

OPTIMAL CREDIT PORTFOLIO

CASE STUDY : (FIRST ALLIED SAVINGS AND LOANS)

BY

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DECLARATION

I hereby declare that this submission is my own work towards the MSc and that, to the best of my knowledge, it contains no material previously published by another person nor material which has been accepted for the award of any other degree of the University, except where due acknowledgement has been made in the text.

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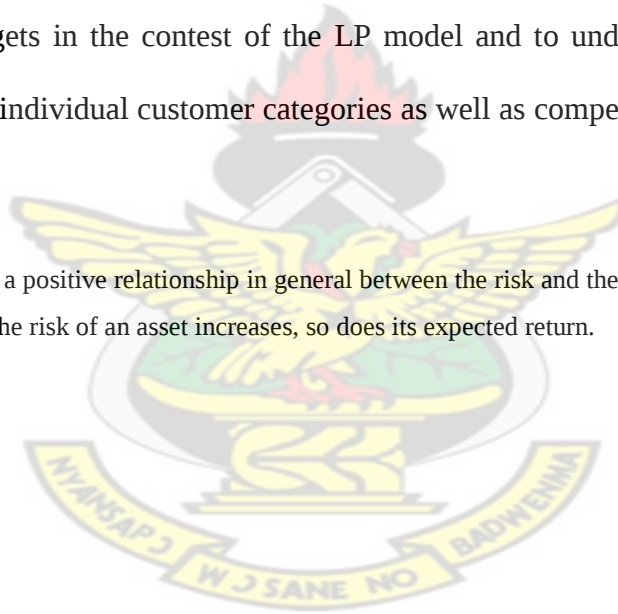
ABSTRACT

With the recent onset of the global credit crisis and the growth of credit portfolio there have been an increased concentrations and credit portfolio volatility (unexpected losses), abysmal returns on risk and capital.

The study sought to formulate and solve a linear programming model to maximize expected credit yield subject to capacity and demand, develop risk related scenarios and interpret results in the light of credit management issues.

Data was obtained from the credit section of First Allied Savings and Loans Limited, Techiman, Ghana and was analyzed using the management scientist software to see how the portfolio fared in achieving its set targets in the contest of the LP model and to understand and quantify the credit characteristics of individual customer categories as well as compensate for risk level of the categories.

The results obtained showed a positive relationship in general between the risk and the expected return of a financial asset. In other words, when the risk of an asset increases, so does its expected return.



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I am grateful to Mr. Edward Kwarteng formerly of First Allied Savings and Loans limited for kindly making available to me all the necessary data needed for my project.



DEDICATION

I dedicate this thesis with all my love and respect to my dear wife, Dorothy Barfi, My Parents, Mr. and Mrs. Owusu-Agyeman and not the least of all, my dearest daughter Lady Alexandra Owusu-Agyeman.

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CHAPTER 1

INTRODUCTION

1.1 Background to the study

The lifeblood of each lending institution is its credit portfolio, and the success of the institutions depends on how well that portfolio is managed (Harold et al, 1998). The credit portfolio is typically the largest asset and the predominate source of revenue. As such, it is one of the greatest sources of risk to a bank's safety and soundness. Whether due to lax credit standards, poor portfolio risk management, or weakness in the economy, credit portfolio problems have historically been the major cause of bank losses and failures.

Effective management of the credit portfolio and the credit function is fundamental to a bank's safety and soundness. Assessing loan portfolio management involves evaluating the steps bank management takes to identify and control risk throughout the credit process. The assessment focuses on what management does to identify issues before they become problems. For decades, good credit portfolio managers have concentrated most of their effort on prudently approving loans and carefully monitoring loan performance. Although these activities continue to be mainstays of loan portfolio management, analysis of past credit problems, such as those associated with oil and gas lending, agricultural lending, and commercial real estate lending in the 1980s, has made it clear that portfolio managers should do more. (Acerbi 2001). Traditional practices rely too much on trailing indicators of credit quality such as delinquency, nonaccrual, and risk rating trends. Banks have found that these indicators do not provide sufficient lead time for corrective action when there is a systemic increase in risk. Effective credit portfolio management begins with oversight of the risk in individual loans. Prudent risk selection is vital to maintaining favorable loan quality. Therefore, the historical emphasis on controlling the

quality of individual loan approvals and managing the performance of loans continues to be essential. But better technology and information systems have opened the door to better management methods (Rockafellar et al, 2000). A portfolio manager can now obtain early indications of increasing risk by taking a more comprehensive view of the loan portfolio.

To manage their portfolios, bankers must understand not only the risk posed by each credit but also how the risks of individual loans and portfolios are interrelated. These interrelationships can multiply risk many times beyond what it would be if the risks were not related. Until recently, few banks used modern portfolio management concepts to control credit risk. Now, many banks view the loan portfolio in its segments and as a whole and consider the relationships among portfolio segments as well as among loans. These practices provide management with a more complete picture of the bank's credit risk profile and with more tools to analyze and control the risk (Garside and Stott, 1999).

Credit is the provision of resources (such as granting a loan) by one party to another party where that second party does not reimburse the first party immediately, thereby generating a debt, and instead arranges either to repay or return those resources (or material(s) of equal value) at a later date. It is any form of deferred payment. The first party is called a creditor, also known as a lender, while the second party is called a debtor, also known as a borrower.

Movements of financial capital are normally dependent on either credit or equity transfers. Credit is in turn dependent on the reputation or creditworthiness of the entity which takes responsibility for the funds.

Credit need not necessarily be based on formal monetary systems. The credit concept can be applied in barter economies based on the direct exchange of goods and services, and some would

go so far as to suggest that the true nature of money is best described as a representation of the credit-debt relationships that exist in society (Ingham, 2004).

Credit is denominated by a unit of account. Unlike money (by a strict definition), credit itself cannot act as a unit of account. However, many forms of credit can readily act as a medium of exchange. As such, various forms of credit are frequently referred to as *money* and are included in estimates of the money supply (Finlay, 2009).

Credit is also traded in the market. The purest form is the credit default swap market, which is essentially a traded market in credit insurance. A credit default swap represents the price at which two parties exchange this risk – the protection "seller" takes the risk of default of the credit in return for a payment, commonly denoted in basis points (one basis point is 1/100 of a percent) of the notional amount to be referenced, while the protection "buyer" pays this premium and in the case of default of the underlying (a loan, bond or other receivable), delivers this receivable to the protection seller and receives from the seller the par amount (that is, is made whole).

1.1.1 Trade credit

The word credit is used in commercial trade in the term "trade credit" to refer to the approval for delayed payments for purchased goods. Credit is sometimes not granted to a person who has financial instability or difficulty. Companies frequently offer credit to their customers as part of

the terms of a purchase agreement. Organizations that offer credit to their customers frequently employ a credit manager.

1.1.2 Consumer credit

Consumer debt can be defined as ‘money, goods or services provided to an individual in lieu of payment.’ Common forms of consumer credit include credit cards, store cards, motor (auto) finance, personal loans (installment loans), retail loans (retail installment loans) and mortgages. This is a broad definition of consumer credit and corresponds with the Bank of England's definition of "Lending to individuals". Given the size and nature of the mortgage market, many observers classify mortgage lending as a separate category of personal borrowing, and consequently residential mortgages are excluded from some definitions of consumer credit - such as the one adopted by the Federal Reserve in the US.

The cost of credit is the additional amount, over and above the amount borrowed, that the borrower has to pay. It includes interest, arrangement fees and any other charges. Some costs are mandatory, required by the lender as an integral part of the credit agreement. Other costs, such as those for credit insurance, may be optional. The borrower chooses whether or not they are included as part of the agreement.

Interest and other charges are presented in a variety of different ways, but under many legislative regimes lenders are required to quote all mandatory charges in the form of an annual percentage rate (APR). The goal of the APR calculation is to promote ‘truth in lending’, to give potential borrowers a clear measure of the true cost of borrowing and to allow a comparison to be made between competing products. The APR is derived from the pattern of advances and repayments

made during the agreement. Optional charges are not included in the APR calculation. So if there is a tick box on an application form asking if the consumer would like to take out payment insurance, then insurance costs will not be included in the APR calculation (Finlay, 2009).

1.1.3 Distinguishing Credit from Loans

While the terms Credit and Loans are often used interchangeably, there is an important distinction between the two. Credit in general refers to promissory notes backed by a promise/contingency. Merchant and commercial banks issue trade credit in amounts that exceed their net worth. These notes are typically backed by future output. The money supply in most Western industrialized nations consists almost entirely of these promissory notes. The important point to make is that they are not backed by a real asset (gold, bank assets). A loan, on the other hand, is the flip side to savings. Consumers and firms deposit a portion of their income (promissory notes) at banks which then loan it out to borrowers.

The key distinction is the very nature of the instrument that the banker uses. In the case of credit, he issues new promissory notes. In the case of loans, he lends out promissory notes deposited by other customers. The current crisis in the U.S. banking sector is credit, not loan related. Banks are more reticent to issue new promissory notes to small and medium size firms, thus hampering the recovery.

1.2 Statement of the Problem

With the recent onset of the global credit crisis and the growth of credit portfolio there have been an increased concentrations and credit portfolio volatility (unexpected losses), abysmal returns on risk and capital.

There are also difficulties in accumulation of information to evaluate borrowers and their projects and problems with encouraging borrowers to repay and difficulties with seizing collateral, and using legal action in collecting bad debts.

Additionally, serious difficulties in sharing information about borrowers among bankers and between bankers and other firms.

1.3 Objectives

The objectives of this project are to;

- 1) Formulate a linear programming model to measure credit yield risk and product
- 2) Solve the model for parameter values based on data from First Allied Savings and Loans Limited.
- 3) Formulate a scenario analysis to accommodate as many entries as possible.
- 4) Interpret results in the light of credit management issues.

1.4 Methodology

The data for this project was obtained from the credit section of First Allied Savings and Loans limited Techiman Branch and to analyze how the portfolio fared in achieving its set targets- profits, loss in the contest of an LP model.

To understand and quantify the credit characteristics of individual customer categories as well as to calculate the risk of their portfolios, hedge the risk of unwanted exposures and understand credit concentrations

Computer software being used is Management Scientist.

1.5 Justification

The goals and objectives of the institution if well addressed is to bring about high profit margin and growth in market share, necessary for the institutions to survival in the constantly changing and competitive financial services industry. In addition, strengths within the institution, such as experience and tenure of staff, lending expertise, cooperative organizational structure, and capital position, may provide opportunities for taking on additional growth or solidifying market share.

Also, strengthened internal control measures will drastically reduce losses thereby helping in a way to strengthen the economy.

CHAPTER 2

LITERATURE REVIEW

2.1 Introduction

In a business firm the objective is to maximize its shareholders' wealth, from the point of view of Finance. In order for this objective to be accomplished all the resources and all the assets of the bank both short-term and long-term should be used as efficiently as possible, with the least cost and the maximum return. In working capital management, which concerns the short term decision of a firm according to Almgren and Chriss (2001) we distinguish four areas of importance from the point of view of practitioners and academicians, that play a very important role in the value of the firm and the owners' wealth. These are cash management, accounts receivable and accounts payable management and inventory management. Liquidity is a major factor that determines the health of a company. Profitability only is not enough. A firm can have successful sales and show profits at the end of the year, but might suffer from lack of cash in the short run, whenever, it is needed. This situation, which is observed very often among Ghanaian banks could lead the management of a company in seeking external sources of funds, which eventually will increase their cost of capital and financial risk, hence lowering their profitability. The financial managers and academicians have realized the importance of liquidity and efficiency in liquid assets management during the last 20 years in the USA and Europe. In Ghana, the emphasis in liquidity has been relatively new, since in general, the emphasis in working capital management is recent. At present, though, it has become obvious, that not only the working capital management strategies which a firm follows affect its shareholders' wealth and the firm's value significantly, but also, that they can be used as a competitive advantage against the other companies of their industry. Furthermore, many cases of financial distress and

bankruptcies are due to mismanagement of working capital. Therefore, we consider this subject as very important and contemporary, especially at this era of markets' globalization.

