OPTIMAL LOCATION OF A HOSPITAL FACILITY

(Case study: Amansie West District)

BY

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DECLARATION

I, hereby declare that this submission is my own work towards the award of MSc and that, to the best of my knowledge, it contains no materials previously published by another person nor material which has been accepted for the award of any other degree of the University, except where due acknowledgement has been made in the text.

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ABSTRACT

The problem of locating a facility on a network to optimize certain objective criteria has been object in the past few years of growing interest for its relevance in the context of minimizing maximum travel distance between demand nodes.

This thesis considers the problem of locating a semi-obnoxious facility (hospital) as a p-center problem under the condition that some existing facilities are already located in the Amansie – West District. The Berman and Drezner (2008) method was used on a 12-node network which had four existing facilities. A new facility location was added. Three sites, namely Manso Atwere, Antoakrom and Moseaso were determined by the method.

Factor rating analysis was used to select Antoakrom and the distance of the farthest patient to the hospital at the new location (Antoakrom) was determined to be 8km.

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DEDICATION

This work is dedicated to the Almighty God who is my source of wisdom, knowledge and strength.

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I am grateful to the Almighty God for he is my source of wisdom, knowledge and strength.

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CHAPTER ONE

INTRODUCTION

1.1 INTRODUCTION

Facility location represents the process of identifying the best location for a service, commodity or production facility.

Location simply refers to the act of putting something in place or position where that thing can be identified. In a very wide sense, location problems deal with finding the right site where one or more new facilities should be placed, in order to optimize some specified criteria, which are usually related to the distance (performance measure) from the facilities to the demand points.

This optimization may vary depending on the particular objective function chosen. The function could be either; to minimize average travel time or cost, minimize average response time, minimize maximum travel time or cost, and or maximize net income (Amponsah, 2007).

A facility is considered as a physical entity that provides services. Facility location problems arise in a wide set of practical applications in different fields of study: management, economy, production planning and many others (Peton, 2002), normally is classified into three categories: desirable (non – obnoxious), semi – obnoxious and obnoxious (non – desirable), (Welch et al., 1997).

In most location problems, we are interested in locating desirable facilities. Ambulances, fire stations, schools, hospitals, post offices, warehouses, and production plants are all considered desirable facilities in this sense. Sometimes, a facility produces a negative or undesirable effect, which may be present even though a high degree of accessibility is required to the facility. If the undesirable effect outweighs the accessibility required, then the facility is said to be obnoxious. Some examples are: airports, recycling centers, prison, waste disposal sites, nuclear power stations, military installations and pollution producing industrial plants. Although necessary for society, these facilities are undesirable and often dangerous to their surroundings, (Erkut et al, 1989).

Brimberg and Juel introduced the term semi-desirable facility in 1998. They argued that the facilities cannot be classified as being purely desirable or purely obnoxious. Sometimes though a facility produces a negative or undesirable effect, this effect may be present even though a high degree of accessibility is required by the facility.

This thesis aims to locate a hospital as an example of the semi – obnoxious facility. Hospitals are useful and necessary for the community, but they are a source of negative effects, such as noise from the hospital's ambulance and also the liquid and solid waste materials from the hospitals that emits unpleasant smell which makes it undesirable. The combination of these two contradicting points makes this facility semi – obnoxious, (Gordillo et al, 2007).

A location problem is considered as a conditional p-center problem, when we are given the locations of q existing service facilities and we are to locate p additional service facilities, so as to minimize the maximum distance between the demand points, each to its nearest service facility, whether existing or new. The conditional location problem then is to locate p new facilities to serve a set of demand points given that q facilities are already located. When q is equal to zero (q = 0), the problem is unconditional. In conditional p –

center problems, once the new p locations are determined, a demand can be served either by one of the existing or by one of the new facilities whichever is the closest facility to the demand (Berman, 2008).

1.2 HISTORICAL BACKGROUND OF HOSPITALS IN GHANA

An institution that provides medical, surgical, or psychiatric care and treatment for the sick or the injured, or a healthcare institution that people go for medical needs and are assisted by nurses, doctors, surgeons and many other people with medical positions is a hospital.

The health Sector of the economy, seeks to improve the health status of all people living in Ghana, through the development and promotion of proactive policies, provision of universal access to basic health service, and the provision of quality and affordable health services.

The early hospitals were mainly set up along the coast of Ghana because of the seat of the British colonial administration. Undeniably, most of the hospitals built in the colonial era were exclusively for European patients, thus ‘Europeans Hospitals’.

Governor Guggisberg’s administration (1919 -1927) saw the need to extend medical service to the other towns to cater for the indigenous population which led to the construction and establishment of the Korle – Bu Teaching Hospital. The hospital was founded in 1923 as the Gold Coast Hospital. The then Governor, Gordon Guggisberg laid the foundation for Korle Bu Hospital in 1921, and it was finally opened on 9th October

1923. Korle – Bu Teaching Hospital is now the leading hospital in Ghana and among the best on the west coast of Africa (Buah, 1980).

In order to maximize the potential health life of all individuals resident in Ghana by reducing the incidence and prevalence of illness, injury and disability, and the prevention of premature death, many other hospitals were established to help achieve this goal.
In 1940, there were two hospital located on the hill over-looking Bantama Township designated African and European Hospitals. As their names implied, the African side treated Africans while the European side treated Europeans. However, on some rare occasions, high-ranking African government officials were given treatment in the European section. By 1952, the need to construct a new hospital to cater for the fast increasing population in Kumasi and therefore Ashanti Region arose. The European Hospital was therefore transferred to the Kwadaso Military Quarters to make way for the new project to begin. In 1955 the new hospital complex was completed and named the Kumasi Central Hospital. The name was later changed to the Komfo Anokye Hospital in honour and memory of the powerful and legendary fetish priest, Komfo Anokye.

European Missionaries and other stakeholders also contributed to the development of the health sector, before and after independent. Each region in Ghana now has its own regional hospital that is helping to improve the efficiency of health services. In addition clinics, health centers, community health planning services and health training schools increase the geographical access to health care.

1.3 BACKGROUND STUDY OF AMANSIE WEST DISTRICT

The Amansie West District (AWDA) is one of the twenty - one political administrative districts in the Ashanti Region of Ghana, created under the Government Decentralization Programme in 1988. It is located within the globe longitude and latitude 6.35°North: 1.40° South and 6.05°West: 2.05°East respectively. The district shares common geographical boundaries with six Districts namely, Atwima Nwabiagya and Atwima Mponua in the West: Bosomtwe – Atwima – Kwanwoma in the North: Amansie East and Amansie Central in the East and Upper Denkyira (in the Central Region) in the South and a regional boundary separating it from the Western Region in the South Western part of the Ashanti Region.

The District covers an area of about 1,364 square kilometers which forms about 5.4% of the total land area of the Ashanti Region. It was curved out of Amansie East (formerly Amansie District). Manso Nkwanta is the district capital.

The Amansie West District Assembly is made up of 300 settlements with the population and its size varying from each other.

According to the 2000 Population and Housing Census all the settlements have a population less than 5000. Out of the 300 settlements Mpatwuam is ranked first in terms of population that is 4140, followed by Pakyi No. 2, Manso Atwere and then Manso Nkwanta. In 2006, the population of the District was estimated at 128,533 living in over 300 settlements with a population density of 7.3 per square kilometer. About 51% of the population is females and 49% males. In light of growth with respect to the population, the District expected to have a total population of 140,043 by the year 2009 (Population census reports and group's projections).

A study was conducted and problems identified were;

1. Inadequate source of portable water
2. Inadequate Health Services
3. Lack of basic educational infrastructure.
4. Improvement in road network

(DMTDP, 2006 -2009)

The district has only one hospital, St. Martian's Hospital at Agoroyesum, with a few other communities with health centers and community clinics.

With well over 100 beds, St Martin's serves as the referral hospital for a wide area. It is 60 miles from Kumasi. The hospital is a specialist centre for the treatment of Buruli ulcers and receives Government funds for this part of its work - though payments are often delayed. As well as general medical and surgical services there are projects helping HIV/AIDS orphans and a small HIV/AIDS support group.

The health system and health problems in the district reflect the level of development of the district. There are five health centers, eight community health compounds, five maternity clinics, one mission outstation in addition to the only hospital, St. Martian's Hospital at Agroyesum which is owned and run by the Catholic Mission and serves as the District Hospital (Annual DHMT Report, Amansie West, 2006).

1.4 PROBLEM STATEMENT

Community health centers and other health facilities in the district only give first aid to patients and then refer patients to the St. Martian's hospital, resulting to congestion at the hospital due to the increasing number of patients. Based on the DMTDP, 2006 -2009 report, a second hospital was recommended for the district. This work therefore seeks to find the optimal site for an additional hospital in the district using the p – center model.

1.5 OBJECTIVES OF THE STUDY

1. To locate an additional hospital in the Amansie West District as a conditional p- center problem
2. To solve the conditional p – center problem using Berman and Drezner algorithm.

1.6 METHODOLOGY

The objective of the study is to locate a hospital in Amansie West using the Conditional P – center model.

Data on road distances between communities were collected and used.

Dijkstra's algorithm was used to find the distance matrix, d (i, j) for all pairs shortest path.

1.7 JUSTIFICATION

Provision of universal access to basic health service, and the provision of quality and affordable health services are of major concern to a critical Sector of the economy, the Ministry of Health. This sector seeks to improve the health status of all people living in Ghana. With an additional hospital in the district, it would in turn help improve on the health status of the people in the community and in the country as a whole. It is hoped that the results of this study would help to inform the health authorities in Amansie west district about the right site to locate a hospital in the district.

1.8 THESIS ORGANIZATION

Chapter one presents the study background, significance, objectives of the study and structure of the thesis.

The second chapter deals with the literature review.

Chapter three present the research methodology.

Data, analysis and discussions is considered in chapter four.

Conclusion and recommendation of the study is in chapter five.

CHAPTER TWO

LITERATURE REVIEW

2.0 INTRODUCTION

In location problem we want to locate specific type of facility. Usually we look for the best way to serve a set of communities whose location and demands are known. This implies one needs to decide on:

i. The number and location of the facilities to serve the demand

ii. Size and capacity of each facility

iii. The allocation of the demand points to open facilities

iv. Optimizing some objective location function.

Most location models deals with desirable facilities, such as warehouse, service and transportation centers, emergency services, etc, which interact with the customers and where distance travel is involved. As a consequence, typical criteria for such decision include minimizing some function of the distance between facilities and/ or clients (i. e., average travel time, average response time, cost function of travel or response time, maximum travel time or cost, etc.). However, during the last two decades, those responsible for the overall development of the area, where the new facility is going to be located (i.e., central government, local authorities) as well as those living in the area (population), are showing an increasing interest in preserving the area's quality of life. Hence, new words have been introduced in the location theory, such as: noxious, obnoxious, semi obnoxious, hazardous, etc. As examples of undesirable facilities we can mention; nuclear and military installations, equipment emitting particular smell or noise, warehouses containing flammable materials, regions containing refuse or waste materials, garbage dumps, sewage plants, correctional centers, etc.

The traditional optimality criterion of closeness (to locate the facility as close as possible to the customers) is replaced by the opposite criterion of how far away from the customers can the facility be placed to ensure accessibility to the demand point. This generates the NIMBY syndrome (NOT-IN-MY-BACK-YARD) (Capitivo and Climaco, 2008).

Buettcher(2004) describe the p-Center problem, as the Min-Max Multicenter problem or the Facility Location problem, to be a famous problem from operations research.

He classified the optimization problem into three different types, depending on which of the restrictions applied.

i. The general optimization problem in which the choice of the distance function d is not restricted in any way.
ii. The metric problem in which d satisfies the triangle inequality.
iii. The metric and symmetric problem in which d(x; y) = d(y; x), and d satisfies the triangle inequality.

It was realized that, the metric, asymmetric p-Center problem had remained unstudied even ten years after its symmetric counterpart had been finally solved (by presenting an algorithm with optimal approximation factor) in 1986. In 1998, the O(log*(n)) approximation algorithm found by Panigrahy and Vishwanathan(1998) was published. Thus, it was clear that - in contrast to the general p-Center problem without any restrictions to the distance function - this problem could be approximated. And the approximation was a very good one, although the algorithm could not guarantee a constant-factor approximation. A few years later, in 2003, it turned out that this is the best approximation ratio possible (unless P = NP), (Halperin et al., 2003). So, the p-Center problem is one of the rare problems for which essentially nothing was known, then a first non-trivial algorithm was found that can approximate the problem, and it was already this very first algorithm that achieves the best approximation ratio possible. This alone is already very exciting. What makes the p-Center problem even more fascinating is the approximability of the problem. log*(n) is one of the functions where the discrepancy between theoretical results and practical consequences becomes very clear. $\mathrm{Log}_2^*(n)$ can be assumed ≤ 6 for all practical purposes. Yet, from a theoretical point of view, there is a clear distinction between constant approximation factor and log*(n).

2.1 SOME APPROACHES TO FACILITY LOCATION PROBLEMS

In Malczewski and Ogryczak (1990) the location of hospitals is formulated as a multi-objective optimization problem and an interactive approach DIN AS, Dynamimic interactive network analysis system (Ogryczak et al., 1989) based on the so called reference point approach (Wierzbicki,1982) is presented. A real application is presented, considering eight sites for potential location and at least four new hospitals to be built, originating in hundred and sixty three alternative location patterns each of them generating many possible allocation schemes. The authors mention that the system can be used to support a group decision - making process making the final decision less subjective. They also observed that during the interactive process the decision – makers have gradually learned about the set of feasible alternatives and in consequence of this leaning process they have change their preference and priorities.

