{Q} = f(\mathbf{0}_{n,n})[f(\mathbf{P} - \mathbf{I}_n)]^{-1}.$$

One thing to notice is that if **P** has an element $\mathbf{P}_{i,i}$ on its main diagonal that is equal to 1 and the $i$th row or column is otherwise filled with 0's, then that row or column will remain unchanged in all of the subsequent powers $\mathbf{P}^k$. Hence, the $i$th row or column of $\mathbf{Q}$ will have the 1 and the 0's in the same positions as in $\mathbf{P}$.

### 3.5.7 Reversible Markov chain

A Markov chain is said to be **reversible** if there is a probability distribution over states, $\boldsymbol{\pi}$, such that

$$\pi_i \Pr(X_{n+1} = j \mid X_n = i) = \pi_j \Pr(X_{n+1} = i \mid X_n = j)$$



for all times $n$ and all states $i$ and $j$. This condition is also known as the detailed balance condition (some books refer the local balance equation). With a time-homogeneous Markov chain, $\Pr(X_{n+1} = j \mid X_n = i)$ does not change with time $n$ and it can be written more simply as $p_{ij}$. In this case, the detailed balance equation can be written more compactly as

$$\pi_i p_{ij} = \pi_j p_{ji}.$$

Summing the original equation over $i$ gives

$$\sum_i \pi_i \Pr(X_{n+1} = j \mid X_n = i) = \sum_i \pi_j \Pr(X_{n+1} = i \mid X_n = j)$$
$$= \pi_j \sum_i \Pr(X_{n+1} = i \mid X_n = j) = \pi_j ,$$



so, for reversible Markov chains, $\boldsymbol{\pi}$ is always a steady-state distribution of $\Pr(X_{n+1} = j \mid X_n = i)$ for every $n$.

If the Markov chain begins in the steady-state distribution, *i.e.*, if $\Pr(X_0 = i) = \pi_i$, then $\Pr(X_n = i) = \pi_i$ for all $n$ and the detailed balance equation can be written as

$$\Pr(X_n = i, X_{n+1} = j) = \Pr(X_{n+1} = i, X_n = j).$$

The left- and right-hand sides of this last equation are identical except for a reversing of the time indices $n$ and $n + 1$.

Reversible Markov chains are common in Markov chain Monte Carlo (MCMC) approaches because the detailed balance equation for a desired distribution $\boldsymbol{\pi}$ necessarily implies that the Markov chain has been constructed so that $\boldsymbol{\pi}$ is a steady-state distribution. Even with time-inhomogeneous Markov chains, where multiple transitions matrices are used, if each such transition matrix exhibits detailed balance with the desired $\boldsymbol{\pi}$ distribution, this necessarily implies that $\boldsymbol{\pi}$ is a steady-state distribution of the Markov chain.

# CHAPTER FOUR

# DATA ANALYSIS AND RESULTS

## 4.0 Introduction

This chapter presents the data obtained and the analyses in accordance with the methods discussed under chapter three above.

## 4.1 Data Collection

The secondary data for this study was collected from a Ghana-based financial institution that operates through 149 branches nationwide . The data was collected over a span of two years from January 2010 to December 2011.



Table 4.0 represents the type of loan and its code

| Housing | Business | Education | Others | Housing |
|---|---|---|---|---|
| HS | BS | ED | OT | HS |

Table 4.1: The data of loan disbursement for the four loan types for the 24 months (GH¢'000)

| | HS | BS | ED | OT | Total |
|---|---|---|---|---|---|
| Jan – 10 | 7,226,494.79 | 826,616.40 | 783,045.10 | 5,948,216.54 | 14,784,372.83 |
| Feb –10 | 8,656,332.62 | 689,182.50 | 725,801.80 | 1,335,422.51 | 11,406,739.43 |
| Mar 10 | 7,288,007.10 | 813,588.42 | 733,785.62 | 1,312,145.29 | 10,147,526.43 |
| Apr –10 | 6,337,540.65 | 631,652.39 | 741,857.26 | 3,425,462.72 | 11,136,513.02 |
| May 10 | 7,341,855.50 | 841,732.52 | 750,017.69 | 1,254,735.57 | 10,188,341.28 |
| Jun – 10 | 7,643,286.00 | 753,841.80 | 758,267.89 | 3,122,536.51 | 12,277,932.20 |
| Jul – 10 | 6,527,476.56 | 871,964.68 | 766,608.83 | 1,145,678.39 | 9,311,728.46 |
| Aug –10 | 8,583,720.90 | 763,826.65 | 593,501.72 | 2,234,225.72 | 12,175,274.99 |
| Sept 10 | 7,581,795.88 | 757,556.25 | 599,436.74 | 3,444,788.33 | 12,383,577.20 |
| Oct –10 | 8,665,316.74 | 798,746.38 | 605,431.10 | 2,423,539.58 | 12,493,033.80 |
| Nov –10 | 8,874,476.00 | 825,618.28 | 611,485.42 | 3,221,562.27 | 13,533,141.97 |
| Dec –10 | 7,754,482.21 | 877,943.65 | 617,600.27 | 5,442,167.86 | 14,692,193.99 |
| Jan – 11 | 7,651,183.32 | 1,986,735.76 | 856,491.50 | 5,432,578.28 | 15,926,988.86 |
| Feb –11 | 9,274,627.25 | 856,554.34 | 1,415,487.60 | 2,326,843.80 | 13,873,512.99 |
| Mar 11 | 8,757,641.70 | 2,985,783.34 | 1,112,173.90 | 3,674,454.32 | 16,530,053.26 |
| Apr –11 | 6,742,217.31 | 2,495,637.32 | 915,116.78 | 4,332,656.86 | 14,485,628.27 |
| May 11 | 7,846,211.66 | 3,445,631.62 | 935,706.91 | 4,332,567.35 | 16,560,117.54 |
| Jun – 11 | 6,861,662.81 | 699,894.64 | 956,760.31 | 3,872,861.45 | 12,391,179.21 |
| Jul – 11 | 8,849,728.50 | 2,133,541.63 | 978,287.42 | 3,256,573.64 | 15,218,131.19 |
| Aug –11 | 9,441,881.13 | 1,332,546.75 | 1,000,298.89 | 4,453,783.25 | 16,228,510.02 |
| Sept 11 | 7,795,643.32 | 934,670.96 | 1,022,805.61 | 3,245,495.24 | 12,998,615.13 |

| | | | | | |
|---|---|---|---|---|---|
| Oct –11 | 8,421,902.48 | 2,185,925.85 | 1,045,818.74 | 4,456,555.66 | 16,110,202.73 |
| Nov –11 | 8,226,575.53 | 1,356,545.23 | 1,069,349.66 | 5,334,765.58 | 15,987,236.00 |
| Dec –11 | 7,652,362.62 | 2,422,356.17 | 1,093,410.03 | 5,284,861.52 | 16,452,990.34 |

## 4.2 Data Processing

**4.3 Estimating the Transition Probability Matrix for the Loan Portfolio:** Following the estimation procedure discussed earlier in chapter three, we need to define the appropriate vectors and matrix. Since $y_j(t)$ is defined as a proportion of loan type *j*, at time *t*, then the actual loan disbursements have to be changed into proportion. This can be done by dividing the individual actual loan disbursement by the total actual loan disbursement for each time $t$ .



