SHORT-TERM TRAFFIC VOLUME PREDICTION IN UMTS NETWORKS: VALIDATION OF KALMAN FILTER-BASED MODEL.

by

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Certification

I hereby declare that this submission is my own work towards the MSc and that, to the best of my knowledge, it contains no material previously published by another person nor material which has been accepted for the award of any other degree of the University, except where due acknowledgment has been made in the text.

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Abstract

Accurate traffic volume prediction in Universal Mobile Telecommunication System (UMTS) networks has become increasingly important because of its vital role in determining the Quality of Service (QoS) received by subscribers on these networks. This study explores traffic volume prediction and, adapts and validates the Kalman filter-based short-term traffic volume prediction model for UMTS networks. In this study, we adapt and validate the Kalman filter-based traffic volume prediction model which is used more in transportation engineering.

The model was adapted based on two key assumptions that make it possible for us to characterize the short-term traffic volume patterns for UMTS networks to suit the Kalman filter algorithm. The model so adapted was carefully fine-tuned and implemented in MATLAB. The model was then validated with traffic volume data collected from a live 3G network using the graphical and r^2 (coefficient of determination) approaches to model validation.

The results indicate that the model performs very well as the predicted traffic volumes compare very closely with the observed traffic volumes on the graphs. The r^2 approach resulted in r^2 values in the range of **0.87** to **0.99** which compare very well with the observed traffic volumes. A little was done on sensitivity analysis of the model parameters, and this has been recommended for future research. The result obtained in this study brings out the fact that, the Kalman filter algorithm is very useful in predicting short-term traffic volumes for UMTS networks.

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List of Abbreviations

2G	Second Generation
3G	Third Generation
3GPP	Third Generation Partnership Project
AR	Auto Regressive
ARMA	Auto Regressive Moving Average
ARIMA	Auto Regressive Integrated Moving Average
BSC	Base Station Controller
BSS	Base Station Subsystem
BST	Base Station Transceiver
CDMA	Code Division Multiple Access
CN	Core Network
DHR	Dynamic Harmonic Regression
EDGE	Enhanced Data GSM Environment
FARIMA	Fractional Auto Regressive Integrated Moving Average
GARCH	Generalized Auto Regressive Conditional Heteroscedasticity
GARMA	Generalized Auto Regressive Moving Average
GoS	Grade of Service
GPRS	General Packet Radio Services
GSM	Global System for Mobile Communication
LRD	Long-Range Dependence
MS	Mobile Station
MSC	Mobile Switching Center
MWN	Multifractal Wavelet Model

NN	Neural Network
O-D	Origin to Destination
PDN	Packet Data Network
PLMN	Public Land Mobile Network
PSTN	Public Switched Telephone Network
QoS	Quality of Service
RAN	Radio Access Network
RNC	Radio Network Controller
SGSN	Serving GPRS Support Node
SMS	Short Message Service
SRD	Short-Range Dependence
TE	Terminal Equipment
UE	User Equipment
UICC	Universal Integrated Circuit Card
UMTS	Universal Mobile Telecommunication Systems
UTRAN	UMTS Terrestrial Radio Access Network
WCDMA	Wideband Code Division Multiple Access

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Dedication

This thesis is dedicated to all my family members, especially to the memory of my late mother, Madam Efua Gbolovi, whose toils and inspirations were the very foundation of this achievement. Rest In Peace mum.

CHAPTER ONE: INTRODUCTION

1.0 Introduction

Deploying and operating telecommunication networks that provide quality service to its subscribers require effective and efficient initial planning. Planning these networks require a lot of insight and forecast into the future of their performance. The central issues to most telecommunication network planners are capacity and coverage planning. Planning telecommunication networks that give high capacity and wide coverage is a very challenging task for network engineers. For these engineers to achieve a high quality network that delivers the desired services to its subscribers, demands that the traffic volume expected on these networks be forecasted with accuracy. This would bring the level of congestion on these networks to the barest minimum. Traffic volume prediction needs tools or models with high level of precision.

Quite a number of techniques have been used in the field of traffic volume prediction in telecommunications, and transportation and highway engineering **[23]**. Most of these techniques make use of either the time series methods or the Kalman filter methods. Apart from these distinct methods, some of the techniques combine the two methods mentioned earlier, what is normally referred to as the hybrid method. The Time Series techniques were developed for data exhibiting seasonality. However, it is known that traffic data show seasonality only in a long term. Short-term traffic volume prediction using time series methods usually has some special requirements on the input data.

The Kalman filter algorithm is developed to produce values that tend to be closer to the true values of the measurements observed over time that contain noise (random variations) and other inaccuracies. The Kalman filter not only works well in practice, but it is theoretically attractive as it is able to minimize the variance of the estimated error. It can be described as an optimal linear estimator.

The Kalman filter has been reported in literature to have been used in traffic volume prediction but mostly in transportation engineering. We are going to adapt the Kalman filter model so used in short-term traffic volume prediction in transportation engineering to traffic volumes for Universal Mobile Telecommunication System (UMTS) networks.

The above mentioned models have performed creditably in their application domains. However, the question still remains on the accuracy of the results produced by these models and hence the need for model validation specifically Kalman filter-based models. The result of this study would contribute significantly to knowledge in short-term traffic volume prediction using the Kalman filter algorithm

1.1 Background of the Study

Accurate and timely estimation of network traffic is increasingly becoming important in achieving guaranteed Quality of Service (QoS) in wireless networks **[7].**

In **[16],** it is reported that, traffic modeling and characterization constitute important steps towards understanding and solving performance-related problems in future wireless and wireline Internet Protocol (IP) networks. The essence of traffic volume prediction in telecommunication systems especially UMTS networks cannot therefore be over emphasized.

The predictability of network traffic is of significant interest in many domains such as congestion control**,** admission control, and network management **[35].**

An accurate traffic prediction model should have the ability to capture the prominent traffic characteristics e.g. short- and long-range dependences, self-similarity in large-time scale, and multifractal in small-time scale **[35].**

Researchers in the field of traffic volume prediction use either time series methods or methods based on the Kalman filter algorithm. These researchers also used hybrid methods for traffic volume prediction in which they combine two or more methods.

A number of forecasting methods are related to time series analysis in which prediction of the future is based on past values of variables. The time series models identify the pattern in the past data and extrapolate that pattern into the future **[16]**.

The Kalman filter is a multi-input, multi-output, recursive digital filter that can optimally estimate, in real time, the states of the system based on its noisy outputs. The estimates are statistically optimal in the sense that they minimize the mean square estimation error **[19]**.

The Kalman filter based prediction algorithm makes significant contributions to various aspects of network traffic engineering and resource allocation. The prediction algorithm with Kalman filter yields superior prediction accuracy than the other algorithms **[29].**

This study adapts and validates a Kalman filter-based model for short-term traffic volume prediction for UMTS networks.

1.2 Problem Statement

Communication network operators are often confronted with the problem of network congestion which invariably results in poor QoS. Ability to predict traffic volume on the network for effective traffic and hence network resource management has been identified as a viable approach to QoS improvement. Traffic modeling, which facilitates traffic volume prediction, captures the statistical properties underlying network packet flow.

Traffic volume prediction problems make it difficult or almost impossible for network operators or planners to anticipate the amount of traffic expected on their networks. When traffic volumes are not estimated accurately, it may lead to network congestions and invariably poor quality of service to network subscribers.

Several approaches for traffic modeling have been proposed for UMTS networks. However, industry and academia do not seem satisfied with results obtained **[1]**. Kalman filtering has been applied to many traffic studies such as motorway traffic state estimation **[31]** freeway travel time estimation, freeway Origin to Destination (O-D) demand matrices, and the prediction of traffic volume and travel time **[6]**. These models have worked well in the area of transportation.

1.3 Objectives of the study

The overall aim of this research is to adapt and validate a traffic model based on the Kalman filter (KF) algorithm for short-term traffic volume prediction for UMTS networks. For this purpose, the following activities will be carried:

1. Adapt the KF-based traffic model developed for vehicular transportation systems to UMTS networks.
2. Validate the adapted model using real-life network traffic data.
3. Carry out sensitivity analysis of the model obtained.

1.4 Justification of the Study

The validation of the adapted model will be useful in predicting traffic volumes on UMTS networks for QoS improvement.

The adapted model can be sensitive to certain network parameters. Sensitivity analysis will permit the identification of these parameters for further work to be carried out.

1.5 Scope of the study

The scope of this research is limited to examining short-term traffic volume prediction in UMTS networks and performing validation of Kalman filter-based model by using live data from a local network operator.

1.6 Thesis Organization

The remainder of this thesis is organized as follows: Chapter 2 presents a comprehensive literature survey on short-term traffic volume prediction in UMTS networks and model validation.

This is followed by Chapter 3 on Kalman filter theory and algorithm, In Chapter 4, we discuss model development and implementation. Chapter 5 is focused on data description and some performance measurement indices of models. In Chapter 6 we present results and corresponding analysis of the study. The conclusion and recommendations are presented in Chapter 7.

CHAPTER TWO: LITERATURE REVIEW

This chapter presents a brief overview of the UMTS network architecture, as well as traffic volume, and short-term traffic volume prediction models developed in various studies. The chapter also gives some theoretical background of the Kalman filter algorithm. We further explore model validation, and conclude with some available techniques for performing effective model validation, and the need for validating models.

2.1 UMTS Network Architecture

The UMTS is a Third Generation (3G) telecommunication system based on Wide-band Code Division Multiple Access Direct Sequence (WCDMA-DS) **[10]**. 3G systems are intended to provide a global mobility with a wide range of services including telephony, paging, messaging, Internet and broadband data access**.** UMTS networks offer teleservices (like speech or SMS) and bearer services, which provide the capability for information transfer between access points**.** The UMTS network architecture offers the following data rates with their corresponding targets.**144 kbits/s** for satellite and rural outdoor, **384 kbits/s** for urban outdoor, and **2048kbits/s** for indoor and low range outdoor **[11].**

UMTS is the recent telecommunication system developed from the existing Global System for Mobile Communication (GSM) system. It is a universal mobile telecommunication system designed to provide seamless telecommunication services with enhancement in quality, data rate, reliability, connectivity, and system interfaces adaptability with current and next generation technologies. The 3rd Generation

Partnership Project (3GPP) is the organization that defines the architecture and operation of the system. Figure 2.1 shows a basic structure of UMTS network. A high level architecture of the UMTS network is also shown in Figure 2.2, with brief discussion of the components in the subsequent sections.

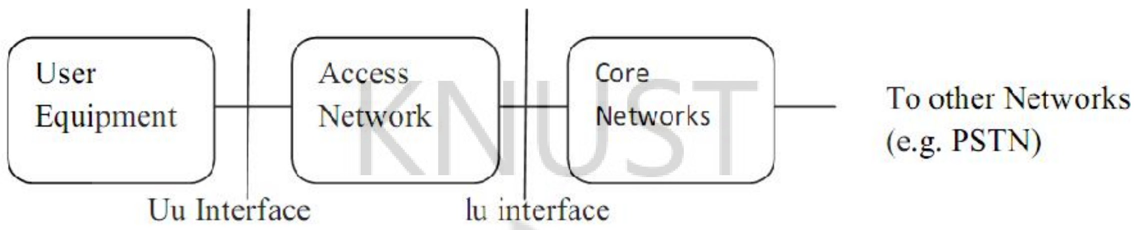


Figure 2.1: Basic Structure of UMTS Network (Source: **[26]**)

2.2 UMTS High Level Network Architecture

The UMTS network architecture has three main parts namely:

- The Core Network (CN)
- The Radio Access Network (RAN)
- The User Equipment (UE)

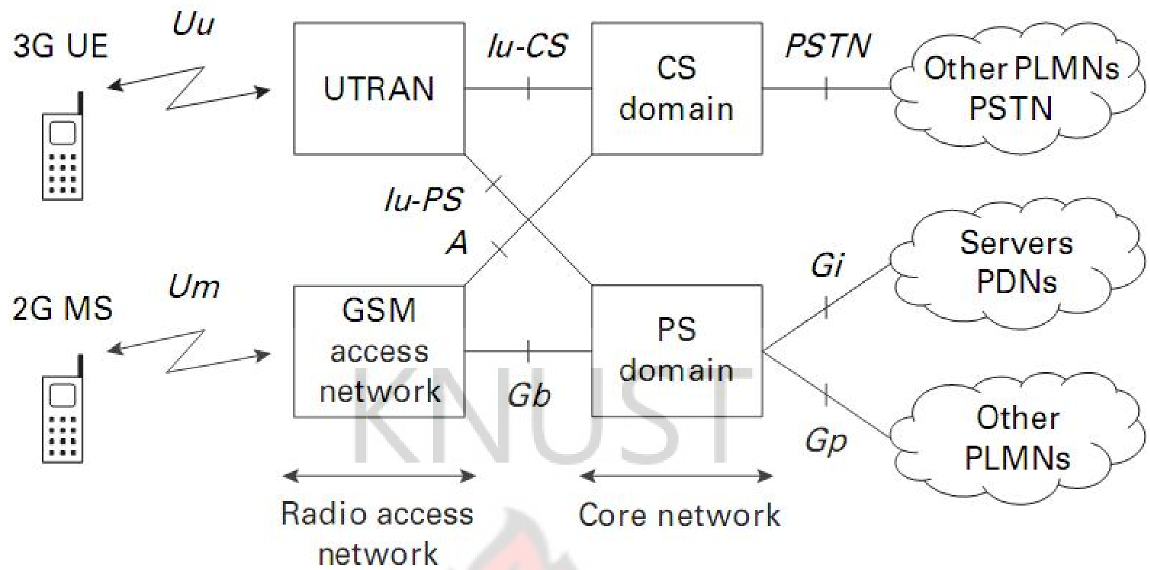


Figure 2.2: High Level Architecture of UMTS Network. (Source: **[3]**)

2.2.1 The Core Network (CN)

The core network contains two domains, the Circuit Switched and Packet Switched domains. The Circuit switched domain transports voice calls using circuit switched technology. It has interfaces to fixed line telephone systems that are known as Public Switched Telephone Network (PSTN), and to circuit switched domains that are run by other network operators.

The packet switched domain transports data streams using packet switching. It communicates with data servers that are controlled by the network operator itself, with external Packet Data Networks (PDNs) like the Internet, and with packet switched domains that are controlled by other network operators. The two domains were carried over from GSM and GPRS respectively, with only a few modifications **[3]**.

2.2.2 The Radio Access Network (RAN)

The fixed network infrastructure that contains components for transmitting over radio is called Radio Access Network (RAN) **[2].**

The radio access network is shown in Figure 2.3. The most important part is the UMTS Terrestrial Radio Access Network (UTRAN), which has two components: the Node B and the radio network controller (RNC). The Iub interface connects a Node B to an RNC, while the Iur interface connects two RNCs. All the interfaces in the figure carry both traffic and signaling **[3].**

Figure 2.4 shows the main components of the UTRAN. The RAN encapsulates all tasks connected with the transmission of information over radio **[2]**. The main components of UTRAN, RNC and Node B are also discussed briefly.

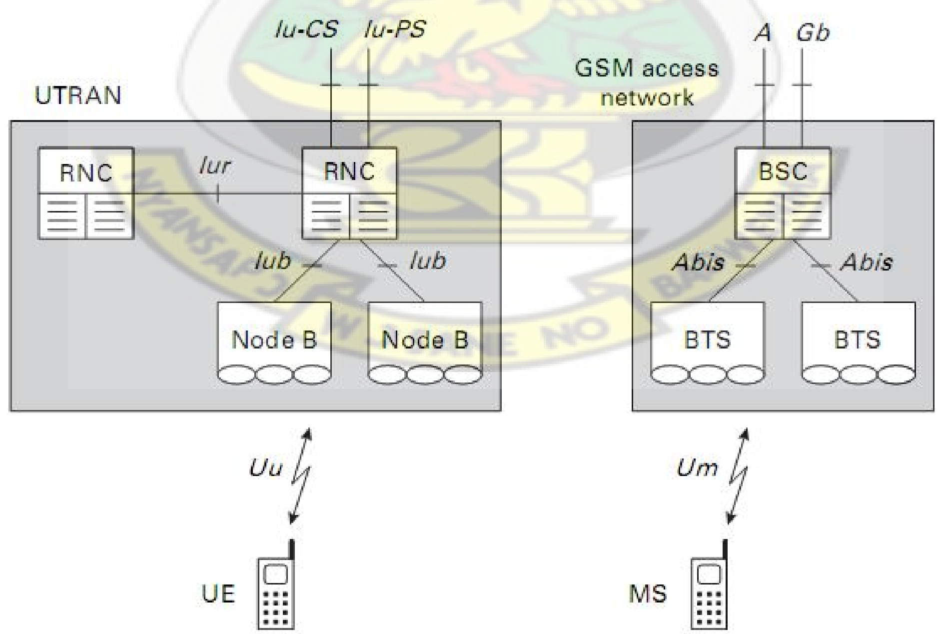


Figure 2.3: Architecture of Radio Access Network in Release 99. (Source: **[3]**)

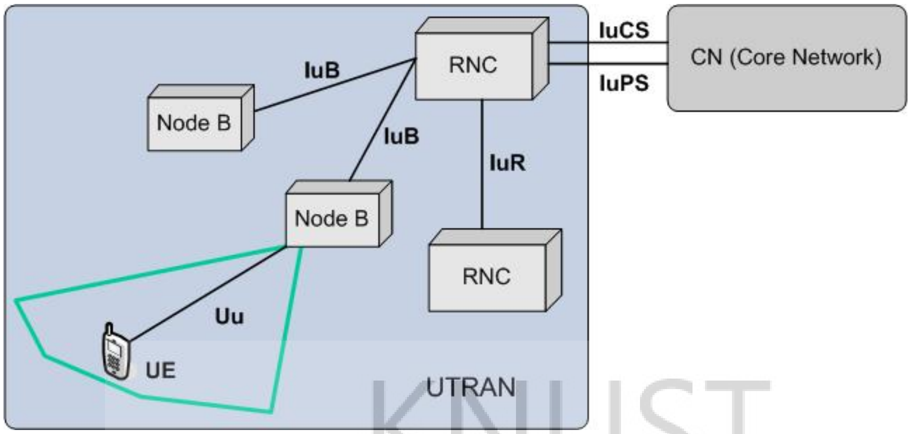


Figure 2.4: UTRAN. (Source: **[12]**)

2.2.2.1 Radio Network Controller (RNC)

The Radio Network Controller (RNC) is the central node in a radio access network. It takes the place of the Base Station Controller (BSC) in GSM and assumes the management of the resources in all attached cells (channel allocation, handover, and power control). A large number of the protocols between UE and RAN are implemented in the RNC. The RNC concurrently communicates over the lu-interface with a maximum of one fixed network node MSC and SGSN at any given time. Thus each RNC is allocated to an MSC and an SGSN. It also has the option of using the Iur-interface to communicate over the CN with neighbouring RNCs **[2]**.