Erkut and Neuman (1992) present a mixed integer linear model for undesirable facility location. The objectives considered are total cost minimization, total opposition minimization and equity minimization.

Caruso et al (1993) present a model for planning an urban solid waste management system. Incineration, composition and recycling are considered for the processing phase and sanitary landfills are considered for the disposal phase. Heuristic techniques (embedded in the reference point approximation) are used to solve the model and, as a consequence, “approximate Pareto solutions” are obtained. By varying the reference point, different solutions can be obtained. The results for a case study (Lombardy region in Italy) are presented and discussed.

Wyman and Kuby (1993, 1995) present a multi-objective mixed integer programming model for the location of hazardous material facilities (including the technologies choice variable) with three objectives functions (cost, risk and equity).

Melachrinoudis et al (1995) propose a dynamic multi-period capacitated mixed integer programming model for the location of sanitary landfills.

Fonseca and Captivo (1996; 2006; 2007) study the location of semi obnoxious facilities as a discrete location problem on a network. Several bi-criteria models are presented considering two conflicting objectives, the minimization of obnoxious effect and the maximization of the accessibility of the community to the closest open facility. Each of these objectives is considered in two different ways, trying to optimize its average value over all the communities or trying to optimize its worst value. The Euclidean distance is used to evaluate the obnoxious effect and the shortest path distance is used to evaluate the accessibility. The obnoxious effect is considered inversely proportional to the weighted Euclidean distance between demand points and open facilities, and demand directly proportional to the population in each community. All the models are solved using Chalmet et al (1986), non- interactive algorithm for Bi-criteria Integer Linear Programming modified to an interactive procedure by Ferreira et al (1994). Several equity measures are computed for each non-denominated solution presented to the decision-maker, in order to increase the information available to the decision –maker about the set of possible solutions.

Ferreira et al (1996) present a bi-criteria mixed integer linear model for the facility location where the objectives are the minimization of total cost and the minimization of environmental pollution at facility sites. The interactive approach of Ferreira et al (1994) is used to obtain and analyze non-dominated solutions.

Giannikos (1998) presents a discrete model for the location of disposal or treatment facilities and transporting hazardous waste through a network linking the population centers that produce the waste and the candidate locations for the treatment facilities method to choose the location for a waste treatment facility in a region of Finland.

Costa et al (2008) develop two bi-criteria models for single allocation hub location problems. In both models the total cost is the first criteria to be minimized. Instead of using capacity constraints to limit the amount of flow that can be received by the hubs, a second objective function is used, trying to minimize the time to process the flow entering the hubs. In the first model, total time is considered as the second criteria and, in the second model, the maximum service time for the hubs are minimized. Non-dominated solutions are generated using an interactive decision-aid approach developed for bi-criteria integer linear programming problems. Both bi-criteria models are tested on a set of instances, analyzing the corresponding non-dominated solutions set and studying the reasonableness of the hubs flow charge for these non-dominated solutions.

Ballou (1998) discusses a selected number of facility location methods for strategic planning. He further classifies the more practical methods into a number of categories in the logistics network, which include single–facility location, multi–facility location, dynamic facility location, retail and service location.

Christopher and Wills (1972) comprehensively present that whether the problem of depot location is static or dynamic, ‘Infinite Set’ approaches and ‘Feasible Set’ approach can be identified. The infinite set approach assumes that a warehouse is flexible to be located anywhere in a certain area. The feasible set approach assumes that only a finite number of known sites are available as warehouse locations. They believe the centre of gravity method is a sort of infinite set model.

Goldengorin et al, (1999) considered the simple plant location problem. This problem often appears as a sub-problem in other combinatorial problems. Several branch and bound techniques have been developed to solve these problems. The thesis considered new approaches called branch and peg algorithms, where pegging refers to assigning

values to variables outside the branching process. An exhaustive computational experiment shows that the new algorithms generate less than 60% of the number of sub-problems generated by branch and bound algorithms, and in certain cases requires less than 10% of the execution times required by branch and bound algorithms.
Firstly, for each sub-problem generated in the branch and bound tree, a powerful pegging procedure is applied to reduce the size of the sub-problem. Secondly, the branching function is based on predictions made using the Beresnev function of the sub-problem at hand. They saw that branch and peg algorithms comprehensively out perform branch and bound algorithms using the same bound, taking on the average, less than 10% of the execution time of branch and bound algorithms when the transportation cost matrix is dense. The main recommendation from the results of the experiment is that branch and peg algorithms should be used to solve SPLP instances.

Ballou (1998) states that exact centre of gravity approach is simple and appropriate for locating one depot in a region, since the transportation rate and the point volume are the only location factors. Given a set of points that represent source points and demand points, along with the volumes needed to be moved and the associated transportation rates, an optimal facility location could be found through minimizing total transportation cost. In principle, the total transportation cost is equal to the volume at a point multiplied by the transportation rate to ship to that point multiplied by the distance to that point.
Furthermore, Ballou outlines the steps involved in the solution process in order to implement the exact centre of gravity approach properly.

2.2 P- CENTRE LOCATION PROBLEM

The conditional location problem is to locate p new facilities to serve a set of demand points given that q facilities are already located. When q is equal to zero (q = 0), the problem is unconditional. In conditional p – center problems, once the new p locations are determined, a demand can be served either by one of the existing or by one of the new facilities whichever is the closest facility to the demand (Berman, 2008).

The *p*-center problem seeks the location of *p* facilities. Each demand point receives its service from the closest facility. The objective is to minimize the maximal distance for all demand points. The *p*-center problem consists of choosing *p* facilities among a set of *M* possible locations and assigning *N* clients to them in order to minimize the maximum distance between a client and the facility to which it is allocated.

Elloumi et al. (2004) , presented a new integer linear programming formulation for this min-max problem with a polynomial number of variables and constraints, and show that its LP relaxation provides a lower bound tighter than the classical one. Moreover, they showed that an even better lower bound *LB**, obtained by keeping the integrality restrictions on a subset of the variables, can be computed in polynomial time by solving at most $O(\log_2(NM))$ linear programs, each having *N* rows and *M* columns. They also show that, when the distances satisfy triangle inequalities, *LB** is at least one third of the optimal value. Finally, they used the *LB** in an exact solution method and report extensive computational results on test problems from the literature. For instances where the triangle inequalities are satisfied, their method out performs the running time of other recent exact methods by an order of magnitude. In addition, it is the first one to solve large instances of size up to $N = M = 1, 817$.

Krumke, 1995 considered the generalization of the p-Center Problem, which is called the α-*Neighbor p-Center Problem* ($p - CENTER^{(\alpha)}$). Given a complete edge-weighted network, the goal is to minimize the maximum distance of a client to it's α nearest neighbor in the set of p centers. He shows that in general finding a O ($2^{poly(|V|)}$)-approximation for $p - CENTER^{(\alpha)}$ is NP-hard(Garey and Johnson, 1979), where $|V|$ denotes the number of nodes in the network. If the distances are required to satisfy the triangle inequality, there can be no polynomial time approximation algorithm with a ($2 - \varepsilon$) performance guarantee for any fixed $\varepsilon > 0$ and any fixed $\alpha \leq p$, unless $P = NP$. For this case, He presented a simple yet efficient algorithm that provides a 4-approximation for $\alpha \geq 2$.

Considering the p-Center Problem with Connectivity Constraint, let G (V, E, W) be a graph with n-vertex-set V and m-edge-set E in which each edge e is associated with a positive distance W (e). Chung-Kung et al. (2006), proposes an additional practical constraint which restricted the p vertices, to be connected. The resulting problem is called the connected p-Center problem (the CpC problem). They first show that the CpC problem is NP-Hard on bipartite graphs and split graphs. Then, an O (n)-time algorithm for the problem on trees is proposed. Finally, the algorithm was extended to trees with forbidden vertices. That is some vertices in V cannot be selected as center vertices, and the time-complexity is also O (n). Meanwhile, it was identified that other variants of the traditional p-Center problem is also a very important issue. For example, just restricting that the *p*-center must be "total", thus, the subgraph induced by the *p*-center has no isolated vertices, is another typical practical variant.

Chen and Chen (2009), presented a new relaxation algorithm for solving the conditional continuous and discrete p-center problems. In the continuous p-center problem, the location of the service facilities can be anywhere in the two-dimensional Euclidean space. In the discrete variant there is a finite set of potential service points to choose from. An analogous representation of the discrete p-center problem is the p-center problem on networks. In the p-center problem on networks, both the demand points and the potential service points are located on a weighted undirected graph, and the distance between any two points is the cost of the shortest path between them. They assumed that, there are a finite number of values for the optimal solution of an unconditional p-center problem. They use the assumption to implement the subroutine Get- Next Bound (Lower-Bound) which returns the smallest value, among the possible values for the optimal solution, which is greater than Lower-Bound. Also the subroutine Find Feasible Solution (Sub, r), which answers the question: ``is there a solution to the sub-problem with value less than r?" (And if so, finds such a solution).

Hassin et al. (2003) introduce a local search strategy that suits combinatorial optimization problems with a min-max (or max-min) objective. According to this approach, solutions are compared lexicographically rather than by their worst coordinate. They apply this approach to the *p*-center problem. Based on a computational study, the lexicographic local search proved to be superior to the ordinary local search. This superiority was demonstrated by a worst-case analysis.

Cheng et al. (2005) worked on the Improved Algorithm for the p-Center Problem on Interval Graphs with Unit Lengths. They presented an O (n) time algorithm for the problem under the assumption that the endpoints of the intervals are sorted, which improves on the existing best algorithm for the problem that has a run time of O (pn). They modeled the network as a graph G = (V, E), where V is the vertex set with |V | = n and E is the edge set with |E| = m. it was assumed that, the demand points coincide with the vertices, and the location of the facilities was restricted to the vertices. Also they assumed that each edge of E has a unit length. It remains an interesting question whether they could develop an approximation algorithm for the p-center problem on interval graphs with general edge lengths.

Finally in the new formulation for the conditional p-median and p-center problems, Berman and Drezner (2007), discuss the conditional p-median and p-center problems on a network. Demand nodes are served by the closest facility whether existing or new. Rather than creating a new location for an artificial facility and force the algorithm to locate a new facility there by creating an artificial demand point, the distance matrix was just modified. They suggested solving both conditional problems by defining a modified shortest distance matrix $\widehat{D}$. The formulation they presented in this paper provided better results than those obtained by the best known formulation. The work presented in this thesis is based on this paper.

CHAPTER THREE

METHODOLOGY

3.0 NETWORK LOCATION MODELS

Network location problems are concerned with finding the right locations to place one or more facilities in a network of demand points, i.e., customer locations represented by nodes in the network, that optimize a certain objective function related to the distance between the facilities and the demand points.

3.1 BASIC FACILITY LOCATION MODELS

This section presents models classified according to their consideration of distance. The maximum distance models and total (or average) distance.

3.1.1 TOTAL OR AVERAGE DISTANCE MODELS

Many facility location planning situations in the public and private sections are concerned with the total travel distance between facilities and demand nodes. An example in the private sector might be the location of production facilities that receive their inputs from established sources by truckload deliveries. In the public sector, one might want to locate a network of service providers such as licensing bureaus in such a way as to minimize the total distance that customers must traverse to reach their closest facility. This approach may be viewed as an “efficiency” objective as opposed to the “equity” objective of minimizing the maximum distance, which is mentioned in other models.

1. **P-median problem**: The p-median model (Hakimi, 1964; 1965) finds the locations of p facilities to minimize the demand-weighted total distance between demand nodes and the facilities to which they are assigned.
2. **The Maxisum Location Problem:** The maxisum location problem seeks the locations of p facilities such that the total demand-weighted distance between demand nodes and the facilities to which they are assigned is maximized.

3.1.2 MAXIMUM DISTANCE MODELS

In some locations problems, an acceptable distance is set a priori. In the facility location literature, a priori acceptable distances such as these are known as "covering" distances. Demand within the covering distance of its closest facility is considered "covered." An underlying assumption of this measure of covering distance is that demand is fully satisfied if the nearest facility is within the coverage distance and is not satisfied if the closest facility is beyond that distance. That is, being closer to a facility more than the covering distance does not improve satisfaction.

1. **Set covering location model:** The objective of this model is to locate the minimum number of facilities required to "cover" all of the demand nodes (Toregas et al., 1971).
2. **Maximal covering location problem:** The objective of the Maximal covering location problem (MCLP) is to locate a predetermined number of facilities, p, in such a way as to maximize the demand that is covered. Thus, the MCLP assumes that there may not be enough facilities to cover all of the demand nodes. If all

nodes cannot be covered, then the model seeks the siting scheme that covers the most demand (Church and ReVelle, 1974).

3. **The p-dispersion problem**: The p-dispersion problem (PDP) is only concerned with the distance between new facilities and the objective is to maximize the minimum distance between any pair of facilities. Potential applications of the PDP include the siting of military installations where separation makes them more difficult to attack or locating franchise outlets where separation reduces cannibalization among stores (Kuby, 1987).

4. **P-Center Problem**: The p-center problem (Hakimi, 1964;1965) addresses the problem of minimizing the maximum distance that demand is from its closet facility given that we are siting a pre-determined number of facilities. There are several possible variations of the basic model. The “vertex” p-center problem restricts the set of candidate facility sites to the nodes of the network while the “absolute” p-center problem permits the facilities to be anywhere along the arcs or the network. Both versions can be either weighted or unweighted. In the unweighted problem, all demand nodes are treated equally. In the weighted model, the distances between demand nodes and facilities are multiplied by a weight associated with the demand node. For example, this weight might represent a node’s importance or, more commonly, the level of its demand.

3.2 THE P-CENTER PROBLEM

The p-center problem is the problem of locating p (facilities) in order to minimizes the maximum response time (the time between a demand site and the nearest facility), using a given number of p.