It is imperative to know the existing situation and conditions that characterize cash and receivables management, before we can make suggestions about the course financial managers should follow, to help their companies survive and succeed in the new globalized competitive market place. (Breen, 1979) provided a method for evaluating investments in accounts receivable consistent with the wealth maximization objective. They determined an objective function of the discounted sum of net cash flows associated with the firms' credit policy and the necessary conditions for maximizing this function, leading to an optimal credit policy that can easily be adopted by practitioners. (Coles and Lowenstein, 1988), developed a model that showed how the cash discount decision in a firm's credit policy can be structured in terms of payments timing, change in sales, variable costs, the firm's cost of capital, the proportion of sales expected to be paid with a discount and the bad debt loss rate. This model determines the optimal discount rate which is the maximum feasible discount rate for a given set of conditions based on the above factors. (Merton, 1969) presented a theoretical model that indicated the effect on the firm value due to the changes in the current assets accounts, and the level of systematic risk that is associated with these changes. He concluded that when a firm keeps a low level of cash for example, it will have higher monitoring costs and higher systematic risk. On the other hand, if the reverse is true, the level of systematic risk will be lower, but the opportunity costs of not investing in more productive investments will be higher. Karatzas et al (1987) surveyed the largest Fortune's 500 US industrial corporations, to determine their current practices of cash management. Their results indicated that during 1978-1981 the average value of the firms' portfolio in short-term investments had been reduced compared to the average value of their total

assets. The results also indicated that the companies preferred to invest their excess cash first in repurchase agreements, and then, in negotiable certificates of deposit, in commercial paper and in Eurodollars. The demand for US T-bills had dropped. The critical factors (in order of preference) for the above investments' selections were found to be capital preservation, the rate of return, the company's philosophy, the investments' liquidity and last the company's creditors. Another issue that was examined was the cash flows forecasting techniques that firms followed. The results indicated that these were the cash budget (the most important), historical trends, zero balance accounts, regression analysis and determination through the firm's bank. Finally, the survey examined the efficiency of their short-term borrowing. The results indicated that there was efficiency since 50% of the sample firms were borrowing at interest rates below prime rates. This result leads the authors to infer that either there was excellent planning, which was questionable, or that there was a deficiency of the prime rate as a benchmark. In general though, this study gave more insights about cash management exercised by USA corporations that was very close to the teachings of the academicians.

More recently, Tsay (2005) examined empirically the cash management practices in the Netherlands. His results indicated that the companies in Netherlands have the tendency to decentralize their cash management decisions to the managers of each subsidiary. They have also a statistically significant difference between short-term and long-term cash flow management. Concerning their payables policy most of the firms use their banks and most are cooperating with more than one credit institution. For domestic cash transfers they use electronic funds transfer systems, while for international ones, they use euro checks. The general tendency for cash management is towards more use of electronic systems and technology for rapid and efficient results.

2.2 Evolution of the Models

Since the 1930s, the development of credit risk evaluation models has gone through comparable analysis, statistical analysis and artificial intelligence. Below is a brief introduction of the key assumptions and values of various credit risk evaluation models.

2.2.1 Comparable Analysis in Credit Risk Management

The traditional credit risk with the default event, the key point is data mining the characteristics of both default and non-default companies to establish the identification equations and categorize the samples. The representative model of this stage is 5C – character, capacity, collateral and condition. People try to make a full qualitative analysis about the obligators willingness and capability of payback from five aspects.

2.2.2 Statistical Analysis in Credit Risk Management

After Fisher's research on heuristics, there developed quickly and enormously credit risk evaluation models based on statistics, of which most represented is Edward Ahman's Z-score. Ahman observed manufacturing companies near or far from bankruptcy in 1968 and took 22 financial ratios to establish the most famous five Z- score based on the mathematical statistical screening.

2.2.3 Artificial Analysis in Credit Risk Management

With the fast development of information technology, recent years have seen large artificial intelligence models that have been incorporated in the credit risk analysis. For instance, neural networks as a self-organizing, self-adapting and self-learning non-parameter method are very robust and accurate in predicting especially does not rigidly follow normal

2.3 Credit Metrics and Other Single Factor Models

Some single factor models which are based on monitoring the changing process of credit from good to bad and building models on credit rating data. The following are the most famous models.

2.4 Portfolio Optimization

The crediting practice has been a vital part of banking operation, through which the resource, i.e., the capital is provided and brokered by banks, is effectively utilized to stimulate the development of businesses and economy. As such, the operation of the crediting business is closely connected to the safety of banking and social resources. The inadequate management of credit not only implicates the burden of excessive credit risk, what is more, the safety of banking operation and the rights of the depositing public could potentially be in jeopardy. Relevant literary sources have extensive discussion on the models of credit-risk assessment as banks grant loans, yet the decision-making model closely related to the asset allocation of loans is rarely mentioned. This study attempts to explore the crediting policies of banks when granting loans to different corporate entities from the perspective of "alternatives of investment combination".

Markowitz (1952) used the so-called "mean-variance model" to interpret how investors define expected returns and risk in the context of alternatives of investment combinations, and employed quadratic programming method to obtain the potentially reachable opportunity set when investors' risk is at minimum, therefore this model is coined "Markowitz risk minimization model". Under Sharpe's (1967) Capital Asset Pricing Model (CAPM) assumption, investors have different preferences in relation to risk, and thus they may reduce the risk by holding risk-

free assets or increase the returns through reverse asset allocation. The CAPM is one of the most commonly used tools in the securities industry in pricing a financial security.

Elton and Cruber (1995) mentioned in various case studies that investors may engage themselves in the risk-free asset loaning behavior to increase the returns of investment combinations within the efficient frontier for them to trade securities with margins, meanwhile having a higher degree of risk preference. Correlations are crucial inputs for portfolio management and risk assessment. Diversifying across various assets with low correlation of returns allows investors to reduce their total portfolio risk, presumably without sacrificing return. Over the past 20 years, a considerable literature has been developed and the dynamics of the correlation of assets has been explored, although the primary focus has been on univariate volatilities and not on correlations. In fact, in the Eagle (2000), Multivariate Generalized Autoregressive Conditional Heteroscedasticity (GARCH) literature one of the most relevant problems is represented by the high number of parameters. In order to solve this difficulty, Bollerslev (1990), proposes a class of multivariate GARCH models in which the conditional correlations are constant and thus the conditional are proportional to the product of the corresponding standard deviations called Constant Conditional Correlation model (CCC). But, in empirical, correlations are not constant. Engle (2002), proposes a new class of models that allow the correlations to change over time. Dynamic Conditional Correlation (DCC) preserves the parsimony of univariate GARCH models of individual assets volatility with a simple GARCH-like time varying correlation. Further, the number of parameters estimated has a considerable improvement over BEKK models. It has been a wide application of the DCC model to allocate assets.

The theory of asset pricing has roots in Arrow (1953), Arrow and Debreu (1954), Samuelson (1965), Black and Scholes (1973), Merton (1973), and Cox and Ross (1976). Harrison and Kreps (1979), Harrison and Pliska (1981), and Kreps (1981) have formalized the earlier models in a general framework. In all these models the security markets are assumed to be complete. The main result of these papers is the martingale definition of the arbitrage-free condition.

In Jouini and Kallal (1995), the arbitrage-free condition is derived in the presence of transaction costs. They show that the arbitrage-free condition is equivalent to the existence of an equivalent probability measure that transforms some process between the bids and ask price processes of traded securities into a martingale. The pricing in incomplete markets is considered, e.g., in Karatzas and Kou (1996), Föllmer and Sonderman (1986), and Föllmer and Scheiwer (1991).

Portfolio optimization under uncertainty originates from the static models of Markowitz (1952, 1958) and Tobin (1958). The discrete multiperiod model can be found in Samuelson (1969). Merton (1969, 1971) analyzes the optimal consumption and portfolio choice problem and its solution using the continuous-time stochastic control in finite and infinite horizon settings. The martingale method to solve the optimal consumption and portfolio choice have been developed in Cox and Huang (1989) and Karatzas, Lehoczky, and Shreve (1987). In these models the utility maximization problem is considered by martingale methods and without the need of imposing any Markovian assumptions.

The single agent's optimality in the presence of transaction costs is studied, e.g., in Constantinides (1979, 1986), Cvitanic and Karatzas (1996), Duffie and Sun (1990), Leland (1985), and Shreve and Soner (1994). For example, they show that in the presence of transaction

costs, the optimal trading strategy involves trading in discrete time intervals and, under particular circumstances, the length of the trading interval may even optimally be chosen as fixed. Cvitanic and Karatzas (1992, 1993) consider, e.g., general closed, convex constraints on portfolio proportions, and different interest rates for borrowing and lending.

Sharpe (1965) and Lintner (1965) derive static equilibrium models for capital asset pricing under uncertainty. The consumption-based capital asset pricing model is derived in Breeden (1979). The basic framework for deriving the security-spot market equilibrium in a continuous-time setting can be found, e.g., in Duffie (1992). Duffie and Zame (1989) have proved the existence of an Arrow-Debreu equilibrium in the case of a smooth-additive utility function. Huang (1987) has derived an equilibrium model with a smooth-additive utility function.

The equilibrium in an incomplete market is studied in Grossman and Shiller (1982). These models start the analysis from the state-price deflators implicitly given by single agents, and then derive the excess expected rates of return on all securities from the covariance of returns with aggregate consumption increments and the 'market risk-aversion' constant.

2.4.1 Credit Portfolio View

Credit Portfolio view uses default probabilities conditional on the current state of the economy. Therefore an obligor rated triple B would have a higher default probability in a recession than in an economic boom. CreditMetrics is the product for quantifying credit risk developed by JP Morgan in 1997. Its idea is like Riskmetrics for quantifying market risk in that both are measuring risk by calculating the VaR (value at risk).

2.5 Credits

Credit is a term used to denote transactions involving the transfer of money or other property on promise of repayment, usually at a fixed future date. The transferor thereby becomes a creditor, and the transferee, a debtor; hence credit and debt are simply terms describing the same operation viewed from opposite standpoints.

