EFFICIENCY OF THE SCHOOL BUS TRANSPORTATION SERVICE: A CASE STUDY OF THE SYSTEM AT WOODBRIDGE SCHOOL COMPLEX (WBSC)

By

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EFFICIENCY OF THE SCHOOL BUS TRANSPORTATION SERVICE: A CASE STUDY OF THE SYSTEM AT WOODBRIDGE SCHOOL COMPLEX, TAKORADI

KNUST

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CERTIFICATION

I hereby declare that this submission is my own work towards the MS. And that, to the best of my knowledge, it contains no material previously published by another person nor material which has been accepted for the award of any other degree of the university, except where due acknowledgement has been made in the text,

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Supervisor's Name	Signature	Date

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Head of Dept. Name	Signature	

DEDICATION

I dedicate this work to my sweetheart Jennifer Padikie Donkor and all my siblings

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The completion of this research represents the realization of a long awaited dream.

This dream could not have been achieved without the guidance, physical and moral support of some very important personalities, whose efforts must be acknowledged.

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ABSTRACT

This research work presents a case study of the School Bus Routing Problem (SBRP). The main objective involves optimizing the total transportation cost by reducing the total lengths of tour made by all buses at the Woodbridge School Complex. The problem is formulated as an Integer Programming Model and solution is presented via the Ant Colony Based Meta-heuristic for the Travelling Salesman Problem. Data on distances were collected from potential picking points and with a Matlab implementation codes, results are obtained. In comparison with the existing routing system at Woodbridge School Complex, the results evince the outperformance of the Ant Colony algorithm in terms of efficiency. In fact, the Ant Colony results reveal a tremendous improvement in the total route length by approximately 33%.

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CHAPTER 1

INTRODUCTION

1.1 BACKGROUND OF THE STUDY

In the Sekondi-Takoradi Metropolis of the Western Region of Ghana, it has become a distinct feature of most private schools to provide transportation service for their students to and from their various schools. Unlike in the advanced countries where most of the School Bus Transportation Services (SBTS) are ran by public transports (Bowerman, Hall and Calamai, 1995). In Ghana and for that matter Sekondi-Takoradi, school transportation service, in most private schools, are executed using school buses provided for the purpose. It is hard to believe that most parents would rather have their wards patronize services executed by taxi drivers or other private transport operators, regardless of the fact that relatively low fees are charged by schools in running transportation services for students.

In trying to find answers to questions that pertains low patronage, any operation researcher would question the degree of efficiency, effectiveness and equity of transportation services that private schools run in Sekondi-Takoradi. A sober reflection, from investigation, revealed that all the systems on which most of the private schools base their transport services, have no scientific basis. This culminated to the inefficient scheduling of school buses to routes or inefficient assignment of students to picking points.

The task of finding an efficient route is an important logistic problem (Bell and McMullen, 2004). When a school is able to reduce the length of its delivery route then it is able to provide better services to student. A typical school bus routing problem includes simultaneously determining the routes for several vehicles from a given school to students and returning to the school without exceeding capacity.

This thesis seeks to analyze the efficiency of school bus transportation service using Woodbridge School Complex as a case study. Savas (1978) outlined efficiency, effectiveness and equity as performance criteria for the provision of school bus transportation services. According to him, efficiency measures the ratio of the level of service to the cost required to provide the service. The level of service required in providing an efficient school bus routing is fixed for a particular situation, hence the main variable in determining the efficiency of a particular solution is the total cost of providing the service in currency units or manpower. Therefore, the efficiency of a solution can be measured by its cost (Bowerman, Hall and Calamai, 1995). In this instance, the problem involves finding the minimum cost of the combined routes for a number of vehicles in order to connect students from the school to a number of locations. Since the cost is directly related to the distance, the school might attempt to find the minimum distance covered by its number of buses in order to satisfy demand. In doing so the school management attempts to minimize cost while increasing or at least maintaining a standard or quality service to students.

It is important to remind readers of the fact that so many research efforts have been geared towards this course. Thus, there exist a whole lot of mathematical models for the School Bus Routing Problem (SBRP) but all these models have their basis from the standard Vehicle Routing Problem (VRP). It is not the aim of this work to come out with a new model but to analyze a real- life situation by applying a model of SBRP. For the purpose of this work, SBTS will be formulated as single objective problem and application centered on the one developed by Patrick Schittekat, Marc Sevauz and Kenneth Sorensen. Unlike in most problems where bus routing formulations involve a given set of bus stops and each connected by a given set of routes, here, a set of potential bus stops are identified. Determining the set of stops to actually visit is a part of the problem formulation. Similar to their work, the real-life situation, as exists in Woodbridge School Complex, is a single School with five (5) school buses. Apart from the fact that the buses are of different capacities (as against their instance), this problem will also be solved using the Ant Colony heuristics for SBTS.

1.2 PROFILE OF WOODBRIDGE SCHOOL COMPLEX AND ITS BUS SYSTEM

The year 1996 marks the opening of Woodbridge School Complex. It is located at Race Course, North Apremdo, near Asakae, Takoradi. Though profit oriented, it was established to provide first class education to children in Sekondi-Takoradi and its catchment areas. The population of the school is 1100. It is currently ranked as first class private school.

SBTS of the school started since its inception. The school started the operation of the SBTS with a 9 passenger bus. As the population begun to grow the 9 passenger bus was replaced by a 16 passenger bus; later a 33 passenger bus was purchased in place of the previous one as it became too small and very weak for the service. Followed by this was an additional 19 passenger bus to supplement the former. The population of the school experienced a remarkable increase in 2006, 2007and 2008. In response to this increase, 45, 45 and 60 capacity buses, respectively, were procured in order to render what they call effective service. All these buses are in operation up to date. 4 out of these 5 operate on regular basis whilst 5 (a 45 capacity) serves as a stand-by bus. 240 out of a total of 1100 patronize the SBTS. There are about 84 potential bus stops for all these buses. The 33 passenger bus (3) visits about 23 potential bus stops as it sets off from the school. The 19 capacity bus (4) visits about 14, while the 45 capacity bus (2) visits 30 potential stops and the 60 capacity bus (1) visits about 16 potential bus stops. On the average, all buses are expected to leave the school around 530 am and return at 730 before morning lessons begin. Lateness sometimes occur under two circumstances. First, it occurs when customers on one hand wait for unreasonably long hours at their various picking points. Secondly, lateness occurs when buses on the other hand, wait for customers at various picking points. Some buses visit some stops more than once. Details about the buses and their tour (Ant sketches) can be seen in Appendix A

Customers (students) are charged on daily basis and customer pays an amount of 70 Ghana pesewa for SBTS. On the average, this is comparatively lower than that charged by other private transport operators (i.e. taxi, trotro, etc).

Management outlined low patronage, high cost of servicing buses, high cost of spare parts, unavailability of genuine parts, poor nature of roads linking stops, etc. as major set-backs of operation.

1.3 STATEMENT OF PROBLEM

Transporting students to and from school is highly recognized in modern societies; hence its relevance cannot be underestimated. More schools are springing up as the campaign on Free and Compulsory Basic Education is on the ascendance to its apex. Following this, many people are getting their children of school going age ready for school. For every school, there is a geographical distribution of students' locations around the given school. Thus, not all length of distances between a given school and location of students' homes can be considered as walking distances for every student. More so, the chaotic traffic jam on our roads in recent times does not favour parents who personally drive their children to school before going to work. It is for the above and other reasons that transportation service for school children has become necessary. In the Sekondi-Takoradi Metropolis, a feature that distinguishes private schools from public ones is the provision of transportation service for their students. It now behooves on parents to decide whether their wards go to school in school buses or engage the services of taxi drivers if they cannot personally drive their wards to school. This decision will definitely be influenced by the overall assessment of the level of transportation services in terms of cost, satisfaction, total riding time and time of arrival of all buses at their origin.

A closer observation has revealed that the transportation service at Woodbridge School Complex suffers low patronage regardless of the fact that service charge per student is comparatively low.

It is evident that total daily fuel consumption of all buses is too high as a result of abnormally long lengths of routes covered by all buses. It is this component of cost that over increases the daily operation cost of transportation service and culminates to losses. The deficit Woodbridge School Complex incurs in her transport operations makes her unable to buy new buses, effectively repair or service faulty buses or replace those ones that are considered dilapidated.

Now the problem to grapple with, involves optimizing total cost of transport operation by reducing the total lengths of tour made by all buses. Providing a desired solution to the routing problem at Woodbridge School Complex forms the core of the objectives of this research whose areas of interests are summarized as follows:

i. Formulation of a mathematical model that takes into account the actual distances between various picking points of the respective buses
ii. Determining the optimal route for each of the buses
iii. Selection of bus stops from a given number of potential bus stops

The statement is not far from the fact that finding solution to the above problems reflects the objective of this research.

1.4 OBJECTIVES

This research is centered on analyzing the SBTS in Woodbridge School Complex. The objectives include:

i. To formulate of a mathematical model that takes into account the actual distances between various picking points of the respective buses
ii. To determine the optimal route for each of the buses
iii. To select bus stops from a given set of potential bus stops

1.5 METHODOLOGY

This section discusses data collection and solution methods.

The data collected from the school is purely primary and quantitative in nature. The researcher personally gathered the information via observation and interview of personnel in charge of transport. Data on the number of buses, student population of the school, number of students that patronize the school bus and number of potential bus stops were obtained through personal interview. Moreover, observation technique was also employed to enable the researcher have a clear picture of the locations of various potential bus stops and their distances from one to the other, given that they are on the same routes. Distances measured from one bus stop to another (according to the order of movements of respective buses) were converted into Cartesian coordinates, which gave rise to the distance matrix (see Appendices B). With regard to the solution method, SBTS will be formulated as single objective problem in order to adopt simple but efficient technique that mirrors the structure of the problem. Since this is a combinatorial problem, the solution grows exponentially with the number of variables. Consequently, no exact algorithm exists for SBTS. That notwithstanding, SBTS will be formulated as an Integer

Programming (IP) and the Ant Colony Optimization (ACO) Heuristics for SBTS will be used and the result tested with real-life data from Woodbridge School Complex. ACO simulates the behaviour of ants as they forage for food and find the most efficient routes from their nest to food source. The decision making process of ants are embedded in the artificial intelligence (AI) algorithm of a group of virtual ants which are used to provide solution to the VRP (Bell and McMullen, 2004). Bell and McMullen established in their paper that the performance of ACO is competitive with other techniques used to generate solutions to the VRP. Dorigo and Gambardella (1997) also concluded in their work that ACO out-performs other nature-inspired algorithms such as simulated annealing and evolutionally computation. Last but not least, Abounacer et al., (2009) came out with the conclusion that ACO performs better than Genetic Algorithm (GA) in terms of cost calculations. However, the performances of both techniques in determining the number of vehicles, required for a given set T of students, are the same. Since the objective function of this thesis is centered on minimizing the total routes length, which reflects the cost of service, the researcher finds it justifiable to adopt ACO technique.

1.6 JUSTIFICATION

The bus routing systems used by almost every private school in the Sekondi-Takoradi Metropolis are non-scientific. It is therefore believed that when the research is successfully completed and recommendations well considered, Woodbridge School Complex will adopt a standard mathematical model for her SBTS.

1.7 SIGNIFICANCE OF THE STUDY

The results of this scientific-based transportation system may be of significance in the following ways as stated below.

i. It will impact positively by reducing the cost of running transport services in the school.
ii. Constructing an efficient route could also lead to improvement in the level of customer satisfaction.
iii. It could also lead to improvement in students' performance since efficiency is closely related to punctuality and reduction of stress, which would have resulted from increased waiting time and long time spent in buses (Abounacer, et al., 2009).
iv. Other schools could also use the model as a basis to improve their SBTS. It will also become the building block for improvement in the services of public transport as well as that of other transport service providers.
v. When this problem is well addressed and the system adopted and improved countrywide, Ghana's national output will increase since a reduction in total lateness will increase the average number of hours per the Ghanaian worker.

1.7 LIMITATIONS

This research was limited to only one school linked up by so many routes. The school serves as the central depot and each route starts and ends at the depot. This choice of the researcher is justified by the fact that in Sekondi-Takoradi, it is very rare to have one bus serving at least two different schools, though these schools might be on the same route. Woodbridge School Complex was chosen because the problem there conforms to the nature of problem the researcher wanted to investigate –that is buses with different capacities.

Like any other academic research, this work was not without constraints. Time and cost posed many challenges. The entire research was to be completed within a given time period. However, time is needed sufficiently well enough for reviewing literature, understanding methodologies, testing alternative algorithms to select efficient ones, etc. The limited time at the researcher's disposal lead to few items being reviewed as literature. The quality of data depends not only on the amount of time one spends in gathering them but partially on how much money one is prepared to spend in gathering them. This work would have considered the routing systems of a given number of schools but the limitations imposed by time and cost influenced the researcher's decision to concentrate on a single school.

The researcher also encountered certain difficulties in connection with data collection. The nature of this study requires the researcher to collect data on distances from one bus stop to the other for all four buses. Other information, like the angle at a particular point with reference to a given geographical direction, etc. are required. Getting a reliable and an efficient measuring instrument did not come with ease. This actually delayed the timely completion of the work.

1.8 ORGANIZATION

The entire organization of this thesis comprises five chapters.

Chapter 1 (Introduction) provides information on the background of the study. It also throws light on the problem statement and objectives of the study, methodology, justification, limitations and organization.

Chapter 2 (Literature Review), reviews related works of some authors and summarizes the Performance Criteria for Providing SBTS.

Chapter 3 (Methodology) presents the description and mathematical formulation of the problem (i.e. the model) with the underlying assumptions. Chapter 4 presents the data collection and analysis. Finally, Chapter 5 is made up of discussions of results, conclusions and recommendations.

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CHAPTER 2

LITERATURE REVIEW

2.0 INTRODUCTION

This work, SBTS is a real-life application of the standard VRP to solving contemporary routing problems in Sekondi-Takoradi, a district in the western region of Ghana. Following the basis of this model, this section is devoted to review literature of related works by some researchers. It also summarizes the criteria for assessing the performance of a school bus transportation service.

2.1 LITERATURE ON RELATED WORKS

Schowenaars et al., (2009) presented a new approach to fuel optimal path planning for multi vehicle using combination of LP and IP. The basic problem formulation was to have the vehicles moving from initials dynamic state to a final state without colliding with each other, while at the same time avoiding other stationary obstacles. They showed that the problem could be rewritten as a linear program with mixed integer program linear constraints that account for the collision avoidance. The problem was solved using the CPLEX optimizatice software with an AMPL/Matlab interface.

Ibraki et al., (2001) introduced a vehicle Routing problem with general time constraints. The problem was to minimize the sum of the distances travelled by a fixed number of vehicles which visit every customer under capacity and time window constraints. The time window constraint was treated as a penalty function, which can be non- convex and discontinuous as long as it is piecewise linear function. First they fixed the order of customers to visit and proceeded to determine the optimal start times to serve customers so that total time penalty of the vehicle is minimized. They proved how the problem could efficiently be solved using dynamic programming which was incorporated into the local search algorithms, In the local search, in addition to standard neighborhoods, they employed a new type of neighborhood called the cyclic exchange neighborhood, whose size generally growths exponentially with the input size. This difficulty was conquered by finding an efficient heuristic to find an improved solution in the cycle exchange neighborhood via the improvement graph. The computation results revealed good prospects for the proposed algorithms.

Sue (1994) studied the problem of permutation routing and sorting on several model of meshes with fixed and reconfigurable row and column buses. He described two fast and fairly simple deterministic algorithms for permutation routing on two-dimensional networks and a more complicated algorithms for multi-dimensional networks. The algorithms were obtained by converting two known off-line routing schemes into deterministic routing algorithms which can be implemented on a variety of different models of meshes with buses. A deterministic algorithm for 1-1 sorting whose running time matches that for permutation routing, and another algorithm that matches the bisection lower bound on reconfigurable networks of arbitrary constant dimension, were introduced.

Bowerman, et al., (1995) proposed a new heuristic for urban school Bus Routing. The problem was formulated as a multi-objective model and a heuristic based on this formulation is developed. The study involves two interrelated problems. One has to do with the assignment of students to their respective bus stops and the second has to do with routing of buses to the bus stops. A problem of these characteristics is a location-routing problem. The nature of the formulation made it possible to organize their study into three layers, where layer one is the school, layer two is the bus stops and layer three the students. School buses routes cause interaction between layers one and two, while movements of student cause interaction between layers two and three. The heuristic approach to this problem involves two algorithms which catered for the multi-objective nature of the model. The first is a districting algorithm which groups students into clusters to be serviced by a unique school bus route. The second is a routing algorithm, which generates a specific school bus route that visits a sub set of potential bus stops sites.

Bell and McMullen (2004) applied a meta-heuristic method of ant colony optimization (ACO) to establish a set of vehicle routing problems. They modified the ACO algorithm used to solve the traditional traveling salesman problem (TSP) in order to allow the search of multiple routes of the vehicle routing problem (VRP). Experimental results exhibited the success of the algorithm in finding solution within 1% of known optimal solution. The usage of multi ant colonies provides a comparatively competitive solution technique especially for larger problems. Also the size of the candidates list used within the algorithm became a significant factor in finding improved solution. The computational times for the algorithm compared favourably with other solution methods.

Dorigo and Gambardella (1997) introduced ant colony systems (ACS) to the TSP. In order to understand the operation of the ACS, experiments were conducted and the results showed that ACS outperforms other nature-inspired algorithms such as simulated annulling and evolutionary computations. They concluded by comparing ACS-3-opt, a version of ACS augmented with a local search procedure to some of the best performing algorithms for symmetric and asymmetric TSP's

Lenstra et al., (1988) under took a survey of solution method for routing problems with time window constraints. The problem they considered include the travelling salesman problem, vehicle routing problem, pickup and delivery problem and the dial-a-ride problem(DARP). They implemented optimization algorithms that use brand and bound, dynamic programming and set partitioning and approximation algorithms based on construction, iterative improvement and incomplete optimization.

Kontoravdis and Bard (1995) proposed a greedy randomized adaptive search procedure (GRASP) to VRP with time window. The objective was to address the problem of finding the minimum number of vehicles required to visit a set of node subject to time window constraints. They also considered a secondary object centered on minimizing the total distance travelled. Feasible solution obtained from GRASP for standard 100 modes data set as well as for a number of real –world problems with up to 417 customers. Experimental results revealed that their proposed procedure out performs techniques existing at the time and requires only a small fraction of time taken by exact method. They gauged the quality of solutions by applying three different lower bounding heuristics. The first considers the "bin parking" aspect of the problem with respect to vehicle capacity; the second is based on the maximum clique associated with customers' incompatibility; the third exploits the time window constraints.

Corberán et al., (2002) addressed the problem of routing school buses in rural areas. They approached this problem with a node routing model with multiple objectives that arise from conflicting viewpoints. From the point of view of cost, the number of buses used to transport students from their homes to school and back is minimized. From the service viewpoint, they minimized the time that a given student spend on route. The multi-objective employs a weighted function to combine individual objective functions into a single one. They developed a solution procedure that considered each objective separately and searched for a set of efficient solution instead of a single optimum. Their solution procedure is based on construction, improving and then combining solutions within the frame work of the evolutionary approach known as scatter search.

Schittekat, et al., (2006) formulated the school bus routing problem using a single objective integer programming model VRP by introducing several other interesting additional features. They considered a set of potential stops as well as a set of students who can walk to one or more of these potential stops. The goal of their routing problem is to select a subset of stops that will actually be visited by the buses; determine which stops each student should walk to; and develop a set of tours that minimize the total distance travelled by all buses. The problem was solved using a commercial integer programming solver and results on small instances were discussed.

Fisher (1981) implemented a Lagragian Relaxation Method for solving integer programming with many side constraints. He considered the dual of the side constraints to obtain a Lagragian problem that is easy to solve and whose optimal value is a lower bound (for a minimization problem) on the optimal value of the original problem. The Lagragian has led to dramatically improved algorithms for a number of important problems in areas of routing, location, scheduling, and assignment and set covering.

Marius (1985) designed and analyzed algorithms for vehicle routing and scheduling problems with time window constraints. He described a variety of heuristics and conducted an extensive computational study of their performance. The problem set include routing and scheduling environment that differ in terms of the type of data used to generate the problem, the percentage of customers with time windows, their tightness and positioning and the scheduling horizon. The observation made was that several heuristics performed well in different problem environ; in particular, an insertion-type heuristics consistently gave very good results.

Desrochers, et al., (1991) proposed the development of a new optimization algorithm for the solution of VRPTW. They solved the LP relaxation of the set portioning formulation of the VRPTW by column generation. By this, feasible columns are added as required by solving a shortest path problem with time window and capacity constraint using dynamic programming. The LP solution obtained generally provides an excellent lower bound that is used in a branch-and-bound algorithm to solve integer set partitioning formulation. Their results indicate the success of the algorithm on a variety of practicalized benchmark VRPTW test problems. The algorithm was capable of optimally solving a 100 customer problems. This problem size is six times larger than any report presented by other published research prior to 1990.

Swersey and Ballard (1984) presented a work on scheduling of school buses. With the scheduling situation considered here, a set of routes each associated with a particular school is given. A single bus is assigned to each route to pickup students and arriving at their school within a specific time window. The problem includes finding the fewest buses needed to cover all routes whiles meeting the time window specifications. They presented two integer programming formulations of the scheduling problem and applied them to actual data from New Heaven, Connecticut for two different years as well as to 30 randomly generated problems. Linear programming relaxation of the integer programs was found to produce integer solutions more than 75% of the time. In the remaining cases, they observed the few functional values can be adjusted to integer values without increasing the number of buses needed. Their method reduces the number of buses needed by about 25% compared to the manual solutions developed by the New Heaven school bus scheduler.

Li and Fu (2002) presented a case study of the bus routing problem. It is formulated as a multi-objective combinatorial optimization problem. The objectives include minimizing the total number of buses required, the total travel time spent by all pupils at all pick-up points and the total bus travel time. They also aimed at balancing the loads and travel times between the buses. They proposed a heuristic algorithm, which was programmed and run efficiently on a PC.

Numerical results were reported using test data from a kindergarten in Hong Kong. This proved to be effective as it save 29% of total travelling times when compared the system under practice.

