KWAME NKRUMAH UNIVERSITY OF SCIENCE AND TECHNOLOGY, KUMASI, GHANA

COLLEGE OF SCIENCES

INSTITUTE OF DISTANCE LEARNING

DEPARTMENT OF MATHEMATICS

APPLICATION OF ASSIGNMENT PROBLEM (WORKFORCE OPTIMIZATION) IN AWARDING JOBS/PROJECTS IN A BIDING TENDER: A CASE STUDY OF SHAMA SENIOR HIGH SCHOOL

THESIS SUBMITTED TO THE INSTITUTE OF DISTANCE LEARNING, KWAME NKRUMAH UNIVERSITY OF SCIENCE AND TECHNOLOGY IN PARTIAL FULFILMENT OF THE REQUIREMENTS FOR THE DEGREE OF MASTER OF SCIENCE IN INDUSTRIAL MATHEMATICS

BY

OKRAH, STEPHEN KWAKU

(PG6323011)

BSC. MATHEMATICS AND STATISTICS (HONS.)

SUPERVISOR:

PROF. S.K. AMPONSAH

APRIL, 2013

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DECLARATION

I hereby declare that this submission is my own work towards the Master of Science degree and that to the best of my knowledge it contains no material previously published by another person nor material which has been accepted for award of any other degree of the university except where due acknowledgement has been made in the text.

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ACKNOWLEDGEMENT

I would like to give thanks to the Almighty God for granting me the strength and knowledge for understanding this course and the completion of this write-up.

I am very grateful to my supervisor, Professor. S. K. Amponsah of the Department of Mathematics, who painstakingly read through every line of the text and offered through his rich experience all the necessary encouragement, direction, guidance, advice and correction for the timely completion of this thesis.

Special mention should be made of my dear guardian, lawyer Augustine Aboagye-Atta Nkrumah, for bringing me this far, through his good counseling and financial support. He has indeed been very helpful to me, and I am grateful.

In sincere appreciation to their ability to endure hardship only to see me through my education, I wish to thank my dear mother, Mad. Veronica Kyebi, my siblings, Mr. Winfred Kofi Amegah Krah, Mr. Richard Antwere, Mad. Diana Afari, Mr. Martin Ntumpah as well as my inspirers, Mr. Kizito Essandoh, Mr. Nathaniel Cobbinah, Mr. Justice Kangah and Mr. Stephen Adjei, not forgetting my late friend, Mr. Michael Kpentey who accommodated me when I was offering this course. May the good Lord grant his soul everlasting rest in peace. Amen.

Finally my sincere thanks go to all who in diverse ways helped in bringing this project to a successful end.God richly bless you all.

DEDICATION

This work is dedicated to my lovely children, Stephen and Vanessa as well as the 'Evergreen Memory' of my father, the late, Mr. S. K. Amegah.

ABSTRACT

Matching highly skilled people to available positions/jobs is a high-stakes task that requires careful consideration by experienced resource managers. A wrong decision may result in significant loss of value due to under-staffing, under-qualification or over-qualification of assigned personnel, and high turnover of poorly matched workers. While the importance of quality matching is clear, dealing with pools of hundreds of jobs and resources in a dynamic market generates a significant amount of pressure to make decisions rapidly. We present a novel solution designed to bridge the gap between the need for high-quality matches and the need for timeliness. This thesis models Shama Senior High School tender - project (Job) assignment problem as an Assignment Problem and solve the problem. The model developed could be adopted for any problem that can be modeled as an assignment problem. The objectives of the study were to: (i) mathematically model Shama Senior High School Tender – project/job assignment problem and solve the problem; (ii) minimize total cost of executing the projects. The Hungarian method was employed to solve the problem. First the algorithm was presented along with relevant examples. A real life problem was analyzed with QM Optimization Software employed to analyze the data obtained from the school. This thesis was organized in to five chapters. Also, related works that have been done in the area of Assignment Problem as well as mathematical theories on Assignment Problems and how they have been solved were reviewed. In the final analysis, it was observed that the total minimum cost of executing the six projects that will maximize total educational quality in the school, or the minimum cost of assigning a tender to a project was GH¢548,000.00. The researcher, therefore, recommended that the assignment problem model should be

adopted by the school and other schools as well as other institutions and agencies for tender-project placement and selection planning.

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CHAPTER 1

1.0 OVERVIEW

Successful organization can be judged much like an engine; the working parts are simply ineffective if they don't fit into place. Similarly, placing the wrong people into mission-critical positions can be expensive and detrimental to corporate success and organizational growth. Maintaining and improving the effectiveness and efficiency of your workforce is essential to the growth and success of any business, hence the reason for solving the assignment problem. This part of the study will look at the introduction, background of the study, statement of the problem, justification and scope of the study as well as the limitations and organization of the thesis.

1.1 INTRODUCTION

In today's competitive business environment, there is a greater need for organizations to manage their operations efficiently to achieve competitive advantage in every part of the organization.

Workforce optimization (Assignment problem) helps companies improve the efficiency and effectiveness of their customer interactions through solutions that capture interactions across all channels, analyze to reveal insights, and drive actions that impact business results and customer experience. Workforce Optimization (Assignment problem) solutions include capabilities for call recording, agent evaluation and coaching, performance management, forecasting and scheduling, interaction analytics and feedback surveys. These capabilities are applied in solutions that enable businesses to harness the data from customer interactions to increase efficiency, improve sales efforts and ultimately improve client experience.

Human resource is the core of any organization as a large part of an organization's workforce is directly involved in running key operations, e.g. labour in manufacturing operations, sales representatives in servicing customers, medical representatives in visiting doctors, contact center employees in handling customer interactions, airline crew in servicing various flight legs, etc. Efficiently managed workforce ultimately adds to the productivity, and thus bottom line of the organization.

The goal of any workforce application is to cover the workload with the resource available, while respecting work constraints, balancing the workload among employees, and minimizing idle time. There are major benefits of better workforce allocation and scheduling such as reduced cost, higher efficiencies, improved customer service, and higher employee satisfaction. For example, any organization with better workforce management can achieve improved operations efficiencies, optimized use of resources, and expanded employee's base. Whereas departure from optimality can make organization:

- Drain its limited budget through high costs-per-call
- Miss opportunities to build revenues
- Reduce customer confidence in their client company and increase customer churn

Workforce planning and scheduling has its applications in many other areas such as:

(i) Project/job/contract allocation

(ii) Staff allocation in departmental stores

(iii) Nurse roistering in hospitals

(iv) Ground staff allocation in airline operations

(v) Work pattern allocation in banks

(vi) Time Tabling of classes and lectures in a university

(vii) Reservation Systems in Airlines, and Railways

(viii) Car, Train, and Bus drivers' route allocation, and scheduling in public transportation

(ix) Shift allocation, and scheduling of labour in manufacturing units, and

(x) Training Schedule preparation.

Workforce optimization problems are highly combinatorial and heavily constrained. No classic algorithm can answer all the constraints involved in workforce optimization such as heavy regulations, economic goals, union rules, individual wishes, staff availability, and skill requirements.

In workforce optimization problems, job requirements such as employee skills, job duration, and job priority are grouped together to give the workload. The activities or tasks of a job need to be scheduled over specific periods of time in order to meet customer requirements. Some of the requirements are quite complex (e.g., skill requirements, various work rules such as shift start times, lengths, break windows, durations, etc.). This correlation of requirements over time often implies sequencing effects that can dramatically affect the cost of jobs.

Manually creating a workforce schedule can be tedious and time-consuming. Researching the countless number of configurations of shifts and breaks becomes quite difficult when the size of the scheduled group is 20 or more, and often best answers and scenario analysis is impossible to find. Workforce solutions can help in coming up with best answers by generating and evaluating all relevant scenarios. This is why optimization model is a key to workforce scheduling and allocation applications. To address workforce-planning problems, a system must take into account the company needs, constraints and objectives, and provide different objects, planning, schedule checking, and reactive planning.

Better workforce planning and scheduling results in increased workforce productivity, feedback and support from the staff, and involvement of support organizations (engineering, stores, operations, etc.), which further results in the improved overall reliability and efficiency of the organization.

In this chapter of the study, an overview of assignment problem would be given; a brief description of the problem statement of the study is also presented together with the objectives, the methodology, the justification, scope, limitation and the organization of the study.

1.2 BACKGROUNDOFTHESTUDY

Workforce planning and scheduling (assignment) entails anticipating supply availability and job requirements in order to have the right people, with the right skill, at the right time, in the right place, at the right cost. The problem of matching future availability of employees with future job requirements is quite complex since hundreds of thousands of employees with thousands of skills are distributed in countries all over the world. The other challenge of matching future availability of employees with future job requirements is that supply and demand is uncertain. Finally, there are no clear costs when matching an employee to a job. There is the problem of discriminating goodmatches from badmatches automatically when matching future availability of employees with future job requirements. In practice, first-line managers know their employees and can assign the right employee to the right job, a good match; however at most institutions like Ghana Education Service (GES) where we have hundreds of thousands of employees and jobs one will like to formulate a model that automates the discrimination of good matches from bad matches.

The assignment problem is a special type of linear programming problem where assignees are being assigned to perform tasks. For example, the assignees might be employees who need to be given work assignments. Assigning people to jobs is a common application of the assignment problem. However, the assignees need not be people. They also could be machines, or vehicles, or plants, or even time slots to be assigned tasks. To fit the definition of an assignment problem, these kinds of applications need to be formulated in a way that satisfies the following assumptions.

(i) The number of assignees and the number of tasks are the same. (This number is denoted by n).

(ii) Each assignee is to be assigned to exactly one task.

(iii) Each task is to be performed by exactly one assignee.

(iv) There is a cost c_{ij} associated with assignee i(i= 1, 2, . . . ,n) performing task j(j = 1, 2, . . . , n).

(v) The objective is to determine how all n assignments should be made to minimize the total cost.

Assignment model comes under the class of linear programming model, which looks alike with transportation model with an objective function of minimizing the time or cost of manufacturing the products by allocating one job to one machine or one machine to one job or one destination to one origin or one origin to one destination only. Basically assignment model is a minimization model.