2.5.1 Types of Credit

The principle classes of credit are as follows :mercantile or commercial credit, which merchants extend to one another to finance production and distribution of goods; investment credit, used by business firms to finance the acquisition of plant and equipment and represented by corporate bonds, long-term notes, and other proofs of indebtedness; bank credit, consisting of the deposits ,loans, and discounts of depository institutions; consumer or personal credit, which comprises advances made to individuals to enable them to meet expenses or to purchase, on a deferred-payment basis, goods or services for personal consumption; public or government credit, represented by the bond issues of national, state, and municipal governments, by the nationals of foreign country, or by international banking institutions, such as the international bank for reconstruction and development

2.5.2 Function of Credit

The principal function of credit is to transfer property from those who own it to those who wish to use it, as in the granting of loans by banks to individuals who plan to initiate or expand a business venture. The transfer is temporary and is made for a price, known as interest, which

varies with the risk involved and also with the demand for, and supply of, credit. Credit transactions have been indispensable to the economic development of the modern world. Credit puts to use property that would otherwise lie idle, thus enabling a country to more fully employ its resources. One of the most significant differences between some nations of Africa, Asia, and South America and the advanced Western nations is the extent to which the use of credit permits the latter to keep their savings continuously at work. The presence of credit institutions rests on the readiness of people to trust one another and of courts to enforce business contracts. The lack of adequate credit facilities makes it natural and necessary for inhabitants of developing countries to hoard their savings instead of putting them to productive and profitable use. Without credit, the tremendous investments required for the development of the large-scale enterprise on which the high living standards of the West are based would have been impossible. The use of credit also makes feasible the performance of the complex operations involved in modern business without the constant handling of money. Credit operations are carried out by means of documents known as credit instruments, which include bills of exchange, money orders, checks, drafts, promissory notes, and bonds. These instruments are usually negotiable; they may legally be transferred in the same way as money. When the party issuing the instrument desires to prevent its use by anyone other than the party to whom it is issued, he or she may do so by inscribing the words “not negotiable” on the instrument.

2.5.3 Issuance of Credit

Creditors sometimes require no other assurance of repayment than the debtor's credit standing, that is, one's record of honesty in fulfilling financial obligations and one's current ability to fulfill similar obligations. Sometimes more tangible security, such as the guarantee of a third party, is

required. Also, the debtor may be obliged to assign the rights to some other property, which is at least equal in value to the loan, as collateral security for payment. Bonds placed on sale by a corporation are often secured by a mortgage on the corporation's property or some part of it. Public borrowing, as by the issuance of bonds of a government, is usually unsecured, resting on the purchaser's confidence in the good faith, taxing power, and political stability of the government. When goods are sold on a deferred-payment plan, the seller may either retain legal ownership of the goods or hold a chattel mortgage until the final payment is made. The depositing of funds in a bank for safekeeping may also be regarded as a form of credit to the bank, as such funds are used for loan and investment purposes, and the bank is legally bound to repay them as an ordinary debtor.

2.5.4 Credit and the Economy

All banking operations, and the methods for controlling them, are part of the credit system of a country. The state of business activity within a country at any given moment may be gauged from the condition of the credit system: expanding credit generally reflects a period of business prosperity, whereas contracting credit usually reflects a period of declining economic activity or depression. Fluctuations in the credit system may affect the level of prices; that is, as credit expands the money supply increases; lending causes prices to rise.

Since the end of World War II, the economic needs of war-devastated countries in many parts of the world have intensified the problems of international credit. Loans for the restoration of international trade and rehabilitation of industries were arranged through the International Bank for Reconstruction and Development and the International Monetary Fund, organized by the United Nations Monetary and Financial Conference at Bretton Woods, New Hampshire, in 1944.

The extension of credit to developing countries of Africa, Asia, and South America by such institutions as the International Bank for Reconstruction and Development contributed to their economic growth.

Two major new credit problems arose in the 1970s and early '80s. The roughly tenfold increase in international oil prices starting in 1973 led many nations to seek credit from any big credit institutions willing to grant it. Borrowing to finance overambitious development plans was another factor in leaving a large number of nations with a heavy debt burden, which then became insupportable when interest rates rose and commodity export prices declined.

Credit Card, is a card that identifies its owner as one who is entitled to credit when purchasing goods or services from certain establishments. They are issued by many businesses serving the consumer, such as oil companies, retail stores and chain stores, restaurants, hotels, airlines, car rental agencies and banks. Some credit cards are honored in a single store, but others are general-purpose cards, for use in a wide variety of establishments. Bank credit cards, are examples of the general purpose card. Establishments dispensing almost every form of product or service are honoring such cards, and it is predicted that credit cards might some day eliminate the need for carrying cash. When a credit card is used, the retailer records the name and account number of the purchaser and the amount of the sale, and forwards this record to the credit card billing office. At intervals, usually monthly, the billing office sends a statement to the cardholder listing all the charged purchases and requesting payment immediately or in installments. The billing office reimburses the retailer directly.

Most of the work involved in credit card operations is now handled by computers. Charges for the use of a credit card are sometimes paid directly by the cardholder, and sometimes borne by

the retail establishments that accept them. In the latter case, the cost is absorbed into the price of the merchandise. Department stores usually charge interest to credit customers who do not settle their bills within a month, but certain credit plans do not charge interest until a bill has been outstanding for several months. Interest rates for overdue balances are regulated by state law. A continuing problem involved in the use of credit cards is the ease with which they can be used fraudulently if stolen or lost, although the liability of the owner is limited.

2.5.5 Letter of Credit

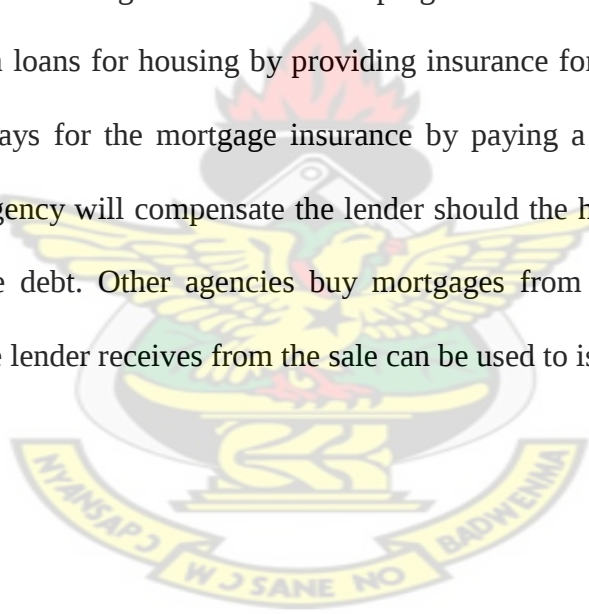
Letter of Credit, document issued by a bank authorizing the bearer to receive money from one of its foreign branches or from another bank abroad. The order is nonnegotiable, and it specifies a maximum sum of money not to be exceeded. Widely used by importers and exporters, the letter of credit is also made available to tourists by their home banks so that they may draw foreign currency while traveling abroad. When the instrument is directed to more than one agent, it is called a circular letter of credit.

Letters of credit are the most common means of small business financing, but they are an important financing tools for companies that engage in international trade.

2.5.6. Types of Mortgage

The two most common mortgages are the fixed-rate mortgage and the adjustable-rate mortgage, with a fixed-rate mortgage, the interest rate stays the same over the life of the loan. With an adjustable-rate mortgage, the interest rate can change at the end of pre-determined intervals, such as every six months or every year. The interest rate is tied to changes in a published index that reflects the current interest rate. One widely used index is the interest rate of the Treasury bonds.

If the index has gone up at the end of the adjustment period, the mortgage rate goes up, and thus the borrower's payment also goes up. Conversely, if the index has gone down, the mortgage rate goes down, and the mortgage payment goes down. Neither the lender nor the borrower can influence or predict in which direction the index will move. Most adjustable-rate mortgages have a maximum interest rate cap. Other, less common mortgages include the balloon mortgage and the graduated payment mortgage. A balloon mortgage is a short-term loan. The borrower makes payments for some period of time and then makes one large payment at the end. The graduated payment mortgage starts out with low monthly payments, which gradually increase over time before stabilizing. Government agencies administer programmes to help low-and moderate – income borrowers obtain loans for housing by providing insurance for lenders against borrower default. The borrower pays for the mortgage insurance by paying a fee to the agency. If the borrower defaults, the agency will compensate the lender should the house sell for less than the amount of the mortgage debt. Other agencies buy mortgages from lenders and sell them to investors. The money the lender receives from the sale can be used to issue additional mortgages



CHAPTER 3

METHODOLOGY

3.1 Introduction

One of the major concepts that most investors should be aware of is the relationship between the risk and the return of a financial asset. Portfolio selection problems involve situations in which a financial manager must select specific investment from a variety of investment alternatives. This type of problem is frequently encountered by managers of banks. The objective function for portfolio selection problems is usually maximization of expected return or minimization of risk. The constraints usually take the form of restrictions on the type of permissible investments, state laws, company policies, maximum permissible risk, and so on.

In this chapter we will use the concept of linear programming to show how a portfolio selection problem can be formulated and solved.

3.2 The Model

A bank has the opportunity to finance a portfolio (continuum) of loans whose size L is derived endogenously. It funds lending out of internal funds (capital) and outside finance from final investors. The supply of outside finance is perfectly elastic at a gross rate of return that is normalized to one (i.e. the risk-free net interest rate is zero). That is, final investors are assumed to make zero profits. The bank acts on behalf of its shareholders (insiders), whose equity holdings constitute the bank's endowment of inside capital.

E.g. the total amount paid for goods X and Y if the price for each is 40 and 10 respectively is given by $40X + 10Y$.

3.2.1 A mathematical program

A mathematical programme is of the form

Maximize (Minimize) $z = f(x_1, x_2, \dots, x_n)$

Subject to

$$g_i(x_1, x_2, \dots, x_n) \leq b_i, \quad 1 \leq i \leq p$$

$$g_i(x_1, x_2, \dots, x_n) \geq b_i, \quad p+1 \leq i \leq k$$

$$g_i(x_1, x_2, \dots, x_n) = b_i, \quad k+1 \leq i \leq m$$

$$x_1 \geq 0$$

where the variables $x_1, x_2, \dots, x_n$ are the unknown and referred to as Decision or Control variables. The function, f to be maximized or minimize is called the objective function of the problem.

The functions $g_i(x_1, x_2, \dots, x_n) \leq b_i \quad 1 \leq i \leq p$

$$g_i(x_1, x_2, \dots, x_n) \geq b_i \quad p+1 \leq i \leq k$$

$$g_i(x_1, x_2, \dots, x_n) = b_i, \quad k+1 \leq i \leq m$$

are referred to as the constraints of the problem and are limitations imposed on the available resources. Constraints of the type $g_i(x_1, x_2, \dots, x_n) = b_i$, are called equality constraints. The constraints of the type $g_i(x_1, x_2, \dots, x_n) \leq b_i$ are called inequality constraints. The $x_j \geq 0$ are additional constraints that require the decision variables to be greater than or equal to zero is referred to as the non-negativity constraints requirement of the problem.

3.3 Linear Programming

It is a mathematical method for determining a way to achieve the best outcome such as maximum profit or lowest cost in a given model for some list of requirements represented as linear equations. Linear programming is a technique for the optimization of a linear objective function, subject to linear equality and linear inequality constraints. Given a polyhedron and a real-valued affine function defined on this polyhedron, a linear programming method will find a point on the polyhedron where this function has the smallest (or largest) value if such point exists, by searching through the polyhedron vertices. Linear programs are problems that can be expressed in canonical form:

$$\text{Maximize } c^T x$$

$$\text{Subject to } Ax \leq b.$$

where x represents the vector of variables (to be determined), c and b are vectors of (known) coefficients and A is a (known) matrix of coefficients. The expression to be maximized or minimized is called the objective function ($c^T x$ in this case). The equations $Ax \leq b$ are the constraints which specify a convex polytope over which the objective function is to be optimized.

3.3.1 Standard form

Standard form is the usual and most intuitive form of describing a linear programming problem.