Tillard et al., (1997) described a tabu search heuristic for the vehicle routing problem with soft time windows. This problem allows lateness at a customer location although a penalty is incurred and added to the objective value. In the tabu search, a neighbourhood of the current solution is created through an exchange procedure that swaps sequences of consecutive customers (segments) between routes. The tabu search also exploits an adaptive memory that contains the routes of the best previously visited solutions. New starting points for the tabu search are produced through combination of routes taken from different solution found in this memory. They reported many best known solutions on classical test problems.

A bounacer et al., (2009) proposed a two population meta-heuristics to the professional staff transportation problem (PSTP). PSTP has to do with building the vehicle routing for transporting staff of one or several companies in order to minimize total cost of transport, taking into account the level of service offered to its users. The meta-heuristics developed here in this paper include ACO and Genetic algorithm (GA). Experimental results proved both techniques to be efficient.

Goel and Gruhn (2006) studied a rich vehicle routing problem incorporating various complexities found in real-life applications. The real life requirements they considered include time window restrictions, heterogeneous vehicle fleets with different travel times, travel cost and capacity, multi-dimensional capacity constraints, vehicle compatibility constraints, orders with multiple pick up, delivery and service location, different start and end locations for vehicles and routes restrictions for vehicles. This problem known as General Vehicle Routing Problem (GVRP) is highly constrained and the search space is likely to contain many solutions such that it is impossible to go in for one solution to another using a single neighbourhood structure. As a result, they proposed iterative improvement approaches based on the idea of changing the neighbourhood structure during the search.

Palmgren, et al., (2001) proposed a column generation algorithm for the Log Truck Scheduling problem. Both pick-up and delivery are included in this problem. Its consist of finding one feasible route for each vehicle in order to satisfy demand of customers and in such way that total transport cost is minimized. They used a mathematical formulation of the log truck scheduling problem that is a generalized set partitioning problem. The column generation algorithm was applied to solving LP relaxed model and a branch and price algorithm for obtaining integer solutions.

Chen and Zhang (2005) introduced adaptive ant colony optimization. Their objective is to improve the critical factor influencing the performance of the parallel algorithm. In their work they proposed strategies for information exchange between processor selections based on sorting and on difference, which makes each processor choose another processor to communicate with and update the pheromone adaptively. In order to increase the ability of the search and avoid early convergence, they also introduced a method of adjusting the time interval of information exchange adaptively in accordance with the diversity of the solution. These techniques were applied to the travelling salesman problem on the massive parallel processors and experimental results revealed high convergence speed, high speed up and efficiency.

Ali, et al., (2009) proposed a solution to the minimum vertex cover problem using ant colony optimization. They introduced a pruning based ant colony algorithm to find approximate solution to the minimum vertex cover problem. The focus was on improving both time and convergence rate of the algorithm as such they introduced a visible set based on pruning paradigm for ant, where in each stop of their traversal, are not forced to consider all the renaming vertices to select the next one for continuing the traversal. Their technique was compared to two existing algorithms based on Genetic Algorithms and computational experiment evinced that ACO Algorithm demonstrates much effectiveness and consistency for solving the minimum vertex cover problem.

Ghiduk (2010) presented ant colony optimization based approach for generating a set of optimal paths to cover all definition –use associations (du-pairs) in the program under test. The objective is to use ant colony optimization to generate suit of test-data for satisfying the generated set paths. They introduced a case study to illustrate their approach and the algorithm proved to be very efficient.

Leeprechanon, et al., (2010) presented a paper which proposes the appreciation of ant colony optimization to solve a static transmission expansion planning (STEP) problem based on DC power flow model. Their major objective is to minimize investment cost of transmission lines added to existing network in order to supply forecasted load as economically as possible and subject to many system constraints-power balance, the generation requirement, line connections and thermal limits. In order to analyze and appraise the feasibility of the ACO, their proposed methodology was applied to the Gaver's six-bus system. Experimental result compared to other conventional approaches of Genetic Algorithm and Tabu search (TS) algorithm revealed the outperformance of ACO in convergence characteristics and computational efficiency.

Nazif and Lee (2010) proposed an "Optimal Crossover Genetic Algorithm for Vehicle Routing Problem with Time Windows". In this work, they considered a set of vehicles with limits on capacity and travel times available to service a set of customers with demands and earliest and latest time for serving. The objective is to find routes for the vehicles to service all customers at minimal cost without violating capacity and travel time constraints of vehicles and time window constraints set by customers. Their proposed algorithm was tested with bench mark instances and also compared with other heuristics in the literature. Results proved the competitiveness of the proposed algorithm in terms of quality of the solution found.

Although this work addresses a contemporary routing problem using ACO, it also considers sensibility analysis on the number of ants needed for optimality. This is an important aspect of the work since it will assist in establishing a relationship between the number of ants and optimality.

2.2 PERFORMANCE CRITERIA FOR THE PROVISION OF SBTS

Three criteria exist for evaluating the provision of public goods and services (Savas, 1978 and Bowerman; Hall and Calamai, 1995). These include efficiency, effectiveness and equity. Each criterion has its own unique set of consideration and objective to satisfy yet there are clear linkages between them in terms of an overall assessment of service provision.

2.2.1 Efficiency Criterion

Efficiency measures the ratio of the level of service to the cost of the resources required to provide the service. Since the level of service in providing efficient school bus transportation is fixed for a particular situation, the main variable in determining the efficiency of a particular solution is the total cost of providing the service in currency units or man power. Therefore the efficiency of a solution is measured by its cost.

There are two main components to consider in the total cost of providing school bus transportation. One cost is the capital cost required to run one school bus for a year. Components of this include payment of bus driver, as well as the cost of vehicle maintenance, purchasing and leasing. The other main cost is the incremental cost or the cost of school bus route per kilometer travelled. It is generally accepted that the capital cost is significantly larger per bus than the incremental cost over year (Kidd, 1991). Therefore, in efficiency terms, a solution that requires fewer routes would be preferred to a solution with fewer routes.

2.2.2 Effectiveness Criterion

The effectiveness of a service is measured by how well the demand for the service is satisfied. In school bus routing measurement of service effectiveness amounts to determining whether bus transportation is available to all eligible student and whether the level of service is acceptable to the public. The question of whether or not a student qualifies for school bus transportation is dependent upon school board specific policies. In Ontario, provincial standards require a minimum level of service provision but local school board trustee determine how these standard are met and to what degree (if any) they are exceeded (Feick, 1991). An example of such a standard is the maximum distances students may walk from their homes to school before being eligible for school bus transportation. In setting this and other standards, the local school board determines the effectiveness of the school bus service by determining which student are eligible for busing.

2.2.3 Equity Criterion

Equity consideration assesses the fairness or impartiality of the provision of the service in question for each eligible student. It believed that the optimization of the efficiency criterion might produce an inexpensive solution. However, such a solution would be unacceptable to the school board due to inequities in the provision of service to students. Following this, several additional goals must be imposed which seek to make the service fair as well as efficient. For example, “first-on / first-off” on a board’s school bus routes can be regarded as equity-related criterion. This policy states that who are picked up first in the morning must also be those who are dropped off first in the afternoon. This ensures that all students on the same route travel on a school bus for approximately the same length of time and that no student has a school day (including travel time) that is significantly longer than any other student. To address this policy, the school bus routes are designed so that they begin and end at the school rather than just beginning or ending at the school. This is considered in the problem formulation of this thesis. Another way to improve equity is to "load-balance" the routes serving an area so that each school bus route transports approximately an equivalent number of students. Satisfaction of this criterion has the added advantage that it reduces the chances of filling routes to over-capacity if additional load is assigned to them during the school year. Such a situation may occur when new students move into a school attendance area or if school attendance areas are redefined when modifying pre-existing bus routes. Though capacity constraint is considered in the problem formulation of this work, emphases are not placed on "load-balance".

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CHAPTER 3

METHODOLOGY

3.0 INTRODUCTION

This chapter involves presentation of Linear Programming (LP) and Integer Programming (IP) models and the problem formulation.

3.1 THE LINEAR PROGRAMMING MODEL

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Linear Programming is the process of transforming a real life problem into a mathematical model which contains variables representing decisions that can be examined and solved for an optimal solution using algorithms (Chibuzor, 2005). Linear programming is described by Sierkma as being "concerned with the maximization or minimization of a linear objective function containing many variables subject to linear equality or inequality constraints." Since its introduction in 1947 as a means for planners to set general objectives and optimize schedules to meet set goals, linear programming and its various extensions have become an important part of not just mathematics but economics, computer science and decision science. This larger superset of linear programming applications is referred to as mathematical programming or optimization because it seeks to maximize (or minimize) a given objective function subject to a linear, nonlinear or integer constraint on the variables.

More formally since a mathematical programming problem is a superset of a linear programming problem, we can safely assume that every solution to the LP problem satisfies the mathematical programming problem. In other words every solution in the LP problem is an admissible solution which solves the mathematical programming problem.

Consider the following problem:

Tough University provides "quality" education to undergraduate and graduate students.

In an agreement signed with Tough's undergraduates and graduates (TUGs), "quality" is defined as follows: every year, each *u* (undergraduate) must take eight courses, one of which is a seminar and the rest of which are lecture courses, whereas each *g* (graduate) must take two seminars and five lecture courses. A seminar cannot have more than 20 students and a lecture course cannot have more than 40 students. The University has a faculty of 1000. The Weary Old Radicals (WORs) have a contract with the University which stipulates that every junior faculty member (there are 750 of these) shall be required to teach six lecture courses and two seminars each year, whereas every senior faculty member (there are 250 of these) shall teach three lecture courses and three seminars each year. The Regents of Touch rate Tough's President at β points per *u* and α points per *g* "processed" by the University. Subject to the agreements with the TUGs and WORs how many *u*'s and *g*'s should the President admit to maximize his rating? Formally then the President faces the following decision problem:

Maximize $\alpha g+\beta u$

Subject to $2g + u \leq 45,000$

$5g + 7u \leq 210,000$

$g \geq 0;\ u \geq 0$ (3.0)

It is convenient to use a more general notation. So let $x = (g; u)^T$; $c = (\alpha, \beta)^T$;

$b = (45000, 210000\ 0)^T$ and let A be the $4x2$ matrix:

$$A = \begin{bmatrix} 2 & 1 \\ 5 & 7 \\ -1 & 0 \\ 0 & -1 \end{bmatrix}$$

Then (2.0) can be rewritten as (2.1)

Maximize $c^T x$

Subject to $Ax \leq b$ (3.1)

In general, a linear programming problem (or LP in brief) is any decision problem of the form (2.2)

Maximize $c_1x_1 + c_2x_2 + \cdots + c_nx_n$

Subject to $a_{i1}x_1 + a_{i2}x_2 + \cdots + a_{in}x_n \leq b_i, \quad 1 \leq i \leq k$

$$a_{i1}x_1 + a_{i2}x_2 + \cdots + a_{in}x_n \geq b_i, \quad k+1 \leq i \leq l$$

$$a_{i1}x_1 + \cdots + a_{in}x_n = b_i, \quad l+1 \leq i \leq m$$

And $x_j \geq 0, \ 1 \leq j \leq p$

$$x_j \geq 0, \ p+1 \leq j \leq q$$

$$x_j arbitrary, \ q+1 \leq j \leq n \tag{3.2}$$

where the c_j, a_{ij} and b_i are fixed real numbers. There are two important special cases:

Case 1: Maximize $\sum_{j=1}^{n} c_jx_j$

$$\text{st} \sum_{j=1}^{n} a_{ij} \leq b_j \ ,1 \leq i \leq m$$

$$x_j \geq 0, \quad 1 \leq j \leq n \tag{3.3}$$

Case 2: Maximize $\sum_{j=1}^{n} c_jx_j$

$$\text{st} \sum_{j=1}^{n} a_{ij}x_j \leq b_j \ ,1 \leq i \leq m$$

$$x_j \geq 0, \quad 1 \leq j \leq n \tag{3.4}$$

3.2 THE BRANCH-AND-BOUND TECHNIQUE TO BINARY INTEGER PROGRAMMING

Because any bounded pure IP problem has only a finite number of feasible solutions, it is natural to consider using some kind of enumeration procedure for finding an optimal solution. Unfortunately, as discussed above, this finite number can be, and usually is, very large. Therefore, it is imperative that any enumeration procedure be cleverly structured so that only a tiny fraction of the feasible solutions actually need be examined. For example, dynamic programming provides one such kind of procedure for many problems having a finite number of feasible solutions (although it is not particularly efficient for most IP problems). Another such approach is provided by the branch-and-bound technique. This technique and variations of it have been applied with some success to a variety of OR problems, but it is especially well known for its application to IP problems.

The basic concept underlying the branch-and-bound technique is to divide and conquer.

Since the original "large" problem is too difficult to be solved directly, it is divided into smaller and smaller sub problems until these sub problems can be conquered. The dividing (branching) is done by partitioning the entire set of feasible solutions into smaller and smaller subsets. The conquering (fathoming) is done partially by bounding how good the best solution in the subset can be and then discarding the subset if its bound indicates that it cannot possibly contain an optimal solution for the original problem.

We shall now describe in turn these three basic steps—branching, bounding, and fathoming—and illustrate them by applying a branch-and-bound algorithm to the prototype example shown below:

(2) $6x_1 + 3x_2 + 5x_3 + 2x4 \leq 10$

(2) $x_3 + x4 \leq 1$

(3) $-x_1 + x_3 \leq 0$

(4) $-x_2 + x_4 \leq 0$ (3.5)

and

(5) x_j is binary, for $j = 1, 2, 3, 4$.

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3.2.1 Branching

When you are dealing with binary variables, the most straightforward way to partition the set of feasible solutions into subsets is to fix the value of one of the variables (say, x_1) at $x_1 = 0$ for one subset and at $x_1 = 1$ for the other subset. Doing this for the prototype example divides the whole problem into the two smaller sub problems shown below.

Sub problem 1:

Fix $x_1 = 0$ so the resulting sub problem is

Maximize $Z = 5x_2 + 6x_3 + 4x_4,$

subject to

(1) $3x_2 + 5x_3 + 2x_4 \leq 10$

(2) $x_3 + x_4 \leq 1$

(3) $x_3 \leq 0$

(4) $-x_2 + x_4 \leq 0$ (3.6)

(5) x_j is binary, for $j = 2, 3, 4$.

Sub problem 2:

Fix $x_1 = 1$ so the resulting sub problem is

Maximize $Z = 9 + 5x2 + 6x_3 + 4x_4$,

subject *to*

(1) $3x_2 + 5x_3 + 2x_4 \leq 4$

(2) $x_3 + x_4 \leq 1$

(3) $x_3 \leq 1$

(4) $-x_2 + x_4 \leq 0$ (3.7)

(5) x_j *is binary, for* $j = 1, 2, 3, 4$.

Figure 3.1 portrays this dividing (branching) into sub problems by a tree (with branches (arcs)) from the all node (corresponding to the whole problem having all feasible solutions) to the two nodes corresponding to the two sub problems. This tree, which will continue "growing branches" iteration by iteration, is referred to as the **solution tree** (or **enumeration tree**) for the algorithm. The variable used to do this branching at any iteration by assigning values to the variable (as with x_1 above) is called the **branching variable.** (Sophisticated methods for selecting branching variables are an important part of some branch-and-bound algorithms but, for simplicity, we always select them in their natural order—$x_1, x_2, \ldots, x_n$ throughout this section.)

Later in the section you will see that one of these sub problems can be conquered (fathomed) immediately, whereas the other sub problem will need to be divided further into smaller sub-problems by setting $x_2 = 0$ or $x_2 = 1$.

For other IP problems where the integer variables have more than two possible values, the branching can still be done by setting the branching variable at its respective individual values, thereby creating more than two new sub problems. However, a good alternate approach is to specify a range of values (for example, $x_j = 0$ or $x_j = 3$) for the branching variable for each new sub problem.

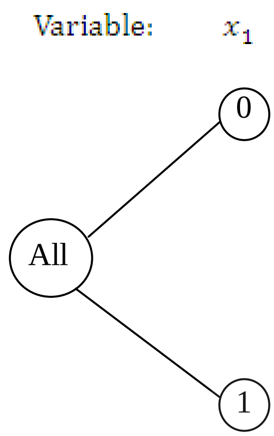


FIGURE 3.1
Figure 3.1 shows a solution tree created by the branching for the first iteration of the BIP branch and bound algorithm for (3.7)

3.2.2 Bounding

For each of these sub problems, we now need to obtain a bound on how good its best feasible solution can be. The standard way of doing this is to quickly solve a simpler relaxation of the sub problem. In most cases, a **relaxation** of a problem is obtained simply by deleting ("relaxing") one set of constraints that had made the problem difficult to solve.

For IP problems, the most troublesome constraints are those requiring the respective variables to be integer. Therefore, the most widely used relaxation is the LP relaxation that deletes this set of constraints.

To illustrate for the example, consider first the whole problem (3.7) Its LP relaxation is obtained by replacing the last line of the model (x_j is binary, for $j = 1, 2, 3, 4$) by the constraints that

$x_j = 1$ and $x_j = 0$ for $j = 1, 2, 3, 4$. Using the simplex method to quickly solve this LP relaxation yields its optimal solution

$(x_1, x_2, x_3, x_4) = \left(\frac{5}{6}, 1, 0, 1\right)$, with $Z = 16\frac{1}{2}$

Therefore, $Z \leq 16\frac{1}{2}$, for all feasible solutions for the original BIP problem (since these solutions are a subset of the feasible solutions for the LP relaxation). In fact, as summarized below, this bound of $16\frac{1}{2}$ can be rounded down to 16, because all coefficients in the objective function are integer, so all integer solutions must have an integer value for *Z*. Bound for whole problem: $Z \leq 16$. Now let us obtain the bounds for the two sub problems in the same way. Their LP relaxations are obtained from the models in the preceding subsection by replacing the constraints that x_j is binary for $j = 2, 3, 4$ by the constraints $0 \leq x_j \leq 1 \; for \; j = 2, 3, 4$. Applying the simplex method then yields their optimal solutions (plus the fixed value of x_1) shown below.

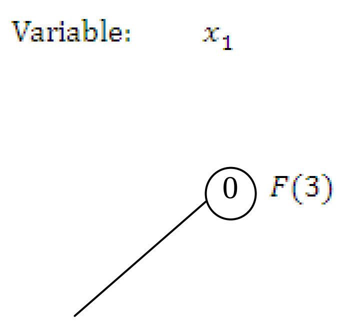
LP relaxation of sub problem 1: $(x_1, x_2, x_3, x_4) = (0, 1, 0, 1)$ with $Z = 9$.

LP relaxation of sub problem 2: $(x_1, x_2, x_3, x_4) = \left(1, \frac{4}{5}, 0, \frac{4}{5}\right)$, with $z = 16\frac{1}{5}$

The resulting bounds for the sub problems then are
Bound for sub problem 1: $Z \leq 9$,

Bound for sub problem 2: $Z \leq 16$.

Figure 3.2 summarizes these results, where the numbers given just below the nodes are the bounds and below each bound is the optimal solution obtained for the LP relaxation.



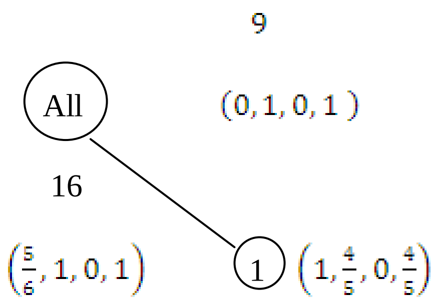


FIGURE 3.2

Figure 3.2 shows the results of bounding for the first iteration of the BIP branch and bound algorithm for (3.7)

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3.2.3 Fathoming

A sub problem can be conquered (fathomed), and thereby dismissed from further consideration, in the three ways described below. One way is illustrated by the results for sub problem 1 given by the $x_1 = 0$ node in Figure 3.7. Note that the (unique) optimal solution for its LP relaxation, $(x_1, x_2, x_3, x_4) = (0, 1, 0, 1)$, is an integer solution. Therefore, this solution must also be the optimal solution for sub problem 1 itself. This solution should be stored as the first **incumbent** (the best feasible solution found so far) for the whole problem, along with its value of *Z*. This value is denoted by

Z^* = value of *Z* for current incumbent, so $Z^* = 9$ at this point. Since this solution has been stored, there is no reason to consider sub problem 1 any further by branching from the $x_1 = 0$ node, etc. Doing so could only lead to other feasible solutions that are inferior to the incumbent, and we have no interest in such solutions. Because it has been solved, we **fathom** (dismiss) sub problem 1 now.

The above results suggest a second key fathoming test. Since $Z^* = 9$, there is no reason to consider further any sub problem whose bound ≤ 9, since such a sub problem cannot have a feasible solution better than the incumbent. Stated more generally, a sub problem is fathomed whenever it's bound $\leq Z^*$.

This outcome does not occur in the current iteration of the example because sub problem 2 has a bound of 16 that is larger than 9. However, it might occur later for **descendants** of this sub problem (new smaller sub problems created by branching on this sub problem, and then perhaps branching further through subsequent "generations"). Furthermore, as new incumbents with larger values of $Z^*$are found, it will become easier to fathom in this way.

The third way of fathoming is quite straightforward. If the simplex method finds that a sub problem's LP relaxation has no feasible solutions, then the sub problem itself must have *no* feasible solutions, so it can be dismissed (fathomed).

In all three cases, we are conducting our search for an optimal solution by retaining for further investigation only those sub problems that could possibly have a feasible solution better than the current incumbent.

Summary of Fathoming Tests

A sub problem is fathomed (dismissed from further consideration) if

Test 1: It's bound $\leq Z^*$,

or

Test 2: Its LP relaxation has no feasible solutions, or

Test 3: The optimal solution for its LP relaxation is integer. (If this solution is better than the incumbent, it becomes the new incumbent, and test 1 is reapplied to all unfathomed

Sub problems with the new larger Z^*.)

Figure 3.3 summarizes the results of applying these three tests to sub problems 1 and 2 by showing the current solution tree. Only sub problem 1 has been fathomed, by test 3, as indicated by $F(3)$ next to the $x_1 = 0$ node. The resulting incumbent also is identified below this node.

The subsequent iterations will illustrate successful applications of all three tests. However, before continuing the example, we summarize the algorithm being applied to this BIP problem. (This algorithm assumes that all coefficients in the objective function are integer and that the ordering of the variables for branching is $x_1, x_2, \dots, x_n$.)

