KWAME NKRUMAH UNIVERSITY OF SCIENCE AND TECHNOLOGY

COLLEGE OF SCIENCES

FACULTY OF PHYSICAL SCIENCE

DEPARTMENT OF MATHEMATICS

OPTIMISING THE TRANSSHIPMENT PROBLEM OF BLUE SKY LIMITED

BY:

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A THESIS SUBMITTED TO THE DEPARTMENT OF MATHEMATICS OF KWAME NKRUMAH UNIVERSITY OF SCIENCE AND TECHNOLOGY IN PARTIAL FULFILLMENT
TO THE REQUIREMENT FOR THE AWARD OF A MASTER OF SCIENCE IN INDUSTRIAL MATHEMATICS

DECLARATION

I hereby declare that this submission is my own work towards the Master of Science degree

and that, to the best of my knowledge it contains no material previously published by another person nor material which has been accepted for award of any other degree of the university except where due acknowledgement has been made in the text.

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ABSTRACT

Transshipment, when possible, can be used as one effective way to reduce total inventory and increase service level in a distribution system. Essentially, transshipment allows the distribution system to take advantage of the risk pooling effect to deal with uncertain demands at different retail locations. Excess inventory at one retail location can be used to cover shortage at another location. Physically, one can interpret inventory stocking at each individual location as being "pooled" together to meet the demands at any other location within the distribution system. As such, the use of transhipment provides more flexibility in deploying the available inventory in the system to meet uncertain customer demand. From a management and operation perspective, a fundamental question in this problem is whether the company should move the product into the demand ports directly from the supply port, or whether the transhipment operation using an established warehouse can help to reduce the total cost to the company and increase responsiveness. From the resource utilization perspective, a transhipment system is certainly preferred as it provides a better utilization of the transporting goods. Currently, as at the time of this work, there is no such method for determining which route to be used in transporting the products by the company. The routes are chosen using guess work and by the discretion of the people in charge. For the data used for our analysis, the company using their crude approach arrived at the following conclusion; shipped the loads of 15000, 5000, 10000, 8000, 2000 and 10000 at a unit costs of GH¢15, GH¢7, GH¢10, GH¢4, GH¢3 and GH¢2 through the routes X - W2 – B, X - W2 – C, Y - W1 – A, Y – W1- B, Y - W2 – D, and Z – W2. Total cost of transporting these products was four hundred and eighteen thousand Ghana cedis (GH¢418,000.00).

ACKNOWLEDGEMENT

I would like to give thanks to the Almighty God for granting me the strength and knowledge for understanding this course and the completion of this write-up.

I am very grateful to my supervisor, Dr. S. K. Amponsah of the Department of mathematics, who painstakingly read through every line of the text and offered through his rich experience all the necessary encouragement, direction, guidance, advice and correction for the timely completion of this thesis.

I will also give thanks to my wife Mrs. Meseseneo Yawa Ametepey, my two daughters Sedinam Akusika Woemenya Ametepey,Dzornunye Manoagbe Yawa Ametepey and my son Malike Dzidula Bubu Kwame Ametepey for their support during the entire course.

Finally my sincere thanks go to all who in diverse ways helped in bringing this project to a successful end.

God richly bless you all.

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CHAPTER ONE

1.0 INTRODUCTION

Consider a distribution system consisting of multiple retail locations. Demands occur at each retail location, which replenishes its inventory from some central warehouse on a periodic basis. Demands at each retail location are first met from the available inventory at the location. When shortage occurs at one location, the shortage can be covered from available inventory at other retail locations through possible lateral transshipment. The objective is to determine the optimal order quantity of each retail location and the resulting optimal transshipment policy after demands are realized at each period so as to minimize the total expected replenishment costs, inventory holding costs, shortage costs and the transshipment costs among the multiple retail locations during some finite time horizon.

Transshipment, when possible, can be used as one effective way to reduce total inventory and increase service level in a distribution system. Essentially, transshipment allows the distribution system to take advantage of the risk pooling effect to deal with uncertain demands at different retail locations. Excess inventory at one retail location can be used to cover shortage at another location. Physically, one can interpret inventory stocking at each individual location as being “pooled” together to meet the demands at any other location within the distribution system. As such, the use of transhipment provides more flexibility in deploying the available inventory in the system to meet uncertain customer demand. Consequently, transshipment can help to reduce the total system inventory and stock-out level at each individual location, at the expense of a higher transportation cost for transshipping the products among the different retail locations. It is interesting yet unclear as to what kind of system configurations and retailer characteristics would benefit most from using transshipment. One objective of this paper is to address a number of managerial issues regarding the impact of transshipment on reducing the costs of the distribution system under different system configurations and retailer characteristics. Our distribution system with transshipment involves a convoluted decision problem consisting of two basic types of decisions that influence each other throughout the finite time horizon. The first type involves the decision for the optimal order quantity at the retail locations during each replenishment cycle. We refer to this decision as the optimal replenishment policy. The second type involves the decision for transshipping the products among the retail locations after demands are observed and shortages occur at different retail locations. We refer to this decision as the optimal transshipment policy.

The combined optimal replenishment problem with transshipment and stochastic demand is generally difficult to solve. The problem is complicated even for the single period model consisting of a two-stage decision problem, where the transshipment decisions are considered as a recourse action to cover shortage after the replenishment quantity has been selected and uncertain demands have been realized. For a finite time horizon, the optimal replenishment policy generally depends on the replenishment and transshipment decisions as well as realized demands in earlier periods. On the other hand, the optimal transshipment policy, which entails the decision of how much as well as from which location the transshipments should come from, also depends on the replenishment policy and realized demands in earlier periods.

This thesis presents a mathematical modeling of a transshipment problem, which can provide useful information to aid decision-makers in their supply chain decision making.

In this chapter of the thesis, an overview of transportation and transshipment model would be given; a brief description of the problem statement of the thesis is also presented together with the objectives, the methodology, the justification and the organization of the thesis.

1.1 BACKGROUND OF STUDY

Supply chain management is one of the main sources of competitive advantage for companies and its importance is increasing due to its effects on solving problems faced by many companies in terms of mismatches between customer demand and supply, which will lead to low levels of customer satisfaction and eventually decreasing sales and market share. Logistics is a critical part of supply chain management, and is used to control the flow of materials, services and information taking into account the cost of these activities on one side and the value created in terms of both the customers and the organization on the other. Supply chain management entails effective replenishment and inventory policies. The use of transshipment points in supply chains renders monitored movement of stocks to intermediary storage locations between two echelon levels. Transshipments are effective policies for correcting discrepancies between demand and inventory available at specific locations, serving as a tool for effective management of stocks that are already procured and delivered into the system. By using transshipments as a tool for utilizing stocks, a company can reduce its costs and improve the level of service without increasing the stock level and bearing additional costs. Transshipment problems are variations of transportation problems in which goods and services are distributed between sources, storage points and destinations. Like in many cases, the objective function of transshipment problems is to minimize cost. Transshipments problems were first defined by Orden (1956) as an extension of transportation problems with transshipment points between the sources and the destinations (Hemaida, 1994). Transshipment models can be used to enhance cost efficient movement of goods and improve the level of customer satisfaction.

In the last couple of decades, the numbers of products offered to the market have generally exploded. At the same time, the product life-time has decreased drastically. The combination of these two trends leads to increased inaccuracy of the demand forecasts, leading to firms facing an increased demand uncertainty resulting in the increase in inventory levels. The role of inventory as a buffer against uncertainty has been established for a long time. However, more recently, the disadvantages of holding inventory have been increasingly recognized, particularly with regard to the adverse impact that this may have on supply chain responsiveness. Increasing globalization has tended to lead to longer supply lead-times, which, by conventional inventory control theory, result in greater levels of inventory to provide the same service levels (Waters, 2002). In lean supply chain thinking, inventory is regarded as one of the seven “wastes” and, therefore, it is considered as something to be reduced as much as possible (Womack and Jones, 1996). Similarly, in agile supply chains, inventory is held at few echelons, with goods passing through supply chains quickly so that companies can respond rapidly to exploit changes in market demand (Christopher and Towill, 2001). There have been various supply chain taxonomies based on these concepts and most stress the need for inventory reduction within each of the classifications.

Interest in the concept of supply chain management has steadily increased since the 1980’s when companies saw the benefits of collaborative relationships within and beyond their own organization. The concept of supply chain is about managing coordinated information and material flows, plant operations, and logistics. It provides flexibility and agility in responding to consumer demand shifts without cost overlays in resource utilization. The fundamental premise of this philosophy is; synchronization among multiple autonomous business entities represented in it. That is, improved coordination within and between various supply-chain members. Increased coordination can lead to reduction in lead times and costs, alignment of interdependent decision-making processes, and improvement in the overall performance of each member as well as the supply chain.

Supply Chain Management (SCM), which is also known as a logistics network (Simchi-Levi et al., 2003) has been extensively studied in recent years. The logistical network consists of facilities and distribution options that perform the functions of procurement of materials, transformation of these materials into intermediate and finished products, and the distribution of these finished products to customers. SCM encompasses the management of all these (process) activities associated with moving goods from raw materials through to the end user.

SCM coordinates and integrates all of these activities into a seamless process. It embraces and links all of the partners in the chain. For this reason, successful SCM is the process of optimizing a company’s internal practices, as well as the company’s interaction with suppliers and customers, in order to bring products to market more efficiently. As we know that firms can no longer effectively compete in isolation of their suppliers and other entities in the supply chain. A typical structure of a divergent inventory system is number of locations which replenish from a central supplier. Due to demand uncertainty inventory investments can be very high in such supply chain systems. A commonly used strategy to introduce flexibility in the system is to establish transshipment links between locations at the same echelon. This means that locations at the same echelon in some sense share inventory. Transshipments, the monitored movement of material between locations at the same echelon, provide an effective mechanism for correcting discrepancies between the locations' observed demand and their available inventory. As a result, transshipments may lead to cost reductions and improved service without increasing system-wide inventories. Lateral transshipments between stocking locations are used to enhance cost efficiency and improve customer service in different ways.