2.2.2.2 The Node B

The term Node B refers to the base station equipment which communicates with the subscriber's handset via the radio link (and of course with the main network via telecoms link). It provides radio resources for a UMTS network and uses UMTS channel allocation to communicate with the handset. It provides all the Radio Frequency (RF) processing enabling transmission and reception of information to and from the mobile terminal. This information is encoded using the W-CDMA scheme **[11]**.

2.2.3 The User Equipment (UE)

The UMTS mobile is known as the user equipment (UE). This is a change in terminology from GSM, where it was known as the mobile station (MS). Most UMTS mobiles are actually dual mode devices that support GSM as well: they communicate using 3G technology in regions of UMTS coverage, but revert to 2G in regions where UMTS base stations have not yet been deployed **[3]**.

Figure 2.5 shows the internal architecture of the user equipment (UE). There are two main components, the mobile equipment (ME) and the Universal Integrated Circuit Card (UICC). The ME is the mobile phone itself, while the UICC is a smart card that plugs into the mobile phone. In a simple mobile phone, the ME is usually a single device, but in data terminals, its functions are often split in two: the mobile termination (MT) handles all the 3G communication functions, while the terminal equipment (TE) is the point where the data streams begin and end.

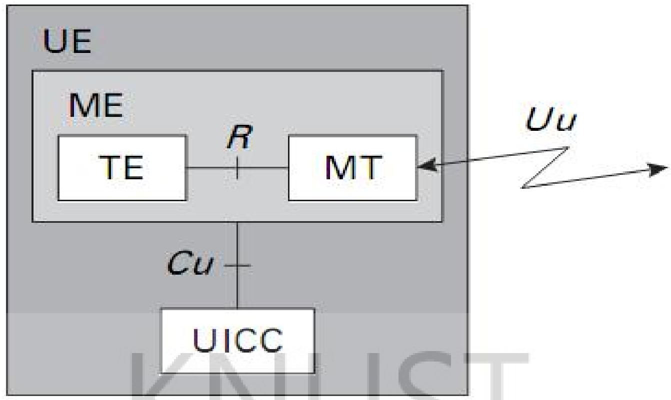


Figure 2.5: UMTS User Equipment. (Source: **[3]**)

2.3 Short-term Traffic Volume Prediction in UMTS Networks

Short-term traffic forecasting can be defined as predicting traffic conditions 20 seconds to several minutes in advance with the hope of employing these forecasts in the design and deployment of telecommunications networks **[6].** The motivation for such forecasting is obvious; one's action in the future should stand better chance of success if one has anticipated the conditions likely to be obtained when the actions are implemented.

2.3.1 Traffic Volume

Traffic volume is defined as the number of calls that pass through a particular switching or routing device in a telecommunication network in a specified period of time

Traffic volume is one of the basic parameters required for the development and operation of all telecommunication systems. **[6]** Traffic volume is also defined as the sum of times for which the routing or switching devices in a telecommunication network are held for a specific period. Traffic volume is expressed as calls per day or calls per hour. The uninterrupted period of 60 minutes during which the traffic is a maximum is known as the *busy hour*. This is generally used as the basics for traffic calculation.

Traffic volume studies are conducted to determine information about the volume of various types and classes of traffic. Traffic volume is used as a quantitative measure of traffic flow or intensity. Traffic flow or intensity is defined as the average number of occupations (calls) occurring during a period equal to one average holding time. This is also defined as the average number of devices held simultaneously during a specified period T. The unit of measurement for traffic volume is erlang (E). Mathematically, traffic volume and intensity are expressed by Equations (2.1) and (2.2) respectively

$$Traffic\ Volume = \sum_{x=1}^{N} xtx \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots.. (2.1)$$

Where tx is sum of times during which exactly x out of N devices are held simultaneously within a period T.

$$Traffic\ Intensity = \frac{Traffic\ Volume}{Duration\ of\ the\ specified\ period} \quad \ldots\ldots\ldots \ (2.2)$$

2.3.2 Traffic Volume Prediction

Short-term traffic forecasting has had an active but somewhat unsatisfying history **[14]**. In the past studies, different methodologies and techniques have been used to forecast traffic volume. A number of forecasting methods are related to time series analysis in which prediction of the future is based on past values of variables. The time series models identify the pattern in the past data and extrapolate that pattern into the future **[16]**. This leads us to the next section that presents some existing works in the field of traffic volume prediction.

2.3.3 Traffic Volume Prediction – Existing Research

Quite a number of techniques have been used in the field of traffic volume prediction in telecommunications, and transportation and highway traffic engineering. Most of the studies conducted in the field were devoted to developing new or improving existing models **[23]**.

Recently, the global wireless industry has created a global partnership project, the Third Generation Partnership Project (3GPP) **[12]**, for standardization of UMTS. Besides the QoS concept and architecture for UMTS networks, several approaches for traffic modeling have been outlined **[16]**. We discuss some traffic volume prediction methods in the subsequent sections.

2.3.4 Time Series Method of Traffic Volume Prediction

A time series is a sequence of data points measured typically at successive time intervals spaced at uniform time intervals. Time series forecasting is the use of a model to predict future values based on previously observed values **[13]**.

The Time Series technique was developed for data exhibiting seasonality. However, it is known that traffic data show seasonality only in a long term. However, in a short time period, the change of traffic characteristic is stochastic rather than seasonal. Therefore, short-term prediction using time series models usually has special requirements on the input data, such as stationarity, white noise, etc **[23]**.

2.3.5 Kalman Filter Method of Traffic Volume Prediction

The Kalman filter algorithm is developed to produce values that tend to be closer to the true values of the measurements observed over time that contain noise (random variations) and other inaccuracies. In **[23]** it was reported that, Okutani et al. (1984) are among the first to use Kalman filtering method for traffic volume prediction. Recently, several studies were reported to combine other algorithms with Kalman Filtering for traffic prediction**.**

In **[33]**, a dynamic short-term corridor travel time prediction model using Kalman filter method is developed. This method involves a multi-step-ahead prediction of traffic condition with a seasonal autoregressive integrated moving average model. In order to adjust the prediction error based on traffic flow data that becomes available in real time, an embedded adaptive Kalman filter is designed. An on-line corridor travel time prediction model was developed. Test results show that this method is able to capture the traffic dynamics and provide more accurate travel time prediction **[23]**.

2.3.6 Hybrid Methods of Traffic Volume Prediction

The hybrid methods use two or more traffic volume prediction techniques to perform traffic volume prediction. In other works by some researchers in the field of network traffic prediction, linear time series models, e.g. Auto Regressive(AR) and Auto Regressive Integrated Moving Average (ARIMA) **[1] [26**] were used**.** The exponential decay of the autocorrelation function of these models gives them the ability to capture the Short-Range Dependence (SRD) characteristics only. However, it has been shown that traffic data exhibited a high degree of Long-Range Dependence (LRD) characteristics in addition to SRD **[17]**. Thus such models cannot characterize the network traffic well and unable for traffic prediction **[17] [26] [27].** In **[14]**, it was shown that Auto Regressive Moving Average (ARMA) and ARIMA models are SRD models and are not considered suitable for modern high speed networks. The Markov model often results in a complicated structure and many parameters when used to model Long range Dependent processes.

The ARMA model is more flexible than AR and MA models in modeling time series, but can only model non-stationary time series **[14]**.

In **[30],** ARMA and Dynamic Harmonic Regression (DHR) were applied to forecast traffic load in a live 3G packet switched core network and the conclusion was that, the DHR model is best in predicting traffic trend on a long term basis as compared to ARMA which performed better on short term basis.

In **[30],** it was reported that the Fractional Auto Regressive Integrated Moving Averaging (FARIMA) captured both SRD and LRD and has been used to model and predict traffic data. However, this model cannot capture the multifractal which has been found in the network traffic in small time scale **[35]**. The introduction of another model called Multifractal Wavelet Model (MWM) as cited in **[35]** was meant to capture the multifractal that the FARIMA model was not able to capture but the MWM model can only capture multifractal but cannot predict traffic.

In order to model both short-range and long-range dependencies in a time series, a generalization of all the regression models was introduced called the Generalized Auto Regressive Moving Average (GARMA) model **[14].** The GARMA model however, involves a more complex methodology in bringing all the regression models under one model for network traffic prediction. The ARIMA with Generalized Auto Regressive Conditional Heteroscedasticity (ARIMA/GARCH) model is a non-linear time series model which combined the linear ARIMA with conditional variance GARCH. The model has the ability to capture both SRD and LRD characteristics. The ARIMA/GARCH model can model and forecast network traffic better than the traditional linear time series model. However, its prediction methodology is more complex and unstable **[35].**

Time Series and Neural Network (NN) models are reported in **[23]** as the most popular models for short-term traffic prediction. Although properly designed NN model with sufficient hidden layers, hidden neurons, and sigmoid transfer function can approximate any continuous function, e.g. the variation of traffic characteristic over time, it is obtained at a cost of estimating large quantity of model parameters, which leads to poor generalization.

A study to combine Wavelet decomposition with Kalman Filter method for short-term traffic prediction was conducted in **[34].** It is reported that by using discrete wavelet decomposition and reconstruction, a complex data series was decomposed into several purer and simpler series. Then, the Kalman filter model was applied to get better prediction accuracy.

The above mentioned models have performed creditably in their application domains. However, the question still remains on the accuracy of the results produced by these models and hence the need for model validation specifically Kalman filter-based models. We now proceed to give some theoretical background of the Kalman filter algorithm.

2.4 Kalman Filter Theory

The Kalman Filter was named after Rudolf Emil Kalman, born in Budapest, Hungary on May 19, 1930. Its purpose is to use measurements observed over time, containing noise (random variations) and other inaccuracies, and produce values that tend to be closer to the true values of the measurements and their associated calculated values **[13]**.

[19], also described the filter as follows: the Kalman filter is a set of mathematical equations that provides an efficient computational (recursive) means to estimate the state of a process, in a way that minimizes the mean of the squared error. The filter is very powerful in several aspects: it supports estimations of past, present, and even future states, and it can do so even when the precise nature of the modeled system is unknown. The Kalman filter is a multi-input, multi-output, recursive digital filter that can optimally estimate, in real time, the states of the system based on its noisy outputs. The estimates are statistically optimal in the sense that they minimize the mean square estimation error. As a result, the Kalman filter has been used to estimate the optimal solution for various real applications.

A number of applications of the filter have being reported in literature, some of which were stated at the earlier sections of this chapter. That notwithstanding, few more applications of the Kalman filter are outlined here.

In **[25]**, some universities use Kalman filter models to play robotic soccer. The Kalman filter was implemented for a robot to locate its own position and that of the ball on the field of play. Another key application of the Kalman filter reported in literature is its use for the real-time identification of distributed parameter in a nuclear power plant **[24]**.

The Kalman filter based prediction algorithm makes significant contributions to various aspects of network traffic engineering and resource allocation. The prediction algorithm with Kalman filter yields superior prediction accuracy than the other algorithms **[29]**. Kalman described his filter using state space techniques, which enables the filter to be used as a smoother, a filter or a predictor.

The Kalman filter addresses the problem of attempting to estimate the state of a discrete-time controlled process. The state is represented by two variables:

- $\hat{\boldsymbol{x}}_{\boldsymbol{k/k}}$, the estimate of the state at time k given observations up to and including time k;
- $\boldsymbol{P}_{\boldsymbol{k/k}}$, the error covariance matrix (a measure of the estimated accuracy of the state estimate).

Discrete time linear systems are often represented in a state variable format given by an input equation:

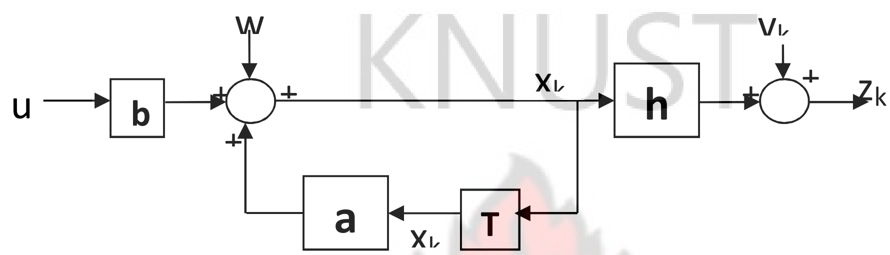
$$x_k = ax_{k-1} + bu_k + w_k \quad ---- (2.3)$$

and a measurement equation:

$$z_k = hx_k + v_k \quad -------- \ (2.4)$$

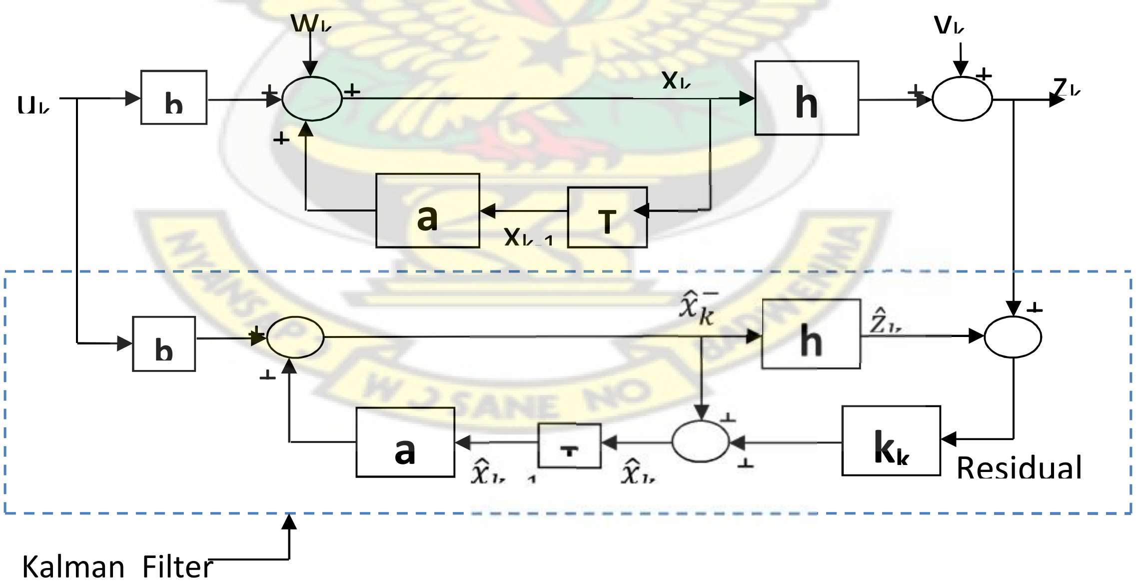
where the state x_k is a vector, a and b are constants and the input u_k is a scalar; k represents the time variable. The noise w_k is a white noise source with zero mean and covariance Q that is uncorrelated with the input. Likewise, the noise v_k is a white noise source with zero mean and covariance R that is uncorrelated with the input. The input u_k usually defaults to zero, so it is sometimes omitted.

Diagrammatically, such a discrete time linear system may be represented as depicted in Figure 2.6 below:



[1]Figure 2.6: Block diagram of discrete time linear system

A Kalman filter for such a system may be thought of as a reconstruction of the same



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[1] Adapted and modified from http://www.swarthmore.edu/NatSci/echeeve1/Ref/Kalman/ScalarKalman.html

system with noise sources ignored. The effects of the noise sources are compensated for by a ***residual term*** $(z_k - \hat{z}_k)$, where $\hat{z}_k$ is the estimate for the actual measured output z_k.

The outlined section of Figure 2.7 represents the Kalman filter for the system of Figure 2.6.