The basic objective of an assignment problem is to assign n number of resources to n number of activities so as to minimize the total cost or to maximize the total profit of allocation in such a way that the measure of effectiveness is optimized. The problem of assignment arises because available resources such as men, machines, etc., have varying degree of efficiency for performing different activities such as job. Therefore, cost, profit or time for performing the different activities is different. Hence the problem is how should the assignment be made so as to optimize (maximize or minimize) the given objective. The assignment model can be applied in many decision-making processes like determining optimum processing time in machine operators and jobs, effectiveness of teachers and subjects, designing of good plant input, etc. This technique is found suitable for routing travelling salesman to minimize the total travelling cost, or to maximize the sales.

The assignment problem is a special case of the transportation problem where all sources and demand are equal to 1. The assignment problems are of two types: (i) balanced and (ii) unbalanced. If the number of row is equal to the number of columns or if the given problem is a square matrix, the problem is termed as a balanced assignment problem. If the given problem is not a square matrix, the problem is termed as an unbalanced assignment problem. If the problem is an unbalanced one, we add dummy rows/columns as required so that the matrix becomes a square matrix or a balanced one. The cost or time values for the dummy cells are assumed as zero.

1.3 STATEMENT OF THE PROBLEM

As a result of the four-year Senior High School program, there is the need for additional classrooms and dormitories in second circle schools in the country of which Shama Senior High School is of no exception. The school also has no sports field for undertaking sporting activities and had to move out of campus to do sporting activities. The toilet facility in the school can no longer support the growing population of the school. Most of the staff members also live far from the school and as a result come to school late. Inadequate energy supply is also a worry to the school since it affects academic work.

In view of these problems, the government through the District Assembly, the school, Parent- Teacher Association and Student Representative Council are awarding these six projects on contract and six tenders have placed their bids to provide the additional classrooms, dormitories, water closet toilet, school sports field, staff bus and a power generating plant.

The problem that this study seeks to find solution to is how to assign the six tenders to one project each in Shama Senior High School in order to minimize total cost and maximize quality of education in the school.

1.4 THE OBJECTIVES OF THE STUDY

The objectives of the study are to:

(i) mathematically model Shama Senior High School Tender - Job (Project) Assignment problem and solve the problem

(ii) minimize total cost of executing the projects by using the Hungarian algorithm so as to maximize quality of education in the school

1.5 METHODOLOGY OF THE STUDY

The Hungarian method shall be employed to solve the problem. First, the algorithm is presented along with relevant examples. A real-life problem will be analyzed. QM Optimization Software shall be employed to analyze the data.

1.6 JUSTIFICATION OF THE STUDY

Successful organization can be judged much like an engine; the working parts are simply ineffective if they do not fit into place. Similarly, placing the wrong people into mission-critical positions can be expensive and detrimental to corporate success and organizational growth. Matching highly skilled people to available jobs is a high-stakes task that requires careful consideration by experienced resource managers. A wrong decision may result in significant loss of value due to understaffing, under-qualification or over-qualification of assigned personnel, and high turnover of poorly matched workers. Maintaining and improving the effectiveness and efficiency of your workforce is essential to the growth and success of any business, hence the reason for solving the assignment problem.

1.7 SCOPE OF THE STUDY

Human resource is the core of any organization as a large part of an organization's workforce is directly involved in running key operations. The scope of the study is Shama Senior High School in the Western Region of Ghana. However, the main focus of this research work was the problem of workforce (job) assignment and management in the Shama Senior High School and also the effect of the projects on performance of the students.

1.8 LIMITATION OF THE STUDY

The study is limited to tenders performance in the project/job assigned to them in Shama Senior High School. This is to make the study manageable given the time and resources available to the researcher to complete the study. The study was also, limited to the perceived effect of inadequate classrooms, dormitories, school sports field, water closet toilet, staff bus and power generating plant on the students' performance in Shama Senior High School.

1.9 ORGANIZATION OF THE THESIS

In Chapter 1, we present the background, the problem statement and objectives of the study. The methodology, justification and limitation of the study were also put forward.

In Chapter 2, related works on Assignment Problem would be discussed.

Chapter 3 is devoted for the research methodology of the study.

Chapter 4 presents data collection and analyses of the study.

Chapter 5 which is the final chapter of the study, presents the conclusion and recommendations of the study.

1.9* SUMMARY

The Assignment Problem is a special case of the transportation problem, where the objective is to minimize the cost or time of completing a number of jobs by a number of persons and Maximize efficiently Revenue, Sales etc. In other words, when the problem involves the allocation of n different facilities to n different tasks, it is often termed as an Assignment Problem. This model is mostly used for planning. The assignment model is also useful in solving problems such as, assignment of machines to jobs, assignment of salesman to sales territories, travelling salesman problem etc. It may be noted that with n facilities and n jobs, there are n! possible assignments. One way of finding an optimal assignment is to write all the n! Possible arrangement, evaluate their total cost and select the assignment with minimum cost. But because of many computational procedures this method is not possible.

The goal of this study is to mathematically model the tender - job placement problem of Shama Senior High School as an assignment problem and solve the problem.

In the next chapter, we shall put forward relevant literature on assignment problems.

CHAPTER 2

LITERATURE REVIEW

2.0 INTRODUCTION

This chapter primarily aimed at reviewing related works that have been done in the area of Workforce Optimization (Assignment problem) as well as Mathematical theories on Assignment Problems and how they have been solved. The researcher will also, seek to understand the meaning and concepts of the Assignment Problem through a review of a selection of materials, books and journals which are directly related to the topic of study. This part of the study will look at the historical and global perspective of the Assignment Problem.

2.1 HISTORICAL AND GLOBAL PERSPECTIVE OF THE ASSIGNMENT PROBLEM

In the standard version of the Assignment Problem, there are no restrictions on possible matching of the bipartite graph. That is, any vertex from X can be matched with a vertex in Y without any restriction. The only objective is to maximize the weights on edges. However, personnel assignment problem is a real-life problem encountered in hierarchical organizations such as military. In such a real-life problem, it is natural to expect constraints on possible matching. For example, as a military organization, the Turkish Armed Forces assign thousands of personnel to vacant positions every year, and want to utilize the personnel to a maximum extent by assigning the right person to the right job, while taking into consideration the hierarchy constraint, (Dinc andOguztuzun, 1998,Cimen (2001).

Toroslu (2003) studied a model having personnel ranks and positional hierarchy structure in hand. This version of the assignment problem is known as the Assignment Problem with Hierarchical Ordering Constraint (APHOC). In this natural variation of the standard assignment problem, the objective remains the same: finding a perfect matching on the bipartite graph that maximizes the weights. The actual distinction in this version is introduced by the single constraint of minimizing the number of hierarchical violations. In his work, Toroslu proves the NP-completeness of this variation of the problem and proposes an efficient approximation algorithm.

During its operational test and evaluation, despite top management support and significant technical achievements, a personnel assignment model implemented for the United States Navy to support assignment decisions experienced overwhelming resistance from the users, the 200 or so enlisted detailers, located at the Bureau of Naval Personnel in Washington, D.C. The research team had neglected to assess the negative impact of the personnel assignment model on an important detailing function: assignment negotiations or bargaining between the detailers and their customers, the service members. By involving the detailers in revising the model and making the failings of the old model the strengths of the new model, Thomas, H. (1991), turned certain failure into a successful program. By managing the behavioral aspects of the implementation with special emphasis on problem identification and requirements structuring, we overcame the difficulties of introducing change to a largely manual and highly decentralized decision process and we compare lessons learned with the experiences of other implementers.

The assignment problem is a well-known graph optimization problem defined on weighted-bipartite graphs. The objective of the standard assignment problem is to maximize the summation of the weights of the matched edges of the bipartite graph. In the standard assignment problem, any node in one partition can be matched with any node in the other partition without any restriction.

Toroslu and Arslanoglu (2012), presented variations of the standard assignment problem with matching constraints by introducing structures in the partitions of the bipartite graph, and by defining constraints on these structures.

According to the first constraint, the matching between the two partitions should respect the hierarchical-ordering constraints defined by forest and level graph structures produced by using the nodes of the two partitions respectively. In order to define the second constraint, the nodes of the partitions of the bipartite graph are distributed into mutually exclusive sets. The set-restriction constraint enforces the rule that in one of the partitions all the elements of each set should be matched with the elements of a set in the other partition. Even with one of these constraints the assignment problem becomes an NP-hard problem. The extended assignment problem with both the hierarchical-ordering and set-restriction constraints becomes an NP-hard multi-objective optimization problem with three conflicting objectives; namely, minimizing the numbers of hierarchical-ordering and set-restriction violations, and maximizing the summation of the weights of the edges of the matching.

Genetic algorithms are proven to be very successful for NP-hard multi-objective optimization problems. The authors also proposed genetic algorithm solutions for different versions of the assignment problem with multiple objectives based on hierarchical and set constraints, and empirically showed the performance of these solutions.

Matching highly skilled people to available positions is a high-stakes task that requires careful consideration by experienced resource managers. A wrong decision may result in significant loss of value due to understaffing, under qualification or over qualification of assigned personnel, and high turnover of poorly matched workers. While the importance of quality matching is clear, dealing with pools of hundreds of jobs and resources in a dynamic market generates a significant amount of pressure to make decisions rapidly.

Naveh et al., (2007), presented a novel solution designed to bridge the gap between the need for high-quality matches and the need for timeliness. By applying constraint programming, a subfield of artificial intelligence, the authors dealt successfully with the complex constraints encountered in the field and reach near-optimal assignments that take into account all resources and positions in the pool. The considerations include constraints on job role, skill level, geographical location, language, potential retraining, and many more. Constraints were applied at both the individual and team levels. The authors introduced a technology and then describe its use by IBM Global Services, where large numbers of service and consulting employees are considered when forming teams assigned to customer projects.

Katta and Jay (2005) presented the problem of allocating a set of indivisible objects to agents in a fair and efficient manner. In a recent paper, Bogomolnaia and Moulin (2001) considered the case in which all agents have strict preferences, and proposed the Probabilistic Serial (PS) mechanism; they define a new notion of efficiency, called ordinal efficiency, and prove that the probabilistic serial mechanism finds an envy-free ordinarily efficient assignment. However, the restrictive assumption of strict preferences is critical to their algorithm. The author’s main contribution was an analogous algorithm for the full preference domain in which agents are allowed to be indifferent between objects. The author's algorithm was based on a reinterpretation of the PS mechanism as an iterative algorithm to compute a flow in an associated network. In addition the authors showed that on the full preference domain it is impossible for even a weak strategy proof mechanism to find a random assignment that is both ordinarily efficient and envy-free.

