OPTIMAL REVENUE COLLECTION USING REVISED SIMPLEX ALGORITHM

(A CASE-STUDY OF GHANA REVENUE AUTHORITY (GRA), SUNYANI WEST DISTRICT)

BY

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DECLARATION

I hereby declare that this submission is my own work towards the M.Sc. and that, to the best of my knowledge, it contains no material previously published by another person or material, which has been accepted for the award of any other degree of the university, except where due acknowledgement has been made in the text.

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ABSTRACT

In spite of the important role taxation plays in Ghana, there have not been any thorough studies to identify the main challenges that the Ghana Revenue Authority (GRA) face in mobilizing revenue in Ghana. Ghana's tax administration plays a key role in improving the country's revenue mobilization and overall fiscal health. This thesis seeks to identify and analyze these challenges and develop an efficient Linear Programming model to optimize revenue for Sunyani West District (SWD). Data was collected from the district Revenue office spanning between 2008 and 2011. The averages of these data were determined and simplified and was modelled into objective functions and subject to some constraints. Matrices generated from this data were run on Matlab package. The Results revealed a significant percentage increase in the district RA effort, which means that the district can raise it revenue collection from GH¢80,694.00 to GH¢90,999.20 annually based on the revised Simplex code.

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CHAPTER ONE

INTRODUCTION

Tax is the levy on all income, goods, services and property of an individual, partnerships, trustees, executors and companies by the government and is backed by law. In Ghana, the law backing the payment of income tax during its introduction in the country is the Income Tax Ordinance No. 27 of 1943.

This law however has been amended several times and the current law backing its payment is the Internal Revenue Act 2000, (Act 592). Tax can be said to be a compulsory contribution extracted from the incomes of individuals and groups by the government to meet the cost of public services.

Tax can also be defined as a compulsory payment to the government according to one's income, property, goods etc., which is used to pay for specific services and perform other social responsibilities by the government, (Rutherford, 2001)

1.1 Background

The role of government revenue and the capacity of government to raise taxes for the purpose of financing economic development have preoccupied economists and policy makers for a long time. Kaldor (1963) raised the very important question of whether underdeveloped countries will "learn to tax", with the underlying view that for these countries to reach higher levels of living standards, they would need to achieve levels of tax effort that are significantly higher than observed at that time.

Kaldor was in fact echoing an earlier call by Sir Arthur Lewis who posited that "the governments of an underdeveloped countries needs to be able to raise revenue of about 11-19 percent of Gross National Product (GNP) in order to give a better than average standard of service".

Indeed, the evidence clearly shows that tax effort rates are much higher for high-income countries than in low-income countries, supporting the notion that performance in tax mobilization is essential for reaching higher levels of income. A low level of government revenue is a constraint on the capacity to finance essential public investment programs and undertake adequate levels of spending on social services, which are essential for improving living standards.

1.1.1 High Level of Revenue Performance

A country or a government is capable of achieving high levels of revenue performance. As Bird, Vazquez and Torgler (2008) believe that, most of the attention in the analyses of tax effort has traditionally been focused on the supply side (or "tax handles" in their words), mainly the availability of readily taxable activities such as trade/commerce and natural resources.

However, as these authors rightly point out, "telling a country that wants to raise its tax levels to find and tax natural resources is not a particularly promising piece of policy advice."

In reality, however, the problem is even much more complicated than presented by Bird and his colleagues. In fact, even finding natural resources does not necessarily guarantee a high level of revenue performance.

Many countries have found natural resources but not all those that were lucky to find a bounty in their underground have been able to take advantage of the resources in raising government revenue.

The problem goes beyond the issue of value addition in the natural resource industry – or moving up the value chain. It also lacks capabilities to innovate within and outside the natural resource value chain.

1.1.2 Historical Background of Taxation In Ghana

Historically, in Ghana the formal sector is not difficult to tax due to the fact that taxes are being deducted at source from their income at the end of the month. However, in the case of the informal sector, it is extremely difficult to tax since many people try to evade tax. The situation is no different from what pertains in the Sunyani West District. This situation can only be reversed when the tax base is broadened by the inclusion of every single income earner in the informal sector. Bagahwa and Naho (1995) states that "public revenue remains weak in numerous sub-Saharan African countries and the tax burden appears to be unevenly distributed. Two sectors are often considered as being under – taxed: the agricultural and the unrecorded urban sectors. Its (i.e. unrecorded sector) under-taxation results in considerable losses in tax revenue. What is worse, the development of unrecorded activities is threatening the official sector which plays a crucial role in collecting government resources". Following this assertion, it is clear that under-taxation of the informal sector is accounting for the imbalances in the tax system.

1.1.3 The Role of Informal Sector in Government Revenue

The economy of Ghana is largely made up of individual and small-scale enterprises. These sectors provide diverse sources of income which, if taxed, could increase government internally generated revenue. The Institute of Statistical, Social and Economic Research (ISSER), 2003, stated that "the main source of employment in Ghana is the informal sector. The sector provides employment opportunities for at least 80% of the labour force".

The estimated size of the informal sector presupposes that, that sector makes significant contribution to the Gross Domestic Product (GDP) and so the sector could equally make major contributions to the tax revenue. A survey conducted in Accra by Jobs and Skills Programs for Africa (JASPA) in 1990/91 established that the informal sector accounts for about 22% of Ghana's real GDP. If the Gross Domestic Product (GDP) contribution could be translated into tax revenue, government revenue could make significant appreciation. With the focus of successive and present governments on the private sector (which is dominated by the informal businesses) as the engine of development, it can be suggested that the sector will witness tremendous expansion and subsequently more tax revenue. However, the government, through the Internal Revenue Service (IRS), has been overly concentrating scarce economic resources on the formal sector with respect to the direct tax collection. The excessive focus on the formal sector and the neglect of the informal sector arguably may be accounting for the abysmal poor collection of tax revenue from the informal sector.

There appears to be no concrete national policy on the way to properly organize and regulate activities of the huge chunk of participants in the informal sector so as to facilitate direct taxation of the income of that sector. This is corroborated by the Institute of Statistical, Social and Economic Research (ISSER) when it states that "data on the activities of the sector are lacking in many respects due to their fragmented, unregulated and unrecorded nature" (ISSER, 2003).

The informal sector relies heavily on the state resources- social, economic and infrastructure to run and make some level of profits. Some of the activities of the informal sector cause so much damage to the environment and so it makes economic and social sense to rope into the tax net all the participants in the informal sector. By taxing the informal sector, the participants at least contribute for the repair and restoration of the environment they damaged.

1.1.4 Revenue Mobilization in Africa

African countries have generally performed poorly in revenue mobilization. The average tax to Gross Domestic Product (GDP) ratio in sub-Saharan Africa increased only moderately over the past two decades. African countries have been unable;

(i) To harness natural resource endowment for the purpose of revenue mobilization.

(ii) To develop their capacity to mobilize resource from the public as revenue.

1.1.5 Challenges in Tax Collection in Sunyani West District

According to the district medium term draft plan 2011, poor data base on rateable items, inadequate qualified tax collectors, inadequate and poor marketing facilities, high rate of tax evasion, inadequate logistics to promote education on the need to pay taxes, lack of permanent internal auditors/local government inspectors, inadequate revenue mobilization capacity and weak tax/revenue collection mechanism are the major problems of the district internal revenue service.

1.1.6 Types of taxes in Ghana

There are two kinds of taxes in Ghana, namely; indirect and direct tax. Indirect tax is levied on goods and services. Examples of such duties include import and export duties, tariffs, Value Added Tax (VAT). Direct tax on the other hand is levied on the income earned by an individual, companies and organizations. Example of indirect tax in Ghana includes personal income tax and corporate tax. To collect direct tax, governments have to specify what counts as income and what kinds of income attract or not attract tax. The Internal Revenue Service (IRS) is the agency responsible for the collection, administration and evaluation of income taxes in the country. It has eleven regional offices, two in the Greater Accra region with the remaining nine in the other nine regional capitals in the country. Beside these regional offices, IRS has fifty (50) tax districts, thirty-eight (38) sub offices of which the Sunyani West District office is one, and twenty –five (25) collection points located at vantage points throughout the country .(Internal Revenue Service Handbook, 2004)

1.1.7 Importance of Tax

In Ghana, taxes are collected for many purposes. Taxes are used to correct market imperfections and to encourage or discourage dumping of goods in a market. The main purpose of taxation is to raise revenues to fund government expenditure and services. Besides these, the government uses taxes to promote economic and social activities of the citizens. Government also collects taxes to achieve the following;

(i) Means of controlling businesses in order to achieve full employment whereby rate of consumption, savings and investments in the country can be influenced by the government.

(ii) Taxes are used to promote economic growth. This can be achieved by imposing a lower rate of tax on company/enterprise incomes that are ploughed back into the business. The government also encourages economic growth by pursuing investment policies generously. It can grant some industries tax concession or impose protective tariffs to encourage the production and purchase of locally made goods.

(iii) Taxation is used to effect the redistribution of income in the society through the introduction of "Pay as you earn (PAYE)".

(iv) Taxes can be used to promote services such as education, health, transportation, national security, social insurance, relief for the poor and other social factors.

1.1.8 District Profile

The Sunyani West District was carved out of the Sunyani Municipal in November, 2007, through Legislative Instrument (L I) 1881, 2007. It was inaugurated on 29th February, 2008.

The Administrated capital of the District is Odumase. It covers a total land area of 394 Square Kilometres. According to the 2000 Population and Housing Census, the District has a population of 62,172 and a growth rate of 2.3 percent per annum .The population is however projected to reach 89,070 in 2013.The predominant occupation in the District is agriculture which employs about 66.1 percent of the active labour force. Services employ 8.4 percent, Industry 4.9 percent and Commerce 0.7 percent. The District shares common boundaries with Sunyani Municipal to the East, Berekum Municipal to the West, Techiman Municipal to the South and North by Wenchi Municipal.

1.1.9 Sources of Revenue in Sunyani West District

The Sunyani West office of GRA is a sub-district office under the Sunyani municipality in the Brong Ahafo Region of Ghana. The office's areas of operation include Odomase, Fiapre, Nsuatre and their environs. These areas have quite a number of small firms/business, hence the need to collect income tax as stated by law to generate revenue for the district for developmental projects. Some of these small firms include; financial institutions, restaurants, telecommunication, transportation, agriculture, education (Private Schools) and auto repairs

"No one doubts that informal sector is heterogeneous as stated by (Lang and Horne, 1987). The above activities indicate a wide variety of informal sector enterprises and entrepreneurs. According to Saunders et al (2000), "dividing the population into a series of relevant strata means that the sample will be more likely to be representative, so as to ensure that each of the strata is represented proportionally within the sample"

1.2 Statement of the Problem

The standard of living in the Sunyani West District keeps on deteriorating as the district is not able to provide the citizens with the basic social amenities such as portable water, better healthcare facilities, quality education, good roads, improved sanitation, infrastructural development and others .The reason being that the district is not able to mobilize sufficient revenue to execute its projects and programmes aimed at improving the lives of the people. The district since its inception in February, 2008 has never met its revenue target, and has to rely heavily on the central government for its basic expenditure financing .