It consists of the following three parts:

- A linear function to be maximized

e.g. maximize $C_1x_1 + C_2x_2$

- Problem constraints of the following form

e.g. $a_{11}x_1 + a_{12}x_2 \leq b_1$

$$a_{21}x_1 + a_{22}x_2 \leq b_2$$

$$a_{31}x_1 + a_{32}x_2 \leq b_3$$

- Non-negative variables

e.g. $x_1 \geq 0$

$$x_2 \geq 0.$$

The problem is usually expressed in matrix form, and then becomes:

$$\text{maximize } c^T x$$

$$\text{subject to } Ax \leq b, x \geq 0.$$

Other forms, such as minimization problems, problems with constraints on alternative forms, as well as problems involving negative variables can always be rewritten into an equivalent problem in standard form.

Example

Suppose that a farmer has a piece of farm land, say A square kilometers large, to be planted with either wheat or rice or some combination of the two. The farmer has a limited permissible amount F of fertilizer and P of insecticide which can be used, each of which is required in different amounts per unit area for wheat (F_1, P_1) and rice (F_2, P_2) . Let S_1 be the selling price of wheat, and S_2 the price of rice. If we denote the area planted with wheat and rice by x_1 and x_2 respectively, then the optimal number of square kilometres to plant with wheat versus rice can be expressed as a linear programming problem:

Maximize $S_1x_1 + S_2x_2$ (maximize the revenue-revenue is the objective function)

Subject to $x_1 + x_2 \leq A$ (limit on total area)

$F_1x_1 + F_2x_2 \leq F$ (limit on fertilizer)

$P_1x_1 + P_2x_2 \leq P$ (limit on insecticide)

$x_1 \geq 0, x_2 \geq 0$ (cannot plant a negative area)

Which in matrix form becomes:

$$\text{Maximize} \quad \begin{bmatrix} S_1 & S_2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$\text{subject to } \begin{bmatrix} 1 & 1 \\ F_1 & F_2 \\ P_1 & P_2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \leq \begin{bmatrix} A \\ F \\ P \end{bmatrix}, \quad \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \geq 0.$$

3.3.2 Augmented form (slack form)

Linear programming problems must be converted into augmented form before being solved by the simplex algorithm. This form introduces non-negative slack variables to replace inequalities with equalities in the constraints. The problem can then be written in the following block matrix form:

Maximize Z in:

$$\begin{bmatrix} 1 & -\mathbf{c}^T & 0 \\ 0 & \mathbf{A} & \mathbf{I} \end{bmatrix} \begin{bmatrix} Z \\ \mathbf{x} \\ \mathbf{x}_s \end{bmatrix} = \begin{bmatrix} 0 \\ \mathbf{b} \end{bmatrix}$$

$$X, X_s \geq 0$$

where X_s are the newly introduced slack variables, and Z is the variable to be maximized.

Example

The example above is converted into the following augmented form:

Subject to $x_1 + x_2 + x_3 = A$ (augmented constraint)

$$F_1 x_1 + F_2 x_2 + x_4 = F \text{ (augmented constraint)}$$

$$P_1 x_1 + P_2 x_2 + x_5 = P \text{ (augmented constraint)}$$

$$x_1, x_2, x_3, x_4, x_5 \geq 0$$

where x_3, x_4, x_5 are (non-negative) slack variables, representing in this example the unused area, the amount of unused fertilizer, and the amount of unused insecticide.

In matrix form this becomes:

Maximize Z in:

$$\begin{bmatrix} 1 & -S_1 & -S_2 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & F_1 & F_2 & 0 & 1 & 0 \\ 0 & P_1 & P_2 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} Z \\ x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} 0 \\ A \\ F \\ P \end{bmatrix}, \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} \geq 0.$$

The standard form of a linear programming problem with n variables and m constraints can be represented as follows

$$\text{Max (min) } z = c_1x_1 + c_2x_2 + \dots + c_nx_n$$

$$\text{s.t } a_{11}x_1 + a_{12}x_2 + \dots + a_{1n}x_n = b_1$$

$$a_{21}x_1 + a_{22}x_2 + \dots + a_{2n}x_n = b_2$$

$$a_{m1}x_1 + a_{m2}x_2 + \dots + a_{mn}x_n = b_n$$

$$x_j \geq 0, j=1, 2, \dots, n$$

3.3.3 Conversion into Standard Form

To develop a general solution method, the linear programming problem must first be recast in the standard form and this could arrive at in the following way. Taking the general form of the linear programming problem that is we incorporate new variable $t \geq 0$ to convert inequality into equations. In order to replace the inequality constraints by equality constraint, we introduce extra variable $t_i \geq 0$ called a slack variable or surplus variable. A variable added to the left hand side of a “less than or equal to” constraint to convert the constraint into an equation is referred to as a slack variable and is given as follows

$$\sum_{j=1}^n a_{ij}x_j + t_i = b_i, 1 \leq i \leq p$$

A variable $t_i \geq 0$ subtracted from the left hand side of a greater than or equal to constraint to convert an inequality constraint into equation is referred to as a surplus

Variable and is given as

$$\sum_{j=1}^n a_{ij}x_j - t_i = b_i, p+1 \leq i \leq k$$

3.3.4 Transformation of General Variables to Satisfy non-negativity Condition

Further, if there exists any variable x_i which does not satisfy the non-negativity condition $x_j \geq 0$, we put

$$x_i^* = \max(x_i, 0)$$

$$x_i^- = -\min(x_i, 0)$$

And replace each x_i by $x_i^+ - x_i^-$ throughout the problem. The result is a linear programming problem in standard form. Before this is solved, the right hand side of the constraint equation is made non-negative by multiplying through each constraint by (-1) if necessary.

3.4 Duality

Every linear programming problem, referred to as a primal problem, can be converted into a dual problem, which provides an upper bound to the optimal value of the primal problem. In matrix form, we can express the primal problem as:

Maximize $c^T x$

subject to $Ax \leq b, x \geq 0$, with the corresponding symmetric dual problem: minimize $b^T y$ subject to $A^T y \geq c, y \geq 0$.

An alternative primal formulation is:

maximize $c^T x$ subject to $Ax \leq b$, with the corresponding asymmetric dual problem; minimize $b^T y$ subject to $A^T y = c, y \geq 0$

There are two ideas fundamental to duality theory. One is the fact that (for the symmetric dual) the dual of a dual linear program is the original primal linear program. Additionally, every feasible solution for a linear program gives a bound on the optimal value of the objective function of its dual. The weak duality theorem states that the objective function value of the dual at any feasible solution is always greater than or equal to the objective function value of the

primal at any feasible solution. The strong duality theorem states that if the primal has an optimal solution, x^* , then the dual also has an optimal solution, y^* , such that $c^T x^* = b^T y^*$.

A linear program can also be unbounded or infeasible. Duality theory tells us that if the primal is unbounded then the dual is infeasible by the weak duality theorem. Likewise, if the dual is unbounded, then the primal must be infeasible. However, it is possible for both the dual and the primal to be infeasible.

Example

Revisit the above example of the farmer who may grow wheat and rice with the set provision of some A land, F fertilizer and P insecticide. Assume now that unit prices for each of these means of production (inputs) are set by a planning board. The planning board's job is to minimize the total cost of procuring the set amounts of inputs while providing the farmer with a floor on the unit price of each of his crops (outputs), S_1 for wheat and S_2 for rice. This corresponds to the following linear programming problem:

minimize $Ay_A + Fy_F + Py_P$ (minimize the total cost of the means of production as the "objective function")

subject $y_A + F_1y_F + P_1y_P \geq S_1$ (the farmer must receive no less than S_1 for his wheat)

to

$y_A + F_2y_F + P_2y_P \geq S_2$ (the farmer must receive no less than S_2 for his rice)

$$y_A \geq 0, y_F \geq 0, y_P \geq 0 \text{ (prices cannot be negative).}$$

Which in matrix form becomes:

$$\begin{aligned} & \text{minimize} \quad [A \quad F \quad P] \begin{bmatrix} y_A \\ y_F \\ y_P \end{bmatrix} \\ & \text{subject to} \quad \begin{bmatrix} 1 & F_1 & P_1 \\ 1 & F_2 & P_2 \end{bmatrix} \begin{bmatrix} y_A \\ y_F \\ y_P \end{bmatrix} \geq \begin{bmatrix} S_1 \\ S_2 \end{bmatrix}, \quad \begin{bmatrix} y_A \\ y_F \\ y_P \end{bmatrix} \geq 0. \end{aligned}$$

The primal problem deals with physical quantities. With all inputs available in limited quantities, and assuming the unit prices of all outputs is known, what quantities of outputs to produce so as to maximize total revenue? The dual problem deals with economic values. With floor guarantees on all output unit prices, and assuming the available quantity of all inputs is known, what input unit pricing scheme to set so as to minimize total expenditure? To each variable in the primal space corresponds an inequality to satisfy in the dual space, both indexed by output type. To each inequality to satisfy in the primal space corresponds a variable in the dual space, both indexed by input type.

The coefficients that bound the inequalities in the primal space are used to compute the objective in the dual space, input quantities in this example. The coefficients used to compute the objective in the primal space bound the inequalities in the dual space, output unit prices in this example.

Both the primal and the dual problems make use of the same matrix. In the primal space, this matrix expresses the consumption of physical quantities of inputs necessary to produce set

quantities of outputs. In the dual space, it expresses the creation of the economic values associated with the outputs from set input unit prices.

Since each inequality can be replaced by equality and a slack variable, this means each primal variable corresponds to a dual slack variable, and each dual variable corresponds to a primal slack variable. This relation allows us to complementary slackness.

Another example

Sometimes, one may find it more intuitive to obtain the dual program without looking at program matrix. Consider the following linear program:

$$\text{minimize } \sum_{i=1}^m c_i x_i + \sum_{j=1}^n d_j t_j \geq g_j, 1 \leq j \leq n$$

$$\text{subject to } \sum_{i=1}^m a_{ij} x_i + e_j t_j \geq g_j, 1 \leq j \leq n$$

$$f_i x_i + \sum b_{ij} t_j \geq h_i, 1 \leq i \leq m$$

$$x_i \geq 0, t_j \geq 0, 1 \leq i \leq m, 1 \leq j \leq n$$

We have $m + n$ conditions and all variables are non-negative. We shall define $m + n$ dual variables: y_j and s_i . We get:

$$\text{Minimize } \sum_{i=1}^m c_i x_i + \sum_{j=1}^n d_j t_j$$

$$\text{Subject to } \sum_{i=1}^m a_{ij}x_i + e_jt_j + y_j \geq g_j, 1 \leq j \leq n$$

$$f_ix_i + \sum_{j=1}^n b_{ij}t_j + s_i \geq h_i, 1 \leq i \leq m$$

$$x_i \geq 0, t_j \geq 0, 1 \leq i \leq m, 1 \leq j \leq n$$

$$y_j \geq 0, s_i \geq 0, 1 \leq j \leq n, 1 \leq i \leq m$$

Since this is a minimization problem, we would like to obtain a dual program that is a lower bound of the primal. In other words, we would like the sum of all right hand side of the constraints to be the maximal under the condition that for each primal variable the sum of its coefficients does not exceed its coefficient in the linear function. For example, x_1 appears in $n + 1$ constraints. If we sum its constraints' coefficients we get $a_{11}y_1 + a_{12}y_2 + \dots + a_{1n}y_n + f_1s_1$. This sum must be at most c_1 . As a result we get:

