

TIME SERIES ARIMA MODELLING OF INFLATION IN GHANA: (1990 – 2009)

BY

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DECLARATION

I hereby declare that apart from certain documentary and other sources which I have cited and duly acknowledged, this submission is the result of my own research and that it has neither in whole nor in part been presented for another degree elsewhere. I also declare that neither my supervisor nor any other person but the author alone is responsible for whatsoever errors and omissions that might appear in the work.

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.....
Supervisor Name Signature Date

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Head of Dept. Name Signature Date

DEDICATION

This work is dedicated to my son Kelvin Kwaku Owusu and the unborn ones.

It also dedicated to my parents and wife Joyce for their love, care and support.

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ABSTRACT

Throughout the world, most central bank policy initiatives have been aimed at achieving and maintaining price stability and the Bank of Ghana is no exception to this rule. This study attempts to outline the practical steps which need to be undertaken in order to use the autoregressive integrated moving average (ARIMA) model for forecasting Ghana's inflation. The main focus of the study is to model inflation and hence used to forecast the monthly inflation on short-term basis, for this purpose, different ARIMA models are used and the candid model is selected based on various diagnostic, evaluation and selection criteria. It can be concluded that the model has sufficient predictive powers and the findings are well in line with those of other studies. Again the study models inflation for the periods of 1990 to 2000 and 2001 to 2009 and it was realized that the inflation model for the period of 1990 to 2000 is ARIMA (1, 2, 2) written as $\hat{y}_t = 18.5770 + 0.455848t - 3.57e^{-0.3}t^2 + 0.7807y_{t-1} - 1.0813\varepsilon_{t-1} + 0.1020\varepsilon_{t-2} + \hat{\varepsilon}_t$. Whilst that of 2001 to 2009 is modelled as ARIMA (2, 2, 1), written as $\hat{y}_t = 34.3958 - 0.637228t + 4.40e^{-0.3}t^2 - 1.3764y_{t-1} - 0.4389y_{t-2} + 0.9860\varepsilon_{t-1} + \hat{\varepsilon}_t$.

It was concluded that inflation for the period of January 2001 to December 2009 was less than that of January 1990 to December 2000. The model is recommended for use by stakeholders because it has a lower error variance of ± 1 which follows closely with the actual data. It is recommended further to be used as the basis for constructing deterministic models such as first and second order differential equations by future researchers

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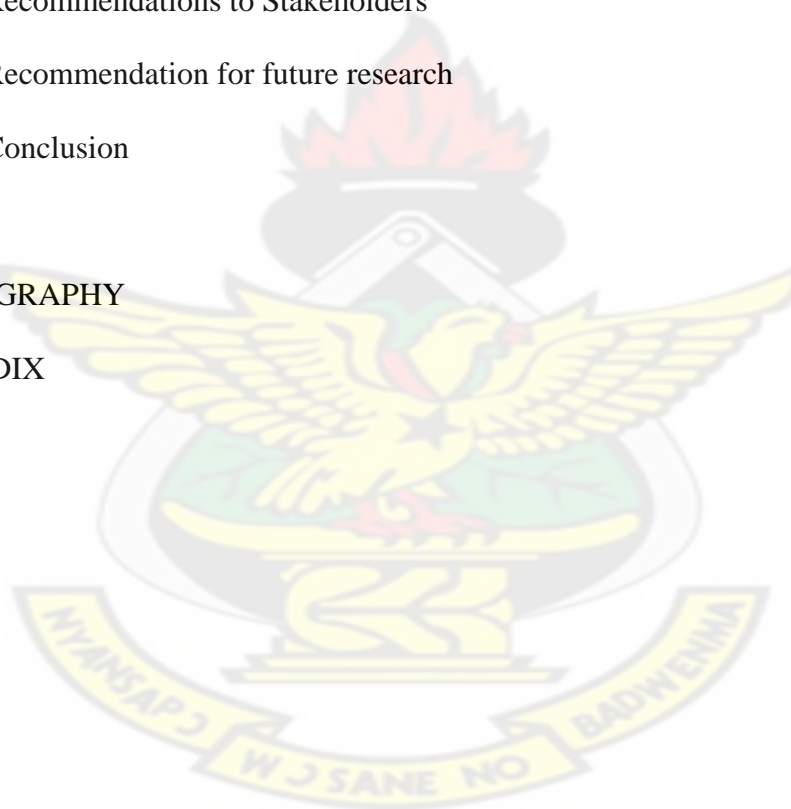
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LIST OF ABBREVIATIONS



ACF	Auto Correlation Function
AIC	Alkaine Information criterion
AR	Auto Regressive
ARMA	Autoregressive Moving Average
ARIMA	Auto Regressive Integrated Moving Average
CEPA	Center for Policy Analysis
CPI	Consumer Price Index
DIFF	Differencing
ERPT	Exchange Rate Pass Through
GDP	Gross Domestic Product
GNP	Gross National Products
HICP	Harmonized Indices of Consumer Prices
IMF	International Monetary Fund
IT	Inflation Targeting
MA	Moving Average
MAD	Mean Absolute Deviation
MAE	Mean Absolute Error
MAPE	Mean Absolute Percentage Error
MSD	Mean Standard Deviation
NBIC	Normalized Bayesian Information criterion
OCED	Organization for Economic Co-operation and Development

PACF	Partial Auto Correlation Function
PRGF	Poverty Reduction and Growth Facility
PSBR	Public Sector Borrowing Requirement
SPSS	Statistical Package for Social Scientist
STSA	Symbolic Time Series Analysis
TS	Time series



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CHAPTER ONE

INTRODUCTION

1.0 BACKGROUND TO THE STUDY

The control of inflation is central to good monetary policy. The concern with inflation emanates not only from the need to maintain overall macroeconomic stability, but also from the fact that inflation hits the poor particularly hard as they do not possess effective inflation hedges. Price stability is a good thing in itself, as inflation injects noise into the price system, makes long-term financial planning more complex, and interacts in perverse ways with imperfectly indexed tax and accounting rules. In the short-to medium term, high inflation and persistent inflation undermines public confidence in the economy and in the management of economic policy generally, with potentially adverse effects on risk-taking, investment, and other productive activities that are sensitive to the public's assessments of the prospects for future economic stability. In the long term, low inflation promotes growth, efficiency, and stability, which supports maximum sustainable economic growth.

Furthermore, low, stable inflation is beneficial from the distributive point of view, because it favors the growth of employment and protects the income of the most vulnerable sectors of the society. Monetary policy may not influence long-term growth beyond this contribution of price stability. The potential consequences of this policy on

economic activity and employment over the short and medium- term arise from the different channels through which changes in monetary policy are transmitted in order to affect inflation. That is why monetary policy should take an anti-cyclical stance, alongside with preserving price stability, in order to avoid extreme variations in overall expenditure or domestic demand. This clearly points to the fact that the accurate measurement on the effects of changes in monetary policy on the economy is essential, both for good policy-making and for choosing among alternative macroeconomic theories.

Inflation dates back as far as to AC 150 in ancient Rome, where government business at that time was mostly done on cash basis using metal currency, variously gold, silver, copper, and bronze. The first recorded inflation after AC 150 was discovered by an English scholar, A.H.M. Jones in 1974, who discovered that the price of a military uniform has increased 166 times from the middle of the second century to the close of the third century. The definition of inflation has undergone lot of changes since 1983 when it appeared in the dictionary for the first time. Inflation was thought of as a cause but as time passed by, the definition and its significance has changed. Economists from different schools differ in their opinion regarding the genesis of inflation. However, inflation is defined as the pervasive and sustained rise in the supply of money which causes devaluation or a decrease in the supply of goods and services.

It is therefore an economic condition wherein the price of goods and services increase steadily, measured against standard level of purchasing power, whereas the supply of goods and services decline along with the devaluation of money . Repetitive price increases erode the purchasing power of money and other financial assets with

fixed values, creating serious economic distortions and uncertainty. Inflation results when actual economic pressures and anticipation of future developments causes goods and services to exceed the supply available at existing prices, or when an available output is restricted by faltering productivity and marketplace constraints. When the upward trend of prices is gradual and irregular averaging only a few percentage points each year, such creeping inflation may not be considered as a serious economic threat and social progress. The illusion of personal income growth beyond actual productivity may encourage consumption; housing investment may increase in anticipation of future appreciation; business investment in plants and equipment may accelerate as prices rise more rapidly than cost, hence resulting in low purchasing power of money. In the mid 1960s a chronic inflationary trend began in most industrial nations from 1965 to 1978. American consumer prices increased at an average annual rate of 5.7 percent, including a peak of 12.2 percent in 1974. This ominous shift was followed by consumer price gains of 13.3 percent in 1979 and 12.4 percent in 1980. Several other industrial nations suffered a similar acceleration of price increases whilst some countries like West Germany, now part of the united federal Republic of Germany avoided the chronic inflation effect.

Evidently inflation in Ghana is caused by both fiscal and non-monetary factors. In the past, Ghana's balance of payment position has been in severe difficulties due to inappropriate trade, fiscal and monetary policies. Excessive money supply is the single most pervasive cause of inflation in Ghana. For instance between 1996 and 1997 inflation was at 25% and 8% respectively, but this was short lived as it shot to 40.5% in December 2000, reflecting fiscal mismanagement. Bernanke et al, (2005) acknowledges

that fact fiscal imbalances were by far a major determinant of inflation: there exist no comprehensive studies on the impact of budget deficit on inflation. He further analyzed the issue for the half year of 2005 and the main findings were that fiscal deficit did matter and contribute to the monetary policy growth of a country.

A non-monetary source of inflation is attributable to poor performance of the agricultural sector between 1995 and 1999. Ghana's agricultural sector grew by 44 percent but dropped again to 1.1 percent in the year 2000. This resulted in high food prices in the country. Food prices alone account for over half of the average household expenditure in Ghana. The other dimension to inflationary trends is that inflation is international. This is because Ghana like the rest of the world is always affected by crude oil hikes, which affects the state of the economy. This has been a major cause of civil strife which belies every military overthrow in the country. For example high inflation rate of 100% in 1979 moderating to 54% led to the removal of Liman's government from office by Jerry John Rawlings in a military putsch in 1981. Rawlings was dubbed "junior Jesus" by the populace on account of his aggressive effort to tackle inflation. His government imposed fiscal and monetary discipline to curb it, but spending unwisely during the 2000 election year shot inflation from 8 percent to 30 percent between 1998 and 2000. John Agyekum Kuffour's new government succeeded in curbing inflation down but again shot up during the 2008 election year from 12.81 percent in January 2008 to 18.4 percent in July 2008. Currently, inflation is still unstable and has assumed a downward decreasing trend from 14.78 percent in January to 14.23 percent in February; it further decreased from 14.23 percent in February to 13.32 percent in March, 2010. Inflation has further dropped from 13.32 percent in

March to an unprecedented value of 11.66 percent in April, representing a drop of 1.7 percent. Recently the inflation rate for July 2010 is 9.46 percent.

During the past three decades, dramatic changes in the inflationary environment have stimulated wealth of studies on the relative accuracy of alternative models of inflation forecasts. Moreover, there has been much work on examining and evaluating different methodologies in forecasting inflation. One approach is associated with the work of (Fama and Gibbons 1997). This approach was extracted from observed nominal interest rates and the market's inherent expectation of inflation. Based on a univariate time-series modeling of the real interest rate, Fama and Gibbons, (1997) found that the interest-rate model yields inflation forecasts with a lower error variance than a univariate model, and that the interest-rate model's forecasts dominate those calculated from the Livingston survey. Meyler and Quin (1998) focused on ARIMA model to forecast Irish inflation and justified that ARIMA models are surprisingly robust with respect alternative (multivariate) model. Based on the effects of inflation, Stockman (1981) develops a model in which an increase in the inflation rate results in a lower steady-state level of output and people's welfare declines. In Stockman's research, money is a complement to capital, accounting for a negative relationship between the steady-state level of output and the inflation rate. Stockman's insight is prompted by the fact that firms frequently put up some cash in financing their investment projects. Sometimes the cash is directly part of the financing package, whereas other times, banks require compensating balances. Stockman modeled this cash investment feature as a cash-in-advance restriction on both consumption and capital purchases.

The study follows simple ARIMA methodology and exclusively focuses on Ghana's monthly inflation. The main focus is to obtain a model for forecasting the monthly inflation on short-term basis, and for this purpose, different ARIMA models are suggested and the best model is selected for short term forecasting of inflation. It is further required to compare inflation between the periods of January 1990 to December 2000 and January 2001 to December 2010. An inflation model is drawn for each of the two periods.

1.1 STATEMENT OF THE PROBLEM

Mundell, (1963) was the first to articulate a mechanism relating to inflation and output growth through something other than the excess demand for commodities. He discovered that, an increase in inflation immediately reduces people's wealth. To accumulate the desired wealth, people save more, thus driving down the real interest rate. This leads to greater savings which causes greater capital accumulation and thus faster output growth. In most of these studies, inflation is found to exhibit high to very high persistence which is close to that of a random walk. This suggests that in order to bring inflation back to its target, the central bank must act more vigorously.

Government expenditure and revenue is raised by inflation. It causes an upward adjustment in government's budget, especially index-linked ones while it simultaneously increases government revenue. Budgetary imbalances have become entrenched and have been the primary cause of resurgence of macroeconomic instability. For example, according to (CEPA, current state of the macro economy

report 1999 and 2000 fiscal year) the non observance of poverty reduction and growth facility (PRGF) conditionality's agreed with international monetary fund (IMF), worsened the state of the economy in 1999 and 2000 fiscal year, as there was total delayance in aid disbursement to the country. Bank of Ghana in an attempt to meet its monetary targets often resulted to bouncing of government cheques. The consequential borrowing by government led to a sharp build up of the banking sector as private sector operatives turned to the banking system for an ace on account of looked up working capital, in spite of enormous interest rates.

Bawumia and Abradu, (2003) indicated that the empirical evidence of a broad monetary growth in Ghana between the period of 1983 and 1999 is suggestive, as theory will predict that a slower money supply growth reduces inflation and the rate of depreciation of the cedi. They then considered the monetary growth trends between the periods of 1983 and 1999 and indicated that broad monetary growth peaked at 62.5 percent in 1985. Thereafter, the pursuit of a tight monetary policy resulted in broad money growth reducing inflation to 20.6 percent by 1990. Over the period of 1983 to 1991 inflation declined from a peak of 122.8 percent in 1983 to 10.2 percent by 1991, whilst monetary growth was 26 percent in 1991. Exchange rate over this period also declined from 93 percent in 1983 to 11.54 percent by 1991.

According to the Ghana macroeconomic preview, a bank of Ghana Source (CEPA, 2001), indicated that the difficulties encountered in the divestiture program deepened the deficit in government budget as net foreign financing declined from 26 percent of GDP in 1998 to 0.9 of GDP in 1995. Net domestic finance also rose from 5 percent of GDP in 1998 to 7 percent of GDP in 1999 and sharply to 10.3 percent of

GDP in the year 2000. As with election years 1992, 1996 and 2000 the fiscal pressures were over bearing as government depleted its deposits of 875 billion cedis from the central bank at the end of July in the same year. By the end of the year a total overdraft of about 900 billion had been extended to the government which caused the monetary authorities to renege on their commitment to IMF, limiting them to financing a domestic loan of not more than 254 billion cedis. This made the government to ask for a waiver from the IMF and further printed new currencies to finance the fiscal deficit.

Economists have suggested three substantive theories of dealing with inflation; the available quantity of money (Monetarist Approach); the aggregate level of income; and supply side productivity and cost variables. Monetarists believe that changes in price levels reflect fluctuating volumes of money available, usually defined as currency demand deposits. They argued that to create stable prices, the money supply should increase at a stable rate commensurate with the economy's real output capacity.

Secondly, the aggregate level of income theory is based on the work of a British economist (Keynes, 1930) who discovered that changes in the national income determines consumption and investment rates; thus government fiscal spending and tax policies should be used to maintain full output and employment levels. The money supply then should be adjusted to finance the desired level of economic growth, while avoiding financial crises and high interest rates that discourage consumption and investment. Government spending and tax policies can be used to offset inflation and deflation by adjusting supply and demand.

The final theory concentrates on supply-side elements, which are related to the significant erosion of productivity. These elements include the long-term pace of capital investment and technological development; changes in the composition and age of the labor force; the shift away from manufacturing activities; the rapid proliferation of government regulations; the diversion of capital investment into non productive uses; the growing scarcity of certain raw material; social and political developments that have reduced work incentives. The over bearing nature of inflation has necessitated this study by using ARIMA Box-Jenkins approach to model inflation from 1990 to 2009

1.2 OBJECTIVES

The study is aimed at achieving four main specific objectives by using ARIMA Box-Jenkins approach to model inflation in Ghana, specifically from January 1990 to December 2009. The following are the specific objectives of the study:

1. To model inflation using the monthly inflation rates from 1990 to 2009;
2. To obtain a model for inflation using the monthly inflation rates from 1990 to 2000;
3. To model inflation using the monthly inflation rates from 2001 to 2009; and
4. Compare inflation rates between the periods of January 1990 to December 2000 and January 2001 to December 2009.

1.2.1 HYPOTHESIS

In order to compare inflation between the periods of January 1990 to December 2000 and January 2001 to December 2009 as in specific objective four, we formulate the following hypothesis for the study:

Ho: There is no significant difference between the mean and variance of the two periods.

H1: There is significant difference between the mean and variance of the two periods.

1.3 METHODOLOGY

The Box-Jenkins ARIMA approach is the methodology used to model inflation from the period of 1990 to 2009, whilst the data for 2010 is reserved for forecasting and validation purposes. Further the study also compares inflation between the periods of January 1990 to December 2000 and January 2001 to December 2009. A model is then provided for each of the two periods. The following statistical software such as; SPSS, Minitab and E-view from Excel were used. In addition, monthly inflation data from secondary sources like, Ghana statistical service, Bank of Ghana and working paper; Bank of Ghana, WP / BOG – 2003 / 05 were also collected to enhance the success of the study.

1.4 JUSTIFICATION

The relationship between inflation and economic growth is one which many economists, have watched with keen interest and is of major concern to stakeholders specifically the Ministry of Finance, central bank of Ghana, the business sector, the manufacturing sectors, export and import sectors etc for planning purposes and to make informed decisions. Therefore modeling inflation using the Box-Jenkins ARIMA approach is plausible to stakeholders because it generates reliable inflation forecast which follows closely with the actual data. Hence the model can be used by stakeholders to plan ahead and make informed decisions in order to reduce risk.

