KWAME NKRUMAH UNIVERSITY OF SCIENCE AND TECHNOLOGY, KUMASI

MODELLING OPTIMAL TRADING STRATEGY FOR A 2^{ND} TIER PENSION FUND MANAGER UNDER ISO-ELASTIC UTILITY FUNCTION

BY

ERIC OPPONG ADJEI

A THESIS SUBMITTED TO THE DEPARTMENT OF MATHEMATICS, KWAME NKRUMAH UNIVERSITY OF SCIENCE AND TECHNOLOGY IN PARTIAL FULFILLMENT OF THE REQUIREMENT FOR THE DEGREE OF M.PHIL ACTUARIAL SCIENCE

JUNE, 2016

DECLARATION

I hereby declare that this submission is my own work towards the award of the M.Phil Actuarial Science degree and that, to the best of my knowledge, it contains no material previously published by another person nor that which had been accepted for the award of any other degree of the university, except where due acknowledgment had been made in the text.

Eric Oppong Adjei	....	....
(Student)	(Signature)	(Date)

Certified by:

Dr. Gabriel A. Okyere	....	....
(Supervisor)	(Signature)	(Date)

Certified by:

Prof. S. K.Amponsah	....	....
(Head of Department)	(Signature)	(Date)

DEDICATION

This thesis is firstly dedicated to God Almighty for his strength, compassion and grace that have sustained me throughout my stay on campus. Secondly, to my mother Miss Leticia Otabil, Rev. and Mrs. Kumah and all loved ones for their support, kindness and encouraging words that motivated me to reach this far. I say God bless you all.

ABSTRACT

The study sought to address portfolio optimization problem of a 2^{nd} Tier Pension Fund Manager in Ghana who wanted to maximize his expected utility from his terminal wealth over all admissible trading strategies on a finite time interval. The research was done within the framework of Iso- elastic utility functions. The novelty of this study was that the interest rate was time dependent and was modeled within the spectral density domain. The drift process was modeled as a Gaussian process. A Monte Carlo simulation was considered for three stocks with a specified covariance structure under different scenarios; (log-normal, normal and exponential) for generating the stock prices. The main result of the simulation study indicated that, for any portfolio under these three scenarios with a log-normally distributed stock, the fund manager should invest larger proportion of their wealth in that stock irrespective of the level of risk aversion coefficient. Likewise, the market data fitted depicted the same results as shown in the simulation studies, hence, improving 2^{nd} Tier Pension Fund Manager's wealth to aid the 2^{nd} Tier Pension contributors when they went on retirement.

ACKNOWLEDGMENT

In the first place I would like to thank the Almighty God for His protection, strength, wisdom and understanding throughout my studies on KNUST campus and as well finishing this thesis.

I am most grateful to my supervisor, Dr. G. A. Okyere for his time, patience, insight, guidance, inspiration and his expert advice throughout my thesis work. Though I was unfavorable, yet you made the work easy for me. God bless you, Dr.

I am also grateful to all the staff members of Mathematics department of KNUST for all the support and advice.

My deepest thanks also go to the staff of Statistical Consulting and Quantitative Skill Development Unit of the Department of Mathematics KNUST, especially Mr. Mowulolo Koku Ameko and 2015/2016 service year group.

I also want to take this opportunity to express my heartfelt appreciation to all siblings and to my parents and family for their prayers and support especially Rev and Mrs. Kumah (CCC, Obuasi) for making this dream a reality and also for their unending encouragement, support and attention, I say God bless you.

Also, many thanks go to all my friends especially Miss. Mary Duah (Global Teachers Co-operative Credit Union, Tamale), my course mates especially Collins Abaitey, for their unceasing support and encouragement.

Lastly, I want to express my special appreciation to Mr. and Mrs. Otieku (Admissions

Office, KNUST), Mr. Robert Kingsley Baffoe (Value Properties, Obuasi), and Mr. Ebenezer Osei-Mensah of the Salvation Army, Ashtown Corps for supporting this worthy course. God bless you all.

CONTENTS

LIST OF ABBREVIATION

IM ....Investment Management

AMWG .... Asset Management Working Group

SSNIT .... Social Security and National Insurance Trust CSAG

.... Civil Servants Association of Ghana

GNAT .... Ghana National Association of Teachers GRNA

.... Ghana Registered Nurses Association

JUSAG ....Judicial Service Staff Association of Ghana DB

.... Defined Benefit

DC ....Defined Contribution GOF

.... Goodness of Fit Test

GES .... Ghana Stock Exchange SPD

....Symmetric Positive Definite

SDE ....Stochastic Differential Equation

GUSSS .... Ghana Universities Staff Superannuation Scheme

OTS ....Optimal Trading Strategy RSA

....Retiree Saving Account

SRI .... Socially Responsible Investing CPT

.... Cumulative Prospect Theory

FFT ....Fast Fourier Transforms PDF .... Probability Density Function

PAL ....Police Assistance Line

LIST OF TABLES

LIST OF FIGURES

CHAPTER 1

INTRODUCTION

1.1 Introduction

The research seeks to address the optimization problem of a 2^{nd} Tier Pension Fund Manager who wants to maximize the expected utility from his terminal wealth under Iso-elastic utility function. Chapter one details with the background of the study, problem statement, objectives, significance of the study, organization of the study among others.

1.2 Background of the Study

Pension is a fund into which a sum of money is added regularly during an employee's employment years, and from which payments are made to support the person's retirement by the government, a former employer or an insurance company in the form of periodic payments. "A pension scheme is a long term investment vehicle whose principal objective is to provide decent and reliable income upon retirement" cited by Kwapong (2013). The pension schemes are designed mainly to provide regular streams of income to persons during retirement, old age or death of the breadwinner. Mostly the scheme could be private, state or occupationally designed. An example of the state pension plan is the Ghana Pension Scheme which is administered by SSNIT, which has approximately 1.2 million-registered members with over 150,000 pensioners according to SSNIT (2015) online report. For most retired persons, the possibility of no money coming in each month is in fact a big blow to them. The continuation of such financial standard of living after retirement is an important issue for workers which resulted in pension plans since 1940 according to Aitken (1994).

Upon the existence of pension plans in Ghana, Ghana's pension scheme has undergone a series of reformation since 1950. Ghana began the pension scheme in 1950 by the British

Colonial Ordinances (Pensions Ordinance No. 42, Chapter 30). The scheme was established for public servants in the Gold Coast which later became known as CAP 30.

Until 2008, the National Pensions Act, 2008 in Ghana instituted a contributory ThreeTier pension Scheme which was to address some of the challenges workers encountered with CAP 30 and SSNIT Scheme. Before the inception of the Pension Schemes in Ghana, Ghanaians knew how to save (or protect themselves) but the savings were made in tangible assets like buildings, lands, equipment etc. The extended family system of protection was the means of support in times of old age, disability or death of a member. The family system was weakened by education and westernization. Ghanaians began to pay much attention to the formal system of social security protection which weakens the traditional system completely according to Kwapong (2013).

In 1965, the Social Security Act (279) was officially endorsed to build contributory Social Security Scheme for the payment of retirement, invalidity, survivors and other benefits for workers in every private sector as well as non-pensionable workers in the public service. The scheme was non-contributory and intended to be a reward for civil servants after a minimum of ten years in service where the incineration of patriotism had been set ablaze in the Gold Coast. The new Pension Ordinance or the CAP 30 Pension scheme also came at a period where the Gold Coast was experiencing an evolution from a colonial dominated civil service to a native civil service. The government gave orders to stop the payment of benefits to members under the Preventive Detention Acts of 1959. The non-contributory factor in the pension scheme hindered the growth of the scheme since government was unable to pay benefits when it was due.

In November 1972, the Social Security Decree (NRCD 127) came to abolish Act (279) of 1965, and established the Social Security and National Insurance Trust (SSNIT) as an independent body to administer a universal social security fund for all Ghanaian workers in the country. Unlike the CAP 30 scheme, the Provident Fund under SSNIT was a contributory scheme from which a lump sum was paid upon retirement. A proposal was made to convert the Provident Fund to Pension Scheme in 1987. In

1991, the Social Security Law 1991 (PNDC Law 247), was revealed to transform the 1972 scheme from a provident fund to a full pension scheme. The sudden shift was to change the lump sum payment to a monthly payment of retirement benefits. The previous schemes showed some disparities as compared to the 1991 scheme. The 1991 scheme had an element of pooling resources to meet unforeseen events. To be able to qualify for benefits, participants must have contributed for at least 240 months to the scheme. The scheme considered only three contingencies; old age, invalidity (disability) and death benefits. Since 1965, the country has been running two major public pension schemes; the CAP 30 and the SSNIT schemes. The discrimination between the two schemes became more noticeable, leading to agitation and protest by some public sector workers on the SSNIT scheme demanding to be placed on the CAP 30 scheme which was considered to be more favorable, particularly the lump-sum element. This brought to light the Three-Tier pension scheme.
In 2008, The National Pensions Act, 2008 in Ghana promulgated a contributory ThreeTier Pension Scheme comprising of the following:

1. A compulsory basic National Social Security Scheme to be managed by SSNIT,
2. A compulsory fully funded and privately managed Occupational Pension Scheme and
3. A voluntary fully funded and privately managed Provident fund and personal pension Scheme.

The 2^{nd} Tier Scheme, to which this thesis is dedicated to, is to be a mandatory occupational Scheme to be run by approved Trustees licensed by the regulatory body but managed by private fund managers. Contribution to the Scheme is 5% of the employee's basic salary. Benefits would be a lump sum payment which is expected to be higher than presently exists under SSNIT and CAP 30. Being a Defined Contribution (DC) Scheme, level of benefits would depend on both level of contribution and returns on investments. Proceeds could be used to purchase annuities to enhance the monthly benefits so as to achieve financial goals set by the retiree.

Among the various activities that prevail in investment, optimal wealth allocation remains extremely an important issue for fund managers and investors who seek to optimize returns. Decisions that are made by fund managers have a direct impact on investor's returns when they go on retirement. In the quest of enhancing fund manager's portfolio, a lot of concentration in the past has been set to the Markowitz meanvariance technique 1952 in his journal "portfolio selection", which without hesitation has provided many handy insights. In his approach, he focuses his work on a singleperiod prospect which curtails the utility theory under consideration. Past researchers have identified that the risk aversion has the potential to largely influence wealth allocation. Since 1950, portfolio management has become more of a science-based discipline. Numerous theoretical advances combined with empirical research have provided portfolio managers with new concepts, insights and techniques for making sound investment decisions.

In line with the decision on benefits payments, labor agitations and protest upsurge gave birth to the Three-Tier pension scheme. It was led by; Civil Servants Association of Ghana (CSAG), Ghana National Association of Teachers (GNAT), the Ghana Registered Nurses Association (GRNA), and the Judicial Service Staff Association of Ghana (JUSAG) during President J. A. Kuffuor's era.

Every reform that came to pass had a particular challenge to solve. One major question needs to be answered; Why the move from CAP 30 to SSNIT Pension Scheme? This was because;

1. The government wanted to make the Scheme a contributory one, since CAP 30 was unfunded or non-contributory.
2. The Scheme was not open to all Ghanaian workers unlike the SSNIT Scheme.

Now a question again is why the move from SSNIT Pension Scheme to Three-Tier Pension Scheme? It is to address the concerns of the Ghanaian workers regarding the conflict in the benefit package to pensioners. Some of the possible issues that led to the shift from SSNIT Pension Scheme to Three-Tier Pension Scheme were:

i. Low monthly pension payments,

ii. Lack of lump-sum payment, and

iii. Hardship of pensioners when accessing their benefits and many others.

The Three-Tier Pension Scheme came as a remedy to the challenges that needed to be addressed in the SSNIT and CAP 30 according to SSNIT (2015) online report. This became one of the motivations for this study.

On 13th November 2014, one of the newspapers in Ghana (The Chronicle) titled its paper that was posted on the Internet as “How to Resolve the Second-Tier Pension Impasse and Improve Pension Administration in Ghana” cited by Ghanaian Chronicle (2014). The researcher was astonished after reading the title. This was because he knew that the new Tier Pension scheme had come to solve the problems of the past. He read further and saw that on 22nd October 2014, twelve of Ghana’s labor unions including public health, local government and education sector workers embarked on an indefinite strike over Government’s failure to pay their 2^{nd} Tier Pension Scheme contributions, and demanded for full disclosure on their funds accrued in the scheme and control over selection of trustee for management of the funds. This revealed the future of the scheme to the researcher that, though on paper 2^{nd} Tier pension fund is performing very well and it is solving the conflicts of the old Schemes with its investments returns and improvement in the access to the informal sector. However, the case is otherwise. This motivated the researcher to look for possible way to help solve future misfortunes.

1.3 Problem Statement

Most fund managers in Ghana seek to allocate wealth in the stock market in order to optimize returns. However the problem they are faced with is what percentage of their wealth must be invested in a particular stock in order to optimize returns.

Again most literature in asset allocation utilizes fixed interest rate in their applications example Lakner (1998). However, in reality varying interest rates are applied. The problem this study seeks to address is to find out if the varying interest rate will perform better than the fixed interest rate, hence the need to consider varying (time dependent) interest rates.

1.4 Objective of the Thesis

This thesis seeks to solve utility maximization problem of a 2^{nd} Tier pension fund manager in Ghana under Iso-elastic utility function, who wants to maximize his expected utility from his terminal wealth over all allowable trading strategies on a finite time interval.

The main objective of this thesis is to work out explicit formula for the optimal trading strategy which is categorized into the following objectives:

1. To identify an optimal trading strategy for a 2^{nd} Tier pension funds in Ghana under Iso-elastic utility function.

2. To observe the behavior of the trading strategy using simulation and market data fitting under the scenarios or observations:

    i. Possible combinations of existing probability distributions with different levels of risk aversion under fixed and varying interest rate for all the stocks.

1.5 Research Questions

The thesis seeks to answer the following questions:

i. What is the optimal trading strategy for 2^{nd} Tier Pension funds in Ghana?

ii. How do investors respond to changes in the variables of the optimal trading strategy?

iii. Does the simulation study produce different result as compared to the market data fitting on the trading strategy?

1.6 Scope of the Study

The thesis focused on 2^{nd} Tier pension scheme in Ghana. The data used to perform the analyses relate to simulation study and market data from yahoo finance stocks and listed and unlisted Ghana Stock Exchange market data. The dataset spans a period of five years from 1^{st} January, 2010 to 31^{st} May, 2015.

1.7 Methodology

The mean-variance portfolio optimization technique by Harry M. Markowitz (1952) theory and power utility theory are used in this thesis. The type of research conducted is quantitative research method. It employed the use of both qualitative and quantitative methods (using mathematical models). The mathematical models used included; utility theory, Fourier series analysis, stochastic differential equations and theorems based on the optimal trading strategy from terminal wealth.