The assignment problem constitutes one of the fundamental problems in the context of linear programming. Besides its theoretical significance, its frequent appearance in the areas of distributed control and facility allocation, where the problems' size and the cost for global computation and information can be highly prohibitive, gives rise to the need for local solutions that dynamically assign distinct agents to distinct tasks, while maximizing the total assignment benefit.

Michael et al., (1979) considered the linear assignment problem in the context of networked systems, where the main challenge is dealing with the lack of global information due to the limited communication capabilities of the agents. The authors addressed this challenge by means of a distributed auction algorithm, where the agents are able to bid for the task to which they wish to be assigned. The desired assignment relies on an appropriate selection of bids that determine the prices of the tasks and render them more or less attractive for the agents to bid for. Up to date pricing information, necessary for accurate bidding, can be obtained in a multi-hop fashion by means of local communication between adjacent agents. The author's algorithm was an extension to the parallel auction algorithm proposed by Bertsekas et al to the case where only local information is available and it is shown to always converge to an assignment that maximizes the total assignment benefit within a linear approximation of the optimal one.

Zhang and Jonathan (2002) studied a multi-period assignment problem that arises as part of a weekly planning problem at mail processing and distribution centers. These facilities contain a wide variety of automation equipment that is used to cancel, sort, and sequence the mail. The input to the problem is an equipment schedule that indicates the number of machines required for each job or operation during the day. This result is then post-processed by solving a multi-period assignment problem to determine the sequence of operations for each machine. Two criteria are used for this purpose. The first is to minimize the number of startups, and the second is to minimize the number of machines used per operation. The problem is modeled as a 0–1 integer program that can be solved in polynomial time when only the first criterion is considered. To find solutions in general, a two-stage heuristic is developed that always obtains the minimum number of startups, but not necessarily the minimum number of machines per operation. In a comparative study, high quality solutions were routinely provided by the heuristic in negligible time when compared to a commercial branch-and-bound code (Xpress). For most hard instances, the branch-and-bound code was not able to even find continuous solutions within acceptable time limits.

In a Processing and Distribution Center (PDC), the equipment is grouped into clusters of identical machines. As such, the scheduling model only provides the number of machines that should be running for each operation during the day, but does not specify the sequence of operations for each machine. To resolve this issue, the schedule must be post-processed by solving a multi-period assignment problem.

Aronson (1986) considered a version of this problem designed to minimize (i) the cost of assigning a person to an activity, and (ii) the cost of transferring a person from one job to another. The latter was assumed to be sequence dependent. The author presented an integer multi-commodity network flow model and developed a specialized branch and bound algorithm to find solutions. Instead of solving the linear programming relaxation, his idea was to solve a set of shortest path sub-problems and branch on jobs that were assigned to more than one machine.

Maxon and Bhadury (2001) studied a multi-period assignment problem with repetitive tasks and tried to introduce a human element into the analysis. Their objective was to minimize a combination of the assignment cost and the ‘‘boredom’’ that results when workers are required to repeat the same task in consecutive periods. A mathematical model was proposed and a simple iterative heuristic was developed and implemented in Excel.

The Airline Crew Assignment Problem (ACA) consists of assigning lines of work to a set of crew members such that a set of activities is partitioned and the costs for that assignment are minimized. Especially for European airline companies, complex constraints defining the feasibility of a line of work have to be respected.

Meinolf et al., (2002) developed two different algorithms to tackle the large scale optimization problem of Airline Crew Assignment. The first is an application of the Constraint Programming (CP) based on Column Generation Framework. The second approach performs a CP based on heuristic tree search. The authors presented how both algorithms could be coupled to overcome their inherent weaknesses by integrating methods from Constraint Programming and Operations Research. Numerical results show the superiority of the hybrid algorithm in comparison to CP based tree search and column generation alone.

“Traffic assignment is the process of allocating a set of present or future trip interchanges, known as Origin-destination (OD) demands, to a specified transportation network” (Easa, 1991). The results of traffic assignment model contribute in many transportation planning and design decisions such as evaluation of what if scenarios for different improvements, environmental and transportation impact analysis, highway design. Traffic assignment models evolved from system level approach to subarea-level approach involving the same elements but with different implementation details. In general the application of traffic assignment models consist of five basic elements including preparing the network, establishing the OD demands, identifying a traffic assignment technique, calibrating and validating a model, and forecasting (Easa, 1991).

If any model concentrates on capacity restraint as a generator of a spread of trips on a network, one should consider a different set of models which usually attempt, with different degrees of success, to approximate to the equilibrium conditions as described by

Wardrop (1952): “Under equilibrium conditions traffic arranges itself in congested networks in such a way that no individual trip maker can reduce his path costs by switching routes”. This principle focuses on the behaviour of individual drivers trying to minimize their own trip costs. Wardrop (1952) proposed an alternative way of assigning traffic onto a network and this is usually referred to as his second principle: “Under social equilibrium conditions traffic should be arranged in congested networks in such a way that the average (or total) travel cost is minimized”. This principle is aimed to achieve an optimum social equilibrium which helps transport planners and engineers trying to manage traffic to minimize travel costs.

Smith and Brennan (1980) investigated the performance of the heuristic assignment techniques currently available to transportation planners in the United States in terms of accuracy for small and medium-sized cities in order to assess the potential for future applications of equilibrium assignment techniques. The study revealed that using the percentage of root mean squared error as the primary accuracy measure/percentage of the accuracy of the assignments in the order of increasing accuracy was all-or-nothing, multipath, and capacity-restrained, and the accuracy of the capacity-restrained assignments appeared to be more sensitive to the assumptions made in computing the peak-hour assigned volumes and capacities than to differences in the capacity restraint techniques.

Meneguzzer (1998) considered the Stochastic User Equilibrium (SUE) assignment problem for a signal-controlled network in which intersection control is flow-responsive and the problem is addressed within a Combined Traffic Assignment and Control (CTAC) modeling framework, in which the calculation of user equilibrium link flows is integrated with the calculation of consistent signal settings. In this model, network users have limited information of travel times and signal control is traffic-actuated. This study solved SUE- based CTAC model algorithmically by means of the so- called Iterative Optimization and Assignment (IOA) procedure and defined a methodological framework for the evaluation of the performance of various traffic-responsive signal control strategies in interaction with different levels of user information, as represented by the spread parameter of the perceived travel time distribution assumed in the SUE assignment sub model.

Shafahi and Ramezani (2007) provided more flexibility to model driver characteristics which affect drivers' route choice by introducing a new fuzzy assignment model. The obtained result of this method is the same as UE results when there are risk-neutral motorists and/or deterministic travel time. The authors also derived mathematical fuzzy user equilibrium condition.

Theoretically, microscopic simulation models can be used to evaluate traffic management strategies in real time but this might not be computationally feasible for large-scale networks and complex simulation models. Chowdhury et al., (2006) presented two Artificial Intelligence (AI) paradigms—Support Vector Regression (SVR) and Case-Based Reasoning (CBR) as alternatives to the simulation models as a decision support tool. They developed two prototype decision support tools to evaluate the likely impacts of implementing diversion strategies in response to incidents on a highway network in Anderson, South Carolina. Then VISSIM, a microscopic simulation model is used to evaluate the performances of the two prototypes by comparing their predictions of traffic conditions.

Haphuoc et al., (2002) proposed an integrated model of modal split and Traffic Assignment, in which the interaction between transit vehicles and the general traffic is modeled explicitly. In this model they applied fuzzy reasoning instead of Logic model for traffic choice behaviour because fuzzy model can describe more precisely the traffic choice behavior compared to Logic model.

Sadek et al., (1998) put forward an architecture for a routing Decision Support System (DSS) based on two emerging artificial intelligence paradigms: case-based reasoning and stochastic search algorithms which is expected to allow the routing DSS to (*a*) process information in real time, (*b*) learn from experience, (*c*) handle the uncertainty associated with predicting traffic conditions and driver behavior, (*d*) balance the trade-off between accuracy and efficiency, and (*e*) deal with missing and incomplete data problems. However, the motivation of this study is to overcome the limitations of real-time traffic flow management.

Hu et al., (1961) aimed at developing simulation-based algorithm for dynamic traffic assignment problems under mixed traffic flow considerations of car, bus, motorcycle, and truck which consists of an inner loop that incorporates a direction finding mechanism for the search process for System Optimization (SO) and User Equilibrium (UE) classes based on the simulation results of the current iteration, including experienced vehicular trip times and marginal trip times. The authors conducted a survey in order to understand trip maker acceptance toward route guidance.

Moreover, the authors conducted numerical experiments in a test network to illustrate the capabilities of the proposed algorithm.

Henry et al., (2001) addressed the fact that the traffic network itself is probabilistic and uncertain and that different classes of travelers respond differently within this uncertain environment given different levels of traffic information considered in their proposed model by capturing the travelers' decision making among discrete choices in a probabilistic and uncertain environment in which both probabilistic travel times and random perception errors that are specific to individual travelers are considered. Travelers' route choices are assumed to be made with the objective of minimizing perceived disutility at each time which depends on the distribution of the variable route travel times, the distribution of individual perception errors and the individual traveler's risk- taking nature at each time instant.

Varia and Dhingra (2004) presented the development of Dynamic User Equilibrium (DUE) traffic assignment model for the congested urban road network with signalized intersections which adopted a simulation-based approach for the case of multiple-origin multiple-destination traffic flows and developed a modified Method of Successive Averages (MSA) to arrive at the user equilibrium condition.

An effective storage location assignment policy, in addition to its potential for optimal usage of warehouse space, reduces travel times related to storage, retrieval and order-picking activities. Moreover, helps to reduce congestions also enhances the balance among different warehouse activities. While previous research works exist regarding warehouse space allocation problem, considering modern concepts of logistics systems and also specific limitations for each case, further research in this respect is needed.

Sanei et al., (2011) studied the problem of space assignment for the products in a warehouse considering various operational constraints. These constraints are mainly set to prevent decentralization of products in storage locations considering more explicit and more exquisite inventory control. A linear integer programming model and a heuristic algorithm based on the branch and bound method is proposed to solve the problem. Further, software has been developed based on proposed algorithm for industrial usage. An experimental study, based on real data from an auto-industry shows the efficiency of the proposed algorithm achieving reasonable solutions.