This problem has been a major headache to the district as it is hampering the effective growth of the district. This research work is basically targeted at developing a mathematical programming model that will help the district to optimize its revenue and its tax collection strategy.

1.3 Objective of the Study

The main objectives of this study are:

(i) to analyze the tax systems in Sunyani West District

(ii) to develop an efficient linear programming model to maximize revenue for Sunyani West District (SWD)

iii) to make recommendation that can address problems of Revenue mobilisation in Ghana.

1.4 Justification of the study

In spite of the important role of taxation in Ghana, there have not been any thorough studies to identify the main challenges that the Ghana Revenue Authority (GRA) face in mobilising revenue for Sunyani West District (SWD).

The study will identify the main challenges facing the District in mobilising taxes. The challenges could be addressed in other to help the district to optimise revenue.

Secondary, the study will enable the district to know how to strategize to mobilize revenue. The study is also aim at developing an efficient linear programming model to maximise revenue for Sunyani West District (SWD).

Finally, the study also aims at providing the District with information on the best way to optimise revenue.

1.5 Methodology

Quarterly secondary data, spanning between 2008 and 2011 shall be collected from the district internal revenue office for the study.

The problem of revenue and tax maximization will be modelled as a linear programming problem. The revised simplex algorithm shall be used to analyze the problem.

Matlab will be used to solve the problem. Sources of information for this project would include the Sunyani West District revenue Office, the internet, library books, journals, and reports. Charts, graphs and other statistical methods will be used for data analysis and presentation.

1.6 Significant of the Study

The Sunyani west district, since its inception has been under performing in its revenue and tax collection efforts. This project is geared towards finding a lasting solution to help the district to optimize its revenue and tax collection so that it can support its inhabitant to improve upon their standard of living with the provision of good social amenities such as schools, hospitals, portable water and so on. It is also envisaged that some other districts in the country with revenue and tax collection challenges can use the findings from this research to improve upon their revenue generation capacity. The research will help to generate data on how the tax systems in the country will not affect the creation and growth of small firms, but rather will create an enabling atmosphere for its administration by tax officials and compliance by taxpayers. This will go a long way to mutually benefit the government and taxpayers by marginally increasing revenue on the part of government for developmental projects and also help to reduce the level of poverty as a result of high levels of unemployment and underemployment in the Brong Ahafo region and the country as a whole through job creation and growth in the small and medium scale industry.

1.7 Limitations of the Study

The study is limited to the Sunyani West District operational area of Revenue Authority. Hence

The major limitations of this study are:

(i) The research will not cover all the districts in Ghana, and therefore might not give a very general outlook of all the districts in the country base on the revenue generation.

(ii) It will sometimes be cumbersome due to the volume of data involved and insufficient time available.

(iii) The research will not cover all the constrains such as Repair Services, Beverages, Food Processing since the revenue generated by some of these constrains are too minimal.

1.8 Organisation of the Study

The study is organised into five chapters, in Chapter one, we considered the background, project statement, objectives and significant of the study. The study was also justify in chapter one including the methodology and limitation of the study. Chapter Two presents relevant literature on tax collection, linear programming and revised simplex algorithm.

In chapter three, we shall put forward the methodology of the study. Chapter four is devoted for the data collection process and analysis of the study.

Chapter five, which is the last chapter of the study, presents the conclusion and recommendation of the study.

CHAPTER TWO

LITERATURE REVIEW

2.0 Introduction

This chapter consist of review of some important literature in the field of taxation and linear programming applications. This study is aimed at maximizing the revenue and tax generated by the Sunyani West District office of the Ghana Revenue Authority focusing on the use of linear programming approach.

2.1 Applications of Linear Programming

Many researchers have solved and continue to solve numerous practical problems using linear programming methods. Some of these landmark works are being reviewed in my research work as follows:

2.1.1 Current Cost of Procurement

Milind et al., (2002) explored a design where, the market signal provided to a supplier was based on the current cost of procurement for the buyer. At the heart of this design lied a fundamental sensitivity analysis of linear programming. Each supplier was required to submit bid proposals that reduced the procurement cost (assuming other suppliers keep their bids unchanged) by some large enough decrement d >a. The authors showed that, for each supplier, generating a profit maximizing bid that decreased the procurement cost for the buyer by at least 'd' could be done in polynomial time. This implied that in designs where the bids were not common knowledge, each supplier and the buyer could engage in an "algorithmic conversation" to identify such proposals in a polynomial number of steps. In addition, they showed that such a mechanism converged to an "equilibrium solution" where all the suppliers were at their profit maximizing solution given the cost and the required decrement 'd'. The United States Air Force's Military Airlift Command (MAC) uses linear programming techniques to solve complex resource-allocation problems that is approximately 1,000 planes (of varying capacity, speed, and range) to ferry cargo and passengers among more than 300 airports scattered around the world. Resource constraints, such as the availability of planes, pilots, and other flight personnel, place limitations or constraints on the capacity of the airlift system.

MAC further determined whether it is more efficient to reduce cargo and top off the fuel tanks at the start of each flight or to refuel at stops along the way and pay for the costs of shipping fuel. The airlift system also requires that cargo handlers and ground crews be available to service the aircraft.

Furthermore, schedulers must be able to deal with disruptions caused by bad weather and emergency changes in shipping priorities. Adding just a couple of percentage points to the efficiency of the airlift system can save the Air Force millions of dollars annually in equipment, labor, and fuel costs. Major commercial airlines, such as American and United force, face similar scheduling problems.

2.1.2 Business and Economics Situation

Linear programming can be applied to various fields of study. Most extensively, it is used in business and economic situation, but can also be utilized in some engineering fields. Some industries that use linear programming model include transportation, energy, telecommunications and production or manufacturing companies. To this extent, linear programming has proved useful in modelling diverse types of problems in planning, routing, scheduling assignment and design. David (1982), Nearing and Tucker (1993) noted that, operational research is a mathematical method developed to solve problems related to tactical and strategic operations. Its origins show its application in the decision-making process of business analysis, mainly regarding the best use for short funds. This shortage of funds is a characteristic of hyper-competive environments. Although the practical application of a mathematical model is wide and complex, it will provide a set of results that enable the elimination of a part of the subjectivism that exists in the decision-making process as to the choice of action alternatives (Bierman and Bonini, 1973).

2.1.3 Profit Maximization Company

Frizzone et al., (2011) developed a separable linear programming model, considering a set of technical factors which might influence the profit of an irrigation project. The model presented an objective function that maximized the net income and specified the range of water availability. It was assumed that yield functions in response to water application were available for different crops and described very well the water-yield relationships. The linear programming model was developed genetically, so that, the rational use of the available water resource could be included in an irrigation project. Specific equations were developed and applied in the irrigation project "Senator Nilo Coelho" (SNCP), located in Petrolina – Brazil. Based on the water-yield functions considered, cultivated land constraints, production costs and products prices, it was concluded that the model was suitable for the management of the SNCP, resulting in optimal cropping patterns.

2.1.4 Linear Programming Techniques in Revenue Generation

The use of linear and other types of mathematical programming techniques has received extensive coverage in the internal revenue service especially maximizing revenue in the form of taxes.

According to Branson and Knox (2001), the annual growth rates of real GDP in New Zealand have varied widely, from 18% to –8%, since World War II. During the period the tax burden (the ratio of tax revenue to GDP) has trended upward from 23% to 35%. The tax mix (the ratio of indirect taxes to direct taxes) has varied between 0.31 and 0.75, having increased recently with the introduction of the goods and services tax. In that paper they estimated a combination of the tax burden and the tax mix which would maximize the rate of growth of real GDP. They found out that such a tax structure would have a time-varying tax burden with a mean of 22.5%, and a time-varying tax mix with a mean of 0.54, which implies a mean share of direct taxes in total tax revenue of 65%. They also found that a move to such a tax structure would generate nearly a 17% increase in real a GDP, and while this increase would yield a 6% reduction in tax revenue to the Treasury, it would deliver a 27% increase in purchasing power to the remainder of the economy.

Velasco et al., (2011) emphasized that a great part of core network operators' revenues comes from the provisioned connectivity services. Taking this premise as their starting point, they first examined the provisioning of differentiated services in current shared-path protection environments. Their analysis revealed that current resource assignment policies were only able to provide a very poor grade of service to the supported best-effort traffic. Aiming to improve this performance, a novel resource partitioning scheme called diff-WS was proposed, which differentiated those wavelengths supporting each class of service in the network. As a major goal of the paper, the benefits of diff-WS over current resource assignment policies were assessed from an economic perspective. For that purpose, the Network Operator Revenues Maximization Problem (NORMA) was presented to design the optical network such that the operator's revenues were maximized. To solve NORMA, the authors derived statistical models to obtain, given a certain grade of service, the highest traffic intensity for each class of service and resource partitioning scheme. These models turn NORMA into a nonlinear problem, which was finally addressed as an iterative approach, solving an Integer Linear Programming (ILP) sub problem at each iteration. The obtained numerical results on several network topologies illustrate that diff-WS maximizes resource utilization in the network and, thus, the network operator's profit.

Sharp (2007) concluded that for linear programming models the effects of income taxes on the optimal activity levels and the optimal values of the dual variables were considered. Neither a fixed percentage tax nor a progressive tax, based on the net pre-tax contribution, changes the optimal activity levels. However, the optimal dual variables were changed in value, proportional to the highest tax rate actually in effect.

2.1.5 Markowitz Approach for Efficient Revenue Generation

Jagannathan (1967) resolved that revenue collection problem faced by institutional managers can be formulated following the Markowitz approach by finding those revenue that were efficient in terms of predicted expected return and standard deviation of return, subject to legal constraints in the form of upper bounds on the proportion of the fund invested in any single security.

He made a suggestion that such problems be re-formulated as parametric linear-programming problems, utilizing a linear approximation to the true (quadratic) formula for managerial risk. Limited empirical evidence suggested that the approximation was acceptable. Moreover, it allowed the use of an extremely simple and efficient special-purpose solution algorithm. With appropriate modifications, the algorithm may prove useful to the managers of an institution with a wide variety of objectives.

2.1.6 Planning, Production, Distribution and Inventory

Martin et al., (1993) presented a linear programming model for planning production, distribution and inventory operations in the glass sector industry. Chen and Wang (2004) proposed a linear programming model to solve integrated supply, production and distribution planning in a supply chain of the steel sector.

Ryu et al., (2004) suggested a bi-level modeling approach comprising two linear programming models, one for production planning and one for distribution planning. These models subsequently consider demand uncertainty, resources and capacities when they are reformulated by multi-parametric linear programming.

2.1.7 Integration of production and Distribution

Oh and Karimi (2006) put forward a linear programming model that integrates production and distribution planning for a multinational firm in the chemical sector in a multi-plant, multi-period and multi-product environment. This model also works with tax and financial data, such as taxes related with the firm's business activity or amortizations.

