DETERMINATION OF OPTIMAL ORDER QUANTITY AND COST USING INVENTORY MODEL WITH BACKORDERS AND LOST SALES UNDER FUZZY COST; CASE STUDY: MANTRAC GHANA LIMITED, ACCRA

BY

VICTOR TEKPOR, B.SC. MINING ENGINEERING

(PG3017209)

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I hereby declare that this submission is my own work towards the Master of Science and that, to the best of my knowledge, it contains no material previously published by another person nor material which has been accepted for the award of any other degree of the University, except where due acknowledgement has been made in the text.

Victor Tekpor (PG3017209) …………………… 20 /05/ 2011

Student Name &ID Signature Date

Tel.: 0248742382

Certified by:

Mr. K.F. Darkwah …………………… ……………………

Supervisor(s) Name Signature Date

Certified by:

Dr. S.K. Amponsah …………………… ……………………

Head of Dept. Name Signature Date

Certified by:

Professor I.K. Dontwi …………………… ……………………

Dean, IDL Signature Date

DEDICATION

THIS PROJECT WORK IS DEDICATED

TO

THE ALMIGHTY GOD,

MY FAMILY

AND

ALL MY FRIENDS AND LOVED ONES

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It would be impossible to scratch the surface in the form of acknowledgement to the many individuals who have helped me throughout the years. However, some have contributed so much that their names literally jumped off the pages of my life and demand recognition.

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ABSTRACT

Continuous review model in which a fraction of demand is backordered and the remaining fraction is lost during the stock out period are considered under fuzzy environment. Fuzziness is introduced by allowing the cost components not clear and vague to certain extent. Trapezoidal fuzzy numbers are used to represent these characteristics. The optimum policy of this model under fuzzy costs are derived. The values are now determined numerically and the sensitivity in the decision variables are highlighted and described.

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CHAPTER ONE

INTRODUCTION

1.1 Background

The design of effective inventory control policies for models with stochastic demands and forecast updates that evolve dynamically over time is a fundamental problem in supply chain management.

In many industrial sectors, manufacturing and supply system usually takes the form of a complex network of suppliers, fabrication/assembly locations, distribution centres and customer locations through which materials, components, products and information flow. (Ettl et al. 2000).

The occurrence of shortage in an inventory system is a phenomenon in real situations. In some cases, while a few customers are ready to wait till the next arrival time of stock, others may be impatient and would persist on satisfying their demand from other sources. The demand during stock out period is normally regarded as complete backorders or lost forever.

A solution to such a model was first derived by Montgomery et al. (1973). Quyang and Wu (1996) also derived a similar model for variable lead time with fixed recorder point analyzed. Also Hariga and Daya (1999) described both periodic and continuous review models with a mixture of backorders and lost sales in case of full and partial demand information. Kumaran et al. (2006) analyzed Kim and Park model under a more general set up assuming the generalized lambda distribution to describe the stochastic lead time demand.

In most stochastic inventory models, the cost parameters are usually treated as crisp variables. However in practical situations, precise values of the cost characteristics are seldom achieved as they may be vague and imprecise (Vijayan and Kumaran, 2008).

The shortage in an inventory system may occur due to different causes such as increase in demand, hike in wages, delayed production, shipping problems. Shortage always results in loss of goodwill and it is very difficult to measure the exact amount of shortage cost. This same problem is experienced in ordering and holding cost.

Hence in inventory systems, the decision maker always allow some flexibility in the cost parameter values in order to tackle the unforeseen circumstances which always fit real situations. These characteristics are better described by the use of fuzzy sets which encompass a range of values than statistical treatment of cost characteristics which is inefficient for these models because of lack of random observations (Vijayan and Kumaran, 2008).

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1.2 Brief History of Mantrac Ghana Limited

Mantrac Ghana Limited, formerly known as Tractor and Equipment Ghana, is the sole authorized dealer for Caterpillar products in Ghana. The Mantrac Group also holds the caterpillar dealership in Kenya, Uganda, Nigeria, Tanzania, Sierra Leone, Egypt, Iraq and parts of Russia.

In addition, the sister company of Mantrac Group, Unatrac, caters for offshore customers through representatives offices in the United Kingdom and Dubai.

Since its establishment in Ghana in 1937, Mantrac Ghana Limited has supplied and supported caterpillar equipment used in many different sectors such as construction, mining, forestry, material handling and power generation.

Apart from Mantrac's long affiliation with caterpillar, the company maintains strategic link with other global suppliers including O&K mining equipment, Olympian generators, MaK marine engines, Perkins engines, Allight lighting towers, Kenworth trucks, Isuzu, Suzuki, General Motors passenger cars and commercial trucks. Mantrac also offers customers' integrated business solutions backed by technical expertise to help customers choose the products and systems for their applications. Finally, Mantrac Ghana deals in forty to fifty thousand land items.

1.3 Problem Statement

The main problem facing the company is the conditional expected marginal holding cost incurred by maintaining excess inventory due to over-ordering; and the conditional expected backlogging cost incurred by not satisfying demand on time due to under ordering.

1.3.0 Objectives

The objectives of the study are:

1. To model Mantrac Ghana Limited's inventory cost using Continuous Review (Q, r) inventory model with all cost components fuzzy.

2. To determine the optimum order quantity, reorder point and optimum cost of Mantrac Ghana Limited L/M Hose-in from inventory model.

1.4. Methodology

The problem under study is the conditional expected marginal holding and backlogging cost incurred as a result of maintaining excess inventory and when demands are not satisfied respectively on time.

The model for the problem is a continuous review (Q, r) inventory model with all cost components fuzzy. A continuous review and periodic review inventory model is considered in which a fraction of demand is backordered and the remaining fraction is lost during the stock out period are considered under fuzzy environment. Trapezoidal fuzzy numbers are used to represent these characteristics.

In carrying out this work, data was obtained from the Inventory Department of Mantrac Ghana Limited. The following data was obtained.

- Stock list,
- Cost per unit item,
- Data on demand,
- Data on supply,
- Inventory holding cost

Stochastic inventory models with a mixture of backorders and lost sales are described by introducing fuzziness into the cost parameters. Several cases of the models with exactly one of the cost components fuzzy and all others crisp as well as with all the cost parameters fuzzy are described. Fuzzifications of the cost parameters for continuous and periodic review inventory models are described.

The signed distance method is adopted to defuzzify the fuzzy cost function. Optimum policies with respect to (Q, r) and (R, T) models under fuzzy cost are derived. Results of numerical computations for optimum parameters of these models under both fuzzy and crisp cases and their comparisons are presented.

Derive 5 and Mat lab are applied to solve the various algorithms.

Materials from the internet, books on inventory from KNUST library, papers and journals on inventory were used in carrying out this project.

1.5 Justification

Economically, because of the cost some companies in Ghana incur as a result of keeping excess inventory in the warehouse, the operating cost of most of this companies keep on increasing as the year goes by. Also the 'promise and fail' attitude of some of these companies to deliver to their costumers on time. This normally happens because of inadequate stocks hence the need for proper investigation into inventory control.

Furthermore due to inadequate spare parts, some transport companies have their fleet of buses reduced thereby affecting the economic and social activities of the population who depended on the services of these companies. In addition, research is the bedrock of every developed and developing countries and as a result there is the need for students to acquire the skills in academic research to help in developing ones country.

Finally, the project is prerequisite to partially fulfill the requirements for the award of M.Sc. in Industrial Mathematics.

1.6 Organization of the thesis.

This thesis has been organized into five chapters. Chapter one introduces the background of the thesis, the problem statement and the objectives of the study. Also the methodology and the justification of the thesis were spelt out.

Chapter two presents the literature review of the thesis whilst chapter three gives the methodology which spells out the mathematical tools that are applied and chapter four deals with data description, analysis and modeling.

Finally, chapter five contains the conclusions and the recommendations.

CHAPTER TWO

LITERATURE REVIEW

Various models have been proposed for stock level dependant inventory systems. Barker and Urban (1988a) investigated a deterministic inventory system in which the demand rate depends on the inventory level described by a polynomial function.

Clark and scarf (1960) considered a multiechelon serial system under continuous review. Svoronos and Zipkin (1991) study continuous-review hierarchical inventory systems with exogenous stochastic replenishment lead times and a one-for-one replenishment policy. By preserving the order of replenishment, the authors were able to approximate the study-state system performance and to bring out the important role played by the lead time variance. Lee and Billington (1993) use a single-node periodic review model as a building block to analyze a decentralized supply chain with normally distributed demands and processing lead time.

More examples on supply chain models were provided by Tayur et. Al. (1999).

An extension of the standard periodic –review model is to impose a capacity limit at each stage on the maximum amount of outputs per time unit. Glasserman and Tayur (1994) demonstrate that in a serial system with an echelon base-stock policy, the inventory and backorders are stable if the mean demand per period is less than the capacity at every node. Glasserman (1997) developed bounds and approximations for setting the base-stock levels in the above system. Glasserman and Wang (1998) use a large deviations approach to obtain an asymptotic linear relationship between lead time and inventory as the fill rate approaches 100%.

Zipkin (2000) provides a systematic discussion of inventory models with stochastic lead times. Based on the system structure, the models are divided into three groups: exogenous sequential systems, parallel systems and limited-capacity systems. Exogenous sequential systems are essentially standard inventory systems with constant lead times replaced by stochastic lead times (Kaplan, 1970). In a parallel system, an infinite-server queue is used to model the supply process. With an unlimited capacity, the order lead times are independent and identically distributed random variables.

The aim of inventory management is to minimize total operating costs while satisfying consumer service requirements. In order to accomplish this objective, an optimal ordering policy will be determined by answering to questions such as when to order and how much to order. The operating costs taken into account are the procurement costs, the holding costs and the shortage costs which are incurred when the demand of a client cannot be satisfied (either lost sales costs or backorder costs).

There exist different inventory policies which are periodic-review policy and the continuous- review policy. The first policy implies that the stock level will be checked a fixed period of time and an ordering decision will be made in order to complete to an upper limit (order up to point), if necessary. In second policy, the stock level will be monitored continuously. A fixed quantity will be ordered when the stock level reaches a recorder point. The order quantity will only be delivered after a fixed lead time and shortage can exist if the inventory is exhausted before the receipt of the order quantity. Those basic policies can be adapted to take into account special situations such as stochastic demands and lost sales or backorder.

Research on Inventory Record Inaccuracy (IRI) has been taking place since 1960s with the report by (Rinehart, 1960). The author stated that this inaccuracy produces a ‘’deleterious effect’’ on operational performance. Following this, it was reported that this divergence between stock record and physical stock results in ‘’warehouse denials’’ (Iglehart and Morey, 1972). Their research took into consideration the frequency and the depth of inventory counts and stocking policy to minimize total inventory and inspection costs.

Moreover, focusing on the significance of measuring IRI, DeHoratius and Raman (2008) show that inventory counts may not impact record inaccuracy and additional buffer stock may not be equally necessary across all items in all stores. In fact, safety stock in the continuous –review lost-sales inventory models is one of the effective inventory management policies for mitigating long run total cost.

Ritchken and Sankar (1984) used a regression-based method to adjust the size of the stock by incorporating an additional safety stock requirement in order to estimate the risk in inventory problems. Persona et al. (2007) propose innovative cost-based analytical models for showing that one can reduce the occurrence of stock-outs by introducing a safety stock of pre-assembled modules or components. On considering the continuous-review lost-sales inventory models with a Poisson demand, Hill (2007) shows that a base-stock policy is ‘’economically’’ optimal and that computing the optimal base-stock and its corresponding cost is quite simple for a backorder model.

However for a lost-sales model, this policy is not optimal. Hence, the author proposes three alternative policies. Two of these involve modifying the optimal base-stock policy by imposing a delay between the placements of successive orders. The third policy is to place orders at pre-determined fixed and regular intervals. However these policies require a lot of complex calculations for lead-times under demand uncertainty.

In addition, quantitative measures were applied and it was found out that the quality of service – level declines in a continuous review (Q, R) inventory policy when there are inventory miscounts and variations in lead-time (Kumar and Arora, 1992). Even though most of the current research focusing on (Q, R) policy often proposes models of operational research, simulation modeling is becoming an effective and timely tool and is capturing the cause and effect relationship in this field (Kang and Gershwin, 2004).

Urban (1995) investigated an inventory system in which the demand rate during stock out periods differs from the in-stock period demand by a given amount. The demand rate depends on both the initial stock and the instantaneous stock. Urban formulates a profit-maximizing model and develops a closed-form solution. Datta and Pal (1990) analyzed an infinite time horizon deterministic inventory system without shortage, which has a level-dependent demand rate up to a certain stock level and a constant demand for the rest of the cycle.

Paul et al. (1996) investigated a deterministic inventory system in which shortages are allowed and are fully back-logged. The demand is stock dependent to a certain level and then constant for the remaining periods. Hwang and Hahn (2000) constructed an inventory model for an item with an inventory-level dependent demand rate and a fixed expiry date. All units that are not sold by their expiry date are regarded as useless and therefore discarded.

The holding cost is explicitly assumed to be varying over time in only few inventory models. Shao et al. (2000) determined the optimum quality target for a manufacturing process where several grades of customers' specifications may be sold. Since rejected goods could be sold later to another customer, variable holding costs are considered in the model. Beltran and Krass (2002) analyzed the dynamic lot sizing problem with positive or negative demands and allowed disposal of excess inventory.

Goh (1994) apparently provides the only existing inventory model in which the demand is stock dependent and the holding cost is time dependent. While Goh (1992) models holding cost variation over time as a continuous nonlinear function, the storage time is divided into a number of distinct periods with successively increasing holding costs. As the storage time extends to the next time period, the new holding cost can be applied either retroactively (to all storage periods) or incrementally (to the new period only).

Fuzzy logic has two different meanings. In a narrow sense, fuzzy logic is a logical system, which is an extension of multivalued logic. However, in a wider sense fuzzy logic (FL) is almost synonymous with the theory of fuzzy sets, a theory which relates to classes of objects with unsharp boundaries in which membership is a matter of degree. In this perspective, fuzzy logic in its narrow sense is a branch of FL. Even in its more narrow definition, fuzzy logic differs both in concept and substance from traditional multivalued logical systems.