$$\text{Maximize } \sum_{j=1}^n g_jy_j + \sum_{i=1}^m h_is_i$$

$$\text{Subject to } \sum_{j=1}^n a_{ij}y_j + f_is_i \leq c_i, 1 \leq i \leq m$$

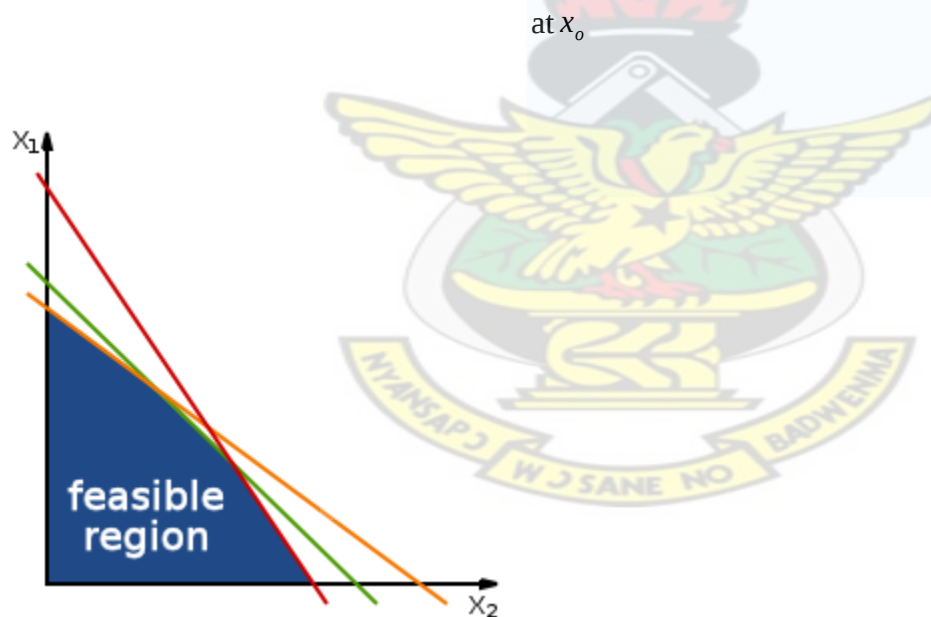
$$g_jy_j + \sum_{i=1}^m b_{ij}s_i \leq d_j, 1 \leq j \leq n$$

$$y_j \geq 0, s_i \geq 0, 1 \leq j \leq n, 1 \leq i \leq m$$

Note that we assume in our calculations steps that the program is in standard form. However, any linear program may be transformed to standard form and it is therefore not a limiting factor.

3.4.1 Feasible Region

The set of points in R^n satisfying all the constraints of the problem is called the feasible region of the problem. Any point in the feasible region is called a feasible point. If x_0 denote a feasible point. If an inequality constraint $g_i(x_1, \dots, x_n) \leq b_i$, is satisfied as equality at x_0 i.e. $g_i(x_1, x_2, \dots, x_n) = b_i$, then the inequality constraint is binding or active at x_0 . If the inequality constraint is satisfied as a strict inequality i.e. $g_i(x_1, x_2, \dots, x_n) < b_i$ then the inequality is inactive or not binding



A series of linear constraints on two variables produces a region of possible values for those variables. Solvable problems will have a feasible region in the shape of a simple polygon.

3.5 Simulated Annealing (SA)

SA consists of three parts: solution space, objective function and initial solution.

1. Solution Space:

It is the group of all possible solutions and it restricts the scope of our choosing the initial solution and the new solution. In many optimization problems, besides objective functions, we also have a set of constraints. Hence; there might be some infeasible solution in the solution space. You can define the solution space exclusive of infeasible solutions or you can allow them by incorporating a penalty function to penalize the occurrence of the infeasible solution.

2. Objective function

It is the mathematical description of the optimization problem. Usually it is constructed as the sum of several optimization requirements and, when infeasible solutions are allowed, objective function needs to incorporate a penalty function.

3. Initial Solution:

It is the starting point of the algorithm. The final solution is independent of the choice of the initial solution.

3.6 The simplex Method

The simplex method refers to the idea of moving from one extreme point to another on the convex set that is formed by the constraint set and non-negativity conditions of the linear programming problem. The simplex method is algebraic in nature and is based upon the Gauss-Jordan elimination procedure.

3.6.1 The simplex Algorithm

The simplex algorithm is an iterative procedure that provides a structured method for moving from one basic feasible solution to another, always maintaining the objective function until an optimal solution is obtained.

Slack variables s_1 and s_2 are restricted to be equal to or greater than zero. However, slack variables have no effect on the objective function. To solve a problem we sequentially generate a set of basic feasible solutions that corresponds to the extreme points of the feasible solution space. Naturally we first determine an initial basic feasible solution. Recall that a basic solution to a set of m equations in n variables ($n > m$) is obtained by setting $(n-m)$ variables equal to zero and solving the resulting system of m equations in m variables.

The m variables are referred to as the basic variables or as the variables “in the basis.” The variables are referred to as the non-basic variables or as the variables “not in the basis.”

A basic feasible solution is defined as being a basic solution where the entire basic variable is non-negative (≥ 0).

A non-degenerate basic feasible solution is defined as being a basic solution, where all m of the basic variables are greater than zero (> 0).

3.6.2 Setting up the Initial Simplex Tableau

In developing a tabular approach for the simplex algorithm, we use an instructive and consistent set of notations that enhances the understanding of the process.

Terms Used

The terms used in the initial simplex tableau are as follows;

c_j =objective function coefficients for variable j.

b_i =right-hand-side coefficients (value) for constraint i

a_{ij} =coefficients of variable j in constraint i

c_B =objective function coefficients of the basic variables.

Notations that will be used extensively in the following development of the simplex method are as follows.

C row, the row of the objective function coefficients

b column, the column of the right-hand-side values of the constraints equations

[A] matrix, the matrix (with m rows and n columns) of the coefficients of the variables in the constraint equations.

Example

Maximize $z = 6x_1 + 8x_2 + 0s_1 + 0s_2$

Subject to $5x_1 + 10x_2 = 60$

$$4x_1 + 4x_2 + 1s_1 = 40$$

$$x_1 \geq 0, x_2 \geq 0, s_1 \geq 0, s_2 \geq 0$$

In this formulation we observe that both slack variables s_1 and s_2 are restricted to be equal to or greater than zero. However, slack variables have no effect on the objective function since they are not associated with real products. To solve this problem algebraically we must be able to sequentially generate a set of basic feasible solutions that correspond to the extreme points of the feasible solution space.

First, we must determine an initial basic feasible solution. A basic solution to a set of m equations in n variables ($n > m$) is obtained by setting $(n-m)$ variables equal to zero and solving the resulting system of m equations in m variables. The m variables are referred to as the basic variables or as the variables “in the basis.” A basic feasible solution is defined as being a basic solution where all m of the basic variables are non-negative (≥ 0). A non-degenerate basic feasible solution is defined as being a basic feasible, where all m of the basic variables are greater than zero (> 0).

Table 3.1 General Form-Initial Simplex Tableau

		Decision variables			Slack variables				
c_j		c_1	c_2	c_n	0	0	0	Solution	Objective function coefficients
c_B	Basic variables	x_1	x_2	x_n	1		s_m		headings
0	s_1	a_{11}	a_{12}	a_{1n}	1	0	0		Constraints coefficient
...	s_2	a_{21}	a_{22}	a_{2n}	0	1	0		

0									
	s_m	a_{m1}	a_{m2}	a_{mn}	0	0	1		
		z_1	z_2	z_{mn}	z_{11}		z_{1m}	Current value of objective function	
	$c_j - z_j$	$c_1 - z_1$	$c_2 - z_2$	$c_{mn} - z_{mn}$	$c_{1_1} - z_{1_1}$	$c_{1_2} - z_{1_2}$	$c_{1_n} - z_{1_n}$		Reduce cost(net contribution/unit)

The initial simplex tableau is a representation of the standard linear programming form, with some supplemental rows and columns. The top row of the table presents the objective function coefficients. Then, there are $c_j - z_j$ in rows, which represent the constraint coefficients. Following is the z_j row which is the net contribution per unit of the j th variable.

The leftmost column in the tableau indicates the values of the objective function coefficients associated with the basic variables, with a set of constraints, the initial column is headed “basic variable”. The next $m+n$ columns contain the constraint coefficients; the final rightmost column displays the solution values of the basic variables.

3.6.3 Special Cases

Some special situations encountered in solving linear problems are:

- 1) Unbounded solutions where there is no limit on some entering variable, and hence there is unbounded profitability
- 2) No feasible solution, when constraints are so restrictive that there is no way of satisfying them.

Both of them can be detected during the simplex procedure. Both generally result from improper formulation of the problem or incorrect data. Multiple solutions to linear programming problems are possible. This is indicated when one of the outside variables has a $c_j - z_j$ value of zero in the optimal solution. Degeneracy occurs when one or more of the solution variables equals zero.

3.6.4 Unbounded Solution

In the case of an unbounded solution the simplex method will terminate with the indication that the entering basic variable can do so only if it is allowed to assume a value of infinity. Specifically, for a maximization problem we will encounter a simplex tableau having a non basic variable whose $c_j - z_j$ row value is strictly greater than zero. And for this same variable all of the a_{ij} elements in its column will be zero or negative. Thus in performing the ratio test for the variable removal criterion, it will be possible only to form ratios having negative numbers or zeros as denominators.

If we have unbounded solution, none of the current basic variables can be driven from solution by the introduction of a new basic variable, even if that new basic variable assumes an infinitely large value. Generally arriving at an unbounded solution indicates that the problem was originally misformulated within the constraints set and needs reformulation.

3.6.5 Multiple Solutions

There may be more than one optimal solution to a linear programming problem. In this case two or more basic solutions will have the same optimum profit or cost. In the optimal tableau of the example the solution has no $c_j - z_j$ value negative, indicating an optimum has been reached. Imagine, however for example variable x_2 , is not a solution variable and $c_j - z_j = 0$. this indicates

that variable x_2 can be brought into the solution without increasing or decreasing the cost. The optimal solution lies within the corner points of the feasible area and any convex combination of these corner points is an optimum solution.

3.6.6 No feasible solution

The lack of a feasible solution can be detected in the simplex table. At some point in the procedure a solution occurs that would appear to be optimal .all the coefficients in the $C_j - Z_j$ rows are non positive if maximizing or nonnegative if minimizing. However one of the solution variables is an artificial variable. Computer programs would stop at this point with a message that no feasible solution exist.

In a business situation the lack of a feasible solution generally indicate an error in formulating the problem or in entering the data.

3.6.7 Degeneracy

Theoretically, if there is a tie between two or more variables to be removed from the basis at some iteration prior to reaching the optimal solutions, a situation known as cycling can occur. Cycling can occur if the arbitrary choice between the tied variables for a variable to be removed from the basis will always generate a degenerate feasible solution in which all of the tied variables reach zero simultaneously s the entering basic variable is increased.

3.7 Modify the constraints

Add a slack variable for each less than or equal to constraint. Add an artificial variable for each equality constraint. Add both an artificial and a surplus variable for each greater than or less than

constraint. For each artificial variable, assign a very large cost (negative profit) in the objective function.

3.7.1 Initial Solution

Identify the initial solution as composed of the slack and artificial variable.

3.7.2 Check for Optimality

The current solution is optimal if: for maximization all coefficients in the $c_j - z_j$ are zero or negative .for minimum all coefficient in $c_j - z_j$ row are zero or positive. If optimum has been reached, stop the simplex procedure.

3.7.3 Entering Variable

This is the non basic variable associated with the largest positive (maximum) or negative (minimum) coefficient in the $c_j - z_j$ row.

3.7.4 Leaving variable

For each row, calculate the ratio of the values in the “solution values column” divided by the coefficient in the entering variable column. Ignore any ratios that are negative .the leaving variable is the row having the smallest ratio.

