KWAME NKRUMAH UNIVERSITY OF SCIENCE AND TECHNOLOGY,

KUMASI – GHANA

COLLEGE OF SCIENCE

FACULTY OF PHYSICAL SCIENCES

DEPARTMENT OF MATHEMATICS

QUEUING THEORY

CASE STUDY: MERCHANT BANK GHANA LIMITED, KUMASI

BY

FRANCIS ADOMAKO

SEPTEMBER, 2009

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DECLARATION

I hereby declare that this submission is my own work towards the MSC and that to the best of my knowledge, it contains no material previously published by another person nor material which has been accepted for the award of any other degree of the University, except where due acknowledgement has been made in the text.

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DEDICATION

I dedicate this work to my entire family, the Adomako family. Most especially my dearest and supportive wife Regina Sasha Goodhead Adomako and my lovely kids Francis Kwadwo Adomako Jnr. And Amisha Emma Nana Yaa Adomako.

Thanks a lot for your support and understanding when the going was tough. May God bless you.

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CHAPTER ONE

INTRODUCTION

1.0 BACKGROUND OF THE STUDY

Recent papers in development economics and finance have began to assign an important role to remittances as key ingredients in the growth prospects of developing nations and having a potentially positive impact as a development tool for developing countries. Remittances are generally defined as that portion of migrants' earnings sent from the migration destination to the place of origin. Although they can also be sent in kind, the term "remittances" is usually limited to refer to monetary and other cash transfers transmitted by migrant workers to their families and communities back home. Remittances reflect the local labour working in the global economy and have been shown to explain partly the connection between growth and integration with the world economy. Remittances improve the integration of countries into the global economy. Remittances have for several generations been an important means of support for family members remaining at home. As migration continues to increase, the corresponding growth of remittances has come to constitute a critical flow of foreign currency into many developing countries and Africa in particular. Policy makers in developing countries have started to streamline financial systems, removing controls and creating incentives, with the aim of attracting remittances especially through official channels.

It's difficult to overstate the importance of remittance income to most African nations and many developing nations. Nworah cites a figure of $300 billion dollars sent from Diasporas to developing nations via remittance. In Africa, the amount of money remitted by diaspora workers - $17 billion per year - is larger than the amount of foreign direct investment in Africa, and rivals official development assistance grants or loans ($25 billion per year). In some African nations, remittance represent as much as 27% of the gross domestic product of some nations. According to the UN's Office of the Special Advisor on Africa, the average African migrant living in a developed nation is sending $200 per month home to his or her family.

Three streams of monetary transfers flowing into countries are included in remittances and published annually by the IMF in its Balance of Payments Statistics Yearbook. These are workers' remittances, compensation of employees and migrant transfers. The term "remittances" has however come to include more than the above in the eyes of a number of states, institutions and experts. For IOM purposes, migrant remittances are defined broadly as monetary transfers that a migrant makes to the country of origin. In other words, financial flows associated with migration. Most of the time, remittances are personal, cash transfers from a migrant worker or immigrant to a relative in the country of origin. They can also be funds invested, deposited or donated by the migrant to the country of origin. The definition could possibly be further broadened to include in-kind personal transfers and donations. International remittances received by developing countries are expected to reach US$ 167 billion in 2005 and have doubled in the last five years (World Bank, 2005). Migrant remittances constitute an important source of foreign exchange, enabling countries to acquire vital imports or pay off external debts. Remittances also play an important role in reducing poverty. There is growing awareness and evidence of the potential that remittances have to contribute to economic development in migrant-sending countries at the local, regional and national levels.

Recent global estimates show that, migrants' remittance flows have assumed a significant prominence. In the developing world, remittances now surpass Official Development Assistance (ODA) receipts (Ratha, 2003). Official development Assistance transfers to developing countries in 2001 stood at about US$52.3 billion (The World Bank, 2004). This figure compares with global remittance flow of about US$77.0 billion the same year, up from US$51.1 billion in 1995 (The World Bank, 2004).

The level of private unrequited transfers increased significantly from US$201.9 million in 1990 to US$1,017.2 million in 2003. Total transfers have increased from just over US$410 million to US$1,408.4 million over the same period reflecting mainly an increase in private unrequited transfers. Private unrequited transfers are estimated to be bigger and more stable than ODA2 and FDI flows into Ghana since 1990. Also positive, though relatively weak, correlation was found between remittances and ODA on one hand, and between remittances and FDI on the other hand, over the period 1990 to 2003. To assess the importance of remittances in the Ghanaian economy, the size of remittances relative to key macroeconomic variables were examined.

The recorded private remittance figures according to some analyst reflects only the "tip of the iceberg" since they do not include remittances sent through informal channels such as hand carriage, families, money couriers or network of informal transfer agents. Based on our estimates the reported figures could represent only about half the actual total. At least as much is transmitted through informal and unrecorded channels which make it impossible to measure the precise amount. However, despite such glaring evidence on the extent of the flow of remittances, gaps still remain in our understanding of how remittances are or can be used to promote development, especially given that existing policy incentives are not generally considered as having been very effective in channelling remittances towards development (Black, 2003). The appreciation of remittances as a development tool is recent and several questions on how best to capture their development impact remain.

Africa lags behind.

While remittances to developing countries have more than doubled in the last decade, they have grown little in Africa, the Bank notes. Total remittances to Africa amounted to nine billion dollars in 1990 and by 2003 had reached fourteen billion dollars and the continent receives about 15 per cent of flows to developing countries. Over the last decade, Egypt and Morocco have been the largest recipients on the continent and North Africa as a whole received more than 60 per cent of total transfers.

In sub-Saharan Africa, Nigeria is the largest recipient, taking between 30 and 60 per cent of the region's receipts. Though official figures are not available, economists believe that money sent home by Nigerians in various parts of the world now exceeds $1.3 bn annually, ranking second only to oil exports as a source of foreign exchange earnings for the country.

For some smaller economies, workers remittances account for a large chunk of national income. Lesotho receives the equivalent of between 30 and 40 per cent of its gross domestic product (GDP) from workers abroad, mainly in neighbouring South Africa. In Eritrea, the World Bank notes, remittances represented 194 per cent of the value of exports and 19 per cent of GDP. During the 1990s, remittances covered 80 per cent of the current account deficit of Botswana.

The challenge facing many African countries that receive substantial income from remittances is how to direct them into programmes that benefit society as a whole. At an African regional meeting of the Global Commission on International Migration (GCIM) held in Cape Town, South Africa, in March 2005, delegates agreed that while remittances could contribute to poverty reduction and development, countries in the region need to do more to enhance remittances' positive effects.

According to the World Bank, the developmental impact of remittances will depend on their continued flow, and that in turn will depend on the ease with which money can be transferred. The Bank estimates that if transaction costs were lowered even by 5 per cent, remittances to developing countries would increase by $3.5 bn a year. In many countries formal money transfers are expensive and at times heavily taxed. US researchers who have examined ways to reduce transfer fees report that average costs amount to 12.5 per cent of the sums transferred, amounting to $10–15 bn annually.

It would also be beneficial for African countries to promote the use of official transfer channels. They could do this by offering incentives to recipients to save more within the formal banking sector. To make formal transfers attractive, countries would also need to offer favourable exchange rates and establish efficient banking systems. In some countries formal banks exist only in urban areas, leaving rural dwellers with no choice but to depend on the informal sector.

1.1 MONEY SENT HOME BY MIGRANTS

1.1.1 Size of Remittance Flows

In principle, there are three ways of measuring remittance inflows in countries. The first approach is the balance of payments estimates, which has been presented in the earlier sections. Other methodologies include micro or household surveys of recipients of such flows e.g. inference from the Ghana Living Standard Survey (GLSS). The third method is through banks or financial institutions in origin countries i.e. focusing on resource transfer institutions. The size of the remittance flows presented here is based on BOP estimates made by Bank of Ghana. The fact that remittances are transmitted through different channels makes it difficult to capture the full amount in the balance of payments statistics of the recipient country, which tends to underestimate the actual flow of remittances. The problem makes it difficult to come up with strong conclusions on the role remittances play in the economy.

Table 1: Private, Official and Total Unrequited Transfers (US$ million) 1983 – 2003

Year	Official	Private	Total
1990	208.6	201.9	410.5
1991	202.4	219.5	421.9
1992	215.3	254.9	470.2
1993	256.2	261.2	517.4
1994	200.8	271.0	471.8
1995	260.0	263.2	523.2
1996	215.6	283.2	497.9
1997	169.7	406.8	576.5
1998	290.5	460.5	751.0
1999	158.0	479.0	637.9
2000	143.1	506.2	649.3
2001	216.1	717.3	978.5
2002	232.4	680.0	912.4
2003	391.2	1,017.2	1,408.4*

Source: Bank of Ghana BOP office
* Provisional*

According to the balance of payments data, total transfers to the Ghanaian economy ranged between US$400 million in 1990 and US$900 million in 2002. Of 12 the total transfers, private unrequited transfers increased from US$201.9 million to almost US$680 million in 2002. Recent estimates based on improved reporting by the financial houses engaged in money transfer suggest that private transfers have increased to about US$1.0 billion. Total transfers at end 2003 were estimated at US$1.4 billion according to the balance of payment estimates. Over the years, private unrequited transfer has gained significant importance in total unrequited transfers. It has actually recorded a persistent growth and clearly represents the main driving force behind the growth of total unrequited transfers. As portrayed in figure 1, while private unrequited transfer jumped up strongly between 2000 and 2003, official transfer rather followed a relatively downward pattern until it reversed to a sluggish upward trend between 2000 and 2003. Another line of analysis, which is worth outlining, is the relationship between private inward unrequited transfers, Overseas Development Assistant (ODA), and Foreign Direct Investment (FDI). These variables are considered in terms of their importance in Gross Domestic Product (GDP). Figure 2 outlines the graphical representation of private unrequited transfers, ODA and FDI each as a percent of GDP. The changing pattern of ODA as a percentage of GDP and the increasing pattern of private unrequited transfers could clearly be observed. Whereas private unrequited transfers maintained a relatively stable upward trend, ODA exhibited a rather unstable trend. Another interesting observation is that private unrequited transfers have over the years stayed above FDI as a percent of GDP.

As a percentage of key macroeconomic variables, remittances have gained economic importance over the years. As depicted in Figure 3 (see also Table 3 in the Appendix 2), remittances as a percentage of GDP increased from 2.24 percent in 1990 to almost 13.4 per cent by 2003. As a percentage of total exports, remittances rose from 22.0 per cent to 39.7 per cent over the same period. This observation suggests that over the period remittances have been increasing more than proportionately compared to GDP, exports and imports. The BOP estimates of private unrequited transfers have significant implications for Gross National Income (GNP) estimates.

Foreign inward remittances have been observed to be one of the least volatile sources of foreign exchange earnings for developing countries. While foreign capital flows tend to rise during favorable economic cycles and fall in bad times, remittances appear to react less violently and as such tend to exhibit remarkable stability over time. For example, remittances to developing countries rose steadily between 1998-2001 when private capital flows declined in the wake of the Asian financial crisis. Even the more stable component of capital flow-FDI and Official Flows-declined in 2000-01 while remittances have continued to rise. Another line of argument is that since remittances are part of current transfers, which is a function of income, it is likely to be less volatile compared to capital flows. Essentially, foreign investors base their investment decisions on pure profit motives, which are generally dictated by the business environment while migrants make their remittances on the basis of pure family ties and other economic commitments between family members. Using a simple historical volatility measures in the case of Ghana, remittance with an annual historical volatility coefficient of 0.21 appears less volatile compared to ODA with a volatility coefficient of 0.60 and FDI with a volatility coefficient of 0.61 over the period 1990-2003.

The literature on remittances has also identified the extent to which remittances correlate with other key macroeconomic variables. The correlation between worker remittances and other capital flows have been found to vary across countries. In this study, a positive correlation coefficient of 0.62 was observed between remittances and ODA and a coefficient of 0.53 was observed between remittances and FDI.

1.2 REGIONAL DISTRIBUTION OF RECEIPTS

In terms of the regional flows of remittances, the USA and Canada appears to be the most important source. Between March and June 2004, a total of about US$100.6m of inward remittances came from USA and Canada. This was followed by the United Kingdom which recorded about US$50.6m, the European Union US$25m, others US$9.2m ECOWAS US$5.8m, and the rest of Africa US$2.8m (See Table 3 and Figure 3).