In Fuzzy Logic Toolbox software, fuzzy logic should be interpreted as FL, that is, fuzzy logic in its wide sense. The basic ideas underlying FL are explained very clearly and insightfully in Foundations of Fuzzy Logic. What might be added is that the basic concept underlying FL is that of a linguistic variable, that is, a variable whose values are words rather than numbers. In effect, much of FL may be viewed as a methodology for computing with words rather than numbers. Although words are inherently less precise than numbers, their use is closer to human intuition. Furthermore, computing with words exploits the tolerance for imprecision and thereby lowers the cost of solution.

Another basic concept in FL, which plays a central role in most of its applications, is that of a fuzzy if-then rule or, simply, fuzzy rule. Although rule-based systems have a long history of use in Artificial Intelligence (AI), what is missing in such systems is a mechanism for dealing with fuzzy consequents and fuzzy antecedents. In fuzzy logic, this mechanism is provided by the calculus of fuzzy rules. The calculus of fuzzy rules serves as a basis for what might be called the Fuzzy Dependency and Command Language (FDCL). Although FDCL is not used explicitly in the toolbox, it is effectively one of its principal constituents. In most of the applications of fuzzy logic, a fuzzy logic solution is, in reality, a translation of a human solution into FDCL.

A trend that is growing in visibility relates to the use of fuzzy logic in combination with neurocomputing and genetic algorithms. More generally, fuzzy logic, neurocomputing, and genetic algorithms may be viewed as the principal constituents of what might be called soft computing. Unlike the traditional, hard computing, soft computing accommodates the imprecision of the real world. The guiding principle of soft computing is: Exploit the tolerance for imprecision, uncertainty, and partial truth to achieve tractability, robustness, and low solution cost. In the future, soft computing could play an increasingly important role in the conception and design of systems whose MIQ (Machine IQ) is much higher than that of systems designed by conventional methods.

Among various combinations of methodologies in soft computing, the one that has highest visibility at this juncture is that of fuzzy logic and neurocomputing, leading to neuro-fuzzy systems. Within fuzzy logic, such systems play a particularly important role in the induction of rules from observations. An effective method developed by Dr. Roger Jang for this purpose is called ANFIS (Adaptive Neuro-Fuzzy Inference System). This method is an important component of the toolbox.

You can use Fuzzy Logic Toolbox software with MATLAB technical computing software as a tool for solving problems with fuzzy logic. Fuzzy logic is a fascinating area of research because it does a good job of trading off between significance and precision—something that humans have been managing for a very long time.

In this sense, fuzzy logic is both old and new because, although the modern and methodical science of fuzzy logic is still young, the concepts of fuzzy logic relies on age-old skills of human reasoning.

Montgomery et al. (1973) propose a continuous review inventory system where a fraction of the unfilled demand is backordered and the remaining fraction is lost. Both the cases of deterministic and stochastic demands are considered, but the stochastic demand case is treated heuristically. Rosenberg (1979) reformulates the above model by introducing a "fictitious demand rate" that simplifies the analysis of the partial backorder policy and gives an economic interpretation of the circumstances under which this policy is optimal.

Kim and Park (1985) extend the Montgomery et al. (1973) stochastic demand model to one in which the cost of a backorder is assumed to be proportional to the length of time for which the backorder exists. Assuming at most one order outstanding at any point in time and an arbitrary continuous density function of lead time demand, they derive the equations from which the optimal order quantity and the reorder point can be iteratively computed. Assuming Poisson demand and an exponential lead time, Woo and Sphicas (1991) formulate a partial backorder model that allows a finite number of orders to be outstanding.

Rabinowitz et al. (1995) analyze a (Q, r) inventory model where a fixed maximum number of backorders b is allowed. During the stockout period, the first b units of incoming demand are backordered and the remainder is lost. Under the assumption of Poisson demand and no more than a single order outstanding, they derive the expected annual cost function and employ an exhaustive search procedure to find the optimal values of Q, r and b. Chu et a!. (2001) generalize the above model by dividing the lead time into two segments and use two backorder control limits, one for each time segment.

Posner et al. (1972) treat the case where backorder customers are willing to wait for a random period of time. The demand process is assumed to be Poisson, and the lead time and how long the customers are willing to wait are assumed to be exponentially distributed. Das (1977) uses an (S-1, S) policy and assumes that customers are willing to wait for a fixed amount of time before canceling their orders. Moinzadeh (1989) also considers an (S-1, S) inventory system with Poisson demand and a constant lead time. Smeitink (1990) proves that Moinzadeh's results hold for an arbitrary lead time and that the steady-state net inventory probabilities depend on the mean of the lead time and not on the shape of its distribution. Chang and Dye (1999) consider a partial backordering system for deteriorating items with the backlogging rate dependent on the length of the waiting time for the next replenishment.

Moon and Gallego (1994) introduce the distribution-free procedures in the analysis of stochastic inventory models. They solve both the continuous review and the periodic review model with a mixture of backorders and lost sales using the minimax distribution-free approach. The treatment of the periodic review model is heuristic.

Porteus (1990) reviews stochastic periodic review models including one where a fraction of the excess demand is backordered. A myopic approximation to this model is provided by Nahmias (1979). For recent findings regarding the computation of optimal solutions to general (s, S) inventory systems with a backorder policy (both periodic and continuous review systems), see Zheng and Federgruen (1991, 1992); for continuous review backorder systems, see Federgruen and Zheng (1992) and for a discussion of the sensitivity of the optimal solutions, see Zheng (1992).

During the lead time there is a cut off point. Before that, if shortage occurs, incoming demands will be filled by emergency orders, and after that all unfilled demands are backordered. Backorder costs are usually time-dependent, that is, they accumulate over time. DeCroix and Arreola-Risa (1998) and Cheung (1998) consider inventory systems that offer economic incentives (time-based price discount) to customers who are willing to wait longer than normal delivery times. Furthermore, Kim and Park (1985) and Park (1989) argue that the time duration of the backorder is a critical factor of the backorder costs and must be considered in an inventory system.

Given the importance of shortening the time duration of the backorder period, it is reasonable to let backorders occur close to the time when replenishment is due to arrive. Although inventory systems are typically customer driven, we do notice that there are many real systems controlled solely by the supplier. In such cases, emergency orders are often adopted instead of the lost sales policy (although they are mathematically the same) in order to maintain customer loyalty. Rabinowitz et al. (1995) consider a model for this type of inventory system. However, in their model, shortages are first backordered and the rest are filled by emergency orders. This may not be the most cost-effective because of the time-dependent cost of backorders. Furthermore, setting the time limit rather than the limit on backorders is operationally more convenient.

The assumption of no more than one outstanding order is commonly made in the exisiting inventory models with emergency orders or lost sales. The usage and plausibility of the assumption has been discussed in detail by Hadley and Whitin (1963), Kim and Park (1985), and Cheung (1998). In particular, Hadley and Whitin (1963) discussed the difficulty in developing exact solutions for the lost sales case when more than one outstanding order is allowed. Hadley and Whitin (1963, p. 198) argued that "If $r < Q$, then there can never be more than a single order outstanding. In the lost sales case then, it is possible to stipulate that there is only a single order outstanding if one requires that $r < Q$."

System Dynamics (SD) methodology aims to model real complex dynamic systems for understanding them and coming up with policies to change the problematic dynamic behavior. The real dynamic problems contain feedbacks, delays, and random noise or uncertainties which make them "complex" (Größler, 2004). Feedbacks and delays are the main reasons why human decision-making behavior results in unwanted behavior in these systems (Sterman 1989a). In most of the cases, the problems that are which SD is interested in have problematic dynamic behavior usually caused by not optimal decisions of humans. To achieve the aim of making valid models of dynamic systems, SD tries to capture human decision making behavior together with feedbacks and delays which are all endogenously included in the model.

In other words, SD models should be able represent 'intended rationality' of human beings (Größler 2004). The words intended rationality or bounded rationality is used to describe the decision making behavior of humans in these complex dynamic systems which are far from optimal. This behavior should not be interpreted as humans acting irrationally (Größler et al. 2004). However, the rationality of decision maker is bounded or limited because of the complexity of many real dynamic systems (Sterman, 2000).

Thus, the modeler should represent the two bounded rationality of the decision maker for the model to be a valid representation of reality.

In order to model human decision-making behavior in a certain system, one must first understand how people behave or decide in that system. Laboratory experiments are conducted where subjects play the role of the decision-maker in the model of the system to capture the behavior of humans. Then their decision behavior is modeled with the help of certain heuristics and rules. Various studies work on generic systems such as stock management problem and use laboratory experiments to come up with decision-making behavior formulation (Sterman 1989a., b., Dogan and Sterman 2000, Barlas and Özevin 2001). Many of these studies base their formulations on anchor and adjustment heuristic which is first proposed by Tversky and Kahneman (1982).

Fuzzy Logic is one of the best tools to model our imprecise and blurred world. The real world is too complicated for precise descriptions to be obtained; therefore approximations (or fuzziness) must be introduced in order to obtain a reasonable, yet traceable, model (Wang 1997). Fuzzy logic is the tool for transforming human knowledge and its decision-making ability into a mathematical formula. In other words, it provides us with meaningful and powerful representation of measurement uncertainties and also with meaningful representation of vague concepts expressed in natural language (Klir & Yuan 1995).

Morgan and Ammentorp (1994) uses the qualitative knowledge of experts on financial risk management to determine decision variables, and numeric ranges of these variables such as what value range is low, normal and high. Then their responses to those ranges were obtained to develop fuzzy logic model. Takahagi (1995) applies fuzzy logic modeling to inventory control model with taking sales as the only decision variable. He claims that the behavior of this fuzzy logic model is similar to human behavior without comparison to any real inventory and order behavior. Although human behavior can be modeled by just contemplating on the reasons, it may cause validation problems. A few other applications make use of fuzzy logic to model human behavior. Sousa-Poza et al. (2003) use survey data to build the fuzzy model of how humans determine job satisfaction. Esmaeli et al. (2006) model electric consumption of low, medium and high income group using fuzzy approach. Ghazanfari et al. (2003) proposes that fuzzy set theory can be applied to model any vague concept in a SD model.

This study proposes fuzzy logic to mimic decision making behavior of humans in Stock Management Problem. Three different types of human behavior are extracted from the data (Barlas and Özevin 2004) and hence, three types of fuzzy logic player are proposed.

The methodology for this project work was based on inventory models with a mixture of backorders and lost sales under fuzzy cost in the European Journal of Operational Research 189 (2008) 105 – 119 , (Vijayan and Kumaran, 2008).

CHAPTER THREE

METHODOLOGY

References are given to different models in the field of inventory models with a mixture of backorders and lost sales under fuzzy cost available in the literature.

Considering a general inventory model where the items are delivered against random demands, the replenishment orders are normally made after a fixed lead time. The items are normally

withdrawn from the inventory in response to the demand. If the entire inventory depletes to zero at a point in time, then a stock out state is said to occur. The stock out period is treated in three different forms;

- Backorders case - the demands that occur during the stock out period are fulfilled from the next delivery of orders.
- Lost sales case - the impatient customers will satisfy their demand from other sources.
- Mixture of backorders and lost sales - some customers who may be willing to wait till the next arrival of stock while a few customers may be impatient.

3.1 Continuous review (Q,r) inventory model

(Q,r) inventory model or the recorder point model is a standard continuous review inventory system in which the inventory position of an item is monitored after every sales and the policy is to order a lot of size Q units when the inventory level drops to the re-order point r. Such models are described in all standard text books on inventory. One such model under the following assumptions are considered below.

Assumptions

(i) The unit cost of the items is a constant independent of the order quantity.

(ii) Shortage cost is fixed for each unit of demand during the stock out period.

(iii) Backorder cost is independent of time.

(iv) Reorder point r is positive. The safety stock = (r- expected lead time demand), is positive.

(v) There is never more than a single order outstanding.

(vi) The stock out period during a cycle (time between the replenish of two consecutive orders) is very small.

More justifications and assumptions on backorders and lost sales models are available in Hadley and Whitin (1963).

Let

D	=	average annual demand
y	=	demand during lead time
θ	=	expected lead time demand
f(y)	=	the probability density function (pdf) of lead time demand
h	=	inventory holding cost per unit per year
C	=	fixed ordering cost per inventory cycle
s	=	fixed shortage cost per unit shortage
β	=	fraction of demand backordered during the stock-out period, $0 < \beta < 1$
Л	=	shortage cost of lost sales including the lost profit

The expected shortage at the end of the cycle is given by:

$$B(r) = \int_{r}^{\alpha} (y-r) f(y) dy,$$

It is referred to as loss function.

α = the expression for annual variable cost under the model (Montgomery et al., 1973) is given by:

$$Z(Q,r) = \frac{CD}{Q} + h\left(\frac{Q}{2} + r - \theta\right) + B(r)\left[h(1-\beta) + \frac{sD}{Q} + \frac{\pi(1-\beta)D}{Q}\right] \text{ ....(1)}$$

The function in Eq. (1) is convex; the optimum values of Q and r are derived by the usual calculus procedure of minimization. This gives

$$Q = \sqrt{\left(\frac{2D\left[C + sB(r) + \pi(1-\beta)B(R)\right]}{h}\right)} \text{ ....(2)}$$

and

$$\int_{r}^{\alpha} f(y) dy = \frac{Qh}{Qh(1-\beta) + sD + \pi D(1-\beta)} \text{ ....(3)}$$

At the extremes, β=1 and β=0 , the above model reduces to the usual backorders and lost sales cases respectively.

3.2 Periodic review (R,T) inventory model

Consider a periodic review policy in which a review of the inventory level and the ordering decision are made at fixed interval of time T and at each review time a sufficient quantity is ordered to bring the inventory position up to level R. Such a policy is often known as a periodic review (R,T) policy. In addition to the assumptions:

(i) The unit cost of the items is a constant independent of the order quantity.

(ii) Shortage cost is fixed for each unit of demand during the stock out period.

(iii) Backorder cost is independent of time.

The following assumptions are also used.

(i) The cost J of making a review is independent of the variables R and T.

(ii) The backorders are incurred in very small quantities.

(iii) The lead time τ is known and constant.