There are basically two main approaches to capture the impact of transshipments between stocking locations. Within the first approach, transshipments are used after the demand is observed but before it is satisfied. If there is excess demand at some of the stocking locations while some have surplus inventory, lateral transshipments between stocking locations can work as a correction mechanism. Moreover, pooling the stocks can be viewed as a secondary source of supply for inventory shortages, especially when transshipments between stocking locations are faster and less costly than emergency shipments from a central depot or backlogging of excess demand. An alternative way of analyzing the impact of transshipments between stocking locations is to consider it as a tool to balance inventory levels of stocking locations during order cycles. To guarantee a certain level of customer service in all stocking locations, it is desirable to keep the inventory position at each location in balance relative to each other.

With increased product specialization and globalization, the shipping industry has experienced steady growth during the past few decades. At the same time, ocean carriers have competed to offer better service at a cheaper price. Carriers are constantly looking for opportunities to introduce new services/ routes to attract and capture more market demand. Operation has indeed become one of the key competitive advantages with optimization-based approaches being expected to play an important role.

Transshipments, the movement of material between locations at the same echelon, provide an effective mechanism for correcting discrepancies between the locations' observed demand and their available inventory. As a result, transshipments may lead to cost reductions and improved service without increasing system-wide inventories.

In today's competitive market, an efficient distribution system is essential to meet rapidly changing demands in a cost-effective and responsive manner. In particular, effective deployment of inventory in a distribution system is necessary to reduce inventory cost, and at the same time, to provide a high customer service level under uncertain market conditions. Transshipment has been considered as one effective way to achieve these goals for a distribution system. Essentially, transshipment allows the distribution system to take advantage of the risk pooling effect to deal with uncertain demands at different locations, where excess inventory at one location can be used to cover shortage at another location. Physically, one can interpret inventory stocking at each individual location as being "pooled" together to meet the demands at any other location within the distribution system. As such, the use of transshipment provides more flexibility in deploying the available inventory in the system to meet uncertain customer demand.

Supply chain management has become an increasingly important consideration for many firms due to its impact on cost, service level, and production quality. Among other issues, it entails defining replenishment and associated inventory policies which are cost effective. One such policy, commonly practiced in multi-location inventory systems, involves movement of stock between locations at the same echelon level. These stock movements are termed lateral stock transshipments, or simply, transshipments. Research efforts have generally viewed transshipments as an emergency recourse when unexpected circumstances have caused a surplus at one location and a shortage at another. One reason for considering only this reason for transshipments is the general lack of consideration of fixed replenishment costs. When these costs are present, we may want to replenish at one location and transship items to another location, in order to save on the fixed costs. Another reason for transshipments is to save on the holding costs, exploiting cases where different locations have different holding costs.

1.2 PROBLEM STATEMENT

The specific form of problem that this thesis seeks to solve is to mathematically model a company's distribution problem as transshipment problem and solve the problem. A transshipment problem allows shipment between supply points and between demand points and it may also contain transshipment points through which goods may be shipped on their way from a supply point to a demand point. Using the following method, a transshipment problem may be transformed into a balanced transportation problem.

1.3 OBJECTIVES

The objectives of the study are:

i. To determine the optimal transshipment cost of Blue Sky Limited.
ii. To model a distribution problem of Blue Sky Limited in Ghana.
iii. To use transshipment related models to solve it.

1.4 JUSTIFICATION

A transportation problem allows only shipments that go directly from a supply point to a demand point. In many situations, shipments are allowed between supply points or between demand points. Sometimes there may also be points (called transshipment points) through which goods can be transshipped on their journey from a supply point to a demand point. Shipping problems with any or all of these characteristics are transhipment problems. Fortunately, the optimal solution to a transshipment problem can be found by solving a transportation problem Hence, the studies of transshipment and transportation problems and their algorithms, has become an important area of research in the contribution to academic knowledge and the benefit of the economy as a whole, hence the reason for solving the transshipment problem.

1.5 METHODOLOGY

Our proposed methodology to our problem would be solved by using the transshipment model with intermediate destinations between the sources and the destinations. The transshipment problem will be converted to a transportation problem and the transportation algorithm will be used to solve it. A real life computational study would be performed and the Excel Solver software will be used to analyze the data.

1.6 LIMITATIONS

Our study is limited to some number of constraints. Management of Blue Limited was not willing to release information on their operations. This made data collection very difficult.

Another limitation was financial difficulty. Getting money for transportation, printing and binding also became a problem.

1.7 ORGANIZATION OF THE THESIS

In chapter one, we presented a background study of transshipment problem model.

In chapter two, related work in the transshipment problem will be discussed.

In chapter three, the transshipment and transportations algorithms by Amponsah and Darkwah (2009) will be introduced and explained.

Chapter four will provide a computational study of the algorithm applied to our transshipment problem instances.

Chapter five will conclude this thesis with conclusions and recommendations

1.8 SUMMARY

In this chapter, we discussed the introduction, background of the study. We also looked at the Objectives, Justifications, Methodology, Limitations and the Organisation of the study.

In the next chapter we shall put forward relevant literature on transshipment.

CHAPTER TWO

LITERATURE REVIEW

Banerjee et al., (2003) examined the effects of transshipments in different operating conditions one of which is based on the concept of inventory balancing via transshipments. Under their redistribution policy, the beginning inventory at each location is equalized. Bertrand and Bookbinder (1998) also use the balancing of the beginning inventory as a redistribution policy for identical retailers.

Tagaras (1989) used the fill rate and the probability of no-stockout to reflect the level of service. For an identical demand structure, balancing the fill rate is equivalent to starting with identical beginning inventory at each location. In a later study, while analyzing the effect of risk pooling in a setting with one central warehouse and three stocking locations, Tagaras (1999) compared random allocation with a 'risk balancing'transshipment policy. In risk balancing, transshipment quantities are determined so as to equalize the probability of a stockout in the following period. For an identical demand structure, risk of stockout will be balanced if each location starts with the same inventory.

Kut (2006) studied a distribution system consisting of multiple retail locations with transshipment operations among the retailers. Due to the difficulty in computing the optimal solution imposed by the transshipment operations and in estimating shortage cost from a practical perspective, we propose a robust optimization framework for analyzing the impact of transshipment operations on such a distribution system. We demonstrate that our proposed robust optimization framework is analytically tractable and is computationally efficient for analyzing even large-scale distribution systems. From a numerical study using this robust optimization framework, we address a number of managerial issues regarding the impact of transshipment on reducing the costs of the distribution system under different system configurations and retailer characteristics. In particular, we consider two system configurations, line and circle, and study how inventory holding cost, transshipment cost, and demand size and variability affect the effectiveness of transshipment operations for the cases of both homogeneous and non-homogeneous retailers. The managerial insights obtained from our robust optimization framework can help to evaluate the potential benefits when investing in transshipment operations.

Krishnan and Rao (1965) examined the transshipment problems with multiple retail locations with identical cost structure. They showed that the optimal stocking quantities satisfy the equal fractile property.

Tagaras (1989) presented a model which extended Krishnan and Rao’s two-location model to allow for different cost structures, and analyzed the pooling effect due to transshipment. His model can also allow for a service constraint on the minimum acceptable fill rates

Taragas and Cohen (1993) studied two-location transshipment model which allow for positive replenishment lead-times. With positive replenishment lead-times, it might be beneficial to hold back stock for future demands, and so it is not necessarily optimal to always transship from the other location (complete pooling) when shortages occur. However, their numerical results showed that complete pooling generally dominates partial pooling.

Herer and Rashit (1999) studied the two-location transshipment problem to include fixed and joint replenishment costs, and derived several properties regarding the structure of the corresponding optimal replenishment and transshipment policies.

Herer and Tzur (2001) considered a dynamic two-location transshipment problem where demands are deterministic and the objective is to minimize the total replenishment, holding and transshipment costs over a finite horizon. The author derived some structural results on the optimal policy and provided a polynomial time algorithm for finding the optimal policy.

Rudi et al., (2001) studied a two location model with decentralized decision making. They analyzed the optimal transshipment prices to maximize the total profit.

Dong and Rudi (2004) analyzed how transshipment can benefit a manufacturer and multiple retailers in settings where the manufacturer can serve as a price setter or a price taking. In their model, the multiple retailers have the same cost structure and complete pooling among retailers is assumed.

Wee and Dada (2005) studied the optimal policies for transshipping inventory in a retail network. They focused on the integrated transshipment decisions instead of the interaction`s among the retailers and the impact of the network structure.

When there are more than two locations in the system and the cost structures are non-identical, the optimal transshipment policy becomes more complex, as one needs to determine from which location, in addition to how many, to transship when a shortage occurs at any location. In general, it is analytically intractable to determine the joint optimal replenishment and transshipment policy. A number of papers studied different heuristic decision rules for lateral transshipment and then evaluated the optimal replenishment policy under these decision rules. This line of research includes the work of Alfredsson and Verrijdt (1999), Archibald, et al. (1997), Axsater (1990, 2003), Dada (1992), Grahovac and Chakravarty (2001), Lee (1987), Minner et al., (2003), and Robinson (1990).

There is a closely related literature where transshipment is allowed in a distribution system periodically as a way to rebalance stock at different locations rather than to cover shortage. This includes the work of Cohen, et al. (1986), Das (1975), Diks and de Kok (1996), Hoadley and Heyman (1977), Jonsson and Silver (1987), and Karmarker and Patel (1977).

Cross docking is a logistic technique which seeks to reduce costs related to inventory holding, order picking, transportation as well as the delivery time. Most of the existing studies in the area are interested in the dock assignment problem and the design of the cross dock transportation networks. Little attention has been given to the transshipment operations inside a cross docking platform. Larbi et al., (2003) studied the transshipment scheduling problem in a simple cross dock with a single strip door and single stack door. The authors proposed a graph based model for the problem. The shortest path in the graph gives the schedule which minimizes the total cost of transshipment operations.