Table 2: Sources of Private Inward Remittances Receipts (US$' million): March-June 2004

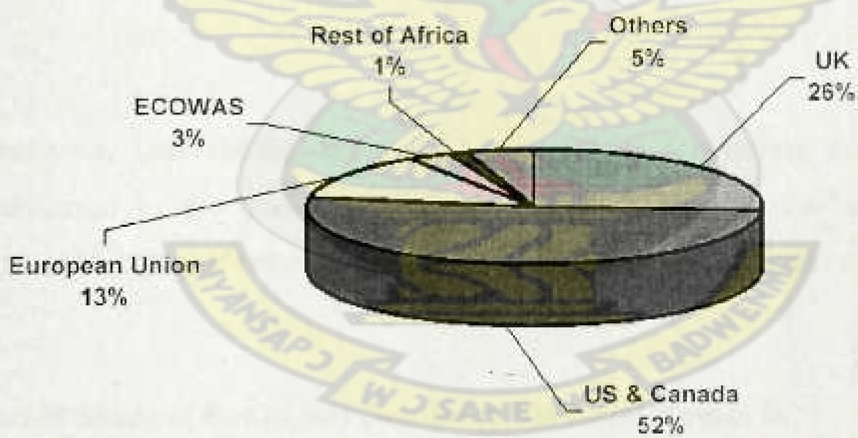
	March	April	May	June*	Total
United Kingdom	11.3	4.7	11.8	22.7	50.5
USA and Canada	29.7	23.5	24.0	23.5	100.7
European Union	7.2	6.1	4.7	7.4	25.4
ECOWAS	0.8	0.5	3.8	0.7	5.8
Rest of Africa	1.1	0.7	0.5	0.5	2.8
Others	4.3	1.6	1.5	1.7	9.1
Total	54.4	37.1	46.3	56.5	194.3

Source: Bank of Ghana, BSD

Note: *As at June 2004, some of the banks were still reporting their returns without indicating the sources.

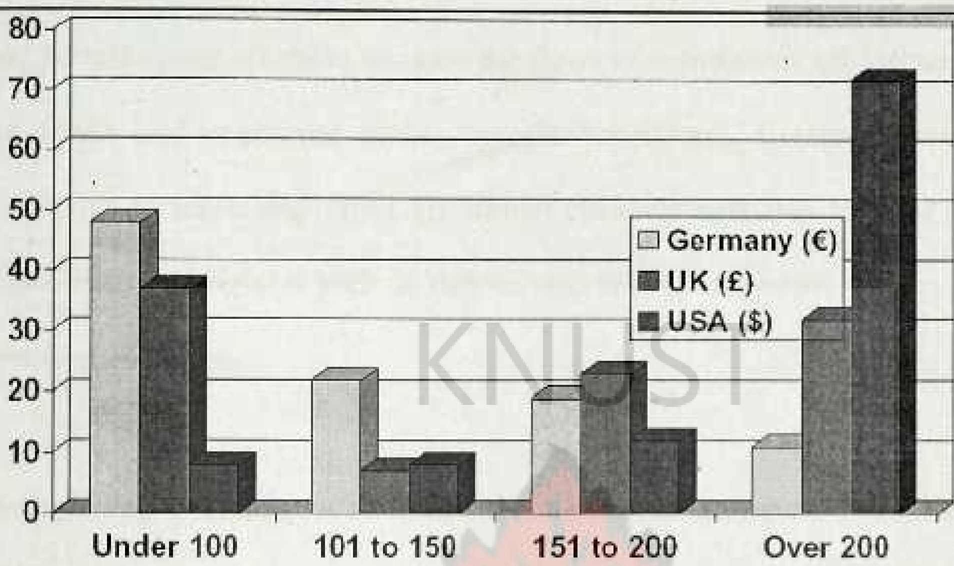
Figure 4.

Sources of Private Inward Remittances: March - June 2004



Within the formal financial set up, banks constitute the dominant channel through which inward transfers are effected. As represented in Table 4, the banks accounted for about 92 percent of private inward transfers between April and June 2004. This figure however compares with a figure of just about 8 percent representing the share of the non-bank financial institutions in the

Distribution of Amount sent by country and respective currency of origin (%)



Average sent:
USA, $ 380 [] Germany, €159 (US$225) [] UK £290 (US$510).

remittance business. One should, however, be careful in concluding that the banks channel is preferred to the non-bank channel. The share of the non-bank financial institutions could be higher if information about transfers through informal channels were available.

Table 3. Market Share of Banks and Non-Bank Financial Houses in Remittances: (US$'million): March-June 2004

	April	May	June	Total Q2
Bank	86.00	101.20	116.80	303.90
Non-Banks	8.50	8.80	8.50	25.90
Total	94.50	110.00	125.30	329.80

Source: Bank of Ghana BSD

1.3 COST OF TRANSFERS

1.3.1 Costs and Distribution Mechanisms

Better information on remittance products and costs will also help government and private sector efforts to increase the flows of remittances via formal channels (ie banks and registered money transfer operators). Creating environments conducive to increasing flows via formal channels will also increase access to other financial products such as deposit and savings accounts, thereby reducing financial exclusion.

Transmitting Operators involved in the transmission of migrant's remittances into Ghana include licensed businesses such as banks and national transfer companies, as well as large international businesses like Western union. The costs of transmitting money actually vary from country to country and also among type of institutions. They reflect the level of involvement of the banking industry and other businesses and the extent to which government involvement facilitated less expensive transfers.

The costs of sending money to recipient countries from the origin country of emigration reflect fees and the commission charged to convert the remittance into local currency. According to information obtained from domestic financial institutions representing the foreign MTOs these costs have decreased over time as remittance flows into Ghana increased. Transmissions by smaller (national) transfer companies currently cost between US$1.50 and US$3.00 for each US$100.00. The same service, however, attracts charges ranging between 1.5 -

2.5 per cent of the value in the case of banks and between 2.0 - 3.5 per cent in the case of major MTOs such as Western Union Financial. The banks and the smaller transfer companies are therefore more competitive than the major companies such as Western Union based on the fee structure done. Ghanaian banks involved in the remittance business also impose charges for paying inward transfers to recipient. Whereas most banks surveyeds do not charge for inward remittances paid out in cedis, those paid in foreign currencies attract high fees. Only two banks among the six banks surveyed charge for foreign transfer payments in cedis (Table 5).

It costs a lot of money to send money overseas. If you're lucky enough to be sending money from your bank account to a bank account in another country, the process is somewhat complex, but not very costly. But use a service like Western Union and you'll pay at least 6% of your money in fees - more, if you're sending small amounts of money. (According to the calculator on Western Union's site, sending \$200 to Nigeria will cost you \$12. Once past the \$12 minimum, WU charges 6% of the amount you're sending, up to \$1000, their maximum amount allowed for a first transfer. Less than \$200 also nets a \$12 charge. Other countries are more expensive - \$200 to Ghana or Mali costs \$22, or 11%.) An article by Dilip Ratha, a senior World Bank Economist, reports that 13% of the average remittance is claimed by transaction fees.

Table 4: Foreign Transfers Charges, June 2003 - Jan 2004

Bank		GCB	SCB	BBG	SSB	ADB	ECO
Inward transfers:							
Pmt in Cedis	Jan-04	Free	\$20 - \$500	Free	Free	Free	\$10
	Jun-03	Free	2.75%	Free	1%	Free	\$10
Pmt in foreign currency	Jan-04	3.25%	\$20 - \$500[2]	\$50	\$10	3%[1]	2.75%
	Jun-03	1%	2.75%	\$50	1.50%	3%	2.75%

Source: Survey Data and Banking Supervision Dept. BIOG

1. Free if the foreign currency is credited to customers accounts
2. Pmt in forex attracts additional 0.5 % on face value s. t. a min.\$20/£15/E20-Max \$500/£350/E500

Remittance receipts payments in foreign currency as at June 2004 attract charges ranging between 2.75 – 3.25 per cent of the face value. Some banks charge scaling fees that ranged between US\$10 – US\$500. In 2003, however, the charges were between 1 – 3 per cent (of face value) or a flat fee of US\$50. Payments in local currency in 2004 attract a minimum fee of US\$10 and a maximum US\$500 but vary across banks. These charges are considered rather high given the fact that transaction fees have already been paid by the remitting customer. Financial houses on the other hand do not charge for these services. This according to the banks is occasioned by the high cost of importing foreign currency, among other factors.

It should be noted, however, that for most banks if not all, inward transfer proceeds, credited to customers' accounts (Foreign) do not attract any charge.

Whereas banks use interbank foreign exchange rate for the conversion of remittance proceeds other financial houses use Forex bureau exchange rates, which are higher thus making them the preferred channel for remittance transfers for some market participants (e.g. for small transaction).

The cost of sending money transfers varies widely:

Cost of sending £100

Destination	**Number of providers**	**lowest percentage cost**	**highest percentage**
Bangladesh	13	2.5%	35%
China	15	5%	35%
Ghana	18	3%	35%
India	18	5%	40%
Kenya	15	5%	35%
Nigeria	15	5%	35%

Fees for sending money transfers of £100 range from £2.50 to £40. This implies a percentage cost of between 2.5% and 40%. Fees for sending £500 range from £4 to £40, giving lower percentage costs between 0.8% and 8%. This is because fixed charges are much higher in percentage terms for smaller transfers. This illustrates the relatively high cost that can be faced by low-income migrants wishing to send small amounts back to families and friends. A few banks and building societies surveyed (four out of the ten) also set minimum fees (the highest being £25). The exchange rate charged on money transfers that are collected in local currency can be a significant additional cost that is often not obvious to those sending money transfers. Money transfer providers tend to guarantee the exchange rate they offer, but not the amount that will be paid out to the receiver, as additional charges may be added at the recipient-end. UK banks and building societies tend to be more expensive for low-value transfers than money transfer operators, although there are exceptions. UK banks generally pitch their remittance services at higher value customers who hold accounts with them. The rates they offer on small money transfers tend to be higher than those offered by money transfer operators.

Money transfer operators (MTOs) tend to offer lower rates than banks for small remittances, as well as convenient services such as longer opening hours. Charges by money transfer operators such as First Remit, Chequepoint and Travelex for sending small remittances of £100 to countries such as Nigeria and Ghana are particularly low. Country-specific financial institutions also sometimes provide services that are cheap, convenient and appropriate to the needs of the customers. For example, Sonali Bank UK charges just £2.50 to send £100 to Bangladesh, and Express Funds and Samba International £3 to send the same amount to Ghana. ICICI Bank, in partnership with Lloyds TSB, as well as Remit2India, charges only £5 whether sending £100 or £500 to India.

1.4 TRANSFER COST OF SENDING REMITTANCES TO GHANA FROM THE U.K. AND THE U.S.

US	MTO	$300	(%)	$500	EXCHANGE RATE	FX COMMISSION
	Western Union	$24.00	8%	$39.00	8,989.00	0.67%
	Vigo	$12.00	4%	$22.00	9,004.00	0.51%
	MoneyGram	$9.99	3%	$9.99	8,995.87	0.60%
	Ria Envia	$15.00	5%	$25.00	8,990.00	0.66%
United Kingdom	MTO	£100	(%)		Exchange Rate	FX COMMISSION
	Express Funds	3	3%		15961	0.0%
	Western Union	14	14%		15864.14	0.28%
	MoneyGram	4.99	5%		15370.39	3.39%

POLICIES MEANT TO REDUCE TRANSFER COSTS

The first type of policy is directed to migrants themselves. Some of these policies are designed to encourage the participation of social organizations to function as exchange houses to reduce transfer costs. In some cases, there have been attempts to negotiate directly with money transfer firms for a reduction in transfer fees. Another policy instrument used in other countries is the promotion of cooperatives and foreign branches of national banks. One joint effort, known as the International Remittance Network (IRN), allows migrants to deposit remittances in United States and withdraw them in their country of origin. All these policy measures are meant to promote competition among transfer firms to reduce excessive transfer fees. A 1997 study by the Mexican senate estimated that the loss due to excessive charges and exchange rate manipulations by transfer firms was about 20 percent of the total flow of remittances. Western Union settled a suit brought by Mexican organization in United States for failure to disclose the true cost of transfers.

1.5 NEED FOR REMITTANCE SERVICES

Safety - If you send money through Western Union it's likely that the money will get to the intended target. Security is the overriding factor in choosing a money transfer provider. Speed and cost are traded off against this. A further decision factor is the distribution network in the receiving country.

Security and speed of transfer are key factors in provider choice. For this reason MTOs dominate the market for sending to countries where MTO brands are well known. By comparison, UK banks are considered highly secure but slow in sending transfers. Country-specific banks are only well trusted among Indians and Bangladeshis, and for them the poor service associated can represent a barrier. Informal providers, in the shape of relatives and friends traveling back home, are seen as an ideal solution for some remittance types, offering security and ultimate sending and receiving convenience for no commission. In contrast, there is ariness about other informal providers in the shape of agents, such as used in 'Chop' or 'Hawala'. These are long established, informal methods of money transfer. Typically, a customer contacts a broker in his or her country, and pays them the money to be transferred. The broker then contacts a broker in the recipient's country. The recipient contacts the local Hawala or Chop dealer to collect the money. Many have had, or know someone who has had, a bad experience. ccording to our research, only those who have no alternative (due to there being no transfer provider in their region at home), or are tempted by low commission rates or higher exchange rates, will take the risk.