3.7.5 Resolve the equation

a) Identify the pivot element as the coefficient in the entering variable row.

b) Divide the entire coefficient in the leaving row by the pivot element.

c) Modify the other rows, possibly including the objective function row as

New row = old row – coefficient of entering variable column x row obtained in step 3.7.6(b).

Calculation of z_j

For the j th column: for each row, multiply the substitution coefficient by the c_j value for that row and sum. The total is z_j , repeat for all columns.

Calculation of $c_j - z_j$

Subtract the z_j from the original objective function coefficient at the top of the table.

3.7.6 Economic interpretation of the $c_j - z_j$ values

First consider the economic interpretation of the $c_j - z_j$ values, the bottom row of the simplex table. Recall that z_j is the opportunity cost of introducing one unit of variable j into the solution .the cost of replacing or subtracting for other solution variables. Since c_j is the unit profit, the $c_j - z_j$ value is the net profit resulting from introducing one unit of j into the solution.

3.8 Convex Region

A region of space is said to be convex if the portion of the line segment between any two points in the region also lies in the region.

3.8.1 Convex function

A function $f(x)$ is said to be convex if the set of points (x, v) where $v \geq f(x)$ form a convex region.

3.8.2 Convex Mathematical Program

A mathematical programming model is said to be convex if it involves the minimization or maximization of a convex function over a convex feasible region. A function which is non-differentiable but is a convex function can be minimized over a convex feasible region.

3.9 Sensitivity Analysis

Sensitivity Analysis is designed to study the effect of changes in the parameters of the linear programming model on the optimal solution. The ultimate objective of the analysis is to obtain information about possible new optimum solutions with minimal additional computations.

An example is the worth of a revenue unit? This problem deals with the study of the sensitivity of the optimum solution to changes in the right hand side of the constraints. If the constraints represent a limited resource, the problem reduces to studying the effect of changing the availability of the resource. The specific goal of this sensitivity problem is to determine the effect of changes in the right hand side of constraints on the optimum objective value. In essence the results are given as predetermined ranges of the right hand side within which the objective optimum value will change at a given constant rate. It enables us to answer such questions as would you like to buy any more of the resource? If so what price should you pay? How many units should you buy at that price? Similar questions can be asked about selling resources, even

though a resource may be currently used in making products, at some price it is worthwhile to forgo products and sell it. These considerations are of interest because they lead to decision that can increase profit or reduce cost.

Another part of the sensitivity analysis deals with the impact of changes of available resources or other conditions which are expressed in the right side of constraints.

Sensitivity analysis involves

- 1) First finding the shadow price, the marginal values for a unit change in the Right-Hand-Side value of the constraint
- 2) Second finding the range of values over which the shadow price holds (R.H.S Ranges)
- 3) Third which objective function coefficient ranges. These ranges indicate the changes in the objective function coefficient within the optimal solution remaining the same. In general, determining the sensitivity of the optimal solution to changes is easiest analytically when only one parameter changes.

3.10 Markowitz programming- Modern portfolio theory (MPT)

Harry Markowitz developed the Modern portfolio theory and it's a theory of finance which attempts to maximize portfolio expected return for a given amount of portfolio risk, or equivalently minimize risk for a given level of expected return, by carefully choosing the proportions of various assets. Although MPT is widely used in practice in the financial industry and several of its creators won a Nobel memorial prize for the theory, in recent years the basic assumptions of MPT have been widely challenged by fields such as behavioral economics.

MPT is a mathematical formulation of the concept of diversification in investing, with the aim of selecting a collection of investment assets that has collectively lower risk than any individual asset. That this is possible can be seen intuitively because different types of assets often change in value in opposite ways. For example, to the extent prices in the stock market move differently from prices in the bond market, a collection of both types of assets can in theory face lower overall risk than either individually. But diversification lowers risk even if assets' returns are not negatively correlated—indeed, even if they are positively correlated.

More technically, MPT models an asset's return as a normally distributed function (or more generally as an elliptically distributed random variable), defines risk as the standard deviation of return, and models a portfolio as a weighted combination of assets, so that the return of a portfolio is the weighted combination of the assets' returns. By combining different assets whose returns are not perfectly positively correlated, MPT seeks to reduce the total variance of the portfolio return. MPT also assumes that investors are rational and markets are efficient.

3.11 Concept

The fundamental concept behind MPT is that the assets in an investment portfolio should not be selected individually, each on their own merits. Rather, it is important to consider how each asset changes in price relative to how every other asset in the portfolio changes in price.

Investing is a tradeoff between risk and expected return. In general, assets with higher expected returns are riskier. For a given amount of risk, MPT describes how to select a portfolio with the highest possible expected return. Or, for a given expected return, MPT explains how to select a

portfolio with the lowest possible risk (the targeted expected return cannot be more than the highest-returning available security, of course, unless negative holdings of assets are possible).

Therefore, MPT is a form of diversification. Under certain assumptions and for specific quantitative definitions of risk and return, MPT explains how to find the best possible diversification strategy.

3.12 History

Harry Markowitz introduced MPT in a 1952 article and a 1959 book. Markowitz classifies it simply as "Portfolio Theory," because "There's nothing modern about it." See also this survey of the history.

3.13 Risk and expected return

MPT assumes that investors are risk averse, meaning that given two portfolios that offer the same expected return, investors will prefer the less risky one. Thus, an investor will take on increased risk only if compensated by higher expected returns. Conversely, an investor who wants higher expected returns must accept more risk. The exact trade-off will be the same for all investors, but different investors will evaluate the trade-off differently based on individual risk aversion characteristics. The implication is that a rational investor will not invest in a portfolio if a second portfolio exists with a more favorable risk-expected return profile – i.e., if for that level of risk an alternative portfolio exists which has better expected returns.

Note that the theory uses standard deviation of return as a proxy for risk, which is valid if asset returns are jointly normally distributed or otherwise elliptically distributed.

3.14 Mathematical model

In some sense the mathematical derivation below is MPT, although the basic concepts behind the model have also been very influential.

This section develops the "classic" MPT model. There have been many extensions since.

Under the model:

- Portfolio return is the proportion-weighted combination of the constituent assets' returns.
- Portfolio volatility is a function of the correlations ρ_{ij} of the component assets, for all asset pairs (i, j) .

In general:

- Expected return:

$$E(R_p) = \sum_i w_i E(R_i)$$

where R_p is the return on the portfolio, R_i is the return on asset i and w_i is the weighting of component asset i (that is, the share of asset i in the portfolio).

- Portfolio return variance:

$$\sigma_p^2 = \sum_i w_i^2 \sigma_i^2 + \sum_i \sum_{j \neq i} w_i w_j \sigma_i \sigma_j \rho_{ij},$$

where ρ_{ij} is the correlation coefficient between the returns on assets i and j . Alternatively the expression can be written as:

$$\sigma_p^2 = \sum_i \sum_j w_i w_j \sigma_i \sigma_j \rho_{ij},$$

where $\rho_{ij} = 1$ for $i=j$.

- Portfolio return volatility (standard deviation):

$$\sigma_p = \sqrt{\sigma_p^2}$$

For a two asset portfolio:

- Portfolio return: $E(R_p) = w_A E(R_A) + w_B E(R_B)$

$$E(R_p) = w_A E(R_A) + w_B E(R_B) = w_A E(R_A) + (1 - w_A) E(R_B).$$

- Portfolio variance: $\sigma_p^2 = w_A^2 \sigma_A^2 + w_B^2 \sigma_B^2 + 2w_A w_B \sigma_A \sigma_B \rho_{AB}$

For a three asset portfolio:

- Portfolio return: $w_A E(R_A) + w_B E(R_B) + w_C E(R_C)$

- Portfolio variance: $\sigma_p^2 = w_A^2 \sigma_A^2 + w_B^2 \sigma_B^2 + w_C^2 \sigma_C^2 + 2w_A w_B \sigma_A \sigma_B \rho_{AB} + 2w_A w_C \sigma_A \sigma_C \rho_{AC} + 2w_B w_C \sigma_B \sigma_C \rho_{BC}$

3.15 Diversification

An investor can reduce portfolio risk simply by holding combinations of instruments which are not perfectly positively correlated (correlation coefficient). In other words, investors can reduce their exposure to individual asset risk by holding a diversified portfolio of assets. Diversification may allow for the same portfolio expected return with reduced risk. These ideals have been

started with Markowitz and then reinforced by other economists and mathematicians such as Andrew Brennan who have expressed ideas in the limitation of variance through portfolio theory.

If all the asset pairs have correlations of 0—they are perfectly uncorrelated—the portfolio's return variance is the sum over all assets of the square of the fraction held in the asset times the asset's return variance (and the portfolio standard deviation is the square root of this sum).

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CHAPTER 4

DATA ANALYSIS AND MODELLING

4.1 INTRODUCTION

In this chapter we provide an introduction to the use of computer for solving our linear programming problem. We begin by explaining the profile of the bank and showing how the management scientist can be used to solve the problem. Then we extend the discussion by formulating and solving the bank's credit portfolio problem with five decision variables and four constraints. We then continue to solve the problem involving five decision variables and various types of constraints.

In discussing the computer solution for each of these problems, we focus on the interpretation of the computer output including both the optimal solution and sensitivity analysis information.

4.2 Profile of the Bank

First Allied Savings and Loans Limited (FASL) was incorporated as a private limited liability company on May 24, 1995 under the Ghana Companies Code, 1963 (Act 179).

FASL was incorporated as a non-bank financial institution to operate a savings and loans business in the country. The Institution was granted an operating license by the Bank of Ghana under the Non-Bank Financial Institutions (NBFI) Law (PNDCL 328) of 1993 on March 27, 1996 to accept deposits from the public and provide credit services to businesses

and consumers.

FASL's authorized business is to carry on savings and loan services.

The Institution was established purposely to engage in micro-financing activities through the mobilization of savings from the retail public – mainly households and small business enterprises – and the provision of credit largely to its target group (micro and small businesses). The target group oriented credits are usually linked to savings.

The Institution has been reaching out to its customers through its branches, agency and a “distance banking” concept. The Institution has been able to position itself as the leader in the savings and loans business through the provision of quality products and the delivery of efficient services to its targeted customers.

FASL started operations on September 25, 1996 at the ground floor of its building at Adum, the capital city of the Ashanti Kingdom, which now houses the Head Office, the Adum branch, and other sections and departments of the Institution.

As of now, FASL could boast of of fourteen (14) branches nationwide; five in Kumasi; three branches in the Brong Ahafo Region: three branches in Accra, two in eastern region and one in Takoradi.

First Allied Savings and Loans Techiman branch has a loan policy involving a total Three million Five Hundred thousand Ghana Cedis (GH¢ 3, 500,000.00). Being a full-service facility, the branch has been tasked to grant loans to different clientele. The following table provides the types of loans, the interest rate charged by the bank, and the probability of bad debt as estimated from past experience.

Table 4.1 Types of Loan and corresponding Interest Rate and Risk

Type of loan	Interest rate	Probability of bad debt
Overdraft	0.410	0.138
Salary Loan	0.280	0.010
Susu Loan	0.390	0.107
Micro Finance Loan	0.390	0.020
Commercial Loan	0.41	0.010

Bad debts are assumed unrecoverable and hence produce no interest revenue.

Competition with other financial institutions in the area requires that the bank

1. Allocate at least 60% of the total funds to Distance loans, Susu loans and commercial loans.
2. Salary loans and Overdraft will have at least 30% of the available funds.
3. The bank also has a stated policy specifying that the overall ration for bad debts on all loans may not exceed 0.02.