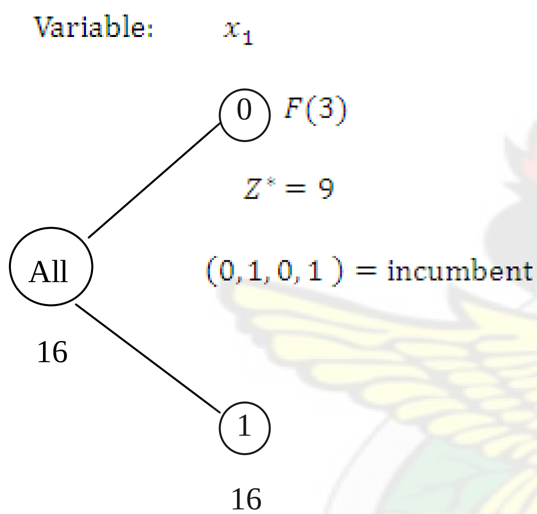

FIGURE 3.3

Figure 3.3 shows the solution tree after the first iteration of the BIP branch and bound algorithm for (3.7)

3.2.4 Summary of the BIP Branch and Bound Algorithm

Initialization: Set $Z^* = -\infty$. Apply the bounding step, fathoming step, and optimality test described below to the whole problem. If not fathomed, classify this problem as the one remaining "sub problem" for performing the first full iteration below.

Steps for each iteration

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1. Branching: Among the remaining (unfathomed) sub problems, select the one that was created most recently. (Break ties according to which the larger has bound.) Branch from the node for this sub problem to create two new sub problems by fixing the next variable (the branching variable) at either 0 or 1.
2. Bounding: For each new sub problem, obtain its bound by applying the simplex method to its LP relaxation and rounding down the value of Z for the resulting optimal solution.
3. Fathoming: For each new sub problem, apply the three fathoming tests summarized above, and discard those sub problems that are fathomed by any of the tests.

Optimality test: Stop when there are no remaining sub problems; the current incumbent is optimal.1 Otherwise, return to perform iteration.

3.3 SOME APPLICATIONS OF BINARY INTEGER PROGRAMMING WITH PRAGMATIC EXAMPLES

Managers frequently face yes or-no decisions. Therefore, binary integer programming (BIP) is widely used to aid in these decisions.

We now will introduce various types of yes-or-no decisions. We also will mention some examples of actual applications where BIP was used to address these decisions.

Each of these applications is fully described in an article in the journal called Interfaces (Hillier and Lieberman, 2001).

3.3.1 Capital Budgeting with Fixed Investment Proposals

Linear programming sometimes is used to make capital budgeting decisions about how much to invest in various projects. However, some capital budgeting decisions do not involve how much to invest, but rather, whether to invest a fixed amount. Specifically, the decisions involve whether to invest the fixed amount of capital required in building a certain kind of facility (factory or warehouse) in a certain location. Management often must face decisions about whether to make fixed investments (those where the amount of capital required has been fixed in advance). Should we acquire a certain subsidiary being spun off by another company? Should we purchase a certain source of raw materials? Should we add a new production line to produce a certain input item ourselves rather than continuing to obtain it from a supplier? In general, capital budgeting decisions about fixed investments are yes-or-no decisions of the following type. Each yes-or-no decision:

Should we make a certain fixed investment?

Its decision variable $= \begin{cases} 1 \text{ } if \text{ } yes \\ 0 \text{ } if \text{ } no \end{cases}$

The July–August 1990 issue of Interfaces describes how the Turkish Petroleum Refineries Corporation used BIP to analyze capital investments worth tens of millions of dollars to expand refinery capacity and conserve energy. A rather different example that still falls somewhat into this category is described in the January–February 1997 issue of Interfaces. A major OR study was conducted for the South African National Defense Force to upgrade its capabilities with a smaller budget. The "investments" under consideration in this case were acquisition costs and ongoing expenses that would be required to provide specific types of military capabilities. A mixed BIP model was formulated to choose those specific capabilities that would maximize the overall effectiveness of the Defense Force while satisfying a budget constraint. The model had over 16,000 variables (including 256 binary variables) and over 5,000 functional constraints.

The resulting optimization of the size and shape of the defense force provided savings of over \$1.1 billion per year as well as vital nonmonetary benefits. The impact of this study won it the prestigious first prize among the 1996 Franz Edelman Awards for Management Science Achievement.

3.3.2 Site Selection

In this global economy, many corporations are opening up new plants in various parts of the world to take advantage of lower labour costs, etc. Before selecting a site for a new plant, many potential sites may need to be analyzed and compared. Each of the potential sites involves a yes-or-no decision of the following type.

Each yes-or-no decision:

Should a certain site be selected for the location of a certain new facility?

For each of the yes-or-no decisions of any of these kinds, its decision variable $= \begin{cases} 1 \text{ } if \text{ } yes \\ 0 \text{ } if \text{ } no \end{cases}$

In many cases, the objective is to select the sites so as to minimize the total cost of the new facilities that will provide the required output. As described in the January–February 1990 issue of Interfaces, AT&T used a BIP model to help dozens of their customers select the sites for their telemarketing centers.

The model minimizes labor, communications, and real estate costs while providing the desired level of coverage by the centers. In one year alone (1988), this approach enabled 46 AT&T customers to make their yes-or-no decisions on site locations swiftly and confidently, while committing to \$375 million in annual network services and \$31 million in equipment sales from AT&T. We next describe an important type of problem for many corporations where site selection plays a key role.

3.3.3 Designing a Production and Distribution Network

Manufacturers today face great competitive pressure to get their products to market more quickly as well as to reduce their production and distribution costs. Therefore, any corporation that distributes its products over a wide geographical area (or even worldwide) must pay continuing attention to the design of its production and distribution network.

This design involves addressing the following kinds of yes-or-no decisions.

Should a certain plant remain open?

Should a certain site be selected for a new plant?

Should a certain distribution center remain open?

Should a certain site be selected for a new distribution center?

If each market area is to be served by a single distribution center, then we also have another kind of yes-or-no decision for each combination of a market area and a distribution center. Should a certain distribution center be assigned to serve a certain market area? For each of the yes-or-no decisions of any of these kinds, its decision variable $= \begin{cases} 1 \text{ if yes} \\ 0 \text{ if no} \end{cases}$

Ault Foods Limited (July–August 1994 issue of Interfaces) used this approach to design its production and distribution center. Management considered 10 sites for plants, 13 sites for distribution centers, and 48 market areas. This application of BIP was credited with saving the company $200,000 per year. Digital Equipment Corporation (January–February 1995 issue of Interfaces) provides another example of an application of this kind. At the time, this large multinational corporation was serving one-quarter million customer sites, with more than half of its $14 billion annual revenues coming from 81 countries outside the United States. Therefore, this application involved restructuring the corporation's entire global supply chain, consisting of its suppliers, plants, distribution centers, potential sites, and market areas all around the world. The restructuring has generated annual cost reductions of $500 million in manufacturing and $300 million in logistics, as well as a reduction of over $400 million in required capital assets.

3.3.4 Dispatching Shipments

Once a production and distribution network has been designed and put into operation, daily operating decisions need to be made about how to send the shipments. Some of these decisions again are yes-or-no decisions. For example, suppose that trucks are being used to transport the shipments and each truck typically makes deliveries to several customers during each trip. It then becomes necessary to select a route (sequence of customers) for each truck, so each candidate for a route leads to the following yes-or-no decision. Should a certain route be selected for one of the trucks? Its decision variable $= \begin{cases} 1 \text{ } if \text{ } yes \\ 0 \text{ } if \text{ } no \end{cases}$

The objective would be to select the routes that would minimize the total cost of making all the deliveries. Various complications also can be considered. For example, if different truck sizes are available, each candidate for selection would include both a certain route and a certain truck size. Similarly, if timing is an issue, a time period for the departure also can be specified as part of the yes-or-no decision. With both factors, each yes-or-no decision would have the form shown below. Should all the following be selected simultaneously for a delivery run:

i. A certain route,
ii. A certain size of truck, and
iii. A certain time period for the departure?

Its decision variable $= \begin{cases} 1 \text{ } if \text{ } yes \\ 0 \text{ } if \text{ } no \end{cases}$

Here are a few of the companies which use BIP to help make these kinds of decisions.

A Michigan-based retail chain called Quality Stores (March–April 1987 issue of

Interfaces) makes the routing decisions for its delivery trucks this way, thereby saving about $450,000 per year. Air Products and Chemicals, Inc. (December 1983 issue of Interfaces) saves approximately $2 million annually (about 8 percent of its prior distribution costs) by using this approach to produce its daily delivery schedules. The Reynolds Metals Co. (January–February 1991 issue of Interfaces) achieves savings of over $7 million annually with an automated dispatching system based partially on BIP for its freight shipments from over 200 plants, warehouses, and suppliers.

3.3.5 Scheduling Interrelated Activities

We schedule interrelated activities in our everyday lives, even if it is just scheduling when to begin our various homework assignments. So too, managers must schedule various kinds of interrelated activities. When should we begin production for various new orders?

When should we begin marketing various new products? When should we make various capital investments to expand our production capacity? For any such activity, the decision about when to begin can be expressed in terms of a series of yes-or-no decisions, with one of these decisions for each of the possible time periods in which to begin, as shown below. Should a certain activity begin in a certain time period? For each of the yes-or-no decisions of any of these kinds, its

decision variable $= \begin{cases} 1 \; if \; yes \\ 0 \; if \; no \end{cases}$

Since a particular activity can begin in only one time period, the choice of the various time periods provides a group of mutually exclusive alternatives, so the decision variable for only one time period can have a value of 1. For example, this approach was used to schedule the building of a series of office buildings on property adjacent to Texas Stadium (home of the Dallas Cowboys) over a 7-year planning horizon. In this case, the model had 49 binary decision variables, 7 for each office building corresponding to each of the 7 years in which its construction could begin. This application of BIP was credited with increasing the profit by $6.3 million.

A somewhat similar application on a vastly larger scale occurred in China recently (January–February 1995 issue of Interfaces). China was facing at least $240 billion in new investments over a 15-year horizon to meet the energy needs of its rapidly growing economy. Shortages of coal and electricity required developing new infrastructure for transporting coal and transmitting electricity, as well as building new dams and plants for generating thermal, hydro, and nuclear power. Therefore, the Chinese State Planning Commission and the World Bank collaborated in developing a huge mixed BIP model to guide the decisions on which projects to approve and when to undertake them over the 15-year planning period to minimize the total discounted cost. It is estimated that this OR application is saving China about $6.4 billion over the 15 years.

3.3.6 Scheduling Asset Divestitures

This next application actually is another example of the preceding one (scheduling interrelated activities). However, rather than dealing with such activities as constructing office buildings or investing in hydroelectric plants, the activities now are selling (divesting) assets to generate income. The assets can be either financial assets, such as stocks and bonds, or physical assets, such as real estate. Given a group of assets, the problem is to determine when to sell each one to maximize the net present value of total profit from these assets while generating the desired income stream. In this case, each yes-or-no decision has the following form. Should a certain asset be sold in a certain time period? Its decision variable $= \begin{cases} 1 \text{ if yes} \\ 0 \text{ if no} \end{cases}$

One company that deals with these kinds of yes-or-no decisions is Homart Development Company (January–February 1987 issue of Interfaces), which ranks among the largest commercial land developers in the United States. One of its most important strategic issues is scheduling divestiture of shopping malls and office buildings. At any particular time, well over 100 assets will be under consideration for divestiture over the next 10 years. Applying BIP to guide these decisions is credited with adding $40 million of profit from the divestiture plan.

3.3.7 Airline Applications

The airline industry is an especially heavy user of OR throughout its operations. For example, one large consulting firm called SABRE (spun off by American Airlines) employs several hundred OR professionals solely to focus on the problem of companies involved with transportation, including especially airlines. We will mention here just two of the applications which specifically use BIP. One is the fleet assignment problem. Given several different types of airplanes available, the problem is to assign a specific type to each flight leg in the schedule so as to maximize the total profit from meeting the schedule. The basic trade-off is that if the airline uses an airplane that is too small on a particular flight leg, it will leave potential customers behind, while if it uses an airplane that is too large, it will suffer the greater expense of the larger airplane to fly empty seats.

For each combination of an airplane type and a flight leg, we have the following yes or-no decision. Should a certain type of airplane be assigned to a certain flight leg?

Its decision variable $= \begin{cases} 1 \text{ } if \text{ } yes \\ 0 \text{ } if \text{ } no \end{cases}$

Delta Air Lines (January–February 1994 issue of Interfaces) flies over 2,500 domestic flight legs every day, using about 450 airplanes of 10 different types. They use a huge integer programming model (about 40,000 functional constraints, 20,000 binary variables, and 40,000 general integer variables) to solve their fleet assignment problem each time a change is needed. This application saves Delta approximately $100 million per year. A fairly similar application is the crew scheduling problem. Here, rather than assigning airplane types to flight legs, we are instead assigning sequences of flight legs to crews of pilots and flight attendants. Thus, for each feasible sequence of flight legs that leaves from a crew base and returns to the same base, the following yes-or-no decision must be made. Should a certain sequence of flight legs be assigned to a crew?

Its decision variable $= \begin{cases} 1 \ if \ yes \\ 0 \ if \ no \end{cases}$

The objective is to minimize the total cost of providing crews that cover each flight leg in the schedule. American Airlines (July–August 1989 and January–February 1991 issues of Interfaces) achieves annual savings of over $20 million by using BIP to solve its crew scheduling problem on a monthly basis.

3.4 PROBLEM DESCRIPTION OF THE PROPOSED MODEL FOR WBSC

The problem confronted here is how to transport a group of students from their homes to school. Residences of the students are geographically dispersed around the school and each school bus is unique in terms of capacity.

The following assumptions are considered:

i. Service is available to only students whose residence is not within a walking distance from the school.
ii. All students to be serviced must walk to an allowed bus stop.
iii. A bus must visits a given stop only once.
iv. Capacities of buses must not be exceeded.

Parameters/Data

K^b = Capacity of bus b

B = Number of buses

C_{ij} = Cost of traversing arc from i to j

S = Set of all potential stops

T_{ti} = Binary variable that shows if a student t can walk to stop i or not

A = Set of all arcs between stops.

D = Set representing the school or the depot

Decision Variables

$N^{(b)}{}_{ij}$ = Number of times bus b traverses arcs from i to j

$$V^{(b)}{}_{i} = \begin{cases} 1, & \text{if bus } b \text{ visit stop } i \\ 0, & \text{otherwise} \end{cases}$$

$$P^{(b)}{}_{it} = \begin{cases} 1, & \text{if vehicle } b \text{ picks up student } t \text{ at stop } i \\ 0, & \text{otherwise} \end{cases}$$

3.5 THE SBTS MODEL

$$\text{Min } f = \textstyle\sum_{i\in s}\sum_{j\in s} C_{ij} \sum_{b=1}^{B} N^{(b)}{}_{ij} \qquad \text{Total routes lengt} \qquad (3.8)$$

$$s.t \quad \sum_{b=1}^{B} V_0^{(b)} \le B\,, \quad b = 1, \dots, B \qquad (3.9)$$

$$\sum_{j\in s} N^{(b)}{}_{ij} = \sum_{j\in s} N^{(b)}{}_{ji} = V^{(b)}{}_{i}\,, \quad \forall\, i \in S, \quad b = 1, \dots, B \qquad (3.10)$$

$$\sum_{i\in T}\sum_{j\notin T} N^{(b)}{}_{ji} \ge V^{(b)}{}_{h}, \quad \forall T \le S\backslash\{0\}, h \in t, \quad b = 1, \dots, B \qquad (3.11)$$

$$\sum_{b=1}^{B} V^{(b)}{}_{i} \le 1, \quad \forall i \in S\backslash\{0\} \qquad (3.12)$$

$$\sum_{b=1}^{B} P^{(b)}{}_{it} \le T_{ti}\,, \forall t \in T, i \in S \qquad (3.13)$$

$$\sum_{i\in S}\sum_{t\in T} P^{(b)}{}_{it} \le K^{(b)}, \quad b = 1, \dots, B \qquad (3.14)$$

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$$P^{(b)}{}_{it} \leq V^{(b)}{}_{i}, \forall i \in S, t \in T, b = 1, \ldots, B \quad (3.15)$$

$$\sum_{i \in s} \sum_{b=1}^{B} P^{(b)}{}_{it} = 1, \quad \forall t \in T \quad (3.16)$$

$$V_i^{(b)} \in \{0,1\}, \forall i \in S, \quad b = 1, \ldots, B \quad (3.17)$$

$$N_{ij}{}^{(b)} \in \{0,1\}, \forall i,j \in S \backslash i \neq j \quad (3.18)$$

$$P_{it}{}^{(b)} \in \{0,1\}, \forall i,j \in S \backslash i \neq j \quad (3.19)$$

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The objective function (3.8) minimizes the total length of all routes covered by all buses. Constraint (3.9) guarantees that all buses start from the school (i.e. D). Constraint (3.10) guarantees that if bus b visits stop i then one arc is traversed by b entering and exiting i. Constrain (3.11) prevents the formation of sub-tours. This means that each cut defined by a customer set T is crossed by a number of arcs not less than the minimum number of buses $n(B)$ required to serve set T. Constraint (3.12) guarantees that a bus visits a particular stop not more than one. Constraint (3.13) ensures that every student walks to his single designated stop only. Constraint (3.14) guarantees that respective capacities of buses are not exceeded. Constraint (3.15) guarantees that a student t designated to stop i is picked up by bus b provided b visits stop i. Constraint (3.16) ensures that all students are picked up only once. Finally,(3.17),(3.18) and (3.19) represent the binary integrality constraints on all decision all decision variables.

3.6 THE WOODBRIDGE SBTS MODEL

Min $f = \sum_{i \in s} \sum_{j \in s} C_{ij} \sum_{b=1}^{4} N^{(b)}{}_{ij}$ $\quad (3.8)^1$

$$s.t \quad \sum_{b=1}^{4} V_0{}^{(b)} \leq 4, \quad b = 1,2,3,4 \quad (3.9)^1$$

$$\sum_{j\in s} N^{(b)}{}_{ij} = \sum_{j\in s} N^{(b)}{}_{ji} = V^{(b)}{}_{i}\ ,\quad \forall\, i \in S, \qquad b = 1,2,3,4 \qquad (3.10)^{1}$$

$$\sum_{i\in T}\sum_{j\notin T} N^{(b)}{}_{ji} \geq V^{(b)}{}_{h}, \quad \forall T \leq S\backslash\{0\}, h \in t, \quad b = 1, \dots, 4 \qquad (3.11)$$

$$\sum_{b=1}^{4} V^{(b)}{}_{i} \ \leq 1, \ \ \forall i \in S\backslash\{0\} \qquad (3.12)^{1}$$

$$\sum_{b=1}^{4} P^{(b)}{}_{it} \ \leq T_{ti}\ , \forall t \ \in T, i \in S \qquad (3.13)^{1}$$

$$\sum_{i\in S}\sum_{t\in T} P^{(b)}{}_{it} \leq K^{(b)}, \ \ b = 1,2,3,4 \qquad (3.14)$$

$$P^{(b)}{}_{it} \leq V^{(b)}{}_{i}, \forall i \in S, t \in T, b = 1,2,3,4 \qquad (3.15)^{1}$$

$$\sum_{i\in s}\sum_{b=1}^{4} P^{(b)}{}_{it} = 1\ , \ \ \forall t \in T \qquad (3.16)^{1}$$

$$V_i^{(b)} \in \{0, 1\}\ , \forall i \in S\ , \ \ b = 1,2,3,4 \qquad (3.17)^{1}$$

$$N_{ij}{}^{(b)} \in \{0, 1\}, \forall i, j \in S\backslash i \neq j \qquad (3.18)^{1}$$

$$P_{it}{}^{(b)} \in \{0,1\}, \forall i, j \in S\backslash i \neq j \qquad (3.19)^{1}$$

Apart from the fact that SBTS is a hard combinatorial problem, the number of customers and that of potential bus stops locations, as exists at Wood Bridge School Complex, are large. Consequent to this, the IP model described above cannot be solved efficiently using the branch-and-bound or any exact polynomial algorithm. A heuristic approach must therefore be used and the one that favours the researcher's choice is the ACO heuristics. The next subsection is devoted to explaining ACO heuristics and it's algorithm for SBTS.

3.7 ANT COLONY OPTIMIZATION

Ant colony optimization is a part of the larger field of swarm intelligence in which scientists study the behavior patterns of bees, termites, ants and other social insects in order to simulate processes.

The ability of insect swarms to thrive in nature and solve complex survival tasks appeals to scientists developing computer algorithms needed to solve similarly complex problems. Artificial intelligence algorithms such as ant colony optimization are applied to large combinatorial optimization problems and are used to create self-organizing methods for such problems. Ant colony optimization is a meta-heuristic technique that uses artificial ants to find solutions to combinatorial optimization problems. ACO is based on the behavior of real ants and possesses enhanced abilities such as memory of past actions and knowledge about the distance to other locations. In nature, an individual ant is unable to communicate or effectively hunt for food, but as a group, ants possess the ability to solve complex problems and successfully find and collect food for their colony. Ants communicate using a chemical substance called pheromone.

As an ant travels, it deposits a constant amount of pheromone that other ants can follow. Each ant moves in a somewhat random fashion, but when an ant encounters a pheromone trail, it must decide whether to follow it. If it follows the trail, the ant's own pheromone reinforces the existing trail, and the increase in pheromone increases the probability of the next ant selecting the path. Therefore, the more ants that travel on a path, the more attractive the path becomes for subsequent ants. Additionally, an ant using a short route to a food source will return to the nest sooner and therefore, mark its path twice, before other ants return. This directly influences the selection probability for the next ant leaving the nest. Over time, as more ants are able to complete the shorter route, pheromone accumulates faster on shorter paths and longer paths are less reinforced.

The evaporation of pheromone also makes less desirable routes more difficult to detect and further decreases their use. However, the continued random selection of paths by individual ants helps the colony discover alternate routes and insures successful navigation around obstacles that interrupt a route. Trail selection by ants is a pseudo-random proportional process and is a key element of the simulation algorithm of ant colony optimization (Dorigo and Gamberdella, 1997).

ACO was first applied to the traveling salesman problem and the quadratic assignment problem (Dorigo, 1992). Ever since, it has been applied to other problems, which include but not limited to the space planning problem (Bland, 1999), the machine tool tardiness problem (Bauer, Bullnbeimer and Hartl, 1999) and the multiple objective JIT sequencing problem (McMullen, 2001).