With the above definition and the decision variable;

$W =$ The maximum distance between a demand node and the facility to which it is assigned

$$y_{i\,j} = \begin{cases} 1 \text{ if the demand node } i \text{ is assigned to a facility at node } j \\ 0 \text{ if not} \end{cases}$$

The p-center problem can be formulated as follows:

Maximize W ……………………………………………….. (1)

Subject to:

$\sum_{j\in J} x_j = p$ …………………………… (2)

$\sum_{j\in J} y_{i\,j} = 1 \quad \forall i \in I$ …………………… (3)

$y_{i\,j} - x_j \leq 0 \quad \forall i \in I, j \in J$ ………………. (4)

$W - \sum_{j\in J} h_i\, d_{i\,j} y_{i\,j} \geq 0 \;\; \forall i \in I$ ……………… (5)

$x_j \in \{0, 1\} \quad \forall j \in J$ …………….. (6)

$y_{i\,j} \in \{0, 1\} \quad \forall i \in I, j \in J$ ………. (7)

The objective function (1) minimizes the maximum demand –weighted distance between each demand node and its closest open facility. Constraint (2) stipulates that p facilities are to be located. Constraint set (3) requires that each demand node be assigned to exactly one facility. Constraint set (4) restricts demand node assignments only to open facilities.

Constraint (5) defines the lower bound on the maximum demand – weighted distance, which is being minimized. Constraint set (6) established the siting decision variable as binary. Constraint set (7) requires the demand at a node to be assigned to one facility only. Constraint set (7) can be replaced by $y_{i\,j} \geq 0\ \forall i \in I; j \in J$ because constraint set (4) guarantees that $y_{i\,j} \leq 1$. if some y_{ij} are fractional, we simply assign node i to its closest open facility (Current et al, 2001)

3.3 THE CONDITIONAL P-CENTER PROBLEM

The conditional location problem is to locate p new facilities to serve a set of demand points given that q facilities are already located. When q = 0, the problem is unconditional. In the conditional p-center problems, once the new p locations are determined, a demand can be served either by one of the existing or by one of the new facilities whichever is the closest facility to the demand.

Consider a network G = (N, L)

Where;

N = the set of nodes, |N| = n

L = the set of links.

Let $d(x\,,y)$ be the shortest distance between any $x, y\ \in\ G$. Suppose that there is a set Q (|Q| = q) of existing facilities. Let $Y\ =\ (Y_1, \ldots, Y_q\,)$ and $X\ =\ (X_1, X_2, \ldots, X_p)$ be vectors of size q and p respectively, where Y_i is the location of existing facility i and X_i is the location of new facility i . Without any loss of generality we do not need to assume that $Y_i\ \in N$. The conditional p-center location problem is to;

$$Min[g(x) = \max_{i=1,\ldots n} \min \{d(X,i), d(Y,i)\}]$$

Where (X, i) and $d(Y, i)$, is the shortest distance from the closest facility in X and Y respectively to the node i,(Berman and Simchi-Levi, 1990).

3.4 BERMAN AND SIMCHI-LEVI ALGORITHM

Berman and Simchi-Levi (1990) suggest to solve the conditional p-center problem on a network by an algorithm that requires one-time solution of an unconditional (p + 1)-center problem.

Step 1 Let D be a distance matrix with rows corresponding to demands and columns corresponding to potential locations. For the p-center problem the columns of D correspond to the set of local centers C. The idea is to create a new potential location representing all existing facilities. If a demand point is utilizing the services of an existing facility, it will use the services of the closest existing facility. Therefore, the distance between a demand point and the new location is the minimum distance calculated for all existing facilities.

Step 2 To force the creation of a facility at the new location, a new demand point is created with a distance of zero to the new potential location and a large distance to all other potential locations. The new distance matrix $\hat{\hat{D}}$ is constructed by adding a new location a_o (a new column) to D so that the columns represent the Q existing locations and a new demand point v_0 with an arbitrary positive weight. For each demand point (node) i, $d(i, a_o) = \min_{k \in Q}\{d_{ik}\}$ and $d(v_0, a_0) = 0$. For each potential location (node) j, $d(v_0, j) = M$ (M is a large number). Again the nodes in Q and potential locations Q are removed.

Step 3 Find the optimal new location using $\hat{\hat{D}}$ for the network with the objective function

$$Min[g(x) = \max_{i=1,\dots n} \min \{d(X,i), d(Y,i)\}]$$

To illustrate the approach, we consider the network of figure 3.1; the numbers next to the links are lengths. Suppose that existing set of facilities are nodes *2* and *3*, and only one facility is to be located ($p = 1$).

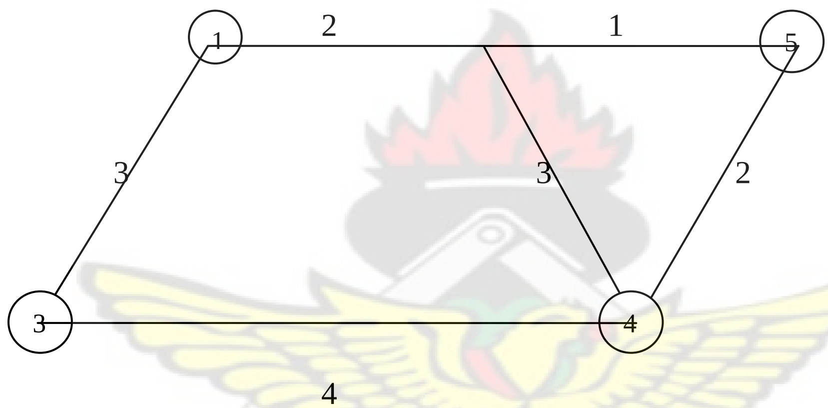


Figure3. 1: Sample network for *p*-center problem

Step 1 By using either the Floyd's algorithm or the Dijkstra's algorithm, we obtained shortest paths matrix (distance matrix) D, for the above network as shown in Table 3.1. In Table 3.1, column 1and row 1 represents the demand nodes and potential location respectively, and each other row represents the interconnected distances.

Table 3.1 All pair shortest path distance matrix, D

Demand nodes	Potential location				
	1	**2**	**3**	**4**	**5**
1	0	2	3	5	3
2	2	0	5	3	1
3	3	5	0	4	6
4	5	3	4	0	2
5	3	1	6	2	0

Step 2 Determine a modified shortest distance matrix, $\widehat{\widehat{D}}$ by: adding a new location a_o (a new column) to D and adding a new demand point v_0 (a new row) with an arbitrary positive weight to the rows. For each demand point (node) i , $d(i, a_o) = \min_{k \in Q}\{d_{ik}\}$ and $d(v_0, a_0) = 0$. For each potential location (node) j , $d(v_0, j) = M$ (M is a large number), and this is shown in Table 3.2.

Table 3.2a the modified Distance Matrix, $\hat{\hat{D}}$

Demand nodes	Potential location					
	1	**2**	**3**	**4**	**5**	$\boldsymbol{a_0}$
1	0	2	3	5	3	2
2	2	0	5	3	1	0
3	3	5	0	4	6	0
4	5	3	4	0	2	3
5	3	1	6	2	0	1
$\boldsymbol{v_0}$	M	M	M	M	M	0

The nodes in Q representing existing nodes are removed, and this is shown in Table 3.2b.

Table 3.2b The Modified Distance Matrix, $\hat{\hat{D}}$ with nodes 2 and 3 removed

Demand nodes	Potential location			
	1	**4**	**5**	$\boldsymbol{a_0}$
1	0	5	3	2
4	5	0	2	3
5	3	2	0	1
$\boldsymbol{v_0}$	M	M	M	0

Step 3 finds the optimal new location using the distance matrices, $\widehat{\widehat{D}}$ and the objective function,

$$Minimize\ [g(x) = \max_{i=1,\dots n} \min\{d(X,i), d(Y,i)\}]$$

Taking the Distance matrix, $\widehat{\widehat{\widehat{D}}}$

$$Minimize\ g(x) = \max_{i=1,\dots n} \min\{d(X,i), d(Y,i)\}$$

$X = \{1, 4, 5, a_0\}$ $Y = \{2, 3\}$

At $X = 1$

$i = 1,$ $d(1,1), d(2,1), d(3,1)$

0, 2, 3

$min = 0$

$i = 2,$ $d(1,2), d(2,2), d(3,2)$

2, 0, 5

$min = 0$

$i = 3,$ $d(1,3), d(2,3), d(3,3)$

3, 5, 0

$min = 0$

$i = 4,$ $d(1,4), d(2,4), d(3,4)$

5, 3, 4

$min = 3$

$i = 5,$ $d(1,5), d(2,5), d(3,5)$

$3, \qquad 1, \qquad 6$

$min = 1$

Therefore at $X = 1$ the maximum = 3, at node 4. The results are then summarized and shown below in Table 3.3; with column 5 representing the maximum distance between demand nodes and rows represent the minimum interconnected distances.

Table 3.3 Optimal location $Min\ (g(x))$, using $\hat{\hat{D}}$

Demand Nodes	**1**	**4**	**5**	***Maximum***
1	0	3	1	3
4	2	0	1	2
5	2	2	0	2
		Minimum $\longrightarrow$		2

From Table 3.3, it is easy to verify that, the optimal new location using $\hat{\hat{D}}$ is node 5 with an objective function value of 2.

3.5 BERMAN AND DREZNER'S ALGORITHM

Berman and Drezner (2008) discuss a very simple algorithm that solves the conditional p-center problem on a network. The algorithm requires one-time solution of an unconditional p-center problem using an appropriate shortest distance matrix. Rather than creating a new location for an artificial facility and force the algorithm to locate a new facility there by creating an artificial demand point, they just modify the distance matrix.

Step 1 Let D be a distance matrix with rows corresponding to demands and columns corresponding to potential locations.

Step 2 solved the conditional problem is by defining a modified shortest distance matrix, from D to $\widehat{D}$

$$\widehat{D} = min\left\{d_{ij} \min_{k \in Q}\{d_{ik}\}\right\} \forall i \in N, j \in C(center),$$

Note that $\widehat{D}$ is not symmetric even when D is symmetric.

The unconditional p-center problem using the appropriate $\widehat{D}$ solves the conditional p-center problem. This is so since if the shortest distance from node i to the new p facilities are larger than $\min_{k \in Q}\{d_{ik}\}$, then the shortest distance to the existing q facilities is utilized. Notice that the size of $\widehat{D}$ is $n \times |C|$ for the conditional p-center.

Step 3 Find the optimal new location using $\widehat{D}$ for the network with the objective function

$$Min[g(x) = \max_{i=1,\dots n} \min\{d(X,i), d(Y,i)\}]$$

To demonstrate the algorithm, a 5-node network depicted in Figure 3.1 is considered where the numbers next to the links are lengths. We solve the 1-center problem. Suppose that the existing set of facilities are Q = {2, 3} and p = 1, the new facility to be located.

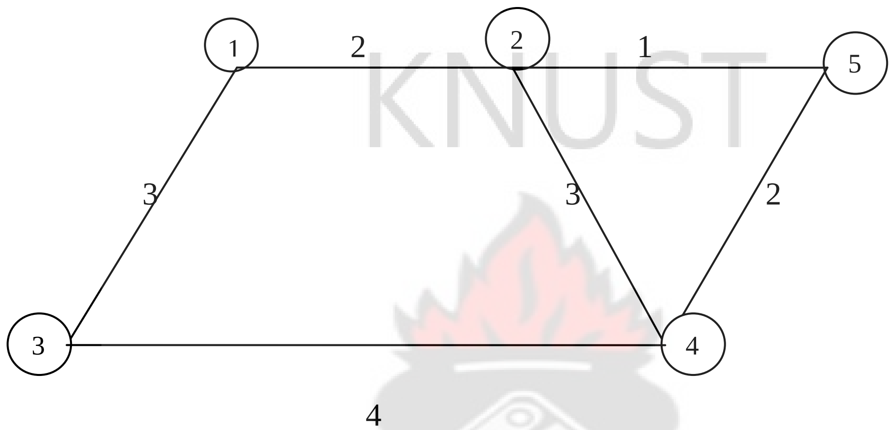


Figure 3.1: Sample network for *p*-center problem

Step 1 By using either the Floyd's algorithm or the Dijkstra's algorithm, we obtained shortest paths matrix (distance matrix) D, for the above network. With column 1and row 1 represents the demand nodes and potential location respectively, and each other row represents the interconnected distances.

Table 3.1: All Pairs Shortest Path Distance Matrix, *D*

Demand nodes	Potential location				
	1	**2**	**3**	**4**	**5**
1	0	2	3	5	3
2	2	0	5	3	1
3	3	5	0	4	6
4	5	3	4	0	2
5	3	1	6	2	0

Step 2 Determine a modified shortest distance matrix by:

$$\widehat{D} = min\left\{d_{ij} \min_{k\in Q}\{d_{ik}\}\right\} \forall i \in N, j \in C(center)$$

For node 1

$i = 1, j = 1$

$$\widehat{D_{11}} = min\left\{d_{11} \min_{\{2,3\}\in Q}\{d_{12}, d_{13}\}\right\}$$

$$\widehat{D_{11}} = min\left\{0 \min_{\{2,3\}\in Q}\{2,3\}\right\} = 0$$

$i = 1, j = 2$

$$\widehat{D_{12}} = min\{d_{12} \min_{\{2,3\}\in Q}\{d_{12}, d_{13}\}\}$$

$$\widehat{D_{12}} = min\left\{2 \min_{\{2,3\}\in Q}\{2,3\}\right\} = 2$$

$i = 1, j = 3$

$$\widehat{D_{13}} = min\left\{d_{13} \min_{\{2,3\}\in Q}\{d_{12}, d_{13}\}\right\}$$

$$\widehat{D_{13}} = min\left\{3 \min_{\{2,3\}\in Q}\{2,3\}\right\} = 2$$

$i = 1, j = 4$

$$\widehat{D_{14}} = min\{d_{14} \min_{\{2,3\}\in Q}\{d_{12}, d_{13}\}\}$$

$$\widehat{D_{14}} = min\left\{5 \min_{\{2,3\}\in Q}\{2,3\}\right\} = 2$$

$i = 1, j = 5$

$$\widehat{D_{15}} = min\left\{d_{15} \min_{\{2,3\}\in Q}\{d_{12}, d_{13}\}\right\}$$

$$\widehat{D_{15}} = min\left\{3 \min_{\{2,3\}\in Q}\{2,3\}\right\} = 2$$

The results are then summarized and shown below in Table 3.4; with column one and row one represent demand node and potential location respectively , and the other rows represents the interconnecting distances.