Table 4. 2 shows the Proportion $y_j(t)$ of loan Disbursement for each type of Loan for a given months $(j)$ of loan disbusement

| t | HS | BS | ED | OT |
|---|---|---|---|---|
| Jan – 10 | 0.4888 | 0.0559 | 0.0530 | 0.4023 |
| Feb – 10 | 0.7589 | 0.0604 | 0.0636 | 0.1171 |
| Mar – 10 | 0.7182 | 0.0802 | 0.0723 | 0.1293 |
| Apr – 10 | 0.5691 | 0.0567 | 0.0666 | 0.3076 |
| May – 10 | 0.7206 | 0.0826 | 0.0736 | 0.1232 |
| Jun – 10 | 0.6225 | 0.0614 | 0.0618 | 0.2543 |
| Jul – 10 | 0.7010 | 0.0936 | 0.0823 | 0.1230 |
| Aug – 10 | 0.7050 | 0.0627 | 0.0487 | 0.1835 |
| Sept – 10 | 0.6122 | 0.0612 | 0.0484 | 0.2782 |
| Oct – 10 | 0.6936 | 0.0639 | 0.0485 | 0.1940 |
| Nov – 10 | 0.6558 | 0.0610 | 0.0452 | 0.2380 |
| Dec – 10 | 0.5278 | 0.0598 | 0.0420 | 0.3704 |
| Jan – 11 | 0.4804 | 0.1247 | 0.0538 | 0.3411 |
| Feb – 11 | 0.6685 | 0.0617 | 0.1020 | 0.1677 |
| Mar – 11 | 0.5298 | 0.1806 | 0.0673 | 0.2223 |
| Apr – 11 | 0.4654 | 0.1723 | 0.0632 | 0.2991 |
| May – 11 | 0.4738 | 0.2081 | 0.0565 | 0.2616 |
| Jun – 11 | 0.5538 | 0.0565 | 0.0772 | 0.3125 |
| Jul – 11 | 0.5815 | 0.1402 | 0.0643 | 0.2140 |
| Aug – 11 | 0.5818 | 0.0821 | 0.0616 | 0.2744 |
| Sept – 11 | 0.5997 | 0.0719 | 0.0787 | 0.2497 |
| Oct – 11 | 0.5228 | 0.1357 | 0.0649 | 0.2766 |
| Nov – 11 | 0.5146 | 0.0849 | 0.0669 | 0.3337 |
| Dec – 11 | 0.4651 | 0.1472 | 0.0665 | 0.3212 |

According to the Bank where data was collected, the transition probability is calculated by $\frac{1}{N}$, where $N$ $is$ the total number of directed arrows leaving a node (state). From this formula it is possible to calculate, for example, the transition probability of a state (HS) moving to another state (BS) is 1/4.

$P_{BS\text{-}BS} = \frac{1}{2} = 0.50$, $P_{BS\text{-}HS} = \frac{1}{4} = 0.250$, $P_{BS\text{-}ED} = \frac{1}{4} = 0.250$, $P_{BS\text{-}OT} = \frac{1}{4} = 0.250$

$P_{HS\text{-}BS} = 0$, $P_{HS\text{-}HS} = \frac{1}{3} = 0.3333$, $P_{HS\text{-}ED} = \frac{1}{3} = 0.3333$, $P_{HS\text{-}OT} = \frac{1}{3} = 0.3333$

$P_{ED\text{-}BS} = 0$, $P_{ED\text{-}HS} = \frac{1}{3} = 0.3333$, $P_{ED\text{-}ED} = \frac{1}{3} = 0.3333$, $P_{ED\text{-}OT} = \frac{1}{3} = 0.3333$

$P_{OT\text{-}BS} = \frac{1}{2} = 0.50$, $P_{OT\text{-}HS} = \frac{1}{4} = 0.250$, $P_{OT\text{-}ED} = 0$ $P_{OT\text{-}OT} = \frac{1}{4} = 0.250$

In the table 4.2 below both the rows and columns represent the transition states. The movement is done from row to column across the table. The values in the table represent the transition probabilities.

Table 4.2 Transition Probability Values

$\boldsymbol{P} =$

| | BS | HS | ED | OT |
|---|---|---|---|---|
| BS | 0.500 | 0.250 | 0.250 | 0.250 |
| HS | 0.0 | 0.3333 | 0.3333 | 0.3333 |
| ED | 0.0 | 0.3333 | 0.3333 | 0.3333 |
| OT | 0.500 | 0.250 | 0.0 | 0.250 |