Supply chain management is one of the main sources of competitive advantage for companies. As a useful tool for inventory and transportation management in supply chains, transshipment points provide an effective mechanism for correcting discrepancies between demand and available inventory. Deniz et al., (2009) studied transshipment problem of a company in the apparel industry with multiple Sub-contractors and customers, and a transshipment depot in between. Unlike a typical transshipment problem that considers only the total cost of transportation, our model also considers the supplier lead times and the customer due dates in the system and it can be used for both supplier selection and timely distribution planning. The proposed model can also be adapted easily by other companies in the industry.

Deniz et al., (2008) considered a supply chain, which consists of *N* stocking locations and one supplier. The locations may be coordinated through replenishment strategies and lateral transshipments, i.e., transfer of a product among locations at the same echelon level. The supplier has limited production capacity. Therefore, the total amount of product supplied to the *N* locations is limited in each time period. When total replenishment orders exceed total supply, not all locations will be able to attain their base stock values. Therefore, different allocation rules are considered to specify how the supplier rations its limited capacity among the locations. We team up the modeling flexibility of simulation with sample path optimization to address the multi-location transshipment problem. The authors solved the sample average approximation problem by random search and by gradient search. With this numerical approach, we can study problems with non-identical costs and correlated demand structures.

Roberto et al., (2009) considered the stochastic capacitated transshipment problem for freight transportation where an optimal location of the transshipment facilities, which minimizes total cost, must be found. The total cost is given by the sum of the total fixed cost plus the expected minimum total flow cost, when the total throughput costs of the facilities are random variables with unknown probability distribution. By applying the asymptotic approximation method derived from the extreme value theory, a deterministic nonlinear model, which belongs to a wide class of Entropy maximizing models, is then obtained. The computational results showed a very good performance of this deterministic model when compared with stochastic one.

Supply chain management emphasizes collaborative relationships between supply chain members. Mangal and Chandna (2007) examined the antecedents of retailer - retailer partnership and to explore its impact on the supply chain performance. The authors considered coordination among stocking locations through replenishment strategies that take explicitly into consideration transshipments, transfer of a product among locations at the same echelon level. A continuous review inventory system has been adopted, in which lateral transshipments are allowed. In general, if a demand occurs at a location and there is no stock on hand, the demand is assumed to be backordered or lost. Lateral transshipments serve as an emergency supply in case of stock out and the rule for lateral transshipments is to always transship when there is a shortage at one location and stock on hand at the other. The aim is to explore the role of lateral transshipment to control inventory and associated cost within supply chain and, from this, to develop an exploratory framework that assists understanding in the area. A simple and intuitive model is presented that enables us to characterize optimal inventory and transshipment policies for ‘n’ locations. The research is based on a case study of a bi-wheeler company in India by using its data and to strengthen its supply chain. This study represents such an effort in that it integrates both inventory and transshipment components in the study of multi-location inventory systems. This study will enable the managers to overcome the uncertainties of demand and lead-time resulting into customer satisfaction and cost reduction.

Banu and Sunderesh (2005) studied a single-item two-echelon inventory system where the items can be stored in each of *N* stocking locations is optimized using simulation. The aim of this study is to minimize the total inventory, backorder, and transshipments costs, based on the replenishment and transshipment quantities. In this study, transshipments which are the transfer of products among locations at the same echelon level and transportation capacities which are the transshipment quantities between stocking locations, are also considered. Here, the transportation capacities among the stocking locations are bounded due to transportation media or the locations' transshipment policy. Assuming stochastic demand, the system is modeled based on different cases of transshipment capacities and costs. To find out the optimum levels of the transshipment quantities among stocking locations and the replenishment quantities, the simulation model of the problem is developed using ARENA 10.0 and then optimized using the Opt Quest tool in this software.

Mabel et al., (2006) developed an analytical framework for studying a two-echelon distribution system consisting of one central warehouse and multiple retail locations with transshipment operations among the retailers. Our framework can be used to model very general distribution systems and analyze the impact of transshipment under different system configurations. The authors demonstrated that their proposed analytical framework is analytically tractable and is computationally efficient for analyzing even large-scale distribution systems. From a numerical study using our framework, we address a number of managerial issues regarding the impact of transshipment on reducing the costs of the distribution system under different system configurations and retailer characteristics. The managerial insights obtained from our analysis can help to evaluate the potential benefits when investing in transshipment operations.

As competition from emerging economies such as China and India puts pressure on global supply chains and as new constraints emerge, it presents opportunities for approaches such as game theory for solving the transshipment problem. Pedro (2004) studied a model which used the well-known Shapley value concept from cooperative game theory as an approach to solve the transshipment problem for maintaining stable conditions in the logistics network. A numerical example was presented to show the usefulness of this approach.

What is the best distribution strategy to ship materials from a source to multiple destinations on a regular basis? This is a common question confronting logistics planners in many industries. Mabel et al., (2004) examined this issue in the context of the ocean freight industry. In particular, the authors tackled the economics of freighting raw materials through a transshipment hub, and propose a method to synchronize the materials flow through the hub. The authors also compared the transshipment hub model with a direct service model, where a vessel is chartered and dispatched directly to bring materials to all the destinations in a single voyage. Our analysis shows that the value of the transshipment hub operation is largely determined by the trade-offs of three factors: (i) the cost of loading/unloading operation at the hub,(ii) cost of detaining material at the hub, and (iii) cost of material/inventory in transit. This conclusion is robust and relatively insensitive to the demand usage at the destinations.

The transportation problem has offered two mathematical facets: (1) as a specialized type of linear programming problem, (2) as a method of representation of some combinatorial problems. Orden (1956) developed a third aspect of the mathematical properties of the transportation problem. It is shown that the same mathematical framework can be extended beyond pair-wise connections, to the determination of optimum linked paths over a series of points. This extension although viewed here as a linear programming problem, takes advantage of the combinatorial aspect of the transportation problem, and applications may arise which, like the assignment problem, appear to be combinatorial problems, but which can be solved by linear programming.

A dynamic network consists of a graph with capacities and transit times on its edges. The quickest transshipment problem is defined by a dynamic network with several sources and sinks; each source has a specified supply and each sink has a specified demand. The problem is to send exactly the right amount of flow out of each source and into each sink in the minimum overall time. Variations of the quickest transshipment problem have been studied extensively; the special case of the problem with a single sink is commonly used to model building evacuation. Similar dynamic network flow problems have numerous other applications; in some of these, the capacities are small integers and it is important to find integral flows. There are no polynomial-time algorithms known for most of these problems. Hoppe and Tardos (1997) presented the first polynomial-time algorithm for the quickest transshipment problem. The author's algorithm provides an integral optimum flow. Previously, the quickest transshipment problem could only be solved efficiently in the special case of a single source and single sink.

Glover et al., (2005) developed a primal simplex procedure to solve transshipment problems with an arbitrary additional constraint. The procedure incorporates efficient methods for pricing-out the basis, determining certain key vector representations, and implementing the change of basis. These methods exploit the near triangularity of the basis in a manner that takes advantage of computational schemes and list structures used to solve the pure transshipment problem. We have implemented these results in a computer code, I/O PNETS-I. Computational results (necessarily limited) confirm that this code is significantly faster than APEX-III on some large problems. We have also developed a fast method for determining near optimal integer solutions. Computational results show that the near optimum integer solution value is usually within 0.5% of the value of the optimum continuous solution value.

Transshipments, monitored movements of material at the same echelon of a supply chain, represent an effective pooling mechanism. With a single exception, research on transshipments overlooks replenishment lead times. The only approach for two-location inventory systems with non-negligible lead times could not be generalized to a multi-location setting, and the proposed heuristic method cannot guarantee to provide optimal solutions. Gong and Yucesan (2006) studied a model that uses simulation optimization by combining an LP/network flow formulation with infinitesimal perturbation analysis to examine the multi-location transshipment problem with positive replenishment lead times, and demonstrates the computation of the optimal base stock quantities through sample path optimization. From a methodological perspective, this paper deploys an elegant duality-based gradient computation method to improve computational efficiency. In test problems, our algorithm was also able to achieve better objective values than an existing algorithm.

Herer and Tzur (2001) investigated the strategy of transshipments in a dynamic deterministic demand environment over a finite planning horizon. This is the first time that transshipments are examined in a dynamic or deterministic setting. The authors considered a system of two locations which replenish their stock from a single supplier, and where transshipments between the locations are possible. Our model includes fixed (possibly joint) and variable replenishment costs, fixed and variable transshipment costs, as well as holding costs for each location and transshipment costs between locations. The problem is to determine how much to replenish and how much to transship each period; thus this work can be viewed as a synthesis of transshipment problems in a static stochastic setting and multi-location dynamic deterministic lot sizing problems. The authors provided interesting structural properties of optimal policies which enhance our understanding of the important issues which motivate transshipments and allow us to develop an efficient polynomial time algorithm for obtaining the optimal strategy. By exploring the reasons for using transshipments, we enable practitioners to envision the sources of savings from using this strategy and therefore motivate them to incorporate it into their replenishment strategies.

A transshipment problem with demands that exceed network capacity can be solved by sending flow in several waves. How can this be done using the minimum number of iterations? This is the question tackled in the quickest transshipment problem. Hoppe and Tardos (1997) describe the only known polynomial time algorithm that finds an integral solution to this problem. Their algorithm repeatedly minimizes sub-modular functions using the ellipsoid method, and is therefore not at all practical. Fleischer presented an algorithm that finds a fully integral quickest transshipment with a polynomial number of maximum flow computations. When there is only one sink, the quickest transshipment problem is significantly easier. For this case, I show how the algorithm can be sped up to return an integral solution using O(k) maximum flow computations, where k is the number of sources. Hajek and Ogier (2003) describe an algorithm that finds a fractional solution to the single-sink quickest transshipment problem.

Virtually all available data on commodity shipments identify the origin and destination but not any transshipment points along the way. Transshipment has large implications for the provision of public infrastructure. A better macroscopic understanding of transshipment is needed. Within travel forecasting models the transshipment problem can be formulated as seeking the probability that a commodity with an origin at location A and a destination at location B has a transshipment point at location C. Transshipment has been studied extensively by researchers in logistics, but almost all those studies relate to improving the activities of an individual firm, rather than on the net effect of many firms acting within a whole economy. Horowitz (2009) studied a model which addressed the above problem.