Table 3.4a Modified Shortest path Distance matrix, $\widehat{D}$

Demand nodes	Potential location				
	1	**2**	**3**	**4**	**5**
1	0	2	2	2	2
2	0	0	0	0	0
3	0	0	0	0	0
4	3	3	3	0	2
5	1	1	1	1	0

Step 2b The existing facility nodes ($Q = \{2,3\}$) are removed the modified shortest path distance matrix, $\widehat{D}$ and this is shown in Table 3.4b below.

Table 3.4b Modified Shortest path Distance matrix, $\widehat{D}$ with existing facility nodes removed

Demand nodes	Potential location		
	1	**4**	**5**
1	0	2	2
4	3	0	2
5	1	1	0

Step 3 Find the optimal new location using $\widehat{D}$ for the network with the objective function

$$Min[g(x) = \max_{i=1,\dots n} \min \{d(X,i), d(Y,i)\}]$$

$X = \{1, 4, 5\}$ $\qquad\qquad\qquad\qquad$ $Y = \{2, 3\}$

At $\boldsymbol{X} = 1$

$i = 1,\quad d(1,1), d(2,1), d(3,1)$

$0, \quad 2, \quad 2$

$min = 0$

$i = 2,\quad d(1,2), d(2,2), d(3,3)$

$0, \quad 0, \quad 0$

$min = 0$

$i = 3,\quad d(1,3), d(2,3), d(3,3)$

$0, \quad 0, \quad 0$

$min = 0$

$i = 4,\quad d(1,4), d(2,4), d(3,4)$

$3, \quad 3, \quad 3$

$min = 3$

$i = 5,\quad d(1,5), d(2,5), d(3,5)$

$1, \quad 1, \quad 1$

$min = 1$

Therefore at $X = 1$ the maximum = 3, at node 4

The results are then summarized and shown in Table 3.5, with column 5 representing the maximum distance between demand nodes and rows represent the minimum interconnected distances.

Table 3.5 Optimal location $Min\ (g(x))$, using $\widehat{D}$

Demand node	**1**	**4**	**5**	***Maximum***
1	0	3	1	3
4	2	0	1	2
5	2	2	0	2
Minimum →				2

From Table 3.5 it is easy to verify that the optimal location is node 5 with an objective function value of 2.

3.6 FACTOR RATING METHOD

In using factor rating method, the following steps must be followed:

1. Develop a list of relevant factors.
2. Assign a weight to each factor to reflect the views of the community.
3. Develop a scale for each factor.
4. Have related people to score each relevant factor, using the scale developed in 3 above.

5. Multiply the score by the weight assigned to each factor and total the score for each location.
6. Make a recommendation based on the maximum point score.

(Amponsah, 2007)

Table 3.6 illustrates an example of the factor rating analysis of which a company must decide among three sites for the construction of a new Satellite Clinic. The firm selected seven factors listed below as a basis for evaluation and have assigned rating weights on each factor.

Table 3.6 Relative scores on factors for a Satellite Clinic

Factor	Factor Name	Rating Weight	Ratio of Rate	Location A	Location B	Location C
1	Land acquisition	5	0.25	25	20	20
2	Power – source availability and cost	3	0.15	12	10.5	15
3	Workforce attitude and cost	4	0.2	6	12	14
4	Population size	2	0.1	1	8	6
5	Community desirability	2	0.1	9	6	8
6	Equipment suppliers in area	3	0.15	7.5	9	13.5
7	Economic Activities	1	0.05	4.5	3	3
Total →				65	68.5	79.5

Clearly from their respective aggregate scores shown in Table 3.6, location C would be recommended since it has the highest aggregate.

CHAPTER FOUR

DATA COLLECTION, ANALYSIS AND RESULTS

4.1 DATA COLLECTION AND ANALYSIS

The length of shortest path between connecting communities is of interest in this study. In view of this the set of distances of roads linking communities was obtained from the Amansie West District Assembly (District Planning Office) and a few others obtained through self survey (example; distance from Abore to Kaniago and then to Adubia).

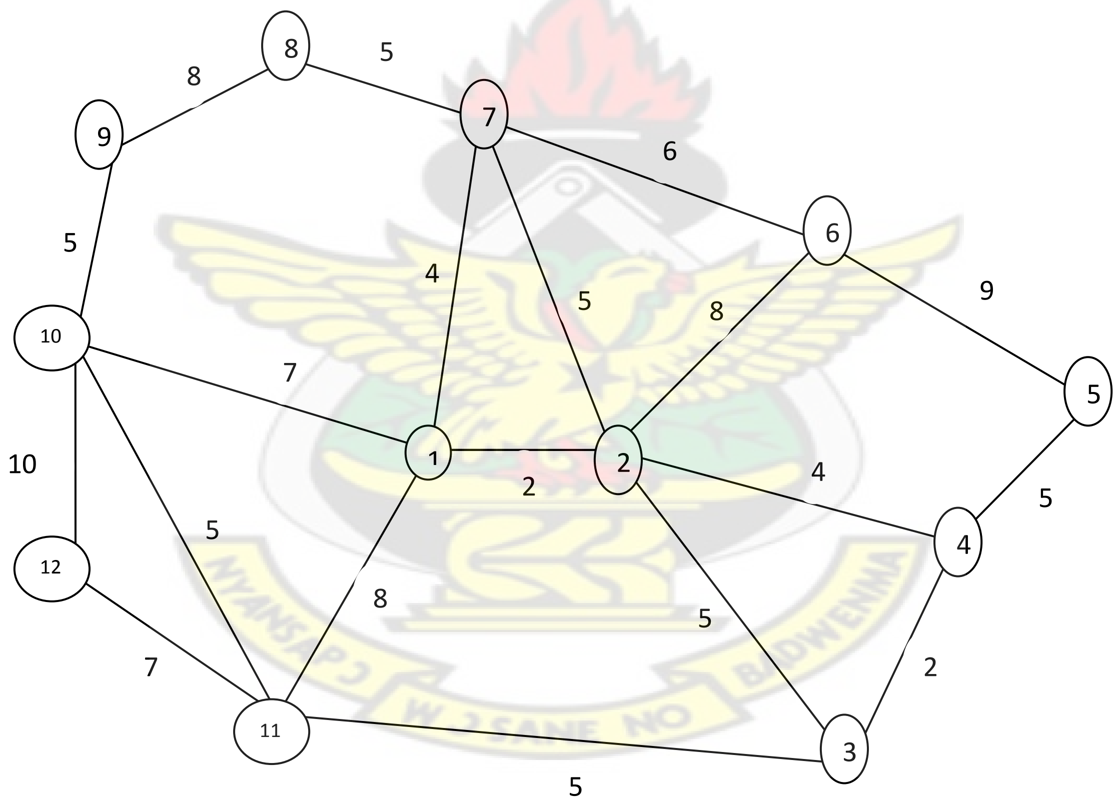
To ensure that the location decisions resulting from the model are not only profitable but also equitable and sustainable, there was a need to develop a twelve node network, thus taking into consideration the twelve major towns. Below are the twelve major towns and their respective assigned nodes, shown in Table 4.1.

Table 4.1 Some Major Towns in Amansie West District

Town	**Node**	**Town**	**Node**
Manso Nkwanta	1	Moseaso	7
Manso Atwere	2	Ahwerewa	8
Agroyesum	3	Mpatuam	9
Mem	4	Abore	10
Odahu	5	Adubia	11
Antoakrom	6	Kaniago	12

With an existing hospital at Agroyesum, a clinic at Manso Nkwanta, and health cnters at Ahwerewa and Adubia. These communities form the set of existing facilities, thus node 3, node 1, node 8 and node 11respectively. The above data is then developed into a network of figure 4.1 below. Numbers in the circles are the nodes representing the twelve major towns and numbers next to the links are the various road distances in kilometers.

Figure 4.1 Network of major towns in Amansie West District



From the network in Figure 4.1 an all pair shortest path distance matrix, *D* is developed and shown in Table 4.2 by using the Dijkstra's algorithm. Column one and row one represents the demand nodes and potential location respectively; the other rows also represent the interconnecting road distances.

Table 4.2 All Pairs Shortest Path Distance Matrix *D*

Demand nodes	Potential location											
	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**
1	0	2	7	6	11	10	4	9	12	7	8	15
2	2	0	5	4	9	8	5	10	14	9	10	17
3	7	5	0	2	7	13	10	15	15	10	5	12
4	6	4	2	0	5	12	9	14	17	12	7	14
5	11	9	7	5	0	9	14	19	22	17	12	19
6	10	8	13	12	9	0	6	11	19	17	18	25
7	4	5	10	9	14	6	0	5	13	11	12	19
8	9	10	15	14	19	11	5	0	8	13	17	23
9	12	14	15	17	22	19	13	8	0	5	10	15
10	7	9	10	12	17	17	11	13	5	0	5	10
11	8	10	5	7	12	18	12	17	10	5	0	7
12	15	17	12	14	19	25	19	23	15	10	7	0

4.2 BERMAN AND DREZNER'S ALGORITHM

At this point, we use the Berman and Drezner's algorithm (2008) to solve the problem.

We begin by formulating the conditional p- center problem as

$$Min[g(x) = \max_{i=1,\dots n} \min \{d(X,i), d(Y,i)\}]$$

Let $d(x, y)$ be the shortest distance between any $x, y \in G$. Suppose that there is a set Q (|Q| = q) of existing facilities. Let $Y = (Y_1, \dots, Y_q)$ and $X = (X_1, X_2, \dots, X_p)$ be vectors of size q and p respectively, where Y_i is the location of existing facility i and X_i is the location of new facility i . Where (X, i) and $d(Y, i)$, is the shortest distance from the closest facility in X and Y respectively to the node i,(Berman and Simchi-Levi, 1990).

Considering figure 4.1 the set of location of new facilities $X = \{2, 4, 5, 6, 7, 9, 10, 12\}$ and the set of location of existing facilities $Y = \{1, 3, 8, 11\}$, then the conditional p- center problem is to:

$$Minimize = \left[g(x) = \max min \begin{Bmatrix} d(2,i), d(1,i), d(3,i), d(8,i), d(11,i), \\ d(4,i), d(1,i), d(3,i), d(8,i), d(11,i), \\ d(5,i), d(1,i), d(3,i), d(8,i), d(11,i), \\ d(6,i), d(1,i), d(3,i), d(8,i), d(11,i), \\ d(7,i), d(1,i), d(3,i), d(8,i), d(11,i), \\ d(9,i), d(1,i), d(3,i), d(8,i), d(11,i), \\ d(10,i), d(1,i), d(3,i), d(8,i), d(11,i), \\ d(12,i), d(1,i), d(3,i), d(8,i), d(11,i), \end{Bmatrix} \right]$$

Where $i = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\}$

4.2.1 THE ALGORITHM

Steps:

1. Let D be a distance matrix with rows corresponding to demands and columns corresponding to potential locations.
2. Solved the conditional problem is by defining a modified shortest distance matrix, from D to $\widehat{D}$

$$\widehat{D} = min\left\{\left(d_{ij}\right), (\min_{k\in Q}\{d_{ik}\})\right\} \forall i \in N, j \in C(center)$$

   Note that $\widehat{D}$ is not symmetric even when D is symmetric.

3. Find the optimal new location using $\widehat{D}$ for the network with the objective function

$$Min[g(x) = \max_{i=1,\ldots n} \min \{d(X,i), d(Y,i)\}]$$

4.3 BERMAN AND DREZNER'S SOLUTION

By considering the all pair shortest path distance for the twelve node network of the Amansie West District in Table 4.2 above, a new shortest path distance matrix is formed. Thus from D to $\widehat{D}$.

4.3.1 MODIFIED SHORTEST DISTANCE MATRIX, $\widehat{D}$

By defining a modified shortest distance matrix$\widehat{D}$:

$$\widehat{D}_{ij} = min\left\{(d_{ij}),(\min_{k\in Q}\{d_{ik}\})\right\}, \forall i \in N, j \in C(center)$$

Table 4.3 summarizes the results of $\widehat{D}$ into a modified shortest path distance matrix with column one representing demand nodes and all other columns representing the minimum interconnecting distance when demand nodes are compared with existing facility nodes. The results of $\widehat{D}$ for the twelve node network under study is elaborated in Appendix 1

Table 4.3a Modified Shortest Distance Matrix, $\widehat{D}$.a

Demand nodes	Potential location											
	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**
1	0	0	0	0	0	0	0	0	0	0	0	0
2	2	0	2	2	2	2	2	2	2	2	2	2
3	0	0	0	0	0	0	0	0	0	0	0	0
4	2	2	2	0	2	2	2	2	2	2	2	2
5	7	7	7	5	0	7	7	7	7	7	7	7
6	10	8	10	10	9	0	6	10	10	10	10	10
7	4	4	4	4	4	4	0	4	4	4	4	4
8	0	0	0	0	0	0	0	0	0	0	0	0
9	8	8	8	8	8	8	8	8	0	5	8	8
10	5	5	5	5	5	5	5	5	5	0	5	5
11	0	0	0	0	0	0	0	0	0	0	0	0
12	7	7	7	7	7	7	7	7	7	7	7	0

It is noted that by comparing road distances with existing set of nodes ($Q = \{1,3,8,11\}$) the minimum is always zero. Hence the set of demand nodes and potential location of the existing facilities is removed from the modified shortest path distance matrix $\widehat{D}$, and this is shown in Table 4.3b.