As stated above, this thesis considered simulation and data fitting study. The researcher applied the trading strategy model of the monthly returns of the stock prices to arrive at the optimal trading strategy for the Fund Manager(s). Sample data from the stock market spanning a period of five years was collected. The simulation study was conducted on Monte Carlo studies under existing distributions (Log-normal, Normal and Exponential distributions) with specified covariance structures which were used to simulate the stock prices.

Distribution check was done on the portfolio and decision was made on which of the stocks followed the existing probability distributions stated.

The computer software package that was used to generate the optimal trading strategy, Fourier analysis output, returns on the portfolio and the stochastic differential equations and other models numerically was the RStudio Development Core Team's "language and environment for statistical computing". The main version adopted was the "R version 3.0.2 (2013-09-25) Frisbee Sail", Platform: x86_64-w64-mingw32/x64 (64-bit)".

Microsoft Office (Excel and Word) 2010 was used in processing the sample data at the initial stages and finally used the R software. The final report was written using the "c 2007-2011, Jonathan Kew, Stefan Loffer version 0.4.0 r.759 (MikTeX 2.9) of TeXworks and TeXnicCenter version 1 Beta 7.50. It is a simple environment for editing, typesetting and previewing TeX documents".

Analysis and numerical simulations of the data was conducted using RStudio and EasyFit software. Matlab software "version R2013a (8.1.0.604), 64-bit (win 64)" was also used to plot the graph of the final data analysis. The resources centers used are the Kwame Nkrumah University of Science and Technology School Library, Department of Mathematics libraries, and the Internet.

1.8 Justification of the Thesis

Agreeing to financial literature (and with Ghana in focus), a very little work has been done on the optimal trading strategy for 2^{nd} Tier Pension funds using Iso-elastic utility theory. The majority of the work done on the pension funds in relation to modeling and forecasting is limited to trend conditions, which only captures the underlying structure of the pension funds reforms and fails to explain or capture mean-variance techniques, this is where this thesis contributes to the existing literatures. It is essential therefore to use utility theories and SDE models to find the optimal trading strategy for 2^{nd} Tier pension fund managers and investors in Ghana. It is also of much value to policy makers, financial analysts, applied economists, researchers and management teams of companies in search for corporate goals to find the best trading strategy for their portfolio, in help of solid financial decisions to prepare for unforeseen future events. It would however, contribute to the economic development and growth of the Ghana pension industry and Ghana at large.

1.9 Thesis Organization

The thesis is structured under five main chapters.

Chapter one which is the introduction deals with the historical and biological background to the thesis, statement of the problem, objectives of the thesis, thesis questions, thesis methodology, thesis justification, and organization of the thesis.

Chapter Two discusses the literature related to the thesis. The review involves theoretical studies done by other researchers on the topic under consideration. Chapter Three describes the methodology used in the thesis to support the trading strategy under Iso-elastic utility functions.

Chapter Four covers the data collection and analysis of the trading strategy model, and the discussion of results.

Finally, the summary, conclusions, recommendations and further studies drawn from the study are presented in Chapter Five.

CHAPTER 2

LITERATURE REVIEW

2.1 Introduction

This section reviews the various literatures linked to the pension fund, 2nd Tier Pension Scheme, Risk Aversion investor and many other considerations. In order to reveal critical evidences and general findings which have already been identified by previous researchers and numerous studies, this thesis will focus on published books, papers, journal, reports, articles and views on the subject matter. Many literature on this topic have stressed the importance of carrying out an activity as this, which Lakner (1998) in his paper "optimal trading strategy for an investor the case of partial information" will serve as the standard reference for this thesis.

2.2 Pension Funds

Universally, the need for societal protection has always been the aim of government, employers, family and others. Before the institution of pension schemes the welfare of the members was on the family as they advanced in age. As the world and families grew in numbers of people the support for the aging from the extended family became a serious problem for the state and the families. Families became fed up with such system, because they could not support the aging again because of high cost of living, expensive health care and individual issues. This led to the formation of pension schemes which among the financial institutions is the largest. Stewart and Yermo (2009) argue that, the importance of pension schemes in Africa for instance, have helped in the alleviation of demographic pressures, poverty amongst the elderly and have provided support for households headed by grandparents.

A pension fund also known as “superannuation fund” in some countries is any plan, fund or scheme which provides retirement income at invalidity, old age and on the death of the breadwinner. The rapid growth in pension plans since 1940 has resulted in pension being a significant factor in the financial markets, where billions of dollars of the pension assets are invested and in personal largest assets by Aitken (1994). This rapid growth can be attributed to several factors including employer participation, union demands, a strong post war economy, government legislation and tax assistance. Moreover, the need for personal retirement savings has increased due to factors such as the decline of the extended family system, the change in attitude toward older workers, and increase in the competition for employees. Today we enter the work force at a later age, leave at an earlier age, and live longer which means we must accrue more retirement funds over a short period of time.

2.3 Design of Pension Plans

There are two basic types of pension plans permitting to how the benefits are securely established. These are: defined contribution plans and defined benefit plans. The fundamental difference between those two plans is the defined parameter.

2.3.1 Defined Contribution Plans

It is what the participant and plan sponsors will contribute in advance each year. The participant receives a benefit at retirement which is a function of the contributions made to his account and fund’s investment returns during and at the end of accrual period. Defined contribution plan, states that contributions are paid into individual account for each member. A defined contribution (DC) plan does not explicitly promise a level of benefit during retirement but instead can be viewed as a regular savings account in which employers and sometimes employees contribute money on a regular basis. Under certain plans the employer fully or partially matches an employee’s contributions. Regardless of the plan design, only the contribution payments are defined. The contributions accumulate over the time at some (random) rate of return. Whatever these funds grow to at the time of retirement will determine the amount of the pension to be received by the participant of the plan. The Ghana pension scheme is an example of a DC plan, which is a 2^{nd} and 3^{rd} Tier pension scheme.

“In a defined contribution plan, the individual bears the investment risk for better or for worse” cited by Aitken (1994) and investment rewards are assumed by each and not by the sponsor or employer. In addition, participants do not necessarily purchase annuities with their savings upon retirement, and bear the risk of surviving their assets. According to Hebb and Clark (2003), defined contribution schemes are classified into two basic types these are; Money Purchase Pension scheme and Profit sharing Pension Scheme.

Money Purchase Plans

It is a type of DC plans that utilizes a formula to predefined required employer contributions; the formula is either a function of the salary or the contributions made by an employee. For example the employer may contribute 6% of participant salaries. Contributions may be reduced by any fine on account of terminated participants with short periods of service. This design is popular with multi-employer plans.

Profit Sharing Plans

This type of DC includes the remaining 5% of the DC plans. A proceed allotment pension plan is a category of defined contribution system where establishment contributions are connected to the profitability of the establishment. The employer’s total annual contribution is determined by using a formula related to profits. Apportionment of proceeds amongst plan members may be based on assigned system, where points are apportioned in relation to service, earnings or on both. “Investment earnings and forfeitures are allocated to employees in proportion to their account balances” cited by Vanderhei et al. (1992). The contribution can be different within an allowable range and can be less in bad financial times and more in good times. For instance, the employer may contribute 10% of the company’s profit before tax to the pension plan allocated by the salaries of the participants.

“Canadian government legislation stipulates a minimum employer contribution of 1% of the earnings of the plan participants” cited by Aitken (1994). This category of DC scheme may act to motivate employees and die to increased productivity. However, the only drawback of this arrangement is that contributions are linked to profit and this increases the uncertainty associated with the level of retirement income. “From the employer’s perspective, costs are linked to the company’s ability to pay” according to Adjei (1999).

2.3.2 Defined Benefits Plans

This type of plan uses a formula to determine in advance the amount of the retirement benefits although this might not be known until retirement. It defines a benefit for an employee upon retirement in which employees contributions are clearly defined. The pension paid to the retiree under this plan is calculated taking into consideration certain factors such as the number of years a person worked, the amount of salary at retirement, age at retirement and a factor known as the accrual rate. SSNIT pension scheme is an example of DB. Where the pension plan allows for early retirement, payments are often reduced to reflect the fact that retirees will receive pension payments for longer periods.
The most common benefit is a unit benefit per year of service. For instance, a pension plan will pay pension benefit of GH40 cedi per month at retirement for each year of service, it is a traditional pension plan that defines a benefit for an employee upon that employee’s retirement is a defined benefit plan stated by Davies (1993). This method is usually practiced by unions due to simplicity of understanding the benefit and arbitration benefit improvement. For example; unions can merely change the unit amount from GH40 to GH45 per month as a benefit improvement thereby providing protection for workers from pre-retirement inflation. This method is advantageous for the employee since it stabilizes the purchasing power of pensions to some extent by Haberman (1990). A DB plan may be classified as funded or unfunded.

These two pension plan designs are combined into HYBRID or FLOOR PLAN, where one is used to institute the minimum and the other provides the accrual above the minimum.

2.4 Some Public Sector Pension Arrangements

This section describes some public sector pension schemes that have been in progress in Ghana, as stated in chapter one. These schemes include:

i. National Pension Act, 2008 Act 766 (The New 3 Tier Pension Scheme)

ii. CAP 30 Pension Scheme Acts 70

iii. Ghana Universities Staff Superannuation Scheme (GUSSS)

iv. The CRIG Pension Scheme

v. Ghana Armed Forces Pension Scheme

vi. The SSNIT Pension Scheme

vii. Act of Parliament (1991), Social Security Law, 1991 (P.N.D.C.L. 247)

2.5 Overview of Investment

According to Sheffrin and Sullivan (2002), investment is the commitment of money or capital to purchase financial instruments or other assets in order to gain profitable returns in the form of interest, income, or appreciation of the value of the instrument. An investment involves the choice by an investor, who can be an individual or an organization, to place or lend money in a vehicle, instrument or financial asset that has a certain level of risk and provides the possibility of generating returns over a period of time. Investment, generally, comes with the risk or uncertainty of the loss of some or the entire principal sum. Investors who can be an organization is called investment management (IM), Investment management is the business of investing other people's money. It is the "buy side" of the broader financial industry. Investment managers, sometimes called asset managers or money managers, put their clients' money to work in common stocks (equities), bonds and other fixed-income securities, commodities, or a combination of any of these. Their clients may be companies, government agencies, non-profit organizations, and individuals in short, or anyone who has money to invest. According to Asset Management Working Group (AMWG), the United

Nations Environment Programmed Finance Initiative and Mercer (2007) “Demystifying Responsible Investment Performance a Review of Key Academic and Broker Research on ESG Factors”.

The investment management industry (IM) manages hedge funds, mutual funds, private equity, venture capital, and other financial investments for third parties, which include companies, pension funds, endowments, insurance companies, private banks, nonprofits, and individuals. The IM industry is also known as the asset management industry. The Trust’s primary investment objective is to achieve the total return necessary to maintain "real value" of assets, with limited risk/volatility, over a long period of time so that the Trust may function in perpetuity without diminished capacity.

2.6 Overview of Cap 30 Pension Scheme

In 1950, the British Colonial Ordinances (Pensions Ordinance No. 42, Chapter 30) established a pension Scheme for public servants in the Gold Coast which later became popularly known as the Cap 30 Scheme. In 1965, the Social Security Act 1965, (Act 279) was enacted to create a contributory Social Security Scheme for payment of superannuation, invalidity, survivors, and other benefits for workers in every establishment employing not less than five workers in the private sector as well as non-pensionable workers in the public service. In 1972 the Social Security

Decree (NRCD 127) repealed Act 279 of 1965, and established the Social Security and National Insurance Trust (SSNIT) to administer a social security fund for all workers in the country. Finally, in 1991, the Social Security Law 1991 (PNDC Law 247), was publicized to change the 1972 Scheme from a provident fund to a full pension Scheme. In effect, since 1965, the country has been functioning two major public pension Schemes; that is, the Cap

30 and the SSNIT Schemes. But over the years disparity between the two Schemes became more obvious, leading to anxiety and demonstration by some public sector workers on the SSNIT Scheme demanding to be placed on the Cap 30 Scheme which was considered more satisfactory, particularly the lump-sum element. The anxieties and protests were led by workers' groups such as the

Civil Servants Association of Ghana (CSAG), Ghana National Association of Teachers (GNAT), the Ghana Registered Nurses Association (GRNA), and the Judicial Service Staff Association of Ghana (JUSAG) cited by Business and Financial times (2013, August 13).

In July 2004, the Government of Ghana under the leadership of President John Agyekum Kufuor found it necessary to appoint a nine-member Presidential Commission on Pensions, chaired by Dr. T. A. Bediako, to examine current pension Schemes in Ghana and guarantee for sustainable pension Scheme(s) that would ensure retirement income security for Ghanaian workers. It is important to underscore the uniqueness of this Commission. This was the first time in the post-independence era that such a Commission had been set up by Government solely to embark on a comprehensive review of public sector pension Schemes in Ghana according to Business and Financial times (2013, August 13).

2.7 The National Pension Act, 2008 Act 766

The national Pension Act, 2008 Act 766 as stated in chapter one was gazette on 12th December, 2008; its objective is to ensure that retirement income is secured for all Ghanaian workers and also to create unified pension structure. The Act introduced a Three-Tier pension scheme which comprises of:

1. First Tier – A mandatory basic social security to be managed by SSNIT.
2. Second Tier – A mandatory fully-funded and privately managed occupational scheme.
3. Third Tier – A voluntary fully-funded and privately managed provident fund and personal pension scheme NPRA (2015).

Developed countries have protection nets inferred in social security systems to avert poverty among elderly people. They are called first-Tier re-distributive schemes provided by the public sector and are mandatory. The second pillar consists of the industry- or company-based mandatory pension plans for employees and the third pillar includes voluntary contributory individual pension and savings plans arranged by individual investors themselves. The first tier basic national social security scheme is a defined benefit scheme; and the second tier occupational pension and personal pension schemes are defined contribution schemes.

Table 2.1 elucidates an overview of the structure of the new pension scheme, it summaries the role of the players and administrators in the managing of the new pension system.

Table 2.1: Organization of the New Pension Institution and their Objectives

INSTITUTIONS	OBJECTIVES
National Pensions Regulatory Authority(NPRA)	The main objective of this organization is to regulate and be an observer to the operations of the scheme(s) and ensure an effective administrations of pension's schemes in the country.
SSNIT	The objective of the SSNIT trust is to operate the basic national social security scheme referred to as the social security scheme such as 1^{st} Tier and other schemes as determined by law on the recommendations of the National Pensions Regulatory Authority.
The Trustees	Responsible to supervise the administration of the 2^{nd} Tier and 3^{rd} Tier schemes.
The Fund Managers	Manage the growth of the pension funds from the occupational scheme (2^{nd} Tier) and the personal private pension scheme.