$\hat{x}_k^-$ represents the ***a priori*** estimate, which is the initial estimate of the state x_k. The a priori estimate is used to predict an estimate for the output $\hat{z}_k$, after which the actual measurement z_k is obtained.

The difference between z_k and $\hat{z}_k$ (known as the residual, as already stated above) is used to refine the a priori estimate after a gain k (known as the Kalman gain) has been applied to it. This difference may be written as

$$residual = z_k - \hat{z}_k \quad ---- (2.5)$$

Given that, from Fig 2,

$$\hat{z}_k = h\hat{x}_k^- \quad ---------- (2.6)$$

equation (2.12) may be expressed as

$$residual = z_k - h\hat{x}_k^- \quad ---- (2.7)$$

(There is no noise term because, as already stated, it is ignored in the Kalman filter and compensated for by the residual term). The residual is thus used as a correction factor for the a priori estimate.

The *refined a priori estimate* is now known as the ***a posteriori*** estimate of the state x_k, and it is represented as $\hat{x}_k$. This a posteriori estimate is then fed back into the system to serve as a *previous* estimate of the system state for the *next* prediction of the Kalman filter.

Thus the Kalman filter basically estimates the state of the system represented by equation (2.3) and refines this estimate using measurement information from z in equation (2.4). Detailed discussion on Kalman filter theory and algorithm is provided in the next chapter of this report.

2.5 Model Verification and Validation

The concepts of model verification and model validation have received much attention in recent years **[9].**

The increased dependence on using computer simulation models in engineering design brings about a critical issue of confidence in modeling and simulation accuracy. Model verification and validation are the primary methods for building and quantifying confidence, as well as for the demonstration of correctness of a model **[9] [21]**.

Model verification is the assessment of the solution accuracy of a mathematical model.

In **[15]**, a model validation was defined as "substantiating that the model, within its domain of applicability, behaves with satisfactory accuracy consistent with the study objectives". Model validation therefore, ensures that the model meets its intended requirements in terms of the methods employed and the results obtained **[15]**.

Model validation has become a primary means to evaluate accuracy and reliability of computational simulations in engineering design **[15]**. Due to uncertainties involved in modeling, manufacturing processes, and measurement systems, the assessment of the validity of a modeling approach must be conducted based on stochastic measurements to provide designers with the confidence of using a model **[32]**.

By definition, no model will ever be a perfect representation of reality. Therefore, it is important to validate models to determine their limits of accuracy and Kalman filter-based models are of no exception **[18]**.

2.5.1 Model Validation – Related Work

Quite a number of researchers in the field of modeling have used model validation in their works **[20]**. These researchers either use the procedure to select existing model, or validate their own models to ascertain the accuracy of these models. In this section of our study, we present a review of some literature on model validation.

The validation of mathematical models is done by comparing model predictions to observed data. Various statistical methods have been suggested and used to assess a model's validity: the Pearson correlation coefficient, the paired t-test, the least-square analysis of slope (=1) and intercept (=0), and the coefficient of variation or the intraclass correlation coefficient. None of these can completely assess the desired reproducibility characteristics. The Pearson correlation coefficient only measures precision of a linear relationship, not accuracy **[20]**. Both the paired t-test and least squares analysis can falsely reject (accept) the hypothesis of high agreement when the residual error is small (large) **[20]**. The coefficient of variation and the intraclass correlation coefficient assume a dependent and an independent variable. More importantly, they fail to recognize the duality (interchangeability) of predictions with observations **[20]**. The relevant question is not whether a model predicts observed data but whether the model and the observation method measure the same **[22]**.

2.6 Model validation Techniques

From the above definition, the question then is, how should a model be validated effectively? Three model validation approaches; graphical comparison, confidence interval and r^2 (coefficient of determination) approaches are discussed in this section of our thesis.

2.6.1 Graphical Comparison Approach

The graphical comparison approach to model validation is the most straightforward approach used in many applications, although the format may be different. One format is to plot the experimental and predicted results with different symbols to distinguish between these two groups on the same graph using model parameters as the axes and then compare the results **[17]**. This format may however, lead to a difficulty if there are many input variables. Another method is an x-y scatter plot by defining the x-axis as experimental results whereas the y-axis as corresponding results predicted (or vice versa). In this method, a straight line from the origin with a slope of one, called the "ideal" line, represents the ideal model.

2.6.2 Confidence Interval Approach

This approach involves uncertainty analysis of model parameters into model validation processes. Probabilistic results from the uncertainty analysis and a specified significance level are used to create an allowable variation, called a confidence interval, of the predicted response corresponding to acceptable variations of input parameters.

For this purpose a widely accepted type I error, designated as α, is used. The type I error, α, is a conditional probability corresponding to the possibility that a null hypothesis designated as H_0) is rejected when it is indeed true, P (reject H_0/H_0) is true. The model validation strategy of this approach is as follows: if all corresponding experimental results are found within the confidence intervals created, the model is not considered to be invalid at the design space tested with the specified significance level. In other words, there is not enough statistical evidence to reject the null hypothesis, i.e., error measure is statistically zero the model is good enough under those conditions. It is worth noting that this approach is applicable to arbitrary response distributions. However, it may not be convenient in a case where there are several design points since there will be several plots to analyze one by one. In addition, the chance to reject a valid model is increased by the number of design points, i.e., the total type I error statistically increases **[17]**.

2.6.3 r^2 (Coefficient of Determination) Approach

To address the concern in the confidence interval approach on the examination of each individual confidence interval at each design point, the r^2 approach eliminates the individual examination at each design point by calculating the distance between physical tests and simulated results of multiple design points all at once in one metric, r^2. Other tests such as χ^2, percentile and Kolmogorov-Smirnov tests can also be performed under the r^2 approach of model validation. In this study, we compute the r^2 to determine the validity of the proposed model **[17]**.

2.7 Need to validate models

From the literature above on model validation, it is clear that a lot of work has been done in the field of model validation. However, almost all the researchers used their chosen techniques mostly in validating a number of models but Kalman filter-based model was not mentioned in literature to the best of our knowledge. It is therefore crucial to validate the Kalman filter-based model developed in this study.

CHAPTER THREE: KALMAN FILTER THEORY AND ALGORITHM

Most practical engineering problems require the estimation of parameters associated with the physical phenomenon based on inaccurate measurements. Many algorithms exist today for parameter estimation but the Kalman filter stands out to be one of the best of such tools and is employed in many engineering processes that can be described by a linear system. In mathematical terms, Kalman filter estimates the states of a linear system. KF not only works in but it is theoretically attractive as it is able to minimize the variance of the estimated error **[28]**. It can be described as an optimal linear estimator. In this chapter, we give the salient features of the Kalman filter that relate to this project.

3.1 Overview of Kalman Filter Theory and Algorithm

The Kalman filter was named after Rudolf Emil Kalman, who first introduced the filter in 1960. The filter has been employed in myriad applications including process control systems, vehicle tracking, marine navigation, geology, demographic estimation and stock price prediction. The filter estimates the instantaneous state of a linear dynamic system perturbed by Gaussian white noise by using measurements that are linearly related to the system state but that are corrupted by Gaussian white noise.

The filter recursively minimizes the mean square estimation error without directly observing the system state or knowing the nature of the modeled system. Since the time of its introduction, the Kalman filter has been the subject of extensive research and application, particularly in the area of autonomous or assisted navigation. This is likely due in large part to advances in digital computing that made the use of the filter practical, but also to the relative simplicity and robust nature of the filter itself. Rarely do the conditions necessary for optimality actually exist, and yet the filter apparently works well for many applications in spite of this situation. Kalman described his filter using state space techniques, which enables the filter to be used as a smoother, a filter or a predictor. The predicting ability of the filter is what this project seeks to make use of.

The Kalman filter addresses the problem of attempting to estimate the state of a discrete-time controlled process. The state is represented by two variables:

- $\hat{x}_{k/k}$, the estimate of the state at time k given observations up to and including time k;
- $P_{k/k}$, the error covariance matrix (a measure of the estimated accuracy of the state estimate). Discrete time linear systems are often represented in a state variable format given by a state equation:

$$x_j = ax_{j-1} + bu_j + w_j \quad ---- (3.1)$$

and a measurement equation:

$$z_j = hx_j + v_j \quad ------- \quad (3.2)$$

where the state x_j is a scalar, a and b are constants and the input u_j is a scalar; j represents the time variable. The noise w_j is a white noise source with zero mean and covariance Q that is uncorrelated with the input. Likewise, the noise v_j is a white noise source with zero mean and covariance R that is uncorrelated with the input. The input u_j

usually defaults to zero, so it is sometimes omitted. The two equations above (equations 3.1 and3.2) form the basis of the Kalman filter algorithm.

3.2 The Kalman Filter Equations

The Kalman filter is essentially a set of mathematical equations that implement a predictor-corrector type estimator that is optimal in the sense that, it minimizes the estimated error covariance when some presumed conditions are met.

The filter has two distinct stages: the Predictor (Time update) and the Corrector (Measurement update) stages.

3.2.1 The Time Update (Predictor) Equations.

At this stage of the Kalman filter algorithm, we project or predict the state of the process X_j under investigation and call this initial estimate, the a priori estimate, $\hat{X}_j^-$

The a priori estimate $\hat{X}_j^-$ is then used to predict an estimate for the output, $\hat{Z}_j$. The difference between the estimated output and the actual output called the residual or innovation is then computed using equation 3.3.

$$Residual = Z_j - \hat{Z}_j = Z_j - h\hat{X}_j^- \quad ------------ (3.3)$$

The residual is then used to refine the initial estimate for the state of X_j to obtain a new estimate called the a posteriori estimate, $\hat{X}_j$

$$\hat{X}_j = \hat{X}_j^- + k(Residual) = \hat{X}_j^- + k\left(Z_j - h\hat{X}_j^-\right) \quad ------ (3.4)$$

Where ***k*,** is the Kalman gain. The measurement update (corrector) equations are then used to correct the estimates of the time update stage.

3.2.2 The Measurement Update (Corrector) Equations.

At this stage of the algorithm, the Kalman gain ***k***, is first computed. The computed gain is then used to update the a posterior estimate via the output Z_j. The error covariance is finally updated.

3.2.3 The Derivation of the Kalman Gain

We begin by defining the two errors of our estimate, the a priori error, e_j^-, and the a posteriori error e_j. The a priori and a posteriori errors are defined as the difference between the actual value of X_j and the a priori and a posteriori estimates respectively.

$$e_j^- = x_j - \hat{x}_j^- \quad \text{------------------------(3.5)}$$

$$e_j = x_j - \hat{x}_j$$

The a priori and the a posteriori errors, are each associated with mean squared errors or variances represented in the equation 3.6 below.

$$p_j^- = E\left\{\left(e_j^-\right)^2\right\} \quad \text{----------------------- (3.6)}$$

$$p_j = E\left\{\left(e_j\right)^2\right\}$$

We start off by substituting equation 3.4 into equation 3.5, and the resultant equation is finally substituted into equation 3.3, yielding equation 3.7.

$$p_j = E\left\{\left(x_j - \hat{x}_j\right)^2\right\}$$

………………………… (3.7)

$$p_j = E\left\{\left(x_j - \hat{x}_j^- + k(z_j - h\hat{x}_j^-)\right)^2\right\}$$

In order to find the value of K, we differentiate the expression obtained in equation 3.7 with respect to k and set the derivative to zero.

$$\frac{\partial p_j}{\partial k} = 0 = \frac{\partial E\left\{\left(x_j - \hat{x}_j^- + k\left(z_j - h\hat{x}_j^-\right)\right)^2\right\}}{\partial k}$$

…………………... (3.8)

$$= 2E\left\{\left(x_j - \hat{x}_j^- + k\left(z_j - h\hat{x}_j^-\right)\right)\left(z_j - h\hat{x}_j^-\right)\right\}$$

$$= 2E\left\{x_j z_j - \hat{x}_j^- z_j + kz_j^2 - kh\hat{x}_j^- z_j - hx_j\hat{x}_j^- + \left(\hat{x}_j^-\right)^2 - khz_j\,\hat{x}_j^- + kh^2\left(\hat{x}_j^-\right)^2\right\}$$

Working through the expression for k yields equation 3.9.

$$k = \frac{E\left\{x_j z_j - \hat{x}_j^- z_j - hx_j\hat{x}_j^- + h\left(\hat{x}_j^-\right)^2\right\}}{E\left\{z_j^2 - 2h\hat{x}_j^- z_j + h^2\left(\hat{x}_j^-\right)^2\right\}}$$

……………………… (3.9)

Looking at the cumbersome nature of the expression, we treat both the numerator and the denominator separately.

Based on the basic assumption that, the measurement noise, v, is uncorrelated to either the input or the a priori estimate of X, we have

$$E\left\{\left(x_j - \hat{x}_j^-\right)v_j\right\} = E\left\{e_j^- \, v_j\right\} = 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots. \; 3.10$$

This simplifies the expression for the numerator to equation 3.11.

$$\text{numerator} = \mathrm{E}\left\{\mathrm{h}x_j^2 - 2\mathrm{h}\hat{x}_j^- x_j + \mathrm{h}\left(\hat{x}_j^-\right)^2\right\}$$

$$= \mathrm{hE}\left\{\left(x_j - \hat{x}_j^-\right)^2\right\} = \mathrm{E}\left\{\left(e_j^-\right)^2\right\} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (3.11)$$

$$= \mathrm{h}p_j^-$$

Using the orthogonal condition for the denominator of the expression in equation 3. 9, the denominator evaluates to equation 3.12.

$$\text{denominator} = \mathrm{E}\left\{\mathrm{h}^2 x_j^2 - 2\mathrm{h}^2\hat{x}_j^- x_j + \mathrm{h}^2\left(\hat{x}_j^-\right)^2 + v_j^2\right\}$$

$$= \mathrm{h}^2\mathrm{E}\left\{x_j^2 - 2\hat{x}_j^- x_j + \left(\hat{x}_j^-\right)^2\right\} + \mathrm{E}\left\{v_j^2\right\} \quad \ldots\ldots\ldots\ldots\ldots \quad (3.12)$$

$$= \mathrm{h}^2 p_j^- + \mathrm{R}$$

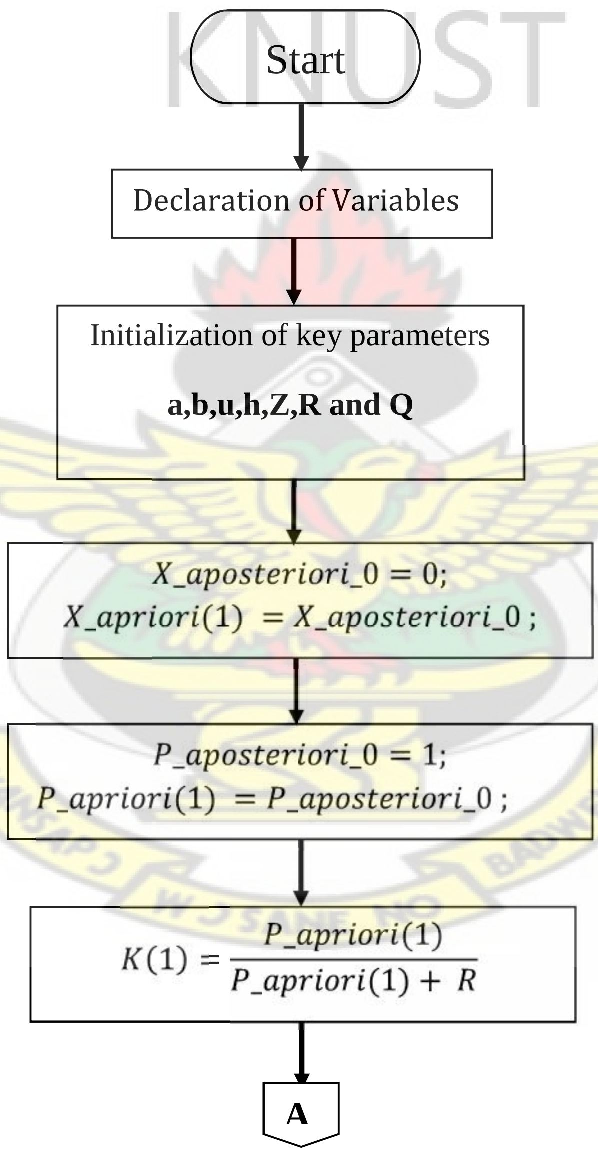
Substituting the new expressions of the numerator and the denominator into equation 9, yields the simplified equation for the Kalman gain as;