Table 4.3b Modified shortest path distance matrix, $\widehat{D}$ with set Q removed

Demand node	Potential location							
	2	**4**	**5**	**6**	**7**	**9**	**10**	**12**
2	0	2	2	2	2	2	2	2
4	2	0	2	2	2	2	2	2
5	7	5	0	7	7	7	7	7
6	8	10	9	0	6	10	10	10
7	4	4	4	4	0	4	4	4
9	8	8	8	8	8	0	5	8
10	5	5	5	5	5	5	0	5
12	7	7	7	7	7	7	7	0

4.3.2 FINDING THE OPTIMAL LOCATION

Taking the optimal new location using the modified shortest distance matrix, $\widehat{D}$ with the objective function;

$$Min\ [g(x) = \max_{i=1,\dots,n} \min\{d(X,i), d(Y,i)\}]$$

$$Y = \{1, 3, 8.11\} \qquad X = \{2, 4, 5, 6, 7, 9, 10, 12\}$$

For node 2

$i = 1,$

$min\{d(2,1), d(1,1), d(3,1), d(8,1), d(11,1)\}$

$min\{0, 0, 0, 0, 0\}$

$min\{0\}$

$i = 2,$

$min\{d(2,2), d(1,2), d(3,2), d(8,2), d(11,2)\}$

$min\{0, 2, 2, 2, 2\}$

$min\{0\}$

$i = 3,$

$min\{d(2,3), d(1,3), d(3,3), d(8,3), d(11,3)\}$

$min\{0, 0,0, 0, 0\}$

$min\{0\}$

$i = 4,$

$min\{d(2,4), d(1,4), d(3,4), d(8,4), d(11,4)\}$

$min\{2,2,2,2,2\}$

$min\{2\}$

$i = 5,$

$min\{d(2,5), d(1,5), d(3,5), d(8,5), d(11,5)\}$

$min\{7,7,7,7,7\}$

$min\{7\}$

$i = 6,$

$min\{d(2,6), d(1,6), d(3,6), d(8,6), d(11,6)\}$

$min\{8, 10, 10, 10, 10\}$

$min\{8\}$

$i = 7,$

$min\{d(2,7), d(1,7), d(3,7), d(8,7), d(11,7)\}$

$min\{4, 4, 4, 4, 4\}$

$min\{4\}$

$i = 8,$

$min\{d(2,8), d(1,8), d(3,8), d(8,8), d(11,8)\}$

$min\{0,0,0,0,0\}$

$min\{0\}$

$i = 9,$

$min\{d(2,9), d(1,9), d(3,9), d(8,9), d(11,9)\}$

$min\{8,8,8,8,8\}$

$min\{8\}$

$i = 10,$

$min\{d(2,10), d(1,10), d(3,10), d(8,10), d(11,10)\}$

$min\{5,5,5,5,5\}$

$min\{5\}$

$i = 11,$

$min\{d(2,11), d(1,11), d(3,11), d(8,11), d(11,11)\}$

$min\{0,0,0,0,0\}$

$min\{0\}$

$i = 12,$

$min\{d(2,12), d(1,12), d(3,12), d(8,12), d(11,12)\}$

$min\{7,7,7,7,7\}$

$min\{7\}$

$Maximum\ at\ node\ 2 = 8$

A summary of the results is presented in Table 4.4 below. With column 10 representing the maximum distance between demand nodes and rows represent the minimum interconnected distances.

Table 4.4 Optimal Location, $Min\ g(x)$ using $\widehat{D}$

Demand Nodes	2	4	5	6	7	9	10	12	*Maximum*
2	0	2	7	8	4	8	5	7	8
4	2	0	5	10	4	8	5	7	10
5	2	2	0	9	4	8	5	7	9
6	2	2	7	0	4	8	5	7	8
7	2	2	7	6	0	8	5	7	8
9	2	2	7	10	4	0	5	7	10
10	2	2	7	10	4	5	0	7	10
12	2	2	7	10	4	8	5	0	10
Minimum →									8

From the results above in Table 4.4, by using the modified shortest distance matrix $\widehat{D}$, it is easy to verify that the optimal new location can be either at node 6, node 7 or node 2, with an objective function value of 8.

4.4 FACTOR RATING METHOD

Based on the Berman and Drenzer's algorithm, three different locations was found to be optimal, thus Manso Atwere (node 2), Antoakrom(node6), and Moseaso (node 7). To decide among the three communities, the factor rating method is used.

Considering the location of a hospital, eight relevant factors listed below is noted as shown in Table 4.5 below with the respective rating weight attached to each factor.

Table 4.5 Relevant Factors and rating weight

Factor	Factor Name	Rating Weight
1	Land acquisition	6
2	Community desirability	2
3	Population Size	4
4	Work force attitude and cost	4
5	Utilities(water, power-source availability and cost)	3
6	Access to public, private and other means of transportation	2
7	Proximity of raw materials and suppliers	3
8	Availability of and proximity to supporting service (example: social services, security and allied health services)	1

Table 4.6 below summarizes the results opinion leaders and other related people score each factor, ranging from one up to hundred for all three locations.

Table 4.6 Location Rate on a 1to 100 basis

Factor	Manso Atwere	Antoakrom	Moseaso
1	60	100	80
2	90	90	90
3	80	100	60
4	30	70	50
5	70	80	50
6	70	90	60
7	60	80	30
8	50	80	30

At this point the ratio of rating weight is multiplied by the rating scores of the communities for each particular factor. The results are then shown in Table 4.7.

Table 4.7 Rating Scores of locations

Factor	Rating Weight	Ratio of Rate	Manso Atwere	Antoakrom	Moseaso
1	6	0.24	14.4	24	19.2
2	2	0.08	7.2	7.2	7.2
3	4	0.16	12.8	16	9.6
4	4	0.16	4.8	11.2	8
5	3	0.12	8.4	9.6	6
6	2	0.08	5.6	7.2	4.8
7	3	0.12	7.2	9.6	3.6
8	1	0.04	2	3.2	1.2
Total →			**62.4**	**88**	**59.6**

Clearly from their respective aggregate scores, Antoakrom or node 6 would be recommended since it has the highest aggregate of 88.

4.5 DISCUSSION

With the algorithm demonstrated above, considering the 12 – node network depicted in Figure 4.1, and solving the conditional 1- center problem with $Q = \{1, 3, 8, 11\}$ and p = 1. It is easy to verify that D are $\widehat{D}$, are the distance matrix shown in Table 4.2 and Table 4.3. The optimal new location using the modified distance matrix $\widehat{D}$, thus by using the Berman and Drezner's algorithm the new hospital can arbitrarily be located at node 6 (Antoakrom), node 2 (Manso Atwere) or node 7(Moseaso), with an objective function value of Min $g(x) = 8$.

Moreover considering the factor rating method, it is clear that Antoakrom has the highest aggregate, hence node 6 or Antoakrom is recommended out of the three communities for the location of a new hospital in the Amansie West District.

CHAPTER FIVE

CONCLUSION AND RECOMMENDATION

5.1 CONCLUSION

The main objective of the study was to use the conditional p – center model to locate an additional hospital in the Amansie West District.

Considering the objective function ($Min[g(x)$]as shown in page 43 and using the method of Berman and Drezner (2008), the facility can be located at any of these three nodes; node 6 (Antoakrom) or node 2 (Manso Atwere) or node 7(Moseaso). By identifying relevant factors for the location of a hospital, determination of rating weights and analysis of scores, the best alternative found among the three communities is Antoakrom, which had the highest aggregate of 88(eighty eight) as compared with Manso Atwere and Moseaso in the using the factor rating method.

The minimum objective function value obtained was 8 kilometers. This implies that the minimum distance travelled by the farthest patient to the new facility at Antoakrom is 8 kilometers.

5.2 RECOMMENDATION

In view of the results obtained in the study, the following recommendations are made;

1. The governments as well as individuals who would like to invest in the establishment of a hospital, in the district are advised to locate it at Antoakrom, or Manso Atwere or Moseaso.
2. Researchers, who will like to do further work, could consider Z. Drenzer's Algorithm that requires solving $0(\log n)$ unconditional p – center problems.

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APPENDIX

$$\widehat{D}_{ij} = min\left\{(d_{ij}), (\min_{k \in Q}\{d_{ik}\})\right\}, \forall i \in N, j \in C(center)$$