2.8 Contributions under Act 766

For the system to function smoothly, the rate of contribution was revised. The rate of contribution simply means how much the employer and the employee should pay into the recipient institutions. The following tables summaries the details of the contributions on the employee's basic salary.

Table 2.2: Contribution rate under the SSNIT Scheme and the Three-Tier Scheme

CONTRIBUTORS	OLD RATE (%)	THREE-TIER NEW RATE (%)	DIFFERENCE (%)
EMPLOYEE	5	5.5	0.5
EMPLOYER	12.5	13.0	0.5
TOTAL	17.5	18.5	1.0

Table 2.3: Contribution Allotment

Recipient Institution	Type of Scheme	Beneficiary	Percentage(%)	Cumulative(%)
SSNIT	1_{st} 1_{st}	SSNIT NHIS	11 2.5	13.5
Private Fund Managers	2_{nd}	Private Fund	5	5
		Total	18.5	18.5

2.9 Benefits under the Act 766

The scheme as being new as it is, the commission that drafted this scheme considered the employees first before any other things. We saw that the CAP 30 or the SSNIT pension schemes had some limitations such as delay in contribution payment by government as he is the largest contributor to the scheme, which this 2^{nd} Tier scheme seeks to address. The 2^{nd} Tier pension scheme has numerous benefits for employees both in formal and informal sectors. Some of the benefits are outlined below;

i. The 2^{nd} Tier was designed to give employees higher lump sum benefits than presently available under the SSNIT pension scheme.
ii. The entry age into the scheme has being amended to 15 years instead of 20 years and survivorship benefits calculation period has also being raised to 15 years instead of 12 years under the 1^{st} Tier basic National Social Security Scheme.
iii. The lump sum serves as a prospect to procure mortgages. This means employees can secure their own house or properties before retirement by using their benefits as warranty.

iv. A prospect for employees to have better control over their pensions benefits, thus those that are privately managed.

v. The newly Scheme is unbiased to all members as the amount accumulated and benefits are completely linked to the contributions made. This will aid employees as an incentive to increased contributions in the 3^{rd} Tier Scheme so as to accumulate more benefits for retirement.

The new 2^{nd} Tier scheme will minimize the hardship encountered by pensioners when they go on retirement and will increase retirement benefit security. It will also improve the standard of living of pensioners as they reach old age. In finances, National savings would also be increased for long term economic development and promotion of growth and development of capital in mortgage and insurance markets.

2.10 Pension Funds Investment and Performance

Pension funds are invested in securities by fund managers in different areas of investment avenues and they are faced with different investments opportunities and where they should possibly invest their wealth. As stated above, a Fund manager of 2^{nd} Tier pension scheme, receives the 5% contribution made by the contributors and are invested in several sectors within and outside Ghana but he must achieve the goal of the funds which he manages. Areas of investment includes; Financial Sectors (Bonds and cash), Manufacturing, Real Estate, Stocks, Derivatives, Energy Sector and Foreign Exchange etc. Kwapong (2013) specified that “SSNIT or the Trust is the largest single institutional investor on the Ghana Stock Exchange and also has investments in equity in unlisted companies and fixed income securities such as bonds and treasury bills. This is definitely an improvement from investing only in government stocks”. Fund managers invest in these areas so as to maximize profit on their investments. Since investors by nature are risk averse, they would want to invest in securities that are less risky; they have to invest in securities that will earn them at least minimal guaranteed returns that would enable them maintain a sufficient liquidity to pay pensioners and as well securing their wealth. But the case is different from other countries, "The case of Chile, with its longer history, is illustrative of the possible evolution as funds mature and tends toward riskier portfolios, even within the very conservative limits set by regulations. At the beginning, most assets were invested in essentially risk free securities, as is the current case in Mexico. As time went by and capital markets developed, funds started to invest in mortgage bonds and corporate securities, to the point that in 1994 these represented a proportion similar to public securities" as stated by Vives (1999).

Some fund managers have also taken the risk to invest in areas like energy, oil, gold and many others. "These investments approaches seek to maximize returns and they have the potential to yield higher returns than those earned from investments in capital markets" cited by Agyeman (2011). Two pension funds in Denmark also invested in an energy sector "Pension Danmark and PKA, two of Denmark's biggest pension funds, acquired a 50 per cent stake in the Anholt wind park to be built by Dong Energy, the Danish utility, off the country's north-east coast. The deal highlights growing interest in the investment opportunities surrounding renewable energy as well as the increasing importance of pension funds as a source of funding for the sector. Claus Stampe, chief investment officer of Pension Danmark, says green energy infrastructure is becoming an attractive asset in an era of low fixed income returns and volatile equity markets. Mr. Stampe says average annual returns from Anholt are expected to be at least double current Danish bond yields of just above 3 percent over the wind farm's 20-year lifespan" by Ward (2011).

Pension funds are increasing in Ghana and across the globe where most of their funds are invested in different classes of securities in search for returns. According to Inderst (2009), he stated that "Pension funds are increasingly moving into new asset classes in a search for yield. Infrastructure is a type of investment being frequently discussed, given its potential to match long-term pension assets and provide diversification. Previously, pension funds exposure to infrastructure has been via listed companies, or via real estate portfolios. However, some larger funds globally are beginning to invest via private equity funds, or occasionally even directly. Australian, Canadian and Dutch pension funds may be considered leaders in this field". The Ghanaian leading pension fund "SSNIT" also invests in real estate sectors, manufacturing sector, hospitality industry, services, banking industry, and private equity funds etc. thus those listed and unlisted on the Ghana Stock Exchange (GSE). Over the world pension institutions are classified as one of the largest and fast growing institution.

Most pension funds lose heavily when there is an economic recession and vice versa. In February 2010, SSNIT was stated to be making little returns on investments with only six out of forty stocks they have invested with. No wonder most firms and pension funds invest most of their wealth in Bonds. Agyeman (2011) commented on the recession as "The global financial crises hit the economies of nations across the world hard with pension funds in different countries coming out as major casualties, however Germany seemed to have escaped these losses. They did so by investing a higher proportion of their pension funds in bonds". In Africa, Nigeria's pension fund also lost a significant portion of its value. In an article on the allafrica.com website entitled Nigeria: Pension Asset and Global Meltdown dated 6th April, 2009 Abubakar Buba stated as cited by Agyeman (2011) that "In Nigeria, 7% of the total contribution to the Retiree Savings Account (RSA) which stood at

N471.77 billion was lost due to crash in the equities market. Pension Fund Asset which had accumulated to an estimated value of N1.1 trillion as at December 2008 was ordinarily supposed to call for celebration for all stakeholders in the pension industry including the apex regulatory body but, the meltdown ensured they never did. In addition, RSA investment in equities which was 15.93% in 2007 crash landed to 9.52% as at the end of December, 2008. Unlike Germany, Nigeria had only about 32% of its total pension funds invested in Federal Government securities or bonds and 11% in real estate". Now Ghana is facing economic crises as most industries are at the verge of closing down, so they are forced to cut down on the number of workers which in turn will affect the pension schemes or pension funds across the country. It will not be a surprise to me if SSNIT will report a loss in 2015/2016 annual report if it continuous investing in some portfolios in Ghana. Whiles assets management companies will be in better position as 2^{nd} Tier pension schemes mangers wish to invest in equities.

2.11 Impact of Act 766

Pension as the name implies, is a fixed amount of money paid regularly to somebody during retirement, either by the government, a former employer or an insurance company. The 2^{nd} Tier came to improve the SSNIT pension scheme which contributors sacrifice today to enjoy tomorrow. Technically as "investment is the sacrifice of current consumption in order to obtain increased consumption at a later date", pension came into being to provide support for employees, civil servants, public servants, workers in private sectors professionals, traders, farmers and self-employed etc. who will suffer the following events;

i. In the event of old age, what will be one's income security? ii. In the event of invalidity, what will be one's income security?

iii. In the event of death of the breadwinner, what will be the income security for one's dependent?

The 1^{st} Tier will care for all these events in case one fall into any. The SSNIT Pension plan wanted people to learn how to sacrifice today, to enjoy tomorrow.

Since pension funds are invested in the listed and unlisted companies on stock exchange, they help government to achieve his developmental objectives. Now, SSNIT has directed some of its wealth into real estate and affordable housing which has aided in solving housing problems in the country thereby bringing development and improvement in the country's infrastructure and as well as employment for the people. Pension funds have also been seen to influence corporate governance in the economy, Ashidam (2011) as cited by Clark and Hebb (2003) identified four factors which facilitate pension funds corporate governance.

i. The use of indexation technique.

ii. The second is the increasing demand by owners for more accountability and transparency.

iii. Pension funds' pressure to undertake socially responsible investing (SRI) iv. To humanize capital with social, moral and political objectives extend pension funds simple concerns for rate of return.

Pension funds investments develop stock markets. Agyeman (2011) as cited in Catalan (2004) stated that "Pension funds could trade frequently, increasing the liquidity of the domestic stock markets, and thus crowding in savings and new investors. Similarly, the intense trading of stock by pension funds and their large size may induce them to seek the introduction of innovations and new financial instruments to lower transaction costs, again attracting additional savings and new market participants" Adjei (1999) as cited by Ashidam (2011) discloses that Pension funds also protect investors and enhance public confidence in the capital market. Pension funds' function in the financial sector which includes the following;

(a) The collection of savings
(b) The investment in securities and other financial assets both locally and internationally,
(c) The payment of annuities and
(d) Provision of forms of insurance.

This leads to more efficient allocation of funds into productive investments in the stock markets thus leading to growth and productivity.

2.12 Utility Theory

Utility functions give fund managers the way to measure the investors' preferences for wealth and the amount of risk that the investor(s) is/are willing to take in the hope of attaining greater wealth. “A utility function measures an investor’s relative preference for different levels of total wealth” by Norstad (2011). In practice, this is common in real life situations and as to how an investor should choose between two choices with uncertain returns and an element of risk. Utility is the satisfaction that an investor derives from a particular investment. Given the uncertainty of returns, a rational investor can no longer maximize his utility with complete certainty.

The story began in St. Petersburg in 1738. There Daniel Bernoulli proposed a solution to the “St. Petersburg Paradox” by introducing the notion of a utility function. Von Neumann and Morgenstern (1947) develops “expected utility” as a base to describe human behavior in a situation when a person has the choice to choose from few perspectives with uncertain outcome. Expected utility is widely used to model the choices of investors. The theory states that “people make a choice based on the value of the outcome (in terms of money or anything else), the probability of its occurrence and the willingness to take higher risk in order to receive a possible higher benefit in the future (personal risk aversion)” as cited by Ivanova (2010).

In standard finance theory, investors’ preferences are assumed to be influenced by their attitude towards risk. Additions to the statements made above it is common to assume that investors are “greedy”, meaning that, they will always prefer more to less as stated by Campbell and Viceira (2002) cited by Pedersen (2013). This hypothesis in economic literature is known as non-satiation so investors aim in portfolio optimization is to maximize their personal utility or wealth. Norstad (2011) used the utility theory to work on portfolio optimization, which he stated that “utility lies in the heart of modern portfolio theory which lay the foundation upon which complicated relevant theories could be developed”. The author developed the basic concepts of the theory through a series of simple example, the author was filling the gap in Robert C. Merton 1990 presentation of his material.

Lakner (1998), which is the main reference paper to this thesis sought to address optimization problem of an investor who wants to maximize the expected utility from terminal wealth. The aim of the paper was that the drift and the Brownian motion appearing in the stochastic differential equations for the security prices are not assumed to be observable for all investor in the market. The paper stated that the interest rate is fixed and the drift process was modeled by Gaussian process which in a special case becomes a mean-reverting process. The paper reported the optimal trading strategy for an investor for a wide range of utility functions under logarithmic and power utility functions.

Ameko and Baah (2014) reviewed the paper of Ocone and Karatzas (1990) and Lakner (1998) within the basis of portfolio management and stochastic differential equations and to identify an optimal trading for an investor under Logarithmic utility function. They applied market data to the model to help observed its dynamics. They took a set of eight risky securities from USA capital market and investigated how much of their wealth should be invested in each of the security. In the paper they assumed short sales and observed that an investor should liquidate 97.8657 in currency of his holdings in the US Bonds. They recommended that their model is to be used by pension funds in Ghana.

Ivanova (2010) studied the elements of the traditional pension scheme in order to select optimal settings for the micro pensions. He saw that most people receive low income and as a result contribute small amount into pension funds, so he worked within the framework of expected utility basis and cumulative prospect theory (CPT) to evaluate the theoretical benefits versus observed behavior. In the first part of the research he compared different contract types, which combined elements like minimum income guarantee, direct investment and trading the upside potential for the lower price of the contract. He used the arbitrage free pricing principle and extended the model used in the literature by incorporating stochastic interest rates. He observed that in the micro pension setting the contracts that include minimum guarantee perform the best. In the second part of the research he looked at the payout phase, and compared an annuity contract versus lump sum withdrawal. He introduced the prediction error that the retiree makes while planning her consumption and evaluate its consequences. He found that annuity is an optimal choice as an insurance against longevity except for the cases of confirmed terminal illness. Loss aversion in CPT explains the choice against annuity that most people often make in the real life. He concluded that both parts provided theoretical guidance for constructing the micro pension contract.

Utility functions have to satisfy these properties non-satiation and risk aversion as most researchers cite when working utility functions. They are expressed by their first and second derivatives which most researchers like Lakner (1998) stated "A function $U : [0,\infty) \longrightarrow R \cup -\infty$ is called a utility function if it is continuous, strictly increasing, strictly concave on its domain, continuously differentiable on

$(0,\infty)$ with derivative function $U^0(.)$ satisfying the relation $\lim_{x \longrightarrow \infty} U^0(x) = 0$. That is the changes of an investors' satisfaction with respect to changes in wealth, is expressed by the first derivative which satisfy the non-satiation principle, while the second derivative shows how investors accept risk with changes in wealth. More on this will be discussed in chapter three. An investigative work was done by Pedersen (2013) to investigate whether an investor can benefit from investing in commodities if commodities are included to the available portfolio in the market. He approached his study from the perspective of myopic investor who seeks to maximize portfolio returns in a single period ahead. An approach within "inter-temporal strategic asset allocation which was chosen, was originally studied by Campbell and Viceira (2002) and cited by Pedersen (2013), the researcher used "Epstein-Zin utility when taxes, transaction costs and borrowing and short sales restrictions are disregarded". The results of the work shown that investors should optimally invest "a large share in bonds and borrow at the short rate irrespective of the risk aversion level and whether commodities are included or not. Another observation was that stocks are more attractive than commodities for lower levels of the relative risk aversion, while the opposite is observed for higher levels". The investor began to appreciate the risk aversion principle and he observed that, holdings of the risky asset decreases as the investor become more risk averse. In general, the portfolio adjustments from including commodities are limited. Nevertheless, the utility value indicated that investors, irrespective of the level of risk aversion and subsamples, can yield welfare gains from investing in commodities, when tested in-sample. It is further established that investors who actively time the market based on the changes in the state variables yield superior returns by including commodities when tested in-sample”. He recommended that since return is time varying any investor who wants to invest should consider investing in commodities.