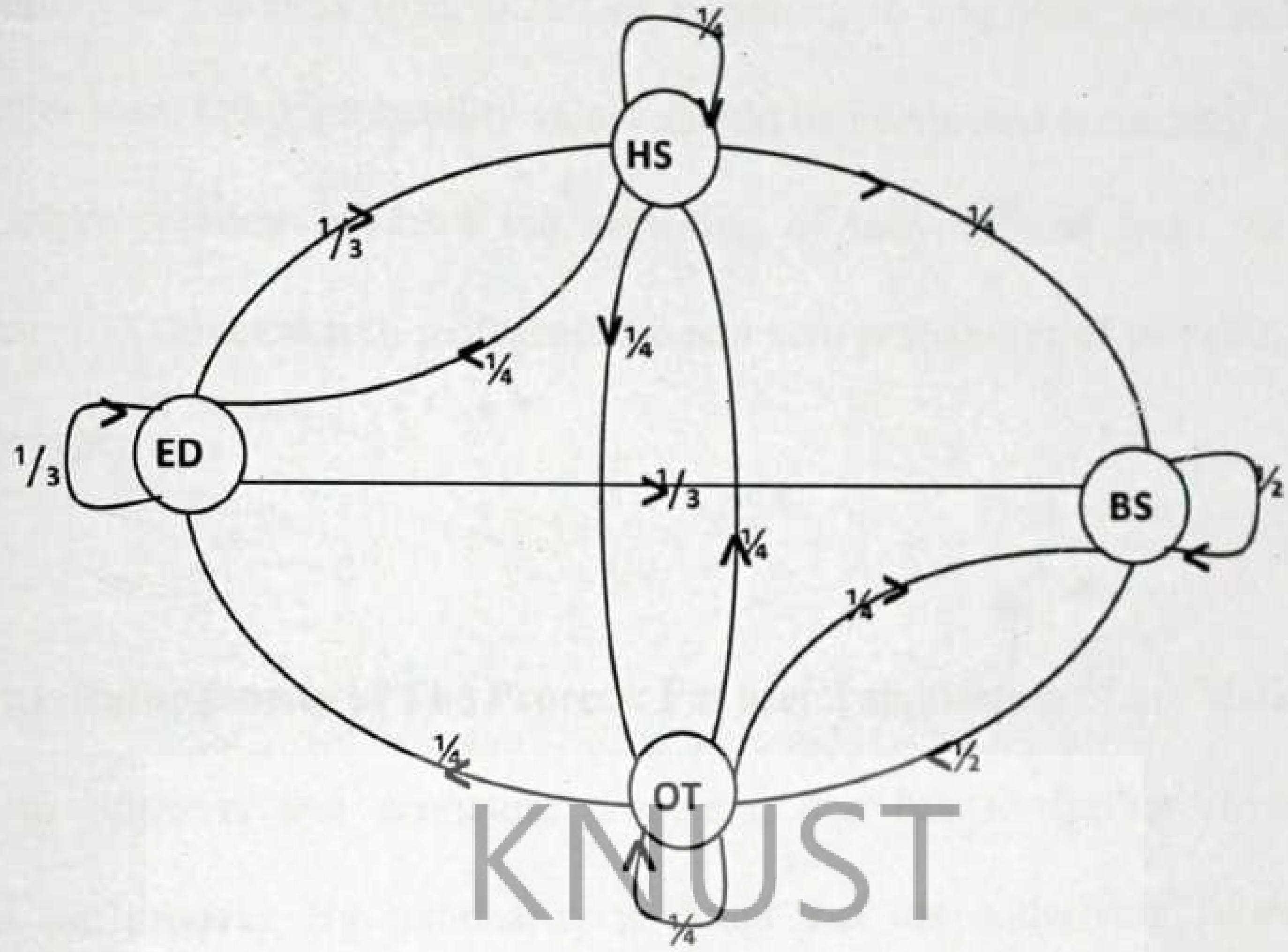




Figure 4.1: Pictorial Representation of the Transition Probability Matrix

**4.4 Transition Probability Matrix:** The transition probability matrix for the loan portfolio is given in Table 4.2 while Figure 4.1 is its pictorial representation. The transition probability matrix shows that the probability of loan switching from business to other loan is quite high (0.500) while loan switching from housing to education is low (0.3333). Probability of no loan switching is quite low for education loan (0.3333) it is the same for other loan (0.3333). Loan switching from housing to business, education to business and other to education cannot be made in one time period due to its zero probability. Loan switching to housing loan is relatively the same from other loan (0.3333) but relatively low from other loan (0.250). The interpretation of this probability values should be made cautiously. Firstly, the probability value gives us the indication of loan switching. It may actually affect the switching or it may not be. If it affects the switching then the probability value gives the probability of switching.

Secondly, the probability value also indicates that if a bank receives a loan application (say a housing loan), then if its allocation is still available, then there is no switching. Otherwise, loan switching is made. The probability value gives the probability of 0.250 no switching, 0.250 of switching to business loan, 0.250 of switching to education loan and 0.250 of switching to other loan. Other probability values should be interpreted accordingly.

The pictorial representation indicates the switching of loan derived from the transition probability matrix. A directed arch represents the non zero probability of switching from one type to another type.

**4.5 Stationarity/Homogeneity of The Process:** For useful application of the Markov process in particular to business and economic problems, one has to further investigate the stationarity of the process. By stationarity it meant that the underlying factors that are responsible for the generation of the data do not change significantly over the sampling period (data collection time) and the forecast periods. This could be verified by analyzing the trend of the backcast proportion of the loan disbursements.

The following matrix operation is use to estimate the values of the backcast proportion or one period forecast proportion of the individual type of loan disbursement.

$\hat{X}(t+1) = X(t)P$ where $\hat{X}(t+1)$ and $X(t)$ are vectors of the Backcast proportion and the actual proportion for all past values of t respectively.

$$
\begin{pmatrix}
0.7589 & 0.0604 & 0.0636 & 0.1171 \\
0.7182 & 0.0802 & 0.0723 & 0.1293 \\
0.5691 & 0.0567 & 0.0666 & 0.3076 \\
0.7206 & 0.0826 & 0.0736 & 0.1232 \\
0.6225 & 0.0614 & 0.0618 & 0.2543 \\
0.7010 & 0.0936 & 0.0823 & 0.1230 \\
0.7050 & 0.0627 & 0.0487 & 0.1835 \\
0.6122 & 0.0612 & 0.0484 & 0.2782 \\
0.6936 & 0.0639 & 0.0485 & 0.1940 \\
0.6558 & 0.0610 & 0.0452 & 0.2380 \\
0.5278 & 0.0598 & 0.0420 & 0.3704 \\
0.4804 & 0.1247 & 0.0538 & 0.3411 \\
0.6685 & 0.0617 & 0.1020 & 0.1677 \\
0.5298 & 0.1806 & 0.0673 & 0.2223 \\
0.4654 & 0.1723 & 0.0632 & 0.2991 \\
0.4738 & 0.2081 & 0.0565 & 0.2616 \\
0.5538 & 0.0565 & 0.0772 & 0.3125 \\
0.5815 & 0.1402 & 0.0643 & 0.2140 \\
0.5818 & 0.0821 & 0.0616 & 0.2744 \\
0.5997 & 0.0719 & 0.0787 & 0.2497 \\
0.5228 & 0.1357 & 0.0649 & 0.2766 \\
0.5146 & 0.0849 & 0.0669 & 0.3337 \\
0.4651 & 0.1472 & 0.0665 & 0.3212
\end{pmatrix}
*
\begin{pmatrix}
0.250 & 0.250 & 0.250 & 0.250 \\
0.0 & 0.3333 & 0.3333 & 0.3333 \\
0.0 & 0.3333 & 0.3333 & 0.3333 \\
0.500 & 0.250 & 0.0 & 0.250
\end{pmatrix}
=
\begin{pmatrix}
0.2483 & 0.2603 & 0.2310 & 0.2603 \\
0.2542 & 0.2766 & 0.2403 & 0.2727 \\
0.2961 & 0.2602 & 0.1833 & 0.2602 \\
0.2418 & 0.2630 & 0.2322 & 0.3076 \\
0.2828 & 0.2602 & 0.1967 & 0.1232 \\
0.2368 & 0.2646 & 0.2338 & 0.2543 \\
0.2680 & 0.2592 & 0.2133 & 0.1230 \\
0.2922 & 0.2591 & 0.0484 & 0.2591 \\
0.2704 & 0.2593 & 0.0485 & 0.2593 \\
0.2830 & 0.2588 & 0.0452 & 0.2588 \\
0.3172 & 0.2584 & 0.0420 & 0.2584 \\
0.2907 & 0.1795 & 0.0538 & 0.1795 \\
0.2510 & 0.2634 & 0.1020 & 0.2634 \\
0.2436 & 0.2706 & 0.0673 & 0.2706 \\
0.2659 & 0.2687 & 0.1939 & 0.2687 \\
0.2493 & 0.2720 & 0.2066 & 0.2720 \\
0.2947 & 0.2611 & 0.2293 & 0.2611 \\
0.2524 & 0.2670 & 0.2135 & 0.2670 \\
0.2827 & 0.2619 & 0.1933 & 0.2619 \\
0.2748 & 0.2625 & 0.2001 & 0.2625 \\
0.2690 & 0.2666 & 0.1975 & 0.2666 \\
0.2955 & 0.2626 & 0.1791 & 0.2626 \\
0.2769 & 0.2677 & 0.1874 & 0.2677
\end{pmatrix}
$$



Table 4.3 gives the value of actual and backcast proportions of the loan disbursements. Fig.4.2 to 4.5 show the trend of actual and backcast proportion.

