MULTI OBJECTIVE NODE ROUTING PROBLEM WITH TIME WINDOWS: AN ALTERNATE APPROACH TO SOLID WASTE COLLECTION AND DISPOSAL IN DEVELOPING COUNTRIES

By

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DECLARATION OF AUTHORSHIP

I, Otoo Dominic declare that this thesis titled, 'Multi-objective node routing problem with time windows an alternative approach to solid waste collection in developing countries' and the work presented in it are my own. I confirm that:

- This work was done wholly or mainly while in candidature for a research degree at the Kwame Nkrumah University of Science and Technology, Kumasi, Ghana.
- Where any part of this thesis has previously been submitted for a degree or any other qualification at this University or any other university/institution, this has been clearly stated.
- Where I have consulted the published work of others, this is always clearly acknowledged.
- Where I have quoted from the work of others, the source is always given. With the exception of such quotations, this thesis is entirely my own work.
- I have acknowledged all main sources of help.

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ABSTRACT

This thesis is concerned with the collection and disposal of solid waste in the developing countries where logistics for collection and improper road network is a challenge. This problem is modeled as a Capacitated Node Routing Problem with Time Windows (CNRPTW), the effect of smell from uncollected waste on the environment and the cost of transportation to the disposal sites. The first part of the study investigates the generation of waste per person per day in third class communities; the second part provides a proposed location model to optimally assign customers unto a zone based on two primary parameters. The third part of the study provides a meta-heuristic method, which takes into account the vehicle capacity and time of collection. This is based on the improved Ant Colony Heuristic, enhanced by three parameters. The introduction of soft windows incorporates vehicle breaking time, crew lunch break and drop-off time. The fourth part of the study modeled the effect of smell from uncollected waste by incorporating factors such time and fraction of biodegradable component, the fifth part considered the implementation of a proposed model on fuel consumption in addition to the existing fuel consumption models. Computational testing is carried out on the test problems used in the literature on our improved models gave competitive results. The final part of the study deals with the implementation of our models on the real life solid waste generation, collection and transportation which exists in one of the nine sub-metropolitan areas in Kumasi Metropolitan Assembly in Kumasi. Competing results were obtained compared with the existing practices on the ground.

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SYMBOLS

f_t	Fuel consumption per unit time	mL/s
v	Vehicle speed	m/s
τ	Acceleration	m/s^2
M_f	Total weight of the vehicle with load	kg
M_e	Weight of the vehicle when empty	kg
ω	Road gradient	rad
F_t	Total fuel consumption	mL
F_a	Total fuel consumption during acceleration	mL
F_d	Total fuel consumption during deceleration	mL
F_c	Total cruise fuel consumption	mL
F_i	Total fuel consumption while idle	mL
F_D	Total fuel consumption during dead speed	mL
t_0	Total time for the journey	s
t_a	Time during acceleration	s
t_d	Time during deceleration	s
t_c	Time during cruising	s
t_i	Idle time	s
t_D	Time during dead speed	s
v_{ia}	Initial acceleration speed	m/s
v_{fa}	Final acceleration speed	m/s
v_c	Cruising speed	m/s
v_{id}	Initial deceleration speed	m/s
v_{fd}	Final deceleration speed	m/s
v_{iD}	Initial dead speed	m/s

v_{fD}	Final dead speed	m/s
D	Total journey distance	km
d_a	Acceleration distance	km
d_d	Deceleration distance	km
d_c	Cruising distance	km
d_D	Dead speed distance	km

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CHAPTER ONE

GENERAL OVERVIEW OF SOLID WASTE

1.0 INTRODUCTION

Solid waste is a continually growing problem at local, regional and global levels. Solid wastes arise from human and animal activities that are normally discarded as useless or unwanted. The term solid waste has been defined differently by various authors. Solid waste is any material that arises from human and animal activities that are normally discarded as useless or unwanted (Tchobanoglous et al., 1993).

According to Zerbock (2003), solid waste includes non-hazardous industrial, commercial and domestic waste including: household organic trash, street sweepings, institutional garbage and construction wastes.

In other words, solid wastes may be defined as the organic and inorganic waste materials produced by various activities of the society and which have lost their value to the first user. As the result of rapid increase in production and consumption, urban societies reject and generate solid material regularly, which leads to considerable increase in the volume of solid waste generated from several sources such as, domestic wastes, commercial wastes, institutional wastes and industrial wastes of most diverse categories.

Management of solid waste may be defined as that discipline associated with the control of generation, storage, collection, transfer and transport, processing, and disposal of solid wastes in a manner that is in accord with the best principles of public health, economics, engineering, conservation, aesthetics, and other environmental considerations. In its scope, solid waste management includes all administrative, financial, legal, planning, and engineering functions involved in the whole spectrum of solutions to problems of solid wastes thrust upon the community by its inhabitants (Tchobanaglous et al., 1977).

Solid wastes have the potential to pollute all the vital components of living environment (i.e., air, land and water) at local and global levels. The problem is compounded by trends in consumption and production patterns and by continuing urbanization of the world. The problem is more acute in developing nations than in developed nations as the economic growth as well as urbanization is more rapid.

1.1.1 Scope of the Study Area

Ashanti region is one of the ten regions in Ghana, and is the second highest populated region in Ghana after Greater Accra region. Ashanti region is centrally located in the middle belt of Ghana, it lies between longitudes 0.15W and 2.25W and latitude 5.50N and 7.46N. The region shares boundaries with four of the ten regions, Brong Ahafo to the north, Eastern region to the east, Central region to the south and Western region to the south west. The population of the region is concentrated in a few districts, Kumasi metropolis alone accounts for nearly one-third of the region's population. The high level of urbanization in the region is due mainly to the high level of concentration of the population in the Kumasi metropolis. The region occupies a land area of 24,389 square kilometres representing 10.2 percent of the total land area of Ghana in which Kumasi alone is 250 square kilometers. It is the third largest region after Northern and Central regions.

Table 1.1: Sub-metropolitan population and percentage of Kumasi

Sub-Metro	Population	Percentage
Kwadaso	251,215	14.6
Nhyiaeso	134,846	7.8
Subin	174,004	10.1
Oforikrom	303,106	17.6
Asokwa	140,161	8.1
Manhyia	152,225	8.8
Tafo	146,024	8.5
Suame	161,199	9.4
Bantama	260,474	15.1
Total	**1,722,894**	**100**

Source: 2010 Population Census

The region has a population density of 148.1 persons per square kilometer, with Kumasi sub-metropolitan area estimated to have a population of 1966109 in 2013 at the time when data was taken from the study area. The people of the region are into farming, mining and trading. Tradition is held very high in the region and blends well with modernity. Residential land use in Kumasi forms about 60% of the total land use in the metropolitan area and they are categorised into three zones namely; the low income, middle income and the high income zone. The municipal area has one teaching hospital, 9 hospitals and some few private hospitals and clinics, two public universities and six private universities. To help improve collection and disposal of waste, the metropolitan assembly have divided the metropolis into nine sub metropolitan assemblies and assigned to private waste management groups to manage the waste. (Waste management agency, KMA, 2012).

1.2 BACKGROUND OF THE STUDY

1.2.1 Solid Waste Management

Solid waste management involves management activities associated with the generation, storage, collection, transfer and transportation, processing and disposal of solid waste, which are environmentally compatible, adopting principles of economy, aesthetics, energy and conservation. It encompasses planning, organization, administration, finance, law, engineering and mathematics aspects involving interdisciplinary relationships.

1.2.2 The Governance of Waste Management in Ghana

In this section, we shall discuss the governance of waste management in Ghana, the environmental hazards associated with improper waste management, and the need to strategically address it.

In Ghana, population redistribution in the form of rapid urbanization is taking place in a context of poor economic performance in rural areas. Local authorities are unable to cope with the consequences of urbanization. One major area where authorities have failed is the management of industrial and domestic waste. In rural setting (i.e. small communities), sanitation is basically an individual affair, and does not depend on elaborate infrastructure to deal with it. Here individual efforts are sufficient for efficient management of the waste. As the size and concentration of the community grows, however, there comes a stage when individual efforts are no longer enough. At this stage, public, public funds have to be committed in a planned and sustained manner for the provision, operation and maintenance of infrastructural facilities for sanitation management.

In most urban areas, especially those in the developing countries, only a percentage of waste generated daily is collected, and safely disposed of, by the authorities concerned. Collection of waste is usually confined to a few areas, the city centre and high-income neighbourhoods, and service is usually irregular. Most wastes are dumped and not properly disposed of. The sight of heaps of stinking, uncollected waste, or waste disposed of by roadsides, on open spaces, in valleys and drains is a common feature of Ghanaian urban areas.

Law and tradition require that waste generated be removed, and disposed of, by urban authorities. Uncollected urban waste is a danger to health, pollutes the environment, is a nuisance, erodes civic morals and can be a major social problem. Thus waste management is an important area of environmental management and urban governance. Private households and firms often consider themselves exempt from any obligation after removing waste from their private domain. Waste management benefits all the community, in that any resident can enjoy and benefit from the service (Atkinson et al., 1999). Thus waste management is placed squarely within the public domain as a public good, and citizens expect the metropolitan assembly to take action and keep their environment clean.

1.2.3 Waste Management in Kumasi

Waste management in Kumasi is a complex issue that has been a major feature on the priority of successive municipal chief executives and waste management groups. Generally, existing facilities including sanitary facility are inadequate to serve the people, the ever escalating volumes of solid waste generated in the Kumasi municipality is overwhelming. Problems are encounted at all levels of waste management namely; poor road network, different housing characteristics making collection in some portion infeasible, increasing waste quantities due to urbanization, inadequate and obsolete waste collection equipment. The situation creates a suitable environment for the bleeding of disease vectors such as mosquitoes, flies, cockroaches and mice. In view of this, some of the inhabitants dispose of rubbish indiscriminately such as drainage channels; in fact the recent advent of polythene bags have even worsen the pride of waste management groups as they are seen everywhere in the city.

1.2.4 Environmental Hazards Associated With Improper Waste Management

Lack of waste collection has forced people to dispose of waste in a haphazard manner on common open spaces, by road sides, in ditches and drains, especially because there is not sufficient space to dispose of waste. Besides being an eyesore and generating foul smell, waste haphazardly disposed encourages the breeding of disease transmitting flies. Unmanaged waste also blocks drains and causes seasonal flooding, leading to deleterious effects such as the damaging of roads. Scattered waste degrades the overall environment and leads to a decrease in land values (Bodin et al., 1989). People often deal with waste by burning it, leading to further air pollution. Uncollected waste reflects badly on society as a whole, and may deter tourist, leading to loss of foreign exchange.

1.2.5 Environment and Development

There are certain basic principles in land use planning that must be adhered to strictly in urban development, in order to provide practical solutions to waste management problems.

Unfortunately, cities and towns in developing countries like Ghana operate no laid down planning policies. The principles are:

- **Access to houses**: For efficient waste management, every house in cities and towns must be accessible by road. This calls for proper layouts in cities and towns in developing countries. It is easy to manage waste and provide services in a well-laid out environment, but in Ghana, haphazardness and spontaneous development characterised by overcrowding and lack of access by road, is the situation in most of the cities and towns. In Kumasi, the second largest city of Ghana, it is the old developed areas, Asafo, Ahodwo Daban, Bantama, Adum and some few other areas that have appreciable good layouts and access to houses. In contrast, the newly developed areas such as Asawasi Estates, Asokore Mampong, Buobai, Nima, Nkontomponi Aferi, Pakuso, Sawaba, Sepe Timpomu and many others do not have good layout and access to houses.
- **Control of population densities**: Research has shown that Kumasi and Accra are not over populated, rather the uneven distribution of population. The over-concentration of the population of people in the new places as mentioned above gives a clear picture of infrastructure break down of planning controls. Such places need proper urban renewal process to be able to keep them clean.

1.2.6 Waste Collection Practices in the Kumasi Metropolis

Kumasi, the capital of Ashanti region has a population of about two million (2,000,000) people and generate about one thousand, three hundred and fifty (1,350) tones of solid waste daily. Two-thirds of this waste is collected in skip containers, positioned at vantage points in the metropolis, mostly in the second and third class communities, (Waste management agency, KMA, 2013).

However, the location of these skip containers are done arbitrary, and this results in either so many clients serving only one facility resulting in over flow of waste or very few people serving one container which makes it to stay for a very long time before it get full resulting in unpleasant smell in the community. Collection of solid waste is estimated to take about 80% of the running cost of the budget estimate of waste management companies. Considering the fact that developing countries are faced with financial constraints, for infrastructure, health services, education and the rest of them, it becomes imperative to put the scant resources to good use.

Proper location of skip containers will ensure that required number of clients serve a container to avoid overflow and its associated risk of disease outbreak. Moreover, minimizing vehicle travel distance will also reduce cost of routing.

1.2.7 Solid Waste Collection Challenges in Kumasi

Kumasi metropolis has two main forms of collection methods; namely door to door collection and skip (community bin) collection. Most of the areas where community skip container collection is practiced are second (2nd) and third (3rd) class communities which are densely populated. Location of containers in these communities' posses a great challenge to inhabitants of these communities in that the containers are placed arbitrary without considering the topology of the area or the average number of people that will serve a particular container. This unscientific way of location and routing method has lead to:

i) some solid waste containers staying for a long time without getting full for collection, thereby generating pageant smell in the neighbourhood, attracting houseflies and breeding of mosquitoes which posses health challenge to humans.

ii) some of the inhabitants to dump their refuse in drains and their backyard, instead of walking long distances to access the facility (skip container).

iii) some of the containers getting full so quickly and thereby compelling inhabitants to dump around the container, because more people are accessing the facility and hence inviting domestic animals to feed on the waste with some eventually dying as a result of swallowing non bio-degradable materials such as rubber bags.

iv) waste collecting agencies uses any available root to collect solid waste and thereby spending a long time to collect few containers in a day leading to high cost of waste collection and also contributing to overstay of containers in the communities.

1.3 PROBLEM STATEMENT

More than two-thirds of all solid waste generated in the metropolis is through community skip collection method, however the method of locating these containers and collection posses a lot of problems to the environment, customers and the service companies alike. Some of these problems are listed below. One, the authorities do not have an exact figure of waste generation per capita per day to aid effective collection plan. Two, the arbitrary location of skip containers in the communities usually contribute to the indiscriminate dumping of solid waste into gutters, streams or burning which has its own environmental problems. Three, transportation of solid waste from source to the dumpsite also comes with a high cost but low productivity due to lack of proper schedule for service collectors. All these problems cumulate to the waste challenges the country faces today with its attended health issues to its citizens.

1.4 OBJECTIVES OF THE STUDY

With the categorization of communities in Kumasi metropolitan area into classes 1, 2 and 3, this study sort to find a more scientific way of collecting waste in class 3 communities where we have low income groups, unplanned housing scheme and poor road network.

The aim of the study is in three fold:

i) To model the location of skip containers to ensure that:

- statistically determine waste generation per capital per day for proper planning of waste collection in third class communities
- the distance from each customer (client) to the disposal point (skip container) is minimised;
- waste generated by these customers will optimally be equal to the capacity the skip container in every three days to reduce smell and outbreak of diseases in the community;

ii) To design a multi-objective routing model with time windows to collect waste from customers to the skip containers using tricycle.

iii) To design a vehicle fuel consumption model for comparative analysis on two time schedules.

1.5 JUSTIFICATION OF THE STUDY

Solid waste management has become a major developmental challenge in all our ten capitals cities in Ghana in recent times in which Kumasi is not an exception. This deserves not only the attention of the Metropolitan Assembly and the waste management institutions but also concerns of corporate organizations and individuals to find a lasting solution to the problem. This is because, vital human resource could be lost through poor waste management which will affect productivity in the Metropolis and the nation at large. It is in this vain that the study intends to use a more scientific approach to model the location of skip containers, collection of waste from customers with time windows and transportation of solid waste to the dumpsite.

Despite the immensity of the problem, very little research on solid waste management has been carried out in the Metropolis. The study will serve as a reference point to the Metropolitan Assembly and waste management institutions as far as solid waste management is concerned. Additionally, the study will contribute to existing body of knowledge on solid waste management and also stimulates further research on the subject in other Metropolitan Areas and Municipalities.

1.6 METHODOLOGY

Collection of solid waste from skip containers placed in strategic points in the community is a Node Routing Problem (NRP). With our objectives set, some organizations in the management of solid waste such as Zoomlion Ghana limited, waste management department of Kumasi Metropolitan Assembly (KMA), Physical Planning Department of KMA (PPD) and Ghana Statistical Service (GSS) were consulted for one information or the other. Not only that some information and references were also gathered from both libraries and the internet. The layout of the study area; Tafo Pankrono were used to determine the number of occupying houses with the help of Geographic Information Software (GIS) from Natural resources department of University of Energy and Natural Resources, Sunyani. The work is proposing the use of Mixed effect models to determine the waste generation per capita per day, Probabilistic Distance Location Model to site skip containers, use an improved Ant Colony System with time windows to collect waste from customers on the skip container and finally use a vehicle fuel consumption models to compare effective collection methods based on two elemental schedule coded in C++ .

1.7 SCOPE OF THE STUDY

Kumasi, the second highest populated city in Ghana has nine (9) sub-metropolitan assemblies. The study area has eleven (11) communities of which seven (7) of them are categorized as third class zones. The area is the smallest of the nine sub metropolitan areas in terms of land area but it is the second highest generator of solid waste after Subin. The area has a population of 157,226 with eleven communities within its domain. Four of these communities are categorised under class two whiles the remaining seven are under class three. The area shares boundary with Manhyia to the east, Suame to the west and Subin to the south. The area generates about eighty-eight tones of solid waste a day. Our study considered five of the seven third class zones namely Old Tafo, Pankrono Dome, Pankrono West, Tafo Adompom and Ahenbronum constituting about 52% of the area population. The area has the second largest market in the Kumasi metropolis, has a number of government assisted schools from basic to secondary levels and one government hospital. The people of the area are mostly traders with some small percentage being government employees.

1.8 LIMITATIONS OF THE STUDY

One of the major problems which militate against smooth running of research work in developing countries in which Ghana is not an exception is lack of data storage and . Collection of information in Ghana for research work is very difficult; this may be due to lack of resources to help acquire equipment to store information, Political reasons, Social, Geographic etc. Another limitation encountered during our study is inability of the stake holders to give research grant. The problems mentioned above were not absent during the time of data acquisition, but with dint of hard work and man support from Zoomlion Ghana limited we were able to overcome those challenges.

1.9 ORGANISATION OF THE STUDY

The study is organized into five chapters. In the first chapter, a background to the solid waste management problem in Ghana and Kumasi and its variant were examined. In chapter two mathematical formulation of Facility Location Problems (FLP), Node Routing Problem (NRP), the literature review of FLP and NRP will be look at. Next, the relevant literature on the waste collection problem, particularly the skip problems and non-skip problems which deal with node routing are reviewed. Finally, a discussion of this chapter is presented. Chapter 3 is divided into four sections. Section one will consider the employment of mixed effect model, section two looks at the mathematical formulation of our improved Probabilistic Distance Location Model, section three, consider the formulation of improved Ant Colony Heuristics, section four will look at the fuel consumption models, finally, a summary of this chapter is presented. Chapter 4 looks at how the data for the study was obtained, implementation our facility location algorithm on our case study. Next, procedures to improve a solution, both in terms of the distance travelled and in terms of the number of vehicles used are presented. Then, computational results for both procedures used on the study area tested. Finally, a summary of this chapter is presented. Summary of findings, conclusions, Recommendations and future works will be presented in chapter five.

1.10 SUMMARY

In this chapter, we looked at the management of solid waste in Ghana as a whole and that of Kumasi. We also looked at the importance of the thesis to society and the effect of uncollected waste to society. In the next chapter, we shall put forward literature on facility location, node routing problem and its variants.

CHAPTER TWO

LITERATURE REVIEW

2.0 INTRODUCTION

In this chapter, we shall present a brief review on location theory, and the contributions to the Reformulation-Linearization Technique (RLT). The literature review will be grouped into three main sub-headings. First, we shall discuss the pure location problem since this is a principal sub-problem, even for solving the more general Location Allocation Problems (LAP). Second, we shall put forward examples of heuristic techniques that have been used to solve the Vehicle Routing Problem (VRP) with time windows for deliveries are presented. This includes previous work dealing with waste collection such as arc routing, as well as node routing; particularly skip problems and non-skip problems. Finally, we shall discuss some literature on environmental hazards of uncollected solid waste.

2.1 FACILITY LOCATION PROBLEMS

In classical facility location problems, the objective is to select the best locations for the facilities from among those available, and to assign customers to open facilities in an optimal manner. Most of these problems are known to be NP-hard (Owen and Daskin,1998). The main difficulty in solving these problems arises from the non-convexity of the objective function and the existence of multiple local minima (Current et al., 2002). In this section, we shall give an introduction to three primary problems in facility location: the $p-$median problem, the uncapacitated plant location problem, and the single source capacitated plant location problem.

2.1.1 The $p-$median Problem

The simplest of facility location problems is the $p-$median problem introduced by Hakimi (1964), but studied by other researchers such as Hansen and Mladenovi´c (1997), Beasley (1985) and Ceselli and Righini (2005). In the $p-$median problem, p facilities are to be selected, such that the sum of demand-weighted distances between the customers, and the facility located nearest is minimized. In these problems, it is assumed that facilities have equal setup cost, and that every facility has enough capacity to serve all of the demand assigned to it.

The p-median problem is usually formulated as an Integer Program (IP). In order to formulate the problem the following notation is used:

$i \in I$: index of demand node,
$j \in J$: index of facility node,
d_i : demand at node i,
C_{ij} : distance between nodes i and j,
p : number of facilities to be located.
The decision variables of the model are

$$p_j = \begin{cases} 1, \text{ if facility } j \text{ is open} \\ 0, \quad \text{otherwise} \end{cases}$$

$$x_{ij} = \begin{cases} 1, \text{ if demand node } i \text{ is served by facility } j \\ 0, \quad \text{otherwise} \end{cases}$$

By using the above notation, the (IP) formulation of the $p-$median problem can be stated as:

$$\text{Minimize } \sum_{i \in I} \sum_{j \in J} d_i C_{ij} x_{ij}$$

Subject to

$$\sum_{j \in J} p_j = p \qquad (2.1)$$

$$\sum_{j \in J} x_{ij} = 1, \; i \in I \qquad (2.2)$$

$$x_{ij} \le p_j \quad i \in I, \; j \in J \qquad (2.3)$$

$$x_{ij}, \; p_j \in \{0, 1\}, \; i \in I, \; j \in J \qquad (2.4)$$

The objective function minimizes the total demand-weighted distances. Constraint (2.1) indicates that exactly p facilities are to be located. Constraint (2.2) ensures that every demand is assigned to exactly one facility, while constraint (2.3) allows assignment only to sites at which facilities have been located. Constraint (2.4) is the integrality constraint. While for fixed values of p (i.e. we want to locate exactly p facilities) the $p-$median problem can be solved in polynomial time, for variable values of p the problem is NP-hard, Garey and Johnson (1979). Thus, for problems with variable p values, meta-heuristics and approximation algorithms such as: tabu search Rosing et al., (1998), simulated annealing, Righini (1985), variable neighborhood search, Hansen and Mladenovi´c (1997), and genetic algorithm, Alp et al., (2003) have been the predominant solution techniques. Exact algorithms such as branch-and-bound have also been proposed for such problems (Galvao and Raggi, 1989).

2.1.2 The capacitated-median model

A capacitated-median model is derived from the p-median model by adding constraints that define the upper and lower capacity of potential facilities and removing (1) the constraint designating that p facilities be built. Subsequently the number of facilities to be open becomes an output of the problem.

Teixeira and Antunes (2008), the endogenous nature of the number of placed facilities is not unlike the solution provided by a Fixed Charge Problem (FCP) which identifies a subset of potential facility locations to minimize the cost of serving a number of demand nodes on a network where there are costs associated with the construction of a new facility

(Nozick, 2001; Daskin, 2008). However, this model differs in that it does not "charge" a cost for the construction of a new facility because it is assumed that the importance of this cost is secondary to the assistance of citizens in need. The cost is only a function of the demands at nodes and their distances to the closest open facility. The formulation of the capacitated-median problem is as follows, where b_j and B_j represent lower and upper capacities respectively:

$$\text{Minimize } \sum_{i\in I}\sum_{j\in J} d_i c_{ij} x_{ij} \qquad (2.5)$$

Subject to

$$\sum_{j\in J} x_{ij} = 1, \quad \forall i \in I \qquad (2.6)$$

$$x_{ij} - p_j \le 0 \quad \forall i \in I,\ j \in J \qquad (2.7)$$

$$\sum_{j} d_i x_{ij} \ge b_j p_j \quad \forall i \in I \qquad (2.8)$$

$$\sum_{j} d_i x_{ij} \le B_j p_j \quad \forall i \in I \qquad (2.9)$$

$$\sum_{k\in J | c_{ik} \le c_{ij}} x_{ik} - p_j \ge 0 \quad \forall i \in I,\ j \in J \qquad (2.10)$$

$$x_{ij},\ p_j \in \{0,1\},\ i \in I,\ j \in J \qquad (2.11)$$

Where

b_j = minimum capacity of facility at site j

B_j = maximum capacity of facility at site j

The objective function (2.5) and the constraints (2.6), (2.7) and (2.11) are identical to those described in the *p*-median model above. Constraints (2.8) and (2.9) represent the minimum and maximum capacities of the facilities in *J*. The capacity constraints introduce a means for incorporating the finiteness of the supplies located at the placed relief facilities. When used in the relief context, constraint (2.10) is added to force assignments to a node's closest facility because this property does not necessarily carry over from the *p*-median model. The capacity constraints can force demand nodes to seek aid at facilities located further away because their closest location is either at capacity or another facility's minimum capacity is yet to be met. It is possible that this constraint's rigidity can make the problem unsolvable, but it keeps assignments from occurring for the sake of meeting some as yet satisfied minimum capacity. Constraint (2.7) in conjunction with constraint (2.10), keep undesirable scenarios in an emergency relief context from occurring, such as sending members from a single neighborhood to different facilities or to a facility that is not the closest. Capacitated-Median models (CMs) have been used sparingly within the literature to this point. One study uses a minimum capacity to act as a lower threshold for placing pharmacies in Catalonia, Spain. Carreras and Serra (1999). This approach is used as a means to efficiently deliver public services to a wide number of consumers. Another paper develops a similar capacitated model, based on the Maximal Coverage Location Problem (MCLP), for placing preventive health care facilities so as to minimize the average distance traveled (Verter and LePierre, 2002). The CM allows planners to provide a more realistic representation of a facility's abilities in an emergency relief distribution context than those models that consider relief supplies to be infinite. For example, each open facility will not have an unbounded amount of ice, food, and supplies (Horner and Downs, 2007, 2008; Horner and Widener, 2009)

2.1.3 Covering models

The Maximal Covering Location Problem (MCLP) was first formulated in the mid-1970s (Church and ReVelle,1974). The MCLP seeks to find the solution to the problem of locating facilities that maximizes the coverage of demand for services within a given acceptable service distance (or response time). Because the MCLP has been shown to be extremely combinatorially complex, a series of heuristic solution procedures have been developed (Galvao et al., 2000). Additionally, the MCLP can be seen as a variant formulation of other prominent location models including the p-Median model and the Location Set Covering model (Church and ReVelle, 1976). Variations of the MCLP have been formulated to include workload capacities, or to maximize coverage and minimize distances to demands outside the maximum covering distance (Church et al.,1991). Covering models have been applied to the location of emergency warning sirens (Current and Okelly, 1992).

2.1.4 Competitive Facility Location Models

One subgroup of location-allocation models deals with the location of plants, warehouses, retail and industrial or commercial facilities which operate in a competitive environment. All competitive location models attempt to estimate the market share capture by each competing facility in order to optimize its location. The best location for a new facility is at the point at which its market share is maximized. The first modern paper on competitive facility location is generally agreed to be Hotelling (1929) on duopoly in a linear market. Hotelling considered the location of two competing facilities on a segment (two ice-cream vendors along a beach trip). The distribution of buying power along is assumed uniform and customers patronize the closest facility. The following developments of competitive location models can be categorized in two groups, as a function of spatial representation:

i) **Continuous competitive location models-** where the potential location of the facilities can be anywhere in the plane.

ii) **Discrete competitive location models**- where facilities are allowed to locate at a finite set of possible locations on a network

2.1.4.1 Continuous Competitive Location Models

Continuous Competitive Location models for retail firms are an extension's of Hotelling's approach in the continuous planar space, where the retail firm is planning to open a chain of outlets in a market in which a chain already exists. In these models, the allocation of customers to facilities is made using Hotelling's proximity assumption-each facility attracts the customers close to it. In these models, the market share attracted by each facility is calculated and then, the best location for the new facility is found. A good representative of these models is the location-allocation Market Share Model (MSM) developed by (Goodchild,1984). In that model, a retail firm is planning to open a chain of outlets in a market in which a competing chain already exists. The entering firm's goal is to maximize the total market share capture by the entire chain. A good review of these type of location-allocation models can be found in (Ghosh and Harche, 1993)

2.1.4.2 Discrete Competitive Location Models

From the late seventies, considerations on the interaction between competitive facilities in discrete space have developed following several approaches. One of the first questions that has been addressed by several authors is the existence or (not) of a set of locations on the vertices of a network that will ensure a Nash equilibrium, that is a position where neither firms have incentives to move. Wendell and Mckelvery (1981) considered the location of two competitive firms with one server each and tried to find a situation where a firm would capture at least 50% of the market regardless of the location of its competitor. Results showed that there was not general strategy for the firm that would ensure this capture if locating at vertices of the network. The authors did not develop a generic algorithm for finding solutions, but they looked at the possible locational strategies. The authors also examined the problem in a tree.

Hakimi (1986) analyzed extensively the problem of competitive location on vertices and proved that, under certain mathematical conditions such as concave transportation cost functions, that there exists a set of optimal locations on the vertices of the network.

2.2 HEURISTICS FOR DELIVERY PROBLEMS

Basically the VRP for delivery problems can be defined as delivering goods to a number of customers who have placed orders for a certain quantity of these goods from a central depot. Due to some constraints such as load, distance and time, a single vehicle may not be able to serve all the customers. The problem then is to determine the number of vehicles needed to serve the customers as well as the routes that will minimize the total distance travelled by the vehicles. Many heuristics have been introduced in the literature for searching for good solutions to the problem.

For instance the savings algorithm of Clarke and Wright (1964), the sweep algorithm of Gillett and Miller (1974), the cluster-first, route-second heuristic of Fisher and Jaikumar (1981), the path scanning heuristic of Golden, De Armon and Bakers (1983), and the route-first, cluster-second heuristic of Beasley (1983). A detailed survey of major developments in heuristics as well as exact algorithms for solving the VRP can be found in the recent paper by Laporte (2009), but this is still growing research area.

2.2.1 Vehicle Routing Problems for Collection

Essentially, the VRP for collection is dealing with the same type of constraints as in a delivery problem when constructing vehicle routes. Thus, this problem also attempts to determine the number of vehicles needed to serve the customers as well as the routes that will minimize the total distance travelled by the vehicles. However, the vehicle for the collection problem is empty when it starts from the depot, whereas the vehicle for the delivery problem begins its route loaded with customers' goods that need to be delivered. In the collection problem vehicles will collect goods from a set of customers and return to the depot at the end of the working day.

Some applications of collection problems that can be found in the literature are cash collection Lambert et al., (1993), collection of raw materials for multi-product dehydration plants Tarantilis and Kiranoudis, (2001a), and milk collection (Caramia and Guerriero, 2010).

2.2.2 Waste Collection Vehicle Routing Problem

Dealing with a waste collection problem is different from the collection problem as discussed in the previous section. There is an additional constraint that needs to be considered in solving this problem. Instead of returning to the depot to unload the collected goods, in a waste collection problem, vehicles need to be emptied at a disposal facility before continuing collecting waste from other customers. Thus, multiple trips to the disposal facility occur in this problem before the vehicles return to the depot empty, with zero waste. A complication in the problem arises when more than one disposal facilities are involved. Here one needs to determine the right time to empty the vehicles as well as to choose the best disposal facility they should go to so that the total distance can be minimized. For example it may not be optimal to allow the collection vehicle to become full before visiting a disposal facility. Increasing quantities of solid waste due to population growth, especially in urban areas, and the high cost of its collection are the main reasons why this problem has become an important research area in the field of vehicle routing. In the next two sections, previous work dealing with waste collection as arc routing problems and as node routing problems are reviewed.

2.2.3 Arc Routing Problems (ARP)

Due to the large number of residential waste that have to be collected, the problem is often dealt with as an arc routing problem, whereas the collection of commercial waste is dealt with as a node routing problem. In this section some of the previous work dealing with arc routing problems for waste collection is reviewed.

Chang, Lu and Wei (1997) applied a revised multi-objective Mixed-Integer Programming model (MIP) for analyzing the optimal path in a waste collection network within a Geographic Information System (GIS) environment. They demonstrated the integration of the MIP and the GIS for the management of solid waste in Kaohsiung, Taiwan. Computational results of three cases particularly the current scenario, proposed management scenario (without resource equity consideration) and modified management scenario (with resource equity requirement) are reported. Both the proposed and the modified management scenarios show solutions of similar quality. On average both scenarios show a reduction of around 36.46% in distance travelled and 6.03% in collection time compared to the current scenario.

Mourao and Almeida (2000) solved a Capacitated Arc Routing Problem (CARP) with side constraints for a refuse collection VRP using two lower-bounding methods to incorporate the side constraints and a three-phase heuristic to generate a near optimal solution from the solution obtained with the first lower-bounding method. Then, the feasible solution from the heuristic represents an upper bound to the problem. The heuristic they developed is a route-first, cluster-second method.

Bautista and Pereira (2004) presented an ant algorithm for designing collection routes for urban waste. To ascertain the quality of the algorithm, they tested it on three instances from the capacitated arc routing problem literature and also on a set of real life instances from the municipality of Sant Boi del Llobregat, Barcelona.

Mourao and Amado (2005) presented a heuristic method for a mixed CARP, inspired by the refuse collection problem in Lisbon. The proposed heuristic can be used for directed and mixed cases. Mixed cases indicate that waste may be collected on both sides of the road at the same time (i.e. narrow street), whereas waste for the directed cases only can be collected on one side of the road. The authors reported computational results for the directed case on randomly generated data and for the mixed case on the extended CARP benchmark problems of Lacomme et al., (2002). Computational results for the directed problem, involving up to 400 nodes show the gap values (between their lower bound and upper bound values computed from their heuristic method) varying between 0.8% and 3%. For the mixed problem, comparison results with four other heuristics namely, extended Path-Scanning, extended Ulusoys, extended Augment-Merge and extended Merge are reported. They stated that they were able to get good feasible solutions with gap values (between the lower bound values obtained from Belenguer et al., (2003) and their upper bound values) between 0.28% and 5.47%.

Li et al., (2008) solved a solid waste collection in Porto Alegre, Brazil which involves 150 neighbourhoods, with a population of more than 1.3 million. They designed a truck schedule operation plan with the purpose of minimizing the operating and fixed truck costs. In this problem the collected waste is discarded at recycling facilities, instead of disposal facilities. Furthermore, the heuristic approach used in this problem also attempts to balance the number of trips between eight recycling facilities to guarantee the jobs of poor people in the different areas of the city who work at the recycling facilities. Computational results indicate that they reduce the average number of vehicles used and the average distance travelled, resulting in a saving of around 25.24% and 27.21% respectively.

Mourao et al., (2009) proposed two-phase heuristics and one best insertion method for solving a Sectoring Arc Routing Problem (SARP) in a municipal waste collection problem. In SARP, the street network is partitioned into a number of sectors, and then a set of vehicle trips is built in each sector that aims to minimize the total duration of the trips. Moreover, workload balance, route compactness and contiguity are also taken into consideration in the proposed heuristics.

Ogwueleka (2009) proposed a heuristic procedure which consists of a route first, cluster second method for solving a solid waste collection problem in Onitsha, Nigeria. Comparison results with the existing situation show that they use one less collection vehicle, a reduction of 16.31% in route length, a saving of around 25.24% in collection cost and a reduction of 23.51% in collection time. In some cases, waste collection problems are solved as node and arc routing problems.

Bautista, Fernandez and Pereira (2008) transformed the arc routing into a node routing problem due to the road constraint such as forbidden turns for solving an urban waste collection problem in the municipality of Sant Boi de Llobregat, Barcelona with 73917 inhabitants using an ant colonies heuristic which is based on nearest neighbour and nearest insertion methods. Computational results show that both methods produce less total distance compared with the current routes. In particular, routes from nearest neighbour and nearest insertion travel 35% and 37% less, respectively.

Santos et al., (2008) presented a Spatial Decision Support System (SDSS) to generate vehicle routes for multi-vehicle routing problems that serve demand located along arcs and nodes of the transportation network. This is mainly due to some streets which are too narrow for standard-sized vehicles to traverse, thus the demand along arcs as well as at network nodes are required for solving waste collection in Coimbra, Portugal.

2.2.4 Node Routing Problems

If the location of every collection point is known when solving the waste collection problem then it is a node routing problem. Vehicles will travel from the depot to a customer and then to another customer, etc, to collect waste based on the sequence of visits on the vehicle route. This sequence includes trips to disposal facilities to empty the vehicle and the last visit would be the depot. In the next section, previous work dealing with node routing problems, particularly the skip problems and non-skip problems are reviewed.

Sbihi and Eglese (2007) discussed the importance attached to waste management and collection in terms of the "green logistics" agenda.

2.2.5 Container/Skip Problems

De Meulemeester et al., (1997) dealt with the problem of delivering empty skips and collecting full skips from customers. Vehicles can carry only one skip at a time, but skips can be of different types. They stated that the problem was first considered by Cristallo (1994). Their solution approach is based on two simple heuristics and an enumerative approach. They reported computational experience with randomly generated problems involving up to 160 customers and a real-world problem involving thirty (30) customers.

Bodin et al., (2000) considered a sanitation routing problem they called the roll on-roll off vehicle routing problem. In this problem trailers, in which waste is collected, are positioned at customers. A tractor (vehicle) can move only a single trailer at a time. Tractor trips involve, for example, moving an empty trailer from the disposal facility to a customer and collecting the full trailer from the customer. A key aspect of their work is that they assumed the set of trips to be operated is known in advance (so the problem reduces to deciding for these trips how they will be serviced by the tractors). The authors presented four heuristic algorithms and gave computational results for problems involving up to 199 trips and a single disposal facility.

Archetti and Speranza (2004) developed a heuristic algorithm called SMART-COLL for a problem motivated by waste collection in Brescia, Italy. In their problem, skips are collected from customers and the vehicle can carry only one skip at a time. They call the problem the 1-skip collection problem. They considered skips of different types and time windows are imposed on both the customers and the disposal facilities. Computational experience was reported for real world data involving fifty-one (51) customers and thirteen (13) disposal facilities.

Teixeira et al., (2004) developed a three-phase heuristic technique to create collection routes for the collection of urban recyclable waste in the central region of Portugal. Three types of waste in separate containers must be collected individually. The collected containers are emptied at two central depots and vehicles start and terminate a route in one of these depots. Computational results show that the total distance travelled of the proposed solution is 29% less than the historical distance.

Baldacci et al., (2006) dealt with an extension of the problem considered by Bodin et al., (2000). The authors considered multiple disposal facilities as well as inventory facilities at which empty trailers are available. They presented an approach based on regarding the problem as a time constrained vehicle routing problem on a directed multi-graph. Computational results for problems involving up to seventy-five (75) customers and two disposal facilities were presented.

Blanc et al., (2006) presented a paper dealing with the collection of containers from end-of-life vehicle dismantlers in the Netherlands. In the problem they considered vehicles can carry two containers at a time. Their heuristic is a two-step procedure, first generating candidate routes, then selecting from these routes using a set partitioning approach. They reported potential cost savings of over 18% compared with the current system.

Even though Aringhieri et al., (2004) solved a skip problem in Perugia, Italy, the authors solved the problem as an arc routing problem due to the different types of containers used to collect different waste. Therefore, the selected arcs for the vehicle to travel depend on the service requests which are characterized by types of waste, container and the collection point.

2.2.6 Non-Skip Problems

The majority of papers in the literature for non-skip problems are case study papers, focusing on results obtained when algorithms are applied to real world data. Only a few of these papers report computational experience with publicly available waste collection test instances.

Chang and Wei (2002) presented a real-life comparative study between a revised heuristic algorithm and an optimization technique particularly, minimum spanning tree and integer programming model, for investigating the effectiveness of vehicle routing and scheduling in a solid waste collection system. To illustrate the comparison of both techniques, a case study in the city of Kaohsiung, Taiwan which involves eight hundred and fifty-four (854) collection points was presented. As expected in terms of cost-saving perspective, a set of near optimal solutions from the heuristic algorithm are not as economic as the optimal solutions from the optimization scheme. Computational results showed that the total number of collection vehicles and the total number of crews needed for the optimal solutions are fourteen (14) and fifty-six (56) respectively whereas, the heuristic solutions require twenty-two (22) vehicles and eighty-four (84) crews in total. However in terms of total routing distance and collection time, the heuristic solutions show reductions of 12.7% and 0.9%, respectively even though they required more vehicles. Moreover, the authors stated that the heuristic algorithms allow the analysis of a much larger service area of interest within the same computational time as compared to the performance of an optimization model. In their view if an improvement in high performance computing comes into reality in the future it may overcome the present computational limitation of the optimization model.

Tung and Pinnoi (2000) proposed a heuristic procedure to solve a waste collection problem in Hanoi, Vietnam. In their problem there are time windows associated with collection from customers and their heuristic first constructs routes based on an approach due to Solomon (1987) and then improves them. They reported computational experience indicating that they can achieve an operating cost saving of 4.6% when compared with the current situation.

Angelelli and Speranza (2002a) presented an algorithm based on tabu search for the periodic version of the problem where routes must be designed over a planning horizon of more than one time period so as to meet customer service requirements. Their approach is based on the tabu search algorithm for vehicle routing ,Cordeau et al.,(1997). Computational results were presented for problems involving between two and six days in the planning horizon.

Angelelli and Speranza (2002b) proposed a model that fits three different waste collection systems to estimate operational costs. Their solution procedure is based on Angelelli and Speranza (2002a) and results were presented relating to two case studies: Val Trompia, Italy and Antwerp, Belgium.

Sahoo et al., (2005) reported how they developed a system called Waste Route to reduce operating costs for a large company involved in waste collection. They gave one example of an area that went from ten routes to nine, improving route productivity (as measured by the amount collected per hour) by some 11%. The heuristic used for the Waste Route system of Sahoo et al., (2005) is fully described in Kim et al., (2006). Customers have time windows for collection, and there are multiple disposal facilities, as well as a driver rest period. The authors extended Solomon’s (1987) insertion heuristic to cope with both multiple disposal facility visits and the driver rest period and used it to construct routes, which are improved using simulated annealing and a local search exchange procedure called CROSS Taillard et al., (1997). As their work is motivated by the practical context reported in Sahoo et al., (2005) they discussed a number of issues with solutions produced by this heuristic: route compactness, workload balancing and computation time. In order to deal with these issues they also presented a heuristic based on capacitated clustering that generates clusters based on the estimated number of vehicles required, and then routes customers within each cluster. Computational results were presented for ten problem instances, derived from real world data, involving up to two thousand one hundred (2100) customers that the authors make publicly available.

Agha (2006) used a mixed integer programming model (MIP) to optimize the routing system for Deir Al-Balah, Gaza Strip. The problem involves 58 pick-up points, one disposal facility and three collection vehicles. Comparison results with the existing routing system are presented in terms of the total distance travelled. The result shows that the solution involves 23.4% less compared to the existing distance. Thus, the monthly cost can be reduced by approximately US$1140.

Ghose et al., (2006) combined both skip and non-skip problems to determine the minimum cost/distance efficient collection paths for transporting solid waste to the landfill for the Asansol Municipality Corporation (AMC) of West Bengal State, India. In total, the problem involves one thousand four hundred and five (1405) collection bins with three different sizes. Three types of vehicles are used for the collection of these bins. The vehicle type-A and the vehicle type-B serve as skip and non-skip problems, respectively. While the vehicle type-C collects the waste from C-type bins and disposes the waste at its nearest A-type bin. The vehicle will repeat the process until all the waste from C-type bins is collected. Then the vehicle will return to the garage from the location of the last A-type bin served. No comparison with the routing system practised by AMC is made. However, they compared the current annual operating cost AMC is spending with their estimated operating cost with respect to the proposed solution. Comparison result indicates that AMC may save about 66.8% every year if the proposed solution is applied.

Martagan et al., (2006) applied classical MIP for a case study in Turkey for transporting metal waste from 17 factories to five potential containers, and from containers to a single disposal centre. The monthly cost of the proposed optimal solution is approximately $48000. Comparison routes with those current practised are not presented. Data is provided by TOSB (TAYSAD Organized Industrial Region) where TAYSAD is an abbreviation of Association of Automotive Parts & Components Manufacturers.

Nuortio et al., (2006) considered a problem based on waste collection in two regions of Eastern Finland. Their problem includes time windows and they solved the problem using Guided Variable Neighbourhood Thresholding.

Apaydin and Gonullu (2007) used Route View ProTM software for constructing waste collection route in Trabzon, Turkey. The collection involves seven hundred and seventy-seven (777) containers and one disposal site. The solution is compared with the present routes in terms of the total route distances, travelling time as well as the monthly cost. Comparison results indicate that their routes outperform the present. Both distance and travelling time are reduced by up to 59% and 67%, respectively whilst the monthly cost is decreased by up to 24.7%.

Karadimas, Papatzelou and Loumos (2007) presented an Ant Colony System (ACS) for determining waste collection routes for the Municipality of Athens (MoA). The collection involves seventy-two (72) loading spots. Comparison results with the empirical method. The route length of the empirical model is 9850, whilst the ACS route is 7328. Thus, the improvement is approximately 25.6%.

Ombuki-Berman et al., (2007) presented a multi-objective genetic algorithm that uses a crossover procedure (Best Cost Route Crossover), they reported results from their approach using the test problems of Kim et al., (2006), but no computation times were given.

Alagöz and Kocasoy (2008) considered health waste collection in Istanbul. They used a commercial vehicle routing package to consider a number of scenarios relating to the type of facility used for waste disposal. McLeod and Cherrett (2008) considered a problem relating to waste collection in the UK. They used a commercial vehicle routing package and reported that vehicle mileage could be reduced by up to 14%.

Coene et al., (2008) proposed several heuristic algorithms for a routing problem of a Belgian company collecting waste at slaughterhouses, butchers and supermarket. The company is responsible for collecting high-risk and low-risk waste categories of animal waste. Both wastes need to be collected separately.

Komilis (2008) presented two mixed integer-linear programming models particularly time based optimization model and cost optimization model for the waste collection problem in Athens. The waste is collected from the source nodes and taken to potential intermediate nodes, namely Waste Production Nodes (WPN) and Waste Transfer Stations (WTS), respectively and finally to the landfill as a sink node. The cost modelling approach used in this work has similarities with the cost optimization model used in

Badran and El-Haggar (2006) particularly in calculating fuel and maintenance costs as well as labour cost. The problem involves seven WPN, three WTS and one landfill. However, WTS may or may not be included in the optimal path, depending on the solution.

Arribas et al., (2010) proposed a methodology for designing an efficient urban waste collection for the west-central zone of the Municipality of Santiago using a combination of mathematical modelling such as linear integer programming and a tabu search algorithm in a GIS environment. The collection involves one thousand six hundred (1600) bins and the comparison results indicate that the proposed routing system manages to reduce 50% of current monthly cost spent on waste collection system with a reduction of 57% in the number of vehicles as well as a reduction of 57% in the number of workers needed to complete the collection.

Chalkias and Lasaridi (2009) used Arc GIS Network Analyst in their work for the waste collection in the Municipality of Nikea (MoN), Athens, Greece. The problem involves 501 collection bins and one disposal site. Besides constructing collection routes, replacing and reallocating the waste collection bins is also taken into consideration. These two scenarios are compared with the current routes. Computational results demonstrate that both scenarios provide savings in terms of collection time and total travel distance. The first scenario (constructing routes with the current location of the waste bins) saves around 3.0% in collection time and 5.5% in distance travelled, whereas the second scenario (constructing routes after replacing and reallocating the waste bins) saves around 17.0% in collection time and 12.5% in distance travelled.

Hemmelmayr et al., (2009) presented a paper motivated by a real world waste collection problem. They consider a periodic problem, where routes must be designed over a multi-day planning horizon so as to meet customer service requirements. The authors considered a number of constraints motivated by their underlying application and in particular in their application the vehicle need not return to the depot empty. They used dynamic programming to sequence disposal facility visits within a variable neighbourhood search approach. Computational results are presented for instances, involving up to 288 customers, derived from vehicle routing problems given in the literature.

Zamorano et al., (2009) attempted to reduce the waste collection management costs in Churriana de la Vega, Spain. This objective includes reducing fuel consumption by minimizing the travel time of the collection routes using Arc GIS Network Analyst. Computational results show reductions of 32.3% in travelling time compared to the current routes.

2.3 ALGORITHMS FOR THE VEHICLE ROUTING PROBLEM (VRP)

Since the VRP is an NP-hard problem, many approximation algorithms have been proposed in the literature. These algorithms can be classified into three groups: construction algorithms, improvement algorithms, and meta-heuristics.

2.3.1 Construction Algorithms

Construction algorithms are used to build an initial feasible solution for the problem. Some authors build a feasible solution by inserting unrouted customers iteratively into current partial routes according to some specific criteria, such as minimum additional distance or maximum savings, until the route's scarce resources (e.g. capacity) are depleted

Cordeau et al., (1999). These types of algorithms are classified as either sequential or parallel algorithms. In a sequential algorithm routes are built one at a time whereas in a parallel algorithm many routes are constructed simultaneously.

2.3.2 Sequential Construction Algorithms

Sequential construction algorithms are mostly based on the *Sweep Heuristic* Gillet and Miller, (1974) and the *Savings Heuristic* Clarke and Wright, (1964). In the sweep heuristic, routes are constructed as an angle sweeps the location of nodes on a 2D space. In the savings heuristic, first routes are constructed in a predefined quantity and then new nodes are added to available nodes in order to obtain maximum savings.

Baker and Schaffer, (1986) proposed the first sequential construction algorithm. The algorithm is based on savings heuristic, and starts with all possible single customer routes in the form of depot – *i* – depot. Then two routes with the maximum saving are combined at each iteration. The saving between customers *i* and *j* is calculated as: $s_{ij} = d_{i0} + d_{0j} - G.d_{ij}$ where *G* is the route form factor and d_{ij} is the distance between nodes *i* and *j*.

Solomon, (1987) proposed *Time Oriented Nearest Neighborhood Heuristic*. Every route is initialized with the customer closest to the depot. At each iteration unassigned customer that is closest to the last customer is added to the end of the route. When there is no feasible customer, a new route is initialized.

2.3.3 Parallel Construction Algorithms

Solomon, (1987) proposed a *Giant-Tour Heuristic*. In this heuristic, first of all, a giant route is generated as a travelling salesman tour without considering capacity and time windows. Then, it is divided into number of routes. Potvin and Rousseau, (1995) proposed parallelization of the *Insertion Heuristics*. Each route is initialized by selecting the farthest customer from the depot as a centre customer. Then, the best feasible insertion place for each not yet visited customer is computed. Customers with the largest difference between the best and the second best insertion place are inserted to the best feasible insertion place.

Antes and Derigs (1995) proposed another parallel algorithm based on the Solomon's heuristic. Offers comes to the customers from the routes, unrouted customers send a proposal to the route with the best offer, and each route accepts the best proposal.

2.4 META HEURISTICS

In order to escape local optima and enlarge the search space, meta heuristic algorithms such as simulated annealing, tabu search, genetic algorithm, and ant colony algorithm have been used to solve the VRP (Bräysy and Gendreau, 2001).

2.4.1 Simulated Annealing

Simulated Annealing (SA) is a stochastic relaxation technique. It is based on the annealing process of solids, where a solid is heated to a high temperature and gradually cooled in order to crystallize Bräysy and Gendreau, (2001). During the SA search process, the temperature is gradually lowered. At each step of the process, a new state of the system is reached. If the energy of the new state is lower than the current state, the new solution is accepted. But if the energy of the new state is higher, it is accepted with a certain probability. This probability is determined by the temperature. SA continues searching the set of all possible solutions until a stopping criterion is reached.

Thangiah et al., (1994) used λ-interchange with λ=2 to define the neighbourhood and decrease the temperature after each iteration. In case the entire neighbourhood has been explored without finding and accepting moves the temperature is increased.

Chiang and Russell (1996) proposed three different SA methods. First one uses modified version of the *k*-node interchange mechanism and second uses λ-interchange with λ=1. The third is based on the concept of tabu list of Tabu Search.