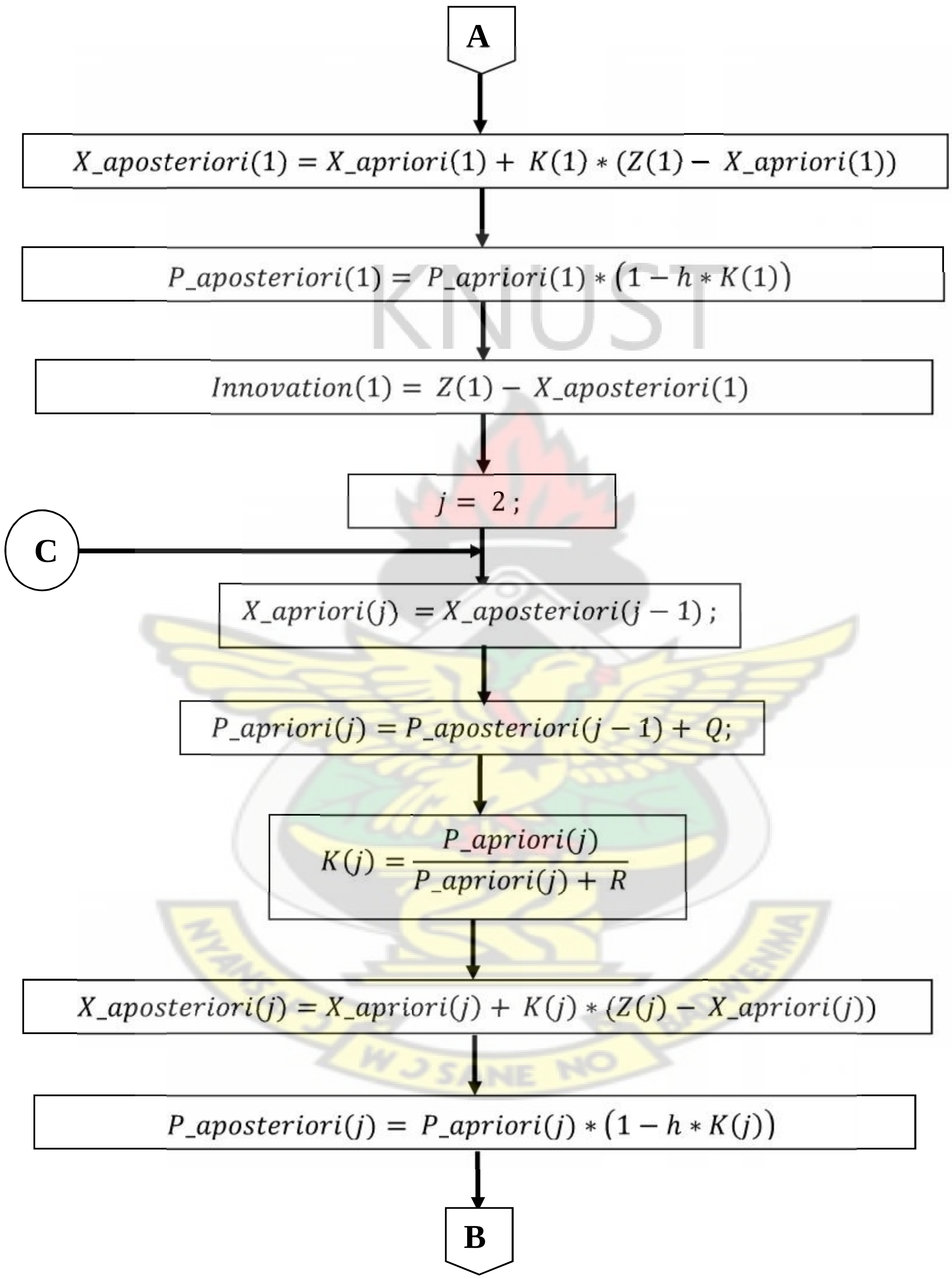
$$k = \frac{hp_j^-}{h^2 p_j^- + R} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (3.13)$$

Similar techniques used for the derivation of the Kalman gain can be employed to derive the covariance errors of the estimation.

3.3 Detailed Flowchart for the Kalman Filter Algorithm

We present a detailed flowchart of the Kalman filter algorithm in this section of the report. This can easily be translated into a program using any relevant programming language.




A
X_aposteriori(1) = X_apriori(1) + K(1) * (Z(1) − X_apriori(1))
P_aposteriori(1) = P_apriori(1) * (1 − h * K(1))
Innovation(1) = Z(1) − X_aposteriori(1)
j = 2 ;
C
X_apriori(j) = X_aposteriori(j − 1) ;
P_apriori(j) = P_aposteriori(j − 1) + Q;
K(j) = P_apriori(j) / (P_apriori(j) + R)
X_aposteriori(j) = X_apriori(j) + K(j) * (Z(j) − X_apriori(j))
P_aposteriori(j) = P_apriori(j) * (1 − h * K(j))
B

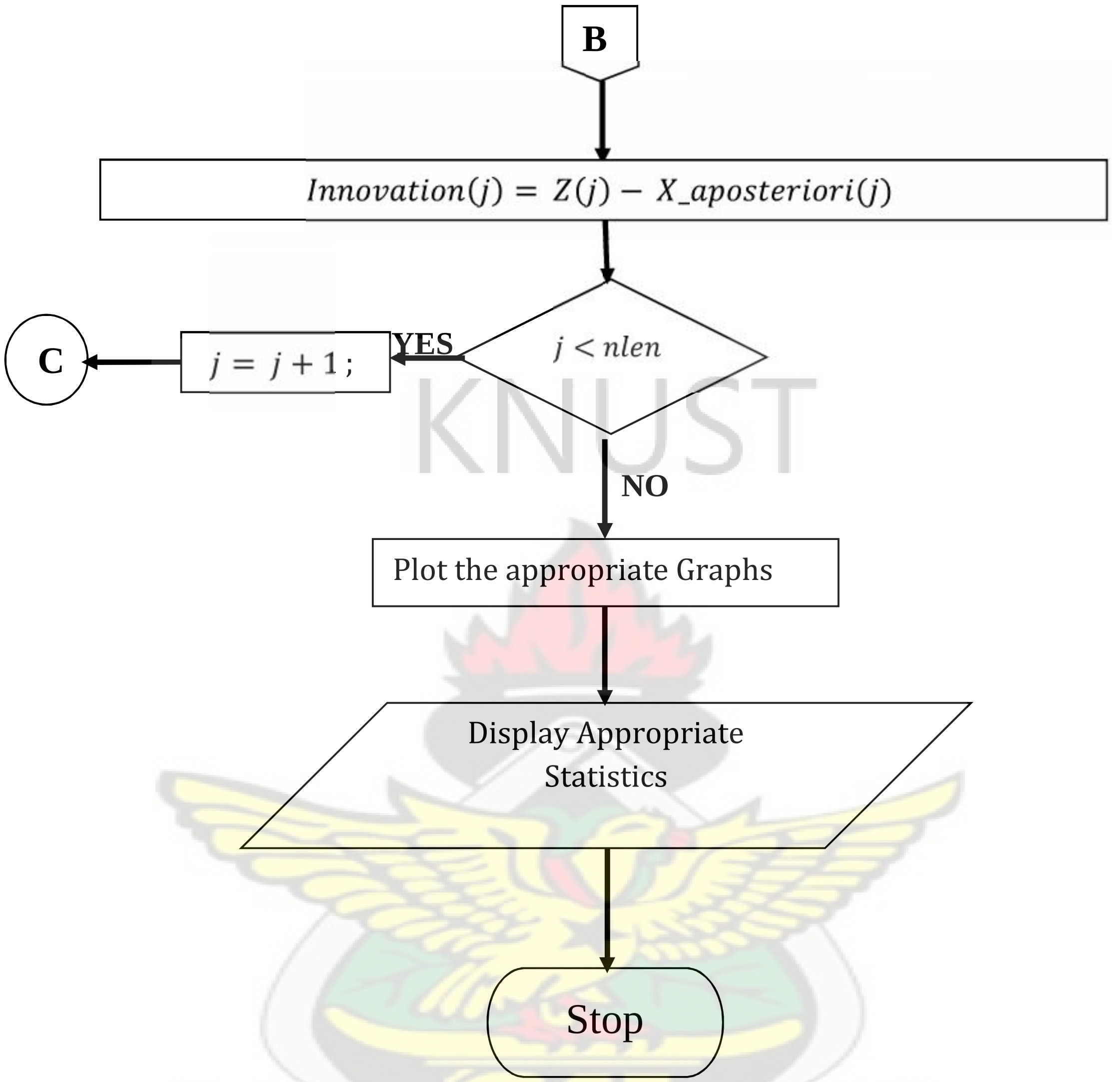


Figure 3.1 : Detailed flowchart for Kalman Filter Algorithm

3.4 A Step – by – Step Walkthrough of the Kalman Filter

In this section we provide a step-by-step approach in explaining the Kalman filter theory.

3.4.1 Step 1 – Building a Model

It is very important at this stage that, we first of all determine that Kalman filtering conditions fit the problem under investigation.

The two equations of the Kalman filter theory below will serve as a guide.

$$x_k = Ax_{k-1} + Bu_k + w_{k-1}$$

$$z_k = Hx_k + v_k \qquad \text{(3.14)}$$

This means that each x_k (our signal values) may be evaluated by using a linear stochastic equation as shown above. Any x_k is a linear combination of its previous value plus a control signal u_k and a process noise w_k. The second equation tells us that any measurement value (whose accuracy we are not sure of) is a linear combination of the signal value and the measurement noise v_k. The process noise and the measurement noise are statistically independent. The entities A, B and H are in general form matrices. We shall assume for the sake of this discussion that they are numerical constants.

3.4.2 Step 2 – Starting the Process

Once the model built at step 1 fits into the Kalman filter equations, the next step is to determine the necessary parameters and initial values. At this stage, we make use of two distinct set of equations namely, the time update (prediction) and the measurement update (correction) equations. Both equations are applied at each k^{th} state of the process.

3.4.2.1 Time Update (Prediction)

$$\hat{x}_k^- = A\hat{x}_{k-1} + Bu_k$$

$$P_k^- = AP_{k-1}A^T + Q \qquad \text{.............................. (3.15)}$$

3.4.2.2 Measurement Update (Correction)

$$K_k = P_k^- H^T (HP_k^- H^T + R)^{-1}$$

$$\hat{x}_k = \hat{x}_k^- + K_k (z_k - H\hat{x}_k^-) \qquad \text{.............................. (3.16)}$$

$$P_k = (I - K_k H)P_k^-$$

Based on the assumption that A, B and H are numerical constants and for this discussion each of value 1, we move on to determine Q and R, which are variances of the process and measurement errors respectively. R is rather simple to find because in general, we are quite sure about the noise in the environment. But finding Q is not so obvious. To start the process effectively, we need to assume the initial estimates of x_0 and P_0 .

3.4.3 Step 3 – Iteration

When all the relevant information are gathered and got the process started, we can now iterate through the estimates. It is important to note here that, the previous estimates will be the input for the current state. The whole iteration process is illustrated in the figure below.

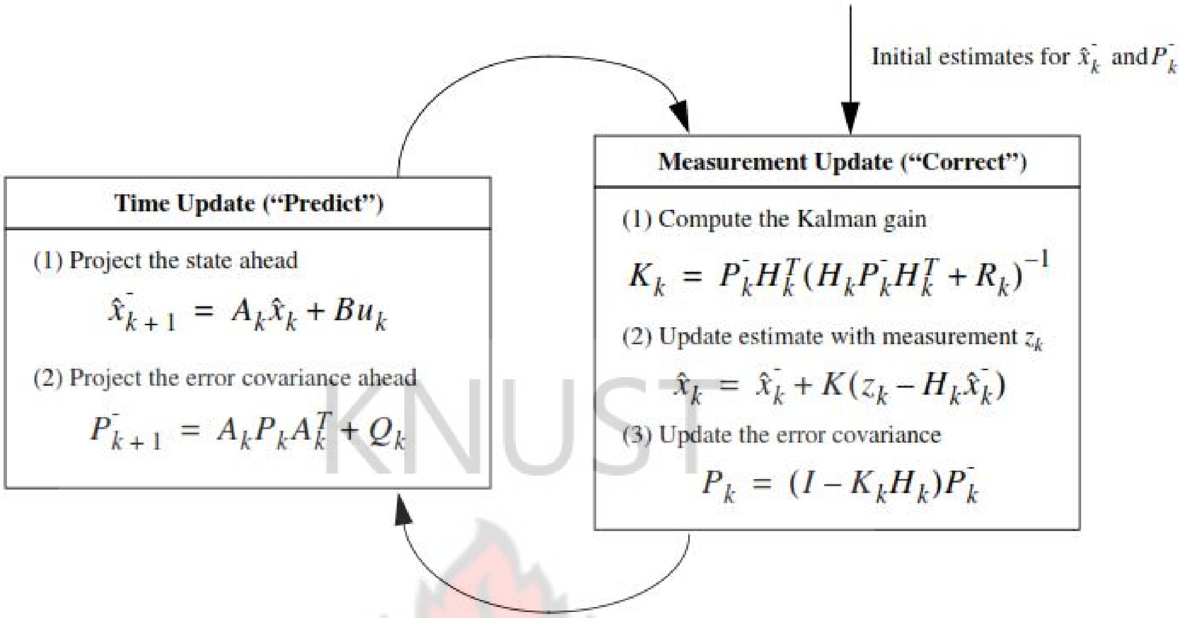


Figure 3.2 : Iteration process for the Kalman Filter Algorithm. (Source**: [8]**)

Here $\hat{x}_k^-$ is the a priori estimate which in a way means the rough estimate before the correction. Also P_k^- is the a priori error covariance. These a priori values will be used in the measurement update equations. In the measurement update equations we really find $\hat{x}_k$ which is the estimate of x at time k. Also, we find P_k which is necessary for estimates at next k step. The Kalman gain K_k is also evaluated at this stage. The values we evaluate at the measurement update stage are also called a posteriori values.

CHAPTER FOUR: METHODOLOGY

As stated earlier under the objectives of this study, the main aim of this study is to develop and validate a model for short-term traffic volume prediction in UMTS networks using the Kalman filter algorithm. In this section, we give a detailed description of the proposed model. This is to present the mathematical and programming backgrounds of the model in clear terms.

4.1 Model Description

Traffic volume prediction problem can be formulated as: given traffic volume history, recorded at discrete time instants, formulate the prediction of traffic volume ***k*** time instants from current.

The proposed model in this study predicts traffic volume in UMTS networks on short-term basis. The short-term basis is chosen because, it allows for certain assumptions as stated below:

(a) For short term forecasting, the state variable transitions may be regarded as a smooth process.

(b) A linear relationship may be assumed between traffic volumes for the current time step and traffic volumes for previous time steps.

From (b) it may be inferred that the traffic volume at any particular time step is a linear combination of the traffic volumes at previous time steps. Assumption (b) is made based on the structure of the Kalman filter equations and also to make it possible to represent the traffic volume accurately.

Let tv_k denote the link traffic volume for the k^{th} time interval to be estimated. Based on the assumption stated in (b) above, there exists the following linear relationship shown in equation (4.1)

$$tv_k = TV_k\theta_k + \varepsilon_k \quad ------------ (4.1)$$

where

$TV_k = [tv_{k-1}, tv_{k-2},...,tv_{k-n}]$, observed traffic volume;

$\theta_k = [\theta_{k-1}, \theta_{k-2},..., \theta_{k-n}]^T$; and

ε_k = noise term.

n = the number of previous traffic volumes taken into account for the computation of the current traffic volume.

Column vector $\boldsymbol{\theta_k}$ is a collection of coefficients for each corresponding observed traffic volume in the row vector $\mathbf{TV_k}$.

In order to implement the Kalman filter model, $\boldsymbol{\theta_k}$ is set as the state vector $\mathbf{x_k}$ in equation (2.3); $\mathbf{TV_k}$ is also set as the measurement matrix $\mathbf{H_k}$ in equation (2.4); $\mathbf{tv_k}$ corresponds to $\mathbf{z_k}$ in equation (2.4); and equation 4.1 is therefore equivalent to the measurement equation shown in equation (2.4).

Assuming now there are n+1 observed traffic volumes: $tv_{k-1}, tv_{k-2},...,tv_{k-n}$, based on the Kalman filter prediction algorithm, first an a priori estimate of $\boldsymbol{\theta_k}$ is calculated as $\hat{\theta}_k^-$, then the observed value of $\mathbf{tv_k}$ is used to update $\hat{\theta}_k^-$ and obtain a posterior estimate $\hat{\theta}_k$. Based on the assumption stated in (a), on the premise that the transition of state vector can be regarded as a smooth process, the traffic volume at the next time interval is then predicted as

$$\widehat{tv}_{k+1} = TV_{k+1}\,\hat{\theta}_k \quad \text{--------------------- (4.2)}$$

4.2 Model Implementation

Before the model is implemented using the recursive Kalman filter prediction process, several parameters need to be decided. The transition matrix **A** is set to an *n x n* identity matrix. This is so because; the transition of state vectors is generally a smooth process for short-term forecasting. Although some measurement errors may be recorded in the course of determining the traffic volumes, the observed traffic volume data are used as the true volumes. The prediction goal in this study is trying to get predicted traffic volumes as close as possible to the true volumes. With this assumption at hand, it is taken here that, there is no observation error and the variance of the measurement noise, **R**, is zero. The above assumption is however not applied to the process error in this study, and this brings the need to compute the variance of the process error, **Q**. The variance of the process error, **Q**, is obtained by minimizing the following negative log likelihood function [4].

$$-\ln\big(L(Q)\big) = \sum_{k=1}^{N}\{\ln\big(X_{k}\big) + Y_k^T X_k^{-1} Y_k\} + C \quad -------------- (4.3)$$

where

$X_k = HP_k^- H^T + R$ (measurement prediction covariance)

$Y_k = z_k - H\hat{x}_k^-$ (measurement residual)

$P_k^- = AP_{k-1}A^T + Q$ (a priori covariance)

N = number of previous observed traffic volumes ‘remembered’ by the system

C = constant

For a likelihood function as presented in equation (3.3), the aim is usually to maximize the function to obtain the most likely value. However, since the negative log is taken to facilitate computation, $\boldsymbol{Q}$ is obtained by *minimizing* rather than *maximizing* the function.