**Table 4.3 Actual and Backcast Proportion of Loan Disbursement**

| | Housing | | Business | | Education | | Others | |
|---|---|---|---|---|---|---|---|---|
| Month | Actual | Backcast | Actual | Backcast | Actual | Backcast | Actual | Backcast |
| Jan – 10 | 0.4888 | _ | 0.0559 | _ | 0.0530 | _ | 0.4023 | _ |
| Feb –10 | 0.7589 | 0.2483 | 0.0604 | 0.2603 | 0.0636 | 0.2310 | 0.1171 | 0.2603 |
| Mar 10 | 0.7182 | 0.2542 | 0.0802 | 0.2766 | 0.0723 | 0.2403 | 0.1293 | 0.2727 |
| Apr –10 | 0.5691 | 0.2961 | 0.0567 | 0.2602 | 0.0666 | 0.1833 | 0.3076 | 0.2602 |
| May -10 | 0.7206 | 0.2418 | 0.0826 | 0.2630 | 0.0736 | 0.2322 | 0.1232 | 0.2630 |
| Jun – 10 | 0.6225 | 0.2828 | 0.0614 | 0.2602 | 0.0618 | 0.1967 | 0.2543 | 0.2602 |
| Jul – 10 | 0.7010 | 0.2368 | 0.0936 | 0.2646 | 0.0823 | 0.2338 | 0.1230 | 0.2646 |
| Aug –10 | 0.7050 | 0.2680 | 0.0627 | 0.2592 | 0.0487 | 0.2133 | 0.1835 | 0.2592 |
| Sept -10 | 0.6122 | 0.2922 | 0.0612 | 0.2591 | 0.0484 | 0.1895 | 0.2782 | 0.2591 |
| Oct –10 | 0.6936 | 0.2704 | 0.0639 | 0.2593 | 0.0485 | 0.2108 | 0.1940 | 0.2593 |
| Nov –10 | 0.6558 | 0.2830 | 0.0610 | 0.2588 | 0.0452 | 0.1993 | 0.2380 | 0.2588 |
| Dec –10 | 0.5278 | 0.3172 | 0.0598 | 0.2584 | 0.0420 | 0.1658 | 0.3704 | 0.2584 |
| Jan – 11 | 0.4804 | 0.2907 | 0.1247 | 0.1795 | 0.0538 | 0.1795 | 0.3411 | 0.1795 |
| Feb –11 | 0.6685 | 0.2510 | 0.0617 | 0.2634 | 0.1020 | 0.2217 | 0.1677 | 0.2634 |
| Mar -11 | 0.5298 | 0.2436 | 0.1806 | 0.2706 | 0.0673 | 0.2150 | 0.2223 | 0.2706 |
| Apr –11 | 0.4654 | 0.2659 | 0.1723 | 0.2687 | 0.0632 | 0.1939 | 0.2991 | 0.2687 |
| May -11 | 0.4738 | 0.2493 | 0.2081 | 0.2720 | 0.0565 | 0.2066 | 0.2616 | 0.2720 |
| Jun – 11 | 0.5538 | 0.2947 | 0.0565 | 0.2611 | 0.0772 | 0.2293 | 0.3125 | 0.2611 |
| Jul – 11 | 0.5815 | 0.2524 | 0.1402 | 0.2670 | 0.0643 | 0.2135 | 0.2140 | 0.2670 |
| Aug –11 | 0.5818 | 0.2827 | 0.0821 | 0.2619 | 0.0616 | 0.1933 | 0.2744 | 0.2619 |
| Sept -11 | 0.5997 | 0.2748 | 0.0719 | 0.2625 | 0.0787 | 0.2001 | 0.2497 | 0.2625 |
| Oct –11 | 0.5228 | 0.2690 | 0.1357 | 0.2666 | 0.0649 | 0.1975 | 0.2766 | 0.2666 |
| Nov –11 | 0.5146 | 0.2955 | 0.0849 | 0.2626 | 0.0669 | 0.1791 | 0.3337 | 0.2626 |
| Dec –11 | 0.4651 | 0.2769 | 0.1472 | 0.2677 | 0.0665 | 0.1874 | 0.3212 | 0.2677 |

It is observed from Table 4.3 that the housing loan proportion has a decreasing trend while the proportions of business, education and other loans has an increasing trend. The same phenomenon is also basically observed for the backcast proportion. Though the trend for the actual and backcast proportion seems to be consistent, the actual proportion in particular the other loan has a fluctuating movement. However for the Backcast proportion, the trend is quite smooth which connotes a stable trend. Thus one would conclude that the estimated transition matrix produces a stable trajectory which will imply homogeneity. Had the trend for the backcast proportion exhibit an erratic movement, then one would obviously conclude that the underlying factors that are responsible for the generation of the data had changed the loan process significantly.

Figure 4.2 shows that the actual housing loan proportion has a decreasing trend while the backcast proportion seems to be consistent. However for the Backcast proportion, the trend is quite smooth which connotes a stable trend.

