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MORTALITY INVESTIGATION OF GHANAIAN MALE PENSIONERS

By

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Declaration

I hereby declare that this submission is my own work towards the award of the M.Sc degree and that, to the best of my knowledge, it contains no material previously published by another person nor material which had been accepted for the award of any other degree of the university, except where due acknowledgement had been made in the text.

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Dedication

I dedicate this work to my loving and supportive parents, MR. and MRS. M. Y. Dadzie and all my siblings. I appreciate you so much.

Abstract

The use of standardized crude rates is seen in most mortality investigations in Ghana. It is necessary to graduate crude rates to obtain smooth mortality rates for actuarial purposes. Logistic regression model with a binomial error structure, was fitted in a statistical software package R, to obtain mortality rates of Ghanaian males who went on retirement in the years 1900, 1995, 2000 and 2005. A total sample of 15,317 pensioners was used for the analysis. The research found that, mortality rates from ages 70 to 75 are high among all groups of male pensioners. Retirement year 2000 is also noted to have high mortality rates compared to the other groups of pensioners. It is recommended that the mortality rates estimates obtained can be used to develop annuity products for pensioners.

Acknowledgment

The success of this work has never been by my efforts alone. I will first thank the Almighty, Most Merciful Allah, for by His Grace this work has come to a successful completion.

My sincere appreciation goes to my supervisor, Nana Kena Frempong for his passionate efforts, knowledgeable contribution, criticisms and guidance throughout this work.

For the unconditional support from my parents and siblings, I express my profound appreciation for their understanding and encouragement.

Contents

List of Abbreviation

AIC .... Akaike Information Criterion AIDS

.... Acquired Immune Defieciency Syndrome

APC ....Age Period Cohort CMIB ....

Continuous Mortality Investigation Bureau MLE ....Maximum

Likelihood Estimation

RF .... Reduction Factor

SSNIT ....Social Security and National Insurance Trust

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Chapter 1

INTRODUCTION

1.1 Background of the Study

In the bid to continue to make improvements in the reduction of frequency of death occurrences, life expectancy has been increasing over the years in several countries. The implication on mortality reduction have been widely recognized and confirmed by various analysts and researchers. However, the predictions of mortality improvements made since the late 1960's have generally underestimated actual mortality experiences, CMI (2004).

Historically this transition to low mortality levels has been seen as integral aspect of a transformation in the development of social and economic environment worldwide. The specific factors influencing low mortality are numerous and complex. Longevity however is dependent on various interacting factors but the two major factors are genetics and life style choices. Lifestyle choices however, are heavily influenced by the environment one finds him or herself as this determines the social and family life, diet, stress factors, available health facilities, disposable income and level of air pollution one is exposed to. It is therefore fair to assume that, how mortality evolves with time varies greatly in different countries due to the factors mentioned.

In earlier years mortality was very high which obviously was not due to genetics but rather epidemics, malnutrition, wars etc. Currently, due to improved health facilities and medical procedures, innovations and technological improvement, life expectancy is increasing and research indicates it will increase further in the near future.

On the other hand the general view that economic development is a key catalyst in the increase in life expectancy has been contradicted. William (1965) explains on the contrary, that urbanization and industrialization were usually associated at the regional level with above average, not below average mortality, and that a worker who is transferred out of low wage agrarian sector into the higher wage industrial or urban sector was sure to have a reduced life expectancy.

At pension, it is assumed the individual is not faced with the stress of work, and may most often be residing in a rather stress free environment usually in their home towns. These pensioners are usually active in social activities like funeral, naming and marriage ceremonies as they are seen as elders of society and are therefore expected to be present at such ceremonies. These categories of people are therefore not usually faced with mortality risk of the youth.

On the other hand, older people find retirement a major problem as the loss of their occupational role makes it difficult for them to readjust their personal situation. This is because occupation forms part of their identity in the society.

Especially after years of working it becomes part of a daily routine.

Work is often the most important element incorporating an individual into society. Identity, lifestyle and general life participation patterns are all determined by work. The process of retirement therefore destabilizes the major social supports of an individual by removing him or her from the world of work in which these supports are rooted.

In Ghana, mortality for older ages is fast improving and it is well known that male mortality improvement is lagging behind female mortality. As Ogwumike and Aboderin (2005) asserted, the percentage of older females in Ghana keep increasing at a rapid rate than the percentage of older males in Ghana.

These assertions on Ghanaian mortality can be appropriately made by estimation and analysis of death probability of the Ghanaian population. The Ghanaian population is made up of different subgroups, and an overall measure that does not take explicit account of the composition of the population is called crude. Crude rates is the total number of death divided by the total population in a specified period of time, Schoenbach and Rosamond (2000).

In most mortality investigations in Ghana, crude rates obtained are adjusted and standardized and used for inferences . Adjustment and standardization are procedures that facilitate the comparison of summary measures across groups. However, an Actuary goes beyond adjustment and standardization of crude rates, by employing graduation methods. Graduation methods are used by actuaries to obtain smooth mortality rates that are created from crude rates, Gavin *et al.*, (1996). This method is important because crude rates show abrupt changes which do not agree with the dependence structure of the true rates, Mazza and Punzo (2012).

1.2 Problem Statement

Crude rates obtained from a mortality investigation are not rates that are used for actuarial purposes. It is imperative to go a step further to use graduation methods to obtain age-specific mortality estimates that are a smooth function of age.

1.3 Objectives of the Study

This research generally intends to use logistic regression based modeling approach to analyze the Ghanaian male pensioners mortality experience. Specifically, to investigate the male pensioners mortality experience in five years intervals.

- To estimate mortality rates by age after pension using a graduation model.
- To analyze the variations of mortality for five year pension intervals from 1990, 1995, 2000, 2005.

1.4 Justification of study

Mortality investigation is an important research area in the insurance and pension industries. This research will provide information for annuity product pricing for pensioners since there are no known annuity products in Ghana. It will also aid insurance companies in premium pricing and pension plans.

This research will inform policy makers about pensioner welfare through mortality experience observed. This will aid the government in implementing appropriate welfare programs for the aged.

This research will also add to literature as there is little relevant literature in this area of study, specifically the Ghanaian experience.

1.5 Brief Methodology

This research is a quantitative research, which will use investigative research design to address research objectives.

Secondary data was obtained from the Social Security and National Insurance Trust.

A total sample of 15,317 male pensioners from the formal sectors will be used. Sample will be from 1990, 1995, 2000 and 2005 pension years.

A logistic regression model with binomial error structure will be fitted using statistical software package R (version 3.1).

The selection of the model is based on the fact that volume of data is large, all pensioners observed are independent of each other and the dependent variable which is age is assumed to be continuous.

1.6 Scope of study

This research intends to estimate the general mortality experience of male pensioners in Ghana, especially those in the formal sector since relevant data can only be obtained from such sectors.

1.7 Limitations of study

The major limitation on this study is the fact that accuracy of data cannot be guaranteed as data obtained are solely secondary data.

Since pensioners in the formal sector form the minority of the population of pensioners in Ghana, this research can only generalize its findings on the formal sectors.

1.8 Organization of the Study

Having introduced the study, and discussed the background, problem statement, research objectives, significance of the study and limitations in chapter one, the rest of the study will be organized in the following sequence.

Chapter two reviews relevant literature. Various mortality models and theories will be reviewed and discussed.

Chapter three gives a detailed description of methodology employed which involves the type of research design employed, the sampling techniques, sources of data and explanation of models used.

Chapter four will focus on data presentation and analysis. In this chapter, model results will be presented and analysis will be based on the findings.

Chapter five concludes the study by discussing the findings, conclusions and recommendations for policy implementations.

Chapter 2

LITERATURE REVIEW

2.1 Introduction

This chapter will review theories on mortality and discuss extensively the factors affecting mortality and deliberate on various literature on mortality in the Ghanaian environment.

2.2 Mortality Theories

The study of mortality has always been a natural subject for actuaries. Gompertz (1825) first introduced his theories which states that, the force of mortality has an exponential growth with age.This was said to be based on "philosophical principles" whereas others preferred to describe his theories as been based scientific and biological principles. Barnett (1968) and Makeham (1860) made some additions to Gompertz theories which is an age independent component, to the exponential growth.

Other theories also followed up which served as bases for various models such as Heligman and Pollard laws, Heligman and Pollard (1980). The GompertzMakeham function described by Forfar *et al.*, (1988) simplifies the original models proposed by Gompertz and Makeham. Renshaw *et al.*, (1997), also employed generalized linear and non-linear models for adjusting the Gompertz and Makeham models, Debon *t al.*, (2006).

Most of these models has it's fundamentals on two component parts of mortality, one being the rate of senescent deaths, and respective rates of different types of anticipated deaths.

2.3 Factors affecting mortality

There are various factors which play important roles in determining the mortality of groups of people. These factors are intermingled and very difficult to disassociate from each other. Yet it is very necessary for the government to identify specific mortality risk factors in order for measures to be put in place to curtail the effects.

Aside age and sex, on a broader note, standard of living can be used to describe the major factor influencing mortality, and this includes various factors such as occupation, nutrition, housing, climate, education and genetics. Some critics do argue that death rate is peculiar to each individual as the various elements contributing to quality of life varies from one person to another. This seems quite plausible as there are some information which may be privy to the individual only and computing mortality based on these known facts may be more accurate. In spite of its merits this approach may be very time wasting when dealing with a large population, where grouping lives into homogeneous groups will be faster in producing results.

2.3.1 Education and Mortality

Various studies have provided evidence that higher educational levels reduce mortality. Sorlie *et al.*, (1995), as cited in Whitehouse and Zaidi (2008), showed that in the US 60% of working aged men with 16 years of education are 60% less likely to die than those with 12 years of education and those with 5-8 years of education are 35% more likely to die, Huisman *t al.*, (2004). Elo and Preston (1996), had similar finding though their percentages do vary. Attanasio and Emmerson (2001) on the other hand argue based on their findings that education affects morbidity but not mortality. Whitehouse and Zaidi (2008) argued that education is related to mortality mainly because it increases income. And higher income implies lower mortality.

On the other hand the relationship between mortality of women and educational level is proven to be quite weak. Huisman et al. (2004). Preston and Elo (1995) also find that the death rates inequalities due to educational levels for men has widened whereas this has contracted for working – age women and has also widened for pensioners than people of working age.

2.3.2 Income and mortality

The effect of income on mortality has been widely researched as majority argue that higher income reduces mortality. For pensioners the major source of income is the monthly pension received.

Pension is an amount promised by an organization to persons on retirement until their death. In order to be able to have an ideal or fair amount of pensions to be paid, the mortality experience should be estimated in order to have a fair idea of life expectancy. However life expectancy has exceeded previous estimates and this poses a major challenge to insurance companies. This implies that mortality in recent times has decreased dramatically as people tend to live longer.