Hundreds of creative efforts are underway across the developing world to solve these problems with remittance. To address safety issues, MoneyGram is offering delivery services of money transfers in the Phillipines, bringing money to your door instead of forcing you to come and collect your funds from an office in town. Alternatively, if your recipient has an ATM card, they will transfer the deposit to her account.

A new remittance strategy - goods and service remittance - addresses the safety, cost and misuse issues simultaneously. Instead of sending money home, make a purchase from a store or website in the US or Europe, and powdered milk, cans of corned beef or a live goat is delivered to your relatives. Manuel Orozco, an economist with the IADB, estimates that as much as 10% of all remittance happens via goods and services.

A pioneer in goods remittance - offers online shoppers the ability to buy supermarket vouchers and mobile phone airtime for relatives in Kenya and Uganda, as well as more conventional gifts like flowers and cards. SuperPlus, Jamaica's largest supermarket chain, goes even further, allowing online shoppers to fill a shopping card for their relatives and arrange for them to pick up the order in one of the SuperPlus stores around the country. SuperPlus is a partner with both Western Union and MoneyGram and has been promoting its supermarket remittance service through Western Union and MoneyGram stores in New York City, home to a large Jamaican diaspora. Goods remittance services generally don't charge a fee, making their profit off goods sales instead.

1.6 CUSTOMER PREFERENCES

Security (trust) was found to be a principal factor in choosing a money transfer provider. People often choose a provider based on the recommendation of someone else that has used them to send a money transfer. Banks are seen as more trustworthy and better known, and thus benefit from this customer preference. Banks also offer a wider range of financial services than money transfer operators. Independent money transfer operators are seen to offer a consistently better service to customers though – perhaps because they are focused on money transfer products, rather than offering the wide range of financial services that banks do. They were reported to have better-polished, speedier procedures and more readily available literature and targeted leaflets. MTOs seem to be more geared to the regular and low-value transfers that are the most common form of remittances to developing countries. Where a transfer needs to be made quickly, customers seem prepared to pay more for the service. Trust, speed and cost are traded off against each other depending on the priority of the customer for that particular money transfer. The distribution network in the receiving country – for example whether the money can reach a relative in a rural area – is another decision factor.

Availability of Local Currency Transfer

MoneyGram do not do local currency transfers in China, where payments are in US$. Travelex Money Transfer pays in

US$ or Niara for Nigeria, and US$ or RMB for China.

1.7 CONTROLS, REGULATIONS AND CUSTOMER SERVICE LEVELS

Controls, regulations and customer service levels in contrast to banks, MTOs do not need their customers to hold accounts with them. Travelex Money Transfer stand out in terms of customer services, offering a loyalty card scheme that gives lower charges after first time use and guaranteed payout amounts to some countries. The typical ID documents required to send any amount are passport, driving licence and utility bill/proof of address. For minimum transfer amounts requiring ID, see page 40. All MTOs are regulated by Customs and Excise, but only one has a customer service charter.

Three types of redress are stated. These are:

- timed deadline by which the MTO must respond to a case;
- consideration on a case-by-case basis, rather than blanket response;
- refund of fee in certain cases.

1.8 REMITTANCE TYPES

1.8.1 Altruistic Motive

The altruism or livelihoods school of thought considers remitting to be an obligation to the household. Remittances are sent out of affection and responsibility towards the family. It has been argued in the poverty literature that the major reason why people migrate to other countries is due to poverty. According to the altruistic model, sending remittances yields a satisfaction to the migrant out of a concern for the welfare of his family

1.8.2 Self- Interest Motive

An opposite motivation is to assume that the migrant is mainly motivated by an economic and financial self-interest, when sending remittances to the home country. The argument behind this theory is that, at every point in time, the successful migrant in the foreign country saves. Then, the need arises on how (in which assets) and where (in which country) to accumulate wealth. An obvious place to invest, at least part of his assets, is in the home country by buying property, land, financial assets, and so on. These assets may earn a higher rate of return than assets in the host country although their risk profile can also be greater. In turn, the family can administer, during the emigration period, those assets for the migrant, thus acting as a trusted agent.

1.8.3 Implicit Family Contract I: Loan Repayment.

The literature has also considered the discussion on the remittance process from the family perspective rather than the individual. In other words, Economic theory has developed explanations of the remittances process that take the family rather than the individual– as the main unit of analysis5. According to the theory, families tend to develop an implicit contract among those who choose to live abroad, the migrant, and those who stay at home. The implicit contract has an inter-temporal dimension, which could last for various years or even decades, as a time horizon. The contract combines elements of investment and repayment. In the loan repayment theory the family invest in the education of the migrant and usually finances the costs of migrating (travel and subsistence costs in the host country).

This is the loan (investment) element of the theory. The repayment part comes after the migrant settles in the foreign country and his income profile starts rising over time and is in a condition to start repaying the loan (principal and interests) back to the family in the form of remittances. This implicitly implies that the family invests in a higher yield "asset"(the migrant) who earns a higher income level in the foreign country than other family members that live and work at home. The amount to be remitted will however, depend among other things, on the income profile of the migrant.

1.8.4 Implicit Family Contract II: Co-Insurance

A variant of the theory of remittances as an implicit family contract between the migrant and those at home relies on the notion of risk diversification. Assuming that economic risks between the sending and foreign country are not positively correlated then it becomes a convenient strategy for the family as a whole, to send some of its members abroad (often the most educated) to diversify economic risks. The migrant, then, can help to support his family in bad times at home. Conversely, for the migrant, having a family in the home country is insurance as bad times can also occur in the foreign country. In this model, migration becomes a co-insurance strategy with remittances playing the role of an insurance claim. As in any contract there is a potential problem of enforcement (e.g. ensuring that the terms of the contract, are respected by the parties). However, we can expect enforcement to be simpler, in principle, due to the fact that these are implicit family contracts, helped by considerations of family trust and altruism (a feature often absent in legally sanctioned contracts).

The theories underlying the motives for remittances transfers suggest that it is only in the Altruistic case that there is a no "quid pro quo". Transfers are made purely out of concern for the family and fits into the standard definition of transfers in the BOP sense. The other motives behind transfers suggest that there may be a quid pro quo as in the case of the implicit family contract, although this may not be immediate or binding. It is safe to conclude that theory raises additional difficulties for definition and measurements.

1.9 DYNAMICS OF CHOOSING FORMAL AND INFORMAL CHANNELS

Most interviewees knew of informal remittance methods, but were skeptical of the procedure. There is a general lack of clarity over the difference in legitimacy between less well known high street providers and agent (eg Hawala) dealers. The two types of businesses can often appear similar to the consumer, operating from smaller shops that may also offer other products and services. There are two informal methods perceived by the consumer – using a middleman or agent, and sending money with friends and relatives travelling overseas. There is an acknowledgment overall that there are providers on the high street that are not well known but claim to transfer money. There is, however, some confusion as to whether these organizations are legitimate operators. They tend to be specific to particular community groups and to operate in a small sphere of awareness and usage. Recommendations are usually word-of-mouth and information is gathered via the local community. There are considerable concerns among consumers surrounding the security of these providers, particularly if the provider is not well known. Where a friend, colleague or relative has recommended them, concerns are abated somewhat. However, it is acknowledged overall that this method represents a risk. *'Some agencies end up moving a week after you sent money and you end up losing it.' Nigerian* Chinese users of middlemen find that charges can be high and there is a high degree of uncertainty, as they do not know the middleman personally. Levels of trust are generally low, but this is dependent on how close the person is and how well known they are to the sender.

Despite the high charges and uncertainty faced, sometimes the middleman is the only choice for people from mainland China. This is particularly the case for those sending money to remote parts. This dependence is compounded by the sense of fear that some may have regarding usage of official channels. Often, they may not have bank accounts, or the proof of identity necessary for a provider such as Western Union. Indeed, there is a perception that few 'official' institutions are capable of sending money to some of the more remote villages in mainland China. Therefore, these middlemen provide a valuable service in a very short space of time, even though this is tempered with a sense of insecurity and lack of trust. For the vast majority of their users, they are the only choice. The method of sending cash via relatives traveling overseas was mentioned by some, and is generally considered an acceptable method. To Ghanaians in particular, this is a favored method as the process of sending the money via relatives conveys an emotional bond and gives a stronger sense of satisfaction.

1.9.1 Summary

This chapter deals with remittances from migrants to their relatives in Ghana. The various channels by which this is done, the cost of transfers and the various remittance types. People go and queue at the Banks for their monies been set from abroad. In the next chapter shall put forward queuing theory and terms associated with it.

CHAPTER TWO

2.0 LITERATURE REVIEW

2.0.1 INTRODUCTION

Queue is a common word that means either a waiting line or the act of joining a line. It occurs whenever arriving customers wait in line to get service at one or more service point. It is formed when the number of customers arriving is greater than the number of customers being served during a period of time.

For example, Vehicles waiting in a petrol bunk, patients waiting for doctor's clinic, planes arriving in an airport for landing, customers waiting at barber shop for hair cut, subscribers waiting for telephone connection, passengers waiting at railway booking counter, customers waiting at ration shop, students waiting at college fee counter. The above situations have some common features like customer's arrival, formation of a queue, waiting for service, provision of service to the customers in a certain order, departure after service etc.

Previous covering models for emergency service consider all the calls to be of the same importance and impose the same waiting time constraints independently of the service's priority. This type of constraint is clearly inappropriate in many context should allow prioritizing the calls for service.

Silva and Serra (2008) formulated a covering model, which considers different priority level. The model heritages its formulation from previous research on Maximum Coverage Models and incorporates results from Queuing Theory, in particular Priority Queuing. The additional complexity incorporated in the model justifies the use of a heuristic procedure.

Gershkov and Schweinzer (2008) addressed the scheduling problem of reordering an existence queue into its order through trade. To that end the authors considered individually rational and balanced budget direct and indirect mechanisms. They also showed that this class of mechanisms allowed them to form efficient queues provided that existing property rights for the service are small enough to enable trade between the agents. In particular, they showed on the one hand that no queue under a fully deterministic service schedule such as first-come, first-serve can be dissolved efficiently and meet their requirements. If, on the other hand, the alternative is service anarchy (i.e. a random queue), every existing queue can be transformed into its efficient order.

The emergence of globally distributed call-center networks, such as Dell International Support Service, has fundamentally increased the challenges to the management. This new trend of networking faces the risk of extensive demand fluctuation both over locations and over time. Existing literature in operation management uses pooling mechanisms to deal with demand fluctuations among different departments, while does not consider system-wide time-varying demand shifts. Queuing literature has developed algorithms to allow the number of servers to change in response to time-varying loads; however, a pooling configuration introduces additional business risk and is thus more complex to change. More sophisticated approaches are needed to accurately describe the reality of call-center operations.

Du et al., (2006) put forward a model generalized from the International Queue run in Dell's globally distributed call-center network and identify its operation risks from both demand fluctuation and pooling management. Then they proposed an economic framework to use a Real-Options Approach to systematically analyze those risks to help the management to hedge the risks to a pre-chosen level and hence secure the associated service quality and the costs.

Wang and Zhu (2005), proposed a dynamic queuing model in which excess demand is cleared through multiple shifts. Due to differences in valuation and costs, some consumers choose to compete in queues for early consumption, while others avoid queues by late consumption. A unique efficient rational expectations equilibrium is shown to exist and some general characteristics of the queuing equilibrium were analyzed. The theory is then applied to some interesting realistic situations such as shopping, highways, and restaurants.

A classic example that illustrates how observed customer behaviour impacts other customers' decisions is the selection of a restaurant whose quality is uncertain. Customers often choose the busier restaurant, inferring that other customers in that restaurant know something that they do not. In an environment with random arrival and service times, customer behaviour is reflected in the lengths of the queues that form at the individual servers. Queue lengths could signal two factors – potentially higher arrivals to the server or potentially slower service at the server. We study the effect of both arrival and service rates on the inference made by an arriving customer.

Senthil and Debo (2008) put forward a model, based on both private information about the service quality and queue length information, customers decide, which queue to join. When the service rates are the same, they confirm that in equilibrium it may be rational to ignore private information and purchase from the service provider with the longer queue when only one additional customer is present in the longer queue. They showed that such congestion driven choice behaviour causes alternating cycles of successful and unsuccessful periods for a service provider. They provided an explanation based on endogenous choice-making for often observed "in" and "out" behaviour of restaurants.

The authors also showed that the success of a service provider depends on the private information of the customer arriving at an empty system and hence, contains a significant factor of luck. When the service rates and unknown service values are negatively correlated our results are strengthened; customers strictly prefer to join longer queues. When the service rates are positively correlated with unknown service values, customers might join shorter queues.