2.4.2 Tabu Search

Tabu Search (TS) presented by Glover (1993) is a memory based local search heuristic. In TS, the solution space is searched by moving from a solution *s* to the best solution in its neighbourhood *N*(*s*) at each iteration. In order to avoid from a local optimum, the procedure does not terminate at the first local optimum and the solution may be deteriorated at the following iteration. The best solution in the neighbourhood is selected as the new solution even if it is poorer. Solutions having the same attributes with the previously searched solutions are put into tabu list and moving to these solutions is forbidden. This usually prevents making a move to solutions obtained in the last *t* iterations. TS can be terminated after a constant number of iterations without any improvement of the overall best solution or a constant number of iteration. Garcia et al., (1994) applied TS to solve VRP for the first time. The authors generated an initial solution using Solomon's insertion heuristic and search the neighbourhood using 2-opt and Or-opt. Garcia et al., (1994) parallelized the TS using partitioning strategy. One processor is used for controlling the TS while the other is used for searching the neighbourhood.

2.4.3 Genetic Algorithm

The Genetic Algorithm (GA) is based on the Darwinian concept of evolution. Solutions to a problem are encoded as chromosomes and based on their fitness; good properties of solutions are propagated to a next generation (Vacic and Sobh, 2002). The creation of the next generations involves four major phases:

i) Representation: The significant features of each individual in the population are encoded as a chromosome.

ii) Selection: Two parent chromosomes are selected from the population.

iii) Reproduction: Genetic information of selected parents is combined by crossover and two offspring of the next generation are generated.

iv) Mutation: The gene sequence of small number of newly obtained is randomly swapped.

A new generation is created by repeating the selection, reproduction, and mutation phases until a specified set of new chromosomes have been created. Then the current population is set to the new population of chromosomes. Thangiah et al., (1991) applied the GA to VRP for the first time. GA is proposed to find good clusters of customer. The routes within each cluster are then constructed with a cheapest insertion heuristic and λ–interchange are applied.

2.4.4 Ant Colony Optimization (ACO)

Ant Colony Optimization is one of the newest metaheuristic for the application to Colony Optimization (CO) problems. The basic ideas of ACO were introduced in Dorigo,

(1992) and successively extended in Dorigo et al., (1999). In this section we present the description of ACO given in Dorigo and Di Caro, (1999). ACO was inspired by the foraging behavior of real ants. This behavior—as described by Deneubourg et al., (1990) enabled ants to find shortest paths between food sources and their nest. Initially, ants explore the area surrounding their nest in a random manner. As soon as an ant finds a source of food, it evaluates quantity and quality of the food and carries some of this food to the nest. During the return trip, the ant deposits a pheromone trail on the ground. The quantity of pheromone deposited, which may depend on the quantity and quality of the food, will guide other ants to the food source. The indirect communication between the ants via the pheromone trails allows them to find the shortest path between their nest and food sources. This functionality of real ant colonies is exploited in artificial ant colonies in order to solve CO problems.

In ACO algorithms the pheromone trails are simulated via a parametrized probabilistic model that is called the *pheromone model*. The pheromone model consists of a set of model parameters whose values are called the *pheromone values*. The basic ingredient of ACO algorithm is a constructive heuristic that is used for probabilistically constructing solutions using the pheromone values.

2.4.4 Variable Neighbourhood Search Method

Variable Neighbourhood Search (VNS) was first introduced in the literature by Hansen and Mladenovic' (1997) for solving combinatorial problems. It is a robust and effective metaheuristic which can be seen from large number of successful applications. The basic idea is the systematic change of neighbourhood within a local search. Contrary to the other metaheuristics based on local search, VNS does not follow a trajectory but explores increasingly distant neighbourhoods of current incumbent solution, and jumps from a solution to a new one if and only if improvement has been made. VNS systematically exploits the concept of neighbourhood change, both in descent to local minima and in escape from the valleys which contains them. Two complementary concepts are combined to achieve this: local search and shaking. First, it is necessary to define a suitable neighborhood structure for the solution space. The method is constructed firstly to find local minima for running neighborhood by using local search. When this is achieved, the neighborhood is changed so that other parts of the solution space can be searched. Let N^k $(k = k_{\min}, \ldots, k_{\max})$ be a finite set of neighborhoods, where $N^k(S)$ is the set of solutions in the kth neighborhood of the solution ***S***. The simplest and most common choice is a structure in which the neighborhoods have increasing cardinality:

$\left|N^{k_{\min}}(S)\right| < \left|N^{k_{\min}+1}(S)\right| < \ldots < \left|N^{k_{\max}}(S)\right|$. Given an incumbent S and an integer

$k \in \{k_{\min}, \ldots, k_{\max}\}$ associated to a current neighborhood, a feasible solution S' is generated in $N^k(S)$, and a local search is then applied to S' in order to obtain a possibly better solution S''. If S'' is better than S' then S'' becomes the new incumbent and the next search begins at the first neighborhood $N^{k_{\min}}$; otherwise, the next neighborhood in the sequence is considered in order to try to improve upon solution S. Should the last neighborhood $N^{k_{\max}}$ be reached without a solution better than the incumbent being found, the search begins again at the first neighborhood $N^{k_{\min}}$ until a stopping condition, e.g., a maximum number of iterations, is satisfied. Research areas below are some of the successful VNS applications:

- mixed integer programming (Lazic et al., 2010);

- minimum labelling Steiner tree (Consoli et al., 2009);
- bandwidth reduction (Mladenovic et al., 2010);
- variable selection and determination (Pacheco et al., 2007);
- container loading (Pareno et al., 2010);
- nurse rostering (Burke et al., 2010);
- quadratic assignment problem (Zhang et al., 2005).
- capacitated arc routing problem (Amponsah and Salhi, 2003)

2.5 ENVIRONMENTAL AND HEALTH IMPACT OF SOLID WASTE DISPOSAL

Uncollected waste is visible to the public and makes waste collection and disposal potentially an explosive political issue. Besides being an eyesore, it generates foul smell and bacteria. Uncollected urban waste is a danger to health, pollutes the environment, is a nuisance, erodes civic morals and can be a major social problem (Amponsah and Salhi, 2004). Solid waste can be classified into different types, depending on their source; household waste is generally classified as municipal waste; industrial waste as hazardous waste, and biomedical waste or hospital waste as infectious waste. The term "solid waste" means any garbage refuse, or sludge from a waste treatment plant, water supply treatment plant, or air pollution control facility and other discarded material, including solid, liquid, semisolid, or contained gaseous material resulting from industrial, commercial, mining, and agricultural operations (US Law-Solid Waste Act 2, 1999).

The term "disposal" means the discharge, deposit, injection, dumping, spilling, leaking, or placing of any solid waste or hazardous waste into or on any land or water so that such solid wastes, hazardous wastes, or any constituent thereof may enter the environment or be emitted into, Journal of Sustainable Development in Africa (Volume 12, No.7, 2010) ISSN: 1520-5509, Clarion University of Pennsylvania, Clarion, Pennsylvania) the air or discharged into any waters, including ground waters, from community activities (US Law-Solid Waste Act 2, 1999). Solid waste disposal sites are found on the outskirts of the urban areas, turning into the child sources of contamination due to the incubation and proliferation of flies, mosquitoes, and rodents; that, in turn, are disease transmitters that affect population's health, which has its organic defenses in a formative and creative state. The said situation produces gastrointestinal, dermatological, respiratory, genetic, and several other kind of infectious diseases. Consequently, dumping sites have a very high economic and social cost in the public health services, and have not yet been estimated by governments, industries, and families. The group at risk from the unscientific disposal of solid waste include – the population in areas where there is no proper waste disposal method, especially the pre-school children; waste workers; and workers in facilities producing toxic, and infectious material. Other high-risk groups include the population living close to a waste dump and those whose water supply has become contaminated, either due to waste dumping or leakage from landfill sites. Uncollected solid waste also increases risk of injury and infection.

2.5.1 Different Ways of Solid Disposal

The UN Environmental Protection Agency (2006) stated that, incineration is the process of destroying waste material by burning it. Incineration is often alternatively named "Energy-from-waste" or "waste-to-energy"; this is misleading as there are other ways of recovering energy from waste that do not involve direct burning. Incineration is carried out both on a small scale by individuals and on a large scale by industries. It is recognized as a practical method of disposing of hazardous waste materials, such as biological medical waste. Many entities now refer to disposal of wastes by exposure to high temperatures as thermal treatment.

Marshal (1995) stated that, waste materials that are organic in nature, such as plant material, food scraps, and paper products, are increasingly being recycled. These materials are put through compost and/or a digestion system to control the biological process to decompose the organic matter and kill pathogens. The resulting stabilized organic material is then recycled as mulch or compost for agricultural or landscaping purposes. What to do with solid waste has long troubled governments, industries, and individuals but, in recent years, for citizens of the United States and other highly developed, rich nations, solid waste disposal has become a source of galloping trouble.

In 1920, the United States cities or towns of public refuse disposal service were responsible for 2.1 pounds of solid waste per day. Then during the 1970s the wiser developed countries began to institute the “polluter pays” principle, in which those who were responsible for environmental degradation were charged with putting it right. This was because there was no proper solution for management of waste Renzoni, (1994).

According to Medina (2002), the major models of disposal of solid waste in the United States are land filling or dumping and incineration. People want their refuse taken away and do not want it disposed of near their habitat, or at least not to see or smell it. However, the European countries have resolved to improving land disposal practices for solid wastes (including sludge), which may reduce the adverse environmental effects of such disposals and other aspects of solid waste disposals on land. This includes means of reducing the harmful environmental effects of earlier and existing landfills, means for restoring areas damaged by such earlier or existing landfills, means for rendering landfills safe for purposes of construction, and other uses and techniques of recovering materials and energy from landfills impact on the surroundings. The closure of existing open dumpsites and the introduction of sanitary landfill is an urgent priority everywhere in the developing world. Even where complementary disposal technologies, such as composting or incineration (waste to energy plants), are practiced, a landfill is still required and is the backbone of any sustainable disposal system.

Matching grants designed to encourage landfill investments and sustainable operations may be an appropriate instrument to consider, primarily because the environmental damages and benefits tend to spill over into neighboring municipalities and regions, or into underlying groundwater resources, Daniel (1999). This statement is true. The reason simply being because waste in the landfills is not properly managed, this results to the impacts on the environment. Medina (2002) supported the US Environmental Protection Agency. The authors states that pollution is not directly transferred from land to people, except in the case of dusts and direct contact with toxic materials. Pollutants deposited on land usually enter the human body through the medium of contaminated crops, animals, food products, or water. Land pollution can also damage terrestrial ecosystems, resulting in the deterioration of the conservation of the environment. Impacts on Residents According to Marshal (1995), open dumpsites are a major problem to the environment, especially on the air that the people inhale. Dumpsites emit obnoxious odours and smoke that cause illness to people living in, around, or closer to them.

According to Wrensh (1990), dumpsites may be a source of airborne chemical contamination via off site migration of gases and the particles and chemicals adhering to dust, especially during the period of active operation of the site. Contamination of soil and groundwater may lead to direct contact or pollution of indoor air for example in the case of volatile organic chemicals into basements of nearby residents and in the case of consumption of home grown vegetables as well. Wrensh (1990) further stated that in some sites, volatile organic chemicals have been detected in odored air of homes nearby dumpsites. In a number of community health surveys, a wide range of health problems, including respiratory symptoms, irritation of the skin, nose, and eyes, gastrointestinal problems, psychological disorders, and allergies, have been discovered. A number of researches have been carried out in response to concerns from the public, often triggered by nuisances caused by emissions of volatile organic compounds.

According to Dolk (1997), dump sites closer to residential areas are always feeding places for dogs and cats. These pets, together with rodents, carry diseases with them to nearby homesteads. The UNEPA (2006) stated that wastes that are not properly managed especially excreta and other liquids and solid wastes from households and the community, are a serious health hazard and could lead to the spreading of diseases. The report further states that unattended wastes lying around attract flies, rats, and other creatures that, in turn, spread diseases. Normally, it is the wet waste that decomposes and releases a bad odor. The bad odor affects the people settled next to the dumpsite, which clearly shows that the dumpsites have serious effects to people settled around or next to them. Wastes from agriculture and industries can also cause serious health risks. Other than this, co-disposal of industrial hazardous wastes with municipal wastes can expose people to chemical and radioactive hazards. Uncollected solid waste can also obstruct storm water runoff, resulting in the forming of stagnant water bodies that become the breeding ground of disease. Wastes dumped near a water source also cause a contamination of the water body or the ground water source. Direct dumping of untreated wastes in rivers, seas, and lakes, result the accumulation of toxic substances in the food chain through the plants and animals that feed on it (Medina, 2002). This clearly shows how waste disposal seriously affects the health of residents located closer to dumpsites. The effect of solid waste disposal in African countries faces a great problem. It is imperative to note that Swaziland is planning to address the issue of solid waste disposal. The major problem of Swaziland is that, they are engaging in a long term plan, while damage is increasing every day.

The National Solid Waste Management Strategy for Swaziland represents a long-term plan up to year 2010 for addressing key issues, needs, and problems experienced with waste management in Swaziland. The strategy attempts to give effects to the National Environmental Policy, the National Environmental Management Act of 2002, and the Waste Regulations Act of 2000. The focus of the strategy is to move towards a holistic approach in waste management, in line with the internationally accepted principles, but taking into account the specific context of Swaziland, with regard to the institutional and legal framework, as well as land tenure and resource constraints. Integrated waste management, thus, represents a move away from waste management through impact management and remediation to a proactive management system that focuses on waste prevention and minimisation (The National Solid Waste Strategy for Swaziland, 2003).

Dumpsites are known for their smelly and unsightly conditions. These conditions are worse in the summer because of extreme temperatures, which speed up the rate of bacterial action on biodegradable organic material. Most developing countries, like Swaziland, use such dumpsites rather than properly managed and environmentally safe landfills. Lack of capital and poor government policies regarding to wastes contributes to such conditions. There is therefore considerable public concern over the possible effects of dumpsites on the health of people living nearby, particularly those where hazardous waste is dumped. Most solid wastes are disposed on the land in open dumps. Disposal of solid waste on the land without careful planning and management can present a danger to the environment and the human health. The environment should be clean and less polluted by all means. This means that waste should be managed at all costs to limit its effects to the environment (US Environmental Protection Agency, 2006).

2.5.2 Location Of Dumpsites/Skip Containers

Mustafa (1993) stated that dumping sites are the most common way of disposal of municipal solid wastes in the cities. Generally, they are found on the outskirts of the urban areas, turning into sources of contamination due to the incubation and proliferation of flies, mosquitoes, and rodents. That, in turn are disease transmitters that affect the child population's health, which has its organic defenses in a formative and creative state.

Mustafa (1993) stated that decomposition of organic compounds by micro organisms is a common phenomenon. Most organic materials, such as food, wood products, or other remnants of plants, decay, and finally return to the environment in the form of simple compounds, such as carbon dioxide, water, or ammonia. Surprisingly, it was found that most synthetic organic polymers, including the majority of plastics, are extremely resistant to biodegradation. This phenomenon starts to create significant economical and environmental problems when landfills sites overflow with plastics.

According to Najem and Strunk (1994), an increasing number of dumpsites are identified in site assessment conducted by the Californian Integrated Waste Management Board (IWMB). The location of dumpsites has proved to be a problem to nearby residents in most parts of the world, particularly in Africa. Swaziland is no exception in the problems associated with waste disposal. These include the development of unofficial dumpsites and littering. In cases where there is a dumpsite, it is either unmonitored or creates an unsightly surrounding.

According to Marc (2006), the location of the dumpsites should be properly planned and managed to avoid risks to human health and the environment, at large. Corrective and management measures are likely to be expensive, complex, and pose serious threats to the environment and its habitants.

2.6 SUMMARY

Location problems and some related problems have been discussed. We provided an overview of some variants on location problems, vehicle routing problems and a comprehensive literature on arc and node routing problems. Further search on the impact of location of skip containers in communities were also reviewed in the chapter. In the next chapter, our proposed maximum capture constructive heuristic for location of skip containers will be looked at, a model on the effect of smell will also be developed and a mixed interior routing and exterior routing algorithms shall be put forward.

CHAPTER THREE

METHODOLOGY

CAPACITATED FACILITY LOCATION PROBLEM

3.0 INTRODUCTION

In this chapter, we shall look at the method of partitioning the entire graph (the customers and their respective demands) into sub-clusters based on the capacity of the skip containers. While considering the capacity of the container, we shall also look at the position of the containers in the communities for the entire graph.

3.1 R^n EUCLIDEAN DISTANCE

A location problem is to locate a facility, or facilities, to serve optimally a given set of customers. The customers are given by their coordinates and demands. The coordinates are points in R^n (usually n = 2), and the demands are positive numbers q_i. Assuming N customers, the data of the problem is a set of points (coordinates) $X = \{X_1, X_2, \ldots, X_N\}$ in R^n and a corresponding set of positive weights (demands) $\{q_1, q_2, \ldots, q_N\}$. Using the Euclidean distance $d(X,Y) = \|X - Y\|$ between two points X, Y in R^n. If the customers are served by M facilities for a given M, we denote X_M to be the set of customers assigned to the M^{th}- facility. The weighted sum of distance travelled by these customers is $\sum_{X_i \in \chi_M} \frac{1}{q_i}\|X_i - c_M\|$ where C_M is the location of the M^{th} - facility. The customers $X = \{X_1, X_2, \ldots, X_N\}$,their demands $\{q_1, q_2, \ldots, q_N\}$ is to be located to facilities with centres $\{c_1, c_2, \ldots, c_M\}$ so as to minimize the weighted sum of distances travelled by all the customers to access the bin. $\min_{c_1,\ldots,c_M} \min_{X_1,\ldots,X_M} \sum_{k=1}^{M} \sum_{X_1 \in \chi_M} \frac{1}{q_i} \left\| X_i - C_M \right\|$ and each container has a capacity Q, such that the sum of demands assigned to the M^{th} – facility cannot exceed it, i.e.

$$\sum_{X_i \in \chi_M} q_i \le Q_K \tag{3.1}$$

3.2 MAXIMUM INSERTION PROBABILISTIC DISTANCE CLUSTERING

Clustering is the process of partitioning a data set into disjoint clusters. Consider data points to be vectors $X = (x_1, x_2, x_3, \ldots, x_p) \in R^p$ and a distance function $d(x, y) = \|x - y\| \ \forall x, y \in R^p$ in a data set D consisting of N data points $\{X_1, X_2, \ldots, X_N\}$ to be partitioned into M clusters where $1 < M < N$ each associated with the centre $C_1, C_2, \ldots, C_M$

3.3 PROPOSED PROBABILISTIC DISTANCE LOCATION MODEL (PDLM)

Let a data set $D \subset R^p$ be partitioned into K clusters $\{c_M : m = 1, 2, \ldots, M\}$, $D = \bigcup_{m=1}^{M} C_m$ and let C_M be the centre of the cluster with capacity Q_M.

With each data point $X \in D$ and a cluster centre C_M, we denote

- a distance $d_m(X, c_m)$ by $d_m(X)$
- a probability of membership in C_m by $P_m(X)$ for each $X \in D$ and each cluster centre C_m,

$$\frac{P_m(X) d_m(X)}{Q_m} = \text{Constant}, \tag{3.2}$$

depending on X and independent of the cluster m.

Thus the cluster membership is thus more probably the closer the data points is the cluster centre.

3.3.1 Probabilities

Given the cluster centres $\{C_1, C_2, \ldots, C_m\}$, let X be a data point and let $\{d_m(X): m = 1, 2, \ldots, M\}$ be its distances from the given centres. The maximum insertion membership probabilities of X are

$$P_m(X) = \frac{\prod_{j \neq m} \frac{d_j(X)}{Q_j}}{\sum_{i=1}^{M} \prod_{j \neq 1} \frac{d_j(X)}{Q_j}}, \quad m = 1, 2, \ldots, M$$

(3.3)

From equation (3.2) using i and m

$$P_i = \frac{P_m(X) \frac{d_m(X)}{Q_m}}{\frac{d_i(X)}{Q_i}}$$

Since $\sum_{i=1}^{M} P_i(X) = 1$,

$$P_m(X) \sum_{i=1}^{M} \left(\frac{\frac{d_m(X)}{Q_m}}{\frac{d_i(X)}{Q_i}} \right) = 1$$

$$P_m(X) = \frac{1}{\sum_{i=1}^{M} \left(\frac{\frac{d_m(X)}{Q_m}}{\frac{d_i(X)}{Q_i}} \right)}$$

$P_m(X) = \dfrac{\prod_{j \neq k} \frac{d_j(X)}{Q_j}}{\sum_{i=1}^{M} \prod_{j \neq i} \frac{d_j(X)}{Q_j}}, m = 1, 2, \ldots, M$, which establishes equation (3.3) .

For two clusters, i.e. $M = 2$

$$P_1 = \frac{\frac{d_2(X)}{Q_2}}{\frac{d_1(X)}{Q_1} + \frac{d_2(X)}{Q_2}}, \quad P_2 = \frac{\frac{d_1(X)}{Q_1}}{\frac{d_1(X)}{Q_1} + \frac{d_2(X)}{Q_2}} \tag{3.4}$$

For three clusters i.e. M = 3

$$P_1 = \frac{\dfrac{d_2(X)d_3(X)}{Q_2 \times Q_3}}{\dfrac{d_1(X)d_2(X)}{Q_1 \times Q_2} + \dfrac{d_1(X)d_3(X)}{Q_1 \times Q_3} + \dfrac{d_2(X)d_3(X)}{Q_2 \times Q_3}} \tag{3.5}$$

$$P_2 = \frac{\dfrac{d_1(X)d_3(X)}{Q_2 \times Q_3}}{\dfrac{d_1(X)d_2(X)}{Q_1 \times Q_2} + \dfrac{d_1(X)d_3(X)}{Q_1 \times Q_3} + \dfrac{d_2(X)d_3(X)}{Q_2 \times Q_3}} \tag{3.6}$$

$$P_3 = \frac{\dfrac{d_1(X)d_3(X)}{Q_2 \times Q_3}}{\dfrac{d_1(X)d_2(X)}{Q_1 \times Q_2} + \dfrac{d_1(X)d_3(X)}{Q_1 \times Q_3} + \dfrac{d_2(X)d_3(X)}{Q_2 \times Q_3}} \tag{3.7}$$

3.3.2 JOINT DISTANCE FUNCTION

The probability function as in equation (3.2) is given by $\dfrac{P_m(X)d_m(X)}{Q_m} = \text{Constant}$.

Let the constant be $D(X)$, function in X, then $\dfrac{P_m(X)d_m(X)}{Q_m} = D(X)$

$$P_m(X) = \frac{D(X)}{\dfrac{d_m(X)}{Q_m}}, \; m = 1, 2, 3, \ldots, M$$

Since $\sum_{i=1}^{M} P_i(X) = 1$

$$D(X) = \frac{\prod_{j=1}^{M} \dfrac{d_j(X)}{Q_j}}{\sum_{i=1}^{M} \prod_{j \neq i} \dfrac{d_j(X)}{Q_j}} \tag{3.8}$$

For two clusters, i.e. $M = 2$; $D(X) = \dfrac{\dfrac{d_1(X)d_2(X)}{Q_1Q_2}}{\dfrac{d_1(X)}{Q_1} + \dfrac{d_2(X)}{Q_2}}$ (3.9)

For three clusters, i.e. $M = 3$;

$$D(X) = \frac{\dfrac{d_1(X)d_2(X)d_3(X)}{Q_1Q_2Q_3}}{\dfrac{d_1(X)}{Q_1}\dfrac{d_3(X)}{Q_3} + \dfrac{d_2(X)}{Q_2}\dfrac{d_3(X)}{Q_3} + \dfrac{d_1(X)}{Q_1}\dfrac{d_2(X)}{Q_2}} \tag{3.10}$$

The joint distance function (JDF) of the entire data set D is the sum of (3.6) over all points, and is a function of the M cluster centers, say $D(c_1, c_2, \ldots, c_M) = \sum_{i=1}^{N} \dfrac{\prod_{m=1}^{M} d_m(X_i, c_m)}{\sum_{t=1}^{M}\prod_{j \neq t} d_j(X_i, c_j)}$ (3.11)

3.4 AN EXTREMA PRINCIPLE

Consider a case of two clusters, using X to be the given data point with distances $d_1(X), d_2(X)$ to the cluster centers and known cluster sizes Q_1 and Q_2. The probabilities in (3.4) are the optimal solution P_1, P_2 of the extrema problem.

$$\text{Minimize } \left\{ \frac{d_1(X)P_1^2}{Q_1} + \frac{d_2(X)P_2^2}{Q_2} \right\} \tag{3.12}$$

Subject to $P_1 + P_2 = 1$

$P_1, P_2 \geq 0$

Note: The explanation for the strange appearance of probabilities squared" above, is that (3.12) is a smoothed version of the "real" clustering problem, namely, min{d_1, d_2} which is non-smooth proposed by Teboulle (2007) for a unified development of smoothed clustering methods.

The Lagrangian of the problem (3.12) is:

$$L(P_1, P_2, \lambda) = \frac{d_1(\mathrm{x})P_1^2}{Q_1} + \frac{d_2(\mathrm{x})P_2^{\,2}}{Q_2} - \lambda(P_2 + P_1 - 1) \qquad (3.13)$$

Setting the partial derivatives with respect to P_1, P_2 to zero, we have $P_1 \frac{d_1(X)}{Q_1} = P_2 \frac{d_2(X)}{Q_2}$

which is the same as the principle in (3.2)

Substitute the probabilities in (3.4) in (3.13) we have $L^*(P_1(\mathrm{x}), P_2(\mathrm{x}), \lambda) = \frac{\frac{d_1(\mathrm{x})d_2(\mathrm{x})}{Q_1Q_2}}{\frac{d_1(\mathrm{x})}{Q_1} + \frac{d_2(\mathrm{x})}{Q_2}}$ which is the same as the distance function obtained in equation (3.9) for $M = 2$

The extrema problem for the entire data set $D = \{X_1, X_2, \ldots, X_N\} \subset R^n$

Minimize $$\sum_{i=1}^{N}\left(\frac{d_1(\mathrm{x}_i)p_1(\mathrm{x}_i)^2}{Q_1} + \frac{d_2(\mathrm{x}_i)p_2(\mathrm{x}_i)^2}{Q_2}\right)$$

(3.14)

Subject to: $p_1(\mathrm{x}_i) + p_2(\mathrm{x}_i) = 1$

$p_1(\mathrm{x}_i),\ p_2(\mathrm{x}_i) \geq 0,\ \ i = 1, 2, 3, \ldots, N$

where $p_1(\mathrm{x}_i), p_2(\mathrm{x}_i)$ are the cluster probabilities at x_i and $d_1(\mathrm{x}_i),\ d_2(\mathrm{x}_i)$ are the corresponding distances. The problem separates into N as in (3.12) and its optimal value is

$$\sum_{i=1}^{N} \frac{\frac{d_1(\mathrm{x}_i)d_2(\mathrm{x}_i)}{Q_1Q_2}}{\frac{d_1(\mathrm{x}_i)}{Q_1} + \frac{d_2(\mathrm{x}_i)}{Q_2}} \qquad (3.15)$$

the sum of the joint distance function of all points.

3.5 CLUSTER CENTERS

Writing equation (3.12) as a function of the cluster centers $C_1,\ C_2$

$$f(C_1, C_2) = \sum_{i=1}^{N}\left(\frac{d_1(\mathrm{x}_i, C_1)p_1(\mathrm{x}_i)^2}{Q_1} + \frac{d_2(\mathrm{x}_i, C_2)p_2(\mathrm{x}_i)^2}{Q_2}\right).$$

For Euclidean distance $d_m(\mathrm{x}, c_m) = \|\mathrm{x} - c_m\|$, $m = 1, 2$ so that

$$f(C_1, C_2) = \sum_{i=1}^{N} \left(\frac{\|\mathrm{x}_i - C_1\| \, p_1(\mathrm{x}_i)^2}{Q_1} + \frac{\|\mathrm{x}_i - C_2\| \, p_2(\mathrm{x}_i)^2}{Q_2} \right) \tag{3.16}$$

and look for centers $C_1,\ C_2$ that minimizes f and the probabilities $p_1(\mathrm{x}_i),\ p_2(\mathrm{x}_i)$ are given for $i = 1, 2, 3, \ldots, N$ and with an assumption that the cluster centers $C_1,\ C_2$ do not coincide with any of the points $i = 1, 2, 3, \ldots, N$

3.5.1 The Gradient Function

The gradient of the Euclidean distance $d_k(\mathrm{x}, c_k) = \|\mathrm{x} - c\|$ with respect to c, for $\mathrm{x} \neq c$

$$\nabla_c \|\mathrm{x} - c\| = -\frac{\mathrm{x} - c}{\|\mathrm{x} - c\|} = -\frac{\mathrm{x} - c}{d(\mathrm{x}, c)}$$

From the assumption above, the gradient of equation (3.14) with respect to C_m is

$$\nabla_c f(c_1, c_2) = -\sum_{i=1}^{N} \frac{\mathrm{x}_i - c_k}{\|\mathrm{x}_i - c_m\|} \times p_m(\mathrm{x}_i)^2$$

$$= -\sum_{i=1}^{N} \frac{\mathrm{x}_i - c_m}{d_m(\mathrm{x}_i, c_m)} \times p_m(\mathrm{x}_i)^2,\ m = 1, 2$$

Setting the gradient to zero, and grouping like terms, we have

$$\sum_{i=1}^{N} \left(\frac{p_m(\mathrm{x}_i)^2}{d_m(\mathrm{x}_i, c_m)} \right) \mathrm{x}_i = \left(\sum_{i=1}^{N} \frac{p_m(\mathrm{x}_i)^2}{d_m(\mathrm{x}_i, c_m)} \right) c_m$$

$$C_m = \sum_{i=1}^{N} \left(\frac{u_m(\mathrm{x}_i)}{\sum_{j=1}^{N} u_m(\mathrm{x}_j)} \right) \mathrm{x}_i \tag{3.17}$$

where $u_m = \dfrac{p_m(\mathrm{x}_i)^2}{d_m(\mathrm{x}_i, c_m)}$ for $\mathrm{m} = 1, 2$

Giving the minimizers as $C_1 = \sum_{i=1}^{N}\left(\dfrac{u_1(\mathrm{x}_i)}{\sum_{j=1}^{N} u_1(\mathrm{x}_j)}\right)\mathrm{x}_i,\;\; C_2 = \sum_{i=1}^{N}\left(\dfrac{u_2(\mathrm{x}_i)}{\sum_{j=1}^{N} u_2(\mathrm{x}_j)}\right)\mathrm{x}_i$ (3.18)

where

$$u_1 = \frac{p_1(\mathrm{x}_i)^2}{d_1(\mathrm{x}_i, c_1)}, \quad u_2 = \frac{p_2(\mathrm{x}_i)^2}{d_2(\mathrm{x}_i, c_2)} \tag{3.19}$$

or using equations (3.4)

$$\left.\begin{aligned} u_1(\mathrm{x}_i) &= \frac{\left(\frac{d_2(\mathrm{x}_i, c_2)}{Q_2}\right)^2 \times \frac{Q_1}{d_1(\mathrm{x}_i, c_1)}}{\left(\frac{d_1(\mathrm{x}_i, c_1)}{Q_1} + \frac{d_2(\mathrm{x}_i, c_2)}{Q_2}\right)^2}, \\ u_2(\mathrm{x}_i) &= \frac{\left(\frac{d_1(\mathrm{x}_i, c_2)}{Q_1}\right)^2 \times \frac{Q_2}{d_2(\mathrm{x}_i, c_2)}}{\left(\frac{d_1(\mathrm{x}_i, c_1)}{Q_1} + \frac{d_2(\mathrm{x}_i, c_2)}{Q_2}\right)^2} \end{aligned}\right\} \tag{3.20}$$

For a function of M cluster centers

$f(C_1, C_2, \ldots, C_M) = \sum_{m=1}^{M}\sum_{i=1}^{N}\left(\dfrac{d_m(\mathrm{x}_i, C_m)\, p_m(\mathrm{x}_i)^2}{Q_m}\right)$ an analog of (3.16)

Then by the results of equation (3.17) the minimizers of f are

$C_m = \sum_{i=1}^{N}\left(\dfrac{u_m(\mathrm{x}_i)}{\sum_{j=1}^{N} u_m(\mathrm{x}_j)}\right)\mathrm{x}_i$ with $u_m = \dfrac{p_m(\mathrm{x}_i)^2}{d_m(\mathrm{x}_i, c_m)}$ for $m = 1, 2, \ldots, M$ (3.21)

3.6 THE PROBABILITY DISTANCE LOCATION ALGORITHM (PDLA)

The results obtained in sub-topics 3.1 and 3.2 are used in the algorithm below. For simplicity, we demonstrate the algorithm for the case of two (2) clusters.

The major steps involved in the formation of the algorithm are described below.

1. **Calculation of number of skip containers**

   The number of skip containers (k) needed in each zone is dependent on demands (q_i) of the customers and the capacity (Q) of the container; $k = \frac{\sum_{i=1}^{N} q_i}{Q}$

2. **Selection of initial cluster centers**

- The x-coordinates of all the customers are arranged in ascending order. The highest x– coordinates is selected with its demand recorded, the next highest x-coordinates value is recorded with its demand until the capacity constraints of the container is satisfied.
- The centroid of these points is found by taken the average of the extreme x-values and that of the y-values.
- The process is continued until all the initial centers are assigned.

Algorithm 3.1: The PDLM Algorithm

Initialization: Given data set D with N points, any two cluster centers C_1, C_2 and the cluster sizes Q_1, Q_2, $Q_1 + Q_2 = Q$, $\varepsilon > 0$

Iteration:

Step I: Compute distances from C_1 and C_2 for all $\mathrm{x} \in D$

Step II: Update the centre C_i' using (equations 3.18 and 3.21)

Step III: If the difference between

$\sum_{i=1}^{K} \left| C_i^{n-1} - C_i^{n-2} \right|$ and $\sum_{i=1}^{K} \left| C_i^{n} - C_i^{n-1} \right|$ is less than ε stop else return to (ii) of C_i'

3.7 ILLUSTRATIVE EXAMPLES

Illustrative Example 1

It is required to locate skip containers with equal capacity of 12 to service 15 customers optimally.

Customers	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
x	2	3	-2	-2	0	-3	2	5	-1	3	6	8	-5	4	3
y	3	4	2	-1	4	-2	-1	4	3	2	6	5	5	3	5
Load (q_i)	1	1	2	2	2	1	2	2	1	1	1	1	2	1	2

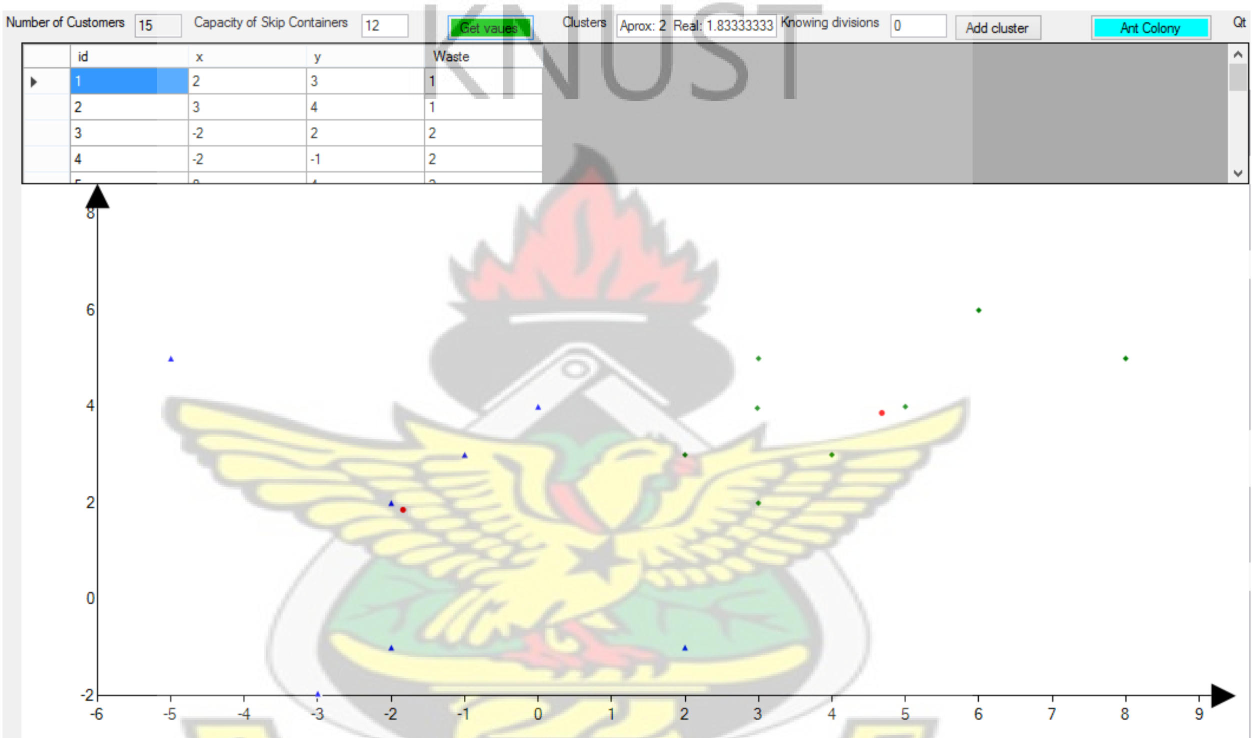


Figure 3.1: 15 Customers with their demands partitioned into two transfer sites (depots) based on the capacity of the skip container

Figure 3.1 shows fifteen (15) customers with their corresponding waste generated to be assigned to a skip container(s) each of capacity 1680 litres (12 of 140 litre bin). The red points indicate the final stable cluster centres to locate the skip container(s) and the green points indicate customers assigned to skip container one (1) and the blue points assigned to skip container two (2).

Table 3.1: Final cluster centres, customers and number of bins assigned to the cluster

Transfer depot number	Stable transfer depot	Customers	Total bins
1	(4.68 , 3.87)	8	10
2	(-1.84 , 1.86)	7	12

Illustrative Example 2

It is required to locate skip containers with equal capacity of 18 to service 32 customers optimally.

Customers	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
x	5	5	6	8	7	8	10	9	11	10	8	13	15	14	13
y	9	25	15	14	10	15	7	7	8	15	13	14	7	6	15
Load (q_i)	2	1	2	2	3	1	2	3	1	2	1	3	1	2	2

16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32
18	1	19	13	17	2	6	8	9	10	7	12	8	15	15	12	1
11	11	15	17	18	7	4	11	4	9	8	1	16	9	11	3	8
3	2	1	2	3	3	2	2	1	3	1	2	2	2	1	3	2

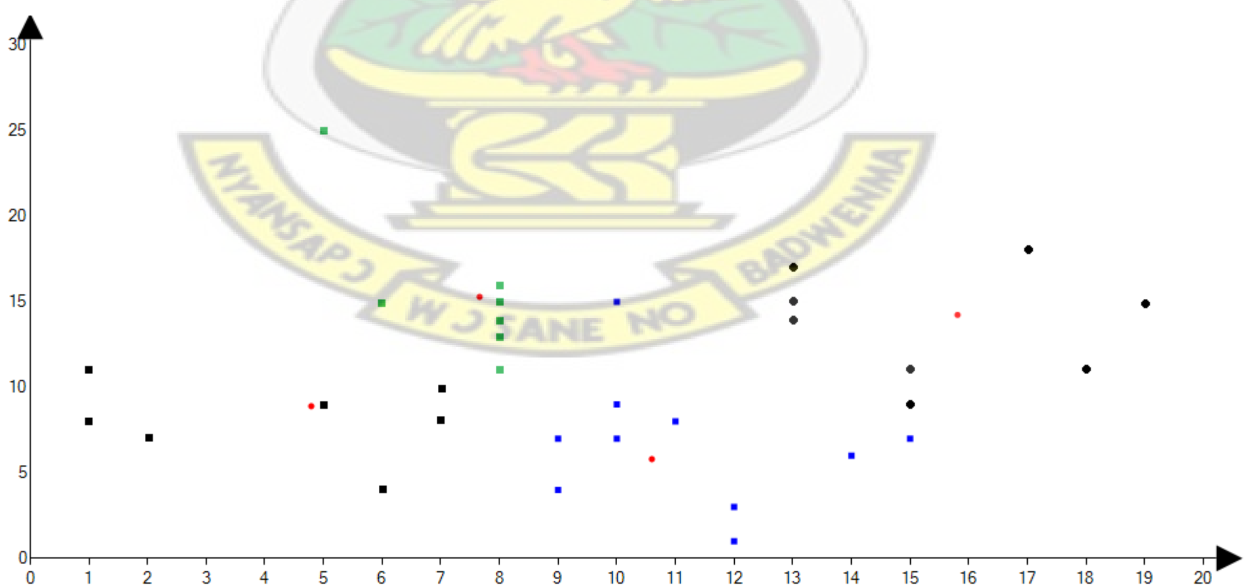


Figure 3.2: 32 Customers with their demands partitioned into four transfer sites (depots) based on the capacity of the skip container

Figure 3.2 shows thirty-two (32) customers with their corresponding waste generated to be assigned to a skip container(s) each of capacity 2520 litres (18 of 140 litre bin). The red points indicate the final stable cluster centres to locate the bin and the different colours indicate points assigned to a particular cluster.

Table 3.2: Final cluster centres, customers and number of bins assigned to the cluster

Transfer depot number	Stable transfer depot	Customers	Total bins
1	(15.78 , 14.31)	8	17
2	(10.65 , 5.72)	10	18
3	(4.91 , 8.99)	7	15
4	(7.96 , 15.07)	7	11

3.8 MATHEMATICAL MODELS FOR SOLID WASTE COLLECTION

In this section, we shall consider the mathematical models of vehicle routing problems on solid waste collection based on the following four variants (parameters):

- An improved Ant Colony algorithm with capacity and angle factor;
- Capacitated vehicle routing problem with time windows, clew lunch break, stopping time and deadheading time;
- Environmentally related Issues on Uncollected Solid Waste;
- Comparative Vehicle Fuel Consumption Analysis on Two Elemental Time Schedule.

3.8.1 Ant Colony Optimization

The basic idea of ACO algorithms was inspired through the observation of swarm colonies and specifically ants Beckers et al., (1989). Insects like ants are social. That means that ants live in colonies and their behaviour is directed more to the survival of the colony as a whole, rather than to that of a single individual. Most species of ants are blind. However, while each ant is walking, it deposits on the ground a chemical substance called pheromone

Dorigo and Di Caro, (1999). Ants can smell pheromone and when choosing their way, they tend to choose, in probability, paths with high pheromone density

Ant Colony Optimisation (ACO) is a meta-heuristic approach designed for solving hard combinatorial optimization problems. Real ant colonies deposit pheromone on the paths they walk while searching for food sources. If other ants searching for food sense the pheromone on a path, they are likely to follow it rather than traveling at random, thus reinforcing the path. As more and more ants follow a path the level of pheromone on that path will enhance, which in turn will increase its selection probability by other ants. On the other hand, the pheromone evaporates over time, reducing the chance of other ants following the path. The longer the path between the nest and the food source the more the pheromone evaporates. Thus, the pheromone levels remain higher on the shorter paths. As a consequence, the level of pheromone laid is basically based on the path length and the quality of the food source. The experimental setting given in Figure 3.3 illustrates the above described behavior of the real ants.

Figure 3.3(a) shows a path that has been formed by ants walking between the food source A and the nest E. When the path is cut off with an obstacle as shown in Figure 3.3 (b) the ants located at point B walking from A to E and those located at point D walking from E to A have to choose either the path passing through point C or the path passing through point H. Since there is no previous pheromone trail on any of the two alternative paths, the selection of either path by the first ants reaching these points is equally likely. Since the path BCD is shorter than the path BHD the ant that has selected the path through point C will arrive at point D before the ant that has selected the path through point H. Hence, an ant returning from E to A and located at point D will find a stronger trail on path DCB. due to the ants that have already selected that path by chance and those walking through BCD. Consequently, the amount of pheromone on path BCD will increase faster than the pheromone on path BHD because of the larger number of ants following path BCD per unit time and the evaporation factor, with time, all ants will select the shorter path. ACO simulates this natural behavior of real ants to solve combinatorial optimization problems by

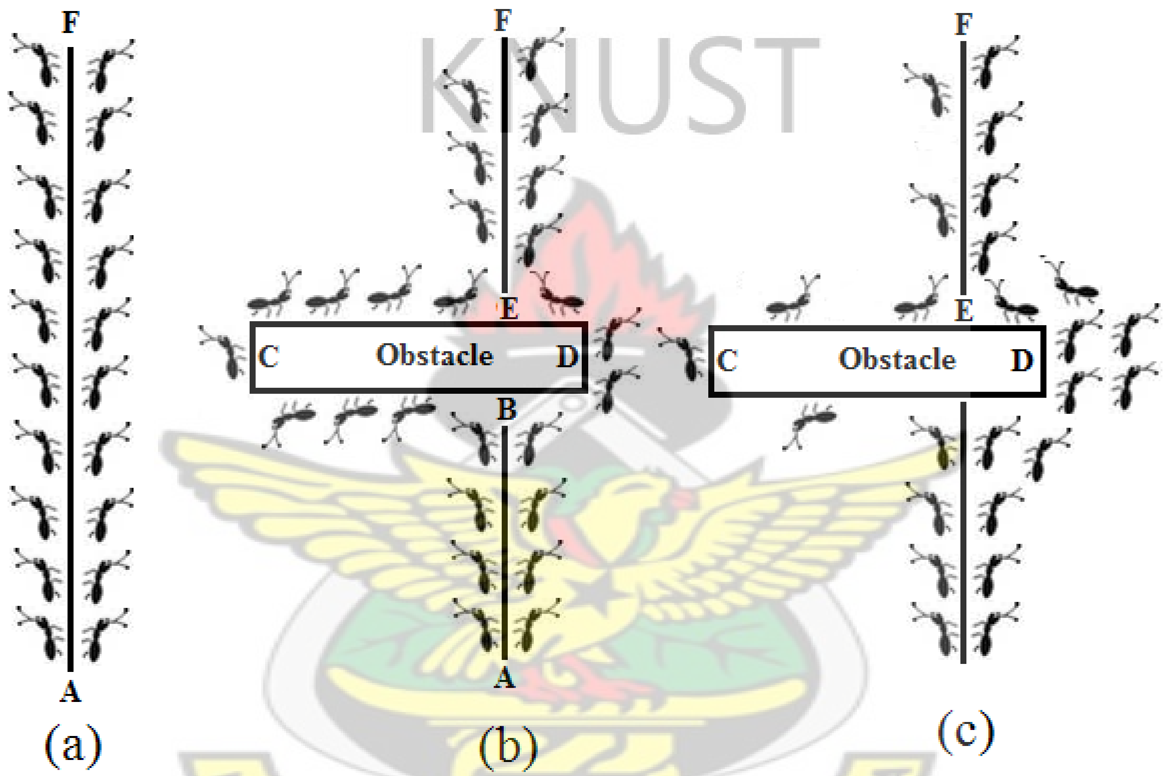


Figure 3.3 An example of the behaviour of real ants

using artificial ants. To apply ACO, the optimization problem is transformed into the problem of finding the best path on a weighted graph. The artificial ants incrementally build solutions by moving on the graph using a stochastic construction process guided by artificial pheromone and heuristic information known as visibility. The amount of pheromone deposited on arcs is proportional to the quality of the solution generated and increases at run-time during the computation.

3.8.2 Artificial Ants

Now in artificial life, the Ant Colony Optimization (ACO) uses artificial ants, called agents, to find good solutions to difficult combinatorial optimization problems (Bonabeau, Press). The behaviour of artificial ants is based on the traits of real ants, plus additional capabilities that make them more effective, such as a memory of past actions. Consider the example in Figure 3.4. The distances between D and H, between B and H, and between B and D are equal to 1. C is positioned in the middle of D and B. 30 new ants come to B from A and 30 to D from E at each time unit. Each ant walks at a speed of 1 per time unit and lays down a pheromone trail of intensity 1 at time *t*. Evaporation occurs in the middle of the successive time interval $(t+1, t+2)$

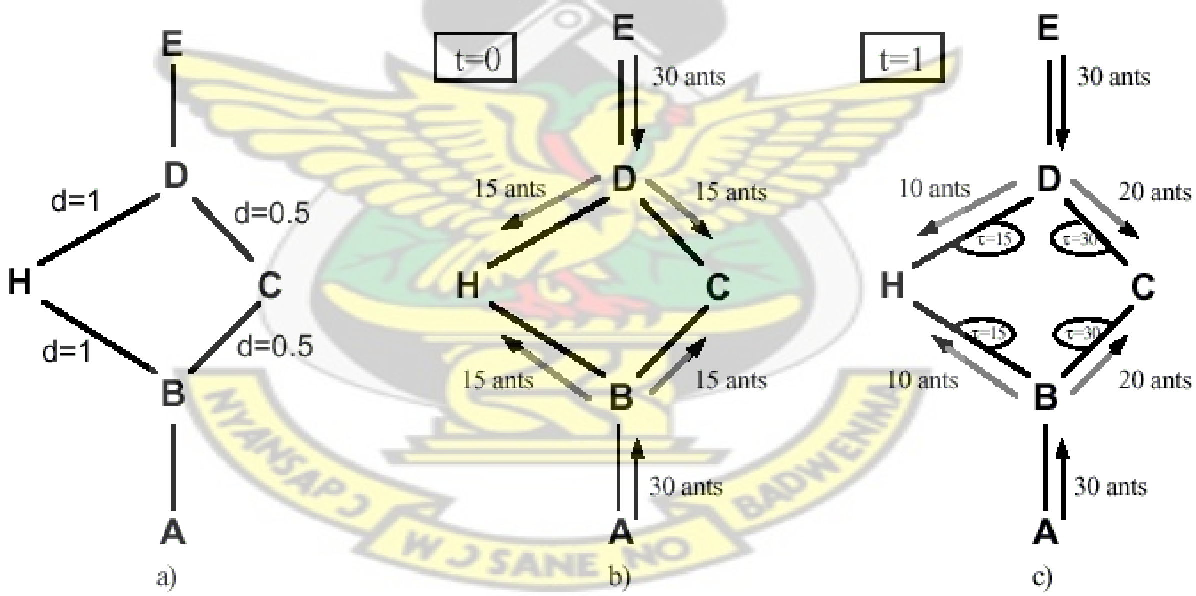


Figure 3.4 The behaviour of artificial ants on a path with time

At $t = 0,$ 30 ants are in B and 30 in D. As there is no pheromone trail they randomly choose the way to go. Thus, approximately 15 ants from each node will go toward H and 15 toward C.

At $t = 1,$ 30 new ants come to B from A, they sense a trail of intensity 15 on the path that leads to H, laid by the 15 ants that went through B-H-D. They also sense a trail of intensity 30 on the path to C, obtained as the sum of the trail laid by the 15 ants that went through B-C-D and by the 15 ants that went through D-C-B. The probability of choosing a path is therefore biased. The expected number of ants going toward C will be the double of those going toward H: 20 versus 10, respectively. The same is true for the new 30 ants in D which came from E. This process continues until all of the ants eventually choose the shortest path.

In brief, if an ant has to make a decision about which path to follow it will most probably follow the path chosen heavily by preceding ants, and the more the number of ants following a trail, the more attractive that trail becomes to be followed.

In the ant meta-heuristic, a colony of artificial ants cooperates in finding good solutions to discrete optimization problems. Artificial ants have two characteristics. On the one hand they imitate the following behaviour of real ants:

- *Colony of cooperating individuals:* Like real ant colonies, ant algorithms are composed of entities cooperating to find a good solution. Although each artificial ant can find a feasible solution, high quality solutions are the result of the cooperation. Ants cooperate by means of the information they concurrently read/write on the problem states they visit.

- *Pheromone trail:* While real ants lie pheromone on the path they visit, artificial ants change some numeric information of the problem states. This information takes into account the ant's current performance and can be obtained by any ant accessing the state. In ant algorithms pheromone trails are the only communication channels among the ants. It affects the way that the problem environment is perceived by the ants as a function of the past history. Also an evaporation mechanism, similar to real pheromone evaporation, modifies the pheromone. Pheromone evaporation allows the ant colony to slowly forget its past history so that it can direct its search towards new directions without being over-constrained by past decisions.
- *Shortest path searching and local moves:* The aim of both artificial and real ants is to find a shortest path joining an origin to destination sites. Like real ants artificial ants move step-by-step through adjacent states of the problem.
- *Stochastic state transition policy:* Artificial ants construct solutions applying a probabilistic decision to move through adjacent states. As for real ants, the artificial ants only use local information in terms of space and time. The information is a function of both the specifications and pheromone trails induced by past ants.

3.8.3 An Improved Ant Colony Heuristic for Capacitated Solid Waste Collection Vehicle Routing Problem

The area of the community where collection of solid waste route are planned is represented by means of a graph $G = (V, E, A)$ where V is associated with street junctions, including dead-ends and the transfer depot E, the nodes and A is associated with existing street sections between junctions. Arcs represent single-way streets while edges represent two way streets. There is a single transfer depot denoted by O, where $O \in V$. Each link

$i \in E \cup A$ with length d_{ij}. The set of links $\{L = E^L \cup A^L\} \in E \cup A$ is defined as the set of links with a minimum of one collection point, where each required $i \in E^L \cup A^L$ has a demand q_i which represents the quantity of solid waste that must be collected. There are m vehicles each with capacity Q to provide service to the required customers in a cluster (link). Each cluster (link) must be serviced by one single vehicle. Each of the serviced vehicles starts and end at the transfer depot, the total waste to be collected by a vehicle in a cluster must not exceed the capacity of the vehicle. Each customer is serviced once by only one vehicle in the time constraint. If the load stored by the ant exceeds the vehicle capacity, the ant must return to the transfer depot, we then obtain a complete route for a vehicle.