Can this be attributed to the fact that either people are receiving higher income hence lower mortality, or are there other factors playing a major role in the mortality experience of recent times. Well, in spite of the widely accepted relationship between income and mortality there are some studies which have different views.

According to Snyder and Evans (2006), lower pension results in low mortality in spite of the positive relationship established between health and mortality. It is generally assumed that the higher pensions should avail the pensioner to better health facilities and healthier lifestyles implying a longer healthier life. There are other views that generally not because of higher income or higher socio economic background that improves mortality but rather implementation of national health insurance which helps provide better health facility for economically developing countries. In contrast to these findings Logue and Blanck (2004) established that higher pensions results in lower mortality which tends to follow majority of other research Pappas *et al.*, (1993) and Schalick *et al.*, (2000). In Logue and Blanck (2004) pension amount was dependent on the level of disability and pensions were computed based on the average pension received between the period 1874 and 1890.

2.3.3 Occupation and mortality

The job people do greatly affects their mortality. Some jobs are highly risky in terms of been prone to accidents or been exposed to toxic chemicals. Some jobs are also not physically engaging leading to poor health. Occupations also reflects the educational level and level of income, therefore it is expected to have a strong relationship with mortality.

Researchers have tried to establish the relationship between occupation and mortality and a strong link has been established by many. Sorlie *et al.*, (1995) and Rogers *et al.*, (2000) show that higher occupations with higher income results in lower death rates.

2.4 Pensioners

Pensioners are a category of people who have retired from active service either due to old age or invalidity.

The population of pensioners are in three categories, according to the reason for pension.They are as follows;

Pensioners who retired at the normal retirement age and are now receiving a pension. In Ghana the normal retirement age is 60 years.

There are pensioners who retired under the ill-health retirement rules of the scheme and are now receiving a pension benefit. This is termed invalidity pension. Lastly, pensioners who retired under the early retirement rules of the scheme and are now receiving a pension benefit. In Ghana the early retirement category fall between ages 55-59 years.

But for the purpose of this research, the focus will be on people who have retired due to old age and early retirement only. In this regard it is prudent to imply that the factors facing mortality of pensioners will be different from the factors influencing mortality of the youth or people in active service.

2.4.1 Pensioner Mortality; the Ghanaian experience

Mortality rates, in recent times has been decreasing globally, hence emphasizing the critical importance of investigation into mortality forecasting and factors influencing this phenomenon. Many researchers have made projections based on falling death and birth rates with most countries having estimates of older persons dominating the population than younger persons in the next few years.

According to Kinsella and Wan (2009), as at 2008 7% of the world's population was 65 years and over. This was projected to increase to 14% by 2040.They further asserted that 62% of the world's older population were living in developing countries.

Ogwumike and Aboderin (2005) made an assessment that the population of Africans who are 60 years and above will increase four fold from 45.7 million to 182.6 million in 2050. The aged in the African population will concentrate more in the sub-Sahara specifically Ghana which will have the highest rise in old age population.

Naturally as the human body ages beyond certain limits it becomes weaker, productivity reduces drastically and become more prone to diseases and consequently death. So by the laws of nature, as we grow older our mortality becomes higher. However, according to Bizz 101 (2015), life expectancy has increased over the past few years, in 1980 life expectancy in Ghana was 51 for men and 53 for women and as at 2012 life expectancy has increased to 62 for men and 64 for women. This brings into light the fact that, the average life expectancy has increased by 11 years over the period of 32 years. But however, as this is worth celebrating for the human race in general, insurance and pension industries may deem this development as draining their profit away since they will keep paying life annuities and pensions as long as the individual is alive. These finding instigated the argument that the legal retirement age should be increased in order for the country to benefit from its experienced older population.

The increasing life expectancy over the years implies that in the near future the population of older ages will keep increasing as stated by Ogwumike and Aboderin

(2005) they gave estimates that in 2005, 5.4% of the Ghanaian population were 60 years and over and projected 7.6% and 14.6% of the population to be 60 and over in 2025 and 2050 respectively.

Pensioners in Ghana however are still in the bracket of high mortality as compared to other countries which has life expectancy way above 62 and 64, and there are various reasons for this. Poverty which affects the physical. emotional and mental well-being of an individual, is a major set back in mortality od pensioners in Ghana . In Ghana and many other African countries, policies to reduce poverty are mainly focused on children, mothers and the youth. Hence the quality of life after pension becomes the sole responsibility of the pensioner with reliance on the meager pension salary and family support which is not even guaranteed. Recently there has been some efforts by the ministry of gender to help senior citizens by providing free medical care for people over 65 years.

According to the 2010 Ghana population and housing census older people in Ghana are in the following age brackets 60 - 74 (young-old), 75 - 84 years (oldold) and 85+ years (very old). These categories of older persons are mostly pensioners, both in the formal and informal sectors of the economy.

The vast majority of the older persons are in the informal sector with minority in the formal sector. Most pension schemes however are focused on the small minority of public and other formal sector workers leaving larger majority with no income security after pension.

In Ghana, aside pension coverage for formal sectors there are provisions for pension coverage for the informal sector which are both managed by the Social Security and National Insurance Trust. This may have contributed to Ghana having a rather increasing aging population as compared to other African counties.

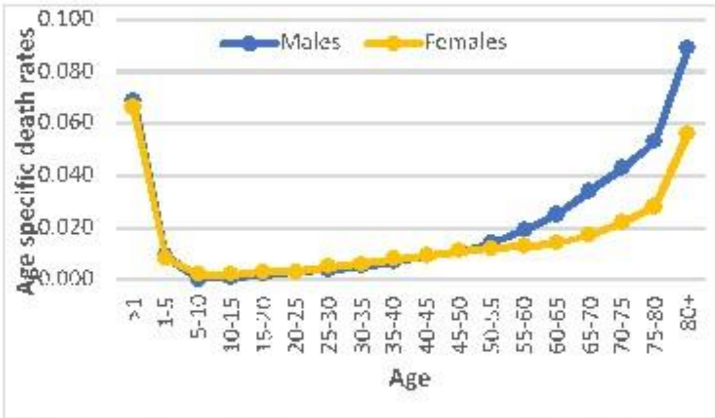
However, it is an established fact that older pensioners have lower income than their younger counterparts and even worse off when compared with younger working age people. Radner (1986) finds that 65-69 year olds had incomes 30% higher on average than 80-84 year olds, in Johnson and Stears (1995) findings from the United Kingdom data,80 year olds were 20% worse off than 65 year olds. Therefore generally poorer populations in most countries constitute majority of pensioners. Having low income has its implications with mortality which is not quite favorable.

Research on the Ghanaian mortality experience is broadly focused on child and maternal mortality. With very few focus on mortality of pensioners.

From the Ghana Statistical Service 2010 census data age specific death rates were computed from ages less than one to 80 years plus, to compare mortality at different ages and sexes. However in all age groups female death rates are lower than male death rates.From the age brackets 50-54 years and above, females have lower death rates than males in urban areas than rural areas. Females in the bracket 75-79 from the rural areas have higher death rates than those in the urban areas. Generally there are not much difference in death rates between ages 15-54, but ages 55 and above have quite large differences in death rates for both males and females.

2.5 Approaches to mortality estimates

Mortality forecasting has been a major research area in recent times due to increasing longevity patterns worldwide. Predicting mortality experience is very complex and a challenging task and many disciplines have interest in this area, hence different models and approaches have been developed. We have the casual models or non-demographic models and the actuarial and demographic models.



Source: 2010 Population and Housing Census; National Analytical Report (2013)

Figure 2.1: Age patterns of mortality by sex (2010)

Keilman (2003), explains that demographic or actuarial models are typically aggregate models which describe how the mortality of groups of individuals evolves over time. It typically uses historical data to estimate changes over time and usually, mortality risks are primarily broken down by demographic variables such as age and sex. Non-demographic or causal models emphasize causal factors in mortality such as health, socio-economic status, environmental conditions and access to health care, Hudson (2006). That is, the focus here is the factors that influence the life chances of the individual.

According to Booth and Tickle (2008), mortality modeling are categorized in three broad approaches, which are expectation, explanation and extrapolative approaches. Generally the expectation approach is subjective and conservative. Explanation approach is restricted to certain causes of death with known determinants and the extrapolative approach makes use of statistical methods of estimation, its emphasis is on age patterns and trends over time.

In most cases of mortality forecasting some aspects of each approach is applied, hence exact distinction between the three approaches is not clearly cut.

2.5.1 Expectation Approach

This approach is purely based on expert opinion, most statistical agencies and actuaries were known to have relied heavily on this approach in the past, but currently are adopting a more complicated extrapolative approach. CMIB (2002, 2004, 2005, 2006, 2007) as cited in Booth and Tickle (2008).

Targeting of life expectancy is one method that was mostly used, here a value is assumed for a future date and a specified path is determined, Olshansky (1988). Targeting has also been conventionally applied to age-specific mortality reduction factors. The Continuous Mortality Investigation Bureau (CMIB) of the United Kingdom Institute and Faculty of Actuaries has projected the proportion of lives of exact age x who die before attaining exact age $x + 1, q_x$, from year 0 by multiplying by the t-year reduction factor $RF(x,t)$:

$$RF(x,t) = a(x) + [1 - a(x)].[1 - f_n(x)]^{\frac{t}{n}} \quad (2.1)$$

$a(x)$ and $f_n(x)$ represent respectively the ultimate reduction factor and the proportion of the total decline $(1 - a(x))$ assumed to occur in n years. The approach embodies an exponential decline in mortality over time to the asymptotic value, $a(x)$ and uses expert opinion to set targets $a(x)$ and $f_n(x)$. Booth and Tickle (2008).

The merits of the expectation approach is that it includes epidemiological and demographic factors, and other relevant information in a qualitative way. This approach is however prone to bias and subjectivity, expert opinion is also subject to "assumption drag". Where expectation trial behind actual experience instead of the other way round.

2.5.2 Explanation approach

Explanatory methods of mortality forecasting are based on structural or causal epidemiological models of certain causes of death involving disease processes and known risk factors, Booth and Tickle (2008).

This approach needs an in-depth understanding of medical conditions and socio economic situation that leads to death. The understanding of causes of death and mortality is yet to be fully developed. Therefore the use of this approach solely is rare. Most often a hybrid of extrapolative, expectation and explanation approach is used.

Most of the models used in this approach are regression based and therefore are within the generalized linear model framework, Tabeau *et al.*, (2001). The difference between this and regression based extrapolative model is that it incorporates explanatory variables or risk factors, which are either lagged or forecast.

Booth and Tickle (2008) further stated that the main advantage of the explanatory approach is that feedback mechanisms and limiting factors can be taken into account.