Preckel et al., (1997) presented economically rational behaviour by fertilizer retailers. Tests were developed to measure efficiency of variable cost minimization, revenue maximization, and profit maximization. These tests included standard linear programming-based nonparametric efficiency tests and simpler but less conclusive tests, which were performed using only simple arithmetic. Results indicated that fertilizer retailers acted as variable cost minimizers, but not as revenue and profit maximizers. Additional tests isolated whether inefficiency in cost minimization was due to a perception of variable input fixity. Management could then take steps to focus efforts on input control.

2.1.8 Planning and control of customer circuit

Gassenfert and Soares (2006) presented a practical proposition for the application of the Linear Programming quantitative method in order to assist planning and control of customer circuit delivery activities in telecommunications companies working with the corporative market. Based upon data provided for by a telecom company operating in Brazil, the Linear Programming method was employed for one of the classical problems of determining the optimum mix of production quantities for a set of five products of that company: Private Telephone Network, Internet Network, Low Speed Data Network, and High Speed Data

Network, in face of several limitations of the productive resources, seeking to maximize the company's monthly revenue. By fitting the production data available into a primary model, observation was made as to what number of monthly activations for each product would be mostly optimized in order to achieve maximum revenues in the company. The final delivery of a complete network was not observed but the delivery of the circuits that made it up, and that was a limiting factor for the study herein, which, however, brought an innovative proposition for the planning of private telecommunications network.

According to Hasan et al., (2010), the Saudi Public Transport Company (SAPTCO) intercity bus schedule comprise a list of 382 major trips per day to over 250 cities and villages with 338 buses. SAPTCO operates Mercedes 404 SHD and Mercedes 404 RI-IL fleet types for the intercity trip. The fleet assignment model developed by American Airlines was adapted and applied to a sample of the intercity bus schedule. The results showed a substantial saving of 29% in the total number of needed buses. This encourages the decision makers at SAPTCO to use only Mercedes 404 SHD fleet type. Hence, the fleet assignment model was modified to incorporate only one fleet type and applied to the sample example. Due to the increase in the problem size, the model was decomposed by stations. Finally, the modified decomposed model was applied to the whole schedule. The model results showed a saving of 16.5% in the total number of needed buses of Mercedes 404 SHD. A sensitivity analysis was carried out and showed that the predefined minimum connection time is critical for model efficiency. A modification to the connection time for 11 stations showed a saving of 14 more buses. Considering their recommendation of performing a field study of the trip connection time for every station, the expected saving of the total number of needed buses would be about 27.4% (90 buses). That would yield a net saving of 16.44 million Saudi Riyals (USD 4.4 million) per year for SAPTCO in addition to owing to the growth of air traffic. Better coordination of hiring new employees.

The revenue analysis showed that these 90 surplus buses would yield about USD 20,744,000 additional revenue yearly.

2.1.9 Route and Level Flight Assignment

Dritan et al., (1998) studied the route and level flight assignment problem aiming at global flight plan optimization, which has already become a key issue all existing flights for all airlines, was becoming an increasingly desirable goal. A number of related problems appeared in the operations research literature, notably vehicle routing, scheduling and other transportation problems. Several studies had been especially devoted to the problem of aircraft scheduling and routing. Aircraft routing requires the generation of non-colliding, time-dependent routes through a specified airspace that they called the airspace network. The problem considered there could be modelled as a specific flow problem in a given space-time network. The study aimed at estimating the effects of routing capabilities at a quantitative level (the congestion level, i.e. the number of potential en-route conflicts), and at a qualitative level (Traffic smoothing). They presented a deterministic model based on a Linear Programming approach for optimizing the level route assignment in a trajectory-based Air Traffic Management (ATM) environment. The problem could be seen as a multi-period (dynamic) problem where the time dimension was an essential ingredient to consider when constructing flight plans. The dynamic problem could be transformed into a static one by using standard technique of time-expanding the underlying network. The authors proposed a model to consider the airspace congestion in a finer way: they considered the number of aircraft involved in potential en-route conflicts rather than the number of aircraft in a sector, sometimes understood as en-route capacities in ATM.

Chung et al., (2008) considered a municipal water supply system over a 15-year planning period with initial infrastructure and possibility of construction and expansion during the first and sixth year on the planning horizon. The uncertainties in water demand and supply were applied on the form of the robust optimization approach of Bertsimas and Sim to design a reliable water supply system. Robust optimization aims to find a solution that remains feasible under data uncertainty. It was found that the robust optimization approach addressed parameter uncertainty without excessively affecting the system. While they applied their methodology to hypothetical conditions, extensions to real-world systems with similar structure were straightforward. Therefore, their study showed that this approach was a useful tool in water supply system design that prevented system failure at a certain level of risk.

Hoesein and Limantara (2010) studied the optimization of water supply for irrigation at Jatimlerek irrigation area of 1236 ha. Jatimlerek irrigation scheme was intended to serve more than one district. The methodology consisted of optimization water supply for irrigation with Linear Programming. Results were used as the guidance in cropping pattern and allocating water supply for irrigation at the area.

A ground water management model based on the linear systems theory and the use of linear programming was formulated and solved by Heidari (1982). The model maximized the total amount of pound water that could be pumped from the system subject to the physical capability of the system and institutional constraints. The results were compared with analytical and numerical solutions. This model was then applied to the Pawnee Valley area of south-central Kansas. The results of this application supported the previous studies about the future ground water resources of the Valley. These results provided a guide for the ground water resources management of the area over the next ten years.

2.1.10 Optimal Water Allocation

Khaled (2004) developed four models of optimal water allocation with deficit irrigation in order to determine the optimal cropping plan for a variety of scenarios. The first model Dynamic Programming model (DP) allocated a given amount of water optimally over the different growth stages to maximize the yield per hectare for a given crop, accounting for the sensitivity of the crop growth stages to water stress. The second model (Single Crop Model) tried to find the best allocation of the available water both in time and space in order to maximize the total expected yield of a given crop. The third model (Multi crop Model) was an optimization model that determined the optimal allocation of land and water for different crops. It showed the importance of several factors in producing an optimal cropping plan. The output of the models was prepared in a readable form to the normal user by the fourth model (Irrigation Schedule Model).

2.2 Economic Assumption of the Linear-Programming Model

In formulating a problem as a linear-programming model, one must understand the economic assumptions that are incorporated into the model. Basically, one assumes that a series of linear (or approximately linear) relationships involving the decision variables exist over the range of alternatives being considered in the problem. For the resource inputs, one assumes that the prices of these resources to the firm are constant over the range of resource quantities under consideration. This assumption implies that the firm can buy as much or as little of these resources as it needs without affecting the per unit cost. Such an assumption would rule out quantity discounts. One also assumes that there are constant returns to scale in the production process. In other words, in the production process, a doubling of the quantity of resources employed doubles the quantity of output obtained, for any level of resources. Finally, one assumes that the market selling prices of the two products are constant over the range of possible output combinations.

These assumptions are implied by the fixed per-unit profit contribution coefficients in the objective function. If the assumptions are not valid, then the optimal solution to the linear-programming model will not necessarily be an optimal solution to the actual decision-making problem. Although these relationships need not be linear over the entire range of values of the decision variables, the linearity assumptions must be valid over the full range of values being considered in the problem.

2.2.1 Maximum Link Utilization

Cherubini et al., (2009) described an optimization model which aims at minimizing the maximum link utilization of IP telecommunication networks under the joint use of the traditional IGP protocols and the more sophisticated MPLS-TE technology. The survivability of the network was taken into account in the optimization process implementing the path restoration scheme. This scheme benefits of the Fast Re-Route (FRR) capability allowing service providers to offer high availability and high revenue SLAs (Service Level

Agreements). The hybrid. IGP/MPLS approach relies on the formulation of an innovative Linear Programming mathematical model that, while optimizing the network utilization, provides optimal user performance, efficient use of network resource, and 100% survivability in case of single link failure. The possibility of performing an optimal exploitation of the network resources throughout the joint use of the IGP and MPLS protocols provides a flexible tool for the ISP (Internet Service Provider) networks traffic engineers. The efficiency of the proposed approach was validated by a wide experimentation performed on synthetic and real networks. The obtained result showed that a small number of ISP tunnels have to be set up in order to significantly reduce the `congestion level of the network while at the same time guaranteeing the survivability of the network.

2.2.2 Separable Linear Programming Model

Frizzone et al., (1997) developed a separable linear programming model, considering a set of technical factors which might influence the profit of an irrigation project. The model presented an objective function that maximized the net income and specified the range of water availability. It was assumed that yield functions in response to water application were available for different crops and described very well the water-yield relationships. The linear programming model was developed genetically, so that, the rational use of the available water resource could be included in an irrigation project. Specific equations were developed and applied in the irrigation project Senator Nilo Coelho" (SNCP), located in Petrolina-Brazil. Based on the water-yield functions considered, cultivated land constraints, production costs and products prices, it was concluded that the model was suitable for the management of the SNCP, resulting in optimal cropping patterns.

2.3 Taxation in the Informal Sector

Ofori (2009) stated that national revenue is raised through various ways. These include direct taxes, indirect taxes, royalties, etc. Both the formal and informal sectors of the economy contribute in mobilizing revenue for national development. While the formal sector is well structured to prevent evasion of taxes, the informal sector is not well organized. Various researchers have also observed that the revenue generated from the informal sector in Ghana has not been the best. In his work he established that the factors that make the informal sector hard-to-tax were very high in Ghana as a result of the predominance of cash transactions, poor record keeping, high illiteracy rate, little or no barriers to entry, lack of laid down procedures, ignorance of tax laws and the peripatetic nature of the informal sector in Ghana.

He therefore stress that, capacity building, tax information and education, simplification of filing procedures, improving tax administration and preparing a master list of informal businesses should be established in other to help to increased the tax base of the informal sector. The authors further argue that among the major challenges confronting the governments of the third world in their forward march to the socio-economic development is how to manage the phenomenon of the fledging informal economy and maximization of income tax from that sector. This assertion was recognized by Anuradha and Ayee (2001), when they state that "how to tax the informal sector remains a pressing question. Fiscally, constrained governments undertaking liberalizing economic reforms are looking for new ways to augment state revenues. The informal sector is an obvious focus of attention as it forms a significant and growing proportion of the economy in the developing countries, yet pays little in the form of taxes".

The researcher is also of the view that taxation of the informal sector in Ghana since independence remains a key challenge to governments. The tax inflow from the informal sector relative to its sheer size can be said to be woefully insufficient. There are many informal sector participants that either deliberately evade tax payment or are not captured by the Internal Revenue Service. The informal sector remains very fluid as there is no concrete regulatory policy in place to monitor activities of the sector.

2.4 Dimensions of Tax Evasion in Ghana.

Tax evasion has been identified as one of the major problems confronting tax administration especially in the developing countries. Evasion of tax is more problematic with respect to the informal sector. Agyei (1984) stated that in Ghana, one of the greatest problems facing tax administration is that of income tax evasion.