Previous emergency service covering models consider all the calls to be of the same importance and impost the same waiting time constraints independently of the service's priority. This type of constraint is clearly inappropriate in many contexts. For example, in urban medical emergency services, calls that involve danger to human life deserve higher priority over calls for more routine incidents. A realistic model in such a context should allow prioritizing the calls for service. Silva and Serra (2002) a formulated a covering model, which considers different priority levels. The model inherits its formulation from previous research in Maximum Coverage Models and incorporates results from Queuing Theory, in particular Priority Queuing. The additional complexity incorporated in the model justifies the use of heuristic procedure. An exhaustive evaluation of the heuristics, based in simulate networks was developed and a numerical example illustrates the functionality of the model.

In many service industries, companies compete with each other on the bases of the waiting time their customers, experience, along with the price they charge for their service. A firm's waiting time standard may either be defined in terms of the expected value or a given, for instance 95%, percentile of the steady state time distribution. Allon and Federgruen, (2006) investigated how a service industry's competitive behaviour depends on the characteristics of the service providers' queuing systems. The authors provided a unifying approach to investigate various standard single stage systems covering the spectrum from M/M/I to general G/GI/s systems, along with open Jackson networks to represent multi-stage service systems. Assuming that the capacity cost is proportional with the service rates we refer to its dependence on:

(i) the firm's demand rate and

(ii) the waiting time standard as the capacity cost function. We show that across the above road spectrum of queuing models, the capacity cost function belongs to a specific four parameter class of function, either exactly or as a close approximation. We then characterize how this capacity cost function impacts on the equilibrium behaviou in the industry.

The authors give separate treatments to the case where the firsms compete in terms of:

(i) prices (only)

(ii) their service level or waiting time standard (only),

(iii) simultaneously in terms of both prices and service levels. The firm's demand rates are simultaneously in terms of both prices and service levels. The firms' demand rates are given by a general system of equations of the prices and waiting time standards in the industry.

Traffic control has always been an important modeling problem. To accomplish such a task, information about each direction's traffic flow is required. Chen, Guralu and Takagi (2006) Modelled the system of pedestrian and cars at a crosswalk in between the HUB and Lowe Hall on UW campus. It predicts how long pedestrian or a car has to wait for the other to pass. This was accomplished by using the theory of Markov Chains as well as a Monte Carlo simulation.

Zheng, Reilly, and Buckley (1992) used both fuzzy optimization and normal simulation methods to solve fuzzy web planning model problems, which are queuing system problems for designing web servers. They applied fuzzy probabilities to the queuing system models with customers arrival rate I and servers, services rate m, and then compute fuzzy system performance variables, including Utilization, Number (of requests) in the System, Throughput, and response Time. For the fuzzy optimization methods, they applied two-steps calculation, first use fuzzy calculation to get the maximum and minimum values of fuzzy steady state probabilities, and then we compute the fuzzy system performance variables. For the simulation method, they used one-step normal queuing systems with a single server and multiple servers, cases, and compare the results of these two cases, giving a mathematical explanation of the difference.

An airport shuttle service is faced with the problem of when to dispatch vans to cities and airports. To determine the best dispatching scheme, Markov chains were used to determine the probabilities of when passengers would arrive at specific locations. Fallen Schwieterman and Vance (1992) built complex systems from smaller systems, a process to create Markov chains that appropriately modeled multiple cities, vans, and passengers was developed. This process was implemented in LISP and was demonstrated to accurately model an airport shuttle dispatching problem.

Owing to the popular of the internet, Online forums have been recognized a good channel for people to discuss and exchange information. Moreover, educators have started to investigate the benefits of applying online forums to the design of learning activities. In order to encourage student to participate on online forum, prompt responses to the questions or opinions of students are very important. Therefore, the investigation of the trade-off between the request-response time and the cost of labours for handling the requests has become a challenging and important issue.

Hwang and Chan (1992) proposed a Queuing theory-based approach proposed to evaluate the equilibrium of responses and the demands in online forums by taking the cost of human resources into consideration. In addition, they put forward an experiment which was conducted by applying the innovative approach to a graduate course in the university to determine the moderate number of assistants so that the requests from the students can be replied within specified number of days, which could provide valuable information in managing online forums for educational purposes.

In the next session, we shall put forward how to construct and solve equations that describe queuing behaviour for the above discussed situations.

2.1 QUEUING SYSTEM

A queuing system consists more servers, an arrival pattern of customer, service pattern, queue discipline, the order in which service is provided and customer behaviour. The word 'queue' is sometimes used to describe the whole system, but it is a part of the system that holds the excess customers who cannot be immediately served. Hence, the number of customer in the system at any given time will equal to the number of customers in the queue plus the number of customers in service. Of course, these numbers will vary with time due to customers' arrival and departure, so they are formally random process.

2.1.1 Arrival Pattern

The pattern of arrival of customers like patients, waiting to enter consulting room, people waiting to make telephone calls, cars at service station are arrival pattern of queue. The arrival pattern is either regular or irregular (or at random intervals of time). A regular arrival pattern is not common in the case of customers coming for service. Generally it is completely random.

In case the number of customers is very large, the probability of an arrival in next interval of time does not depend upon the number of customers already in the system. Hence the arrival is completely random and it follows Poisson process with mean equals average number of arrivals per unit of time. Also, we can view the times between arrivals as independent negative exponentially distributed random arrivals. The same process, viewed another way, gives a Poisson distributed random variables for the number of customers arriving in any time interval. We use Poisson arrival process as convenient one in queuing models because the data requirements are minimal. A single parameter, the arrival rate λ, is enough to describe the entire process. The reciprocal of λ gives the mean time between arrivals.

Another arrival process is renewal process which means that times between arrivals are independents random variables.

2.1.2 Service Pattern

The service pattern is similar to arrival pattern, but there are some important differences. Strictly speaking, it is not a random because it is interrupted by the idle periods when no customers arrives.

The servers may work fast when the queue is long. In this case, the easiest service time distribution to work with is the negative exponential distribution, which is fully characterized by a single parameter, the mean service rate μ or its mean service time $^{1}/_{\mu}$.

2.1.3 Service Arrangements

For providing service to the incoming customers one or more service points (or services) are established. The number depends on the number of customers, rate of arrivals, time taken for providing service to a single customer, etc. Depending on these variables, a service channel is either a single service channel or a multiple service channel.

Single Servers

In the case of a single channel service, there is one queue and service counter, where one customer gets service at one.

Pictorial representation of a queuing system are shown in the figure.

Queue Figure 2.1

Series Channels

A customer must pass successively through all the ordered channels before service is completed. For example, in public offices where part of the service are at different service counters.

Figure 2.2

Parallel Channels

A number of channels providing identical service facilities so that several customers may be serviced simultaneously.

For example, in a barber shop, many chairs are considered as different service channels and the customers may wait in a single line until one of the service channels is ready to serve.

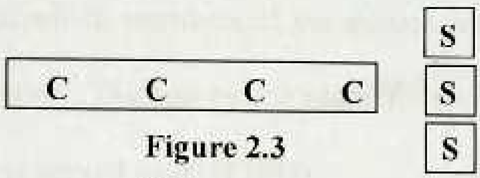


Figure 2.3

Three servers, one queue

Multiple-Server Model

A queuing system is called a multiple-server model when the system has a number of parallel channels with one server.

Examples include: Ticket counters at railway station, airports, etc.

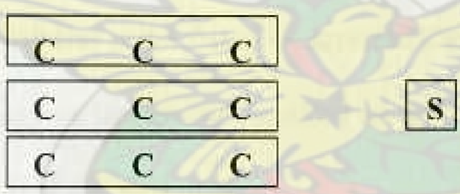


Figure 2.4

2.1.4 Service Time

The time required for servicing a customer is called service time. Service time may be either a constant or it vary with the customer. However, for sake of simplicity of queue model, it is assumed that time required for servicing is constant for all customers. Moreover, since the arrival pattern is assumed to be random, the service time is also taken as random. Hence the service time follows exponential distribution with mean equal to reciprocal of the mean rate of service.

In case where the assumption of service time follows exponential fails, Erlaing distribution is applied.

2.2 QUEUE DISCIPLINE

The order in which members of the queues are selected for service is known as queue discipline. This can be one of the following types:

First in First served (out) (FIFO)

Here the selection of customers for services is done in the order in which they arrive and form the queue. Customers served in ration shop is an example.

Last in First out (LIFO)

Selection of customers for service is done in the reverse order in which they arrive and the person entering last is the fist to be selected for service.

Example: When bags of cement are packed in a store-room, it is the last one packed which is brought out first.

Service in Random Order (SIRO)

The selection of customer for servicing is random at any particular time.

Service in Priority (SIP)

Certain customers are given priority over others in selection for service. It is of two types: Non-emptive priority and emptive priority.

(i) **Non-emptive Priority**

The customer already getting service is allowed to continue till it is completed even if a priority customer comes mid-way during the service.

(ii) **Emptive Priority**

The service to non-priority customer is stopped as soon as a priority customer arrives. E.g. Repairing of some critical machine when it breaks in a factory.

2.3 CUSTOMER BEHAVIOUR

Customer generally behaves in four (4) ways:

(i) **Balking:** Some customers show reluctance for waiting in the queue. They do not join the queue at their correct position and attempt to jump the queue and reach the service centre by passing other ahead of them.

(ii) **Reneging:** Some customers after waiting some time in the queue leaves the queue without getting the service due to impatience.

(iii) **Collusion:** Some of the customers join together and only one of them, instead of all, stay in the queue. However, when their time comes for service, the customers who were in collusion demand service.

(iv) **Joekeying:** In case ~~there~~ is more than one queue for similar type of service, some customers keep on shifting from one queue to another queue to improve their position and to get immediate service.

Output: The output per unit of time will depend upon the total time spent in the system, both while waiting and for getting service. The output is inversely proportional to the total system time. The output rate or production rate of the queuing system is the mean number of service completions per unit and is denoted by R.

2.4 DEFINITION OF TRANSIENT AND STEADY STATE

A system is said to be in **transient state** when its operating characteristics (like input, output, mean queue length etc.) are dependent upon time.

A system is said to be in steady state when the behaviour of the system is independent of time. If $P_n(t)$ denotes the probability that there are n customers in the system at time (t), then in the steady state case;

$$\lim \; P_n(t) \text{ (independent) of } t)$$

This implies:

$$\lim \quad \frac{d}{dt} P_n(t) = 0$$

The queue models can be of the following:

(i) **Probability Models:** When both arrival rate and service rate are unknown and are assumed to be random variables. The probability distribution obtained is based on past experience.

(ii) **Deterministic Models:** When both arrival and service rate are exactly known.

(iii) **Mixed Models:** Either the arrival rate or the service rate is exactly known and the other is not known.

Terminology used in the Text

We shall use the following terms in this and the subsequent chapters.

(i) **Customer:** A unit coming for service to the service station is called a customer. For Example, persons, machines, telephone calls, demand for some commodity.

(ii) **Waiting Line:** A line formed by customers waiting to receive service is known as waiting line.

(iii) **Arrival Rate:** The arrival rate is calculated by dividing the total number of arrivals by the total units of time. It is denoted by λ.

If the rate of arrivals is in Poisson distribution, then the distribution of time between two arrivals will be exponential.

(iv) **Service rate:** Average number of customers being served per unit of time. It is denoted by μ

(v) **Traffic intensity or Utilisation Rate:**

$$\text{Utilisation rate } (\rho) = \frac{\textit{Means arrival rate}}{\textit{Mean service rate}} = \frac{\lambda}{\mu}$$

This is the probability of having to queue on arriving. If $\rho > 1$, the traffic intensity is very high and consequently the waiting time will be more.

(vi) **Idle Rate:** Idle rate is $\rho_0 = 1 - \rho = 1 - \frac{\lambda}{\mu}$

Expected idle time (for service facility)

$$= 1 - \frac{\lambda}{\mu} \text{ x Total service time}$$

(vii) Expected number of customers in the system, both waiting and in service

$$E(n) = \frac{\lambda}{\mu - \lambda} = \frac{\rho}{1 - \rho}$$

(viii) Expected number of customer in queue or waiting line *E(m)*.

$$E(m) = E(n) - \rho = \frac{\lambda}{\mu - \lambda} - \frac{\lambda}{\mu} = \frac{\lambda}{\mu(\mu - \lambda)}$$

(ix) Expected time spent by the customer in system $E(T)$

$E(T)$ = expected time spent while waiting for service + the service time.

$$= E(w) + \frac{1}{\mu}$$

Since the rate of arrivals is λ, the expected number of customers in the system $E(n)$ will be equal to λ x $E(T)$. So

$$E(T) = \frac{E(n)}{\lambda} = \frac{\frac{\lambda}{\mu - \lambda}}{\lambda} = \frac{1}{\mu - \lambda}$$

(x) Expected waiting time in the queue $E(w)$:

$$E(w) = E(T) - \frac{1}{\mu}$$

$$= \frac{1}{\mu-\lambda} - \frac{1}{\mu} = \frac{\lambda}{\mu(\mu-\lambda)}$$

(xi)(a) Probability that there are n customers in the system at any time t, both waiting and in service is denoted by $P_n(t)$.