2.5.3 Extrapolative Approach

As noted earlier this approach makes use of historical data and the assumption here is that past trends will continue into the future. This assumption cannot always hold as some exceptions do occur that will alter trends. Situation such as the outbreak of Ebola, AIDS or other epidemics or natural disasters which can claim the lives of large numbers of the youth. This approach is used for long term forecasting hence large data samples is most appropriate.

In earlier years mortality forecasting was quite simple and it involved some degree of subjective judgment, Pollard (1987). Until quite recently, more complicated approaches were developed of which the Lee Carter model has been predominant, Stoeldraijer *et al.*, (2013). This method summarizes age specific mortality and period for a single population and how mortality evolves over time. Lee and Carter (1992), Stoeldraijer et al. (2013).

The strengths of the extrapolative approach hence the Lee Carter model includes its robustness in age specific mortality with linear trends, Booth *et al.*, (2006). The Lee Carter model can also be seen as a special case of log–linear model for contingency tables Bishop *et al.*, (1975) and King (1989).

2.6 Some models for mortality estimates

Ronald D. Lee and Lawrence Carter introduced the Lee Carter (LC) model in 1992 using US data to forecast age-specific mortality from 1990 to 2065, (Lee and Carter (1992)). This method is most widely used for forecasting age-specific mortality today.

Girosi and Gary (2007) show that this model is a special case of a considerably simpler, and less often biased, random walk with drift model.

The first step of the Lee carter model consists of modeling mortality rates using $\log m_{xt}$ in terms of vectors a and b along the age dimension and T along the time dimension such that

$$logm_{xt} = a_x + b_x T_t \quad (2.2)$$

Where a_x and b_x are parameters to be estimated. The constraints here are such that:

$$\sum_x b_x = 1 \quad (2.3)$$

$$\sum_t T_t = 0 \quad (2.4)$$

Some extensions on the Lee Carter model includes the Generalized Lee Carter model by Haberman and Renshaw (1996). Here a cohort effect is added to the original model given as:

$$logm(x,t) = a_x + b_x k_t + c_x y_{k-t} \quad (2.5)$$

This model is also exposed to identification problem hence the following constraints applies, Cairns *et al.*, (2009):

$$\sum_t k_t = 0 \tag{2.6}$$

$$\sum_x b_x = 1 \tag{2.7}$$

$$\sum_x c_x = 1 \tag{2.8}$$

$$\sum_{k-t} y_{k-t} = 0 \tag{2.9}$$

Currie *et al.*, (2006), introduces the Age Period Cohort (APC) model

$$logm(t,x) = a_x + \frac{1}{n_a}k_t + \frac{1}{n_a}y_{t-x} \tag{2.10}$$

Where n_a is the number of ages in the data set. The APC model has its origins in medical statistics. The constraints imposed here are:

$$\sum_t k_t = 0 \tag{2.11}$$

$$\sum_{t-x} y_{t-x} = 0 \tag{2.12}$$

According to Cairns *et al.*, (2009), Renshaw and Haberman (RH), Lee Carter and Currie all have the underlying assumptions that qualitatively ,age, period and cohort effect are different in nature and hence the need for them to be modeled in different ways. The continuous mortality investigation bureau (CMIB) employed the extrapolating formulae derived from past mortality data to make predictions for future mortality improvements. Cairns (2000) identified three uncertainties involved in using this formulae

Model uncertainty - the appropriate statistical model to be adopted may be unclear

Parameter uncertainty - uncertainty in the parameters to be used in the model Stochastic uncertainty - the exact outcome is partly dependent on random fluctuations, hence the outcome of these models are deemed stochastic.

2.7 Calculating Crude rates

The choice of models used depends on whether the interest lies in the continuous rates, that is force of mortality or discrete rates, that is probability of death.

2.7.1 Poisson model

This model estimates the force of mortality $\mu_{x+\frac{1}{2}}$ at time $x+\frac{1}{2}$. The distribution for the number of death Dx at age x

$$Dx \sim Poisson(E_x \mu_{x+\frac{1}{2}})$$

With expectation $E_x \mu_{x+\frac{1}{2}}$, $\mu_{x+\frac{1}{2}}$ is the expected number of deaths for age $(x,x+1)$ and E_x is the number at risk for age x. The force of mortality here is computed by

$$u = \frac{d_x}{E_x^c} \tag{2.13}$$

where E_x^c is the central exposed to risk given by

$$E_x^c = E_x - \frac{1}{2} d_x \tag{2.14}$$

2.7.2 Binomial model

The binomial model is an important probability model used where there are two possible outcomes in a discrete data set. In probability of death estimates, the major two outcomes are either an observed life dies or stays alive.

This model is used to estimate the probability of death q_x. The number of death d has a distribution of $D \sim Binomial(N,q_x)$. Where N is the number of lives observed.

There are three assumption governing the binomial distribution;

- Each replication of the process results in one of two possible outcomes,that is success or failure.
- The probability of success is the same for each replication.
- The replications are independent.

The estimate of probability of death under this model is given by

$$q_x = \frac{d}{N} \tag{2.15}$$

with an unbiased mean of q_x and variance $q_x(1 - q_x)/N$; $0 < q_x < 1$

The assumptions underlying this model are that;

- Uniform distribution of deaths: this implies an increasing force of mortality

$$_tq_x = tq_x \tag{2.16}$$

- The Balducci assumption: this implies a decreasing force of mortality between integer ages

$$_{1-t}q_{x+t} = (1 - t)q_x \quad (0 \le t \le 1) \tag{2.17}$$

- Constant force of mortality

$$_tq_x = 1 - e_{-ut} \quad (0 \le t \le 1) \tag{2.18}$$

2.8 Graduation

Graduation is defined by Haberman and Renshaw (1996), as the methods by which crude probabilities are fitted to provide estimate that are a smooth function of age.

A desirable graduation should have features such as smoothness, adherence to data and should be suitable for the purpose.

Smoothness and adherence to data do conflict with each other. When graduated rates show little adherence to data, yet achieves a smooth curve, this will result in over graduation and the vice versa, where there is more adherence to data but inadequate smoothness is achieved, this will result in undergraduation. The purpose for which the work is done serves as a guideline to determine certain expectations.

For example in the pension or annuity companies it will be detrimental to overestimate death as it will imply that pension or annuity cost estimations will be based on overestimated death, and Payments may be made for longer periods than expected.

2.8.1 Methods of Graduation

Graduation methods are classified in two, we have the parametric methods and nonparametric methods, and this depends on whether they adjust the data to a function or directly achieves smoothness, Debon *et al.*, (2006).

However the two main approaches to graduation includes graduation by parametric formula, and graduation nonparametric formula.

- Graduation by Parametric formula

  The main widely used models are the Gompertz (1825) and Makeham (1860) models. These are best fitted with older age groups. The Gompertz assumes force of mortality grows exponentially with age whereas Makeham adds a constant (which could be accidental death which has no relation with age) to the exponential growth of mortality (which is inclined to senescence death.

  Over time as mortality among the young and middle age increased, it became difficult to use Makeham formula to obtain desirable graduation, Debon *et al.*, (2006). This led to the introduction of new models known as the Heligman and Pollard laws, Heligman and Pollard (1980). The Gompertz and the Makeham models has the following formulae; Gompert (1825):

$$\mu_x = Bc^x \tag{2.19}$$

Makeham (1860):

$$\mu_x = A + Bc^x \tag{2.20}$$

where μ_x is the force of mortality and A, B and c are parameters to be estimated.

- Graduation by non-parametric formula

  By this methods graduated rates are estimated by smoothing crude rates calculated from the original data. This is best used when some information on data used is unavailable, since the age-dependent function assumption is not a requirement as it is in parametric methods.

  According to Tomas and Planchet (2013), there are two approaches to smooth crude rates under this methods.

  – The Exogenous Approaches

    This is where actual mortality experience is compared with a standard table. This becomes a reference table where which is adjusted to the experience of a given data

  – The Endogenous Approaches

    This involves analyzing information on the crude rates and using that to obtain a smooth set of rates which can be used to make realistic projections. It is best used for large volume of data and can produce biased results when a small data sample is used.

  According to Debon *et al.*, (2006), some of the well-known methods includes the kernel smoothing and splines smoothing.

  – Kernel smoothing

    This was initially developed to estimate density functions

$$\omega_x = \frac{k(\frac{x-x_i}{b})}{\sum_{i=1}^{r} k(\frac{x-x_i}{b})} \tag{2.21}$$

    where b is the bandwidth and k is the kernel function.

– Splines smoothening

The objective here is to find the estimator q_x by minimizing

$$w = \sum_{j=1}^{n} w_j (q_j - q_j)^2 + \delta \sum_{j=1}^{n-m} (\Delta^m q_j)^2 \tag{2.22}$$

The smoothening parameter is defined as;

$$\Delta q_j = q_j - q_{(j-1)} \tag{2.23}$$

Δ^m is the difference operator applied m times.

2.9 Desirable properties of theoretical models

There are certain criteria that are expected to be met when modeling, and mortality rates modeling is no exception. These criterion are in other words desirable properties which are expected from a good model. Cairns (2000) further explains some of these properties.

Parsimony in this context implies a model with fewer parameters. This is more preferable than a model with numerous parameters. Though more parameters indicates more individual homogeneous groups, it makes the model complex and prone to errors and oversights.

Transparency also means that the model should be clear and uncomplicated to avoid inappropriate use. No part of the output or process should be obscured whatsoever.

Thirdly, whether the model has the ability to generate sample paths for underlying and unobservable death rates and whether cohort effects are allowed for, should they be present.

Finally, the ability of a model to produce a non-trivial correlation structure between the year-on-year changes in mortality rates at different ages.

He further states that none of the three models explained have a clear cut distinctions as to whether they possess the property of parsimony and transparency but clearly all the above models have the ability to generate sample paths. With the exception of LC, the other two models, that is RH and Currie do incorporate cohort effects. The non-trivial correlation structure is obviously not present in all three models.

Chapter 3

METHODOLOGY

3.1 Introduction

This chapter is solely focused on how this research was carried out, it explains further the type of data used how it was analyzed by use of methods and models employed.

3.2 Research Design

This is a quantitative research employing investigative research design to find out the mortality experience of male pensioners in Ghana.

3.3 Sources of Data

Secondary data was obtained from the Social Security and National Insurance Trust (SSNIT). Information was obtained on pensioners date of birth and death if any, year of retirement. At the time of study pensioners were observed from year of pension to June 2014, which is the last date life certificates were updated as at the time data was retrieved. Hence any pensioner whose life certificate had not been updated as at that date was assumed dead until otherwise proved.

3.4 Population sampling techniques

The general population of pensioners includes invalidity pensioners, hazardous workers pensioners, old age pensioners, and early retirees.