1.5 LIMITATIONS

The following problems were encountered in the course of the study;

- Limited access to extensive data set of variables:
- Unwillingness on the part of agencies and bodies concerned to give reliable data: and
- Non-availability of some powerful mathematical and econometrics software's for data analysis.

1.6 ORGANISATION

The study has five chapters and it is organized as follows:

Chapter One deals with the introduction. Chapter Two highlights on the empirical and theoretical reviewing of literature by considering the theories on inflation and its effects. Chapter Three deals with the methodology preferably times series concept on ARIMA. Chapter Four discusses and analyses the results obtained. Finally, Chapter Five deals with the outcome, summary and conclusion of the study.



CHAPTER TWO

LITERATURE REVIEW

2.0 INTRODUCTION

This chapter reviews the relevant theories associated with the study and the need to analyze the impact of inflation on growth. The chapter is divided into two main headings namely: Theories on inflation and its effect, and mathematical application.

2.1 THEORIES ON INFLATION AND ITS EFFECT

The recent macroeconomic debate on inflation and economic growth is concentrated on the impact of inflation on real output and its costs on welfare. Fischer (1930) hypothesis reveals that inflation is the main determinant of interest rate since a one percent increase in the rate of inflation, results to a corresponding one percent increase in the interest rate. Bailey (1956) argued that inflation has negative effects on the economy through its cost on welfare. He further stated that the cost associated with unanticipated inflation are; the distributive effects from creditors to debtors, increasing uncertainty affecting consumption, savings, borrowing and investment decisions.

It is acknowledged that the above debate evolved from a controversial notion between the structuralists and the monetarists. This made Mundell (1965) to predict a

positive relationship between the rate of inflation and the rate of capital accumulation, which in turn, implies a positive relationship to the rate of economic growth. He argued that since money and capital are substitutable, an increase in the rate of inflation increases capital accumulation by shifting the portfolio from money to capital, thereby stimulating a higher rate of economic growth.

Conversely, Fischer and Modigliani (1978) suggested a negative and non-linear relationship between the rate of inflation and economic growth. Sargent and Wallace (1981) in their contribution to the debate indicated an unpleasant monetarist arithmetic that, the more increase in the cost of borrowing, the harder to finance this debt stock and the more condensed the expectations of economic agents for the possibility of monetization by money authority. Stockman (1981) also developed a model of which an increase in inflation rate results to a lower steady-state level of output called the stockman effect. He stated that money is complement to capital, accounting for a negative relationship between the steady-state level of output and inflation rate. Viren (1987) examines the time series properties of inflation and interest rates variables using monthly data from six OECD countries covering the period 1972 to 1984. The analysis focuses on the hypothesis that real rates of interest are constant over time and that movements in nominal rates can be explained by inflation only. These hypotheses are tested by applying both formal and informal test procedures and by carrying out tests both in the time and in the frequency domain. He concluded that the empirical evidence is at variance with the hypotheses, except in the case of the United States where the results lend some support for the existence of the Fisher relationship. Shee (1989) examines the patterns of impact resulting from the Fed's 1979 regime change on

inflation expectations and real economic activity. Using time series intervention analysis, no significant impacts were found on the expected inflation rate, the unemployment rate, and the growth rate of real GNP. In addition, inflation rates were somewhat modest in most countries before the 1970's and after which the rates became higher. Therefore, most empirical studies conducted before the 1970's shows evidence of a positive relationship between inflation and economic growth. A negative relationship between the two indicates a severe inflation hike, (Sarel, 1995).

Barro (1995) estimated a negative relationship between inflation and growth; for each one percentage point increase in the U.S.A inflation, the annual growth rate is reduced by 0.223 percent. Whilst Smyth (1995) also estimated 10 percent increase in the inflation rate of growth of a total factor of 0.025 percent. This was followed with a test to ascertain the robustness of the above estimations and the results are suggestive, that there is no connection between inflation and the level of productivity. (Camerom et al; 1996). In addition to Camerom's contribution, Stanners (1996) remarked on a paper by W. R. J. Alexander who concluded, on the basis of econometric analysis involving variables additional to the two principal ones, that a decrease in inflation rate would result in a significant gain in the growth rate of national output. He justified that the assertion by Alexander did not show any verbal conclusion and also did not follow from the results of the algebraic analysis which precedes it, and more generally, time-series analysis, with or without additional variables, is unlikely to be able to coin the conclusion of simple two-parameter, cross-section correlation studies: (that the growth rates of countries are not correlated with their inflation rates).

Cunningham et al., (1997) also discovered that the positive relationship between inflation uncertainty and unemployment is dependent on three significant factors. First, the existence of a positive relationship between inflation and unemployment only begins to manifest in mid-1970s. Second, the inflation uncertainty-unemployment relationship is not applicable in every single digit SIC firms. Thirdly, the relationship between inflation uncertainty and unemployment exists only on low-frequency components. Bruno and Easterly (1998) further argued that the negative long-run relationship between inflation and growth found in the above literature is only present with high frequency data and with extreme inflation observations. They examined discrete high inflation crises and concluded that growth falls sharply during discrete high inflation crises and recovers rapidly and strongly after inflation falls. Coorey et al., (1998), also confirmed that relative prices did have a significant impact on inflation in the transition economies and that this impact was not necessarily temporary.

Next, Cati et al., (1999) used the Brazilian as a case study from January 1974 to June 1993; a time frame characterized by great influence of the effects resulted from the implementation of stabilization plans. They concluded that the macroeconomic interpretation of the results is in line with the inflationary inertia hypothesis, which states that inflation perturbations are extremely persistent.

Ng and Perron (2001) continued with the study by using a technique which generates the same conclusion reached by Cati et al., (1999) without the use of dummy variables and concluded that inflation rate is non stationary during the same period under analysis. We also consider Albacete (2001) who forecasted inflation in the European Monetary Union. They realized that Inflation in the European Monetary

Union is measured by the Harmonized Indices of Consumer Prices (HICP), which is analyzed by breaking down the aggregate index in two different ways. One refers to the breakdown into price indexes corresponding to big groups of markets throughout the European countries and other HICP countries. They concluded that the breakdown by group of markets improves the European inflation forecasts and constitutes a framework in which general and specific indicators can be introduced for further improvements. Batini and Yates (2003) investigated the properties of monetary regimes that combine price-level and inflation targeting. They considered both, at an optimal control and a simple rule characterization of these regimes. They also derived the numerical results by modelling the economy as a small-scale open-economy RE model calibrated on UK data, and the conclusion was that: one, the relative merits of price-level and inflation targeting, as well as combination of the two, depend on a particular modelling and policy assumptions; and finally, these merits do not always change gradually or monotonically as we move from one regime to another.

Recently, Batini (2006) used empirical analysis to classify inflation persistence into three main groups namely; disparity between systematic monetary policies and their greatest effect on inflation, and concluded that monetary policies are related to exogenous shocks derived from the private sector. Secondly, inflation lagged response to non-systematic economic policy shock. He related this to the number of disparities necessary to make inflation respond to political shock. Finally, Positive serial correlation in inflation he said is strongly based on price control. Darne and Ferrara (2009) focused on the acceleration cycle in the euro area, namely the peaks and troughs of the growth rate which delimitate the slowdown and acceleration phases of the

economy. Their aim was in two folds: First, was to put forward a reference turning point chronology of this cycle on a monthly basis, based on gross domestic product and industrial production index. Secondly, they assessed a new turning point indicator, based on business surveys, which is carefully watched by central banks and short-term analysts.

Finally Jayasooriya (2009), contributed to the effects on inflation by using empirical investigation which includes causality, co-integration and error correction models, which reveals the existence of a long-term equilibrium relationship between minimum wages and inflation, and a one-way causality between the two variables. He concluded by suggesting the following that: an interruption in equilibrium leads not only to a significant adjustment process but also to structural changes in long-run equilibrium. Finally, macroeconomic stability is established through the impulse response function in a situation, where shocks are applied to both minimum wages and inflation. He further recommended that policy-making entities should contemplate a minimum wage adjustment process in a climate of unstable inflation.

2.2 MATHEMATICAL APPLICATION

Numerous studies have investigated the relative accuracy of alternative inflation forecasting model. Some of these models used in forecasting inflation among others are Vector Auto Regressive (VAR), Bayesian Vector Auto Regressive (BVAR), Structural Vector Auto Regressive (SVAR), Seasonal Auto Regressive Integrated Moving

Average (SARIMA), Simple Auto Regressive (SAR), random walk and Auto Regressive Fractionally Integrated Moving Average (ARFIMA). The rest are the Expectations-augmented Philip curve, P-Star, Leading indicators and Traditional monetarist model. Among these models is the Auto Regressive Integrated Moving Average (ARIMA) model, used in this study to model inflation from 1990 to 2009 and also forecast inflation on short-term basis from January to December 2010.

Stein (1970) started it, and discovered that equally plausible models yield qualitatively different predictions for the relationship between inflation rate and per capita output. However, when the inflation rate is initially steady at zero and then increases permanently, there is no ambiguity and that the average person suffers a welfare loss. Thus, policy makers may face a dilemma: reducing inflation may raise the average person's welfare, but the growth rate of per capital output may fall.

Fama (1997) compared the accuracy of survey respondents' inflation forecast relative to univariate time series models. This was extended again by Fama and Gibbons (1982, 1984), which was based on a univariate time-series modeling on the real interest rate. They observed that the interest rate model yields inflation forecast with a lower error variance than a univariate model, and that the interest rate model's forecast dominate those calculated from the Livingston survey. Stock and Watson (1989) showed that going beyond a single model such as the Phillip's curve and including a wide set of potential explanatory variables, leads to a better model in terms of forecast accuracy. Monetary variables, e.g., based on a money demand function can serve as additional explanatory variables in an inflation-forecasting model.

Barro (1995) then explores the inflation – economic growth relationship using a large sample covering more than 100 countries from 1960 to 1990. His empirical findings indicated that there exist a statistically negative relationship between inflation and economic growth, if a certain number of the country's characteristics (education, fertility rate etc) are held constant. He finally suggested that there is at least some reason to consider that higher long-term inflation reduces economic growth.

This contribution investigates time series data for the purpose of modelling asset class returns in the German capital markets. As a broader model class, the vector auto regressions with possible cointegration vectors are considered. Motivated by the dividend discount model on the market level cointegration of dividend yields, bond yields and inflation rates are tested. Both the methodology of Johansen and Engel Granger is applied. Model one is a pure vector autoregressive model and Model two incorporates the possible cointegration of bond yields and inflation rates. (Thomas: 1995). This is followed closely by Bruno and Easterly (1995) who examined the determinants of economic growth. They used the annual consumer price index (CPI) of 26 countries that experienced inflation crises during the period between 1961 and 1992. They considered an inflation rate of 40 percent and over as the threshold level for inflation crisis. They observed an inconsistent or inconclusive relationship between inflation and economic growth below the threshold level, whilst countries with high inflation crisis are excluded from the sample. The empirical analysis suggested a temporal negative relationship between inflation and economic growth beyond the threshold level. They concluded that countries recover their pre-crisis economic growth rates, following successful reduction of high inflation.

Malla (1997) therefore conducted an empirical analysis using a small sample of Asian countries and countries belonging to the organization for economic Co-operation and Development (OECD) separately. After controlling for labour and capital inputs, the estimated results suggested that for OECD countries, there exist a statistically significant negative relationship between economic growth and inflation. Besides, the relationship is not statistically significant for the developing Asian countries. The conclusion was that the cross country relationship between inflation and long term economic growth experienced some fundamental problems, hence an inclusive relationship between inflation and economic growth.

Again Ling and Li (1997) considered fractionally integrated moving Average (FIMA) models with conditional heteroscedasticity, which combined with popular generalized auto regressive conditional heteroscedasticity (GARCH) and (ARIMA) models. This is supported by Drost and Klaassen (1997) who argued that financial data set exhibit conditional heteroscedasticity and as a result GARCH – type model are often used to model this phenomenon.

Meyer et al; (1998) considered the autoregressive integrated moving average (ARIMA) for forecasting Irish inflation and justified that ARIMA models are surprisingly robust with respect to alternative (multivariate) model. Gudmundsson (1998) also used the Variable regression coefficient time lags as the source of randomness to find the relationships between economic time series. This was modelled here by means of variable regression coefficients. The model entails heteroscedastic residuals with a negative serial correlation and can be estimated by the Kalman filter. He realized that the extension of the traditional regression model is highly significant

for the relationship between quarterly values of wages and prices in Iceland. Another approach was used by Dahl and Hansen (1998), who applied recently developed tools designed to select among regime-switching models from a broad class of linear and nonlinear regression models and provided a discussion of the impact on the formation of inflation expectations in the presence of multiple and recurrent changes in inflation regimes. The empirical findings gave a plausible explanation as to why the rational-expectations, hypothesis based on direct measures of inflation expectations from survey series is typically rejected because of large systematic differences between actual and expected inflation rates. Another dimension on inflation analysis is the use of time series to assess price of items as seen in the work of Peterloy and Weaver. Peterloy and Weaver, (1998), used Time series analysis of retail food prices in Russian markets to provide estimate of anticipated and unanticipated inflation, as well as inflation uncertainty derived from a GARCH-M model. The results indicated that distortions in relative prices were induced by the anticipated inflation rate, rather than by unanticipated inflation or a measure of inflation uncertainty. No support was found for the Lucas hypothesis that a positive relationship exists between the relative price structure and the unanticipated rate of inflation. They used time series analysis of retail food prices in Russian markets to provide estimate of anticipated and unanticipated inflation, as well as inflation uncertainty derived from a GARCH-M model. The results indicated that distortions in relative prices were induced by the anticipated inflation rate, rather than by unanticipated inflation or a measure of inflation uncertainty. No support was found for the Lucas hypothesis that a positive relationship exists between the relative price structure and the unanticipated rate of inflation.

In another development Cati et al; (1999) used a standard unit root test for the Brazilian case from the period between January 1974 and June 1993. They realized that the series are stationary and the observed perturbations have temporary effect. The conclusion was that macroeconomic interpretation of the results are in line with the inflationally inertia hypothesis. In relation to the above Chan (1999), adopted the multiple time-series modelling approach suggested by Tiao and Box (1981) to construct a stochastic investment model for price inflation and share dividends. They observed that the method has the advantage of being direct and sequential with respect to iterative steps of tentative specification, estimation and diagnostic checking, parallel to those of the well-known Box-Jenkins method in the univariate time-series analysis. It does not specify any a prior causality as compared to those of other stochastic asset models in the literature. Fari and carneiro (2001) also used the vector auto regression (VAR) to analyze a bivariate time series model for the annual Brazilian data from the period of 1980 to 1995. They concluded that, there exist a negative relationship between inflation and economic growth in the short run; whilst it does not affect economic growth in the long run.

Campêlo and Cribari (2003) showed that, the use of robust standard unit root tests can generate the same inference about the series' order of integration, without using dummy variables, which takes into consideration the existence of in liners. The authors use two monthly series related to the Brazilian inflation rate. The first is similar to Cati et al: (1999) and the second covers the period between February 1944 and February 2000. The main result indicates the presence of inflation inertia, which is small as opposed to the previous results. Chulho and Shambora (2003) analyzed

macroeconomic effects of inflation targeting policy in New Zealand using Markov switching model with one time permanent break. The results showed that the inflation targeting policy has significantly changed the inflation dynamics in New Zealand economy. The Markov switching model clearly detects a structural break date that is very close to the actual date of the policy change. The volatility in the inflation rate shows a considerable reduction after the structural break date. Again the results also showed that the inflation targeting policy led to a structural change in real GDP growth rate. The policy change significantly reduced the volatility of real GDP growth rate after the break date. They concluded that there was a lag of about one year and six months between the monetary policy change and its actual effect on output growth.

The Auto regressive Fractionally Integrated moving average (ARFMA) models was used by Doornik and Ooms (2004) to draw inferences concerning the British and North American inflationary process. The observation was that the quarterly sample of the American case was 0.32 percent of inflation indicating a stationary series, whilst the British case was between 0.47 and 0.59 percent of inflationary rate, showing the possibility of a non stationary process. Gil-Alana (2005) applied the ARFIMA model for the analysis of the U.S.A inflation rate and concluded that the results vary significantly according to low perturbation that are specified in the model. For example, in the white noise specification, the regression showed a stationary inflation with a fractionally integrated parameter equal to 0.25 percent. This is followed closely by Dossche and Everaert (2005), who used the structural time series approach and measured different sorts of inflation persistence allowing for an unobserved time-varying inflation target. The unobserved components were identified using the Kalman

filtering and smoothing techniques. They concluded that inflation persistence, ranges from one quarter in case of a cost-push shock, or several years for a shock to long-run inflation expectations.

Next is Batini (2006) who researched into the European Union by focusing on the lagged impact of monetary policy actions in relation to inflation response. His conclusion was that, the greatest effect of monetary action takes place one year after their implementation. Owing to the forecasting of inflation and growth, Marcellino (2006), used the benchmark to forecast growth and inflation by finding out whether these complicated time series models can really outperformed the standard linear models for GDP growth and inflation. He conducted the model comparison based on the out of sample forecasting performance by considering a large variety of models and evaluation criteria, using real time data and a sophisticated bootstrap algorithm to evaluate the statistical significance of the results. The main conclusion was that in general, linear time series models can hardly be beaten if they are carefully specified, and therefore still provide a good benchmark for theoretical models of growth and inflation. However, he also identified some important cases where the adoption of a more complicated benchmark can alter the conclusions of economic analyses about the driving forces of GDP growth and inflation. Hence, comparing theoretical models also with more sophisticated time series benchmarks can guarantee more robust conclusions. Another contribution to the models used by other researchers is the method of Symbolic Time Series Analysis (STSA) to a series of inflation from a group of Latin-American economies by Brida and Garrido (2006), they started with a partition of two inflation regimes, by using data symbolization for identifying temporal patterns. Afterwards the

statistical information obtained from the patterns was used to estimate the parameters of a nonlinear model proposed by Brida (2000). They compared the performance of the model against a naive benchmark predictor to verify its power to anticipate the qualitative behavior of the inflation time series. Their conclusion was that when the use of STSA is made through pure optimization criteria, the performance of the model is poor. Whilst, the performance of the model increases considerably when the partition of the space of states is made according to economics intuition,

Currently, Candelaria et al., (2007), analyzed a set of countries which adopted inflation targeting (IT) as a policy tool and modelled the pre-IT period with ARMA and GARCH methods. They conducted the one-step ahead forecasting for the remainder of the times series data by comparing the actual and forecasted inflation levels for each country. They concluded that, even though the actual inflation levels are lower than the forecasted ones, there is no statistical evidence to suggest that the adoption of IT causes a structural break in the inflation levels of the countries which adopt IT. The aim of the study is to deal with the empirical aspects of the ‘new’ monetary policy framework, known as Inflation Targeting.