3.8.3.1 Proposed Ant Colony Heuristic

Our proposed Ant Colony system (ACS) is a modification based on the ACS proposed by Dorigo and Gamberdella (1997). The usual ant colony system algorithm is of slow in convergence and is easy to fall into local optimum and stagnation in the planning of vehicle path. Our improved Ant Colony Algorithm sought to improve the existing model whiles introducing additional three factors such as the weight factor, saving factor and the angle factor in the transition model to improve solution quality and convergence.

The ACS is based on four major steps:

i) Set parameters and initialize the pheromone trials;

ii) Build the ant solution by using the state transition rule and carry out the local pheromone update

iii) Apply local search to improve solution build by an Ant;

iv) Update the global pheromone information.

Each artificial ant has a memory called tabu list. The tabu list forces the ant to make legal tours. It saves the cities (customers) already visited and forbids the ant to move already visited cities (customers) until a tour is completed.

After all customers are visited, the tabu list of each ant will be full. The shortest path found is computed and saved. Then, tabu lists are emptied. This process is iterated for a user-defined number of cycles. If there are n nodes and b_i is the number of ants at city i, our Improved Ant Colony Heuristics for the thesis has the following parameter notations:

$K = \sum_{i=1}^{n} b_i$: Total number of ants

N: Set of customers to be visited

$tabu_k$: Tabu list of the k-th ant

$tabu_k(s)$: s-th customer visited by the k-th ant in the tour

τ_{ij} (t): Intensity of trail on edge between customer i and customer j at time t

η_{ij} : Visibility of edge between customer i and customer j

$\omega_{ij}(t)$: Weight factor, which is the ratio of the current weight including weight of customer j to the capacity of the service vehicle

$u_{ij}(t)$: Saving heuristic and

$\psi_{ij}(t)$: Angle factor

3.8.3.2 Pheromone Trial Initialization

The initial trial pheromone level on each edge is given by $\tau_{ij}(0) = \dfrac{1}{(n+1)L}$ (3.21) where n is the number of nodes in a given route (link) and L is the tour length generated by the nearest neighbor heuristic which is an improvement over the usual initial trial level $\tau_{ij}(0) = \frac{1}{n}$. The nearest neighbor heuristic is obtained by the following steps;

i) Randomly start with one node that has not been visited at the beginning of a route;

ii) Select the not yet visited closest feasible customer as the next customer to be visited.

iii) Repeat step 2, until the vehicle capacity is violated, then go back to the transfer depot.

3.8.3.3 STATE TRANSITION PROBABILITY FUNCTION

The state transition rule that ants uses to select the next affected node are modified, which is calculated as follows. An ant k, positioned on a vertex i, chooses the next vertex j to visit by applying the probabilistic rule p_{ij}^{k} and is given by

$$P_{ij}^{k}(t) = \begin{cases} \dfrac{\left[\tau_{ij}(t)\right]^{\alpha} \times \left[\eta_{ij}\right]^{\beta} \times \left[U_{ij}(t)\right]^{\gamma} \times \left[w_{ij}(t)\right]^{\gamma} \times \left[\psi_{ij}(j)\right]}{\sum_{k \in \text{allowed}_k} \left[\tau_{ik}(t)\right]^{\alpha} \times \left[\eta_{ik}\right]^{\beta} \times \left[U_{ik}(t)\right]^{\gamma} \times \left[w_{ik}(t)\right]^{\gamma} \times \left[\psi_{ik}(k)\right]}, & \text{if } j \in \text{allowed}_k \\ 0 & \text{otherwise} \end{cases} \quad (3.22)$$

where α, β and γ are adjustable parameters that determine the relative influence of the pheromone and the visibility in the transition probabilities and allowed_k denote the neighbor affected node of affected node i that the kth ant has not yet visited.

3.8.3.4 Visibility Function

The visibility function η_{ij} from customer i to customer j is given by $\eta_{ij} = \left(\dfrac{1}{d_{ij} + d_{oj}}\right)$ (3.23) replacing the traditional visibility function $\eta_{ij} = \frac{1}{d_{ij}}$ where d_{ij} s the distance between customer i and customer j and the value of d_{oj} is the distance between the affected node j and the transfer depot. The modification of visibility function is to enhance the ants perception of transfer depot node from the present affected node, which will help guide the ants to move and therefore avoid falling into local optimum.

3.8.3.5 Saving Heuristic

The saving heuristic takes in account the distance from the current customer i from the transfer depot and that of the next probable customer j and weight factor of the next probable customer. The saving heuristic is given by $U_{ij} = d_{oi} + d_{oj} - \frac{1}{(q_j+1)} d_{ij}$ (3.24)

where q_j is the number of waste bins to be collected from customer j which is also an improvement from the usual saving heuristic $U_{ij} = d_{oi} + d_{oj} - f \times d_{ij}$ where f is an arbitrary constant.

3.8.3.6 Weight Factor

Weight factor has been factored into the probabilistic rule to guide the ant of the capacity constraints of the service vehicle. The weight factor is given by

$$w_{ij}(t) = \left(\frac{Q_i + q_j}{Q} \right) \tag{3.25}$$

where q_j is the number of waste to be collected from customer j, Q_i is the total capacity of waste in the ant tabu list after servicing customer i.

3.8.3.7 The Angle Factor

$\psi_{ij}(t)$ is the angle factor, which is the angle between the vector $\left[\overrightarrow{\text{node}_i^1\text{node}_i}, \overrightarrow{\text{node}_i\text{node}_j}\right]$ where node_i^1 is the previous node before visiting the affected node_i, node_j represents the current visiting affected node after node_i. For escape of an ant when it faced a dead end, an angle in the range $0 \le \theta \le \frac{\pi}{2}$ is assigned a constant value of 0.001 and an angle in the range $\frac{\pi}{2} < \theta \le \pi$ its value is $\dfrac{\theta}{\pi}$

$$\text{and } \theta = \cos^{-1}\left(\frac{\overrightarrow{\text{node}_i^1\text{node}_i} \cdot \overrightarrow{\text{node}_i\text{node}_j}}{\left|\overrightarrow{\text{node}_i^1\text{node}_i}\right|\left|\overrightarrow{\text{node}_i\text{node}_j}\right|}\right) \tag{3.26}$$

Ants chooses the next affected node *j* in accordance with the equation

$$j = \begin{cases} \underset{u \in JJ_i^k}{\text{argmax}} \left\{ \left[\tau_{iu}(t)\right]^{\alpha} \times \left[\eta_{iu}(t)\right]^{\beta} \times \left[U_{iu}(t)\right]^{\gamma} \times \left[w_{iu}(t)\right]^{\gamma}, & \text{if } q \le q_0 \\ JJ, & \text{if } q > q_0 \end{cases}$$

Where $JJ = P_{ij}^k(t)$ and q, q_0 are random variables such that $q, q_0 \in (0,1)$. When $q > q_0$ the ant chooses the next node in accordance with JJ

3.8.3.8 Local Search (2-Opt) Heuristic

When an ant has completed a tour, we use a 2-optimal heuristic to improve the solution in each tour. In each iteration, the algorithm examines each two distinct arcs $a_i \to a_{i+1}$ and $a_j \to a_{j+1}$ in the route *R*. These two arcs are replaced by the arcs $a_i \to a_j$ and $a_{i+1} \to a_{j+1}$ provided the distance decreases and there is actually an arc joining them, then the original arcs are replaced by the arcs $a_i \to a_j$ and $a_{i+1} \to a_{j+1}$.

Consider a capacitated vehicle routing problem with optimal route as shown in Figure 3.5.

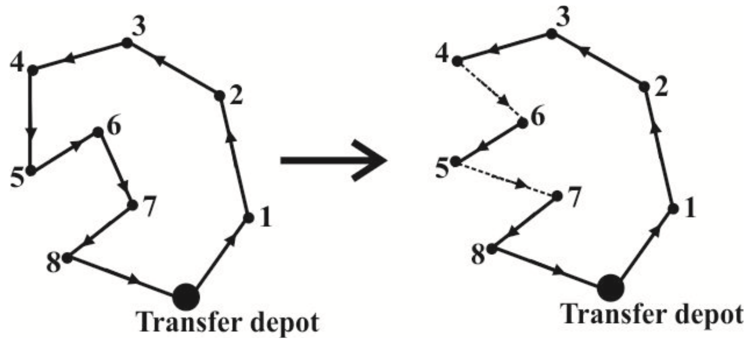


Figure 3.5: Optimal route by 2-opt heuristic

The tour has a length $d_{01}+d_{12}+d_{23}+d_{34}+d_{45}+d_{56}+d_{67}+d_{78}+d_{80}$

When the tour is improved by 2-opt heuristic, the optimal tour becomes

$$d_{01}+d_{12}+d_{23}+d_{34}+d_{46}+d_{65}+d_{57}+d_{78}+d_{80}$$

Basically, when we have cluster centre consisting of three (3) customers then the improved optimal tour is given by $\min\left\{d_{01}+d_{13}+d_{32}+d_{20}, d_{02}+d_{21}+d_{13}+d_{30}, d_{03}+d_{32}+d_{21}+d_{10}\right\}$.

3.8.3.9 Local and Global Pheromone Updates

In order to avoid the probability of selecting a customer repeatedly, the amount of pheromone on an arc is reduced through evaporation. This is done by applying a local pheromone updating rule given by $\tau_{ij}^{\text{new}}=(1-\rho)\tau_{ij}^{\text{old}}+\rho\tau_{ij}(0)$ where $\rho\in(0,1)$ (3.26)

and ρ is the trial evaporation constant, τ_{ij}^{new} is the pheromone on the link (*i*, *j*), τ_{ij}^{old} is the pheromone on the link (*i, j*) before updating. When no feasible customer is available due to vehicles capacity constraints then the transfer depot is chosen and a new route is started. This process is executed until all customers have been visited. When all ants construct their tours, the best ant tours are chosen and the global pheromone updating rule is applied by the rule given by $\tau_{ij}^{\text{new}}=(1-\rho)\tau_{ij}^{\text{old}}+\rho\sum_{r=1}^{m}\Delta\tau_{ij}^{r}$ (3.27) where $\Delta\tau_{ij}^{r}$ is the increased in pheromone on link (i, j) of route r found by the ant. The pheromone increment updating rule is given by

$$\Delta\tau_{ij}^{r} = \begin{cases} \dfrac{P}{\omega \times \sum_{r} D^{r}} \times \dfrac{D^{r} - d_{ij}}{m^{r} \times D^{r}} & \text{if link } (i,\ j) \text{ is on the } r\text{th route} \\ 0 & \text{otherwise} \end{cases}$$

Where P is a constant, L the total length of all routes in the solution, D^{r} is the length of the rth route in the solution, d_{ij} is the length of link (i, j) and m^{r} the number of customers in the rth route and ω is the number of routes in the solution and $\omega > 0$.

ILLUSTRATIVE EXAMPLE

Consider the route obtained by ants in its tour

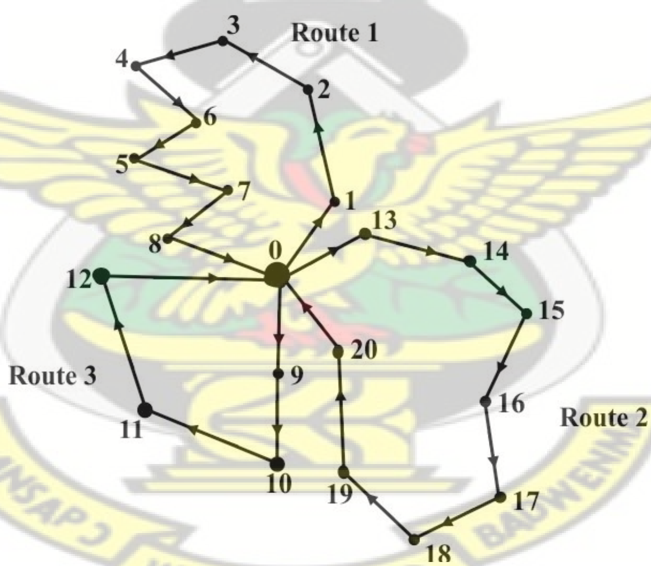


Figure 3.6: An example of capacitated Vehicle Routing Problem

The parameters for updating the increased pheromone on the links are as shown in the solution below.

		Customer							
Route	1	1	2	3	4	5	6	7	8
	2	13	14	15	16	17	18	19	20
	3	9	10	11	12				

		D^r	m^r
Route	1	$D^1 = d_{0,1} + d_{1,2} + d_{2,3} + d_{3,4} + d_{4,5} + d_{5,6} + d_{6,7} + d_{7,8} + d_{8,0}$	8
	2	$D^2 = d_{0,13} + d_{13,14} + d_{14,15} + d_{15,16} + d_{16,17} + d_{17,18} + d_{18,19} + d_{19,20} + d_{20,0}$	8
	3	$D^3 = d_{0,9} + d_{9,10} + d_{10,11} + d_{11,12} + d_{12,0}$	4
$\sum$		$L = D^1 + D^2 + D^3$	

		$\Delta\tau_{ij}^r$
Route	1	$\Delta\tau_{0,1}^1 = \frac{P}{3L} \times \frac{D^1 - d_{0,1}}{8D^1}$ • • • • • • •
	2	• $\Delta\tau_{13,14}^2 = \frac{P}{3L} \times \frac{D^2 - d_{13,14}}{8D^2}$ • • • • • •
	3	• • • $\Delta\tau_{12,0}^3 = \frac{P}{3L} \times \frac{D^3 - d_{12,0}}{4D^1}$
$\sum$		$\sum_r \Delta\tau_{ij}^r = \frac{P}{3 \times L}$

Using the results from table 3.1 and 3.2, the best ant tour is as follows. First, we consider the fifteen customers, where we had two transfer depots (sites) with their centers C_1 and C_2 indicated with a red colour and the customers with a blue colour.

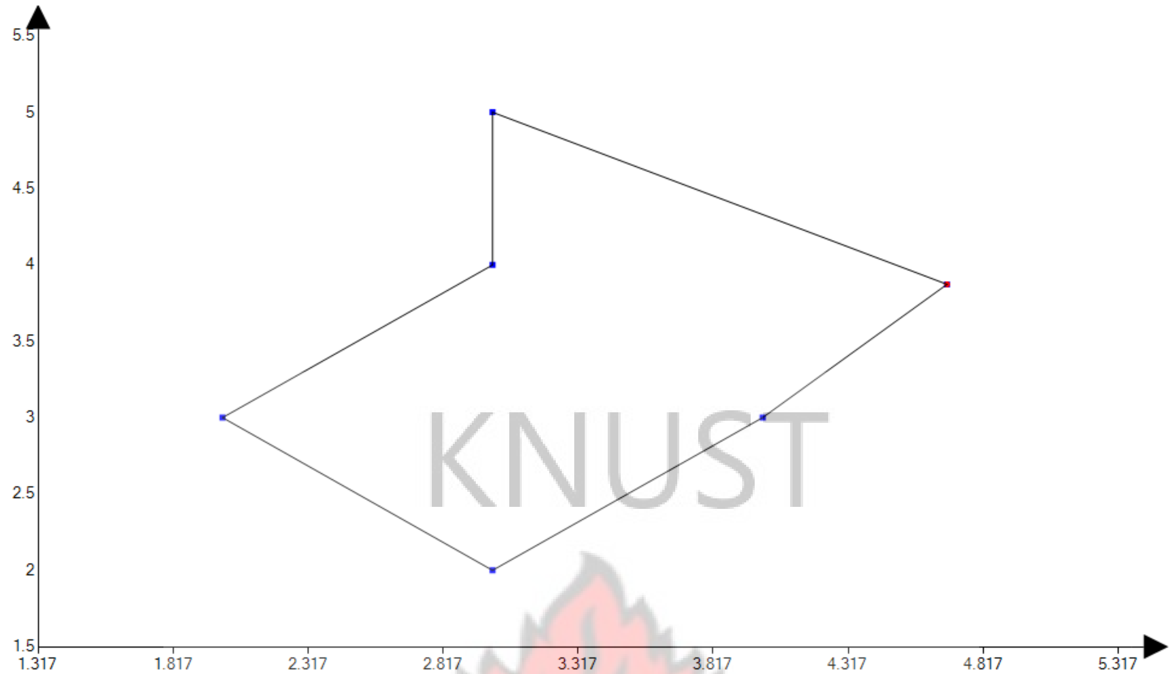


Figure 3.7: First Ant best tour from sub-cluster one in transfer depot one

With a tricycle of capacity 840 litres (6 of 140 litre bins), sub-clusters 1 and 2 would need two rounds of routing each to complete collection of waste from the customers. Applying our model on Ant colony system the results on Ant best tour, customers visited, number of bins emptied and distance covered in a tour are shown in the table below.

Table 3.3: Summary of routing from sub-clusters in transfer depots one and two

Sub-cluster One			
Customer ID, ant best tour	Customers	Total bins	Length (m)
$0 \rightarrow 15 \rightarrow 2 \rightarrow 1 \rightarrow 10 \rightarrow 14 \rightarrow 0$	5	6	8.375
$0 \rightarrow 8 \rightarrow 12 \rightarrow 11 \rightarrow 0$	3	4	8.241
Sub-cluster Two			
$0 \rightarrow 6 \rightarrow 4 \rightarrow 7 \rightarrow 9 \rightarrow 0$	4	6	15.861
$0 \rightarrow 3 \rightarrow 13 \rightarrow 5 \rightarrow 0$	3	6	12.377

For the 32 customers, we had four transfer depots and two routes for each sub-cluster

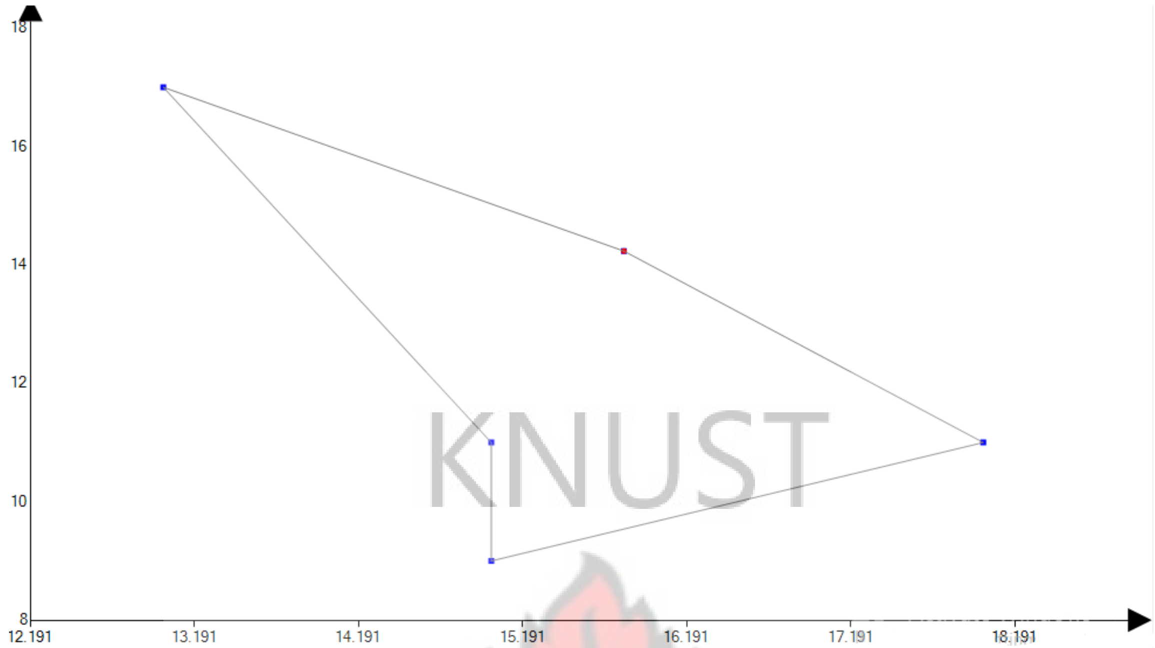


Figure 3.8: Second Ant best tour from sub-cluster one in transfer depot one

Using a tricycle of capacity 1260 litres (9 of 140 litre bins), the four sub-clusters would need two rounds of routing each to complete collection of waste from the customers. Applying our model on Ant colony system the results on Ant best tour, customers visited, number of bins emptied and distance covered in a tour are shown in the table below.

Table 3.4: Summary of routing from sub-cluster one in transfer depots one and two

Sub-cluster One			
Customer ID, ant best tour	Customers	Total bins	Length (m)
$0 \to 12 \to 15 \to 20 \to 18 \to 0$	4	9	15.707
$0 \to 19 \to 30 \to 29 \to 16 \to 0$	4	8	19.779
Sub-cluster Two			
$0 \to 7 \to 25 \to 8 \to 22 \to 24 \to 0$	5	9	15.227
$0 \to 9 \to 13 \to 14 \to 27 \to 31 \to 0$	5	9	18.29
Sub-cluster Three			
$0 \to 1 \to 21 \to 32 \to 17 \to 0$	4	9	12.505
$0 \to 5 \to 23 \to 26 \to 0$	3	6	9.244
Sub-cluster Four			
$0 \to 2 \to 3 \to 11 \to 4 \to 10 \to 6 \to 0$	6	9	28.558
$0 \to 28 \to 0$	1	2	1.72

3.9 PROPOSED CAPACITATED VEHICLE ROUTING PROBLEM WITH TIME WINDOWS, CREW LUNCH BREAK, STOPPING TIME AND DEAD HEADING TIME

3.9.0 INTRODUCTION

Vehicle Routing Problem (VRP) is a class of well known NP-hard combinatorial optimisation problem. VRP is concerned with the design of optimal routes, used by a fleet of identical vehicles stationed at a central depot to serve a set of customers with known demands. When the capacity constraint is considered, the problem is considered as a Capacitated VRP (CVRP) with the objective of minimizing total cost (distance) of routes.

The Capacitated Vehicle Routing Problem with Time Windows (CVRPTW), is a generalization of the CVRP. In the CVRPTW, the vehicles must comply with constraints of time windows associated with each customer in addition to the capacity constraints.

3.9.1 Capacitated Vehicle Routing Problem with Time Windows

The collection of solid waste vehicle routing problem is mainly solved on residential or commercial (Industrial) waste. Both residential and commercial collection problems can be classified as variants of vehicle routing problem with time windows (VRPTW) but with additional constraints. A VRP comprises a set of vehicles, customer stops and a depot. Each vehicle starts from the depot, visits a number of customers and ends at the depot. A VRPTW is an extension of VRP by an additional time constraints associated with each customer. Solid waste vehicle routing problem with time windows can be summarized as follows:

Objectives

- Minimize number of vehicles;
- Minimize total travel time;
- Balance workload among collecting vehicles.

Constraints

- Vehicle capacity (volume, weight)
- Route capacity (maximum number of residential customers a vehicle can handle per day)
- Routing time limit per vehicle
- Time windows of the stopping times at customer point and the transfer depot.
- Clews' lunch break.

3. 9.2 Problem Formulation

To explain the problem of VRPTW, we present a mathematical programming model for a simplified version by minimizing the travel and service time. We adopted the basic VRPTW model by Cordeau et al., (2002) and modified it by incorporating the transfer depot, clews' lunch time, stopping time and deadheading time. A simplified solid waste VRPTW is defined on the network $G = (V, A)$ where $A = \{(V_i, V_j); i \neq j$ and $i, j \in V)$ is an arc set and the vertex set $V = \{v_0, v_1, \ldots, v_{n+m}, v_{n+m+1})$ where v_0 and v_{n+m} denote the transfer depot at which vehicle of capacity Q start and end their tour and v_{n+m+1} is the node for the lunch break. Each vertex in V has an associated demand $q_i \geq 0$, a service time $t_s \geq 0$ and a service time window $\eta_i[t_i, T_i]$; where η_i is the number of bins to be emptied at customer i and the transfer depot time window $[T_0, T_t]$ representing earliest possible departure time from the transfer depot and the latest possible arrival time at the transfer depot. In particular, the transfer depot has $t_0 = 0$ and $q_0 = 0$. The set of service points $C = \{v_1, v_2, \dots, v_n\}$ specifies a set of *n* customers. The arrival time of a vehicle at customer $i, i \in C$ is denoted by t_{ai} and its departure time t_{pi}. An arc (V_i, V_j) has an associated distance $d_{ij} \geq 0$ and a travel time $t_{ij}(t_{pi})$ a function of the departure time from customer *i*. The set of available vehicles is denoted by *K*. The objective function is the minimization of total time. There are two decision variables; x_{ij}^k is a binary decision that indicates whether vehicle *k* travels between customers *i* and *j*. The real decision variable t_{sti} indicates service start time for customer *i* served by vehicle *k*. The VRPTW model is formulated as follows.

$$\text{Min} \sum_{k \in K} \sum_{(i,\, j) \in A} t_{ij}^k x_{ij}^k + \sum_{k \in K} \sum_{j \in C} (t_{s,n+ms}^k - t_{s0}^k) x_{0j}^k$$

Subject to

$$\sum_{i \in C} q_i \sum_{j \in V} x_{ij}^k \leq Q, \quad \forall k \in K \tag{3.28}$$

$$\sum_{k \in K} \sum_{j \in V} x_{ij}^k = 1, \quad \forall i \in C \tag{3.29}$$

$$\sum_{i \in V} x_{il}^k - \sum_{j \in V} x_{lj}^k = 0, \quad \forall l \in C, \forall k \in K \tag{3.30}$$

$$x_{i0}^k = 0, x_{n+m,i}^k = 0, \forall i \in V, \forall k \in K \tag{3.31}$$

$$\sum_{j \in V} x_{0j}^k = 1, \quad \forall k \in V \tag{3.32}$$

$$\sum_{j \in V} x_{j,m+n}^k = 1, \quad \forall k \in V \tag{3.33}$$

$$\sum_{i=1}^{n+m} x_{i,n+m+1i}^k = 1, \quad \forall k \in K \tag{3.34}$$

$$\sum_{j=1}^{n+m} x_{n+m+1,j}^k = 1, \quad \forall k \in K \tag{3.35}$$

$$t_i \sum_{j \in V} x_{ij}^k \le t_{sti}^k, \ \forall i \in V, \forall k \in K \tag{3.36}$$

$$\eta_i (T_i - t_i) \sum_{j \in V} x_{ij}^k, \ \forall k \in K \tag{3.37}$$

$$x_{ij}^k (t_{sti}^k + t_{si} + t_{ij} (t_{sti}^k + t_{si}) \le t_{stj}^k, \ \forall (i,\ j) \in A, \forall k \in K \tag{3.38}$$

$$\sum_{i=0}^{n+m} t_{ij} x_{ij}^k + \sum_{i=0}^{n+1} t_{si}^k x_{ij}^k + \sum_{i=1}^{n} T_{di} x_{ij}^k \le (T_t - T_0) \tag{3.39}$$

$$x_{ij}^k \in \{0,1\}, \forall (i,\ j) \in A, \forall k \in K \tag{3.40}$$

$$t_{sti}^k \in R, \forall i \in V, \forall k \in K \tag{3.41}$$

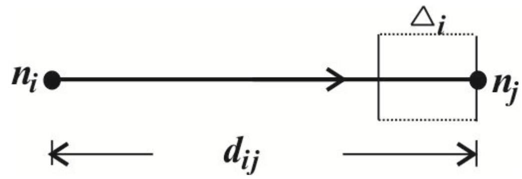
Constraint (3.28) impose the rule that the vehicles capacity cannot be exceeded, (3.29) ensures that all customers are served, if a vehicle arrives at a customer it must also depart from that customer (3.30), route must start and end at the depot (3.31), each vehicle leaves from and returns to the transfer depot once (3.32) and (3.33) respectively. Constraints (3.34) and (3.35) are introduced to add the lunch break for each route; service times must satisfy time window start (3.36) and ending (3.37) times; and service start time must allow for travel time between customers (3.38). Constraint (3.39) ensures that the transfer depot time window is not violated. Decision variables type and domain are indicated in (3.40) and (3.41)

3.9.3 Stopping Criteria

The speed of a service vehicle depends on the nature of road, the present weight of the vehicle and the distance between two adjacent customers. In this thesis we shall consider the time spend by a driver when he/she apply breaks to stop at a customer for service, while assuming a constant speed of the vehicle.

Consider the speed of the vehicle as $\alpha \mathrm{ms}^{-1}$

Final velocity at customer j is zero;



The distance from customer j when breaks are applied is given as Δ_i

Applying Newton's law of motion

$$0^2 = \alpha^2 + 2a\Delta_i$$

$$a = -\frac{\alpha^2}{2\Delta_i}$$

Also

$$0 = \alpha + \left(-\frac{\alpha^2}{2\Delta_i}\right)t$$

$$t = \frac{2\Delta_i \alpha}{\alpha^2}$$

$$= \frac{2\Delta_i}{\alpha}$$

Time taken to travel from customer i to customer j $(t_{ij}) = \dfrac{d_{ij} - \Delta_i}{\alpha} + \dfrac{2\Delta_i}{\alpha}$ (3.42)

a_0 = time taken to alight from the vehicle before the start of service and get onto the vehicle after service

Time taken to service customer i is $t_{si} = \eta_i t'_{si} + a_0$ (3.43)

Time for deadheading T_d (traversing an edge i to j without collection) is $T_d = \dfrac{d_{ij}}{\alpha}$

3.10 ENVIRONMENTALLY RELATED ISSUES ON UNCOLLECTED SOLID WASTE

3.10.0 INTRODUCTION

Uncollected solid waste is visible to the public and makes collection and disposal of solid waste potentially an explosive political issue. Besides being an eyesore, it generates foul smell and bacteria. Uncollected urban waste is a danger to health, pollutes the environment, erodes civic morals and can be a major social problem. Uncollected solid waste has the potential of deterring tourist, leading to loss of income to the local authorities and the government at large.

3.10.1 Heptagonal Signed Graph of Impact of Solid Waste

Waste and its associated impact to the growth of a society is simplified below. The seven (7) controlling factors are: *b* = amount of bacteria; *r* = rate of migration into the city; *n* = number of diseases, *m* = amount of modernization; *p* = population size; *s* = sanitation facilities and
w = quantity of waste.

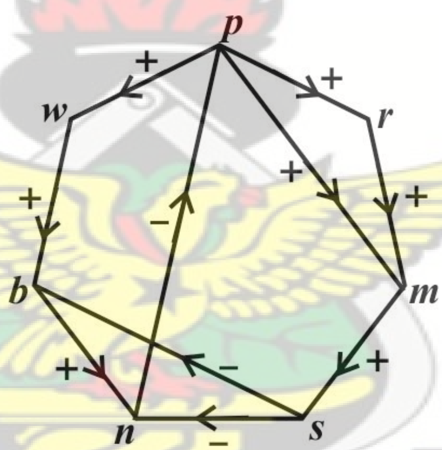


Figure 3.9: Signed Digraph of Solid Waste Impact

The arc from *w* to *b* is marked positive (+) since an increase in waste leads to an increase in bacteria, whereas the arc from *s* to *n* is marked negative (-) since an improvement in sanitation facilities leads to a decrease in the number of diseases. An increase in population (*p*) results in an increase in waste generation (*w*) and in turn increases the bacteria and a decrease in number of diseases (*n*) reduces population. An increase in population (*p*) increases the pressure towards modernization (*m*), leading to improvement in sanitation facilities (*s*).

3.10.2 Characteristics of Solid from Developing Country

Characteristics of solid waste especially in development countries depend on a number of factors such as food habits, cultural traditions, socio-economic and climate conditions. The organic matter in solid waste from developing countries is usually higher, because most of the food comes in their primary state. This larger fraction tends to decompose at a faster rate due to high temperatures usually experienced in Africa. Table (4.3) shows the constituents of the refuse in Kumasi the second largest city of Ghana.

Table 3.5: Refuse analysis- City of Kumasi, Ghana

Type of Waste	House to House (%)	Market refuse (%)	Public dumping area (%)
Putrescible	62.0	73	71.0
Paper	12.8	5.3	6.8
Metal	1.5	0.7	1.3
Textile	2.7	1.6	2.5
Glass	1.9	1.8	1.7
Plastics	18.1	15.0	16.2
Others	1.0	2.6	0.5

3. 10.3 Proposed Mathematical Model for Spread of Smell

Servicing a street segment depends on the quantity of solid waste (q_{ij}) along that street and the cost of servicing it. In this model, the cost of servicing a customer j is the distance from customer i to customer j (d_{ij}). Amponsah and Salhi (2004) had first proposed model for the spread of smell using the as $\delta_{ij} = \frac{(q_{ij})^{\alpha}}{(C_{ij})^{1-\alpha}}, 0 \le \alpha \le 1$. However, in this thesis, we introduce an improvement version of the spread of smell, based on the capacitated routing of the service vehicle, and a time factor, since the spread of smell is the level of decomposition of the organic matter in the container.

Our proposed model for the spread of smell is given by

$$\delta_{ij} = \frac{\left\{ \eta \times \sum_{i=1}^{n} q_{ij} \right\}^{\alpha(t)}}{\left\{ \sum_{i=1}^{n} d_{ij} \right\}^{1-\frac{1}{\alpha(t)}}}, \; 0 \le \alpha(t) \le t_{\max} \tag{3.44}$$

where η is the fraction of waste which is degradable. The term $\alpha(t)$ is time dependent and can be expressed as

$$\alpha(t) = \begin{cases} \varepsilon & 0 \le t \le t_1 \\ 2\varepsilon & t_1 < t \le t_2 \\ 3\varepsilon & t_2 < t \le t_3 \\ \ldots & \ldots \end{cases} \tag{3.45}$$

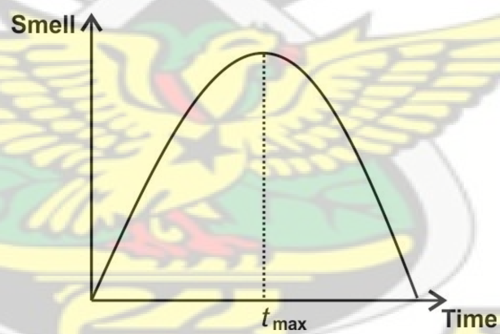


Figure 3.10: Graph of smell against time

ILLUSTRATIVE EXAMPLE

Testing our models on vehicle routing problem with time windows and smell on the fifteen (15) and thirty-two customer problem in Tables 3.1 and 3.2, the results are summarized in the table below. The first four columns of tables 3.6 and 3.7 were obtained using the improved Ant Colony System, the fifth and sixth columns were obtained using our improved model on VRPTW and model on smell respectively.

Table 3.6: Summary of routing time and smell from the 15 customer problem

Sub-cluster One					
Customer ID, ant best tour	Customers	Total bins	Length (m)	Time (sec)	Smell
$0\rightarrow15\rightarrow2\rightarrow1\rightarrow10\rightarrow14\rightarrow0$	5	6	8.375	255.94	12.39
$0\rightarrow8\rightarrow12\rightarrow11\rightarrow0$	3	4	8.241	90.51	3.71
Sub-cluster two					
$0\rightarrow6\rightarrow4\rightarrow7\rightarrow9\rightarrow0$	4	6	15.861	215.25	8.08
$0\rightarrow3\rightarrow13\rightarrow5\rightarrow0$	3	6	12.377	167.62	9.54

Table 3.7: Summary of routing time and smell from the 32 customer problem

Sub-cluster One					
Customer ID, ant best tour	Customers	Total bins	Length (m)	Time (sec)	Smell
$0\rightarrow12\rightarrow15\rightarrow20\rightarrow18\rightarrow0$	4	9	15.707	307.94	27.49
$0\rightarrow19\rightarrow30\rightarrow29\rightarrow16\rightarrow0$	4	8	19.779	266.02	16.52
Sub-cluster two					
$0\rightarrow7\rightarrow25\rightarrow8\rightarrow22\rightarrow24\rightarrow0$	5	9	15.227	316.22	28.03
$0\rightarrow9\rightarrow13\rightarrow14\rightarrow27\rightarrow31\rightarrow0$	5	9	18.29	313.04	24.79
Sub-cluster three					
$0\rightarrow1\rightarrow21\rightarrow32\rightarrow17\rightarrow0$	4	9	12.505	279.31	31.98
$0\rightarrow5\rightarrow23\rightarrow26\rightarrow0$	3	6	9.244	206.00	11.60
Sub-cluster four					
$0\rightarrow2\rightarrow3\rightarrow11\rightarrow4\rightarrow10\rightarrow6\rightarrow0$	6	9	28.558	301.97	18.39
$0\rightarrow28\rightarrow0$	1	2	1.72	96.20	1.33

3.11.0 Comparative Vehicle Fuel Consumption Analysis on two Elemental Time Schedule

In this section, we shall look at our proposed dead speed fuel consumption model in addition to an improved version of four elemental fuel consumption models first proposed by Bowyer et al., (1985).

3.11.1 Proposed Dead Speed Model and Improved Fuel Consumption Models for Day Service

Transportation of goods and services within cities has become a challenge due to urbanization, leading to low productivity and high consumption of fuel. High consumption of fuel by inter-city service vehicles depends on several factors. Some of these factors include: traffic conditions, nature of road, Weight of the vehicle, energy efficiency parameters, drag force, rolling resistance, tyre pressure, vehicle frontage area, air resistance and others. In literature, one of the mostly used fuel consumption models was described by Bowyer et al. (1985). The model estimates fuel consumption on four modes: namely Idle, Cruise, Acceleration and Deceleration. In this study, we have proposed an additional model to the existing models called Dead speed fuel consumption model. This model comes into play when the vehicle is in motion but it is such that the speed of the vehicle is in the range of $5\text{kmh}^{-1} \le V \le 20\text{kmh}^{-1}$.

The five elemental models uses constant parameters such as idle fuel rate, fuel consumption per unit of energy, fuel consumption per unit of energy-acceleration, rolling drag force, rolling aerodynamics force and instantaneous acceleration. The variable parameters such as speed of the vehicle, mass of the vehicle, cruise speed, time for each of the mode, distance for each of the modes would have to be determined before the model is applied. The time for each of the mode is given by: $T_D = t_a + t_d + t_D + t_i + t_C$ where

$T_D =$ Total time for the entire journey;
$t_a =$ time during acceleration;
$t_d =$ time during deceleration;
$t_D =$ time during dead speed mode;
$t_i =$ time during idle mode;
$t_C =$ time during cruising.

and their corresponding distance is given by $d_a + d_d + d_D + d_C = D.$ The model consist of five functions F_a, F_d, F_C, F_i and F_D which correspond to fuel consumption estimations in milli-litres (ML) for acceleration, deceleration, Cruise, Idle and Dead mode respectively. The mathematical equations are described below.

(a) Acceleration fuel consumption model is given by:

$$F_a = \varphi^{-1}\left\{\alpha + (AC_r + 0.1Bk_1\bar{V} + \beta_1 ME_k + k_2\beta_2 ME_k^2 + 0.000981\beta_1 M\omega)d_a\right\} \tag{3.46}$$

where

$$k_1 = 0.312 + 0.000544V_{fa} - 0.0171\sqrt{V_{ia}}$$

$$k_2 = 1.115 + 0.00215V_{fa} + 0.00538V_{ia}$$

$$\bar{V} = \tfrac{1}{2}(V_i + V_f)$$

$$E_k = \frac{0.3858\times10^{-4}(V_{fa}^2 - V_{ia}^2)}{d_a}$$

(b) Deceleration fuel consumption model is given by:

$$F_d = \varphi^{-1}\left\{\alpha + (0.087Ak_x + Bk_1^{'}k_y\bar{V}_d + k_a\beta_1 ME'_k + k_x\beta_1 ME'^2_k + 0.00981\beta_1 M\omega)d_d\right\} \tag{3.47}$$

where

$$k_1^{'} = 11.1661 + 0.78V_i - 0.189\sqrt{V_f}$$

$$k_x = 0.046 + \tfrac{100}{M} + 0.0042V_i + 0.0026V_f + 0.0544\omega$$

$$k_y = \sqrt[4]{k_x^3}$$

$$k_a = k_x^{3.81}(1350 - k_x^{3.81})$$

$$E'_k = -\frac{0.3858\times10^{-4}(V_{fd}^2 - V_{id}^2)}{d_d}$$

(c) Cruise fuel consumption model is given by:

$$F_C = \varphi^{-1}\left\{\frac{f_i}{V_C} + (AC_r + 0.02BV_C^2 + \tau_c{}'\beta;_1 ME_C + 0.00371\beta_2 k_2 ME_C^2 + 0.000981\tau_c{}''\beta'_1 M\omega)d_C\right\} \tag{3.48}$$

where

$$\tau_c' = \tfrac{0.040}{V_C} + 1.3\times10^{-5}V_C^2$$

$$E_C = 0.258 - 0.001V_C$$

$$\tau_c'' = 0.4582 + 3.78\times10^{-3}V_C$$

(d) Dead speed fuel consumption model is given by:

$$F_D = \varphi^{-1}\left\{\begin{array}{l}\dfrac{f_i}{V_D} + (AC_r + 0.02BV_D^2 + \tau_c'''\beta'_1 ME'_D + 0.00371\beta_2 k_2 ME'^2_D + \\ 0.000981\tau_D''\beta'_1 M\omega)d_D\end{array}\right\} \tag{3.49}$$

where

$$\tau_D''' = \tfrac{0.3125}{V_D} - 0.00013V_D^2$$

$$E'_D = 0.0443 + 0.0018V_D$$

$$\tau_D'' = 0.4582 - 3.78\times10^{-3}V_D$$

(e) Idle fuel consumption model is given by: $F_i = \alpha t_i$ (3.50)

Total fuel consumption for a distance *D* and total time T_D under the five elemental models in (mL) is given by

$$F_T = \int_0^{t_a} F_a dt + \int_0^{t_d} F_d dt + \int_0^{t_c} F_C dt + \int_0^{t_D} F_D dt + \int_0^{t_i} F_i dt \tag{3.51}$$

3.11.2 Night Service

In the night, most of our roads in the urban centers are free from traffic congestion and therefore vehicle fuel consumption models on Idle and Dead speed modes are absent. The consumption model for vehicles in the night is usually made up of three namely; Acceleration, Deceleration and Cruise models. The total fuel consumption of a service vehicle is given by

$$F_T = \int_0^{t_a} F_a dt + \int_0^{t_d} F_d dt + \int_0^{t_c} F_C dt \tag{3.52}$$

Table 3.8: Notation used in the model

Notation	Description	Values
A	Function Parameter (mL/km)	21 – 100
B	Function Parameter (mL/km)/(km/h)^2	0.0055 – 0.018
α	Constant Idle fuel rate	(0.375 – 0.556)mL/s
$\beta_1,\ \beta'_1$	Fuel consumption per unit of energy	(0.005 – 0.16)mL/KJ
β_2	Fuel consumption per unit of energy-acc.	(0.01 – 0.05)mL/(KJm/s^2)
φ	Efficiency parameter for diesel engines	0.79 – 0.95
ω	Road gradient	$(0.0056-0.0167)\pi$rad
C_r	Coefficient of rolling resistance	0.010
k_1	Integration coefficient	
k_2	Integration coefficient	
k_x	An energy related parameter	
k_y	An energy related parameter	
$\tau_c^{'}$	Calibration parameter during cruising	
$\tau_c^{''}$	Calibration parameter during cruising	
$\tau_c^{'''}$	Calibration parameter during cruising	
$E_D^{'}$	An energy relation parameter during dead speed	
$\tau_D^{''}$	Calibration parameter during dead speed	
$\tau_D^{'''}$	Calibration parameter during dead speed	

CHAPTER FOUR

DATA COLLECTION AND ANALYSIS

4.0 INTRODUCTION

In this chapter, we shall adopt our proposed methods on probabilistic location distance location model, Ant Colony heuristics with time windows to solve a real case of solid waste generation per capita, collection and transportation which exist in Tafo Pankrono, a third class communities in Kumasi Metropolitan Assembly, Ghana. The solid waste generation per capita per day are presented in the first section. Section two is devoted to the probabilistic clustering and location of skip containers in the communities mentioned above. The third section of this chapter will consider multiple vehicle routing with time windows using improved ant colony heuristics and the impact of smell due to uncollected solid waste in the communities. The last but not least section of the chapter looks at comparative analysis of fuel consumption for transportation of solid waste during day and night to the dump site.

4.1 DATA COLLECTION AND ANALYSIS OF SOLID WASTE GENERATION PER PERSON PER CAPITA PER DAY

The data for the analysis of waste generation per person per capita per day was a mixture of primary and secondary. The area has put into five (5) zones by Statistical service in terms of population. Each zone made up of different number of houses:

Zone one (1) 424 houses, Zone two (2) 325, Zone three (3) 542, Zone four (4) 561 and Zone five (5) 623 houses. A sample of 10 houses was selected from each zone using remainder linear systematic sampling method, since none of the zones is a multiple of the sample size.

4.1.1 Remainder Linear Systematic Sampling

The sample interval for systematic sampling is given by $k = \frac{N}{n}$, if N is an integer multiple of the sample size n with $N = nk$. For a population which is not a multiple of the sample size $N = nk + r$ where $0 < r < n$ and N, n, k, r are integers.

The population is divided into two strata such that the first stratum contains the front $(n-r)k$ units and the second stratum contains the hind $(k+1)r$ units. The sampling procedure is as follows:

(a) Select a random start k_1 from 1 to k, and every k-th number thereafter from the first $(n - r)$ groups as the first stratum sample. That is, the k_1 -th unit of each group of the first stratum is in the sample. Denote the sample set as s' with indices $kl + k_1, l = 0, 1, \ldots, (n-r)-1$

(b) Select a random start k_2 from 1 to $(k+1)$ starting with the $[(n-r)k + k_2]$-th unit and every $(k + 1)$ -th number thereafter from the r groups as the second stratum sample. Denote the sample set from the second stratum as s'' with indices $(k+1)l' + (n-r)k + k_2,\ l' = 0, 1, \ldots, r-1$

The method was used to obtain the sample from each of the five (5) zones. The waste generated from the selected families was observed repeatedly every three days for a month and the average recorded for month one (1). The procedure is repeated for six month as shown below.

Applying the formula $(n-r)k$, the first strata have 1-252 members, and the second strata have 253-424 members.

By simple random sampling $k_1 = 19$, and using the formula $kl + k_1, l = 0, 1, \ldots, (n-r)-1$, the sample set for stratum 1is $s' = \{19, 61, 103, 145, 187, 229\}$

By simple random sampling $k_2 = 8$, and using the formula $[(n-r)k + k_2]$ gives the start value as 259. The sample set for stratum 2 is $s'' = \{259, 302, 345, 388\}$.

Table 4.1: Average waste generation per day per family size

Zone 1 (First month)		
House ID	Family size	Average waste per day(kg)
19	4	2.6
61	3	2.2
3a	3	2.0
45a	4	2.5
87a	5	3.0
29b	4	1.8
59b	3	1.6
2c	2	1.0
45c	4	2.8
88c	5	3.1

Table 4.2: Average waste generation per day per family size over six month period

Zone 1 (Average waste generation per day per family size (kg))							
House ID	Family size	Month 1	Month 2	Month 3	Month 4	Month 5	Month 6
19	4	2.6	2.5	2.3	2.4	2.4	2.4
61	3	2.2	1.8	1.9	2.0	1.6	1.7
3a	3	2.0	2.2	1.9	2.2	1.7	1.9
45a	4	2.5	3.0	2.6	2.8	2.3	2.3
87a	5	3.0	3.4	2.8	2.6	2.9	2.8
29b	4	1.8	2.3	2.6	2.2	2.4	2.6
59b	3	1.6	1.9	1.9	1.8	1.6	2.0
2c	2	1.0	1.3	1.4	1.0	1.0	1.2
45c	4	2.8	2.6	2.3	2.2	2.2	2.4
88c	5	3.1	2.9	2.9	3.0	3.0	2.8

Similar approach was used to obtain the sample for zones 2 though 5, as shown in the Appendix

After testing the data with four different models, the best one with the lowest AIC was found to be Model 4: varying intercept and varying slope model without random effects

$y_{ij} = \beta_{0[j]} + \beta_{1[j]} x_i + e_{ij}$, where $e_{ij} \sim iid\ N(0, \sigma)$

Fitting a linear model $y_i = \beta_0 + \beta_1 x_i + e_i$, where $e_i \sim iid\ N(0, \sigma)$ gave an average waste generation per person per day as 0.597kg

4.2 CAPACITATED LOCATION OF SEMI-OBNOXIOUS FACILITY (SKIP CONTAINERS)

The proposed probabilistic method for facility location is applied on the study area to optimally find the number of 14m^3 bins that needs to be located in the area based on their waste generation and the capacity of the container. As per the objective of the study, the research concentrated on the third class communities of the area. Table 4.3 show the number of communities in Tafo

Table 4.3: Communities in Tafo Sub-metro

NO.	COMMUNITY	CLASS
1.	Adabraka	2
2.	New Tafo	2
3.	Pankrono estates	2
4.	Tafo Nhyiaeso	2
5.	Old Tafo West	3
6.	Old Tafo East	3
7.	Pankrono Atafoa	3
8.	Pankrono Dome	3
9.	Pankrono West	3
10.	Tafo Adompom	3
11.	Tafo Ahenbronum	3

The study area has eleven (11) communities of which seven (7) of them are categorized as third class zones. The study considered eight (8) of these third class communities namely; Old Tafo East, Pankrono Dome, Pankrono West, Tafo Adompom and Tafo Ahenbronum. The area under study was physically divided into four zones due to physical boundaries such as streams, huge bridges which cannot be used by vehicles. Zone one (1) has seven hundred and forty-nine (749) houses, zone two (2) with five hundred and sixty-one (561) houses, zone three (3) has five hundred and forty-two (542) houses and zone four had six hundred and twenty-three (623) houses. Data obtained from Ghana Statistical service and our own field work provided the number of people in the houses in the study area. These numbers were used to determine the average waste a house can generate over a three day period before collection. With these data, zone one is to be served with one thousand one hundred (1100) of one hundred and forty (140) litre waste bins, Eight hundred and twenty-eight (828) of one hundred and forty (140) litre waste bins to zone two, seven hundred and ninety-two (792) and seven hundred and eighty-nine (789) bins in zones 3 and four respectively.

4.2.1 Raw Data Points of the Study Area

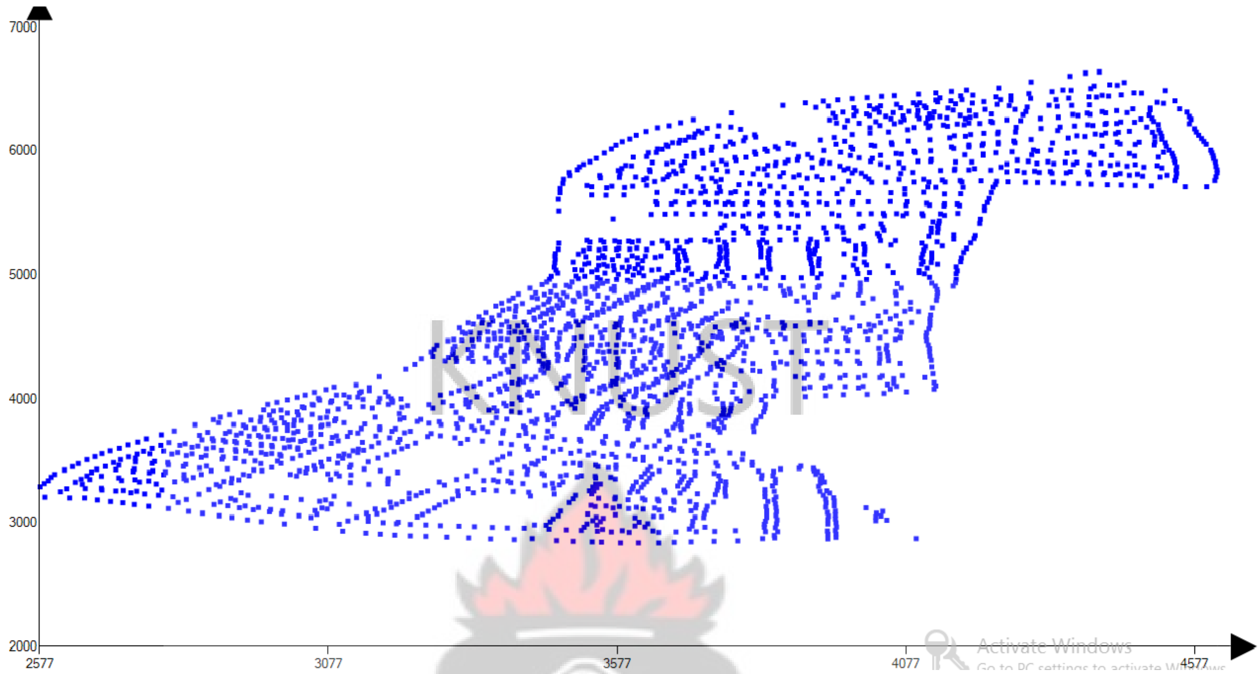


Figure 4.1: customer points in the study area

4.2.2 Cluster Centres from the Zones

A 14m^3 skip containers were used for the location centres, with a compaction factor of 0.4, a container can hold 166 of 140 litre bins. With the number of bins in zone one, the number of bins needed is $\frac{\sum_{i=1}^{n} q_i}{v_j} = \frac{1100}{166} = 6.62 \cong 7$

Coordinates of all the houses were taken and presented as shown in the diagrams below with the blue dots representing the houses and the red dot as the cluster centre.