With the ongoing discussion of our model and its implementation in this study, the traffic volume prediction process using the Kalman filter algorithm is presented in the following pseudo codes.

Begin

Let k = n + 1

Initialize $\hat{x}_{k-1}$ and P_{k-1}. Customarily, P_{k-1} is set to be a matrix with very small values. For the purpose of this study, $\hat{x}_{k-1}$ is set to be $[1/n, 1/n, 1/n, ..., 1/n]^T$ and $P_{k-1} = 10^{-2} \cdot \mathbf{I}_{nxn}$.

Compute the values of $\hat{x}_k^-$ and P_k^- using the predictor stage equations of the Kalman filter equations.

Let $H_k = [tv_{k-1}, ..., tv_{k-n}]$ and $y_k = tv_k$.

Compute the Kalman gain K, $\hat{x}_k$, and P_k using the corrector stage equations of the Kalman filter equations.

Let $H_{k+1} = [\, tv_k, ..., tv_{k-(n-1)}]$, and compute the predicted traffic volume as

$$\widehat{tv}_{k+1} = H_{k+1}\hat{x}_k$$

Increment k; that is let k = k + 1

Repeat steps 3 through to 7.

End

The MATLAB code listing for the full implementation of the proposed model is detailed in appendix A

The Kalman filter prediction model thus developed and implemented in this chapter, is further validated with traffic volume data collected from a live mobile telecommunication network in the subsequent chapter.

CHAPTER FIVE: DATA DESCRIPTION AND MODEL VALIDATION

In this chapter, we give a good description of the data used in the validation of the proposed model. The chapter then concludes with the description of some performance indices used to measure the model's performance.

5.1 Data Description

Traffic volume data obtained from chosen live cell sites of a mobile telecommunication network operator are used to validate the proposed Kalman filter prediction model. These data were collected from three different operating areas representing residential, urban/commercial and rural areas. The locations are Nhyiaeso, Adum and Aborfor representing residential, urban/commercial and rural areas respectively, all in the Kumasi metropolis of the Ashanti region of Ghana.

Three different data sets were obtained from these locations. Each data set has 25 days of traffic volume data (July 1, 2010 – July 25, 2010). In order to capture effective traffic volume data, the data were collected between the hours of 12:00 am midnight and 11:00pm inclusive, constituting 24 samples of traffic volume data for each day under investigation. Out of these data, an average traffic volume is computed for each day, narrowing down the average traffic volume samples to 25. In order to determine the traffic volume pattern on days of the week basis, 7 average traffic volume samples were also derived from the data sets representing Monday through to Sunday inclusive.

5.1.1 Graphical Representation of Data sets.

Average traffic volumes of the locations and data sets described above are presented in tables and corresponding graphs in this section of the study.

The average traffic volumes for each day for the 25 days are presented here for some sectors of the locations where traffic volume data were collected.

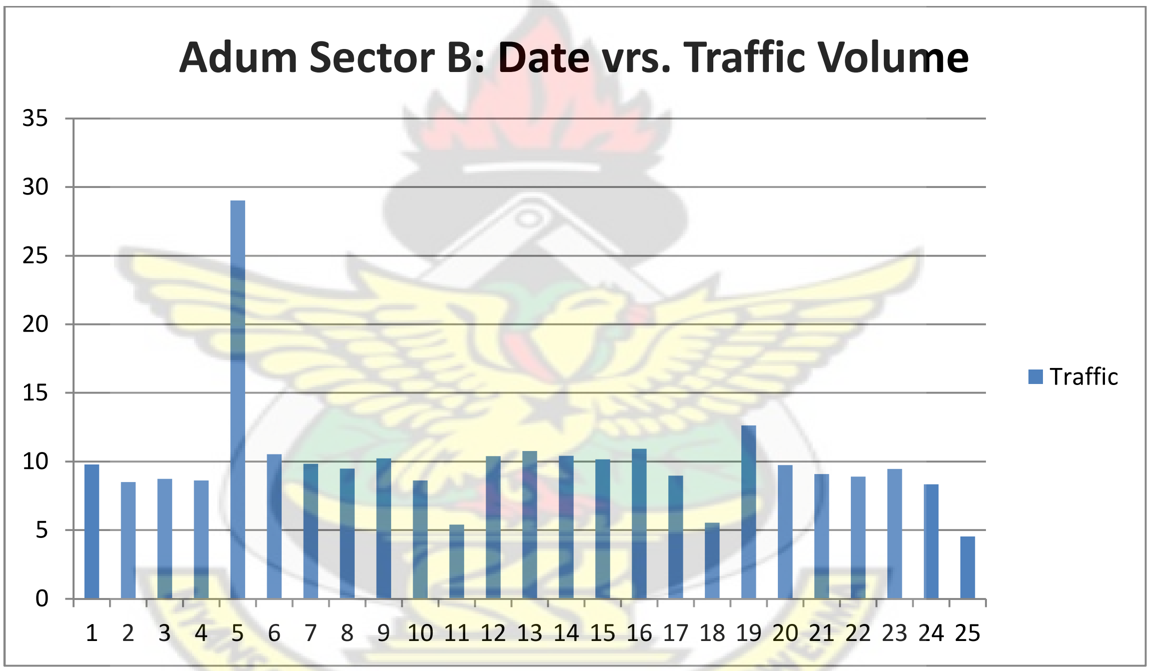


Figure 5.1: Average Daily Traffic Volume for Adum Sector B.

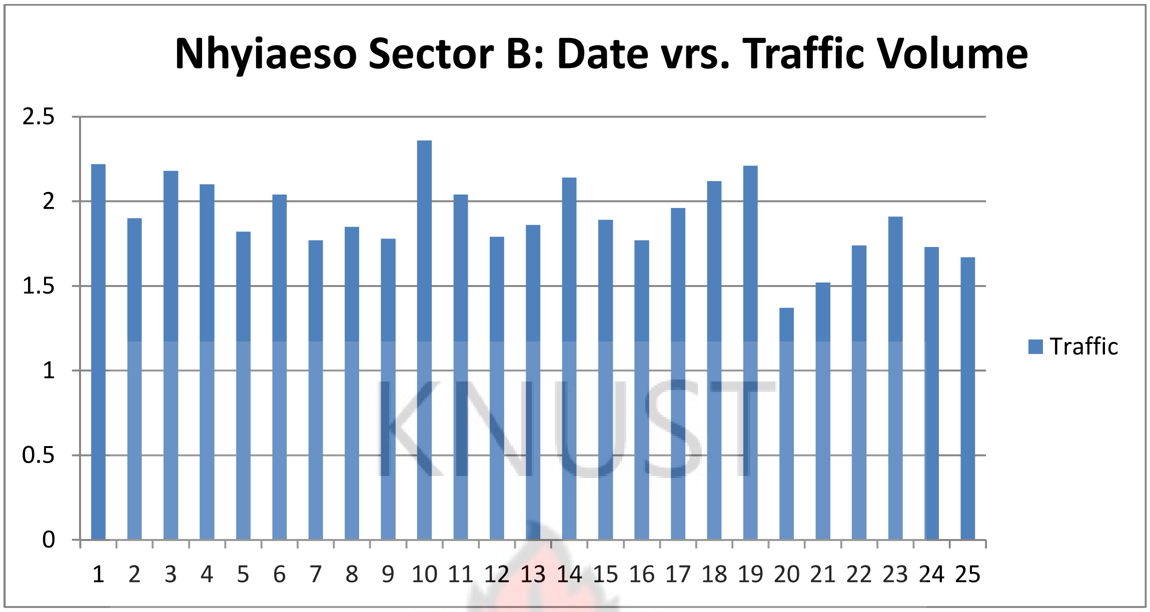


Figure 5.2: Average Daily Traffic Volume for Nhyiaeso Sector B.

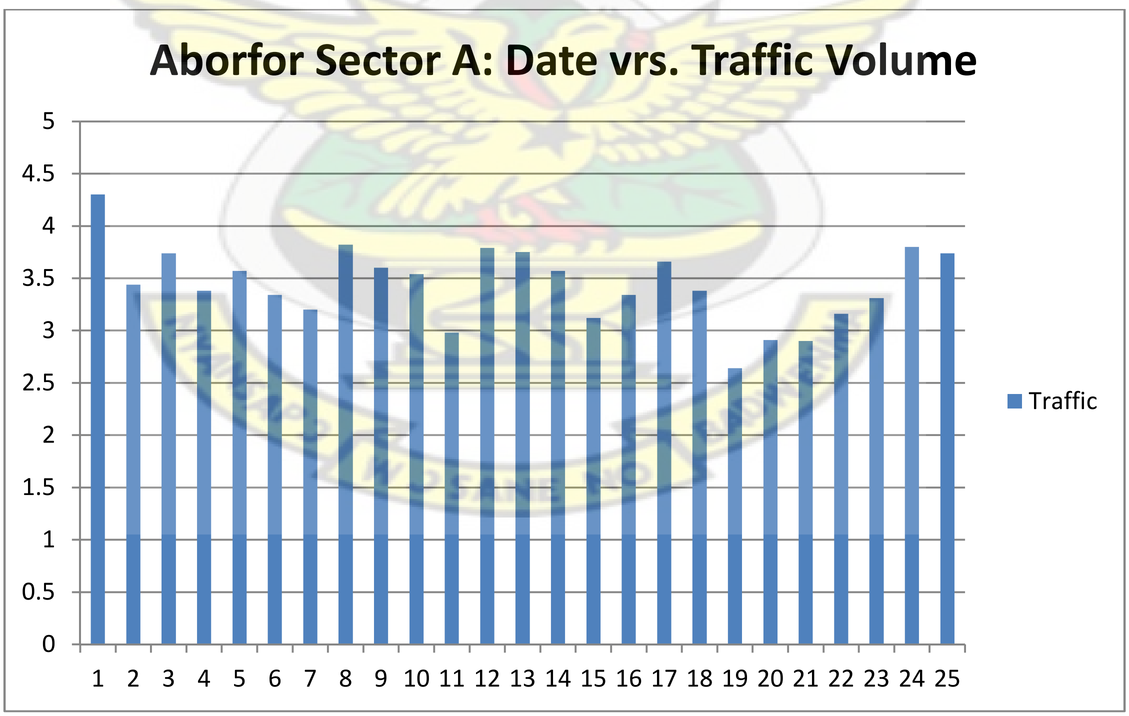


Figure 5.3: Average Daily Traffic Volume for Aborfor Sector A.

We also present average traffic volumes for each day of the week for the three chosen locations here.

Table 5.1: Average Day of Week Traffic Volume for Adum Sector B.

Day	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday	Sunday
Traffic	17.36	10.34	9.77	9.52	10.21	8.64	5.16

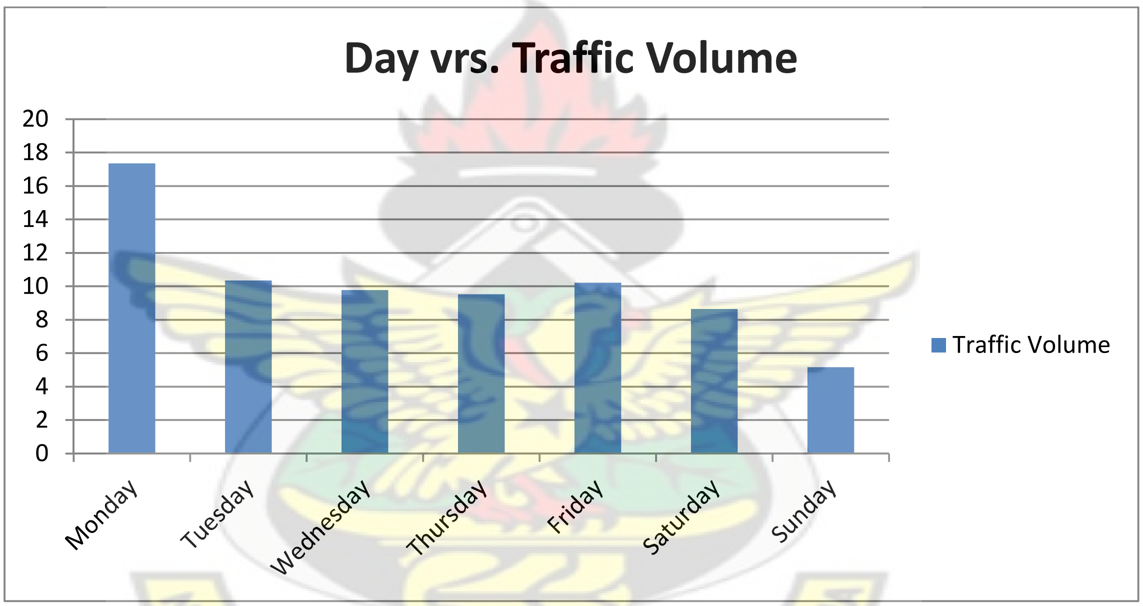


Figure 5.4: Average Day of Week Traffic Volume for Adum Sector B.

Table 5.2: Average Day of Week Traffic Volume for Nhyiaeso Sector B

Day	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday	Sunday
Traffic	2.81	2.39	2.45	2.58	2.49	2.97	2.91

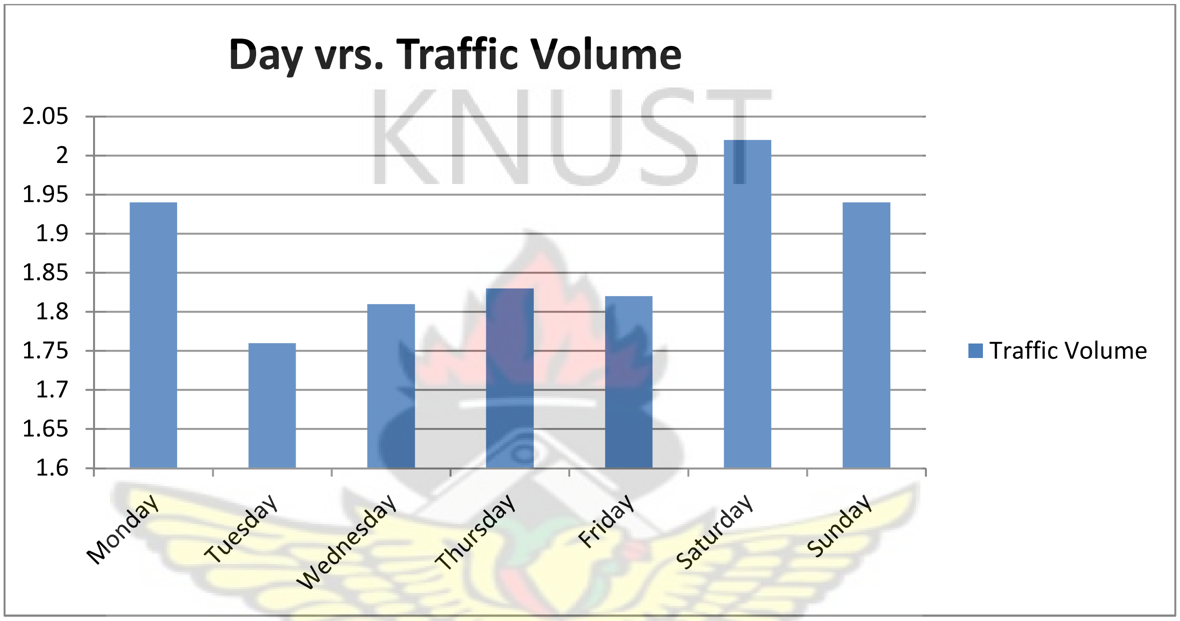


Figure 5.5: Average Day of Week Traffic Volume for Nhyiaeso Sector B

Table 5.3: Average Day of Traffic Volume for Aborfor Sector A.

Day	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday	Sunday
Traffic	3.33	3.33	3.22	3.37	3.42	3.67	3.38

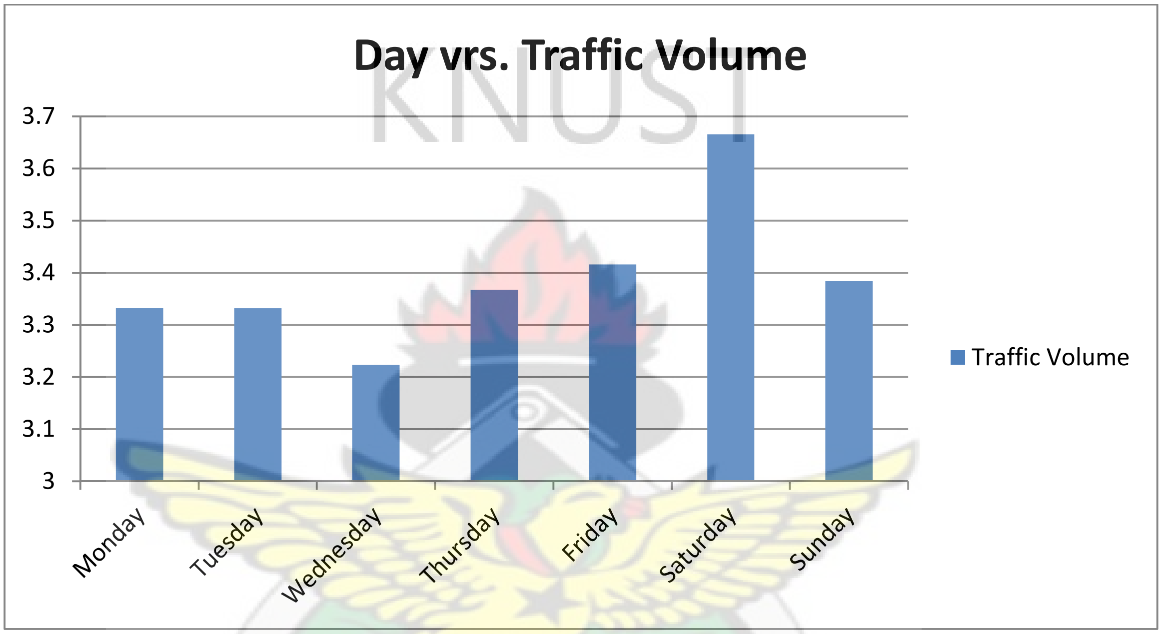


Figure 5.6: Average Day of Week Traffic Volume for Aborfor Sector A.