Another contribution to the mathematical models of this review is the use of intervention analysis by Angeriz and Arestist (2008), they applied Intervention Analysis to a multivariate structural time series models, which avoids certain biases encountered in the use of conventional regression estimators, new empirical evidence is produced in the case of a number of OECD countries. The results demonstrate that although Inflation Targeting has gone hand-in-hand with low inflation, the strategy was introduced well after inflation had begun its downward trend. But, then, Inflation

targeting ‘locks in’ low inflation rates. This was supported by Genc (2009), who used the Zivot-Andrews test to analyze a set of countries which adopted inflation targeting (IT) as a policy tool. The conclusion was that there is no statistical evidence to suggest that the adoption of IT causes a structural break in the inflation levels of the countries which adopt IT. This is followed by Shintani et al., (2009), who considered the relationship between the exchange rate pass through, (ERPT) and inflation by estimating a nonlinear time series model. They used a simple theoretical model of ERPT determination, and realized that the dynamics of ERPT can be well-approximated by a class of smooth transition autoregressive (STAR) models with inflation as a transition variable. Their conclusion was that a decline in the ERPT during the 1980s and 1990s are associated with lowered inflation.

Finally, Byrne et al.,(2010), contrasted the time-series properties of aggregate and disaggregate of UK inflation and came with the following suggestions; that the aggregate inflation is found to be non-stationary, the unit root rejection frequencies are increasing when we use more disaggregate data. Structural break analysis suggests that structural shifts in monetary policy could alter inflation persistence. Next, panel evidence indicates that the unit root hypothesis can be rejected for sectoral inflation rates. Finally, the persistence properties of UK inflation findings showed a statistically significant difference between aggregate and disaggregate series.

2.3 CONCLUSION

The chapter has dealt with the reviewing of the relevant literature organized under the following heading; theories on inflation and its effect and mathematical application. The next chapter, which is chapter three, deals with the methodology by strictly considering concepts on ARIMA models.



CHAPTER THREE

METHODOLOGY

3.0 INTRODUCTION

This chapter deals with the basic concepts on time series, stationary and non stationary time Series, ARIMA model, (autoregressive integrated moving average), Principles of ARIMA Modeling (Box-Jenkins 1976) and Conclusion. The objectives of the study are achieved alongside with statistical software such as Minitab, SPSS and E-view from Excel.

3.1 BASIC CONCEPTS ON TIME SERIES

Time series by definition is a collection of observations made sequentially according to the time of their outcome. It is a sequence of data points, measured typically at successive times, spaced often at uniform time intervals. Time series analysis is composed of methods that attempt to understand the underlying context of the data points by making forecasts or predictions. It involves the use of a model to forecast future events based on known past events, hence one is able to forecast future data points before they are measured. A standard example in econometrics is the opening price of a share of stock based on its past performance.

The term time series analysis is used to distinguish a problem, firstly from more ordinary data analysis problems (where there is no natural ordering of the context of individual observations), and secondly from spatial data analysis where there is a context that, observations relate to geographical locations. There are additional possibilities in the form of space-time models (often called spatial-temporal analysis). A time series model will generally reflect the fact that observations close together in time will be more closely related than observations further apart. In addition, it often makes use of the natural one-way ordering of time, so that values in a series for a given time will be expressed as been derived from past values, rather than from future values.

Methods for time series analyses are often divided into two classes: frequency-domain methods and time-domain methods. The former centers on spectral analysis and recently wavelet analysis, and can be regarded as model-free analyses well-suited to exploratory investigations. Time-domain methods have a model-free subset consisting of the examination of auto-correlation and cross-correlation analysis, which involves specified time series models. A time series plot may reveal various features of the data such as: Trend, Periodicity and Unusual features.

- Trend indicates a long term change in the mean level as well as the upward or downward movement that characterizes a time series over a period of time. Thus trend reflects the long-run growth or decline in the time series
- Periodicity shows pattern repeating in time variations. The periodic patterns in a time series complete themselves within a calendar year and are then repeated

on a yearly basis .This is usually caused by such factors as weather and customs.

- Unusual features, refers to the irregular fluctuations of a time series, this is in line with the erratic movements that follow no recognizable or regular pattern. Many irregular fluctuations in a time series are caused by events that cannot be forecasted such as earthquakes, wars, and hurricanes.

These features could be modeled in an additive form as

$$y_t = m_t + s_t + \varepsilon_t \quad t = 0, 1, \dots, n \quad (1.0)$$

Where M_t , called trend is usually a slowly changing function of time S_t is a periodical function of time and ε_t is the random noise component.

The trend (M_t) can exist as linear, Quadratic and polynomial. A linear trend is defined as: $M_t = \beta_0 + \beta_1 t$ (1.1)

A Quadratic trend is also defined as $M_t = \beta_0 + \beta_1 t + \beta_2 t^2$ (1.2)

Lastly a polynomial trend of degree $K \geq 1$ is defined as

$$M_t = \beta_0 + \beta_1 t + \beta_2 t^2 + \dots + \beta_k t^k \quad (1.3)$$

As shown by Box and Jenkins (1970) and further in (1976), models for time series data can have many forms and represent different stochastic processes such as autoregressive (AR) model of order (p), moving average (MA) model of order (q) and Autoregressive moving average (ARMA) model of order (p,q). These three classes of models depend linearly on previous data points, hence a combination of the above

models produce the Autoregressive integrated moving average (ARIMA) model. The use of the autoregressive fractionally integrated moving average (ARFIMA) model generalizes the former three. Among other types of non-linear time series models, also exist models that represent the changes of variance along with time (heteroskedasticity). These models are called autoregressive conditional heteroskedasticity (ARCH) and this collection comprises a wide variety of representation such as: (GARCH, TARCH, EGARCH, FIGARCH, CGARCH, etc). Here changes in variability are related to, or predicted by, recent past values of the observed series.

3.2 STATIONARY AND NON STATIONARY TIME SERIES

There are two types of Stationarity to be discussed under this heading namely: Stationary and Non stationary time series.

3.2.1 STATIONARY TIME SERIES

A series is said to be stationary if the mean and autocovariances of the series do not depend on time. The theory behind ARIMA estimation is based on stationary time series. Stationary series are made up of Strict and weak Stationary series. Strict stationary series is made up of three assumptions as follows:

- (i) The mean $u(t) = E(y_t)$

(ii) The variance $\sigma^2(t) = \text{var}(y_t) = \gamma(0)$

(iii) The autocovariances $\gamma(t_1, t_2) = \text{Cov}(y_{t_1}, y_{t_2})$

Hence a time series is said to be strictly stationary if the joint distribution of any set of n observations $y(t_1, t_2) = \text{cov}(y_{t_1}, y_{t_2})$ is the same as the joint distribution of $y(t_1), y(t_2) \dots y(t_n)$ for all n and k . For a strictly stationary time series the distribution of $y(t)$ is independent of t . Thus it is not just the mean and variance that are constant but also, all higher order moments are independent of t .

The auto covariance function can be written $\gamma(t_1, t_2)$ as $\gamma(\tau)$ where $k = t_2 - t_1$ called the lag. Hence the auto covariance function can be defined with respect to τ as $\gamma(\tau) = \text{Cov}[y(t), y(t + \tau)]$. The defined (ACVF) above has coefficient at lag k . Therefore the Autocorrelation coefficient $\rho(\tau)$ at lag k is given as:

$$\rho(\tau) = \frac{\gamma(\tau)}{\gamma(0)} \quad (1.4)$$

Where $\rho(\tau)$ is called the autocorrelation function abbreviated as ACF. Therefore a plot of $\rho(\tau)$ against k is called the correlogram.

3.2.2 WEAKLY STATIONARY

A time series is said to be weakly stationary if its mean is constant and its autocovariance function depends only on the lag. This is based on two assumptions as follows:

$$E[y(t)] = \mu \quad \text{And} \quad (1.5)$$

$$\text{Cov} [y(t), y(t + \tau)] = \gamma(\tau) \quad (1.6)$$

It must be noted however that no assumptions are made about higher order moments. A given series $y(t_1), y(t_2), \dots, y(t_n)$ follows a multivariate normal distribution; since the multivariate normal distribution is completely characterized by the first (population mean) and the second moments (population variance). It must again be noted here that the two concepts of strict Stationarity and weak Stationarity are equivalent.

3.2.3 NON STATIONARY TIME SERIES

Most of the time series we encounter are nonstationary. Any series that is not stationary is said to be nonstationary and hence must pass through the due process of the Box- Jenkins approach to make it stationary. A simple nonstationary time-series model is given by;

$$y_t = \mu_t + e_t \quad (1.7)$$

Where the mean μ_t is a function of time and e_t is a weakly stationary series. A random noise process y_t defined as

$$y_t = y_{t-1} + \varepsilon_t \quad (1.8)$$

The random noise process defined above in equation (1.8) is a nonstationary series, where ε is called the stationary random disturbance term. Assume that y_0 is equal to zero. Then the process evolves as follows:

$$y_1 = \varepsilon_1 \quad \text{Hence}$$

$$y_2 = y_1 + \varepsilon_2 = \varepsilon_1 + \varepsilon_2 \quad (1.9)$$

This is generalized by successive substitution in terms of “t” as:

$$y_t = \sum_{j=1}^t \varepsilon_j \quad (2.0)$$

Hence the expectation of the mean and variance of a random noise process are defined as $E(y_t) = t \mu$ and $Var(y_t) = t \sigma^2$ since the mean and variance change with time t , the process is therefore a nonstationary process.

3.3 ARIMA MODEL

ARIMA model (autoregressive integrated moving average) is a generalization of the simple AR model that uses three tools for modeling the serial correlation in the disturbance. It is called an integrated model because the stationary ARMA model that is fitted to the differenced data has to be summed or “integrated” to provide a model for the nonstationary data. A difference stationary series is said to be integrated and is denoted as $I(d)$ where d is the order or integration. Suppose we difference ‘d’ times, to make the series

stationary then the process is integrated of order d or $I(d)$ and it has d unit roots.

This is modelled as ARIMA (p, d, q)

The order of integration is the number of unit roots contained in the series, or the number of differencing operations it takes to make the series stationary. Each integration order corresponds to differencing the series being forecasted. A first-order integrated component means that the forecasted model is differenced once with ($d=1$) of the original series. A second order integrated component means the forecasted model is differenced twice with ($d=2$). The three tools for modeling the serial correlation as contained in ARIMA models are as follows:

- Autoregressive model of order p $AR(p)$
- Moving average model of order q $MA(q)$
- ARMA (p, q)

3.3.1 AUTOREGRESSIVE MODEL $AR(p)$

Autoregressive model $AR(P)$ is a type of random process which is often used to model and predict various types of natural phenomena. The idea behind the autoregressive models is to explain the present value of the series X_t , by a function of (p) past values such as $X_{t-1}, X_{t-2}, X_{t-3} \dots, X_{t-p}$.

Therefore an Autoregressive process of order p is written as

$$y_t = \varphi_1 y_{t-1} + \varphi_2 y_{t-2} + \dots + \varphi_p y_{t-p} + \varepsilon_t \quad (2.1)$$

where $\{\varepsilon_t\}$ is white noise, i.e., $\{\varepsilon_t\} \sim \text{WN}(0, 2)$, and ε_t is uncorrelated with X_s for each $s < t$. Since AR is autoregressive, writing equation (2.1) in terms of the lag operator L , gives equation (2.2) as shown below

$$y_t = (\varphi_1 L + \varphi_2 L^2 + \dots + \varphi_p L^p) \cdot y_t + \varepsilon_t \quad (2.2)$$

Now using the backward shift operator on equation 2.2 we obtain

$$Z_t = X_t - \varphi X_{t-1} - \varphi_2 X_{t-2} - \dots - \varphi_p X_{t-p} \quad (2.3)$$

It is noted however that y_t is replaced by x_t , and y_{t-1} is also replaced by x_{t-1} and so on. This simplifies to:

$$Z_t = X_t (1 - \varphi_1 B - \varphi_2 B^2 - \dots - \varphi_p B^p) \quad (2.4)$$

Suppose we let $Z_t = \varphi(B)X_t$,

Therefore equation 2.4 becomes, $X_t = \frac{1}{\varphi(B)} Z_t$

since $Z_t = \varphi(B)X_t$ then an AR of order P can be simplified as

$$\varphi(B) = 1 - \varphi_1 B - \varphi_2 B^2 - \dots - \varphi_p B^p \dots \quad (2.5)$$

By definition AR of order (1) is defined as

$$X_t = \varphi y_{t-1} + \varepsilon_t \quad (2.6)$$

Where $\varepsilon_t \sim \text{WN}(0, \sigma^2)$ and $\varphi = \text{constant}$

We define AR (2) as

$$y_t = \phi y_{t-1} + \phi_2 y_{t-2} + \varepsilon_t \quad (2.7)$$

This continues with AR (3) up to AR of order (p) as in the case of equation 2.3.

To test for Stationarity of AR (1) let $X_t = \phi X_{t-1} + Z_t$ and $Z_t = \varepsilon_t$ is a white noise process, then $X_t = \phi(\phi X_{t-2} + Z_{t-1}) + Z_t$ where $X_t = \phi X_{t-1} + Z_t$, by expanding the above equation we have $X_t = \phi^2 X_{t-2} + \phi Z_{t-1} + Z_t$. This simplifies to:

$$\phi^K X_{t-k} + \sum_{j=0}^{K-1} \phi Z_{t-j} \quad (2.8)$$

the later simplifies to

$$\phi^K X_{t-k} = X_t - \sum_{j=0}^{K-1} \phi Z_{t-j} \quad (2.9)$$

as $K \rightarrow \infty$ Then the limit as K approaches infinity on equation (2.9) is:

$$K \xrightarrow{lin} \infty E(X_t - \sum_{j=0}^{K-1} \phi Z_{t-j})^2 \text{ This reduces to}$$

$$K \xrightarrow{lim} \infty \phi^{2K} E(X_{t-k}^2) = 0 \quad (3.0)$$

If $\phi < 1$ and the variance of X_t is bounded then

$$X_t = \sum_{j=0}^{\infty} \phi^j Z_{t-j} \quad (3.1)$$

Where in the mean sense ϕ_j is defined as:

$$\phi_j = \begin{cases} \phi^j & \text{for } j \geq 0, \\ 0 & \text{for } j < 0 \end{cases}$$

It can so far be stated based on the assumptions on equations 2.8, 2.9, 3.0 and 3.1 that AR (1) is stationary and has a linear process with mean

$$EX_t = \sum_{j=0}^{\infty} \varphi^j E(Z_{t-j}) = 0 \text{ and variance } \gamma(0) = \frac{\sigma^2}{1-\varphi^2}$$

3.3.2 MOVING AVERAGE MA (q)

Moving average is a term in the model consisting of a model parameter times a model forecast error. This is used to specify stationary time series and is defined as

$$y_t = \theta_1 \varepsilon_{t-1} + \theta_2 \varepsilon_{t-2} + \dots + \theta_p \varepsilon_{t-p} + \varepsilon_t \quad (3.2)$$

Where $\varepsilon_t \sim WN(0, \sigma^2)$, $\theta_1, \dots, \theta_q$ are constant and $t = 0 \pm 1, \pm 2, \dots$

It can be remarked that X_t is a linear combination of $q+1$ white noise variables, such that X_t and $X_{t+\tau}$ are uncorrelated for all lags $\tau > q$. It can also be remarked that ε_t is an i.i.d process defined as $(\varepsilon_t, \dots, \varepsilon_{t-q})$ and $(\varepsilon_{t+\tau}, \dots, \varepsilon_{t-q})$. By definition an MA 1 is defined as;

$$y_t = \theta_1 \varepsilon_{t-1} + \varepsilon_t$$

Consider an MA (2) process which is also defined as

$$y_t = \theta_1 \varepsilon_{t-1} + \theta_2 \varepsilon_{t-2} + \varepsilon_t \quad (3.3)$$

$$\text{This simplifies to } y_t = (1 + \theta_1 B + \theta_2 B^2) \varepsilon_t \quad (3.4)$$

As a result MA (2) has a zero mean $E(X_t) = 0$ and expectation defined as:

$E(X_t) = E(Z_t + \theta_1 Z_{t-1} + \theta_2 Z_{t-2}) = 0$. The error term ε_t is represented by Z_t .

To test for stationary it is demonstrated first that MA of order (1) is stationary and does not depend on time (t) but rather on the lag. Consider MA (1) defined as $X_t = \theta_1 \varepsilon_{t-1} + \varepsilon_t$ where $t = 0, \pm 1, \pm 2, \dots$ and $X_t = y_t$. Then expectation of X_t becomes

$$E(X_t) = E(\varepsilon_t + \theta Z_{t-1}) = 0 \quad (3.5)$$

Where $\varepsilon_t = Z_t$

By finding the covariance of X_t and Z_{t-1} in terms of X_t and $X_{t+\tau}$ we obtain the

Covariance of X_t and $X_{t+\tau}$ as

$$\gamma(\tau) = \text{cov}(X_t, X_{t+\tau}) = \begin{cases} (1 + \theta^2) & \text{if } \tau = 0 \\ \theta\sigma^2 & \text{if } \tau = \pm 1 \\ 0 & \text{if } |\tau| > 1 \end{cases} \quad (3.6)$$

The Auto-covariance and Auto-correlation function is expressed by dividing $\gamma(\tau)$ by $\gamma(0)$ to obtain the autocorrelation function as shown below

$$\rho(\tau) = \frac{\gamma(\tau)}{\gamma(0)} = \begin{cases} 1 & \tau = 0 \\ \frac{\theta}{1+\theta^2} & \tau = \pm 1 \\ 0 & |\tau| > 1 \end{cases} \quad (3.7)$$

It can hence be generalized from equations 3.5, 3.6 and 3.7 that MA of order (1) is weakly stationary.