A more precise technique to understand an investor’s investment behavior is through the theories of Absolute Risk Aversion (ARA) and Relative Risk Aversion (RRA) presented by Pratt (1964) and Arrow (1970). William Sharpe stated about ARA and RRA which was cited by Norstad (2011) that “the assumption of constant relative risk aversion seems much closer to the preferences of most investors than does that of constant absolute risk aversion. Nonetheless, it is by no means guaranteed to reflect every Investor’s attitude. Some may wish to take on more risk as their wealth increases. Others may wish to take on less.

Many analysts counsel a decrease in such risk as one age. Some strategies are based on acceptance of more or less risk, based on economic conditions and so on.

For these and other reasons it is important to at least consider strategies in which an Investor’s risk tolerance changes from time to time. However, such changes, if required at all, will likely be far more gradual than those associated with a constant risk tolerance expressed in terms of end-of-period value”.

One can still allocate wealth without considering utility theory which Darko (2012), used Markowitz model to construct stock portfolio in Ghana, he analyzed the applicability of the classical Markowitz model on the Ghana stock Exchange, which he further determined which of the stocks indexes will give a better profit for an investor who wants to invest in the GSE (all shares index, non-financial index and financial index) when given the nod. Historical monthly data was used to conclude his work. He concluded that an investor should invest 83.44% of his wealth into non-financial index while investing 16.56% in the financial index which he stated that Ghanaian stock market obeys the principle of the Markowitz model. Markowitz approach to optimizing a portfolio considers only the mean and the variance of the stock prices to determine which of the stocks is highly volatile than the other with returns.

Pulley (1981) indicated that the mean variance formulation offers a very good local approximation to expected utility for more general utility functions using both monthly and semi-annual return data. The researcher also suggested that the mutual fund manager should choice portfolios which maximize utility for a wide class of different investors having different utility functions and wealth levels regardless of the actual form of their utility functions.

2.13 Fourier Analysis

In the early nineteenth century, Joseph Fourier, while studying the problem of heat flow, developed a cohesive theory of such series. Consequently, the series was named after him. "Fourier analysis of a time series is a decomposition of the series into a sum of sinusoidal components (the coefficients of which are the discrete Fourier transform of the series)" stated by Bloomfield (1976), Bloomfield (2000). It is also known as harmonic analysis or harmonic regression. It is a data analysis procedure that is used to measure the fluctuations in a time series by comparing them with sinusoids. The oscillations in the transformed data can be described in sinusoidal terms as "spectrum analysis".

The method of fitting sinusoids of frequencies to a time series data became a major problem for most of the researchers who used Fourier series, Beveridge (1921) presented wheat prices in Western Europe which shown that there were a succession of peaks in the data, but there were no evident of tendency of the data occurring regularly. Most of the researchers like Knott (1897) who claimed to have discovered the components in the Japanese earthquakes with periods related to the lunar cycle but Schuster (1987) opposed to the method and showed that its magnitude was not large enough to be statistically significant; he also came with the "Periodogram" theory which has become popular in this era. Most of the researchers came out with methods to estimating the frequencies but some of them disagreed to the methods proposed by their learned friends and came out with their methods. Stroke (1879) in his comment on Stewart and W.Dodgson (1879) report pointed out that Thomson (1876) harmonic analysis could also be used to determine the unknown period of a component in a data which this thesis will settle on. Until 1940s and 1950s where there was great interest shown in the estimating spectrum of a series, where Tukey (1950) and others proposed a slightly different procedure, in which there was a rapid development of the theory and practice of spectrum estimation.

The essence of Fourier analysis is to represent a set of data in sinusoidal forms, most electrical engineers and Telecom engineer use the advantage of Fourier series to transform voltage waveform in sinusoidal form. Lewis et al. (2002) used Fourier analysis to forecasting the inbound call time series of a call center of the New South Wales Police Assistance Line (PAL). The PAL is a 24-hour inbound telephone call center available to the police and the community. They identify that the scheduling of the staff and resources to meet the incoming call was a problem. Hughes (1995); Klungle (1997,1998); Bianchi et al. (1993) revealed that there has not been work done in using Fourier analysis to develop a process for forecasting a call center's incoming calls. The call arrival process of an inbound call that contains seasonal patterns, cyclic patterns and trends was used together with the Fast Fourier Transform (FFTs) for the analysis of the data. They used controlled examples to assess the utility of the process which was subsequently applied to the PAL data. They were able to use the model to identify call patterns which would be used to help the scheduling of staffing and resources in the PAL.

Ekpenyong et al. (2014) worked on the application of Periodogram and Fourier Series Analysis to modeling all-items monthly inflation rates in Nigeria from 2003 to 2011. The main objectives of their paper were to identify inflation cycles, fit a suitable model to the data and make forecasts of future values. For them to achieve their objectives, monthly all-items inflation rates data for the period was obtained from the Central Bank of Nigeria (CBN) website. Periodogram and Fourier series methods of analysis were used to analyze the data. Based on the analysis, they observed that, the inflation cycle within the period was fifty one (51) months, which narrates to the two government administrations within the period. An appropriate significant Fourier series modeling comprising the trend, seasonal and error components was fitted to the data and the model was used to forecasting the inflation rates of Nigeria for thirteen months and this forecast compared to the actual data prove to be favorable for the thirteen months. The paper concluded that Fourier series model could also be used to model inflation rate because of its advantage of identifying inflation cycles and establishing a suitable model of the series.

2.14 Simulation Study

Simulation simple means the reproduction of the essential features of something that is made to behave or look like the original to aid in research or study usually by the help of computer programs. According to Banks (2000), simulation is the problem solving method which is very useful when the systems become so complex that common sense and simple calculations are not enough. The aim of the simulation models is to provide a researcher with enough information that the behavior of the simulated system will appear and be as similar to the real system as possible Banks (1998) as cited by Sandstrom and Zulumovski (2012). Simulations are considered if the system is unpredictable over time.

2.14.1 Types of Simulation Models

There are two kinds of simulation model structures that are used in academia and real-world. These are;

(a) Stochastic and deterministic model and

(b) Static and dynamic model

The stochastic model is based on the fact that random event will occur whiles the deterministic is based on the fact that no random event will occur. The dynamics of the stock prices which is to be used exhibits this kind of simulation study. The stochastic model was classified as Static and Dynamic which Monte Carlo simulation took the form of Static while Continuous and Discrete-event simulation took the form of Dynamic by Sandstrom and Zulumovski (2012).

2.15 Monte Carlo Methods

Monte Carlo methods are nothing more than a computer based utilization of the law of large numbers to estimate a certain probability distribution. The method generally is a random number generator procedure that produces an infinite stream of random variables that are independent and identically distributed (i.d.d) according to some probability distribution.

Kyng and Konstandatos (2014), studied on the multivariate Monte-Carlo simulation and Economic Valuation of complex financial contracts excel based implementation method to generate the value of the contract. They stated that the economic valuation of complex financial contracts is often done using Monte Carlo simulation. They studied the standard single European options and then extended it to exotic multi-asset and multi-period features on single asset pricing. They used the Executive Stock Option to motivate their discussion which was studied in their previous paper using novel theory. They demonstrated the simulation of the multivariate normal distribution and the multivariate Log-Normal distribution using the Cholesky Square Root of a covariance matrix for replicating the correlation structure in the multi-asset, multi period simulation required for estimating the economic value of the contract. They did their simulation within the context of standard Black Scholes framework with constant parameters. Excel implementation was also provided for transparency and simplicity for students. They concluded that their approach has relevance to industry due to the widespread use of Excel by practitioners as well as graduates who may desire to work in the finance industry. This allows students to be able to price complex financial contracts for which an analytic approach is intractable.

CHAPTER 3

METHODOLOGY

3.1 Overview

This section presents some useful mathematical background materials, used throughout this thesis which includes the definitions of the various mathematical concepts such as random variables, stochastic process, martingales, Brownian motion, Fourier series, diversification, etc. this section provides the basic foundation to the structures that are built across this thesis for a better appreciation of the mathematical formulas herein.

3.2 Basics of Measure Theory

3.2.1 Probability Space

A probability space is a triplet $(\Omega, \mathrm{F}, \mathrm{P})$ consisting of:

i. Ω, a nonempty set called the sample space, which contains all possible outcomes of some random experiment.

ii. F, a $\sigma - algebra(field)$ of subsets of Ω with Boolean algebra properties.

iii. P a probability measure on (F), i.e., a function which assigns to each set $A \in \mathrm{F}$ a number $\mathrm{P}(A) \in [0,1]$, which represents the probability that the outcome of the random experiment lies in the set Shreve (1997).

3.2.2 σ-Algebra

let Ω be a non empty set. A σ-Algebra is a collection of f subsets of Ω with the following three properties;

i. $\varphi,\Omega \in F$ ii. If $A \in F$, then it's compliment $A^c \in F$ iii. If $A_1,A_2,...$ is

a sequence of sets in f, then $\bigcup_{k=1}^{\infty} A_k \in f$.

3.2.3 Filtration

Given a probability space (Ω,F,P), we call a sequence of σ-algebras $\{F_t\}_{t\in T}$ a filtration if $F_s \subset F_t \subset F$ for all $s,t \in T$ such that $s \leq t$. Or simply filtration is the flow of information.

3.2.4 Random Variable

Let Ω be a non empty finite set, a random variable X is a function that assigns a numerical value to each state of the world. i.e; $X : \Omega \to <$

3.2.5 Probability Measure

The measure of chance of an occurrence of an event, P : $F \to [0,1]$ which satisfies the following properties;

i. $P(\Omega) = 1$ ii.

$P(\Phi) = 0$ iii. $0 \leq$

$P \leq 1$ iv. For

any

mutually

disjoint

events

$A_1,A_2,....$

$\in$ F,

$$P(A_1,\cup A_2,.....) = P(A_1) + P(A_2) + ...$$

3.2.6 Stochastic Process

A stochastic process on the probability space $(\Omega,\mathrm{F},\mathrm{P})$ is a family of random variables X_t parameterized by $t \in T$, where $T \subset <$. A stochastic process is called adapted to the filtration f_t if X_t is f_t predictable , for any $t \in T$. f_t predictable means given any two numbers $a,b \in <$, then all the states of the world for which X takes values between a and b forms a set that is an event.

3.2.7 General Brownian Motion

A general Brownian Motion is a stochastic process $(W_t)_{t\geq 0}$ which satisfies the following properties ;

i. $W_0 = 0$ ii. If $0 \leq s < t$, then $W_t - W_s \sim N(\mu(t-s),\sigma^2(t-s))$ iii. Future changes are independent of past and present ; if $0 \leq r \leq s < t$, then the random variable $W_t - W_s$ and W_r are independent. iv. The paths $t \rightarrow W_t$ are continuous with probability of 1

Where μ is the drift, σ^2 is the variance of the Brownian motion. μ and σ^2 are constants. In the case where $\mu = 0$ and $\sigma^2 = 1$, we get the normalized Brownian motion or the Standard Brownian Motion called the Weiner Process. i.e ; the condition iii now becomes if $0 \leq s < t$, then $W_t - W_s \sim N(0, t - s)$.

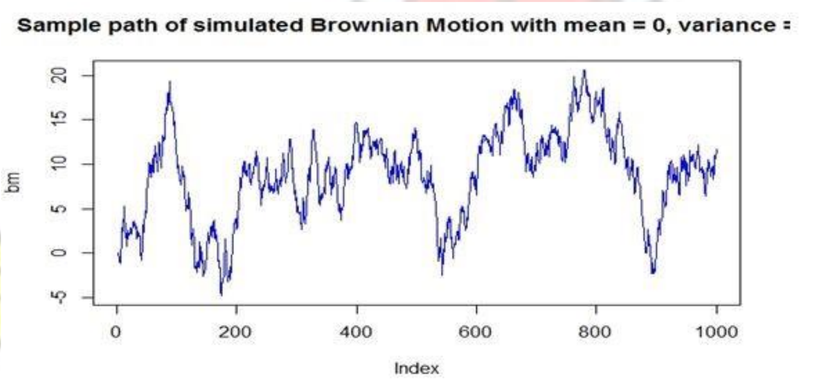


Figure 3.1: Sample Path of Brownian motion or Wiener process

The Brownian motion is the most fundamental continuous time stochastic process. It is both a martingale of the type considered in chapter three section 3.2 and a Gaussian process as considered in section 3.6. It has also continuous sample path, independent increments, and the strong Markov property. Having all these beautiful properties allows for a rich mathematical theory. For example, many probabilistic computations involving the Brownian motion can be made explicit by solving partial differential equations. Further, the Brownian motion is the corner stone of diffusion theory and of stochastic integration. As such it is the most fundamental object in applications to and modeling of natural and man-made phenomena.

3.2.8 Geometric Brownian Motion

If the stochastic process X_t is a Brownian motion with drift μ and variance rate σ^2, the process $\{Y_t = \exp X_t, t \geq 0\}$ is called a geometric Brownian motion, or the exponential Brownian motion.

3.2.9 Martingale

A martingale is a stochastic process such that ;

i. $E[Z_t] < \infty\ \forall\ t$ ii. $E[Z_t/\mathrm{F}_s]$

$= Z_s \forall\ s < t$ iii. $E[Z_0] =$

$E[Z_n] \forall n$

In general a martingale is process whose expected future value is equal to its current value and the process has no drift and has a constant mean.

3.2.10 Itô Process

The stochastic process $X = X_t, t \geq 0$ that solves

$$X_t = X_0 + \int_0^t a(X_s, s)ds + \int_0^t b(X_s, s)dW_s$$

is an Itô process. The corresponding stochastic differential is given by

$$dX_t = a(X_t,t)dt + b(X_t,t)dW_t$$

3.2.11 Itô Lemma

Suppose B_t follows a Brownian motion and $f(x)$ is twice differentiable, then by Taylor theorem of the second order

$$df(B_t) = f'(B_t)dB_t + \frac{1}{2}f''(B_t)(dB_t)^2$$

but $(dB_t)^2 = dt$ and this implies that

$$df(B_t) = f'(B_t)dB_t + \frac{1}{2}f''(B_t)dt.$$

This is the Itô lemma for the functions of Brownian motion.