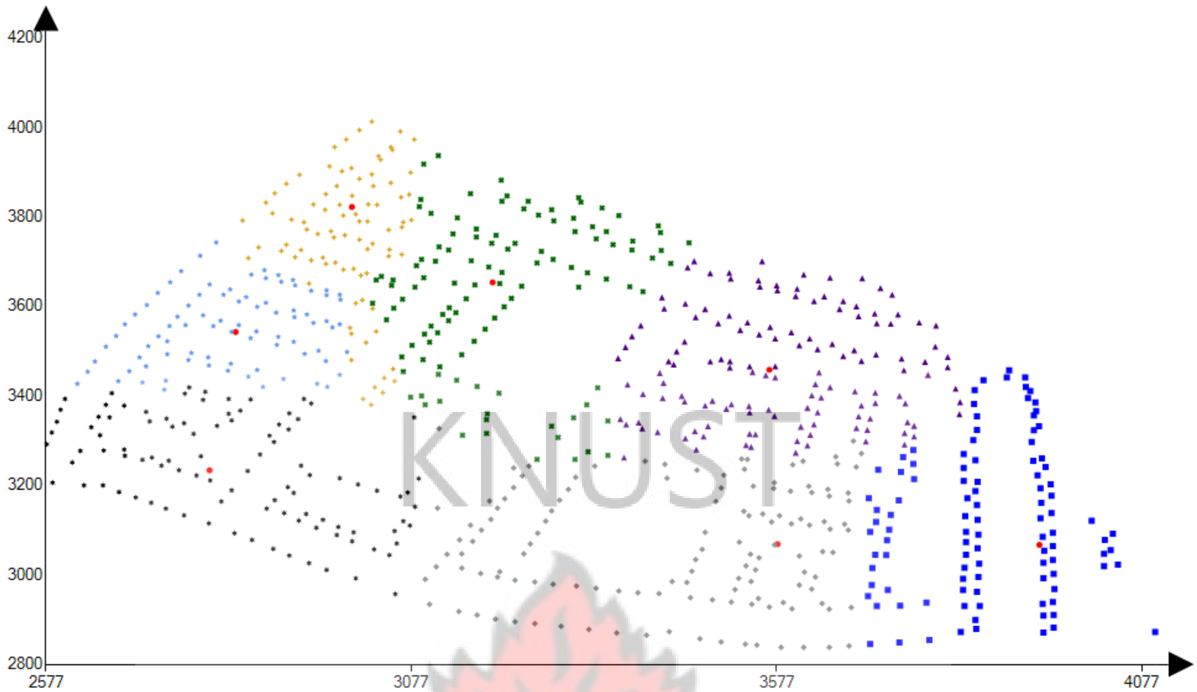


Figure 4.2: Final cluster centres in zone 1

Figure 4.3: Cluster centre and members in zone one sub-cluster one

The number of houses assigned to each sub-cluster, number of bins and the final cluster centers from zone one is as shown in Table 4.4.

Table 4.4: Summary of sub-cluster and members in zone 1

Sub-cluster	Cluster centre	Number of customers	Number of bins (140*l*)
1	(2836.52, 3542.69)	83	165
2	(2806.72, 3212.50)	99	161
3	(2995.70, 3822.21)	85	165
4	(3188.21, 3653.63)	113	166
5	(3566.97, 3458.67)	136	166
6	(3578.23, 3068.46)	130	152
7	(3936.45, 3067.07)	103	125
Total		**749**	**1100**

With the number of bins in zone two, the number of bins needed is $\frac{\sum_{i=1}^{n} q_i}{v_j} = \frac{828}{166} = 4.99 \cong 5$

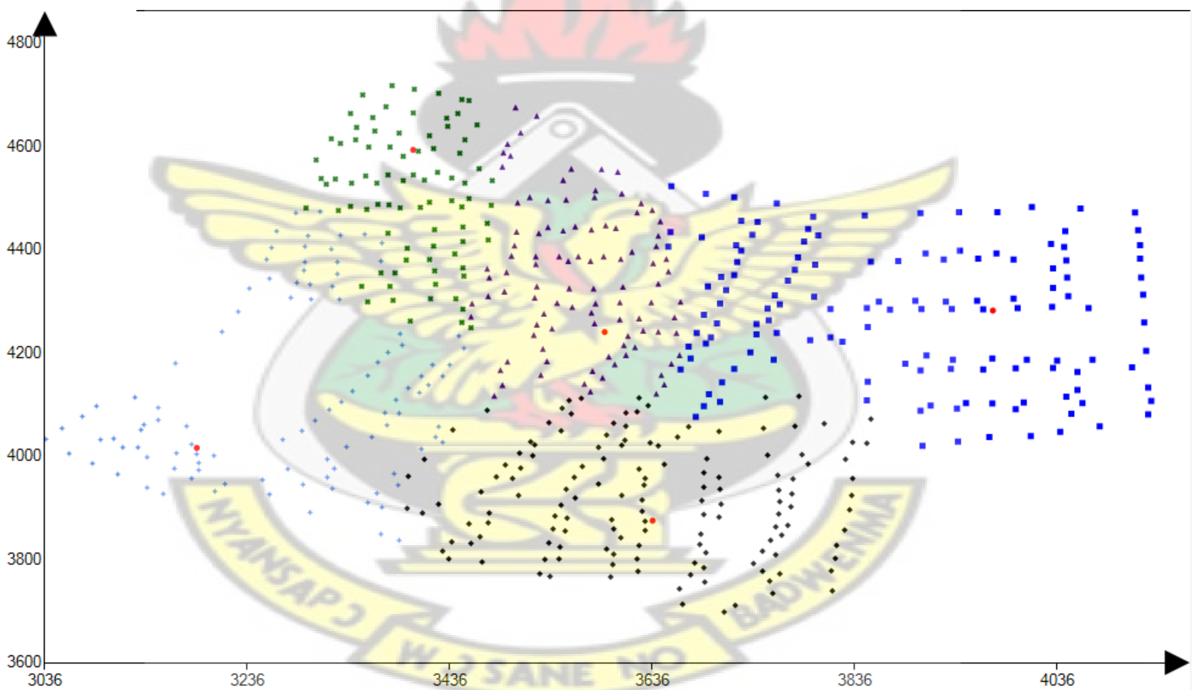


Figure 4.4: Final cluster centres in zone 2

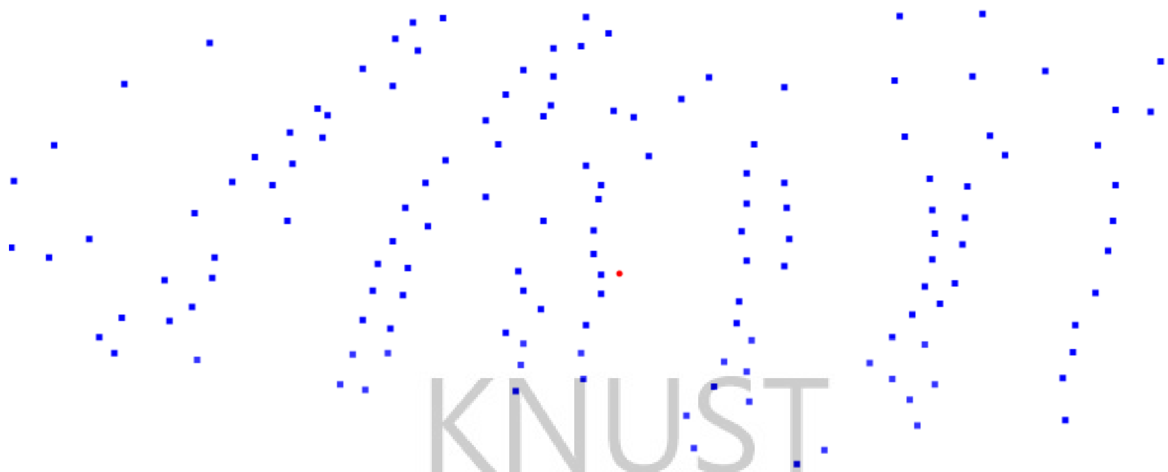


Figure 4.5: Cluster centre and members in zone two sub-cluster one

Table 4.5: Summary of sub-cluster members in zone 2

Sub-cluster	Cluster centre	Number of customers	Number of bins (140l)
1	(3636.30, 3874.95)	141	165
2	(3962.67, 4287.58)	134	166
3	(3589.01, 4240.61)	99	166
4	(3399.84, 4593.46)	84	166
5	(3186.11, 4016.00)	103	165
Total		**561**	**828**

With the number of bins in zone three, the number of bins needed is $\frac{\sum_{i=1}^{n} q_i}{v_j} = \frac{792}{166} = 4.77 \cong 5$

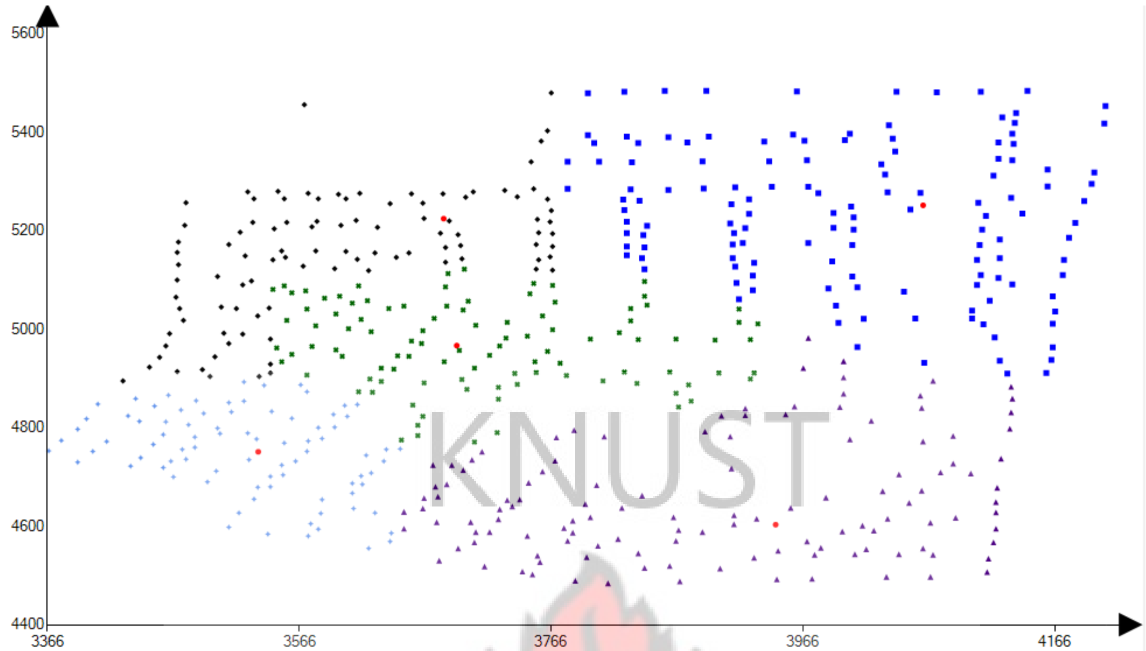


Figure 4.6: Final cluster centres in zone 3

Figure 4.7: Cluster centre and members in zone three sub-cluster one

Table 4.6: Summary of sub-cluster and members in zone 3

Sub-cluster	Cluster centre	Number of customers	Number of bins (140*l*)
1	(3533.30, 4751.13)	84	165
2	(4061.14, 5251.59)	134	139
3	(3680.54, 5224.59)	98	166
4	(3944.08, 4603.17)	124	164
5	(3690.92, 4966.87)	102	158
Total		**542**	**792**

With the number of bins in zone four, the number of bins needed is $\frac{\sum_{i=1}^{n} q_i}{v_j} = \frac{789}{166} = 4.75 \cong 5$

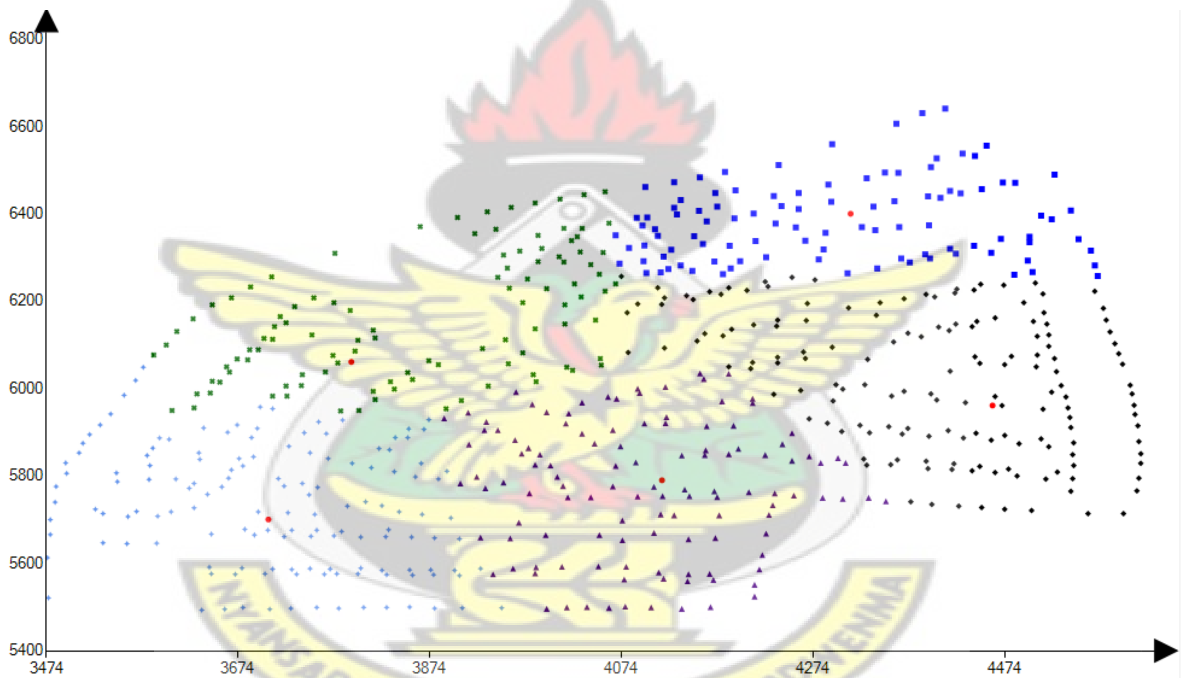


Figure 4.8: Final cluster centres in zone 4

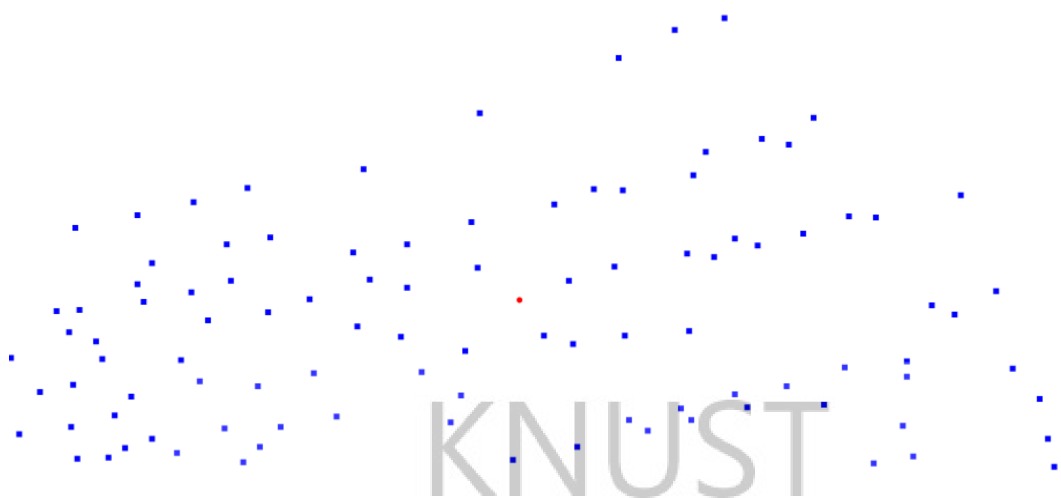


Figure 4.9: **Cluster centre and members in zone four sub-cluster one**

Table 4.7: **Summary of sub-cluster and members in zone 4**

Sub-cluster	Cluster centre	Number of customers	Number of bins (140*l*)
1	(4313.19, 6400.44)	102	158
2	(4470.51, 5961.12)	156	166
3	(4116.35, 5790.22)	119	135
4	(3792.16, 6061.22)	106	166
5	(3705.55, 5700.23)	140	164
Total		**623**	**789**

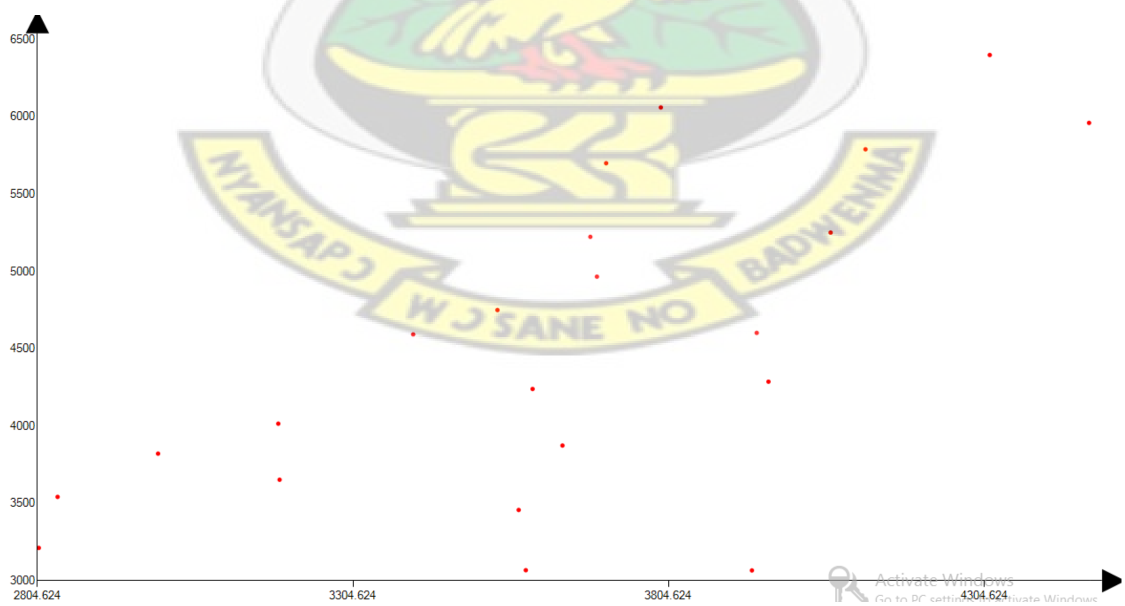


Figure 4.10: Final location centre of skip containers in the study area

4.3 CAPACITATED VEHICLE ROUTING PROBLEM WITH TIME WINDOWS AND SPREAD OF SMELL

Having located the skip containers with defined members to service it, we then use a tricycle with a capacity of $3.5m^3$ and can hold 35 of the 140 litre bins to collect waste from customers and dispose it into the skip container. We apply our improved Ant Colony heuristic to route the service customers. The problem instance for the proposed ant heuristics are $\alpha = \beta = 1.0$, $\gamma = 17$, $\rho = 0.85$ and m = number of ants = number of customers in the sub-sector.

The vehicle routing is done with a soft time window; service time for a customer is 49seconds and a preparation time of 6seconds. The best improved ant tour length by 2-opt heuristics is used to calculate the smell.

4.3.1 OPTIMAL VEHICLE ROUTING WITH TIME WINDOWS FOR ZONE ONE

Zone one has 749 customers and 1100 bins, by the capacity of the tricycle, there will be seven sub-clusters to be toured by a tricycle. From our probabilistic location model, sub-cluster one had 83 customers with 165 bins, a tricycle with a capacity of 35 bins will have to do 5 tours to completely service all customers in the cluster. Sub-cluster two had 99 customers with 161 bins giving us 5 tours, sub-cluster three had 85 customers and 165 bins giving 5 tours, sub-cluster four had 113 customers and 166 bins giving 5 tours, sub-cluster five had 136 customers and 166 bins giving 5 tours, sub-cluster six had 130 customers and 152 bins giving 5 tours and the seventh sub-cluster had 103 customers and 125 bins giving 4 tours. Each tricycle is allowed 4 minutes (240 seconds) offloading time, this time is added to the route time to obtain total time for the particular route

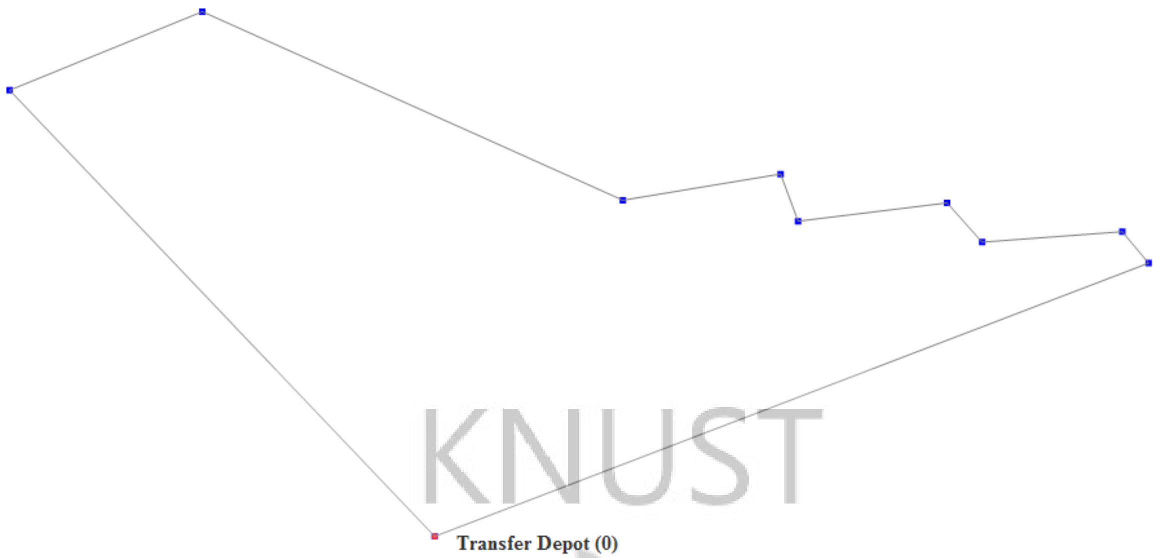


Figure 4.11: First Ant best tour from sub-cluster one in zone one

The first Ant best tour obtained from sub-cluster one is as shown above with a total collection time of 1336.81sec, a tour length of 534.659m, visited 9 customers and emptied 35 bins from the customers.

$$0 \to 26a \to 37a \to 42a \to 38a \to 24a \to 39a \to 23a \to 64a \to 63a \to 0$$

Table 4.8: Summary of routing from sub-cluster one in zone one

Sub-cluster One					
Tour	Customers	Total bins	Length (m)	Time (sec)	Smell
$0 \to 26a \to \ldots \to 63a \to 0$	9	35	534.659	1636.81	152.00
$0 \to 31 \to \ldots \to 75 \to 0$	21	35	896.562	1712.18	107.44
$0 \to 74 \to \ldots \to 27a \to 0$	17	35	783.728	1687.67	117.57
$0 \to 73 \to \ldots \to 30 \to 0$	20	35	774.556	1707.28	118.50
$0 \to 95 \to \ldots \to 94 \to 0$	16	25	477.837	1382.67	59.67
Total	**83**	**165**	**3467.342**	**8126.61**	

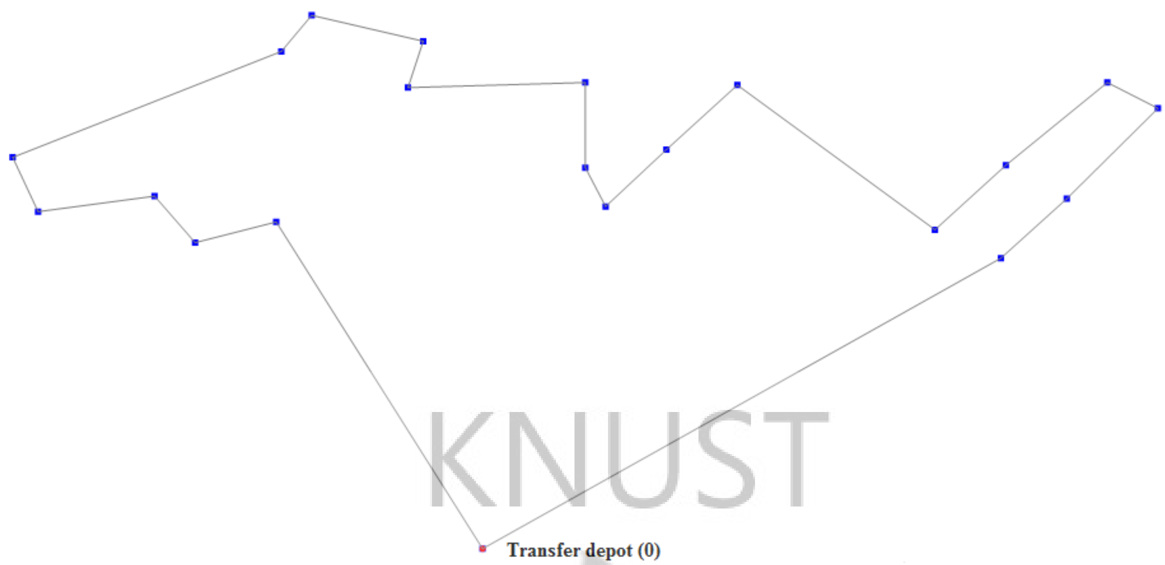


Figure 4.12: Third Ant best tour from sub-cluster two in zone one

The third Ant best tour obtained from sub-cluster two is as shown above with a total collection time of 1707.16sec, a tour length of 844.371m, visited 20 customers and emptied 35 bins from the customers.

$$0 \rightarrow 68c \rightarrow 69c \rightarrow 70c \rightarrow 81 \rightarrow 82 \rightarrow 83 \rightarrow 79 \rightarrow 78 \rightarrow 77 \rightarrow 61 \rightarrow 70 \rightarrow 62 \rightarrow 69 \rightarrow 68 \rightarrow 63 \rightarrow 57 \rightarrow 55 \rightarrow 58 \rightarrow 54 \rightarrow 59 \rightarrow 0$$

Table 4.9: Summary of routing from sub-cluster two in zone one

Sub-cluster two					
Tour	Customers	Total bins	Length (m)	Time (sec)	Smell
$0 \rightarrow 62c \rightarrow \ldots \rightarrow 83c \rightarrow 0$	23	35	1488.572	1724.88	76.49
$0 \rightarrow 2 \rightarrow \ldots \rightarrow 47 \rightarrow 0$	17	35	917.236	1686.42	105.81
$0 \rightarrow 68c \rightarrow \ldots \rightarrow 59 \rightarrow 0$	20	35	844.371	1707.16	111.84
$0 \rightarrow 81c \rightarrow \ldots \rightarrow 12 \rightarrow 0$	25	35	1204.497	1738.60	88.15
$0 \rightarrow 11 \rightarrow \ldots \rightarrow 10 \rightarrow 0$	14	21	736.460	1250.13	26.48
Total	**99**	**161**	**5191.136**	**8107.19**	

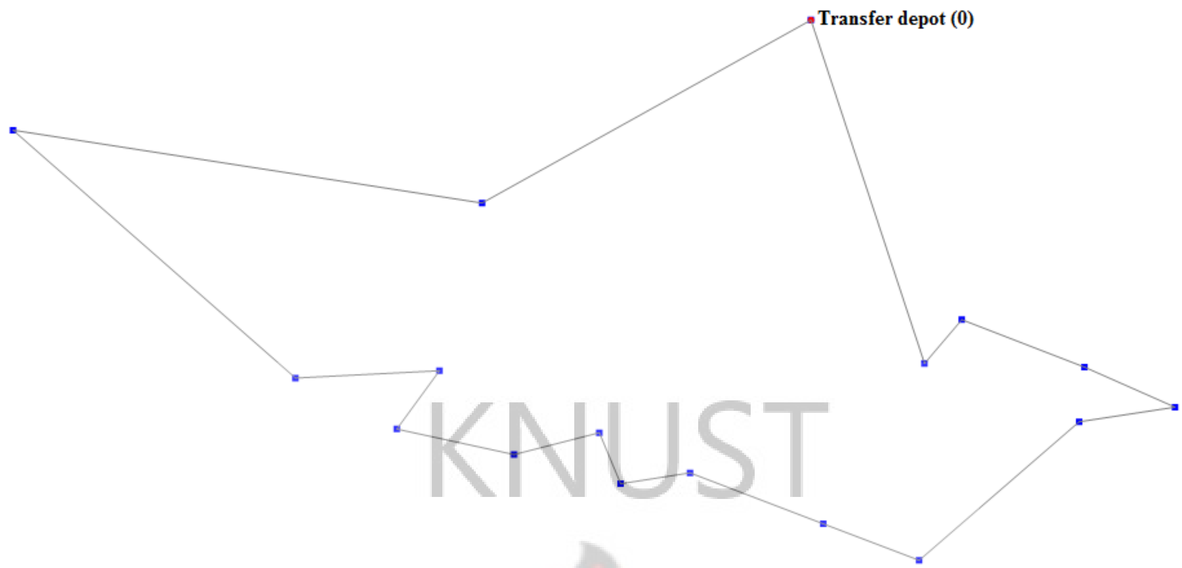


Figure 4.13: **Second Ant best tour from sub-cluster three in zone one**

The second Ant best tour obtained from sub-cluster three is as shown above with a total collection time of 1741.94seconds, a tour length of 642.299m, visited 16 customers and emptied 35 bins from the customers.

$$0 \to 60a \to 84a \to 85a \to 86a \to 94a \to 52a \to 52ai \to 53a \to 45a \to 54a \to 44a \to 43a \to 55a \to 42a \to 65a \to 57a \to 0$$

Table 4.10: **Summary of routing from sub-cluster three in zone one**

Sub-cluster three					
Tour	Customers	Total bins	Length (m)	Time (sec)	Smell
$0 \to 82a \to \ldots \to 40a \to 0$	20	35	1222.836	1766.18	87.27
$0 \to 60a \to \ldots \to 57a \to 0$	16	35	642.299	1741.94	134.34
$0 \to 94a \to \ldots \to 5b \to 0$	18	35	723.667	1756.00	124.02
$0 \to 1b \to \ldots \to 98a \to 0$	18	35	619.432	1754.16	137.64
$0 \to 4b \to \ldots \to 100a \to 0$	13	25	384.970	1422.66	68.99
Total	**85**	**165**	**3593.204**	**8440.94**	

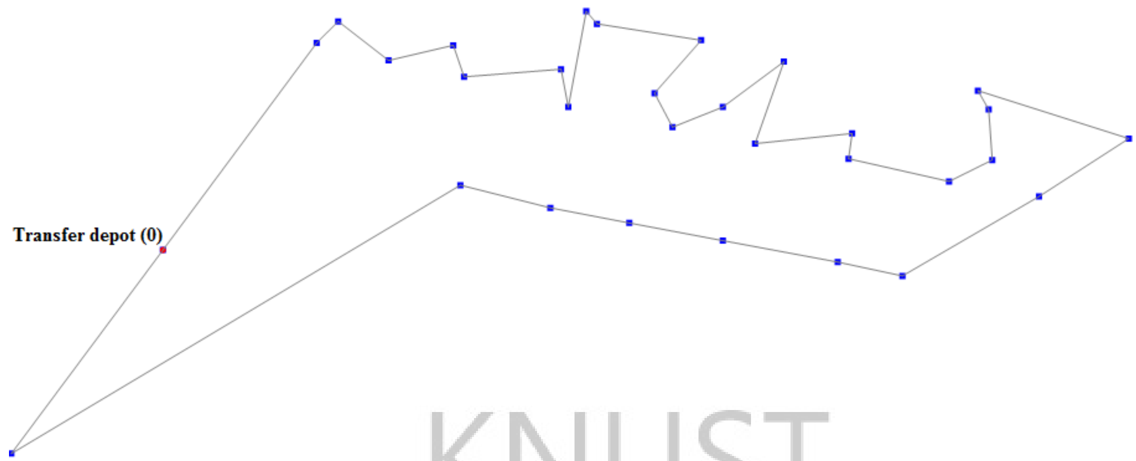


Figure 4.14: Third Ant best tour from sub-cluster four in zone one

The third Ant best tour obtained from sub-cluster four is as shown above with a total collection time of 1770.29seconds, a tour length of 1539.704m, visited 30 customers and emptied 35 bins from the customers.

$$0 \to 37c \to 28b \to 29b \to 30b \to 78b \to 80b \to 81b \to 56b \to 53b \to 55b \to 54b \to 79b \to 32b \to 52b \to 33b \to 51b \to 50b \to 34b \to 49b \to 48b \to 47b \to 45b \to 35b \to 46b \to 36b \to 44b \to 37b \to 43b \to 38b$$

Table 4.11: Summary of routing from sub-cluster four in zone one

Sub-cluster four					
Tour	Customers	Total bins	Length (m)	Time (sec)	Smell
$0 \to 21d \to \ldots \to 54c \to 0$	21	35	1501.295	1711.80	76.06
$0 \to 39d \to \ldots \to 55c \to 0$	21	35	1729.982	1711.25	69.17
$0 \to 37c \to \ldots \to 38b \to 0$	30	35	1539.704	1770.29	74.78
$0 \to 25b \to \ldots \to 39c \to 0$	23	35	883.396	1726.84	108.51
$0 \to 20c \to \ldots \to 22c \to 0$	18	26	702.733	1423.39	51.85
Total	**113**	**166**	**6357.11**	**8343.57**	

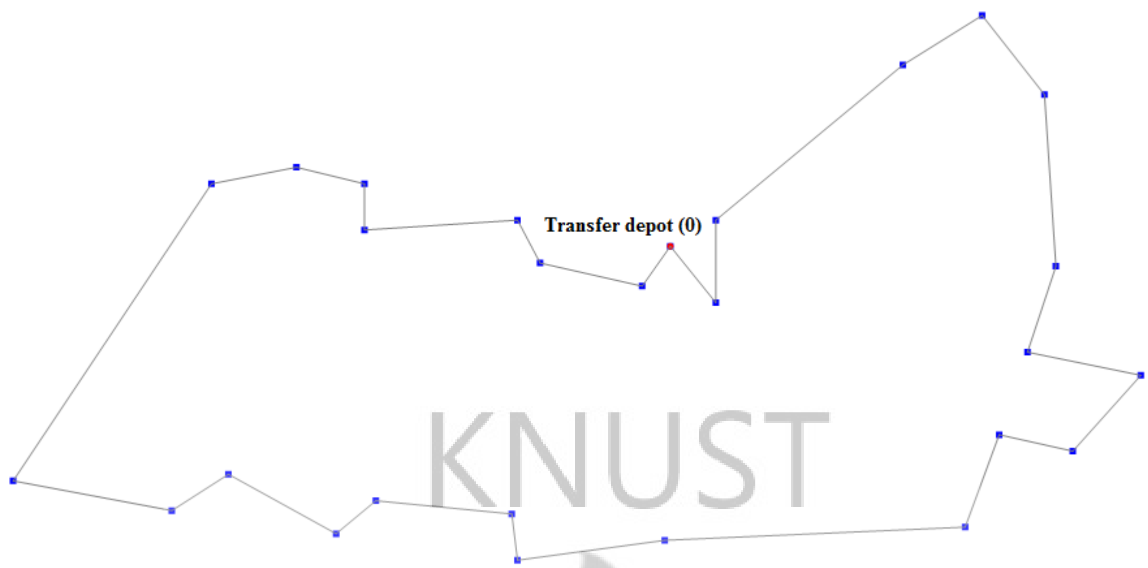


Figure 4.15: Fifth Ant best tour from sub-cluster five in zone one

The fifth Ant best tour obtained from sub-cluster five is as shown above with a total collection time of 1477.88sec, a tour length of 729.788m, visited 26 customers and emptied 26 bins from the customers.

$$0 \to 11e \to 10e \to 9e \to 98b \to 8e \to 7e \to 6e \to 30e \to 31e \to 5e \to 4e \to 3e \to 56e \to 2e \to 1e \to 55e \to 100d \to 82d \to 83d \to 15e \to 23e \to 22e \to 14e \to 21e \to 13e \to 12e \to 0$$

Table 4.12: Summary of routing from sub-cluster five in zone one

Sub-cluster five					
Tour	Customers	Total bins	Length (m)	Time (sec)	Smell
$0 \to 76b \to \ldots \to 94b \to 0$	22	35	1220.419	1719.03	87.38
$0 \to 68e \to \ldots \to 2c \to 0$	28	35	1294.256	1757.57	84.01
$0 \to 99b \to \ldots \to 57e \to 0$	31	35	1610.948	1776.81	72.55
$0 \to 64e \to \ldots \to 99d \to 0$	29	35	1251.813	1764.08	85.91
$0 \to 11e \to \ldots \to 12e \to 0$	26	26	729.788	1477.88	50.55
Total	**136**	**166**	**5191.136**	**8107.19**	

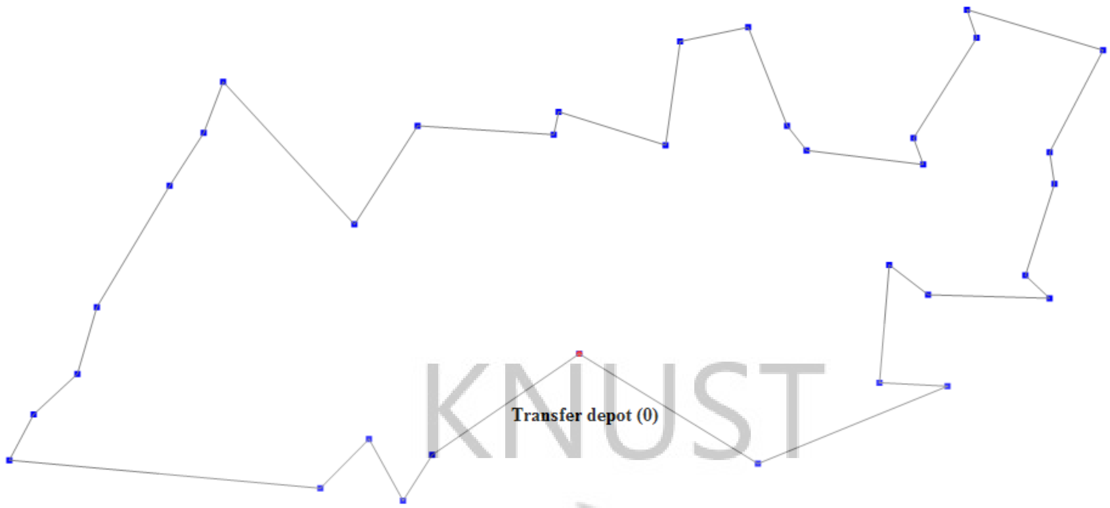


Figure 4.16: **Fourth Ant best tour from sub-cluster six in zone one**

The fourth Ant best tour obtained from sub-cluster six is as shown above with a total collection time of 1796.46sec, a tour length of 1353.738m, visited 34 customers and emptied 35 bins from the customers.

$$0 \to 4g \to 85f \to 86f \to 71f \to 81f \to 82f \to 70f \to 67f \to 73f \to 78f \to 36e \to 77e \to 72e \to 66f \to 65f \to 71e \to 76e \to 69e \to 64f \to 70e \to 63f \to 62f \to 75f \to 60e \to 61f \to 76f \to 89f \to 90f \to 91f \to 92f \to 95f \to 96f \to 100f \to 99f \to 0$$

Table 4.13: **Summary of routing from sub-cluster six in zone one**

Sub-cluster six					
Tour	Customers	Total bins	Length (m)	Time (sec)	Smell
$0 \to 19d \to \ldots \to 18d \to 0$	22	35	1864.403	1718.00	76.49
$0 \to 52d \to \ldots \to 45e \to 0$	31	35	1840.000	1776.18	105.81
$0 \to 94f \to \ldots \to 1g \to 0$	31	35	1260.544	1776.80	111.84
$0 \to 4g \to \ldots \to 99f \to 0$	34	35	1353.738	1796.46	88.15
$0 \to 5g \to \ldots \to 87f \to 0$	12	12	374.643	967.68	122.57
Total	**130**	**152**	**6693.328**	**8035.12**	

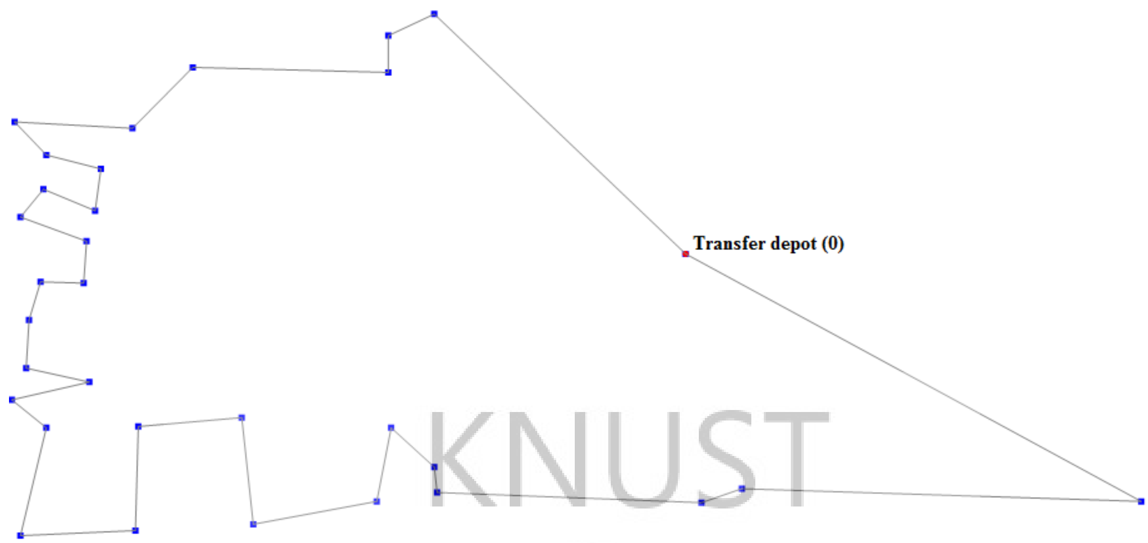


Figure 4.17: Second Ant best tour from sub-cluster seven in zone one

The second Ant best tour obtained from sub-cluster seven is as shown above with a total collection time of 1775.57sec, a tour length of 1768.622m, visited 31 customers and emptied 35 bins from the customers.

$$0 \to 15f \to 91e \to 92e \to 81e \to 75e \to 74e \to 68f \to 12g \to 13g \to 69f \to 83f \to 14g \to 15g \to 84f \to 11g \to 10g \to 16g \to 9g \to 32g \to 39g \to 38g \to 33g \to 34g \to 37g \to 36g \to 35g \to 59f \to 60f \to 46f \to 44f \to 45f \to 0$$

Table 4.14: Summary of routing from sub-cluster seven in zone one

Sub-cluster seven					
Tour	Customers	Total bins	Length (m)	Time (sec)	Smell
$0 \to 62c \to \ldots \to 83c \to 0$	25	35	1179.574	1740.24	89.40
$0 \to 2 \to \ldots \to 47 \to 0$	31	35	1768.622	1775.57	68.15
$0 \to 68c \to \ldots \to 59 \to 0$	28	35	1179.954	1757.54	77.58
$0 \to 11 \to \ldots \to 10 \to 0$	19	20	699.008	1250.26	23.68
Total	**103**	**125**	**4827.158**	**6823.61**	

Table 4.15: Summary of Ant best tour, total collection time and distance in zone one

Zone one	Customers	Total bins	Length (m)	Time (sec)
Sub-cluster one	83	165	3467.342	8126.61
Sub-cluster two	99	161	5191.136	8107.19
Sub-cluster three	85	165	3593.204	8440.94
Sub-cluster four	113	166	6357.110	8343.57
Sub-cluster five	136	166	5191.136	8107.19
Sub-cluster six	130	152	6693.328	8035.12
Sub-cluster seven	103	125	4827.158	6823.61
Total	**749**	**1100**	**35320.414**	**55984.24**
Average	**95**	**158**	**5045.77**	**7997.75**

4.3.2 OPTIMAL VEHICLE ROUTING WITH TIME WINDOWS FOR ZONE TWO

Zone two has 561 customers and 828 bins, by the capacity of the tricycle, there will be five sub-clusters to be toured by a tricycle. From our probabilistic location model, sub-cluster one had 141 customers with 165 bins, a tricycle with a capacity of 35 bins will have to do 5 tours to completely service all customers in the cluster. Sub-cluster two had 134 customers with 166 bins giving us 5 tours, sub-cluster three had 99 customers and 166 bins giving 5 tours, sub-cluster four had 84 customers and 166 bins giving 5 tours and sub-cluster five had 103 customers and 165 bins giving 5 tours. Each tricycle is allowed 4minutes (240 seconds) offloading time, this time is added to the route time to obtain total time for the particular route.

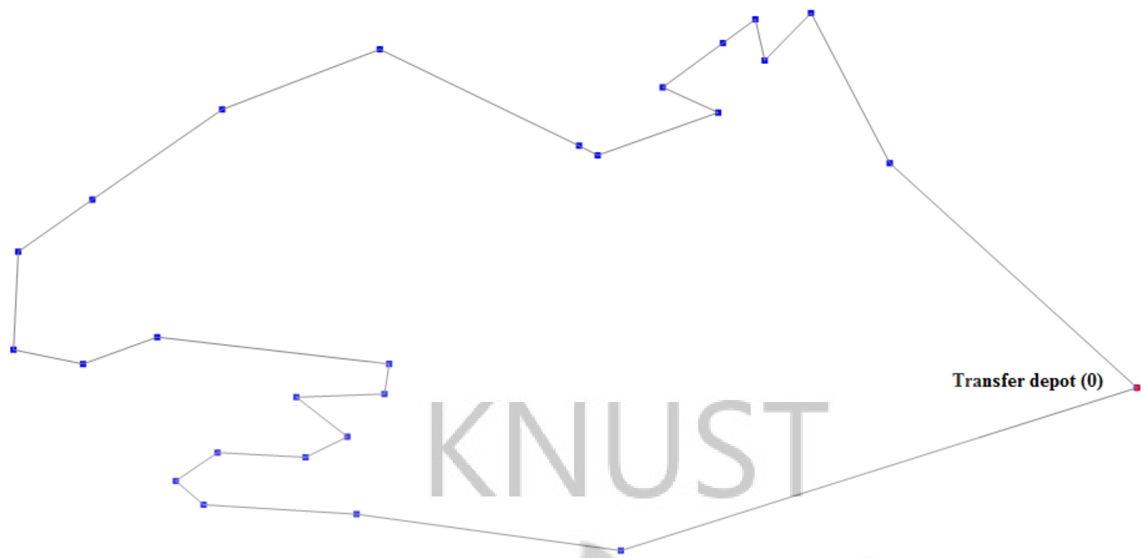


Figure 4.18: First Ant best tour from sub-cluster one in zone two

The first Ant best tour obtained from sub-cluster one is as shown above with a total collection time of 1745.8sec, a tour length of 1182.989m, visited 26 customers and emptied 35 bins from the customers.

$$0 \to 12i \to 79j \to 76j \to 33i \to 32i \to 31i \to 75j \to 8i \to 7i \to 17i \to 92h \to 84h \to 83h \to 76h \to 75h \to 81h \to 74h \to 72h \to 73h \to 71h \to 70h \to 67h \to 66h \to 68h \to 69h \to 93h \to 0$$

Table 4.16: Summary of routing from sub-cluster one in zone two

Sub-cluster One					
Tour	Customers	Total bins	Length (m)	Time (sec)	Smell
$0 \to 12i \to \ldots \to 93h \to 0$	26	35	1182.898	1745.80	89.23
$0 \to 95h \to \ldots \to 36j \to 0$	30	35	1530.597	1769.84	75.08
$0 \to 74j \to \ldots \to 88i \to 0$	34	35	1407.955	1797.10	79.40
$0 \to 4j \to \ldots \to 61i \to 0$	29	35	975.018	1765.16	101.56
$0 \to 76i \to \ldots \to 77i \to 0$	22	25	635.387	1421.10	87.42
Total	**141**	**165**	**5731.855**	**8499.00**	

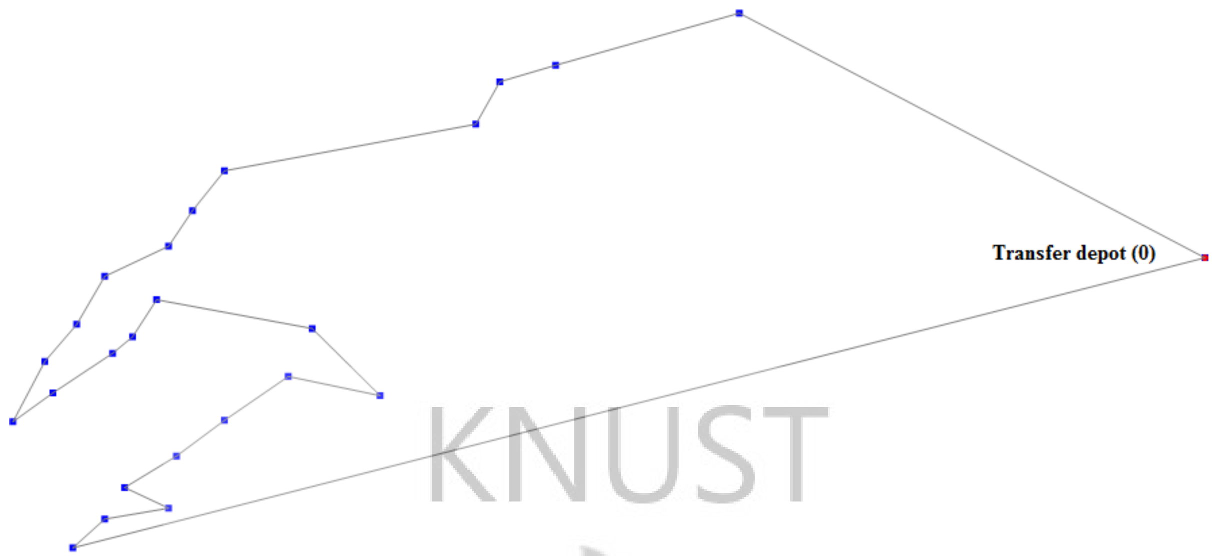


Figure 4.19: **Second Ant best tour from sub-cluster two in zone two**

The second Ant best tour obtained from sub-cluster two is as shown above with a total collection time of 1732.41sec, a tour length of 1310.328m, visited 24 customers and emptied 35 bins from the customers.

$$0 \to 79k \to 80k \to 86k \to 97k \to 92k \to 49j \to 50j \to 95j \to 92j \to 89j \to 61j \to 58j \to 57j \to 54j \to 53j \to 55j \to 38j \to 56j \to 59j \to 60j \to 62j \to 37j \to 65j \to 66j \to 0$$

Table 4.17: **Summary of routing from sub-cluster two in zone two**

Sub-cluster Two					
Tour	Customers	Total bins	Length (m)	Time (sec)	Smell
$0 \to 84k \to \ldots \to 39j \to 0$	18	35	1094.934	1693.41	93.97
$0 \to 79k \to \ldots \to 66j \to 0$	24	35	1310.328	1732.02	83.32
$0 \to 4k \to \ldots \to 45j \to 0$	32	35	1767.747	1782.09	68.17
$0 \to 48k \to \ldots \to 30k \to 0$	35	35	1392.26	1803.02	80.00
$0 \to 41k \to \ldots \to 53k \to 0$	25	26	819.235	1475.88	46.79
Total	**134**	**166**	**6384.504**	**8486.42**	

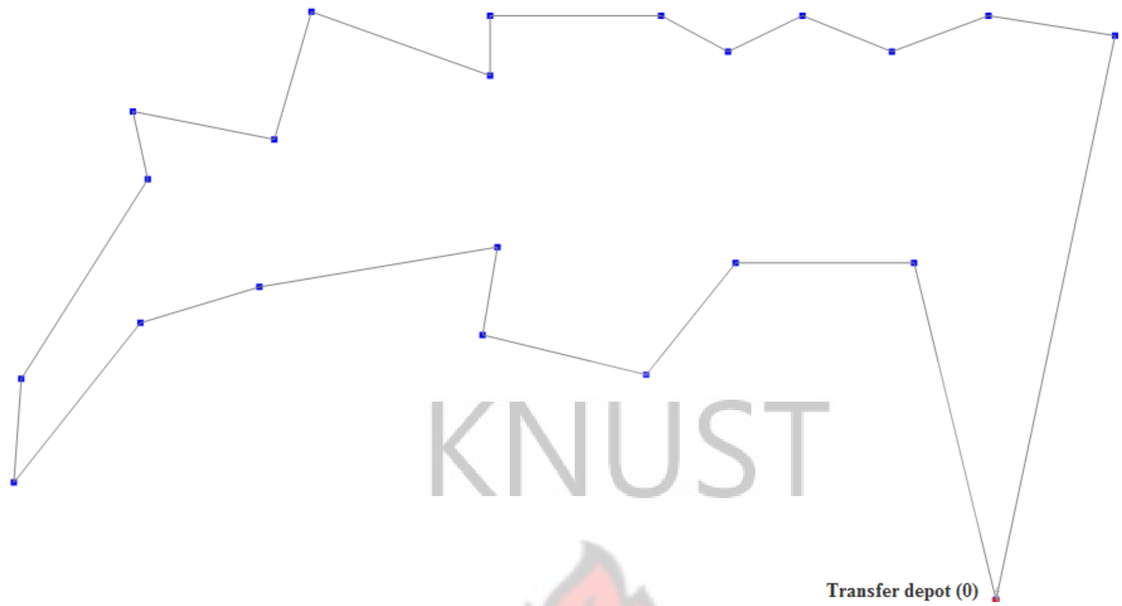


Figure 4.20: Third Ant best tour from sub-cluster three in zone two

The third Ant best tour obtained from sub-cluster two is as shown above with a total collection time of 1714.65sec, a tour length of 713.596m, visited 21 customers and emptied 35 bins from the customers.