Ashayeri and Golders (1985) studied warehouse assignment design and offered two ways to optimize design of the storage. The authors also presented a step design algorithm and design for warehouse. Also in literature a step method for the design of the warehouse and several solved examples, assumption for warehouse design, hierarchical design method were presented.

Muppant (2007) presented important factors for assigning storage and proposed a Meta-heuristic slow freezing algorithm that COI has been considered as a selected criterion for selecting the locations. The author proposed an algorithm based on branch and bound in order to assign places for storage.

Semih et al., (2008) presented a distribution-type warehouse assignment problem that various types of products were collected from different suppliers for storing in the warehouse for a determined period and for delivery to different customers. The aim of their study was to design a multiple-level warehouse shelf configuration which minimized the annual carrying costs. Since proposed mathematical model was shown to be NP-hard, a Particle Swarm Optimization algorithm (PSO) as a novel heuristic was developed for determining the optimal layout.

Jinxiang et al., (2010) presented a detailed survey of the research on warehouse assignment design, performance evaluation, practical case studies, and computational support tools. The authors presented an extensive review on warehouse operation planning problems. The problems were classified according to the basic warehouse functions, i.e., receiving, storage, order picking, and shipping. Their purpose was to provide a bridge between academic researchers and warehouse practitioners, explaining what planning models and methods were currently available for warehouse operations, and what were the future research opportunities.

Liong et al., (2000) studied a special Quadratic Assignment Problem (QAP) which consists of deciding the assignment of customers to loading positions as well as fulfilling their demands, i.e. a double-assignment problem. Brief introductions about QAP and its applications are also given. In this work, the main questions were (i) where a customer should be assigned in a list of possible loading positions, (ii) from which storage areas should the customer be served, and (iii) how many lifting trucks should be assigned to each loading position in order to minimize cost and residence time. We have applied the Greedy Algorithm to get a good initial solution, and then a modified Genetic Algorithm to find the best solution to the problem. We explored the nearest neighbour using recombination procedure and maintaining the elements with the lower cost. The best solution found is always saved. Comparison with previous work and suggestions for further work has also been included.

Ahmad and Nima (2002) studied a problem of assigning a set of applicants to the service stations, which can be state as follows: A set of geographically scattered applicants must be served from a set of service stations so that the total cost of services is minimized. The authors considered two capacity for each service station, i.e. usual capacity and extra capacity. The set of applicants partitioned in two sets, special and ordinary applicants. A Mixed Integer Programming (MIP) formulation is given and a genetic algorithm (GA) proposed for solving the problem. The authors solved some randomly generated instances of introduced problem with the GA.

Dimitri et al., (1992) proposed auction algorithms for solving several types of assignment problems with inequality constraints. Included are asymmetric problems with different numbers of persons and objects, and multi-assignment problems, where persons may be assigned to several objects and reversely. A central new idea in all these algorithms is to combine regular auction, where persons bid for objects by raising their prices, with reverse auction, where objects compete for persons by essentially offering discounts. Reverse auction can also be used to accelerate substantially (and sometimes dramatically) the convergence of regular auction for symmetric assignment problems.

It is with the aim of solving scheduling problems with irregular cost functions that Sourd (2004) studied the continuous assignment problem. It consists in partitioning a d dimensional region into sub-regions of prescribed volumes so that the total cost is minimized. The dual problem of the continuous assignment problem is an unconstrained maximisation of a non-smooth concave function. The pre-emptive variant of the scheduling problem with irregular cost functions corresponds to the one-dimensional continuous assignment problem and a lower bound for the non-pre-emptive variant can be derived. It is computationally tested in a branch-and-bound algorithm.

The formulation of Facility Layout Problems (FLPs) as Quadratic Assignment Problems (QAPs) has gained substantial attention from researchers. The main reason is that, QAPs provide possibilities to solve FLPs computationally. To date, there are two common approaches used to solve FLPs formulated as QAPs, that is, exact methods and approximate methods (also known as heuristics). In recent years, there is an increasing interest in solving QAPs using the general extension of heuristic methods called meta-heuristics. Ant Colony Optimisation (ACO) has currently emerged as a new and promising meta-heuristic.

Phen et al., (2008) presented a model aimed to provide a comprehensive review of the concepts of ACO and its application in solving QAPs. In addition, the various ACO algorithms or variants developed to solve them are critically analysed and discussed. It is shown that these existing algorithms still possess many limitations and weaknesses. Finally, useful strategies and research directions are provided to improve these weaknesses.

Assignment problems are defined with two sets of inputs, i.e. set of resources and set of demands. Assignment of each resource to each demand has its own cost. Exactly one resource has to be assigned to each of the demands in such way, that maximal cost of the assignment is minimal when comparing to other assignments.

Hungarian algorithm (also known as Kuhn-Munkres algorithm) is able to find an optimal solution of assignment problems in polynomial time, but is only able to solve assignment problems with precisely defined demands and resources. This presents a major problem in many real-life scenarios while the nature of these problems is such that inputs are commonly defined only vaguely (i.e. fuzzily). In order to solve them, their precise formalization is needed. Formalization of their properties is normally far from being a straightforward procedure and can present large costs in the meaning of time and money. Fuzzy logic on the other hand successfully copes with the processing of imprecise data.

Miha (2009) presented an extension of the Hungarian algorithm with the introduction of fuzzy logic methods – fuzzy Hungarian algorithm. Vaguely defined resources and demands can be easily described with fuzzy values which present an input to fuzzy Hungarian algorithm.

The extended version of the algorithm is therefore able to cope with vaguely defined assignment problems, can be used more efficiently (i.e. with no further formalization of vaguely defined terms) and in a wider scope of assignment problems than the basic approach. Basic version of the Hungarian algorithm which was firstly presented by Harold Kuhn is presented in this article. Its extension with fuzzy logic methods is described and its usage on an example of vaguely defined assignment problem is demonstrated. Its benefits were also justified by the comparison of the results between the basic version of Hungarian algorithm and the fuzzy version of Hungarian algorithm on the same problem.

The channel-assignment problem is important in mobile telephone communication. Since the usable range of the frequency spectrum is limited, the optimal channel-assignment problem has become increasingly important. Omid (2010) presented a model and the goal of this is to find a channel assignment to requested calls with the minimum number of channels subject to interference constraints between channels. This algorithm consists of: (i) the fixed channel assignment stage; (ii) the neural network stage. In the first stage, the calls in a cell determining the lower bound on the total number of channels are assigned channels at regular intervals; then the calls in adjacent six cells are assigned channels by a cluster heuristic method sequentially. In the second stage, the calls in the remaining cells are assigned channels by a binary neural network. The performance is verified through solving well-known benchmark problems. Especially for Sivarajan's benchmark problems, my algorithm first achieves the lower bound solutions in all of the 12 instances.

Mingfang et al., (2010) studied the weapon-target assignment (WTA) problem which has wide applications in the area of defense-related operations research. This problem calls for finding a proper assignment of weapons to targets such that the total expected damaged value of the targets to be maximized. The WTA problem can be formulated as a nonlinear integer programming problem which is known to be NP-complete. There does not exist any exact method for the WTA problem even small size problems, although several heuristic methods have been proposed. In this paper, Lagrange relaxation method is proposed for the WTA problem. The method is an iterative approach which is to decompose the Lagrange relaxation into two sub-problems, and each sub-problem can be easy to solve to optimality based on its specific features. Then we use the optimal solutions of the two sub-problems to update Lagrange multipliers and solve the Lagrange relaxation problem iteratively. Our computational efforts signify that the proposed method is very effective and can find high quality solutions for the WTA problem in reasonable amount of time.

Assignment problem (AP) is a well known topic and is used very often in solving problems of engineering and management science. In this problem a_{ij} denotes the cost for assigning the j^{th} job to the i^{th} person. The cost is usually deterministic in nature.

Nagarajan and Solairaju (2010) presented studies which a_{ij} was considered to be trapezoidal and triangular numbers denoted by a_{ij} which are more realistic and general in nature. Robust's ranking method has been used for ranking the fuzzy numbers. The fuzzy assignment problem has been transformed into crisp assignment problem in the linear programming problem form and solved by using Hungarian method; Numerical examples show that the fuzzy ranking method offers an effective tool for handling the fuzzy assignment problem.

The Assignment Problem (AP) and Bottleneck Assignment Problem (BAP) are well studied in operational research literature. Abraham and Aneja (1993) considered two related problems which simultaneously generalize both AP and BAP. Unlike AP and BAP, these generalizations are strongly NP-complete. The authors propose two heuristics to solve these generalized problems: one based on a greedy principle and the other based on tabu search. Computational results are presented which show that these heuristics, when used together, produce good quality solutions. Their adaptation of tabu search also gave some new insight into the application of tabu search on permutation problems.

Wang and Liu (2010) presented a new algorithm on a special assignment problem in which the real assigned jobs are less than or equal to both the total persons and the total jobs. To this special assignment problem the authors posed the concept of reserve point, discussed the character of reserve point and accessed to relevant conclusion a new method to solve this special assignment problem is given through increasing reserve points finally.

One-sided assignment problems combine important features of two well-known matching models. First, as in roommate problems, any two agents can be matched and second, as in two-sided assignment problems, the payoffs of a matching can be divided between the agents.

Bettina and Alexandru (2009) presented a similar approach to one-sided assignment problems as Sasaki (1995) for two-sided assignment problems and we analyze various desirable properties of solutions including consistency and weak pair-wise monotonicity. The authors showed that for the class of solvable one-sided assignment problems (i.e., the subset of one-sided assignment problems with a non-empty core), if a sub-solution of the core satisfies (indifference with respect to dummy agents, continuity, and consistency) or (Pareto indifference and consistency), then it coincides with the core. However, the authors also prove that on the class of all one-sided assignment problems (solvable or not), no solution satisfies consistency and coincides with the core whenever the core is non-empty. Finally, the authors commented on the difficulty in obtaining further positive results for the class of solvable one-sided assignment problems in line with Sasaki's (1995) characterizations of the core for two-sided assignment problems.

Anshuman and Rudrajit (2006) solved the generalized "Assignment problem" through genetic algorithm and simulated annealing. The generalized assignment problem is basically the "N men- N jobs" problem where a single job can be assigned to only one person in such a way that the overall cost of assignment is minimized. While solving this problem through Genetic Algorithm (GA), a unique encoding scheme is used together with Partially Matched Crossover (PMX). The population size can also be varied in each iteration. In Simulated Annealing (SA) method, an exponential cooling schedule based on Newtonian cooling process is employed and experimentation is done on choosing the number of iterations (m) at each step. The source codes for the above have been developed in C language and compiled in GCC. Several test cases have been taken and the results obtained from both the methods have been tabulated and compared against the results obtained by coding in AMPL.