3.2.12 Diffusion Process

Diffusion process is a stochastic process X_t that satisfy the stochastic differential equation of the form

$$dX_t = \mu(X_t)dt + \sigma(X_t)dB_t.$$

This is process is classified as an itô process.

3.2.13 Expectation

A random variable $X : \Omega \to <$ is called integrable if $\mathrm{R}_{\Omega} |x(w)|dp(w) =$

$\int_R |x|p(x)dx < \infty$.where $p(x)$ is the probability density function of x.

Then the expectation (mean)of the integrable random variable X is defined by

$$E[X] = \int_\Omega X(w)dp(w) = \int_R Xp(x)dx$$

.

3.2.14 Conditional Expectation

Let X be a random variable on the probability space (Ω,F,P) and G be a sub σ- algebra of f. Then $E\left[\frac{X}{G}\right]$ is defined to be any random variable Y that satisfies ;

i. Y is G-measurable.

ii. $\int_A E\left[\frac{X}{G}\right] dp = \int_A X dp$, for all $A \in G$.

Then $E\left[\frac{X}{G}\right]$ is called the conditional expectation of X given G.

3.2.15 Stochastic Differential Equation

A stochastic differential equation (or SDE) is a differential equation in which one or more of the terms is a stochastic process, thus resulting in a solution which is itself a stochastic process.

3.2.16 Markov Process

Let $(\Omega,\mathcal{F},P)$ be a probability space. Let $\{f_t\}_{t=0}^{n}$ be a filtration under f. Let $\{X_t\}_{t=0}^{n}$ be a stochastic process on $(\Omega,\mathcal{F},P)$. This process is said to be a Markov if :

i. The stochastic process X_t is adapted to the filtration $\{F_t\}$, and ii. For each $t = 0,1,2,...,n-1$, the distribution of X_{t+1} conditioned on F_t is the same as the distribution of X_{t+1} conditioned on X_t

That is to say :

A Markov process is a stochastic process where only the present value of the variable is relevant for predicting the future.

3.2.17 Radon Nikodym Theorem

Theorem

Let P and $\tilde{P}$ be two probability measures on a space $(\Omega,\mathcal{F})$. Assume that for every $A \in \mathcal{F}$ satisfying $P(A) = 0$, we also have $\tilde{P}(A) = 0$. Then we say that $\tilde{P}$ is absolutely continuous with respect to P. Under this assumption, there is a non-negative random variable Z such that

$$\tilde{P}(A) = \int_A Z\,dP, \quad \forall A \in \mathcal{F} \tag{3.1}$$

and Z is called the Radon-Nikodym derivative of $\tilde{P}$ with respect to P. Equation 3.1 implies the apparently stronger condition

$$\tilde{E}(X) = E[XZ] \quad (3.2)$$

for every random variable X for which $E[XZ] < \infty$

if $\tilde{P}$ is absolutely continuous with respect to P, and P is absolutely continuous with respect to $\tilde{P}$, we say that P and $\tilde{P}$ are equivalent if and only if $P(A) = 0$ exactly when $\tilde{P} = 0 \; \forall \; A \in F$.

If P and $\tilde{P}$ are equivalent and Z is the Radon-Nikodym derivative of $\tilde{P}$ w.r.t P then $\frac{1}{Z}$ is the Radon-Nikodym derivative of P w.r.t $\tilde{P}$. i.e.,

$$\tilde{E}(X) = E[XZ] \forall X, \quad (3.3)$$

$$\mathbb{E}(Y) = \tilde{\mathbb{E}}(Y\frac{1}{Z})\forall Y \quad (3.4)$$

According to Shreve (1997)

3.2.18 Girsanov Theorem

Let W(t); $0 \leq t \geq T$ be a Brownian Motion on a probability space (Ω,F,P). Let $F(t), 0 \leq t \leq T$, be the accompanying filtration, and let $\theta(t), 0 \leq t \geq T$ be a process adapted to this filtration. For $0 \leq t \leq T$, define

$$\tilde{W}(t) = \int_0^t \theta(u)du + W(t), \quad (3.5)$$

$$Z(t) = exp - \int_0^t \theta(u)dW(U) - \frac{1}{2}\int_0^t \theta^2(u)du, \quad (3.6)$$

and define a new probability measure by

$$\tilde{P}(A) = \int_A Z dP \quad \forall A \in F \tag{3.7}$$

Under $\tilde{P}$ the process $\tilde{W}(t), 0 \le t \le T$ is a Brownian Motion.

Given a condition on the size of the θ if

$$\mathbb{E}exp\frac{1}{2}\int_0^T \theta^2(u)du < \infty \tag{3.8}$$

Certainly, Z(t) is a martingale

Remarks

$$dZ(t) = -\theta(t)Z(t)dW(t) + \frac{1}{2}\theta^2(t)Z(t)dW(t) - \frac{1}{2}\theta^2(t)Z(t)dt$$

$$dZ(t) = -\theta(t)Z(t)dW(t)$$

If $\tilde{P}$ is a probability measure, since Z(0) = 1, E$Z(t)$ = 1 for every $t \ge 0$

$$\tilde{P}(\Omega) = \int Z(t)dP = EZ(t) = 1 \tag{3.9}$$

Furthermore, if X is a nonnegative random variable, then

$$\tilde{E}[Z] = \tilde{E}[ZX] \qquad \forall A \in F \tag{3.10}$$

and if Z is almost surely strictly positive, we also have $\mathbb{E}[Y] = \tilde{\mathbb{E}}[\frac{Y}{Z}]$.

Moreover, Two measures P and $\tilde{P}$ on the same probability space which have the same measure-zero sets are said to be equivalent

Recall that

$$\tilde{P}(A) = \int Z(T)\,dP \qquad \forall \in F,$$

if $P(A) = 0$ then $\int Z(T)\,dP = 0$ for every ω,

but $\mathbb{P}(A) = \int \frac{1}{Z(T)} d\mathbb{P}, A \in F$ if $P(A) = 0$

then $\int \frac{1}{Z(T)} d\mathbb{P} = 0$

3.2.19 Law of Large Numbers

Definition

Let $X_1, X_2, \dots$ be a sequence of independent, identically distributed random variables each with expected value μ and variance σ^2, defined by:

$$Y_n \triangleq \frac{X_1 + X_2 + \dots + X_n}{n}, \qquad for\ n = 1, 2, 3, \dots \tag{3.11}$$

Then Y_n converges to the expected value almost surely as n approaches infinity according to Shreve (1997), this method will be applied in chapter four under simulation study.

3.3 Optimization and Utility Theory

This section is to present the underlying theories within utility theory which is the main backing of asset allocations in this thesis. Our main theory in this thesis is power utility function also known as Isoelastic utility functions, with a risk adverse investor. If a fund manager decides to allocate wealth, utility functions provide a ranking to judge uncertain situations as to which stocks should he invest more or less.

3.3.1 Utility Theory

In the application of utility theory to finance, it is assumed that a numerical value call the utility can be assigned to each possible value of the investors' wealth by what is known as preference utility.

Theorem:

The expected utility theorem states

i. A function U(X) can be constructed representing an investor's utility of wealth, X

ii. The investor is faced with uncertainty by making decisions on the basis of maximizing the expected value of utility.

The expected utility of an investor can be derived from the following axioms if the behavior of the investor is consistent and he is rational in his decision making as stated by Von Neumann and Morgenstern

(1947).

Axioms of Utility Theory

1. Comparability: $U(A) > U(B)$,$U(B) > U(A) and U(A) = U(B)$ implies the fund manager can preference between all available certain outcomes. A is preferred to B, B preferred to A, or the fund manager is indifferent between A and B.

2. Transitivity: $U(A) > U(B)$,$U(B) > U(C) \Rightarrow U(A) > U(C)$ if A is preferred to B and B is preferred to C than A is preferred to C. Also $U(A) = U(B)$ and $U(B) = U(C) \Rightarrow U(A) = U(C)$ this implies that the fund manager is consistent in his ranking of outcome.

3. Independence: if $U(A) = U(B)$ and of course U(C) is equal to itself, then $pU(A) + (1 - p)U(C) = pU(B) + (1 - p)U(C)$ Thus, the choice between any two outcomes is independent of all the other outcomes, because the fund manager is indifferent between two certain outcomes.

4. Certainty equivalence: if $U(A) > U(B) > U(C)$ then there exist a unique $p(0 < p < 1)$ such that $pU(A) + (1 - P)U(C) = U(B)$ in this the B is called the certainty equivalent which is the maximum price that the fund manager would be willing to pay. It represent the certain outcome of wealth that yields the same certain utility as the expected utility yielded by A and C.

3.3.2 Optimization Problem

In mainstream finance theory, investors' preferences are assumed to be influenced by their attitude towards risk. In addition to the above stated axioms, it is common to assume that fund managers are "greedy", meaning that they will always prefer more to less stated by Campbell and Viceira (2002) cited by Pedersen (2013). This assumption in economic literature is known as non-satiation as stated in chapter two section 2.12. With this in mind, the optimization problem of a fund manager can be written as follows.

$$maxE[U(X)] = \sum_i p_i U(X_i) \tag{3.12}$$

where X is the wealth and p_i is the Probability of X_i

3.3.3 Properties of Utility Functions

The utility functions are assumed to satisfy these properties typically of a fund manager who acts in the aspect of uncertainty.

1. Non- satiation: $U_0 > 0$ this property assumes that fund managers will prefer more wealth to less. The derivative of utility with respect to wealth is often referred to as the marginal utility of wealth. Nonsatiation is therefore equivalent to an assumption that the marginal utility of wealth is strictly positive.

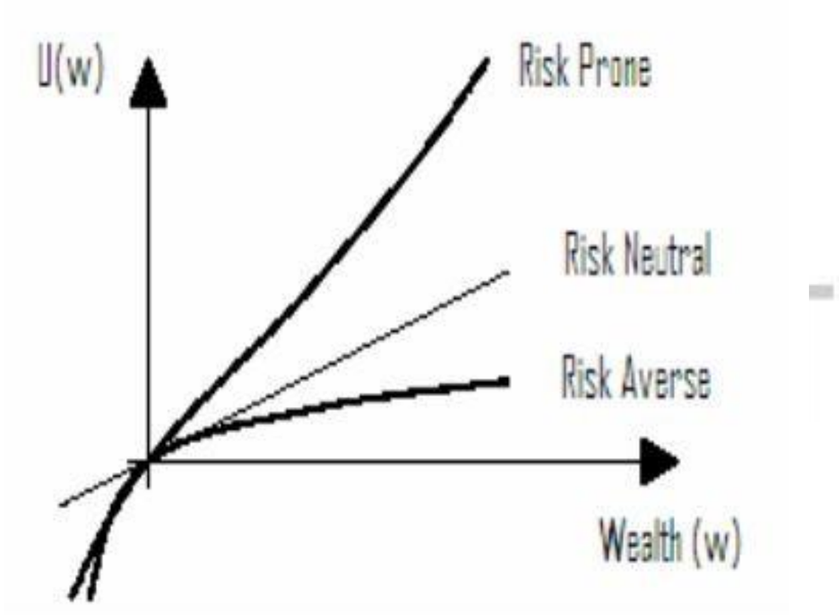
2. Risk aversion: $U^{00} < 0$ this property is used to express the attitudes of fund managers towards risk which indicate that the fund manager's utility curve of the function is strictly concave of wealth.

Fundamentally an investor can be categorized by three different risk attitudes using the second derivative of their utility functions.

i. Risk lover (Risk prone) $U^{00} > 0$, thus a risk lover investor values an incremental increase in wealth more highly than an incremental decrease and will seek fair gamble (that is a gamble that leaves the expected wealth of the investor unchanged or equal to zero). ii. Risk averse $U^{00} < 0$, thus a risk averse investor values an incremental increase in wealth less highly than an incremental decrease in wealth and will reject a fair gamble.

iii. Risk neutral $U^{00} = 0$, a risk neutral investor is indifferent between a fair gamble and the state quo.

In practice it is believed that investors are risk averse by nature which will be the main concentration of this thesis.



Source: Darko (2012)

Figure 3.2: Attitude of Investors towards Risk

3.3.4 Arrow- Pratt Measure

A more defined technique used to interpret an investor's investment behavior is through the theories Absolute Risk Aversion (ARA) and Relative Risk Aversion (RRA) presented by Pratt (1964) and Arrow (1970) measured by the functions;

$$ARA(X) = -\frac{U''(X)}{U'(X)} \tag{3.13}$$

$$ARA(X) = -X\frac{U''(X)}{U'(X)} \tag{3.14}$$

With the use of the first derivatives of ARA(X) and RRA(X), it is feasible to determine a fund manager's attitude towards risk with changes in wealth. An increasing absolute risk aversion $ARA^{0}(X) > 0$ is described as reducing the amount of wealth invested in the risky asset in the portfolio when his wealth increases, while decreasing absolute risk aversion $ARA_0(X) < 0$ is described as increasing the amount of wealth invested in the risky asset when wealth increases. If the absolute risk aversion is constant $ARA_0(X) = 0$ the investor will keep a fixed amount of wealth invested in the risky asset. It is commonly thought that absolute risk aversion should decrease, as this seems as the most plausible behavior stated by Campbell and Viceira (2002) which was cited by Pedersen (2013). Similarly, increasing relative risk aversion $RRA_0(X) > 0$ is measured as investing a smaller portion of wealth in the risky asset when wealth increases, while decreasing relative risk aversion $RRA_0(X) < 0$ implies that a larger portion of wealth is invested in risky asset when wealth increases. Constant relative risk aversion $RRA_0(X) = 0$ describes the behavior where the investor continually adjusts his investment in order to hold the portion invested in the risky asset constant stated by Pedersen (2013).

Arrow-Pratt measures can be used to differentiate between the commonly used types of utility functions for economic and financial optimizations.

3.3.5 Common Utility Functions

A number of functions can be used as utility functions, since they capture the characteristics of the value of more wealth and risk aversion. The thesis lists four utility functions and their properties used to fit the behavior of a rational fund manager(s) with terminal wealth W(X) since optimal portfolio allocation is based on these;

1. The exponential function: $U(X) = -e^{-aX}$, a > 0
2. The logarithmic function: $U(X) = \ln(X)$
3. The power functions: $U(X) = \dfrac{X^{\lambda} - 1}{\lambda}$
4. The quadratic functions: $U(X) = X + dX^2$, d < 0

From section 3.3.3 the thesis stated that a utility function should satisfy the following properties

i. A utility function must be an increasing continuous function: $U^0(X) > 0$ that is non-satiation ii. A utility function must be concave: $U^0(X) < 0$ that is the risk aversion iii. Decreasing absolute relative risk aversion $ARA^0(X) < 0$ iv. Constant relative risk aversion $RRA^0(X) = 0$

If the above axioms and the properties of utility functions are satisfied, equation 3.12 describes the maximization problem of a rational investor.