Consider also the test for stationarity for MA of order (2) where:

$$E(X_t) = E(Z_t + \theta_1 Z_{t-1} + \theta_2 Z_{t-2}) = 0$$

Hence expressing X_t and $X_{t+\tau}$ to find the covariance we have

$$\gamma(\tau) = \text{cov}(X_t, X_{t+\tau}) = \begin{cases} (1 + \theta_1^2 + \theta_2^2)\sigma^2 & \tau = 0 \\ (\theta_1 + \theta_1\theta_2)\sigma^2 & \tau = \pm 1 \\ \theta_2\sigma^2 & \tau = \pm 2 \\ 0 & |\tau| > 2 \end{cases} \quad (3.8)$$

Dividing $\gamma(\tau)$ by $\gamma(0)$ we obtain the auto correlation function for an MA (2) process as

$$\rho(\tau) = \begin{cases} 1 & \tau = 0 \\ \frac{\theta_1 + \theta_1\theta_2}{1 + \theta_1^2 + \theta_2^2} & \tau = \pm 1 \\ \frac{\theta_2}{1 + \theta_1^2 + \theta_2^2} & \tau = \pm 2 \\ 0 & |\tau| > 2 \end{cases} \quad (3.9)$$

It implies from equations 3.8 and 3.9 that MA of order (2) is weakly stationary.

Again considering the invertibility of an MA process, then MA of order (1) is invertible if it can be expressed in terms of the lagged values, by substituting repeatedly for lagged values of Z_t so that $Z_t = X_t - \theta Z_{t-1}$. The substitution yields

$$Z_t = X_t - \theta(X_{t-1} - \theta Z_{t-2})$$

This simplifies to

$$Z_t = X_t - \theta X_{t-1} + \theta^2 X_{t-2} - \theta^3 Z_{t-3} \quad (4.0)$$

So that

$$Z_t = X_t - \theta X_{t-1} + \theta^2 X_{t-2} - \theta^3 X_{t-3} + \theta^4 X_{t-4} + \dots + (-\theta)^n Z_{t-n} \quad (4.1)$$

However if $|\theta| < 1$ then

$$E = (Z_t - \sum_{j=0}^{n-1} (-\theta)^j X_{t-j})^2 = E(\theta^{2n} Z_{t-n}^2) \rightarrow \alpha_{n \rightarrow \infty} \quad (4.2)$$

Hence the sum is convergent in mean square sense and we obtain a representation of the model as:

$$Z_t = \sum_{j=0}^{\infty} (-\theta)^j X_{t-j} \quad (4.3)$$

Where $|\theta| < 1$ is defined as an invertible process.

3.3.3 AUTOREGRESSIVE MOVING AVERAGE: ARMA (p, q)

ARMA model is a combination of the simple AR and MA model of order (p,q) called autoregressive moving average (ARMA). It is defined mathematically as

$$y_t = \phi_1 y_{t-1} + \phi_2 y_{t-2} + \dots + \phi_p y_{t-p} + \theta_1 \varepsilon_{t-1} + \theta_2 \varepsilon_{t-2} + \dots + \theta_q \varepsilon_{t-q} + \varepsilon_t \quad (4.4)$$

Where ε_t is a purely random process with mean zero and variance σ^2

Using the lag operator L , we can write an ARMA process as

$$\phi(L)y_t = \theta(L)\varepsilon_t \quad (4.5)$$

Where $\Phi(L)$ and $\theta(L)$ are polynomials of orders p and q , respectively, defined as

$$\phi(L) = (1 - \phi_1 L - \phi_2 L^2 - \dots - \phi_p L^p) \quad (4.6)$$

Simplifying 4.6 gives equation 4.7 as shown below

$$\theta(L) = (1 - \theta_1 L + \theta_2 L^2 + \cdots + \theta_q L^q) \quad (4.7)$$

Form equation 4.4 ARMA of order (1, 1) can be defined as

$$y_t = \phi_1 y_{t-1} + \theta_1 \varepsilon_{t-1} + \varepsilon_t$$

In terms of the lag operator L ARMA of order (1, 1) is written as

$$(1 - \phi L)y_t = (1 + \theta_1 L)\varepsilon_t$$

$$\text{Hence } y_t = \left(\frac{1 + \theta_1 L}{1 - \phi L} \right) \varepsilon_t \quad (4.8)$$

Since ε_t is a purely random process with variance σ^2 , then equation 4.8 becomes

$$\text{Var}(y_t, y_{t-1}) = \frac{(\phi + \theta)(1 + \phi\theta)}{1 - \phi} \sigma^2 \quad (4.9)$$

Where

$$\rho(1) = \frac{\text{cov}(y_t, y_{t-1})}{\text{Var}(y_t)} = \frac{(\phi + \theta)(1 + \phi\theta)}{1 + \theta^2 + 2\phi\theta} \quad (5.0)$$

Successive values of $\rho(k)$ can be obtained from the recurrence relation

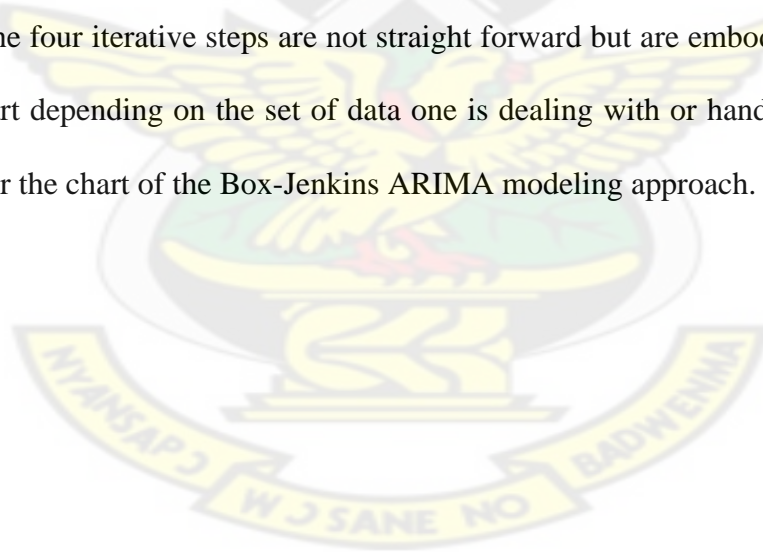
$$\rho(k) = \phi \rho(k-1) \text{ for } k > 2$$

Thus, the ACF for ARMA of order (1, 1) is such that the magnitude of ρ_1 depends on both ϕ and θ .

3.4 PRINCIPLES OF ARIMA MODELLING (BOX-JENKINS 1976)

Box-Jenkins forecasting models are based on statistical concepts and principles and are able to model a wide spectrum of time series behavior. It has a large class of models to choose from and a systematic approach for identifying the correct model form. There are both statistical tests for verifying model validity and statistical measures of forecast uncertainty. In contrast, traditional forecasting models offer limited number of models relative to the complex behavior of many time series with little in the way of guidelines and statistical tests for verifying the validity of the selected model. It consists of four iterative procedures such as: Model Identification, Model Fitting, Model Diagnostics and Forecasting.

The four iterative steps are not straight forward but are embodied in a continuous flow chart depending on the set of data one is dealing with or handling. See figure 3.1 below for the chart of the Box-Jenkins ARIMA modeling approach.



Box-Jenkins Modeling Approach

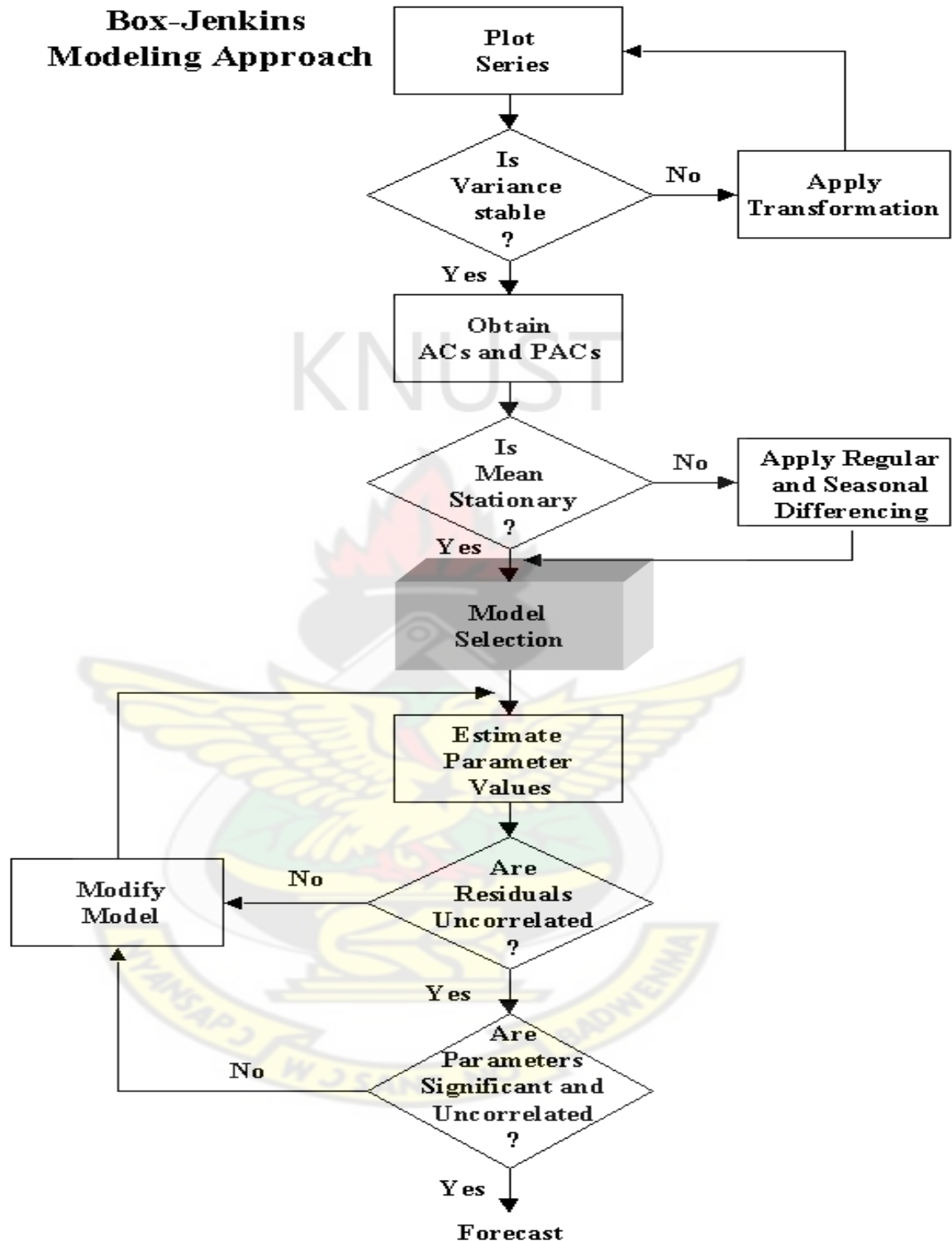


Figure 3.1 Flow chart for Box-Jenkins ARIMA modeling

Referring to the above chart, it should be noted that the variance of the errors of the underlying model must be invariant (i.e. constant). This means that the variance for each subgroup of data used must be the same and should not depend on the level or the point in time. If this is violated then one can remedy this by stabilizing the variance and also making sure that, there are no deterministic patterns in the data.

3.4.1 MODEL IDENTIFICATION

A preliminary Box-Jenkins analysis with a plot of the initial data should be run as the starting point in determining an appropriate model. The input data must be adjusted to form a stationary series, one whose values vary more or less uniformly about a fixed level over time. Apparent trends can be adjusted by having the model apply a technique of "regular differencing," a process of computing the difference between every two successive values, computing a differenced series which has overall trend behavior removed. If a single differencing does not achieve stationarity, it may be repeated, although rarely if ever, are more than two regular differencing required. Where irregularities in the differenced series continue to be displayed, log or inverse functions can be specified to stabilize the series such that the remaining residual plot displays values approaching zero and without any pattern. Given a set of time series data like inflation under consideration, one can calculate the mean, variance, autocorrelation function (ACF), and partial autocorrelation function (PACF) of the time series. This calculation enables one to look at the estimated ACF and PACF which gives one an idea about the correlation between the observations, indicating the sub-

group of models to be entertained. This process is done by looking at the cutoffs in the AC and PACF. At the identification stage for this inflation data, we try to match the estimated ACF and PACF with the theoretical ACF and PACF as a guide for tentative model selection, but the final decision cannot be made until the model is estimated and diagnosed.

Judge (1985) points out that when the PACF has a cutoff at p while the ACF tails off, it gives us an autoregressive of order p (AR (p)). If the ACF has a cutoff at q while the PACF tapers off, it gives a moving-average of order q (MA (q)). However, when both ACF and PACF tail off, it suggests the use of the autoregressive moving-average of order p and q (ARMA (p, q)). Sometimes the ACF doesn't die out quickly, which may suggest that our stochastic process is nonstationary. This situation suggests the use of the ARIMA (p, d, q) to difference the data (d) times, once or twice, until stationarity is obtained. This is the error term, equivalent to pure, white noise. Consider for example the time series plot of Ghana's monthly inflation from the period of January 1990 to December 2009 as shown in figure 3.2 in the next page.

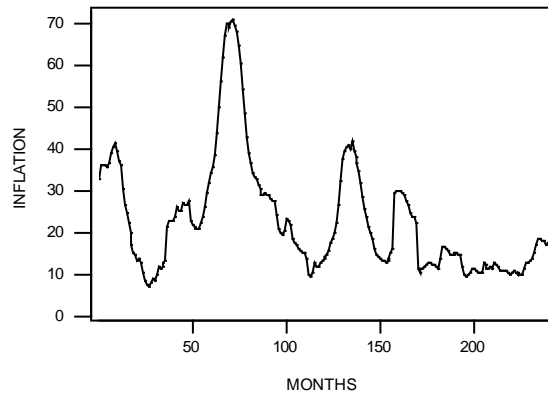


Figure 3.2 General trend of Ghana's Monthly Inflation: period: 1990-2009

The figure above for example requires differencing due to an unstable mean and variance, which has made the data non stationary. A time series is said to be stationary (second-order stationary) if the statistical properties such as the mean (first moment) and the variance (second moment) of the time series are essentially constant throughout time. From the plot of the time series values, if the observed values of a time series seem to fluctuate with constant variation around a constant mean, then it is reasonable to believe that the time series is stationary, otherwise, it is nonstationary.

Differencing is a trend removing operation by using special kind of linear filter with weights $(-1, 1)$. The first lag (lag 1) differencing operator is denoted by ∇ ,

$$\text{Thus } \nabla X = X_t - X_{t-1}. \quad (5.1)$$

The backward shift operator denoted by B is then used to obtain

$$BX_t = X_{t-1} \quad (5.2)$$

Therefore equation 5.1 becomes

$$\nabla X_t = X_t - BX_t \quad (5.3)$$

Considering the second lag operator at $K = 2$

$$\text{Then } \nabla^2 X_t = \nabla(\nabla X_t)$$

This simplifies to

$$\Delta^2 X_t = X_t - 2X_{t-1} + X_{t-2} \quad (5.4)$$

We substitute $X_t = M_t + Y_t$ and obtain

$$\nabla^2 X_t = M_t + Y_{t-2} (M_{t-1} + Y_{t-1}) + M_{t-2} + Y_{t-2} \quad (5.5)$$

Expressing equation (5.4) in terms of $\beta_0, \beta_1 \dots \beta_k$

We have

$$\nabla^2 X_t = \beta_0 + \beta_1 t + \beta_2 t^2 - 2 [\beta_0 + \beta_1 (t-1) + \beta_2 (t-1)^2] + \beta_0 + \beta_1 (t-2) + \beta_2 (t-2)^2 + \nabla^2 Y_t \quad (5.6)$$

The terms are simplified further to obtain

$$\nabla^2 X_t = 2\beta_2 + \nabla^2 Y_t \quad (5.7)$$

Hence the generalized polynomial trend of degree K is

$$\nabla^K X_t = K! \beta_k + \nabla^K Y_t \quad (5.8)$$

The k th differencing trend of degree k gives a stationary process with mean about $K!$ and β_K , since the mean fluctuates about zero. Now differencing at lag d using the differencing operator on equation 5.1 which is $\nabla X_t = X_t - X_{t-1}$ becomes

$$\nabla_d X_t = X_t - X_{t-d} \quad (5.9)$$

By using the backward operator $BX_t = X_{t-d}$ on equation 5.9 we obtain

$$\nabla_d X_t = X_t - B^d X_t$$

This simplifies to

$$\nabla_d X_t = (1 - B^d) X_t \quad (6.0)$$

Further, applying the lag $-d$ operator on equation (5.1) becomes

$$\nabla_d X_t = (M_t + S_t + Y_t) - (M_{t-d} + S_{t-d} + Y_{t-d})$$

Hence

$$\nabla_d X_t = M_t - M_{t-d} + Y_t - Y_{t-d} \quad (6.1)$$

The operator at lag d removes the seasonal effect. Consider for example the time series plot of the first and second difference of the inflation data which was made stationary at the second stage as shown in figure 3.2 and 3.3 respectively at page 51.

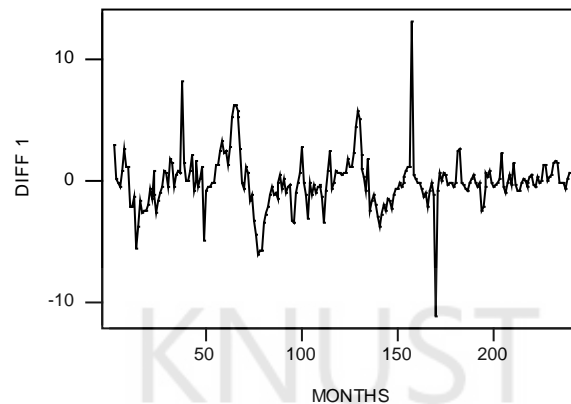


Figure 3.3: First difference of inflation data

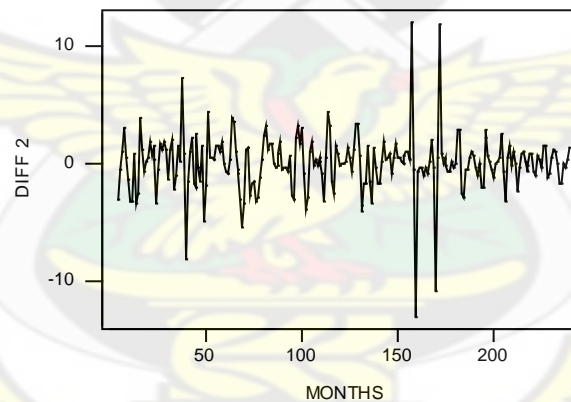


Figure 3.4: Second difference of inflation data

It must be noted that if a single differencing does not achieve stationarity, it may be repeated, although rarely if ever, are more than two regular differencing required. Where irregularities in the differenced series continue to be displayed, log or inverse functions can be specified to stabilize the series such that the remaining residual plot displays values approaching zero and without any pattern.