$$0 \to 3l \to 22l \to 6l \to 18l \to 8l \to 21l \to 16l \to 9l \to 13l \to 12l \to 86l \to 49l \to 25i \to 24i \to 48l \to 11l \to 10l \to 42i \to 43i \to 7l \to 5l \to 0$$

Table 4.18: Summary of routing from sub-cluster three in zone two

Sub-cluster Three					
Tour	Customers	Total bins	Length (m)	Time (sec)	Smell
$0 \to 27l \to \ldots \to 60\# \to 0$	18	35	1122.167	1694.01	92.44
$0 \to 94k \to \ldots \to 15l \to 0$	19	35	863.243	1701.45	110.20
$0 \to 3l \to \ldots \to 5l \to 0$	21	35	713.596	1714.65	125.19
$0 \to 28i \to \ldots \to 4l \to 0$	25	35	922.729	1738.56	105.39
$0 \to 38i \to \ldots \to 40i \to 0$	16	26	407.751	1412.32	74.67
Total	**99**	**166**	**3329.486**	**8260.99**	

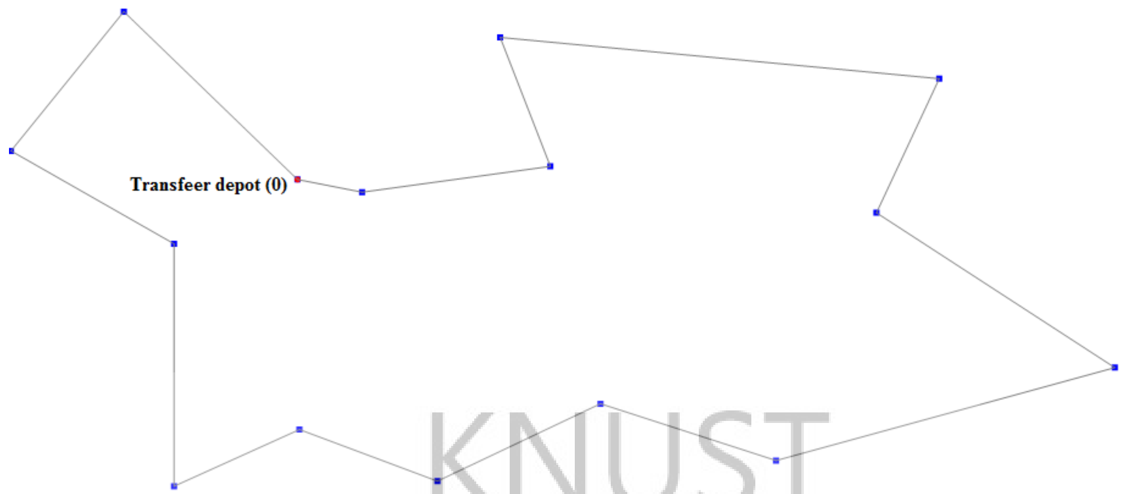


Figure 4.21: Fifth Ant best tour from sub-cluster four in zone two

The Fifth Ant best tour obtained from sub-cluster two is as shown above with a total collection time of 1460.30sec, a tour length of 377.597m, visited 14 customers and emptied 28 bins from the customers.

$$0 \rightarrow 37m \rightarrow 22m \rightarrow 23m \rightarrow 7m \rightarrow 20m \rightarrow 10m \rightarrow 19m \rightarrow 11m \rightarrow 15m \rightarrow 17m \rightarrow 50m \rightarrow 38m \rightarrow 18m \rightarrow 21m \rightarrow 0$$

Table 4.19: Summary of routing from sub-cluster four in zone two

Sub-cluster Four					
Tour	Customers	Total bins	Length (m)	Time (sec)	Smell
$0 \rightarrow 57l \rightarrow \ldots \rightarrow 13m \rightarrow 0$	18	35	1209.299	1693.63	87.92
$0 \rightarrow 1m \rightarrow \ldots \rightarrow 14m \rightarrow 0$	18	35	807.353	1693.98	115.25
$0 \rightarrow 44m \rightarrow \ldots \rightarrow 46m \rightarrow 0$	17	34	747.395	1658.33	111.26
$0 \rightarrow 49m \rightarrow \ldots \rightarrow 24m \rightarrow 0$	17	34	562.313	1658.69	134.63
$0 \rightarrow 37m \rightarrow \ldots \rightarrow 21m \rightarrow 0$	14	28	377.597	1460.30	98.18
Total	**84**	**166**	**3703.957**	**8164.93**	

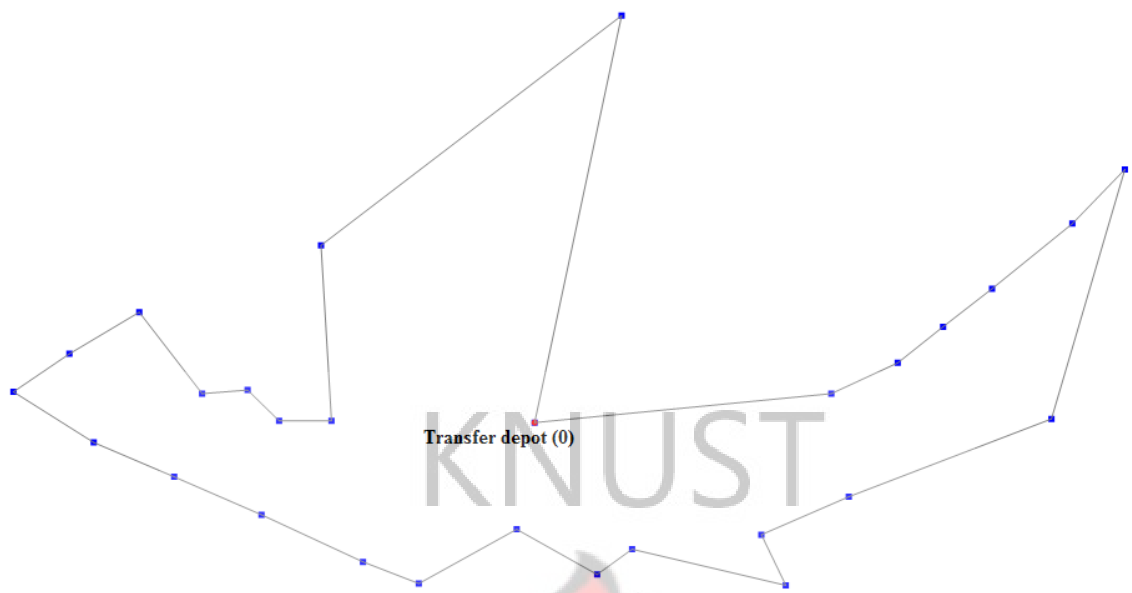


Figure 4.22: Fourth Ant best tour from sub-cluster five in zone two

The Fourth Ant best tour obtained from sub-cluster two is as shown above with a total collection time of 1750.73sec, a tour length of 1438.276m, visited 27 customers and emptied 35 bins from the customers.

$$0 \to 56h \to 16h \to 20h \to 11h \to 19h \to 12h \to 14h \to 13h \to 1h \to 2h \to 3h \to 4h \to 5h \to 6h$$
$$\to 8h \to 7h \to 30h \to 58h \to 31h \to 32h \to 36h \to 43h \to 42h \to 38h \to 37h \to 35h \to 33h \to 0$$

Table 4.20: Summary of routing from sub-cluster five in zone two

Sub-cluster Five					
Tour	Customers	Total bins	Length (m)	Time (sec)	Smell
$0 \to 50h \to \ldots \to 15h \to 0$	18	35	1328.635	1693.26	92.44
$0 \to 14i \to \ldots \to 57h \to 0$	19	35	1147.538	1699.98	110.20
$0 \to 60h \to \ldots \to 55h \to 0$	26	35	1500.682	1745.02	125.19
$0 \to 56h \to \ldots \to 33h \to 0$	27	35	1438.276	1750.73	105.39
$0 \to 8h \to \ldots \to 27h \to 0$	13	25	342.172	1363.97	74.67
Total	**103**	**165**	**5757.303**	**8252.96**	

Table 4.21: Summary of Ant best tours, total collection time and distance covered in zone two

Zone Two	Customers	Total bins	Length (m)	Time (sec)
Sub-cluster one	141	165	5731.855	8499.00
Sub-cluster two	134	166	6384.504	8486.42
Sub-cluster three	99	166	3329.486	8260.99
Sub-cluster four	84	166	3703.957	8164.93
Sub-cluster five	103	165	5757.303	8252.96
Total	**561**	**828**	**24907.105**	**41664.30**
Average	**113**	**166**	**4981.421**	**8332.86**

4.3.3 Optimal Vehicle Routing With Time Windows For Zone Three

Zone three has 542 customers and 792 bins, by the capacity of the tricycle, there will be five sub-clusters to be toured by a tricycle. From our probabilistic location model, sub-cluster one had 84 customers with 139 bins, a tricycle with a capacity of 35 bins will have to do 4 tours to completely service all customers in the cluster. Sub-cluster two had 134 customers with 165 bins giving us 5 tours, sub-cluster three had 98 customers and 166 bins giving 5 tours, sub-cluster four had 124 customers and 164 bins giving 5 tours and sub-cluster five had 102 customers and 158 bins giving 5 tours. Each tricycle is allowed 4minutes (240seconds) offloading time, this time is added to the route time to obtain total time for the particular route.

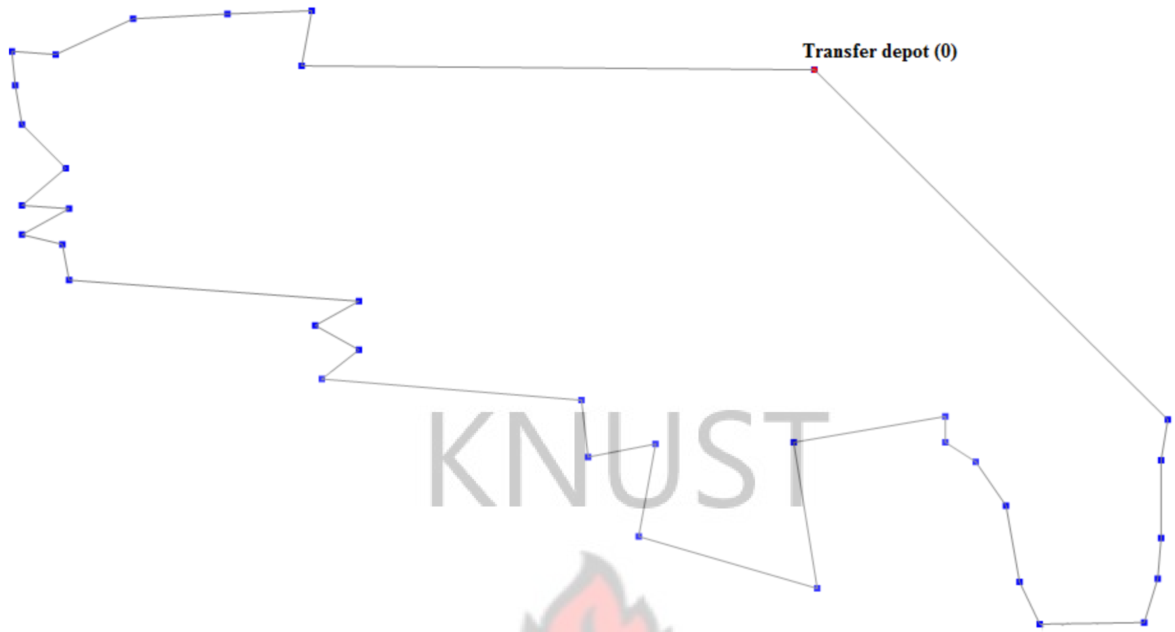


Figure 4.23: Second Ant best tour from sub-cluster one in zone three

The second Ant best tour obtained from sub-cluster one is as shown above with a total collection time of 1803.25seconds, a tour length of 1525.993m, visited 35 customers and emptied 35 bins from the customers.

$$0 \to 22r \to 41s \to 40s \to 39s \to 23r \to 64r \to 62r \to 61r \to 27r \to 57r \to 28r \to 54r \to 31r \to 33r \to 11r \to 35r \to 10r \to 36r \to 8r \to 6r \to 5r \to 69q \to 68q \to 4r \to 77q \to 78q \to 79q \to 64q \to 62q \to 60q \to 59q \to 61q \to 63q \to 70q \to 71q \to 0$$

Table 4.22: Summary of routing from sub-cluster one in zone three

Sub-cluster One					
Tour	Customers	Total bins	Length (m)	Time (sec)	Smell
$0 \to 92s \to \ldots \to 24r \to 0$	30	35	1548.889	1769.90	74.48
$0 \to 22r \to \ldots \to 71q \to 0$	35	35	1525.993	1803.25	75.23
$0 \to 79s \to \ldots \to 82s \to 0$	35	35	1445.739	1804.03	78.00
$0 \to 83s \to \ldots \to 97q \to 0$	34	34	1296.528	1766.01	76.92
Total	**134**	**139**	**5817.149**	**7143.19**	

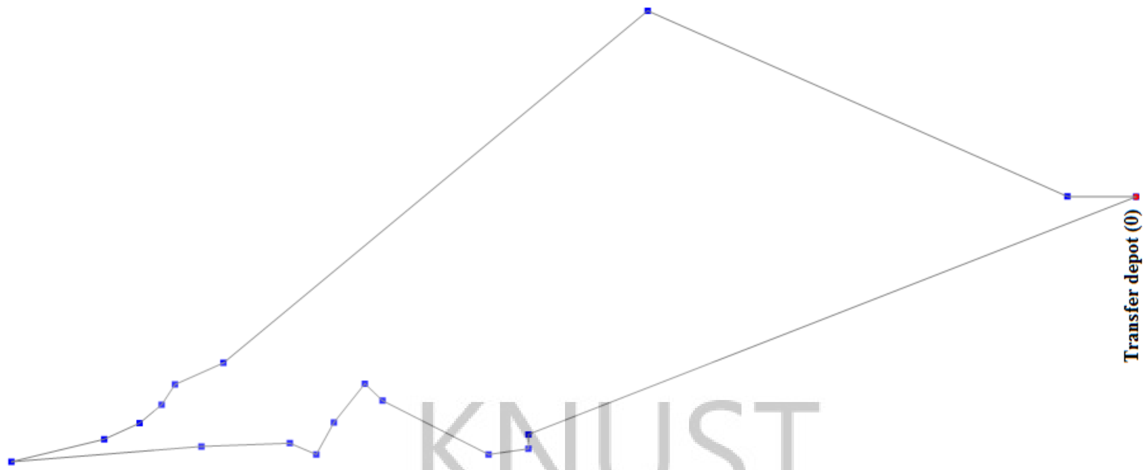


Figure 4.24: First Ant best tour from sub-cluster two in zone three

The first Ant best tour obtained from sub-cluster two is as shown above with a total collection time of 1447.67seconds, a tour length of 1467.445m, visited 17 customers and emptied 35 bins from the customers.

$$0 \rightarrow 53p \rightarrow 56p \rightarrow 52p \rightarrow 54p \rightarrow 70p \rightarrow 51p \rightarrow 44p \rightarrow 50p \rightarrow 45p \rightarrow 39p \rightarrow 46p \rightarrow 47p \rightarrow 48p \rightarrow 49p \rightarrow 71p \rightarrow 60s \rightarrow 16s \rightarrow 0$$

Table 4.23: Summary of routing from sub-cluster two in zone three

Sub-cluster Two					
Tour	Customers	Total bins	Length (m)	Time (sec)	Smell
$0 \rightarrow 53p \rightarrow \ldots \rightarrow 16s \rightarrow 0$	17	35	1467.445	1447.67	77.23
$0 \rightarrow 68p \rightarrow \ldots \rightarrow 98r \rightarrow 0$	19	35	1118.367	1460.16	92.65
$0 \rightarrow 84r \rightarrow \ldots \rightarrow 29s \rightarrow 0$	18	35	776.211	1454.69	90.68
$0 \rightarrow 61s \rightarrow \ldots \rightarrow 18s \rightarrow 0$	28	35	1231.234	1519.28	87.44
$0 \rightarrow 69r \rightarrow \ldots \rightarrow 67r \rightarrow 0$	16	26	483.513	1172.82	66.61
Total	**98**	**166**	**5076.77**	**7054.62**	

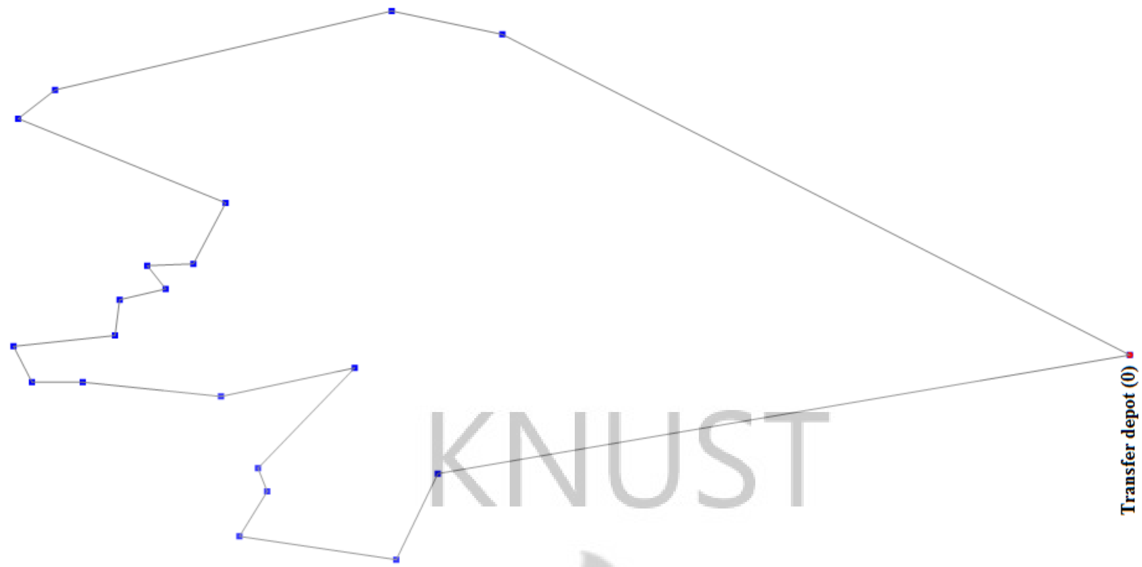


Figure 4.25: **Second Ant best tour from sub-cluster three in zone three**

The second Ant best tour obtained from sub-cluster three is as shown above with a total collection time of 1467.98 seconds, a tour length of 976.555m, visited 20 customers and emptied 35 bins from the customers.

$$0 \to 84m \to 85m \to 80m \to 81m \to 78m \to 45n \to 77m \to 76m \to 71m \to 47n \to 46n \to 50n \to 43n \to 51n \to 42n \to 54n \to 56n \to 58n \to 62n \to 61n \to 0$$

Table 4.24: **Summary of routing from sub-cluster three in zone three**

Sub-cluster Three					
Tour	Customers	Total bins	Length (m)	Time (sec)	Smell
$0 \to 79m \to \ldots \to 34n \to 0$	18	35	883.110	1454.81	108.53
$0 \to 84m \to \ldots \to 61n \to 0$	20	35	976.555	1467.98	101.46
$0 \to 50q \to \ldots \to 44n \to 0$	31	35	1696.209	1535.14	70.09
$0 \to 46q \to \ldots \to 39q \to 0$	32	35	1533.987	1542.62	74.97
$0 \to 21n \to \ldots \to 24n \to 0$	23	24	779.096	1156.37	38.06
Total	**124**	**164**	**5868.957**	**7156.92**	

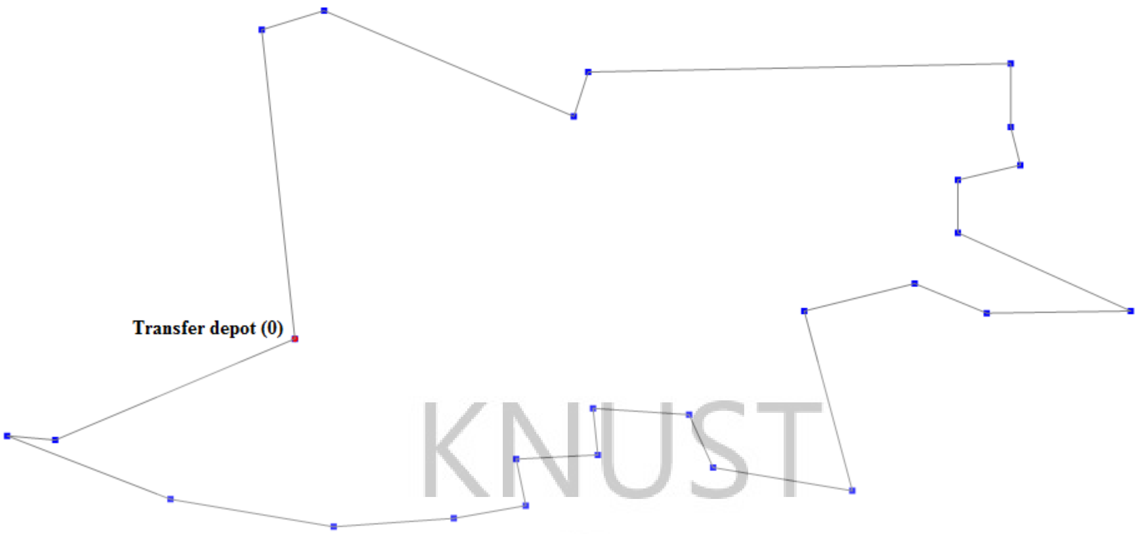


Figure 4.26: Third Ant best tour from sub-cluster four in zone three

The third Ant best tour obtained from sub-cluster four is as shown above with a total collection time of 1499.61seconds, a tour length of 986.132m, visited 25 customers and emptied 35 bins from the customers.

$$0 \rightarrow 83r \rightarrow 77r \rightarrow 50r \rightarrow 51r \rightarrow 34r \rightarrow 37r \rightarrow 38r \rightarrow 45r \rightarrow 44r \rightarrow 29q \rightarrow 26q \rightarrow 41r \rightarrow 25q \rightarrow 8q \rightarrow 9q \rightarrow 24q \rightarrow 17q \rightarrow 10q \rightarrow 11q \rightarrow 4q \rightarrow 3q \rightarrow 98p \rightarrow 99q \rightarrow 63p \rightarrow 1q \rightarrow 0$$

Table 4.25: Summary of routing from sub-cluster four in zone three

Sub-cluster Four					
Tour	Customers	Total bins	Length (m)	Time (sec)	Smell
$0 \rightarrow 18q \rightarrow \ldots \rightarrow 60p \rightarrow 0$	25	35	1209.655	1398.74	87.90
$0 \rightarrow 100p \rightarrow \ldots \rightarrow 88r \rightarrow 0$	19	35	745.162	1461.43	121.61
$0 \rightarrow 83r \rightarrow \ldots \rightarrow 1q \rightarrow 0$	25	35	986.132	1499.61	100.80
$0 \rightarrow 79p \rightarrow \ldots \rightarrow 77p \rightarrow 0$	23	35	763.759	1486.64	119.62
$0 \rightarrow 16q \rightarrow \ldots \rightarrow 14q \rightarrow 0$	10	18	265.031	894.36	33.07
Total	**102**	**158**	**3969.739**	**6840.78**	

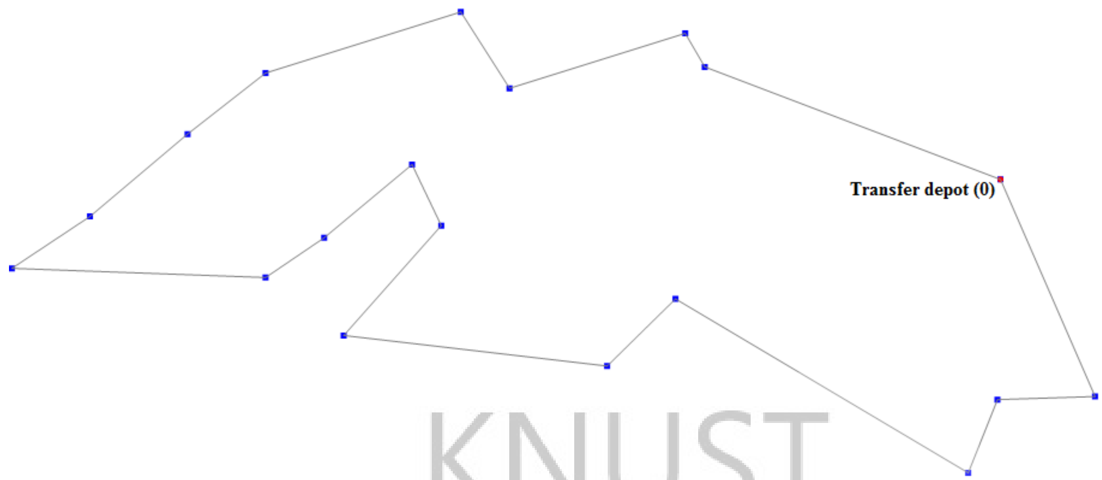


Figure 4.27: Fourth Ant best tour from sub-cluster five in zone three

The fourth Ant best tour obtained from sub-cluster five is as shown above with a total collection time of 1455.20 seconds, a tour length of 553.288m, visited 18 customers and emptied 35 bins from the customers.

$$0 \rightarrow 76n \rightarrow 91n \rightarrow 58m \rightarrow 97n \rightarrow 95n \rightarrow 96n \rightarrow 98n \rightarrow 17p \rightarrow 16p \rightarrow 15p \rightarrow 19p \rightarrow 20p \rightarrow 21p \rightarrow 22p \rightarrow 34p \rightarrow 18p \rightarrow 5p \rightarrow 2p \rightarrow 0$$

Table 4.26: Summary of routing from sub-cluster five in zone three

Sub-cluster Five					
Tour	Customers	Total bins	Length (m)	Time (sec)	Smell
$0 \rightarrow 70n \rightarrow \ldots \rightarrow 56m \rightarrow 0$	17	35	711.647	1448.98	125.42
$0 \rightarrow 90n \rightarrow \ldots \rightarrow 25p \rightarrow 0$	18	35	744.097	1454.28	121.73
$0 \rightarrow 81n \rightarrow \ldots \rightarrow 57m \rightarrow 0$	18	35	845.750	1454.39	111.72
$0 \rightarrow 76n \rightarrow \ldots \rightarrow 2p \rightarrow 0$	18	35	553.288	1455.20	148.46
$0 \rightarrow 100n \rightarrow \ldots \rightarrow 4p \rightarrow 0$	13	25	379.321	1123.06	69.67
Total	**84**	**165**	**3234.103**	**6935.91**	

Table 4.27 Summary of Ant best tours, total collection time and distance covered in zone three

Zone Three	Customers	Total bins	Length (m)	Time (sec)
Sub-cluster one	134	139	5817.149	7443.19
Sub-cluster two	98	166	5076.770	7054.62
Sub-cluster three	124	164	5868.957	7156.92
Sub-cluster four	102	158	3969.739	6840.78
Sub-cluster five	84	165	3234.103	6935.91
Total	**542**	**792**	**23966.718**	**35431.42**
Average	**109**	**159**	**4793.344**	**7086.28**

4.3.4 Optimal Vehicle Routing With Time Windows for Zone Four

Zone four has 623 customers and 789 bins, by the capacity of the tricycle, there will be five sub-clusters to be toured by a tricycle. From our probabilistic location model, sub-cluster one had 102 customers with 158 bins, a tricycle with a capacity of 35 bins will have to do 5 tours to completely service all customers in the cluster. Sub-cluster two had 156 customers with 166 bins giving us 5 tours, sub-cluster three had 119 customers and 135 bins giving 4 tours, sub-cluster four had 106 customers and 166 bins giving 5 tours and sub-cluster five had 140 customers and 164 bins giving 5 tours. Each tricycle is allowed 4minutes (240seconds) offloading time, this time is added to the route time to obtain total time for the particular route.

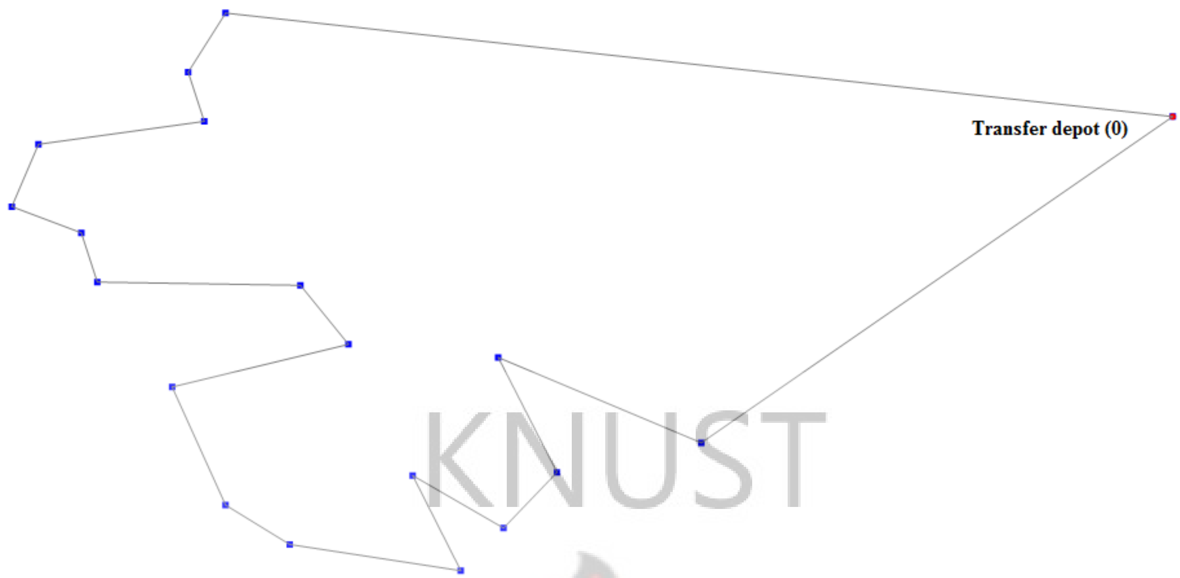


Figure 4.28: Second Ant best tour from sub-cluster one in zone four

The second Ant best tour obtained from sub-cluster five in zone four is as shown above with a total collection time of 1454.48 seconds, a tour length of 766.574m, visited 18 customers and emptied 35 bins from the customers.

$0 \rightarrow 9\text{x} \rightarrow 3\text{x} \rightarrow 2\text{x} \rightarrow 4\text{x} \rightarrow 75w \rightarrow 1\text{x} \rightarrow 99w \rightarrow 100w \rightarrow 97w \rightarrow 98w \rightarrow 94w \rightarrow 93w \rightarrow 95w \rightarrow 96w \rightarrow 26\text{x} \rightarrow 25\text{x} \rightarrow 20\text{x} \rightarrow 24\text{x} \rightarrow 0$

Table 4.28: Summary of routing from sub-cluster one in zone four

Sub-cluster One					
Tour	Customers	Total bins	Length (m)	Time (sec)	Smell
$0 \rightarrow 47\text{x} \rightarrow \ldots \rightarrow 81w \rightarrow 0$	15	35	955.801	1434.10	102.93
$0 \rightarrow 9\text{x} \rightarrow \ldots \rightarrow 24\text{x} \rightarrow 0$	18	35	766.574	1454.48	119.32
$0 \rightarrow 50\text{x} \rightarrow \ldots \rightarrow 11\text{x} \rightarrow 0$	27	35	1630.914	1510.23	71.95
$0 \rightarrow 32\text{x} \rightarrow \ldots \rightarrow 21\text{x} \rightarrow 0$	29	35	1390.535	1524.49	80.07
$0 \rightarrow 34\text{x} \rightarrow \ldots \rightarrow 33\text{x} \rightarrow 0$	13	18	489.456	911.69	21.92
Total	**102**	**158**	**5233.28**	**6834.99**	

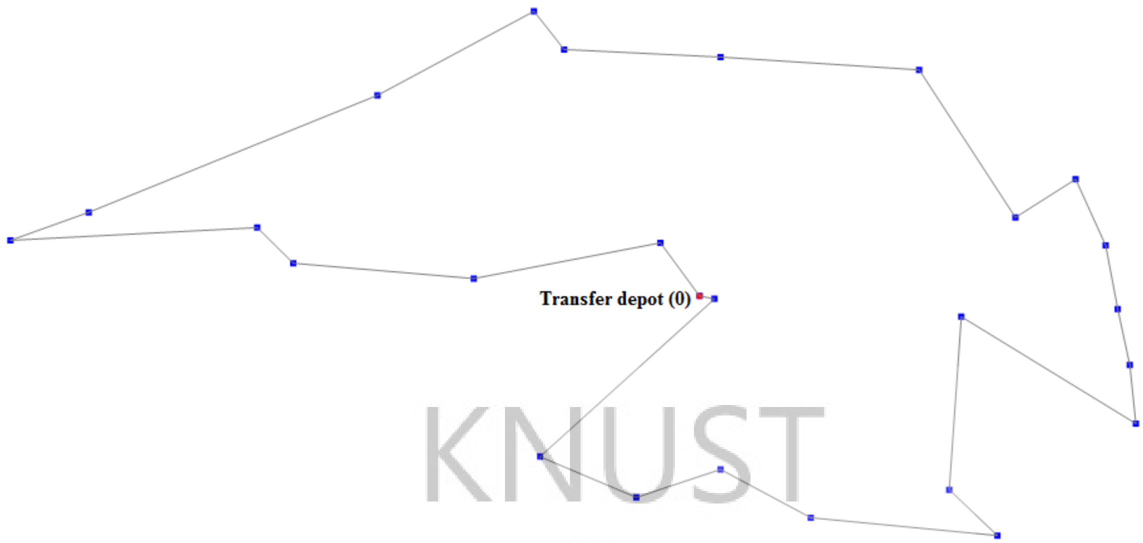


Figure 4.29: Fifth Ant best tour from sub-cluster two in zone four

The fifth Ant best tour obtained from sub-cluster five in zone four is as shown above with a total collection time of 1232.28 seconds, a tour length of 825.128m, visited 25 customers and emptied 26 bins from the customers.

$$0 \to 36y \to 33y \to 76y \to 32y \to 79y \to 80y \to 31y \to 35y \to 28y \to 25y \to 24y \to 21y \to 20y$$
$$\to 39y \to 40y \to 41y \to 42y \to 43y \to 53y \to 55y \to 58y \to 56y \to 57y \to 37y \to 38y \to 0$$

Table 4.29: Summary of routing from sub-cluster two in zone four

Sub-cluster Two					
Tour	Customers	Total bins	Length (m)	Time (sec)	Smell
$0 \to 35w \to \ldots \to 95x \to 0$	26	35	1337.017	1506.20	82.20
$0 \to 15y \to \ldots \to 27w \to 0$	35	35	1450.754	1564.62	77.82
$0 \to 16y \to \ldots \to 2y \to 0$	35	35	1572.900	1563.32	73.72
$0 \to 17y \to \ldots \to 23y \to 0$	35	35	1214.697	1564.23	87.66
$0 \to 36y \to \ldots \to 38y \to 0$	25	26	825.128	1232.28	46.56
Total	**156**	**166**	**6400.496**	**7430.65**	

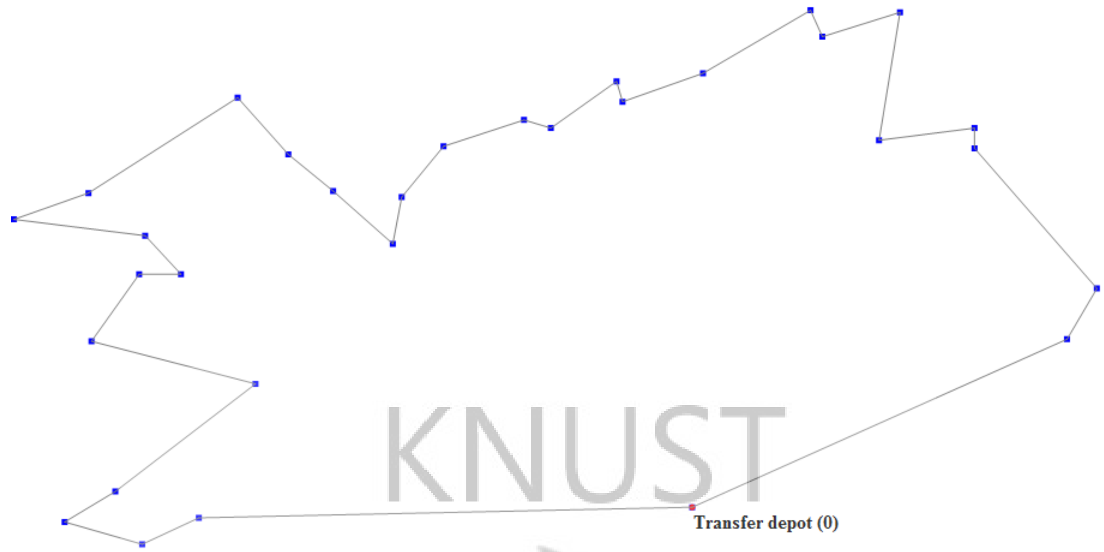


Figure 4.30: First Ant best tour from sub-cluster three in zone four

The first Ant best tour obtained from sub-cluster three in zone four is as shown above with a total collection time of 1531.56 seconds, a tour length of 1290.177m, visited 30 customers and emptied 35 bins from the customers.

$$0 \rightarrow 2w \rightarrow 4w \rightarrow 5w \rightarrow 10w \rightarrow 7w \rightarrow 9w \rightarrow 8w \rightarrow 17w \rightarrow 95v \rightarrow 94v \rightarrow 85v \rightarrow 93v \rightarrow 86v \rightarrow 89v \rightarrow 90v \rightarrow 94u \rightarrow 95u \rightarrow 98u \rightarrow 99u \rightarrow 100u \rightarrow 83u \rightarrow 97u \rightarrow 96u \rightarrow 79u \rightarrow 78u \rightarrow 74u \rightarrow 75u \rightarrow 67u \rightarrow 69u \rightarrow 72u \rightarrow 0$$

Table 4.30: Summary of routing from sub-cluster three in zone four

Sub-cluster Three					
Tour	Customers	Total bins	Length (m)	Time (sec)	Smell
$0 \rightarrow 2w \rightarrow \ldots \rightarrow 72u \rightarrow 0$	30	35	1290.177	1531.56	84.19
$0 \rightarrow 9z \rightarrow \ldots \rightarrow 52t \rightarrow 0$	30	35	1562.684	1529.70	74.04
$0 \rightarrow 15z \rightarrow \ldots \rightarrow 97v \rightarrow 0$	29	35	1453.617	1523.49	77.72
$0 \rightarrow 23z \rightarrow \ldots \rightarrow 98v \rightarrow 0$	30	30	1299.366	1380.37	52.76
Total	**119**	**135**	**5605.844**	**5965.12**	

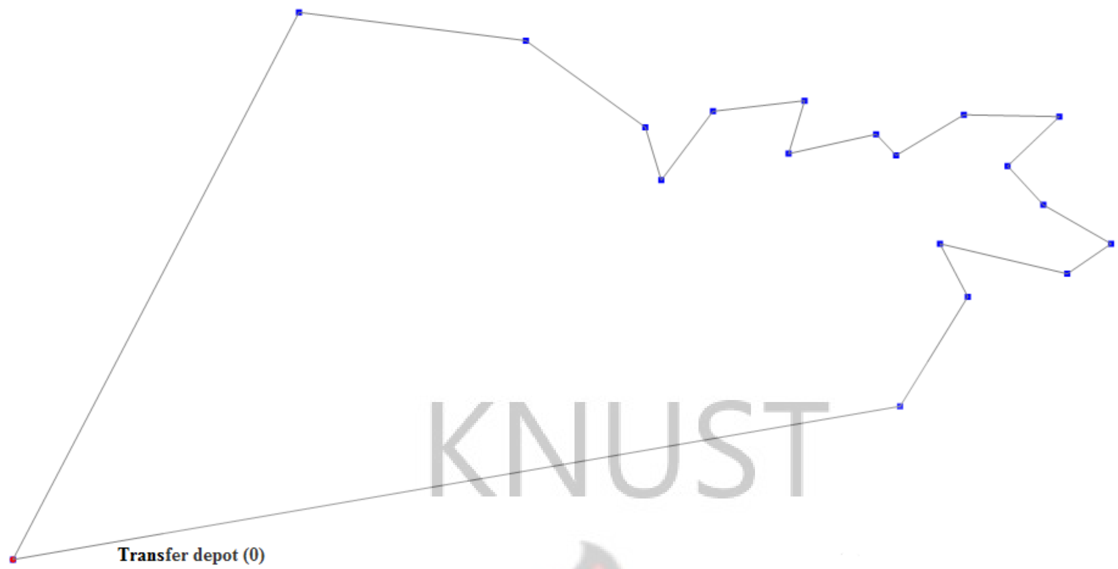


Figure 4.31: **Second Ant best tour from sub-cluster four in zone four**

The second Ant best tour obtained from sub-cluster four in zone four is as shown above with a total collection time of 1453.42 seconds, a tour length of 1123.457m, visited 18 customers and emptied 35 bins from the customers.

$$0 \to 72v \to 59w \to 58w \to 51w \to 57w \to 56w \to 52w \to 53w \to 48w \to 54w \to 78w \to 47w \to 46w \to 83w \to 42w \to 45w \to 41w \to 80v \to 0$$

Table 4.31: **Summary of routing from sub-cluster four in zone four**

Sub-cluster Four					
Tour	Customers	Total bins	Length (m)	Time (sec)	Smell
$0 \to 69v \to \ldots \to 60w \to 0$	16	35	1152.162	1440.56	90.82
$0 \to 72v \to \ldots \to 80v \to 0$	18	35	1123.457	1453.42	92.36
$0 \to 22v \to \ldots \to 68v \to 0$	23	35	1404.140	1484.78	79.55
$0 \to 47v \to \ldots \to 6v \to 0$	28	35	1212.607	1519.76	87.76
$0 \to 26v \to \ldots \to 25v \to 0$	21	26	808.525	1203.77	47.20
Total	**106**	**166**	**5700.891**	**7102.29**	

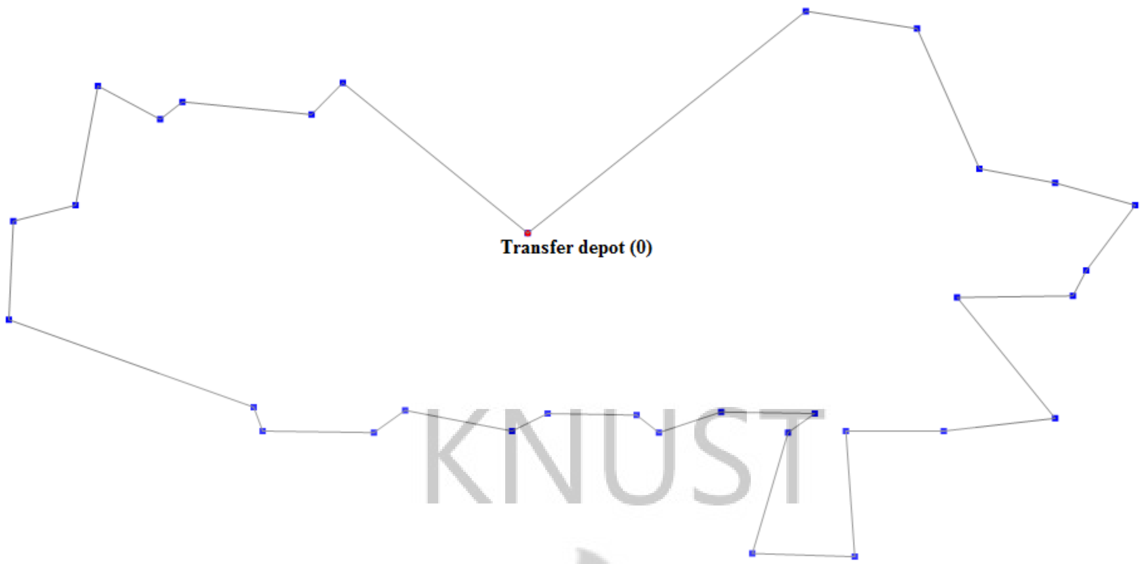


Figure 4.32: Fourth Ant best tour from sub-cluster five in zone four

The fourth Ant best tour obtained from sub-cluster five in zone four is as shown above with a total collection time of 1544.98 seconds, a tour length of 1308.425m, visited 32 customers and emptied 35 bins from the customers.

$$0 \to 56u \to 57u \to 61u \to 62u \to 65u \to 34u \to 78t \to 81t \to 79t \to 13t \to 12t \to 11t \to 10t \to 84t \to 83t \to 85t \to 8t \to 86t \to 87t \to 7t \to 91t \to 4t \to 3t \to 92t \to 3u \to 2u \to 4u \to 24u \to 5u \to 25u \to 46u \to 49u \to 0$$

Table 4.32: Summary of routing from sub-cluster five in zone four

Sub-cluster Five					
Tour	Customers	Total bins	Length (m)	Time (sec)	Smell
$0 \to 33v \to \ldots \to 20u \to 0$	21	35	1581.904	1471.73	73.44
$0 \to 66u \to \ldots \to 9t \to 0$	33	35	1641.390	1549.84	71.65
$0 \to 26u \to \ldots \to 14t \to 0$	33	35	1491.513	1550.15	76.39
$0 \to 56u \to \ldots \to 49u \to 0$	32	35	1308.425	1544.98	83.40
$0 \to 40u \to \ldots \to 44u \to 0$	21	24	673.210	1144.18	41.97
Total	**140**	**164**	**6696.442**	**7260.88**	

Table 4.33: Summary of Ant best tours, total collection time and distance covered in zone four

Zone Four	Customers	Total bins	Length (m)	Time (sec)
Sub-cluster one	102	158	5233.28	6834.99
Sub-cluster two	156	166	6400.496	7430.65
Sub-cluster three	119	135	5605.844	5965.12
Sub-cluster four	106	166	5700.891	7102.29
Sub-cluster five	140	164	6696.442	7260.88
Total	**623**	**789**	**29636.953**	**34593.93**
Average	**125**	**158**	**5927.391**	**6918.79**

4.4.0 IMPLEMENTATION OF FUEL CONSUMPTION MODEL

In this section we shall apply our five elemental fuel consumption models to calculate the fuel the waste vehicle (SINOTRUCK) will consume per trip during the day. The same model will be used to determine the fuel consumption during the night in addition to overtime payment to the crew members. In the last section, we shall do the comparison analysis of the two time schedules.

4.4.1 Fuel Consumption Dynamics During the Day

Records obtained from weeks of field trips with the crew members gave the following information

Average distance range from transfer depot to dump site (Dompoase): $(17.56-18.48)\text{km}$

Average time for in-trip: 52minutes

Average time for out-trip: 46minutes

Average time for half trip: 49minutes

Initial and final velocities during acceleration: $20\text{kmh}^{-1},\ 49\text{kmh}^{-1}$

Initial and final velocities during deceleration: $10\text{kmh}^{-1},\ 30\text{kmh}^{-1}$

Cruising speed: 52kmh^{-1}

Average dead speed: 12.5kmh^{-1}

Average time during acceleration: 16minutes

Average time during deceleration: 5minutes

Average cruising time: 5 minutes

Average dead speed time: 15 minutes

Average idle time: 8 minutes

4.4.2 Acceleration Model

Table 4.34: Constants used in Acceleration Model

A	B	k_1	α	C_r	M_f	E_k	k_2	β_1	β_2	ω
25	0.0067	0.2791	0.39	0.01	16500	6.4742×10^{-4}	1.1742	0.06	0.025	0.008π

$\bar{V}$	φ	d_a	t_a
9.5835	0.92	9.20016	960

$$F_a = \int_0^{960}\left[\varphi^{-1}\left\{\alpha + (AC_r + 0.1Bk_1\bar{V} + \beta_1 M_f E_k + k_2\beta_2 M_f E_k^2 + 0.000981\beta_1 M_e \omega)d_a\right\}\right]dt$$
$$= 3830.6016\text{mL}$$

4.4.3 DECELERATION MODEL

Table 4.35: Constants used in Deceleration Model

$k_1^{'}$	$\bar{V}_d$	k_x	k_y	$E_k^{'}$	k_a	d_d
17.3508	5.5555	0.0956	0.1719	1.42887×10^{-3}	0.1305	1.6665

$$F_d = \int_0^{300}\left[\varphi^{-1}\left\{\alpha + (0.087Ak_x + Bk_1^{'}k_y\bar{V}_d + k_a\beta_1 M_f E'_k + k_x\beta_1 M_f E'^2_k + 0.00981\beta_1 M_f \omega)d_d\right\}\right]dt$$
$$= 449.2251\text{mL}$$

4.4.4 Cruising Model

Table 4.36: Constants used in Cruising Model

f_i	V_c	V_C	E_c	β'_1	$\tau_c^{'}$	$\tau_c^{''}$	d_c
13	14.444	52	0.2436	0.01	0.005481	0.5128	4.3332

$$F_C = \int_0^{300}\left[\varphi^{-1}\left\{\begin{array}{l}\dfrac{f_i}{V_C} + (AC_r + 0.02BV_c^2 + \tau_c{'}\beta'_1 M_f E_C + 0.00371\beta_2 k_2 M_f E_C^2 + \\ 0.000981\tau_c{''}\beta'_1 M_f \omega)d_C\end{array}\right\}\right]dt$$
$$= 964.509$$

4.4.5 Dead Speed Model

Table 4.37: Constants used in Dead Speed Model

f_i	V_D	E'_D	β'_1	τ''_D	τ'''_D	d_D
3.125	3.4722	0.0505	0.01	0.4451	0.0234	3.12498

$$F_D = \int_0^{900} \left[\varphi^{-1} \left\{ \begin{array}{l} \dfrac{f_i}{V_D} + (AC_r + 0.02BV_D^2 + \tau_c{'''} \beta'_1 M_f E'_D + 0.00371\beta_2 k_2 M_f E'^2_D + \\ 9.81\times10^{-4} \tau_D{''} \beta'_1 M_f \omega) d_D \end{array} \right\} \right] dt$$
$$= 2063.2509\text{mL}$$

4.4.7 Idle Model

$$F_i = \int_0^{480} (\varphi^{-1}\alpha)dt$$
$$= 203.4782\text{mL}$$

Total fuel consumption for waste collection truck during in-trip during day service:

$$F_{IN} = 3830.6016 + 449.225 + 964.509 + 2063.2509 + 203.4782$$
$$= 7511.0647\text{mL}$$

4.4.8 Fuel Consumption During Out-Trip

The distance traveled by the waste collection vehicle is the same, but the weight of the vehicle differs. Records taking by the weighing bridge at the dumpsite gives an out-trip weight of the vehicle as 9400kg. The out-trip weight of the vehicle will be used as M_e in place of M in the acceleration, deceleration, cruise and the dead speed models.

$$F_a = \int_0^{960} \left[\varphi^{-1} \left\{ \alpha + (AC_r + 0.1Bk_1\bar{V} + \beta_1 M_e E_k + k_2\beta_2 M_e E_k^2 + 0.000981\beta_1 M_e \omega) d_a \right\} \right] dt$$
$$= 3463.035\text{mL}$$

$$F_d = \int_0^{300} \left[\varphi^{-1} \left\{ \alpha + (0.087Ak_x + Bk_1^{'}k_y\bar{V}_d + k_a\beta_1 M_e E'_k + k_x\beta_1 M_e E'^2_k + 0.00981\beta_1 M_e \omega) d_d \right\} \right] dt$$
$$= 385.2251\text{mL}$$

$$F_C = \int_0^{300} \left[\varphi^{-1} \left\{ \begin{array}{l} \dfrac{f_i}{V_C} + (AC_r + 0.02BV_c^2 + \tau_c\,'\beta'_1 M_e E_C + 0.00371\beta_2 k_2 M_e E_C^2 + \\ 0.000981\tau_c\,''\beta'_1 M_e \omega) d_C \end{array} \right\} \right] dt$$

$$= 735.259\text{mL}$$

$$F_D = \int_0^{900} \left[\varphi^{-1} \left\{ \begin{array}{l} \dfrac{f_i}{V_D} + (AC_r + 0.02BV_D^2 + \tau_c\,'''\beta'_1 M_e E'_D + 0.00371\beta_2 k_2 M_e E'^2_D + \\ 9.81\times10^{-4}\tau_D\,''\beta'_1 M_e \omega) d_D \end{array} \right\} \right] dt$$

$$= 1611.691\text{mL}$$

$$F_i = \int_0^{480} (\varphi^{-1}\alpha) dt$$

$$= 203.4782\text{mL}$$

Total fuel consumption for waste collection truck during out-trip during day service:

$$F_{OUT} = 3463.035 + 385.225 + 735.259 + 1611.691 + 203.4782$$

$$= 6398.6882\text{ml}$$

Total fuel consumption for waste collection truck for a trip during day service:

$$F_T = (7511.0647 + 6398.6882)\text{ml}$$

$$= 13909.7529\text{ml}$$

$$= 13910\text{ml}$$

4.4.9 Service Time And Cost Of Day Routing

With the 22 transfer sites proposed by our probabilistic distance location model, and an additional transfer depot at Moro market, the study area has 23 transfer sites. Based on our waste generation per capita per day, service of customers are to be done after every three days i.e. a service on Monday resumes on Thursday, a service on Tuesday resumes on Friday and a service on Wednesday resumes on Saturday. An analysis of the number of trips a vehicle can make in a day including lunch break and the total fuel consumption is summarized in the tables below

Table 4.38: Summary of service time per trip per day

Event	Time (minutes)	3 trips (minutes)
Loading	10	30
Off-loading	20	60
Routing (in and out)	98	294
Lunch break	-	60
Total	**128**	**444**
	2.133hrs	**7.4hrs**

The summary results from the table above shows that the optimal number of trips a vehicle can service in a day is 3trips. With 23 bins in the community, the study area needs three (3) vehicles to service the area. Table 4.39 gives the summary of the analysis on consumption in terms of volume and money.