(b) Probability of zero customers waiting.

$$Po = 1 - \rho = 1 - \frac{\lambda}{\mu}$$

(c) Probability of 1, 2, ... *n persons waiting in the queue for service are:*

$$\rho_1 = \rho_0\left[\frac{\lambda}{\mu}\right] \quad \rho_2 = \rho_0\left[\frac{\lambda}{\mu}\right]^{0}$$

$$\rho_n = \rho_0\left[\frac{\lambda}{\mu}\right]^{n}$$

2.5 DISTRIBUTION OF ARRIVALS

The model in which only arrivals are counted and no departure takes place are called pure birth model. The number of arrivals in time interval (o, t) is a random variable which follows a Poisson distribution with parameter.

$$\lambda(t) \text{ is } \rho_n(t) = \frac{(\lambda t)^n}{n!} e^{-\lambda t}, \; for \; n \geq 0$$

Distribution of Inter-arrival Times (Exponential Process)

Inter-arrival times are defined as the time intervals between two successive arrivals. If the arrival process follows the Poisson distribution, an associated random variable defined as the time between successive arrivals follows the exponential distribution $f(t) = \lambda e$ and vice versa. That is;

Let the random variable T be the time between successive arrivals; then

$$
\begin{aligned}
P(T > t) &= P\text{ (no arrival in time t)} \\
&= \rho_o(t) \\
&= e^{n}\, e^{-\lambda t}
\end{aligned}
$$

The cumulative distribution function of T is ....

$$
\begin{aligned}
F(t) = P(T \leq t) &= 1 - P(T > t) \\
&= 1 - e^{-\lambda t} \qquad 0t \leq t \leq \infty
\end{aligned}
$$

The density function $f(t) =$ of inter-arrival times is:

$$
f(t) = \frac{d}{dt} F(t) = \begin{cases} \lambda e^{-\lambda t}, & 0 \leq t \leq \infty \\ 0, & t < 0 \end{cases}
$$

The expected time of inter-arrival is given by:

$$
E(t) = \int_0^{\infty} t f(t)\, dt
$$

$$
= \int_0^{\infty} \lambda t e^{-\lambda t}\, dt = \frac{1}{\lambda}
$$

Thus T has the exponential distribution with mean $\frac{1}{\lambda}$.

2.6 DISTRIBUTION OF SERVICE TIMES

The time t to complete the service on a customer follows an exponential distribution which is given by

$$
s(t) = \begin{cases} \mu e^{-\lambda t}, & 0 \leq t \leq \infty \\ 0, & t < 0 \end{cases}
$$

where μ is the mean service rate for a particular service channel.

2.7 MARKOVIAN QUEUING-MODELS

To be a Markovian queuing system, both inter-arrival times and service times must be negative exponentially distributed, so that it is fully characterized by only one parameter, which is related to the mean.

When the queuing model is Markovian, we can solve a set of linear equations whose solution provides the steady-state distribution of the number of customers in the system. All of the other performance measures we have mentioned (v) – (xi) can be computed. The simplest Markovian model to set up and solve is the single server, infinite capacity model.

Kendal's Notation for Representing Queuing Models

Generally, queuing model may be completely specified in the following symbolic form:

$$(a/b/c) : (d/e)$$

The first and second symbols denote the type of distribution of inter-arrival times and of inter-service times respectively. The third symbol specifies the number of servers, whereas fourth symbol stands for the capacity of the system and the last symbol denotes the queue discipline.

If we specify the following letters as:

$\mu \equiv$ Poisson arrival or departure distribution

$E \equiv$ Erlang or Gamma inter-arrival or service time distribution

$GI \equiv$ General input distribution

$G \equiv$ General service time distribution, Then $(M/Ek/I)$: (∞/ FIFO) defines a queuing system in which arrivals follow Poisson distribution, service time are Erlang, single server, infinite capacity and first in first out' queue discipline.

M/M/I Queuing System: This queuing system deals with the process in which arrivals and services occur randomly overtime. Arrivals can be considered as **births** to the system, since if the system is in state *En and an arrival occurs, the state is changed to* E_{n+1}. If departure occurs, the state E_n is changed down to E_{n-1} and can be looked upon as death. This type process is a birth-death process.

Model I (M/M/I): (∞/FIFO): This model deals with a queuing system having single server, Poisson arrival, exponential services and infinite capacity and the service is on the basis "first in First out".

2.8 RELATIONS BETWEEN AVERAGE QUEUE LENGTH AND AVERAGE WAITING TIME-LITTLE'S FORMULA

Mean number equals to the mean number in the queue plus the mean number in service. That is:

$$E(n) = E(m) + \rho$$

A similar relation in terms of time is:

$$E(T) = E(w) + \frac{1}{\mu}$$

Another relation is $\quad U = \frac{\rho}{c}$

Where the average utilization equals the mean number of busy servers divided by number of services c. If there is only one server c = 1, then

$$U = \rho = 1 - \rho_0$$

If the arrival rate is constant λ and if every arrival is accepted, then the output rate

$$R = \lambda$$

However, full capacity will not accept every arrival. Only those customers who arrive when the system is less than full are allowed to the system. In such cases, the pure arrival rates have to be adjusted to get an effective arrival rate.

If arrivals are random state, the pure arrival rates that are successful is $1 - \rho_N$. Hence, in this case;

$$R = \lambda (1 - P_N)$$

Little's formula gives the relationship between $E(n)$, $E(T)$ and R as

$$E(n) = R\,E(T)$$

If any two of these quantities are fixed, the third can be determined. In an infinite capacity system, where

$$R = \lambda$$

$$E(n) = \lambda E(T)$$

Any system that transforms input to output overtime and possesses a steady state conditions corresponding to $E(n)$, R, $E(T)$ will obey Little's formula.

Model II (M/M/I): (∞/SIRO). This model is the same as Model I, except the service discipline follows the SIRO rule (service in random order) instead of FIFO-rule. For SIRO-case, we must have;

$$P_n = (1 - \rho)\rho^n,\ n \geq 0$$

The average number of customers in the system is $E(n)$ and P_n *remains unchanged.* The three queue disciplines FIFO, LIFO, SIRO differ only on the distribution of waiting time where the probability of long and short waiting time change depending upon the discipline used.

2.9 LIMITED QUEUE CAPACITY

Model III (M/M/I): C/FIFO). When waiting space is limited, a number of minor modifications must be made to the previous models. In this model the maximum number of customers in the system is limited to C.

The steady-state equations are identical to those used previously with additional difference equation for $n = C$, then

$$P_c(t+\Delta t) = P_{c(t)}[1-\mu\Delta t] + P_{c-1}(t)(\lambda\Delta t)(1-\mu\Delta t) + O(\Delta t)$$

This gives after simplification, the equation.

$$\frac{d}{dt} P_c(t) = -\mu P_c(t) + \lambda P_{c-1}(t)$$

From which the steady-state equations for this model, therefore, can be written as

$$\mu P_1 = \lambda P_0$$

$$\mu p_{n+1} = (\lambda+\mu)P_n - \lambda p_{n-1}, 1 \le n \le c-1,$$

$$\mu p_C = \lambda p_{C-1}$$

Using the iterative procedure (as was done in Model I), the first two difference equations give:

$$P_n = (\lambda/\mu)^n \quad P_{0,}{}^n \le C$$

Applying the normalizing equation to determine P_o, $n \le C$

$$1 = P_o + P_1 + P_2 + \ldots + P_o + \left(\frac{\lambda}{\mu}\right) P_o + \ldots\ldots + \left(\frac{\lambda}{\mu}\right)^c P_o$$

$$= P_o\left(1 + \left(\frac{\lambda}{\mu}\right) + \left(\frac{\lambda}{\mu}\right) + \ldots + \left(\frac{\lambda}{\mu}\right)^c\right) = \left(\frac{1 - \left(\frac{\lambda}{\mu}\right)^{C+1}}{1 - \left(\frac{\lambda}{\mu}\right)}\right)$$

$$\Rightarrow \quad P_o = \frac{1 - \left(\frac{\lambda}{\mu}\right)}{1 - \left(\frac{\lambda}{\mu}\right)^{C-1}} = \frac{1 - P}{1 - P^{C+1}} \quad for\ \rho \neq 1$$

$$\text{Thus} \quad P_n = \begin{cases} \dfrac{(1-\rho)\rho^n}{1 - \rho^{c+1}} & \rho \neq 1 \\[2ex] \dfrac{1}{C+1} & \rho = 1 \end{cases}$$

(i) Average number of customers in the system is given by

$$E(n)\sum_{n=0}^{c} n P_n = P_0 \sum_{n=0}^{c} nP^n = P_0\rho \sum_{n=0}^{c} \frac{d}{d\rho}(\rho^n) = P_0\rho \frac{d}{d\rho}\sum_{n=0}^{c} \rho^n$$

$$= P_0\rho \frac{d}{dp}\left(\frac{1 - p^{C+1}}{1 - p}\right) \quad \textit{by using geometric series}$$

Expansion:

$$= P_0\rho \frac{1 - (C+1)p + Cp}{(1-\rho)^2}$$

$$= P_0\rho \frac{1 - (C+1)\rho^c + C^{\rho c+1}}{(1-\rho)(1-\rho^{c-1})} \quad \text{for } p \neq 1$$

$$= \frac{C}{2} \quad for\ p = 1$$

(ii) Average queue length is given by

$$E(m = \sum_{n=s}^{C} (n-1)\, p_n = E(n) - \sum_{n=s}^{C} Pn = E(n) - (1-Po) = E(n) - \left[1 - \left(\frac{1-p}{1-p^{C+1}}\right)\right]$$

$$= E(n) - \left[\frac{p - p^{C+1}}{1 - p^{C-1}}\right] = E(n) - \left[\frac{p(1-p^{C})}{1-p^{C+1}}\right]$$

$$= \rho^2 \left[\frac{1 - C + (C-1)p^{C}}{(1-p)(1-p)^{C-1}}\right], \quad p \neq 1$$

(iii) The average waiting time in the system can be obtained by using little's formula

$$E(T) = \frac{1}{\lambda} E(n)$$

Where λ is the mean rate of customer entering the system and is equal to $\lambda(1-P_n)$

(iv) The average waiting time in the queue is

$$E(w) = E(T) - \frac{1}{\mu} \text{ or } E(w) = E(m) / \lambda$$

2.10 MULTI-SERVER QUEUES

Multi-server system is designated by *M/M/s*, where s is the number of servers. The system capacity C would certainly have to be at least s. If $C = s$ then the maximum number of customers permitted in the system equals the number of servers. In this case there is no need to wait for service.

Assume that all servers operate at the same mean rate μ. So, the service to customers take place on a number of customers simultaneously, but departures will occur one at a time. (The probability of two or more services being completed at exactly the same instant is zero).

If there is only one customer in the system then he will receive from one of servers at the rate μ. If there are two customers present, they will both receive service, each at the rate μ. So the rate at which this serves works is twice as much as the rate at which a single server works of 2μ. Similarly, is s customers are present, they all will receive service each at the rate μ. So the rate which this server works is s times as much as the rate at which a single server works or $s\mu$.

State diagram:

The steady-state equations are:

$$0 = \lambda p_o + \mu p_1$$

$$0 = \lambda p_o - (\lambda + \mu) p_1 + 2\mu p_2$$

$$0 = \lambda p_1 - (\lambda + 2\mu) p_2 + 3\mu p_3$$

$$0 = \lambda p_{s-1} - s\mu p_s$$

These can be solved in the usual birth-death equations as:

$$P_j = \frac{1}{j!} \; \frac{\lambda}{\mu} \; P_o, \quad for\ j = 1, 2, \ldots\ldots, s$$

Using the normalization equation to evaluate P_o, we find

$$P_o = \frac{1}{\sum_{i=0}^{x} \frac{\rho}{i!}}, \quad where\ \rho = \lambda / \mu$$

If s is very large relation to ρ, so that the remaining terms in the infinite series are small, so we write ρ_0 as e^{-P}, *and hence*

$$\rho_j = \frac{1}{j} \rho^j e^{-\rho}$$

Is approximately, the distribution of the number of occupied servers.