Consider for example the time series plot of the Auto correlation function (ACF) of the differenced inflation data as well as the Partial auto correlation function (PACF). The Autocorrelation Function (ACF) and partial Autocorrelation Function (PACF) are estimated via ordinary least square (OLS) method as shown below in figure 3.5 and figure 3.6 respectively.

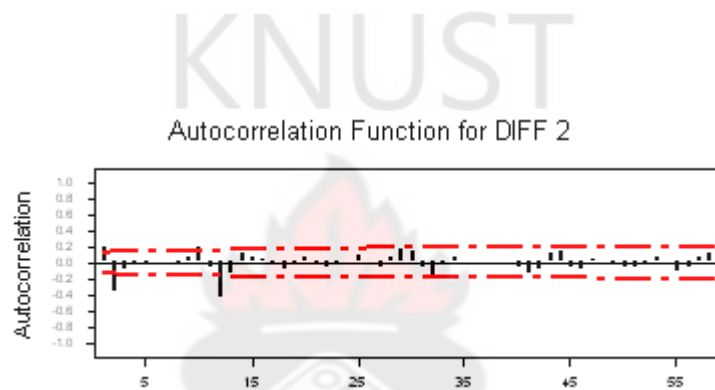


Figure 3.5: Autocorrelation Function for Diff 2

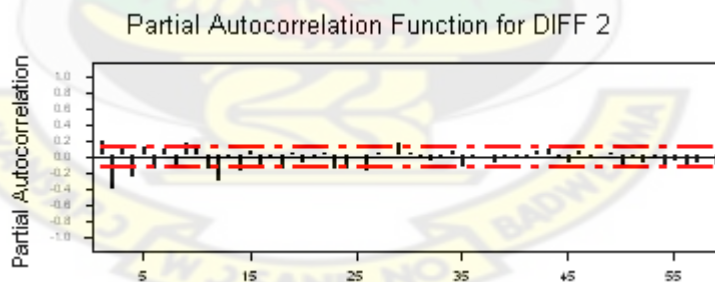


Figure 3.6 Partial Autocorrelation Function for Diff 2

When the PACF cutoff at lag p while the ACF tails off gives an indication of autoregressive of order p ($AR(p)$). If the ACF cutoff at lag q while the PACF tapers off, it gives a moving-average of order q ($MA(q)$). However, when both ACF and

PACF tail off, it suggests the use of the autoregressive moving-average of order p and q (ARMA (p, q)). Sometimes the ACF doesn't die out quickly, which may suggest that our stochastic process is nonstationary. This situation suggests the use of the ARIMA (p, d, q) to difference the data (d) times. A good autoregressive model of order p (AR (p)) has to be stationary, and a good moving average model of order q (MA (q)) has to be invertible. Invertibility and stationarity will give a constant mean, variance, and covariance. Anderson (1976), Chatfield (1984), and Judge (1985) pointed out that it is possible to show how the AR and MA processes are equivalent, thus causing one to expect when a low-order-model of one type adequately explains a series and in the same manner a higher-order-model explaining another series. This expectation is valid only if the sum of the coefficients is less than one. Nevertheless, the principle of parsimony requires the model builder to choose the low-order-model, where the smallest possible number of parameters is employed for adequate representation.

Finally, quality of the coefficients has to meet two requirements. They must be statistically significant, and the correlation between the coefficients must be less than 0.9. The estimated ARIMA model must have a significant t -statistic for each coefficient of the estimated model. The correlation matrix measures the correlation between the estimated coefficients. The coefficients of the ARIMA model are correlated. However, if the absolute correlation coefficient between the two estimated ARIMA coefficients is 0.9 or more, such a coefficient value may suggest that the estimated coefficients are unstable and of poor quality. Under this condition the estimate could be inappropriate for future time periods, unless the behavior of future observations is the same as the behavior of a given realization.

3.4.2 MODEL FITTING

Model fitting consists of finding the best possible estimates for the parameters of the tentatively identified model. In this stage, methods of estimation such as the method of moments, least-squares estimators and maximum likelihood estimators are considered to estimate the parameters. A Box-Jenkins model is considered not invertible, if the weights placed on the past z -observations when expressing z_t as a function of these observations do not decline as we move further into the past. A model which is invertible on the other hand, implies that these weights do decline. Intuitively, this condition should hold, since it seems only logical that a recent observation should count more heavily than a more distantly past observation. The condition of stationarity and invertibility implies that the parameters used in the model under consideration satisfy certain criteria. When we obtain the final least squares point estimates of the parameters in our model, we should verify that these point estimates satisfy the stationarity and invertibility conditions. The model will be considered inadequate if those conditions are not met.

3.4.3 MODEL DIAGNOSTICS

In model diagnostics, various diagnostics such as the method of autocorrelation of the residuals and the Ljung-Box-Pierce statistic are used to check the adequacy of the tentatively identified model. If the model is found to be inappropriate, we would return back to model identification and cycle through the steps until, ideally, an acceptable

model is found. In order to achieve an acceptable model we test whether the estimated model conforms to the specifications of a stationary univariate process. In particular, the residuals should be independent from each other and constant in mean and variance over time. (Plotting the mean and variance of the residuals over time and performing a Ljung-Box test or plotting autocorrelation and partial autocorrelation of the residuals are helpful to identify misspecification). If the estimation is inadequate, we have to return to step one and attempt to build a better model which can be used to forecast future time series values.

The following tests are used in selecting the best or candid model:

- a. Correlogram of the residuals
- b. Normality test of the residuals: where under the normality test we consider the following
 - i. histogram
 - ii. Swilk (Shapiro-Wilk)
 - iii. Sktest (Skewness-Kurtosis)
- c. Akaike information criterion (AIC) given by the relation

$$AIC = 2K \ln(L) \quad (6.2)$$

Where K is the number of parameters in the model, and L is the maximized value of the likelihood function for the estimated model.

d. Normalized Bayesian Information Criterion being one of the model selection criteria indicates that the model with the least NBIC value is selected among the other proposed models.

The ACF of the residuals can be examined in two ways. First, the ACF can be scanned to see if any individual coefficients fall outside some specified confidence interval around zero. Approximate confidence intervals can be computed. The correlogram of the true residuals (which are unknown) is such that r_k is normally distributed with mean

$$E(r_k) = 0 \quad (6.3)$$

And variance

$$Var(r_k) = \frac{1}{N} \quad (6.4)$$

Where (r_k) is the autocorrelation coefficient of the ARMA residuals at lag k . The appropriate confidence interval for r_k can be found by referring to a normal distribution (CDF). For example, the 0.975 probability point of the standard normal distribution is 1.963. The 95% confidence interval for r_k is therefore $\pm 1.96 / N$. For the 99% confidence interval, the 0.995 probability point of the normal CDF is 2.57. The 99% CI is therefore $\pm 2.57 / N$. An r_k outside this CI is evidence that the model residuals are not random. A subtle point that should be mentioned is that the correlogram of the estimated residuals of a fitted ARMA model has somewhat different properties than the correlogram of the true residuals which are unknown because the true model is unknown.

A different approach to evaluating the randomness of the ARMA residuals is to look at the ACF “as a whole” rather than at the individual r_k separately (Chatfield, 2004). The test is called the Portmanteau Lack of Fit Test and the test statistic is

$$Q = N \sum_{k=1}^r r_k^2 \quad (6.5)$$

This statistic is referred to as the *portmanteau statistic*, or “*Q*” *statistic*. The *Q* statistic, computed from the lowest *K* autocorrelations, say at lags $k = 1, 2, \dots, 20$, follows a Chi-square distribution with $(K - p - q)$ degrees of freedom, where *p* and *q* are the AR and MA orders of the model and *N* is the length of the time series. If the computed *Q* exceeds the value from the Chi-square table for some specified significance level, the null hypothesis that the series of autocorrelations represents a random series is rejected at that level. The *p*-value gives the probability of exceeding the computed *Q* by chance alone, given a random series of residuals. Thus non-random residuals give *high Q* and *small p-value*. A significance level greater than 99%, for example, corresponds to a *p*-value *smaller* than 0.01

3.4.4 FORECASTING

The Box-Jenkins methodology requires that the model to be used in describing and forecasting a time series should be both stationary and invertible. Thus, in order to tentatively identify a Box-Jenkins model, we must first determine whether the time series we wish to forecast is stationary. If it is not, we must transform the time series into a series of stationary time series values through the process of differencing. A time

series is said to be stationary (second-order stationary) if the statistical properties such as the mean (first moment) and the variance (second moment) of the time series are essentially constant through time. From the plot of the time series values, if the observed values of a time series seem to fluctuate with constant variation around a constant mean, then it is reasonable to believe that the time series is stationary, otherwise, it is said to be nonstationary. Notwithstanding, after scrutinizing the estimated time series model through all the diagnostic checks, then the model is fit for forecasting.

3.5 CONCLUSION

Chapter three is crafted into four main sections and specifically deals with the basic concepts on time series, Stationary and Non stationary time Series, ARIMA model, (autoregressive integrated moving average), and Principles of ARIMA Modeling (Box-Jenkins 1976) The next chapter which is chapter four discusses the results of the study in a more detailed and concise manner.

CHAPTER FOUR

ANALYSIS AND DISCUSSION OF RESULTS

4.0 INTRODUCTION

This chapter displays, discusses and interpret the results obtained from the study. It has further been organized into Preliminary analysis, Model fitting, Model diagnostic, Evaluating the accuracy of the forecast, Inflation model for the period of 1990 to 2000, Inflation model for the period of 2001 to 2009, Comparing inflation rates between the two periods of January 1990 to December 2000 and January 2001 to December 2009 and Conclusion.

4.1 PRELIMINARY ANALYSIS

It is recommended that a lengthy time series data is required for univariate time series forecasting. Meyler et al, (1988), recommended that at least 50 observations should be used for such a univariate time series forecasting. This could be problematic if few observations are used. However when using a long time series data, it could be possible that the series contains a structural break which may necessitate only examining a sub-section of the entire data series or alternatively using intervention analysis or dummy variables. This is because, there may be some conflict between the

need for sufficient degrees of freedom for statistical robustness and having a shorter data sample to avoid structural breaks. The series should be plotted against time to assess whether any structural breaks, outliers or data errors occurred. This step may also reveal whether there is significant seasonal pattern in the times series or not. A dimension of the preliminary analysis for examining nonstationarity of the data is by considering the time series plot of inflation from 1990 to 2009 as shown below in figure 4.1.

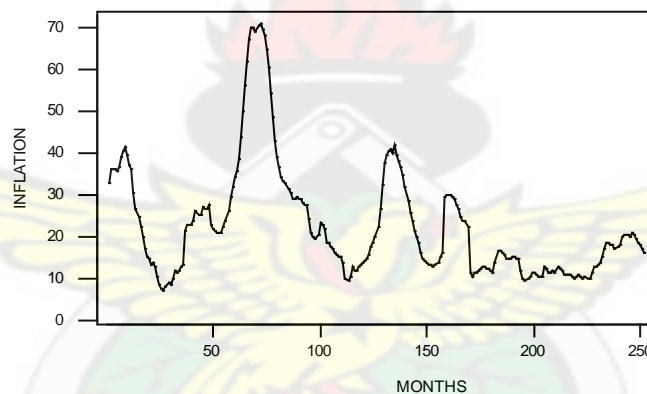


Figure 4.1 General trend of Ghana's Monthly Inflation: period: 1990-2009

It is revealed from figure 4.1 above that inflation rate for the period of 1990 to 2009 is nonstationary due to an unstable mean which increase and decrease at certain points. The mean and variance ought to be adjusted to form stationary series, so that the values vary more or less uniformly about a fixed level over time. The mean is not constant throughout the series as it assumes a downward trend by decreasing from the highest peak to the lowest peak. There are sudden swings around 1995, 2003, and 2004 after which the mean stabilizes in the remaining years whilst the variance reduces from

the highest swing it attained, hence the mean and variance are non stationary. A normality test performed on the mean and variance using the Anderson-Darling Normality Test at 95% confidence interval see table 4.1 and figure 4.2 below, revealed that the mean is rightly skewed with a mean value of 22.995 and variance of 14.0004. The coefficient of skewness and kurtosis are 1.61881 and 2.6340 respectively. It is evident at 5% significant level that there are large swings in the data indicating non stationarity.

Table: 4.1 Descriptive Statistics of Inflation (Anderson-Darling Normality Test)

ITEM	OBSERVATION
A-Squared	11.325
P-Values	0.000
Mean	22.9995
Standard Deviation	14.0004
Variance	196.012
Skewness	1.61881
Kurtosis	2.6340

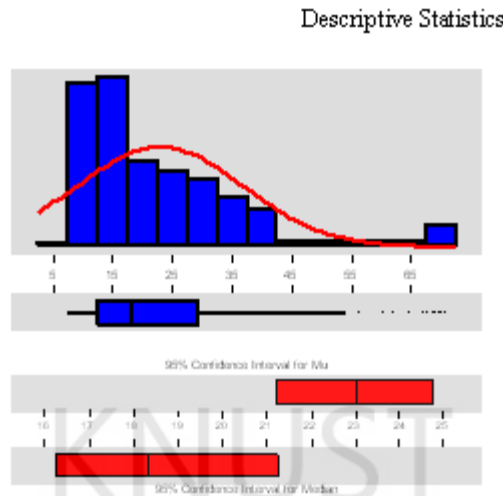


Figure: 4.2 Anderson –Darling Normality Plot for inflation from 1990-2009.

Due to the nonstationarity of the data above which we observe from the time series plot and the Anderson Darling Normality test we also apply the unit root test and precisely the Augmented Dickey Fuller Test as shown below in Table 4.2.

TABLE: 4.2 Augmented Dickey Fuller Unit Root Test on Inflation.

ADF Test Statistic	-2.0408	1% Critical Value*	-3.4807
		5% Critical Value	-2.8833
		10% Critical Value	-2.5783

*Mackinnon Critical values for rejection of hypothesis of a unit root.

The alternative hypothesis of a unit root test is rejected in favour of the null hypothesis if the absolute value of the test statistic is less than the critical value; therefore the null hypothesis of the unit root test above is accepted at a critical value of

1% and even at a critical value of 5% and 10%. From table 4.2 above the absolute value of the test statistic at 1% is less than the critical value, thus $-2.0408 / < / -3.4807 /$. Again at 5% and 10% the absolute value of the test statistic is less than the absolute value of the critical values, hence we accept the null hypothesis and indicate that, unit root exist and as a result the data is non stationary.

Before differencing, it is revealed from figure 4.4 in appendix one that the model under consideration has a quadratic trend with a less MSD of 151.654. It also has MAPE and MAD of 48.180 and 9.170 respectively which are comparatively low. Therefore by virtue of the MSD, MAPE and MAD associated with the model under consideration, the suggested model is of the form:

$$m_t = 28.6837 + 1.60e^{-0.2t} - 3.70e^{-0.4t^2} \quad (6.6)$$

Differencing is the process of computing the difference between every two successive values by computing a differenced series which has overall trend behavior removed. This is a trend removing operation by using special kind of linear filter with weights (-1, 1). The data is differenced for the first time as shown in figure 4.5 in appendix two. At first difference inflation rate was very high with the mean fluctuating around the centroid as it decreases and increases at certain points in the year under review. The continual changes in the mean due to large swings associated with the first difference of the time series plot, makes the mean non-stationary and hence making the variance non-stationary. Since single differencing does not achieve stationarity, differencing is repeated, although rarely if ever, are more than two regular differencing

required, see figure 4.6 in appendix two for the second differencing to make the data stationary.

Evidently from figure 4.6 in appendix two, the variance and mean looks stationary as compared to the first difference, but this is not enough to test for stationarity with respect to the mean and variance. A confirmatory test on stationarity is followed through using the Augmented Dickey Fuller unit root test on the first and second difference of inflation as shown below in Table 4.3.

TABLE: 4.3 Unit Root Test (After First and Second Difference of Inflation).

ADF Test Statistic	-4.2371	1% Critical Value*	-2.5747
		5% Critical Value	-1.9422
		10% Critical Value	-1.6159

*Mackinnon Critical values for rejection of hypothesis of a unit root.

After the first and second difference it is observed from table 4.3 that the absolute value of the test statistic is greater than the critical value at 1%, thus/ $-4.2371 / > / -2.5747 /$. Hence we accept the alternative hypothesis and reject the null hypothesis of a unit root test of non stationarity and conclude that the data is stationary and is due for both the ACF and PACF analysis. In order to obtain a fair idea about the correlation between the observations, indicating the sub-group of models to be entertained, the cutoffs in the ACF and PACF are considered as a guide for tentative

model selection as shown below in figure 4.7 and 4.8 respectively. Figure 4.7 and 4.8 show the ACF and PACF for Diff two.

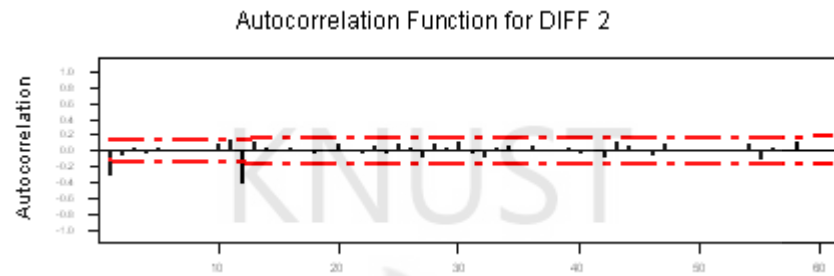


Figure 4.7 Auto correlation Function for inflation: period: 1990-2009

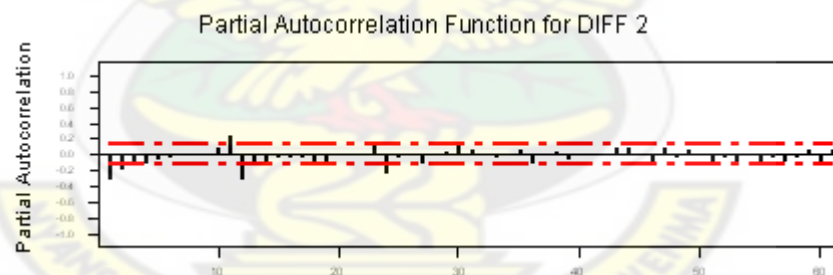


Figure 4.8: Partial Auto correlation Function for inflation: period: 1990-2009.