Table 4.39: Fuel consumption analysis per week per study area during day service

Day	**Number of Vehicles**	**Total Trips**	**Fuel consumed (m*l*)**	**Fuel consumed (*l*)**	**Amount at GH¢3.267per *l***
Monday	3	9	125190	125.19	409.00
Tuesday	3	9	12590	125.19	409.00
Wednesday	2	5	69550	69.55	227.22
Thursday	3	9	12590	125.19	409.00
Friday	3	9	12590	125.19	409.00
Saturday	2	5	69550	69.55	227.22
Total		**46**			**2090.44**

4.5. 0 NIGHT SERVICE

The night service make use of three of the five elemental fuel consumption models; namely, Acceleration, deceleration and cruising. In this section we shall use these three elemental models to evaluate the fuel consumption and the total time needed to completely lift the waste from the study area.

4.5.1 Fuel Consumption Dynamics During the Night

Records obtained from weeks of field trips with the night crew members gave the following information

Average distance range from transfer depot to dump site (Dompoase): $(17.56-18.48)\text{km}$

Average time for in-trip: 28minutes

Average time for out-trip: 23.4minutes

Average time for half trip: 25.7minutes

Initial and final velocities during acceleration: 5kmh^{-1}, 58kmh^{-1}

Initial and final velocities during deceleration: 10kmh^{-1}, 45kmh^{-1}

Cruising speed: 60kmh^{-1}

Average time during acceleration: 12.2minutes

Average time during deceleration: 3.5minutes

Average cruising time: 10 minutes

4.5.2 Acceleration Model

Table 4.40: Constants used in Acceleration Model during night service

A	B	k_1	α	C_r	M_f	E_k	k_2	β'_1	β_2	ω
25	0.0067	0.3006	0.39	0.01	16500	3.753×10^{-6}	1.15711	0.06	0.025	0.008π

$\bar{V}$	φ	d_a	t_a
8.75	0.92	6.405	732

$$F_a = \int_0^{732}\left[\varphi^{-1}\left\{\alpha + (AC_r + 0.1Bk_1\bar{V} + \beta_1 M_f E_k + k_2\beta_2 M_f E_k^2 + 0.000981\beta_1 M_e\omega)d_a\right\}\right]dt$$
$$= 1843.31\text{mL}$$

4.5.3 Deceleration Model

Table 4.41: Constants used in Deceleration Model

k_1'	$\bar{V}_d$	k_x	k_y	E_k'	k_a	d_d	t_d
20.6011	9.1389	0.11315	0.19509	3.57259×10^{-3}	0.334785	1.60517	210

$$F_d = \int_0^{210} \left[\varphi^{-1} \left\{ \alpha + (0.087 A k_x + B k_1^{'} k_y \bar{V}_d + k_a \beta_1 M_f E'_k + k_x \beta_1 M_f E'^{2}_k + 0.00981 \beta_1 M_f \omega) d_d \right\} \right] dt$$
$$= 697.501\text{mL}$$

4.4.4 Cruising Model

Table 4.42: Constants used in Cruising Model

f_i	V_c	V_C	β'_1	E_c	$\tau_c^{'}$	$\tau_c^{''}$	d_c	t_c
15	16.6667	60	0.01	0.24133	0.00601	0.52120	10.00	600

$$F_C = \int_0^{600} \left[\varphi^{-1} \left\{ \begin{matrix} \frac{f_i}{V_C} + (AC_r + 0.02 B V_c^2 + \tau_c ' \beta'_1 M_f E_C + 0.00371 \beta_2 k_2 M_f E_C^2 + \\ 0.000981 \tau_c '' \beta'_1 M_f \omega) d_C \end{matrix} \right\} \right] dt$$
$$= 4283.68\text{mL}$$

Total fuel consumption for waste collection truck during in-trip during night service:

$$F_{IN} = 1843.31 + 697.50 + 4283.68$$
$$= 6824.49\text{mL}$$

4.5.8 Fuel Consumption During Out-Trip

The distance traveled by the waste collection vehicle is the same, but the weight of the vehicle differs. Records taking by the weighing bridge at the dumpsite (Dompoase) gives an out-trip weight of the vehicle as 9400kg. The out-trip weight of the vehicle will be used as M_e in place of M in the acceleration, deceleration and cruising models.

$$F_a = \int_0^{732} \left[\varphi^{-1} \left\{ \alpha + (AC_r + 0.1 B k_1 \bar{V} + \beta_1 M_e E_k + k_2 \beta_2 M_e E_k^2 + 0.000981 \beta_1 M_e \omega) d_a \right\} \right] dt$$
$$= 1770.54\text{mL}$$

$$F_d = \int_0^{300} \left[\varphi^{-1} \left\{ \alpha + (0.087 A k_x + B k_1^{'} k_y \bar{V}_d + k_a \beta_1 M_e E'_k + k_x \beta_1 M_e E'^{2}_k + 0.00981 \beta_1 M_e \omega) d_d \right\} \right] dt$$
$$= 506.86\text{mL}$$

$$F_C = \int_0^{600} \left[\varphi^{-1} \left\{ \begin{matrix} \frac{f_i}{V_C} + (AC_r + 0.02 B V_c^2 + \tau_c ' \beta_1 M_e E_C + 0.00371 \beta_2 k_2 M_e E_C^2 + \\ 0.000981 \tau_c '' \beta_1 M_e \omega) d_C \end{matrix} \right\} \right] dt$$
$$= 3316.59\text{mL}$$

Total fuel consumption for waste collection truck during out-trip during night service:

$$F_{OUT} = 1770.54 + 506.86 + 3316.59$$
$$= 5593.99\text{ml}$$

Total fuel consumption for waste collection truck for a trip during night service:

$$F_T = (6824.49 + 5593.99)\text{ml}$$
$$= 12418.48\text{ml}$$
$$= 12419\text{ml}$$

4.5.9 Service Time and Cost of Night Routing

With the 22 transfer sites proposed by our probabilistic distance location model, and an additional transfer depot at Moro market, the study area has 23 transfer sites. Based on our waste generation per capita per day, service of customers are to be done after every three days i.e. a service on Monday resumes on Thursday, a service on Tuesday resumes on Friday and a service on Wednesday resumes on Saturday. An analysis of the number of trips a vehicle can make in a day including lunch break and the total fuel consumption is summarized in Tables 4.43 and 4.44.

Table 4.43: Summary of service time per trip per night

Event	Time (minutes)	5 trips (minutes)
Loading	10	50
Off-loading	20	100
Routing (in and out)	51.4	257
Rest period	-	60
Total	**81.4**	**467**
	1.3567hrs	**7.7833hrs**

The summary of results from Table 4.43 shows that the optimal number of trips a vehicle can service in a night is 5. With 23 bins in the community, the study area needs an average of three (3) vehicles to service the area. Table 4.44 gives the summary of the analysis on consumption in terms of volume and money.

Table 4.44: Fuel consumption analysis per week per study area during night service

Day	Number of Vehicles	Total Trips	Fuel consumed (m*l*)	Fuel consumed (*l*)	Amount at GH¢3.267per *l*
Monday	3	15	186285	186.285	608.59
Tuesday	2	8	99352	99.352	324.58
Thursday	3	15	186285	186.285	608.59
Friday	2	8	99352	99.352	324.58
Total		**46**			**1866.34**

4.6 SUMMARY

In this chapter we put forward our proposed methods to solve for the waste generation, collection and transportation as pertained in the study area. The chapter had four main sections. In section one, a mixed effect models were used to model the waste generation per person per day, in the second section our proposed probabilistic distance location model was used to optimally assign customers unto a particular service bin, taking into consideration the distance and weight of a customer from the transfer depot (skip container) and the capacity of the skip container. The third section of this chapter was devoted to our proposed Ant Colony Heuristics to collect waste from customers with a capacity and time window constraints, the minimum distance travelled and effect of smell, the last but not least section of the chapter was for the comparison of fuel consumed by waste collection vehicles during day and night using our improved five elemental fuel consumption models.

CHAPTER FIVE

CONCLUSIONS, RECOMMENDATIONS AND SUGGESTIONS FOR FUTURE RESEARCH

5.0 INTRODUCTION

This chapter provides a summary of the new ideas put forward to solve a real world problem as it exist in the study area in respect of solid waste generation, collection and transportation. Findings from the solution of our proposed models will be outlined and further research areas, which may be worthwhile pursuing in the future, would be mentioned.

5.1 CONCLUSIONS

5.1.0 EMPIRICAL RESULTS FROM MIXED EFFECT MODELS

What cannot be measured cannot be managed, so the need to actually determine the solid waste generation per person per day is very important to waste management. The results obtained from the third class communities in the study area gave a value of 0.597kg per person per day which is about 3% higher than the perceived value of 0.56 as used in the metropolis. The total waste generation for the population in the Kumasi Metropolis which is about 1350 tonnes a day gives an under estimated value of about 90 tonnes a day per our 6.61% increase in waste generation per person per day assuming equal generation for the entire population.

5.1.1 Results from Probabilistic Distance Location Model

The adoption of the improved probabilistic distance location model to the study area of 2475 houses and about 3509 of 140 litre bins has been able to optimally assign customers unto a zone. The results shows that the study area needs about 23 skip containers to be emptied after every three days. The results provides an organized way of waste collection as compared to the current procedure of haphazard way of call for service, which renders most drivers redundant low work schedules and increased cost of payment to the service companies.

5.1.2 Capacitated Vehicle Routing With Time Windows

With the proposed collection method of half–day location which ensures that a skip container does not sit for more than 12 hours, our improved method on Ant Colony gave the following results:

The optimal routing time obtained by a tricycle in zone one (1) had seven sub-clusters with a total collection time of 933.07 minutes and an average sub-cluster time of 133.30 minutes. Total distance covered by a tricycle in zone one is 35.32km and an average distance per sub-cluster is 5.046km. The optimal route obtained by a tricycle in zone two (2) had five sub-clusters with a total collection time of 694.41 minutes and an average sub-cluster time of 138.88 minutes. Total distance covered by a tricycle in zone two is 24.907km and an average distance per sub-cluster is 4.981km. Optimal route obtained by a tricycle in zone three (3) had five sub-clusters with a total collection time of 590.53minutes and an average sub-cluster time of 118.11 minutes. Total distance covered by a tricycle in zone three is 23.966km and an average distance per sub-cluster is 4.793km.The final optimal route obtained by a tricycle in zone four (4) had five sub-clusters with a total collection time of 576.57minutes and an average sub-cluster time of 115.31minutes. Total distance covered by a tricycle in zone four is 29.637km and an average distance per sub-cluster is 5.927km.

With the proposed three (3) sub-cluster collection per tricycle per day, a service company needs 4 tricycles for optimum collection of solid waste from 12 sub-clusters in zones one (1) and two (2) on day one, for example on Monday and three (3) tricycles on day two (2), for example on Tuesday to collect waste from zones three and four which has 10 sub-clusters. The same schedule is repeated after three days of service, thus on Thursday and Friday, after which the cycle repeats on Monday.

5.1.3 Results on Fuel Consumption Models

Taken into accounts the normal working hours as eight (8) less one hour (1) of lunch break, a service truck can do a maximum of three trips of waste in a day which translate to a fuel consumption of GH¢45.44 per trip. For total lifting of 46 trips, the service company needs an average of three (3) vehicles per day for six days to completely lift the waste in the study area, resulting in total fuel consumption of GH¢2090.44.

However, for the night service a service truck can do five trips, translating to a fuel consumption of GH¢40.57 per trip. To lift the total waste of 46 trips, the service company needs an average of three (3) vehicles for four (4) days in the week to completely lift the waste from the study area resulting in GH¢1866.34.

5.2 RECOMMENDATIONS

We strongly recommend our findings 0.595 kg of waste generation per person per day in a third class community to the Waste Management Agency of Kumasi Metropolitan Assembly (KMA) for proper forecast of waste generation of solid waste in the metropolis.

Secondly, we recommend our results on location and routing models with time windows to the service company in the study area and any other service company that operates in a third or second class communities, since it will help reduce indiscriminate dumping of waste in water bodies which leads to pollution of water bodies, dumping of waste in drains that can lead to flooding when it rains, burning of waste which increases ozone depletion.

Thirdly, we recommend night collection of waste in the metropolis since it has the potential of cutting back service cost of about GH¢224.10 per week, saves about six (6) vehicles and its crew members for other purposes. Apart from these added advantages of night service, it also saves road users and passersby from the unfriendly smell that accompany waste collection vehicles during the day.

5.3 POSSIBLE RESEARCH AVENUES

The avenues for future research which we feel may be worthwhile considering are outlined in this section.

- Due to the high transportation cost of waste collection vehicles due to the smaller capacity, research must be taken on location of a transfer station within the metropolis that can be assessed optimally in terms of distance from each sector of the metropolis for much bigger compaction vehicle to lift to the dump site.
- Research must be taken in the area of waste generation from all the classes of communities to properly know the waste generation in a day for proper planning of waste.
- A method that determines the location of dumpsites to reduce the cost of transportation of waste with factors such as land requirement, surface water pollution, soil texture and land topology to be considered.

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APPENDIX A

Appendix A shows the results of remainder systematic sampling method used to select customers in zones two (2) to zone five (5) for the waste generated per family size per day in kilograms.

Table A 1: Average waste generation per day per family size over six month period for zone 2

Zone 2 (Average waste generation per day per family size (kg))							
House ID	Family size	Month 1	Month 2	Month 3	Month 4	Month 5	Month 6
40d	6	3.2	3.4	3.4	3.5	3.2	3.1
72d	5	3.0	3.0	2.8	2.9	3.0	2.9
87d	4	2.7	2.9	2.4	2.6	2.8	2.6
20e	4	2.2	3.4	2.6	2.9	2.5	2.8
53e	5	3.1	2.4	3.0	3.2	3.1	3.1
86e	3	1.9	2.5	1.9	2.1	2.0	1.9
19f	3	3.0	2.6	2.1	2.4	1.6	1.8
52f	4	2.6	2.9	3.0	2.2	2.2	2.1
85f	3	1.8	2.2	2.4	1.8	1.9	1.6
18g	2	1.4	2.0	1.3	0.8	0.9	1.1

Table A 2: Average waste generation per day per family size over six month period for zone 3

Zone 3 (Average waste generation per day per family size (kg))							
House ID	Family size	Month 1	Month 2	Month 3	Month 4	Month 5	Month 6
42h	2	1.0	1.2	1.3	0.9	1.0	1.3
98h	6	3.2	3.4	3.1	3.1	3.2	3.2
54i	5	3.2	3.1	2.8	2.4	3.0	3.0
10j	4	2.6	2.6	2.4	2.2	2.7	2.5
66j	3	2.0	2.1	1.8	1.6	2.0	2.0
22k	3	1.5	1.9	2.0	1.9	1.9	2.0
78k	2	1.6	1.1	1.3	1.0	1.1	1.2
34l	2	1.1	2.0	1.4	1.1	1.2	1.0
74l	6	3.4	3.6	3.5	2.8	3.3	3.0
31m	7	3.8	3.8	3.8	3.5	3.8	3.4

Table A 3: Average waste generation per day per family size over six month period for zone 4

Zone 4 (Average waste generation per day per family size (kg))							
House ID	Family size	Month 1	Month 2	Month 3	Month 4	Month 5	Month 6
62m	5	3.0	2.9	3.1	2.8	3.0	3.0
16n	4	2.4	2.4	2.6	2.6	2.4	2.6
70n	3	1.8	1.9	1.8	1.9	1.6	2.0
24p	5	3.0	2.8	3.0	2.8	2.8	3.1
78p	6	3.4	3.2	3.6	3.2	3.4	3.4
32q	8	3.9	3.6	4.1	4.3	4.0	3.9
86q	5	3.1	3.2	2.8	3.1	2.6	3.1
79r	4	2.8	3.1	2.4	2.2	2.2	2.6
34s	4	3.0	2.8	2.3	2.6	2.0	2.2
89s	3	2.9	2.6	2.0	2.0	1.8	1.8

Table A 4: Average waste generation per day per family size over six month period for zone 5

Zone 5 (Average waste generation per day per family size (kg))							
House ID	Family size	Month 1	Month 2	Month 3	Month 4	Month 5	Month 6
13t	4	2.8	2.6	2.6	2.4	2.1	2.2
75t	2	1.2	1.1	1.2	2.1	1.1	1.0
37u	6	3.5	2.9	3.4	3.2	3.3	3.4
99u	3	1.3	2.1	1.7	1.6	1.9	1.8
61v	4	2.2	2.8	2.5	2.1	2.6	2.3
23w	6	2.9	3.3	3.0	3.0	3.4	3.0
79w	5	3.2	3.2	2.8	2.7	2.9	2.6
42x	5	2.9	3.5	2.9	3.2	3.1	2.8
5y	3	1.9	2.1	2.1	1.8	1.8	1.9
68y	5	2.7	3.6	3.1	2.6	2.7	2.7

Identifier	Minimum	Mean	Maximum	Values	Missing
Family_Size	2.000	4.120	8.000	300	0

Identifier	Minimum	Mean	Maximum	Values	Missing
Month	1.000	3.500	6.000	300	0

Identifier	Minimum	Mean	Maximum	Values	Missing
Waste_p_size	0.09500	2.457	4.320	300	0

Identifier	Minimum	Mean	Maximum	Values	Missing
Waste_p_p	0.04750	0.6071	1.055	300	0

Model 1: Mean model or intercept only model

$y_{ij} = \beta_0 + e_{ij}$, where $e_{ij} \sim iid\, N(0,\sigma)$

R code: lm(formula = y ~ 1, data = waste)

Coefficients:

	Estimate	Std. Error	t value	Pr(>|t|)
Intercept	2.45705	0.04191	58.63	<2e-16 ***

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
Residual standard error: 0.7259 on 299 degrees of freedom

AIC = 662.16

Model 2: one-factor fixed effect ANOVA model: fixed variations within periods

$y_{ij} = \beta_{0[j]} + e_{ij}$, where $e_{ij} \sim iid\, N(0,\sigma)$

R code: lm(formula = y ~ period - 1, data = waste_grp)

Coefficients:

	Estimate	Std. Error	t value	Pr(>|t|)
period2	2.6274	0.1028	25.57	<2e-16 ***
period1	2.4759	0.1028	24.09	<2e-16 ***
period6	2.3960	0.1028	23.31	<2e-16 ***
period5	2.3656	0.1028	23.02	<2e-16 ***
period3	2.4828	0.1028	24.16	<2e-16 ***
period4	2.3946	0.1028	23.30	<2e-16 ***

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

AIC = 726.0851

The estimated residual standard error is $\hat{\sigma} = 0.727$

Model 3: one-factor fixed effect ANOVA model with random intercepts

$y_{ij} = \beta_{0[j]} + b_{0[j]} + e_{ij}$, where the residuals $e_{ij} \sim iid\, N(0,\sigma)$ and the random effects $b_{0[j]} \sim iid\, N(0,\sigma_{b_0})$

R code: lme(y~period-1,data=waste2,random=~1|period)

AIC = 686.0851, indicates better model performance after random effects $b_{0[j]}$ introduced.

Estimates of random effects standard deviation parameters are $\hat{\sigma}_{b_0} = 0.274$ and $\hat{\sigma} = 0.727$

$y_i = \beta_0 + \beta_1 x_i + e_i$, where $e_i \sim iid\, N(0,\sigma)$

Model 4: varying intercept and varying slope model without random effects

$y_{ij} = \beta_{0[j]} + \beta_{1[j]} x_i + e_{ij}$, where $e_{ij} \sim iid\, N(0,\sigma)$

R code: lme(y~x,data=waste2) or lmList (y~x,data=waste2)

Varying intercepts

Period	Estimate	Std. Error	t value	Pr(>|t|)
2	0.9558942	0.1285052	7.438564	1.175948e-12
1	0.4528883	0.1285052	3.524280	4.937397e-04
6	0.4378757	0.1285052	3.407455	7.491630e-04
5	0.1626280	0.1285052	1.265537	2.067019e-01
3	0.5030898	0.1285052	3.914938	1.129411e-04
4	0.4222754	0.1285052	3.286057	1.141738e-03

Varying slopes or coefficients for the predictor variable household-size

Period	Estimate	Std. Error	t value	Pr(>|t|)
2	0.4057053	0.02957466	13.71801	0
1	0.4910223	0.02957466	16.60280	0
6	0.4752729	0.02957466	16.07028	0
5	0.5347019	0.02957466	18.07973	0
3	0.4805122	0.02957466	16.24743	0
4	0.4787196	0.02957466	16.18682	0

The residual standard error $\hat{\sigma} = 0.289$ highly reduced.
AIC = 119.66

Model 5: varying intercept and varying slope model with random effects

$y_{ij} = \beta_{0[j]} + b_{0[j]} + \left(\beta_{1[j]} + b_{1[j]}\right) x_i + e_{ij}$ where the residuals $e_{ij} \sim iid\ N\left(0, \sigma\right)$,the random intercepts $b_{0[j]} \sim iid\ N\left(0, \sigma_{b_0}\right)$ and random slopes $b_{1[j]} \sim iid\ N\left(0, \sigma_{b_1}\right)$

AIC = 136.07

APPENDIX B

Appendix B shows the results of our proposed probabilistic distance location model to assign customers unto a cluster centres in each of the four zones.