Solving the state assignment problem means finding the optimum assignment for each state within a sequential digital circuit. These optimum assignments will result in decreasing the hardware realization cost and increasing the reliability of the digital circuit. Unfortunately, the state assignment problem belongs to the class of Nondeterministic Polynomial time problems (NP complete) which requires heavy computations. Different attempts have been made towards solving the problem with reasonable recourses.

Walid (2009) presented a methodology for solving the state assignment problem, the methodology conducted a neighbourhood search while using a heuristic to determine the fitness of solution. To avoid being trapped at a local optimum solution, a meta-heuristic (simulated annealing) was utilized for deciding whether a new solution should be accepted. A case study was included to demonstrate the proposed procedure efficiency. The proposed approach finds the optimum assignment for the case study. The authors explored the usage of a stochastic search technique inspired by simulated annealing to solve the problem of the state assignment problem. This proved the efficiency of the methodology.

Wan (2001) studied the component assignment problem in PCB assembly, where assigning components to appropriate machines, in order to get a minimum assembly time for the assembly line, can be formulated as an integer linear programming model. In order to obtain the optimal solution to the component assignment problem, the branch-and-bound method can be applied. However, it is not efficient. The author proposed the tabu search heuristic approach to the component assignment problem. The procedure of the tabu search to the problem is presented, and a numerical example is provided. Finally, the performance of the tabu search is analyzed with the example.

Odior et al., (2010) addressed the problem of effectiveness of feasible solutions of a multi-criteria assignment problem and this was done in two steps. In the first step, the authors determine whether or not a given feasible solution of a multi-criteria assignment problem is a real efficient one. In the second step, if the feasible solution is not real efficient, the authors provided a real efficient solution that dominates that not real efficient solution, using their proposed method which consists of transforming the original problem into an assignment problem.

The Generalized Assignment Problem consists in assigning a set of tasks to a set of agents with minimum cost. Each agent has a limited amount of a single resource and each task must be assigned to one and only one agent, requiring a certain amount of the resource of the agent. Helina and Daniel (1998) presented new meta-heuristics for the generalized assignment problem based on hybrid approaches. One meta-heuristic is a MAX-MIN Ant System (MMAS), an improved version of the Ant System, which was recently proposed by Stutzle and Hoos to combinatorial optimization problems, and it can be seen as an adaptive sampling algorithm that takes in consideration the experience gathered in earlier iterations of the algorithm. Moreover, the latter heuristic is combined with local search and tabu search heuristics to improve the search. A Greedy Randomized Adaptive Search heuristic (GRASP) is also proposed. Several neighborhoods are studied, including one based on ejection chains that produce good moves without increasing the computational effort. The authors presented computational results of the comparative performance, followed by concluding remarks and ideas on future research in generalized assignment related problems.

Assignment problems are used throughout many research disciplines. Most assignment problems in the literature have focused on solving a single objective. Mark and Garry (2008) studied assignment problems that have multiple objectives that need to be satisfied. In particular, this chapter looks at how multi-objective evolutionary algorithms have been used to solve some of these problems. Additionally, the authors examined many of the operators that have been utilized to solve assignment problems and discuss some of the advantages and disadvantages of using specific operators.

The extended usage of Distributed Computing Systems (DCS) has made the task assignment strategies more attractive. Various types of algorithms have been developed for the Task Assignment Problem (TAP) in distributed computing systems along the different definitions of cost function. The final goal of task assignment algorithms is the assignment of some cooperative tasks to a set of interconnected processors. This assignment must minimize the total system cost and obtain a reasonable amount of load balancing. Abbas and Nasrollah (1992) studied a fair cost functions for task assignment problem in distributed computing systems are defined in order to satisfy some system requirements appropriately. Then, by the employment of the linear programming (LP) concepts, a polynomial approximation algorithm for task assignment problem is designed and the validity of the proposed algorithm is proven by theoretical analysis. Finally, the results of the execution of this algorithm on several problem instances are provided.

The classical Generalized Assignment Problem (GAP) may be stated as finding a minimum-cost assignment of tasks to agents such that each task is assigned to exactly one agent and such that each agent's resource capacity is honoured. This NP-hard problem has applications that include job scheduling, routing, loading for flexible manufacturing systems, and facility location. Due to the difficulty in solving "hard" GAPs to optimality, most recent papers either describe heuristic methods for generating "good" solutions or, in the case of optimizing methods, computational results are limited to 500 to 1,000 binary variables.

Nuass (2003) described a special purpose branch-and-bound algorithm that utilizes linear programming cuts, feasible-solution generators, Lagrangean relaxation, and sub-gradient optimization. The author presented computational results for solving "hard" problems with up to 3,000 binary variables. An unanticipated benefit of the algorithm is its ability to generate good feasible solutions early in the process whose solution quality generally dominates the solutions generated by two recently published heuristics. Furthermore, the computation time required is often less than the time taken by the heuristics. Thus, the authors have an optimizing algorithm that can be used quite effectively as a heuristic when proof of optimality is not an absolute requirement.

Terry and Andris(1979)studied the components and characteristics of an important class of models called weighted assignment models and identify these elements in a number of existing and potential applications. The weighted assignment model represents problems with the following characteristics: A set of tasks must be divided among a set of agents, and each task must be completed by only one agent. Tasks may be completed at one of several predetermined levels. When an agent completes a task at some level, a resource possessed by the agent is consumed and a system disutility is incurred. The weighted assignment model may be used to determine the best assignment of tasks to agents and the task completion levels so as to minimize total system disutility while satisfying the agent resource constraints. In the first part of the paper, a general formulation is presented, and its relationship to assignment models, transportation models, knapsack models and various fixed charge models is established. In the second part of the paper, a number of applications are described which demonstrate the usefulness of weighted assignment models. These applications include machine loading problems, personnel assignment problems and districting problems. Computational results for several of the applications are presented to document the tractability of the models.

Franses and Gerhard (2003) studied an assignment problem particular to the personnel scheduling of organisations such as laboratories. Here the authors have to assign tasks to employees. The authors focused on the situation where this assignment problem reduces to constructing maximal matchings in a set of interrelated bipartite graphs. The authors described in detail how the continuity of tasks over the week is achieved to suit the wishes of the planner. Finally the authors discussed the implementation of the algorithm in the package IPS. Its main characteristic is the introduction of profiles, which easily allows the user to steer the algorithm.

2.2 SUMMARY

In this chapter, we put forward pertinent literature on assignment problems.

The next chapter is devoted for the research methodology of the study.

CHAPTER 3

METHODOLOGY

3.0 INTRODUCTION

This chapter provides discussions of the proposed method for solving our proposed Assignment Problems.

Assignment Problem is a special type of transportation problem which is also a resource allocation problem. Here we have n jobs to perform with n persons and the problem is how to distribute the job to the different persons involved. Depending on the intrinsic capacity or merit or potential of the individual, he will be able to accomplish the task in different times. Then the objective function in assigning the different jobs to different persons is to find the optimal assignment that will minimize the total time taken to finish all jobs by the individuals.

The problem may be stated formally as follows: Given an n x n array of real numbers representing the individual return associated with assigning one item to one person. We have to find the best assignment so that the total return is optimal.

The general problem is modeled as;

Maximize $Z = \sum_{i=1}^{m} \sum_{j=1}^{n} C_{ij} X_{ij}$

Subject to $\sum_{j=1}^{n} X_{ij} = 1$

$$\sum_{i=1}^{m} X_{ij} = 1$$

$x_{ij} \in \{0, 1\}$, where $m = n$

A great deal of research has been performed to improve the solving times and ease of assignment problem. One of the major areas which research has been done in is the bipartite matching and graph theory.

3.1 GRAPH THEORY

DEFINITION: A graph is an ordered pair $G = (V, E)$ consisting of a finite set and a subset E of elements of the form(x, y) where x and y are in V. The set V are called the vertices of the graph and the set E are called the edges.

DEFINITION: A graph G is said to be a bipartite (or bicolored) graph if the vertices can be partitioned into two mutually disjoint sets X and Y so that there is no edge of the form (x, x') with x and x' in X or of the form $(y, y')$with y and y' in Y. A bipartite graph will be denoted by $G = (\{X, Y\}, E)$.

NOTATION: The cardinality of a set X will be denoted by $|X|$.

Bipartite graphs $G = (\{X, Y\}, E)$ are represented by matrices. The X vertices are for example used for row indices and the Y vertices are used as column indices. Generally, the existence of an edge (x, y) is indicated by a 1 in the x, y cell of the $|X| \times |Y|$ matrix; no edge is indicated by 0. For the assignment problem, we are representing an edge by 0and no edge by a nonzero number.

DEFINITION: A matching for a bipartite graph $G = (\{X, Y\}, E)$ is a subset M of E such that no two elements of Mhave a common vertex.

DEFINITION: If $G = (\{X, Y\}, E)$ is a bipartite graph, set

$\rho(G) = \max \{|M| \mid M \text{ is a matching of } G\}$.

A matching M such that $|M| = \rho(G)$ will be called a maximal matching.

DEFINITION: A set of vertices V′ is said to be a cover of a set of edges E′ if every edge in E′ is incident on one or more of the vertices of V′. A set of vertices S will be called a cover of the bipartite graph $G = (\{X, Y\}, E)$ if every edge of G is incident on one or more of the vertices of S.

DEFINITION: If $G = (\{X, Y\}, E)$ is a bipartite graph, set

$c(G) = \min \{|S| \mid S \text{ is a cover of } G\}$.

A cover S such that $|S| = c(G)$ will be called a minimal cover of G.

THEOREM: If $G = (\{X, Y\}, E)$ is a bipartite graph, then $\rho(G) \leq c(G)$.

PROOF: Let S be a cover with $|S| = c(G)$. Let M be a matching. Then each e in M has at least one of its vertices in S. If $|M| > S$, then by the pigeon hole principle, two edges e_1 and e_2 meet the same vertex v in S. This contradicts the definition of a matching. So we have that $|M| \leq |S| = c(G)$.