5.2 Performance of the Proposed Model

Rarely do the results predicted by deterministic models match the observed results [5].The performance of the prediction models mainly depends on the accuracy of these models. To ensure that the model described in Chapter 3 represents the reality sufficiently well, it needs to be validated. Results of model validation and other performance measures of the proposed model are presented in Chapter 5 of this thesis.

Based on the data described in the previous sections of the chapter, we proceed to give a brief detail on some key model performance evaluation criteria. The commonly used criteria in evaluating the performance of the proposed Kalman filter prediction model are discussed briefly.

5.2.1 Mean Absolute Percentage Error (MAPE)

The first criterion used is Mean Absolute Percentage Error (MAPE) as defined in Equation (5.1) **[34]**.

$$MAPE = \frac{1}{N}\sum_{j=1}^{N}\left|\frac{\widehat{tv}_j - tv_j}{tv_j}\right| \times 100\% \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots.. \quad (5.1)$$

where

$\widehat{tv}_j$ is the predicted traffic at time index j

tv_j is the measured traffic volume at time index j

N is the number of iterations or number of traffic volume data samples used

5.2.2 Root Mean Square Error (RMSE)

The second performance evaluation technique used is Root Mean Square Error (RMSE) as defined in Equation (5.2) **[34]**

$$RMSE = \sqrt{\frac{1}{N}\sum_{j=1}^{N}(\widehat{tv}_j - tv_j)2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (5.2)$$

CHAPTER SIX: RESULTS AND ANALYSIS

In this chapter, we present the results and analysis of the performance of the proposed Kalman filter prediction model of the study.

6.1 Data Description

Table 6.1: Data description for Adum (commercial/urban area).

Data Set	Mean	STD	Cell ID
1	5.07	5.77	KP0308A
2	9.95	4.36	KP0308B
3	4.88	2.90	KP0308C

Table 6.2: Data description for Nhyiaeso (residential area).

Data Set	Mean	STD	Cell ID
1	2.36	0.35	KP0344A
2	1.91	0.23	KP0344B
3	2.71	0.35	KP0344C

Table 6.3: Data description for Aborfor (rural area).

Data Set	Mean	STD	Cell ID
1	3.44	0.37	AR0628A
2	2.85	0.29	AR0628B
3	1.63	0.12	AR0628C

6.2 Model Validation Results

The Kalman filter model developed in Chapter 3 of this study needs to be validated in order to ascertain its representation of reality. Based on the data sets described in Chapter 5, we used one of the three model validation techniques presented earlier in Chapter 2 of this study. The model validity from the graphical comparison technique can be inferred as follows:

6.2.1 Model Validation with Direct Graphical Comparison

We followed the procedures of the graphical comparison approach to model validation discussed earlier in section 2.6.1 of Chapter 2. As a result, we plot the observed (Exact), the initially predicted (a Priori) and the updated (a Posteriori) traffic volumes shown in Red, Blue and Green respectively on the graphs below. We used three different scenarios with varying values of R, P_o, X_0 and Q representing: covariance of measurement noise, initial estimate for the estimation error, a priori estimate of the traffic volumes, and the covariance of process noise respectively. And for each of these scenarios we presented three graphs representing the three locations chosen for data collection in this study. We now present the graphs with their corresponding interpretations after running the MATLAB program codes presented in Appendix A.

6.2.1.1 Scenario 1($R = 0.1$, $P_o = 1$, $X_0 = 0$ and $Q = 0.5$)

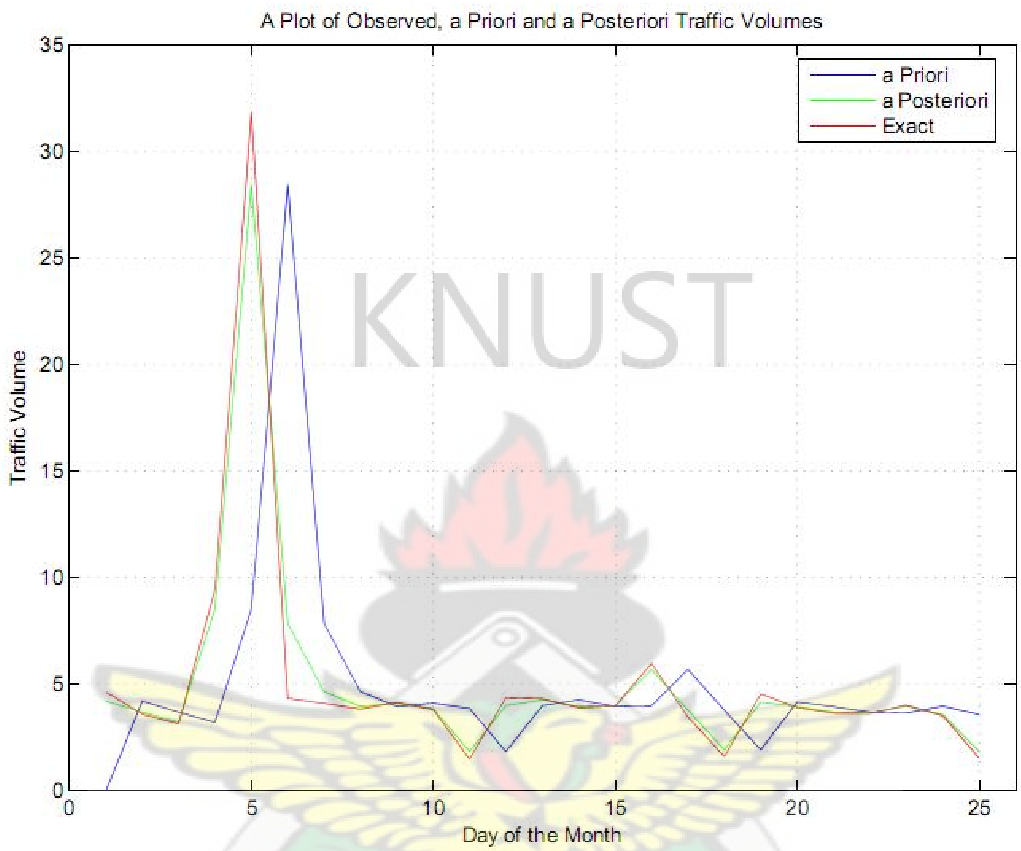


Figure 6.1: Observed, a Priori and Predicted Traffic Volume for Adum Sector A.

From the plot above, we make the following conclusion: A closer look at the plot indicates that, although the a priori estimate deviates a little bit from the observed traffic volume, the a posteriori estimates however, gives a value that compares creditably with the observed traffic volumes. This indicates that, the initial estimates are corrected using the relevant Kalman equations. We therefore conclude that, the model sufficiently represents a reality based on the graphical display shown in Figure 6.1 above.

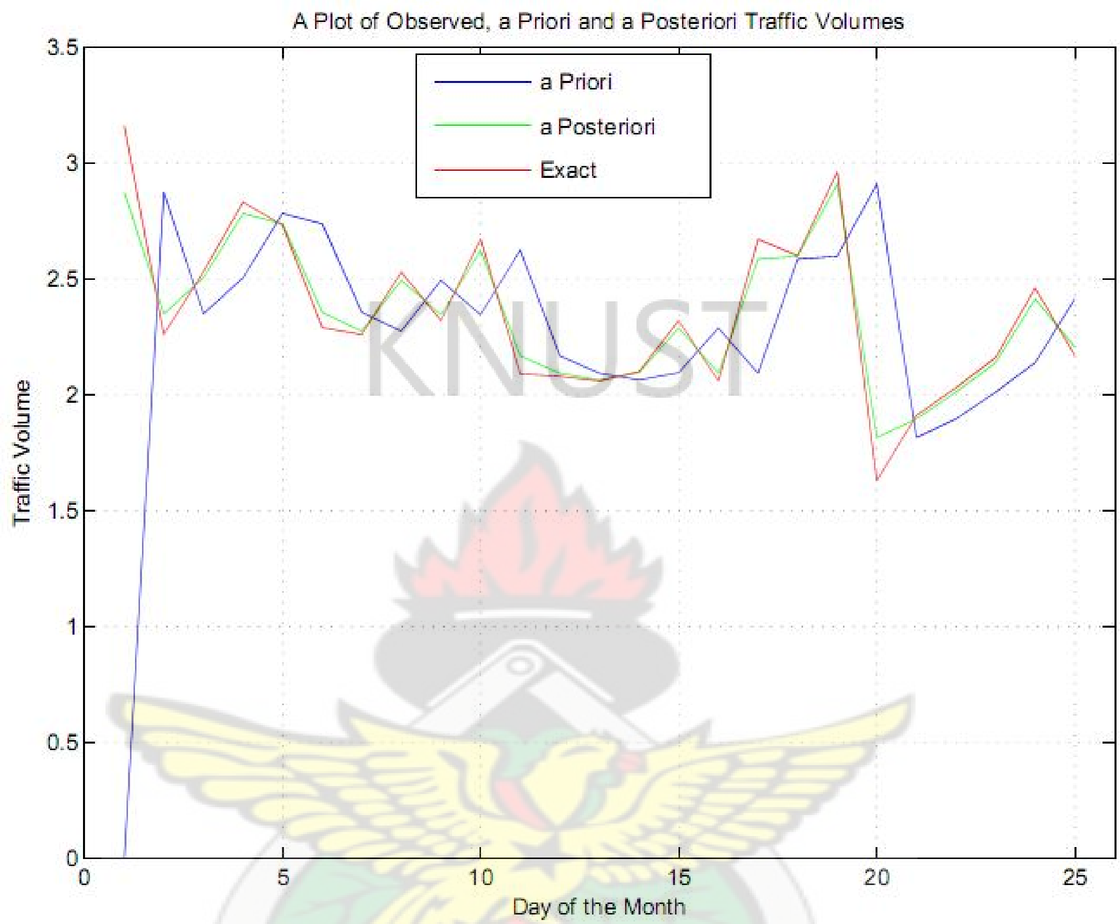


Figure 6.2: Observed, a Priori and Predicted Traffic Volumes for Nhyiaeso Sector A.

Though the a priori traffic volume shown in blue deviates from the exact traffic volume shown in red, the a posteriori traffic volume shown in green gets closer and closer to the observed traffic volume over time. This indicates that, the Kalman filter model's performance gets better as the number of iterations increases. The model's performance can be said to be quite accurate.

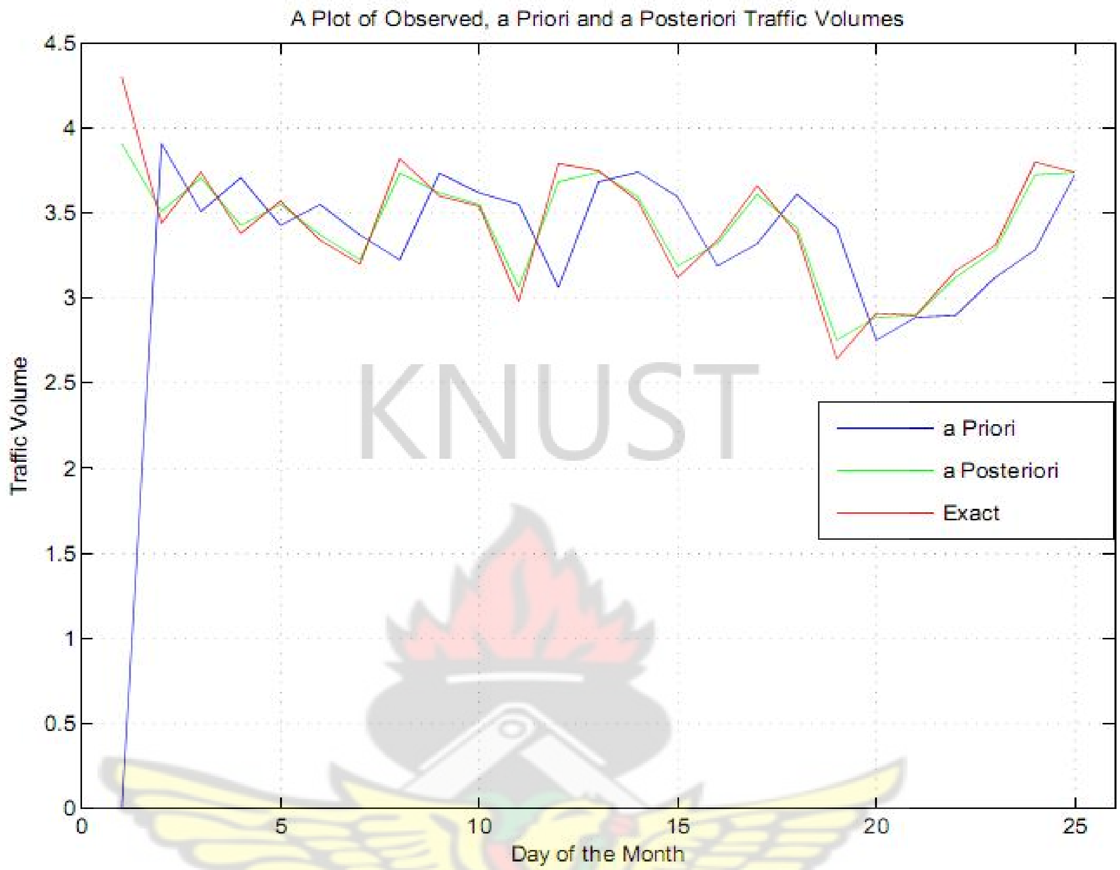


Figure 6.3: Observed, a Priori and Predicted Traffic Volumes for Aborfor Sector A.

A closer look at the plot shown in Figure 6.3, the predicted traffic volume does not deviate so much from the observed or the measured traffic volume. Though the model may need a little adjustment of some parameters to attain perfection, one can conclude that, the model is very reliable for traffic volume prediction in UMTS networks on short term basis.

6.2.1.2 Scenario 2 (R = 0.05, P_o = 0.5, $X_0 = 0.5 * Z_1$ and Q = 0.5)

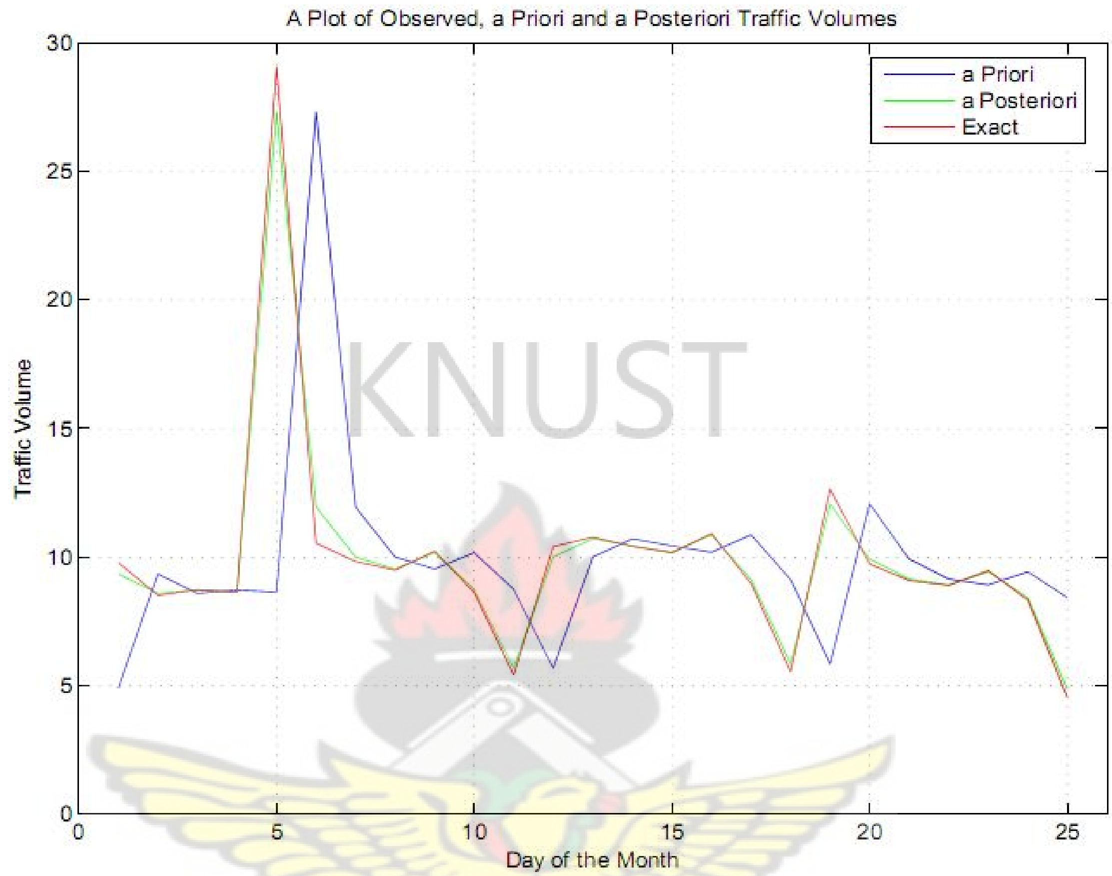


Figure 6.4: Observed, a Priori and Predicted Traffic Volumes for Adum Sector B.