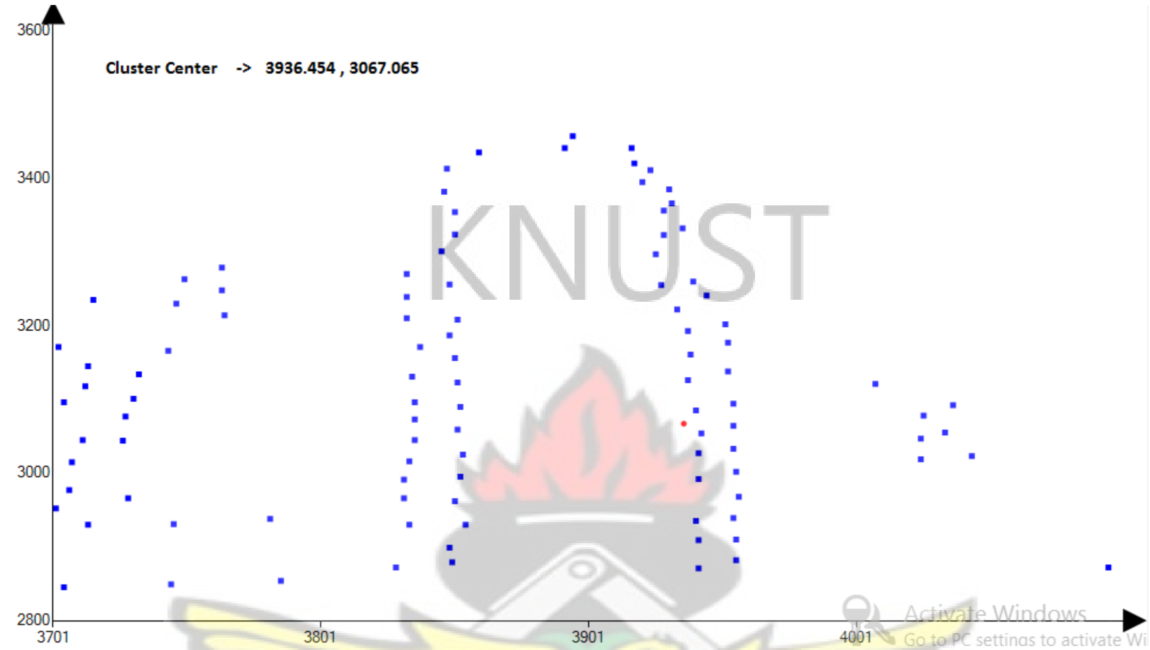


Figure B1: Cluster centre and members in zone one sub-cluster seven

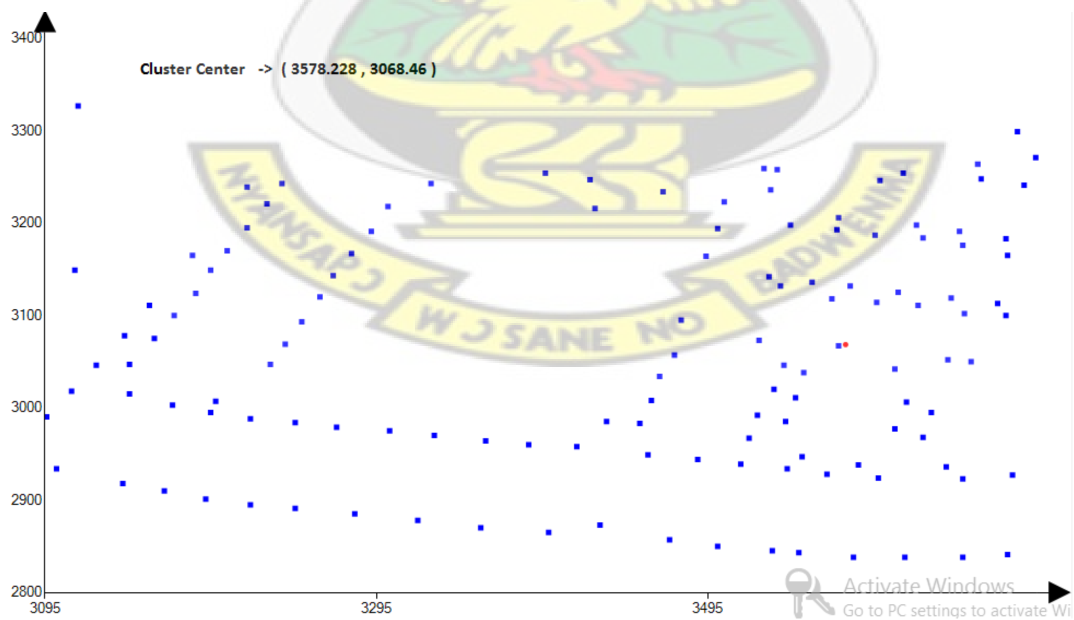


Figure B2: Cluster centre and members in zone one sub-cluster six

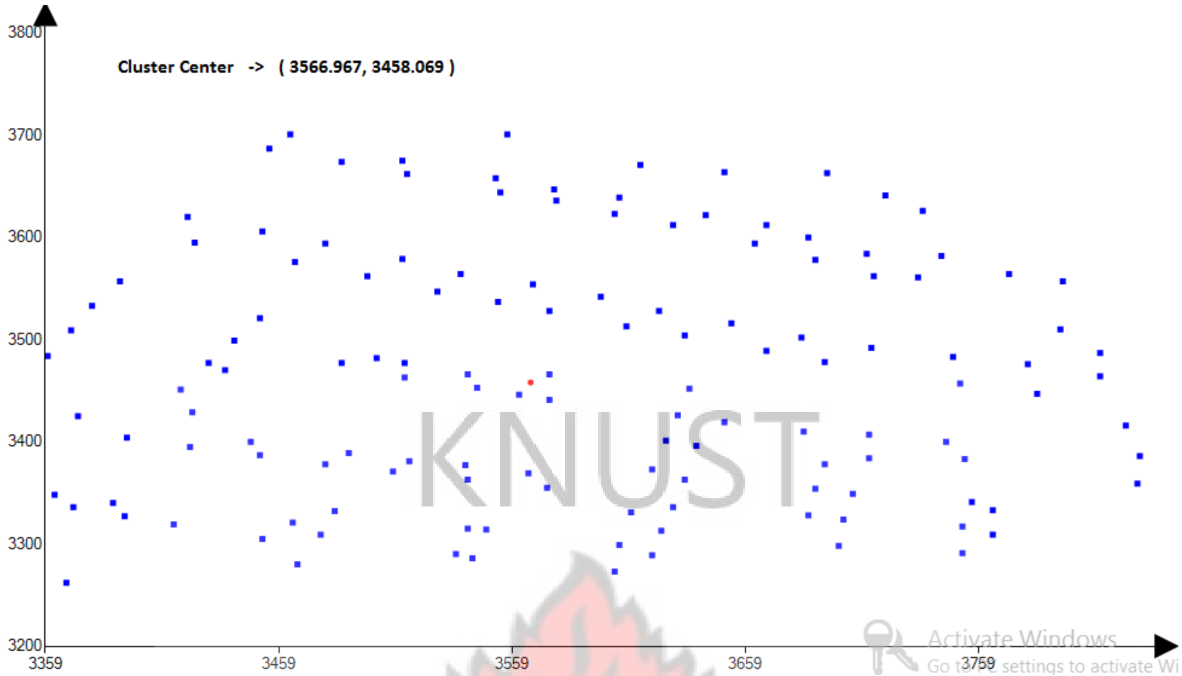


Figure B3: Cluster centre and members in zone one sub-cluster five

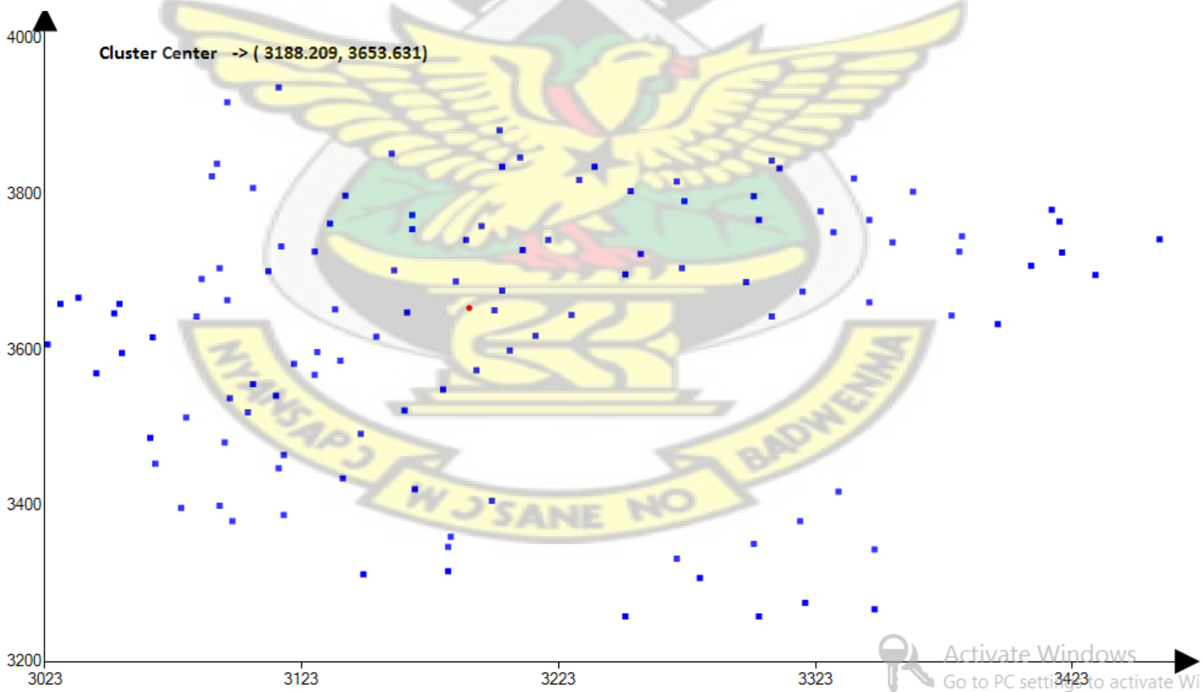


Figure B4: Cluster centre and members in zone one sub-cluster four

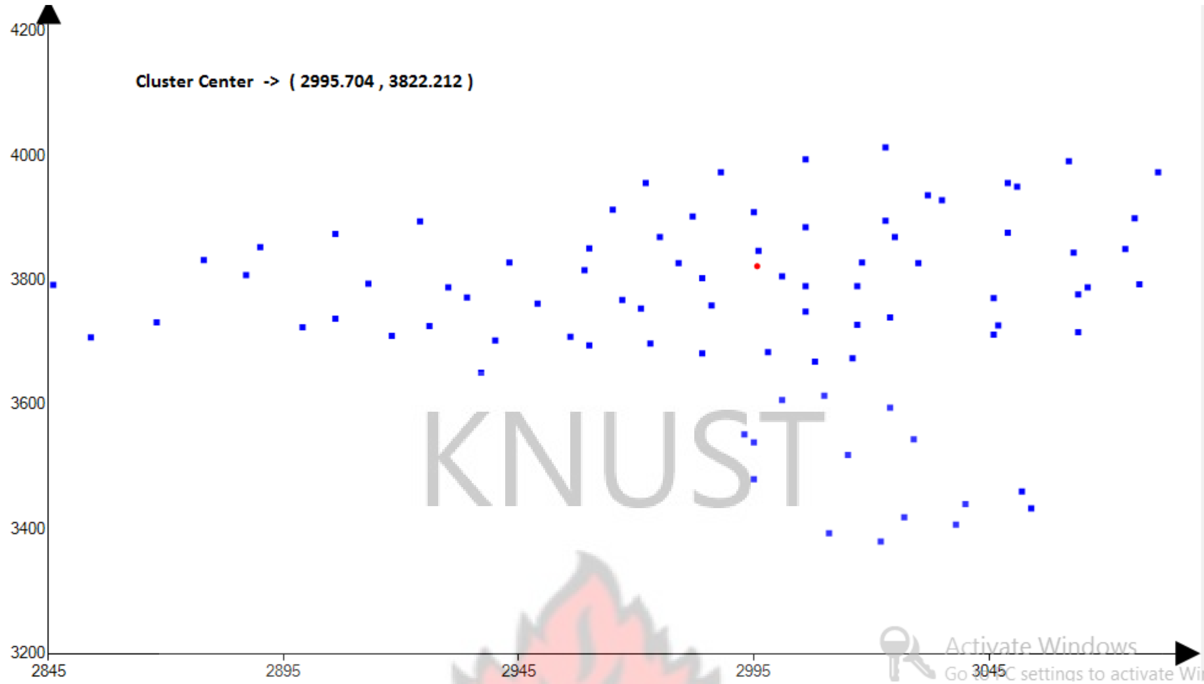


Figure B5: Cluster centre and members in zone one sub-cluster three

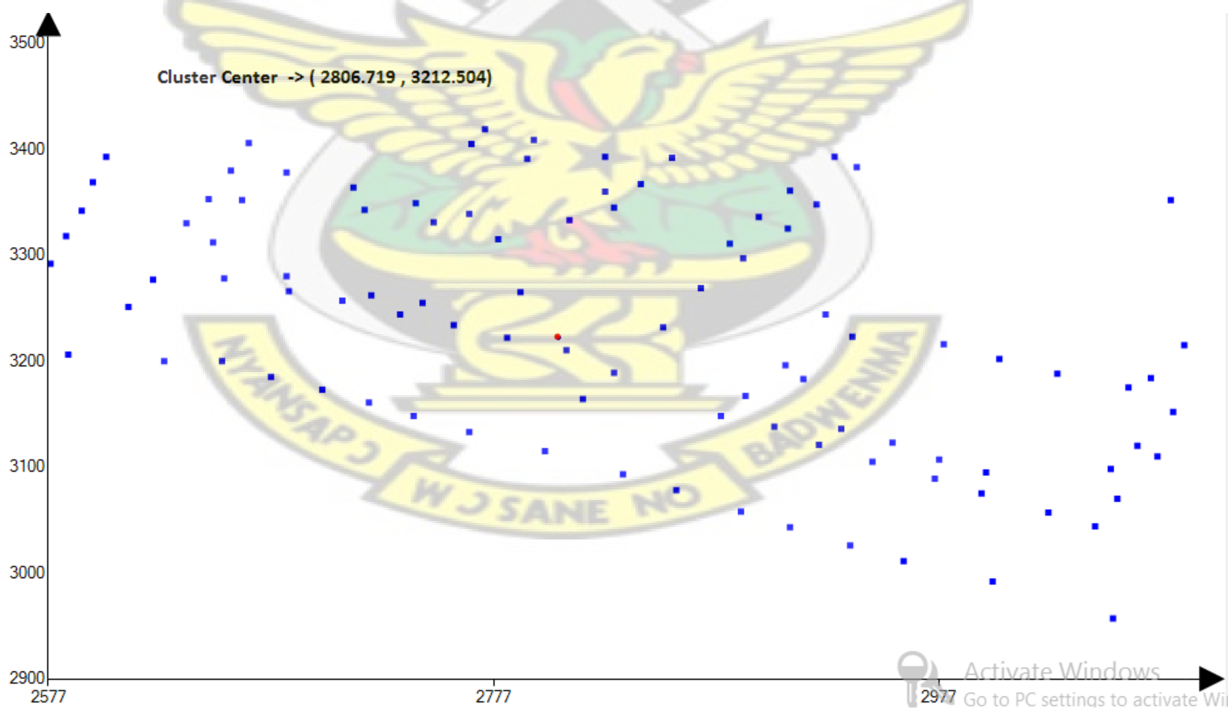


Figure B6: Cluster centre and members in zone one sub-cluster two

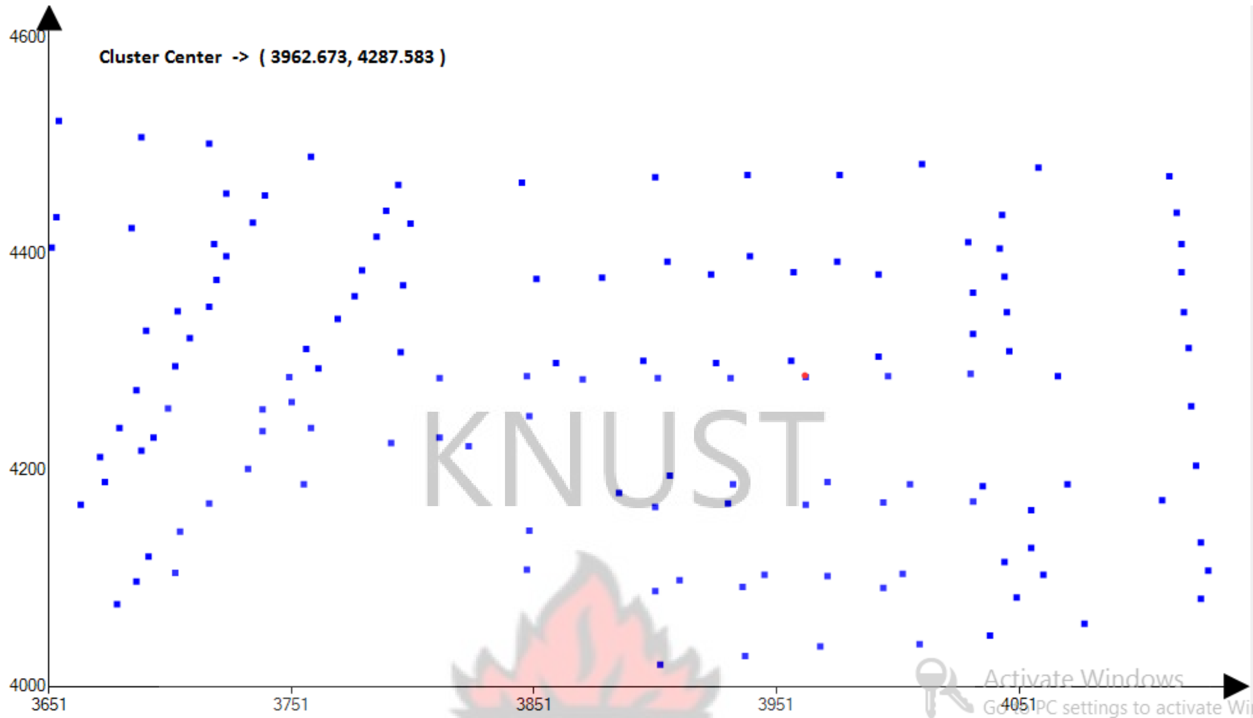


Figure B7: Cluster centre and members in zone two sub-cluster two

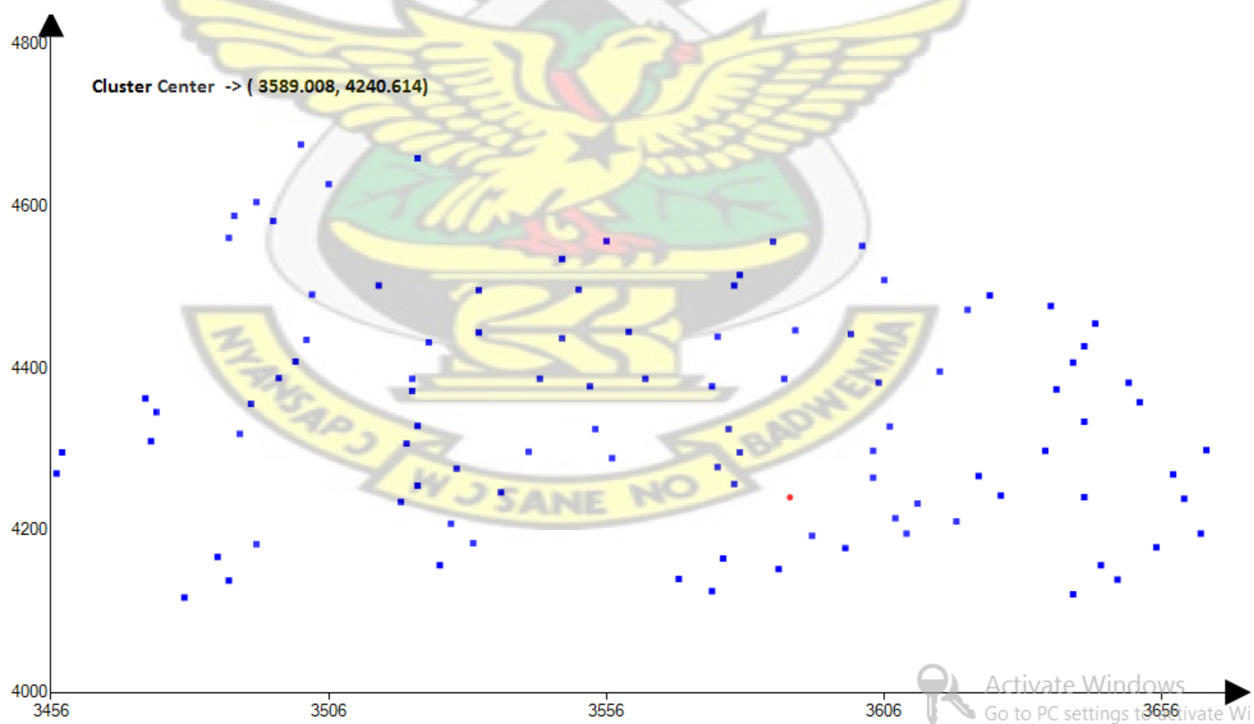


Figure B8: Cluster centre and members in zone two sub-cluster three

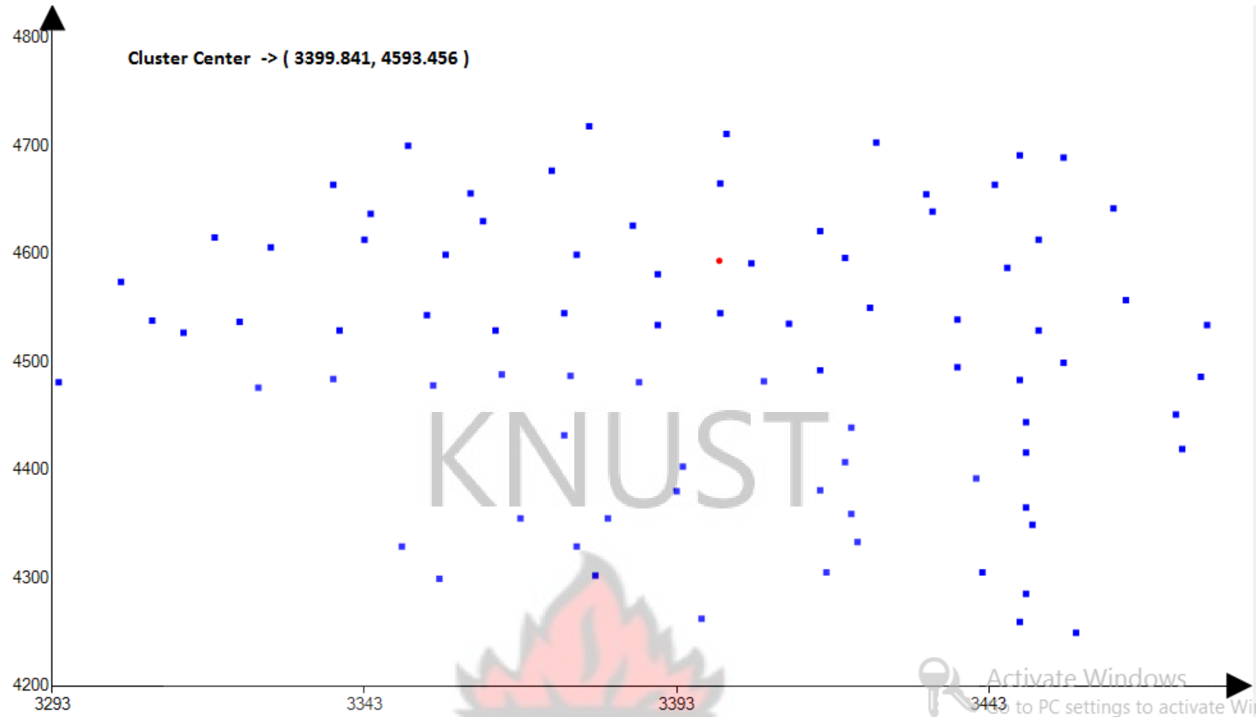


Figure B9: Cluster centre and members in zone two sub-cluster four

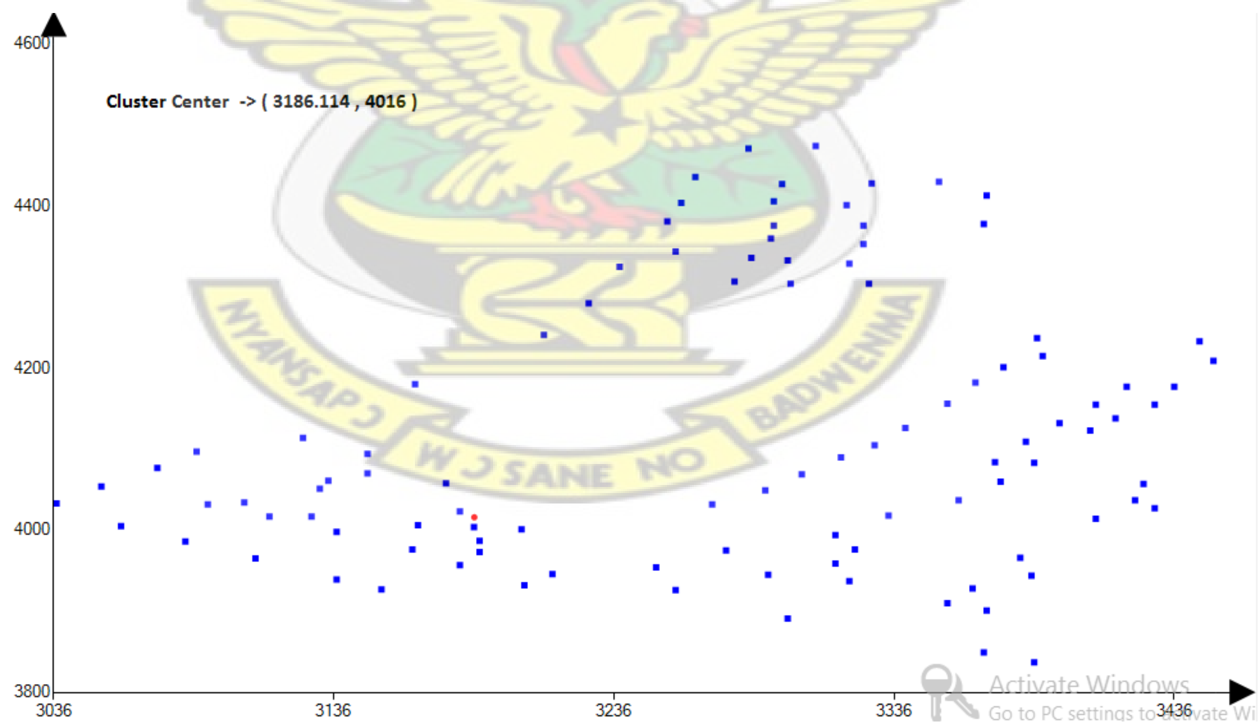


Figure B10: Cluster centre and members in zone two sub-cluster five

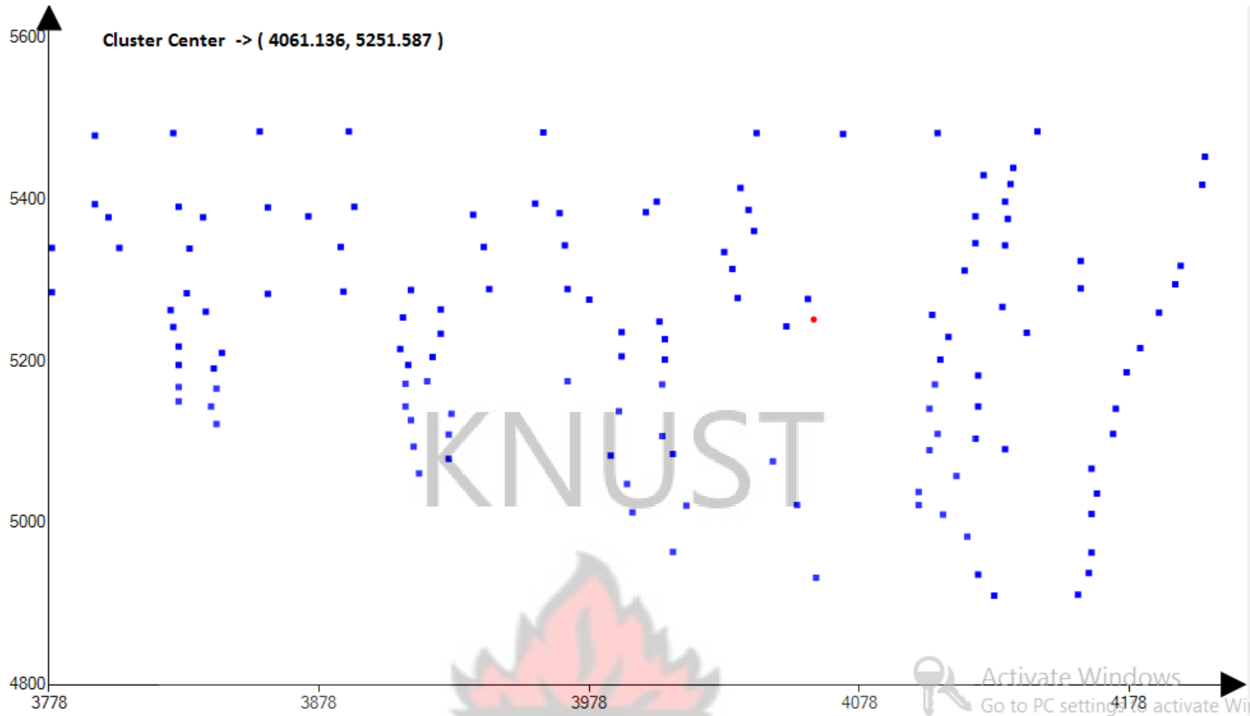


Figure B11: Cluster centre and members in zone three sub-cluster two

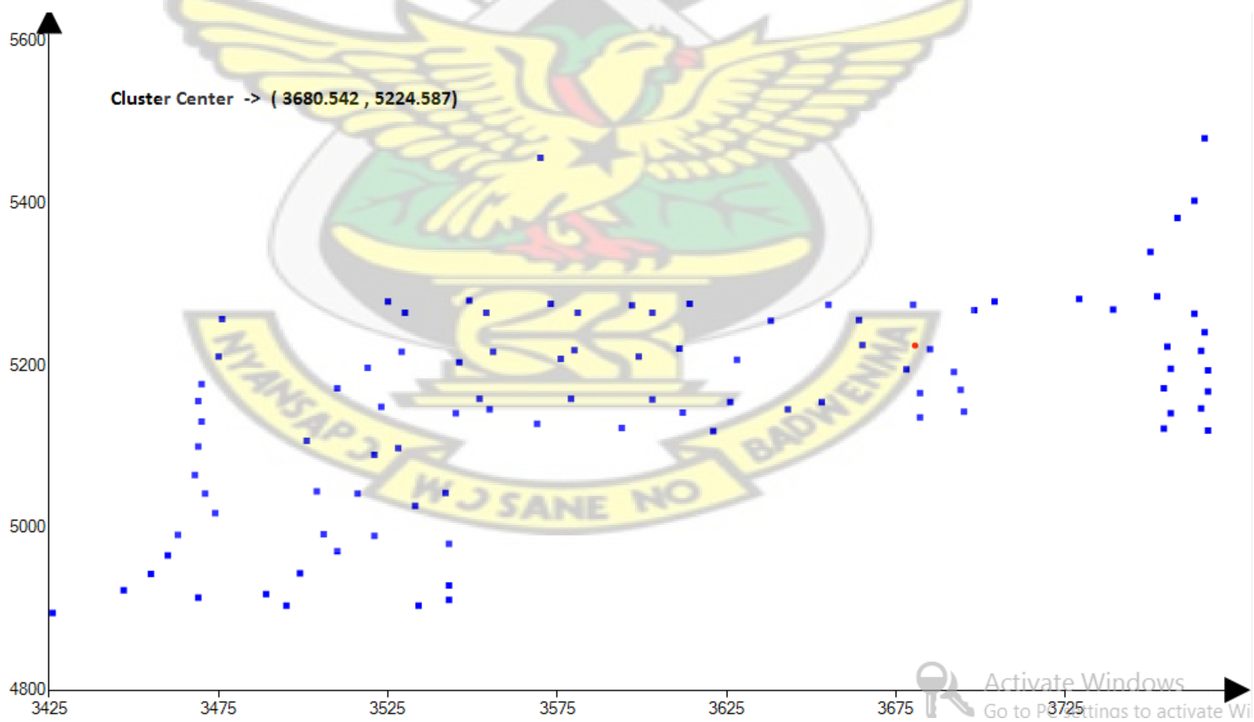


Figure B12: Cluster centre and members in zone three sub-cluster three

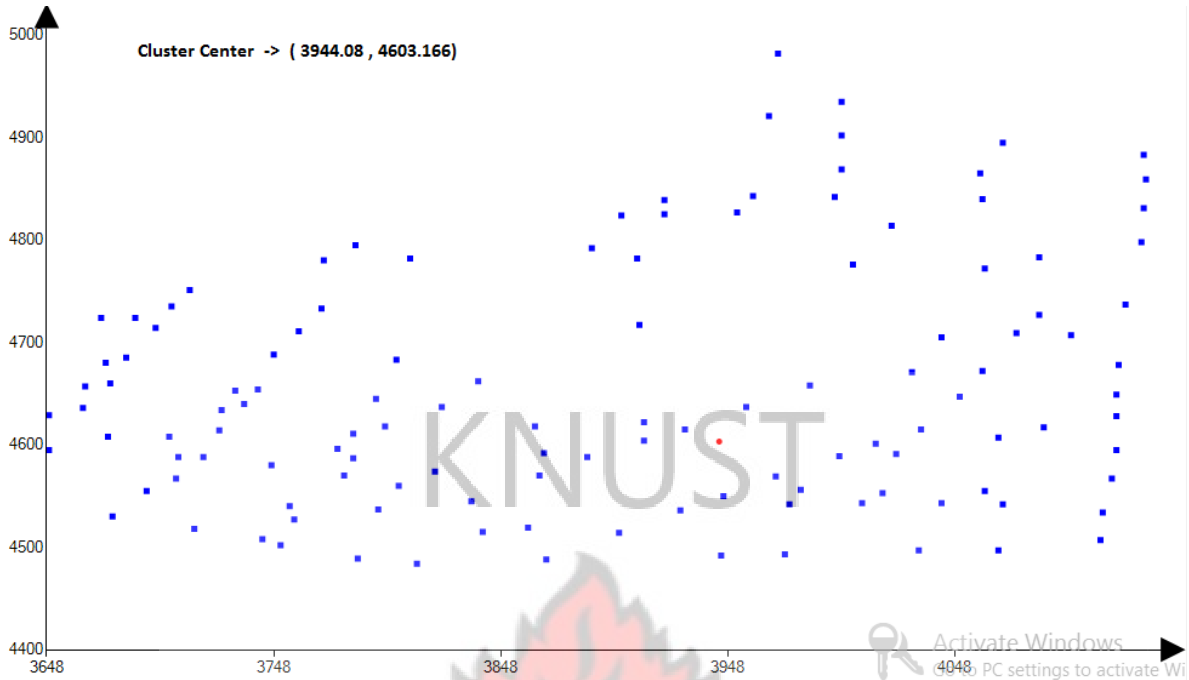


Figure B13: Cluster centre and members in zone three sub-cluster four

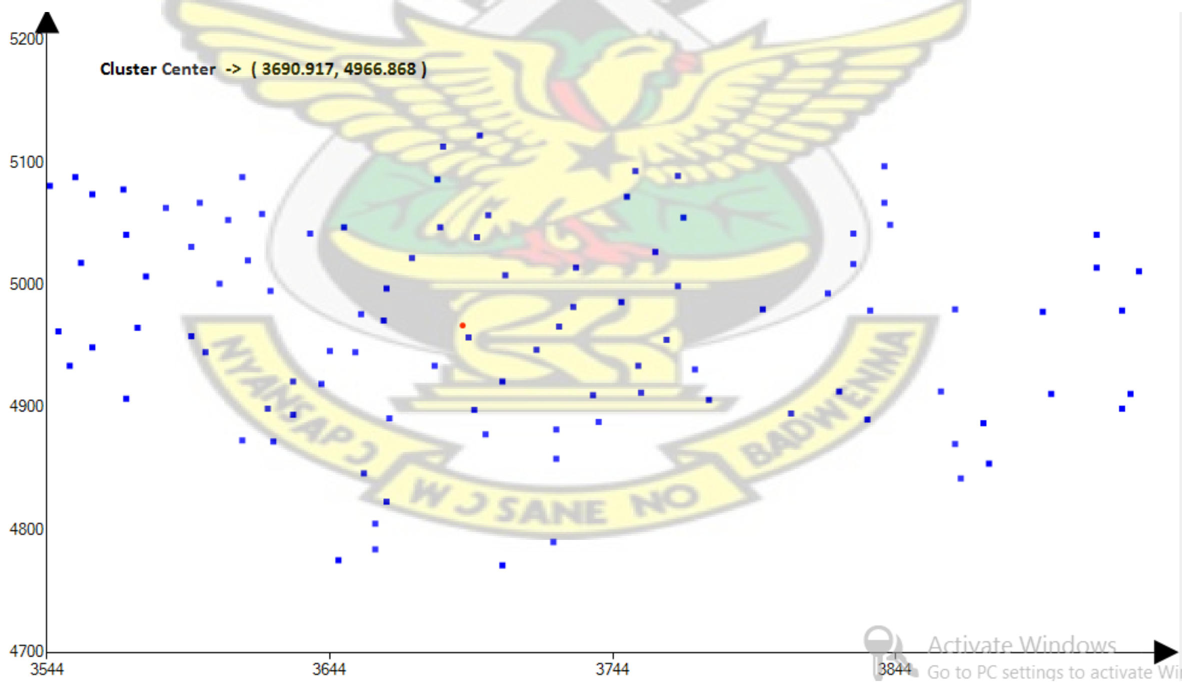


Figure B14: Cluster centre and members in zone three sub-cluster five

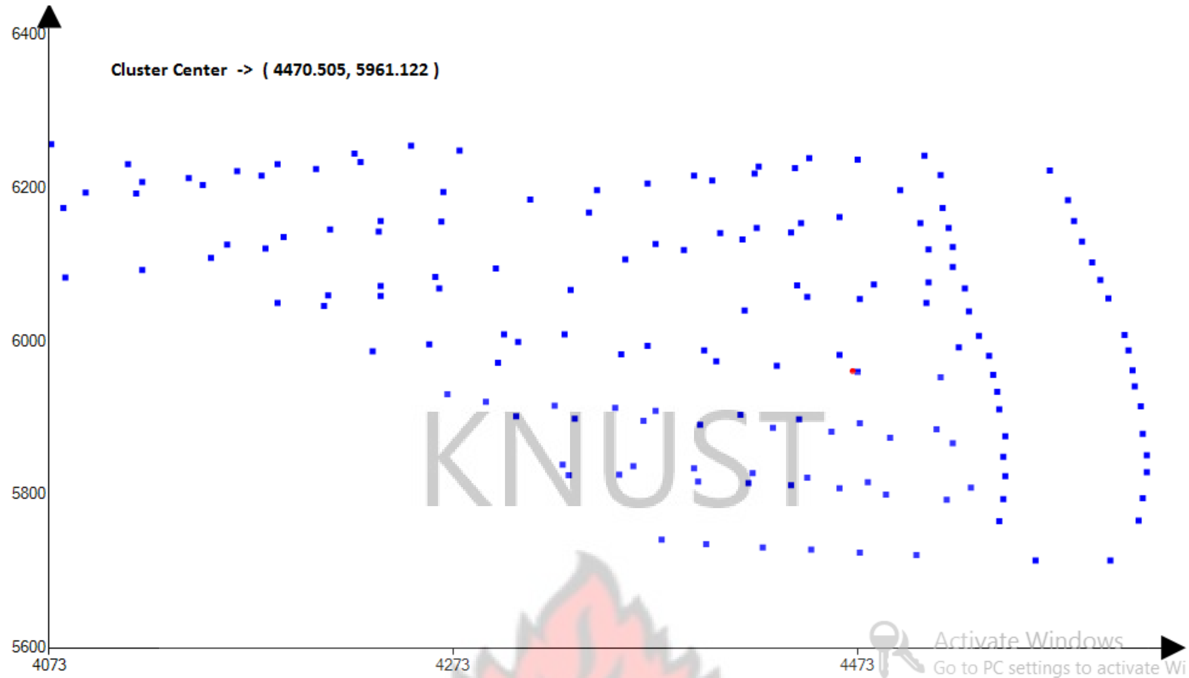


Figure B15: Cluster centre and members in zone four sub-cluster two

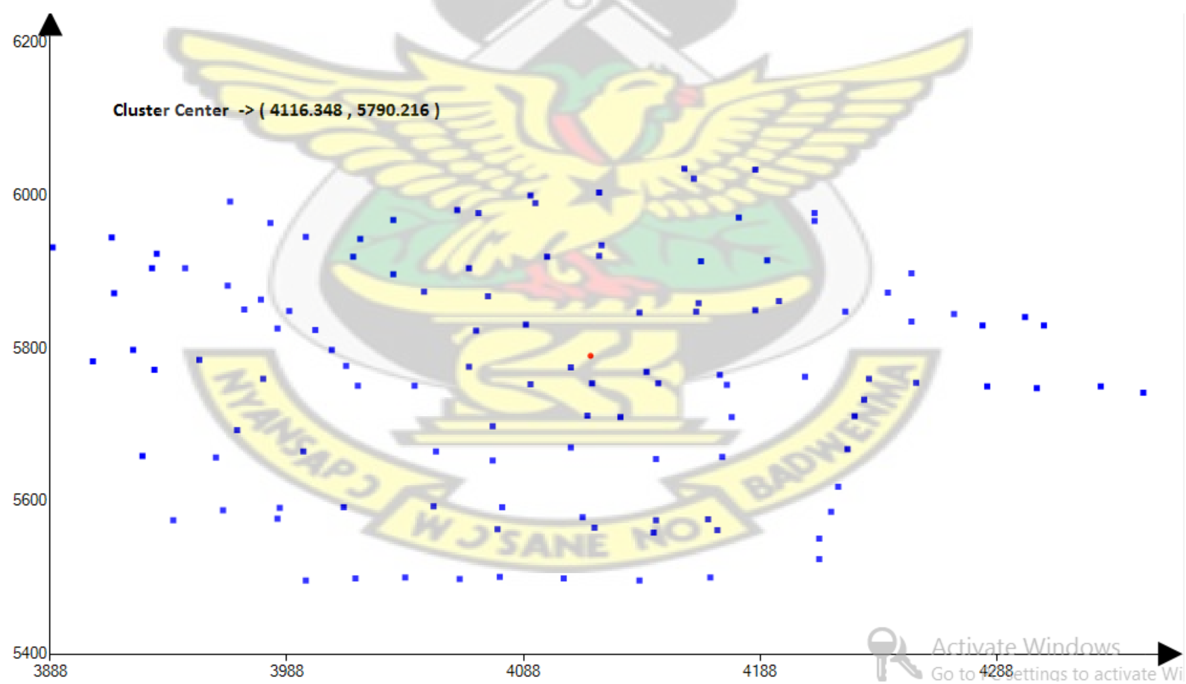


Figure B16: Cluster centre and members in zone four sub-cluster three

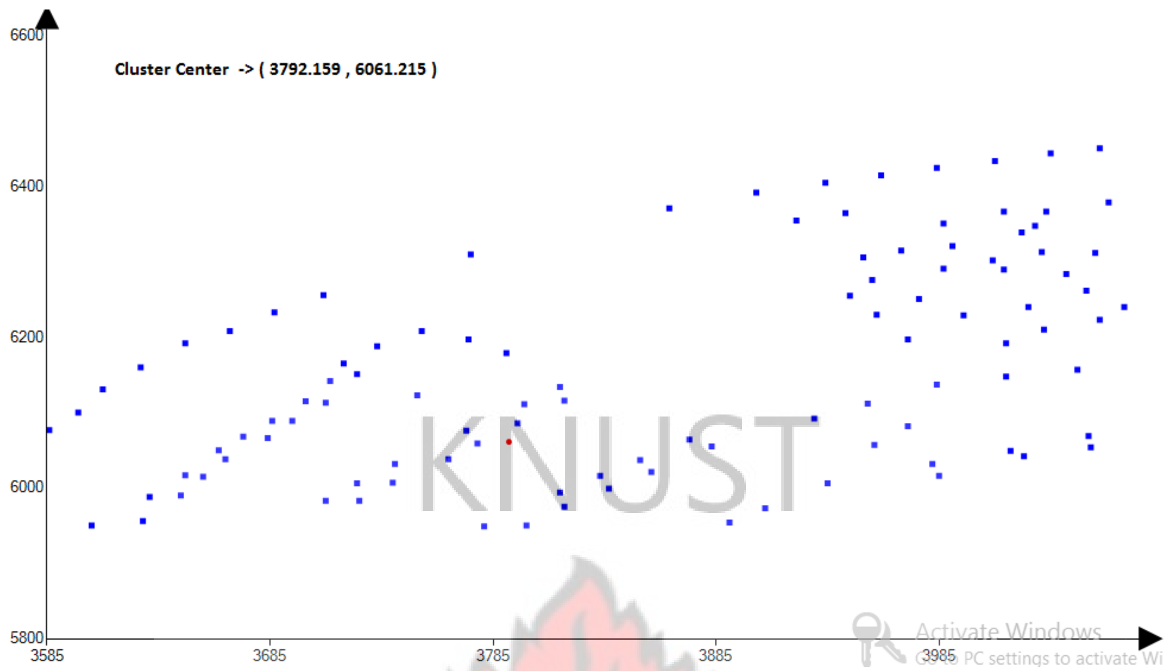


Figure B17: Cluster centre and members in zone four sub-cluster four

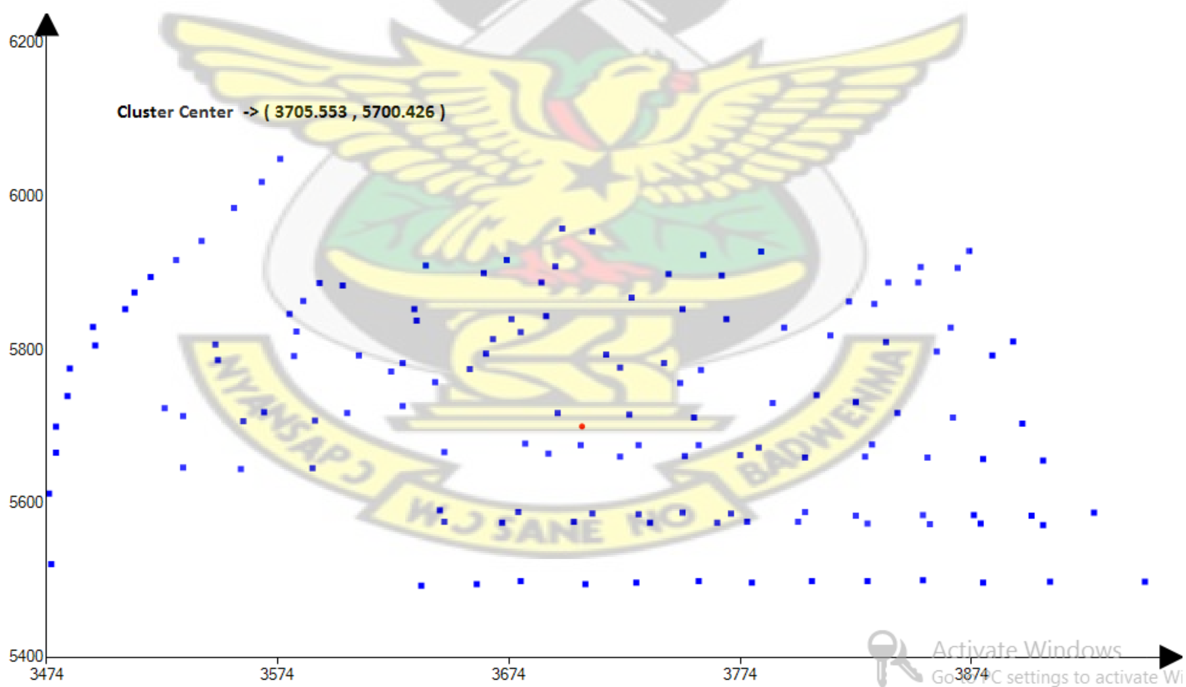


Figure B18: Cluster centre and members in zone four sub-cluster five

APPENDIX C

Appendix C gives the final Ant route obtained from each of the zones and its sub-clusters from the results obtained from our improved Ant Colony System.