In order to determine which of the four utility functions are credible and satisfy the properties of utility functions? The tables 3.1 and 3.2 determine the credibility of the various utilities functions.

Table 3.1: First and Second Derivative of the utility Functions

Utility Names	U(X)	$U^{0}(X)$	$U^{00}(X)$
Quadratic Utility	$U(X) = X + dX^2$	$1 + 2dX$	$-2d$
Exponential Utility	$U(X) = -e^{-aX}$	ae_{aX}	$-a_2e_{-aX}$

Power Utility	$U(X) = \frac{X^{\lambda} - 1}{\lambda}$	$X^{\lambda-1}$	$(\lambda - 1)X^{\lambda-2}$
Log Utility	$U(X) = \ln(X)$	$\frac{1}{X}$	$\frac{1}{X^2}$

Table 3.2: ARA and RRA of the Utility Functions

Utility	ARA(X)	$ARA'(X)$	RRA(X)	$RRA'(X)$
Quadratic Utility	$\frac{-2d}{1+2dX}$	$\frac{4d^2}{(1+2dX)^2}$	$\frac{-2dX}{1+2dX}$	$\frac{4d^2X}{(1+2dX)^2}$
Exponential Utility	a	0	aX	a
Power Utility	$-\frac{(\lambda-1)}{X}$	$\frac{(\lambda-1)}{X^2}$	$-(\lambda - 1)$	0
Log Utility	$\frac{1}{X}$	$-\frac{1}{X^2}$	1	0

From Table 3.2 in order for the power utility to satisfy the four properties the thesis impose the restriction that $\lambda < 1$ whiles the log utility function satisfies the three of the properties thus; non-satiation, decreasing ARA and constant RRA.

3.4 Spectral Analysis of the Interest Rate

The idea of this section is to decompose the time series data of the interest rate into a combination of sinusoids with random and uncorrelated coefficients. It is the analysis in the frequency domain that is in contrast to the time domain approach. The frequency domain approach considers regression on sinusoids while the time domain approach considers regression on the past values of the time series data.

3.4.1 Fourier Series Analysis

In 1822, Joseph Fourier completed his work on the Analytical Theory of Heat (Théorie Analytique de la Chaleur) in which he introduced the series as a solution D.J.S (2002) as stated in chapter two sections 2.11. This process became known as Fourier analysis in which he stated that any periodic function can be represented in sinusoidal form. The interest rate in the paper as used by Lakner (1998) was transformed by Fourier series analysis. It is also known as harmonic analysis or harmonic regression. It is a data analysis procedure that is used to measure the fluctuations in a time series by comparing them with sinusoids. We shall consider a three-parameter “sinusoid plus constant” model given by

$$x_t = \mu + A\cos wt + B\sin wt + e_t \tag{3.15}$$

where; w is the angular frequency measured in radians, given by $2\pi f$ f is the fundamental frequency or the periodicities of the data

μ is the trend equation

A and B are the coefficients of Fourier series

x_t is the series e_t is the error term

The series consists of trend, seasonal and error components. The figure below shows the trend and oscillations of the historical data from GSE.

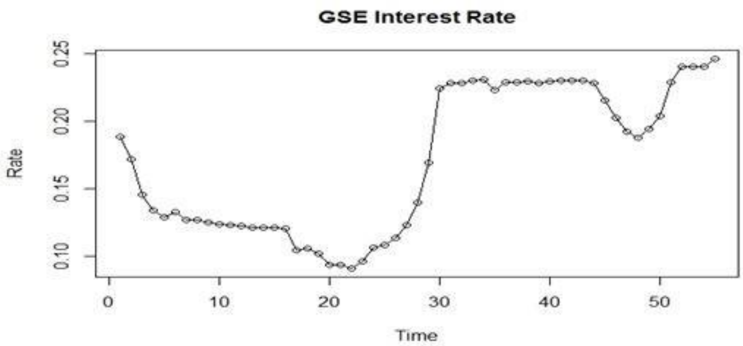


Figure 3.3: Plot of Historical Interest Rate

3.4.2 Least Squares Fitting Of Sinusoids

The least squares estimation of A, B and μ leads to the minimization of

$$S(A,B,\mu) = \sum_{i=0}^{N-1}(x_t - \mu - A\cos wt - B\sin wt)^2$$

With its derivatives given as

$$\frac{\partial S}{\partial \mu} = -2\sum_{i=0}^{N-1}(x_t - \mu - A\cos wt - B\sin wt)$$

$$\frac{\partial S}{\partial A} = -2\sum_{i=0}^{N-1}\cos wt(x_t - \mu - A\cos wt - B\sin wt)$$

$$\frac{\partial S}{\partial B} = -2\sum_{i=0}^{N-1}\sin wt(x_t - \mu - A\cos wt - B\sin wt)$$

The solutions for the above equations by equating them to zero are given by;

$$\tilde{\mu} = \frac{1}{n}\sum_{t=1}^{N} X_t$$

$$\tilde{A} = \frac{2}{N}\sum_{t=1}^{N} x_t \cos wt$$

$$\tilde{B} = \frac{2}{N}\sum_{t=1}^{N} x_t \sin wt$$

for

w = $2\pi f$ f

= $\frac{1}{K}$

Where K is the periodicity of period in units
The fitted equation of the series is given by

$$r_t = \tilde{\mu} + \tilde{A}\cos wt + \tilde{B}\sin wt \tag{3.16}$$

Therefore

$$r_t = \tilde{\mu} + \tilde{A}\cos 2\pi ft + \tilde{B}\sin 2\pi ft$$

$$r_t = 0.17112 - 0.00073\cos 2\pi ft + 0.00038\sin 2\pi ft \tag{3.17}$$

The figure below shows the estimated interest rate using the Fourier analysis.

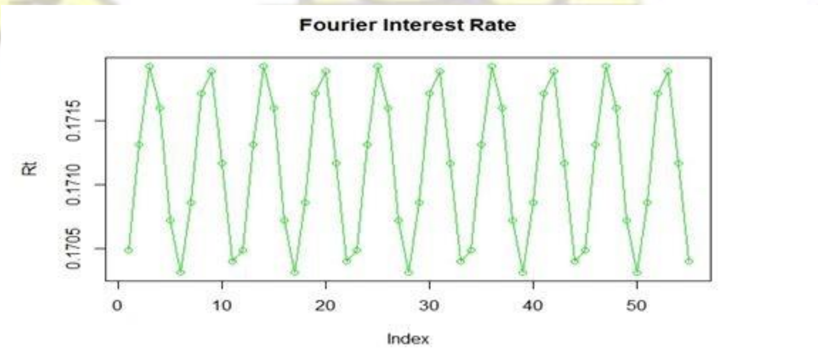


Figure 3.4: Plot of the transformed Interest Rate

3.4.3 Properties of Fourier Analysis

1. The most basic property of the sinusoids that makes them generally suitable for the analysis of time series is their simple behavior under a change of time scale. A sinusoid of frequency f (in cycles per unit time) or period $\frac{1}{f}$ (in units of time) may be written as

$$c(t) = R\cos 2\pi(ft + \varphi)$$

2. Sum of sinusoids with a common frequency is another sinusoid with the same frequency is another sinusoid with the same frequency. Thus any sinusoid with frequency f is a linear combination of the two basic functions $\cos 2\pi ft$ and $\sin 2\pi ft$.

$$R\cos 2\pi(ft + \varphi) = R\cos 2\pi ft \cos 2\pi\varphi - R\sin 2\pi ft R\sin 2\pi\varphi$$

3.4.4 Comparison of the Interest Rates

The essence of the Fourier analysis is the representation of a mathematical curve that describes a smooth repetitive oscillation. It is a function of time as shown in equation 3.17.

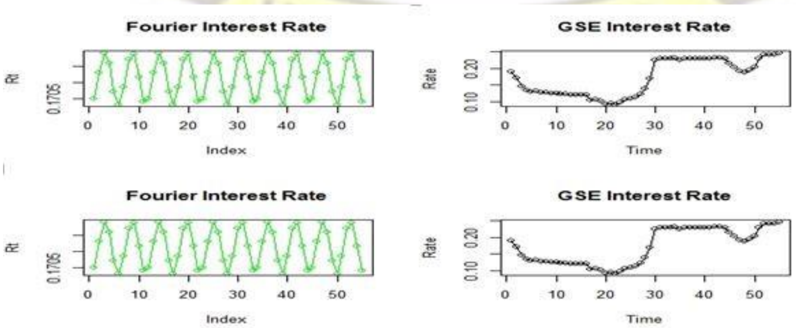


Figure 3.5: Comparison of the Interest Rates

3.5 Simulation Study

The simulation study will consider a set of existing probability distributions, to aid this researcher to generate stock prices and also help to achieve the objectives set for the work. These distributions are;

1. Log-normal distribution
2. Normal distribution
3. Exponential distribution

The problem of generating correlated and independent stock prices under the existing distributions is solved in Cholesky decomposition. The cholesky decomposition will be used to transform the distributions and simulate multivariate random variable of the stock prices with a given covariance structure matrix (variance-covariance matrix and it must be a genuine matrix). Together with the other theories and models that will be discussed in this chapter.

3.5.1 The Lognormal Distribution

The log-normal (or lognormal) distribution is a continuous probability distribution of a random variable whose logarithm is normally distributed it is related to normal distribution. Thus, if the random variable X is log-normally distributed, then $Y = \ln(X)$ has a normal distribution. Likewise, if Y has a normal distribution, then $X = \exp(Y)$ has a lognormal distribution. A random variable which is lognormally distributed takes only positive real values.

The distribution is occasionally referred to as the Galton distribution or Galton's distribution, after Francis Galton. Given a log-normally distributed random variable X, it is a distribution of two parameters μ and σ, respectively, the mean and standard deviation of the variable's natural logarithm. Its pdf is denoted by;

$$f(x) = \frac{1}{x\sigma\sqrt{2\pi}} \exp^{-\frac{(\ln x - \mu)^2}{2\sigma^2}}, \ for x > 0 \qquad (3.18)$$

Mean = $\exp(\mu + \sigma^2)$

Variance = $\exp(2\mu + 2\sigma^2) - \exp(2\mu + \sigma^2)$

3.5.2 The Normal Distribution

The normal distribution is a continuous distribution with two parameters μ and σ. The two parameters are the mean and standard deviation respectively. It produces symmetric pdf with a distinctive shape sometimes called bell-curve. The probability density function is denoted by

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp^{-\frac{(x - \mu)^2}{2\sigma^2}}, \ for - \infty < x < \infty \qquad (3.19)$$

Mean = μ

Variance = σ^2

3.5.3 The Exponential Distribution

The exponential distribution is one of the important continuous distributions which take it value between $(0,\infty]$. It has a memoryless property its PDF is denoted by

$$f(x) = \frac{1}{\theta} e^{\frac{-x}{\theta}} \quad for x > 0 \tag{3.20}$$

Mean = θ

Variance = θ^2

3.6 Determining Covariance and Correlation

To determine the risk of a portfolio, we need to know more than the risk and return of the component part. Two statistical measures that allow us to measure the co-movement of the stocks returns are covariance and correlation.

3.6.1 Covariance

The covariance is the expected product of the deviations of the two returns from the stocks means. It is a measure of the tendency among the returns to move either oppositely or together. The covariance between returns X and Y is denoted by:

$$Cov(X,Y) = E[(X - E(X))(Y - E(Y))]$$

When estimating from historical data, we use

$$Cov(X,Y) = \frac{1}{T-1}\sum_{i=1}^{T} E[(X - E(X))(Y - E(Y))] \quad (3.21)$$

Properties

1. var(X) = cov(X, X)
2. cov(X, Y) = cov(Y, X)
3. cov(X, Y) = 0, if X and Y are independent

3.6.2 Correlation

The correlation between two stocks is the measure of how the returns move in relation to each other. It is between +1 (returns always move together) and -1 (returns always move oppositely). Independent risks have no tendency to move together and have zero correlation. It is defined as covariance of the returns divided by their standard deviation.

$$Corr(X,Y) = \frac{Cov(x,Y)}{SD(X)SD(Y)} \quad (3.22)$$

Properties

1. $-1 \leq \rho \leq$

2. $\rho = 0$, if X and Y are independent

3.7 Cholesky Decomposition

Given A a symmetric positive definite (SPD) matrix, the cholesky decomposition method consists of decomposing A as a product of $A = C^{0}C$ where C is the upper triangular matrix and C^{0} is the transpose matrix (lower triangular matrix). A is the variance-covariance matrix.

3.7.1 Definition

A symmetric positive definite matrix A is given by:

$$A = L^{T}DL \tag{3.23}$$

where L is an upper triangular matrix and D is the diagonal matrix with positive diagonal elements.

The Cholesky decomposition is one of the few numerically stable matrix algorithm used. The variance-covariance matrix,Σ, is a symmetric positive definite matrix.Therefore,

$$\Sigma = L^{T}DL \tag{3.24}$$

$$\Sigma = (L^{T}\sqrt{D}L)(\sqrt{D}L) = (\sqrt{D}L)^{T}(\sqrt{D}L) \tag{3.25}$$

The matrix $Q = \sqrt{D}L$ therefore satisfies $Q^{T}Q = \Sigma$. It is called the Cholesky Decomposition of Σ.

3.8 The Model

Let (Ω, F, P) , $\mathrm{F} = \mathrm{F}_t; 0 \leq t \leq T$ be a complete filtered probability space with a fixed terminal time $T > 0$. There are N risky securities on this space with the N-dimensional price process

$$S = S_t = (S_1(t), S_2(t), ..., S_N(t))'; t\epsilon[0,1]$$

in the market.

The dynamics of these processes are determined by the system of stochastic differential equations.

$$dS_i(t) = \mu_i(t)S_i(t)dt + \sum_{j=1}^{N} \sigma_{ij} dW_j^{(1)}(t) \tag{3.26}$$

where i=1,2,...,N

In the above equation the drift

$$\mu = \mu_t = (\mu_1(t), \mu_2(t), ..., \mu_N(t))'; t\epsilon[0,1]$$

is an adapted, measurable N-dimensional process such that

$$\int_0^T \|\mu_u\|^2 du < \infty, a.s \tag{3.27}$$

where k.k is the Euclidean norm. The process

$$w^1 = w_t^{(1)} = (w_1^{(1)}(t), w_2^{(1)}(t), ..., w_N^{(1)}(t))'; t\epsilon[0,1]$$

is an N-dimensional Brownian motion and $\sigma = (\sigma_{i,j})i,j = 1,N$ is a nonsingular matrix of constants (volatility matrix). Let r_t be a deterministic time dependent interest rate.