Ozdemir et al., (2003) studied a supply chain model, which consists of N retailers and one supplier. The retailers may be coordinated through replenishment strategies and lateral transshipments, that is, movement of a product among the locations at the same echelon level. Transshipment quantities may be limited, however, due to the physical constraints of the transportation media or due to the reluctance of retailers to completely pool their stock with other retailers. The authors introduced a stochastic approximation algorithm to compute the order-up-to quantities using a sample-path-based optimization procedure. Given an order-up-to S policy, we determine an optimal transshipment policy, using an LP/network flow framework. Such a numerical approach allows us to study systems with arbitrary complexity.

The decentralized transshipment problem is a two-stage decision making problem where the companies first choose their individual production levels in anticipation of random demands and after demand realizations they pool residuals via transshipment. The coordination will be achieved if at optimality all the decision variables, i.e. production levels and transshipment patterns, in the decentralized system are the same as those of centralized system. Hezarkhani and Kubiak (2009) studied a model with the coordination via transshipment prices. The authors proposed a procedure for deriving the transshipment prices based on the coordinating allocation rule introduced by Anupindi et al, (2006). With the transshipment prices being set, the companies are free to match their residuals based on their individual preferences. The authors drew upon the concept of pair-wise stability to capture the dynamics of corresponding matching process. As the main result of this paper, we show that with the derived transshipment prices, the optimum transshipment patterns are always pair-wise stable, i.e. there are no pairs of companies that can be jointly better off by unilaterally deviating from the optimum transshipment patterns.

De Rosa et al., (2001) studied the Arc Routing and Scheduling Problem with Transshipment (ARPT), a particular Arc Routing Problem whose applications arise in garbage collection. In the ARPT, the demand is collected by specially equipped vehicles, taken to a transfer station, shredded or compacted and, finally, transported to a dump site by means of high-capacity trucks. A lower bound, based on a relaxation of an integer linear formulation of the problem, is developed for the ARPT. A tailored Tabu Search heuristic is also devised. Computational results on a set of benchmark instances are reported.

Asmuth et al., (1979) studied a multi-commodity transshipment problem where the prices at each location are an affine function of the supplies and demands at that location and the shipping costs are an affine function of the quantities shipped. A system of prices, supplies, demands, and shipments is defined to be equilibrium, if there is a balance in the shipments, supplies, and demands of goods at each location, if local prices do not exceed the cost of importing, and if shipments are price efficient. Lemke's algorithm is used to compute equilibrium.

Perincherry and Kikuchi (1990) presented a transshipment problem in which the projected demand and supply at different locations on different days are known in fuzzy quantities. The formulation of the model follows that of fuzzy linear programming in that the solution is a shipment schedule which satisfies the objective at a `reasonable cost'. Priorities for satisfying requirements at demand points and supply points on selected days are incorporated by multiplying corresponding weights to h, the level of satisfaction. The presentation follows from the general to the specific formulation with an example.

The multi-location replenishment and transshipment problem is concerned with several retailers facing random demand for the same item at distinct markets, that may use transshipments to eliminate excess inventory/shortages after demand realization. When the system is decentralized so that each retailer operates to maximize their own profit, there are incentive problems that prevent coordination. These problems arise even with two retailers who may pay each other for transshipped units. Hanany et al., (2010) presented a new mechanism based on a transshipment fund, which is the first to coordinate the system, in a fully non-cooperative setting, for all instances of two retailers as well as all instances of any number of retailers. Moreover, our mechanism strongly coordinates the system, i.e., achieves coordination as the unique equilibrium. The computation and information requirements of this mechanism are realistic and relatively modest. We also present necessary and sufficient conditions for coordination and prove they are always satisfied with our mechanism. Numerical examples illustrate some of the properties underlying this mechanism for two retailers.

Dahan (2009) studied a model, which considered two retailers between which transshipments can take place at the end of the period. The retailers differ in cost and demand distributions, operate in a single period, and cooperate to minimize joint costs. The authors work differs from previous analyses as it considers the possibility that customers are not always willing to wait for transshipments. Instead, only some customers are willing to wait and return to the retailer for transshipments. The objective of the research was to find the replenishment levels and transshipment quantities that minimize the total expected system cost. The authors considered two cases - a partially deterministic case, and a fully stochastic case. In the partially deterministic case, the number of returning customers is a known fraction of those that could not be satisfied off-the-shelf. The fully stochastic case treats the number of returning customers as a random variable whose probability density function is known and whose expected value is a fraction of the customers that could not be satisfied off-the-shelf. In the partially deterministic case, we show that the transshipment decision has a form similar to complete pooling. Thereafter, we prove that the objective function is convex in the replenishment levels, and suggest numerical methods for finding the optimal replenishment levels. In the fully stochastic case the number of returning customers is unknown. Thus, the transshipment decision is a stochastic planning problem. The authors have a newsvendor problem (the optimal transshipment quantity) nested within a larger newsvendor problem (the optimal replenishment levels). We show that the optimal transshipment quantity is found by solving a capacitated newsvendor problem. Thereafter, the authors analyze the convexity of the objective function with respect to the replenishment levels. We illustrate the analysis with a probability density function of returning customers which is normally distributed. the authors showed that for this distribution, the objective function is not convex in the decision variables. Two approximations to the objective function are presented, and shown to be convex. The authors proposed a solution methodology which utilizes numerical methods on the objective function and the two approximations.

Topkis (1984) developed a complement and substitution principles applicable to sittings in transhipment dual stage problems such as those encountered in factories and warehouses. Direct examination of the basic property of this transportation problem suggests that two locations of a similar nature would be reasonable substitutes. Such elements may not apply to location pairs where there is one or more warehouses. Where no warehouse is present, complement and substitution principles are functional. Model illustrations of factory warehouses and demand centre locations are highlighted.

Huang and Greys (2008) studied a newsvendor game with transshipments, in which n retailers face a stochastic demand for an identical product. Before the demand is realized, each retailer independently orders her initial inventory. After the demand is realized, the retailers select an optimal transshipment pattern and ship residual inventories to meet residual demands. Unsold inventories are salvaged at the end of the period. The authors compared two methods for distribution of residual profit—transshipment prices (TPs) and dual allocations (DAs)—that were previously analyzed in literature. TPs are selected before the demand is known, and DAs, which are obtained by calculating the dual prices for the transshipment problem, are calculated after observing the true demand. The authors first studied the conditions for the existence of the Nash equilibrium under DA and then compared the performance of the two methods and show that neither allocation method dominates the other. The author's analysis suggests that DAs may yield higher efficiency among "more asymmetric" retailers, whereas TPs work better with retailers that are "more alike," but the difference in profits does not seem significant. The authors also linked expected dual prices to TPs and use those results to develop heuristics for TPs with more than two symmetric retailers. For general instances with more than two asymmetric retailers, the authors proposed a TP agreement that uses a neutral central depot to coordinate the transshipments (TPND). Although DAs in general outperform TPND in our numerical simulations, its ease of implementation makes TPND an attractive alternative to DAs when the efficiency losses are not significant (e.g., high critical fractiles or lower demand variances).

Lateral transshipments in multi-echelon stochastic inventory systems imply that locations at the same echelon of a supply chain share inventories in some way, in order to deal with local uncertainties in demands. While the structure of a transshipment policy will depend on many important factors, a commonly observed phenomenon at the retail level, called "customer switching", may be of some significance. Under such a phenomenon, a customer, who cannot obtain a desired product at a specific location, may visit one or more other retail locations in search of the item. Liao (2010) studied the inventory replenishment and transshipment decisions in the presence of such stochastic "customer switching" behavior, for two firms which are either under centralized control, or operate independently. The first model adopted in this study considers two retailers that sell the same product to retail customers. After demand is realized, transshipments occur if only one location has insufficient inventory. Under this circumstance, a random fraction of the unfulfilled demand from the stocked out firm (which we refer to as the "shortage firm") may switch to the other firm with surplus inventory (which we refer to as the "surplus firm"). We examine the impact of such customer switching behavior on the firms' inventory decisions. The authors identified situations when the firm with surplus inventory is willing to (1) transship the entire quantity requested ("complete pooling policy"), (2) transship a portion of the amount requested ("inventory keeping policy"), or (3) transship nothing ("no-shipping policy") to the shortage firm. The authors demonstrated that a unique pair of optimal order quantities exists if the two firms are centeredly coordinated. When the firms operate independently, we derive a sufficient condition for the existence of a unique equilibrium replenishment order quantity pair. The authors also explored the optimal shortage or excess reporting policy when inventory information is asymmetric. Since the firm with a surplus makes a transshipment decision based on the magnitude of the shortage at the other location, it is possible that the shortage firm reports to the surplus firm some desired shortage quantity, instead of the real shortage. The authors proved that there is a possibility that the under-reporting situation exists.

Herer et al., (2006) considered coordination among stocking locations through replenishment strategies that take explicitly into consideration transshipments, that is, transfer of a product among locations at the same echelon level. They incorporate transportation capacity such that transshipment quantities between stocking locations are bounded due to transportation media or the location's transshipment policy. They model different cases of transshipment capacity as a capacitated network flow problem embedded in a stochastic optimization problem. Under the assumption of instantaneous transshipments, they develop a solution procedure based on infinitesimal perturbation analysis to solve the stochastic optimization problem, where the objective is to find the policy that minimizes the expected total cost of inventory, shortage, and transshipments. Such a numerical approach provides the flexibility to solve complex problems. Investigating two problem settings, they show the impact of transshipment capacity between stocking locations on system behavior. They observe that transportation capacity constraints not only increase total cost, but also modify the inventory distribution throughout the network.

Zhaowei et al., (2009) studied a new type of transshipment problem, the flows through the cross dock are constrained by fixed transportation schedules and any cargos delayed at the last moment of the time horizon of the problem will incur relative high inventory penalty cost. The problem is known to be NP-complete in the strong sense. The authors therefore focused on developing efficient heuristics. Based on the problem structure, we propose a Genetic Algorithm to solve the problem efficiently. Computational experiments under different scenarios show that GA outperforms CPLEX solver.