When s is finite and the approximation is not appropriate, the steady state distribution is :

$$P_j = \frac{\frac{\rho^j}{j!}}{\sum_{i=0}^{x} \frac{\rho}{i!}}$$

And this distribution is called a **truncated poison distribution.**

$$P_c = \frac{\frac{\rho^c}{j!}}{\sum_{I=0}^{x} \frac{\rho^j}{i!}}$$

This formula is known as Erlang's lost formula or first Erlang function. The mean number of occupied servers or customers in the system is given by $\rho(1 - \rho c)$.

The solution discussed above were for "No waiting case". Suppose some waiting capacity is available, the steady-state equations are:

$$0 = -\lambda\rho_0 + \mu\lambda$$

$$0 = -\lambda\rho_{j-1} - (\lambda + j\mu)\rho_j + (j+1)\mu\rho_{j+1} \qquad \text{for } j = 1, 2, \ldots\ldots s-1$$

$$0 = \lambda\rho_{j-1} - (\lambda + s\mu)\rho_j + s\mu\rho_{j+1} \qquad \text{for } j = s, s+1, \ldots\ldots$$

If the total system capacity C is finite, there would be one equation for the last state

$$0 = \lambda\rho_{c-1} - s\mu\rho_c$$

Hence we get models:

M/M/s: (C/FIFO) maximum number in the system is limited to C where C /s.

$$\lambda_j = \begin{cases} \lambda 0 \le j < C \\ 0 j \ge C \end{cases}$$

$$\mu_j = \begin{cases} j\mu 0 \le j < s \\ s\mu s \le j \le C \end{cases}$$

We get

$$P = \begin{cases} \frac{I}{j!}\left(\frac{\lambda}{\mu}\right) P_0, & s \le j < s \\ \frac{I}{s!} \; \frac{I}{s^{i-s}} \left(\frac{\lambda}{\mu}\right)^j + P_0, & s \le j \le C \end{cases}$$

Were

$$P = \left\{ \sum_{j=0}^{s-1} \frac{I}{j!} \left(\frac{\lambda}{\mu}\right)^j + \sum_{j=s}^{c} \frac{1}{S^{i-s} \; S!} \left(\frac{\lambda}{\mu}\right)^j \right\}^{-1}$$

Provided $\frac{\lambda}{s\mu} < 1.$

(i) Average queue length is given by:

$$E(m) = \sum_{n=s}^{c} (n-s) P_n = \sum_{n=s}^{c} (n-s).\frac{\left(\frac{\lambda}{\mu}\right)^n}{S!\,S^{n-s}} P_0$$

$$= \frac{I}{s!}\left(\frac{\lambda}{\mu}\right)^s P_0 \; P \sum_{X=0}^{C-S} \cancel{xP^{x-1}}, \quad \text{for } P = \frac{\lambda}{s\mu}$$

$$= \frac{I}{s!}\left(\frac{\lambda}{\mu}\right)^s P_0\rho \sum_{X=0}^{c-x} \frac{d}{dp}(P^x) = \frac{(sp)^s}{s!} P_0\rho \frac{d}{dp}\left(\sum_{X=0}^{cs} p^x\right)$$

$$= \frac{(sp)PoP}{s!} \frac{d}{dp}\left(\frac{1-p^{c-s+1}}{1-p}\right)$$

$$= \frac{P_0\,(sp)^s\; p[1-p^{c-s+1} - (1-p)(c-s+1)\,p^{c-s}]}{(1-p)^2.\; s!}$$

(ii) Average number of customers in the system is given by:

$$E(\mathrm{n}) = \sum_{n=0}^{c} \mathrm{n}\, P_n + \sum_{n=0}^{s-1} \mathrm{n}\, P_n = \sum_{n=s}^{c} n\, P_n + \sum_{n=0}^{s-1} n\, P + E(m) + s\sum_{n=s}^{c} P_n$$

$$= \boldsymbol{E(m)} + \mathrm{s}\left[\sum_{n=0}^{c} P_n - \sum_{n=0}^{s-1} P_n\right] + \sum_{n=0}^{s-1} n\, P_n$$

$$= \boldsymbol{E(m)} + \mathrm{s} - \boldsymbol{P_0} \sum_{n=0}^{s-1} \frac{(s-n)(ps)^n}{\mathbf{n!}}$$

(iii) Average waiting time in the system can be obtained by using Little's formula, that is

$$\boldsymbol{E(T)} = \frac{E(n)}{\lambda}, \text{ where } \lambda = \lambda(1-Po)$$

Average waiting time in the queue can be obtained by using

$$E(w) = E(T) - \frac{I}{\mu} = \frac{E(T)}{\mu(1-Pc)}$$

(M/M/s) : (∞ / FIFO): This model is the generalization of *(M/M/I)*: **(∞ / FIFO)** in the sense that service channels is 's' in number instead of a single server. Clearly, since the input is Poisson, and the service is exponential, we have a birth-death process. The mean arrival rate is given by $\lambda_n = \lambda$ for all n if there are more is customers in the system, all the servers 's' remain busy and each is of mean rate μ, and hence the mean service rate is $s\mu$. Other hand, if there are fewer customers than 's' in the system, n < s, only n of the s servers are busy and thus the mean rate is n μ. Hence

$$\mu_n = \begin{cases} n\mu, & 1 \le n < s \\ s\mu, & n \ge s \end{cases}$$

The steady state solution is:

$$P_n \begin{cases} \dfrac{1}{n!}\left(\dfrac{\lambda}{\mu}\right)^n P_0, & 1 \le n < s \\ \dfrac{1}{s^{n-s}}\left(\dfrac{\lambda}{\mu}\right)^n P_0, & n \ge s \end{cases}$$

Making use of the boundary condition $\sum_{n=0}^{\infty} P_n = 1$, *we find the value of* P_0 *as*

$$P_0 = \left(\sum_{n=0}^{s-1} \frac{1}{n!}\left(\frac{\lambda}{\mu}\right)^n + \frac{1}{n!}\left(\frac{\lambda}{\mu}\right)^s \frac{s\mu}{s\mu - \lambda} \right)^{-1}$$

(i) Average waiting time in the queue is $\dfrac{\mu \left(\dfrac{\lambda}{\mu}\right)^s P_0}{(s-1)!(s\mu - \lambda)}$

(ii) Average queue length is given by

$$E(m) = \frac{\mu \left(\dfrac{\lambda}{\mu}\right)^s P_0}{(s-1)!(s\mu - \lambda)^2} = \left(\frac{\lambda}{\mu}\right) \frac{1}{s!} P_0 \frac{\dfrac{\lambda}{\mu s}}{\left(1 - \dfrac{\lambda}{\mu s}\right)^2}$$

(iii) Average number of customers in the system is:

$$E(n) = \mathrm{E(m)} + \frac{\lambda}{\mu}$$

$$= \frac{\lambda\mu \left(\frac{\lambda}{\mu}\right)^{s} P_0}{(s-1)!\,(s\mu - \lambda)^2} + \frac{1}{\mu}$$

(iv) Expected waiting time that a customer spend in the system is $\frac{E(m)}{\lambda} + \frac{1}{\mu}$

$$= \frac{1}{(s-1)!}\left(\frac{\lambda}{\mu}\right)^{s} \frac{\mu}{(s\mu-\lambda)^2} P_0 + \frac{1}{\mu}$$

2.11 M/G/1 QUEUING SYSTEM

So far we discussed the queuing processes which are either birth and death or non-birth and death processes. They are in either case Markovian and the theory of Markov chains and processes could be applied in their studies. Now we consider the models where the distributions of the inter-arrival time or the service time do not possess the memory less property. That is not of exponential distribution. The process *N(t)* giving the state of the system or system size at time *t* will then be no longer Markovian. However, the analysis of the process can be based on the associated process, which is Markovian. Two techniques is by means of imbedded Makov chains. The second technique is supplementary variable technique, due to Keilson and Kohavian. Not that in the queue *M/M/I*. *G* represents the service time distribution is general, other things are same as *M/M/I* queue.

We discuss the technique of Imbedded Markov chains. Consider a single-server queuing system whose arrival process is Poisson with the average arrival rate λ. The service times by β, are independent and identically distributed with mean $\frac{1}{\mu}$ and the distribution function is denoted by F_B and its pdf, when it exists, by $f_B = F_B$. The discipline followed is FIFO. In particular, if we let F_B be the exponential distributed with parameter μ, then we obtain *M/M/1* queuing system. If the service times are assumed to be a constant, then we get the *M/D/1* queueing system.

Let $n = 1, 2, \ldots\ldots$ $(t_0 = 0)$ be the n^{th} departure epoch. That is, the instant at which the n^{th} unit completes his service and leaves the system. These points t_n are called the **regeneration points** of the process $\{ N(t) \}$. Let X_n be the number of units in the system at t_n, so that

$$X_n = N(t_n), \qquad n = 1, 2, \ldots\ldots$$

The stochastic process $\{ X_n,\ n = 1, 2, \ldots\ldots \}$ is a discrete parameter Markov chain, known as the **imbedded Markov chain** of the continuous-parameter stochastic process $\{N(t) : t \geq 0 \}$

Let A_n be the random variable giving the number of units that arrive during the service time of the n^{th} unit, then

$$X_{n+1} = \begin{cases} X_{n+1} + A_{n+1} & \text{if } X_n > 1 \\ A_{n+1} & \text{if } X_n = 0 \end{cases}$$

That is, if $X_n = 0$ the next unit to arrive is the $(n+1)^{st}$; during its service time if A_{n+1} Units arrive, then the $(n+1)^{st}$ unit departs at time t_{n+1}, leaving A_{n+1} units behind.

$X_n > 0$, then the number of units left behind by the $(n+1)^{st}$ units equals $X_{n-1} + A_{n+1}$

Now the service times of all the units have same distribution so that

$A_n \equiv A$ *for* $n = 1, 2, \ldots\ldots$ we

$$A(A=r) = \int_0^{\infty} P(A=r) \text{ service time of a unit is t) } dB(t)$$

Where P(A = r/service time of a unit is t) = $\frac{e^{-\lambda t} (\lambda t)^r}{r!}$

And so $a_r = R\{A = R\} = \int_0^{\infty} e^{\frac{-\lambda t(\lambda t)}{r!}} f_B(t)dt$

Gives the distribution of A, the number of arrivals during the service time of a unit.

Note :

$\sum_{i=1}^{\infty} aj = 1$. The probabilities

$$Pij = P\{X_{n+1} = j / Xn = i\} = \begin{cases} P(A = j - i + 1 = a_{j-i+1}, & i \geq 1, j \geq 1 - 1 \\ P(A = j) = a_j, & i = 0, j \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

The above relation clearly indicate that $\{X_n ; n \geq 0\}$ *is* Markov chain having transition probability matrix

$$\begin{pmatrix} a_0 & a_1 & a_2 & \cdots \\ a_0 & a_1 & a_2 & \cdots \\ 0 & a_0 & a_1 & \cdots \\ 0 & 0 & a_0 & \cdots \\ .. & . & . & \cdots \end{pmatrix}$$

As every state can be reached from every other state, the Markov chain $\{X_n\}$ is irreducible. Again as $P_{ii} \geq 0$, the chain is a periodic. When the traffic intensity $\rho = \lambda / \mu < 1$, the chain is persistent, non-null and hence ergodic. By applying the ergodic theorem of Markov chain (see chapter 6), the limits

$$V_k = \lim_{n \to \infty} P_{jk}\ (n),\ k = 0, 1, 2, \ldots\ldots$$

Exists and are independent of the initial state j.

The probability $V = (v_0, v_1, \ldots\ldots), \sum v_{j=1}$ are given as the unique solutions of $v = vp$. Let $G(z) = \sum_{j=0}^{\infty} a_j z^j$ and $V(z) = \sum_{j=0}^{\infty} v_j z^j$ denote the generating functions of the distributions of $\{a_j\}$ and $\{v_j\}$ respectively. We have

$$G(z) = \sum_{j=0}^{\infty} a_j z^j \text{ and } V(z) = \sum_{j=0}^{\infty} v_j z^j \left(\int_0^{\infty} \frac{e^{-\lambda t}(\lambda t)^j}{j!} f_B(t)dt \right) = \int_0^{\infty} e^{-\lambda t} \left(\sum_{j=0}^{\infty} \frac{(\lambda tz)^j}{j!} \right) f_B(t)dt$$

$$= \int_0^{\infty} e^{-\lambda t} . e^{\lambda t} f_B(t)dt. = \int_0^{\infty} e^{-\lambda t(1-z)} f_B(t)dt$$

$$= F(\lambda(1-z))$$

Not that the Laplace transform a function $f(t)$ is denoted by $F(s)$ and is $F(s) = \int e^{st} f(t)dt$. In this situation, $F(\lambda(1-z)$ is the Laplace transform of service time distribution evaluated as $s = \lambda(1-z)$. Hence

$$E(A) = G'(1) = \left(\frac{d}{dz} G(z) \right)_{z=1} = \left(\frac{d}{dt} F(\lambda(1-z)) \right)_{z=1} = \left\{ \left(\frac{dF}{ds} \right) \left(\frac{ds}{dz} \right) \right\}_{s=0}$$

$$= \left(\frac{dF}{ds} \right)_{s=0} (-\lambda = -\lambda \int_0^{\infty} \frac{d}{ds} (e^{st}) f_B(t)dt as F(s) = \int_0^{\infty} e\ \ f_B(t)dt$$

$$= -\lambda \int_0^{\infty} -t.e^{st} f_B(t)dt = + \lambda E(B) = \frac{\lambda}{\mu} = \rho$$

Hence the reciprocal of the service rate μ of the server equal the average service time $E(B)$ as mentioned earlier.