For the purpose of this study, the target population comprises of both old age and early retirees. By old age retirees we mean individuals who go on retirement at the normal retirement age of 60 years. The early retirees include individuals who opt go on retirement from the ages 55 to 59 years.

The sample size chosen depends on the number of retirees in each retirement year. For the retirement year of interest, all lives who went on retirement in those particular years were sampled.

3.5 Sampling

The sample size chosen is the total number of pensioners who retired in particular years of interest which are 1990, 1995, 2000 and 2005.

The five year interval between retirement years observed, is a reasonable period to make justified comparisons. Life expectancy of Ghanaian males has increased from 59 years in 2009, to an average of 63.5 years in 2010, Ghana Health Service (2010) and GSS (2010). It is expected that there will be a significant change in mortality of male pensioner within this period.

Purposive sampling was employed on the total population of 102,118 male pensioners, which spans from 1990 to 2014 retirement years. This method was employed in order sample pensioners who retired in the particular years of interest.

The total sample obtained consists of 15,317 male pensioners. For the pension year 1990 we have 670 pensioners,1995 pension year we have 4,317 pensioners, pension year 2000 we have 4,748 pensioners and 5,582 pensioners for 2005 pension year.

3.6 Data collection

In order to have a complete set of accurate data,data collected was cleaned up,missing and incomplete entries were removed.

Pensioners who had not renewed their life certificate as at the time of research and had their pension payments ceased were assumed dead as at the date of last update.

For the purpose of confidentiality member identification numbers were removed and data were regrouped to have three essential details, date of retirement, date of death or last update and current age if still alive.

For each group of pensioners in a retirement year, data was sorted to obtain the age at pension, number of deaths at each age, and the exposed to risk at each age.

Pensioners were exposed to investigation from the year they retired to June 2014. From the data, after the age 80, reported deaths were very scanty and to avoid distorted or misleading results the investigation ended at age 80.

3.7 Data analysis techniques

The data was analyzed using the logistic regression model with a binomial error structure, fitted using the statistical software package R(version 3.1).

3.7.1 Binomial model

The binomial model is a very simple method of estimating crude rates, used only when the number of death is known at a certain time within the year or period of investigation.

This model is used to estimate the probability of death q_x. The number of death dx has a distribution of $D \sim Binomial(N,q_x)$. Where N is the number of lives observed and q_x is the probability of death. The estimate of probability of death under this model is given by

$$q_x = \frac{d}{N} \tag{3.1}$$

with an unbiased mean of q_x and variance $\frac{q_x(1-q_x)}{N}$.

3.7.2 Logistic Regression model

Logistic regression with binomial errors will be used to fit the observed data. β_o and β_1 are parameters to be estimated. These estimates will be obtained as output from the statistical software package R.

$$\log\left(\frac{q_x}{1-q_x}\right) = \beta_o + \beta_1 x \tag{3.2}$$

with a suitable transformation for log odds given as: $\log(q_x/1 - q_x)$ which transforms q_x on to the interval $(-\infty,\infty)$ this is done to enable the functions to be used in the smoothing process.

The assumptions underlying the logistic regression are as follows:

Logistic regression call handle all sorts of relationships between variables. It does not necessarily need a linear relationship between the dependent variables, because it applies a non-linear log transformation to predicted odds ratio The logistic regression requires each observation to be independent. All pensioner lives observed are independent of each other.

The independent variables do not need to be multivariate normal- although multivariate normalty produces a more stable solution. Lastly it can handle ordinal and nominal data as independent variables. The independent variables do not need to be metric (interval or ratio scaled).

Logistic regression assumes linearity of independent variables and log odds. Though it does not require the dependent and independent variable to be related linearly.

Lastly it requires a large sample sizes, to provide sufficient numbers in both categories of the response variable.

These assumptions are requirement for the data to be used and all these assumptions are in line with data structure. The sample size to be used is large, variables in the data are independent of each other, and all observed deaths are independent of each other.

3.7.3 Maximum likelihood estimation

According to Myung (2003), the maximum likelihood estimator is the parameter value that best relates to the probability density function that makes the observed data most probable.This is an alternative way of computing the population variable q_x

Suppose we have $x_1,x_2,...,x_n$ of n independent and identically distributed observations

$$f\gamma(x_1,x_2,...,x_n) = f(x_1,x_2,...,x_n|\gamma)$$

The likelihood function of γ is the function;

$$L(\gamma) = f(x_1, x_2, ..., x_n|\gamma) \tag{3.3}$$

$$\gamma = \prod_{i=1}^{n} f(x_i|\gamma) \tag{3.4}$$

Myung (2003) further states that the optimal properties of the maximum likelihood estimator (MLE) includes sufficiency, consistency, efficiency, and parameterization invariance.

3.7.4 Akaike Information Criterion (AIC)

AIC assesses the quality of statistical models, it does this by measuring the quality of each model used, and hence it serves as a means for model selection. In a scenario where various models are applied, the best model will be the one with the minimum AIC value.

AIC serves as a criterion for model estimation and selection.

Given that y is the number of estimated parameters, and L the maximized value of the likelihood function, the AIC is generally given as;

$$AIC = 2y - 2ln(L) \tag{3.5}$$

Supposing we have a true model $m(y)$ and an approximating model $s(y|\theta_k)$ with fitted model as $s(y|\breve{\theta}_k)$:

$$AIC = 2lns(y|\check{\theta}_k) + 2k \tag{3.6}$$

However;

$$E(AIC) + 0(1) = \delta k \tag{3.7}$$

where E represents expectation and δk is the expected Kullback discrepancy, it reflects the average separation between the true model and the fitted model.

Chapter 4

DATA PRESENTATION ANALYSIS AND DISCUSSION

4.1 Introduction

This chapter is solely dedicated to presentation and interpreting of data collected for the study. Analysis of data will be done on each cohort for retirement year observed. Crude rates, and resulting graduated rates will be computed, in order to assess observed findings and answer research questions raised.

4.2 Analysis of Death data

Deaths were recorded over the period, and total pensioners who died at each age were computed. Appendix A shows the number of death for each group observed from year of retirement. From the data 670 lives retired in 1990 and 55.37% died within the observed period, in 1995, 4,317 lives retired and 55.59% died within observation period. 4,748 lives retired in 2000 and the percentage of recorded death was 82.62% and in 2005 the percentage of recorded death was 54.37% of 5,582 lives who retired in that year.

Table 4.1: Recorded death of Male pensioners

	1990	1995	2000	2005
Sample	670	4,317	4,748	5,582
Percentage of Deaths	55.37 %	55.59%	82.62%	54.37%

4.3 Analysis of initial exposed to risk

Exposed to risk is the number of pensioners who were at risk of death at a particular age. This was computed by adding the cumulative survivors from previous years to the number at risk for the current year. From Appendix B, in 1990 the exposed to risk increases for ages 55 to 68 as the number of deaths are less than the new entrants into the observation period. From ages 69 to 80 the number of deaths were more than the new entrants hence the number exposed at these ages keeps decreasing.

The new entrants into the observation period are lives who retire at various ages from 55 to 80 years. It is quite normal to have people retiring at ages 55 to 65. But peculiar to the data collected there are lives who go on retirement as late as 70 years and above. The explanation here is that, though some individuals had attained the normal retirement age, they may have reported for pension claims at an age way above the normal retirement age and had started receiving pension at that age, based on the date of birth initially reported. These new entrants are quite a few though and would not affect data analysis in any way as they were also observed for the same period.

4.4 Estimating initial probability of death

The binomial model was employed since its best suitable for older ages. Under this model the distribution for number of death d_x is given as;

$$D_x \sim B(E_x, q_x)$$

with expectation $E_x q_x$ and variance $E_x q_x(1 - q_x)$.

The estimate of probability of death is given by;

$$q_x = \frac{d_x}{E_x} \tag{4.1}$$

From the data crude rates q_x,that is, the probability of death at age x for ages x = 55 to x = 80 where d_x is the number of death and E_x is the initial exposed to risk representing the number of pensioners exposed to risk at age x. For the four retirement years, the plots below shows graphically the observed probability of death for each cohort. In Fig 4.1, it is observed that the crude rates have a

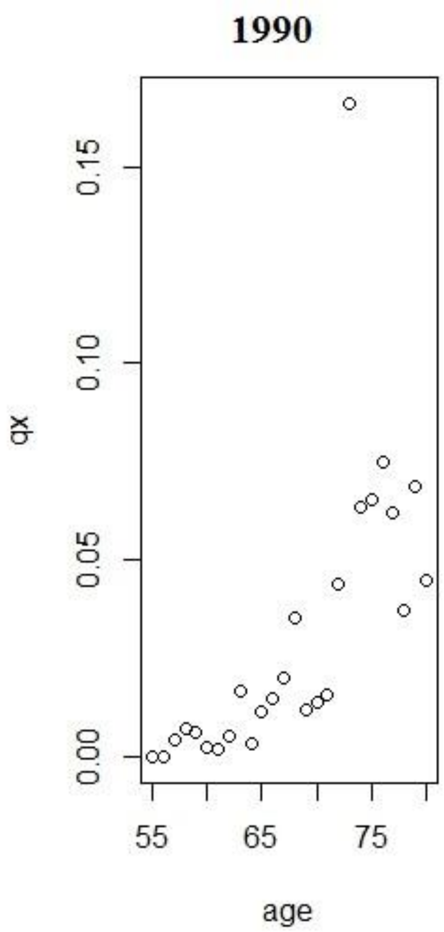


Figure 4.1: Plot of crude rates for 1990 male pensioners

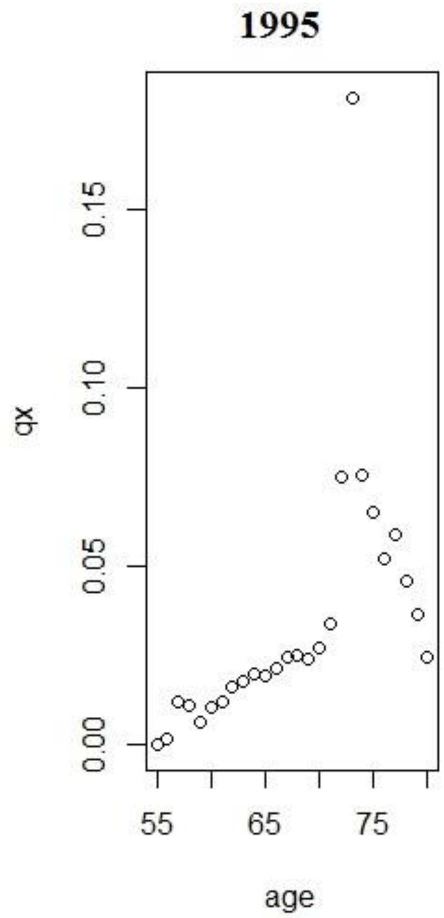


Figure 4.2: Plot of crude rates for 1995 male pensioners

consistent flow from age 55 to 71, it gets quite scattered at older ages, with an out-lier at age 73. The out-lier indicates that there are high observed deaths at that age. Plots in Fig 4.2 indicates a consistency in rates increment from age 55 to 71 and increases at a diminishing rate from 74 to 80 years. Interestingly, an outlier is observed at age 73 which is similar to the out-lier in the plot for 1995 group of retirees. Fig 4.3 shows a similar trend to the plots discussed above, the