Herer and Tzur (1998) investigated the strategy of transshipments in a dynamic deterministic demand environment over a finite planning horizon. This is the first time that transshipments are examined in a dynamic or deterministic setting. The authors considered a system of two locations which replenish their stock from a single supplier, and where transshipments between the locations are possible. Our model includes fixed (possibly joint) and variable replenishment costs, fixed and variable transshipment costs, as well as holding costs for each location and transshipment costs between locations. The problem is to determine how much to replenish and how much to transship each period; thus this work can be viewed as a synthesis of transshipment problems in a static stochastic setting and multi-location dynamic deterministic lot sizing problems. The authors provided interesting structural properties of optimal policies which enhance our understanding of the important issues which motivate transshipments and allow us to develop an efficient polynomial time algorithm for obtaining the optimal strategy. By exploring the reasons for using transshipments, the authors enable practitioners to envision the sources of savings from using this strategy and therefore motivate them to incorporate it into their replenishment strategies

Belgasmi et al., (2008) studied a multi-location inventory system where inventory choices at each location are centrally coordinated. Lateral transshipments are allowed as recourse actions within the same echelon in the inventory system to reduce costs and improve service level. However, this transshipment process usually causes undesirable lead times. The authors proposed a multi-objective model of the multi-location transshipment problem which addresses optimizing three conflicting objectives: (1) minimizing the aggregate expected cost, (2) maximizing the expected fill rate, and (3) minimizing the expected transshipment lead times. We apply an evolutionary multi-objective optimization approach using the strength Pareto evolutionary algorithm (SPEA2), to approximate the optimal Pareto front. Simulation with a wide choice of model parameters shows the different trades-off between the conflicting objectives.

Transshipments, monitored movements of material at the same echelon of a supply chain, represent an effective pooling mechanism. Earlier papers dealing with transshipments either do not incorporate replenishment lead times into their analysis, or only provide a heuristic algorithm where optimality cannot be guaranteed beyond settings with two locations. Gong and Yucesan (2010) presented a method that uses infinitesimal perturbation analysis by combining with a stochastic approximation method to examine the multi-location transshipment problem with positive replenishment lead times. It demonstrates the computation of optimal base stock quantities through sample path optimization. From a methodological perspective, this study deploys a duality-based gradient computation method to improve computational efficiency. From an application perspective, it solves transshipment problems with non-negligible replenishment lead times. A numerical study illustrates the performance of the proposed approach.

One of the most important problems in supply chain management is the distribution network design problem system which involves locating production plants and distribution warehouses, and determining the best strategy for distributing the product from the plants to the warehouses and from the warehouses to the customers. Vahidreza et al., (2009) studied a model which allows for multiple levels of capacities available to the warehouses and plants. The authors developed a mixed integer programming model for the problem and solved it by a heuristic procedure which contains 2 sub-procedures.

The authors used harmony-search meta-heuristic as the main procedure and linear programming to solve a transshipment problem as a subroutine at any iteration of the main procedure.

Glover et al., (1974) presented a primal simplex procedure to solve transshipment problems with an arbitrary additional constraint. The procedure incorporates efficient methods for pricing-out the basis, determining representations, and implementing the change of basis. These methods exploit the near triangularity of the basis in order to take full advantage of the computational schemes and list structures used in solving the pure transshipment problem. Also reported is the development of a computer code, I/O PNETS-I for solving large scale singularly constrained transshipment problems. This code has demonstrated its efficiency over a wide range of problems and has succeeded in solving a singularly constrained transshipment problem with 3000 nodes and 12,000 variables in less than 5 minutes on a CDC 6600. Additionally, a fast method for determining near optimal integer solutions is also developed. Computational results show that the near optimum integer solution value is usually within a half of one percent of the value of the optimum continuous solution value.

Cheng and Karimi (2002) addressed a special case of the general chemical transhipment problem, namely the tanker lightering problem. When tankers are fully loaded with crude oil, they may not be able to enter the shallow channels or refinery ports due to the draft limitation. Under such circumstances, it is necessary to transfer some part of the crude oil from the tanker to lightering vessels in order to make the tanker “lighter”. After such transhipment operation, the tanker can travel to the refinery port, which it previously cannot. And, the lightering vessels also travel to the refinery port to deliver the lightered crude oil. With tanker lightering operation, large tankers can also deliver crude oil to shallow-draught refinery ports. Furthermore, it helps to reduce the demurrage costs of tankers as well as inventory holding costs (Chajakis, 2000) at the refinery. During congested time, tankers could spend days awaiting lightering service. Since the demurrage costs of tankers are extremely high, effective scheduling of lightering operation is crucial for minimizing the system cost by reducing the waiting times of tankers and increasing the utilization of lightering vessels.

Chajakis (1997) considered a scheduling problem faced by a shipping company that provides lightering services to multiple refineries clustered in a region. The company operates a fleet of multi-compartment lightering vessels with a mix of different configurations such as numbers of compartments, sizes, speeds, heating equipment, and so on. When a tanker arrives at the lightering location, one lightering vessel pumps off crude oil from one side of the tanker. Therefore, at most two lightering services can take place simultaneously for a tanker, one at each side of the tanker. And, these multi-compartment lightering vessels can pick up multiple types of crude from the same/different tankers during a voyage. After enough crude oil has been offloaded, the tanker leaves the lightering system and travels to its designated refinery port. However, lightering vessels travel to the refinery ports, deliver the crude oils, and then return to the lightering location to continue their service. In other words, the lightering vessels make multiple voyages among the refinery ports and lightering location in order to service multiple tankers. Furthermore, we consider a two-stage lightering practice for large tankers, first stage at an offshore location farther from the refinery and the second stage at the lightering location closer to the refinery.

Lin et al. (2003) addressed a limited form of the two-stage lightering practice for large tankers, first stage at an offshore location farther from the refinery and the second stage at the lightering location closer to the refinery using an event-based approach. They assumed single-compartment vessels, did not restrict the number of simultaneous services for a single tanker, did not allow pickups from more than two tankers within one voyage of a lightering vessel, ignored differences in crude densities, and did not allow the freedom to select lightering crudes. In this paper, we develop a new continuous-time MILP formulation that addresses all of the above drawbacks. Thus, we allow multi-compartment lightering vessels, restrict the number of simultaneous transfers to two, allow more than two pickups in one voyage for any lightering vessel, consider the impact of varying crude densities, select optimally the right lightering crudes, and most importantly use a realistic cost-based scheduling objective. Often, these features are real and important in the tank lightering problem. In contrast to the general chemical transhipment problem, the volumes and assignments to lightering vessels in this case are decided by the optimization model. In addition, the system cost here is an indicator of the customer satisfaction level as well as the utilization of fleet of lightering vessels. Our MILP model generates optimal lightering schedule with lightering volumes, sequence, times, and assignments, which minimizes the operating costs of lightering vessels, the demurrage costs of tankers as well as the delivery times of crude oil from the lightering location to refinery ports.

Mues et al., (2005) stated that the transhipment Problems and Vehicle Routing Problems with Time Windows (VRPTW) are common network flow problems and well studied. Combinations of both are known as intermodal transportation problems. This concept describes some real world transportation problems more precisely and can lead to better solutions, but they are examined rarely as mathematical optimization problems.

According to White, (1972) the movement of vehicles and goods in a transportation system can be represented as flows through a time-dependent transhipment network. An inductive out-of-kilter type of algorithm is presented which utilizes the basic underlying properties of the dynamic transhipment network to optimize the flow of a homogeneous commodity through the network, given a linear cost function.

Hsu and Bassok (1999) considered a single period problem with one input resulting in a random yield of multiple, downward substitutable products. They showed how the network structure of the problem can be used to devise an efficient algorithm.

McGillivray and Silver (1978) considered a case where products had identical costs and there is a fixed probability that a customer demand for a stocked-out product can be substituted by another available product.

Gilbert et al., *(*1997) identified some of the main issues in freight transportation planning and operations, and presented appropriate Operations Research models and methods, as well as computer-based planning tools.

Allahviranloo and Afandizadeh (2008) formulated a model to determine the optimum investment on port development from a national investment prospective. On the other hand, costs and benefits are calculated from consumer and investor view point.

CHAPTER THREE

METHODOLOGY

3.0 INTRODUCTION

In this chapter we shall put forward the transportation and the transshipment problems and their solution procedures.

3.1.1 The Transportation Problem

The transportation problem seeks the determination of a shipping plan of a single commodity from a number of sources, m, say, to a number of destinations, n, say, at a minimum total cost, while satisfying the demand at all destination.

The standard scenario where a transportation problem arises is that of sending units of a product across a network of highways that connect a given set of cities. Each city is considered either as a "source," (supply route) or a "sink," (demand route). Each source has a given supply, each sink has a given demand, and each highway that connects a source-sink pair has a given transportation cost per unit of shipment. This can be visualized in the form of a network, as depicted in Fig 3.1.