3.1.1 GRAPHS FOR THE ASSIGNMENT PROBLEM

The assignment problem corresponds to a bipartite graph. The vertices V may be partitioned into two sets: (1)the assigned tasks and (2) the assignees. The edges consist of (unordered) pairs connected the assigned tasks and the assignees. Since we allow the theoretic possibility of assigning any task to any assignee, the graph for the assignment problem consists of vertices $V = (X, Y)$ where $X = \{x_1, \ldots, x_m\}$ and $Y = \{y_1, \ldots, y_m\}$ and edges consisting of all combinations $E = \{(x_i, y_j) \mid 1 \leq i \leq m, 1 \leq j \leq m\}$. We can denote the incidence matrix of this graph as an $m \times m$ matrix consisting entirely of 1's and we can use the matrix as a substitute for the graph. This has some disadvantage in that an index, say 1, can denote x_1 or y_1 and so the row and column indices should be kept separate.

Now consider subgraphs of the assignment problem. For example, let (a_{ij}) be the $m \times m$ cost matrix of the assignment problem and let G′ be the sub graph whose edges are all (i, j) with $a_{ij} = 0$ and all vertices incident on any of the edges. If we are considering i as row indices and j as column indices, the vertices will consist of all rows of A which have a 0 entry and all columns of A which have a zero entry.

3.1.2 VERIFICATION OF THE ASSIGNMENT ALGORITHM

We verify the assignment algorithm terminates at some stage with a cover of size m. The proof is obtained by noting that sum of the current cost matrices is strictly decreasing at each stage provided there is a minimal cover of size less than m. We use the view point of the previous section.

THEOREM: Let be $A = (a_{ij})$ be an $m \times m$ matrix of positive entries. Suppose that there is a cover S of the set E′ of edges (i, j) with $a_{ij} = 0$ of size less than or equal to $m - 1$.

Let $D = \{ (i, j) \in E' \mid \text{both } i \text{ and } j \text{ are in } S\}$. Let a_0 be the minimum of $\{a_{ij} \mid i \notin S, j \notin S\}$. Suppose that

$$\begin{cases} a_{ij} + a_0 (i, j) \in D \\ a_{ij} - a_0\ (i, j) \notin E' \\ a_{ij} \text{ otherwise} \end{cases}$$

Then $B = (b_{ij})$ is a positive matrix with $\Sigma\, a_{ij} > \Sigma b_{ij}$.

REMARK: We can paraphrase the theorem as follows. Suppose that 0's of A are crossed by crossing out the rows and columns containing a 0. Suppose that the minimal number of crossed out rows and columns necessary to cross out all the 0's of A is less than m - 1. Suppose that the minimal entry in the entries not crossed out by the minimal number of crossed out rows and columns is added to all doubly crossed out entries of A and subtracted from all non crossed out entries to give a matrix B = (b_{ij}). Then B is a positive matrix and $\Sigma a_{ij} > \Sigma b_{ij}$.

PROOF: Let s and t be any integers with $0 \leq s + t \leq m - 1$ and suppose that s rows and t columns are crossed out,i.e., the cover C consists of s vertices of X and t vertices of Y. Then there are st doubly crossed out elements, m(s + t) – st crossed out elements, and m(s + t) singly crossed out elements. So there are $m^2 - m(s + t) + st$ entries that are not crossed out. Now suppose that the minimal (non zero) entry in the uncrossed out part of A is r.

Since all the zero entries are crossed out r, we get that r > 0. So we get

$\Sigma a_{ij} - \Sigma b_{ij} = - rst + r(m^2 - m(s + t) + st) = rm(m - (s + t)) > 0$

since m > s + t. Also note that all $b_{ij} \geq 0$ since the minimal entry is subtracted.

The process of adding to double crossed out entries and subtracting from non crossed out entries is a combination of operations that does not change the optimal solution of the assignment problem.

THEOREM: Suppose that the first three steps in the assignment algorithm is implemented. Then the optimal solution of the assignment problem with the new cost matrix does not change.

PROOF: For first step note that the optimal solution x* does not change if a constant is subtracted from any row or column. To see this suppose that c is subtracted from every element in the first row of the cost coefficient matrix.

Then the problem becomes

Minimize $Z= \Sigma_j (c_{1j} - c) x_{1j} + \Sigma_{i \geq 2, j} c_{ij}x_{ij}$

Subject to the constraints

$\Sigma_j x_{ij} = 1 \ (1 \leq i \leq m)$

$\Sigma_i x_{ij} = 1 \ (1 \leq i \leq n)$

$0 \leq x_{ij} \leq 1.$

But we note that

$\Sigma_j (c_{1j} - c) x_{1j} + \Sigma_{i \geq 2, j} c_{ij}x_{ij} = \Sigma_{i, j} c_{ij}x_{ij} - \Sigma_j c x_{1j} = \Sigma_{i, j} c_{ij}x_{ij} - c.$

So the optimal solution for the original problem is exactly the optimal solution for the perturbed problem. The same holds for all other rows and columns. So the first step of the algorithm does not change the optimal solution.

Now suppose c is added to each cost of a doubly covered entry and c is subtracted from each cost of an uncovered entry. This is equivalent to adding c to each covered column and subtracting c from each uncovered row. To see this suppose c is added to the covered column 1 and subtracted from the uncovered row 1. Suppose column 2 is uncovered and row 2 is covered. Then we get the northwest 2 × 2 corner is

	cov(+ c)	uncov(no action)
uncov (– c)	c – c = 0	– c
cov(no action)	+ c	0

This covers all 4 cases. So the action in the third step is a series of action in the first step and the optimal solution does not change.

3.1.3 MAXIMAL MATCHINGS USING THE HUNGARIAN ALGORITHM

Let G = ((X, Y}, E) be a bipartite graph. Let M be a matching for G. The Hungarian Algorithm either shows that M is a maximal matching for G or finds a matching M′ for G with |M′| = |M| + 1.

(i). Label all vertices in X with (*) when the vertices do not meet an edge of M and call all vertices untested.

(ii). If in the previous step no new labels have been given to a vertex of X, then STOP. Otherwise, go to III.

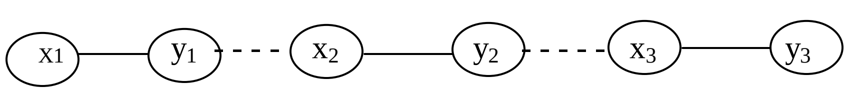
(iii). While there is a labelled but untested vertex x_i of X, label with x_i all vertices y_j of Y that have not yet been labelled and that can be connected to x_i with an edge NOT IN M. The vertex x_i is now tested (even if no edge is added) and the vertices y_j are now labelled.

(iv). If no new label has been given in III, then STOP. Otherwise, find an untested but labelled vertex y_j of Y and label with y_j any unlabeled vertex x_k of X which is joined to y_j by an edge IN M. The vertex y_j is now tested (even if no edge is added) and vertex x_k is now labelled. If y_j cannot be connected to an unlabeled vertex in X, then STOP and an Augmenting Tree has been found.

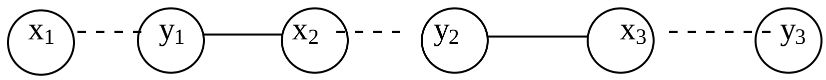
(v). Return to II.

The algorithm stops after a finite number of iterations since each vertex is receiving at most one label and each vertex is tested at most once. There are two possibilities:

(i). (Augmenting Tree) There is a labelled vertex of Y that does not meet an edge of M. This has the following diagram

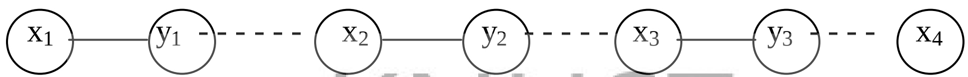


Where the dashed lines represent edges in M and the solid lines represent edges not in M. In this case the size of the matching can be increased by switching edges to look like



where an additional vertex x_1 has been matched to y_1.

(ii). (Hungarian Tree) A diagram of the form



where no increase in the matching to include x1 is possible.

The termination is obtained when all unmatched elements in X have been matched or produce Hungarian trees. Elements that produce Hungarian trees are called Hungarian acorns.

THEOREM: Suppose that G = ({X, Y}, E) be a bipartite graph and that M is a matching for G. Suppose that an Augmenting Tree $\{x_1, y_1, \ldots, x_n, y_n\}$ has been found using the Hungarian algorithm. Let S be the set of edges given by

$S = \{(y_1, x_2), (y_2, x_3), \ldots, \{y_{n-1}, x_n)\}$ and let M_1 be the set of edges

$M_1 = \{(x_1, y_2), (x_2, y_2), \ldots, (x_n, y_n)\}$.Then the set of edges M′ given by $M' = S \cup (M - M_1)$ is a matching for G with $|M'| = |M| + 1$.

PROOF: First note that yn cannot be attached to a labelled vertex x by an edge $e = (y_n, x)$ in M; otherwise, the vertex y_n would have to be a labelled and an unlabeled vertex at the stage when $\{x_1, y_1, \ldots, x_n\}$ is formed. To see that y_n is labelled at this stage, we see that the vertex x cannot be a * vertex (since * vertices are incident on no edges of M), and therefore, x must have been labelled by some y using an edge (y, x) in M. This would mean that $y = y_n$ since both (y_n, x) and (y, x) are in the matching M and have a vertex in common. So x must have been labelled by y_n and this implies that y_n must have been labelled. To see y_n is unlabeled, we see that the vertex y_n is labelled by x_n in the algorithm only if y_n is unlabeled.

Now we show that M′ is a matching by showing each vertex z is incident on at most one edge in M′. If z is in$\{y_1, x_2, \dots, y_{n-1}, x_n\}$, then z is incident on an edge (y_i, x_{i+1}) in M_1 and no other edge in M. So z is incident on one edge in M′. If z= x_1, then z is *-vertex and z is incident on no edge in M and incident only on the edge (x1, y1) inM′. If z = y_n, then z is incident on one edge (x_n, y_n) since y_n is incident on no edge of M; otherwise, (y_n, x) would be in M but x cannot be labelled by the preceding paragraph or unlabeled by the fact that the Hungarian Algorithm has stopped on y_n. So we get that the Augmenting Tree produces a matching M′.

We note that the matching M′ has one more edge than M.