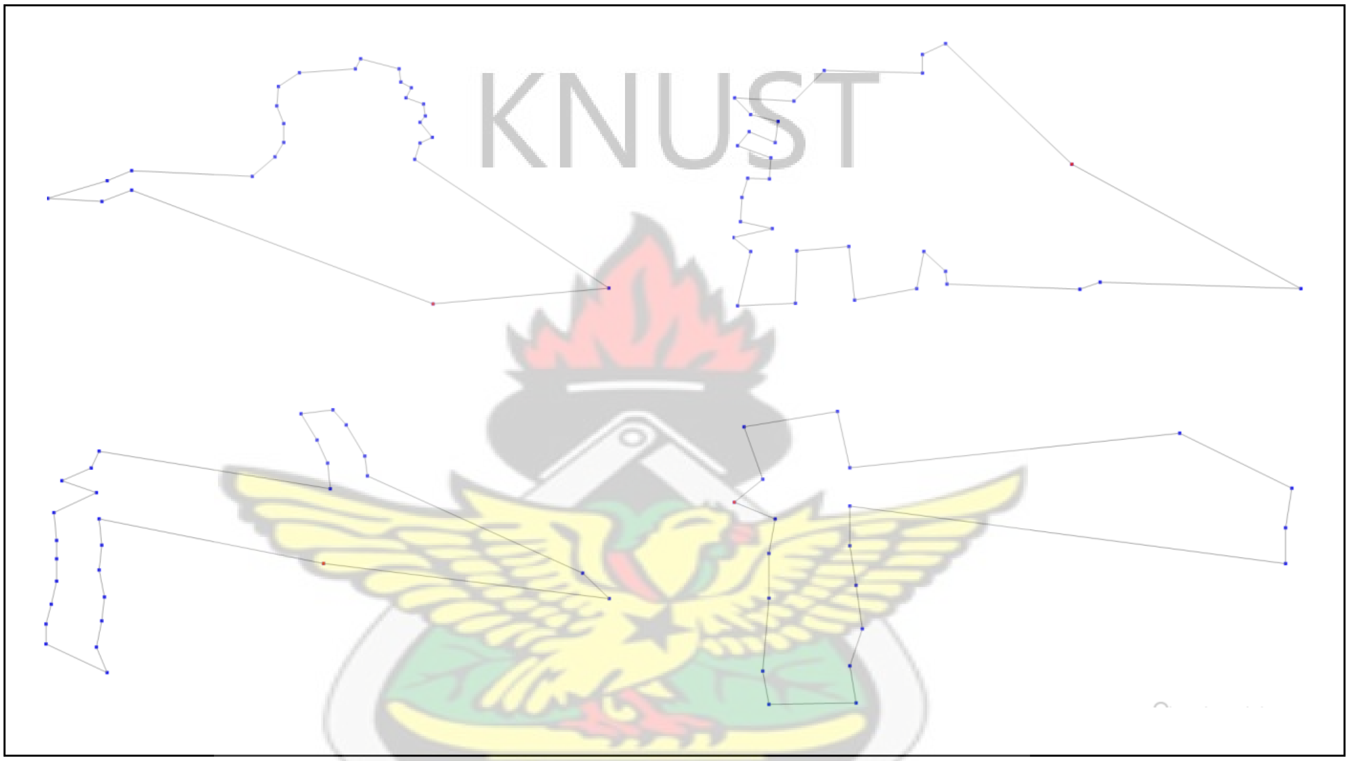


Figure C 1: Final ant route of zone one sub-cluster one

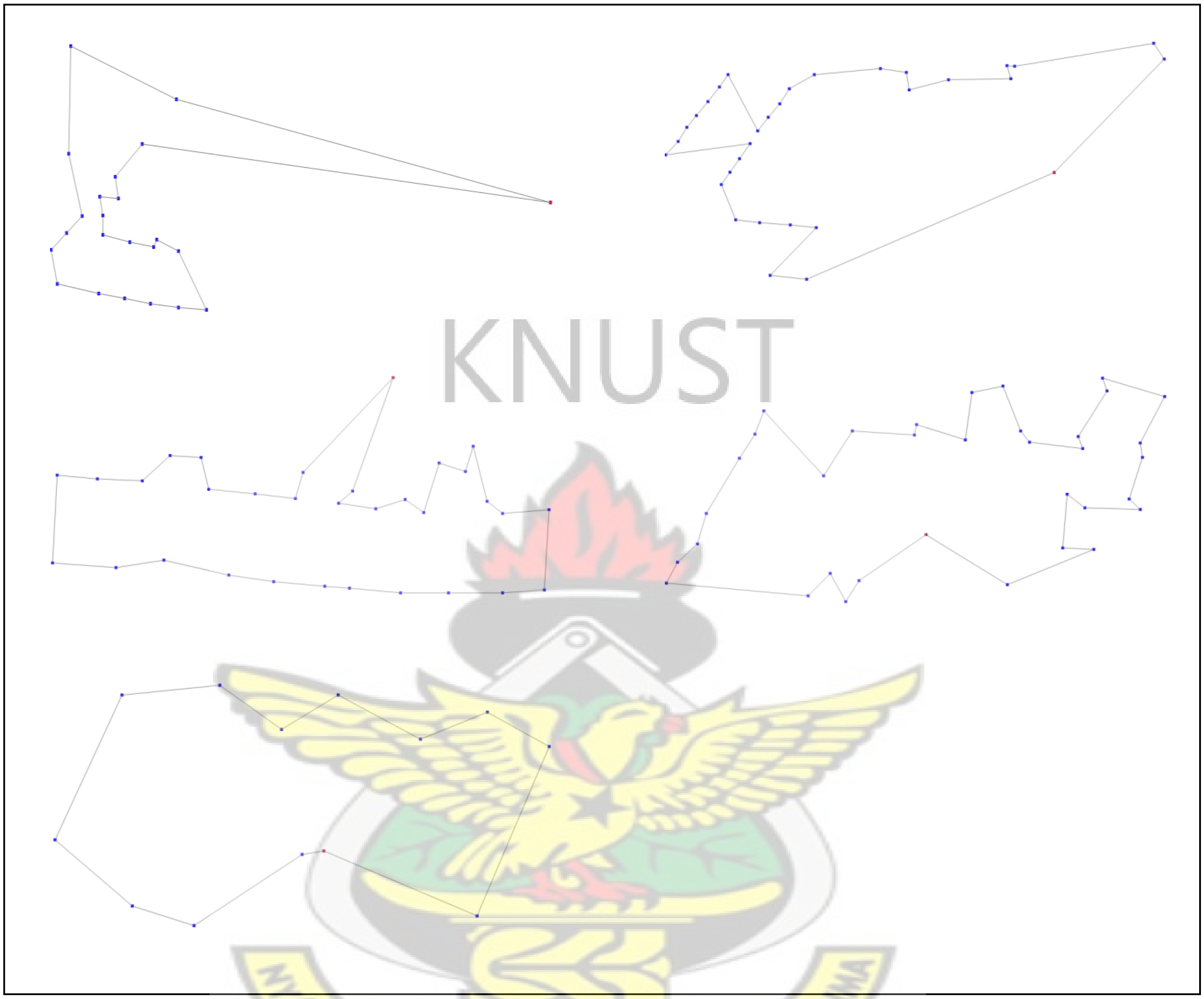


Figure C 2: Final ant route of zone one sub-cluster two

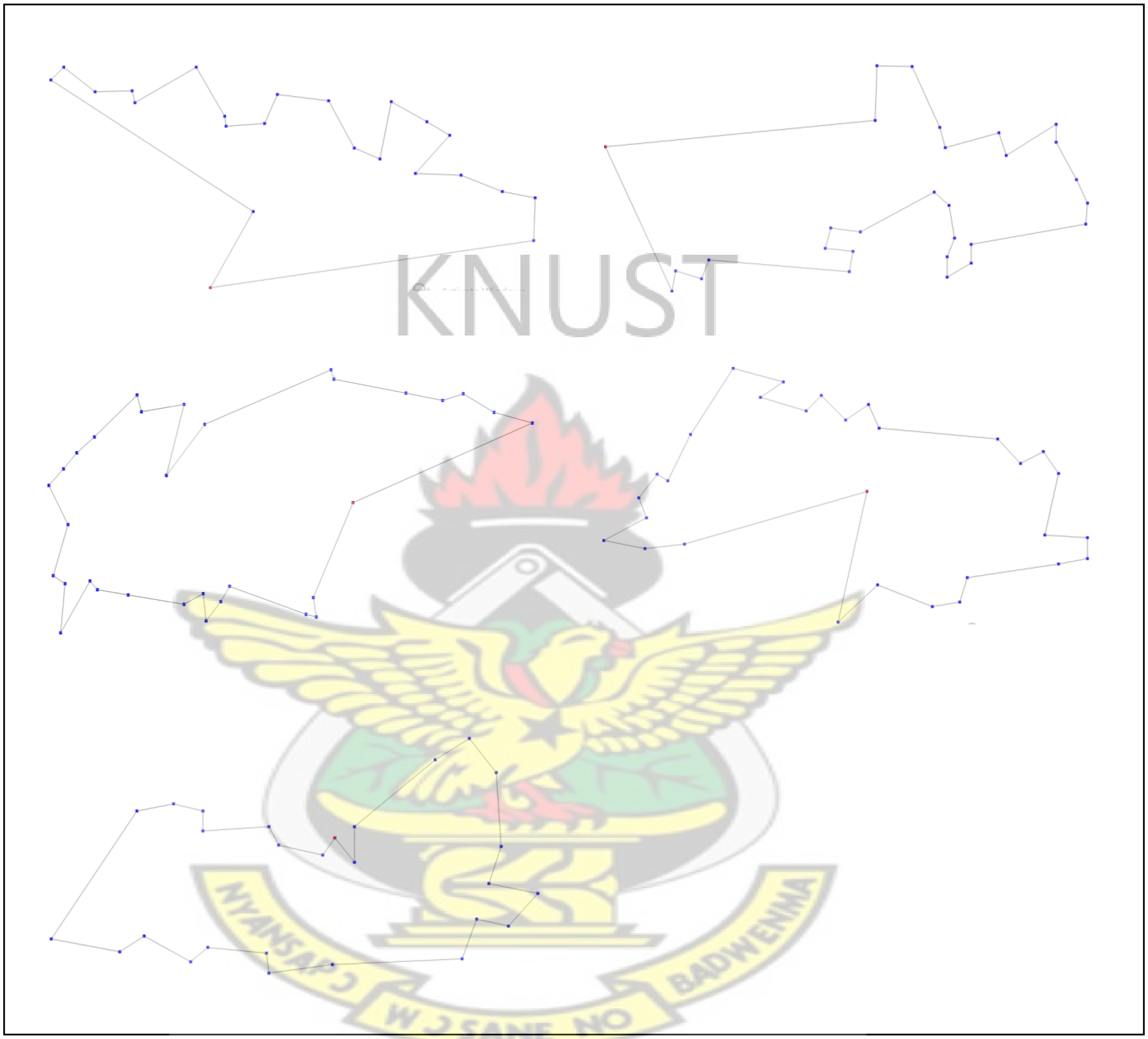


Figure C 3: Final ant route of zone one sub-cluster three

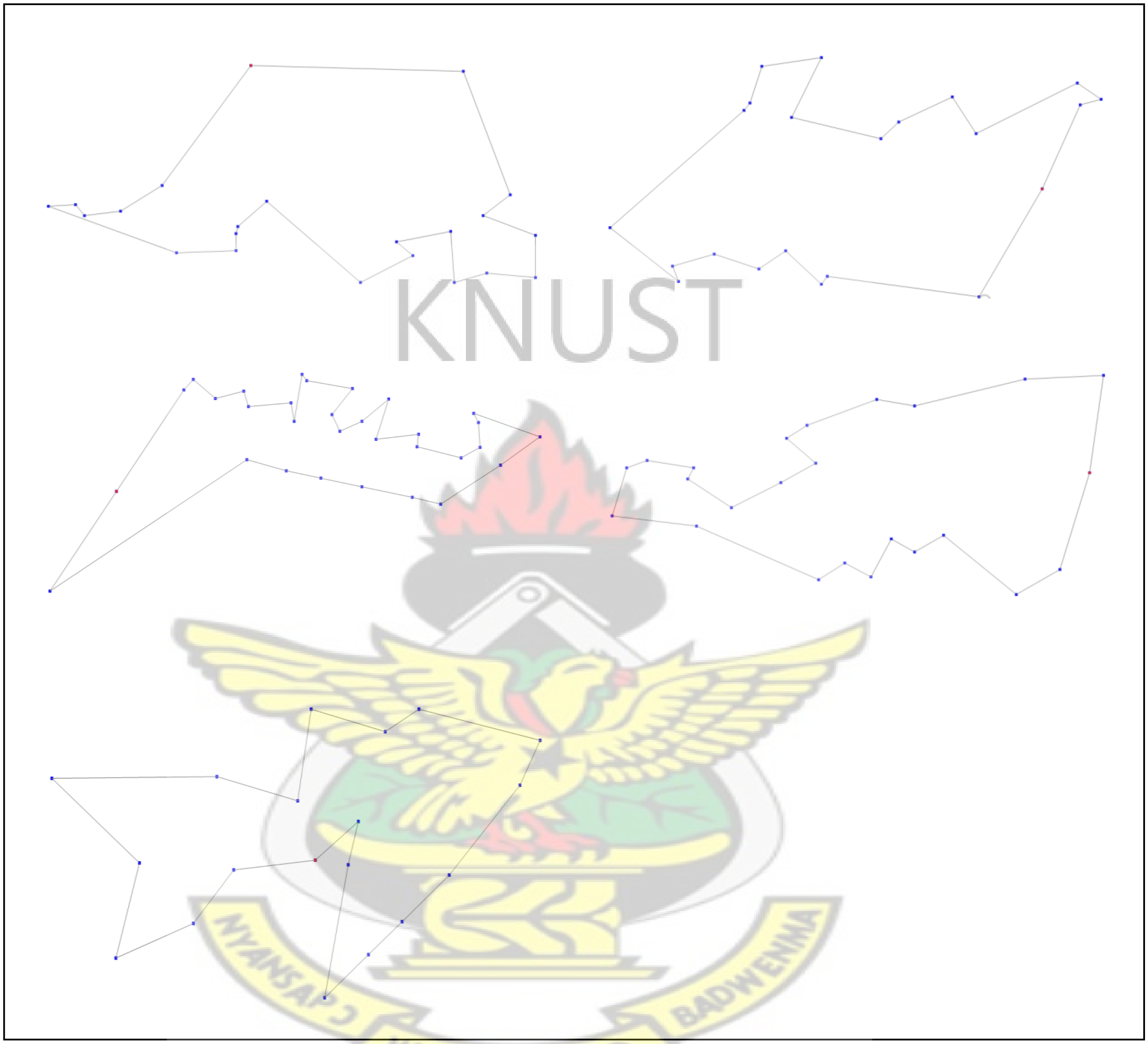


Figure C 4: Final ant route of zone one sub-cluster four

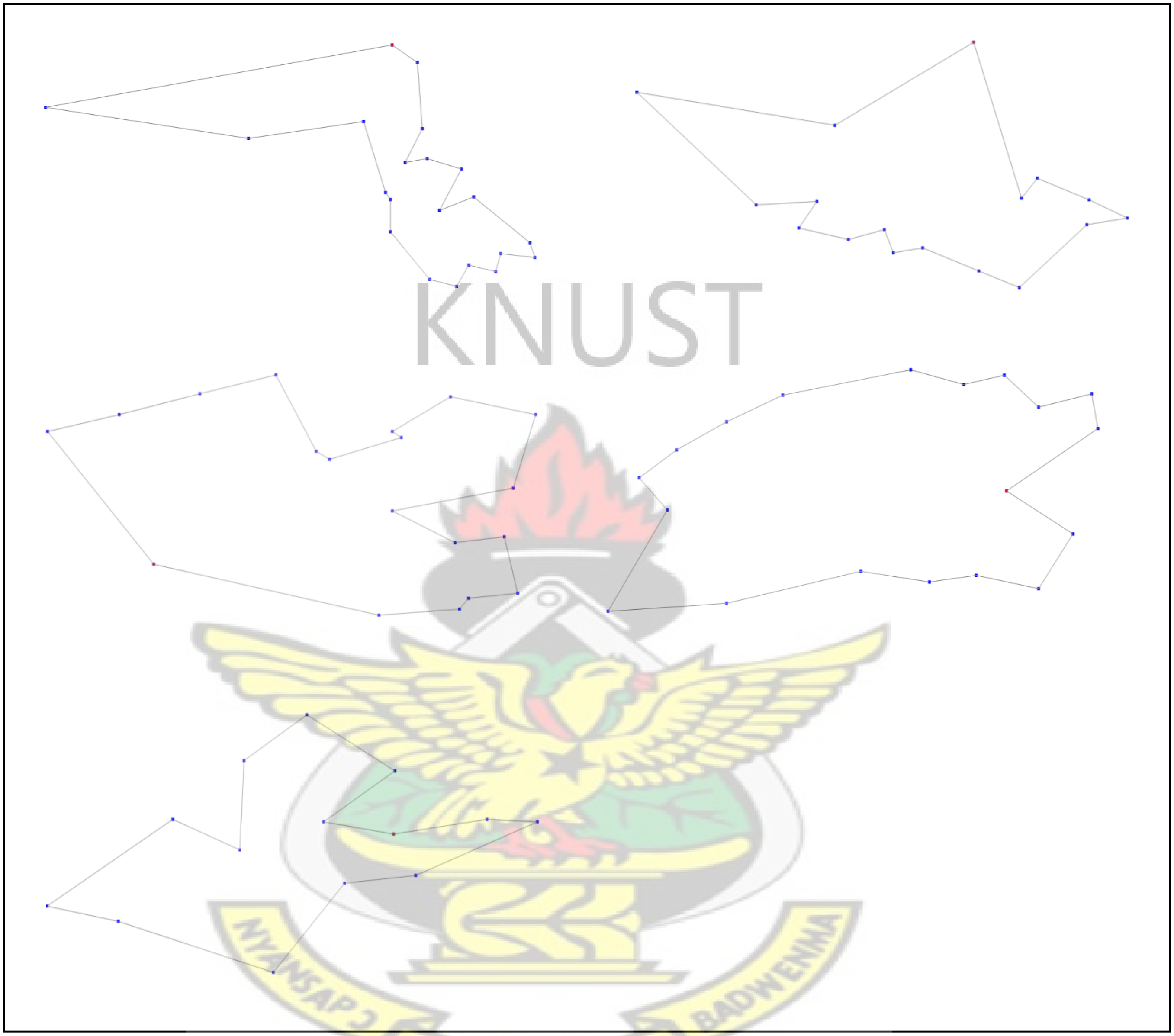


Figure D 5: **Final ant route of zone one sub-cluster five**

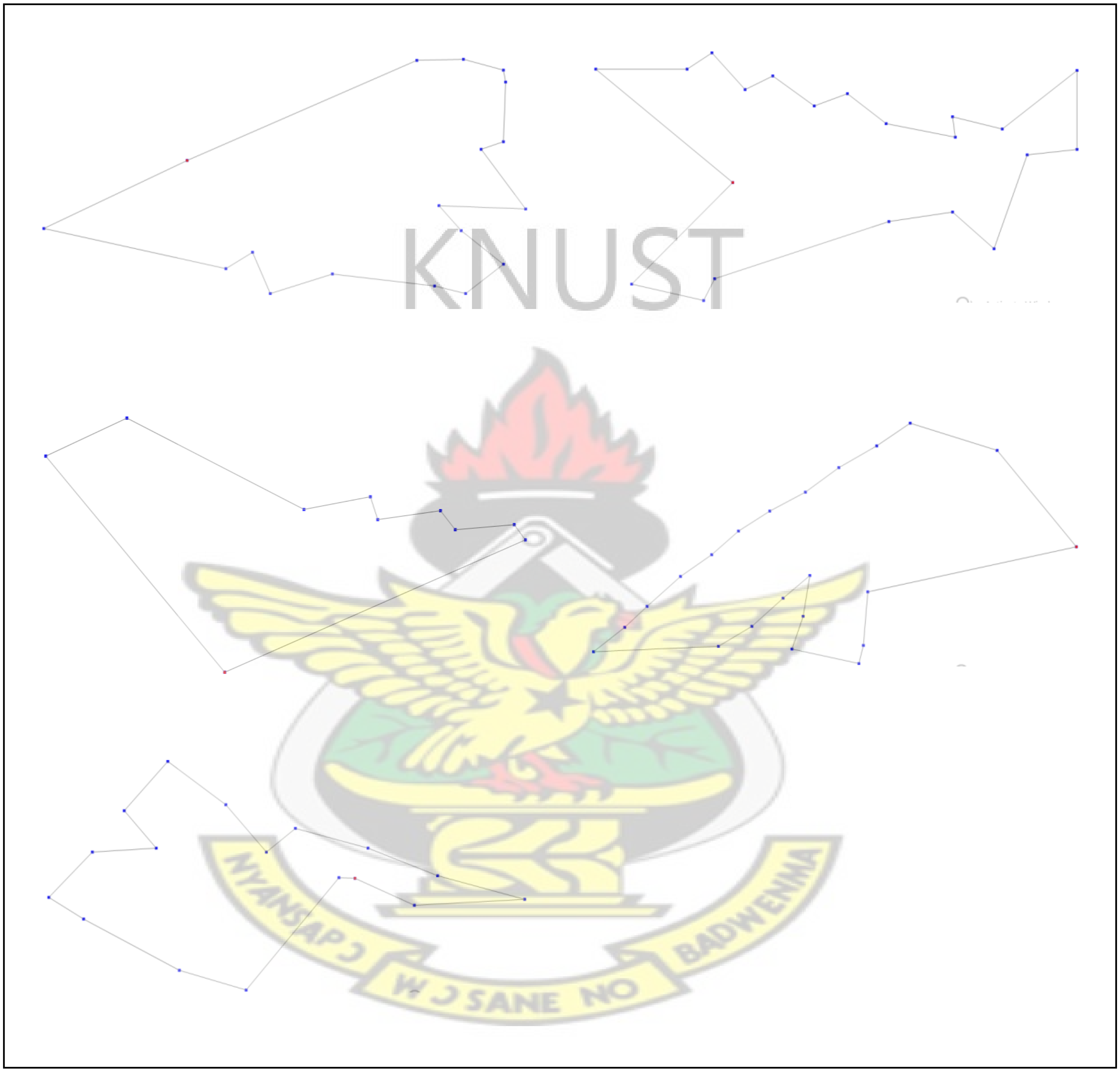


Figure C 6: Final ant route of zone one sub-cluster six

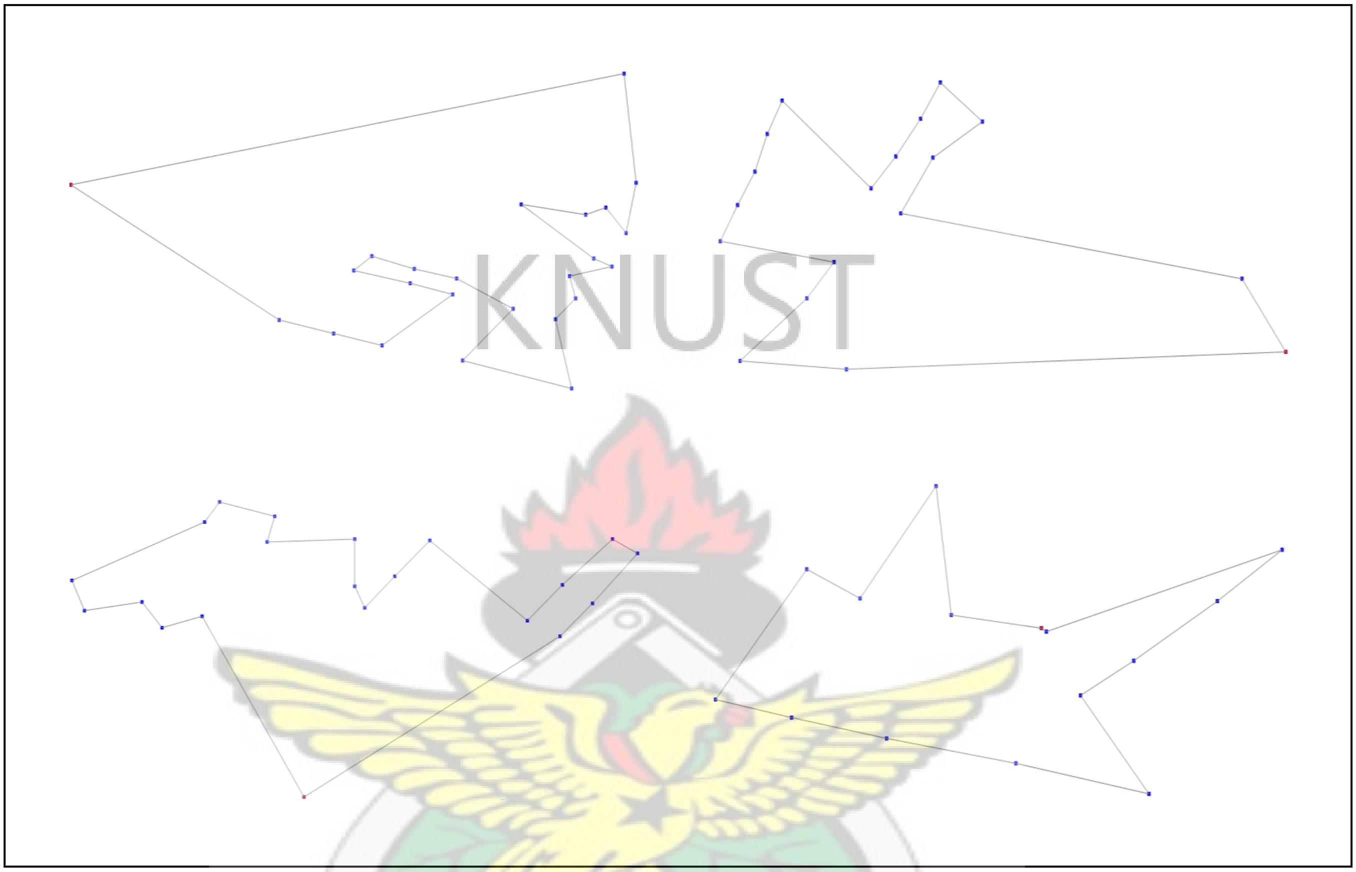


Figure C 7: Final ant route of zone one sub-cluster seven

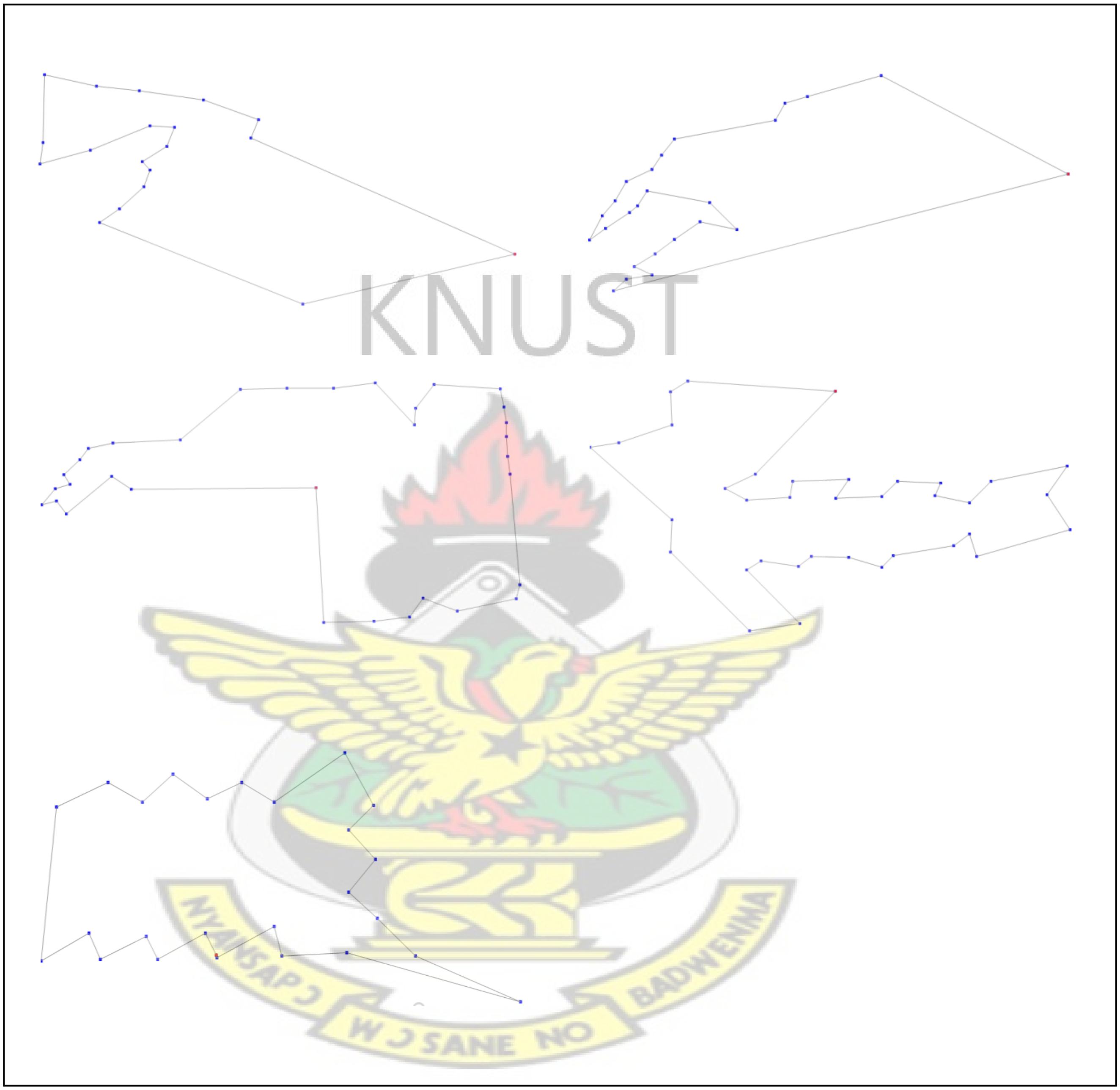


Figure C 8: Final ant route of zone two sub-cluster one

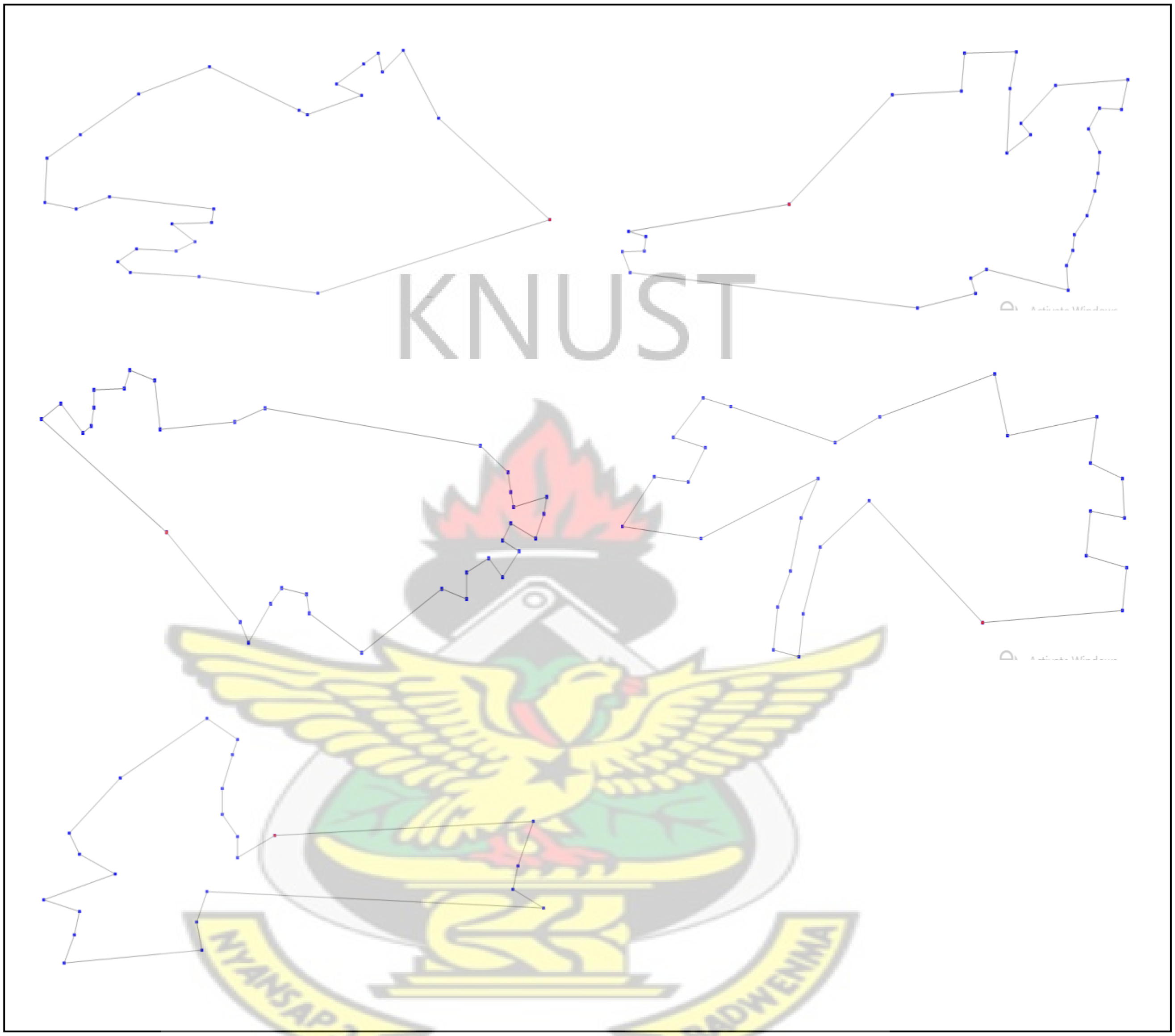


Figure C 9: Final ant route of zone two sub-cluster two

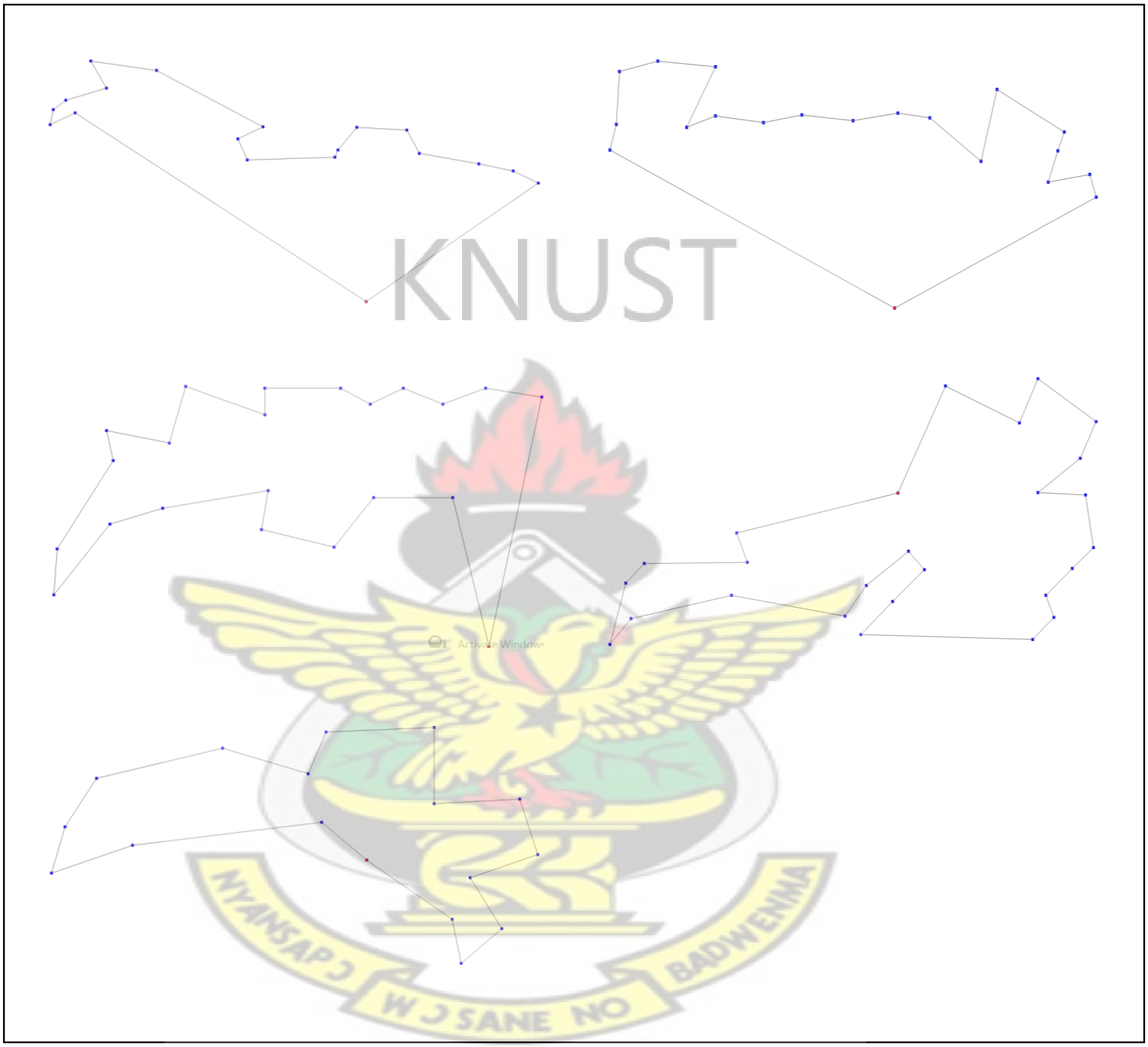


Figure C 10: Final ant route of zone two sub-cluster three

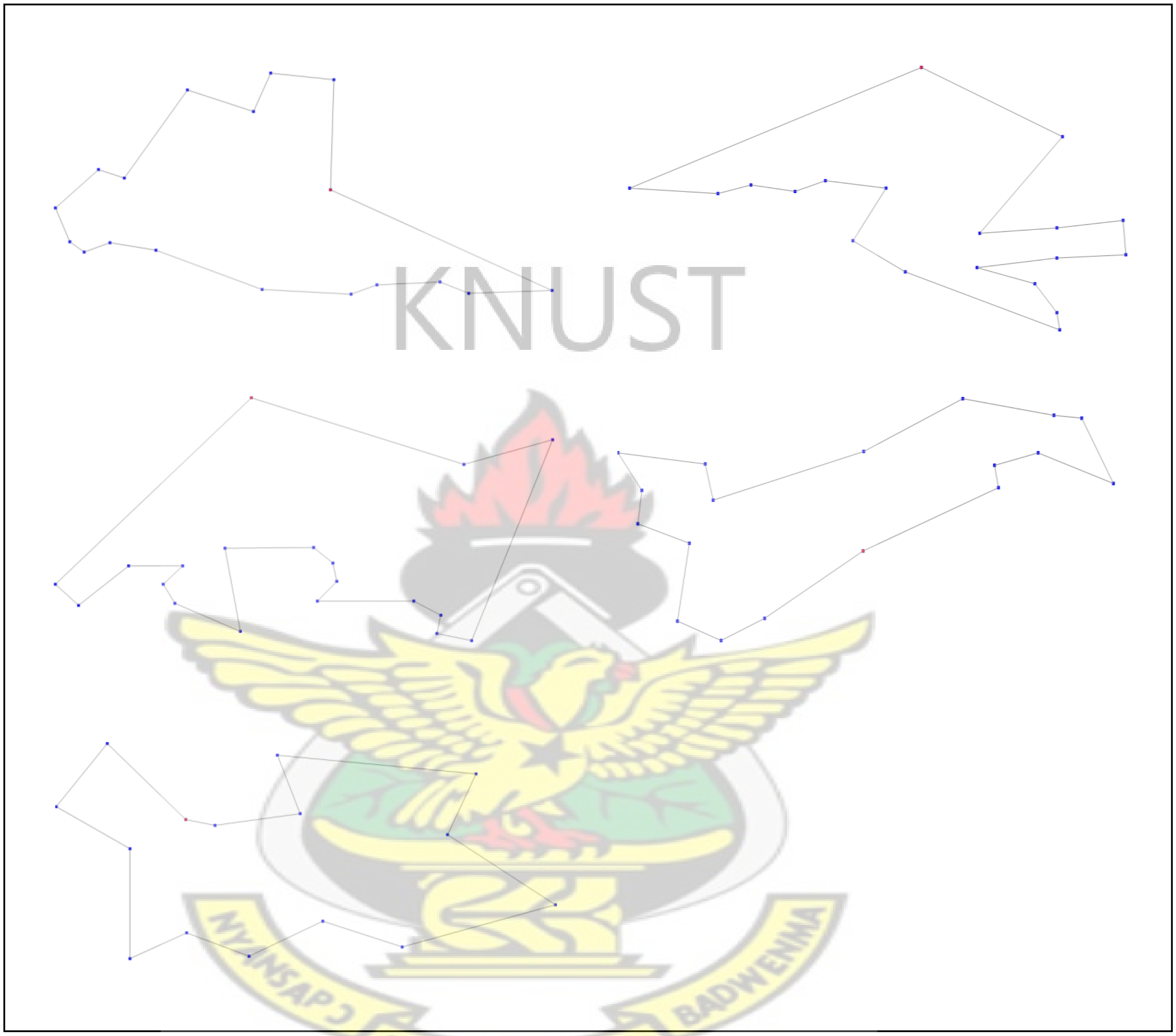


Figure C 11: Final ant route of zone two sub-cluster four

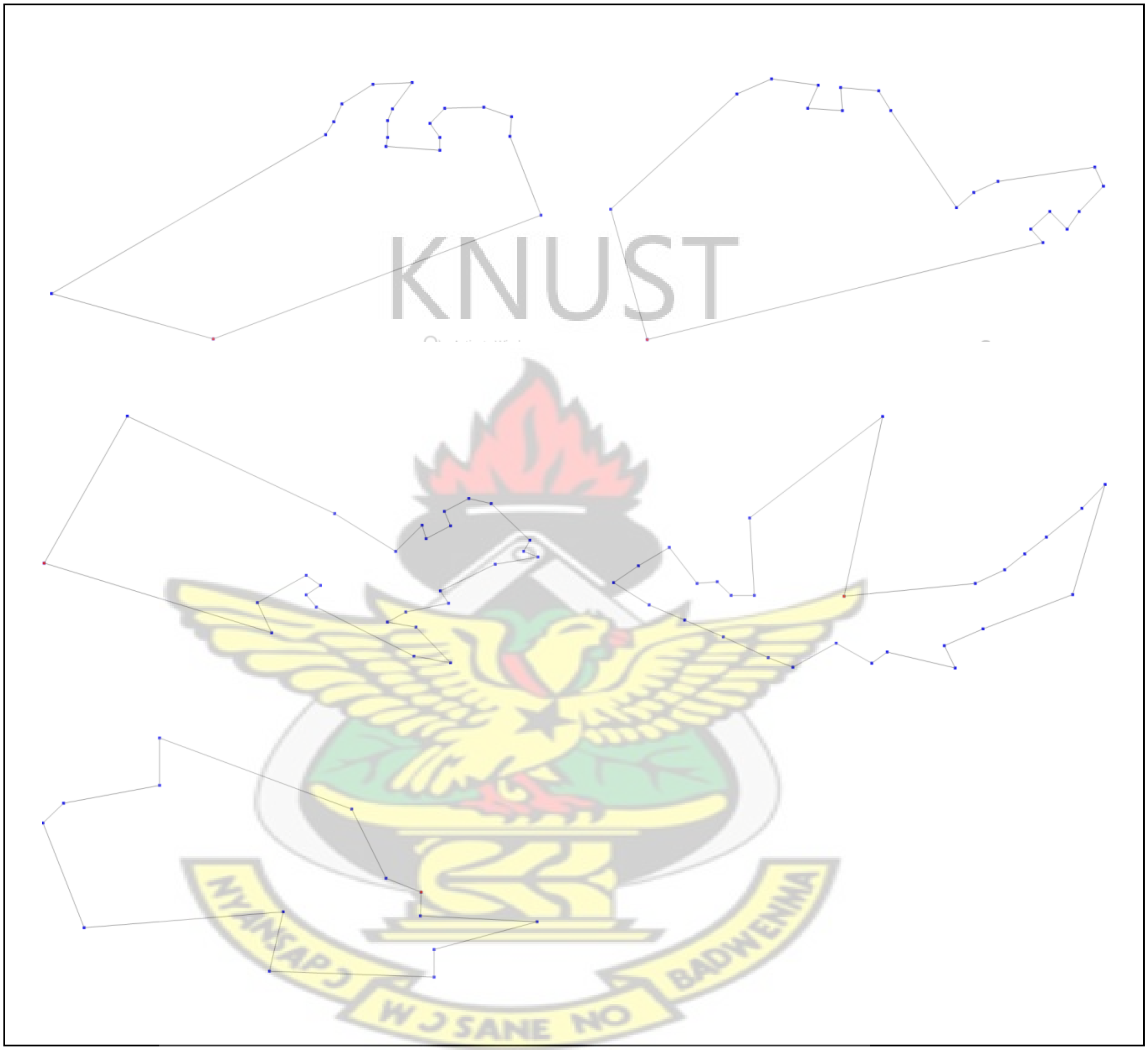


Figure C 12: Final ant route of zone two sub-cluster five

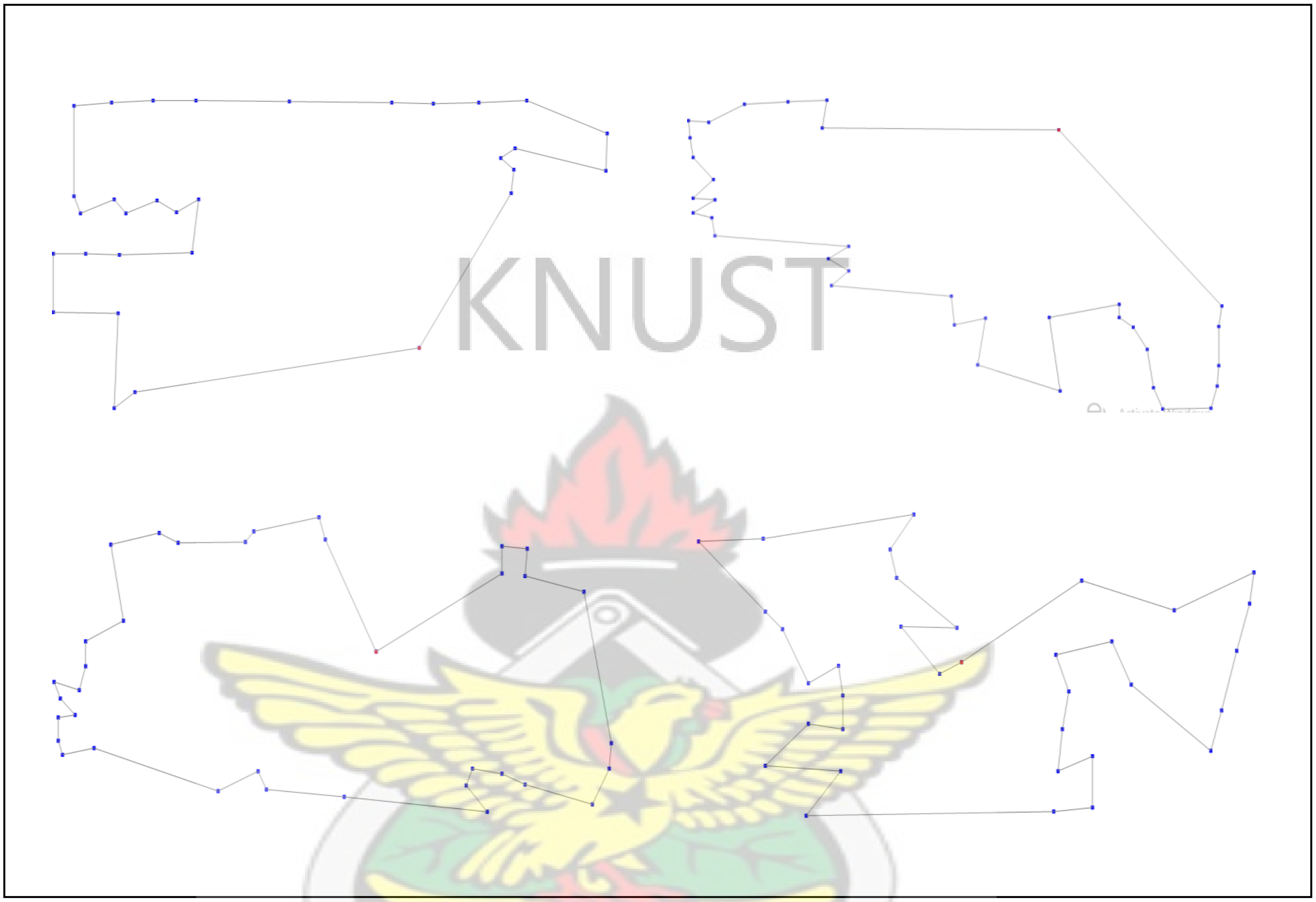


Figure C 13: Final ant route of zone three sub-cluster one

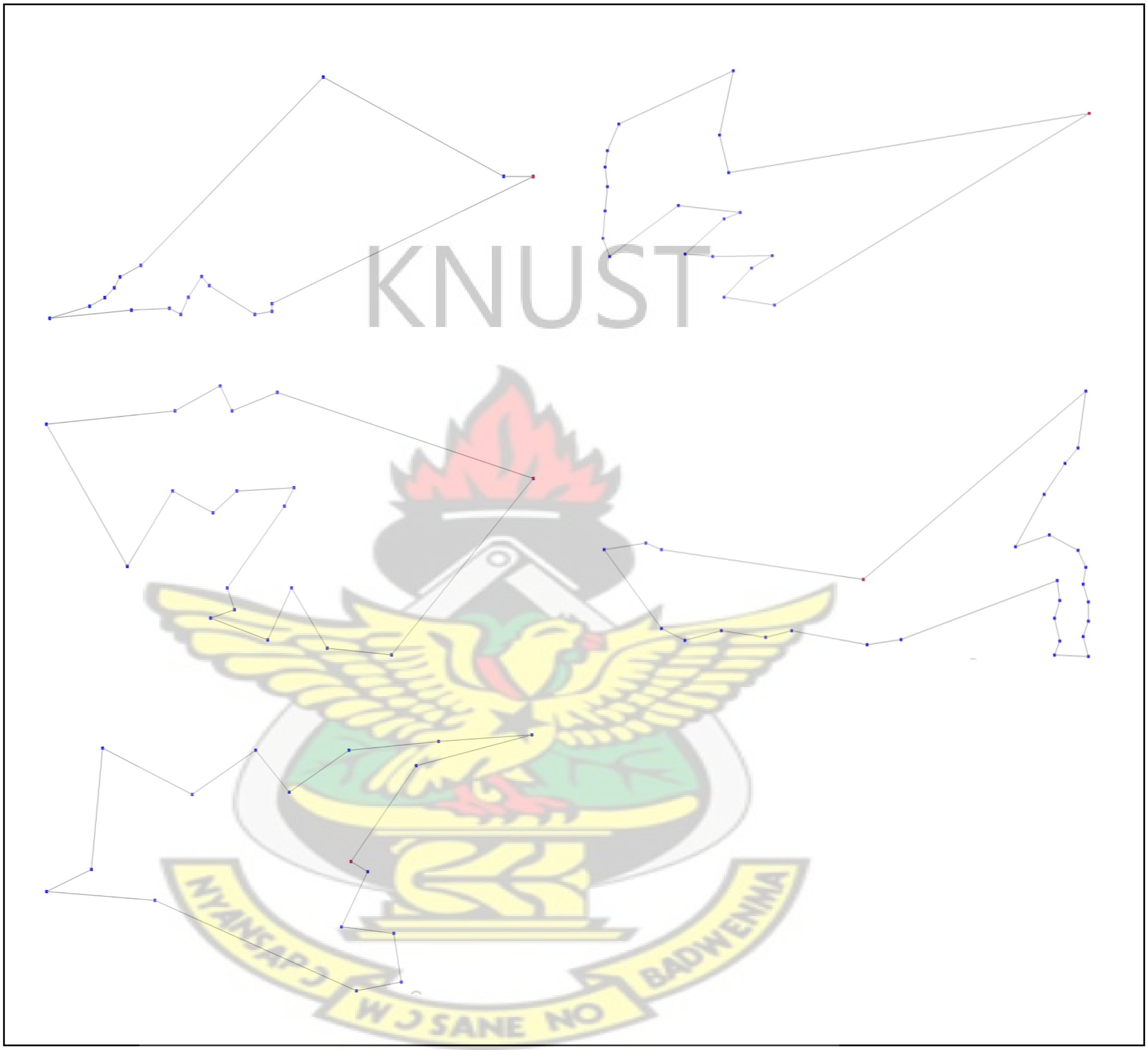


Figure C 14: Final ant route of zone three sub-cluster two

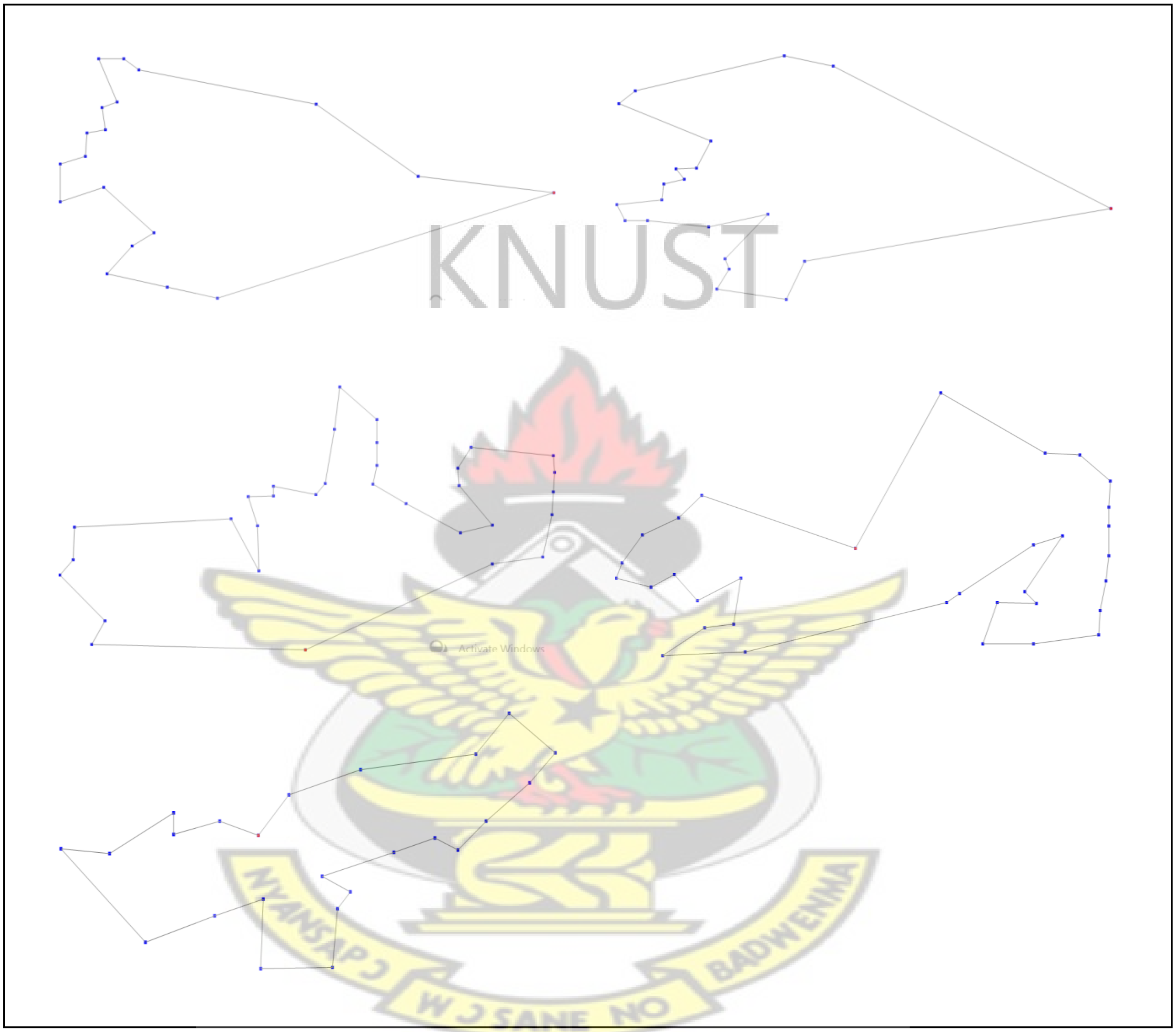


Figure C 15: Final ant route of zone three sub-cluster three

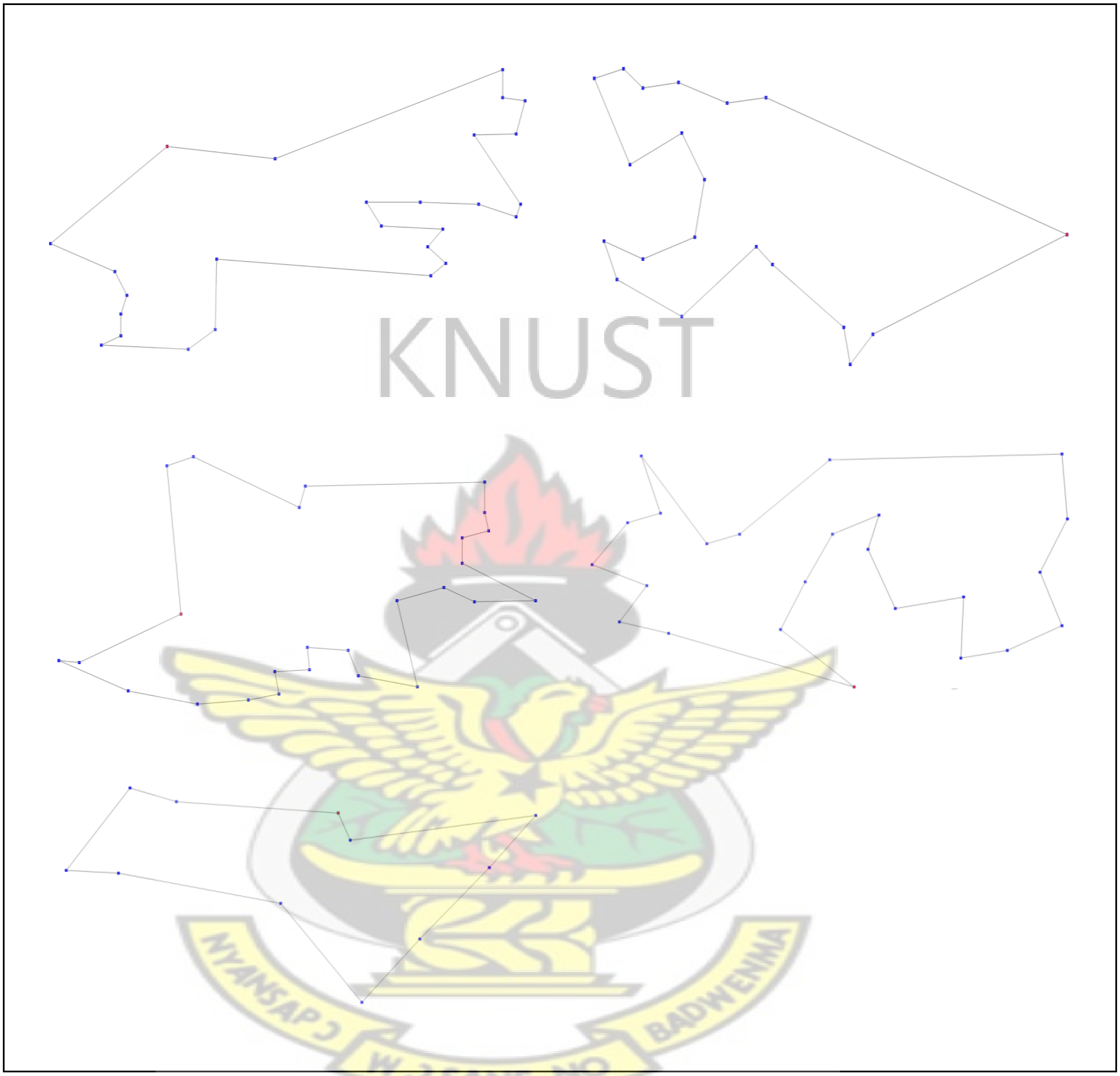


Figure C 16: Final ant route of zone three sub-cluster four

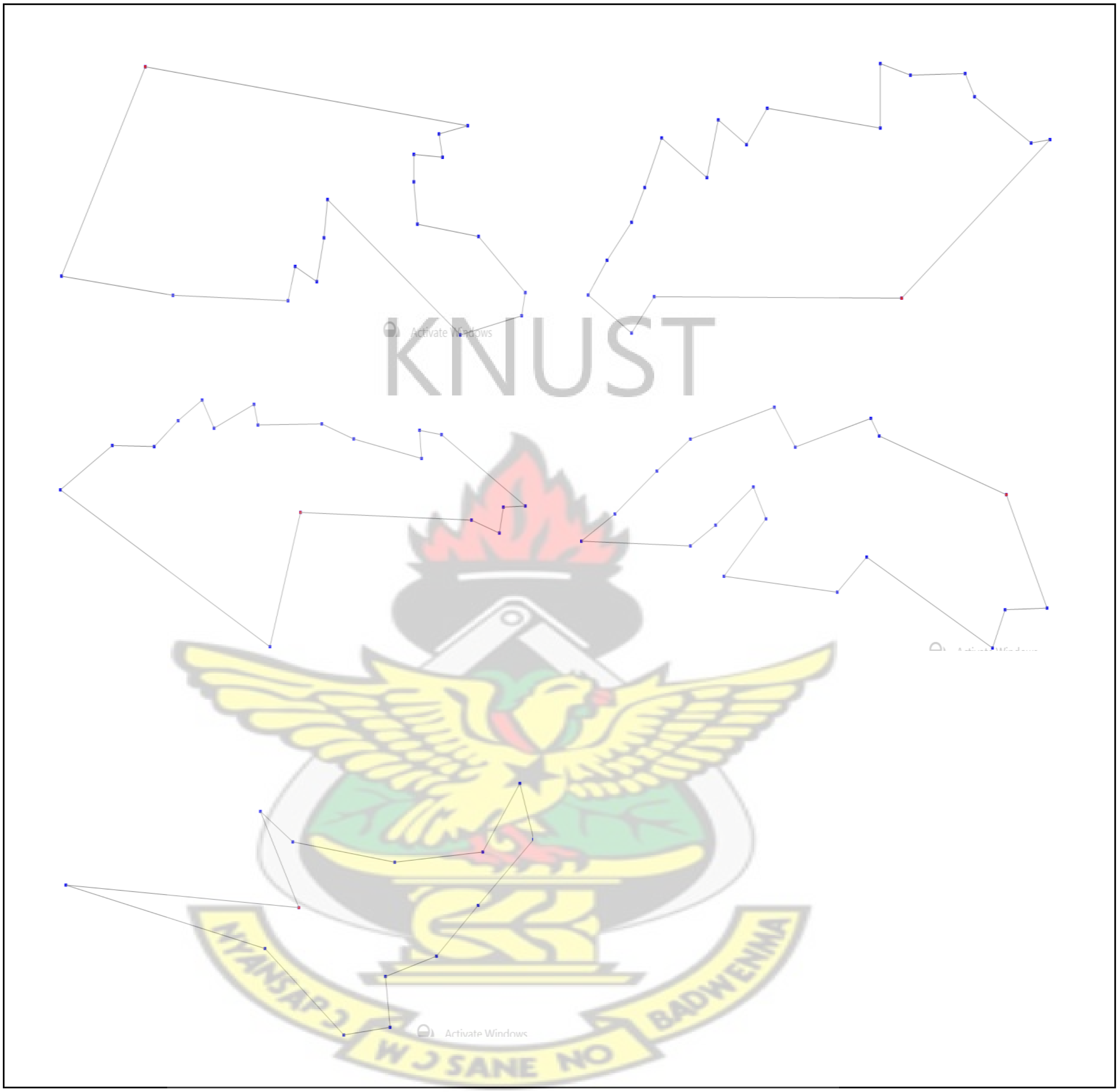


Figure C 17: Final ant route of zone three sub-cluster five

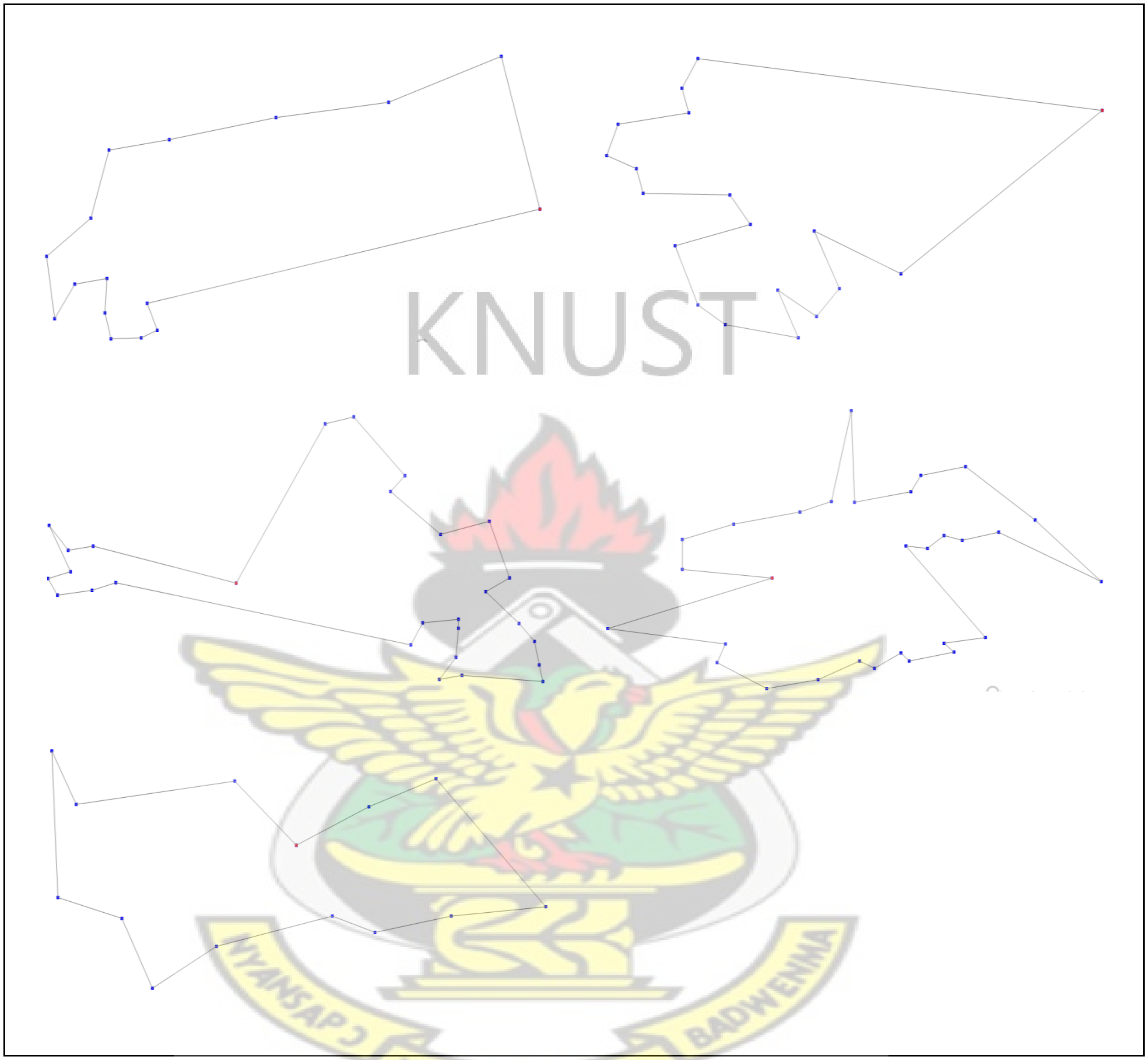


Figure C 18: Final ant route of zone four sub-cluster one

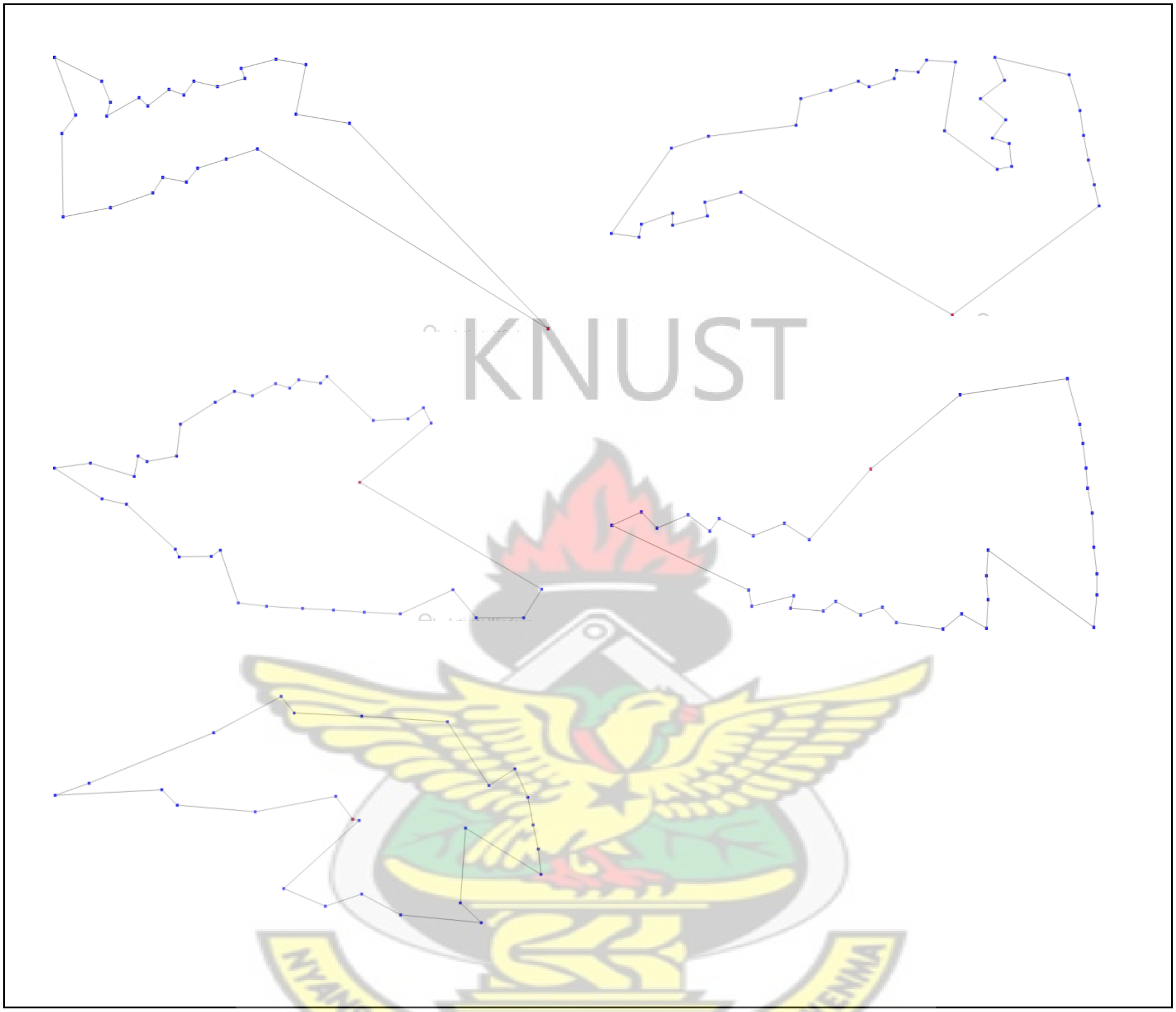


Figure C 19: **Final ant route of zone four sub-cluster two**

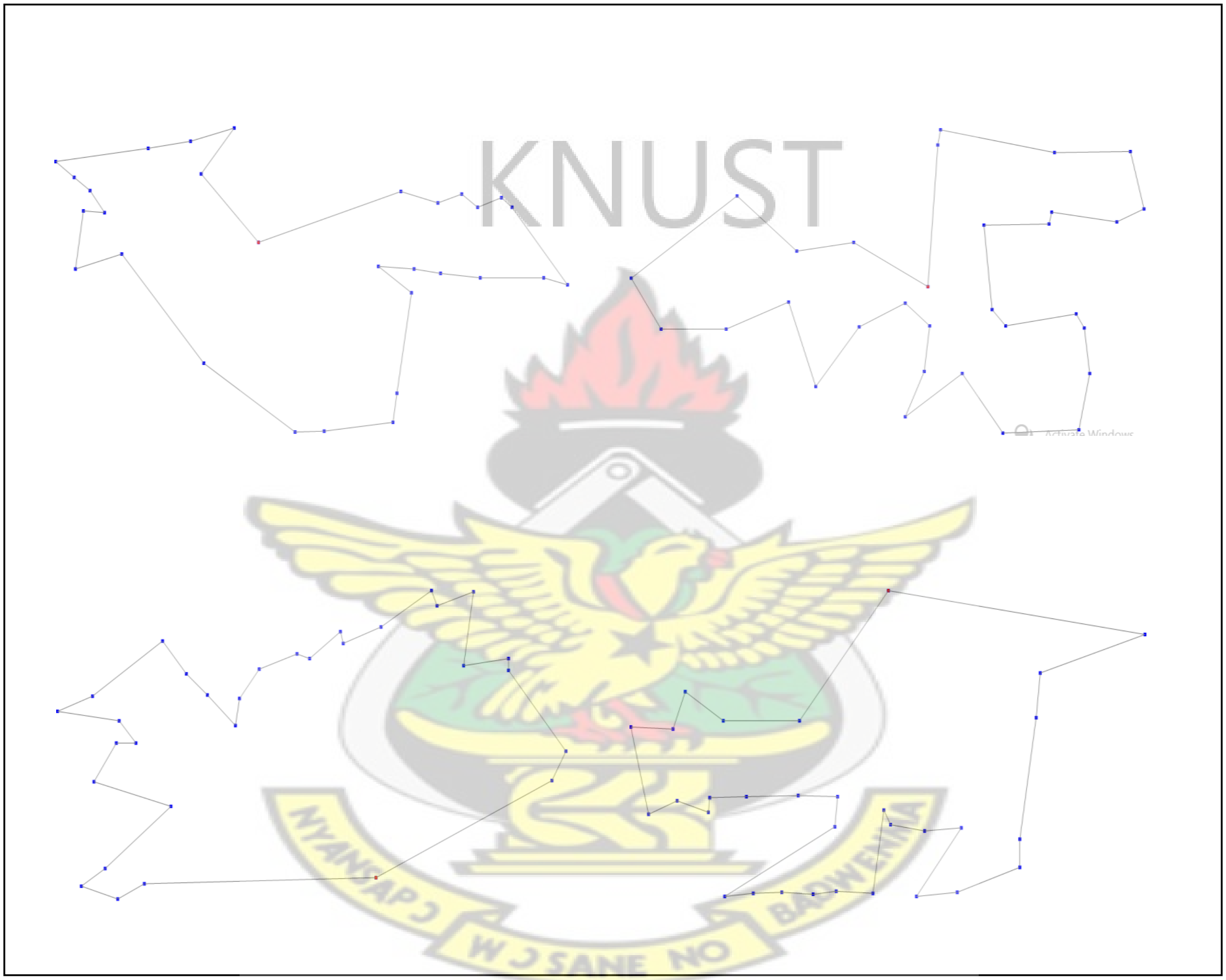


Figure C 19: Final ant route of zone four sub-cluster three

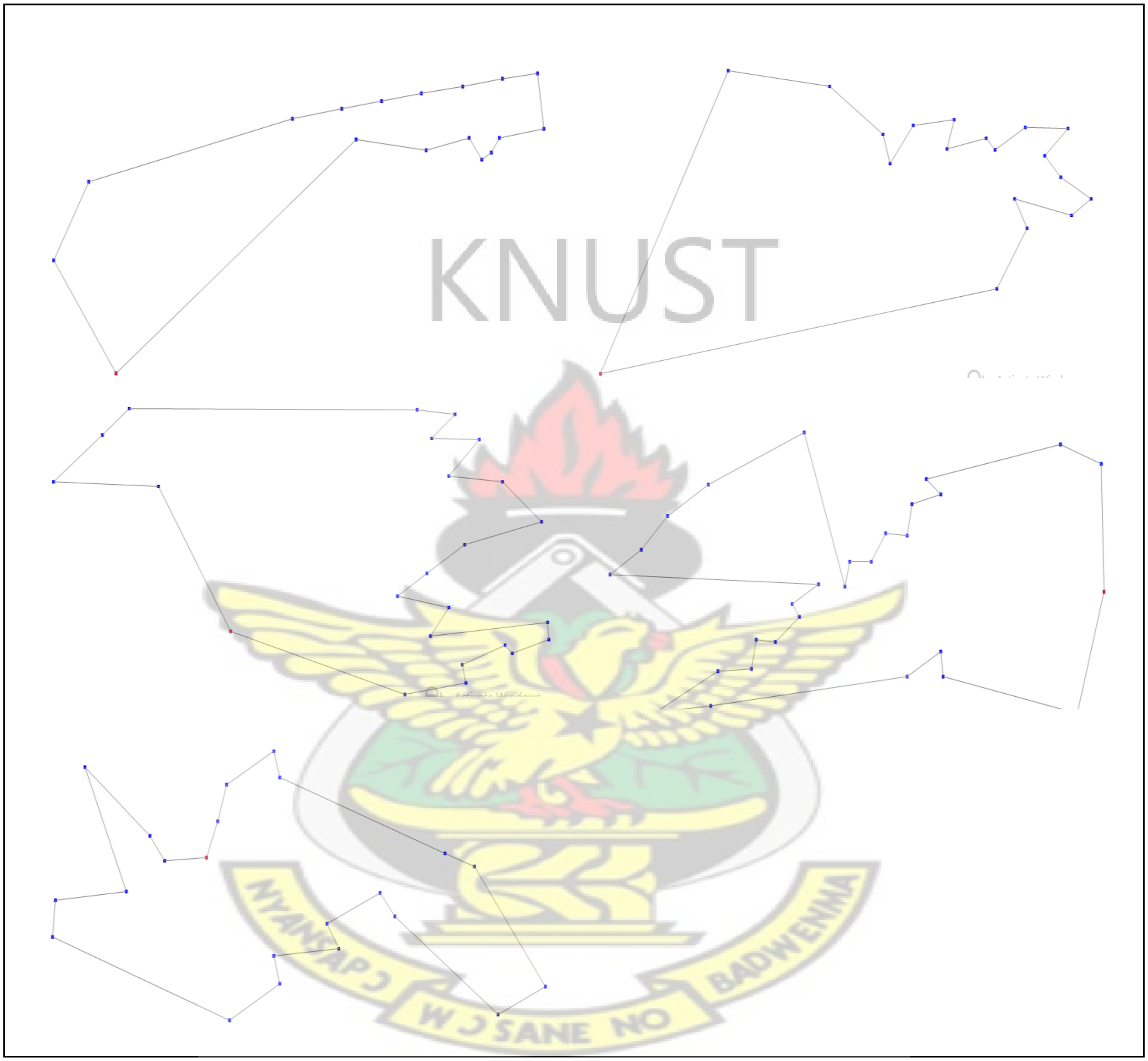


Figure C 20: Final ant route of zone four sub-cluster four

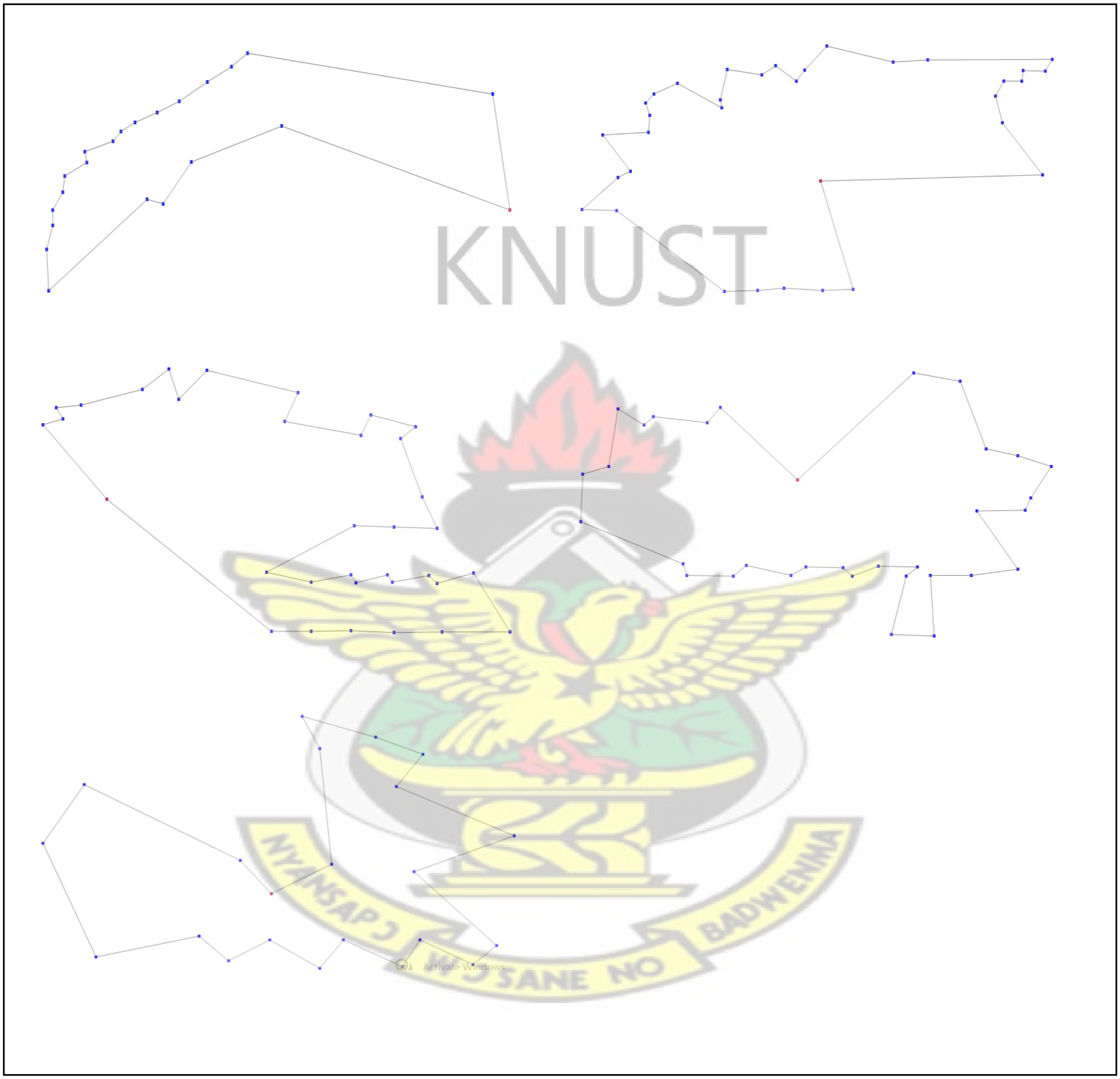
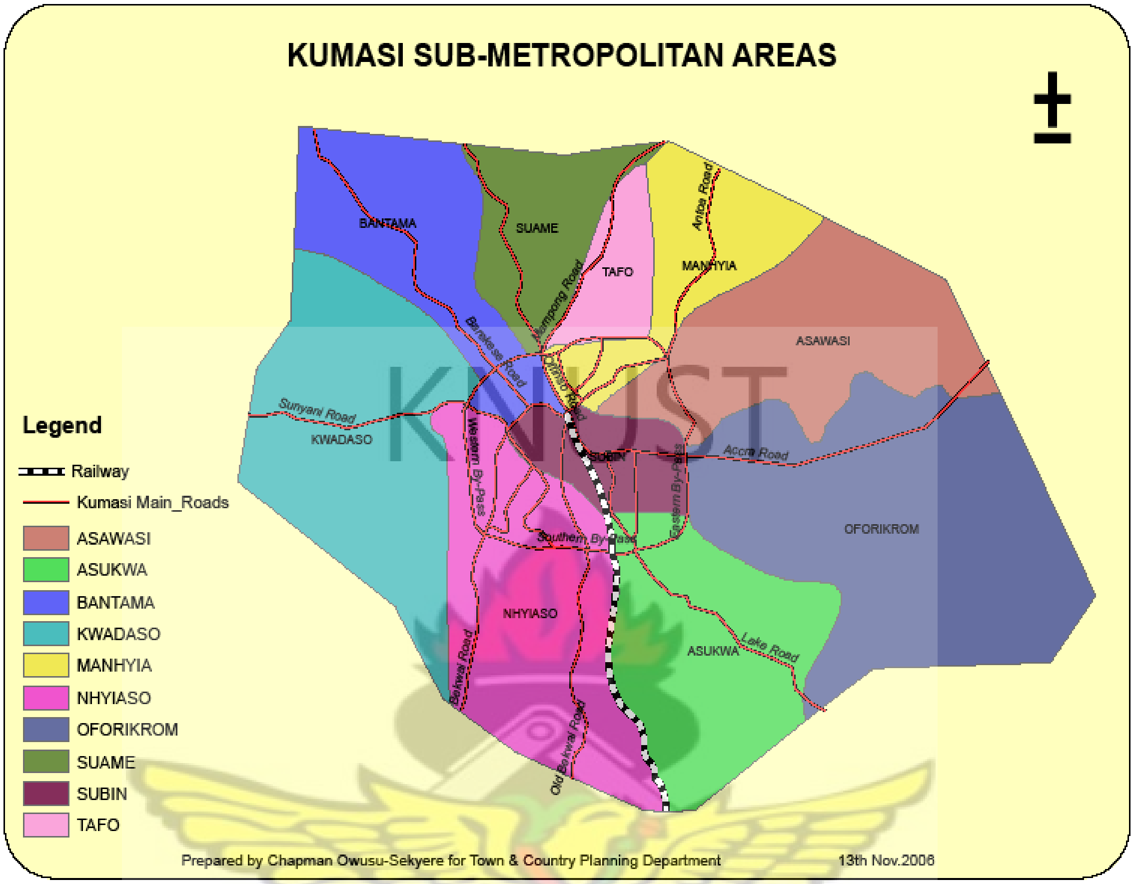


Figure C 21: Final ant route of zone four sub-cluster five



A tricycle for collecting waste for customers

A skip container located at a transfer depot to receive waste from tricycle

RESEARCH MATERIAL SUBMITTED FOR PUBLICATION

This appendix contains information relating to the research material submitted for journal publication by the author.

- A paper entitled "**Analysis of solid waste generation per person per day using mixed effect models**" This is currently under review. Material presented based on chapters three and four sections one.

- A paper entitled "**Location of semi-obnoxious facility using probabilistic distance location model**" This is currently under review. Material presented based on chapter three and four sections two.

- A paper entitled "**A real world application of Vehicle Routing Problem in solid waste collection using Improved Ant Colony System**" This is currently under review. Material presented based on chapters three and four sections three.

- A paper entitled "**Capacitated vehicle routing problem with time windows, stopping time, dead heading time and effect of smell**" This is currently under review. Material presented based on chapters three and four sections three.

- A paper entitled "**Comparative vehicle fuel consumption analysis on two elemental time schedules**" This is currently under review. Material presented based on chapters three and four sections four.