For node 1

$i = 1 \quad j = 1 \quad Q = \{1,3,8,11\}$

$\widehat{D}_{11}$

$= min\{d_{11}, min\{d_{11}, d_{13}, d_{18}, d_{1\,11}\}\}$

$= min\{0, min\{0,7,9,8\}\}$

$= 0$

$i = 1 \quad j = 2 \quad Q = \{1,3,8,11\}$

$\widehat{D}_{12}$

$= min\{d_{12}, min\{d_{11}, d_{13}, d_{18}, d_{1\,11}\}\}$

$= min\{2, min\{0,7,9,8\}\}$

$= 0$

$i = 1 \quad j = 3 \quad Q = \{1,3,8,11\}$

$\widehat{D}_{13}$

$= min\{d_{13}, min\{d_{11}, d_{13}, d_{18}, d_{1\,11}\}\}$

$= min\{7, min\{0,7,9,8\}\}$

$= 0$

$i = 1 \quad j = 4 \quad Q = \{1,3,8,11\}$

$\widehat{D}_{14}$

$= min\{d_{14}, min\{d_{11}, d_{13}, d_{18}, d_{1\,11}\}\}$

$= min\{6, min\{0,7,9,8\}\}$

$= 0$

$i = 1 \quad j = 5 \quad Q = \{1,3,8,11\}$

$\widehat{D}_{15}$

$= min\{d_{15}, min\{d_{11}, d_{13}, d_{18}, d_{1\,11}\}\}$

$= min\{11, min\{0,7,9,8\}\}$

$= 0$

$i = 1 \quad j = 6 \quad Q = \{1,3,8,11\}$

$\widehat{D}_{16}$

$= min\{d_{16}, min\{d_{11}, d_{13}, d_{18}, d_{1\,11}\}\}$

$= min\{10, min\{0,7,9,8\}\}$

$= 0$

$i = 1 \quad j = 7 \quad Q = \{1,3,8,11\}$

$\hat{D}_{17}$

$= min\{d_{17}, min\{d_{11}, d_{13}, d_{18}, d_{1\,11}\}\}$

$= min\{4, min\{0,7,9,8\}\}$

$= 0$

$i = 1 \quad j = 8 \quad Q = \{1,3,8,11\}$

$\hat{D}_{18}$

$= min\{d_{18}, min\{d_{11}, d_{13}, d_{18}, d_{1\,11}\}\}$

$= min\{9, min\{0,7,9,8\}\}$

$= 0$

$i = 1 \quad j = 1 \quad Q = \{1,3,8,11\}$

$\hat{D}_{19}$

$= min\{d_{19}, min\{d_{11}, d_{13}, d_{18}, d_{1\,11}\}\}$

$= min\{12, min\{0,7,9,8\}\}$

$= 0$

$i = 1 \quad j = 10 \quad Q = \{1,3,8,11\}$

$\hat{D}_{1\,10}$

$= min\{d_{1\,10}, min\{d_{11}, d_{13}, d_{18}, d_{1\,11}\}\}$

$= min\{7, min\{0,7,9,8\}\}$

$= 0$

$i = 1 \quad j = 11 \quad Q = \{1,3,8,11\}$

$\hat{D}_{1\,11}$

$= min\{d_{1\,11}, min\{d_{11}, d_{13}, d_{18}, d_{1\,11}\}\}$

$= min\{8, min\{0,7,9,8\}\}$

$= 0$

$i = 1 \quad j = 12 \quad Q = \{1,3,8,11\}$

$\hat{D}_{1\,12}$

$= min\{d_{1\,12}, min\{d_{11}, d_{13}, d_{18}, d_{1\,11}\}\}$

$= min\{15, min\{0,7,9,8\}\}$

$= 0$

For node 2

$i = 2 \quad j = 1 \quad Q = \{1,3,8,11\}$

$\hat{D}_{21}$

$= min\{d_{21}, min\{d_{21}, d_{23}, d_{28}, d_{2\,11}\}\}$

$= min\{2, min\{2,5,10,10\}\}$

$= 2$

$i = 2 \quad j = 2 \quad Q = \{1,3,8,11\}$

$\widehat{D}_{22}$

$= min\{d_{22}, min\{d_{21}, d_{23}, d_{28}, d_{2\,11}\}\}$

$= min\{0, min\{2, 5, 10, 10\}\}$

$= 0$

$i = 2 \quad j = 3 \quad Q = \{1,3,8,11\}$

$\widehat{D}_{23}$

$= min\{d_{23}, min\{d_{21}, d_{23}, d_{28}, d_{2\,11}\}\}$

$= min\{5, min\{2, 5, 10, 10\}\}$

$= 2$

$i = 2 \quad j = 4 \quad Q = \{1,3,8,11\}$

$\widehat{D}_{24}$

$= min\{d_{24}, min\{d_{21}, d_{23}, d_{28}, d_{2\,11}\}\}$

$= min\{4, min\{2,5,10,10\}\}$

$= 2$

$i = 2 \quad j = 5 \quad Q = \{1,3,8,11\}$

$\widehat{D}_{25}$

$= min\{d_{25}, min\{d_{21}, d_{23}, d_{28}, d_{2\,11}\}\}$

$= min\{9, min\{2,5, 10, 10\}\}$

$= 2$

$i = 2 \quad j = 6 \quad Q = \{1,3,8,11\}$

$\widehat{D}_{26}$

$= min\{d_{26}, min\{d_{21}, d_{23}, d_{28}, d_{2\,11}\}\}$

$= min\{8, min\{2,5, 10, 10\}\}$

$= 2$

$i = 2 \quad j = 7 \quad Q = \{1,3,8,11\}$

$\widehat{D}_{27}$

$= min\{d_{27}, min\{d_{21}, d_{23}, d_{28}, d_{2\,11}\}\}$

$= min\{5, min\{2,5, 10, 10\}\}$

$= 2$

$i = 2 \quad j = 8 \quad Q = \{1,3,8,11\}$

$\widehat{D}_{28}$

$= min\{d_{28}, min\{d_{21}, d_{23}, d_{28}, d_{2\ 11}\}\}$

$= min\{10, min\{2,5, 10, 10\}\}$

$= 2$

$i = 2 \quad j = 9 \quad Q = \{1,3,8,11\}$

$\widehat{D}_{29}$

$= min\{d_{29}, min\{d_{21}, d_{23}, d_{28}, d_{2\ 11}\}\}$

$= min\{14, min\{2,5, 10, 10\}\}$

$= 2$

$i = 2 \quad j = 10 \quad Q = \{1,3,8,11\}$

$\widehat{D}_{2\ 10}$

$= min\{d_{2\ 10}, min\{d_{21}, d_{23}, d_{28}, d_{2\ 11}\}\}$

$= min\{9, min\{2,5, 10, 10\}\}$

$= 2$

$i = 2 \quad j = 11 \quad Q = \{1,3,8,11\}$

$\widehat{D}_{2\ 11}$

$= min\{d_{2\ 11}, min\{d_{21}, d_{23}, d_{28}, d_{2\ 11}\}\}$

$= min\{10, min\{2,5, 10, 10\}\}$

$= 2$

$i = 2 \quad j = 12 \quad Q = \{1,3,8,11\}$

$\widehat{D}_{2\ 12}$

$= min\{d_{2\ 12}, min\{d_{21}, d_{23}, d_{28}, d_{2\ 11}\}\}$

$= min\{17, min\{2,5, 10, 10\}\}$

$= 2$

For node 3

$i = 3 \quad j = 1 \quad Q = \{1,3,8,11\}$

$\widehat{D}_{31}$

$= min\{d_{31}, min\{d_{31}, d_{33}, d_{38}, d_{3\ 11}\}\}$

$= min\{7, min\{7,0, 15, 5\}\}$

$= 0$

$i = 3 \quad j = 2 \quad Q = \{1,3,8,11\}$

$\widehat{D}_{32}$

$= min\{d_{32}, min\{d_{31}, d_{33}, d_{38}, d_{3\ 11}\}\}$

$= min\{5, min\{7,0, 15, 5\}\}$

$= 0$

$i = 3 \quad j = 3 \quad Q = \{1,3,8,11\}$

$\widehat{D}_{33}$

$= min\{d_{33}, min\{d_{31}, d_{33}, d_{38}, d_{3\ 11}\}\}$

$= min\{0, min\{7,0, 15, 5\}\}$

$= 0$

$i = 3 \quad j = 4 \quad Q = \{1,3,8,11\}$

$\widehat{D}_{34}$

$= min\{d_{34}, min\{d_{31}, d_{33}, d_{38}, d_{3\ 11}\}\}$

$= min\{2, min\{7,0, 15, 5\}\}$

$= 0$

$i = 3 \quad j = 5 \quad Q = \{1,3,8,11\}$

$\widehat{D}_{35}$

$= min\{d_{35}, min\{d_{31}, d_{33}, d_{38}, d_{3\ 11}\}\}$

$= min\{7, min\{7,0, 15, 5\}\}$

$= 0$

$i = 3 \quad j = 6 \quad Q = \{1,3,8,11\}$

$\widehat{D}_{36}$

$= min\{d_{36}, min\{d_{31}, d_{33}, d_{38}, d_{3\ 11}\}\}$

$= min\{13, min\{7,0, 15, 5\}\}$

$= 0$

$i = 3 \quad j = 7 \quad Q = \{1,3,8,11\}$

$\widehat{D}_{37}$

$= min\{d_{37}, min\{d_{31}, d_{33}, d_{38}, d_{3\ 11}\}\}$

$= min\{10, min\{7,0, 15, 5\}\}$

$= 0$

$i = 3 \quad j = 8 \quad Q = \{1,3,8,11\}$

$\widehat{D}_{38}$

$= min\{d_{38}, min\{d_{31}, d_{33}, d_{38}, d_{3\ 11}\}\}$

$= min\{15, min\{7,0, 15, 5\}\}$

$= 0$

$i = 3 \quad j = 9 \quad Q = \{1,3,8,11\}$

$\widehat{D}_{39}$

$= min\{d_{39}, min\{d_{31}, d_{33}, d_{38}, d_{3\ 11}\}\}$

$= min\{15, min\{7,0, 15, 5\}\}$

$= 0$

$i = 3 \quad j = 10 \quad Q = \{1,3,8,11\}$

$\widehat{D}_{3\,10}$

$= min\{d_{3\,10}, min\{d_{31}, d_{33}, d_{38}, d_{3\,11}\}\}$

$= min\{10, min\{7,0, 15, 5\}\}$

$= 0$

$i = 3 \quad j = 11 \quad Q = \{1,3,8,11\}$

$\widehat{D}_{3\,11}$

$= min\{d_{3\,11}, min\{d_{31}, d_{33}, d_{38}, d_{3\,11}\}\}$

$= min\{5, min\{7,0, 15, 5\}\}$

$= 0$

$i = 3 \quad j = 12 \quad Q = \{1,3,8,11\}$

$\widehat{D}_{3\,12}$

$= min\{d_{3\,12}, min\{d_{31}, d_{33}, d_{38}, d_{3\,11}\}\}$

$= min\{12, min\{7,0, 15, 5\}\}$

$= 0$

For node 4

$i = 4 \quad j = 1 \quad Q = \{1,3,8,11\}$

$\widehat{D}_{41}$

$= min\{d_{41}, min\{d_{41}, d_{43}, d_{48}, d_{4\,11}\}\}$

$= min\{6, min\{6, 2, 14, 7\}\}$

$= 2$

$i = 4 \quad j = 2 \quad Q = \{1,3,8,11\}$

$\widehat{D}_{42}$

$= min\{d_{42}, min\{d_{41}, d_{43}, d_{48}, d_{4\,11}\}\}$

$= min\{4, min\{6, 2, 14, 7\}\}$

$= 2$

$i = 4 \quad j = 3 \quad Q = \{1,3,8,11\}$

$\widehat{D}_{43}$

$= min\{d_{43}, min\{d_{41}, d_{43}, d_{48}, d_{4\,11}\}\}$

$= min\{2, min\{6, 2, 14, 7\}\}$

$= 2$

$i = 4 \quad j = 4 \quad Q = \{1,3,8,11\}$

$\widehat{D}_{44}$

$= min\{d_{44}, min\{d_{41}, d_{43}, d_{48}, d_{4\,11}\}\}$

$= min\{0, min\{6, 2, 14, 7\}\}$

$= 0$

$i = 4 \quad j = 5 \quad Q = \{1,3,8,11\}$

$\widehat{D}_{45}$

$= min\{d_{45}, min\{d_{41}, d_{43}, d_{48}, d_{4\,11}\}\}$

$= min\{5, min\{6, 2, 14, 7\}\}$

$= 2$

$i = 4 \quad j = 6 \quad Q = \{1,3,8,11\}$

$\widehat{D}_{46}$

$= min\{d_{46}, min\{d_{41}, d_{43}, d_{48}, d_{4\,11}\}\}$

$= min\{12, min\{6, 2, 14, 7\}\}$

$= 2$

$i = 4 \quad j = 7 \quad Q = \{1,3,8,11\}$

$\widehat{D}_{47}$

$= min\{d_{47}, min\{d_{41}, d_{43}, d_{48}, d_{4\,11}\}\}$

$= min\{9, min\{6, 2, 14, 7\}\}$

$= 2$

$i = 4 \quad j = 8 \quad Q = \{1,3,8,11\}$

$\widehat{D}_{48}$

$= min\{d_{48}, min\{d_{41}, d_{43}, d_{48}, d_{4\,11}\}\}$

$= min\{14, min\{6, 2, 14, 7\}\}$

$= 2$

$i = 4 \quad j = 9 \quad Q = \{1,3,8,11\}$

$\widehat{D}_{49}$

$= min\{d_{49}, min\{d_{41}, d_{43}, d_{48}, d_{4\,11}\}\}$

$= min\{17, min\{6, 2, 14, 7\}\}$

$= 2$

$i = 4 \quad j = 10 \quad Q = \{1,3,8,11\}$

$\widehat{D}_{4\,10}$

$= min\{d_{4\,10}, min\{d_{41}, d_{43}, d_{48}, d_{4\,11}\}\}$

$= min\{12, min\{6, 2, 14, 7\}\}$

$= 2$

$i = 4 \quad j = 11 \quad Q = \{1,3,8,11\}$

$\widehat{D}_{4\,11}$

$= min\{d_{4\,11}, min\{d_{41}, d_{43}, d_{48}, d_{4\,11}\}\}$

$= min\{7, min\{6, 2, 14, 7\}\}$

$= 2$

$i = 4 \quad j = 12 \quad Q = \{1,3,8,11\}$

$\widehat{D}_{4\,12}$

$= min\{d_{4\,12}, min\{d_{41}, d_{43}, d_{48}, d_{4\,11}\}\}$

$= min\{14, min\{6, 2, 14, 7\}\}$

$= 2$

For node 5

$i = 5 \quad j = 1 \quad Q = \{1,3,8,11\}$

$\widehat{D}_{51}$

$= min\{d_{51}, min\{d_{51}, d_{53}, d_{58}, d_{5\,11}\}\}$

$= min\{11, min\{11,7,19,12\}\}$

$= 7$

$i = 5 \quad j = 2 \quad Q = \{1,3,8,11\}$

$\widehat{D}_{52}$