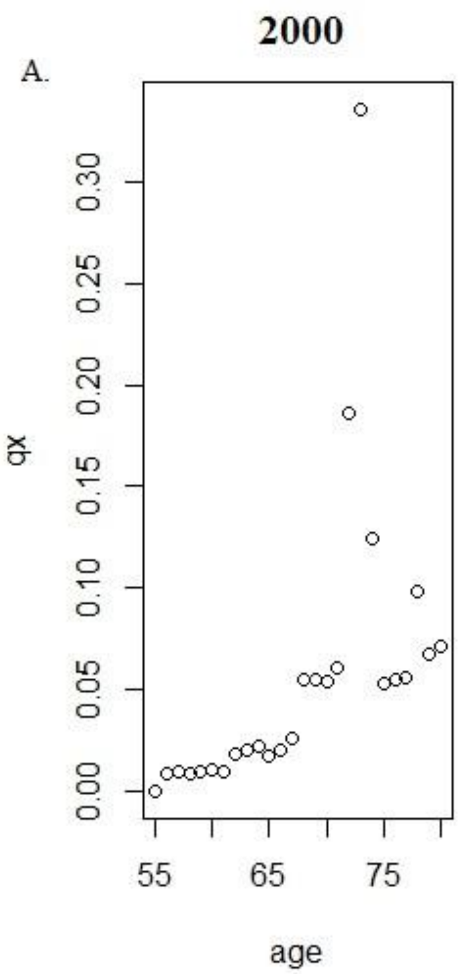


Figure 4.3: Plot of crude rates for 2000 male pensioners

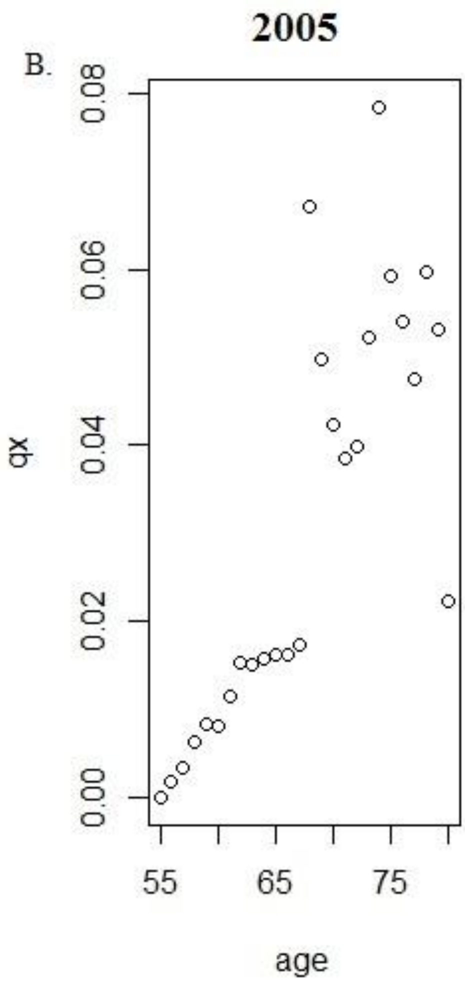


Figure 4.4: Plot of crude rates for 2005 male pensioners

consistency of increment in rates is from 55 to 71 years with an out-lier at 73. Ages 72 and 74 also have rates high above the consistent flow though not as high as at 73 years. From age 75 the plots fall back in line and continues the consistent incremental flow to 80 years.

For Fig 4.4 Crude plots for 2005 pensioners is quite scattered at older ages from age 68 to 80 years, the incremental consistency in rates starts from age 55 to age 67. At age 68 the rates starts decreasing gets to 71, increases to the highest point at age 74 and starts decreasing once again.

4.5 Graduation

Debon et al. (2006), made assertions that because the sequence of crude death probabilities generally presents abrupt changes, it is necessary to graduate crude rates in order to correspond with the hypothesis that the probabilities of death for two consecutive ages should be very close.

Graduation therefore needs to be done to obtain graduated estimates of crude rates that are a smooth function of age, as well as to remove sampling errors. It is necessary to employ suitable graduation methods to transform the discrete state of the crude rates to continuous form of graduated rates.

A logistic regression model with a binomial error structure is the most appropriate model to be fitted, this is given as;

$$\log(\frac{q_x}{1-q_x}) = \beta_o + \beta_1 x \tag{4.2}$$

where x =age, β_o and $\beta_1 x$ are parameters to be estimated.

In order to obtain an appropriate function for smoothing, the log-odds of the probability of death q_x is estimated, since probability is the function to be estimated. The log-odds is defined on the range $[-\infty,\infty]$ with the binomial model;

$$\log(\frac{\tilde{q}_x}{1-\tilde{q}_x}) = \tilde{\beta}_o + \tilde{\beta}_1 x \tag{4.3}$$

The model was fitted in a statistical software package R. A two column matrix was used, the matrix for each retirement year in question mort.90, mort.95, mort.2000, mort.2005 has two columns $c(1,2)$ the first column contains d_x and the second is E_x. The age x = 55 to x = 80 can be found in the column vector x. The odds ratio is given as $F_x = \frac{q_x}{1-q_x}$, the output for the log of the odds are displayed in Fig 4.5 to Fig 4.8 for all four observed cohorts. With a negative intercept for all four models the plots for the log-odds are less than one. For each of the years the parameters

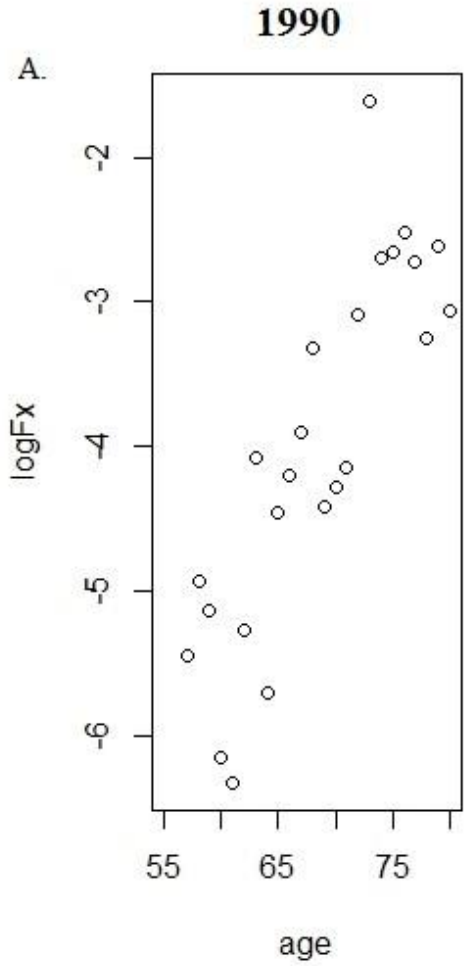


Figure 4.5: Plot of log odds against age for pensioners in the year 1990

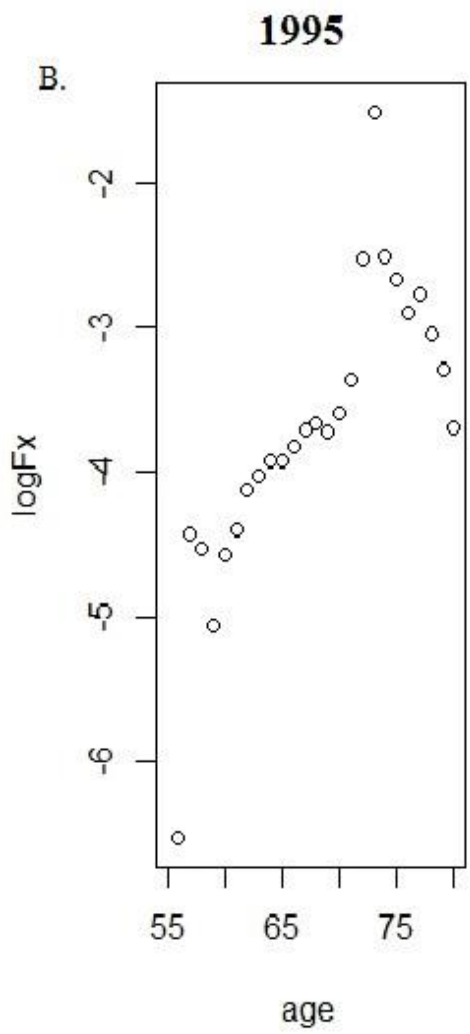


Figure 4.6: Plot of log odds against age for pensioners in the year 1995

$\tilde{\beta}_0$ and $\tilde{\beta}_1$ are obtained from the output of the statistical software R. For the four retirement years been investigated the readings obtained are as follows From Table 4.2, age is significant in all the four logistics models. The intercepts for all