We suppose the initial prices $S_i(0); i = 1,...,N$ are deterministic positive constants.

Let $\mathrm{F}^s = \mathrm{F}_t^s; t \leq T$ be the augmented filtration generated by the price process S. It shall be assumed that:

1. F^s-adapted processes are observable
2. Agents in this market do observe the Brownian motion $w^{(1)}$ and the drift process μ
3. The interest rate r_t, the initial price vector S_0 and the volatility matrix σ are known to all agents acting in the market.

We define the positive local martingale by the equation $Z = Z_t; t \leq T$ by the equation

$$dZ_t = -(\mu_t - r_t 1)'(\sigma')^{-1} Z_t dw_t^1, \quad (3.28)$$

$$Z_0 = 1 \quad (3.29)$$

where 1 is the N-dimensional vector with all entries equal to 1.

3.28 and 3.29 have the unique solution.

$$Z_t = exp - \int_0^t (\mu_t - r_t 1)'(\sigma')^{-1} dw_u^1 - \frac{1}{2}\int_0^t \|\sigma^{-1}(\mu_u - r_t 1)\|^2 du \tag{3.30}$$

From Assumption 2.1 of Lakner (1998), Z is a martingale.

Next, we shall define a trading strategy for an agent acting in this market. Let $\pi_i(t)$ be the amount of wealth invested in the i-th security at time t.

Definition : A trading strategy π = $\pi_t = (\pi_1(t), \pi_2(t), ..., \pi_N(t))^0$; $0 \leq t \leq T$ is an N-dimensional measurable, F^S-adapted process such that:

$$\int_0^T \|\pi_t\|^2 du < \infty, a.s \tag{3.31}$$

We emphasize that a trading strategy is required to be F^S-adapted, thus investors indeed observe the security prices only, not the drift or the Brownian motion.

Let X_t be the wealth at time t of an agent who follows the trading strategy π. The initial wealth $X_0 = x_0$ is a deterministic constant.

The process $X = X_t; t\epsilon[0,T]$ is assumed to grow according to the dynamics

$$dX_t = \pi_t' \mu_t dt + \pi_t' \sigma dw_t^{(1)} + (X_t - \pi' 1) r_t dt \tag{3.32}$$

By applying Ito's rule the discounted wealth $e^{-r_t t} X_t$ has the form

$$d(e^{-r_t t} X_t) = e^{-r_t t} \pi' \sigma d\tilde{w}_t, \tag{3.33}$$

where

$$\tilde{w}_t = w_t^1 + \int_0^t \sigma^{-1}(\mu_u - r_t 1)du \tag{3.34}$$

By Girsanov Theorem and Assumption 2.1 of Lakner (1998), the Ndimensional process

$$\tilde{w} = \tilde{w}_t = (\tilde{w}_1(t),...,\tilde{w}_N(t)); 0 \leq t \leq T$$

is a Brownian motion under the probability measure $\tilde{P}$ where

$$\frac{d\tilde{P}}{dP} = Z_T \tag{3.35}$$

We denote by $\tilde{E}$ the expectation operator corresponding to the measure
$\tilde{P}$

3.8.1 Definition

A trading strategy π is called admissible if. $X_t \geq 0$, a.s., t ϵ $[0,T]$

3.8.2 Definition

A function U : $[0,\infty]$ $7\rightarrow$ $R \cup -\infty$ is called a utility function if it is continuous, strictly increasing, strictly concave on it domain, continuously differentiable on $(0,\infty)$ with derivative function $U^0(.)$ satisfying the relation

$$lim_{x\to\infty}U^0(0) = 0, \tag{3.36}$$

Our optimization problem is to maximize the expected utility from terminal wealth at time T, i.e.,

$$maxE[U(X_T)]$$

Over all admissible trading strategies. We define the N-dimensional return process

$$R = R_t = (R_1(t), R_2(t), ..., R_N(t)); t\epsilon[0, T]$$

by

$$dS_i(t) = S_i(t)dR_i(t), i = 1,...,N \tag{3.37}$$

So we have the following decompositions for the return process

$$dR_t = \mu_t dt + \sigma dw_t^{(1)}, \tag{3.38}$$

and

$$dR_t = r_t 1dt + \sigma d\tilde{w}_t^{(1)}, \tag{3.39}$$

Equation (3.37) - (3.39) imply that S, R and $\tilde{w}$ each generate the same filtration. Thus, F^s is continuous Karatzas and Shreve (1988), corollary 2.7.8.

Let $\zeta = \zeta_t; t\epsilon[0, T]$ be the optional projection of the P-martingale Z to F^s, so that

$$\zeta_t = E[Z_t|\mathcal{F}^s], a.s, t\epsilon[0, T] \tag{3.40}$$

We note that ζ is a martingale with respect to (P,$\mathcal{F}^s$) and for every $\mathcal{F}^s$ measurable random variable V and $\mathcal{F}_u$-measurable random variable W with $0 \leq t \leq \mu \leq T$

$$\tilde{E}V = E\zeta_t V \quad (3.41)$$

$$\tilde{E}[Y|\mathcal{F}_t^s] = \frac{1}{\zeta_t}E[Z_u Y|\mathcal{F}_t^s] \quad (3.42)$$

and

$$\tilde{E}[W|\mathcal{F}_t^s] = \frac{1}{\zeta_t}E[Z_u W|\mathcal{F}_t^s] \quad (3.43)$$

The last identity implies that $\frac{1}{\zeta}$ is a ($\tilde{P}$, $\mathcal{F}^s$)-martingale. Since $\mathcal{F}^s$ is generated by $\tilde{w}$, so $\frac{1}{\zeta}$ and also ζ, must be continuous.

Let the function $I : (0,\infty) 7\to [0.\infty)$ be the pseudo inverse function of the strictly decreasing derivative of the utility function:

$$I(y) = infx \geq 0 : U^0 \leq y \quad (3.44)$$

Equation(3.44) defined function I actually becomes the inverse function of U_0 if $lim_{x\to 0}U^0(x) = \infty$. However, we did not make this assumption.We recall the following theorem from Lakner (1998)

3.8.3 Theorem

Suppose that for every constant $x\epsilon(0,\infty)$,

$$\tilde{E}[I(x\zeta_T)] < \infty \tag{3.45}$$

then the optimal level of terminal wealth is

$$\tilde{X}_T = I(ye_{-r_t T}\zeta_T) \tag{3.46}$$

where the constant y is uniquely determined by

$$\tilde{X}_T = [e_{-r_t t I(ye^{-r_t T}\zeta_T)}] = x_0 \tag{3.47}$$

The optimal wealth process $\tilde{X}$ and the trading strategy $\tilde{\pi}$ is implicitly determined by:

$$e^{-r_t t}\hat{X}_t = \tilde{E}[e^{-r_t T} I(ye^{-r_t T}\zeta_T)|\mathcal{F}_t^S] = x_0 + \int_0^T e^{-r_t t}\hat{\pi}_t' \sigma d\tilde{W}_t \tag{3.48}$$

3.9 Explicit Representation of the Optimal Trading Strategy

3.9.1 The Drift

We assume that the drift process μ_t for the various securities follows the following process:

$$d\mu_t = \alpha(\delta - \mu_t)dt + \beta\ dw_t^{(2)}, \tag{3.49}$$

where $w_t^{(2)}$ is an N-dimensional Brownian motion with respect to (F,P), independent of $w_t^{(1)}$ under P, α and β are known $N * N$ matrices of real numbers, and δ is a known N-dimensional vector of real numbers. We shall assume that β is invertible and that μ_0 follows an Ndimensional normal distribution with mean vector m_0 and covariance matrix γ_0. The vector m_0 and the matrix γ_0 are assumed to be known to all agents in the market. We note that if α is a diagonal matrix with positive entries in the diagonal, then μ will be an N-dimensional Orstein-Uhlenbeck process with mean-reverting drift. We shall also assume that $tr(\gamma_0)$ and $\|\beta\|$ are small.

Now we are ready to state the main theorem of the thesis:

3.9.2 The Main Theorem of the Thesis

Theorem

Suppose that U is twice continuously differentiable on $(0,\infty)$ and

$$I(x) < K_2(1 + x^{-5}) \tag{3.50}$$

$$-I^0(x) < K_2(1 + x^{-2}) \tag{3.51}$$

for some $K_2 > 0$. Then the optimal trading strategy is

$$\hat{\pi}_t = H(t)\frac{1}{\zeta_t}E[I'(ye^{-r_tT}\zeta_T)\zeta_T^2(-\gamma(t)(\phi'(t))^{-1}\int_t^T \phi'(u)(\sigma')^{-1}d\bar{w}_u - m_t + r_t1)|\mathcal{F}_t^S] \tag{3.52}$$

where

$$H(t) = e^{r_t(t-2T)y(\sigma\sigma')^{-1}} \tag{3.53}$$

with ζ and m_t by equation 3.5 and 4.9 respectively of Lakner (1998), also the constant y is uniquely determined by 3.47

3.9.3 The Utility Function

Finally, we will consider an investor with an Iso-elastic utility function as used by Lakner (1998), given by:

$$U(x) = \frac{1}{\lambda} x^{\lambda}, \tag{3.54}$$

where $\lambda < 0$ the equation (3.54) satisfy the properties of utility functions given at section 3.3.3

In this case

$$I(x) = x^{\frac{1}{\lambda - 1}} \tag{3.55}$$

and

$$-I^{'}(x) = \frac{1}{1-\lambda} x^{\frac{1}{\lambda-1} - 1} \tag{3.56}$$

It is clear that equation 3.50 and 3.51 holds and condition (3.45) is also satisfied because

$$\tilde{E}[I(x\zeta_T)] = x^{\frac{1}{\lambda-1}} \tilde{E}[\zeta_T^{\frac{1}{\lambda-1}}] = x^{\frac{1}{\lambda-1}} E[\zeta_T^{\frac{\lambda}{\lambda-1}}] \leq x^{\frac{1}{\lambda-1}} (E[\zeta_T])^{\frac{\lambda}{\lambda-1}} = x^{\frac{1}{\lambda-1}} < \infty \tag{3.57}$$

Then the optimal trading strategy becomes

$$\hat{\pi} = \frac{1}{1-\lambda} (\sigma\sigma^{'})^{-1} (m_t - r_t 1) \hat{X}_t + G_t \tag{3.58}$$

where

$$G_t = y^{\frac{1}{1-\lambda}} \frac{1}{1-\lambda} \exp(r_t(t + \frac{T\lambda}{1-\lambda})) \frac{1}{\zeta_t} (\sigma\sigma')^{-1} \gamma(t)(\phi(t))^{-1} \times E[\zeta_T^{\frac{\lambda}{\lambda-1}} \int_t^T \phi'(u)(\sigma')^{-1} d\tilde{w}_u | \mathcal{F}_t^S] \tag{3.59}$$

Thus the optimal trading strategy under Iso-elastic utility function is given by:

$$\pi_t = \frac{1}{1-\lambda}(\sigma\sigma')^{-1}(\mu_t - r1)X_t \tag{3.60}$$

This was used by Lakner (1998), formula (4.32).

3.9.4 The Trading Strategy Model

Now the optimal trading strategy for power or Iso-elastic utility function under full information under time dependent interest rate is given by:

$$\hat{\pi}_t = \frac{1}{1-\lambda}(\sigma\sigma')^{-1}(\mu_t - r_t 1)X_t \tag{3.61}$$

Since $1 - \lambda$ in the last equation is greater than 0, we will let $1 - \lambda$ be equal to A. Where A is called coefficient of risk aversion. The more A becomes greater, the more investors become risk averse and the reverse is true. We will use A as a measure of how risk averse an investor is and it will be used in the next chapter for the data analysis.

CHAPTER 4

ANALYSIS AND RESULTS

4.1 Introduction

This chapter presents, discusses and interpretation of results obtained in the analysis. It gives a practical illustration of the methodology outlined in the Chapter three. Here, the computations that were involved in the modeling process will be outlined numerically and graphically. The study looked at stock prices from the year 1^{st} January, 2010 to 31^{st} May, 2015 under the scenarios; Simulation study and Market Data fitting. R software was used to perform the simulations and formulation of the model 3.61

4.1.1 Assumptions for the study

1. There are N risky securities available in the market but only three of the securities were considered.
2. The interest rate is not always fixed at the horizon for all the stocks in the market as we can see from most papers.
3. Fund managers will observe only the stock prices and the interest rate.
4. The drift and the Brownian motion cannot be observed.
5. A deterministic covariance matrix was used.
6. A trading strategy for next period of investment is calculated.
7. A constant wealth is considered in this study.
8. Short sales or borrowing restrictions are not considered.

4.2 Scenario Study

4.2.1 Trading Strategies

This chapter is studied for trading strategies under four scenarios; in the first scenario the investor had the opportunity to invest in stocks that are uncorrelated with fixed interest rate. In the second scenario the investor got the prospect to invest in stocks that are uncorrelated with varying interest rate. Third and fourth scenarios assume that stocks are correlated with fixed and time dependent interest rate for all the stocks. All the scenarios are studied for an investor who does not accept any short sales or borrowing restrictions, and have a finite investment horizon and whose preferences are described by Iso-elastic utility. All the Trading Strategies are in long positions and are stated in percentages form. Common for the four scenarios were that, they were studied at different levels of risk aversions ($\lambda = -0.5, -5 \ and - 20$) and with 5% of monthly contribution from SSNIT contributors basic salary as 2^{nd} Tier contribution which is also fixed at each time of the investment period, in this thesis it was assumed to be GH 1000 cedis.

In this research, different levels of simulations (N = 500, 1000, 2000 and 10000) were considered.

As assumed from section 4.1.1 that, the Fund manager of 2^{nd} Tier pension scheme wants to invest a sum of money into three different stocks at a time, so as to make good use of the stocks at maturity of investment. As the saying goes "do not put all your eggs in one basket", if a fund manager decides to invest all his funds into one stock, he can lose all his investments in times of bad investment or if the price of the stock begins to fall. On the other hand, if the investor decides to invest in two or more stocks, the investor can see the true state of his investments by using Markowitz's theory like the correlation of the stocks returns.

Additions to the scenarios stated above, the fund manager may want to consider the scenarios where stock prices followed existing probability distributions in the market. The investor considered three different distributions these are:

1. Log-normal distribution.
2. Normal distribution.
3. Exponential distribution.

4.2.2 Assumptions for the simulation study

1. The order of the combination(s) of the distributions is not important.
2. Discussions are based purely on N = 10000 from all the Tables for all the stocks.

The tables shown in this section are the various possible combinations of the distributions for the three (3) stocks, for example, there can be an instance where all the three (3) stocks can follow;

i. Log-normal,

ii. Normal or

iii. Exponential.