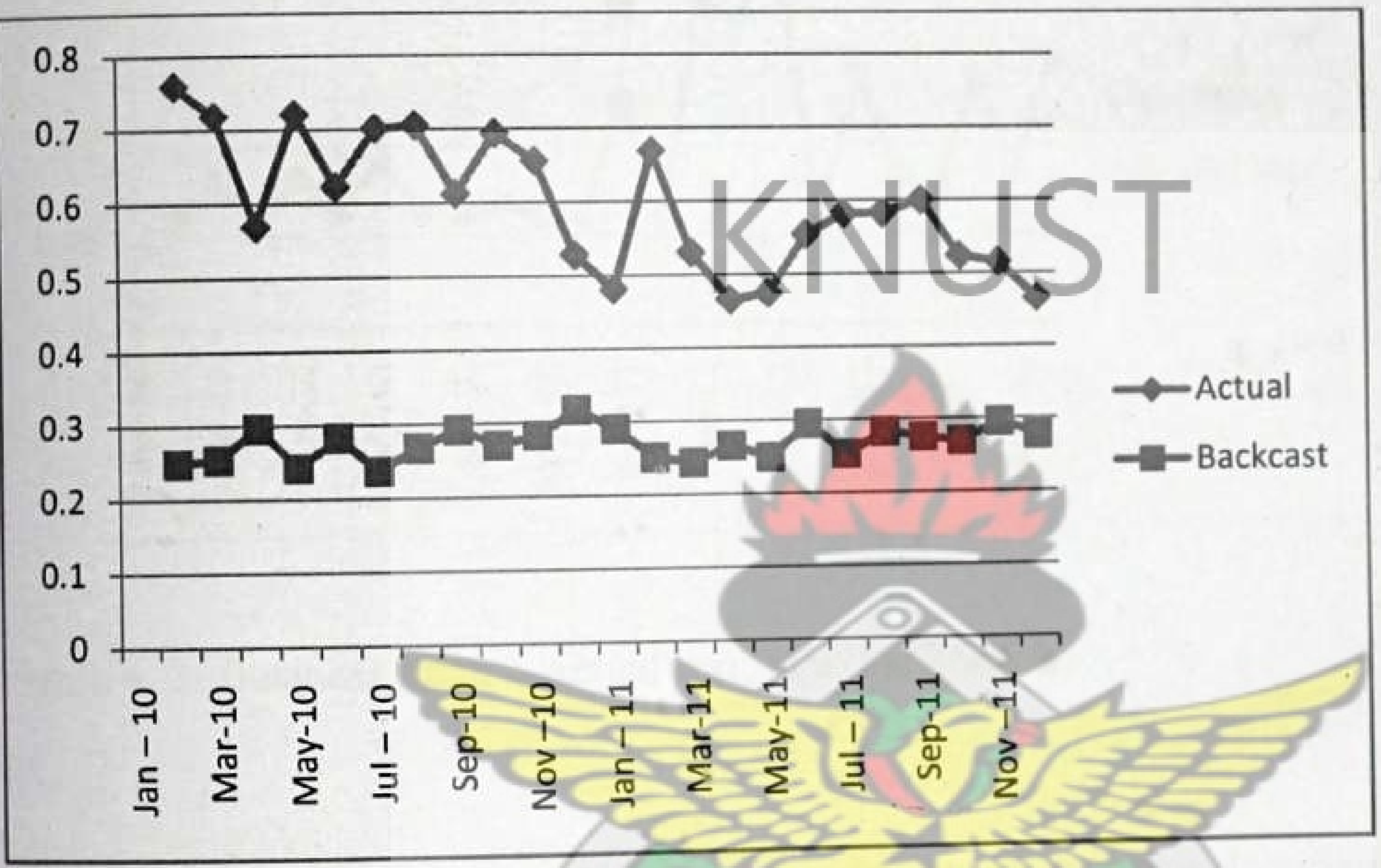



Figure 4. 2: Housing Loan: Actual and Backcast Proportion

Figure 4.3 shows that the actual Business loan proportion has a increasing trend while the backcast proportion seems to be consistent. However for the Backcast proportion, the trend is quite smooth which connotes a stable trend.

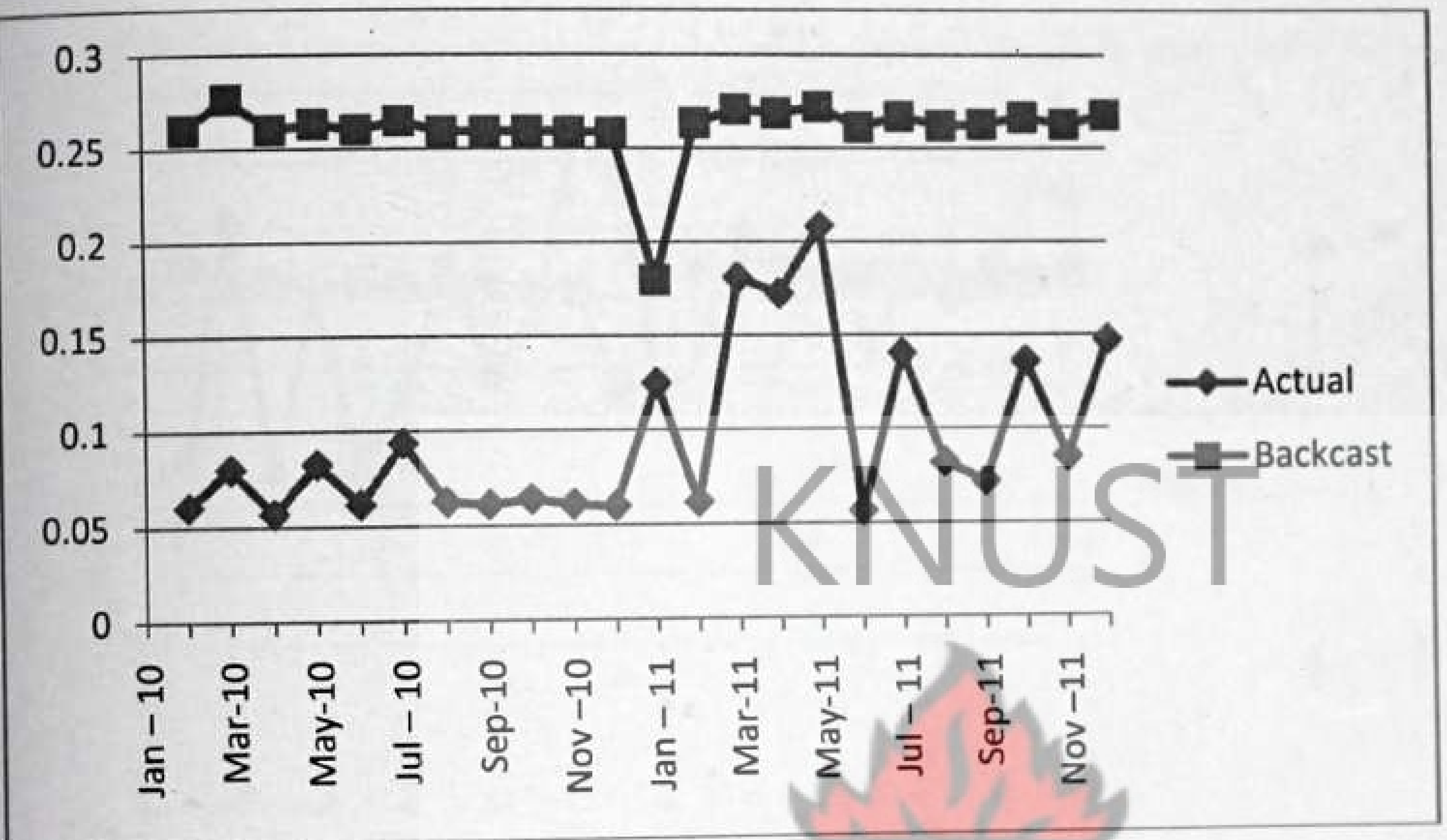



Figure 4. 3: Business Loan – Actual and Backcast Portfolio

Figure 4.4 the actual Education loan proportion seems to has a decreasing trend while the backcast proportion seems to be consistent. However for the Backcast proportion, the trend is quite smooth which indicate a stable trend.