3.8 ACO HEURISTICS FOR ROUTING BUSES TO STOPS

3.8.1 Route construction

Using ACO, an individual ant simulates a vehicle, and its route is constructed by incrementally selecting customers until all customers have been visited. Initially, each ant starts at the depot and the set of customers included in its tour is empty. The ant selects the next customer to visit from the list of feasible locations and the storage capacity of the vehicle is updated before another customer is selected. The ant returns to the depot when the capacity constraint of the vehicle is met or when all customers are visited. The total distance L is computed as the objective function value for the complete route of the artificial ant. The ACO algorithm constructs a complete tour for the first ant prior to the second ant starting its tour. This continues until a predetermined number of ants m each construct a feasible route.

Using ACO, each ant must construct a vehicle route that visits each customer. To select the next customer j, the ant uses the following probabilistic formula (Dorigo and Gamberdella, 1997)

$$j = \begin{cases} argmax\{(T_{iu})(\eta_{iu})^{\beta}\} \; for \; u \notin M_k, q \leq q_0 \\ S \; otherwise \end{cases} \qquad (3.20)$$

Where T_{iu} is equal to the amount of pheromone on the path between the current location i and possible locations u. The value η_{iu} is defined as the inverse of the distance between the two customer locations and the parameter β establishes the importance of distance in comparison to pheromone quantity in the selection algorithm $(\beta > 0)$. Locations already visited by an ant are stored in the ants working memory M_k and are not considered for selection. The value q is a random uniform variable $[0,1]$ and the value q_0 is a parameter. When each selection decision is made, the ant selects the arc with the highest value from (3.13) unless q is greater than q_0. In this case, the ant selects a random variable (S) to be the next customer to visit based on the probability distribution of P_{ij}, which favors short paths with high levels of pheromone:

$$P_{ij} = \begin{cases} \dfrac{(T_{iu})(\eta_{iu})^{\beta}}{\sum_{u \notin M_k}(T_{iu})(\eta_{iu})^{\beta}} & \text{if } j \notin M_k \\ 0 \; otherwise \end{cases} \qquad (3.21)$$

Using formulas (3.20) and (3.21) each ant may either follow the most favorable path already established or may randomly select a path to follow based on a probability distribution established by distance and pheromone accumulation. If the vehicle capacity constraint is met, the ant will return to the depot before selecting the next customer. This selection process continues until each customer is visited and the tour is complete.

3.8.2 Trail updating

To improve future solutions, the pheromone trails of the ants must be updated to reflect the ant's performance and the quality of the solutions found. This updating is a key element to the adaptive learning technique of ACO and helps to ensure improvement of subsequent solutions (Bell and McMullen, 2004). Trail updating includes local updating of trails after individual solutions have been generated and global updating of the best solution route after a predetermined number of solutions m has been accomplished.

First, local updating is conducted by reducing the amount of pheromone on all visited arcs in order to simulate the natural evaporation of pheromone and to ensure that no one path becomes too dominant. This is done with the following local trail updating equation,

$$T_{ij} = (1 - \alpha)\, T_{ij} + (\alpha)\, T_0 \qquad (3.22)$$

Where α is a parameter that controls the speed of evaporation and T_0 is equal to an initial pheromone value assigned to all arcs in graph G. For this study, T_0 is equal to the inverse of the best known route distances found for the particular problem. After a predetermined number of ants m construct a feasible route, global trail updating is performed by adding pheromone to all of the arcs included in the best route found by one of m ants. Global trail updating is accomplished according to the following relationship,

$$T_{ij} = (1 - \alpha)\, T_{ij} + \alpha L^{-1} \qquad (3.23)$$

This updating encourages the use of shorter routes and increases the probability that future routes will use the arcs contained in the best solutions. This process is repeated for a predetermined number of iterations and the best solution from all of the iterations is presented as an output of the model and should represent a good approximation of the optimal solution for the problem.

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CHAPTER 4

DATA COLLECTION AND ANALYSIS

4.0 INTRODUCTION

This chapter displays ant colony results of the data taken on the various buses. Since four buses are involved, the chapter is divided into four sections, where each section provides ACO results for each bus. For each section, four or five results are displayed for different ant numbers. The results are obtained by an ant colony programme written in Matlab implementation codes. Each figure comprises two panels viz, lower and upper. The upper pannel shows the distance covered by various ants at every iterative point whilst the lower displays the complete optimal tour of the best ant (for the different ant numbers).

The data involves length of distances between various picking points. The conversion of the distances into cartesian coordinates is what gave rise to the distance matrix as given in Appendix C. This was made possible by recordig the angles at various picking points with reference to the magnetic north (call it directional conversion). The distances were recoded with the aid of a car that reads distances digitally, while the angles were obtained with the help of an i-phone. The i-phone also provided the GPS information at the respective picking points. This information on longitudes and lattitudes could also be converted to coordinates. One possibility is the use of a software called the eye-calculator (call it GPS conversion). Preliminary experimentation proved results of the the different distance matrices to be the same. The results in this section are based on the directional conversion, which was chosen arbitrary. The data on the original tour of the various buses is given in 'Ant Sketches' (see Appendix A).

4.1 ACO OUTPUT FOR BUS 1

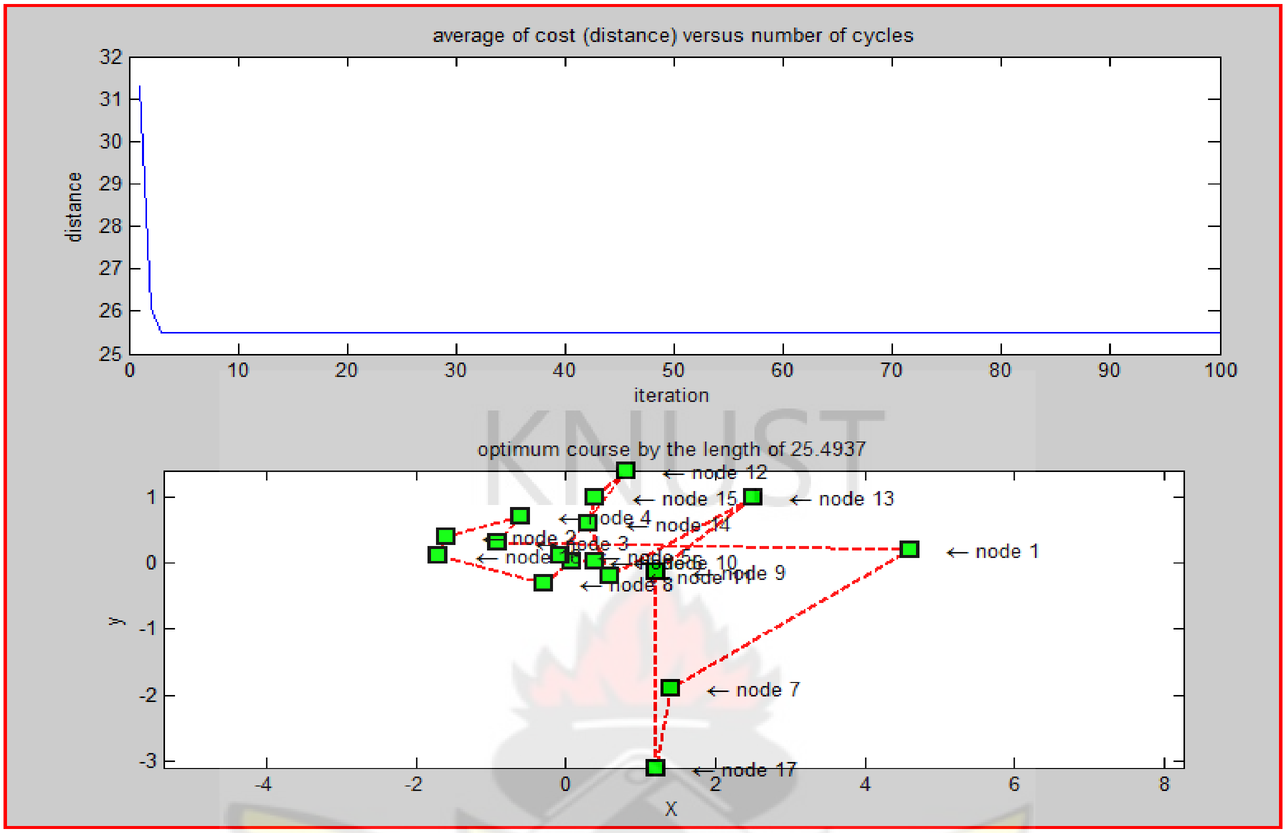

Figure 4.1.1: Ant colony results for 10 ants.

Figure 4.1.1 depicts the ant colony results of bus 1 for 10 ants. From the figure, the tour of the best ant out of the 10 is shown by the lower panel. Thus, the optimal route length covered by the best ant is about 25km. This is worse compared to the actual distance covered by bus 1.

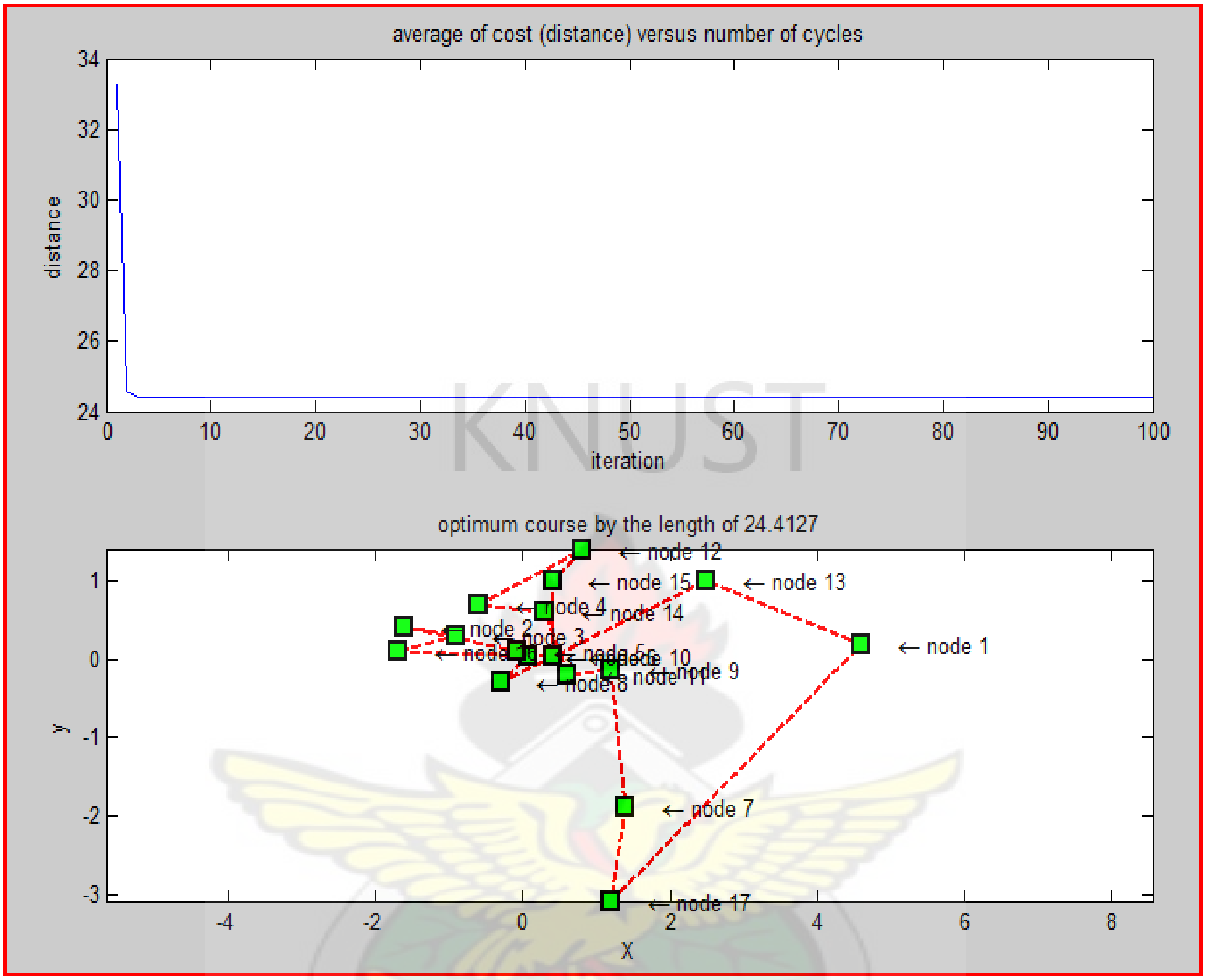

Figure 4.1.2: Ant colony result for 15 ants.

Figure 4.1.2 shows that for 15 ants, the best ant would cover an optimal route length of about 24km. This is equivalent to the actual route length covered by bus 1. The direction from the source node is displayed by the lower panel.

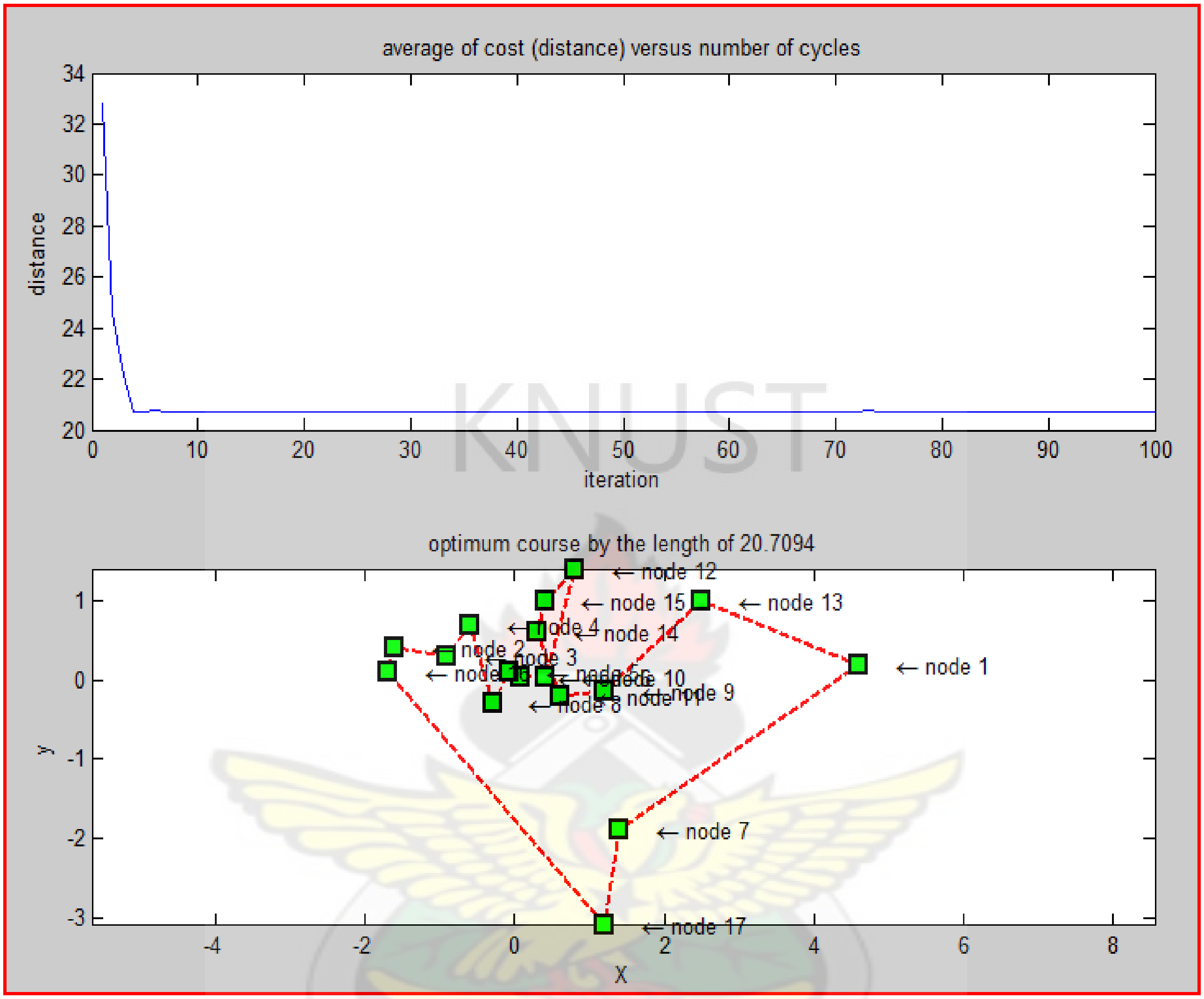

Figure 4.1.3: Ant colony result for 50 ants.

Figure 4.1.3 shows that for 50 ants, the optimal route length recorded by the best ant is approximately 21km, which is better than that covered by bus 1. The complete tour and order of movement is displayed in the lower panel.

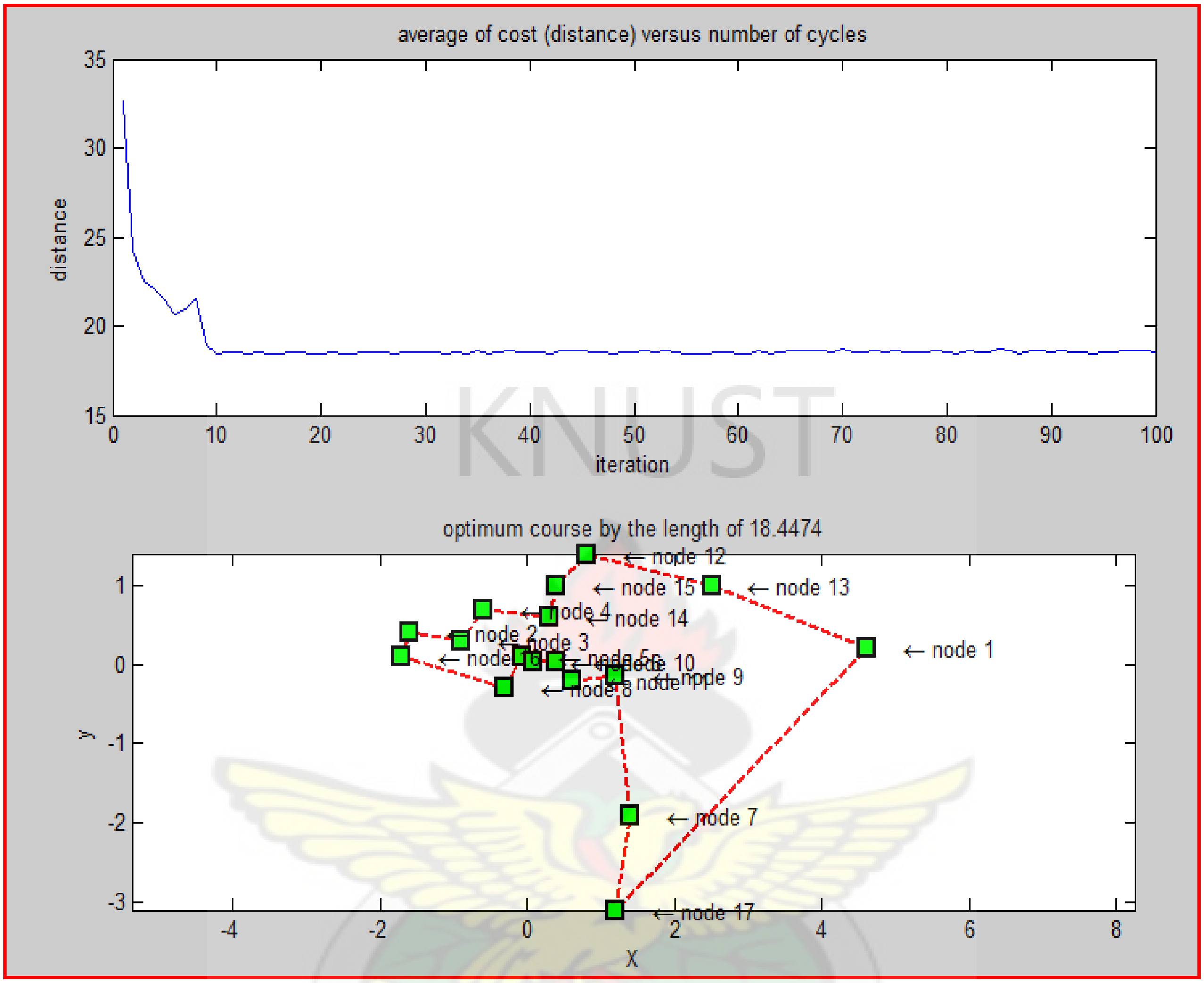

Figure 4.1.4: Ant colony result for 100 ants.

Figure 4.1.4 shows that for 100 ants, the optimal route length covered by the best ant as it starts from the source node is approximately 18km. This represents an improvement of the one covered by the 50 ants. The course of the best ant from the source node is shown in the lower panel.