The output of the ACF and PACF in figure 4.7 and 4.8 respectively indicate that both ACF and PACF tail off after cutting at lag two, thus they both tend to decay consistently as the lags die out at the later part of the output. The ACF cuts at lag two (2) and further around lag twelve (12), there after it decays and dies off at the later part

of the output in figure 4.7. The PACF also cuts at lag two (2) and further around lag twelve (12), after which it decays and dies off at the later part of the output in figure 4.8. This suggests the use of an Autoregressive Integrated moving-average of order (p, d, q), (ARIMA (p, d, q)). On this account several models such as ARIMA (3, 2, 1), ARIMA (4, 2, 1) and ARIMA (2, 2, 0) models are suggested for tentative model selection.

4.2 MODEL FITTING

Having obtained some suggested models we find the best possible estimates for the parameters, by considering the final estimates of parameter, the modified - pierce (Ljung – Box) chi-square and the model selection criteria. Consider the final estimates of parameter, the modified - pierce (Ljung – Box) chi-square and the model selection criteria for ARIMA (3, 2, 1) as shown below in table 4.4, 4.5 and 4.6 respectively.

Table 4.4 Final Estimates of Parameter for ARIMA (3, 2, 1)

TYPE	COEFFICIENT	SE COEFF	T VALUE	P VALUE
AR 1	-1.3602	0.0628	-21.66	0.000
AR 2	-0.5920	0.1002	-5.91	0.000
AR 3	-0.1844	0.0629	-2.93	0.004
MA 1	-0.9833	0.0001	-13.074.13	0.000

Table 4.5: Modified Box-Pierce (Ljung-Box) Chi-Square Statistics.

Lag	12	24	36	48
Chi-square	57.1	62.8	82.0	95.5
DF	7	19	31	43
P-Value	0.000	0.000	0.000	0.000

Table 4.6 Model Selection Criteria for ARIMA (3, 2, 1).

MODEL	R-Squared	RMSE	MAPE	MAE	Max APE	Max AE	Normalized BIC
ARIMA (3, 2, 1)	0.983	1.818	5.797	1.125	91.790	12.385	1.329

From Table 4.4 above which deals with the final estimate of parameter for ARIMA (3, 2, 1), it is observed that the p-values of AR (1), AR (2), AR (3) and MA (1) are less than 0.05 and are therefore significant. It is observed further that the coefficients of AR (1), AR (2), AR (3) and MA (1) being -0.3602, -0.5920, -0.1844 and -0.9833 respectively Sum up to -2.1366 which is far less than 1.0. This indicates that inflation has a downward trend and continues to decrease at a minimal rate without rising up. The parameters ϕ_1, ϕ_2, ϕ_3 and θ_1 are strongly significant and the p-values of the Ljung-Box test in table 4.5, above, suggest that more of the groups of the residuals are correlated and represent a white noise variable.

Consider also the second suggested model being ARIMA (2, 2, 0) by looking at the final estimates of parameter, the modified - pierce (Ljung – Box) chi-square and the model selection criteria for ARIMA (2, 2, 0) as shown below in table 4.7, 4.8 and 4.9 respectively.

Table 4.7 Final Estimates of Parameter for ARIMA (2, 2, 0).

TYPE	COEFFICIENT	SE COFF	T VALUE	P VALUE
AR 1	-0.3964	0.0620	-6.39	0.000
AR 2	-0.2181	0.0621	-3.51	0.001

Table 4.8 Modified Box-Pierce (Ljung-Box) Chi-Square Statistics (ARIMA 2, 2 0).

Lag	12	24	36	48
Chi-square	60.8	66.1	84.3	98.7
DF	9	21	33	45
P-Value	0.000	0.000	0.000	0.000

Table 4.9 Model Selection Criteria for ARIMA (2, 2, 0)

MODEL	R-Squared	RMSE	MAPE	MAE	Max APE	Max AE	Normalized BIC
ARIMA (2, 2, 0)	0.981	1.895	6.286	1.195	89.731	12.070	1.367

Table 4.7 above on final estimate of parameter for ARIMA (2, 2, 0) shows a significant p-value of 0.00 which is less than 0.05. The coefficients of AR (1) and AR (2) are -0.3964 and -0.2181 respectively which sum up to -0.6145. The sum of -0.6145 is far less than 1.0 and therefore means that inflation has a downward trend and continues to decrease at a minimal rate without rising up. The parameters ϕ_1 , and ϕ_2 are strongly significant and the p-values of the Ljung-Box test in table 4.8, above, suggest that more of the groups of the residuals are correlated and represent a white noise variable.

Finally consider the last suggested model, ARIMA (4, 2, 1) by looking at the final estimates of parameter, the modified - pierce (Ljung – Box) chi-square and the model selection criteria for ARIMA (4, 2, 1) as shown below in table 4.10, 4.11 and 4.12 respectively.

Table 4.10 Final Estimates of Parameter for ARIMA (4, 2, 1)

TYPE	COEFFICIENT	SE COFF	T VALUE	P VALUE
AR 1	-1.3874	0.0634	-21.89	0.000
AR2	-0.6763	0.1062	-6.37	0.000
AR 3	-0.3759	0.1063	-3.54	0.000
AR 4	-0.1401	0.0635	-2.21	0.028
MA 1	-0.9840	0.0001	-17949.93	0.000

Table 4.11 Modified Box-Pierce (Ljung-Box) Chi-Square Statistics

Lag	12	24	36	48
Chi-square	51.8	57.7	76.1	87.7
DF	6	18	30	42
P-Value	0.000	0.000	0.000	0.000

Table 4.12: Model Selection Criteria for ARIMA (4, 2, 1)

MODEL	R-Squared	RMSE	MAPE	MAE	MaxAPE	MaxAE	Normalized BIC
ARIMA (4, 2, 1)	0.982	1.876	6.125	1.186	89.432	12.310	1.413

It can be deduced from Table 4.10 and 4.11 above (final estimate of parameter and Ljung-Box test for ARIMA (4, 2, 1)) that the parameters of $\varphi_1, \varphi_2, \varphi_3, \varphi_4$ and θ_1 are strongly significant and the p-values of the Ljung-Box test in table 4.11, above, suggest that more of the groups of the residuals are correlated and represent a white noise variable.

4.3 MODEL DIAGNOSTIC

The model selection criteria which consist of the NBIC, R-square, RMSE, MaxAPE, and MaxAE from tables 4.6, 4.9 and 4.12 are used in selecting the candid model from the suggested models as well as evaluating the accuracy of the forecast. It is observed from ARIMA (3, 2, 1), ARIMA (2, 2, 0) and ARIMA (4, 2, 1), being the suggested models, that the final estimate of parameters from tables 4.4, 4.7 and 4.10 as well as the modified box-pierce (Ljung-box) chi-square statistics are significant. It is further observed that more of the groups of the residuals are correlated and represent a white noise variable. However the Normalized BIC test reveals that the model with the least Normalized BIC is better in terms of forecasting performance than the one with a large Normalized BIC. ARIMA (3, 2, 1) has the least Normalized BIC of 1.329 compared to 1.367 and 1.413 of ARIMA (2, 2, 0) and ARIMA (4, 2, 1) in tables 4.6, 4.9 and 4.12 respectively above. The R-Squared, RMSE, MAPE, MAE, MaxAPE, and MaxAE are further taken into consideration as it measures the accuracy of the fitted time series model. RMSE and MAE serve as measures for comparing forecast of the same series across different models and hence the smaller the error, the better the forecasting ability of the model. Empirically from tables 4.6, 4.9, and 4.12 above ARIMA (3, 2, 1) has RMSE of 1.818, 1.895 for ARIMA (2, 2, 0) and 1.876 for ARIMA (4, 2, 1), which indicates a better forecasting ability and a smaller forecasting error for ARIMA (3, 2, 1) than the other suggested models.

The MAE, MAPE, MaxAPE, MaxAE from tables 4.6, 4.9, and 4.12 above gives an indication of a smaller error and a better forecasting ability for ARIMA (3, 2, 1) than

the other suggested models. Based on the supporting model selection criteria and the forecasting evaluation criteria, it is proposed that the best model among the three suggested models as stated above is ARIMA (3, 2,1) written as:

$$y_t = \varphi_1 y_{t-1} + \varphi_2 y_{t-2} + \varphi_3 y_{t-3} + \theta_1 \varepsilon_{t-1} + \varepsilon_t \quad (6.7)$$

Substituting the values of $\varphi_1, \varphi_2, \varphi_3$ and θ_1 into equation 6.7 we obtain

$$\hat{y}_t = -1.3602y_{t-1} - 0.5920y_{t-2} - 0.1844y_{t-3} + 0.9833\varepsilon_{t-1} + \hat{\varepsilon}_t \quad (6.8)$$

Where $\hat{\varepsilon}_t$ is the realization of a WN $(0, \delta^2)$

Now substituting the trend m_t in equation 6.8 above then the model for inflation between the periods of 1990 to 2009 becomes

$$\begin{aligned} \hat{y}_t = & 28.6837 + 1.60e^{-0.2}t - 3.70e^{-0.4}t^2 + -1.3602y_{t-1} - 0.5920y_{t-2} - \\ & 0.1844y_{t-3} + 0.9833\varepsilon_{t-1} + \hat{\varepsilon}_t \end{aligned} \quad (6.9)$$

4.4 EVALUATING THE ACCURACY OF THE FORECAST

To assess the out-of-sample forecasting ability of the model it is advisable to retain some observations at the end of the sample period which are not used to estimate the model. The standard approach of forecast accuracy analysis is to investigate the basis of the forecasts, their efficiency in terms of incorporating all available information and their relative performance compared with other forecast of the same indicator. Test of forecast efficiency, determines whether the forecast utilizes all the available

information at a given point in time. It is further to check whether the forecast efficiency is related to the question of the impact of potential determinants of inflation on the forecast performance. In order to estimate the model recursively and forecast ahead a 12- month's inflation for the year 2010, we retain inflation figures from January to April 2010 for the purposes of estimating and validation of the model, whilst inflation figures from January 1990 to December 2009 was used to obtain the model. Found below is a 12-months ahead inflation forecast for the year 2010, starting from January to December 2010 alongside the existing actual values from January to April 2010 in table 4.13 (12-months inflation forecast from January to December 2010).

Table 4.13 12-Months Forecasted Inflation for 2010 (January - December)

PERIOD	FORECASTED INFLATION	ACTUAL DATA	FORECASTED ERROR	LOWER LIMIT	UPPER LIMIT
January	15.05	14.78	-0.3	11.36	18.74
February	14.10	14.23	0.1	7.06	21.13
March	13.10	13.32	0.2	2.45	23.75
April	12.14	11.66	-0.5	-2.84	27.13
May	12.00	10.98	-1.0	-8.57	31. 69
June	11. 77	NA	-	-14.73	36. 63
July	11.11	NA	-	-21.27	41.48
August	10. 78	NA	-	-28.19	47.06
September	10.20	NA	-	-35.45	52. 68
October	9.86	NA	-	-43.05	58.91
November	9.15	NA	-	-50.96	65.04
December	8.92	NA	-	-59.18	71.96

It should be noted however that a good model has a low forecasting error, therefore when the magnitude of the difference between the forecasted and actual values are low then the model has a good forecasting power and if the difference is high, then the model has a low forecasting power. On the contrary if the mean error (ME), for example of all the twelve forecasts for 2010 are all positive, then the model is forecasting too low on the average, whilst the model will be forecasting too high on the average if the mean error (ME), of all the twelve forecasts for 2010 are all negative. When the ME is of the same magnitude as the Mean Absolute Error (MAE), this would also indicate that the model is either forecasting consistently too low (if ME is positive) or too high (if ME is negative), the Root Mean Square Error (RMSE) will always be at least as large as the MAE. They will only be equal if all errors are exactly the same. The ME, MAE and RMSE all vary depending on the dimensions or scale of measurement of the dependent variable. It could be revealed from table 4.13 above (forecasted inflation for 2010), that the differences between the forecasted and actual values under the forecasted error are -0.3, 0.1, 0.2 -0.5 and -1.0 which are extremely low and lie between ± 1 .

In addition the forecasted values from January to December 2010 from table 4.13 above follows closely with the actual values from January to April 2010 when they are placed side by side. The forecasted errors of -0.3, 0.1, 0.2 -0.5 and -1.0 are combinations of both positive and negative errors which shows that, the model is not forecasting too low on the average or too high on the average. Hence, from the ongoing assessment per the actual and the forecasted inflation in table 4.13 above, it could be suggested that inflation is expected to decrease steadily from the first half year of 2010

through the second half year of 2010 from 15.05 percent to 11.77 percent. The third half year of 2010 will see inflation going up down around 10 to 11 percent with a percentage decrease of 0.66, 0.33 and 0.58 percent respectively. Due to the downward trend assumed by the forecasted values in table 4.13 above, a single inflation digit of 8 to 9 percent with a percentage decrease of 0.34, 0.71 and 0.23 percent will be assumed by the fourth quarter of 2010.

It can therefore be projected that a single inflation digit of 8 to 9 percent for the fourth quarter of 2010 will be assumed visa vice the Bank of Ghana's projection for inflation which is to move towards 10 percent in the fourth quarter of 2010 and possibly attain a single digit range of 7 to 11 percent early 2011. It must however be acknowledged that the above projection based on table 4.13, is likely to face uncertainties like the behavior of Crude oil prices, which has been highly volatile in recent times, exchange rate depreciation and its effect on consumer prices, particularly on the prices of imported goods and finally the impact of the recession on commodity terms of trade.

4.5 INFLATION MODEL FOR THE PERIOD OF 1990 TO 2000

The time series plot of inflation from the period of January 1990 to December 2000 see figure 4.9 in appendix three, has an unstable mean which is not uniform about a fix point resulting to an unstable variance. In the process of detrending the data it was observed that the period from January 1990 to December 2000 had MAPE of 56.618, MAD of 12.030 and MSD of 225.941, see figure 4.10 in appendix three.

We apply differencing, which is the process of computing the difference between every two successive values by computing a differenced series which has overall trend behavior removed. See figure 4.11, and 4.12 in appendix four for the first and second differenced data. The data was differenced twice before stationarity was obtained. This gives an integrated ARIMA with d equal to two (2). In order to obtain a fair idea about the correlation between the observations, indicating the sub-group of models to be entertained, we look at the cutoffs in the ACF and PACF which is to serve as a guide for tentative model selection see figure 4.13 and 4.14 in appendix five. From the output as indicated in figure 4.13 and 4.14 in appendix five, both ACF and PACF tail off after cutting at lag two, thus they both tend to decay consistently as the lags die, however the ACF do not decay at the later part of the output. The suggested models are ARIMA (2, 2, 3) and ARIMA (1, 2, 2). Tables 4.14, 4.15 and 4.16 below show the final estimates of parameter, the modified - pierce (Ljung – Box) chi-square and the model selection criteria for ARIMA (1, 2, 2).

Table 4.14 Final Estimates of Parameter for ARIMA (1, 2, 2)

TYPE	COEFFICIENT	SE COFF	T VALUE	P VALUE
AR 1	0.7807	0.0645	0.0358	0.000
MA 1	1.0813	0.0032	338.81	0.000
MA 2	-0.1020	0.0358	-2.85	0.005

Table 4.15 Modified Box-Pierce (Ljung-Box) Chi-Square Statistics (ARIMA 1, 2, 2)

Lag	12	24	36	48
Chi-square	23.9	31.7	52.3	65.6
DF	8	20	32	44
P-Value	0.002	0.031	0.007	0.004

Table 4.16 Model Selection Criteria for ARIMA (1, 2, 2)

MODEL	R-Squared	RMSE	MAPE	MAE	MaxAPE	MaxAE	Normalized BIC
ARIMA (1, 2, 2)	0.989	1.714	5.643	1.232	36.552	7.859	1.265

On the other side the final estimates of parameter, the modified - pierce (Ljung – Box) chi-square and the model selection criteria for ARIMA (2, 2, 3) are shown below in tables 4.17, 4.18 and 4.19

Table 4.17 Final Estimate of Parameter for ARIMA (2, 2, 3)

TYPE	COEFFICIENT	SE COFF	T VALUE	P VALUE
AR 1	1.4762	0.0640	23.07	0.000
AR 2	-0.8810	0.0619	-14.24	0.000
MA 1	1.7810	0.0398	44.8	0.000
MA 2	-1.1956	0.0206	-57.92	0.000
MA 3	0.1474	0.0210	7.01	0.000

Table 4.18 Modified Box-Pierce (Ljung-Box) Chi-Square Statistics (ARIMA 2, 2, 3)

Lag	12	24	36	48
Chi-square	21.8	30.7	50.2	61.6
DF	6	18	30	42
P-Value	0.000	0.000	0.000	0.000

Table 4.19 Model Selection Criteria for ARIMA (2, 2, 3)

MODEL	R-Squared	RMSE	MAPE	MAE	MaxAPE	MaxAE	Normalized BIC
ARIMA (2, 2, 3)	0.989	1.712	5.635	1.236	36.157	7.774	1.338

From the final estimate of parameter and the modified box-pierce (Ljung-box) chi-square statistics as in tables 4.14, 4.15, 4.17, and 4.18 above, it was observed that the parameters of ARIMA (2, 2, 3) and ARIMA (1, 2, 2) been the suggested models are significant and that more of the groups of the residuals are correlated and represents a white noise variable. However the Normalized BIC test reveals that the model with the least Normalized BIC is better in terms of forecasting performance than the one with large Normalized BIC. ARIMA (1, 2, 2) has the least Normalized BIC of 1.265 compared to 1.338 of ARIMA (2, 2, 3). The R-Squared, RMSE, MAPE, MAE, MaxAPE, and MaxAE are further taken into consideration as it measures the accuracy of the fitted time series values. RMSE and MAE serve as measures for comparing forecast of the same series across different models and hence the smaller the error, the better the forecasting ability of the model. Based on the model selection criteria as in tables 4.16 and 4.19, above it is proposed that the best model among the lot is ARIMA (1, 2, 2). From the numerical output of tables 4.14, 4.15 and 4.16 above the proposed model for ARIMA (1, 2, 2) is written as

$$y_t = \varphi_1 y_{t-1} + \theta_1 \varepsilon_{t-1} + \theta_2 \varepsilon_{t-2} + \varepsilon_t \quad (7.0)$$

Substituting the values of φ_1 , θ_1 and θ_2 into equation 7.0 we obtain

$$\hat{y}_t = 0.7807 y_{t-1} - 1.0813 \varepsilon_{t-1} + 0.1020 \varepsilon_{t-2} + \hat{\varepsilon}_t \quad (7.1)$$

Where $\hat{\varepsilon}_t$ is the realization of a WN $(0, \delta^2)$, substituting the trend m_t into equation 7.1 the final model for the period of 1990 to 2000 is written as

$$\hat{y}_t = 18.5770 + 0.455848t - 3.57e^{-0.3}t^2 + 0.7807y_{t-1} - 1.0813\varepsilon_{t-1} + 0.1020\varepsilon_{t-2} + \hat{\varepsilon}_t \quad (7.2)$$

Where $\hat{\varepsilon}_t$ is the realization of a WN $(0, \delta^2)$

4. 6 INFLATION MODEL FOR THE PERIOD OF 2001 TO 2009

The time series plot of inflation from the period of January 1990 to December 2000 see figure 4.15 in appendix six, has an unstable mean which is not uniform about a fix point resulting to an unstable variance. In the process of detrending the data it was observed that the period from January 2001 to December 2009 had MAPE of 21.5464, MAD of 3.6996 and MSD of 24.2529, see figure 4.16 in appendix six. We apply differencing, which is the process of computing the difference between every two successive values by computing a differenced series which has overall trend behavior removed. See figure 4.17 and 4.18 in appendix seven for the first and second differenced data. The data was differenced twice before stationarity was obtained.