We see that the Hungarian algorithm terminates after a finite number of iterations. On each level the points are tested one after the other and after all the points are tested or perhaps before the algorithm ends. As a whole the algorithm either terminates when the only X elements left are Hungarian acorns.

THEOREM: Suppose M is a matching of the bipartite graph G = ({X, Y}, E) and suppose the Hungarian algorithm produces only Hungarian trees for all unmatched X. Let X^{un} be all unlabeled X vertices and Y^{lab} be all labelled Y vertices. Then

1) $S = X^{un} \cup Y^{lab}$ is a minimal cover of G, and

2) |M| = |S| and M is a maximal matching of G.

PROOF: First we show S is a cover. We argue by contradiction. Suppose $(x, y) \in E$ and assume $x \in X^{lab}$ and $y \in Y^{un}$. We show that such an edge does not exist. First assume $(x, y) \in M$. Since x is labelled and (x, y) is inM, the vertex cannot be the first vertex in a path labelled with a (*) due to I of the algorithm. So x must be part of chain of the form

x_1------- y_1 - - - - x_2 ------- y_2 - - - - ... - - - - x_k ------- y_k - - - - $x_{k+1} = x$.

where the solid lines are not in M and the dashed lines are in M. But there is at most one edge in M incident of x and this edge is (x, y). So we must have that (x, y) = (x, y_k) and y = y_k. The vertex y_k is labelled and this contradicts the assumption that y is not labelled. Now suppose the (x, y) $\notin$ M. Since x is labeled, it follows that y must be labelled which is a contradiction. So we have a contradiction in all cases. This means that S is a cover.

Now we show that |M| = |S|. This shows that M is a maximal matching and S is a minimal cover. We find a one–one function f of S onto M. If y $\in$ Y^{lab}, then y cannot be a termination of some tree starting at a (*); otherwise, there would be an augmenting step. So y meets an edge {y, x} of M. Since M is a matching, {y, x) is the unique edge of M that y meets. Note that x gets a label from y and so x is not in X^{un}. So we set

f(y) = (y, x) ∈ M. Now letx′ ∈ X^{un}. Note that x′ meets an edge (x′, y′) of M; otherwise, the vertex x′ would have the label (*). As before the edge (x′, y′) is the only edge in M that x meets. Finally, the element y′ is unlabeled. Indeed, if y′ were labelled, then there would be a labelled x″ with an edge (x″, y′) not in M and with (y′, x′) in M and so there would be an augmenting tree through y′. We let f(x′) = (x′, y′).

Now we see that the function f is one-one. In fact, f(y) = f(y′) implies that y = y′ since y is the unique y element on which f(y) is incident. Also f(x) = f(x′) implies x = x′ and finally f(x) = f(y) is not possible as we have already shown. So f is one-one.

Since f is one-one, we have that |S| ≤ |M|. But we have already seen that |M| ≥ |S|. So we get that |S| = |M|.

COROLLARY: Let G = ({X, Y}, E) be a bipartite graph. Then $\rho(G) = c(G)$.

PROOF: We have that $\rho(G) \le c(G)$. But let M be a maximal matching for G, i.e., a matching with $|M| = \rho(G)$. Then the Hungarian algorithm terminates on M without a break through and the preceding theorem implies that there is acover S with $|S| = |M|$. So we get that $c(G) = |S| = |M| \leq \rho(G)$. So we get that $\rho(G) = c(G)$.

.**COROLLARY**: Let $G = (\{X, Y\}, E)$ be a bipartite graph with $|X| = |Y|$. Suppose that G has a minimal cover S with $|S| = |X|$. Then there is a matching M such that M is incident on all vertices of G.

PROOF: Since S is a minimal cover, there is a matching M with $\rho(M) = |S| = |X|$. Since no vertex of G is incident on more than one edge of M, we must have that M is incident on $2|X|$ vertices which must mean the M is incident on all vertices of G.

Now back to the algorithm for the assignment problem running on a cost matrix of dimension $m \times m$. The first part of the algorithm runs until a minimal cover of size M is for the zeroes in the cost matrix. The first part of the algorithm goes to termination since the sum of all the costs decreases at every stage when the minimal cover has less that m vertices. When the minimal cover is reached with m vertices, we run the Hungarian algorithm on the bipartite graph defined by the zeroes in the final cost matrix. A matching of size m is obtained to give the minimal co

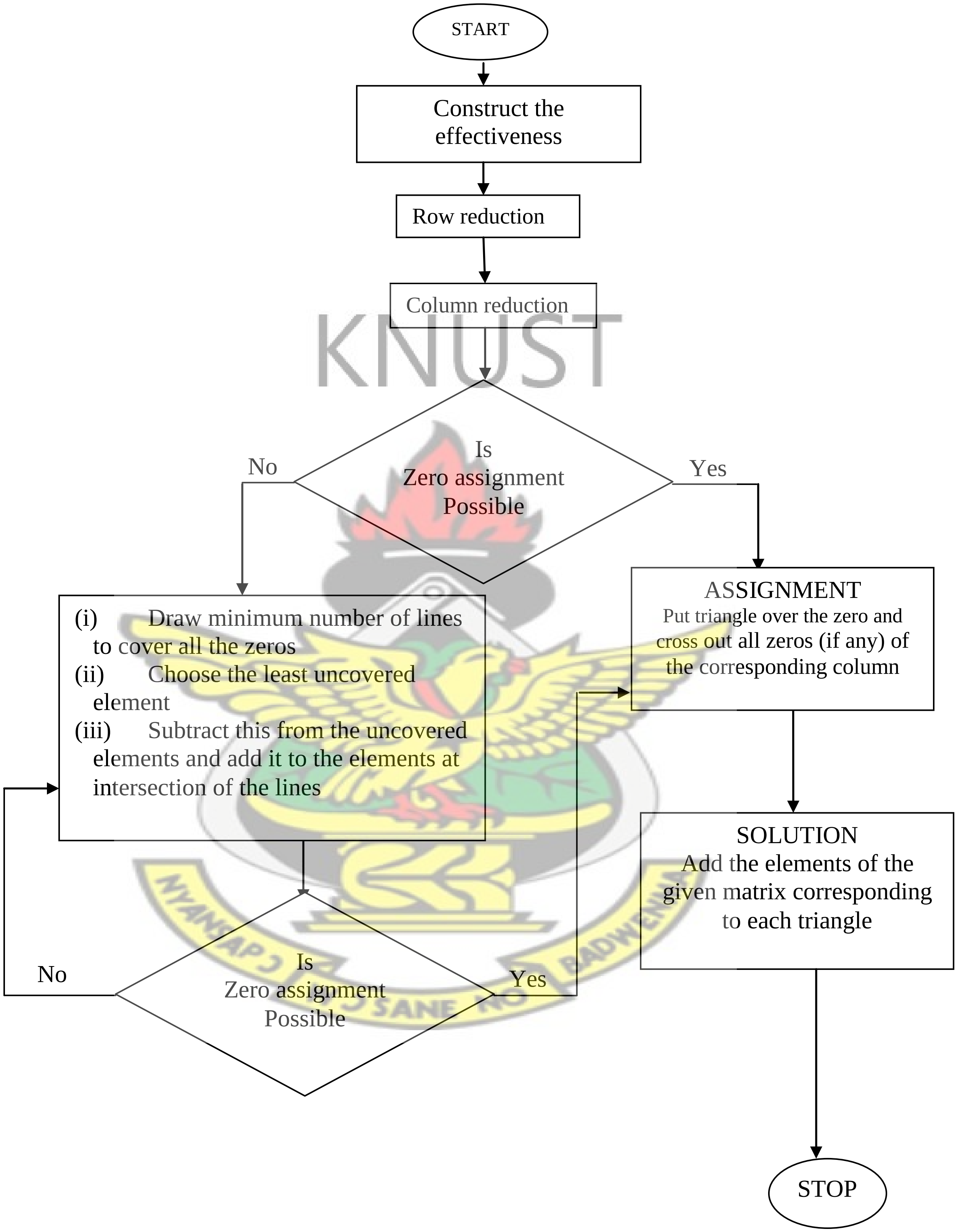


Figure 3.1 Flow chart for Assignment problem

3.1.4 HUNGARIAN METHOD: ALGORITHM

Step 1: Prepare Row reduced Matrix by selecting the minimum values for each row and subtracting it from other elements of the row

Step 2: Prepare column reduced Matrix by subtracting minimum value of the column from the other values of that column

Step 3: First row-wise assign a zero if there is only one zero in the row and cross (X) other zeros in that column.

Step 4: Now assign column wise if there is only one zero in that column and cross other zeros in that row.

Repeat Step 3 and 4 till all zeros are either assigned or crossed. If the number of assignments made is equal to number of rows present, then it is the optimal solution otherwise proceed as follows.

Step 5: Mark (P) the row which is not assigned. Look for crossed zero in that row. Mark the column containing the crossed zero. Look for assigned zero in that column. Mark the row containing assigned zero. Repeat this process till all makings are over.

Step 6: Draw straight lines through unmarked rows and marked column. The number of straight lines drawn will be equal to number of assignments made.

Step 7: Examine the uncovered elements. Select the minimum.

a. Subtract it from uncovered elements.

b. Add it at the point of intersection of lines.

c. Leave the rest as it is.

Prepare a New Table.

Step 8: Repeat Steps 3 to 7 till number of allocations = Number of rows.

3.2 SUMMARY

In this chapter, the researcher presented the Hungarian algorithm for solving the Assignment Problem.

Data collection and analysis of the problem will be presented in the next chapter.

CHAPTER 4

DATA COLLECTION AND ANALYSIS

4.0 INTRODUCTION

In this chapter, we shall consider a computational study of the Hungarian algorithm for solving Shama Senior High School Tender/Contractor – Job Assignment Problem.

The aim is to determine the best assignment policy in the school in order to minimize the cost of execution of the six projects. The general practice is that the school does not have a well structured plan on how to assign contracts to various tenders. Tenders are assigned by the discretion of the school authorities. These methods are faulted, and are basically inefficient.