$= min\{d_{52}, min\{d_{51}, d_{53}, d_{58}, d_{5\,11}\}\}$

$= min\{9, min\{11,7,19,12\}\}$

$= 7$

$i = 5 \quad j = 3 \quad Q = \{1,3,8,11\}$

$\widehat{D}_{53}$

$= min\{d_{53}, min\{d_{51}, d_{53}, d_{58}, d_{5\,11}\}\}$

$= min\{7, min\{11,7,19,12\}\}$

$= 7$

$i = 5 \quad j = 4 \quad Q = \{1,3,8,11\}$

$\widehat{D}_{54}$

$= min\{d_{54}, min\{d_{51}, d_{53}, d_{58}, d_{5\,11}\}\}$

$= min\{5, min\{11,7,19,12\}\}$

$= 5$

$i = 5 \quad j = 5 \quad Q = \{1,3,8,11\}$

$\widehat{D}_{55}$

$= min\{d_{55}, min\{d_{51}, d_{53}, d_{58}, d_{5\,11}\}\}$

$= min\{0, min\{11,7,19,12\}\}$

$= 0$

$i = 5 \quad j = 6 \quad Q = \{1,3,8,11\}$

$\widehat{D}_{56}$

$= min\{d_{56}, min\{d_{51}, d_{53}, d_{58}, d_{5\,11}\}\}$

$= min\{9, min\{11,7,19,12\}\}$

$= 7$

$i = 5 \quad j = 7 \quad Q = \{1,3,8,11\}$

$\widehat{D}_{57}$

$= min\{d_{57}, min\{d_{51}, d_{53}, d_{58}, d_{5\,11}\}\}$

$= min\{14, min\{11,7,19,12\}\}$

$= 7$

$i = 5 \quad j = 8 \quad Q = \{1,3,8,11\}$

$\widehat{D}_{58}$

$= min\{d_{58}, min\{d_{51}, d_{53}, d_{58}, d_{5\,11}\}\}$

$= min\{19, min\{11,7,19,12\}\}$

$= 7$

$i = 5 \quad j = 9 \quad Q = \{1,3,8,11\}$

$\widehat{D}_{59}$

$= min\{d_{59}, min\{d_{51}, d_{53}, d_{58}, d_{5\,11}\}\}$

$= min\{22, min\{11,7,19,12\}\}$

$= 7$

$i = 5 \quad j = 10 \quad Q = \{1,3,8,11\}$

$\widehat{D}_{5\,10}$

$= min\{d_{5\,10}, min\{d_{51}, d_{53}, d_{58}, d_{5\,11}\}\}$

$= min\{17, min\{11,7,19,12\}\}$

$= 7$

$i = 5 \quad j = 11 \quad Q = \{1,3,8,11\}$

$\widehat{D}_{5\,11}$

$= min\{d_{5\,11}, min\{d_{51}, d_{53}, d_{58}, d_{5\,11}\}\}$

$= min\{12, min\{11,7,19,12\}\}$

$= 7$

$i = 5 \quad j = 12 \quad Q = \{1,3,8,11\}$

$\widehat{D}_{5\,12}$

$= min\{d_{5\,12}, min\{d_{51}, d_{53}, d_{58}, d_{5\,11}\}\}$

$= min\{19, min\{11,7,19,12\}\}$

$= 7$

For node 6

$i = 6 \quad j = 1 \quad Q = \{1,3,8,11\}$

$\widehat{D}_{61}$

$= min\{d_{61}, min\{d_{61}, d_{63}, d_{68}, d_{6\,11}\}\}$

$= min\{10, min\{10, 13, 11, 18\}\}$

$= 10$

$i = 6 \quad j = 2 \quad Q = \{1,3,8,11\}$

$\widehat{D}_{62}$

$= min\{d_{62}, min\{d_{61}, d_{63}, d_{68}, d_{6\,11}\}\}$

$= min\{8, min\{10, 13, 11, 18\}\}$

$= 8$

$i = 6 \quad j = 3 \quad Q = \{1,3,8,11\}$

$\widehat{D}_{63}$

$= min\{d_{63}, min\{d_{61}, d_{63}, d_{68}, d_{6\,11}\}\}$

$= min\{13, min\{10, 13, 11, 18\}\}$

$= 10$

$i = 6 \quad j = 4 \quad Q = \{1,3,8,11\}$

$\widehat{D}_{64}$

$= min\{d_{64}, min\{d_{61}, d_{63}, d_{68}, d_{6\,11}\}\}$

$= min\{12, min\{10, 13, 11, 18\}\}$

$= 10$

$i = 6 \quad j = 5 \quad Q = \{1,3,8,11\}$

$\widehat{D}_{65}$

$= min\{d_{65}, min\{d_{61}, d_{63}, d_{68}, d_{6\,11}\}\}$

$= min\{9, min\{10, 13, 11, 18\}\}$

$= 9$

$i = 6 \quad j = 6 \quad Q = \{1,3,8,11\}$

$\widehat{D}_{66}$

$= min\{d_{66}, min\{d_{61}, d_{63}, d_{68}, d_{6\,11}\}\}$

$= min\{0, min\{10, 13, 11, 18\}\}$

$= 0$

$i = 6 \quad j = 7 \quad Q = \{1,3,8,11\}$

$\widehat{D}_{67}$

$= min\{d_{67}, min\{d_{61}, d_{63}, d_{68}, d_{6\,11}\}\}$

$= min\{6, min\{10, 13, 11, 18\}\}$

$= 6$

$i = 6 \quad j = 8 \quad Q = \{1,3,8,11\}$

$\widehat{D}_{68}$

$= min\{d_{68}, min\{d_{61}, d_{63}, d_{68}, d_{6\,11}\}\}$

$= min\{11, min\{10, 13, 11, 18\}\}$

$= 10$

$i = 6 \quad j = 9 \quad Q = \{1,3,8,11\}$

$\widehat{D}_{69}$

$= min\{d_{69}, min\{d_{61}, d_{63}, d_{68}, d_{6\,11}\}\}$

$= min\{19, min\{10, 13, 11, 18\}\}$

$= 10$

$i = 6 \quad j = 10 \quad Q = \{1,3,8,11\}$

$\widehat{D}_{6\,10}$

$= min\{d_{6\,10}, min\{d_{61}, d_{63}, d_{68}, d_{6\,11}\}\}$

$= min\{17, min\{10, 13, 11, 18\}\}$

$= 10$

$i = 6 \quad j = 11 \quad Q = \{1,3,8,11\}$

$\widehat{D}_{6\,11}$

$= min\{d_{6\,11}, min\{d_{61}, d_{63}, d_{68}, d_{6\,11}\}\}$

$= min\{18, min\{10, 13, 11, 18\}\}$

$= 10$

$i = 6 \quad j = 12 \quad Q = \{1,3,8,11\}$

$\widehat{D}_{6\,12}$

$= min\{d_{6\,12}, min\{d_{61}, d_{63}, d_{68}, d_{6\,11}\}\}$

$= min\{25, min\{10, 13, 11, 18\}\}$

$= 10$

For node 7

$i = 7 \quad j = 1 \quad Q = \{1,3,8,11\}$

$\widehat{D}_{71}$

$= min\{d_{71}, min\{d_{71}, d_{73}, d_{78}, d_{7\,11}\}\}$

$= min\{4, min\{4, 10, 5, 12\}\}$

$= 4$

$i = 7 \quad j = 2 \quad Q = \{1,3,8,11\}$

$\widehat{D}_{72}$

$= min\{d_{72}, min\{d_{71}, d_{73}, d_{78}, d_{7\,11}\}\}$

$= min\{5, min\{4, 10, 5, 12\}\}$

$= 4$

$i = 7 \quad j = 3 \quad Q = \{1,3,8,11\}$

$\widehat{D}_{73}$

$= min\{d_{73}, min\{d_{71}, d_{73}, d_{78}, d_{7\ 11}\}\}$

$= min\{10, min\{4, 10, 5, 12\}\}$

$= 4$

$i = 7 \quad j = 4 \quad Q = \{1,3,8,11\}$

$\widehat{D}_{74}$

$= min\{d_{74}, min\{d_{71}, d_{73}, d_{78}, d_{7\ 11}\}\}$

$= min\{9, min\{4, 10, 5, 12\}\}$

$= 4$

$i = 7 \quad j = 5 \quad Q = \{1,3,8,11\}$

$\widehat{D}_{75}$

$= min\{d_{75}, min\{d_{71}, d_{73}, d_{78}, d_{7\ 11}\}\}$

$= min\{14, min\{4, 10, 5, 12\}\}$

$= 4$

$i = 7 \quad j = 6 \quad Q = \{1,3,8,11\}$

$\widehat{D}_{76}$

$= min\{d_{76}, min\{d_{71}, d_{73}, d_{78}, d_{7\ 11}\}\}$

$= min\{6, min\{4, 10, 5, 12\}\}$

$= 4$

$i = 7 \quad j = 7 \quad Q = \{1,3,8,11\}$

$\widehat{D}_{77}$

$= min\{d_{77}, min\{d_{71}, d_{73}, d_{78}, d_{7\ 11}\}\}$

$= min\{0, min\{4, 10, 5, 12\}\}$

$= 0$

$i = 7 \quad j = 8 \quad Q = \{1,3,8,11\}$

$\widehat{D}_{78}$

$= min\{d_{78}, min\{d_{71}, d_{73}, d_{78}, d_{7\ 11}\}\}$

$= min\{5, min\{4, 10, 5, 12\}\}$

$= 4$

$i = 7 \quad j = 9 \quad Q = \{1,3,8,11\}$

$\widehat{D}_{79}$

$= min\{d_{79}, min\{d_{71}, d_{73}, d_{78}, d_{7\ 11}\}\}$

$= min\{13, min\{4, 10, 5, 12\}\}$

$= 4$

$i = 7 \quad j = 10 \quad Q = \{1,3,8,11\}$

$\widehat{D}_{7\,10}$

$= min\{d_{7\,10}, min\{d_{71}, d_{73}, d_{78}, d_{7\,11}\}\}$

$= min\{11, min\{4, 10, 5, 12\}\}$

$= 4$

$i = 7 \quad j = 11 \quad Q = \{1,3,8,11\}$

$\widehat{D}_{711}$

$= min\{d_{7\,11}, min\{d_{71}, d_{73}, d_{78}, d_{7\,11}\}\}$

$= min\{12, min\{4, 10, 5, 12\}\}$

$= 4$

$i = 7 \quad j = 12 \quad Q = \{1,3,8,11\}$

$\widehat{D}_{7\,12}$

$= min\{d_{7\,12}, min\{d_{71}, d_{73}, d_{78}, d_{7\,11}\}\}$

$= min\{19, min\{4, 10, 5, 12\}\}$

$= 4$

For node 8

$i = 8 \quad j = 1 \quad Q = \{1,3,8,11\}$

$\widehat{D}_{81}$

$= min\{d_{81}, min\{d_{81}, d_{83}, d_{88}, d_{8\,11}\}\}$

$= min\{9, min\{9, 15, 0, 17\}\}$

$= 0$

$i = 8 \quad j = 2 \quad Q = \{1,3,8,11\}$

$\widehat{D}_{82}$

$= min\{d_{82}, min\{d_{81}, d_{83}, d_{88}, d_{8\,11}\}\}$

$= min\{10, min\{9, 15, 0, 17\}\}$

$= 0$

$i = 8 \quad j = 3 \quad Q = \{1,3,8,11\}$

$\widehat{D}_{83}$

$= min\{d_{83}, min\{d_{81}, d_{83}, d_{88}, d_{8\,11}\}\}$

$= min\{15, min\{9, 15, 0, 17\}\}$

$= 0$

$i = 8 \quad j = 4 \quad Q = \{1,3,8,11\}$

$\widehat{D}_{84}$

$= min\{d_{84}, min\{d_{81}, d_{83}, d_{88}, d_{8\,11}\}\}$

$= min\{14, min\{9, 15, 0, 17\}\}$

$= 0$

$i = 8 \quad j = 5 \quad Q = \{1,3,8,11\}$

$\widehat{D}_{85}$

$= min\{d_{85}, min\{d_{81}, d_{83}, d_{88}, d_{8\ 11}\}\}$

$= min\{19, min\{9, 15, 0, 17\}\}$

$= 0$

$i = 8 \quad j = 6 \quad Q = \{1,3,8,11\}$

$\widehat{D}_{86}$

$= min\{d_{86}, min\{d_{81}, d_{83}, d_{88}, d_{8\ 11}\}\}$

$= min\{11, min\{9, 15, 0, 17\}\}$

$= 0$

$i = 8 \quad j = 7 \quad Q = \{1,3,8,11\}$

$\widehat{D}_{87}$

$= min\{d_{87}, min\{d_{81}, d_{83}, d_{88}, d_{8\ 11}\}\}$

$= min\{9, min\{9, 15, 0, 17\}\}$

$= 0$

$i = 8 \quad j = 8 \quad Q = \{1,3,8,11\}$

$\widehat{D}_{88}$

$= min\{d_{88}, min\{d_{81}, d_{83}, d_{88}, d_{8\ 11}\}\}$

$= min\{0, min\{9, 15, 0, 17\}\}$

$= 0$

$i = 8 \quad j = 9 \quad Q = \{1,3,8,11\}$

$\widehat{D}_{89}$

$= min\{d_{89}, min\{d_{81}, d_{83}, d_{88}, d_{8\ 11}\}\}$

$= min\{8, min\{9, 15, 0, 17\}\}$

$= 0$

$i = 8 \quad j = 10 \quad Q = \{1,3,8,11\}$

$\widehat{D}_{8\ 10}$

$= min\{d_{8\ 10}, min\{d_{81}, d_{83}, d_{88}, d_{8\ 11}\}\}$

$= min\{13, min\{9, 15, 0, 17\}\}$

$= 0$

$i = 8 \quad j = 11 \quad Q = \{1,3,8,11\}$

$\widehat{D}_{8\ 11}$

$= min\{d_{8\ 11}, min\{d_{81}, d_{83}, d_{88}, d_{8\ 11}\}\}$

$= min\{17, min\{9, 15, 0, 17\}\}$

$= 0$

$i = 8 \quad j = 12 \quad Q = \{1,3,8,11\}$

$\widehat{D}_{8\,12}$

$= min\{d_{8\,12}, min\{d_{81}, d_{83}, d_{88}, d_{8\,11}\}\}$

$= min\{23, min\{9, 15, 0, 17\}\}$

$= 0$

For node 9

$i = 9 \quad j = 1 \quad Q = \{1,3,8,11\}$

$\widehat{D}_{91}$

$= min\{d_{91}, min\{d_{91}, d_{93}, d_{98}, d_{9\,11}\}\}$

$= min\{12, min\{12, 15, 8, 10\}\}$

$= 8$

$i = 9 \quad j = 2 \quad Q = \{1,3,8,11\}$

$\widehat{D}_{92}$

$= min\{d_{92}, min\{d_{91}, d_{93}, d_{98}, d_{9\,11}\}\}$