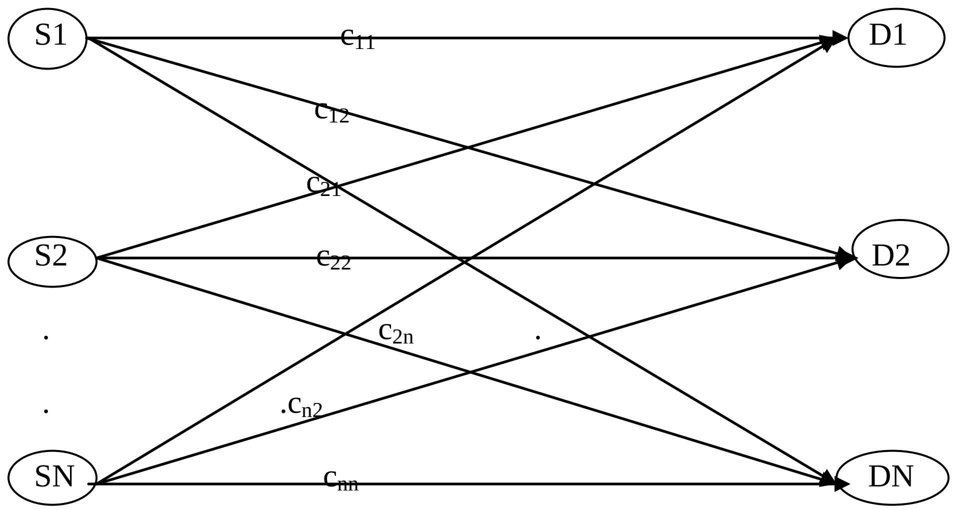

Fig. 3.1 The Shipment from sources to sinks

Given such a network, the problem of interest is to determine an optimal transportation scheme that minimizes the total cost of shipments, subject to supply and demand constraints. Problems with this structure arise in many real-life situations. The transportation problem is a linear programming problem, which can be solved by the regular simplex method but due to its special structure a technique called the transportation technique is used to solve the transportation problem. It got its name from its application to problems involving transporting products from several sources to several destinations, although the formation can be used to represent more general assignment and scheduling problems as well as transportation and distribution problems. The two common objectives of such problems are either to:

- minimize the total transportation cost of shipping a single commodity from *m* sources to *n* destinations, or

- maximize the profit of shipping from *m* sources to *n* destinations.

3.1.2 Characteristics of a Transportation Problem

(i) Objective function is to reduce the transportation cost to the minimum.

(ii) Maximum quantity available at the sources is limited. This is a constraint.

(iii) Maximum quantity required at the destination is specified. This cannot be exceeded, this is another constraint.

(iv) Transportation cost is specified for each item.

(v) Sum of the products available from all sources is equal to sum of the products distributed at various destinations

Maximum quantity available at the source, maximum quantity required at the destination and the cost of transportation, all refer to a single product.

3.1.3 Degeneracy in Transportation Problem

Transportation with m-origins and *n*-destinations can have (m+n-1) positive basic variables, otherwise the basic solution degenerates. So whenever the number of basic cells is less than (m + n-1), the transportation problem is degenerate.

3.1.4 How to resolve degeneracy in transportation problem

To resolve the degeneracy, the positive variables are augmented by as many zero-valued variables as is necessary to complete (m +n –1) basic variables.

3.2 MATHEMATICAL FORMULATION

Let the cost of transporting one unit of goods form i^{th} origin to j^{th} destination be C_{ij} , i= 1,2, ….m, j=1,2,….n. If $x_{ij} \geq 0$ be the amount of goods to be transported from i^{th} origin to j^{th} destination, then the problem is to determine x_{ij} so as to

$$\text{Minimize } \sum_{i=1}^{m}\sum_{j=1}^{n} XijCij$$

Subject to the constraint

$$\sum_{j=1}^{n} Xij \text{ , (i = 1, 2, ..., m)}$$

$$\sum_{i=1}^{m} Xij \text{ , (j = 1, 2, ..., n)}$$

and $x_{ij} \geq 0$, for all i and j.

3.2.1 Feasible Solution

A set of non-negative allocations, x which satisfies the row and column restrictions is known as feasible solution. A feasible solution to an m-origin and n-destination problem is said to be basic feasible solution if the number of positive allocations are (m+n–1).

3.2.2 Non – Degenerate Basic Feasible Solution

A basic feasible solution of an (m × n) transportation problem is said to be non-Degenerate if it has following two properties: (a) Initial basic feasible solution must contain exactly (m+n–1) number of individual allocations.

(b) These allocations must be in independent positions. Independent positions of a set of allocations mean that it is always impossible to form any closed loop through these allocations.

Definition (Loop)

In a transportation table, an ordered set of four or more cells is said to form a loop if:

(i). Any two adjacent cells in the ordered set lie in the same row or in the same column.

(ii). Any three or more adjacent cells in the ordered set do not lie in the same row or in the same column.

3.2.3 Degenerate Basic Feasible Solution A basic feasible solution that contains less than (m + n – 1) non – negative allocations is said to be degenerate basic feasible solutions.

3.3 BALANCED TRANSPORTATION PROBLEM If total supply equals total demand, the problem is said to be a balanced transportation problem: that is

$$\sum_{i=1}^{m} ai = \sum_{j=1}^{n} bj$$

3.3.1 Unbalanced Transportation Problem

If the transportation problem is known as an unbalanced transportation problem then, there are two cases.

Case (1).

Here, $\sum_{i=1}^{m} ai > \sum_{j=1}^{n} bj$

To solve this, we first balance it by introducing a dummy destination in the transportation table. The cost of transporting to this destination is all set equal to zero. The requirement at this destination is assumed to be equal to

$$\sum_{i=1}^{m} ai - \sum_{j=1}^{n} bj$$

Case (2) .

Here, $\sum_{i=1}^{m} ai \ < \ \sum_{j=1}^{n} bj$

To solve this, we first balanced it by introducing a dummy origin in the transportation table; the costs associated with are set equal to zero. The availability is

$$\sum_{j=1}^{n} bj \ = \sum_{i=1}^{m} ai$$

3.4 METHODS OF FINDING INITIAL BASIC FEASIBLE SOLUTION FOR A BALANCED TRANSPORTATION PROBLEM

The three basic methods are:

- The Northwest Corner Method
- The Least Cost Method
- The Vogel's Approximation Method

3.4.1 Northwest-Corner Method

The steps below are used in the Northwest- Corner method

Step (1) The first assignment is made in the cell occupying the upper left-hand (North West) corner of the transportation table. The maximum feasible amount is allocated there, i.e.; x_{11} = min (a_1, b_1) .

Step (2) If $b_1 > a_1$, the capacity of origin O_1 is exhausted but the requirement at D_1 is not satisfied. So move downs to the second row, and make the second allocation: x_{21} = min (a_2 , $b_1 - x_{11}$) in the cell (2,1). If $a_1 > b_1$, allocate x_{12} = min ($a_1 - x_{11}$, b_2) in the cell (1,2) . Continue this until all the requirements and supplies are satisfied.

3.4.2 Least-Cost Method

The least cost method uses shipping costs in order to come up with a basic feasible solution that has a lower cost. To begin the minimum cost method, first we find the decision variable with the smallest shipping cost x_{ij}. Then assign x_{ij} its largest possible value, which is the minimum of s_i and d_j . After that, as in the Northwest Corner Method we should cross out row i and column j and reduce the supply or demand of the non crossed-out row or column by the value of x_{ij}, then we will choose the cell with the minimum cost of shipping from the cells that do not lie in a crossed-out row or column and we will repeat the procedure.

3.4.3 Vogel'S Approximation Method (VAM)

Step 1 For each row of the transportation table, identify the smallest and the next to-smallest costs. Determine the difference between them for each row. Display them alongside the transportation table by enclosing them in parenthesis against the respective rows. Similarly compute the differences for each column.

Step 2 Identify the row or column with the largest difference among all the rows and columns. If a tie occurs, use any arbitrary tie breaking choice. Let the greatest difference correspond to i^{th} row and the minimum cost be C_{ij} . Allocate a maximum feasible amount $x_{ij} = \min (a_i , b_j)$ in the $(i, j)^{th}$ cell, and cross off the i^{th} row or j^{th} column.

Step 3. Re compute the column and row differences for the reduced transportation table and go to step 2. Repeat the procedure until all the rim requirements are satisfied.

3.5 OPTIMAL SOLUTION

A feasible solution (not necessarily basic) is said to be optimal if it minimizes the total transportation cost.

3.5.1 Theorem for Testing Optimality.

If we have a B.F.S. consisting of $m + n - 1$ independent positive allocations and a set of arbitrary number u and v ($i = 1,2,...m$; $j = 1,2,...n$) such that $c_{rs} = u_r + v_s$ for all occupied cells (r,s) then the evaluation d_{ij} corresponding to each empty cell (i, j) is given by

$$d_{ij} = c_{ij} - (u_i + v_j)$$

3.5.2 Solution to Optimality

As mentioned above, the solution method for transportation problems is a streamlined version of the Simplex algorithm. As such, the solution method also has two phases. In the first phase, the aim is to construct an initial basic feasible solution; and in the second phase, to iterate to an optimal solution. For optimality, we need a method, like the simplex method, to check and obtain the optimal solution. The two methods used are:

i. Stepping-stone method

ii. Modified distributed method (MODI)

3.5.3 Stepping Stone

(i). Consider an initial tableau

(ii). Introduce a non-basic variable into basic variable (iii). Add the minimum value of all the negative cells into cells that has " positive sign", and subtracts the same value to the "negative" cells

(iv). Repeat this process to all possible non-basic cells in the tableau until one has the minimum cost. If it does not give the optimal solution or yield a good results, Introduce the MODI method for optimality

3.5.4 Modified distributed method (MODI)

It is a modified version of the stepping stone method

MODI determines if a tableau is the optimal, tells which non-basic variable should be firstly considered as an entry variable, and makes use of stepping-stone to get its answer of next iteration

Procedure (MODI)

Step 1: let u_i, v , c_{ij} variables represent rows, columns, and cost in the transportation tableau, respectively

Step 2: (a) form a set of equations that uses to represent all basic variables $u_i + v_j = c_{ij}$

(b) Solve variables by assign one variable = 0

Step3: (a) form a set of equations use to represent non-basic variable (or empty cell) as

Such $c_{ij} – u_i – v_j = k_{ij}$

(b) Solve variables by using step 2b information

Step 4: Select the cell that has the most negative value in 3b

Step 5: Use stepping-stone method to allocate resource to cell in Step 4

Step 6: Repeat the above steps until all cells in 3a has no negative Value.