Otieku (1988), in his later study of the problems facing tax administration in Ghana also identified tax evasion as a problem. Tax evasion is the deliberate distortion of the facts relating to an assessment after the tax liability has been incurred so as to reduce the liability. According to Otieku, (1988), "any deliberate attempts by a taxpayer, his agent or tax officer to reduce the ultimate tax liability of the taxpayer by the use of any unlawful means constitute tax evasion. It is the deliberate attempt by the taxpayer to distort facts relevant for an objective ascertainment of his liability". The author identify that tax evasion is as a result of factors such as large scale of illiteracy among populace, ignorance of tax laws, dominance of difficult-to-identify sole-proprietor, inadequate number of tax offices and officers, complex tax laws etc (Otieku, 1988).

Evidence of income tax evasion is difficult to obtain in any country, particularly the developing countries, since it is an illegal activity. However, in developed countries, more is known about tax evasion than in developing ones. This is due to the fact that there is refinement of statistics and wealth of research resources in the developed countries (Agyei, 1984). Tax evasion, apart from resulting in loss of revenue to the government undermines confidence in the fairness of the tax law.

2.5 Approach to Taxable Income

Establishing the values of revenue-maximizing ETIs is important for a number of reasons. Firstly, despite the large number of empirical studies, it has proved difficult to obtain reliable estimates of the elasticity of taxable income, even where the focus of attention has been specific sub-sets of taxpayers such as those at the top of the income distribution: For the top marginal tax rate for example, Saez et al., (2009, p. 58) conclude that, 'the most reliable longer-run estimates range from 0.12 to 0.4, suggesting that the U.S. marginal top rate is far from the top of the Laffer curve'. In fact, analyzing this suggests that such low Elasticity of Taxable Income (ETI) estimates are, at least in principle, quite consistent with revenue-reducing top marginal rate responses.

Werning (2007) and Saez et al., (2009) have argued that the set of welfare-improving tax reforms is closely related to whether an increase in a particular marginal tax rate is expected to produce an increase in revenue of some minimum amount.

Werning (2007), for example, demonstrates that for a tax reform to generate a Pareto superior tax structure, it is required to reduce all tax rates but yield the same or more revenue overall, even though some taxpayers may respond in ways that reduce revenue while others' responses enhance revenues. Hence, Pareto efficiency requires the tax system to be on the 'right' (revenue-increasing) side of the Laffer curve. They put forward that, identifying the 'right' side of the Laffer curve in this context is more complex than establishing where the elasticity of taxable income with respect to the tax rate equals minus one. However, the key components of the revenue-maximising elasticity of taxable income can readily be calculated using relatively little information, namely the details of the marginal tax rates and income thresholds describing the complete structure. Furthermore, with information on the complete distribution of taxable income, revenue-maximising ETI values at aggregate levels can be obtained.

2.6 The Framework of Tax Noncompliance in the Informal Sector.

Admittedly, tax noncompliance is very high in the informal sector as compared to the formal sector businesses which are far more visible to IRS officials. Noncompliance arises not only by the actions and inactions of the taxpayers but also by the tax law makers. "Noncompliance could be said, therefore, to be a product of the decisions of rule makers as of the actions of the taxpayers" (Kidder and Craig, 2000). At any single point in time, compliance and noncompliance are the result of decisions of political, legal, and administrative actors and of the behavior of taxpayers. This suggests that tax noncompliance is not only the function of the taxpayers but also the actions and inactions of the "powers that be" in charge of tax administration as a whole.

In studying the tax noncompliance behavior of the informal sector entrepreneurs and businesses, this thesis adopts the "typology of noncompliance" presented by Kidder and Craig, (2000) as noncompliance type variables examined below:

(i) Procedural noncompliance- this results from failure to follow rules about when to file and which forms to file. Such violations do not necessarily result in understatement of tax liability but have to do with the procedures by which the taxpayer declares income and deductions. Compliance with IRS rules and

procedures is time and resources consuming hence it is only a few informal sector entrepreneurs who can afford to comply.

(ii) Unknowing noncompliance- this involves underpayment of taxes through ignorance of complex, changing, and sometimes ambiguous rules. Since the enactment of the IR Act in 2000 there has been a series of amendments in every year as the government fiscal statement is presented.

(iii) Lazy noncompliance- this occurs when individuals discover that they cannot document legitimate expenses for business or health costs or fail to keep track of outside earnings for which there is no withholding. Indeed, recording keeping practices in the informal sector seems to be poor hence the noncompliance.

(iv) Asocial noncompliance- this occurs when the income earner arranges his business activities so as to be invisible to the IRS officials. This is done by transacting business in cash and moving from one place to another so as not to attract attention of the tax man. Many informal sector entrepreneurs engage in this form of noncompliance

(v) Symbolic noncompliance- takes place to protest for perceived unfairness and inequities in tax laws. Refusal to pay tax as a protest against what one perceives as unfair tax laws and abuse of tax revenue by the government officials.

(vi) Social noncompliance- this results from peer pressure and social influence on the taxpayer not to comply with the tax laws. This is where network of individuals collectively refuse to pay tax hence anybody in that network must conform to the group norm.

(vii) Brokered noncompliance- this takes place upon the advice of a knowledgeable expert such as accountant, lawyer, or bond dealer. The taxpayer is advised by the accountant hired to prepare the accounts not to pay tax and in return account may be compensated for nonpayment of tax.

(viii) Habitual noncompliance- this emerges over time as the taxpayer establishes a pattern of non-declaration of income or under declaration of income.

2.7 Summary

In this chapter, other research works done by some scholars in connection with Linear programming problems and tax collections were reviewed.

From the various literatures, the researcher is of the view that an efficient linear programming model can be developed to maximize revenue collection for Sunyani West District (SWD)

The next chapter presents the research methodology of the study.

CHAPTER THRE

METHODOLOGY

3.1 Introduction

This chapter presents the methodology used for developing optimal revenue generation model. The first part of this chapter deals with theories of linear programming models, theoretical methods used in solving it with particular emphases on the Simplex method, the graphical method and the revised simplex algorithm.

3.2 Linear programming

In this sub section, we shall consider the various theories of linear programming algorithms. We shall also consider the formation of LP model and its various forms

3.2.1 Definition

A linear programming is the problem of optimizing a linear objective function in the decision variables $x_1, x_2 \ldots x_n$ subject to linear equality or inequality constrains on x_i's.

It is also a mathematical method for determining a way to achieve the best outcome (such as maximum profit or lowest cost) in a given mathematical model for some list of requirements represented as linear equations.

Linear Programming deals with special mathematical problems by developing rules and relationships that aim at distribution of limited funds under the restrictions imposed by either technological or practical aspects when an attribution decision has to be made (Andrade, 1990). It also specifies a class of mathematical problems, in which a linear function is maximized (or minimized) subject to given linear constraints. This problem class is broad enough to encompass many interesting and important applications, yet specific enough to be tractable even if the number of variables is large (Michael, 1997).

Linear programming, sometimes known as linear optimization, is the problem of maximizing or minimizing a linear function over a convex polyhedron specified by linear and non negativity constraints. Simplistically, linear programming is the optimization of an outcome based on some set of constraints using a linear mathematical model.

3.3 Forms of Linear Programming Problems

A linear programming may take one of the following forms:

(i) Matrix form

(ii) General form and

(iii) Standard form

3.3.1 Linear Programming in the Matrix Form

Linear programs are problems that can be expressed in canonical form as:

$$\text{Maximize } C^T x$$

$$\text{Subject to } Ax \leq b$$

Where x represents the vector of variables (to be determined), c and b are vectors of (known) coefficients and A is a (known) matrix of coefficients. The expression to be maximized or minimized is called the objective function ($C^T x$ in this case).

The inequalities $Ax \leq b$ are the constraints which specify a convex poly-tope over which the objective function is to be optimized.

A Linear programming model may simply be presented in the matrix vector form as;

$$\text{Maximize (Minimize) } c^T x$$

$$\text{Subject to: } Ax \leq b,\ x \geq 0$$

Given an m - vector, b = $(b_1,...,b_m)^T$, an n - vector, $c = (c_1,...,c_n)^T$, and an $m \times x$ matrix,

$$A = \begin{pmatrix} a_{11} & a_{12} & . & . & . & a_{1n} \\ a_{21} & a_{22} & . & . & . & a_{2n} \\ . & . & . & & & . \\ . & . & & . & & . \\ . & . & & & . & . \\ a_{m1} & a_{m2} & . & . & . & a_{mn} \end{pmatrix}$$

3.3.2 The general form of a linear program is

The general form of linear programming is;

$$\text{Maximize } c_1x_1 + \cdots + c_nx_n$$

$$\text{Subject to } a_{11}x_1 + \cdots + a_{1n}x_n + \leq b_1$$

$$a_{m1}x_1 + \cdots + a_{mn}x_n + \leq b_m$$

$$x_1 \geq 0, \ldots x_n \geq 0$$

Here $c_1, \dots, c_n, b_1, \dots, b_m$ and $a_{11}, \dots, a_{mn}$, are given numbers, and $x_1, \dots, x_n$, are variables whose values are to be determined, maximizing the given objective subject to the given constraints. There are n variables and m constraints, in addition to the non negativity restrictions on the variables. The constraints are called linear because they involve only linear functions of the variables.

3.3.3 Standard Form of a Linear Programming Problem

A linear programming problem is in **standard form** if it seeks to maximize the objective Function

$$Z = c_1x_1 + c_2x_2 + \dots + c_nx_n$$

subject to the constraints

$$\begin{aligned} a_{11}x_1 + a_{12}x_2 + \dots + a_{1n}x_n &\leq b_1 \\ a_{21}x_1 + a_{22}x_2 + \dots + a_{2n}x_n &\leq b_2 \\ \vdots \quad & \quad \vdots \\ a_{m1}x_1 + a_{m2}x_2 + \dots + a_{mn}x_n &\leq b_m \end{aligned}$$

Where $x_1 \geq 0$ and $b_1 \geq 0$ After adding slack variables, the corresponding system of **constraint equations** is

$$a_{11}x_1 + a_{22}x_2 + \dots + a_{1n}x_n + s_1 = b_1$$

$$a_{21}x_1 + a_{12}x_2 + \dots + a_{2n}x_n + s_2 = b_2$$

$$a_{m1}x_1 + a_{m2}x_2 + \dots + a_{mn}x_n + s_m = b_n$$

where $\cancel{s_i \geq 0}$

3.3.3.1 The Standard Maximum Problem

In general, a maximum Linear Programming (LP) is formulated as follows;

Maximize

$$c^T x = c_1x_1 + ... + c_nx_n$$

subject to the constraints

$$a_{11}x_1 + a_{12}x_2 + ... + a_{1n}x_n \le b_1$$

$$a_{21}x_1 + a_{22}x_2 + ... + a_{2n}x_n \le b_2$$

$$\vdots \qquad \vdots$$

$$a_{m1}x_1 + a_{m2}x_2 + ... + a_{mn}x_n \le b_m$$

and $\quad x_1 \ge 0, x_2 \ge 0, ..., x_n \ge 0$

3.3.3.2 The Standard Minimum Problem

In general, minimum Linear Programming is formulated as follows;

Minimize

$$y^T b = y_1b_1 + ... + y_mb_m$$

subject to the constraints

$$y_1a_{11} + y_2a_{12} + ... + y_ma_{1m} \ge c_1$$

$$y_1a_{12} + y_2a_{22} + ... + y_m a_{m2} \geq c_2$$

$$y_1a_{1n} + y_2a_{2n} + ... + y_m a_{mn} \geq c_n \text{ and}$$

$$y_1 \geq 0, y_2 \geq 0, ..., y_m \geq 0$$

3.4 Types of Linear Programming

In this section, we shall discuss the following types of linear programming

3.4.1 Integer programming problem

A mixed-integer program is the minimization or maximization of a linear function subject to linear constraints. More explicitly, a mixed-integer program with n variables and m constraints has the form:

Maximize

$$\sum_{j=1}^{n} c_j x_j$$

Subject to

$$\sum_{j=1}^{n} a_{ij}\, x_j = bi$$

(i=1,2…m)

$x_j \geq 0$ (j=1,2…n)

x_j integer (for some or all $j = 1, 2, \ldots, n$).