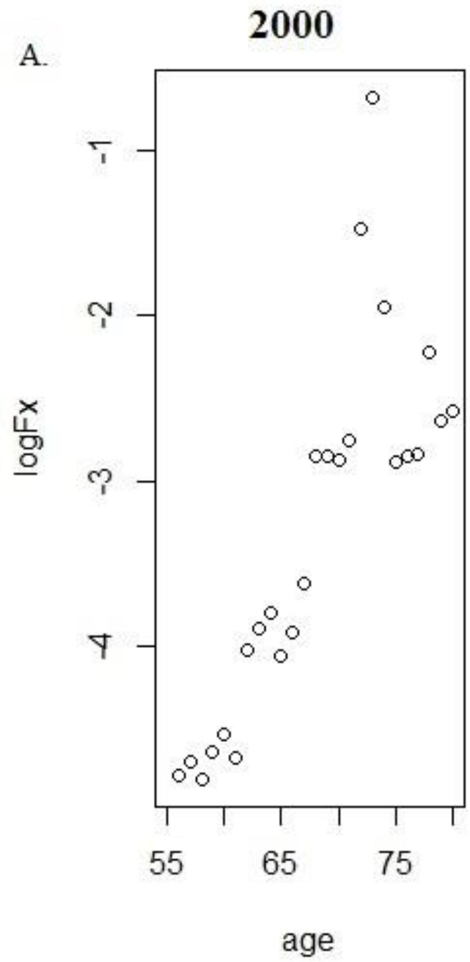


Figure 4.7: Plot of log odds against age for pensioners in the year 2000

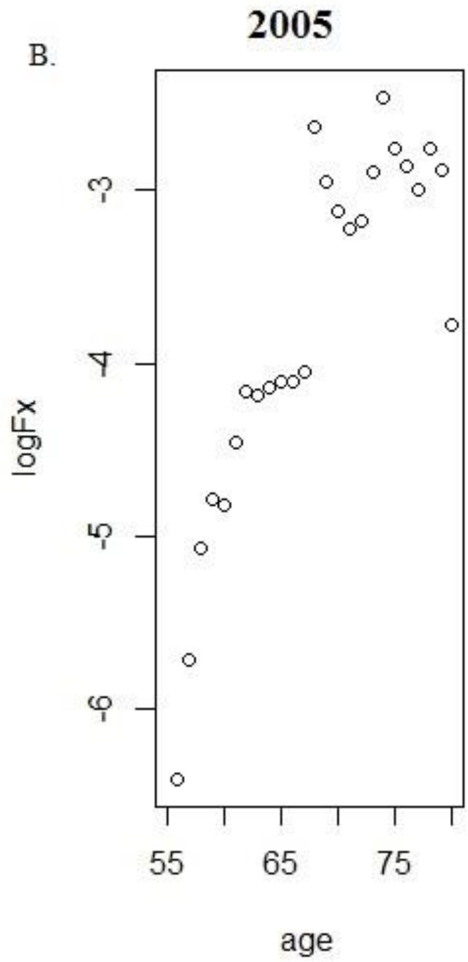


Figure 4.8: Plot of log odds against age for pensioners in the year 2005

Table 4.2: Parameter estimates table(standard error)

	1990	1995	2000	2005
Intercept	-12.646(0.0695)	-9.371(0.253)	-11.334(0.221)	-8.958(0.225)
Age	0.130(0.0095)	0.087(0.0035)	0.122(0.0031)	0.080(0.0031)

the four different groups are negative throughout as expected. retirement year 2005 has the highest intercept value. From the model this indicates the highest constant independent of age ($\tilde{\beta}_o$). 1990 has the least constant independent of age value. The values for age x are positive throughout, and cohort year 2005 has the highest value. From the model these represent the parameter $\tilde{\beta}_1$. Using the

Table 4.3: Goodness of fit test estimates

	1990	1995	2000	2005
Null deviance	390.10	1522.37	3580.4	1230.71
Residual deviance	174.96	888.16	1931.4	558.04
Chi-square test	1.042957e-48	1.03568e-48	1.04523e-48	1.012480e-48
AIC	273.99	1039.3	2088.8	713.93

*AIC=Akaike Information Criterion estimates of $\tilde{\beta}_o = -12.646325$ and $\tilde{\beta}_1 = 0.130196$ the graduated estimates for

1990 retirement year $\tilde{q}_x$ are given by;

$$\log(\frac{\tilde{q}_x}{1-\tilde{q}_x}) = \tilde{\beta}_o + \tilde{\beta}_1 x \tag{4.4}$$

$$\tilde{q}_x = \frac{1}{1+\exp(12.464325 - 0.130196x)} \tag{4.5}$$

This is done for all ages $x = 55$ to $x = 80$ to obtain the graduated rates for 1990 as indicated in the plots below. The graduated rates show a smooth curve which increases with age with some few plots outside the range. The age 73 plot is a significant outliers because of how far is it outside the range.

The graduated rates for 1995 is computed by using $\tilde{\beta}_o = -9.37074$ and $\tilde{\beta}_1 = 0.08666$. From Table 4.2 the resulting rates provide a smooth curve as indicated in Fig 4.10, the curve depicts the trend at younger ages better than at older ages.

$$\tilde{q}_x = \frac{1}{1+\exp(9.3704 - 0.08666x)} \tag{4.6}$$

The resulting graduated rates for the retirement year 2000 are computed by using $\tilde{\beta}_o = -11.334365$ and $\tilde{\beta}_1 = 0.121963$ from Table 4.2. The curve in Fig 4.11

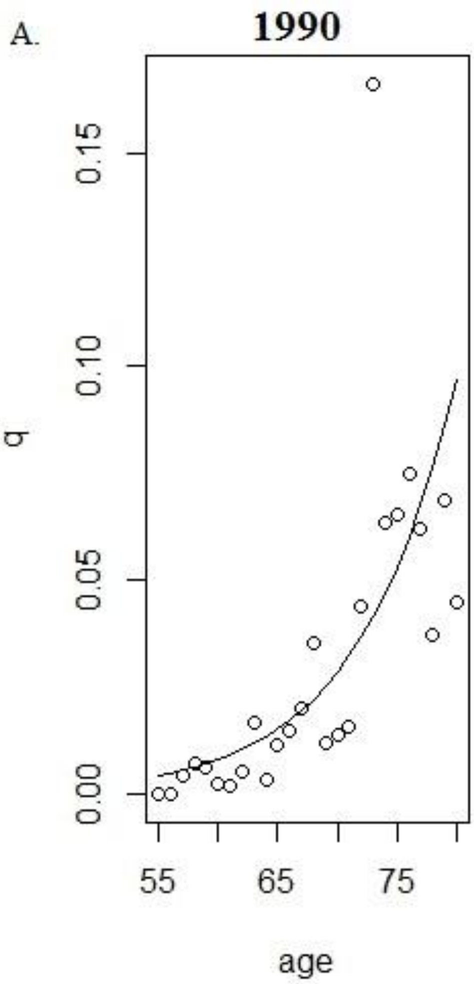


Figure 4.9: Plot of graduated estimate of the probability of death against age for pensioners in 1990

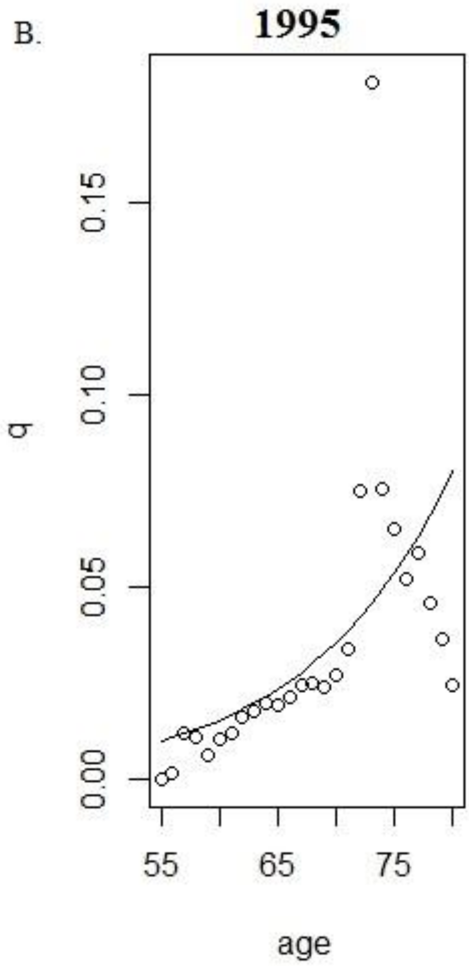
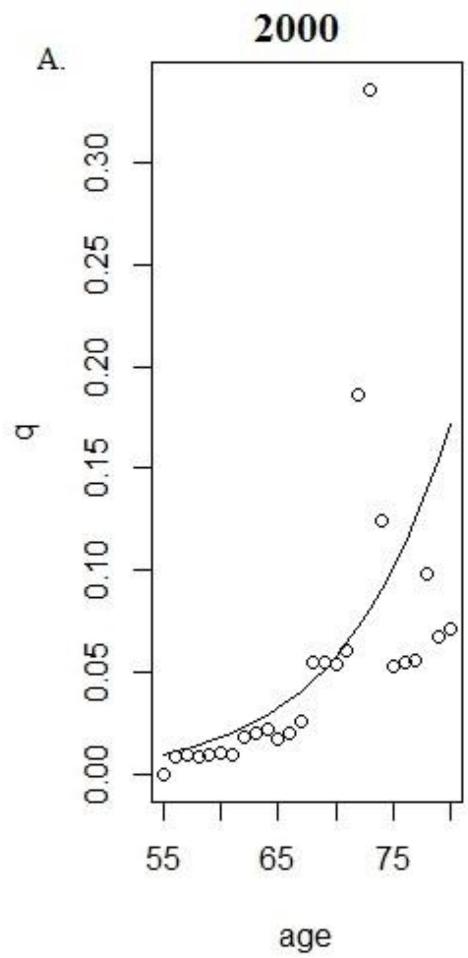


Figure 4.10: Plot of graduated estimate of the probability of death against age for pensioners in 1995



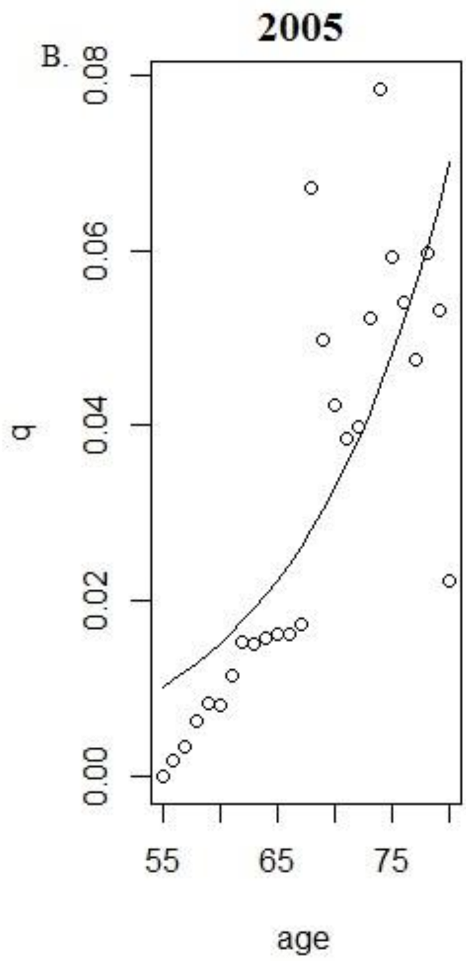


Figure 4.11: Plot of graduated Figure 4.12: Plot of graduated estimate of the probability of death estimate of the probability of death against age for pensioners in 2000 against age for pensioners in 2005

is a smooth curve which will indicate mortality estimate for all integer ages. The resulting parameters for the lives who retired in year 2005 are given as $\tilde{\beta}_0 = -8.95817$ and $\tilde{\beta}_1 = 0.07967$. Fig 4.12 shows the resulting graduated rates as a smooth curve.