The annual ordering and review cost is given by W=C+J. Let $\int f(y,T)dy$ denote the probability that y units are demanded in time T. Expected number of demands short per review period is given by:

$$B(R,T) = \int_{R}^{\alpha} (y-R)\, f(y,T)dy \; .$$

Using the same notations:

D = average annual demand

y = demand during lead time

θ = expected lead time demand

f(y) = the probability density function (pdf) of lead time demand

h = inventory holding cost per unit per year

C = fixed ordering cost per inventory cycle

s = fixed shortage cost per unit shortage

β = fraction of demand backordered during the stock-out period, $0 < \beta < 1$

Л = shortage cost of lost sales including the lost profit, to describe the characteristics, the average annual variable cost under the model is given by:

$$Z(R,T) = \frac{W}{T} + h\left(R - D\tau - \frac{DT}{2}\right) + B(R,T)\left[h(1-\beta) + \frac{s+\pi(1-\beta)}{T}\right] \text{…………………(4)}$$

Using calculus, Montgomery et al.(1973) gave the optimal value of R for a given T, as the solution of the equation:

$$\int_R^{\alpha} f(y,T)dy = \frac{hT}{hT(1-\beta)+s+\pi(1-\beta)} \text{……………………………………………...(5)}$$

Different values of T produce different optimal values of R. We choose the pair (R, T) which minimizes the cost function in Eq .(4)

3.3 Preliminary concepts of fuzzy set theory

Some preliminary concepts of fuzzy set theory required in the development of our models are described below.

3.3.1 Fuzzy preliminaries

The fuzzy set theory was introduced by (Prof. Lotfi A. Zadeh, 1965) and fuzzy logic in 1973 to deal with problems in which fuzzy phenomena exist. In a universe of discourse X, a fuzzy subset Ǎ of X is defined by the membership function μ Ǎ(x) which maps each element x in X to a real number in the interval [0,1].The function value of μ Ǎ(x) denotes the grade of membership. A fuzzy set is normal if the largest grade obtained by any element in that set is 1. A fuzzy set Ǎ on X is convex if and only if

$u_{\tilde{A}}(\lambda x_1 + (1-\lambda)x_2) \geq \min.(u_{\tilde{A}}(x_1), u_{\tilde{A}}(x_2))$ for all $x_1, x_2 \in X$ and $\lambda \in [0,1]$, where min denotes the minimum operator.

A fuzzy number is a fuzzy subset of the real line which is both normal and convex. For a fuzzy number $\tilde{A}$, its membership function can be denoted by $u_{\tilde{A}}(x) = \begin{cases} l(x), x < m, \\ 1, m \leq x \leq n, \\ u(x), x > n \end{cases}$

where l(x) is upper semi continuous, strictly increasing for x < m and there exist $m_1 < m$ such that l(x) = 0 for $x \leq m_1, u(x)$ is continuous, strictly decreasing function for x > n and there exist $n_1 \geq n$ such that u(x) = 0 for $x \geq n_1, l(x)$ and $u(x)$ are called the left and right reference functions respectively.

The diagram below shows a block diagram of a Fuzzy system.

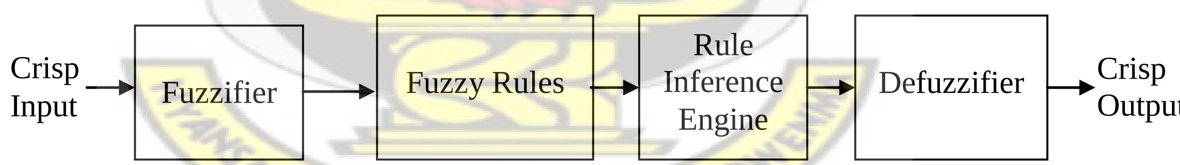

Figure 3.1: A Block Diagram of a Fuzzy System

3.3.2 Trapezoidal fuzzy number

The fuzzy number $\tilde{A}$ is said to be a trapezoidal fuzzy number if it is fully determined by (a_1, a_2, a_3, a_4) of crisp numbers such that $a_1 < a_2 < a_3 < a_4$, whose membership function representing a trapezoid can be denoted by:

$$u_{\tilde{A}}(x) = \begin{cases} \frac{x-a_1}{a_2-a_1}, a_1 \le x \le a_2, \\ 1, a_2 \le x \le a_3, \\ \frac{x-a_4}{a_3-a_4}, a_3 \le x \le a_4 \\ 0, otherwise \end{cases}$$

where a_1, a_2, a_3 and a_4 are lower limits, lower mode, upper mode and upper limit respectively of the fuzzy number $\tilde{A}$.

The interval $[a_1, a_4]$ is called the support of the fuzzy number and it gives the range of all possible values of $\tilde{A}$ that are at least marginally possible or plausible. The interval $[a_2, a_3]$ corresponds to the core of the fuzzy number and gives the range of most plausible values. The intervals $[a_1, a_2]$ and $[a_3, a_4]$ are called penumbra of the fuzzy number $\tilde{A}$ (Zimmerman,1991).

Let $\tilde{A}_1 = (a_{11}, a_{12}, a_{13}, a_{14})$, $\tilde{A}_2 = (a_{21}, a_{22}, a_{23}, a_{24})$, be two trapezoidal fuzzy numbers, then

$\tilde{A}_1 + \tilde{A}_2 = (a_{11} + a_{21}, a_{12} + a_{22}, a_{13} + a_{23}, a_{14} + a_{24})$ and for all $b \ge 0$, b$\tilde{A}_1$ = $(ba_{11}, ba_{12}, ba_{13}, ba_{14})$.

The set $A(\alpha) = \{x : u_{\tilde{A}}(x) \ge \alpha\}$, $\alpha \in [0,1]$ is called the α cut of $\tilde{A}$. $\tilde{A}(\alpha)$ is a non empty bounded closed interval contained in the set of real numbers and it can be denoted by $\tilde{A}(\alpha) = [\tilde{A}_l(\alpha). \tilde{A}_u(\alpha)]$. $\tilde{A}_l(\alpha)$ and $\tilde{A}_u(\alpha)$ are respectively the left and right limits of $\tilde{A}(\alpha)$ and are usually known as the left and right α cuts of $\tilde{A}$.

For a trapezoidal fuzzy number $\tilde{A} = (a_1, a_2, a_3, a_4)$, $\tilde{A}_l(\alpha) = a_1 + (a_2 - a_1)\alpha$ and $\tilde{A}_u(\alpha) = a_4 - (a_4 - a_3)\alpha$. The fuzzy set [a, b, α] on $(-\alpha, \alpha)$ is called a level α fuzzy interval, if its membership function is $u_{[a,b:\alpha](x)} = \begin{cases} \alpha, 0 \le \alpha \le 1, a \le x \le b \\ 0, otherwise \end{cases}$

3.4 Signed distance method

In order to defuzzify the fuzzy cost function, we need to consider some distance measures as in Yao and Wu (2000). The signed distance between the real numbers a and 0, denoted by d_o (a, 0) is given by d_o (a, o) =a. Hence the signed distance of $\tilde{A}_l(\alpha)$ and $\tilde{A}_u(\alpha)$ measured from 0 are

$d_o(\tilde{A}_l(\alpha), 0) = \tilde{A}_l(\alpha)$ and $d_o(\tilde{A}_u(\alpha), 0) = \tilde{A}_u(\alpha)$ respectively.

The signed distance of the interval $(\tilde{A}_l(\alpha), \tilde{A}_u(\alpha))$ measured from the origin 0 is given by:

$d_o((\tilde{A}_l(\alpha), \tilde{A}_u(\alpha)), 0) = \frac{1}{2}[d_o(\tilde{A}_l(\alpha), 0) + d_o(\tilde{A}_u(\alpha), 0)] = \frac{1}{2}(\tilde{A}_l(\alpha) + A_u(\alpha))$, where $\tilde{A}_l(\alpha)$ and $\tilde{A}_u(\alpha)$ exist and are integrable for $\alpha \in [0,1]$.

For each $\alpha \in [0,1]$, the crisp interval $[\tilde{A}_l(\alpha), \tilde{A}_u(\alpha)]$ and the level α fuzzy interval $[[\tilde{A}_l(\alpha), \tilde{A}_u(\alpha)]:\alpha]$ are in one to one correspondence. The signed distance from

$[[\tilde{A}_l(\alpha), \tilde{A}_u(\alpha)]:\alpha]$ to Ồ (where Ồ is the 1 level fuzzy point which maps to the origin) is d([[$\tilde{A}_l(\alpha), \tilde{A}_u(\alpha)$]:α], Ồ) = d_o (($\tilde{A}_l(\alpha), \tilde{A}_u(\alpha)$),0) = $\frac{1}{2}(\tilde{A}_l(\alpha) + A_u(\alpha))$, . The signed distance of $\tilde{A}$ measured from Ồ defined as

$$d(\tilde{A}, \text{Ồ}) = \frac{1}{2}\int_0^1 (\tilde{A}_l(\alpha) + A_u(\alpha))d\alpha, \quad \text{……………………………………………………….(6)}$$

where $\tilde{A}_l(\alpha)$ and $\tilde{A}_u(\alpha)$ are respectively left and right cuts of the fuzzy number $\tilde{A}$.

Let $\tilde{A}_i, i = 1, 2, 3, \ldots\ldots\ldots\ldots, N$ be N fuzzy numbers and bi, i = 1, 2, 3… N are real crisp constants. Then

$$d\left(\sum_{i=1}^{N} b_i \tilde{A}_i, \tilde{O}\right) = \sum_{i=1}^{N} b_i d\left(\tilde{A}_i, \tilde{O}\right) \quad \text{……………………………………………………….(7)}$$

The signed distance formula in Eq. (6) is considered when ranking fuzzy numbers.

(Yao and Wu, 2000). Fuzzy numbers $\tilde{A}_1$ and $\tilde{A}_2$ are ranked as $\tilde{A}_1 < \tilde{A}_2$ if:

$$d\left(\tilde{A}_1, \tilde{O}\right) < d\left(\tilde{A}_2, \tilde{O}\right) \quad \text{………………………………………………………………(8).}$$

3.5. Continuous review (Q, r) model under fuzzy cost parameters.

Four different cases of the continuous review inventory model are considered under fuzzy cost parameters.

3.5.1 (Q, r) model with all cost components fuzzy.

Assuming that the cost components c, h, s and π are all fuzzy, we represent them by trapezoidal fuzzy numbers given below.

$\tilde{C} = \left(c-\delta_1, c-\delta_2, \tilde{c}+\delta_3, c+\delta_4\right),$

$\tilde{h} = \left(h-\delta_5, h-\delta_6, h+\delta_7, h+\delta_8\right),$

$\tilde{S} = \left(s-\Delta_1, s-\Delta_2, s+\Delta_3, s+\Delta_4\right)$ and

$\tilde{\pi} = \left(\pi-\Delta_5, \pi-\Delta_6, \pi+\Delta_7, \pi+\Delta_8\right)$, where δ_i and Δ_i, i = 1, 2, 3, ………., 8 are arbitrary positive numbers under the following restrictions.

$C > \delta_1 > \delta_2, \delta_3 < \delta_{4,} h > \delta_5 > \delta_6$ and $\delta_7 < \delta_8$.

$S > \Delta_1 > \Delta_2, \Delta_3 < \Delta_4, \pi > \Delta_5 > \Delta_6$ and $\Delta_7 < \Delta_8$.

The left and right limits of α cuts of $\tilde{C}, \tilde{h}, \tilde{S}$ and $\tilde{\pi}$ are given below.

$\tilde{C}_L(\alpha) = c - \delta_1 + (\delta_1 - \delta_2)\alpha,\ \tilde{C}_u(\alpha) = c + \delta_4 - (\delta_4 - \delta_3)\alpha,$

$\tilde{h}_L(\alpha) = h - \delta_5 + (\delta_5 - \delta_6)\alpha,\ \tilde{h}_u(\alpha) = h + \delta_8 + (\delta_8 - \delta_7)\alpha,$

$\tilde{S}_L(\alpha) = s - \Delta_1 + (\Delta_{1_1} - \Delta_2)\alpha,\ \tilde{S}_u(\alpha) = s - \Delta_4 + (\Delta_4 - \Delta_3)\alpha$

$\tilde{\pi}_L(\alpha) = \pi - \Delta_5 + (\Delta_5 - \Delta_6)\alpha,\ \ \tilde{\pi}_u(\alpha) = \pi + \Delta_8 + (\Delta_8 - \Delta_7)\alpha$

The annual variable cost of the model under all cost components being fuzzy is given by:

$$\tilde{Z}(Q,r) = \frac{\tilde{C}D}{Q} + \tilde{h}\left(\frac{Q}{2} + r - \theta\right) + B(r)\left[h(1-\beta) + \frac{\tilde{S}D}{Q} + \frac{\tilde{\pi}(1-\beta)D}{Q}\right] \qquad \text{………………….(9)}$$

The left and right α cuts of the fuzzified cost function are respectively given by the following:

$$\tilde{Z}_L(Q,r)=\frac{\tilde{C}_L(\alpha)D}{Q}+\tilde{h}_L(\alpha)(\frac{Q}{2}+r-\theta)+B(r)[\tilde{h_L}(\alpha)(1-\beta)+\frac{\tilde{S}_L(\alpha)D}{Q}+\frac{\tilde{\pi}_L(\alpha)(1-\beta)D}{Q}]$$
………………………..(10)

and

$$\tilde{Z}_u(Q,r)=\frac{\tilde{C}_u(\alpha)D}{Q}+\tilde{h}_u(\alpha)(\frac{Q}{2}+r-\theta)+B(r)[\tilde{h_u}(\alpha)(1-\beta)+\frac{\tilde{S}_u(\alpha)D}{Q}+\frac{\tilde{\pi}_u(\alpha)(1-\beta)D}{Q}]$$
………………………..(11)

Using equations (6), (7), (10) and (11), defuzzified value of $\tilde{Z}(Q,r)$ is given by:

$$d(\tilde{Z}(Q,r),\tilde{O})=\frac{k_1 D}{Q}+k_2(\frac{Q}{2}+r-\theta)+B(r)[k_2(1-\beta)+\frac{k_3 D}{Q}+\frac{k_4(1-\beta)D}{Q}]$$ ………..(12)

where

$$k_1=\frac{\left(4c-\delta_1-\delta_2+\delta_3+\delta_4\right)}{4},$$

$$k_2=\frac{\left(4h-\delta_5-\delta_6+\delta_7+\delta_8\right)}{4},$$

$$k_3=\frac{\left(4s-\Delta_1-\Delta_2+\Delta_3+\Delta_4\right)}{4}$$ and

$$k_4=\frac{\left(4\pi-\Delta_5-\Delta_6+\Delta_7+\Delta_8\right)}{4}.$$

The defuzzified valued $d(\tilde{Z}(Q,r),\tilde{O})$ is taken as the estimate of fuzzy cost function in Eq. (9) denoted by $E(\tilde{Z}_{Q,r})$. The estimate in Eq. (12) is a convex function of Q and r (similar to Eq.(1) and a unique minimum of $E(\tilde{Z}_{Q,r})$ is obtained by equating to zero the first order partial derivatives of $E(\tilde{Z}_{Q,r})$ with respect to Q and r. That is

$$\frac{\partial E(\tilde{Z}_{Q,r})}{\partial Q} = 0 \quad \ldots\ldots(13)$$

and

$$\frac{\partial E(\tilde{Z}_{Q,r})}{\partial r} = 0 \quad \ldots\ldots(14)$$

Solving Equations (13) and (14), we get

$$Q = \sqrt{\left(\frac{2D[k_1 + k_3 B(r) + k_4(1-\beta)B(r)]}{k_2}\right)} \quad \ldots\ldots(15)$$

and

$$\int_r^{\alpha} f(y)dy = \frac{Qk_2}{Qk_2(1-\beta) + k_3 D + k_4 D(1-\beta)} \quad \ldots\ldots(16)$$

Since Equations (15) and (16) are not explicit to get the optimum values of Q and r, the iterative procedure suggested by (Hadley and Whitin ,1963) is used to solve the equations. Initially put

B (r) = 0 in Eq.(15) which gives $Q_o = \sqrt{\left(\frac{2Dk_1}{k_2}\right)}$ ……(17)

Replacing Q by Q_o, Equation (16) gives a value of r, say r_o. Using these initial values, the iteration continued to arrive at the optimum solution.