This problem is called the (linear) integer-programming problem. It is said to be a *mixed* integer program when some, but not all, variables are restricted to be integer, and is called a pure integer program when all decision variables must be integers.

3.4.2 Branch and Bound

The most widely used method for solving integer programs is branch and bound. Sub problems are created by restricting the range of the integer variables. For binary variables, there are only two possible restrictions: setting the variable to 0, or setting the variable to 1. More generally, a variable with lower bound l and upper bound u will be divided into two problems with ranges l to q and q+1 to u respectively. Lower bounds are provided by the linear-programming relaxation to the problem: keep the objective function and all constraints, but relax the integrality restrictions to derive a linear program. If the optimal solution to a relaxed problem is (coincidentally) integral, it is an optimal solution to the sub problem, and the value can be used to terminate searches of sub problems whose lower bound is higher.

3.4.3 Quadratic Linear Programming

The general quadratic program can be written as

$$\text{Minimize } f(\mathbf{x}) = C\mathbf{x} + \frac{1}{2}X^{T}QX$$

$$\text{Subject to } AX \leq b$$

$$X \geq 0$$

where **c** is an *n*-dimensional row vector describing the coefficients of the linear terms in the objective function, and Q is an (n ′ n) symmetric matrix describing the coefficients of the quadratic terms. If a constant term exists it is dropped from the model. As in linear programming, the decision variables are denoted by the *n*-dimensional column vector x, and the constraints are defined by an (m ′ n) A matrix and an m-dimensional column vector b of right-hand-side coefficients. We assume that a feasible solution exists and that the constraint region is bounded. When the objective function $f(x)$ is strictly convex for all feasible points the problem has a unique local minimum which is also the global minimum. A sufficient condition to guarantee strictly convexity is for Q to be positive definite.

3.4.3.1 Karush-Kuhn-Tucker Conditions

the Lagrangian function for the quadratic program is;

$$L(x, \mu) = cx + \frac{1}{2} X^T QX + \mu (Ax - b), \tag{3.1}$$

Where μ is an m-dimensional row vector. The Karush-Kuhn-Tucker conditions for a local minimum are given as follows.

$$\frac{\partial L}{\partial xj} \geq 0, j=1,\ldots,n \qquad c + x^T Q + \mu A \geq 0 \tag{3.2}$$

$$\frac{\partial L}{\partial \mu i} \leq 0, i=1,\ldots m \qquad Ax - b \leq 0 \tag{3.3}$$

$$x_j \frac{\partial L}{\partial xj} = 0, j=1,\ldots,n \qquad x^T(c^T + Qx + A^T\mu) = 0 \tag{3.4}$$

$$\mu_i g_i(x) = 0, i = 1, \ldots, m \qquad \mu(Ax - b) = 0 \tag{3.5}$$

$$x_j \geq 0, j = 1, \ldots, n \qquad x \geq 0 \tag{3.6}$$

$$\mu_i \geq 0, i = 1, \ldots, m \qquad \mu \geq 0 \tag{3.7}$$

to put (3.2) – (3.7) into a more manageable form we introduce nonnegative surplus variables $y \in \mathcal{R}^n$ to the inequalities in (3.2) and nonnegative slack variables $v \in \mathcal{R}^m$ to the inequalities in (3.3) to obtain the equations

$$c^T + Qx + A^T\mu^T - y = 0 \text{ and } Ax - b + v = 0. \tag{3.8}$$

The KKT conditions can now be written with the constants moved to the right-hand side.

$$Qx + A^T\mu^T - y = -c^T \tag{3.9}$$

$$Ax + v = b \tag{3.10}$$

$$x \geq 0,\ \mu \geq 0,\ y \geq 0,\ v \geq 0 \tag{3.11}$$

$$y^Tx = 0,\ \mu v = 0 \tag{3.12}$$

The first two expressions are linear equalities, the third restricts all the variables to be nonnegative, and the fourth prescribes complementary slackness.

3.4.3.2 Augmented Lagrangian methods

They are certain class of algorithms for solving constrained optimization problems. They have similarities to penalty methods in that they replace a constrained optimization problem by a series of unconstrained problems; the difference is that the augmented Lagrangian method adds an additional term to the unconstrained objective. This additional term is designed to mimic a Lagrange multiplier. The augmented Lagrangian is not the same as the method of Lagrange multipliers.

Let us say we are solving the following constrained problem:

$$\min f(\mathbf{x}) \tag{3.13}$$

subject to

$$c_i(\mathbf{x}) = 0 \; \forall i \in I. \tag{3.14}$$

This problem can be solved as a series of unconstrained minimization problems. For reference, we first list the penalty method approach:

$$\min \Phi_k(\mathbf{x}) = f(\mathbf{x}) + \mu_k \sum_{i \in I} c_i(\mathbf{x})^2 \tag{3.15}$$

The penalty method solves this problem, then at the next iteration it re-solves the problem using a larger value of μ_k (and using the old solution as a the initial guess or "warm-start").

The augmented Lagrangian method uses the following unconstrained objective:

$$\min \Phi_k(\mathbf{x}) = f(\mathbf{x}) + \frac{\mu_k}{2} \sum_{i \in I} c_i(\mathbf{x})^2 - \sum_{i \in I} \lambda_i c_i(\mathbf{x}) \tag{3.16}$$

and after each iteration, in addition to updating μ_k, the variable $\lambda$is also updated according to the rule

$$\lambda_i \leftarrow \lambda_i - \mu_k c_i(\mathbf{x}_k) \tag{3.17}$$

where $\mathbf{x}_k$ is the solution to the unconstrained problem at the k^{th} step, i.e.

$$\mathbf{x}_k = \operatorname{argmin} \Phi_k(\mathbf{x}) \tag{3.18}$$

The variable λ is an estimate of the Lagrange multiplier, and the accuracy of this estimate improves at every step. The major advantage of the method is that unlike the penalty method, it is not necessary to take $\mu \rightarrow \infty$ in order to solve the original constrained problem. Instead, because of the presence of the Lagrange multiplier term, μ can stay much smaller.

The method can be extended to handle inequality constraints.

3.4.3.2.1 The Optimal Solution

The simplex algorithm can be used to solve (3.9) – (3.12) by treating the complementary slackness conditions (3.12) implicitly with a restricted basis entry rule. The procedure for setting up the linear programming model follows.

(i) Let the structural constraints be Equations (3.9) and (3.10) defined by the KKT conditions.

(ii) If any of the right-hand-side values are negative, multiply the corresponding equation by –1.

(iii) Add an artificial variable to each equation.

(iv) Let the objective function be the sum of the artificial variable

(v) Put the resultant problem into simplex form.

The goal is to find the solution to the linear program that minimizes the sum of the artificial variables with the additional requirement that the complementarily slackness conditions be satisfied at each iteration. If the sum is zero, the solution will satisfy (3.9) – (3.12).

To accommodate (3.12), the rule for selecting the entering variable must be modified with the following relationships in mind. x_j and y_j are complementary for $j = 1, \ldots, n$ while

m*i* and vi are complementary for $i = 1, \ldots, m$.

The entering variable will be the one whose reduced cost is most negative provided that its complementary variable is not in the basis or would leave the basis on the same iteration. At the conclusion of the algorithm, the vector x defines the optimal solution and the vector μ defines the optimal dual variables.

This approach has been shown to work well when the objective function is positive definite, and requires computational effort comparable to a linear programming problem with m + n constraints, where m is the number of constraints and n is the number of variables in the QP. Positive semi-definite forms of the objective function, though, can present computational difficulties. Van De Panne (1975) presents an extensive discussion of the conditions that will yield a global optimum even when f(x) is not positive definite. The simplest practical approach to overcome any difficulties caused by semi-definiteness is to add a small constant to each of the diagonal elements of Q in such a way that the modified Q matrix becomes positive definite. Although the resultant solution will not be exact, the difference will be insignificant if the alterations are kept small.

3.4.3.3 Primal-dual Interior Point Method for Nonlinear Optimization

The primal-dual method's idea is easy to demonstrate for constrained nonlinear optimization. For simplicity consider the all-inequality version of a nonlinear optimization problem:

$$\text{minimize } f(x) \text{ subject to } \quad c(x) > 0 \quad x \in \mathcal{R}^m. \tag{3.19}$$

The logarithmic barrier function associated with (3.1) is

$$B(x,\mu) = f(x) - \mu \sum_{i=1}^{m} \ln(c_i(x)) \tag{3.20}$$

Here μ is a small positive scalar, sometimes called the "barrier parameter". As μ converges to zero the minimum of $B(x,\mu)$ should converge to a solution of (3.19).

The barrier function gradient is

$$g_h = g - \mu \sum_{i=1}^{m} \frac{1}{C_i(x)} \nabla c_i(x) \tag{3.21}$$

where g is the gradient of the original function $f(x)$ and ∇c_i is the gradient of c_i.

In addition to the original ("primal") variable x we introduce a Lagrange multiplier inspired dual variable $\lambda \in \mathbb{R}^m$ (sometimes called "slack variable")

$$\forall_{i=1}^{m} c_i(x)\lambda_i = \mu \tag{3.22}$$

(3.22) is sometimes called the "perturbed complementarily" condition, for its resemblance to "complementary slackness" in KKT conditions.

We try to find those (x_μ, λ_μ) which turn gradient of barrier function to zero.

Applying (3.22) to (3.21) we get equation for gradient:

$$g - A^T\lambda = 0 \tag{3.23}$$

where the matrix A is the constraint $c(x)$ Jacobian.

The intuition behind (3.23) is that the gradient of $f(x)$ should lie in the subspace spanned by the constraints' gradients. The "perturbed complementarities" with small μ (3.22) can be understood as the condition that the solution should either lie near the boundary $c_i(x) = 0$ or that the projection of the gradient g on the constraint component $c_i(x)$ normal should be almost zero.

Applying Newton's method to (3.22) and (3.23) we get an equation for (x, λ) update (p_x, p_λ):

$$\begin{pmatrix} W & -A^T \\ \Lambda A & C \end{pmatrix} \begin{pmatrix} p_x \\ p_\lambda \end{pmatrix} = \begin{pmatrix} -g + A^T\lambda \\ \mu 1 - C\lambda \end{pmatrix}$$

where W is the Hessian matrix of $f(x)$ and Λ is a diagonal matrix of λ.