3.6 THE TRANSSHIPMENT PROBLEM

We may come across a certain situation that a company (or companies) may be producing the product to their capacity, but the demand arises to these products during certain period in the year or the demand may reach the peak point in a certain period of the year. This is particularly true that products like Cool drinks, Textbooks, Notebooks and Crackers, etc. The normal demand for such products will exist, throughout the year, but the demand may reach peak points during certain months in the year. It may not possible for all the companies put together to satisfy the demand during peak months. It is not possible to produce beyond the capacity of the plant. Hence many companies have their regular production throughout the year, and after satisfying the existing demand, they stock the excess production in a warehouse and satisfy the peak demand during the peak period by releasing the stock from the warehouse. This is quite common in the business world. Only thing that we have to observe the inventory carrying charges of the goods for the months for which it is stocked is to be charged to the consumer. Take for example crackers; though their production cost is very much less, they are sold at very high prices, because of inventory carrying charges. When a company stocks its goods in warehouse and then sends the goods from warehouse to the market, the problem is known as Transshipment problem.

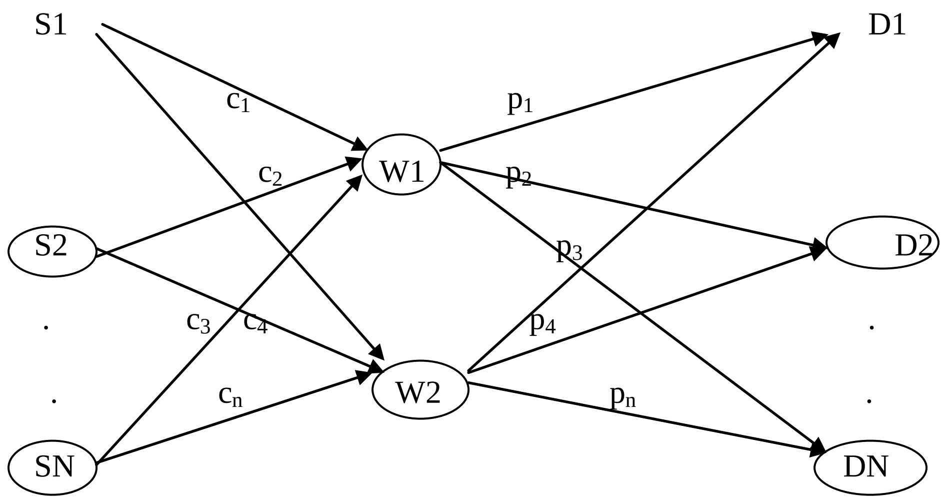

Figure 3.2: The Transshipment Problem

The transshipment problem is an extension of the framework of the transportation problem. The extension is in allowing the presence of a set of transshipment points that can serve as intermediate stops for shipments, possibly with a net gain or loss in units. Any given transshipment problem can be converted into an equivalent transportation problem. Hence, the procedure for solving the transportation problems can be applied to the solution of transshipment problem as well.

A transshipment problem (or network flow problem) consists of finding the cheapest way of shipping goods through a network of routes so that all given demands at all points of the network is satisfied.

Given:

- a network of routes as a graph
- a set of nodes which act as sources (supplies)
- a set of nodes which act as sinks (demands)
- the amount of supply and demand at each node
- the cost of each transport route (edge)

The transshipment problem is similar to the transportation problem except that in the transshipment problem it is possible to ship both into and out of the same node (point). It is an extension of the transportation problem in which intermediate nodes, referred to as transshipment nodes are added to source as well as sink nodes to account for locations such as warehouses. In this more general type of distribution problem, shipments may be made between any pair of the three general types of nodes: origin nodes, transshipment nodes and destination nodes. For example (i) transshipment problems permits shipments of goods from origins to transshipment nodes and on to destinations, (ii) From one origin to another origin, (iii) From one transshipment location to another, (iv) from one destination location to another and (v) directly from origins to destinations.

THE MODEL

The general linear programming model of a transshipment problem is

$$\text{Min } \sum_{allarcs} CijXij$$

Subject to

$$\sum_{arcout} Xij - \sum_{arcin} Xij = S_i \quad \text{nodes Origin i}$$

$$\sum_{arcout} Xij - \sum_{arcin} Xij = 0 \quad \text{Transshipment nodes}$$

$$\sum_{arcin} Xij - \sum_{arcout} Xij = D_i \quad \text{demand nodes j}$$

Where

x_{ij} = amount of units shiped from node i to node j

C_{ij} = cost per unit of shipping from node i to node j

S_i = supply at origin node i

Dj = demand at origin node j

The following steps describe how the optimal solution to a transshipment problem can be found by solving a transportation problem.

Step1: If necessary, add a dummy to a demand point or supply points (with a supply of 0 and a demand equal to the problems excess supply) to balance the problem. Shipments to the dummy and from a point to itself will be zero. Let s = total available supply and d= total demand.

Step2: Construct a transportation tableau as follows: A row in the tableau will be needed for each supply point and transshipment point, and a column will be needed for each demand point and transshipment point. Each supply point will have a supply equal to its original supply, and each demand point will have a demand to its original demand. Let s = total available supply and d = demand. Then each transshipment point will have a supply equal to (point's original supply) + s and a demand equal to (point's original demand) + *s*. This ensures that any transshipment point that is a net supplier will have a net outflow equal to point's original supply and a net demander will have a net inflow equal to point's original demand. Although we do not know how much will be shipped through each transshipment point, we can be sure that the total amount will not exceed *s*.

Step 3: Solve the transportation table of step 2 by the transportation technique.

CHAPTER FOUR

DATA COLLECTION AND ANALYSIS

4.0 INTRODUCTION

In this chapter, we shall present data collection and analysis of the data. Data from the Blue Sky Limited shall be examined.

4.1 Data Collection and Analysis

A company has three factories *X*, *Y* and *Z* producing product *P* and two warehouses to stock the goods and the goods are to be sent to four market centres *A*, *B*, *C* and *D* when the demand arises. Figure 4.1 shows the cost of transportation from factories to warehouses and from warehouses to the market centres, the capacities of the factories, and the demands of the market centres. Formulate a transportation matrix and solve the problem for minimizing the total transportation cost.

Figure 4.1 depicts the cost of transportation from factories to warehouses and from warehouses to the market centres (in thousand Ghana cedis).

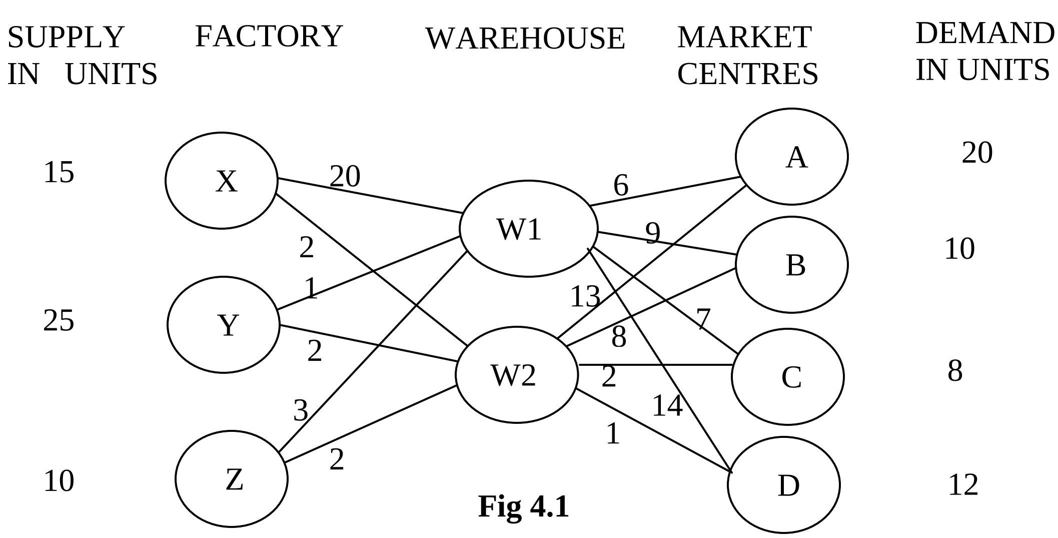

Fig 4.1

To formulate a transportation problem for three factories and four market centers, we have to find out the cost coefficients of cells. For this, if we want the cost of the cell *XA*, the cost of transportation from *X* to warehouse *W*1 + Cost transportation from *W*1 to market center *A* are calculated and as our objective is to minimize the cost, the least of the above should be entered as the cost coefficient of cell *XA*. Similarly, we have to workout the costs and enter in the respective cells.

Cell *XA*: Route *X*-*W*1-*A* and *X*- *W*2- A minimum of these two (26 and 15) i.e. 15

Cell *XB* Route *X* - *W*1 - *B* and *X* - *W*2 - *B* Minimum of the two is (29, 10) i.e. 10

Cell *XC* Route *X* - *W*1 - *C* and *X*- *W*2 - *C* Minimum of the two is (27, 4) i.e. 4

Cell *XD* Route *X*- *W*1 - *D* and *X*- *W*2 - *D* Minimum of the two is (34, 3) i.e. 3

Cell *YA*: Route *Y*-*W*1-*A* and *Y*- *W*2- A minimum of these two (7 and 15) i.e. 7

Cell *YB* Route *Y* - *W*1 - *B* and *Y* - *W*2 - *B* Minimum of the two is (10, 10) i.e. 10

Cell *YC* Route *Y* - *W*1 - *C* and *Y*- *W*2 - *C* Minimum of the two is (8, 4) i.e. 4

Cell *YD* Route *Y*- *W*1 - *D* and *Y*- *W*2 - *D* Minimum of the two is (15, 3) i.e. 3

Cell *ZA*: Route *Z*-*W*1-*A* and *Z*- *W*2- A minimum of these two (9 and 14) i.e. 9

Cell *ZB* Route *Z* - *W*1 - *B* and *Z* - *W*2 - *B* Minimum of the two is (12, 9) i.e. 9

Cell *ZC* Route *Z* - *W*1 - *C* and *Z*- *W*2 - *C* Minimum of the two is (10, 3) i.e. 3

Cell *ZD* Route *Z*- *W*1 - *D* and *Z*- *W*2 - *D* Minimum of the two is (17, 2) i.e. 2

The required transportation problem is shown in Table 4.2:

	A	B	C	D	SUPPLY
X	15	10	4	3	15
Y	7	10	4	3	25
Z	9	9	3	2	10
DEMAND	20	10	8	12	**50**

To formulate the problem, form transportation tableau, let

i= product to be shipped.

j = destination of each product.