The display from this scenario from all indications really brings a great improvement upon the first scenario. The a posteriori traffic volumes shown in green and the observed traffic volumes shown in red are almost the same except for some minor deviations. The model parameters chosen in this scenario improves greatly upon its performance.

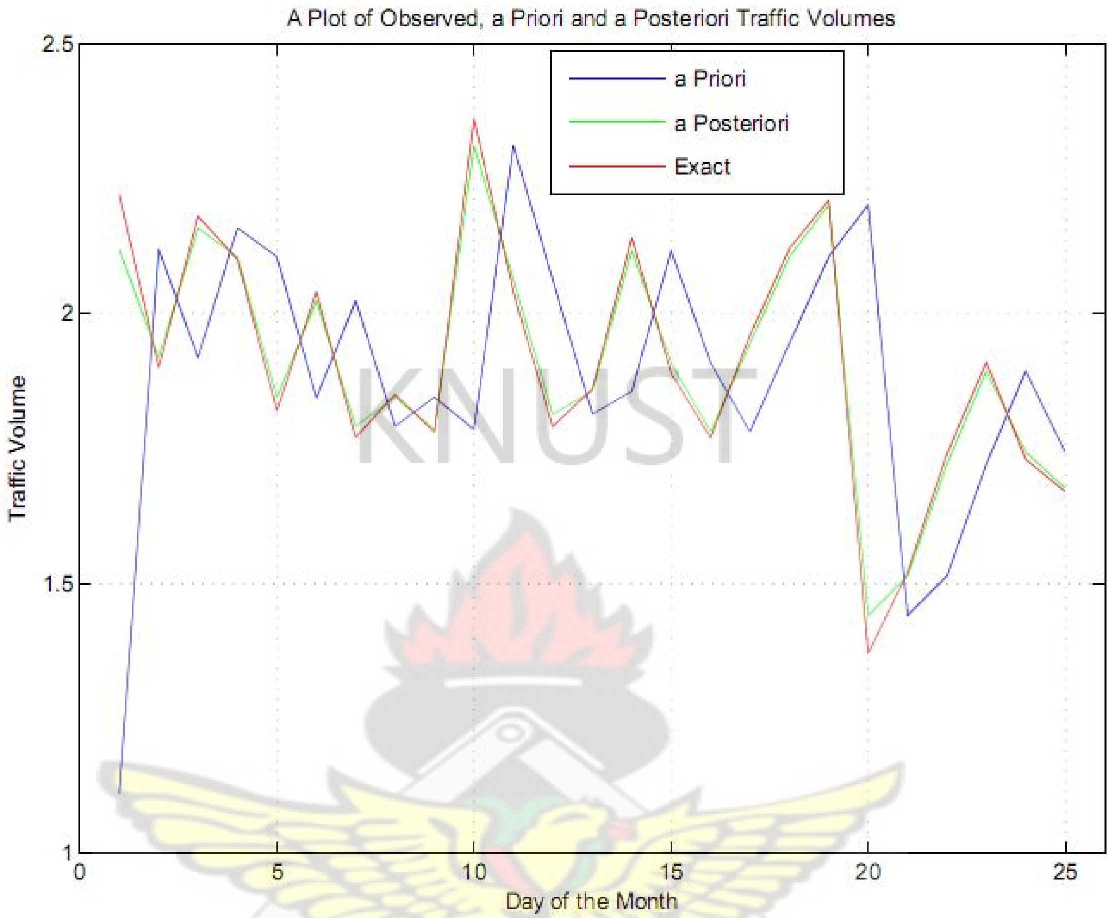


Figure 6.5: Observed, a Priori and Predicted Traffic Volumes for Nhyiaeso Sector B.

The traffic volume prediction pattern shown on this plot is not so different from the one shown in Figure 6.4. We can therefore conclude that, the predictive power of the proposed model is quiet consistent. This is so because; the traffic characteristics of rural and residential areas were not so different in this study.

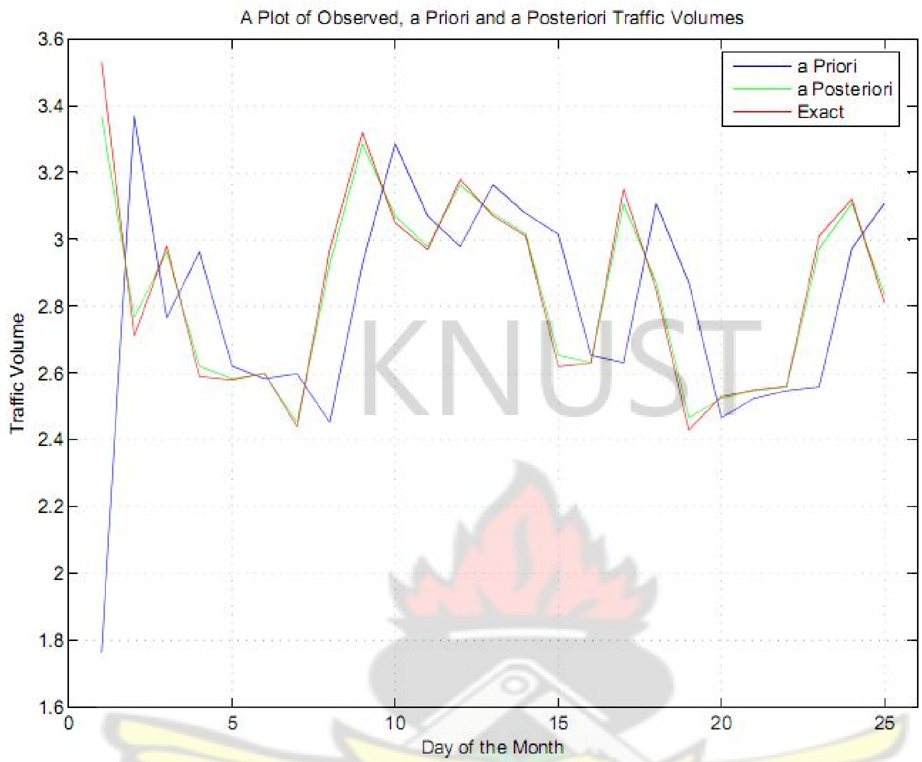


Figure 6.6: Observed, a Priori and Predicted Traffic Volumes for Aborfor Sector B.

The pattern shown in the figure above is not so different from those that are described already under this same scenario. The model can be said to be capable of predicting with high accuracy.

6.2.1.3 Scenario 3 ($R = 0.25$, $P_o = 0.005$, $X_0 = Z_1$ and $Q = 0.5$)

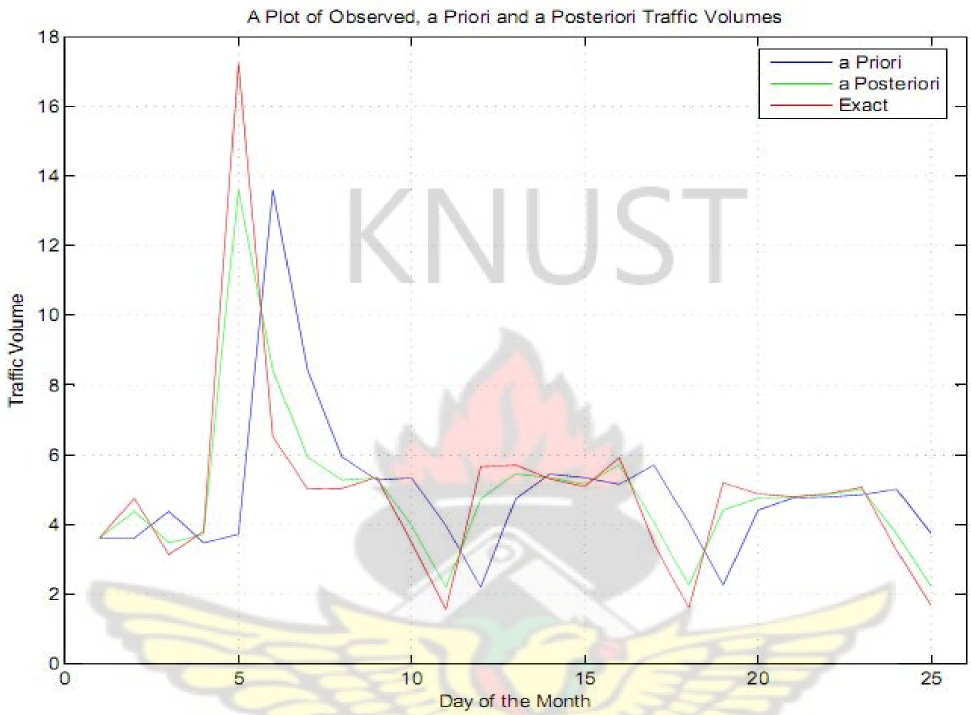


Figure 6.7: Observed, a Priori and Predicted Traffic Volumes for Adum Sector C.

Although, there are some deviations along the line, the model's performance under this scenario is not a bad representation of reality.

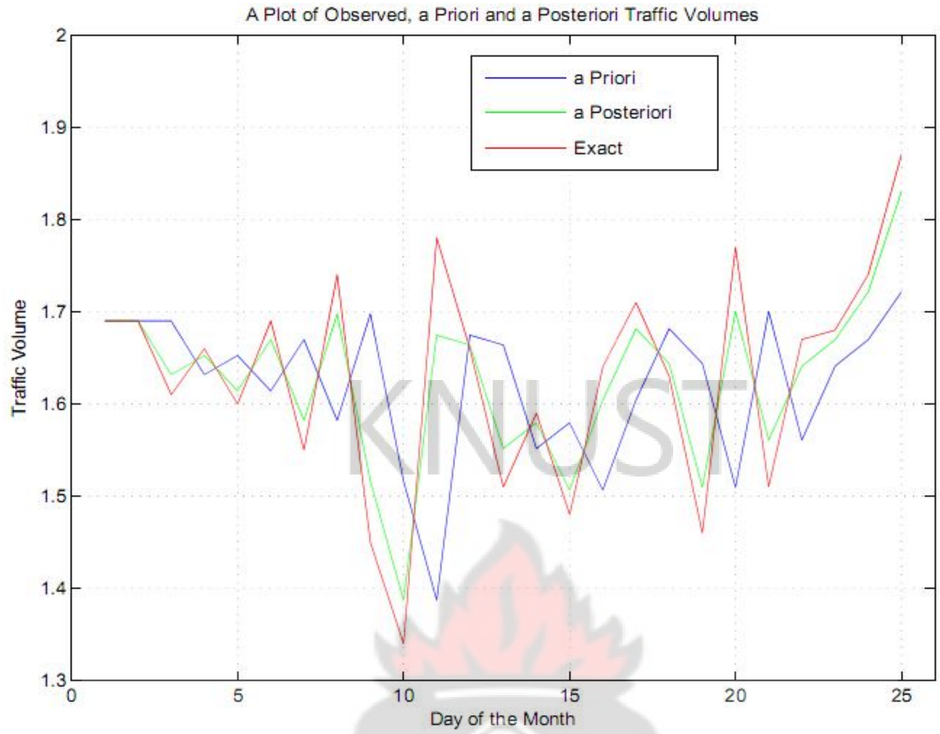


Figure 6.8: Observed, a Priori and Predicted Traffic Volumes for Nhyiaeso Sector C.

The plot shows some consistency at the initial stages of the prediction process but, deviated at the middle of the process. However, the model's performance improves towards the end of the process signifying that, the model's performance improves as the number of iterations increases.

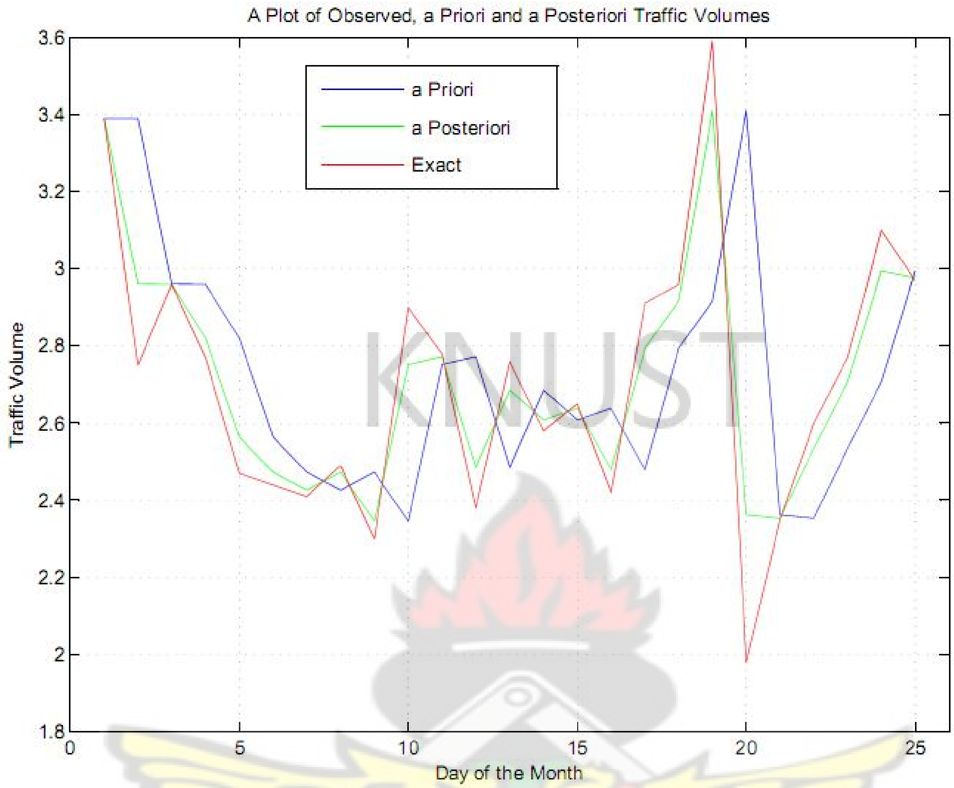


Figure 6.9: Observed, a Priori and Predicted Traffic Volumes for Aborfor Sector C.

Based on the graphs shown for all the scenarios under the graphical comparison approach of model validation, the model sufficiently represents reality. The model is therefore useful for traffic volume prediction in UMTS networks on short term basis.

6.2.2 Model Validation with the r^2 Approach

We run the various data sets in MATLAB in order to compute r^2 (a measurement of the model validity). The computation of the r^2 is based on the assumption that, the probability density function of the final configuration is normally distributed. Key performance indicators of model validation and sensitivity analysis are presented in the tables below representing each of the three scenarios used under the graphical comparison approach.

6.2.2.1 Scenario 1 (R = 0.1, P_o = 1, X_0 = 0 and Q = 0.5)

For Scenario 1, we set the covariance of measurement error to 0.1 (i.e. R=0.1), the covariance of estimation error to 1(P_0 = 1), a priori estimate of system's state to 0 (i.e. X_0= 0) and the covariance of process error to 0.5(i.e. Q= 0.5).

The table below shows the results obtained from model validation using the r^2 approach under scenario 1.

Table 6.4: Model Performance for Scenario 1

Location	Mean(μ)	Std(σ)	RMSE	MAPE	r^2
Adum	5.07	5.77	1.02	9.42	0.97
Nhyiaeso	2.36	0.35	0.08	2.17	0.93
Aborfor	3.44	0.37	0.09	1.59	0.91

6.2.2.2 Scenario 2 (R = 0.05, P_o = 0.5, X_0 = 0.5 * Z_1 and Q = 0.5)

For Scenario 2, we set the covariance of measurement error to 0.05 (i.e. R= 0.05), the covariance of estimation error to 0.5 (i.e. P_0 = 0.5), a priori estimate of system's state to half of the first value of the observed traffic volume (i.e. X_0 = 0.5 * Z_1) and the covariance of process error to 0.5(i.e. Q = 0.5). Z_1 is the first value of the observed traffic volume.

The table below shows the results obtained from model validation using the r^2 approach under scenario 2.