4.3 MODELLING

4.3.1 Mathematical model

The variables of the model can be defined as follows

x_1 = Overdraft amounts

x_2 = Salary loan amounts

x_3 = Micro Finance loan amounts

x_4 = Susu loan amounts

x_5 = Commercial loan amounts

The objective of the institution is to maximize the net return comprised of the difference between the revenue from interest and lost funds due to bad debts. Since bad debts are not recoverable, both as principal and interest, the objective function may be written as

Let

x_j = units of returns for the jth group.

P_j = probability of bad debt for the jth group.

$I_j(1 - P_j) x_j$ = units of revenue for the jth group.

$$\text{Net revenue} = \sum_{j=1}^n I_j (1 - P_j) x_j$$

Maximize: $\sum_{j=1}^n I_j (1 - P_j) x_j = \text{objective function}$

Incorporating data from Table 4.1 the objective function can be written as:

$$Z = 0.41(1-0.14) x_1 + 0.28 (1-0.01) x_2 + 0.39 (1-0.11) x_3 + 0.39 (1-0.02) x_4 + 0.41(1-0.01) x_5$$

$$Z = 0.35x_1 + 0.28x_2 + 0.35x_3 + 0.38x_4 + 0.41x_5$$

The problem has four constraints:

In accordance with current policy of the bank the four constraints may be constructed as follows.

(1) Total funds: the total amount allocated for all loans is 3.5 million cedis. We can thus give out not more than 3.5 million cedis

$$x_1 + x_2 + x_3 + x_4 + x_5 \leq 3.5 \text{ (in millions)}$$

(2) Salary loans and overdraft

$$x_1 + x_2 \geq 1.05$$

(3) Commercial Loans, Susu loans and Distance Loans

$$x_3 + x_4 + x_5 \geq 2.1$$

(4) 4. The bank also has a stated policy specifying that the overall ratio for bad debts on all loans should not exceed 0.02. This means

$$\underline{0.14x_1 + 0.01x_2 + 0.02x_3 + 0.11x_4 + 0.01x_5} \leq 0.02$$

$$x_1 + x_2 + x_3 + x_4 + x_5$$

Simplifying to

$$0.12 x_1 + - 0.01 x_2 + 0.00 x_3 + 0.09 x_4 - 0.01 x_5 \leq 0.02$$

Non-negativity

$$x_1 \geq 0, x_2 \geq 0, x_3 \geq 0, x_4 \geq 0, x_5 \geq 0$$

4.3.2 Scenario 1: All the four constraints

$$\text{Maximize } z = 0.35x_1 + 0.28x_2 + 0.35x_3 + 0.38x_4 + 0.41x_5$$

Subject to

$$x_1 + x_2 + x_3 + x_4 + x_5 \leq 3.5$$

$$x_1 + x_2 \geq 1.05$$

$$x_3 + x_4 + x_5 \geq 2.1$$

$$0.12 x_1 + - 0.01 x_2 + 0.00 x_3 + 0.09 x_4 - 0.01 x_5 \leq 0.02$$

A subtle assumption in the formulation above is that all loans are issued at approximately the same time. This assumption allows us to ignore the differences in the time values of the funds allocated to the different loans.

The solution is given in Table 4.2 below

Table 4.2 SOLUTIONS

Variable	Value
X ₁	0.159
X ₂	0.891
X ₃	0.000
X ₄	0.000
X ₅	2.450

Objective Function Value = 1.310

Sensitivity analysis was done and table 4.4 gives the objective function coefficients ranges corresponding to the maintenance of the solution given in Table 4.3.

Table 4.3 OBJECTIVE COEFFICIENT RANGES

Variable	Lower Limit	Current Value	Upper Limit
X ₁	0.280	0.350	No Upper Limit
X ₂	No Lower Limit	0.280	0.350
X ₃	No Lower Limit	0.350	0.413
X ₄	No Lower Limit	0.380	0.442
X ₅	0.348	0.410	No Upper Limit

The objective function value indicates that the optimal solution to the problem will provide a maximum profit of 1.310 million cedis. The value of the variables are Overdraft is 0.159 millions

cedis, salary loan is 0.891 million cedis, micro finance loan is 0.00, susu loan is 0.0 and commercial loan is 2.450 .This shows that only Overdraft, Salary loan and Commercial loans are recommended. Of the remaining, that is Susu and microfinance loans are less attractive.

4.3.3 Scenario 2: Five constraints

If Management of the bank will not consider any solution that does not include Susu and Microfinance loans such as the previous solution in which Susu loans and Microfinance loans had no allocations, then it means we should add an additional constraint. Regarding the 60% allocation of the total funds to Susu loans, Microfinance loans and Commercial loans, only Commercial loans took the entire allocation. It is therefore reasonable to add an additional constraint so that all loans have allocation. Management will then have to add the requirement that the amount of loans allocated to microfinance and susu must be at least 40% of the total loan portfolio. We then have $x_3 + x_4 \geq 1.4$

Adding this new constraint to the problem and resolving we obtain the optimal solution shown below.

Maximize $0.35x_1 + 0.28x_2 + 0.35x_3 + 0.38x_4 + 0.41x_5$

Subject to

$$1) \quad 1x_1 + 1x_2 + 1x_3 + 1x_4 + 1x_5 \leq 3.5$$

$$2) \quad 1x_1 + 1x_2 \geq 1.05$$

$$3) \quad 1x_3 + 1x_4 + 1x_5 \geq 2.1$$

$$4) \quad 0.21x_1 - 0.01x_2 + 0.09x_4 - 0.01x_5 \leq 0$$

$$5) 1x_3 + 1x_4 \geq 1.4$$

The solution is given in Table 4.4 below

Table 4.4 Solution for Five Constraints

Variable	Value
X ₁	0.000
X ₂	1.050
X ₃	1.167
X ₄	0.233
X ₅	1.050

Objective Function Value = 1.222

Sensitivity analysis was done and table 4.4 gives the objective function coefficients ranges corresponding to the maintenance of the solution given in Table 4.5.

Table 4.5 OBJECTIVE COEFFICIENT RANGES

Variable	Lower Limit	Current Value	Upper Limit
X ₁	No Lower Limit	0.350	0.353
X ₂	0.277	0.280	0.410
X ₃	No Lower Limit	0.350	0.351
X ₄	0.379	0.380	No Upper Limit
X ₅	0.347	0.410	No Upper Limit

The objective function value section shows that the optimal solution to the problem will provide a maximum profit of 1.222 million cedis. The optimal value of the decision variables are given by $x_1=0.00$, $x_2=1.050$, $x_3=1.167$, $x_4=0.233$ and $x_5=1.050$. Thus the bank should allocate 1.050 million cedis to Salary loans, 1.167 million cedis to microfinance loans, 0.233 million cedis to Susu loans and 1.050 million cedis to Commercial loans. Overdraft facility should be discarded.

4.3.4 Scenario 3: Six constraints.

If management wishes to include overdraft facility which had no allocations, it is therefore reasonable to add an additional constraint so that all loans have allocation Management will then have to add the requirement that the amount of loans allocated to overdraft facility at most 15% of the total loan portfolio. We then have $x_1 \leq 0.525$

Adding this new constraint to the problem and resolving we obtain the optimal solution shown below.

Maximize $0.35x_1+0.28x_2+0.35x_3+0.38x_4+0.41x_5$

Subject to

- 1) $1x_1+1x_2+1x_3+1x_4+1x_5 \leq 3.5$
- 2) $1x_1+1x_2 \geq 1.05$
- 3) $1x_3+1x_4+1x_5 \geq 2.1$
- 4) $0.21x_1-0.01x_2+0.09x_4-0.01x_5 \leq 0$
- 6) $1x_1 \leq 0.525$

The solution is given in Table 4.6 below

Table 4.6 Solution for six constraints

Variable	Value
X ₁	0.000
X ₂	1.050
X ₃	1.167
X ₄	0.233
X ₅	1.050

Objective Function Value = 1.222

Sensitivity analysis was done and table 4.7 gives the objective function coefficients ranges corresponding to the maintenance of the solution given in Table 4.6.

Table 4.7 OBJECTIVE COEFFICIENT RANGES

Variable	Lower Limit	Current Value	Upper Limit
X ₁	No Lower Limit	0.350	0.353
X ₂	0.277	0.280	0.410
X ₃	No Lower Limit	0.350	0.351
X ₄	0.379	0.380	No Upper Limit
X ₅	0.347	0.410	No Upper Limit

The objective function value section shows that the optimal solution to the problem will provide a maximum profit of 1.222 million cedis. The optimal value of the decision variables are given

by $x_1=0.00$, $x_2=1.050$, $x_3=1.167$, $x_4=0.233$ and $x_5=1.050$. Thus the bank should allocate 1.050 million cedis to Salary loans, 1.167 million cedis to microfinance loans, 0.233 million cedis to Susu loans and 1.050 million cedis to Commercial loans. Overdraft facility should be discarded.

4.3.5 Scenario 4: Increased interest rates for the most risky product.

Management wishes to include overdraft in the overall allocations so we increase the interest rates of the most risky product, overdraft by one percent each and resolving we obtain the optimal solution shown below.

Maximize $0.36x_1+0.28x_2+0.35x_3+0.38x_4+0.41x_5$

Subject to

- 1) $1x_1+1x_2+1x_3+1x_4+1x_5 \leq 3.5$
- 2) $1x_1+1x_2 \geq 1.05$
- 3) $1x_3+1x_4+1x_5 \geq 2.1$
- 4) $0.21x_1-0.01x_2+0.09x_4-0.01x_5 \leq 0$
- 5) $1x_3+1x_4 \geq 1.4$
- 6) $1x_1 \leq 0.525$

The solution is given in Table 4.8 below

Table 4.8 Increased interest rates for the most risky product.

Variable	Value
x_1	0.095
x_2	0.955
x_3	1.400

X ₄	0.000
X ₅	1.050

Objective Function Value = 1.222

Sensitivity analysis was done and table 4.9 gives the objective function coefficients ranges corresponding to the maintenance of the solution given in Table 4.8.

Table 4.9 OBJECTIVE COEFFICIENT RANGES

Variable	Lower Limit	Current Value	Upper Limit
X ₁	0.353	0.360	No Upper Limit
X ₂	No Lower Limit	0.280	0.287
X ₃	0.347	0.350	0.414
X ₄	No Lower Limit	0.380	0.383
X ₅	0.346	0.410	No Upper Limit

The objective function value section shows that the optimal solution to the problem will provide a maximum profit of 1.222 million cedis. The optimal value of the decision variables are given by $x_1=0.095$, $x_2=0.955$, $x_3=1.400$, $x_4=0.000$ and $x_5=1.050$. Thus the bank should allocate 0.095 million cedis to Overdraft loans, 0.955 to Salary loans, 1.400 million cedis to microfinance loans and 1.050 million cedis to Commercial loans. Susu loans should be discarded.

4.3.6 Scenario 5: Increased interest rate for the least risky product

We increase the interest rates of the least risky product and resolving we obtain the optimal solution shown below.

Maximize $0.35x_1 + 0.29x_2 + 0.35x_3 + 0.38x_4 + 0.41x_5$

Subject to

- 1) $1x_1 + 1x_2 + 1x_3 + 1x_4 + 1x_5 \leq 3.5$
- 2) $1x_1 + 1x_2 \geq 1.05$
- 3) $1x_3 + 1x_4 + 1x_5 \geq 2.1$
- 4) $0.21x_1 - 0.01x_2 + 0.09x_4 - 0.01x_5 \leq 0$
- 5) $1x_3 + 1x_4 \geq 1.4$
- 6) $1x_1 \leq 0.525$

The solution is given in Table 4.14 below

Table 4.10 Increased interest rates for the least risky product.