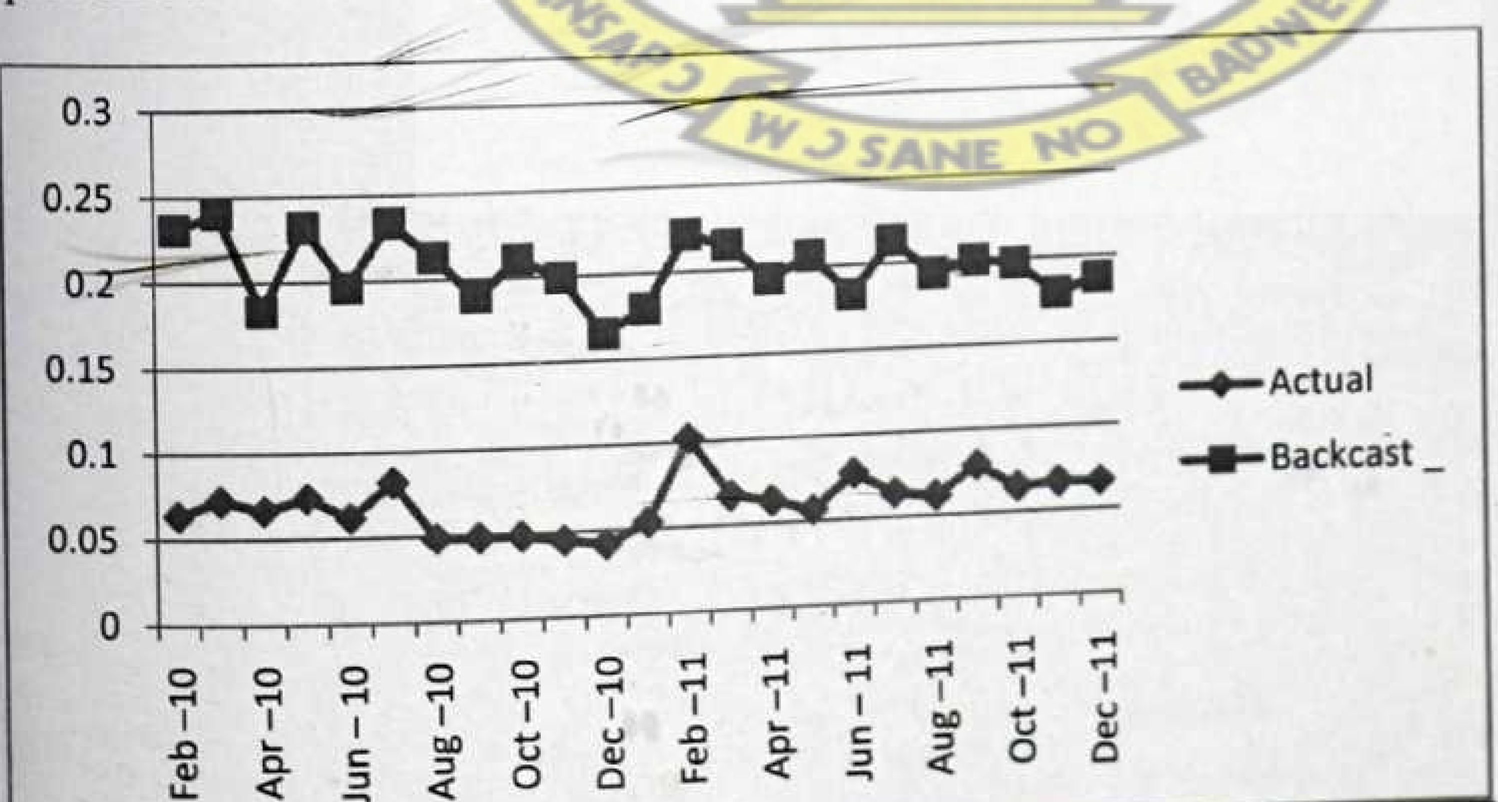



Figure 4.4: Education Loan – Actual and Backcast Proportion

Figure 4.5 shows that the actual housing loan proportion has a increasing creasing trend while the backcast proportion seems to be consistent. the Backcast proportion, shows that the trend is quite smooth and a stable.

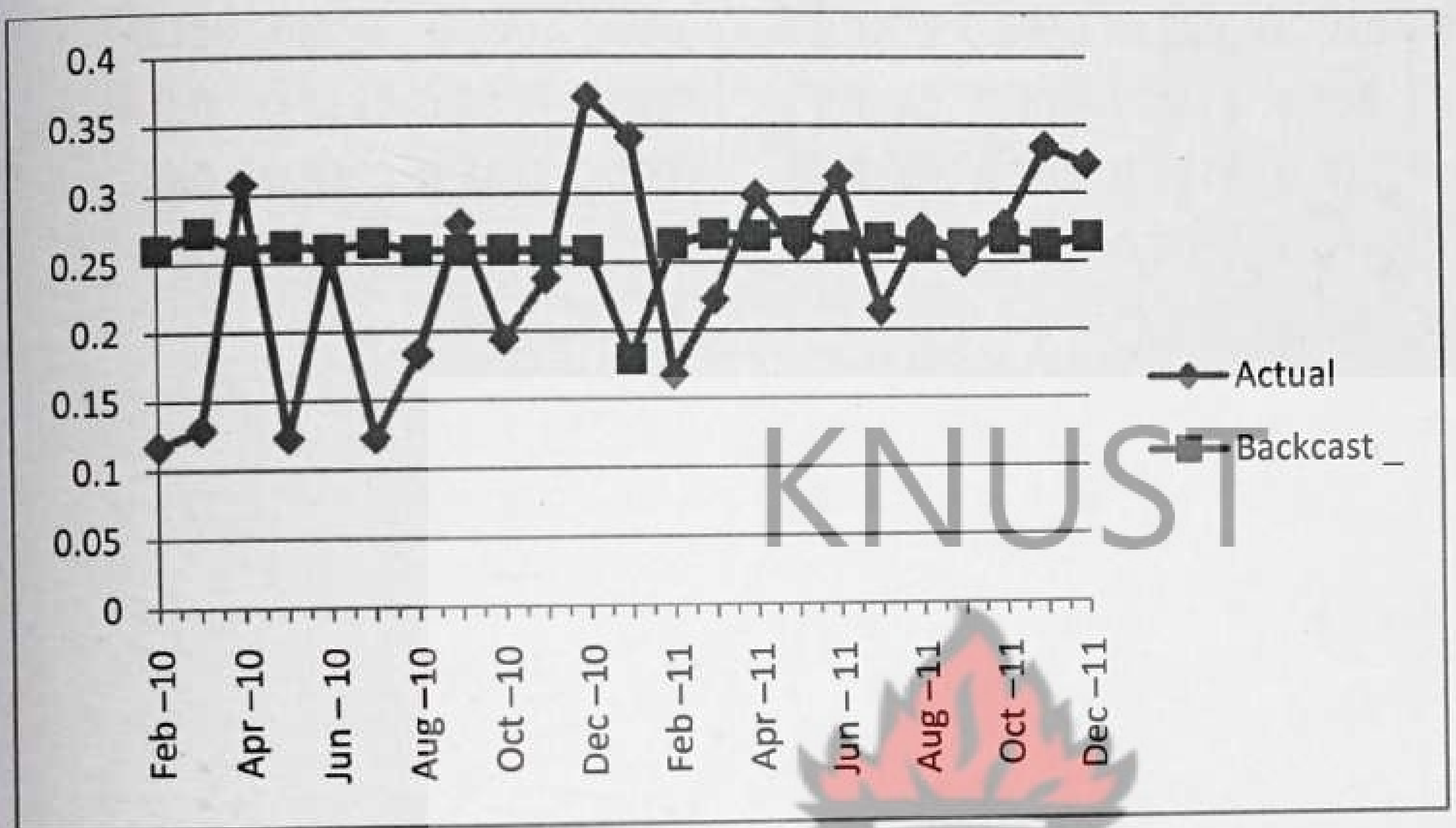



Figure 4.5: Other Loan – Actual and Backcast Proportion

**4.6 Steady State Distribution and The First Passage Time:** The long term proportion of the loan disbursements is indicate by steady state distribution of proportion of the loan disbursements which in turn be used to estimate the optimal loan portfolio mix.

In this study, the estimated steady state distribution is given as follows:

From table 4.2 in section 4.2

$$P = \begin{pmatrix} 0.250 & 0.250 & 0.250 & 0.250 \\ 0.0 & 0.3333 & 0.3333 & 0.3333 \\ 0.0 & 0.3333 & 0.3333 & 0.3333 \\ 0.500 & 0.250 & 0.0 & 0.250 \end{pmatrix}$$

$$P^2 = \begin{pmatrix} 0.250 & 0.250 & 0.250 & 0.250 \\ 0.0 & 0.3333 & 0.3333 & 0.3333 \\ 0.0 & 0.3333 & 0.3333 & 0.3333 \\ 0.500 & 0.250 & 0.0 & 0.250 \end{pmatrix}^2 = \begin{pmatrix} 0.1875 & 0.2915 & 0.229 & 0.2915 \\ 0.1665 & 0.305 & 0.2218 & 0.305 \\ 0.1665 & 0.305 & 0.2218 & 0.305 \\ 0.250 & 0.2708 & 0.2083 & 0.2708 \end{pmatrix}$$

. . . . . . . . .