$s_i =$ the capacity of source node *i*,

$d_j =$ the demand of destination *j*,

x_{ij}= the total capacity from source *i* to destination *j*

C_{ij}= the per unit cost of transporting commodity from *i* to destination *j*.

The problem can be modeled as:

$$\text{Minimize} \quad 15x_{11} + 10\,x_{12} + 4\,x_{13} + 3\,x_{14}$$

$$7x_{21} + 10\,x_{22} + 4\,x_{23} + 3\,x_{24}$$

$$9x_{31} + 9\,x_{32} + 3\,x_{33} + 2\,x_{34}$$

Subject to

$$x_{11} + x_{12} + x_{13} + x_{14} = 15$$

$$x_{21} + x_{22} + x_{23} + x_{24} = 25$$

$$x_{31} + x_{32} + x_{33} + x_{34} = 10$$

$$x_{11} + x_{21} + x_{31} = 20$$

$$x_{12} + x_{22} + x_{32} = 10$$

$$x_{13} + x_{23} + x_{33} = 8$$

$$x_{14} + x_{24} + x_{34} = 12$$

Using the West Corner rule we get the initial basic solution.

The solution tableau is as shown in Table 4.3 below,

	A		B		C		D		SUPPLY
X	15	15		10		4		3	15
Y	5	7	10	10	8	4	2	3	25
Z		9		9		3	10	2	10
DEMAND	20		10		8		12		**50**

The initial basic feasible solution is;

$\bar{x} = (x_{B11}, x_{12}, x_{13}, x_{14}, x_{B21}, x_{B22}, x_{B23}, x_{24}, x_{31}, x_{32}, x_{33}, x_{B34})$

This from the table is given as;

$\bar{x}$ = (15, 0, 0, 0, 5, 10, 8, 2, 0, 0, 0, 10) in thousands with the total transportation cost of

Cost = (15000*15) + (5000*7) + (10,000*10) + (8,000*4) + (2,000*3) + (10,000*2)

Total Cost = GH¢418,000.00

Now, we use the optimality conditions to improve upon our solution by the MODI method.

Now we find the cost equation of the basic cell;

$C_{ij} = u_i + v_j$

Thus, $u_1 + v_1 = 15$

$u_2 + v_1 = 7$

$u_2 + v_2 = 10$

$u_2 + v_3 = 4$

$u_2 + v_4 = 3$

$u_3 + v_4 = 2$

Letting $u_1 = 0$, from the equations we have;

$u_1 = 0$

$v_1 = 15$

$u_2 = -8$

$v_2 = 18$

$v_3 = 12$

$u_3 = -9$

$v_4 = 11$

We find the net evaluation factor or the reduced costs for the non-basic variables.

$e_{ij} = C_{ij} - u_i - v_j$

$e_{12} = 10 - u_1 - v_2$

$e_{13} = 4 - u_1 - v_3$

$e_{14} = 3 - u_1 - v_4$

$e_{31} = 9 - u_3 - v_1$

$e_{32} = 9 - u_3 - v_2$

$e_{33} = 3 - u_3 - v_3$

Hence,

$e_{12} = -8$

$e_{13} = -8$

$e_{14} = -8$

$e_{31} = 3$

$e_{32} = 0$

$e_{33} = 0$

The presence of negative values for the reduced cost signifies non optimality; hence we readjust. From the above, the minimum reduced costs for the non-basic variable is x_{13}. Therefore x_{13} should enter the basis since it is the most negative reduced cost.

We then move on to next improvement iteration.

At the end of this stage of iteration, the basic feasible solution is:

$\bar{x}$ = (7, 0, 8, 0, 13, 10, 0, 2, 0, 0, 0, 10) in thousands

Now we find the cost equation of the basic cell;

$C_{ij} = u_i + v_j$

Thus,

$u_1 + v_1 = 15$

$u_1 + v_3 = 4$

$u_2 + v_1 = 7$

$u_2 + v_2 =10$

$u_2 + v_4 = 3$

$u_3 + v_4 = 2$

Letting $u_1 = 0$, from the equations we have;

$u_1 = 0$

$v_1 = 15$

$u_2 = -8$

$v_2 = 18$

$v_3 = 4$

$u_3 = -9$

$v_4 = 11$

We find the net evaluation factor or the reduced costs for the non-basic variables.

$e_{ij} = C_{ij} – u_i - v_j$

$e_{12} = 10 – u_1 – v_2$

$e_{14} = 3 – u_1 – v_4$

$e_{23} = 4 – u_2 – v_3$

$e_{31} = 9 – u_3 – v_1$

$e_{32} = 9 – u_3 – v_2$

$e_{33} = 3 – u_3 – v_3$

Hence,

$e_{12} = -8$

$e_{14} = -8$

$e_{23} = 8$

$e_{31} = 3$

$e_{32} = 0$

$e_{33} = 8$

The presence of negative values for the reduced cost signifies non optimality; hence we readjust. From the above, the minimum reduced costs for the non-basic variable is x_{12}. Therefore x_{12} should enter the basis since it is the most negative reduced cost.

We then move on to next improvement iteration.

At the end of this stage of iteration, the basic feasible solution is:

$\bar{x} = (0, 7, 8, 0, 20, 3, 0, 2, 0, 0, 0, 10)$ in thousands

Now we find the cost equation of the basic cell;

$C_{ij} = u_i + v_j$

Thus,

$u_1 + v_2 = 10$

$u_1 + v_3 = 4$

$u_2 + v_1 = 7$

$u_2 + v_2 = 10$

$u_2 + v_4 = 3$

$u_3 + v_4 = 2$

Letting $u_1 = 0$, from the equations we have;

$u_1 = 0$

$v_1 = 7$

$u_2 = 0$

$v_2 = 10$

$v_3 = 4$

$u_3 = -1$

$v_4 = 3$

We find the net evaluation factor or the reduced costs for the non-basic variables.

$e_{ij} = C_{ij} – u_i - v_j$

$e_{11} = 15 – u_1 – v_1$

$e_{14} = 3 – u_1 – v_4$

$e_{23} = 4 – u_2 – v_3$

$e_{31} = 9 – u_3 – v_1$

$e_{32} = 9 – u_3 – v_2$

$e_{33} = 3 – u_3 – v_3$

Hence,

$e_{11} = 8$

$e_{14} = 0$

$e_{23} = 0$

$e_{31} = 3$

$e_{32} = 0$

$e_{33} = 0$

Since all the reduced cost for the non-basic variables are all positive, it implies the optimal solution is reached.

We then proceed to find our optimal route and calculate our total cost of shipment from the routes to the various destinations.

The optimal allocation is:

Cell	Route	Load	Cost in GH¢
XB	X - W2 - B	7000	10
XC	X - W2 – C	8000	4
YA	Y - W1 – A	20000	7
YB	Y – W1- B	3000	10
YD	Y - W2 - D	2000	3
ZD	Z – W2 –D	10000	2

Cost = (7000*10) + (8000*4) + (20,000*7) + (3,000*10) + (2,000*3) + (10,000*2)

Total Cost = GH¢278,000.00

CHAPTER FIVE

CONCLUSIONS AND RECOMMENDATIONS

5.0 INTRODUCTION

With increased product specialization and globalization, the shipping industry has experienced steady growth during the past few decades. At the same time, ocean carriers have competed to offer better service at a cheaper price. Carriers are constantly looking for opportunities to introduce new services/routes to attract and capture more market demand. Operation has indeed become one of the key competitive advantages with optimization-based approaches being expected to play an important role.

Our study is largely motivated by a practical problem faced by a logistics subsidiary of a shipping company. The company has three factories *X*, *Y* and *Z* producing product *P* and two warehouses to stock the goods and the goods are to be sent to four market centres *A*, *B*, *C* and *D* when the demand arises. The challenge for the logistics subsidiary is to help the client to determine the most cost effective distribution strategy to move the products from the supply port to the demand ports.

From a management and operation perspective, a fundamental question in this problem is whether the company should move the product into the demand ports directly from the supply port, or whether the transhipment operation using an established warehouse can help to reduce the total cost to the company and increase responsiveness. From the resource utilization perspective, a transhipment system is certainly preferred as it provides a better utilization of the transporting goods.

5.1 CONCLUSIONS

This thesis seeks to solve a real-life problem of a Company in Ghana using transhipment models. It was observed that the route that gave minimum achievable transportation cost was

X - W2 – B, X - W2 – C, Y - W1 – A, Y – W1- B, Y - W2 – D, and Z – W2 –D at the unit transportation cost of GH¢10, GH¢4, GH¢7, GH¢10, GH¢3 and GH¢2 with a shipping loads of 7000, 8000, 20000, 3000, 2000 and 10000. This means that the company should spend a total cost of two hundred and seventy-eight thousand Ghana cedis (GH¢278,000.00) to transport its products from the production centers through the warehouse to the market centers using the above routes.

Currently, as at the time of this work, there is no such method for determining which routes to be used in transporting the products by the company. The routes are chosen using guess work and by the discretion of the people in charge.

For the data used for our analysis, the company using their crude approach arrived at the following conclusion; shipped the loads of 15000, 5000, 10000, 8000, 2000 and 10000 at a unit costs of GH¢15, GH¢7, GH¢10, GH¢4, GH¢3 and GH¢2 through the routes

X - W2 – B, X - W2 – C, Y - W1 – A, Y – W1- B, Y - W2 – D, and Z – W2.

Total cost of transporting these products was four hundred and eighteen thousand Ghana cedis (GH¢418,000.00).

5.2 RECOMMENDATIONS

The use of computer application in computation gives a systematic and transparent solution as compared with an arbitrary method. Operation has become one of the key competitive advantages with optimization-based approaches being expected to play an important role. Using optimization-based approaches to model industrial problem gives a better result. Management may benefit from the proposed approach for transporting the goods from the manufacturing centers to the various market centers in order to minimized transportation cost. We therefore recommend that our transshipment model should be adopted by the company for its transshipment planning.

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