Table 6.5: Model Performance for Scenario 2

Location	Mean(μ)	Std(σ)	RMSE	MAPE	r^2
Adum	9.95	4.36	0.49	2.53	0.99
Nhyiaeso	1.91	0.23	0.03	1.14	0.98
Aborfor	2.85	0.29	0.04	0.85	0.97

6.2.2.3 Scenario 3($R = 0.25$, $P_o = 0.005$, $X_0 = Z_1$ and $Q = 0.5$)

For Scenario 3, we set the covariance of measurement error to 0.25 (i.e. R= 0.25), the covariance of estimation error to 0.005 (i.e. $P_0 = 0.005$), the a priori estimate of system's state to the first value of the observed traffic volume (i.e. $X_0 = Z_1$) and the covariance of process error to 0.5 (i.e. Q = 0.5).

The table below shows the results obtained from model validation using the r^2 approach under scenario 3.

Table 6.6: Model Performance for Scenario 3

Location	Mean(μ)	Std(σ)	RMSE	MAPE	r^2
Adum	4.88	2.90	0.93	12.05	0.89
Nhyiaeso	2.71	0.35	0.11	2.92	0.87
Aborfor	1.63	0.12	0.04	1.95	0.89

From the tables above, we conclude that the performance of the model is of high accuracy based on the various values produced for r^2 in the tables. The second scenario gives the highest performance index of the model.

With the values of r^2 ranging between 0.87 and 0.99, we can confidently say that the predictive power of the model can be trusted under the above model parameter values.

We bring the entire study to a conclusion and make some recommendations for future research in the subsequent Chapter.

CHAPTER SEVEN: CONCLUSION AND RECOMMENDATIONS

In this chapter we bring the study to a conclusion by giving a summary of the entire study, major contributions by this research and directions for future research.

7.1 Summary

This study investigates modeling and validation of a Kalman filter-based model for short term traffic volume prediction in UMTS networks. The Kalman filter-based models were used in traffic volume prediction by many researchers in the field of traffic volume prediction. These researchers used the Kalman filter-based models predominantly in the area of transportation engineering. In this study, we adapted the Kalman filter-based model to suit the characteristics of short-term traffic volume for UMTS networks. For the adaptation to be effective, and represent our system to suit the Kalman filter algorithm, we made the following assumptions:

(a) For short term forecasting, the state variable transitions may be regarded as a smooth process.

(b) A linear relationship may be assumed between traffic volumes for the current time step and traffic volumes for previous time steps.

Inferring from the assumptions stated above the system was thus modeled in the equation below: $\boldsymbol{tv_k = TV_k\theta_k + \ \varepsilon_k}$ as already explained in Chapter 4 of this thesis. The model of the system was carefully studied and converted into pseudo codes and a comprehensive flowchart. We finally implemented the model using MATLAB as a programming language. The MATLAB codes for the full implementation of the model are shown in Appendix A. Traffic volume data collected from Adum, Nhyiaeso and Aborfor all in and around the Kumasi metropolis of the Ashanti region of Ghana were used to test the performance of the model

The model thus adapted and implemented, was also validated using the graphical and. r^2 approaches to model validation. The graphical comparison to model validation gave very good performance values indicating that, the model could be used to predict real traffic volume sufficiently. This is further deepened by the results obtained using the r^2 approach to model validation. r^2 value as high as 0.99 indicating 99% accuracy was recorded under some set of model parameters. We made few adjustments to the values of some key model parameters and ascertained their sensitivity. This was done by varying the values of R, P_o, X_o and Q representing covariance of measurement noise, initial estimate for the estimation error, a priori estimate of the traffic volumes, and the covariance of process noise respectively during the model validation scenarios illustrated in Chapter 6 of this thesis.

7.2 Major Contributions

This study makes some significant contribution to knowledge in the area of short-term traffic volume prediction using a Kalman filter-based model for UMTS networks

The Kalman filter theory has been explained very well without complex mathematical concepts. This study also brought to the fore that, Kalman filter based models can be adapted and implemented for traffic volume prediction for UMTS networks. It is also very important to ascertain the accuracy of models by performing a rigorous model validation with live traffic volume data using appropriate techniques. Traffic volume data for testing and validating traffic volume prediction models must represent residential, urban/commercial and rural areas for a total representation of subscribers' traffic

We therefore conclude that, the model adapted and validated in this study has a highly accurate predictive power. Further adjustments to key model parameters such as varying the values of R, P_o, X_o and Q representing covariance of measurement noise, initial estimate for the estimation error, a priori estimate of the traffic volumes, and the covariance of process noise respectively would fine tune its performance, and we are very confident that the model performs very well in short-term traffic volume prediction in UMTS networks. This model is therefore recommended for use with little fine tuning for short-term traffic volume forecasting during cell planning for telecommunication networks.

7.2 Future Directions

We adapted, modeled and implemented in MATLAB a Kalman filter-based model for short-term traffic volume prediction for UMTS networks. The model thus adapted was validated with data sets from three different operating areas; Nhyiaeso, Adum and Aborfor all in the Kumasi metropolis of the Ashanti region of Ghana representing residential, urban/commercial and rural areas respectively. We did little work on the sensitivity of model and network parameters on the performance of the model.

Future works in this area of this research could consider a more rigorous sensitivity analysis of all model parameters as well as the network parameters and measure their impacts on the performance of the model. The data set could be extended to cover all the operating regions of telecommunications operators in Ghana to further the performance of the model. Finally, the model could also be adapted for traffic volumes exhibiting long-term characteristics as well.

References

[1]. Adas, A., "Traffic models in broadband networks", *IEEE Comm. Mag*., vol. 35, pp. 82-89. July 1997.

[2]. B. Walke, P. S. (2003). *UMTS - The Fundamentals.* John Wiley & Sons, Ltd.

[3]. Cox, C. (2008). *Essentials of UMTS.* New York: Cambridge University Press.

[4]. Digalakis, V,et al(1993). ML estimation of stochastic linear system with the EM algorithm and its application to speech recognition. IEEE Transactions on Speech and Audio Processing,1(4):431-442.

[5]. Douglas D.Mooney, R. J. (1999). *A Course in Mathematical Modeling.* USA: Mathematical Association of America.

[6]. Gopalakrihnan, S.(2000). *Prediction of Short-Term Traffic Volume for Applications in Intelligent Transportation Systems*, MSc. Thesis, University of Regina: Canada

[7]. Gowrinshankar, S.(2008), A Time Series Modeling and prediction of wireless Network Traffic' pages 40-52 [online].Available at: http://gesj.internet-academy.org.ge/download.php?id=1397.pdf&t=1 , Last accessed: 7th November, 2010.

[8]. Gary, B., Welch, G. "An Introduction to the Kalman Filter", University of North Carolina at Chapel Hill, TR 95-041, 24-July 2006.

[9]. Hills, G. R., and Trucano, T. G., "Statistical Validation of Engineering and Scientific Models: Background", SAND99-1256, 1999.

[10]. http://www.huawei.com/technologies_az.do Last accessed: 18th July, 2010

[11]. http://www.umtsworld.com/technology/overview.htm Last accessed: 16th July, 2010

[12]. http://www.3gpp.org

[13]. http://www.en.wikipedia.org

[14]. Iarina, M. et al., 'A Comparative study of the Statistical Methods Suitable for Network Traffic Estimation' [online].Available at: www.wseas.us/e-library/conferences/2009/rodos/ COMMUNICATIONS /COMMUNICATIONS15.pdf , Last accessed: 14th October, 2010.

[15]. Jian, C., et al., "Approaches for Model Validation: Method and Illustration on a Sheet Metal Flanging Process". *ASME* , vol. no.128, pp. 588-597, May 2006.

[16]. Klemm, A., et al. 'Traffic Modeling and Characterization for UMTS Networks', [Online], Available at: http://citeseerx.ist.psu.edu/viewdoc/download?doi=10.1.1.4.2737&rep=rep1&type=pdf Last accessed: 17th October, 2010

[17]. Leland,W.E. et al., "On the self-similar nature of Ethernet traffic (Extended Version)", *IEEE/ACM Trans. Net*., vol.2, no.1, pp. 1-15, Feb 1994

[18]. McCarthy, A.M and Broome L.S , A method for validating stochastic models of population viability: a case study of the mountain pygmy-possum (Burramys parvus)

[19]. Mingook Kim, CAPTAIN, ROKA "STOCHASTIC ESTIMATION AND CONTROL OF QUEUES WITHIN A COMPUTER NETWORK" MSc. Thesis

[20]. N.R. *St-Pierre* Validating Mathematical Models of Biological Systems: Application of the Concordance Correlation Coefficient

[21]. Oberkampf, W. L., and Trucano, T. G., "Validation methodology in computational fluid dynamics", AIAA 2000-2549, 2000, pp. 1-33.

[22]. Oravec, M, et al. (2008), 'Video Traffic Prediction Using Neural Networks' [online], Available at: http://Uni-obuda.hu/journal/Oravec_Petras_Pilka_16.pdf, Last accessed: 27th July, 2010

[23]. Qi, Y.(2010). *Probabilistic models for short term traffic conditions prediction*, Ph.D. Thesis, Louisiana State University: U.S.A

[24]. R.P., EDWARDS R.M. "Kalman filter application for distributed parameter estimation in reactor systems" Nuclear Science and Engineering, July 1996

[25]. Rick Middleton. “Applications of the Kalman Filter Algorithm to Robot Localization and World Modelling” Electrical Engineering Final Year Project 2002, University of Newcastle, 2002

[26]. Sang, A. and Li,S., “A predictability analysis of network traffic”, *Computer networks*, vol. 39, pp. 329345, 2002.

[27]. Shu Y., et al., “Traffic prediction using FARIMA models”, *ICC’99*,[Online] Available at: Available at: http://ieeexplore.ieee.org/iel5/6202/16571/00765402.pdf, *Last accessed:* 27th October, 2010.

[28]. Simon, D. (2001), ‘Kalman Filtering’ [online]. Available at: http://calpso.inesc-id.pt/FCUL/psm/docs/kalman-dan-simon.pdf. Last accessed: July 17, 2010

[29]. Sung-Joo, Won, You-Jip, Seong, Byeong-Chan “On-line Prediction Algorithm for Non-stationary VBR Traffic” Journal of KISS:Information Networking, 2007

[30]. Svoboda, P. et al. ‘Forecasting of Traffic Load in a Live 3G Packet Switched Core Network’ [Online]. Available at:http://ieeexplore.ieee.org/iel5/4599592/4610700/04610775.pdf, *Last accessed:* 27th October, 2010.

[31]. Wang, Y.,et al., ’Motorway Traffic State Estimation based on Extended Kalman Filter’[Online]. Available at: http://www.google.com.gh/url?sa=t&source=web&cd=1&ved=0CBUQFjAA&url=http%3A%2F%2Fwww.nt.ntnu.no%2Fusers%2Fskoge%2Fprost%2Fproceedings%2Fecc03%2Fpdfs%2F329.pdf&rct=j&q=Motorway%20traffic%20state%20estimation&ei=gS_lTMPSGILP4wa9x6jwDg&usg=AFQjCNETj69lGoEAc7DR21IWHyxC-X1AQA&cad=rja . Last accessed 17th November, 2010.

[32]. Wei Chen, MODEL VALIDATION VIA UNCERTAINTY PROPAGATION AND DA TRANSFORMATIONS * Integrated Design Automation Laboratory (IDEAL) Department of Mechanical Engineering Northwestern University Lusine Baghdasaryan

[33]. Xia, J. and M. Chen, “Dynamic Freeway Corridor Travel Time Prediction Using Single Inductive Loop Detector Data”, Transportation Research Board, Transportation Research Board 88th Annual Meeting, Washington D.C., 2009.

[34]. Xie, Y., Zhang, Y., and Ye, Z., "Short-Term Traffic Volume Forecasting Using Kalman Filter with Discrete Wavelet Decomposition", Transportation Research Board, Transportation Research Board 85th Annual Meeting, Washington D.C., 2006.

[35]. Zhou B. et al., 'Network Traffic Modeling and Prediction with ARIMA/GARCH', *Third International Working Conference: Performance Modeling and Evaluation of Heterogeneous Networks*, IIkley, West Yorkshire, U.K., July 18 – 20 , 2005. [online].Available at: http://citeseerx.ist.psu.edu/viewdoc/download?doi=10.1.1.107.3031&rep=rep1&type=pdf , Last accessed: 7th November, 2010.

APPENDIX A: MATLAB Codes

```
%*********************************************************************************
%***MALLAB CODES FOR IMPLEMENTING KALMAN FILTER PREDICTION MODEL *****
%*********************************************************************************
%Declaration of Variables

%Initialization of Key Parameters
a = 1;
u = 0;
h = 1;
R = 0.1; %0.25; %0.05; %0.1;
Q = 0.5;
%K = 4.62;

Z =
[9.78,8.51,8.73,8.63,29.03,10.54,9.83,9.49,10.23,8.62,5.41,10.4,10.77,10.41,10.17,10.92,8.97,
5.54,12.64,9.73,9.08,8.9,9.47,8.33,4.54];
nlen = length(Z);
zmean = mean(Z);
zstd = std(Z);

ME = 0;
RM = 0;
errorsquare = 0;
devsquare = 0;
%Estimation of Initial state of the system
X_aposteriori_0 = 0
X_apriori(1) = X_aposteriori_0;

%Estimation of Initial error covariance of the system
P_aposteriori_0 = 1;
P_apriori(1) = P_aposteriori_0;

%Computation of Initial Kalman Gain
```

```
if((P_apriori(1)== 0)) % Guarding against Division by Zero

     K(1) = 0;

 else

     K(1) = P_apriori(1)/(P_apriori(1) + R);
end

%Computation of the a Posteriori State of the system
X_aposteriori(1) = X_apriori(1) + K(1) * (Z(1) - X_apriori(1));

P_aposteriori(1) = P_apriori(1) * (1 - h * K(1));

residual(1) = abs(Z(1) - X_aposteriori(1));

%Perform iterations to get the rest of the system values.
j = 2;

for j = 2 : nlen,

  %Calculate the state and the output

  %x(j) = a * x(j-1) + w(j);
  %z(j) = h * x(j) + v(j);

  %Predictor equations
  X_apriori(j) = a * X_aposteriori(j-1);
  P_apriori(j) = a* a * P_aposteriori(j-1) + Q;

  %Corrector equations

 if((P_apriori(j)== 0)) % Guarding against Division by Zero

     K(j) = 0;
```

```
  else

      K(j) = P_apriori(j)/(P_apriori(j) + R);
 end

   X_aposteriori(j) = X_apriori(j) + K(j) *(Z(j) - X_apriori(j));
   P_aposteriori(j) = P_apriori(j) * (1 - h * K(j));
   residual(j) = abs(Z(j) - X_aposteriori(j));
   deviation(j) = Z(j) - zmean;

end

%Plotting of Appropriate Figures

j = 1: nlen;

%Plot States and State Estimates

figure;
h1 = plot(j,X_apriori,'b');
grid;
hold on;

h2 = plot(j ,X_aposteriori,'g');
grid;
h3 = plot(j, Z, 'r');
grid;
hold off;

%Make formatting
legend([h1(1) h2(1) h3(1)],'a Priori', 'a Posteriori', 'Exact');
title('A Plot of Observed, a Priori and a Posteriori Traffic Volumes');
ylabel('Traffic Volume');
xlabel('Day of the Month');
xlim = [0 length(j) + 1];
set(gca, 'XLim',xlim);
```

```
%Plot Covariance
figure;
h1 = plot(j,P_apriori,'b');
grid;
hold on;
h2 = plot(j ,P_aposteriori,'g');
grid;
hold off;
legend([h1(1) h2(1)],'a Priori', 'a Posteriori');
title('Calculated  a Priori and a Posteriori covariance');
ylabel('Covariance');
xlabel('Day of the Month');
set(gca, 'XLim',xlim);

%Plot errors
figure;
h1 = plot(j,Z - X_apriori,'b');
hold on;
h2 = plot(j ,Z - X_aposteriori,'g');
hold off;
legend([h1(1) h2(1)],'a Priori', 'a Posteriori');
title('Actual a Priori and a Posteriori error');
ylabel('Errors');
xlabel('Day of the Month');
set(gca, 'XLim',xlim);

%Plot Kalman gain, K
figure;
h1 = plot(j,K,'r');
hold on;
legend([h1(1)],'Kalman Gain');
title('Kalman Gain');
ylabel('Kalman gain K');
xlabel('Time Index');
set(gca, 'XLim',xlim);
```

```
for j = 1 : nlen % Computation of MAPE and RMSE
   ME = ME + abs(X_aposteriori(j) - Z(j))/Z(j);
   RM = RM + (X_aposteriori(j) - Z(j))^2;
   errorsquare = errorsquare + (residual(j))^2;
   devsquare = devsquare + (deviation(j))^2;
end
display(zmean);
display(zstd);
%COMPUTATION OF COEFFICIENT OF DETERMINATION(r2)
SSE = errorsquare;
SST = devsquare;
MAPE = ((1/nlen) * ME) * 100;
RMSE = sqrt(((1/nlen) * RM));
totalerror = SSE/SST;
rsquare = 1 - totalerror;
display(MAPE);  display(RMSE); display(SSE); display(SST);
display(rsquare); display('Observed Traffic'); display('Predicted Traffic');
display('Kalman Gain');display('Innovation'); display('Error Covariance');

for j = 1 : nlen % Displaying Actual and Predicted Traffic Volumes
   display(j);    display(Z(j));      display(X_aposteriori(j));
   display(K(j));     display(residual(j));
   display(P_aposteriori(j));
end
```