3.5.2 (Q,r) model under fuzzy ordering cost

Representing the ordering cost C in section 3.1 as the trapezoidal fuzzy number $\tilde{C}$ in section 3.5.1 and keeping all other components crisp constants, the cost function in Eq. (1) becomes

$$\tilde{Z}(Q,r) = \frac{\tilde{C}D}{Q} + h(\frac{Q}{2} + r - \theta) + B(r)[h(1-\beta) + \frac{S\,D}{Q} + \frac{\pi(1-\beta)D}{Q}] \ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(18)}$$

The left and right α cuts of the function in Eq. (18) are respectively given by:

$$\tilde{Z}_L(Q,r) = \frac{\tilde{C}_L\,D}{Q} + h(\frac{Q}{2} + r - \theta) + B(r)[h(1-\beta) + \frac{S\,D}{Q} + \frac{\pi(1-\beta)D}{Q}] \ldots\ldots\ldots\ldots\ldots\ldots \text{(19)}$$

and

$$\tilde{Z}_u(Q,r) = \frac{\tilde{C}_u\,D}{Q} + h(\frac{Q}{2} + r - \theta) + B(r)[h(1-\beta) + \frac{S\,D}{Q} + \frac{\pi(1-\beta)D}{Q}] \ldots\ldots\ldots\ldots\ldots\ldots \text{(20)}$$

Using Equations (6), (7), (19) and (20), the estimate of cost function under fuzzy ordering cost is given as:

$$d(\tilde{Z}(Q,r),\tilde{O}) = \frac{k_1 D}{Q} + h(\frac{Q}{2} + r - \theta) + B(r)[h(1-\beta) + \frac{sD}{Q} + \frac{\pi(1-\beta)D}{Q}] \ldots\ldots\ldots\ldots \text{(21)}$$

The optimum values of Q and r are obtained from the following expressions:

$$Q = \sqrt{\left(\frac{2D[k_1 + sB(r) + \pi(1-\beta)B(r)]}{h}\right)} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(22)}$$

and

$$\int_r^{\alpha} f(y)dy = \frac{Qh}{Qh(1-\beta) + sD + \pi D(1-\beta)} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(23)}$$

3.5.3 (Q, r) model under fuzzy holding cost

We consider the model in Equation (1) with holding cost, h alone assumed as fuzzy. With the trapezoidal fuzzy number $\tilde{h}$ denoting the fuzzy holding cost, the signed distance value (defuzzified value) of total cost is given by:

$$d(\tilde{Z}(Q,r),\tilde{O}) = \frac{CD}{Q} + k_2(\frac{Q}{2} + r - \theta) + B(r)[k_2(1-\beta) + \frac{sD}{Q} + \frac{\pi(1-\beta)D}{Q}] \text{.............(24)}$$

The optimum values of Q and r are obtained from the relations:

$$Q = \sqrt{\left(\frac{2D[c + sB(r) + \pi(1-\beta)B(r)]}{k_2}\right)} \text{...(25)}$$

and

$$\int_r^\alpha f(y)dy = \frac{Qk_2}{Qk_2(1-\beta) + sD + \pi D(1-\beta)} \text{.. (26)}$$

3.5.4 (Q, r) Model under fuzzy shortage costs

The model in Equation (1) with the shortage costs as fuzzy and all other cost components crisp is considered. The model in Equation (1) involves two shortages costs, s and π corresponding to the demands backordered and lost demands respectively. Both s and π are allowed to be fuzzy. The cost function in Equation (1) under fuzzy shortage costs $\tilde{s}$ and $\tilde{\pi}$ is given by:

$$\tilde{Z}(Q,r) = \frac{CD}{Q} + h(\frac{Q}{2} + r - \theta) + B(r)[h(1-\beta) + \frac{\tilde{S}D}{Q} + \frac{\tilde{\pi}(1-\beta)D}{Q}] \text{.....................(27)}$$

The left and right α cuts for the above cost function are respectively

$$\tilde{Z}_L(Q,r) = \frac{CD}{Q} + h(\frac{Q}{2} + r - \theta) + B(r)[h(1-\beta) + \frac{\tilde{S}_L(\alpha)D}{Q} + \frac{\tilde{\pi}_L(\alpha)(1-\beta)D}{Q}] \text{.........(28)}$$

and

$$\tilde{Z}_u(Q,r) = \frac{C\,D}{Q} + h(\frac{Q}{2} + r - \theta) + B(r)[h(1-\beta) + \frac{\tilde{S}_u(\alpha)D}{Q} + \frac{\tilde{\pi}_u(\alpha)(1-\beta)D}{Q}] \text{.........(29)}$$

The defuzzified value of cost function in Equation (27) using signed distance formula (6) is given by:

$$d(\tilde{Z}(Q,r),\tilde{O}) = \frac{CD}{Q} + h(\frac{Q}{2} + r - \theta) + B(r)[h(1-\beta) + \frac{k_3 D}{Q} + \frac{k_4(1-\beta)D}{Q}] \text{..............(30)}$$

3.6 Periodic review (R, T) model under fuzzy cost parameters

Four different cases of the periodic review inventory model is considered.

3.6.1 (R, T) model under all cost components fuzzy

In addition to the fuzzy cost parameters $\tilde{c}, \tilde{h}, \tilde{s}$ and $\tilde{\pi}$, the review cost J is also fuzzified by the trapezoidal fuzzy number as $\tilde{J} = (J-\zeta_1, J-\zeta_2, J+\zeta_3, J+\zeta_4)$, where ζ_i, i = 1, 2, 3, 4 are arbitrary positive numbers which satisfy $J > \zeta_1 > \zeta_2$ and $\zeta_3 < \zeta_4$.

For the trapezoidal fuzzy number:

$\tilde{W} = \tilde{C} + \tilde{J} = \left(W-\delta_1-\zeta_1, W-\delta_2-\zeta_2, W+\delta_3+\zeta_3, W+\delta_4+\zeta_4\right)$, the left and right α cuts are respectively $\tilde{W}_L(\alpha) = W-\delta_1-\zeta_1+(\delta_1-\delta_2+\zeta_1-\zeta_2)\alpha$ and

$\tilde{W}_u(\alpha) = W+\delta_4+\zeta_4-(\delta_4-\delta_3+\zeta_4-\zeta_3)\alpha$.

The cost function in Equation (4) under the above set of fuzzy cost parameters becomes:

$$\tilde{Z}(R,T) = \frac{\tilde{W}}{T} + \tilde{h}\left(R - D\tau - \frac{DT}{2}\right) + B(R,T)\left[\tilde{h}(1-\beta) + \frac{\tilde{s} + \tilde{\pi}(1-\beta)}{T}\right] \text{..................(33)}$$

The left and right α cuts of the fuzzified cost function in Equation (33) are respectively given by:

$$\tilde{Z}_L(R,T) = \frac{\tilde{W}_L(\alpha)}{T} + \tilde{h}_L(\alpha)\left(R - D\tau - \frac{DT}{2}\right) + B(R,T)\left[\tilde{h}_L(\alpha)(1-\beta) + \frac{\tilde{s}_L(\alpha) + \tilde{\pi}_L(\alpha)(1-\beta)}{T}\right]$$

…………………(34)

and

$$\tilde{Z}_u(R,T) = \frac{\tilde{W}_u(\alpha)}{T} + \tilde{h}_u(\alpha)\left(R - D\tau - \frac{DT}{2}\right) + B(R,T)\left[\tilde{h}_u(\alpha)(1-\beta) + \frac{\tilde{s}_u(\alpha) + \tilde{\pi}_u(\alpha)(1-\beta)}{T}\right]$$

………………….(35)

Using equations (6), (7), (34) and (35), the estimate value of $\tilde{Z}_L(R,T)$ denoted by $E(\tilde{Z}_{R,T})$ is given by:

$$E(\tilde{Z}_{R,T}) = d(\tilde{Z}(R,T),\tilde{O}) = \frac{k_5}{T} + k_2\left(R - D\tau - \frac{DT}{2}\right) + B(R,T)\left[k_2(1-\beta) + \frac{k_3 + k_4(1-\beta)}{T}\right]$$

…………………(36), where

$$k_5 = \frac{\left(4W - \delta_1 - \delta_2 - \zeta_1 - \zeta_2 + \delta_3 + \delta_4 + \zeta_3 + \zeta_4\right)}{4}$$

For a given T, the value of R which minimizes Eq. (36) satisfies $\frac{\partial d\left(\tilde{Z}(R,T),\tilde{O}\right)}{\partial R} = 0$, which reduces to

$$\int_r^\alpha f(y)dy = \frac{k_2 T}{k_2 T(1-\beta) + k_3 + k_4(1-\beta)} \quad \text{…………………………………………….……(37)}$$

Equation(37) gives the optimum value of R for a specified value of T. The optimal value of T is determined using the trial and error procedure as advocated in (Hadley and

Whitin, 1963). Three particular cases of the periodic review model in Equation (33) are considered below, where one of the cost components is assumed as fuzzy.

3.6.2 (R, T) model under fuzzy ordering and review costs.

Under fuzzy ordering and review costs, the cost function in Equation (4) becomes

$$\tilde{Z}(R,T) = \frac{\tilde{W}}{T} + h\left(R - D\tau - \frac{DT}{2}\right) + B(R,T)\left[h(1-\beta) + \frac{s+\pi(1-\beta)}{T}\right] \ldots\ldots\ldots\ldots\ldots\ldots(38)$$

The signed distance $d\left(\tilde{Z}(R,T),\tilde{O}\right)$ of fuzzified function in Equation (38) is given by

$$d\left(\tilde{Z}(R,T),\tilde{O}\right) = \frac{k_5}{T} + h\left(R - D\tau - \frac{DT}{2}\right) + B(R,T)\left[h(1-\beta) + \frac{s+\pi(1-\beta)}{T}\right] \ldots\ldots\ldots(39)$$

and the optimal value of R for a given value of T is obtained from the expression

$$\int_R^{\alpha} f(y,T)dy = \frac{hT}{hT(1-\beta) + s + \pi(1-\beta)} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (40)$$

3.6.3 (R, T) model under fuzzy holding cost

Keeping the cost components C, J, s and π constant, under the fuzzy holding cost, the signed distance value and optimal value of R are respectively given by the following equations

$$d(\tilde{Z}(R,T),\tilde{O}) = \frac{W}{T} + k_2\left(R - D\tau - \frac{DT}{2}\right) + B(R,T)\left[k_2(1-\beta) + \frac{s+\pi(1-\beta)}{T}\right] \ldots\ldots\ldots(41)$$

and

$$\int_R^{\alpha} f(y,T)dy = \frac{k_2T}{k_2T(1-\beta) + s + \pi(1-\beta)} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(42)$$

3.6.4 (R, T) model under fuzzy shortage costs

The cost function in Equation (4) under fuzzy shortage costs $\tilde{s}$ and $\tilde{\pi}$ is given by

$$\tilde{Z}(R,T) = \frac{W}{T} + h\left(R - D\tau - \frac{DT}{2}\right) + B(R,T)\left[h(1-\beta) + \frac{\tilde{s} + \tilde{\pi}(1-\beta)}{T}\right] \ldots\ldots\ldots\ldots\ldots\ldots(43)$$

The defuzzified value of fuzzified cost function in Equation (43) is given by

$$d(\tilde{Z}(R,T),\tilde{O}) = \frac{W}{T} + h\left(R - D\tau - \frac{DT}{2}\right) + B(R,T)\left[h(1-\beta) + \frac{k_3 + k_4(1-\beta)}{T}\right] \ldots\ldots..(44)$$

Optimum value of R is obtained from the expression

$$\int_R^{\alpha} f(y,T)dy = \frac{hT}{hT(1-\beta) + k_3 + k_4(1-\beta)} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots..(45)$$

The final equations giving optimal values of the decision variables Q and R in all the four models described above are of the same type. Hence Eqs (22) and (23), (25) and (26), and (31) and (32) are solved using the iterative procedure as described to solve Eqs (15) and (16). However, neither a theoretical comparison of the solutions or an assessment of the impact of the level of fuzziness in the decision variables is possible.

3.7 Fuzzy Logic Approach to Mimic Decision-Making Behavior of Subjects

The patterns of ordering behavior of subjects are divided into three basic classes by (Barlas and Özevin, 2004);

(i) smooth, continuous-oscillatory or non-oscillatory-damping orders,

(ii) alternating large-and-zero discrete orders, like a high frequency signal,

(iii) long periods of constant orders punctuated by a few sudden large ones.

Anchor and adjustment rule is only a valid representation of (i) type of players.
The rest cannot be mimicked by this rule. Then several other decision rules are evaluated to see if they can generate an ordering pattern that belongs to the rest of the classes of interest.
The study proposes fuzzy logic as an alternative modeling tool for decision making behavior of humans to commonly used heuristics and rules. We believe that fuzzy logic provides a valid representation of human behavior in Stock Management Game. Unlike other proposed heuristics, it provides not only the desired pattern, but also provides us with the reasons why humans act the way they do which is in full consensus with System Dynamics modeling methodology.