. . . . . . . . .

. . . . . . . . .



$$P^7 = \begin{pmatrix} 0.250 & 0.250 & 0.250 & 0.250 \\ 0.0 & 0.3333 & 0.3333 & 0.3333 \\ 0.0 & 0.3333 & 0.3333 & 0.3333 \\ 0.500 & 0.250 & 0.0 & 0.250 \end{pmatrix}^7 = \begin{pmatrix} 0.1946 & 0.2918 & 0.2188 & 0.2918 \\ 0.1944 & 0.2914 & 0.2185 & 0.2914 \\ 0.1944 & 0.2914 & 0.2185 & 0.2914 \\ 0.1947 & 0.2914 & 0.2188 & 0.2918 \end{pmatrix}$$

$$P^8 = \begin{pmatrix} 0.250 & 0.250 & 0.250 & 0.250 \\ 0.0 & 0.3333 & 0.3333 & 0.3333 \\ 0.0 & 0.3333 & 0.3333 & 0.3333 \\ 0.500 & 0.250 & 0.0 & 0.250 \end{pmatrix}^8 = \begin{pmatrix} 0.1946 & 0.2918 & 0.2188 & 0.2918 \\ 0.1944 & 0.2914 & 0.2185 & 0.2914 \\ 0.1944 & 0.2914 & 0.2185 & 0.2914 \\ 0.1947 & 0.2914 & 0.2188 & 0.2918 \end{pmatrix}$$

Since $P^n$ tends to a limit as $n$ becomes large, it implies that the long term probabilities tend to a stationary state. That is $\lim_{n\to\infty} P^n = \begin{pmatrix} 0.194 & 0.291 & 0.218 & 0.291 \\ 0.194 & 0.291 & 0.218 & 0.291 \\ 0.194 & 0.291 & 0.218 & 0.291 \\ 0.194 & 0.291 & 0.218 & 0.291 \end{pmatrix}$



$$\pi = \begin{array}{cccc} HS & BS & ED & OT \\ 0.194 & 0.291 & 0.218 & 0.291 \end{array}$$

This means that in the long run, the housing loan will be 19.4% of the total loan, 29.1% for the business loan, 21.8% for the education loan and 29.1% for the other loan. This information also indirectly indicates the relative importance of the various loan type, besides the information on the first passage time.

**4.7 Forecast on Loan Disbursement Proportion:** One of the advantages of using a Markov model in analyzing the loan portfolio mix, besides understanding its basic characteristic is its ability to make forecast on the proportion. Forecasting the monthly proportion of all loan types is again based on the following relation $\hat{X}(T+\tau)=X(T)P^{\tau}$ where $\hat{X}\ (T+\tau\ )$ and $X(T)$ are vector of forecast proportion for $\tau$ period ahead and actual proportion respectively. For example $\hat{X}\,(25)$ is a vector of forecast proportion for January 2012. Table 4 gives the vector of monthly forecast proportion for the year 2012. The forecast proportion for the housing loan is at 19.5% for January 2012 while the proportion for business, education and other loans is at 29.2%, 21.8% and 29.2% respectively. The forecast Proportions remain the same for the rest year 2012 of the months for the housing loan is forecasted to drop at a level of 34.66% in February 2012 and finally settled down at 32.05% in July 2012 onwards.

Below is a sample calculation of the forecast. Table 4.5 gives forecast of monthly loan proportion for the year 2012.

$$\begin{pmatrix} 0.4804 & 0.1247 & 0.0538 & 0.3411 \\ 0.6685 & 0.0617 & 0.1020 & 0.1677 \\ 0.5298 & 0.1806 & 0.0673 & 0.2223 \\ 0.4738 & 0.1723 & 0.0632 & 0.2991 \end{pmatrix} * \begin{pmatrix} 0.194 & 0.291 & 0.218 & 0.291 \\ 0.194 & 0.291 & 0.218 & 0.291 \\ 0.194 & 0.291 & 0.218 & 0.291 \\ 0.194 & 0.291 & 0.218 & 0.291 \end{pmatrix} = \begin{pmatrix} 0.194 & 0.291 & 0.218 & 0.291 \\ 0.194 & 0.291 & 0.218 & 0.291 \\ 0.194 & 0.291 & 0.218 & 0.291 \\ 0.194 & 0.291 & 0.218 & 0.291 \end{pmatrix}$$

Table 4.5: Forecast of Monthly Loan Proportion for the Year 2012

| t | Month | Loan Type | | | |
|---|---|---|---|---|---|
| | | Housing | Business | Education | Other |
| 1 | January | 0.194 | 0.291 | 0.218 | 0.291 |
| 2 | February | 0.194 | 0.291 | 0.218 | 0.291 |
| 3 | March | 0.194 | 0.291 | 0.218 | 0.291 |
| 4 | April | 0.194 | 0.291 | 0.218 | 0.291 |
| 5 | May | 0.194 | 0.291 | 0.218 | 0.291 |
| 6 | June | 0.194 | 0.291 | 0.218 | 0.291 |
| 7 | July | 0.194 | 0.291 | 0.218 | 0.291 |
| 8 | August | 0.194 | 0.291 | 0.218 | 0.291 |
| 9 | September | 0.194 | 0.291 | 0.218 | 0.291 |
| 10 | October | 0.194 | 0.291 | 0.218 | 0.291 |
| 11 | November | 0.194 | 0.291 | 0.218 | 0.291 |
| 12 | December | 0.194 | 0.291 | 0.218 | 0.291 |



### 4.8 Discussion

The ~~trend~~ for the forecast proportion is marginally an upward trend for the business, education and other loan types. Business loan proportion for the year 2012 is forecasted at 29.1%. For education and other loan, the corresponding forecast is at 21.8% and 29.1% respectively. One obvious observation is that, the forecasted proportions for individual loan remains the same. This phenomenon is to be expected as Markov Chain model is a short term forecasting model. The forecast values discussed above give the policy maker an indication on the average proportion of different types of loan. In practice forecasts have to be updated as current data are available, and it is recommended that at the beginning of a month forecasts could be made when the previous month data are known. This will further improve the accuracy of the forecasts. Moreover if forecasts on the total allocation for retail banking loan is available, one would easily compute the individual loan allocation using the updated proportion forecasts. The major findings of this study are as follows:

1. The implicit characteristic of the disbursement process derived from the transition probability matrix shows that loan switching is possible in the retail banking unit. The existence of non absorbing loan further indicates that the aggregate loan disbursement data is the best proxy of the individual movement of loan disbursement among its type. Non zero probability values of switching from any loan type to business loan indicate that business loan allocation is not fully utilized. Thus signifying that business loan is of less important to retail banking

2. The rate of convergence to the equilibrium state is the measurement of how fast the process reach its equilibrium state. One would analyze the behaviour of the loan proportion forecasts as given in Table 3. It is observed that the forecast proportions beginning January 2012 for all loan types are the same. One would view matured loan demand process as the ability of the bank to declassify the loan disbursement according to its types. Thus, shorter period means that the bank is able to declassify it without much difficulty.

3. This study further stimulates the expected income on interest by using the Markov proportions and the forecast on the value of loans in each type. It had been proved that loan allocation using Markov proportions yields higher expected income on interest and considered superior to the existing policy.