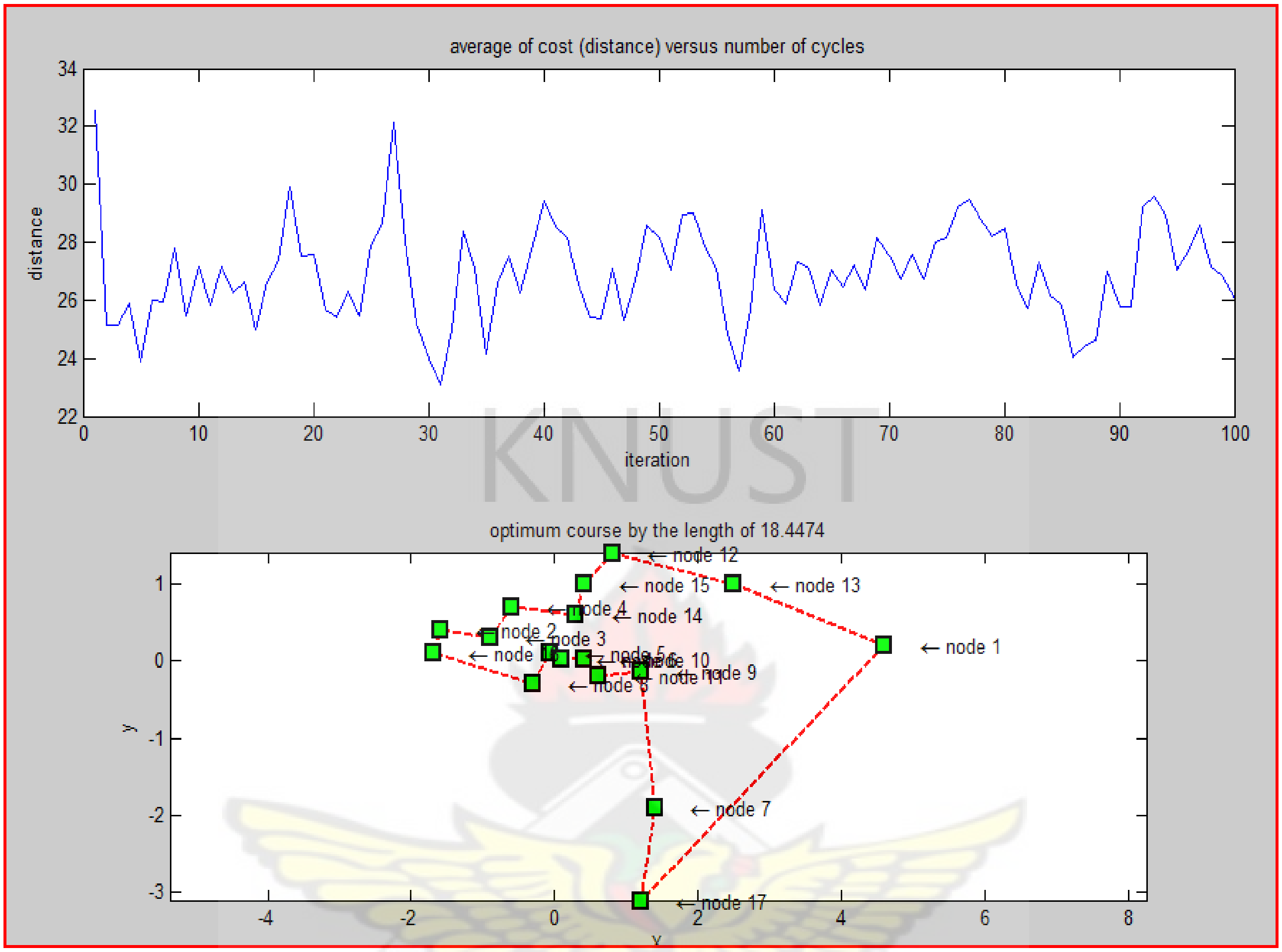

Figure 4.1.5: Ant colony result for 200 ants.

Figure 4.1.5 shows that for 200 ants the best ant will cover an optimum course by the length of about 18km, which is the same as that which was covered by 100 ants. This means that the optimal route length displaced by bus 1 using ant colony is approximately 18km, and is given by the optimal route is given by the following order of tour:

17→1→13→12→15→14→4→3→2→16→8→5→6→10→11→9→7→17 (See Appendix B for names corresponding to nodes).

4.2 ACO OUTPUT FOR BUS 2

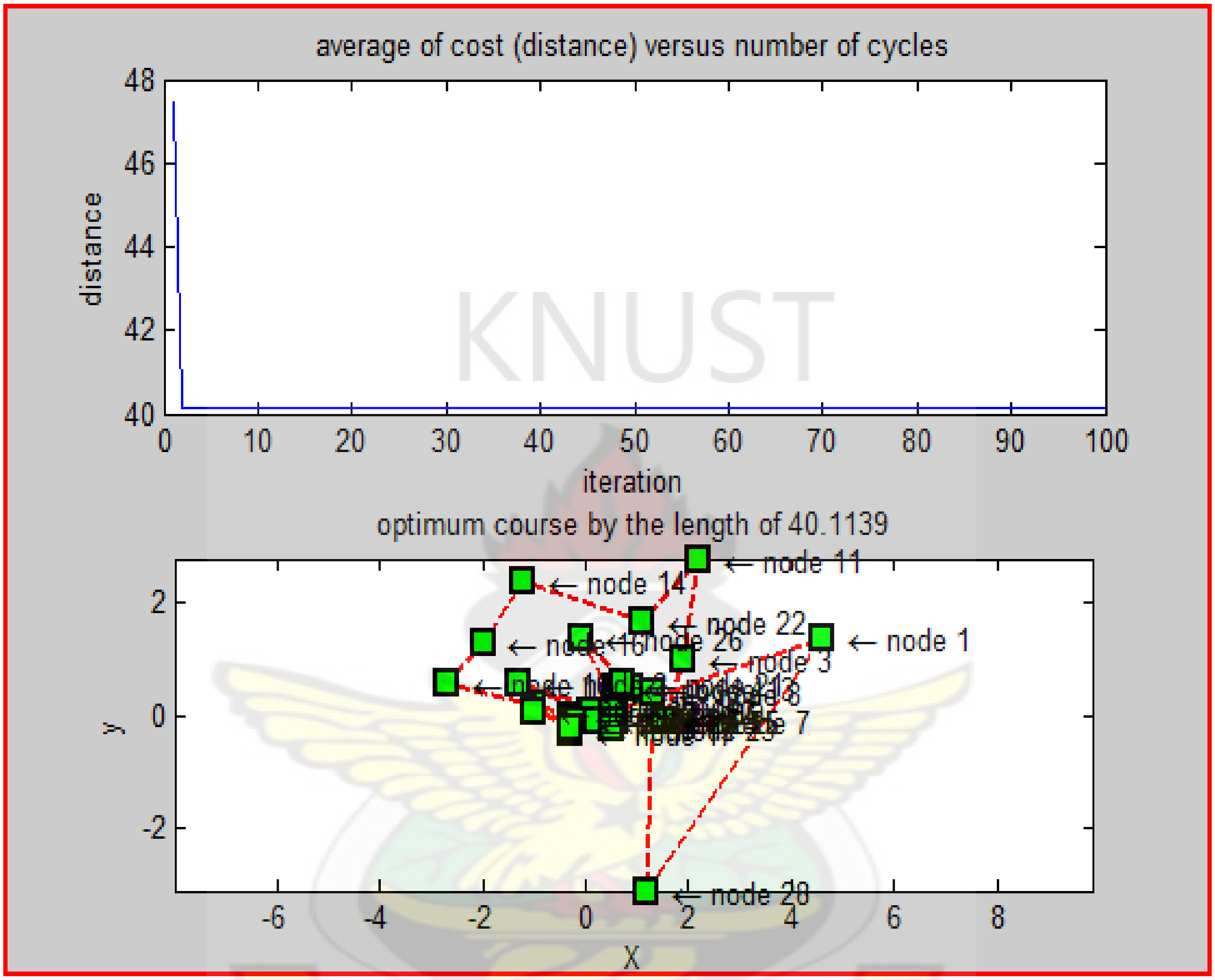

Figure 4.2.1: Ant colony results for 8 ants.

Figure 4.2.1 depicts the ant colony results for bus 2 with the number of ants equal to 8. It can be seen from the figure that the optimal route length covered by the best out of the 8 ants is approximately 40km. This is very close to the actual distance covered by bus 2 (i.e. 40.7km). The course of the best ant is shown by the lower panel.

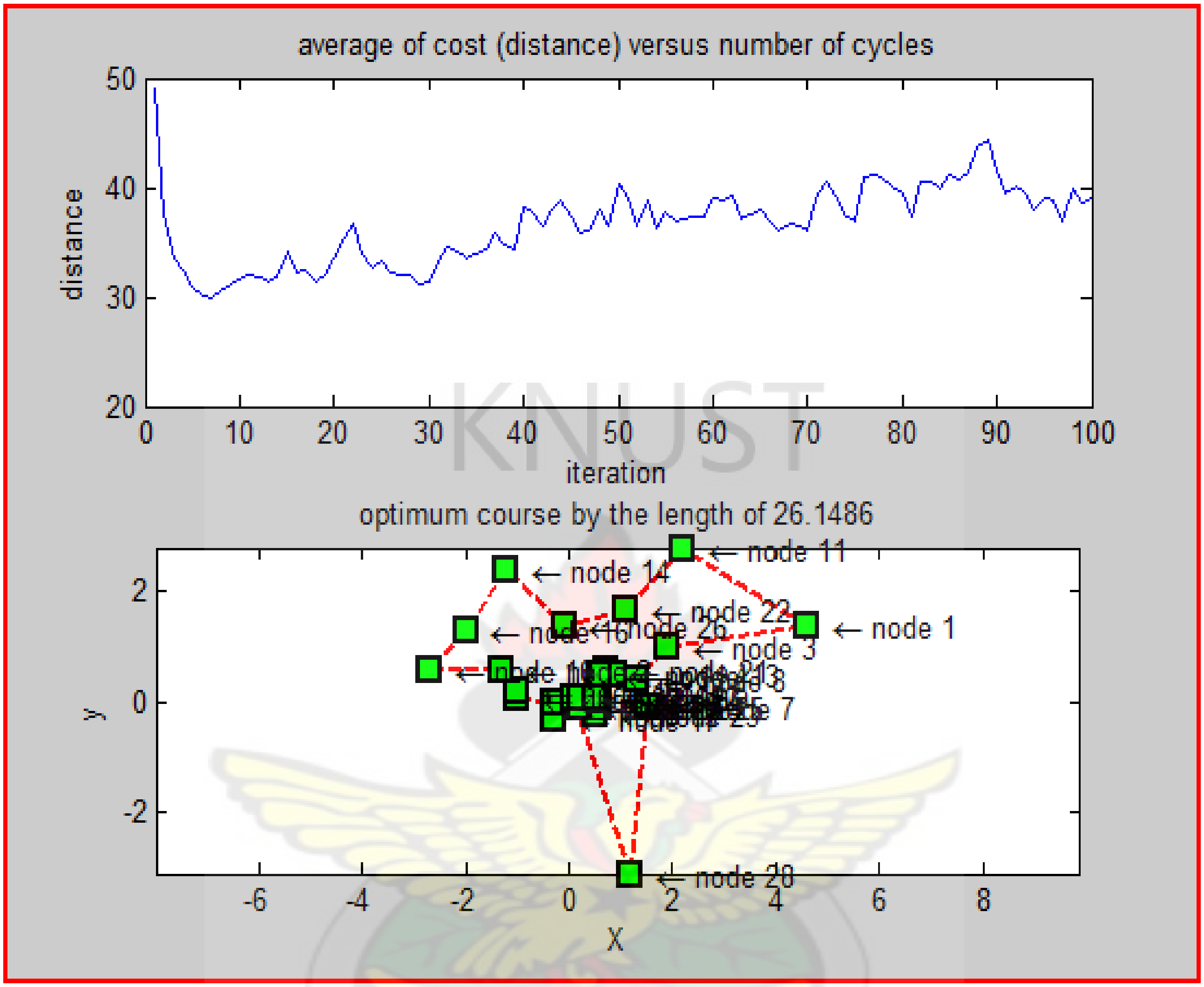

Figure 4.2.2: Ant colony results for 50 ants.

Figure 4.2.2 shows that for 50 ants the total distance covered by the best ant is approximately 27km, representing an improvement from the previous 8 ants. The course of the best ant from the source node is shown by the lower panel.

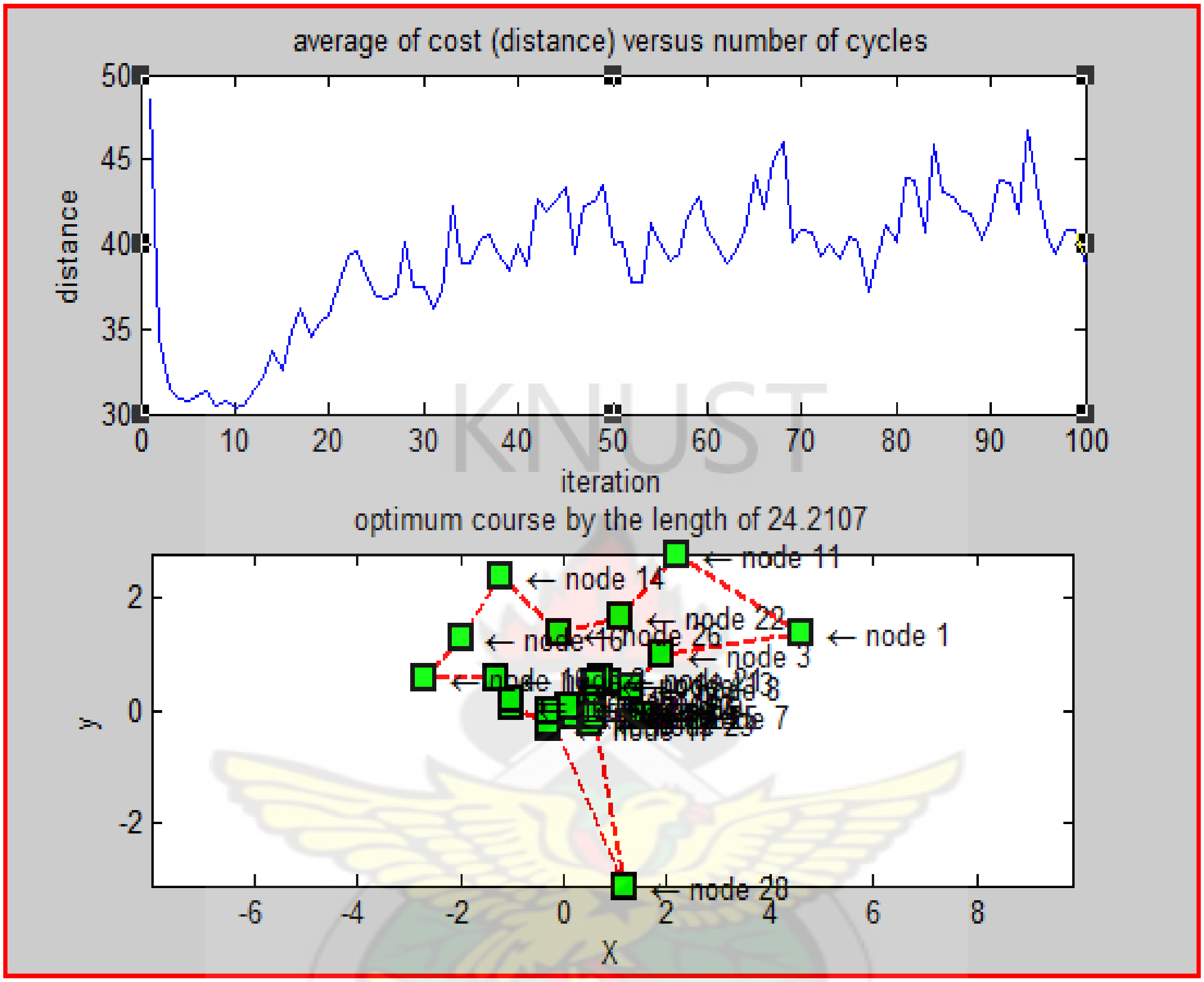

Figure 4.2.3: Ant colony results for 100 ants.

From figure 4.2.3, it can be seen that the optimal route length covered by the best out of the 100 ants is by about 24km, which also represent an improvement from the previous 50 ants. The tour of the best ant from the source node is shown by the lower panel.

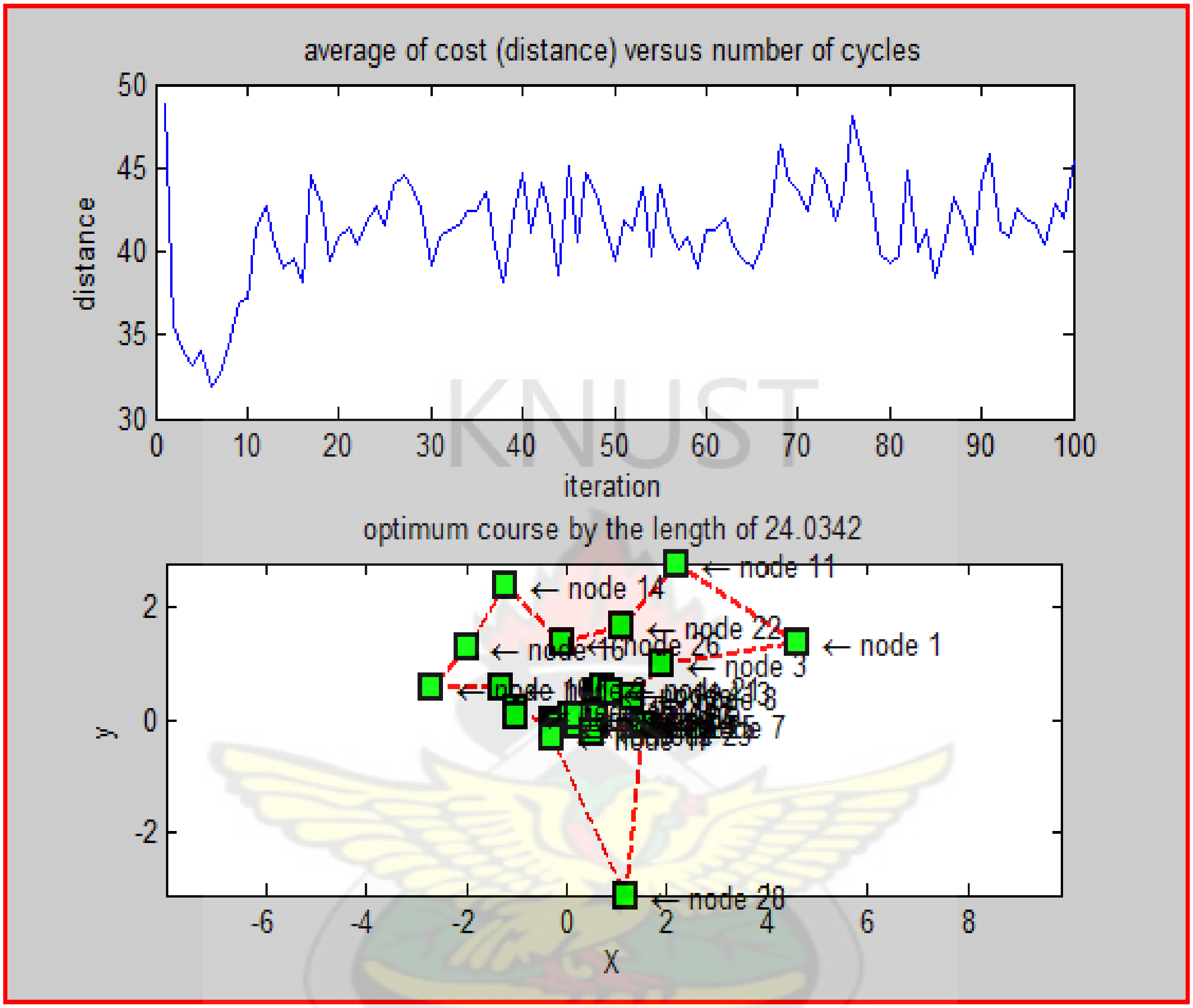

Figure 4.2.4: Ant colony results for200 ants.

Similarly, in Figure 4.2.4, the best ant out of 200 will displace an optimum course by the length of approximately 24km. This is equivalent to that which was covered by the previous 100 ants. Thus, the optimal course of bus 2 via ant colony optimization is about 24km. The optimal route and order of movement (clockwise) from the source node (28) is displayed in the lower panel.

4.3 ACO OUTPUT FOR BUS 3

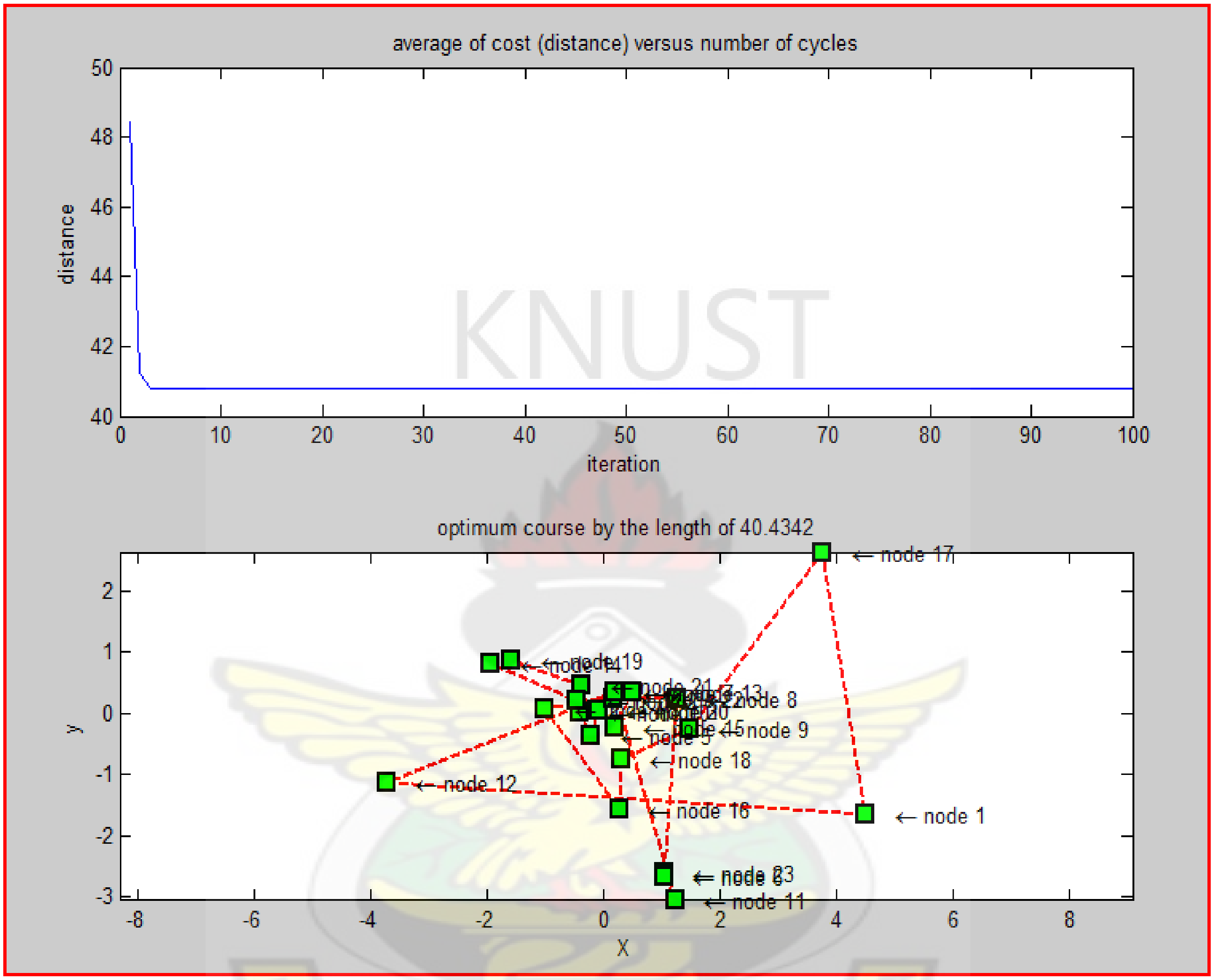

Figure 4.3.1: Ant colony results for7 ants

Figure 4.3.1 depicts the ant colony result for bus 3 using 7 ants. It is discernible from the figure that the optimal route length covered by the best out of 7 ants is close to 40km, which is also equivalent to the actual distance completed by bus 3 (39.94km) in its tour. The course of the best ant from the source node is displayed by the lower panel.