In order to be able to use fuzzy logic in modeling the decision making behavior of subjects, their reasoning should be well understood. For this aim, the game has been played several times, to make the "If...then" rules clear. Clear understanding of "If...then" rules helped to understand what kind of inputs subjects ignore and how they interpret the information that they take into account. In Barlas and Özevin (2004), some rules are only tested with continuous exponential delay and other rules only with discrete delay. In this paper, after different types of ordering behavior is modeled, the models are tested with both delay types. The reason for testing fuzzy logic models with both delay types is to observe how the logic works on the delay that it is not designed for. Singleton Sugeno type of fuzzy logic has been used to model each class of ordering behavior. Fuzzy Logic modeling is done by utilizing Matlab Fuzzy Toolbox (1995). The orders of the fuzzy logic player are placed in Stella manually and also the inputs to the fuzzy logic players are made manually from Stella.

Furthermore, the outputs are in real numbers whereas the orders for the Stock Management Game in Stella could only be adjusted in increments of five. Thus the orders of the Fuzzy Logic player are rounded up to the nearest number, multiple of five. Throughout this section, we use data from the short, step-up-and-down customer demand game with either exponential or discrete delay.

3.7.1 Modeling (i) Type Subjects

The ordering behavior of Type (i) subjects is explained as smooth, continuous oscillatory or non-oscillatory—damping orders. In this type, the subjects do usually not take the supply line levels into consideration. They try to order as much as the demand so as to keep inventory at the initial level. Even though, subjects order as much as the demand all the time, usually backordering occurs because of exponential delay and step up in customer demand. When the inventory is below zero, subjects order greater than demand. In this situation, they can not avoid making a peak or an overflow in the inventory level because of both step-down in customer demand and receiving delay. Here is the fuzzy logic model of the subjects with (i) type ordering behavior:

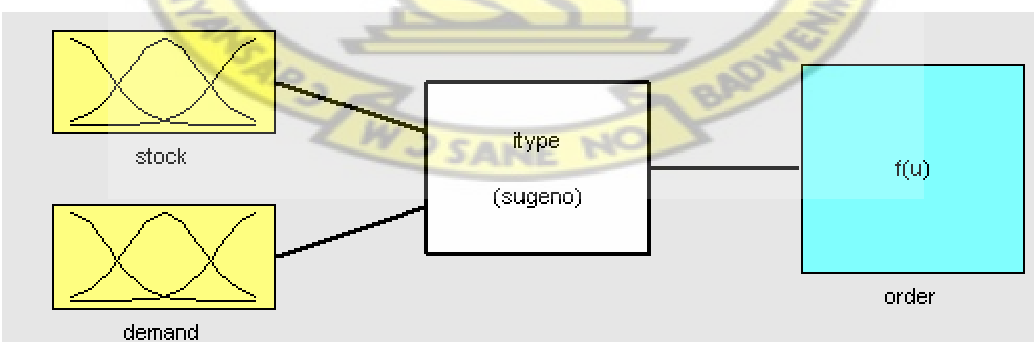

Figure 3.2: Fuzzy Logic Model for (i) type subjects

The inputs are stock level and demand. Several fuzzy logic models were designed to mimic (i) type decision making behavior and the ones that mimic (i) type players best are the ones that don't take supply line level as a factor in decision making. The ranges of the decision factors are obtained from the experimental data on subjects with (i) type of ordering behavior.

Table 3.1: Inference rule table of the fuzzy logic model for (i) type subjects

1. If (stock is NB) and (demand is high) then (order is 70) (1)
2. If (stock is NB) and (demand is med) then (order is 50) (1)
3. If (stock is NB) and (demand is low) then (order is 40) (1)
4. If (stock is NS) and (demand is high) then (order is 55) (1)
5. If (stock is NS) and (demand is med) then (order is 35) (1)
6. If (stock is NS) and (demand is low) then (order is 20) (1)
7. If (stock is Z) and (demand is high) then (order is 50) (1)
8. If (stock is Z) and (demand is med) then (order is 30) (1)
9. If (stock is Z) and (demand is low) then (order is 15) (1)
10. If (stock is PS) and (demand is high) then (order is 35) (1)
11. If (stock is PS) and (demand is med) then (order is 20) (1)
12. If (stock is PS) and (demand is low) then (order is 10) (1)
13. If (stock is PB) and (demand is high) then (order is 25) (1)
14. If (stock is PB) and (demand is med) then (order is 15) (1)
15. If (stock is PB) and (demand is low) then (order is 5) (1)

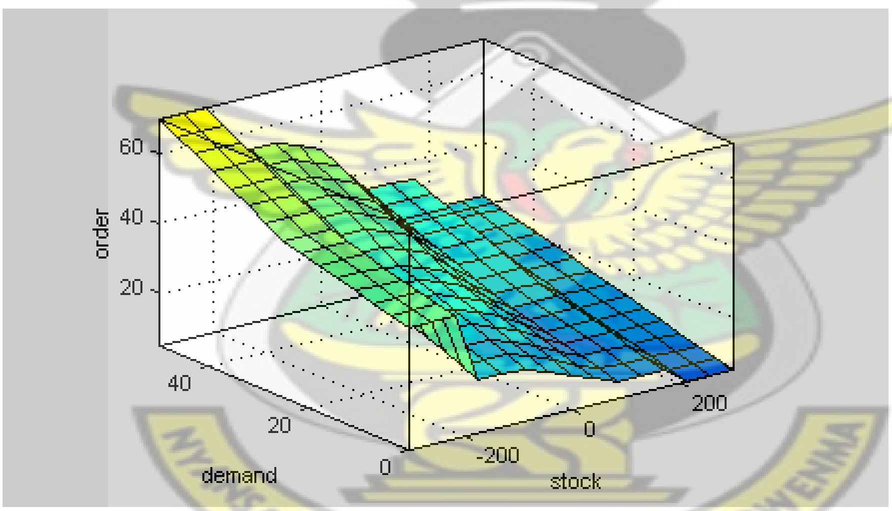

Figure 3.3: Surface of the fuzzy logic model for (i) type subjects

The ordering response to changes in stock level and demand is smooth. There is a slightly sharper increase in order when the inventory level moves from the membership functions 'PS' to 'PB' or from 'NS' to 'NB'. Transitions of the inventory level from 'Z' to 'NS' or 'PS' are quite smooth and close to linear. This is obtained by keeping the ranges that membership functions intersect small, at the same time by choosing the output values of consecutive membership functions close. For example,

IF stock is NS and demand is med then order is 35.

IF stock is Z and demand is med then order is 30.

IF stock is PS and demand is med then order is 20.

Looking at the inference rules given above, when the demand is exactly medium, which is 25, and stock level is exactly zero then the order will be 30. Similarly when the demand is exactly medium, which is 25, and stock level is in NS, which happens between -135 and -165, then the order will be 35. This will cause a gradual increase because of exponential delay.

3.7.2 Modeling (ii) Type Subjects

This ordering behavior is explained as alternating large-and-zero discrete orders like a high frequency signal. (ii) type subjects are usually observed in discrete delay games. In discrete delay the inflow is simply lagged by the given delay time. Hence, in the discrete game, a placed order is received in 4 days as exactly the same amount. In the experimental data, the subjects capture the dynamics of the game as the game unfolds. So an idea about the delay type can quickly be developed by observing supply line level. If the players do not consider supply line level as a decision factor, usually an oscillatory and unstable stock level results. The inventory of the

(ii) type subjects usually endures zigzagging stock levels since a large order reaches the inventory after a discrete delay of 4 days. Then the inventory begins to drop gradually till the next large order reaches the inventory.

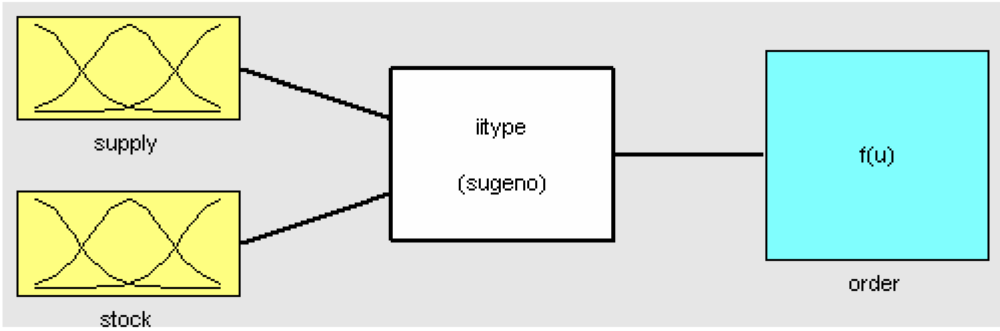

Figure 3.4: Fuzzy Logic Model for (ii) type subjects

The decision making factors for the model is supply line and stock level. The demand is not considered as decision making factor. Even though the order amount should be the total of four day delays' demand, the subjects usually make the large orders looking at the level of the inventory or randomly.

The order range is between 0 and 300 where the maximum order level is from the experimental data of subjects with (ii) type ordering behavior.

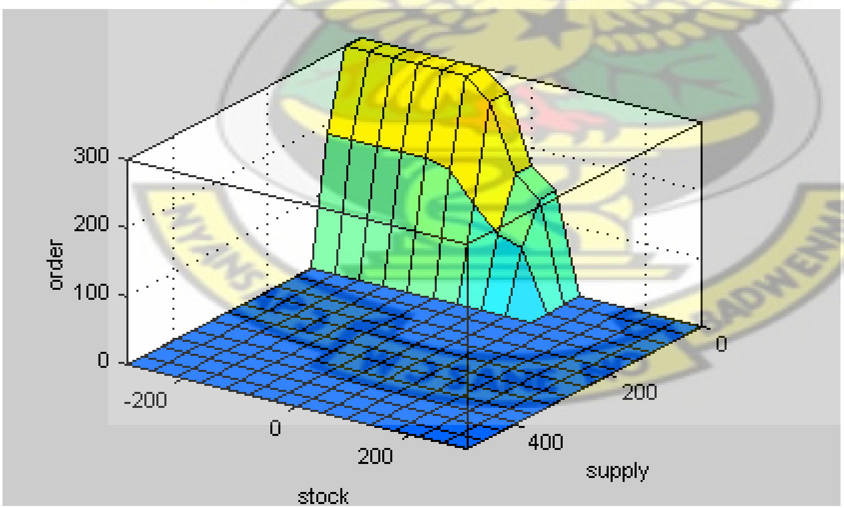

Figure 3.5: Surface of the fuzzy logic model for (ii) type subjects

The surface is very steep which means the order is zero or a large amount. The inference rules suggest that if stock is 'P' or the supply line is 'B' then the order is zero.

If the supply line level is 0, then it is known, because of discrete delay, there will be no inflow to the stock for at least four days. If the stock level is not too high, the model should avoid supply line level to drop to zero. As soon as the supply line level is ‘S’ and the stock level is not ‘P’, a large order is placed according to the stock level.

3.7.3 Modeling (iii) Type Subjects

This ordering behavior is explained as long periods of constant orders punctuated by a few sudden large ones. This (iii) type subjects are common in continuous exponential delay games. Some players do not order smoothly like (i) type. They order smoothly when the inventory level is around the desired level. When the inventory level is below the desired level, in other words when backordering occurs, subjects tend to give large orders to compensate the discrepancy in the inventory level quickly. Similarly, when the inventory level is above the desired level, they cease ordering.

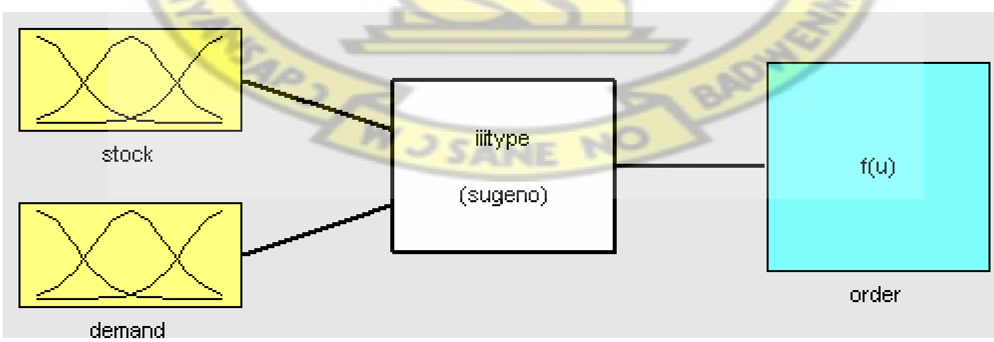

Figure 3.6: Fuzzy Logic Model for (iii) type subjects

The decision making factors for (iii) type players are stock level and demand. Demand is needed as an input for fuzzy logic model to obtain a smooth ordering around desired range of inventory to counteract the demand. (iii) type players order in high amounts only when backordering occurs or cease ordering only when there is overflow of goods. Hence, supply line is not a decision making factor for this type of players.

Table 3.2: Inference rule table of the fuzzy logic model for (iii) type subjects

1. If (stock is P) and (demand is high) then (order is 10) (1)
2. If (stock is P) and (demand is med) then (order is 0) (1)
3. If (stock is P) and (demand is low) then (order is 0) (1)
4. If (stock is Z) and (demand is high) then (order is 40) (1)
5. If (stock is Z) and (demand is med) then (order is 25) (1)
6. If (stock is Z) and (demand is low) then (order is 10) (1)
7. If (stock is NS) and (demand is high) then (order is 150) (1)
8. If (stock is NS) and (demand is med) then (order is 100) (1)
9. If (stock is NS) and (demand is low) then (order is 60) (1)
10. If (stock is NB) and (demand is high) then (order is 300) (1)
11. If (stock is NB) and (demand is med) then (order is 250) (1)
12. If (stock is NB) and (demand is low) then (order is 200) (1)

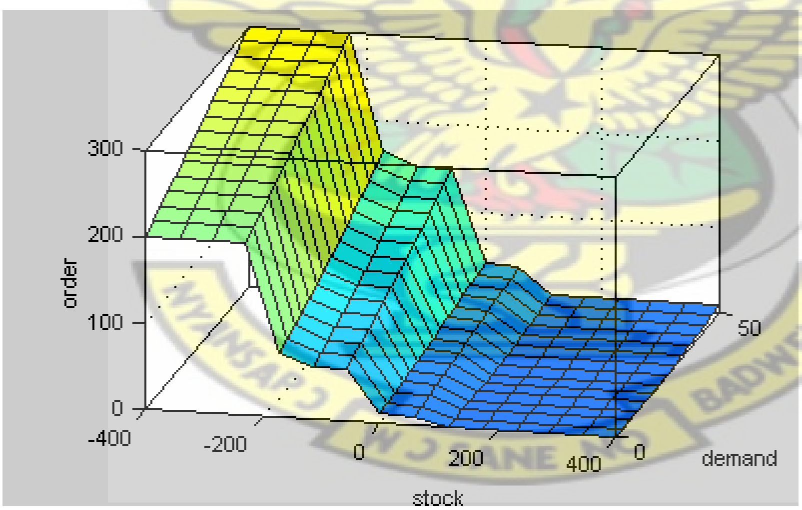

Figure 3.7: Surface of the fuzzy logic model for (iii) type subjects

Looking at the surface, it is seen that the ordering response to changes in stock level and demand is smooth between 0 and 100. When the stock level is above 100, the model does not order. There is a sharp increase in order when the inventory level moves from the membership functions ‘Z’ to ‘NS’ or from ‘NS’ to ‘NB’. The order increases gradually as the demand increases no matter what the inventory level is. Few sudden large orders or the leaps in orders are obtained when the inventory level goes below 0 or below -200.