$= min\{14, min\{12, 15, 8, 10\}\}$

$= 8$

$i = 9 \quad j = 3 \quad Q = \{1,3,8,11\}$

$\widehat{D}_{93}$

$= min\{d_{93}, min\{d_{91}, d_{93}, d_{98}, d_{9\,11}\}\}$

$= min\{15, min\{12, 15, 8, 10\}\}$

$= 8$

$i = 9 \quad j = 4 \quad Q = \{1,3,8,11\}$

$\widehat{D}_{94}$

$= min\{d_{94}, min\{d_{91}, d_{93}, d_{98}, d_{9\,11}\}\}$

$= min\{17, min\{12, 15, 8, 10\}\}$

$= 8$

$i = 9 \quad j = 5 \quad Q = \{1,3,8,11\}$

$\widehat{D}_{95}$

$= min\{d_{95}, min\{d_{91}, d_{93}, d_{98}, d_{9\,11}\}\}$

$= min\{22, min\{12, 15, 8, 10\}\}$

$= 8$

$i = 9 \quad j = 6 \quad Q = \{1,3,8,11\}$

$\widehat{D}_{96}$

$= min\{d_{96}, min\{d_{91}, d_{93}, d_{98}, d_{9\,11}\}\}$

$= min\{19, min\{12, 15, 8, 10\}\}$

$= 8$

$i = 9 \quad j = 7 \quad Q = \{1,3,8,11\}$

$\widehat{D}_{97}$

$= min\{d_{97}, min\{d_{91}, d_{93}, d_{98}, d_{9\,11}\}\}$

$= min\{13, min\{12, 15, 8, 10\}\}$

$= 8$

$i = 9 \quad j = 8 \quad Q = \{1,3,8,11\}$

$\widehat{D}_{98}$

$= min\{d_{98}, min\{d_{91}, d_{93}, d_{98}, d_{9\,11}\}\}$

$= min\{8, min\{12, 15, 8, 10\}\}$

$= 8$

$i = 9 \quad j = 9 \quad Q = \{1,3,8,11\}$

$\widehat{D}_{99}$

$= min\{d_{99}, min\{d_{91}, d_{93}, d_{98}, d_{9\,11}\}\}$

$= min\{0, min\{12, 15, 8, 10\}\}$

$= 8$

$i = 9 \quad j = 10 \quad Q = \{1,3,8,11\}$

$\widehat{D}_{9\,10}$

$= min\{d_{9\,10}, min\{d_{91}, d_{93}, d_{98}, d_{9\,11}\}\}$

$= min\{5, min\{12, 15, 8, 10\}\}$

$= 5$

$i = 9 \quad j = 11 \quad Q = \{1,3,8,11\}$

$\widehat{D}_{9\,11}$

$= min\{d_{9\,11}, min\{d_{91}, d_{93}, d_{98}, d_{9\,11}\}\}$

$= min\{10, min\{12, 15, 8, 10\}\}$

$= 8$

$i = 9 \quad j = 12 \quad Q = \{1,3,8,11\}$

$\widehat{D}_{9\,12}$

$= min\{d_{9\,12}, min\{d_{91}, d_{93}, d_{98}, d_{9\,11}\}\}$

$= min\{15, min\{12, 15, 8, 10\}\}$

$= 8$

For node 10

$i = 10 \quad j = 1 \quad Q = \{1,3,8,11\}$

$\widehat{D}_{10\,1}$

$= min\{d_{10\,1}, min\{d_{10\,1}, d_{10\,3}, d_{10\,8}, d_{10\,11}\}\}$

$= min\{7, min\{7, 10, 13, 5\}\}$

$= 5$

$i = 10 \quad j = 2 \quad Q = \{1,3,8,11\}$

$\widehat{D}_{10\ 2}$

$= min\{d_{10\ 2}, min\{d_{10\ 1}, d_{10\ 3}, d_{10\ 8}, d_{10\ 11}\}\}$

$= min\{9, min\{7, 10, 13, 5\}\}$

$= 5$

$i = 10 \quad j = 3 \quad Q = \{1,3,8,11\}$

$\widehat{D}_{10\ 3}$

$= min\{d_{10\ 3}, min\{d_{10\ 1}, d_{10\ 3}, d_{10\ 8}, d_{10\ 11}\}\}$

$= min\{10, min\{7, 10, 13, 5\}\}$

$= 5$

$i = 10 \quad j = 4 \quad Q = \{1,3,8,11\}$

$\widehat{D}_{10\ 4}$

$= min\{d_{10\ 4}, min\{d_{10\ 1}, d_{10\ 3}, d_{10\ 8}, d_{10\ 11}\}\}$

$= min\{12, min\{7, 10, 13, 5\}\}$

$= 5$

$i = 10 \quad j = 6 \quad Q = \{1,3,8,11\}$

$\widehat{D}_{10\ 5}$

$= min\{d_{10\ 5}, min\{d_{10\ 1}, d_{10\ 3}, d_{10\ 8}, d_{10\ 11}\}\}$

$= min\{17, min\{7, 10, 13, 5\}\}$

$= 5$

$i = 10 \quad j = 6 \quad Q = \{1,3,8,11\}$

$\widehat{D}_{10\ 6}$

$= min\{d_{10\ 6}, min\{d_{10\ 1}, d_{10\ 3}, d_{10\ 8}, d_{10\ 11}\}\}$

$= min\{17, min\{7, 10, 13, 5\}\}$

$= 5$

$i = 10 \quad j = 7 \quad Q = \{1,3,8,11\}$

$\widehat{D}_{10\ 7}$

$= min\{d_{10\ 7}, min\{d_{10\ 1}, d_{10\ 3}, d_{10\ 8}, d_{10\ 11}\}\}$

$= min\{11, min\{7, 10, 13, 5\}\}$

$= 5$

$i = 10 \quad j = 8 \quad Q = \{1,3,8,11\}$

$\widehat{D}_{10\ 8}$

$= min\{d_{10\ 8}, min\{d_{10\ 1}, d_{10\ 3}, d_{10\ 8}, d_{10\ 11}\}\}$

$= min\{13, min\{7, 10, 13, 5\}\}$

$= 5$

$i = 10 \quad j = 9 \quad Q = \{1,3,8,11\}$

$\widehat{D}_{10\ 9}$

$= min\{d_{10\ 9}, min\{d_{10\ 1}, d_{10\ 3}, d_{10\ 8}, d_{10\ 11}\}\}$

$= min\{5, min\{7, 10, 13, 5\}\}$

$= 5$

$i = 10 \quad j = 10 \quad Q = \{1,3,8,11\}$

$\widehat{D}_{10\ 10}$

$= min\{d_{10\ 10}, min\{d_{10\ 1}, d_{10\ 3}, d_{10\ 8}, d_{10\ 11}\}\}$

$= min\{0, min\{7, 10, 13, 5\}\}$

$= 0$

$i = 10 \quad j = 11 \quad Q = \{1,3,8,11\}$

$\widehat{D}_{10\ 11}$

$= min\{d_{10\ 11}, min\{d_{10\ 1}, d_{10\ 3}, d_{10\ 8}, d_{10\ 11}\}\}$

$= min\{5, min\{7, 10, 13, 5\}\}$

$= 5$

$i = 10 \quad j = 12 \quad Q = \{1,3,8,11\}$

$\widehat{D}_{10\ 12}$

$= min\{d_{10\ 12}, min\{d_{10\ 1}, d_{10\ 3}, d_{10\ 8}, d_{10\ 11}\}\}$

$= min\{10, min\{7, 10, 13, 5\}\}$

$= 5$

For node 11

$i = 11 \quad j = 1 \quad Q = \{1,3,8,11\}$

$\widehat{D}_{11\ 1}$

$= min\{d_{11\ 1}, min\{d_{11\ 1}, d_{11\ 3}, d_{11\ 8}, d_{11\ 11}\}\}$

$= min\{8, min\{8, 5, 17, 0\}\}$

$= 0$

$i = 11 \quad j = 2 \quad Q = \{1,3,8,11\}$

$\widehat{D}_{11\ 2}$

$= min\{d_{11\ 2}, min\{d_{11\ 1}, d_{11\ 3}, d_{11\ 8}, d_{11\ 11}\}\}$

$= min\{10, min\{8, 5, 17, 0\}\}$

$= 0$

$i = 11 \quad j = 3 \quad Q = \{1,3,8,11\}$

$\widehat{D}_{11\,3}$

$= min\{d_{11\,3}, min\{d_{11\,1}, d_{11\,3}, d_{11\,8}, d_{11\,11}\}\}$

$= min\{5, min\{8, 5, 17, 0\}\}$

$= 0$

$i = 11 \quad j = 4 \quad Q = \{1,3,8,11\}$

$\widehat{D}_{11\,4}$

$= min\{d_{11\,4}, min\{d_{11\,1}, d_{11\,3}, d_{11\,8}, d_{11\,11}\}\}$

$= min\{7, min\{8, 5, 17, 0\}\}$

$= 0$

$i = 11 \quad j = 5 \quad Q = \{1,3,8,11\}$

$\widehat{D}_{11\,5}$

$= min\{d_{11\,5}, min\{d_{11\,1}, d_{11\,3}, d_{11\,8}, d_{11\,11}\}\}$

$= min\{12, min\{8, 5, 17, 0\}\}$

$= 0$

$i = 11 \quad j = 6 \quad Q = \{1,3,8,11\}$

$\widehat{D}_{11\,6}$

$= min\{d_{11\,6}, min\{d_{11\,1}, d_{11\,3}, d_{11\,8}, d_{11\,11}\}\}$

$= min\{18, min\{8, 5, 17, 0\}\}$

$= 0$

$i = 11 \quad j = 7 \quad Q = \{1,3,8,11\}$

$\widehat{D}_{11\,7}$

$= min\{d_{11\,7}, min\{d_{11\,1}, d_{11\,3}, d_{11\,8}, d_{11\,11}\}\}$

$= min\{12, min\{8, 5, 17, 0\}\}$

$= 0$

$i = 11 \quad j = 8 \quad Q = \{1,3,8,11\}$

$\widehat{D}_{11\,8}$

$= min\{d_{11\,8}, min\{d_{11\,1}, d_{11\,3}, d_{11\,8}, d_{11\,11}\}\}$

$= min\{17, min\{8, 5, 17, 0\}\}$

$= 0$

$i = 11 \quad j = 9 \quad Q = \{1,3,8,11\}$

$\widehat{D}_{11\,9}$

$= min\{d_{11\,9}, min\{d_{11\,1}, d_{11\,3}, d_{11\,8}, d_{11\,11}\}\}$

$= min\{10, min\{8, 5, 17, 0\}\}$

$= 0$

$i = 11 \quad j = 11 \quad Q = \{1,3,8,11\}$

$\widehat{D}_{11\,11}$

$= min\{d_{11\,11}, min\{d_{11\,1}, d_{11\,3}, d_{11\,8}, d_{11\,11}\}\}$

$= min\{0, min\{8, 5, 17, 0\}\}$

$= 0$

$i = 11 \quad j = 12 \quad Q = \{1,3,8,11\}$

$\widehat{D}_{11\,12}$

$= min\{d_{11\,12}, min\{d_{11\,1}, d_{11\,3}, d_{11\,8}, d_{11\,11}\}\}$

$= min\{7, min\{8, 5, 17, 0\}\}$

$= 0$

For node 12

$i = 12 \quad j = 1 \quad Q = \{1,3,8,11\}$

$\widehat{D}_{12\,1}$

$= min\{d_{12\,1}, min\{d_{12\,1}, d_{12\,3}, d_{12\,8}, d_{12\,11}\}\}$

$= min\{15, min\{15, 12, 23, 7\}\}$

$= 7$

$i = 12 \quad j = 2 \quad Q = \{1,3,8,11\}$

$\widehat{D}_{12\,2}$

$= min\{d_{12\,2}, min\{d_{12\,1}, d_{12\,3}, d_{12\,8}, d_{12\,11}\}\}$

$= min\{17, min\{15, 12, 23, 7\}\}$

$= 7$

$i = 12 \quad j = 3 \quad Q = \{1,3,8,11\}$

$\widehat{D}_{12\,3}$

$= min\{d_{12\,3}, min\{d_{12\,1}, d_{12\,3}, d_{12\,8}, d_{12\,11}\}\}$

$= min\{12, min\{15, 12, 23, 7\}\}$

$= 7$

$i = 12 \quad j = 4 \quad Q = \{1,3,8,11\}$

$\widehat{D}_{12\,4}$

$= min\{d_{12\,4}, min\{d_{12\,1}, d_{12\,3}, d_{12\,8}, d_{12\,11}\}\}$

$= min\{14, min\{15, 12, 23, 7\}\}$

$= 7$

$i = 12 \quad j = 5 \quad Q = \{1,3,8,11\}$

$\widehat{D}_{12\,5}$

$= min\{d_{12\,5}, min\{d_{12\,1}, d_{12\,3}, d_{12\,8}, d_{12\,11}\}\}$

$= min\{19, min\{15, 12, 23, 7\}\}$

$= 7$

$i = 12 \quad j = 6 \quad Q = \{1,3,8,11\}$

$\widehat{D}_{12\ 6}$

$= min\{d_{12\ 6}, min\{d_{12\ 1}, d_{12\ 3}, d_{12\ 8}, d_{12\ 11}\}\}$

$= min\{25, min\{15, 12, 23, 7\}\}$

$= 7$

$i = 12 \quad j = 7 \quad Q = \{1,3,8,11\}$

$\widehat{D}_{12\ 7}$

$= min\{d_{12\ 7}, min\{d_{12\ 1}, d_{12\ 3}, d_{12\ 8}, d_{12\ 11}\}\}$

$= min\{19, min\{15, 12, 23, 7\}\}$

$= 7$

$i = 12 \quad j = 8 \quad Q = \{1,3,8,11\}$

$\widehat{D}_{12\ 8}$

$= min\{d_{12\ 8}, min\{d_{12\ 1}, d_{12\ 3}, d_{12\ 8}, d_{12\ 11}\}\}$

$= min\{23, min\{15, 12, 23, 7\}\}$

$= 7$

$i = 12 \quad j = 9 \quad Q = \{1,3,8,11\}$

$\widehat{D}_{12\ 9}$

$= min\{d_{12\ 9}, min\{d_{12\ 1}, d_{12\ 3}, d_{12\ 8}, d_{12\ 11}\}\}$

$= min\{15, min\{15, 12, 23, 7\}\}$

$= 7$

$i = 12 \quad j = 10 \quad Q = \{1,3,8,11\}$

$\widehat{D}_{12\ 10}$

$= min\{d_{12\ 10}, min\{d_{12\ 1}, d_{12\ 3}, d_{12\ 8}, d_{12\ 11}\}\}$

$= min\{10, min\{15, 12, 23, 7\}\}$

$= 7$

$i = 12 \quad j = 11 \quad Q = \{1,3,8,11\}$

$\widehat{D}_{12\ 11}$

$= min\{d_{12\ 11}, min\{d_{12\ 1}, d_{12\ 3}, d_{12\ 8}, d_{12\ 11}\}\}$

$= min\{7, min\{15, 12, 23, 7\}\}$

$= 7$

$i = 12 \quad j = 12 \quad Q = \{1,3,8,11\}$

$\widehat{D}_{12\ 12}$

$= min\{d_{12\ 12}, min\{d_{12\ 1}, d_{12\ 3}, d_{12\ 8}, d_{12\ 11}\}\}$

$= min\{0, min\{15, 12, 23, 7\}\}$

$= 0$