Because of (3.19) and (3.22) the condition $\lambda \geq 0$ should be enforced at each step. This can be done by choosing appropriate α:

$$(x, \lambda) \to (x + \alpha p_x, \lambda + \alpha p_\lambda).$$

3.5 Solution Techniques for Linear Programming Problem

The solutions to the various Linear Programming Problem (LPP) include

(a) Graphical Method

(b) Simplex algorithm

(c) QSB Package

(d) Matlab Package

(e) YE's Interior Point algorithm

(f) Microsoft Excel 2003

The most convenient and effective technique in use now is the simplex and the revised simplex algorithm

3.5.1 The Graphical Method

A linear programming problem involving one or two variables can be solved by using the graphical approach.

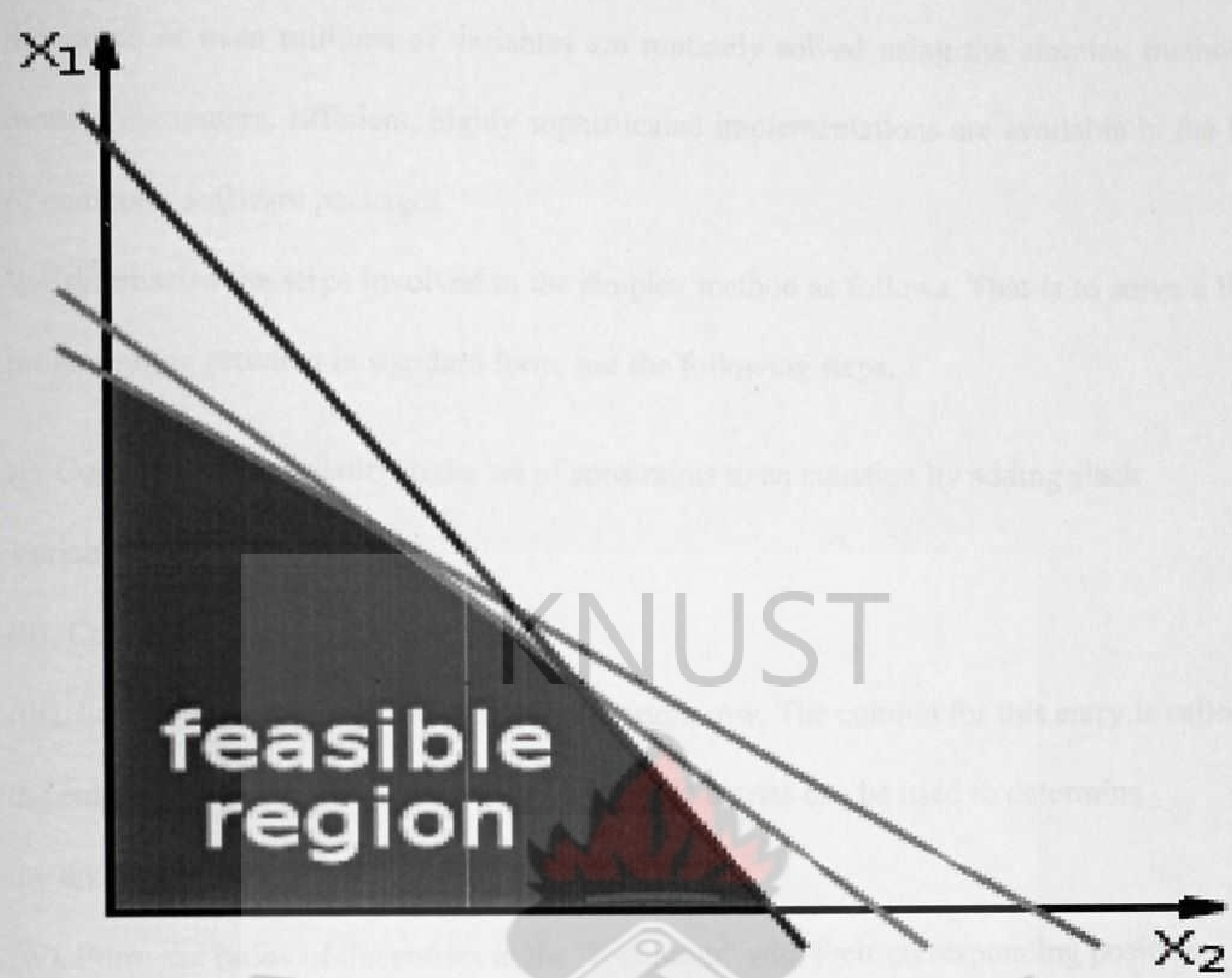
Steps involved in the Graphical Method

(i) Write down the inequalities according to the construction of the problem.

(ii) Represent the inequalities as equations.

(iii) Draw linear graphs from the equations to obtain the corner points satisfying the region of intersection where the solution set will be determined from.

(iv) Substitute the coordinates of the corner points into the objective function to obtain the maximum or minimum value of the problem.



The shaded region represents all set of points which satisfy all the constraints

3.5.2 The Simplex Method

The simplex method has been the standard technique for solving a linear program since the 1940's. In brief, the simplex method passes from vertex to vertex on the boundary of the feasible polyhedron, repeatedly increasing the objective function until either an optimal solution is found, or it is established that no solution exists. In principle, the time required might be an exponential function of the number of variables, and this can happen in some contrived cases. In practice, however, the method is highly efficient, typically requiring a number of steps which is just a small multiple of the number of variables. Linear programs in thousands or even millions of variables are routinely solved using the simplex method on modern computers. Efficient, highly sophisticated implementations are available in the form of computer software packages

We summarize the steps involved in the simplex method as follows. That is to solve a linear programming problem in standard form, use the following steps.

(i). Convert each inequality in the set of constraints to an equation by adding slack Variables.

(ii). Create the initial simplex tableau.

(iii). Locate the most negative entry in the bottom row. The column for this entry is called the entering column. (If ties occur, any of the tied entries can be used to determine the entering column.)

(iv). Form the ratios of the entries in the "b-column" with their corresponding positive entries in the entering column. The departing row corresponds to the smallest nonnegative ratio bi/aij. (If all entries in the entering column are 0 or negative, then there is no maximum solution. For ties, choose either entry.) The entry in the departing row and the entering column is called the pivot.

(v). Use elementary row operations so that the pivot is 1, and all other entries in the entering column are 0. This process is called pivoting.

(vi). If all entries in the bottom row are zero or positive, this is the final tableau. If not, go back to (iii).

(vii). If you obtain a final tableau, then the linear programming problem has a maximum solution, which is given by the entry in the lower-right corner of the tableau.

Note that the basic feasible solution of an initial simplex tableau is

$(x_1, x_2, \ldots, x_n, s_1, s_2, \ldots, s_m) = (0, 0, \ldots, 0, b_1, b_2, \ldots, b_m)$

This solution is basic because at most m variables are nonzero (namely the slack variables).It is feasible because each variable is nonnegative.

3.5.3 The Revised Simplex Method

The revised simplex method is a concise and efficient implementable algebraic representation of the simplex method. Only a small part of the condensed tableau is actually calculated. These entries are needed to completely determine a pivot step and the resulting economy of computation has proved the key to practical software implementation of the simplex method. Revised simplex method is also an efficient computational procedure for digital computers. At each iteration, fewer entries are needed whiles in the simplex, total entries required. Furthermore revised simplex generates the inverse of the current basis matrix automatically Instead of representing the whole tableau explicitly, we manipulate the basic and non basic variables. Let us consider the following LP problem

Max $Z = CX$

Sub to: $AX \leq b$

$X > 0$

Initial constraints become (standard form)

$$[A \quad I]\begin{pmatrix} X_S \\ X_S \end{pmatrix} = b$$

X_S = slack variable

Let B = 1

Where I is the identity matrix that appeared in the solution of a given problem

X_B = basic variable value

$$X_B = \begin{pmatrix} X_{B1} \\ \vdots \\ \vdots \\ \vdots \\ X_{BM} \end{pmatrix}$$

At any iteration non-basic variable = 0

$BX_B = b$

Therefore $X_B = B^{-1}b$

At any iteration given the original b vector and the inverse matrix, X_B (current R.H.S) can be calculated.

$Z = C_BX_B$

Where C_B = objective coefficient of basic variables.

Steps in Revised Simplex Method

1. Determine entering variable x_j, with associated vector p_j

(i) Compute $Y = C_BB^{B-1}$

(ii) Compute $Z_J - C_J = YP_j - C_J$ for all non basic variables

(iii) Choose largest negative value (maximization). If non stop

2. Determine the leaving variable X_r, with associated vector P_r

(i) Compute $bx_B = b^{-1}$ (current RHS)

(ii) Compute current constraints coefficient of entering variable

$a^j = B^{-1} P_J$

X_r Is associated with $\Theta = \{ (X_B)_K \backslash a^J_k, a^j_k > 0 \}$

3. Determine next basis (calculate B^{-1})

Go to step 1

CHAPTER FOUR

Data collection, Analysis and Interpretation of Results

4.0 Introduction

This chapter presents analysis and interpretation of LP model. We shall also put forward the solution to the LP using Matlab package.

4.1 Sources of data collection

The Sunyani West District Revenue Authority has its own way of collecting revenue on taxable items which focuses on taxes grouped according to the following headings; Pay as you earn (P), Companies (C), Self-employed (S) and Miscellaneous (M). Each category has sub taxes which constitute the group. For instance pay as you earn which is one of the main categories is constituted by manufacturing, transport hotel and restaurant and so on. Each of the tax type falls within the categories mention above. (See Appendix A)

4.1.1 Type of Data

The data for this project work is a secondary quarterly data obtained from the offices of the Sunyani west District RA in the Brong Ahafo region of Ghana, and it spans between 2008 and 2011. The various taxable agents were designated as: Agriculture and forestry (X_1), Electricity, and water (X_2), construction and construction equipment (X_3), Wholesale and retail trade (X_4), Hotel and Restaurant (X_5), Other Professional And Business Activities (X_6), Education (X_7), Health And Social Work (X_8), Registration Fee (X_9), and penalties (X_{10})

Table 4.1: Average Revenue (AR) generated by the District RA for the year 2008-2011.

TAX TYPE	PAY AS YOU EARN (P)	CAMPANIES (C)	SELF EMPLOYED (S)	MISCELLA NEOUS- (M)	TOTAL (T)
X_1	2200	1100	2300	1100	6700
X_2	1542	400	1242	1000	4184
X_3	5000	1000	4000	1000	11000
X_4	1780	1168	180	168	3296
X_5	4288	2941	1924	5232	14385
X_6	3264	1934	1700	2121	9019
X7	1236	764	894	962	3856
X_8	1890	790	290	590	3560
X_9	5732	2619	1890	1002	11243
X_{10}	5336	3834	2781	1500	13451
TOTAL	32268	16550	17201	14675	
BUDGET	35000	20000	18000	17000	

Source: Sunyani West District Revenue Authority Office

4.1.2 Statistical Analysis of the Raw Data

The averages of the expenditure were determined from the quarterly report to get the average revenue (A.R) for each year. The various revenue sub-heads were assigned variable names and were grouped into sub headings based on the quarterly reports. This presents a simplified form of table as shown in Table 4.1. The summation of the actual revenue became objective function. The unit charge for each of the items was also determined and that yields the constrains. This formed the coefficient matrix.