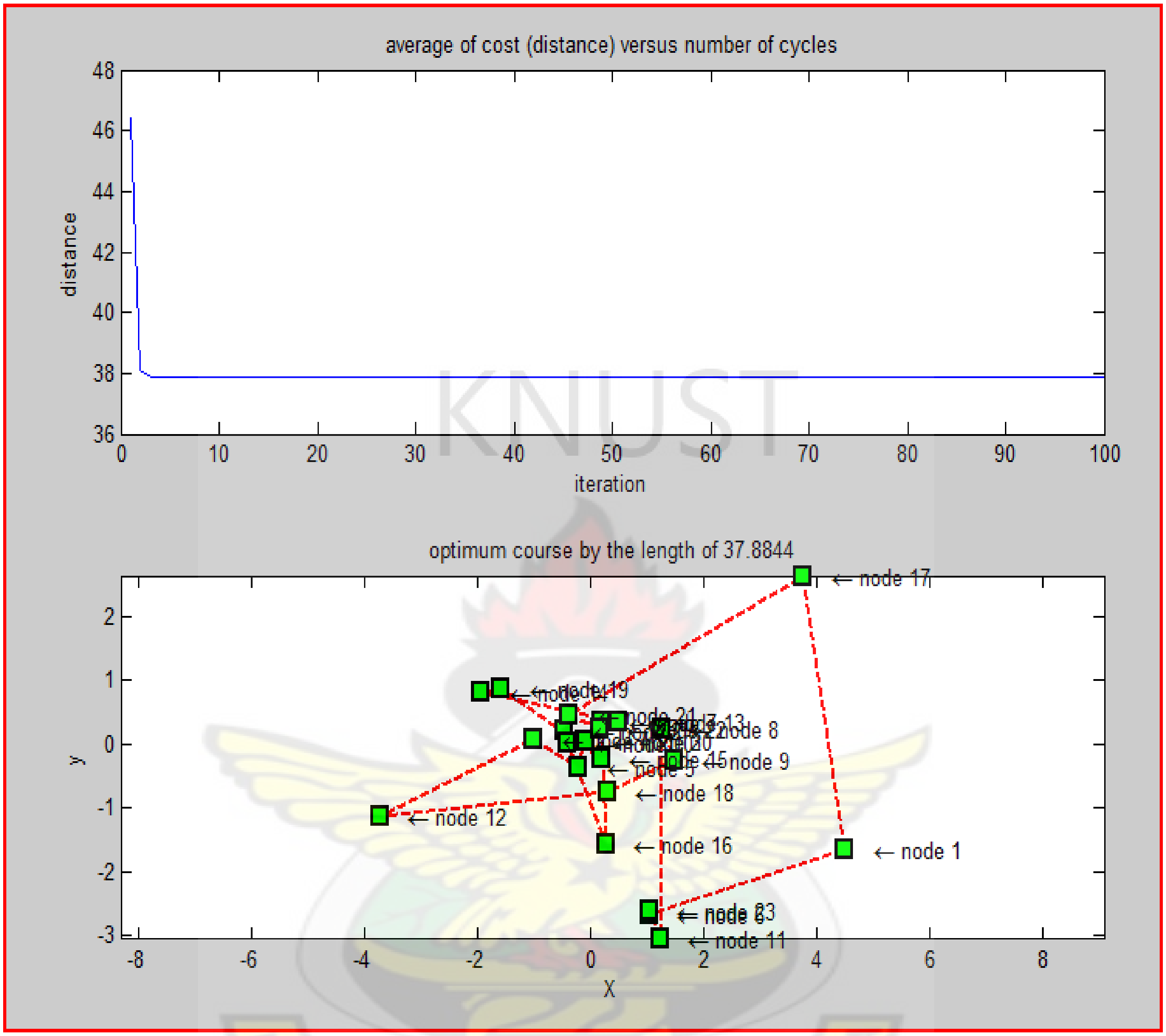

Figure 4.3.2: Ant colony results for 10 ants

Figure 4.3.2 shows that for 10 ants, the best ant out of 10 will cover an optimal route length of about 37km, representing an improvement in that which was covered by the previous 7 ants. The route and order of movement from the source node is shown in the lower panel.

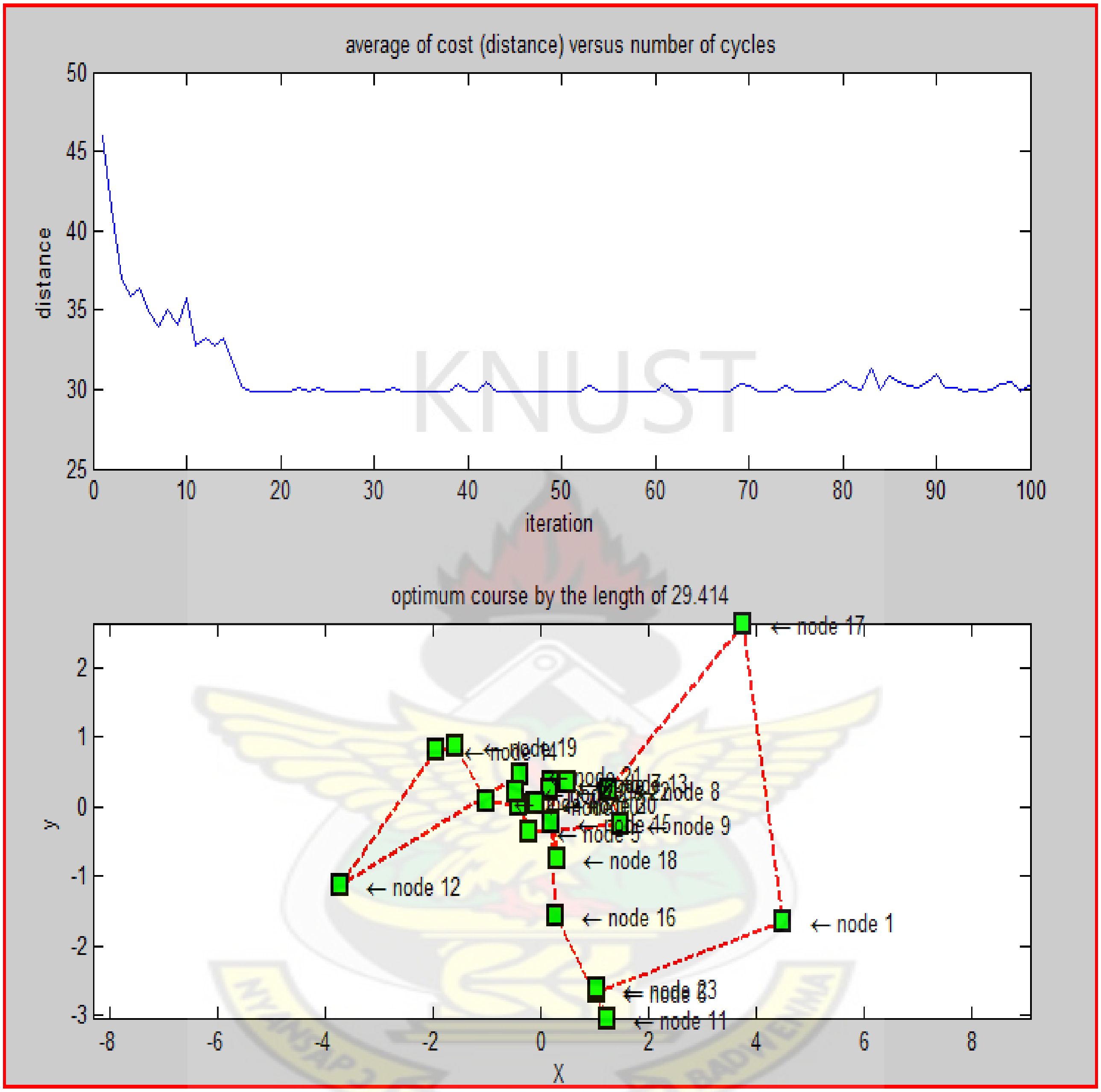

Figure 4.3.3: Ant colony results for 50 ants

It can be seen from Figure 4.3.3 that for 50 ants, the best tour will record the length of about 29km. This represents an improvement of the previous solution. The route and order of movement of the best ant is shown in the lower panel.

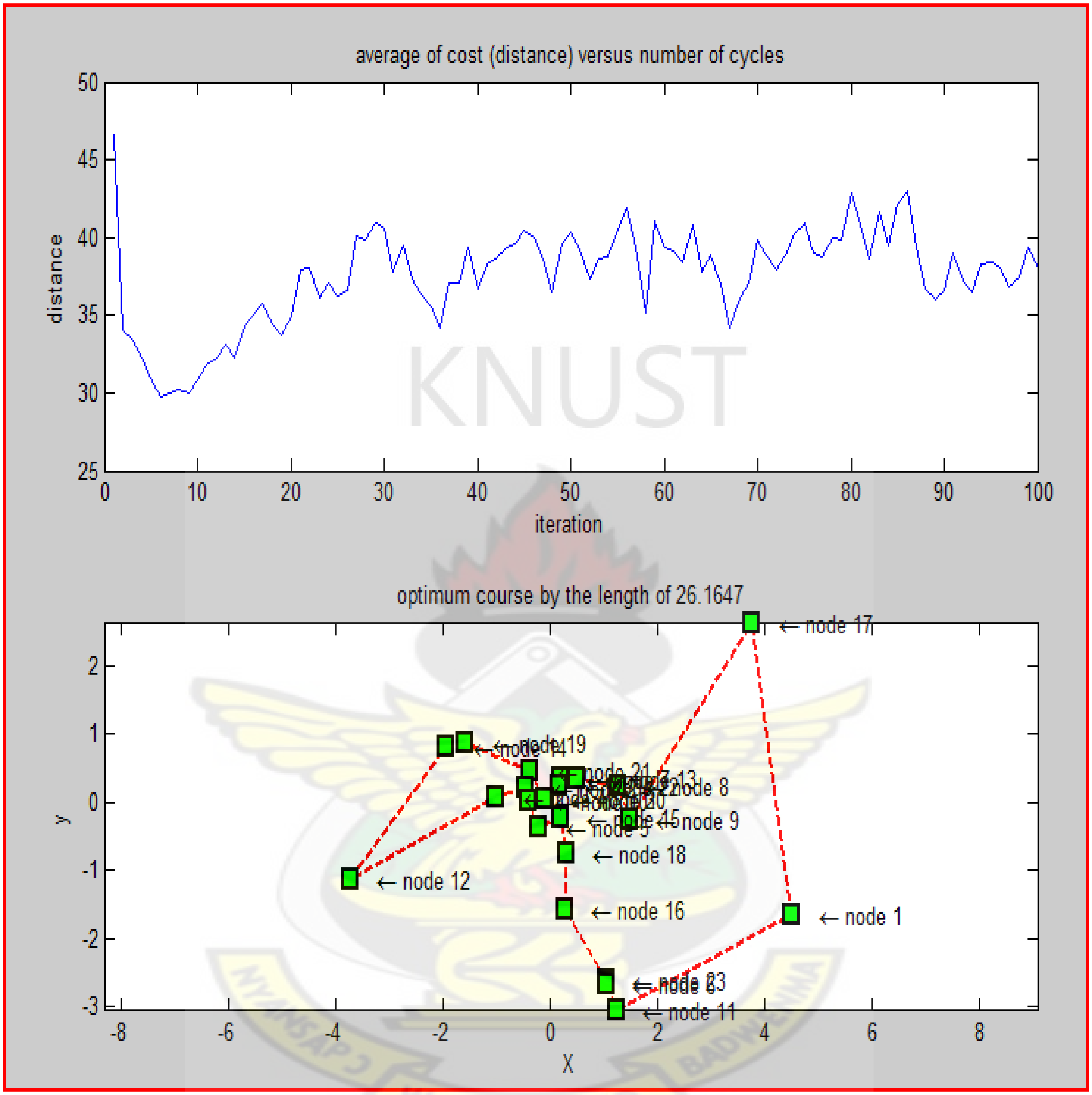

Figure 4.3.4: Ant colony results for 100 ants

Figure 4.3.4 also reveals that for 100 ants, the best ant will complete a tour with an optimum route length of about 26km. This still shows an improvement in the optimal solution. The course of the best ant is captured by the lower panel.

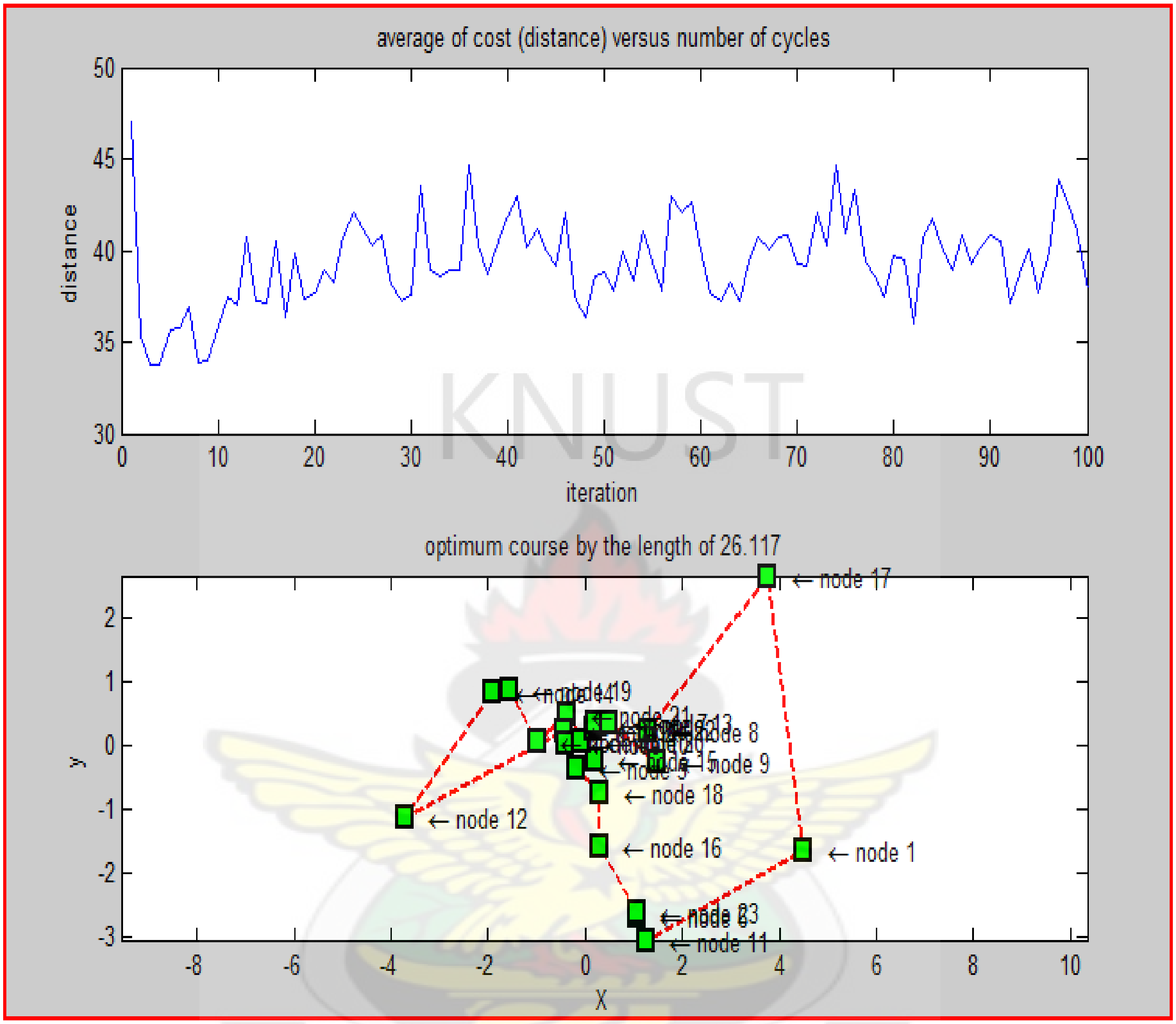

Figure 4.3.5: Ant colony results for 200 ants

Figure 4.3.5 shows that for 200 ants, the optimum length completed by the best out of 200 ants is approximately 26km. This is approximately the same as that completed by the best out of the previous 100 ant. Hence, the optimum course for bus 3 using ant colonies is approximately by the length of 26km. The optimal course from the source node is given by 23→6→11→1→17→13→8→9→7→22→21→2→10→3→4→19→14→12→20→15→5→18→16→23.

4.4 ACO OUTPUT FOR BUS 4

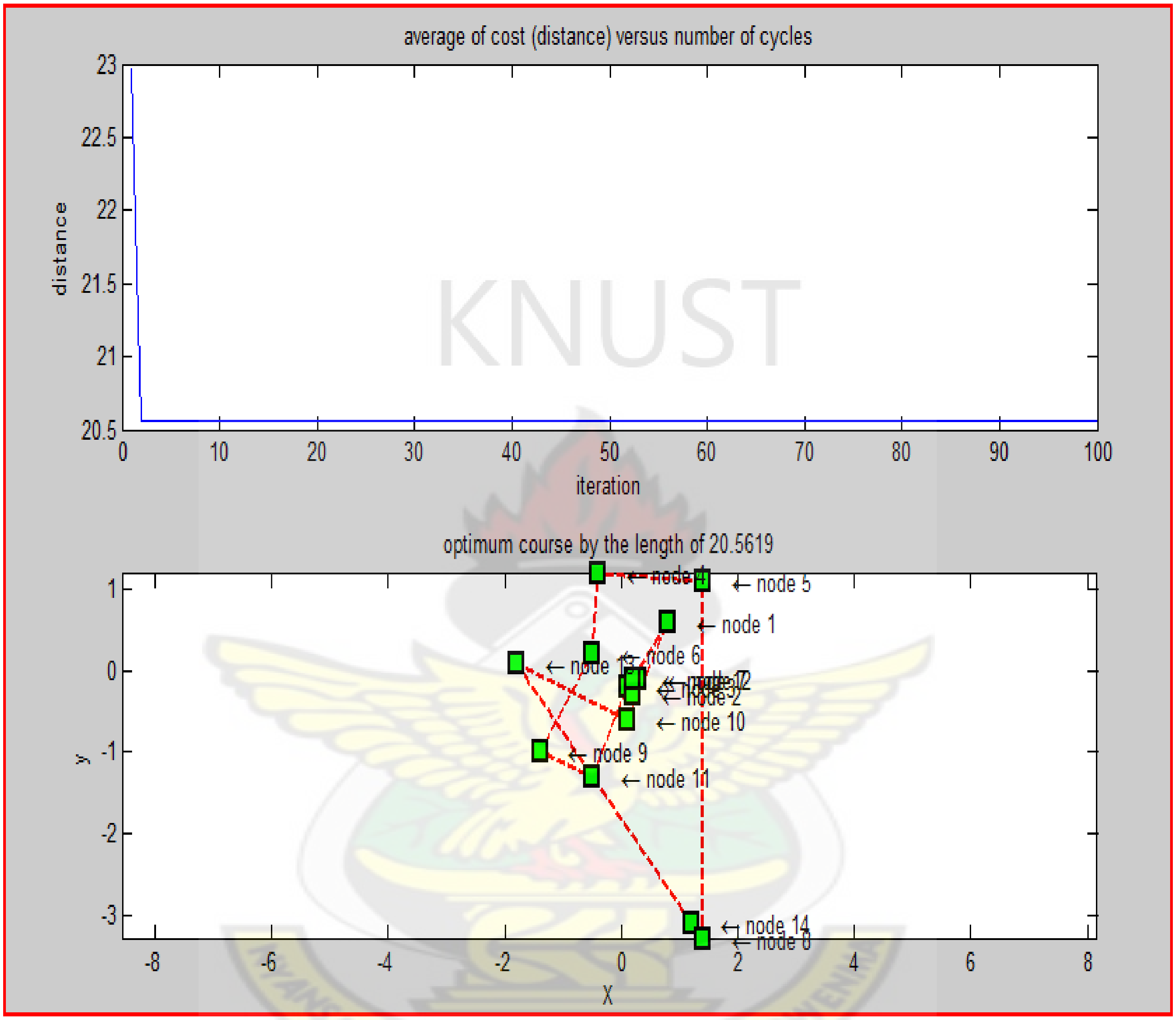

Figure 4.4.1: Ant colony results for 5 ants

Figure 4.4.1 depicts the ant colony results for bus 4 using 5 ants. It can be observed from the figure that the optimal route length covered by the best out of 5 ants is approximately 21km, which is worse in comparison with the actual distance covered by bus 4 (17.8km). The tour of the best ant from the source node is shown in the lower panel.