Now $V = VP$ give an infinite system of equations. Hence we get

$$v_j = v_0 a_j + \sum_{i=0}^{j+1} v a_{j-i+1}$$

$$\Rightarrow \sum_{i=0}^{\infty} v_j z^j = \sum_{i=0}^{\infty} a_0 a_j z^j + \sum_{i=0}^{\infty}\left(\sum_{j=1}^{\infty} v_i a_{j-1+1}\right) z^j$$

$$\Rightarrow \quad v(z) = v_0 \sum_{i=0}^{\infty} a_j z^j + \sum_{i=0}^{\infty}\left(\sum_{i=i-1}^{\infty} v_i a_{j-1+1} z^j\right) \text{interchanging the order of summation}$$

$$= v_0 \sum_{i=0}^{\infty} a_j z^j + \sum_{i=0}^{\infty}\left(\sum_{i=0}^{\infty} v_i a_j z^{k+i-1}\right)$$

$$= v_0 \sum_{i=0}^{\infty} a_j z^j + \frac{1}{z}\left(\sum_{i=0}^{\infty} v z^i \sum_{k=0}^{\infty} a_k z^k\right) = v_0 G(z) + \frac{1}{z}[V(z)-V(0)]G(z)$$

$$\Rightarrow \quad V(z) - \frac{1}{z} G(z)V(z) = v_0 G(z) - \frac{1}{z} v_0 G(z)$$

$$\Rightarrow \quad V(z)\; 1 - \frac{1}{z} G(z) = v_0 G(z)\; 1 - \frac{1}{z}$$

$$\Rightarrow \quad V(z) = \frac{v_0 G(z)(z)-1}{z - G(z)}$$

Since $V(1) = \sum_{i=0}^{\infty} v_\cdot = 1 = G(1)$, we can use L'Hospital rule to obtain

$$V(1) = V(1) = \lim v_0 \frac{G'(z)(z)-1) + G(z)}{1 - G'(z)} = \frac{v_0 G(1)}{1 - G'(1)} = \frac{v_0}{1 - G'(1)}$$

Provided $G'(1)$ is finite and < 1. Taking $p = G'(1)$, it follows that

$$1 = \frac{v_0}{1-p} \Rightarrow v_0 = 1 - p$$

And, since v_0 is the probability that the server is idle, p is the utilization limit.

$$V(z) = \frac{(1-p)(z-1)\,F(\lambda(l-z))}{Z - F(\lambda(l-z))}$$

This is known as **Pollaczek-Khinchin (P.K) formula**

Remark

The average number of units in the system, in the steady state, is determined by taking Derivative of $V(z)$ with respect to z and taking limit $z \to 1$,

$$E(N) - \lim E(Xn) = \sum jv_j = \lim V'(z) = \frac{p}{1-p}$$

Using the derivation of generating function. More generally,

$$E(N) = \frac{\rho + \lambda 2\, E(B)2}{2(1-\rho)}$$

This is known as the Pollaczek-khinchin (P-K) mean-value formula.

Note: The result hold true for all scheduling disciplines under the following conditions:

(i) The server is not idle wherever a unit is waiting for service.

(ii) The scheduling is non-pre-emptive; that is once a unit under service, it should be completed without interruption.

2.11 MODEL: $(M/E_R/1:FIFO)$

This is a queueing model with Poisson arrivals, Erlang service with k phases, single channel, first come, first served discipline and infinite population. In this model, one unit is served in k phases. The arrival or departure of one unit, therefore means an increase or decrease of k phases in the system and the completion of service of one phase of a unit will mean the decrease of one phase in the system. If at any time the m units waiting in the queue with one unit in service which has to still pass through s phases, then the total number of phases n in the system at that will be given by

$$n = mk + s$$

If μ denotes the number of units served per unit time, then $k\mu$ will be the number of phases served per unit time and therefore

$$\mu = \lambda$$

$$\mu_n = \mu k$$

When P_n denotes the steady state probabilities of n phases in the system,

$$\lambda P_{n-k} + k\mu P_{n+1} = \lambda P_n + k\mu P_n \qquad n \geq 1$$

$$k\mu P_1 = \lambda P_0$$

The steady state equation gives:

$$0 = \lambda P_0 + k\mu P_1, \qquad n = 0$$

$$0 = \lambda Pk + \lambda P_0 \qquad n = k$$

$$0 = k\mu P_n + k\mu P_{n+1}, \quad 1 \leq n < k$$

$$\Rightarrow \qquad P_k = \frac{\lambda}{k\mu} Po, \; for\; n = k,$$

$$P_{n+1} = P_k \; for \leq n < k$$

(i) Average number of units in the system

$$E(n) = \frac{k+1}{2k} \frac{\lambda}{\mu} \frac{\lambda}{\mu - \lambda} + \frac{\lambda}{\mu}$$

(ii) Average number of units in the queue

$$E(m) = \frac{k+1}{2k} \; \frac{\lambda}{\mu}\left(\frac{\lambda}{\mu-\lambda}\right)$$

(iii) Average time spend by a unit in the system

$$E(T) = \frac{k+1}{2k} \; \frac{\lambda}{\mu(\mu-\lambda)}$$

For constant service time, taking limit k tends to ∞, we get

$$E(n) = \frac{1}{2} \; \frac{\lambda^2}{\mu(\mu-\lambda)} + \frac{\lambda}{\mu}$$

$$E(m) = \frac{1}{2} \; \frac{\lambda}{\mu} \; \frac{\lambda}{(\mu-\lambda)}$$

$$E(T) = \frac{1}{2} + \frac{\lambda}{\mu(\mu-\lambda)} + \frac{\lambda}{\mu}$$

$$e(w) = \frac{1}{2} \; \frac{\lambda}{\mu} \; \frac{\lambda}{(\mu-\lambda)}$$

When $k = 1$, the Erland service time distribution reduces to exponential distribution and the values of

$$E(n),\ E(m),\ E(T),\ E(w)$$

Are same as that Model 1.

Various formulae for (m)/G/1:(∞/GD) are:

Average number of customers in the system

$$= \frac{\lambda 2 \ \sigma 2 + p2}{2(1-p)} + p, \textit{ where } \sigma 2 = \text{variance of service time}$$

Average waiting time of a customer in the queue.

$$= \frac{\lambda \sigma^2 + \rho^2}{2\lambda(1-\rho)}$$

Average waiting time that a customer spends in the system

$$= \frac{\lambda \sigma^2 + \rho^2}{2\lambda(1-\rho)} + \frac{1}{\mu}$$

2.12 SUMMARY

In this chapter, the arrival pattern, service pattern and service arrangements at the bank were considered. The way a customer behaves at the bank, the relation between average Queue Length and average waiting time and various queue disciplines were also put forward. In this next chapter, we shall consider Bank as a care study for our work.

CHAPTER THREE

3.0 THE OPERATIONS OF MERBAN MONEY TRANSFER

When life brings its opportunities and challenges, emergencies and the unexpected money may be required urgently.

Money transfer operators provides the means to solve ones financial needs by receiving money from relatives, friends and business partners abroad through their network service. Receiving money from anywhere across the globe has never been quick and simple. MERBAN Money Transfer (MMT) is the money transfer service of merchant Bank. With over 11 agencies spread across the world, MERBAN Money Transfer brings money from relatives abroad to beneficiaries.

Indeed SIMPLICITY and SPEED are the hallmark of MMT. MMT has internet based system which means monies from most agencies are available to be collected within minutes MMT make sure their cherished beneficiaries go through simple and hassle-free process to receive their money. Nonetheless, for the safety of their own monies, beneficiaries and required to show appropriate identity anytime they call to cash transfer.

BENEFITS

This means of transferring money is convenient, reliable, safe, efficient and gives direct access to funds from any of the Banks funds net branches.

3.1 BASIC TIPS REQUIRED TO SEND MONEY

(i) One can easily send money through any operator even though the person may not hold bank account abroad.

(ii) The money can be received either instantly or some few hours after the transaction is completed.

(iii) The beneficiary does not pay fees. All fees are paid by the sender.

(iv) The sender's name, country or city origin and the amount sent in local currency is then communicated to the beneficiary.

(v) Any transfer to Africa needs a test question for security reasons. Some money transfer operators do not require a test question.

3.2 BASIC TIPS REQUIRED TO RECEIVE MONEY

The following are the important information one needs to know to collect money from any money transfer operator:

1. The beneficiary needs to know the exact transaction code.

2. The beneficiary needs to know:

   (a) The name ~~of the sender~~.

   (b) Country/City of transfer's origin.

   (c) Expected amount of money to be received.

3. The beneficiary should have a valid identification card (ID card) and sometimes the answer to a test question.

   The identification card (ID card) can be either a passport, voter's identification card or driver's license or any valid card which the beneficiary can be identified.

4. One does not need to have a bank account before he/she can collect the money. The beneficiary receives the money in cash.

3.3 SUMMARY

In this chapter, the operation and the procedure required to send and receive money were also considered.

In the next chapter, we shall consider the data collection and analysis.

CHAPTER FOUR

DATA COLLECTION AND ANALYSIS

4.0 INTRODUCTION

This chapter deals with the data collection and analysis of the data. The collection was based on the number of customers who wanted to withdraw money and the time each customer has served at the various tellers, the time for each teller to serve a customer was recorded as the customer's service time and time for a customer to the queue for a service as the customers arrival time. The data was collected at Merchant Bank Ltd, Nhyiaeso – Kumasi.

We collected the data with the help of three of my colleagues. We got to the bank around 6:00a.m. and the time that each customer arrived was taken. Each of my friends was assigned to a teller and the time that a customer spends at each teller was also taken.

4.1 METHODOLOGY

Let the total service time for:

Teller 1 : T1 = 27,129s = 452.15 min = 7.535833333 hours.

Teller 2 : T2 = 28,880s = 464.666666 min. = 7.744444444 hours.

Teller 3 : T3 = 26,596s = 443.2666667 min. = 7.387777778 hours.

Total be T1 + T2 + T3 = 81,605s = 1360.08333 min = 22.66805556 hours.

Total number of customers:

TC = 97 + 132 + 111 = 340 Customers

$\therefore$ Average service rate (Customers / Hours)

$$\mu = \frac{340}{22.66805556} = 14.99908094$$

$$= 15 \text{ customer/hour}$$

For arrival time = 29,855s = 497.5833333 mins. = 8.293055556 Hours.

$$\lambda = \frac{340}{8.293055556} = 40.99815776$$

$$= 41 \text{ customer/hr}$$

Number of tellers, $M = 3$

The probability, P_0 of zero people in the system is

$$P_0 = \frac{1}{\left(\sum_{n=0}^{n-1} \frac{1}{n!} \left[\frac{\lambda}{\mu}\right]^n + \frac{1}{M!} \left[\frac{\lambda}{\mu}\right]^M \times \frac{M\mu}{M\mu - \lambda} \right)}, \text{ for } M\mu > \lambda \text{ for all } M\mu > \lambda$$

$$P_0 = \frac{1}{\left(\sum_{n=0}^{2} \frac{1}{n!} \left[\frac{\lambda}{\mu}\right]^n + \frac{1}{3!} \left[\frac{\lambda}{\mu}\right]^3 \times \frac{3 \times 15}{3 \times 15 - 41} \right)}$$

$$P_0 = \frac{1}{\sum_{n=0}^{2} \frac{1}{n!}\left(\frac{41}{15}\right)^n + \frac{1}{3!}\left(\frac{41}{15}\right)^3 \times \frac{3 \times 15}{3 \times 15 - 41}} = \frac{1}{45.75833333}$$

$$= 0.021853942$$

$\therefore \quad P_0 = 0.02$

The average number of people in the system, Ls, is

$$\text{Ls} = \frac{\lambda\mu\left(\lambda/\mu\right)^m}{(M-1)!\ (M\mu - x)^2} P_0 + \frac{\lambda}{\mu}$$

$$= \frac{41 \times 15 \left(\frac{41}{15}\right)^3}{(3-1)!(3(15)-41)^2} \times 0.021853942 + \frac{41}{15}$$

$$= \frac{12558.93778 \times 0.021853942}{32} + 2.733333333$$

$$= 0.576947126 + 2.733333333$$

$$= 11.31028046$$

The average number of customer in the system is 12.