From the graduated rates computed, the lines in each plot (Fig 4.9, Fig 4.10, Fig 4.11, Fig 4.12) show a smooth curve this was computed by using the first degree term in age x. Further computations were made by using higher degree polynomials in the model in other to have a better fit, but this did not give any significant improvement. So the linear model was maintained for all the four retirement years.

4.6 Analysis of graduated estimate of probability of death

From Table 4.3, it is interesting to note however that, taking lives born in 1935, from ages 55, 60, 65 and 70 years mortality increases from 0.0041 to 0.0152 and further increase from 0.0321 to 0.0329 respectively. The rate of change in rates increases steadily from ages 55 to 65 years and increases at a reduced rate from 65 years to 70. Comparing graduated rates by retirement years, there is an observation that mortality rates tend to reduce with time for older ages than for younger ages. Taking 1990 group of retirees for example, mortality rates for older ages are higher than 1995 group of retirees, as the rates from ages 76 to 80 indicates in Table 4.4. Just as well, rates for 2005 group of retirees tends to be lower at all ages when compared with 2000 group of retirees. However retirement year 2000 did not follow this trend. There are exceptionally high rates for this group hence all rates with the exception of rates for age 55 are higher than 1995 group of retirees.

On a general note, all lives observed have death rates below 0.02 for ages 55 to 60. The odds ratio is the probability of success divided by the probability of Table 4.4: Graduated rates for each gruop of pensioners

Age	1990	1995	2000	2005
55	0.0041	0.0099	0.0097	0.0102
56	0.0047	0.0108	0.0109	0.0110
57	0.0054	0.0118	0.0123	0.0119
58	0.0061	0.0128	0.0139	0.0129
59	0.0069	0.0140	0.0157	0.0140
60	0.0079	0.0152	0.0177	0.0151
61	0.0090	0.0166	0.0199	0.0163
62	0.0102	0.0180	0.0225	0.0177
63	0.0116	0.0196	0.0253	0.0191
64	0.0132	0.0214	0.0285	0.0206
65	0.0150	0.0233	0.0321	0.0223
66	0.0171	0.0253	0.0361	0.0241
67	0.0194	0.0275	0.0406	0.0261
68	0.0220	0.0299	0.0456	0.0282

69	0.0250	0.0326	0.0512	0.0304
70	0.0284	0.0354	0.0575	0.0329
71	0.0322	0.0385	0.0645	0.0355
72	0.0365	0.0418	0.0722	0.0383
73	0.0414	0.0455	0.0808	0.0414
74	0.0469	0.0494	0.0904	0.0447
75	0.0531	0.0536	0.1009	0.0482
76	0.0600	0.0582	0.1125	0.0520
77	0.0678	0.0631	0.1253	0.0560
78	0.0765	0.0684	0.1393	0.0604
79	0.0862	0.0742	0.1546	0.0651
80	0.0970	0.0803	0.1712	0.0701
Averages Rates	0.0309	0.0345	0.0597	0.0317

Table 4.5: Odds ratio between retirement year groups

	Odds Ratio		
Age(x)	1990/1995	1995/2000	2000/2005
55	0.4143	1.0222	0.9513
56	0.4327	0.9868	0.9924
57	0.4520	0.9525	1.0353
58	0.4721	0.9195	1.0800
59	0.4931	0.8876	1.1267
60	0.5151	0.8568	1.1753
61	0.5380	0.8271	1.2261
62	0.5619	0.7984	1.2791
63	0.5869	0.7707	1.3344
64	0.6130	0.7440	1.3920
65	0.6403	0.7182	1.4521
66	0.6688	0.6933	1.5149
67	0.6986	0.6692	1.5803
68	0.7297	0.6460	1.6486
69	0.7621	0.6236	1.7198
70	0.7960	0.6020	1.7941
71	0.8315	0.5811	1.8717
72	0.8685	0.5609	1.9525
73	0.9071	0.5415	2.0369
74	0.9475	0.5227	2.1249
75	0.9896	0.5046	2.2167
76	1.0337	0.4871	2.3124
77	1.0797	0.4702	2.4123
78	1.1277	0.4539	2.5166
79	1.1779	0.4381	2.6253
80	1.2303	0.4229	2.7387

failure. In our case the odds ratio is defined as probability of death divided by the probability of survival. The idea here is to compare the odds between two cohorts with five years intervals. So from Table 4.5 we have the odds ratio between 1990 and 1995,1995 and 2000, and lastly 2000 and 2005. The odds computed increases with age for all groups compared, implying that older ages have higher odds.

- Comparing 1990 and 1995 group of retirees:

  From the variations computed for these two groups it has become quite clear that lower ages have higher death occurrences than older ages. The odds ratio for these two groups are less than one for ages 55 to 75 and from 76 to 80 years the odds ratio are all greater than one.

- Comparing 1995 and 2000 gruop of retirees:

  As already noted the year 2000 group of retirees have exceptionally high estimated probability of death, so it is expected that the comparison between 1995 and 2000 groups clearly indicate this observation. For all ages the odds ratio is less than one, with the exception of age 55 where we have a higher estimated death rates for 1995 than in 2000. For the rest of the other ages having less than one odds ratio implies the obvious, that death rates are higher in 2000 than in 1995.

- Comparing 2000 and 2005 group of retirees:

  The relationship between these two groups show a better picture of improvement in death rates. This is because all odds computed are greater than one with the exception of ages 55 and 56 where we have less than one odds. The odds ratio increases steadily as age increases to the highest odds of 2.7387 at age 80. It implies therefore that at age 80 there are higher deaths of 2.7387 times in 2000 than in 2005.

Fig 4.13 shows graphically that, the mortality rates for lives who retired in 1990 are lower for ages 55 to 70 than the other observed retirement years. From ages 55 to 74, the mortality for the 1990 group of retirees are better off as the rates

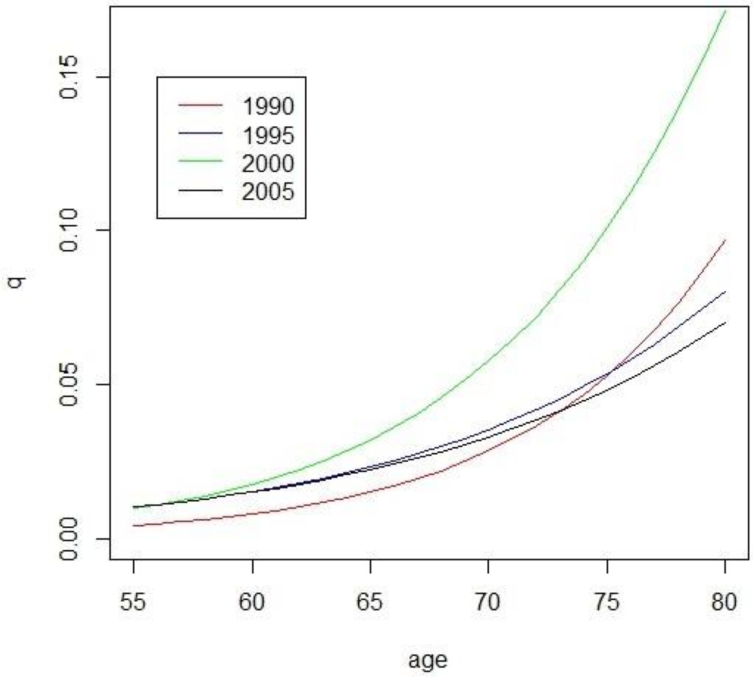


Figure 4.13: Plots of graduated rates for each group of lives observed are

higher for pensioners in 1995.

4.7 Discussions on expected versus actual death

Expected death are the death expected to occur based on the graduated estimates computed. Therefore expected death is computed by multiplying the graduated rate computed at each age by the number exposed to risk at that age. That is

$$Expected \quad death = \tilde{q}_x . E_x \tag{4.7}$$

This outcome is compared with the actual deaths occurred and deviations are obtained by

$$Expected \quad death - Actual \quad death$$

From Table 4.5 the expected death are higher at older ages than at younger ages. For 1990 group of retirees age 63 has a negative deviation indicating a lower expected death compared to the actual death. Also at age 68 the number of deaths occurred is 21 but the expected death computed was 13, giving us a Table 4.6: Deviations between actual and expected deaths 1990 and 1995

	1990			1995		
	Death			Death		
Age(x)	Expected	Actual	Deviation	Expected	Actual	Deviation
55	0	0	0	3	0	3
56	1	0	1	7	1	6
57	1	1	0	12	12	0
58	2	2	0	17	14	3
59	2	2	0	22	10	12
60	4	1	3	36	24	12
61	5	1	4	49	36	13
62	6	3	3	58	51	7
63	7	10	-3	67	60	7
64	8	2	6	75	68	7
65	9	7	2	83	69	14
66	10	9	1	91	77	14
67	12	12	0	99	87	12
68	13	21	-8	106	89	17
69	15	7	8	115	83	32
70	17	8	9	122	93	29
71	19	9	10	130	113	17
72	21	25	-4	132	235	-103
73	23	91	-68	121	484	-363
74	21	29	-8	123	188	-65
75	23	28	-5	126	153	-27
76	24	30	-6	130	117	13
77	25	23	2	134	125	9
78	27	13	14	139	93	46
79	29	23	6	146	71	75
80	30	14	16	154	47	107

deviation of -8. From ages 72 to 76 negative deviation were obtain indicating that at these ages there were higher actual deaths than expected deaths.

Regarding 1995 group of retirees, Table 4.5 from ages 55 to 71 there were positive computed deviations as actual death for this range were lower than the expected deaths computed. Form ages 72 to 75 negative deviations were computed with the widest been -363 at age 73. From age 76 to 80 positive deviations were obtained. For the retirement year 2000 however, Table 4.6 Ages 55 to 67 all had

Table 4.7: Deviations between actual and expected deaths 2000 and 2005

	2000			2005		
	Death			Death		
Age(x)	Expected	Actual	Deviations	Expected	Actual	Deviations
55	3	0	3	3	0	3
56	7	5	2	7	1	6
57	11	8	3	11	3	8
58	17	10	7	17	8	9
59	24	15	9	24	14	10
60	48	29	19	53	28	25
61	71	33	38	72	50	22
62	85	67	18	82	71	11
63	100	79	21	91	72	19
64	114	88	26	100	76	24
65	130	69	61	110	80	30
66	147	80	67	120	80	40
67	164	106	58	129	85	44
68	182	220	-38	138	328	-191
69	196	209	-13	139	228	-89
70	210	195	15	144	186	-42
71	224	209	15	150	162	-12
72	238	611	-374	156	162	-6
73	217	902	-685	162	205	-43
74	162	223	-62	166	292	-125
75	159	83	76	166	204	-39
76	168	82	86	168	175	-7
77	178	79	99	172	146	26
78	187	132	55	177	175	2
79	187	81	106	179	146	33
80	194	80	114	183	58	124

positive deviations as the number of actual deaths which occurred at these ages are lower than expected deaths computed. At ages 68 and 69 negative deviations were obtained. At 70 and 71 positive deviations were obtained with the same value of 15. Ages 72 to 74 registered negative deviations with widest been -685 at age 73. This is expected as age 73 was noted to be an extreme out-lier in Fig 4.11 positive deviations were computed for ages 75 to 80.