In order to obtain a fair idea about the correlation between the observations, indicating the sub-group of models to be entertained, we look at the cutoffs in the ACF and PACF which is to serve as a guide for tentative model selection see figure 4.19 and 4.20 in appendix eight. From the output of the PACF and ACF as indicated in figure 4.19 and 4.20 in appendix eight, both ACF and PACF tail off after cutting at lag two, thus they both tend to decay consistently as the lags die out, hence ARIMA (2, 2, 1) and ARIMA (3, 2, 1) are suggested among the lot. Tables 4.20, 4.21 and 4.22 below show

the final estimates of parameter, the modified - pierce (Ljung – Box) chi-square and the model statistic for ARIMA (2, 2, 1)

Table 4.20 Final Estimates of Parameter for ARIMA (2, 2, 1)

TYPE	COEFFICIENT	SE COFF	T VALUE	P VALUE
AR1	-1.3764	0.0842	-16.34	0.000
AR2	-0.4389	0.0844	5.20	0.000
MA 1	-0.9860	0.0020	-490.39	0.000

Table 4.21 Modified Box-Pierce (Ljung-Box) Chi-Square Statistics (ARIMA, 2, 2, 1)

Lag	12	24	36	48
Chi-square	44.6	53.1	59.8	62.9
DF	8	20	32	44
P-Value	0.000	0.000	0.000	0.000

Table 4.22 Model Selection Criteria for ARIMA (2, 2, 1)

MODEL	R-Squared	RMSE	MAPE	MAE	MaxAPE	MaxAE	Normalized BIC
ARIMA (2, 2, 1)	0.932	1.894	5.845	0.935	95.711	13.066	1.479

Further, we consider the final estimates of parameter, the modified - pierce (Ljung – Box) chi-square and the model statistics for ARIMA (3, 2, 1) as shown below in tables 4.23, 4.24 and 4.25.

Table 4.23 Final Estimates of Parameter for ARIMA (3, 2, 1)

TYPE	COEFFICIENT	SE COEFF	T VALUE	P VALUE
AR1	-1.5364	0.0905	-16.97	0.000
AR 2	-0.8406	14.91	5.64	0.000
AR 3	-0.2716	0.0905	-3.02	0.003
MA 1	-0.9939	0.0002	-6117.91	0.000

Table 4.24 Modified Box-Pierce (Ljung-Box) Chi-Square Statistics (ARIMA, 3, 2, 1)

Lag	12	24	36	48
Chi-square	39.7	49.7	57.5	59.5
DF	7	19	31	43
P-Value	0.000	0.000	0.000	0.001

Table 4.25 Model Selection Criteria for ARIMA (3, 2, 1)

MODEL	R-Squared	RMSE	MAPE	MAE	MaxAPE	MaxAE	Normalized BIC
ARIMA (3, 2, 1)	0.933	1.897	5.880	0.939	94.666	13.119	1.523

From the final estimate of parameter and the modified box-pierce (Ljung-box) chi-square statistics as in tables 4.20, 4.21, 4.23, and 4.24, it was observed that the parameters of ARIMA (2, 2, 1) and ARIMA (3, 2, 1) been the suggested models are significant. It is further observed that more of the groups of the residuals are correlated and represents a white noise variable. However the Normalized BIC test reveals that the model with the least Normalized BIC is better in terms of forecasting performance than the one with a large Normalized BIC. ARIMA (2, 2, 1) has the least Normalized BIC of 1.479 compared to 1.523 of ARIMA (3, 2, 1). The R-Squared, RMSE, MAPE, MAE, MaxAPE, and MaxAE are further taken into consideration as it measures the accuracy of the fitted time series values. RMSE and MAE serve as measures for comparing forecast of the same series across different models and hence the smaller the error, the better the forecasting ability of the model. Based on the model selection criteria as in tables 4.22 and 4.25 above, it is proposed that the best model from the two suggested models among the lot is ARIMA (2, 2, 1). From the numerical output in tables 4.20 and 4.21 the suggested model is written as

$$y_t = \varphi_1 y_{t-1} + \varphi_2 y_{t-2} + \theta_1 \varepsilon_{t-1} + \varepsilon_t \quad (7.3)$$

Substituting the values of φ_1, φ_2 and θ_1 into equation 7.3 the inflation model for the period of 2001 to 2009 is written as

$$\hat{y}_t = -1.3764y_{t-1} - 0.4389y_{t-2} + 0.9860\varepsilon_{t-1} + \hat{\varepsilon}_t \quad (7.4)$$

Where $\hat{\varepsilon}_t$ is the realization of a WN $(0, \delta^2)$, substituting the quadratic trend m_t from figure 4.15 in appendix six into equation 7.4 then the general model for inflation between the period of 2001 to 2009 is written as

$$\hat{y}_t = 34.3958 - 0.637228t + 4.40e^{-0.3}t^2 - 1.3764y_{t-1} - 0.4389y_{t-2} + 0.9860\varepsilon_{t-1} + \hat{\varepsilon}_t \quad (7.5)$$

Where $\hat{\varepsilon}_t$ is the realization of a WN $(0, \delta^2)$

4. 7 COMPARING INFLATION RATES BETWEEN THE TWO PERIODS (1990-2000 AND 2001-2009)

This section compares inflation rates between the two periods of January 1990 to December 2000 and January 2001 to December 2009 by looking at the trend analysis, differencing for each period, mean difference, standard deviation and standard error. The hypothesis below is formulated in response to the above:

Ho: there is no significant difference between the mean and variance of the two periods.

H1: there is significant difference between the mean and variance of the two periods.

To start with, it could be stated from sections 4.5 and 4.6 respectively in pages 80 and 84 that the period of January 1990 to December 2000 is modelled as ARIMA (1, 2, 2), written as $\hat{y}_t = 18.5770 + 0.455848t - 3.57e^{-0.3}t^2 + 0.7807y_{t-1} - 1.0813\varepsilon_{t-1} + 0.1020\varepsilon_{t-2} + \hat{\varepsilon}_t$ whilst that of January 2001 to December 2009 is also modelled as ARIMA(2, 2, 1), written as $\hat{y}_t = 34.3958 - 0.637228t + 4.40e^{-0.3}t^2 - 1.3764y_{t-1} - 0.4389y_{t-2} + 0.9860\varepsilon_{t-1} + \hat{\varepsilon}_t$

Secondly we take a look at the trend analysis of the two periods by considering figure 4.10 in appendix three and figure 4.16 in appendix six respectively which deals with the trend analysis between the two periods. A look at the trend analysis for the two periods reveal that both the mean and variance in each case are not stable due to large swings and sudden shot within the trend, hence rendering the mean and variance non stationary. It is also observed that the period of January 1990 to December 2000 see figure 4.10 in appendix three and figure 4.16 on the trend analysis in appendix six, had MAPE of 56.618, MAD of 12.030 and MSD of 225.941, compared to MAPE of 21.4207, MAD of 3.7000 and MSD of 23.6172 for the period of January 2001 to December 2009. The comparison of the accuracy measures, reveal a less forecasting error for the second period (2001-2009). Again from the trend analysis, the smaller accuracy measures amounting from the MAPE, MAD and MSD in the case of 2001 to 2009 leads to smaller fluctuations in the mean and variance than the period of 1990 to 2000, indicating a significant difference between the two periods.

Consider also the time series plot of 'diff' two for the two periods of January 1990 to December 2000 and January 2001 to December 2009 see figure 4.12 in appendix four and 4.18 in appendix seven. It could be observed from the period of

January 1990 to December 2000 that apart from January 1990 where inflation was as high as 33.0 percent and rose spontaneously to 35.9 percent in December 1990, the mean and variance look stable and fluctuated around -5 to 5, until it peaked itself around 26.0 percent in June 1993 to 27.7 percent in December 1993. For the second periods of January 2001 to December 2009, apart from the sudden swings in the mean and variance at 29.9 percent in March 2003 which rose to 30.0 percent in April and decrease slightly to 29.0 percent in July 2003, the residuals of the plotted data in figure 4.18 in appendix seven fluctuates well around -10 and 10 and looks more smoothen than that of figure 4.12 in appendix four, for the period of January 1990 to December 2000. It could possibly be suggested from the time series plot of 'diff' two in figure 4.12 in appendix four and 4.18 in appendix seven that, inflation rate from 2001 to 2009 was stable in terms of the mean and variance than 1990 to 2000.

Statistically, consider the mean difference, the standard deviation and the standard error of the two periods from January 1990 to December 2000 and January 2001 to December 2009 by using the “ T ” test on the sample statistics at 5 % significant level as shown below in tables 4.26.

Table: 4.26 Sample Statistics of the Two Periods

Year	Data Size	Mean Difference	Std. Deviation	Std. Error Mean
2001-2009	114	17.16	7.914	0.741
1990-2000	132	27.92	15.806	1.376

It can be said of table 4.26 (Sample statistics of the two periods) that at 5% significance level the standard error mean associated with the mean and variance for the period of January 2001 to December 2009 is 0.741 which is less than 1.367 for the period of January 1990 to December 2000. Again considering the mean difference of the two periods, it is observed that the period of 2001 to 2009 has less mean of 17.16 compared to 27.92 for the period of 1990 to 2000. Further the standard deviations for the two periods (2001 to 2009 and 1990 to 2000) are 17.16 and 27.92 respectively. This implies that the mean is stable for the period of 2001 to 2009 than the period of 1990 to 2000. In respect to the ongoing comparison between the two periods and the formulated hypothesis, it is therefore realized that the mean is stable for the period of 2001 to 2009 than 1990 to 2000. We however reject the null hypothesis of no significant difference between the mean and variance of the two periods, in favour of the alternative hypothesis at 5% significant level, and conclude that there is significant difference between the two periods. Hence inflation rate for the period of January 2001 to December 2009 was less than that of January 1990 to December 2000. It should however be noted that election years of 1992, 1996, 2000, 2004 and 2008 had significant impact on inflation due to governments spending in the two periods under consideration.

4.8 CONCLUSION

This chapter among other chapters deals with the analysis and discussion of results. The discussion was done under the following headings such as: Preliminary

analysis, Models fitting, Model diagnostic, evaluating the accuracy of the forecast, inflation model for the period 1990 to 2000, inflation model for the period of January 2001 to December 2009 and comparing inflation rates between the two periods.

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CHAPTER FIVE

SUMMARY, RECOMMENDATIONS AND CONCLUSION

5.0 INTRODUCTION

Price stability is the best contribution monetary policy can make to economic growth and prosperity. It is now universally accepted that price stability is a cornerstone of modern well-functioning economies. Inflation is costly in a social justice sense, because it arbitrarily redistributes wealth among different groups of people in a society. Not only does inflation blunt the link between effort and reward, it typically hits hardest those who least can afford it. Inflation is also costly because it obscures the relative price signals that must come through clearly if the economy is to adapt to change and make the most of opportunities for growth. Overall macroeconomic stability, however, also depends on a sound overall government policy framework which does not itself contribute to economic fluctuations. It also depends strongly on what is happening beyond Ghana borders. This chapter which is the last but not the least is organized into the following headings: Summary of chapter, Recommendations and Conclusion.

5.1 SUMMARY OF FINDINGS

This research is an attempt to select the best and accurate model among various ARIMA estimated models which possess high power of predictability (forecasting

power). A framework for ARIMA modeling which includes the following steps: data collection and examination; determining the order of integration; model identification; diagnostic checking; model stability testing; and forecasting performance evaluation has been identified. We have adopted the traditional Box-Jenkins approach of forecasting known as ARIMA modeling, in which a time series is expressed in terms of past values of itself (the autoregressive component) plus current and lagged values of a 'white noise' error term (the moving average component). The primary purpose behind this study is to find out which ARIMA model is more accurate and appropriate for forecasting purposes in the real world situation, keeping in view the cost of model building.

A general rule of thumb for univariate forecasting is to test for all the stages of the ARIMA process. ARIMA models are theoretically justified and can be surprisingly robust with respect to alternative (multivariate) modeling approaches. Indeed, Stockton and Glassman (1987,) upon finding similar results for the United States commented that "it seems somewhat distressing that a simple ARIMA model of inflation should turn in such a respectable forecast performance relative to the theoretically based specifications. The study is based on Ghana's monthly inflation data, which was used to estimate various possible ARIMA models and the best model, was selected based on the NBIC and other supporting statistics such as RMSE, MAPE, MAE, MaxAPE and MaxAE. It was concluded that the inflation model for the period of 1990 to 2009 is ARIMA (3, 2, 1) which is surprisingly robust with respect to the alternative model. Further inflation between the periods of January 1990 to December 2000 and January 2001 to December 2009 was compared on the basis of the mean and variance using

MAPE, MAD and MSE from the trend analysis as well as the time series plot of the first and second difference. It was concluded that inflation rate for the period of January 2001 to December 2009 was less than that of January 1990 to December 2000. The period of 1990 to 2000 was modelled as ARIMA (1, 2, 2) whilst that of 2001 to 2009 was modelled as ARIMA (2, 2, 1)

5.2 RECOMMENDATIONS

Stabilization measures are necessary in offsetting the distortions inflation causes to normal economic activities, among such measures is the inflation targeting approach. The contemporary use of Inflation Targeting Monetary Policy as an instrumental regulatory mechanism by the central banks of many economies around the world is something worthy of notice. In recent times, inflation targeting monetary policy, coupled with non-artificial Central Bank independency has been a source of great success for the management and stabilization of macro-economic variables in countries like Norway, Sweden, Israel, Iceland, Denmark, New Zealand, United Kingdom etc. In lieu of the above, the following suggestions among others are recommended to Stakeholders and Researchers who may further work on this study in the near future.

5.2.1 RECOMMENDATION TO STAKEHOLDERS

Inflation is of major concern to stakeholders specifically the Ministry of Finance, central bank of Ghana, financial institutions, the business sector, etc for planning purposes and to make informed decisions. Therefore modeling inflation using the Box-Jenkins ARIMA approach is plausible to stakeholders because it generates reliable inflation forecast which follows closely with the actual data. The model has sufficient predictive powers and a less error margin of ± 1 , which makes it reliable for use by stakeholders for planning well ahead of time.

5.2.2 RECOMMENDATIONS FOR FUTURE RESEARCH

The inflation model obtained is stochastic in nature and is therefore recommended for use by future researcher's as basis for constructing deterministic models such as first and second order stochastic differential equation, using current economic trend. The model can further be used for prediction and explanation purposes by connecting it to the macroeconomic theory.

Finally, it is recommended for future researchers to look beyond one model, by considering two or more models such as the Philip curve model, P-star model, leading indicators model and the Price equation model in addition to the ARIMA model. This will account well for the error margin associated with inflation forecasting and give room for comparative analysis of errors associated with different methods.

5.3 CONCLUSION

This study is an attempt to select the best and accurate model among various ARIMA models which possess high power of predictability for forecasting Ghana's inflation from the period of January 1990 to December 2009 and also compare inflation between the period of January 1990 to December 2000 and January 2001 to December 2009. It is further to model inflation between the periods of 1990 to 2000 and 2001 to 2009. The main focus was to forecast the monthly inflation on short-term basis, and for this purpose, different ARIMA models were used and the candid model was proposed based on various diagnostic, selection and evaluation criteria. A framework for ARIMA forecasting was drawn up on the basis of in-sample and out-of-sample forecast. It was concluded that the model has sufficient predictive powers based on the associated error margins and the findings are well in line with those of other studies. Further inflation between the periods of January 1990 to December 2000 and January 2001 to December 2009 were compared on the basis of the mean and variance, using accuracy measures such as MAPE, MAD and MSE from the trend analysis as well as the time series plot of the first and second difference. It was concluded that inflation rate for the period of January 2001 to December 2009 was less than January 1990 to December 2000. The model for 1990 to 2009 is ARIMA (3, 2, 1), whilst the period of 1990 to 2000 is ARIMA (1, 2, 2) and ARIMA (2, 2, 1) for 2001 to 2009.