CHAPTER FOUR

DATA COLLECTION, ANALYSIS AND MODELING

4.1 Data Collection and Description

Data for this project was obtained from Inventory Department of Mantrac Ghana Limited covering a period of six years on monthly basis.

The data comprises the following:

- Monthly data on stock, demand and supply from January 2005 to December 2010.

The table below displays the stock, demand and supply data for January, 2005 to December, 2006

YEAR	**2005**			**2006**		
MONTH	**STOCK**	**DEMAND**	**SUPPLY**	**STOCK**	**DEMAND**	**SUPPLY**
JANUARY	2400	1804	1804	3500	1405	1405
FEBRUARY	2060	2045	2045	2600	1975	1975
MARCH	2500	1750	1750	4000	4128	4000
APRIL	1500	251	251	4500	2145	2145
MAY	1400	468	468	4800	2694	2694
JUNE	900	390	390	3000	1945	1945
JULY	1000	1005	1000	4100	3056	3056
AUGUST	2000	1065	1065	3500	2043	2043
SEPTEMBER	935	50	50	1000	632	632
OCTOBER	3500	3567	3500	2000	1024	1024
NOVEMBER	4000	1157	1157	1200	380	380
DECEMBER	3000	4309	3000	1500	1902	1500
AVERAGE	**2099.58**	**1488.42**		**2975**	**1944.08**	

Table 4.1 Stock, demand and supply data for January, 2005 to December, 2006.

The stock, demand and supply data for January, 2007 to December 2010 is displayed in appendix A. The average stock and demand for each year was displayed in the last rows of the table. The average monthly stock for the five years period was **2684** and that of demand was **1932**.

- Inventory holding cost per unit per year, fixed ordering cost per inventory cycle, fixed shortage cost per unit shortage and shortage cost of lost profits as at the year 2011 are displayed in the table below.

COST	**AMOUNT (GH¢)**
Fixed ordering cost per inventory cycles (C)	58.00
Holding cost per item per year (h)	2.90
Fixed shortage costs per unit short (s)	5.80
Shortage cost of lost sales including the lost profit (π)	8.70

Table 4.2 Data on cost components

The costs are in Ghana cedis.

4.2 Stock, Demand and Supply data compared

Figure 4.1 below displays the trajectory of stock, demand and supply data from January 2005 to December 2010.

The visual pattern of the graph shows that during most of the periods, the stock was more than the demand and supply. However, a careful observation of the pattern of the graph shows the incidence of periodicity with high and low points. During some months, the demand was more than the stock and supply and this will result into backorders and lost sales which is one of the problems of the company.

The high demands during this period is due to some factors such as the low humidity which cause the hose to wear off very early because those periods are in the harmattan. Also because the grounds are very hard during those periods, the hydraulics systems of the excavators which make use of the hose do wear off quickly because of the difficulties the excavators experienced when excavating.

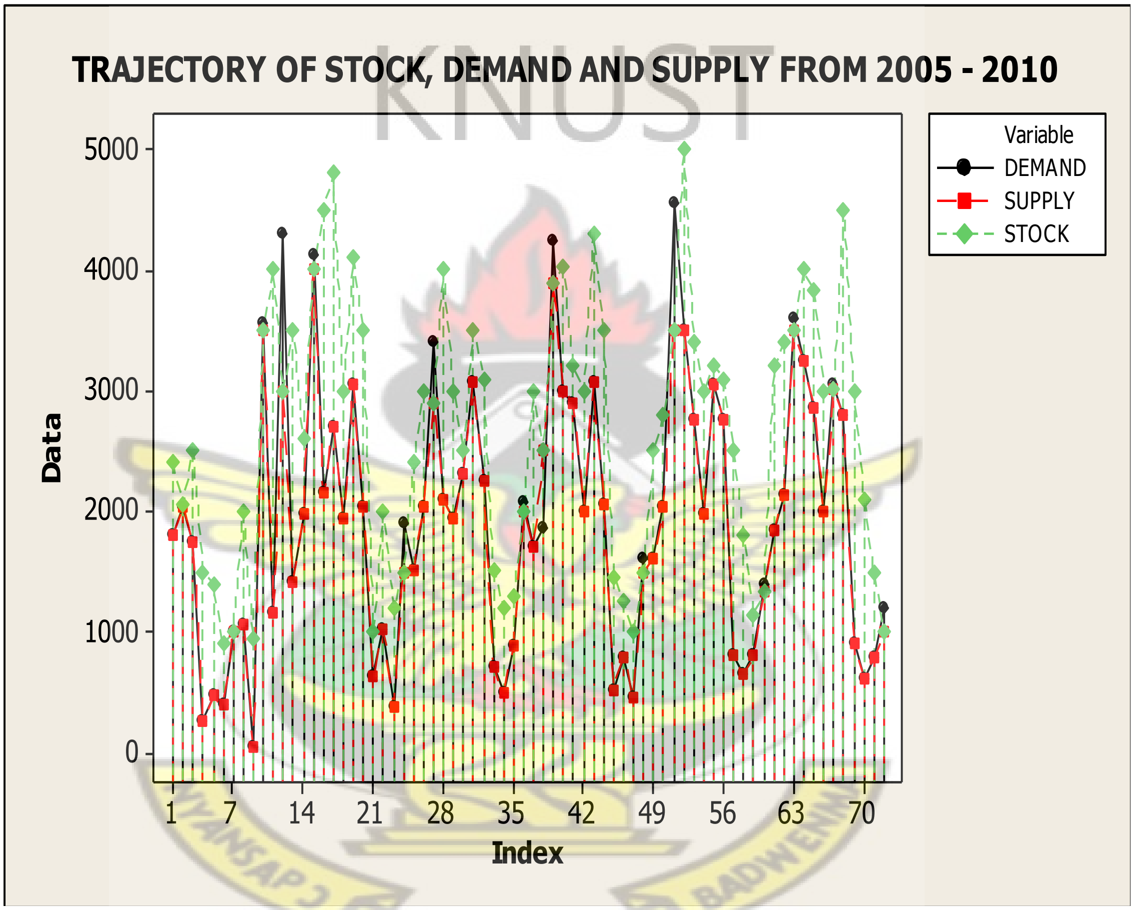

Figure 4.1

N.B.: Index stands for number of months.

4.2.1 Description of trajectory of stock data

Figure 4.2 on the next page describes the trajectory of stock data from January, 2005 – December, 2010. The visual pattern of stock shown in the figure is indicative of periodicity. The average monthly stock level was about **2684** and majority of the stock were below indicating stock out period. Limiting stock out period is one of the concerns of this study.

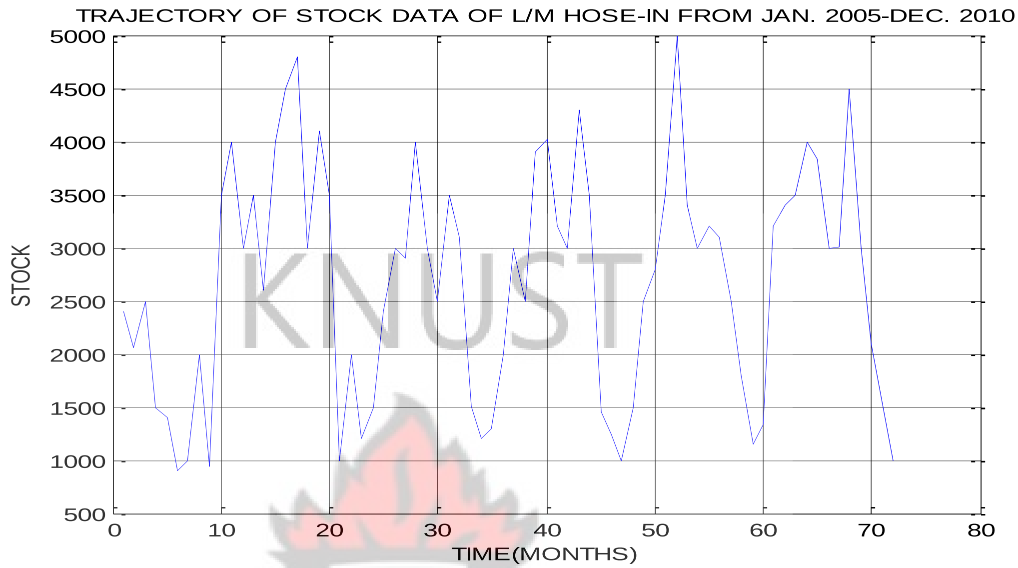

Fig. 4.2

4.2.2 Description of trajectory of demand data

Figure 4.3 below describes the trajectory of demand data from January, 2005 to December, 2010. The visual pattern of demand as shown in figure 4.3 is of similar pattern as that of the stock levels shown in figure 4.2 indicating that the demand depends on the stock. If the demand exceeds the stock, backorders and lost sales may occur.

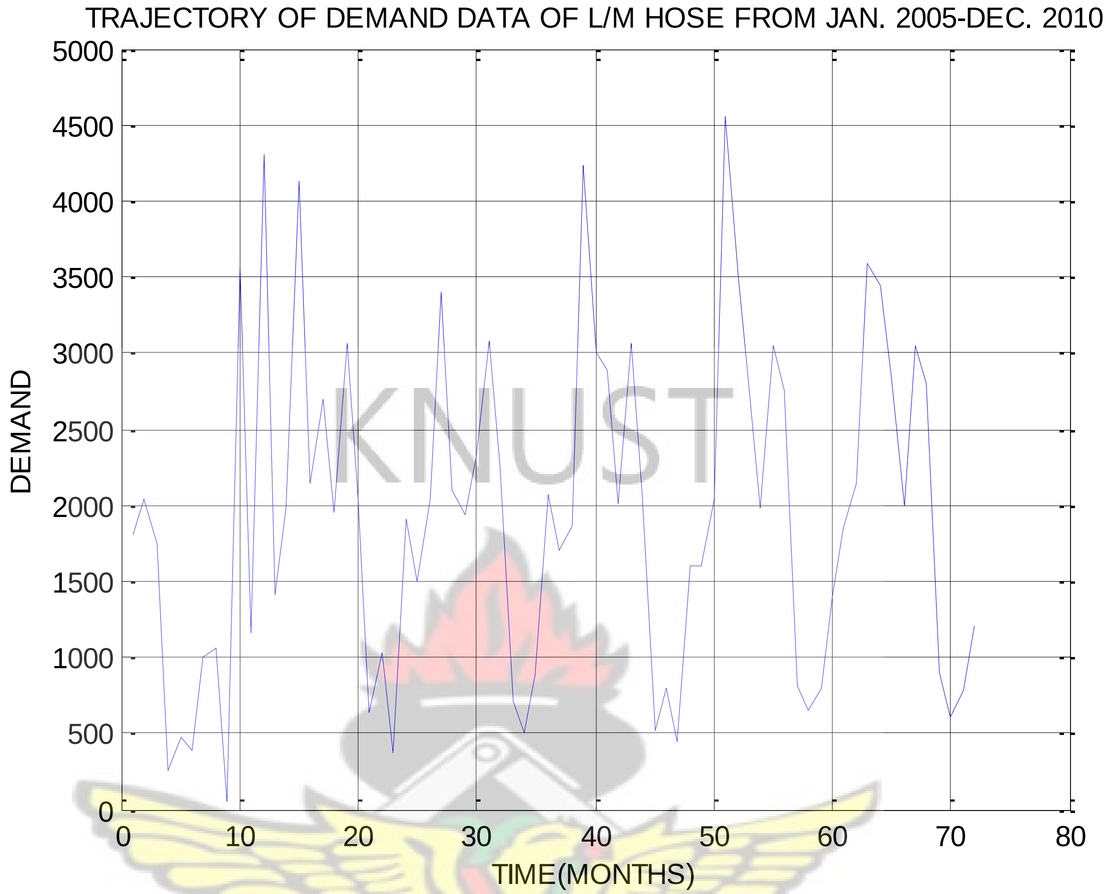

Figure 4.3

The average monthly demand was **1932 units** and it was observed that majority of the demands were above the average.

4.3 Computational procedures

The following values were used in the computations.

Average annual demand (D)	=	1931.86 x 12
	=	2683.985 units
Fixed ordering cost per inventory cycle (C)	=	GH¢58.00
Inventory holding cost per unit year (h)	=	GH¢2.90

Fixed shortage costs per unit short (s)	=	GH¢5.80
Shortage cost of lost sales including lost profit (π)	=	GH¢8.70
Fraction of demand backordered during stock out period (β)	=	0.1
Expected lead time demand (θ)	=	$\frac{50}{0.5} = 100$
The expected shortage at the end of the cycle B(r)	=	0.088

Assuming that the cost components c, h, s and π are all fuzzy, we represent them by trapezoidal fuzzy numbers given below.

$$\tilde{C} = \left(c-\delta_1, c-\delta_2, c+\delta_3, c+\delta_4\right),$$

$$\tilde{h} = \left(h-\delta_5, h-\delta_6, h+\delta_7, h+\delta_8\right),$$

$\tilde{S} = \left(s-\Delta_1, s-\Delta_2, s+\Delta_3, s+\Delta_4\right)$ and

$\tilde{\pi} = \left(\pi-\Delta_5, \pi-\Delta_6, \pi+\Delta_7, \pi+\Delta_8\right)$, where δ_i and Δ_i, i = 1, 2, 3, …………, 8 are arbitrary positive numbers under the following restrictions.