4.1 DATA COLLECTION AND ANALYSIS

Six projects were to be undertaken in Shama Senior High School. These projects were put on tender and six contractors tendered in their bids. After appropriate introspection and evaluation by the school's tender board, it was realised that, each of these six tenders who placed their bids on these projects can do any of the six projects in the school. The bids (in thousands of Ghana cedis) are given in Table 4.1

Table 4.1: Estimated cost (in thousands of Ghana cedis) of tenders to the various projects

TENDER	PROJECTS					
	CLASSROOM BLOCK	DORMITORY BLOCK	SCHOOL FIELD	SRC W.C TOILET	STAFF BUS	POWER PLANT
A	130	200	80	10	100	50
B	125	205	75	13	94	43
C	123	200	76	7	99	39
D	129	203	86	1	93	48
E	122	208	82	6	101	47
F	126	199	81	11	100	46

The problem now is how the projects should be assign to the tenders so as to maximise educational quality in the school and minimise total cost of undertaking the six projects. The problem can be modeled as:

$$\text{Minimize} \quad Z = \sum_{i=1}^{m} \sum_{j=1}^{n} C_{ij} X_{ij}$$

$$\text{Subject to} \quad \sum_{j=1}^{n} X_{ij} = 1$$

$$\sum_{i=1}^{m} X_{ij} = 1$$

$$x_{ij} \in \{0,1\}, \text{ where } m = n,$$

x_{ij} = the assignment of tender i to job j

C_{ij} = the cost or time of assigning tender i to job j

Hence the problem becomes,

$$\text{Minimize } Z = 130x_{11} + 200x_{12} + 80x_{13} + 10x_{14} + 100x_{15} + 50x_{16}$$
$$+125x_{21} + 205x_{22} + 75x_{23} + 13x_{24} + 94x_{25} + 43x_{26}$$
$$+123x_{31} + 200x_{32} + 76x_{33} + 7x_{34} + 99x_{35} + 39x_{36}$$
$$+129x_{41} + 203x_{42} + 86x_{43} + x_{44} + 93x_{45} + 48x_{46}$$
$$+122x_{51} + 208x_{52} + 82x_{53} + 6x_{54} + 101x_{55} + 47x_{56}$$
$$+126x_{61} + 199x_{62} + 81x_{63} + 11x_{64} + 100x_{65} + 46x_{66}$$

Subject to

$$x_{11} + x_{12} + x_{13} + x_{14} + x_{15} + x_{16} = 1$$
$$x_{21} + x_{22} + x_{23} + x_{24} + x_{25} + x_{26} = 1$$
$$x_{31} + x_{32} + x_{33} + x_{34} + x_{35} + x_{36} = 1$$
$$x_{41} + x_{42} + x_{43} + x_{44} + x_{45} + x_{46} = 1$$
$$x_{51} + x_{52} + x_{53} + x_{54} + x_{55} + x_{56} = 1$$
$$x_{61} + x_{62} + x_{63} + x_{64} + x_{65} + x_{66} = 1$$
$$x_{11} + x_{21} + x_{31} + x_{41} + x_{51} + x_{61} = 1$$
$$x_{12} + x_{22} + x_{32} + x_{42} + x_{52} + x_{62} = 1$$
$$x_{13} + x_{23} + x_{33} + x_{43} + x_{53} + x_{63} = 1$$
$$x_{14} + x_{24} + x_{34} + x_{44} + x_{54} + x_{64} = 1$$
$$x_{15} + x_{25} + x_{35} + x_{45} + x_{55} + x_{65} = 1$$
$$x_{16} + x_{26} + x_{36} + x_{46} + x_{56} + x_{66} = 1$$

$$x_{ij} = \begin{cases} 1 & if\ job\ i\ is\ to\ person\ j \\ 0 & otherwise \end{cases}$$

A walk through the Hungarian algorithm with the above model gives the following computational table values.

Table 4.2: Row reduced cost matrix (in thousands of Ghana cedis)

TENDER	PROJECTS					
	CLASSROOM BLOCK	DORMITORY BLOCK	SCHOOL FIELD	SRC W.C TOILET	STAFF BUS	POWER PLANT
A	120	190	70	0	90	40
B	112	132	62	0	81	30
C	116	193	69	0	92	32
D	128	202	85	0	92	47
E	116	202	76	0	95	41
F	115	188	70	0	89	35

Table 4.3: Column reduced cost matrix (in thousands of Ghana cedis)

TENDER	PROJECTS					
	CLASSROOM BLOCK	DORMITORY BLOCK	SCHOOL FIELD	SRC W.C TOILET	STAFF BUS	POWER PLANT
A	8	58	8	0	9	10
B	0	0	0	0	0	0
C	4	61	7	0	11	2
D	12	70	23	0	11	17
E	4	70	14	0	14	11
F	3	56	8	0	18	5

This result is not optimal, so we cannot make assignment.

The algorithm will then compute the next iterative stage, with the table values shown below

Table 4.4: Reduced cost matrix (in thousands of Ghana cedis)

TENDER	PROJECTS					
	CLASSROOM BLOCK	DORMITORY BLOCK	SCHOOL FIELD	SRC W.C TOILET	STAFF BUS	POWER PLANT
A	6	56	6	0	7	8
B	0	0	0	2	0	0
C	2	59	5	0	9	0
D	10	68	21	0	9	15
E	2	68	12	0	12	9
F	1	54	6	0	16	3

This result is not optimal, so we cannot make assignment. The algorithm will then compute the next iterative stage, with the table values shown below

Table 4.5: Reduced cost matrix (in thousands of Ghana cedis)

TENDER	PROJECTS					
	CLASSROOM BLOCK	DORMITORY BLOCK	SCHOOL FIELD	SRC W.C TOILET	STAFF BUS	POWER PLANT
A	5	55	5	0	6	7
B	0	0	0	3	0	0
C	2	59	5	1	9	0
D	9	67	20	0	8	14
E	1	67	11	0	11	8
F	0	53	5	0	15	2

This result is not optimal, so we cannot make assignment. The algorithm will then compute the next iterative stage, with the table values shown below

Table 4.6: Reduced cost matrix (in thousands of Ghana cedis)

TENDER	PROJECTS					
	CLASSROOM BLOCK	DORMITORY BLOCK	SCHOOL FIELD	SRC W.C TOILET	STAFF BUS	POWER PLANT
A	5	50	0	0	1	7
B	5	0	0	8	0	5
C	2	54	0	1	4	0
D	9	62	15	0	3	9
E	1	62	6	0	6	8
F	0	48	0	0	10	2

This result is not optimal, so we cannot make assignment. The algorithm will then compute the next iterative stage, with the table values shown below

Table 4.7: Reduced cost matrix (in thousands of Ghana cedis)

TENDER	PROJECTS					
	CLASSROOM BLOCK	DORMITORY BLOCK	SCHOOL FIELD	SRC W.C TOILET	STAFF BUS	POWER PLANT
A	4	49	0	0	0	6
B	5	0	1	9	0	1
C	2	54	1	2	4	0
D	8	61	15	0	2	8
E	0	61	6	0	5	7
F	0	48	0	0	10	2

This result is optimal, so we make our assignment.

Tender A→Staff Bus

Tender B → Dormitory Block

Tender C →Power Plant

Tender D → S.R.C W.C toilet

Tender E → Classroom Block

Tender F →School Field

The total minimum cost that will maximize total educational quality is given as:

Total = 100 + 205 + 39 + 1+ 122+ 81

Total = 548 (in thousands of Ghana cedis)

Minimize Z= 130(0) + 200(0) + 80(0) + 10(0) + 100(1) + 50(0)

+125(0) + 205(1) +75(0) + 13(1) +94(0) + 43(0)

+123(0) + 200(0) + 76(0)+ 7(0) + 99(0)+ 39(1)

+129(0) + 203(0) + 86(0)+ 1(1)+ 93(0)+ 48(0)

+122(1) + 208(0) + 82(0)+ 6(0)+ 101(0)+ 47(0)

+126(0) + 199(0) + 81(1)+ 11(0)+ 100(0)+ 46(0)

Z = 548 (in thousands of Ghana cedis)

By applying their criteria of assignment to this data, the assignment below were obtained:

Tender A→Classroom Block

Tender B → Dormitory Block

Tender C →School Field

Tender D → S.R.C W.C toilet

Tender E →Staff Bus

Tender F →Power Plant

The total minimum cost that will maximize total educational quality is given as:

Total = 130 + 205 + 76 + 1+ 101+ 46

Total = 559 (In thousands of Ghana cedis)

4.2 SUMMARY

This chapter discussed the data collection and analysis for the study.

The next chapter, which is the last chapter of the study, presents conclusions and recommendations for the study.

CHAPTER 5

CONCLUSIONS AND RECOMMENDATIONS

5.0 INTRODUCTION

We have described the tender /contractor - project placement and selection problem of Shama Senior High School as an Assignment Problem. The main focus of this research work was the problem of tender- job/project assignment and management in the Shama Senor High School and also the effect of the projects on the academic performance of the students. The study was, however, limited to the tenders/contractors performance in the job/project assigned to them in Shama Senior High School. This was to make the study manageable, given the time and the resources available to the researcher to complete the study. The study was also limited to the perceived effect of inadequate classrooms, dormitories, etc on the students' academic performance in the school. We applied the Hungarian algorithm to solve the tender/contractor - project placement and selection problem. It can, however, be applied to any situation that can be modeled as an assignment problem by management of the school.

5.1 CONCLUSIONS

This thesis seeks to solve a real-life problem of Ghana Education Service (GES) tender/contractor - project placement and selection problem using the Hungarian assignment algorithm. After a careful analysis of the data, it was observed that the total minimum cost of executing the six projects that will maximize total educational quality in the school, or the minimum cost for assigning a contractor or tender to a project was 548 (in thousands of Ghana cedis). Hence, the contractors or tenders were assigned as follows:

Tender/Contractor A →Staff Bus

Tender/Contractor B → Dormitory Block

Tender/Contractor C →Power Plant

Tender/Contractor D → S.R.C W.C toilet

Tender/Contractor E → Classroom Block

Tender/Contractor F →School Field

It was also realised that using the Hungarian method to solve the problem, the school would have saved Gh¢11,000.00.

5.2 RECOMMENDATIONS

The use of computer application in computation gives a systematic and transparent solution as compared to an arbitrary method. Using the more scientific assignment problem model for the placement and selection of the tenders/contractors to the various projects gives a better result. Management may benefit from the proposed approach for placement and selection of tenders/contractor to guarantee optimal results from tenders/contractor. The researcher, therefore, recommends that the assignment problem model should be adopted by the school and other schools as well as other institutions and agencies for tender/contractor placement and selection planning. It is also recommended that further research should be done in this area by other researchers.

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