The average time a customer spends in the system, Ws, is given by

$$\text{Ls} = \frac{\mu\left(\lambda/\mu\right)^m}{(M-1)!\ (M\mu - \lambda)^2} P_0 + \frac{1}{\mu} = \frac{\text{Ls}}{\lambda}$$

$$\therefore \quad W_s = \frac{11.31028046}{41} \; hrs$$

$$= 0.275860499 \; hrs$$

$$\Rightarrow W_s = 16.55162994 \text{ min.}$$

On the average, an individual spends about 17 minutes in the system.

The number of people in the queue, Lq, is

$$Lq = L_s - \frac{\lambda}{\mu} = L_s - \frac{41}{15}$$

$$= 11.31028046 - 2.733333333$$

$$= 8.576947126$$

Averagely, there are 9 peoples found in the queue waiting for service.

On the average, a person spends Wq time waiting in the queue for service, where,

The average time a customer spends in the queue writing for service is

$$W_q = W_s - \frac{L}{\mu} = \frac{Lq}{\lambda}$$

$$= \frac{8.576947126}{41} \; hrs$$

$$= 0.209193832 \; hrs$$

$$\Rightarrow W_q = 12.55162994 \text{ min.}$$

Approximately 13 minutes is spent in the queue waiting for service.

PEAK SEASON

Let total service time for

Teller 1 - T_1 = 31856s = 530.9333min = 8.848888889 *hrs*

Teller 2 - T_2 = 29948s = 499.1333 min = 8.318888880 *hrs*

Teller 3 - T_3 = 30224s = 503.7333 min = 8.395555556 *hrs*

Total - $T_1 + T_2 + T_3$ = 92,028s = 1533.8 min = 25.56333333 *hrs*

Total number of customers,

TC = 146 + 152 + 163

= 461 customers

∴ Average service rate (customers/hr.)

$$\mu = \frac{461}{25.56333333} = 18.0336194$$

$$= 18 \text{ customers / hr.}$$

If arrival rate is λ then from

Total arrival time = 31915s = 531.91667 min = 8.86527778 hr.

$$\lambda = \frac{461}{8.86527778} = 52.0006267$$

$$= 52 \text{ customers/hr}$$

Number of tellers, M = 3

The probability Po, of zero people in the system is

$$P_o = \frac{1}{\sum_{n=0}^{m-1} \frac{1}{n!}\left[\frac{\lambda}{\mu}\right] + \frac{1}{M!}\left[\frac{\lambda}{\mu}\right]^M \times \frac{M-\mu}{M\mu-\lambda}} \quad \text{fall all } M\mu > \lambda$$

$$P_o = \frac{1}{\sum_{n=0}^{2} \frac{1}{n!}\left(\frac{52}{18}\right)^n + \frac{1}{3!}\left(\frac{52}{18}\right) \times \frac{3x18}{3(18)-52}} = \frac{1}{1+2.888889+4.1728395+108.493831}$$

$$= \frac{1}{116.5555556}$$

$$= 0.008579599619$$

$$\Rightarrow P_o = 0.009$$

The average number of people n the system, Ls, is

$$L_s = \frac{\lambda\mu\left(\frac{\lambda}{\mu}\right)^M}{(M-1)!\,(M\mu-\lambda)^2} P_o + \frac{\lambda}{\mu}$$

$$= \frac{52 \times 18\left(\frac{52}{18}\right)^3}{2!(3(18)-52)^2} \times 0.008579599619 + \frac{52}{18}$$

$$= \frac{22{,}566.71605 \times 0.0008579599619 \times 2.888888889}{8}$$

$$= 24.20167355 + 2.888888889$$

$$= 27.09056244$$

The average number of customers in the system is 28.

The average time a customer speeds in the system Ws, is

$$W_s = \frac{\lambda\mu \left(\frac{\lambda}{\mu}\right)}{(M-1)!\,(M\mu-\lambda)^2} P_0 + \frac{1}{\mu} = \frac{L_s}{\lambda}$$

$$W_s = \frac{27.09056244}{52} \; hrs$$

$$= 0.520972354 \; hrs.$$

$$\Rightarrow W_s = 31.25834128 \text{ min.}$$

On the average, an individual spends 31 minutes in the system.

The number of people in the queue, Ls, is

$$L_q = L_s - \frac{\lambda}{\mu} = L_s - \frac{52}{18}$$

$$= 27.09056244 - 2.888888889$$

$$= 24.20167355$$

Averagely, an approximate of 25 people are found in the queue waiting for service.

On the average, the time a person spends waiting in the queue for service is

$$W_q = W_s - \frac{1}{\mu} = \frac{L_q}{\lambda}$$

$$= \frac{24.20167355}{52} \; hrs$$

$$= 0465416799 \; hrs$$

$$\Rightarrow W_q = 27.92500794 \text{ min}$$

~~That is,~~ a person spends approximately 28 minutes waiting in the queue for service.

CHAPTER FIVE

CONCLUSIONS AND RECOMMENDATIONS

5.0 INTRODUCTION

The aims of this chapter are to provide an overall summary of the main findings presented in this project, identify the problems and come along with detailed suggestions for future investigations.

5.1 CONCLUSION

From our analysis, it was observed that the multi-channel system has a drastic effect on almost all the characteristic. In particular the following results were observed:

(i) The probability that there are zero customers in the Bank is 0.02 during off peak season and 0.009 during peak season.

(ii) There are approximately 12 customers in the Bank during off peak season and 28 customers in the Bank during peak season.

(iii) Customers spend an average of 17 minutes in the bank at any given time during off peak season and 31 minutes in the Bank during peak season.

(iv) There are 9 customers found in the queue at any time during off peak season and 25 customers in the queue waiting service during peak season.

(v) A customer spends 13 minute in the queue during off peak season and 28 minutes in the queue during peak season.

5.2 RECOMMENDATION

From the analysis above, we want to make the following recommendations.

1. There should be no need to increase the number of tellers. But they can increase the number of tellers during the peak season especially during Easter and Christmas periods.

2. Also there is the need for the bank to increase the security due to the number of customers that come into the bank.

3. Also there should be a teller whose main responsibility is to check whether a customer's transaction number is correct and that the money has arrived at the bank to save time.

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APPENDIX

PEAK SEASON

TABLE 1 **SERVICE TIME (SECONDS)**

150	131	149	171	168	120	188	210	201	99	217	312
230	218	111	240	211	221	214	197	189	209	222	314
311	305	175	90	115	210	101	350	422	100	173	89
331	289	215	333	321	287	371	401	189	193	228	105
109	117	231	210	203	501	387	411	288	271	101	273
211	171	181	119	190	151	132	157	203	322	471	132
112	151	171	175	182	134	211	282	217	219	222	192
178	185	205	231	247	187	213	231	220	312	105	198
239	255	273	161	271	283	142	322	115	182	255	337
82	195	193	148	271	119	272	165	300	211	301	219
217	224	251	173	112	230	102	199	44	257	202	283
199	232	193	241	282	121	210	205	231	244	171	189

Total = 31856s

Average = 218.19178

PEAK SEASON

TABLE 2 **SERVICE TIME (SECONDS)**

121	115	132	140	103	101	109	111	157	182	107	120
205	211	201	197	181	155	99	230	189	222	311	297
241	221	301	110	97	215	1981177		201	224	312	251
279	101	230	114	203	214	182	135	157	201	221	314
204	113	322	317	301	297	185	271	200	213	204	101
182	201	205	192	176	163	145	210	115	190	202	183
197	119	205	234	312	440	103	335	216	241	189	170
125	195	181	105	111	109	495	401	382	367	291	205
322	305	110	105	124	187	192	264	275	251	233	201
199	189	291	270	109	121	141	207	241	257	300	199
205	186	171	150	131	197	182	177	175	138	151	167
172	195	121	202	215	141	145	182	104	121	196	182
137	141	221	251	201	100	98	140				

Total = 29948s

Mean (Average) = 190.4

PEAK SEASON

TABLE 3 SERVICE TIME (SECONDS)

205	121	221	104	197	181	120	109	110	150	112	115
125	103	118	112	141	153	157	281	97	231	254	303
105	215	218	107	177	179	205	192	174	224	235	162
151	244	191	189	331	201	104	125	159	309	182	177
164	199	257	289	294	151	142	137	105	341	300	106
121	147	155	163	149	205	241	281	203	125	190	149
231	242	133	189	205	217	197	176	351	297	121	146
163	171	329	195	1857	159	312	115	137	120	330	270
104	89	151	170	411	132	205	187	121	106	113	111
397	303	215	317	204	111	157	198	183	161	307	213
205	291	280	143	127	161	200	105	105	129	161	175
190	199	205	180	167	146	170	154	162	168	208	131
124	189	191	169	195	162	184					

Total = 30224

Mean (Average) = 185,4

(OFF PEAK)

TABLE 4 SERVICE TIME (SECONDS)

321	115	402	128	265	292	187	199	255	312	259	298
301	200	209	331	279	281	273	333	442	291	98	389
327	463	277	265	198	291	305	321	317	292	281	334
263	251	387	275	413	389	249	277	365	312	290	266
289	327	183	287	294	268	371	169	270	281	294	336
268	271	293	285	241	191	222	303	245	399	287	283
289	271	266	182	327	199	293	307	263`257		249	236
301	271	224	281	266	251	307	241	403	233	272	104
311											

Total = 27,129

Mean (Average) = 279.68

(OFF PEAK)

TABLE 5 **SERVICE TIME (SECONDS)**

105	146	218	139	172	133	195	200	177	202	158	169
166	193	251	204	111	189	196	271	109	82	301	215
203	211	153	142	292	220	235	206	263	174	289	233
241	132	199	357	312	294	266	205	118	173	244	241
201	222	187	363	294	281	204	190	198	335	127	77
195	311	286	196	214	225	203	160	384	283	261	194
176	181	225	242	215	228	197	256	249	238	205	220
193	261	188	254	209	221	183	162	255	207	300	281
115	194	232	102	141	204	225	197	281	253	264	222
231	198	176	181	205	221	132	270	234	202	193	151
163	154	215	257	189	240	238	199	201	200	181	

Total = 27,880

Mean (Average) = 211.21

(OFF PEAK)

TABLE 6 **SERVICE TIME (SECONDS)**

241	186	192	203	237	261	207	153	181	283	244	177
161	301	251	288	221	237	251	149	311	148	267	214
271	281	195	179	257	268	168	189	194	337	265	297
251	198	234	277	281	265	269	285	310	205	78	321
189	203	299	208	271	255	261	274	201	196	165	327
322	197	284	243	222	205	294	179	261	201	155	197
309	264	338	289	217	226	259	263	314	287	270	131
125	345	205	271	291	244	175	304	188	246	237	312
266	183	331	143	232	129	257	199	281	295	225	180
194	223	200									

Total = 26,596

Mean Average = 239.60

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Merchant Bank Location and Addresses of Payment Centres

1. **HEAD OFFICE/ACCRA MAIN**
   44 Kwame Nkrumah Avenue
   P. O. Box 401
   Accra – Ghana
   Tel: +233 (21) 670464/7011718 (Ext. 259)
   Fax: +233 (21) 670464

2. **KUMASI**
   1 Rain Tree Avenue
   (Opp. Ridge Police Station)
   P. O. Box 8618
   Ahensan – Kumasi
   Tel: +233 (51) 29293/26021
   Fax: +233 (21) 7011975

3. **KONONGO**
   Kumasi Road
   Mines Road Junction
   Tel: +233 (531) 24367

4. **RIDGE**
   57 Examination lap Accra
   P. O. Box 401
   Accra – Ghana
   Tel: +233 (21) 251131-4
   Fax: +233 (21) 7011975

5. **TAKORADI**
   Ground Floor
   SSNIT Office Complex
   P. O. Box 22
   Takoradi
   Tel: +233 (31) 23373/24554
   Fax: +233 (31) 24884

Collaboration with Rural Banks for Payments in Brong-Ahafo

1. **SUNYANI**
   Nsoatreman Rural Bank
   Grand Floor
   Cocoa House Area One
   Sunyani
   Tel: +233 (61) 27057/23669
   Fax: +233 (61) 24166

2. **BEREKUM**
   Wanfie Rural Bank
   Near Berekum Lorry Park
   Berekum - Zongo
   Tel: +233 (642) 22226

3. **DORMAA AHENKRO**
   Wamfie Rural Bank
   Near Fredie Video City
   Ahenkro
   MS Street,
   Dormaa Ahenkro
   Tel: +233 (648) 22100