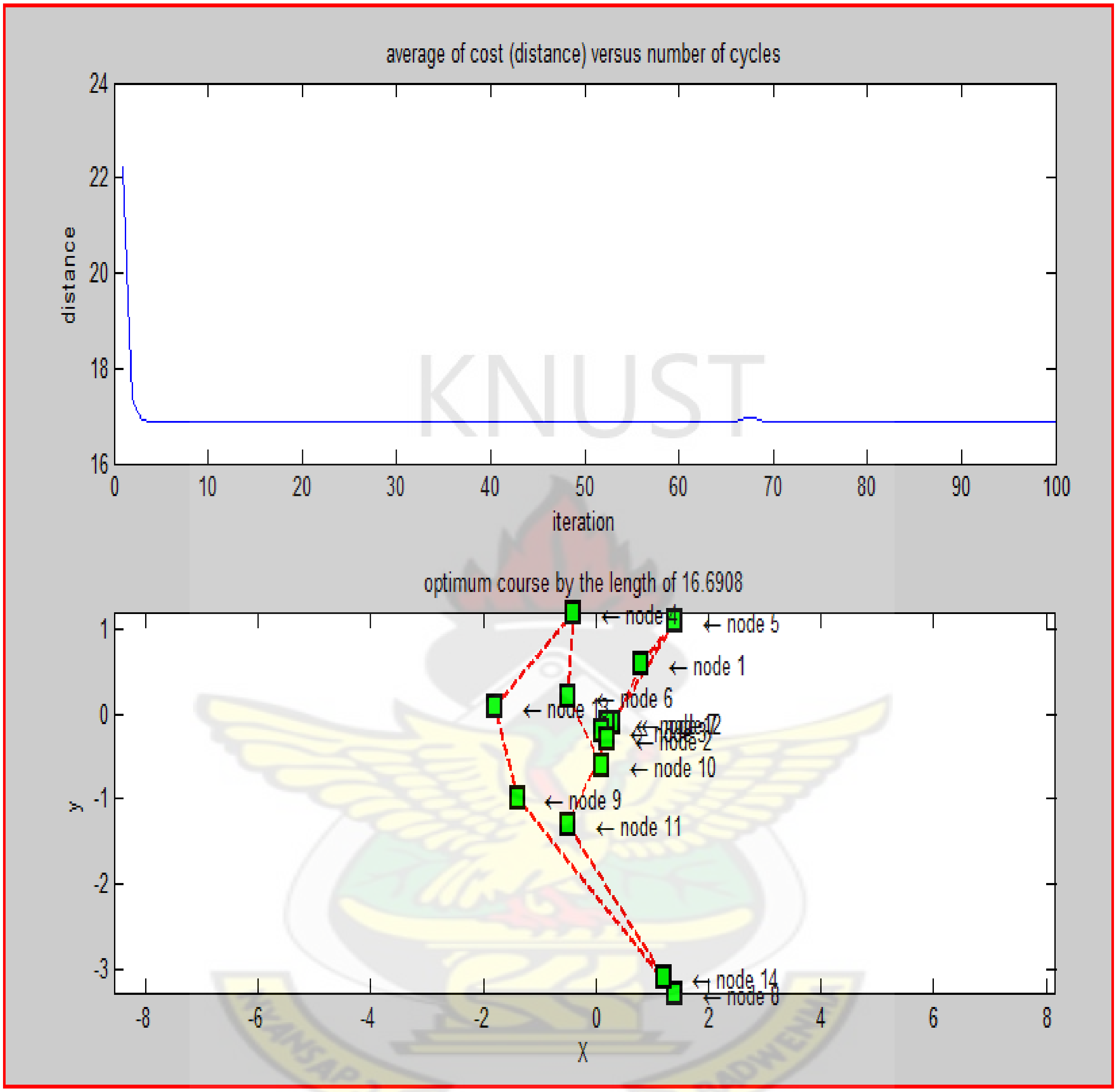

Figure 4.4.2: Ant colony results for 15 ants

Figure 4.4.2 reveals that for 15 ants, the optimal course covered by the best out of 15 ants is by the length of about 17km, which is about the same distance covered by bus 4. The course of the best ant from the source node is shown by the lower panel.

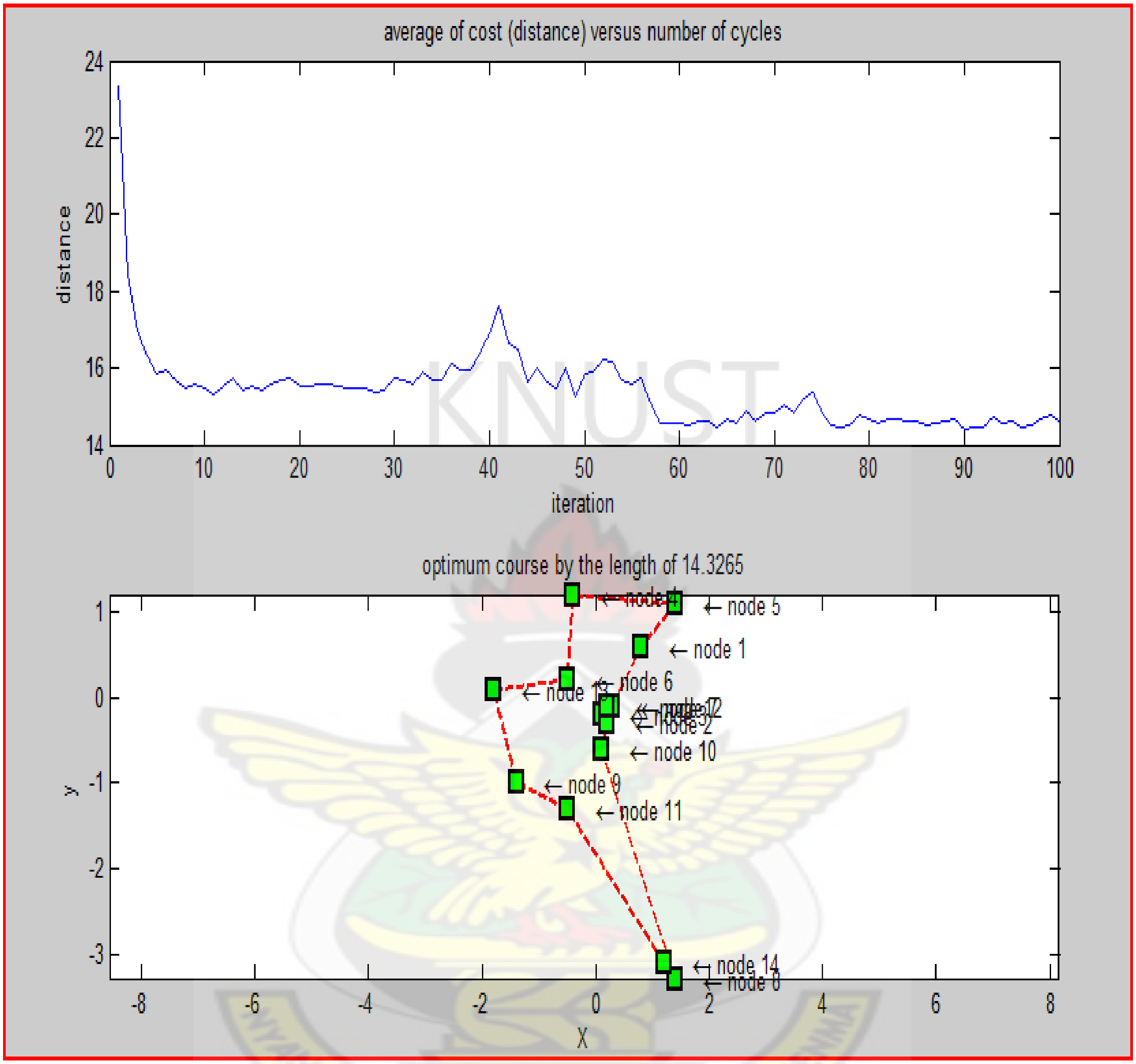

Figure 4.4.3: Ant colony results for 50 ants

It is discernible from Figure 4.4.3 that for 50 ants, the optimum course of the best ant is by the length of about 14km, representing an improvement in previous solution. The course of the best ant from the source node is shown in the lower panel.

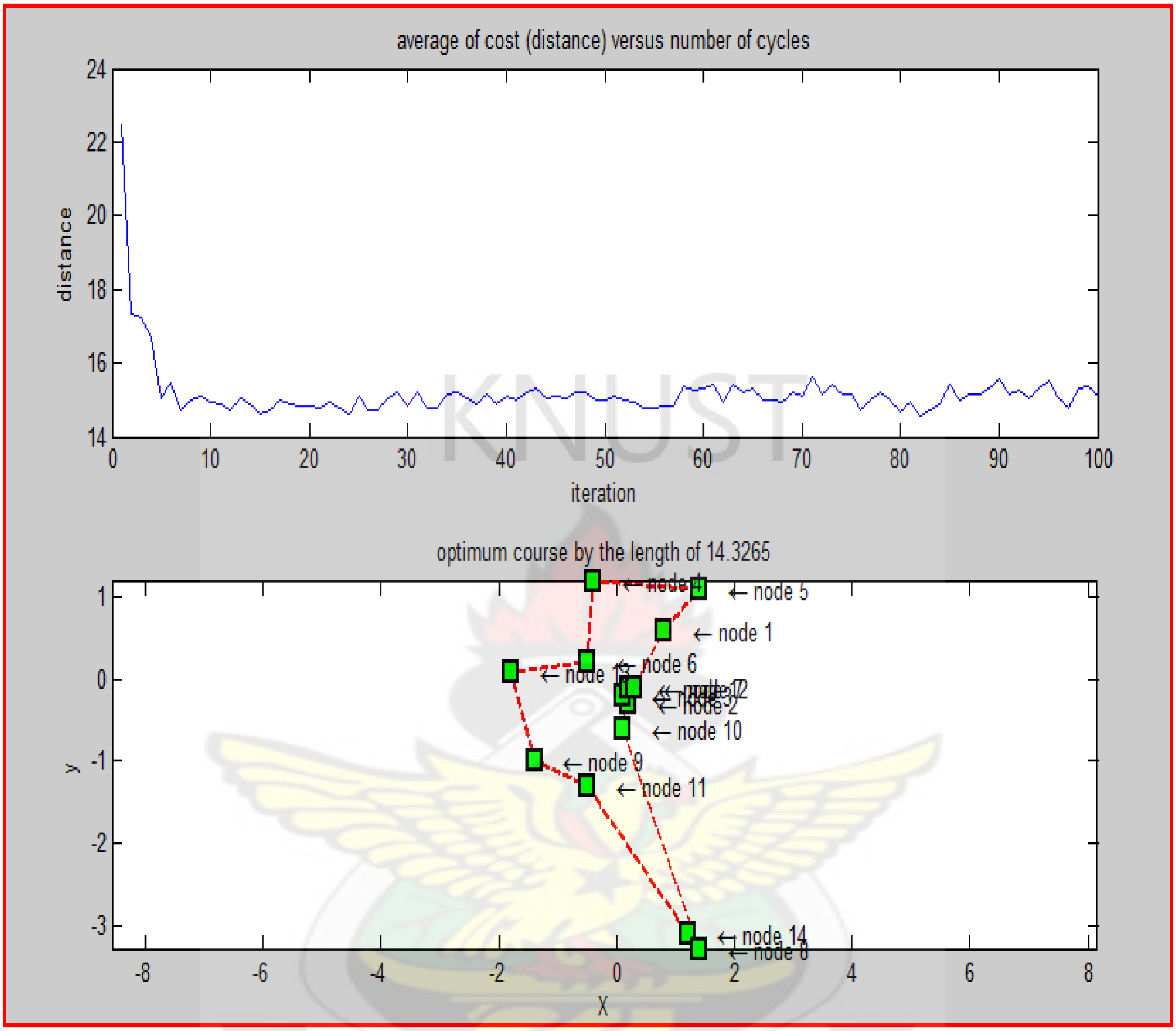

Figure 4.4.4: Ant colony results for 100 ants

From Figure 4.4.4, the best out of 100 ants records an optimal route length of approximately14km, representing about the same optimal distance covered by the best out of the previous 50 ants. It can therefore be summarized that the optimum course for bus 4 using ant colonies is by the length of approximately 14km. The optimal route and order of movement of bus 4 from the source node is given by 14→ 8→ 10→2→3→7→ 12→1→5→4→6→13 →9→11→14.

Table 4.1: SUMMARY OF ANT COLONY RESULTS

BUS	ORIGINAL LENGTH (KM)	NUMBER OF ANTS	OPTIMAL LENGTH (KM)	PERCENTAGE CHANGE
1	24.4	10	25.4937	4.4824
		15	24.4127	-0.0520
		50	20.7094	-15.1254
		100	18.4474	-24.3959
		200	18.4474	-24.3959
2	40.7	8	40.1139	-1.4400
		50	26.1486	-35.7528
		100	24.2107	-40.5143
		200	24.0342	-40.9479
3	39.94	7	40.4342	0.0310
		10	37.8844	-5.1467
		50	29.414	-26.3545
		100	26.1647	-34.4900
		200	26.117	-34.6094
4	17.8	5	20.5619	15.5163
		15	16.6908	-6.2315
		50	14.3265	-19.5140
		100	14.3265	-19.5140

Source: Author's field survey. May, 2011

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Table 4.1 presents a summary of the ant colony results for buses 1, 2, 3 and 4. The original lengths of tour by the four buses are shown in column 2, whilst the optimal route lengths obtained from ant colony results for the various ants numbers are displayed in column 4. The last column shows the percentage change in the original route lengths. The negative changes depict improvement in optimal solution whilst the positive changes shows otherwise.

For bus 1, the percentage changes in routes lengths corresponding to ants' numbers 10, 15, 50, 100 and 200 are 4.4824, -0.0520, -15.1254, -24.3959 and -24.3959. Also, for bus 2, the percentage changes in the lengths of routes for ants' numbers 8, 50, 100 and 200 are -1.4400, -35.7528, -40.5143 and -40.9479, in that order. Similarly, observing the lengths of routes for ants' numbers 7, 10, 50, 100 and 200; one could find the percentage changes for bus 3 to be respectively, -0.0310, -5.1467, -26.3545, -34.4900 and -34.6094. Finally, for bus 4 the percentage change in the lengths of tour for ants' numbers 5, 15, 50 and 100 are 15.5163, -6.2315, -19.5140 and -19.5140 correspondingly.

CHAPTER 5

DISCUSSIONS, CONCLUSIONS AND RECOMMENDATIONS

5.0 INTRODUCTION

This chapter comprises discussion of the experimented results, drawing conclusions and recommendations based on the outcome of the experimentation.

5.1 DISCUSSION OF RESULTS

The focus of this work is centered on constructing routes for vehicles using the school bus transportation system of Woodbridge school complex as a case study. In the work, the routes are constructed by simulating the behaviour of artificial ants (naturally know as ants). Since ants naturally move in "colonies" the methodology is based on the idea that a group of ants are allowed to explore various routes that link a given number of points in a given area. As these ants are put to work different ants will take route that link a given numbers of point and the result of the best ant will be considered as the optimal route length. Thus, the results shown in the graphs are based the tour of the best ants. In order to ascertain convergence of the optimal solution, the experimentation involves variation in the number of ants. The school has four buses that operate on different routes and for each bus different ants' numbers are considered. However, it is important to note that the research performed several experiments on more than those presented, but for the sake of the objectives of this work only few different cases for each bus is presented.

The results in section 4.1 show the optimal route lengths that will yield the optimal (reduced) cost for bus 1. From figure 4.11, ants' numbers corresponding to 10 will be inefficient since it increases the original tour length for bus 1 by approximately 4.5%. If 15 numbers of ants are considered, the optimal cost will be given by a route length of about 24 kilometers (see figure 4.12). This shows a result close to the actual distance covered by bus 1, which could mean that the school operates based on a route constructed by about 10 ants. However for ant numbers 50, 100 and 200 the optimal course for bus 1 is by the length of approximately 20km, 18km and 18km, respectively. It can therefore be concluded that the optimal course for bus 1 is by the length of about 18km, which represent a reduction in cost by about 25% (1/4).

The experimented results in section 4.2 shows the optimal curse for the tour of bus 2. 8 ants will yield an optimal route length of about 40, which is somehow close to the original tour length covered by bus 2. For 50, 100 and 200 ants the route length will be reduced by about 36%, 41% and 41%, respectively (refer to figures 4.21-4.24). Hence, the optimal course for bus 2 is by length of close to 24 kilometers.

The optimal course for bus 3 is depicted by the figures in section 4.3. For 7 ants the optimal course is by the length of about 40km. Thus, the existing length of tour covered by the bus 3 depicts the results of 7 ants. This implies that for ants' number greater than 7, the optimal course taken by bus 3 will begin to improve. Ants' numbers 10 and 50 reduces the cost by close to 5% and 26% respectively. Slimily 100 and 200 ants reduce the route length for bus 3 by the same percentage of 35%. As a result, the optimal route length for the bus 3 is approximately 26km.

From section 4.4 (see figures 4.41- 4.44), the tour constructed by 5 ants will be inefficient for consideration by bus 4 since it increases the original tour length constructed by bus 4 by approximately 16%. 15 ants construct an optimal course by length of about 17km, which also represents about 6%. This suggests that the optimal route length will improve for number of ant greater than 15.For 50 and 100 ants the optimal route length converges to approximately 14km representing about 20% optimization of cost of transport service rendered by bus 4.

It could be calculated from Table 4.1 that in general, the ant colony optimization has performed creditably well by reducing the original route length by about 40km, which is a reduction from 122.86km to 86.9251km and represents about 33% reduction in total cost.

It also could also be discerned from the figures and table that the number of ants required for route length to begin to improve varies with the problem size. For example in figures 4.11 to 4.14 the route length will begin to improve for at least 16 ants. Also for bus 2, at least 8 ants are required for route length to begin to improve. For buses 3 and 4 at least 8 ants and 15 are required. Since the number of nodes for the buses 1, 2, 3 and 4 respectively are 17, 28, 23 and 14, it is logic to contend that for smaller number of nodes, high number of ants are required for optimal route length to begin to improve than are required for large number of nodes.

It is also part of the objective to select bus stops from a given number of potential bus stops. For this research work, every point that a bus stops and picks student is considered potential and some will be eliminated. For bus 1 some bus stops (nodes) are very close to each other and they will be merged resulting to the creation of 13 bus stops out of 17. For bus 2, 15 bus stops will be selected out of 28. Moreover, for buses 3 and 4, 10 and 9 bus stops will be selected from 23 and 14 bus stops respectively.

5.2 CONCLUSIONS

The ant colony algorithm has proven itself to be a powerful tool for solving strong combinatorial optimization problems like the VRP. The results evince the possibility of the ant colony optimization heuristic to converge the solution to optimality.

Based on the experimented results and the discussions above, the following conclusions can be made.

i. Optimal route lengths for buses 1, 2, 3 and 4 are approximately 18km, 24km 26km and 14km respectively.

ii. Cost of services rendered by buses 1, 2, 3 and 4 are reduced by 25%, 41%, 35% and 20%, respectively.

iii. In general, the total cost of transportation is reduced by approximately 33% and the optimal routes are displayed in figures 4.1.5, 4.2.4, 4.3.5 and 4.4.4.

iv. Number of ants required for the commencement of improvement in optimal route length varies inversely with the number of nodes.

v. The number of bus stops that will be selected from 17, 28, 23 and 14 bus stops as applicable to buses 1, 2, 3 and 4 are about 13, 15, 10 and 9 respectively.

As we compare the results of the ant colony to the existing system at Woodbridge school complex, we may argue rightly that the school bus transportation service as presently exists at Woodbridge School Complex is inefficient. Hence, to run an efficient service, the school must adopt a system that is scientific- based, especially the ant colony heuristic for the school bus routing problem.

5.3 RECOMMENDATIONS

i. A clear definition of distance between two points must be considered when creating a picking point from one point to the other. This must be done by defining a lower bound on the distance between the two points.

ii. Bad routes that join shortest paths between two points must be developed.

iii. Back-tour must not be made on already visited routes.

iv. New rules governing the transport service must involve the condition that all student walk to an allowed bus stops.

v. The school should purchase three additional buses of capacities 60, 33 and 19 to support the operation of the existing ones of the respective capacities.

vi. The 45 capacity bus serving as stand-by should be released to support the operation of bus 2 as this will ensure comfortability of customers during ride time.

vii. In order to avoid lateness, traffic must be avoided; this can only be ensured if all customers walk to their designated bus stops in time.

viii. Limit must be imposed on the time at which a bus must enter and exit and a given bus stop.

ix. Since the school has no policy that ensures that members of staff are transported to and from work, the buses should not wait at un-designated stops to pick staff members.

x. In addition, the researcher recommends that any further research based on this study should consider first, the assignment of buses to various tours before constructing their respective routes. A lower bound restriction must be imposed on the distance between two points before considering them as eligible picking points and this must also be included in the ant colony algorithm.