The 2005 group of retirees had positive deviations for ages 55 to 67, and from ages 68 to 76 negative deviations were obtained. Age 77 upwards had positive deviations.

Chapter 5

SUMMARY OF FINDINGS, CONCLUSION AND RECOMMENDATIONS

5.1 Introduction

In spite of the general view on how mortality evolves over time there may be peculiar situation that may alter the status quo and would be worth investigating. This section summarizes the findings from data analyzed conclude the research and suggest recommendations

5.2 Summary of findings

From the data analyzed, it has come to light that in spite of the widely accepted knowledge that mortality rates decline with time, there are certain situations that do not follow the status quo.

As observed, the pensioners in 1990 have a lower death rate than pensioners in 1995 for ages 55 to 73 and higher for ages 73 upwards for both 1995 and 2005 retirement years. The lives retired in year 2000 have an exceptionally high death rates but the retirees in year 2005 have lower death rates than retirees in year 2000.

On further observation there is a clear indication that irrespective of the cohort retirement year an individual belonged to the mortality rates increases by age. But the fact is that rate of change varies greatly depending on the year of birth and age at retirement.

On a general note, it has been observed that each group of lives retiring in a particular year have peculiar characteristics which determines their age specific mortality over time.

The variations between the actual and observed death are wider at older ages be it positive of negative. With regards to the variations observed the death recorded at older ages is not consistent with the numbers exposed to risk.

5.3 Conclusion

From the results, generally, males who retire early,that is between the ages 55 to 59 years have lower mortality than males who retire at older ages.

Male retirees in 1990 who retired at older ages, that is age 76 upwards have higher mortality than those who retired in 1995 and are within the same age range. From ages 74 upwards,retirees in 1990 have a higher mortality than retirees in 2005 who are in the same age range.

For the year 2000,males who retired at age 55 have lower mortality than those who retired in 1995 and 2005 at the same age. At age 56, males who retired in year 2000 have lower mortality than those who retired in 2005 at the same age. From 57 to 80 years retirees in year 2000 have the highest mortality as compared to the other group of retirees, that is 1900,1995 and 2005. But one very glaring observation is that taking a group of people born in particular year, mortality increases by age irrespective of the group belonged to. Also, in all groups of retirees, high estimated rates of mortality is observed from ages 70 to 75.

5.4 Recommendation

Based on the objectives and major findings of the study, it is important for the government and other stakeholders to implement policies that will carter for the welfare of our senior citizens and hence contribute to the improvement of pensioner mortality.

5.4.1 Future annuity product pricing for pensioners

It is recommended that graduated rates of mortality computed can be used by insurance companies to develop annuity products for pensioners in Ghana.

5.4.2 Recommendation for future research

- Modeling Ghanaian male pensioner mortality using outlier-adjusted model
- Modeling the effects of Socio-economic factors on mortality of pensioners in Ghana
- Investigating rural urban mortality experience for pensioners in Ghana
- National health insurance and mortality trends of pensioners in Ghana

REFERENCES

Attanasio, O. P. and Emmerson, C. (2001). Differential mortality in the uk. *National Bureau of Economic Research*, No. w8241.

Barnett, H. A. R. (1968). A thoery of mortality. *Journal of the Institute of Actuaries (1886-1994)*, 94(2):19–171.

Bizz 101 (2015). Should ghana's pension age be increased. *http://tonywelcomes.blogspot.com/*.

Booth, H. and Tickle, L. (2008). Mortality modelling and forecasting: A review of methods. *Annals of actuarial science*, 3(1-2):3–43.

Cairns, A. J. G. (2000). A discussion of parameter and model uncertainty in insurance. *Insurance: Mathematics and Economics*, 27(3):313–330.

CMI (2004). Projecting future mortality. *Continuous Mortality Investigation Mortality Sub-committee.*, 7.

Debon, A., Montes, F., and Sala, R. (2006). A comparison of nonparametric methods in the graduation of mortality: Application to data from the valencia region (spain). *International Statistical Review*, 74(2):215–233.

Elo, I. T. and Preston, S. H. (1996). Educational differentials in mortality: United states 1979-1985. *Social science and medicine*, 42(1):47–57.

Gavin, J., Haberman, S., and Verall, R. (1996). *Graduation by Kernel and adaptive kernel methods with a boundary correction. Insurance Mathematics and Economics*. 3(17), 235.

Ghana Health Service (2010). The health sector in ghana facts and figures 2010.

Girosi, F. and Gary, K. (2007). Understanding the lee-carter mortality forecasting method.

Gompertz, B. (1825). On the nature of the function of the law of human mortality and on a new mode of determining the value of life contingencies. *Philosophical Transactions of The Royal Society*, 115:513–585.

GSS (2010). Population and housing census. *Ghana Statistical Service*.

Haberman, S. and Renshaw, A. (1996). Generalized linear models and actuarial science. *The Statistician*, 45:407–436.

Heligman, L. and Pollard, J. (1980). The age pattern of mortality. *Journal of the Institute of Actuaries*, 107:49–80.

Hudson, R. (2006). The mortality risk of pensions-methods of control and policy implications for the uk. *Journal of Financial Regulation and Compliance*, 14(4):363–374.

Huisman, M., Kunst, A. E., Andersen, O., Bopp, M., Borgan, J. K., Borrell, C., and Mackenbach, J. P. (2004). Socioeconomic inequalities in mortality among elderly people in 11 european populations. *Journal of Epidemiology and Community Health*, 58(6):468–475.

Johnson, P. and Stears, G. (1995). Pensioner income inequality. *Fiscal Studies*, 16(4):69–93.

Keilman, N. (2003). *Types of models for projecting mortality*.

King, G. (1989). *Unifying political methodology: The likelihood theory of statistical inference.* University of Michigan Press.

Kinsella, K. and Wan, H. (2009). An aging world. *US Department of Commerce, Economics and Statistics Administration, US Census Bureau*, page 37.

Lee, R. D. and Carter, L. R. (1992). Modeling and forecasting us mortality. *Journal of the American statistical association*, 87(419):659–671.

Logue, L. M. and Blanck, P. (2004). There is nothing that promotes longevity like a pension: Disability policy and morality of civil war union army veterans. *Wake Forest L. Rev.*, 39:49.

Makeham, W. (1860). On the law of mortality. *Journal of the Institute of Actuaries*, 13:325–358.

Mazza, A. and Punzo, A. (2012). Dkgrad: An r package for mortality rates graduation by fixed and adaptive discrete beta kernel techniques. arXiv preprint arXiv::1211.1184.

Myung, I. (2003). Tutorial on maximum likelihood estimation. *Journal of mathematical Psycology*, 47(1):90–100.

Ogwumike, F. O. and Aboderin, I. (2005). Exploring the links between old age and poverty in anglophone west africa: evidence from nigeria and ghana. *Generations Review*, 15(2):7–15.

Olshansky, S. J. (1988). On the biodemography of aging: A review essay. *Population and Development Review*, 24:381–393.

Pollard, J. H. (1987). Projection of age-specific mortality rates. *Population Bulletin of the United Nations*, 21:55–69.

Preston, S. H. and Elo, I. T. (1995). Are educational differentials in adult mortality increasing in the united states? *Journal of aging and health*, 7(4):476– 496.

Radner, D. (1986). *Changes in the Money INcome of the Aged and Nonaged*.

Schoenbach, V. J. and Rosamond, W. D. (2000). Understanding the fundamentals of epidemiology: an evolving text. *Chapel Hill: North Carolina*.

Snyder, S. E. and Evans, W. N. (2006). The effect of income on mortality: evidence from the social security notch. *The Review of Economics and*

Statistics, 88(3):482–495.

Stoeldraijer, L., Duin, V. C., Wissen, V. L., and Janssen, F. (2013). Impact of different mortality forecasting methods and explicit assumptions on projected future life expectancy: The case of the netherlands. *Demographic Research*, 29.

Tomas, J. and Planchet, F. (2013). A new non-parametric approach to smoothing and forecasting of mortality. *Cahiers de Recherche de lISFA*, pages 1–33.

Whitehouse, E. and Zaidi, A. (2008). *Socio-Economic Differences in Mortality.*

William, J. (1965). Regional inequality and the process of national development. *Economic development and cultural change 13 no 4 part 11 supplement 206-247. 1984, British mortality and the value of life 1781-1931 Population studies.*, 38:157–172.

Appendix A

Table 5.1: Number of deaths (d_x) at each age x

Age(x)	1990	1995	2000	2005	Total
55	0	0	0	0	0
56	0	1	5	1	7
57	1	12	8	3	24
58	2	14	10	8	34
59	2	10	15	14	41
60	1	24	29	28	82
61	1	36	33	50	120
62	3	51	67	71	192
63	10	60	79	72	221
64	2	68	88	76	234
65	7	69	69	80	225
66	9	77	80	80	246
67	12	87	106	85	290
68	21	89	220	328	658
69	7	83	209	228	528
70	8	93	195	186	482
71	9	113	209	162	493
72	25	235	611	162	1033
73	91	484	902	205	1682
74	29	188	223	292	732
75	28	153	83	204	566
76	30	117	82	175	469
77	23	125	79	146	442
78	13	93	132	175	413
79	23	71	81	146	334
80	14	47	80	58	180
Total	371	2400	3923	3035	9729
Percentage change (%)	3.81	24.67	40.32	31.2	

Appendix B

Table 5.2: Number exposed to risk at each retirement age (E_x)

Age(x)	1990	1995	2000	2005
55	88	308	299	315
56	177	689	595	607
57	233	1013	880	921

58	281	1308	1220	1287
59	340	1599	1554	1702
60	472	2362	2701	3494
61	559	2980	3565	4385
62	586	3197	3769	4642
63	601	3393	3938	4788
64	601	3493	4003	4860
65	611	3560	4052	4923
66	610	3610	4081	4960
67	606	3601	4051	4943
68	599	3555	3997	4885
69	585	3517	3823	4573
70	586	3452	3654	4379
71	581	3367	3481	4212
72	573	3144	3289	4066
73	548	2671	2688	3916
74	458	2492	1790	3722
75	429	2347	1574	3436
76	402	2243	1497	3237
77	372	2123	1417	3066
78	349	2034	1342	3924
79	336	1964	1212	2750
80	313	1917	1131	2605