The percentages of the data was further determined using the relation below;

Usage Ratio = $\frac{Tax\ type}{Total} \times 100$, which yields Table 4.2

Table 4.2 Percentage Usage Ratio by the District RA for the year 2008-2011

TAX TYPE	PAY AS YOU EARN (P)	CAMPANIES (C)	SELF EMPLOYED (S)	MISCELLANEOUS- (M)
X_1	32.836	16.418	34.328	16.418
X_2	36.855	9.56	23.901	23.901
X_3	45.455	9.091	36.364	9.091
X_4	54.005	35.437	5.461	5.097
X_5	29.809	20.445	13.375	36.371
X_6	36.19	21.444	18.849	23.517
X7	32.054	19.813	23.185	24.948
X_8	53.09	22.191	8.146	16.573
X_9	50.983	23.294	16.81	8.912
X_{10}	39.67	28.503	20.675	11.152
BUDGET	35000	20000	18000	17000

Source: Sunyani West District Revenue Authority Office

4.2 Formation of LP Model

Generally, there are four main steps that need to be followed in formulating a linear programming model.

(i) Identify the decision variables and assign symbols x and y to them. These decision variables are those quantities whose values we wish to determine.

(ii) Identify the set of constraints and express them as linear equations/ inequalities in terms of the decision variables. These constraints are the given conditions.

(iii) Identify the objective function and express it as a linear function of decision variables. It might take the form of maximizing profit or production or minimizing cost.

(iv) Add the non-negativity restrictions on the decision variables, as in the physical problems, negative values of decision variables have no valid interpretation.

4.2.1 The proposed LP Model For Maximising Tax Collection In Sunyani West District

The proposed LP model for optimal Revenue collections in Sunyani West District is as follows

Objective function:

We formulate the objective function as:

$$\text{Maximize } Z=\delta X_1+\delta X_2+\delta X_3+\delta X_4+\delta X_5+\delta X_6+\delta X_7+\delta X_8+\delta X_9+\delta X_{10}$$

Where δ is a percentage constant.

X_1= Agriculture and forestry

X_2= Electricity, and water

X_3= construction and construction equipment

X_4= Wholesale and retail trade

X_5= Hotel and Restaurant

X_6= Other Professional And Business Activities

X_7= Education

X_8= Health And Social Work

X_9= Registration Fee

X_{10}= penalties

Decision Variables:

The constrains are:

$$P_{11}X_1+P_{12}X_2+P_{13}X_3+P_{14}X_{4+}P_{15}X_5+P_{16}X_6+P_{17}X_7+P_{18}X_8+P_{19}X_9+P_{10}X_{10}\leq B_P$$

$$C_{11}X_1+C_{12}X_2+C_{13}X_3+C_{14}X_{4+}C_{15}X_5+C_{16}X_6+C_{17}X_7+C_{18}X_8+C_{19}X_9+C_{10}X_{10}\leq B_C$$

$$S_{11}X_1+S_{12}X_2+S_{13}X_3+S_{14}X_{4+}S_{15}X_5+S_{16}X_6+S_{17}X_7+S_{18}X_8+S_{19}X_9+S_{10}X_{10}\leq B_S$$

$$M_{11}X_1+M_{12}X_2+M_{13}X_3+M_{14}X_{4+}M_{15}X_5+M_{16}X_6+P_{17}X_7+M_{18}X_8+M_{19}X_9+M_{10}X_{10}\leq B_M$$

$X_1\geq0,\ X_2\geq0,\ X_3\geq0,\ X_4\geq0,\ X_5\geq0,\ X_6\geq0,\ X_7\geq0,\ X_8\geq0,\ X_9\ X1\geq0, X_{10}\geq0.$

where X_i are taxes for the various categories and

$P_{ij,}\ C_{ij},\ S_{ij}$ and M_{ij} represents the various coefficients of tax categories. And

B_P= Budget for tax category of Pay as you earn

B_C=Budget for tax category of Companies

B_S= Budget for tax category of Self employed

B_M = Budget for tax category of Miscellaneous

4.2.2 Assumptions

Linear programming is based on four mathematical assumptions. An assumption is a simplifying condition taken to hold true in the system being analyzed in order to render the model mathematically tractable (solvable). The first three assumptions follow from a fundamental principle of LP: the linearity of all model equations. (This applies to constraint inequalities as well, since the addition of slack and surplus variables convert all inequalities into equations.) Lineari2ty means that all equations are of the form: $ax + by + ... + cz = d$, where a, b, c, d are constants.

Base on this, a problem can be realistically represented as a linear program if the following assumptions hold:

(a) The constraints and objective function are linear.

(i) This requires that the value of the objective function and the response of each resource expressed by the constraints are proportional to the level of each activity expressed in the variables.

(ii) Linearity also requires that the effects of the value of each variable on the values of the objective function and the constraints are additive. In other words, there can be no interactions between the effects of different activities; i.e., the level of activity X_1 should not affect the costs or benefits associated with the level of activity X_2.

(b) Divisibility: the values of decision variables can be fractions. Sometimes these values only make sense if they are integers; then we need an extension of linear programming called integer programming.

(c) Certainty: the model assumes that the responses to the values of the variables are exactly equal to the responses represented by the coefficients.

(d) Data: formulating a linear program to solve a problem assumes that data are available to specify the problem.

4.3 Implementation of LP Model

A linear programming model was generated for the objective function Z, from table 4.2 based on the broad categories of the taxes collected as;

Max $Z = X_1+X_2+X_3+X_4+X_5+X_6+X_7+X_8+X_9+X_{10}$

Subject to:

(1) Pay as you earn:

Total budget for pay as you earn=35000

$32.836x_1+36.855x_2+45.455x_3+54.005x_4+29.809x_5+36.19x_6+32.054x_7$

$+53.09x_8+50.983x_9+39.67x_{10} \leq 35000$

(2) Companies

Total budget for Companies=20000

$16.418x_1+9.56x_2+9.091x_3+35.437x_4+20.445x_5+21.444x_6+19.813x_7$

$+22.191x_8+23.294x_9+28.503x_{10} \leq 20000$

(3) Self-employed

Total budget for Self-employed=18000

$34.328x_1+23.901x_2+36.364x_3+5.461x_4+13.375x_5+18.849x_6+23.185x_7$

$+8.146x_8+16.81x_9+20.675x_{10}\leq18000$

(4). Miscellaneous

Total budget for Miscellaneous=17000

$16.418x_1+23.901x_2+9.091x_3+5.097x_4+36.371x_5+23.517x_6+24.948x_7$

$+16.573x_8+8.912x_9+11.152x_{10}\leq17000$

(5) Non-Negativity

$X_1\geq0,\ X_2\geq0,\ X_3\geq0,\ X_4\geq0,\ X_5\geq0,\ X_6\geq0,\ X_7\geq0,\ X_8\geq0,\ X_9\geq0,\ X10\geq0.$

That is,

$Z= X_1+X_2+X_3+X_4+X_5+X_6+X_7+X_8+X_9+X_{10}$

Subject to:

$32.836x_1+36.855x_2+45.455x_3+54.005x_4+29.809x_5+36.19x_6+32.054x_7+53.09x_8+50.983x_9+39$

$.67x_{10}\leq35000$

$16.418x_1+9.56x_2+9.091x_3+35.437x_4+20.445x_5+21.444x_6+19.813x_7+22.191x_8+23.294x_9+28.$

$503x_{10}\leq20000$

$34.328x_1+23.901x_2+36.364x_3+5.461x_4+13.375x_5+18.849x_6+23.185x_7+8.146x_8+16.81x_9+20.$

$675x_{10}\leq18000$

$16.418x_1+23.901x_2+9.091x_3+5.097x_4+36.371x_5+23.517x_6+24.948x_7+16.573x_8+8.912x_9+11.152x_{10} \leq 17000$

$X_1 \geq 0, X_2 \geq 0, X_3 \geq 0, X_4 \geq 0, X_5 \geq 0, X_6 \geq 0, X_7 \geq 0, X_8 \geq 0, X_9 \geq 0, X10 \geq 0.$

4.4 Interpretation of the proposed model

At this point we seek to interpret the proposed model

4.4.1 Computational Procedure

The coefficients of the various categories, left-hand side constrain inequalities and right-hand side constrains were written in matrices form. Matlab algorithm programme code of Revised simplex was obtained and modified. See Appendix C

The matrices were put in the Matlab program code and ran on AMD Anthlon™ 64× 2 Dual-core processor TK-57,32-bit operating system, 1.90GHz speed, Windows Vista Toshiba laptop computer.

4.4.2 Matrix Formulation

Using A,B and F for the matrices of left-hand side inequalities, right-hand side constraints and the objective functions respectively, we have:

$$F = [1 \quad 1 \quad 1 \quad 1 \quad 1 \quad 1 \quad 1 \quad 1 \quad 1 \quad 1]$$

$$A = \begin{bmatrix} 32.83 & 36.85 & 45.45 & 54.00 & 29.80 & 36.20 & 32.10 & 53.10 & 50.90 & 39.67 \\ 16.42 & 9.56 & 9.09 & 35.40 & 20.50 & 21.40 & 19.80 & 22.20 & 23.30 & 28.50 \\ 34.33 & 23.90 & 36.36 & 5.50 & 13.40 & 18.90 & 23.20 & 8.20 & 16.80 & 20.67 \\ 16.42 & 23.90 & 9.09 & 5.10 & 36.40 & 23.50 & 24.90 & 16.60 & 8.90 & 11.15 \end{bmatrix}$$

$$B = \begin{bmatrix} 35000 \\ 20000 \\ 18000 \\ 17000 \end{bmatrix}$$

4.5 Solution to the Model using Matlab package

Table 4.3 showing the solution to the model using matlab package

TAX TYPE	DECISION VARIABLE	UNIT COST	TOTAL CONTRIBUTION	REDUCED COST
X1:Agric. & Forestry	0	1	0	0.0324464
X2:Elect. & Water	20504.9	1	20504.90	0
X3:Const. & Const. Equip.	1905.89	1	1905.89	0
X4:Wholesale & Retail	5643.04	1	5643.04	0
X5:Hotel and Restaurant	13826.7	1	13826.70	0
X6:Prof. & Busi. Act.	3125.80	1	3125.80	0
X7:Education	0	1	0	0.0129429
X8:Health & soc. Work	0	1	0	0.0427262
X9:Registration fee	17319.07	1	17319.07	0
X10:Penalties	28673.8	1	28673.80	0

4.5.1 Optimal solution

Optimal solution found after 5 iteration is Z = GH¢ 90,999.20

4.6 Interpretation of the Results

The variables show that funds to achieve Revenue target should be allocated to X_2, X_3, X_4, X_5, X_6, X_9, X_{10}, which corresponds to Electricity and water, Construction and construction equipment, Wholesale and retail trade, Hotel and Restaurant, Other professional and

Business Activities, Registration fees and penalties which is the last one respectively. Whiles there should be a reduced cost of tax funds to X_1, X_7 and X_8 which means that, Agriculture and forestry, Education, Health And Social Work should have a reduce tax respectively.