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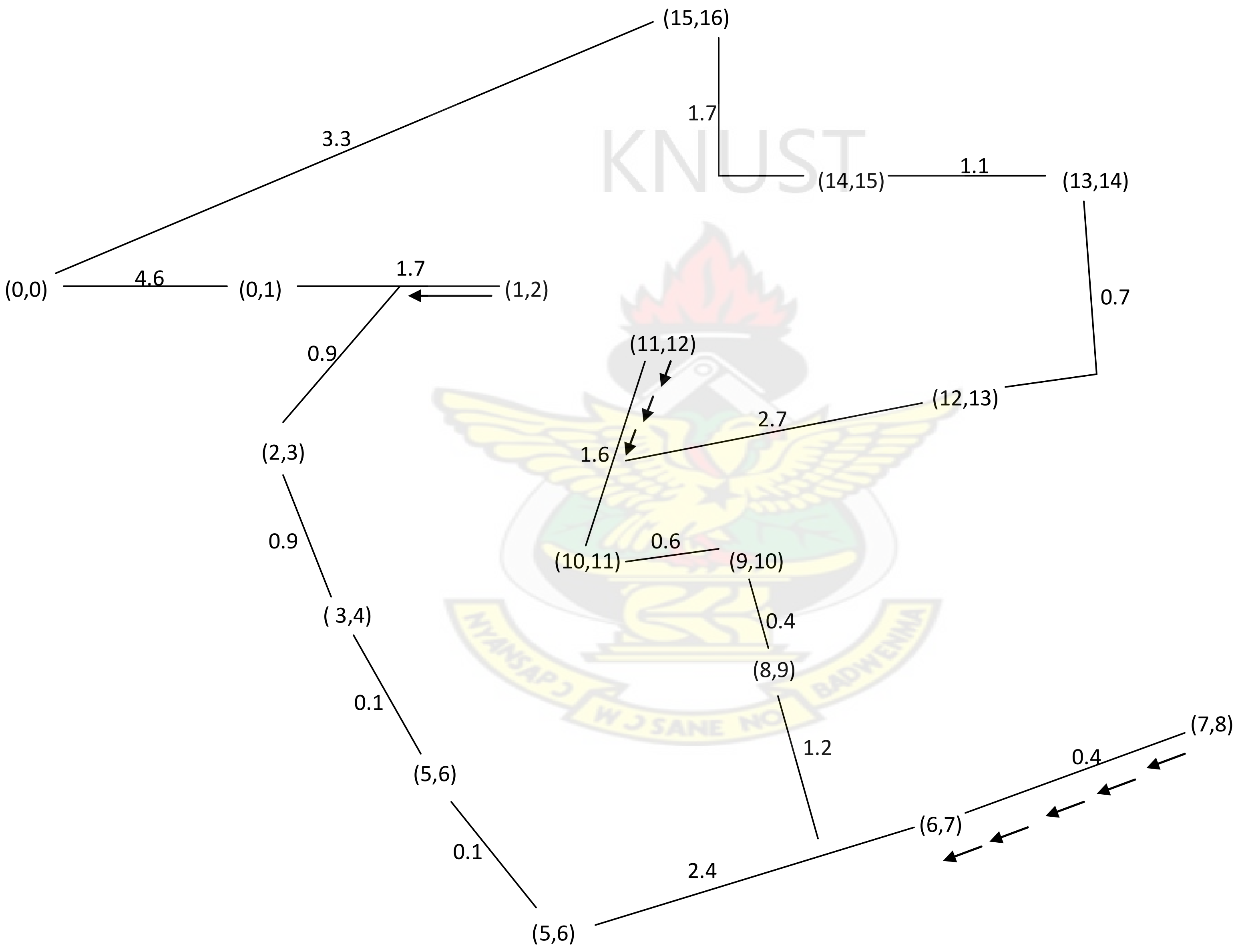
APPENDIX A: BUS TOUR (ANT SKECHES) **Bus 1**



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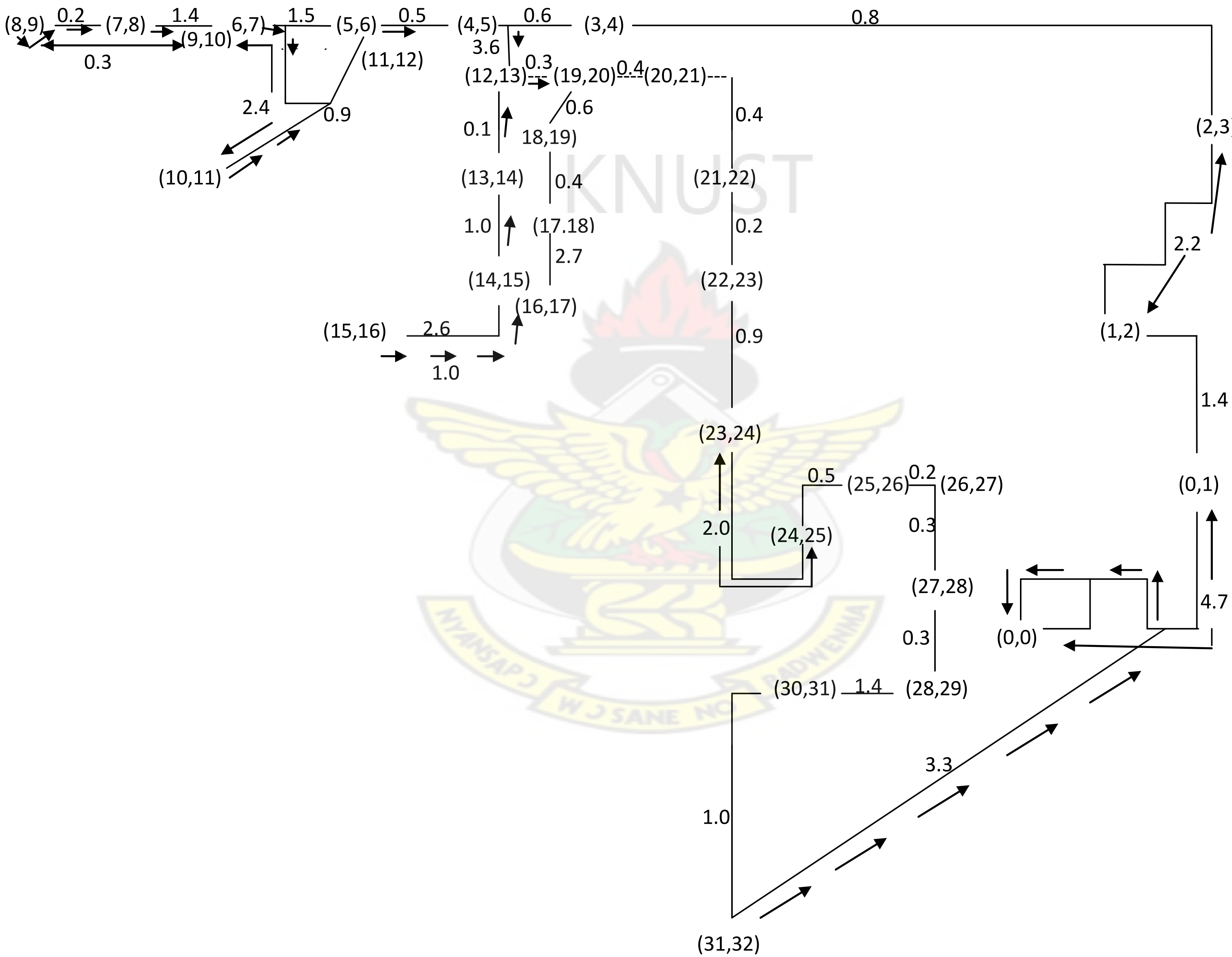
Bus 2


(8,9)
0.2
(7,8)
1.4
6,7)
1.5
(5,6)
0.5
(4,5)
0.6
(3,4)
0.8
(9,10)
0.3
3.6
(11,12)
(12,13)
0.3
(19,20)
0.4
(20,21)
2.4
0.9
0.6
0.4
(2,3)
0.1
18,19)
(10,11)
(13,14)
0.4
(21,22)
1.0
(17.18)
0.2
2.2
2.7
(14,15)
(22,23)
(16,17)
(15,16)
2.6
(1,2)
0.9
1.0
1.4
(23,24)
0.5
(25,26)
0.2
(26,27)
(0,1)
0.3
2.0
(24,25)
(27,28)
4.7
0.3
(0,0)
(30,31)
1.4
(28,29)
3.3
1.0
(31,32)

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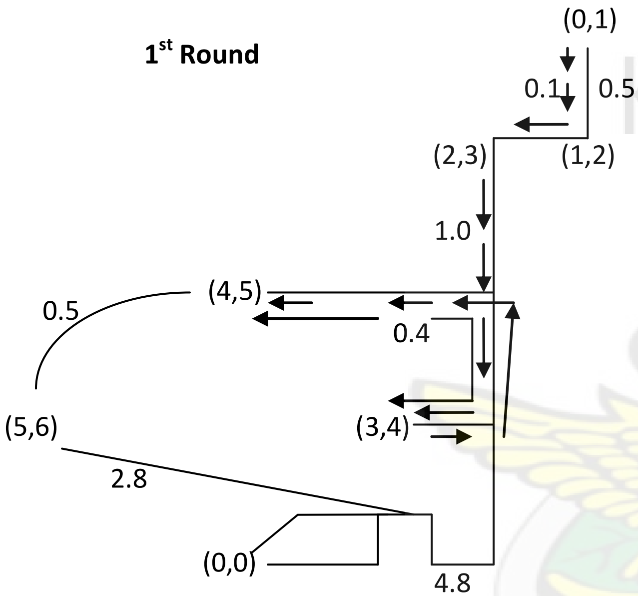
Bus 3

1st Round



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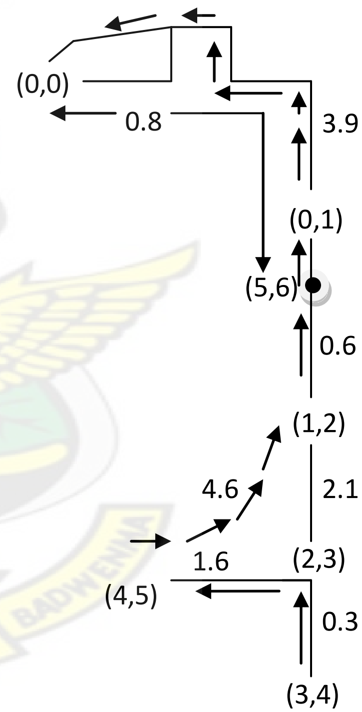
3rd Round



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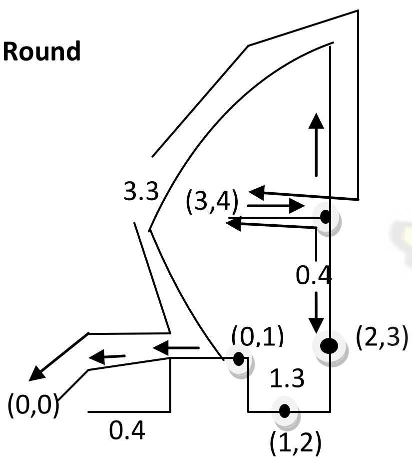
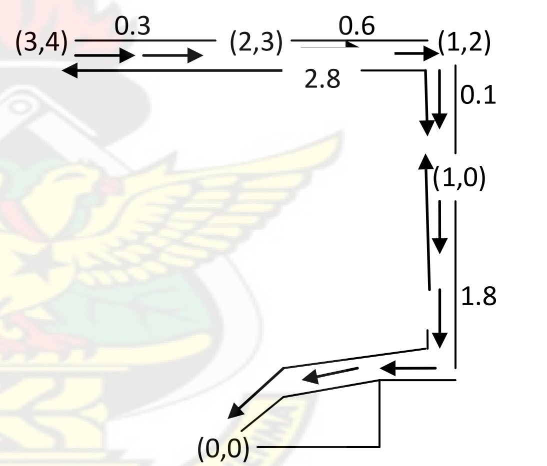
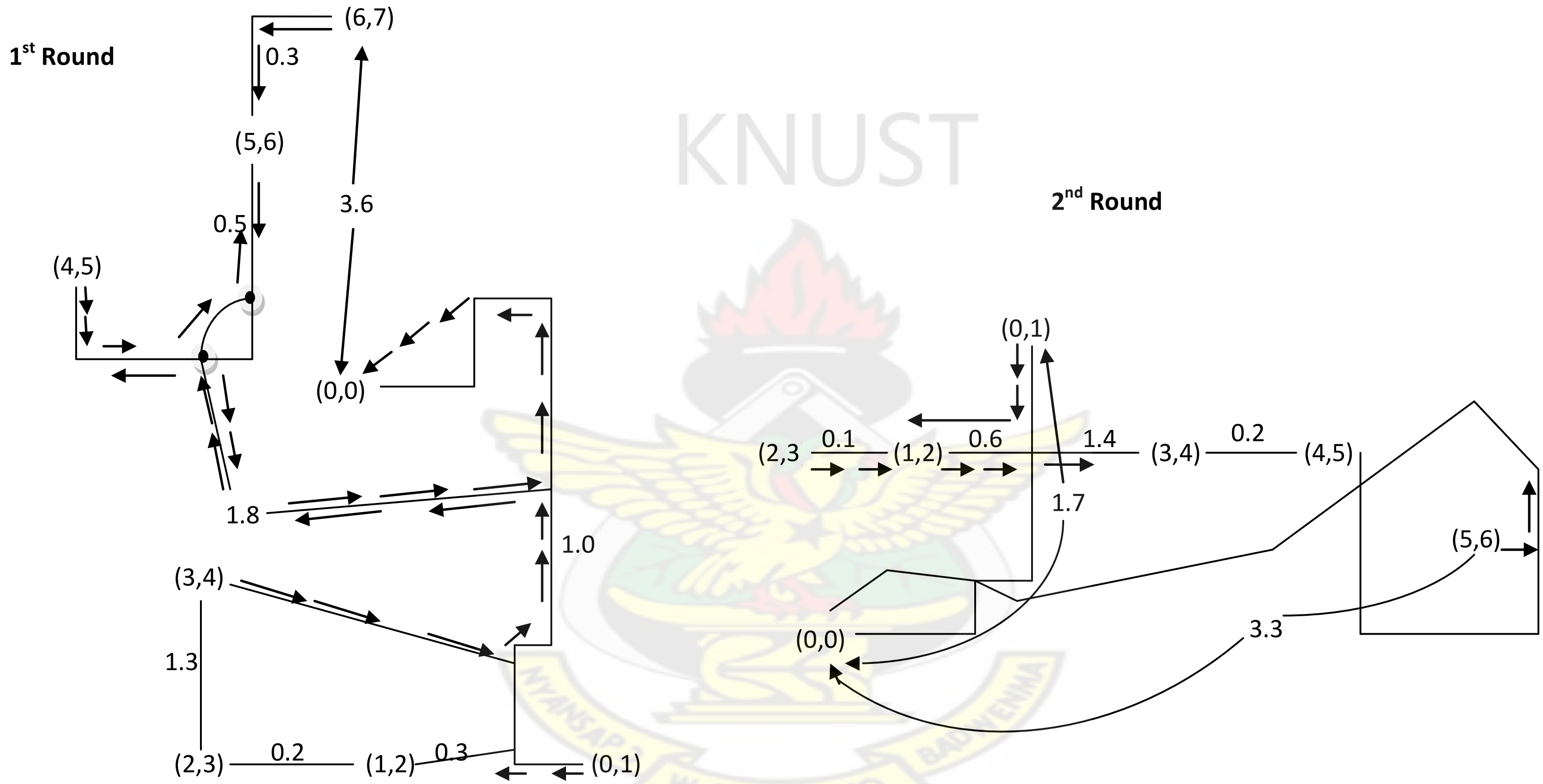
2nd Round



4th Round



Bus 4


1st Round
(6,7)
0.3
(5,6)
3.6
0.5
(4,5)
(0,0)
1.8
1.0
(3,4)
1.3
(2,3)
0.2
(1,2)
0.3
(0,1)
2nd Round
(0,1)
(2,3
0.1
(1,2)
0.6
1.4
(3,4)
0.2
(4,5)
1.7
(5,6)
(0,0)
3.3

APPENDIX B: NAMES OF VARIOUS PICKING POINTS

S/N	BUS 1	BUS 2	BUS 3	BUS 4
1	Cape Coast station	Adom Electricals	TTI	Apollo
2	New Takoradi	Bishop Porters Primary	West line Station	Apollo Dupaul gate 1
3	1st Stop Pharmacy	Gov't Propt. Sekondi Rd.	Bertsack Ventures	Apollo Dupaul gate 2
4	All Needs Supermarket	Twincity Sign Post	Lagos Town School	P & O Press Ltd
5	Segoe Junction	Bubbles	Doctor Ogoo	Barracks 1
6	Timber Bar	Kweikuma junction	Doctor Ogoo 2	Barracks 2 (All Shall Pass Ent)
7	Air Force 1	Church of Jesus Christ of Latter day Saint	Woodbridge School	Barracks 3 (AQ Coy Blk Q)
8	Air Force 2	Jendu Park	Galaxy Oil	Woodbridge School.
9	31st DWM	Sekondi Prisons	Onat K.	I Adu
10	Methodist Church	Bakaikyir	Lagos Town Mkt	Paul Essah Ave. Sign Post
11	Central Police Reserve	Adiembra Round About	Lagos Town Sch.	Uncompleted Building Opp. MTN stand
12	Chapel Hill	Opp. Kweikuma Junction	Woodbridge School	Ahantaman Rural Bank
13	Davi Ama	Fijai Junction	Apowa Estate	Lagos Town Sch.
14	Pass Gya Tires	May's Spot	Apowa Bus stop	Woodbridge Sch.
15	Cemetery K Road	Hill Top Executive House	God is Great Academy	
16	Lagos Town School	Ahantaman Sec. Sch.	Alberta Lodge opp. Beahu	
17	Woodbridge School	Agric. Station	Apowa Highway Estate	
18		Opp. Boy Boison	Apremdo	
19		Fijai Ridge Rd. Betty Link	Woodbridge Sch.	

					Saloon				
		BUS 1			**BUS 2**		**BUS 3**		**BUS 4**
20					My Redeemer Liveth Fashion		Ground Plaza		
21					Hasko Enterprise Block Factory		Above All Ent.		
22					Justice Bar & Spot		AOG cavalry		
23					Queen of Peace		Assakae Station		
24					Ck Man Junction		Woodbridge School		
25					Ecowas Link photo Lab				
26					No. 9 Taxi Rank				
27					VIP				
28					Bertsack Venture				
29					Lagos Town School				
30					Woodbridge School				

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APPENDIX C: DISTANCE MATRIX TABLE

S/N	BUS 1		BUS 2		BUS 3		BUS 4	
	X	*Y*	*X*	*Y*	*X*	*Y*	*X*	*Y*
1	4.6	0.2	4.6	1.4	4.5	-1.6	0.8	0.6
2	-1.6	0.4	-1.3	0.6	-0.5	0.2	0.2	-0.3
3	-0.9	-0.3	1.9	1.0	-0.1	0.1	0.1	-0.2
4	-0.6	0.7	0.6	0.5	-1.0	0.1	-0.4	1.2
5	-0.1	0.1	0.6	0.2	-0.2	-0.4	1.4	1.1
6	0.1	0.0	0.5	0.1	1.1	-2.6	-0.5	0.2
7	1.4	-1.9	1.5	-0.1	0.2	0.4	0.3	-0.1
8	-0.3	-0.3	1.3	0.4	1.3	0.2	1.4	-3.3
9	1.2	-0.1	0.2	0.1	1.5	-0.3	-1.2	-1.0
10	0.4	0.0	-2.7	0.6	-0.4	0.0	0.1	-0.6
11	0.6	-0.2	2.2	2.8	1.2	-3.1	-0.5	-1.2
12	0.8	1.4	0.1	0.1	-3.7	-1.1	0.2	-0.1
13	2.5	1.0	0.9	0.5	0.5	0.3	-1.8	0.1
14	0.3	0.6	-1.2	2.4	-1.9	0.8	1.2	-3.1
15	0.4	1.0	-1.0	0.2	0.2	-0.2		
16	-1.7	0.1	-2.0	1.3	0.3	-1.6		
17	1.2	-3.1	-0.3	-0.3	3.8	2.6		
18			-0.3	0.0	0.3	-0.7		
19			-0.3	-0.2	-1.6	0.9		

S/N		BUS 1			BUS 2			BUS 3			BUS 4	
		X	*Y*		*X*	*Y*		*X*	*Y*		*X*	*Y*
20					-0.2	-0.1		-0.1	0.1			
21					0.7	0.6		-0.4	0.5			
22					1.1	1.7		0.2	0.3			
23					0.5	0.2		1.1	-2.6			
24					0.2	-0.1						
25					0.6	-0.1						
26					-0.1	1.4						
27					-1.0	0.1						
28					1.2	-3.1						

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APPENDIX D: MATLAB PROGRAMME FOR TSP ANT COLONY-BASED

```
function [cost,f]=ants_cost(m,n,d,at,el);
for i=1:m
    s=0;
    for j=1:n
        s=s+d(at(i,j),at(i,j+1));
    end
    f(i)=s;
end
cost=f;
f=f-el*min(f);%elimination of common cost
function [at]=ants_cycle(app,m,n,h,t,alpha,beta);
for i=1:m
    mh=h;
    for j=1:n-1
        c=app(i,j);
        mh(:,c)=0;
        temp=(t(c,:).^beta).*(mh(c,:).^alpha);
        s=(sum(temp));
        p=(1/s).*temp;
        r=rand;
        s=0;
        for k=1:n
            s=s+p(k);
            if r<=s
                app(i,j+1)=k;
                break
            end
        end
    end
end
at=app;% generation of ants tour matrix during a cycle.
function [x,y,d,t,h,iter,alpha,beta,e,m,n,el]=ants_information;
iter=100;%number of cycles.
m=200;%number of ants.
x=[8 0 -1 2 4 6 3 10 2.5 -5 7 9 11 13];
y=[2 4 6 -1 -2 0.5 0 3.7 1.8 1 0 4 3 2];%take care not to enter iterative points.
n=length(x);%number of nodes.
for i=1:n%generating link length matrix.
    for j=1:n
        d(i,j)=sqrt((x(i)-x(j))^2+(y(i)-y(j))^2);
    end
end
e=.1;%evaporation coefficient.
alpha=1;%order of effect of ants' sight.
beta=5;%order of trace's effect.
for i=1:n%generating sight matrix.
    for j=1:n
        if d(i,j)==0
            h(i,j)=0;
        else
            h(i,j)=1/d(i,j);
        end
```

```
    end
end
t=0.0001*ones(n);%primary tracing.
el=.96;%coefficient of common cost elimination.
function [app]=ants_primaryplacing(m,n);
rand('state',sum(100*clock));
for i=1:m
    app(i,1)=fix(1+rand*(n-1));%ants primary placing.
end
function [t]=ants_traceupdating(m,n,t,at,f,e);
for i=1:m
    for j=1:n
        dt=1/f(i);
        t(at(i,j),at(i,j+1))=(1-e)*t(at(i,j),at(i,j+1))+dt;%updating traces.
    end
end

[x,y,d,t,h,iter,alpha,beta,e,m,n,el]=ants_information;
for i=1:iter
    [app]=ants_primaryplacing(m,n);
    [at]=ants_cycle(app,m,n,h,t,alpha,beta);
    at=horzcat(at,at(:,1));
    [cost,f]=ants_cost(m,n,d,at,el);
    [t]=ants_traceupdating(m,n,t,at,f,e);
    costoa(i)=mean(cost);
    [mincost(i),number]=min(cost);besttour(i,:)=at(number,:);
    iteration(i)=i;
end
subplot(2,1,1);plot(iteration,costoa);
title('average of cost (distance) versus number of cycles');
xlabel('iteration');
ylabel('distance');
[k,l]=min(mincost);
for i=1:n+1
    X(i)=x(besttour(l,i));
    Y(i)=y(besttour(l,i));
end
subplot(2,1,2);plot(X,Y,'--rs','LineWidth',2,...
                'MarkerEdgeColor','k',...
                'MarkerFaceColor','g',...
                'MarkerSize',10)
xlabel('X');ylabel('y');axis('equal');
for i=1:n
    text(X(i)+.5,Y(i),['\leftarrow node ',num2str(besttour(l,i))]);
end
title(['optimum course by the length of ',num2str(k)])
```