# CHAPTER FIVE

# CONCLUSION AND RECOMMENDATIONS

## 5.1 Conclusion

The main aims of the study are to estimate the transition matrix using a sample of observed monthly loan movements between delinquency states and to obtain optimal loan allocation mix policy, which could be used as guiding principle on future allocation purpose.

The steady state transition matrices and the steady state probability vectors were computed for each loan type.



The discussions can be concluded in the following lines that among the four types of loans, housing loan is expected to constitute 19.4 % of the retail banking. This is followed by other type 29.1%, business loan 21.8 % and education loan 29.1%.

## 5.2 Recommendations

It is recommended that, this work could be helpful to business organizations, Agro industries and Agricultural insurance companies. In this way they would be able to advice their stake holders or clients as to what time to invest or not to invest.

The bank should train the more staff on how to use computer software for calculating loans.

The bank should employ at least on mathematician to work on loan calculations.

REFERENCE

1. Acerman, M.C., (1985). Predicting the mean annual flood from basin characteristics in Scotland Hydrological Science Journal. 30, 37-49

2. Acerman, M.C. and Sinlair, C.D., (1986). Classificatiion of drainage basins according to their characteristics: an application for flood frequency analysis in Scotland, Jornal of Hydrology, 84,365-380

3. Andrade, RFS., Schellnhuber, H.J., Claussen, M., (1998). Analysis of rainfall records possible relation to self-organized criticality. Physica A 254,557-568.

4. Austin, P.H. and Houze, RA. (1972). Analysis of the structure of precipitation patterns in New England. N. Appl. Met, 11,92-35.

5. Berger, A.N., and G.F.Udel, (1996). Universal Banking and the Future of Small Business Lending. In: Sauders, A., Walter, I., (eds), Universal Banking: Financial System Design Reconsidered. Irwin, Chicago, IL, pp.558-627.

6. Boer, R., Perdinan, M.S. Faquih, A. (2004). The use of climatic data generator to cope with daily climatic data scarcity in simulation studies. In: Proceedings of the Fourth International Crop Science Congress. Brisbane, Australia, 26 September- 1 October (available online at http://www.regional.org.au/au/cs/2004poster/2/6/1369-boerr.htm). 23/10/2011

7. Chandler, R.E. (1996b). The second-order spectral analysis of spatial-temporal rainfall models. Technical report, no. 158, Department of Statistical Science, University College London, London WCIE 6BT. http://ww.ucl.ac.uk/Stats/research/abstract.html.20. 23/10/2011

8. Chandler, R.R. (1997). A spectral method for estimating parameters in rainfall models. Benoulli, 3, No. 1,1(22).

9. Cox, D.R and Isham, V. (1994); Stochastic models of precipitation, In: Statistics for the Environment – 2: Water Related Issues (Eds V Barnet and K. Turkman), pp.3 (18) Wiley, Chichester.

10. Eguiluz, VM., Chialvo, D., Cecchi, GA., Baliki, M., Apkarian, A.V., (2005). Scale-free brain functional networks, Phys. Rev. Lett. 94,18 102.

11. Kakou, A. (1997). In preparation. PhD theses, Department of Statistical Science, University College London

12. Kittel, T.G.F., Resenbloom, N.A., Royle, J.A., Daly, C., Gibson, W.P., Fisher, H.H., Thornton, P., Yates, D.N., Aulenbach, S., Kaufman, C., McKeown, R., Bachelet, D., Schimel, D.S., (2004), VEMAP Phase 2 Bioclimatic Databse. I, Gridded historical (20th century) climate for modeling ecosystem dynamics across the conterminous, USA: Clim. Res 27, 151-170.

13. Le Cam, L,. (1961). A stochastic description of precipitation. In Proceedings of the Fourth Berkeley Symposium on Mathematical Statistics and Probability, 3, (ed. J. Neyman), pp. 165 (86. Berkeley, California.

14. Liu, X., Ren, L., Yuan, F., Yang, B. and Nanjing, P.R., (2009). Meteorological drought forecasting using Markov Chain model, in 11th International Conference on Environmental Science and Information Application Technology, Chania, Crete, Greece, 23-26.

15. Lohani, V.K., Loganathan, G.V, (1997). An early warning system for drought management using the Palmer drought index, Journal of American Water Resources Association 33,1375-1386

16. Lohani, V.K., Loganathan, G.V, and Mostaghimi, S., (1998). Long-term analysis and short-term forecasting of dry spells by Palmer Drought Severity Index, Nordic Hydrology 29(1), 21-40.

17. Lorrai, M. and Sechi, G.M., (1995). Neural nets for modeling rainfall-runoff transformation, Water Resources Management 9 (4),299-313.

18. Luk, K.C., Ball, J.E and Sharma, A., (2000). A study of optimal madel lag and spatial inputs to artificial neural network for rainfall forecasting, Journal of Hydrology 227(1-4), 56-65

19. Luckett, D.G., 1984. Money & Banking (3rd.Edition). New York: McGraw Hill. stochastic weather generators. Agric, For. Meteorol. 109, 289-296.

20. Mavromatis, T., Jones P.D. (1998). Comparison of climate change scenario construction methodologies for impact assessment studies. Agric, For Meteorol 91,51-67

21. Mavromatis, T. Jones, P.D., (1999). Evaluation of HADCM2 and direct use of daily GCMdata in impact assessment studies. Clim. Change 41,583—614.

22. McCullagh, P. and Nelder, J.A. (1989), Generalized Linear Models (second edition). Chapman and Hall, London.

23. Mearns, L.O., Giorgi, F., McDaniel. L., Shields, C., (1995). Analysis of daily variability of precipitation in a nested regional climate model: comparison with observations and doubled CO2 results. Glob. Planetary Change 10,55-78.

24. Mearns, L. O., Rosenzweig, C., Goldberg, R., (1996). The effects of changes in daily and interannual climatic variability of CERESWheat: a sensitivity study, Clim. Change 32, 257 -292.

25. Mearns, L. O., Rosenzweig, C., Goldberg, R., (1997). Mean and variance change in climate scenarios: methods, agricultural applications, and measures of uncertainty. Clim, Change 35, 367 – 396.

26. Miranda, J.G.V., Andrade, R.F.S,. (1999). Rescaled range analysis pluviometric records in northeast Brazil. Theor. Appl. Climatoal. 63, 79-88. Miranda, J.G.V., Andrade, RFS., da Silva, A.B.Ferrerira, C.S. Gonzalex, A.P., Carrera.

27. Mosley, M.P. (1981). Delimitaton of New Zealand hydrology regions. Journal of Hydrology. 49:173 -192.

28. Northrop, P.J. (1996). Modelling and statistical analysis of spatial-temporal rainfall elds, PhD thesis, Department of Statistical Science, University College London.

29. Osborn, T., Hulme, M., (1997). Development of a relationship between station and grid-box rainday frequencies for climate model validation. J.Clim 10, 1885 -1909

30. Paulo, A.A. and Pereira, L.S., (2007). Prediction SPI drough calss transitions using Markov chains. Water Management 21 (10), 1813 -18-27

31. Waymire, E., Gupta, V.K., and Rodriguez-Iturbe, I. (1984). A spectral theory of rainfall intensity at the meso-scale. Water Resources Research, 20,1453{65.