$C > \delta_1 > \delta_2, \delta_3 < \delta_4, h > \delta_5 > \delta_6$ and $\delta_7 < \delta_8$.

$S > \Delta_1 > \Delta_2, \Delta_3 < \Delta_4, \pi > \Delta_5 > \Delta_6$ and $\Delta_7 < \Delta_8$.

Five sets of the fuzzy components used for the computation based on arbitrary choices of Δ_i and δ_i, i = 1,2,3,…………,8 are displayed in table 4.3 on the next page.

d_i	1st	2nd	3rd	4th	5th	a_i	1st	2nd	3rd	4th	5th
d_1	50	48	50	50	50	a_1	5.2	5.0	4	5	3
d_2	45	42	47	43	42	a_2	4.2	3.0	2	3	2
d_3	5	7	10	8	6	a_3	1.2	1.5	1.5	1.5	1.8
d_4	10	12	14	15	20	a_4	1.4	1.8	2	4	5
d_5	2	2.2	2.3	2.1	2.1	a_5	7.7	5.2	4.2	3.2	2
d_6	1.5	1.3	1.4	1.4	1.5	a_6	7.4	4.8	2.4	2.2	1.5
d_7	1	1.2	1.5	1.3	1.6	a_7	1.1	1.6	2.1	1.8	2.5
d_8	1.3	1.5	1.7	2	2	a_8	1.4	1.9	2.5	3	2.8

Table 4.3 Table of Arbitrary values of $\partial_i = d_i$ and $\Delta_i = a_i$, where i= 1,2,3,………,8 used in the computations.

4.3.1 Calculation of fuzzified costs

For the first set of values, we obtained the fuzzified costs as follows:

$$\tilde{C} = (58-50,58-45,58+5,58+10)$$
$$\tilde{C} = (8,13,63,68),$$

$$\tilde{h} = (2.9-2,2.9-1.5,2.9+1,2.9+1.3)$$
$$\tilde{h} = (0.9,1.4,3.9,4.2),$$

$$\tilde{s} = (5.8-5.2,5.8-4.2,5.8+1.2,5.8+1.4)$$ and
$$\tilde{s} = (0.6,1.6,7,7.2)$$

$$\tilde{\pi} = (8.7-7.7,8.7-7.4,8.7+1.1,8.7+1.4)$$
$$\tilde{\pi} = (1,1.3,9.2,10.1)$$

The procedure is repeated for the 2nd, 3rd, 4th and 5th sets and the results are displayed in table 4.4 on the next page.

Ĉ	ĥ	ŝ	$\tilde{\pi}$
(8,13,63,68)	(0.9,1.4,3.9,4.2)	(0.6,1.6,7,7.2)	(1,1.3,9.2,10.1)
(10,16,65,70)	(0.7,1.6,4.1,4.4)	(0.8,2.8,7.3,7.6)	(3.5,3.9,10.3,10.6)
(8,11,68,72)	(0.6,1.5,4.4,4.6)	(1.8,3.8,7.3,7.8)	(4.5,6.3,10.8,11.2)
(8,15,66,73)	(0.8,1.5,4.2,4.9)	(0.8,2.8,7.3,9.8)	(5.5,6.5,10.5,11.7)
(8,16,64,78)	(0.8,1.4,4.5,4.9)	(2.8,3.8,7.3,10.8)	(6.7,7.2,11.2,11.5)

Table 4.4 Table of fuzzified cost components

4.3.2 Calculation of signed distances

For the first set of fuzzy cost components, the signed distances k_1= d(Ĉ, Ô), k_2 = d(ĥ, Ô), k_3 = d(ŝ, Ô) and k_4 = d($\bar{\pi}$, Ô) were determined using the following equations:

d(Ĉ, Ô) = $0.25(4C - \delta_1 - \delta_2 + \delta_3 + \delta_4)$,
d(ĥ, Ô) = $0.25(4h - \delta_5 - \delta_6 + \delta_7 + \delta_8)$,
d(ŝ, Ô) = $0.25(4s - \Delta_1 - \Delta_2 + \Delta_3 + \Delta_4)$ and
d($\bar{\pi}$, Ô) = $0.25(4\pi - \Delta_5 - \Delta_6 + \Delta_7 + \Delta_8)$. (Yao and Wu, 2000)

By substituting in the values, we obtained the following:

d(Ĉ, Ô) = $0.25(4 \times 58 - 50 - 45 + 5 + 10)$ = **38.00** ,

d(ĥ, Ô) = $0.25(4 \times 2.9 - 2 - 1.5 + 1 + 1.3)$ = **2.60,**

d(ŝ, Ô) = $0.25(4 \times 5.8 - 5.2 - 4.2 + 1.2 + 1.4)$ = **4.10** and

d($\bar{\pi}$, Ô) = $0.25(4 \times 8.7 - 7.7 - 7.4 + 1.1 + 1.4)$ = **5.55**

The procedure is repeated for the 2nd, 3rd, 4th and 5th sets and the results are displayed in table 4.5 on the next page. (See appendix D.1 for Matlab output).

$d(\hat{C}, \tilde{O})$	$d(\hat{h}, \hat{O})$	$d(\hat{s}, \tilde{O})$	$d(\tilde{\pi}, \tilde{O})$
38.00	2.60	4.10	5.55
40.25	2.70	4.63	7.08
39.75	2.78	5.18	8.20
40.50	2.85	5.18	8.55
41.50	2.90	6.25	9.15

Table 4.5 Table of signed distances

4.3.3 Calculation of percentage increase P_c, P_h, P_s and P_π

The percentage increase P_c, P_h, P_s and P_π in C, h, s and π respectively under fuzzy cases are determined using the following equations:

$$P_C = \left(\frac{d(\tilde{C}, \tilde{O}) - C}{C} \right) \times 100,$$

$$P_h = \left(\frac{d(\tilde{h}, \tilde{O}) - h}{h} \right) \times 100,$$

$$P_s = \left(\frac{d(\tilde{s}, \tilde{O}) - s}{s} \right) \times 100 \text{ and}$$

$$P_\pi = \left(\frac{d(\tilde{\pi}, \tilde{O}) - \pi}{\pi} \right) \times 100.$$ (Vijayan, Kumaran, 2008).

For the first set of values, we obtained the percentage increase in the cost parameters as follows:

$$P_C = \left(\frac{(38 - 58}{58} \right) \times 100 = -34.48,$$

$$P_h = \left(\frac{2.60 - 2.90}{2.90} \right) \times 100 = -10.34,$$

$$P_s = \left(\frac{4.10 - 5.80}{5.80}\right) \times 100 = -29.31 \text{ and}$$

$$P_\pi = \left(\frac{5.55 - 8.70}{8.70}\right) \times 100 = -36.21.$$

The procedure is repeated for the 2nd, 3rd, 4th and 5th sets and the results are displayed in table 4.6 below. (See appendix D.1 for Matlab output).

P_C	P_h	P_s	P_π
-34.48	-10.34	-29.31	-36.21
-30.60	-6.90	-20.26	-18.68
-31.47	-4.31	-10.78	-5.75
-30.17	-1.72	-10.78	-1.72
-28.45	0	+7.76	+5.17

Table 4.6 Table of percentage increase in the costs components

4.3.4 Calculation of the optimum quantity, reorder point and total cost

The optimum quantity (Q), the reorder point (r) and the corresponding total cost $E(\tilde{Z}_{Q,r})$ are determined using the following equations:

$$Q = \sqrt{\left(\frac{2D[k_1 + k_3 B(r) + k_4(1-\beta)B(r)]}{k_2}\right)} \quad \ldots\ldots \quad (\alpha)$$

$$r = \left(\frac{Qk_2}{Qk_2(1-\beta) + k_3 D + k_4 D(1-\beta)}\right) \times D \quad \ldots\ldots \quad (\beta)$$

$$E(\tilde{Z}_{Q,r}) = \frac{k_1 D}{Q} + k_2\left(\frac{Q}{2} + r - \theta\right) + B(r)\left(k_2(1-\beta) + \frac{k_3 D}{Q} + \frac{k_4(1-\beta)D}{Q}\right) \quad \ldots\ldots \quad (\gamma)$$

(Vijayan, Kumaran, 2008)

By substituting the various values into equation (α) the optimum quantity (Q) is

$$Q = \sqrt{\left(\frac{2 \times 23182.32[38 + 4.10 \times 0.088 + 5.55(1-0.1) \times 0.088]}{2.60}\right)}$$

Q = 831.81 units.

Also the reorder point r is obtained by substituting the various values into equation (β).

$$r = \left(\frac{831.81 \times 2.60}{831.81 \times 2.60(1-0.1) + 4.1 \times 23182.32 + 5.55 \times 23182.32(1-0.1)}\right) \times 23182.32$$

r = 235.62

From

$$\frac{k_1 D}{Q} = \frac{38 \times 23182.32}{831.81} = 1059.05 \text{ ....(a)}$$

$$k_2\left(\frac{Q}{2} + r - \theta\right) = 2.6\left(\frac{831.81}{2} + 235.62 - 100\right) = 1433.965 \text{ .... (b)}$$

$$B(r)\left(k_2(1-\beta) + \frac{k_3 D}{Q} + \frac{k_4(1-\beta)D}{Q}\right) =$$

$$0.088\left(2.6(1-0.1) + \frac{4.1 \times 23182.32}{831.81} + \frac{5.55(1-0.1) \times 23182.32}{831.81}\right) = 22.511737 \text{ ....(c)}$$

The total cost $E(\tilde{Z}_{Q,r})$ is obtained by substituting the values in equations (a), (b) and (c) into equation (γ) giving;

$E(\tilde{Z}_{Q,r})$ = 1059.05 + 1433.965 + 22.511737

$E(\tilde{Z}_{Q,r})$ **= 2515.50**

The procedure is repeated for the 2nd, 3rd, 4th and 5th sets and the results are displayed in table 4.7 below. (See appendix D.1 for Matlab output).

Sets of values	**Q**	**r**	$E(\tilde{Z}_{Q,r})$
1st	831.81	235.62	2515.50
2nd	841.30	205.00	2555.20
3rd	826.20	181.33	2518.60
4th	822.98	180.96	2576.50
5th	826.97	164.51	2585.50
Average	**829.85**	**193.48**	**2550.30**

Table 4.7 Table of optimal (Q, r) policy with all cost components fuzzy

4.4 Discussion of Results

Table 4.8 below reveals that the optimum values of the decision variables Q and R and the corresponding total cost will vary with respect to the changes in the level of fuzziness in all cost components.

P_C	P_h	P_s	P_π	Q	r	$E(\tilde{Z}_{Q,r})$
-34.48	-10.34	-29.31	-36.21	831.81	235.62	2515.50
-30.60	-6.90	-20.26	-18.68	841.30	205.00	2555.20
-31.47	-4.31	-10.78	-5.75	826.20	181.33	2518.60
-30.17	-1.72	-10.78	-1.72	822.98	180.96	2576.50
-28.45	0	+7.76	+5.17	826.97	164.51	2585.50

Table 4.8 Table of percentage increase in cost components and corresponding optimum quantities and total costs.

While the optimum order quantity experiences slight changes from one cost component to the other, the reorder point experiences higher changes from one cost component to the other. Also the total cost experiences slight changes from one cost component to the other. Generally, the reorder point records considerable decrease when the fuzzy cost increases.

The percentage change in C, h, s and π under fuzzy case (based on signed distance values) from their crisp values increases down the columns.

The total inventory cost often depend on the lead time demand, the expected shortage cost at the end of the period, the fraction of demand backordered during stock out period and the quantity demanded by the customers.

A continuous review (Q, r) inventory model was constructed for the study of the impact and sensitiveness of the impreciseness of the cost components in the decision variables and the total cost.

The continuous review model with all cost parameters fuzzy are useful and efficient because in practical situations, precise values of the cost characteristics are seldom achieved as they may be vague and imprecise to certain extent. For example, the shortage in an inventory system may occur due to different causes such as sudden increase of demand, transportation problems, unforeseen incidents, hike in wages, delayed production e.t.c.

Shortage brings loss of goodwill and it is difficult to the exact amount of shortage cost. The same problem is experienced in the case of the ordering and holding costs hence in inventory, the decision maker may allow some flexibility in the cost parameter values in order to tackle the uncertainties which always fit the real situations.

Also statistical treatment of the cost characteristics is inefficient for these models because of the lack of random observations. As such, these characteristics are better described by the use of fuzzy sets which encompass a specific range of values.

CHAPTER FIVE

CONCLUSIONS AND RECOMMENDATIONS

5.1 Conclusions

- An inventory model to model Mantrac Ghana Limited's inventory cost of L/M Hose-in using a continuous review (Q, r) inventory model with all cost components was developed.

- Considering the optimal (Q, r) policy with all cost components fuzzy for the L/M Hose-in, the average optimum order quantity was **829.85** units, the reorder point was **193.48 units** and the average total cost for the period was **GH¢2550.30**.

- The fuzziness in the cost components are represented by the trapezoidal fuzzy numbers which are $\hat{C}$, $\hat{h}$, $\hat{s}$ and $\tilde{\pi}$.

5.2 Recommendations

Based on the findings so far arrived at, in order to ensure proper inventory control systems at Mantrac Ghana Limited, the following recommendations are made.

- The continuous review (Q, r) inventory model with all the cost components fuzzy should be used to model the inventory of Mantrac Ghana Limited.

- The optimum quantity and reorder point should be determined at the end of each year to guide in the New Year to minimize the backorders and lost sales since this brings loss of goodwill.
- In this research, the continuous review (Q, r) inventory model was used with all cost components fuzzy. There should be further research study using the periodic review (R, T) inventory model with all cost components fuzzy.