Variable	Value
x_1	0.000
x_2	1.050
x_3	1.167
x_4	0.233
x_5	1.050

Objective Function Value = 1.232

Sensitivity analysis was done and table 4.11 gives the objective function coefficients ranges corresponding to the maintenance of the solution given in Table 4.10.

Table 4.11 OBJECTIVE COEFFICIENT RANGES

Variable	Lower Limit	Current Value	Upper Limit
x_1	No Lower Limit	0.350	0.363

x_2	0.277	0.290	0.410
x_3	No Lower Limit	0.350	0.355
x_4	0.375	0.380	No Upper Limit
x_5	0.347	0.410	No Upper Limit

The objective function value section shows that the optimal solution to the problem will provide a maximum profit of 1.232 million cedis. The optimal value of the decision variables are given by $x_1=0.00$, $x_2=1.050$, $x_3=1.167$, $x_4=0.233$ and $x_5=1.050$. Thus the bank should allocate 1.050 million cedis to Salary loans, 1.167 million cedis to microfinance loans, 0.233 million cedis to Susu loans and 1.050 million cedis to Commercial loans. Overdraft facility should be discarded.

4.3.7 Scenario 6: Reduced mandates- Two Constraints

We reduce the mandates to two and maintain the interest rates of the products and resolving we obtain the optimal solution shown below.

Maximize $0.35x_1+0.28x_2+0.35x_3+0.38x_4+0.41x_5$

Subject to

$$1) \ 1x_1+1x_2+1x_3+1x_4+1x_5 \leq 3.5$$

$$2) \ 1x_1+1x_2 \geq 1.05$$

The solution is given in Table 4.22 below

Table 4.12 Reduced mandates for the products-Two Constraints

Variable	Value
x_1	1.050

X ₂	0.000
X ₃	0.000
X ₄	0.000
X ₅	2.450

Objective Function Value = 1.372

Sensitivity analysis was done and table 4.13 gives the objective function coefficients ranges corresponding to the maintenance of the solution given in Table 4.12.

Table 4.13 OBJECTIVE COEFFICIENT RANGES

Variable	Lower Limit	Current Value	Upper Limit
x ₁	0.350	0.350	0.410
x ₂	No Lower Limit	0.280	0.410
x ₃	No Lower Limit	0.350	0.410
x ₄	No Lower Limit	0.380	0.410
x ₅	0.380	0.410	No Upper Limit

The objective function value section shows that the optimal solution to the problem will provide a maximum profit of 1.372 million cedis. The optimal value of the decision variables are given by $x_1=1.050$, $x_2=0.000$, $x_3=0.000$, $x_4=0.000$ and $x_5=2.450$. Thus the bank should allocate 1.050 million cedis to Overdraft, 1.167 million cedis and 2.450 million cedis to Commercial loans. Susu loans, Microfinance and Salary loans should be discarded.

4.3.8 Scenario 7: Reduced mandates-One Constraint

We reduce the mandates to one and maintain the interest rates of the products and resolving we obtain the optimal solution shown below.

Maximize $0.35x_1 + 0.28x_2 + 0.35x_3 + 0.38x_4 + 0.41x_5$

Subject to

$$1) \quad 1x_1 + 1x_2 + 1x_3 + 1x_4 + 1x_5 \leq 3.5$$

The solution is given in Table 4.24 below

Table 4.14 Reduced mandates for the products-One Constraint

Variable	Value
x_1	0.000
x_2	0.000
x_3	0.000
x_4	0.000
x_5	3.500

Objective Function Value = 1.435

Sensitivity analysis was done and table 4.15 gives the objective function coefficients ranges corresponding to the maintenance of the solution given in Table 4.14.

Table 4.15 OBJECTIVE COEFFICIENT RANGES

Variable	Lower Limit	Current Value	Upper Limit
x_1	No Lower Limit	0.350	0.410
x_2	No Lower Limit	0.280	0.410
x_3	No Lower Limit	0.350	0.410
x_4	No Lower Limit	0.380	0.410
x_5	0.380	0.410	No Upper Limit

The objective function value section shows that the optimal solution to the problem will provide a maximum profit of 1.435 million cedis. The optimal value of the decision variables are given by $x_1=0.000$, $x_2=0.000$, $x_3=0.000$, $x_4=0.000$ and $x_5=3.500$. Thus the bank should allocate all the funds to Commercial loans and discard the rest of the products.

4.3.9 Scenario 8: Reduced mandates-One Constraint and increased interest rates

We reduce the mandates to one and increase the interest rates of the products and resolving we obtain the optimal solution shown below.

Maximize $0.35x_1+0.30x_2+0.35x_3+0.38x_4+0.41x_5$

Subject to

$$1) \quad 1x_1+1x_2+1x_3+1x_4+1x_5 \leq 3.5$$

The solution is given in Table 4.26 below

Table 4.16 Reduced mandates for the products-One Constraint

Variable	Value
x_1	0.000
x_2	0.000
x_3	0.000
x_4	0.000
x_5	3.500

Objective Function Value = 1.435

Sensitivity analysis was done and table 4.17 gives the objective function coefficients ranges corresponding to the maintenance of the solution given in Table 4.16.

Table 4.17 OBJECTIVE COEFFICIENT RANGES

Variable	Lower Limit	Current Value	Upper Limit
x_1	No Lower Limit	0.350	0.410
x_2	No Lower Limit	0.280	0.410
x_3	No Lower Limit	0.350	0.410
x_4	No Lower Limit	0.380	0.410
x_5	0.380	0.410	No Upper Limit

The objective function value section shows that the optimal solution to the problem will provide a maximum profit of 1.435 million cedis. The optimal value of the decision variables are given by $x_1=0.000$, $x_2=0.000$, $x_3=0.000$, $x_4=0.000$ and $x_5=3.500$. Thus the bank should allocate all the funds to Commercial loans and discard the rest of the products.

CHAPTER FIVE

CONCLUSION AND RECOMMENDATIONS

5.1 SUMMARY OF FINDINGS

Below, in tables 5.1, 5.2 and 5.3 is a summary of all the results obtained in the last chapter.

Table 5.1 Summary of the Scenarios with Original mandate

Scenario	Interest Rate	Objective Function value	No of allocations
One	Remains the same	1.310	3
Two	Remain the same	1.222	4
Three	Remains the same	1.222	4

Table 5.2 Summary of Scenarios with increased interest rate for most and least risky products respectively

Scenario	Interest Rate	Objective Function value	No of allocations
Four	One percent Increase for most risky product	1.222	4
Five	One percent Increase for least risky product	1.232	4

Table 5.3 Summary of the Scenarios with reduced mandates

Scenario	Interest Rate	Objective Function value	No of allocations
Six	Interest remains same	1.372	2
Seven	Interest remains same	1.435	1
Eight	Two percent increase in salary loan	1.435	1

Looking at tables 5.1, 5.2 and 5.3, and the scenarios given it can be observed that the scenario with only one constraint produced the highest profit margin but it has only one allocation which will not be conducive for the clients in the catchment area as other customer's businesses are likely to suffer if management thinks of profit alone.

To satisfy most of the clients while making good profits the other scenarios one to five produced four allocations with good profits. Considering the one with original mandate produced a profit of 1.310 million cedis but only three allocations. Susu and microfinance loans were left out because of their comparatively low interest rates and their high risks. One constraint was added and the profit dropped to 1.222 million cedis with four allocations and additional constraint

produced the same profit and same allocations. This case, overdraft facility was left out because of its high interest rate and high risk.

An increase in interest rate for the more risky products were employed to see if we get allocations for all the products but it remained four allocations with an increase in profits with increasing interest rates microfinance being left out because of its relatively high risk. Again, an increase in the interest rate for the least risky product resulted in four allocations and a slight increase in the overall profits. Overdraft was omitted so this suggest that if we increase the interest rates of the less risky products it will give more profit and that can be used to compensate for the losses to be incurred in the most risky product.

Therefore we suggest management considers the ones with six constraints which have good profits and four allocations each. This shows that if Management is interested in profit only then all other mandates and policies must be dropped. Obviously, management could implement either of the solutions shown above and maximize profit.

5.2 CONCLUSION

We formulated and solved the bank's model, and illustrated how linear programming involving a number of scenarios can be used to assist in the decision-making process. The bank offers Overdraft facilities, Salary loan, Susu loan, Distance loans and Commercial loans. The most profitable loan to the bank is Susu loan.

Also there is a positive relationship between the risk and the expected return of a financial asset. In other words, when the risk of an asset increases, so does its expected return. What this means is that if an investor is taking on more risk, he/she is expected to be compensated for doing so with a higher return. Similarly, if the investor wants to boost the expected return of the

investment, he/she needs to be prepared to take on more risk. This was seen when the interest rate of salary loans was increased. It could also be observed that in all scenarios some of the products were picked and had high values and this is because of the high interest rate and low risk.

Our model shows that if the Bank adapts to the model they will be able to make very good profits on loans alone. I therefore conclude that the scientific method used to develop our proposed model can have a dramatic increase in the profit margin of the Bank should they adapt to it. I also conclude that more loans be granted to Susu, microfinance, salary and Commercial loans and little percentage or nothing is allocated to Overdraft. This will increase our credit margins and consequently reduce loan losses.

5.3 RECOMMENDATIONS

It is recommended that First Allied Savings and Loans limited should improve upon its profitability by advancing more loans to customers on reasonable terms in view of the competitive nature of banking business in recent times and management should exercise appropriate control over expenditure.

It is recommended that the bank should continue its program to increase its deposit mobilization to expand its operation.

Again it is recommended that Management should consider increasing the interest rates of salary loans and overdraft.

It is recommended that a risk department is also set up to work on the defaulters alone.

Lastly, it is recommended that apart from loan disbursement, banks and other financial institutions should employ scientific methods and mathematical methods in most of the businesses they conduct.

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APPENDIX 1 DEFINITION OF SOME TERMS

Constraint An equation or inequality that rules out certain combinations of decision variables as feasible solutions.

Constraint function The left-hand side of a constraint (i.e.,the portion of the constraint containing the variables).

Objective function All linear programs have a linear objective function that is to be either maximized or minimized.It is used to measure the profit or cost of a particular solution.

Solution Any set of values for the variables.

Optimal solution A feasible solution that maximizes or minimizes the value of the objective function.

Nonnegativity constraints A set of constraints that requires all variables to be nonnegative.

Mathematical model A representation of a problem where the objective and all the constraint conditions are described by mathematical expressions.

Linear program A mathematical model with a linear objective function,a set of linear constraints and nonnegative variables.

Linear functions A mathematical expressions in which the variables appear in separate terms and are raised to the first power.

Feasible solution A solution that satisfies all the constraints

Feasible region The set of all feasible solutions

Slack variable A variable added to the left-hand side of a less-than or equal to constraint to convert the constraint into an equality.

Unboundedness A maximization linear programming problem is said to be unbounded if the value of the solution may be made infinitely large without violating any of the constraints.

Sensitivity analysis The evaluation of how changes in the coefficients of a linear programming problem affect the optimal solution to the problem.

Range of optimality The range of values over which an objective function coefficient may vary without causing any change in the values of the decision variables in the optimal solution.

Dual price The improvement in the value of the optimal solution per unit increase in a constraint right-hand-side value.

Degeneracy When one or more of the basic variables have a value of zero.

Simplex tableau A table used to keep track of the calculations made when the simplex method is employed.