4.7 Findings

Analysis of tax in SWD was carried out and an efficient LP model was developed to optimized revenue for SWD. The analysis of the data reviewed that the District RA should strategised it plans to apportion taxes to the various tax types in order for the District to achieve a realistic target of revenue mobilization in the various tax categories. From the analysis, the contributions for penalties recorded GH¢28673.80 which is significantly higher than all the other tax types. This pre-supposes that most people evade tax and therefore the District IRS should increase the target for penalties so that those who evade tax would be pushed to pay more penalties. Electricity and water recorded GH¢20504.90 whiles registration fee also recorded GH¢17319.07. Hotel and restaurant brought to bear a reasonable amount of GH¢13826.70 whiles wholesale and retail trade recorded GH¢5643.89.the last two which recorded the least was other professional and business activities and construction and construction equipment with a numerical value of GH¢3125.80 and GH¢1905.89 respectively.

CHAPTER FIVE

SUMMARY, CONCLUSIONS AND RECOMMENDATIONS

5.1 Introduction

In this final chapter, we present the summary of the project, discussion and conclusion and some recommendations will be put forward.

5.2 Summary

The primary aims of this thesis are to develop optimization models based on Linear programming algorithm and use it to determine the optimal revenue collection and Analysis of tax system for Sunyani west District RA.

To achieve this aims, secondary data were collected from the District RA office spanning from 2008-2011. Based on these empirical data, LP model was formulated and a revised simplex algorithm was used to analysed the data with the help of matlab package. It was found out that the average total revenue by the District for the past four years has been GH¢ 80,694.00. The optimal value of GH¢ 90,999.20 annually on the Revised simplex algorithm code revealed an appreciable increase in revenue. Hence with this research work, the District IRS can raise its revenue to GH¢ 90,999.20 annually which represent 12.77% increase in the District IRS effort.

This can be achieved if we spread the tax net to cover areas such as Electricity and water, Construction and construction equipment, Wholesale and retail trade, Registration fees, other professional and Business Activities, Hotel and Restaurant, penalties

5.3 Discussions and Conclusions

The revenue data collected from the Sunyani west District Revenue Service was modelled into Linear Programming problem. This data was run on a matlab software code. The analysis done in chapter four using revised simplex algorithm showed that average annual revenue generated by the District GRA between 2008 and 2012 was GH¢ 80,694.00 Based upon this research work, the District GRA can raise its revenue to GHS GH¢ 90, 999.20 which represents a 12.77% increase in the District RA collection.

In conclusion, the researcher believes that taxation is a complex issue that calls for further research especially on the factors that make some sectors hard-to-tax and tax noncompliance behaviour of the informal sector entrepreneurs.

5.4 Recommendations

Based on the study, the following recommendations are made.

The work should serve as basis for further research works in improving revenue mobilization exercise by Revenue Services in Ghana. The government must take active role in building the capacity of the informal sector entrepreneurs in terms of training them to be equipped with basic financial and accounting skills so as to be able to keep the basic accounting and financial records that will enable GRA make objective assessment of income tax. These training programmed should be developed by the GRA in conjunction with the Ministries of Finance and Local Government. The programmed should be decentralized to the unit Committee levels so that all major identifiable informal entrepreneurs are trained to be able to keep adequate financial records which will track all business activities of the informal businesses. Tax education should be pursued vigorously and on a sustained basis which with time is likely to encourage voluntary compliance from the informal sector. The GRA must embark on nationwide exercise to establish master list of all the informal sector businesses that are easily identifiable. This could be done by employing the services of national service personnel and a large number of unemployed graduates out there. This exercise may be up-dated annually. Researcher believes that there is more room for improvement. The GRA should continue to expand its offices to everywhere in Ghana where there are a good number of informal businesses. However, this should only be done taking into account the cost-benefit analysis. The research work also reveals that the contribution of Electricity and water, Hotel and Restaurant yielded higher rate of tax which should be focused in other to achieve the tax target for the District RA.

The researcher is of the view that the District RA will benefit a lot by way of addressing revenue leakages if they can acquire automatic tax collection machines for tax collection by the District.

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APPENDIX A

Averages of Raw Data 2008-2011

TAX TYPE	PAY AS YOU EARN	CAMPANIES	SELF-EMPLOYED	MISCELLANEOUS	TOTAL
Agric, Hunting and forestry	1200	113	1200	190	2703
Manufacturing	1000	979	1090	909	3978
Electricity, gas and water	15.42	4.00	12.42	10.00	41.84
Construction and const equip	5000	1000	4000	1000	11000
Wholesale and retail trade	1780	1168	180	168	329600
Hotel And Restaurant	1054	234	678	1398	3364
Transport	2340	1230	439	2673	6682
Financial Intermediation	104	790	458	308	1660
Computer, Research And Development	790	687	349	853	2679
Other Professional And Business Act.	1023	590	890	906	3409
Public Administration And Defence	874	978	320	309	82481
Education	1236	764	894	962	3856
Health And Social Work	1890	790	290	590	3560
Other Social And Personal Act.	1367	580	490	906	3343
Private Household Employment	2560	1805	980	496	5841
Registration Fees	5732	2619	1890	1002	11243
Stamp Duty	2434	1903	1127	789	6253

Penalties	342	12000	674	2.15	13.57
TOTAL	32268	16750	18191	13674	
BUDGET	33000	17000	18950	15678	

TAX TYPE	PAY AS YOU EARN	CAMPANIES	SELF-EMPLOYED	MISCELLANEOUS	TOTAL
Agric, Hunting and forestry	1300	1131	1200	180	2703
Manufacturing	1000	979	1090	909	3978
Electricity, gas and water	1542	4.00	12.42	10.00	41.84
Construction and const equip	5000	1000	4000	1000	11000
Wholesale and retail trade	1780	1168	180	168	329600
Hotel And Restaurant	1054	234	678	1398	3364
Transport	2340	1230	439	2673	6682
Financial Intermediation	104	790	458	308	1660
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2Public Administration And Defence	874	978	320	309	2481
Education	1236	764	894	962	3856
Health And Social Work	1890	790	290	590	3560
Other Social And Personal Act.	1367	580	490	906	3343
Private Household Employment	2560	1805	980	496	5841

Registration Fees	5732	2619	1890	1002	11243
Stamp Duty	2434	1903	1127	789	6253
Penalties	342	12000	674	215	1357
TOTAL	32268	16750	18191	13674	
BUDGET	33000	17000	18950	15678	

Appendix B

Number of iteration ran for the optimal value.

iter 1: mu = 2.908792e+012, resid = 6.874749e+008

iter 2: mu = 3.389059e+011, resid = 8.010009e+007

iter 3: mu = 4.006529e+010, resid = 9.524913e+006

iter 4: mu = 5.647018e+009, resid = 2.056123e+006

iter 5: mu = 1.513498e+009, resid = 1.192457e+006

APPENDIX C

Matlab code

```
function revised(c,b,a,inq,maximize)
if nargin<3||nargin>5
   fprintf('\nError:Number of input arguments are inappropriate!\n');
else
   n=length(c);m=length(b);j=max(abs(c));
   if nargin<4
      maximize=0;
      inq=-ones(m,1);
   elseif nargin<5
      maximize =0;
   end
   if ~isequal(size(a),[m,n])||m~=length(inq)
      fprintf('\nError: Dimension mismatch!\n');
   else2
      if maximize ==1
         c=-c;
      end
      count=n;nbv=1:n;bv=zeros(1,m);av=zeros(1,m);
      for i=1:m
         if b(i)<0
            a(i,:)=-a(i,:);
            b(i)=-b(i);
         end
         if inq(i)<0
            count=count+1;
            c(count)=0;
            a(i,count)=1;
            bv(i)=count;
         elseif inq(i)==0
            count=count+1;
            c(count)=-10*j;
            a(i,count)=1;
            bv(i)=count;
            av(i)=count;
         else
            count=count+1;
            c(count)=0;
            a(i,count)=-1;
            nbv=[nbv count];
            count=count+1;
            c(count)=-10*j;
```

```
        a(i,count)=1;
        av(i)=count;
        bv(i)=count;
    end
end
A=[-c;a];
B_inv=eye(m+1,m+1);
B_inv(1,2:m+1)=c(bv);
x_b=B_inv*[0; b'];
fprintf('\n.............The initial tablaue................\n')
fprintf('\t z');disp(bv);
fprintf('------------------------------------------------------\n')
disp([B_inv x_b])
flag=0;count=0;of_curr=0;
while(flag~=1)
    [s,t]=max(B_inv(1,:)*A(:,nbv));
    y=B_inv*A(:,nbv(t));count=count+1;
    if(any(y(2:m+1)>0))
        fprintf('\n.............The %dth   tablaue................\n',count)
fprintf('\t z');disp(bv);
        fprintf('------------------------------------------------------\n')
        disp([B_inv x_b y])
        if count>1 && of_curr==x_b(1)
            flag=1;
            if maximize ==1
                x_b(1)=-x_b(1);
            end
        fprintf('\nThe given problem has degeneracy!\n');
        fprintf('\nThe current objective function value=%d.\n',x_b(1));
            fprintf('\nThe current solution is:\n');
            for i=1:n
                found=0;
                for j=1:m
                    if bv(j)==i
                        fprintf('x%u = %d\n',i,x_b(1+j));found=1;
                    end
                end
                if found==0
                    fprintf('x%u = %d\n',i,0);
                end
            end
        else
            of_curr=x_b(1);
            if(s>=0)
                flag=1;
                for i=1:length(av)
```

```
            for j=1:m
               if av(i)==bv(j)
                fprintf('\nThe given LPP is infeasible!\n');
                  return
               end
            end
         end
         if maximize ==1
            x_b(1)=-x_b(1);
         end
      fprintf('\nReqiured optimization has been achieved!\n');
   fprintf('\nThe optimum objective function value=%d.\n',x_b(1));
         fprintf('\nThe optimum solution is:\n');
         for i=1:n
            found=0;
            for j=1:m
               if bv(j)==i
                  fprintf('x%u = %d\n',i,x_b(1+j));found=1;
               end
            end
            if found==0
               fprintf('x%u = %d\n',i,0);
            end
         end
         if (s==0 && any(y(2:m+1)>0))
        fprintf('\nThe given problem has alternate optima!\n');
         end
      else
         u=10*j;
         for i=2:m+1
            if y(i)>0
               if (x_b(i)/y(i))<u
                  u=(x_b(i)/y(i));
                  v=i-1;
               end
            end
         end
         temp=bv(v);bv(v)=nbv(t);
         nbv(t)=temp;
         E=eye(m+1,m+1);
         E(:,1+v)=-y/y(1+v);
         E(1+v,1+v)=1/y(1+v);
         B_inv=E*B_inv;
         x_b=B_inv*[0; b'];
      end
   end
```

```
        else
            fprintf('\nThe given problem has unbounded solution\n')
            return
        end
    end
  end
end
```