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APPENDIX ONE

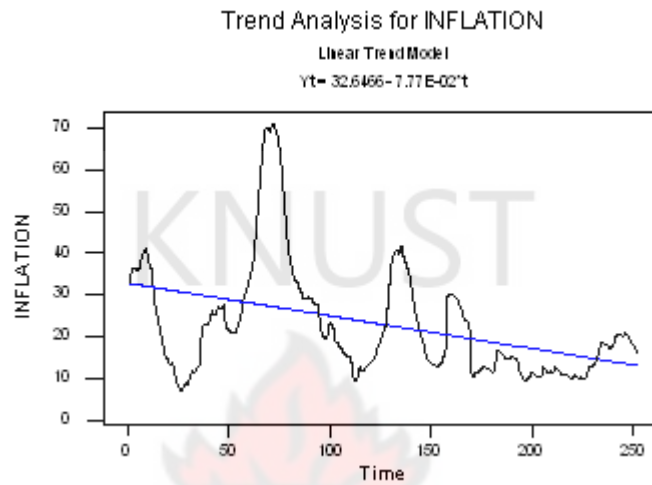


Figure 4.3 Trend Analysis for Inflation (Linear type)

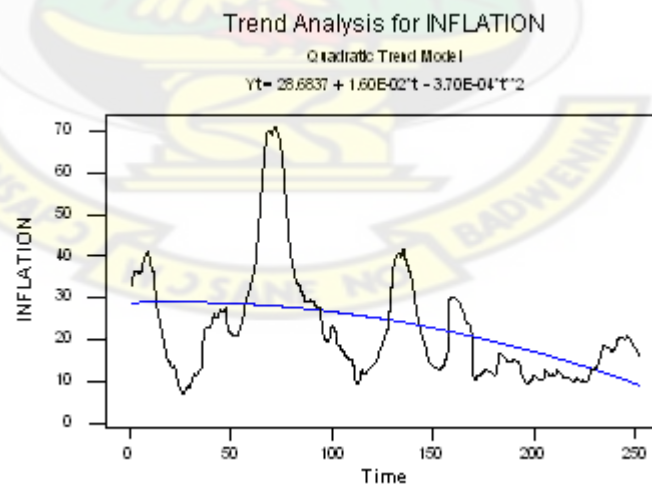


Figure 4.4 Trend Analysis for Inflation (Quadratic type)

APPENDIX TWO

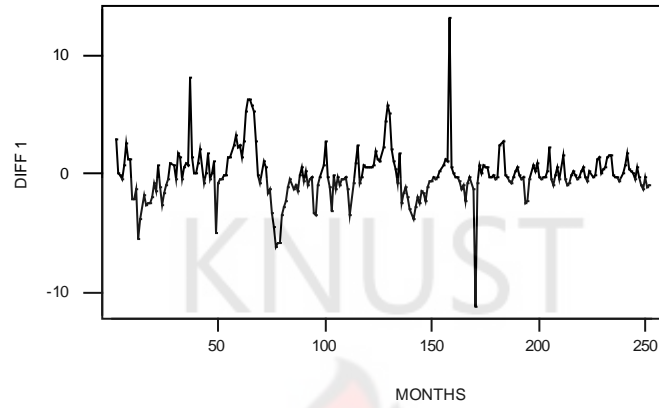


Figure 4.5 First Difference of inflation: period 1990 to 2009

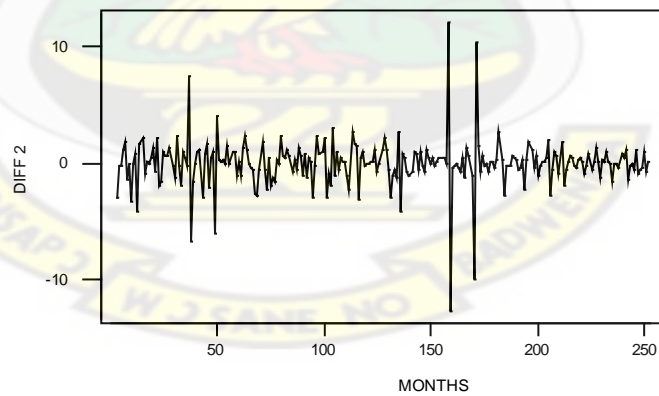


Figure 4.6 Second Difference of inflation: period 1990 to 2009

APPENDIX THREE

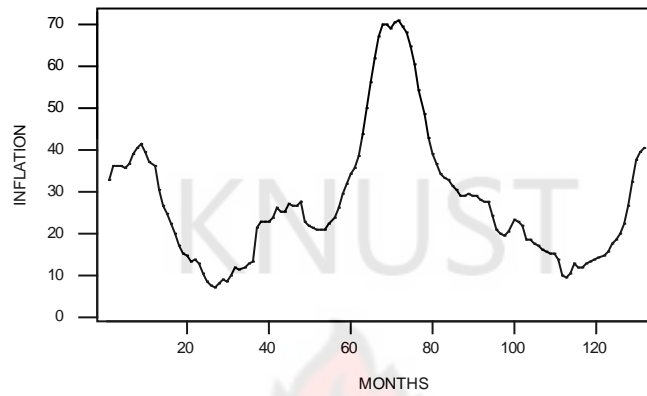


Figure: 4.9 General trend of inflation: Period 1990 to 2000

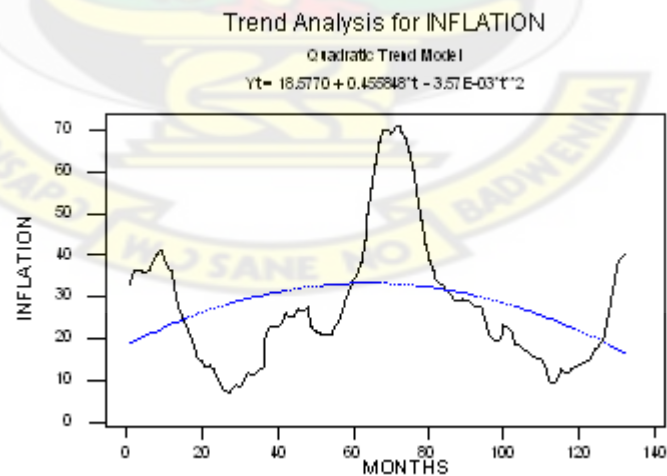


Figure: 4.10 Trend Analysis for inflation: Period 1990 to 2000

APPENDIX FOUR

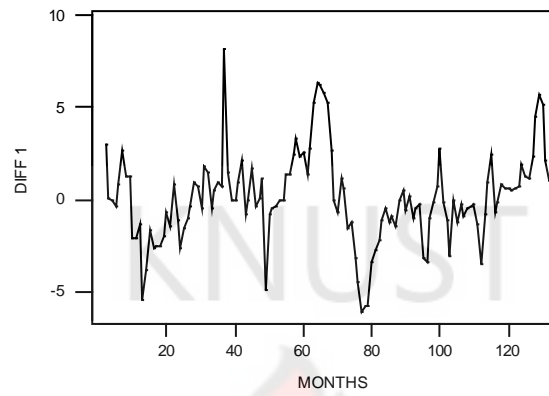


Figure 4.11 First difference of inflation period: 1990 to 2000

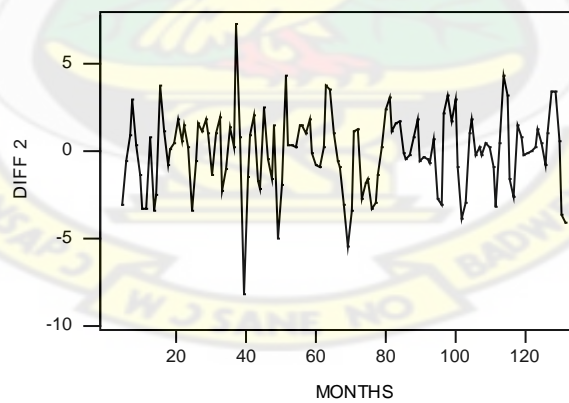


Figure 4.12 Second difference of inflation: period: 1990 to 2000

APPENDIX FIVE

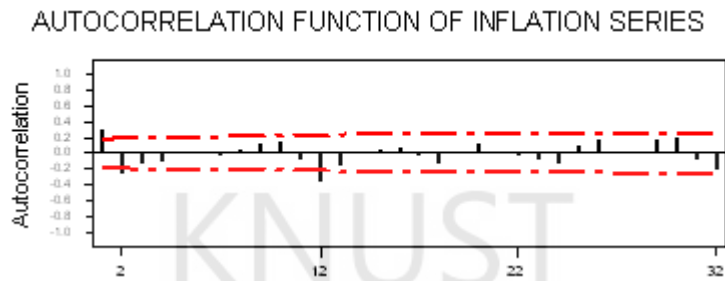


Figure 4.13 ACF of diff 2: period: 1990 to 2000

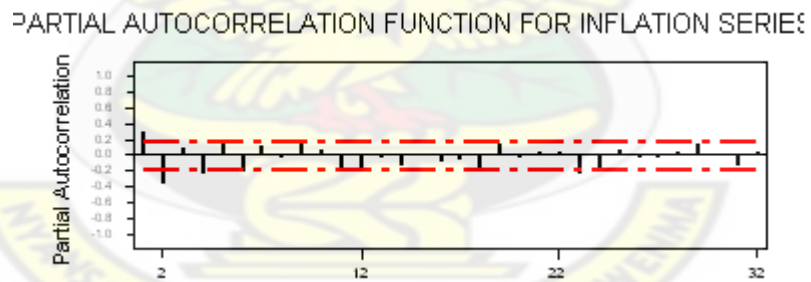


Figure 4.14 PACF of diff 2: period: 1990 to 2000

APPENDIX SIX

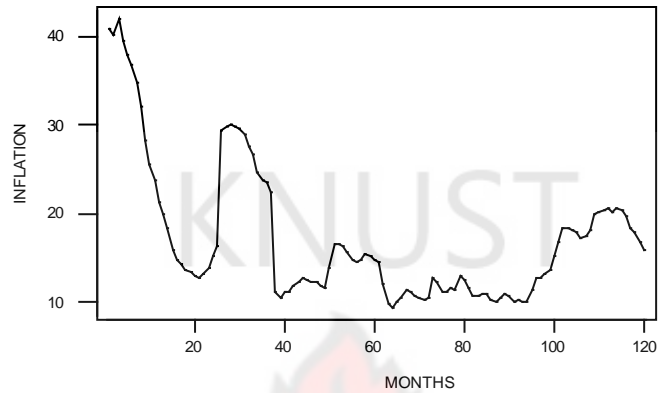


Figure 4.15 General Trend of Ghana's monthly inflation: period 2001 to 2009

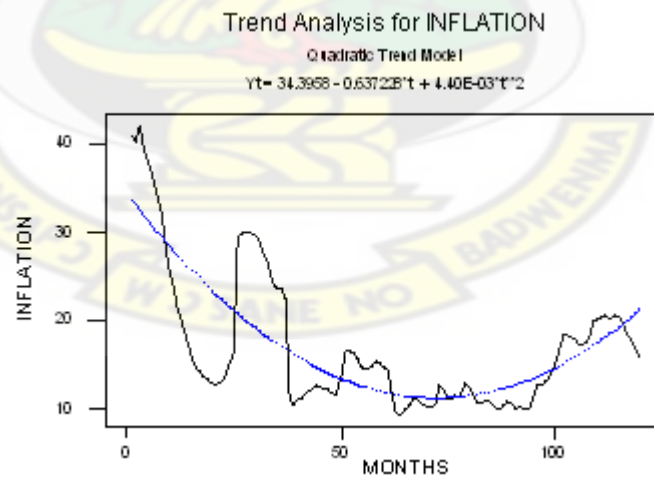


Figure: 4.16 Trend Analysis for inflation: Period 2001 to 2009

APPENDIX SEVEN

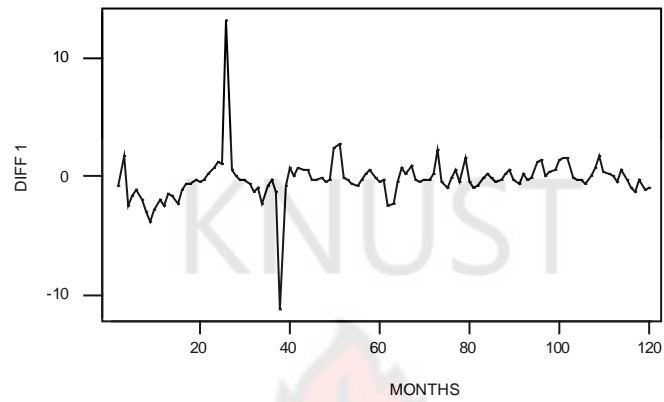


Figure 4.17 First difference of inflation: period: 2001 to 2009

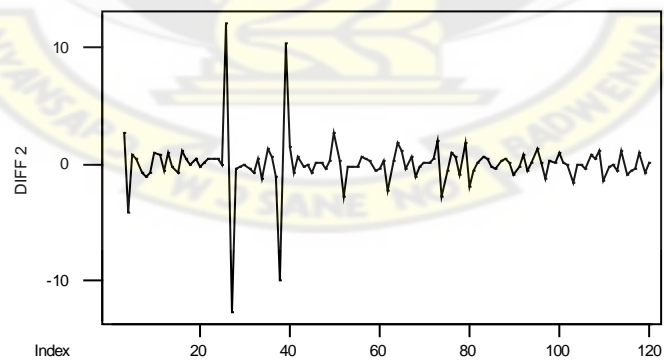


Figure 4.18 Second difference of inflation: period: 2001 to 2009

APPENDIX EIGHT

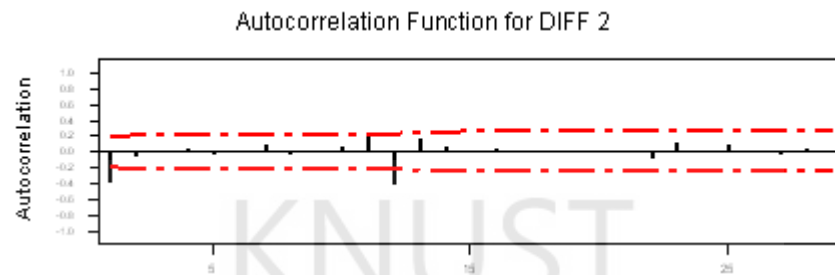


Figure 4.19 ACF of diff 2: period: 2001 to 2009

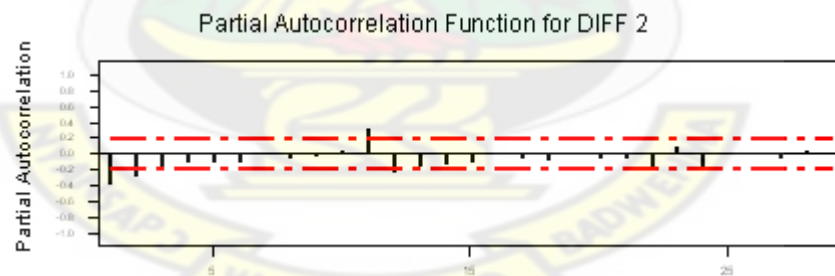


Figure 4.20 PACF of diff 2: period: 2001 to 2009

APPENDIX NINE

Table 4.7 Ghana's Monthly Inflation Data from 1990 to 2009

MONTH	INFLATION	MONTH	INFLATION
1	33	34	11.7
2	36	35	12.6
3	36.1	36	13.3
4	36	37	21.5
5	35.6	38	23
6	36.4	39	23
7	39	40	23
8	40.2	41	23.9
9	41.4	42	26
10	39.3	43	25.2
11	37.2	44	25.2
12	35.9	45	26.9
13	30.4	46	26.5
14	26.6	47	26.6
15	24.9	48	27.7
16	22.3	49	22.8
17	19.8	50	22
18	17.3	51	21.5
19	15.3	52	21.1
20	14.6	53	21
21	13.2	54	20.9
22	14	55	22.3
23	12.9	56	23.7
24	10.3	57	26.1
25	8.7	58	29.4
26	7.7	59	31.7
27	7.3	60	34.2
28	8.2	61	35.6
29	8.9	62	38.4
30	8.4	63	43.6
31	10.2	64	49.9
32	11.7	65	56.1
33	11.2	66	61.9

MONTH	INFLATION
67	67.2
68	69.9
69	69.8
70	69.1
71	70.2
72	70.8
73	69.2
74	68
75	64.8
76	60.3
77	54.2
78	48.4
79	42.6
80	39.2
81	36.5
82	34.3
83	33.2
84	32.7
85	31.5
86	30.6
87	29.2
88	29.1
89	29.6
90	29
91	29.2
92	28.2
93	27.7
94	27.4
95	24.2
96	20.8
97	19.8
98	19.6
99	20.3
100	23.1
101	22.9
102	21.8
103	18.7
104	18.6
105	17.4
106	17.1

MONTHS	INFLATION
107	16.2
108	15.7
109	15.3
110	15
111	13.7
112	10.2
113	9.4
114	10.3
115	12.7
116	12
117	11.8
118	12.6
119	13.2
120	13.8
121	14.3
122	14.9
123	15.6
124	17.5
125	18.7
126	19.8
127	22.1
128	26.6
129	32.3
130	37.4
131	39.5
132	40.5
133	40.9
134	40.1
135	41.9
136	39.5
137	37.9
138	36.8
139	34.9
140	32
141	28.3
142	25.6
143	23.7
144	21.3
145	19.9
146	18.3

MONTHS	INFLATION
147	16
148	14.9
149	14.3
150	13.7
151	13.5
152	13.1
153	12.9
154	13.2
155	14
156	15.2
157	16.3
158	29.4
159	29.9
160	30
161	29.8
162	29.6
163	29
164	27.7
165	26.8
166	24.6
167	23.8
168	23.6
169	22.4
170	11.3
171	10.5
172	11.2
173	11.2
174	11.9
175	12.4
176	12.9
177	12.6
178	12.4
179	12.3
180	11.8
181	11.6
182	14
183	16.7
184	16.6
185	16.3
186	15.7

MONTHS	INFLATION
187	14.9
188	14.7
189	14.9
190	15.4
191	15.3
192	14.8
193	14.6
194	12.1
195	9.9
196	9.5
197	10.2
198	10.5
199	11.4
200	11.2
201	10.8
202	10.5
203	10.3
204	10.5
205	12.76
206	12.27
207	11.28
208	11.21
209	11.75
210	11.39
211	12.91
212	12.56
213	11.67
214	10.87
215	10.7
216	10.92
217	10.89
218	10.42
219	10.19
220	10.5
221	11.02
222	10.69
223	10.14
224	10.41
225	10.19
226	10.14

MONTHS	INFLATION
227	11.4
228	12.75
229	12.81
230	13.21
231	13.79
232	15.29
233	16.88
234	18.41
235	18.31
236	18.1
237	17.89
238	17.3
239	17.44
240	18.13
241	19.86
242	20.34
243	20.53
244	20.56
245	20.06
246	20.74
247	19.5
248	18.37
249	18.04
250	16.92
251	15.9

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