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APPENDIXES

APPENDIX A DATA ON STOCK, DEMAND AND SUPPLY FOR L/M HOSE-IN

TABLE A1 Data on stock, demand and supply from Jan. 2005 – Dec. 2006

YEAR	2005			2006		
MONTH	**STOCK**	**DEMAND**	**SUPPLY**	**STOCK**	**DEMAND**	**SUPPLY**
JANUARY	2400	1804	1804	3500	1405	1405
FEBRUARY	2060	2045	2045	2600	1975	1975
MARCH	2500	1750	1750	4000	4128	4000
APRIL	1500	251	251	4500	2145	2145
MAY	1400	468	468	4800	2694	2694
JUNE	900	390	390	3000	1945	1945
JULY	1000	1005	1000	4100	3056	3056
AUGUST	2000	1065	1065	3500	2043	2043
SEPTEMBER	935	50	50	1000	632	632
OCTOBER	3500	3567	3500	2000	1024	1024
NOVEMBER	4000	1157	1157	1200	380	380
DECEMBER	3000	4309	3000	1500	1902	1500
AVERAGE	**2099.58**	**1488.42**		**2975**	**1944.08**	

TABLE A.2 Data on stock, demand and supply from Jan. 2007 – Dec. 2008

YEAR	2007			2008		
MONTH	**STOCK**	**DEMAND**	**SUPPLY**	**STOCK**	**DEMAND**	**SUPPLY**
JANUARY	2400	1504	1504	3000	1704	1704
FEBRUARY	3000	2040	2040	2500	1865	2500
MARCH	2900	3400	2900	3895	4234	3895
APRIL	4000	2100	2100	4022	3001	3001
MAY	3000	1934	1934	3200	2890	2890
JUNE	2500	2310	2310	3000	2003	2003
JULY	3500	3078	3078	4300	3070	3070
AUGUST	3100	2250	2250	3500	2065	2065
SEPTEMBER	1510	712	712	1450	520	520
OCTOBER	1200	500	500	1250	792	792
NOVEMBER	1300	890	890	1000	450	450
DECEMBER	2000	2068	2000	1500	1600	1500
AVERAGE	**2534.17**	**1898.83**		**2718.08**	**2016.17**	

TABLE A.3 Data on stock, demand and supply from Jan. 2009 – Dec. 2010

YEAR	**2009**			**2010**		
MONTH	**STOCK**	**DEMAND**	**SUPPLY**	**STOCK**	**DEMAND**	**SUPPLY**
JANUARY	2500	1607	1607	3200	1850	1850
FEBRUARY	2800	2045	2045	3400	2135	2135
MARCH	3500	4560	3500	3500	3590	3500
APRIL	5000	3500	3500	4000	3250	3250
MAY	3400	2750	2750	3840	2850	2850
JUNE	3000	1984	1984	3000	2000	2000
JULY	3200	3055	3055	3010	3045	3010
AUGUST	3100	2750	2750	4500	2800	2800
SEPTEMBER	2500	810	810	3000	900	900
OCTOBER	1800	655	655	2100	610	610
NOVEMBER	1145	798	798	1490	780	780
DECEMBER	1340	1400	1340	1000	1200	1000
AVERAGE	**2773.75**	**2159.5**		**3003.33**	**2084.17**	

AVERAGE ANNUAL DEMAND (D) FOR THE PERIOD - 23182.32 units
AVERAGE ANNUAL STOCK - 32207.88 units

APPENDIX B

TABLE B.1 Cost of a unit L/M Hose - in

YEAR	**COST PER ITEM (GH¢)**
2005	12.40
2006	12.70
2007	12.90
2008	13.50
2009	13.60
2010	14.00
TOTAL	**79.10**
AVERAGE	**13.18**

TABLE B.2 Data on cost components

COST	AMOUNT (GH¢)
Fixed ordering cost per inventory cycles (C)	58.00
Holding cost per item per year (h)	2.90
Fixed shortage costs per unit short (s)	5.80
Shortage cost of lost sales including the lost profit (π)	8.70

APPENDIX C

C.1 MATLAB CODE FOR TRAJECTORY OF STOCK CURVE

```
    A=[];
for i=1:72
    a=input('enter values of [t,X]:');
    A=[A;a];
end
t=A(:,1);
X=A(:,2);
plot(t,X,'-')
title('TRAJECTORY OF STOCK DATA OF L/M HOSE-IN FROM JAN. 2005-DEC.
2010')
xlabel('TIME(MONTHS)')
ylabel('SUPPLY')
grid
```

C.2 MATLAB CODE FOR TRAJECTORY OF DEMAND CURVE

```
A=[];
for i=1:72
    a=input('enter values of [t,X]:');
    A=[A;a];
end
t=A(:,1);
X=A(:,2);
plot(t,X,'-')
title('TRAJECTORY OF DEMAND DATA OF L/M HOSE-IN FROM JAN. 2005-DEC.
2010')
xlabel('TIME(MONTHS)')
ylabel('SUPPLY')
grid
```

APPENDIX D

MATLAB OUTPUT OF OPTIMAL SOLUTION UNDER CONTINUOUS REVIEW (Q, r) MODEL WITH ALL COST COMPONENTS FUZZY

D.1 1ST SET VALUES

```
>> D=23182.32

D =

  2.3182e+004

>> C=58

C =

    58

>> h=2.9

h =

    2.9000

>> s=5.8

s =

    5.8000

>> p=8.7

p =

    8.7000

>> b=0.1

b =

    0.1000

>> t=100

t =

   100
```

```
>> Br=0.088

Br =

    0.0880

>> d1=50

d1 =

    50

>> d2=45

d2 =

    45

>> d3=5

d3 =

     5

>> d4=10

d4 =

    10

>> d5=2

d5 =

     2

>> d6=1.5

d6 =

    1.5000

>> d7=1

d7 =
```

```
     1

>> d8=1.3

d8 =

    1.3000

>> a1=5.2

a1 =

    5.2000

>> a2=4.2

a2 =

    4.2000

>> a3=1.2

a3 =

    1.2000

>> a4=1.4

a4 =

    1.4000

>> a5=7.7

a5 =

    7.7000

>> a6=7.4

a6 =

    7.4000

>> a7=1.1

a7 =
```

```
    1.1000

>> a8=1.4

a8 =

    1.4000

>> k1=0.25*(4*C-d1-d2+d3+d4)

k1 =

    38

>> pc=((k1-C)/C)*100

pc =

  -34.4828

>> k2=0.25*(4*h-d5-d6+d7+d8)

k2 =

    2.6000

>> ph=((k2-h)/h)*100

ph =

  -10.3448

>> k3=0.25*(4*s-a1-a2+a3+a4)

k3 =

    4.1000

>> ps=((k3-s)/s)*100

ps =

  -29.3103

>> k4=0.25*(4*p-a5-a6+a7+a8)
```

```
k4 =

    5.5500

>> pp=((k4-p)/p)*100

pp =

  -36.2069

>> Q=sqrt((2*D*(k1+k3*Br+k4*(1-b)*Br))/k2)

Q =

  831.8110

>> r=((Q*k2)/(Q*k2*(1-b)+k3*D+k4*D*(1-b)))*D

r =

  235.6158

>> EZQR=(k1*D/Q)+k2*((Q/2)+r-t)+Br*(k2*(1-b)+((k3*D)/Q)+(k4*(1-b)*D)/Q)

EZQR =

  2.5155e+003
```

D.2 2^{ND} SET VALUES

```
>> d1=48

d1 =

    48

>> d2=42

d2 =

    42

>> d3=7

d3 =

     7
```

```
>> d4=12

d4 =

    12

>> d5=2.2

d5 =

    2.2000

>> d6=1.3

d6 =

    1.3000

>> d7=1.2

d7 =

    1.2000

>> d8=1.5

d8 =

    1.5000

>> a1=5

a1 =

     5

>> a2=3.0

a2 =

     3

>> a3=1.5

a3 =
```

```
   1.5000

>> a4=1.8

a4 =

   1.8000

>> a5=5.2

a5 =

   5.2000

>> a6=4.8

a6 =

   4.8000

>> a7=1.6

a7 =

   1.6000

>> a8=1.9

a8 =

   1.9000

>> k1=0.25*(4*C-d1-d2+d3+d4)

k1 =

  40.2500

>> pc=((k1-C)/C)*100

pc =

 -30.6034

>> k2=0.25*(4*h-d5-d6+d7+d8)

k2 =
```

```
    2.7000

>> ph=((k2-h)/h)*100

ph =

   -6.8966

>> k3=0.25*(4*s-a1-a2+a3+a4)

k3 =

    4.6250

>> ps=((k3-s)/s)*100

ps =

  -20.2586

>> k4=0.25*(4*p-a5-a6+a7+a8)

k4 =

    7.0750

>> pp=((k4-p)/p)*100

pp =

  -18.6782

>> Q=sqrt((2*D*(k1+k3*Br+k4*(1-b)*Br))/k2)

Q =

  841.3013

>> r=((Q*k2)/(Q*k2*(1-b)+k3*D+k4*D*(1-b)))*D

r =

  204.9975

>> EZQR=(k1*D/Q)+k2*((Q/2)+r-t)+Br*(k2*(1-b)+((k3*D)/Q)+(k4*(1-b)*D)/Q)
```

EZQR =

2.5552e+003

D.3 3RD SET VALUES

```
>> d1=50

d1 =

    50

>> d2=47

d2 =

    47

>> d3=10

d3 =

    10

>> d4=14

d4 =

    14

>> d5=2.3

d5 =

    2.3000

>> d6=1.4

d6 =

    1.4000

>> d7=1.5

d7 =

    1.5000
```

```
>> d8=1.7

d8 =

    1.7000

>> a1=4

a1 =

     4

>> a2=2

a2 =

     2

>> a3=1.5

a3 =

    1.5000

>> a4=2

a4 =

     2

>> a5=4.2

a5 =

    4.2000

>> a6=2.4

a6 =

    2.4000

>> a7=2.1

a7 =
```

```
    2.1000

>> a8=2.5

a8 =

    2.5000

>> k1=0.25*(4*C-d1-d2+d3+d4)

k1 =

   39.7500

>> pc=((k1-C)/C)*100

pc =

  -31.4655

>> k2=0.25*(4*h-d5-d6+d7+d8)

k2 =

    2.7750

>> ph=((k2-h)/h)*100

ph =

   -4.3103

>> k3=0.25*(4*s-a1-a2+a3+a4)

k3 =

    5.1750

>> ps=((k3-s)/s)*100

ps =

  -10.7759

>> k4=0.25*(4*p-a5-a6+a7+a8)

k4 =
```

```
   8.2000

>> pp=((k4-p)/p)*100

pp =

  -5.7471

>> Q=sqrt((2*D*(k1+k3*Br+k4*(1-b)*Br))/k2)

Q =

  826.1972

>> r=((Q*k2)/(Q*k2*(1-b)+k3*D+k4*D*(1-b)))*D

r =

  181.3268

>> EZQR=(k1*D/Q)+k2*((Q/2)+r-t)+Br*(k2*(1-b)+((k3*D)/Q)+(k4*(1-b)*D)/Q)

EZQR =

  2.5186e+003
```

D.4 4TH SET VALUES

```
>> d1=50

d1 =

    50

>> d2=43

d2 =

    43

>> d3=8

d3 =

     8
```

```
>> d4=15

d4 =

   15

>> d5=2.1

d5 =

   2.1000

>> d6=1.4

d6 =

   1.4000

>> d7=1.3

d7 =

   1.3000

>> d8=2

d8 =

    2

>> a1=5

a1 =

    5

>> a2=3

a2 =

    3

>> a3=1.5

a3 =

   1.5000
```

```
>> a4=4

a4 =

     4

>> a5=3.2

a5 =

    3.2000

>> a6=2.2

a6 =

    2.2000

>> a7=1.8

a7 =

    1.8000

>> a8=3

a8 =

     3

>> k1=0.25*(4*C-d1-d2+d3+d4)

k1 =

   40.5000

>> pc=((k1-C)/C)*100

pc =

  -30.1724

>> k2=0.25*(4*h-d5-d6+d7+d8)

k2 =
```

```
    2.8500

>> ph=((k2-h)/h)*100

ph =

   -1.7241

>> k3=0.25*(4*s-a1-a2+a3+a4)

k3 =

    5.1750

>> ps=((k3-s)/s)*100

ps =

  -10.7759

>> k4=0.25*(4*p-a5-a6+a7+a8)

k4 =

    8.5500

>> pp=((k4-p)/p)*100

pp =

   -1.7241

>> Q=sqrt((2*D*(k1+k3*Br+k4*(1-b)*Br))/k2)

Q =

  822.9768

>> r=((Q*k2)/(Q*k2*(1-b)+k3*D+k4*D*(1-b)))*D

r =

  180.9639

>> EZQR=(k1*D/Q)+k2*((Q/2)+r-t)+Br*(k2*(1-b)+((k3*D)/Q)+(k4*(1-b)*D)/Q)

EZQR =
```

2.5765e+003

D.5 5TH SET VALUES

d1 =

50

>> d2=42

d2 =

42

>> d3=6

d3 =

6

>> d4=20

d4 =

20

>> d5=2.1

d5 =

2.1000

>> d6=1.5

d6 =

1.5000

>> d7=1.6

d7 =

1.6000

>> d8=2

```
d8 =

     2

>> a1=3

a1 =

     3

>> a2=2

a2 =

     2

>> a3=1.8

a3 =

    1.8000

>> a4=5

a4 =

     5

>> a5=2

a5 =

     2

>> a6=1.5

a6 =

    1.5000

>> a7=2.5

a7 =

    2.5000

>> a8=2.8
```

```
a8 =

    2.8000

>> k1=0.25*(4*C-d1-d2+d3+d4)

k1 =

   41.5000

>> pc=((k1-C)/C)*100

pc =

  -28.4483

>> k2=0.25*(4*h-d5-d6+d7+d8)

k2 =

    2.9000

>> ph=((k2-h)/h)*100

ph =

     0

>> k3=0.25*(4*s-a1-a2+a3+a4)

k3 =

    6.2500

>> ps=((k3-s)/s)*100

ps =

    7.7586

>> k4=0.25*(4*p-a5-a6+a7+a8)

k4 =

    9.1500
```

```
>> pp=((k4-p)/p)*100

pp =

   5.1724

>> Q=sqrt((2*D*(k1+k3*Br+k4*(1-b)*Br))/k2)

Q =

  826.9663

>> r=((Q*k2)/(Q*k2*(1-b)+k3*D+k4*D*(1-b)))*D

r =

  164.5071

>> EZQR=(k1*D/Q)+k2*((Q/2)+r-t)+Br*(k2*(1-b)+((k3*D)/Q)+(k4*(1-b)*D)/Q)

EZQR =

  2.5855e+003
```

D.6 AVERAGE VALUES

```
>> Avg(Q)=(831.81+841.30+826.20+822.98+826.97)/5
>> AvgQ=(831.81+841.30+826.20+822.98+826.97)/5

AvgQ =

  829.8520

>> Avgr=(235.62+205+181.33+180.96+164.51)/5

Avgr =

  193.4840

>> AvgTC=(2515.5+2555.2+2518.6+2576.5+2585.5)/5

AvgTC =

  2.5503e+003
```