That is stock one; stock two; and stock three; can be log-normally, normally and exponentially distributed prices.

Another possible combination can also be resulted in two of the stocks having the same distribution, and the third stock following a different distribution. For example stock one (1) and (2) can both be normally distributed whereas stock three (3) exponentially distributed.

We can also have the instance where all the three (3) stocks can follow three different distributions, for example stock one can be Normally distributed, stock two Log-normally and stock three Exponentially distributed.

The following tables (Table 4.1, 4.2, 4.3, 4.4, and Appendix A) as stated in percentages represent the mean possible optimal trading strategies for an investor, and the investor can meet these combinations in the market world assuming the above scenarios hold.

As indicated in 3.61, the optimal trading strategy for an investor is determined by the following formula.

$$\hat{\pi}_t = \frac{1}{1-\lambda}(\sigma\sigma')^{-1}(\mu_t - r_t 1)X_t$$

Where

- $\hat{\pi}_t$ represents the investment strategy.

- λ represents the level of risk aversion of the investor.
- σ represents the volatility matrix of the N securities.
- ^{0} represents a transpose.
- μ_t represents the N dimensional vector of drift of the various securities.
- r_t is the time dependent interest rate
- 1 represents N-dimensional vector of entries 1
- X_t represents the wealth at time t in Ghana cedis

The investor applied the proposed model to the simulation study and observed the behavior of the trading strategy. With

- GH X_t = 1000 cedis as Wealth
- r_t = 17.19% as fixed interest rate for all the stocks
- $\lambda = -0.5$, -5 and -20 as the levels of risk aversion for the scenarios

  One and Three and varying interest rate for scenarios

Two and Four

4.3 Scenario One

This table 4.1 reports the average wealth allocation in percentages of uncorrelated stock returns for an investor with a monthly wealth = 1000 in Ghana cedis, with fixed Interest Rate = 17.19% and varying level of risk aversion ($\lambda = -0.5, -5, -20$) under Log-normally, Normally and

Exponentially distributed stock prices. From Table (4.1), one can observe that the trading strategy for all levels of risk aversions consisted of long position for all the stocks. Since our objective is to invest in all the three stocks at a time for diversification purposes and also to avoid the danger of losing all your wealth should you invest in only one stock, the study used N = 500, 1000, 2000 and 10000 for all the Tables as stated in (4.2.2), the reason being that the thesis assumed the law of large numbers to the simulation work as stated at section 3.2.14. The investor observed that the total allocation for N = 10000 sum to 100 and all the allocations are stated in percentages.

Table 4.1: Log-normally, Normally and Log-normally Distributed Trading Strategies

levels of risk aversion $\lambda = -0.5$	number of simulations			
	N=500	N=1000	N=2000	N=10000
stock 1 (Log-normal)	79.25	79.34	78.76	77.92
stock 2 (Normal)	15.14	15.02	15.13	15.98
stock 3 (Exponential)	5.61	5.63	6.10	6.10
levels of risk aversion $\lambda = -5$	number of simulations			
	N=500	N=1000	N=2000	N=10000
stock 1 (Log-normal)	79.48	77.90	77.56	78.21
stock 2 (Normal)	14.78	15.95	16.36	15.88
stock 3 (Exponential)	5.74	6.16	6.07	5.95
levels of risk aversion $\lambda = -20$	number of simulations			
	N=500	N=1000	N=2000	N=10000
stock 1 (Log-normal)	79.24	78.96	77.73	78.29
stock 2 (Normal)	15.13	15.67	16.09	15.84
stock 3 (Exponential)	5.63	5.37	6.17	5.87

From table 4.1, the more A becomes greater the more fund manager became risk averse as stated in section (3.9.4), thus allocated more of his wealth into the risk-less ventures that is log-normally stock. From Table (4.1), the investor invested (78.21%) in stock 1 for $\lambda = -5$, followed by stock 2 (15.88%) and stock 3 (5.92%). One could see from Table (4.1) that the whole numbers remained constant but the decimals were changing as risk aversion coefficient decreases to -20. From the Table regardless of the level of risk aversion the investor preferred investing more in stock 1; followed by stock 2 and stock 3 respectively.

A similar observation can be seen about Table (A.1a and A1.b) from the (Appendix A), for two of the stocks been Log-normally distributed with the other one been normally distributed stock prices. It can be seen that, the investor invested larger proportion of his wealth into stock 1 (56.43%), followed by stock 2 with (36.07%) and stock 3 (6.5%) respectively for $\lambda = -0.5$. Risk Averse fund manager prefers to have a stable consumption throughout time, which implies that he seeks to avoid risk of negative future returns. Thus he wants to invest in stocks which will have positive returns in the future. Thus, it is advisable to allocate wealth in this proportion if the stocks follow the stated combinations.

4.4 Scenario Two

Table (4.2) shows uncorrelated stock returns which follow a Lognormally, Normally and Exponentially Distributed stocks with varying interest rate, "the degree to which the stock returns move together or oppositely" and "how the stocks face common risks" can help to optimal portfolio or asset allocation. Table (4.2) exhibits similar result as showed in Table (4.1)

Table 4.2: Log-normally, Normally and Log-normally Distributed Trading Strategies

levels of risk aversion $\lambda = -0.5$	number of simulations			
	N=500	N=1000	N=2000	N=10000
stock 1 (Log-normal)	96.57	96.28	96.15	96.23
stock 2 (Normal)	2.02	2.34	2.44	2.38
stock 3 (Exponential)	1.41	1.39	1.41	1.4
levels of risk aversion $\lambda = -5$	number of simulations			
	N=500	N=1000	N=2000	N=10000
stock 1 (Log-normal)	96.37	96.38	96.38	96.24
stock 2 (Normal)	2.26	2.22	2.24	2.35
stock 3 (Exponential)	1.37	1.41	1.39	1.41
levels of risk aversion $\lambda = -20$	number of simulations			
	N=500	N=1000	N=2000	N=10000
stock 1 (Log-normal)	96.00	96.62	96.03	96.28
stock 2 (Normal)	2.59	1.97	2.58	2.32
stock 3 (Exponential)	1.41	1.41	1.39	1.40

Since this study seeks to observe the behavior of the Trading Strategy, for N = 10000 at risk aversion level $\lambda = -0.5$, it was observed that stock one had the highest allocation of wealth (96.23%) followed by stock two (2.38%) and stock three (1.40%). The percentage margin between stock two and three is not all that huge as compared to stock one and stock two and stock one and three respectively. This can be accounted as a result of variance in the interest rate. The same behavior is seen from

Table (4.1). One can see the same result from APPENDIX (A2). Thus it is advisable to allocate Wealth in this proportion if the stocks are Log-normally, Normally and Exponentially Distributed under time dependent interest rate.

4.5 Scenario Three

This section accounts for correlated stock returns with fixed interest rate together with the varying risk aversion with a given Wealth by;

- GH X_t = 1000 cedis as Wealth
- r_t as time dependent interest rate for all the stocks
- $\lambda = -0.5, -5\ and - 20$ as the levels of risk aversion

Table (4.3) reports one of the possible combinations of the distributions where two of the stocks are log-normally and the other one is normally distributed. Until now, all the scenarios discussed so far mainly based on uncorrelated stocks with fixed and varying interest rate. We applied this scenario to the model (3.61) and observed the behavior of the model.

Table 4.3: Log-normally, Normally and Log-normally Distributed Trading Strategies

levels of risk aversion $\lambda = -0.5$	number of simulations			
	N=500	N=1000	N=2000	N=10000
stock 1 (Log-normal)	29.68	29.48	29.95	30.53
stock 2 (Normal)	25.29	25.62	24.57	24.88
stock 3 (Log-normal)	45.03	44.90	45.48	44.59
levels of risk aversion $\lambda = -5$	number of simulations			

	N=500	N=1000	N=2000	N=10000
stock 1 (Log-normal)	29.43	31.54	30.46	29.98
stock 2 (Normal)	25.73	25.35	24.58	24.90
stock 3 (Log-normal)	44.84	43.10	44.96	45.12
levels of risk aversion $\lambda = -20$	number of simulations			
	N=500	N=1000	N=2000	N=10000
stock 1 (Log-normal)	30.61	29.75	29.83	29.58
stock 2 (Normal)	24.66	26.08	24.80	25.20
stock 3 (Log-normal)	44.72	44.17	45.37	45.21

This scenario is similar to the ones already discussed. Table 4.3 displays that an investor goes a long position for each level of risk aversion. No matter the level of risk aversion the fund manager invested the same amount of wealth percentages for N = 10000 in the stock at different levels of risk aversion and apportioned higher amounts into log-normally stock prices that is stock three, followed by another log-normally stock, that is stock one. Table 4.3 shows that an investor should invest a greater part of their wealth in one of the lognormally stock prices as it was highlighted in the previous scenarios.

APPENDIX (A3) shows similar result.

4.6 Scenario Four

This is also similar to what we have discussed in scenario three; it reports possible combinations of correlated stocks returns with stock two been normal, and stock one and three been log-normal with varying interest rate for all of the stocks. The objective of this thesis is to observe the behavior of the trading strategies as we vary the risk aversion parameter together with our interest rate. As we saw from Table (4.3), the more the fund manager becomes risk aversive the more he invest in less risky asset that is log-normal.

Table 4.4: Log-normally, Normally and Log-normally Distributed Trading Strategies

levels of risk aversion $\lambda = -0.5$	number of simulations			
	N=500	N=1000	N=2000	N=10000
stock 1 (Log-normal)	22.68	20.31	19.55	20.53
stock 2 (Normal)	30.22	29.96	29.91	29.67
stock 3 (Log-normal)	47.09	49.72	50.54	49.80
levels of risk aversion $\lambda = -5$	number of simulations			
	N=500	N=1000	N=2000	N=10000
stock 1 (Log-normal)	20.93	20.55	21.13	20.67
stock 2 (Normal)	20.65	29.58	29.35	29.99
stock 3 (Log-normal)	49.42	49.88	49.52	49.34
levels of risk aversion $\lambda = -20$	number of simulations			
	N=500	N=1000	N=2000	N=10000
stock 1 (Log-normal)	20.28	20.11	20.73	20.51
stock 2 (Normal)	29.84	29.57	30.74	29.42
stock 3 (Log-normal)	49.88	50.32	48.53	50.07

From the table 4.4 at N = 10000 it reveals that at risk levels $\lambda = -5$ and -20, one can see that stocks which follows log-normal distribution (stock 3) had the highest percentages followed by normal distribution (stock 2) then log-normal distribution (stock 1) with (49.34%), (29.99%) and (20.67%) for $\lambda = -5$ and (50.07%), (29.42%) and (20.51%) for $\lambda = -20$ respectively but the decimals were varying as we changed risk aversion coefficient.

4.7 Conclusion on the Simulation Study

Taking a fastidious look at the tables 4.1 - 4.4 and those in Appendix A, one can see that regardless of the level of Risk Aversion coefficient, if the model (3.61) is run for 500, 1000, 2000 or 10000 times and their averages are found, the same percentages are reported as we vary the Risk aversion coefficient together with our fixed and varying interest rate for all the stocks. It was observed that any combination that had Log-normally distributed stock, the fund manager invested larger percentage in one of the Log-normally distributed stocks under the scenarios studied in the simulation section of this thesis.

4.8 Fitting Market Data to the Model

4.8.1 Distribution Fitting

This part of the thesis seeks to fit market data to the model (3.61) and observe the behavior of the trading strategies of a 2^{nd} Tier Fund Manager or any investor who is a risk averse to make investment decisions. The dataset as sourced from Ghana Stock Exchange, Yahoo Finance and Bank of Ghana is presented below. The data consisted of three stocks taken from GSE (2015), yahoo finance (2015) and the interest rate from Bank of Ghana (2015), which span a period of 5 years from 1^{st} January 2010 to 31^{st} May, 2015 of monthly stock prices for analysis. The stocks were taking from companies that deals in these commodities;

1. Gold,
2. Oil and
3. Bonds.

The selected companies are unlisted companies of GSE (2015). The paper will refer to the stocks as Gold (stock 1), Oil (stock 2) and Bonds (stock 3) accordingly. The Descriptive statistics, goodness of fit test concerning the data and the model results of the data will be reported in this section.

4.8.2 Statement of Hypotheses

H_0: The statistical distribution provides an accurate statistical model for the data

H_1: The statistical distribution does not provide an accurate statistical model for the data

4.9 Maximum Likelihood Estimates

The purpose of this section is to come out with a statistical distribution that best fits the stock prices obtained. On this basis, it is important to define what the parameters for distributions that are chosen. Since we are only attempting to liken the data to an existing statistical distribution, we can only estimate the parameters chosen. Upon the estimation of such parameters, we literally define the distribution. Many theories exist for estimating the parameters for the distributions, based on the data available. In this study, we use the maximum likelihood estimator to state these parameters. The maximum likelihood estimates of the selected distributions (Log-normal, Normal and Exponential) on Gold are organized in Table (4.5). R software and Curve fitting software, EasyFit were relied upon heavily for the curve fitting procedure which includes the determination of the maximum likelihood estimates. From Table (4.5) and Appendix (B1) μ is the mean (location) of the data, σ is the standard deviation (shape) and λ mean the (scale).

Table 4.5: Fitted Parametric Distribution of the Gold Prices

Distribution	Parameters
	λ=0.01184
	σ=15.525 $\mu = 84.471$
	μ = 4.4202

4.10 Goodness of Fit Test

4.10.1 Charts of GOF

The goodness of fit (GOF) tests is a statistical model which describe how well a data fits a set of observations of a random sample with a theoretical probability distribution function. In other words, these tests shows how well the distribution we selected fits our data. Measures of goodness of fit is typically the summarize of the discrepancy between observed values and the values expected under the model in question.

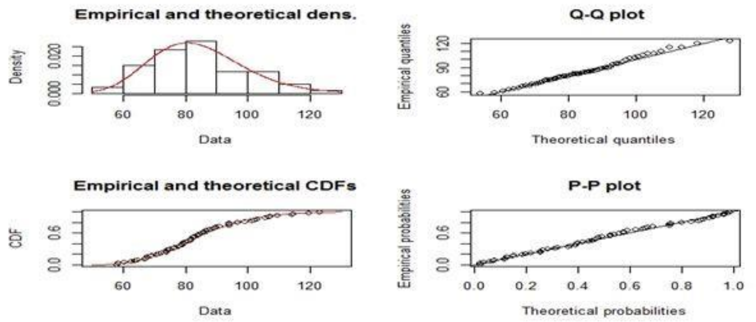


Figure 4.1: plot of PDF, CDF, Q-Q plot and P-P plot of the fitted Gold prices under lognormal distribution.

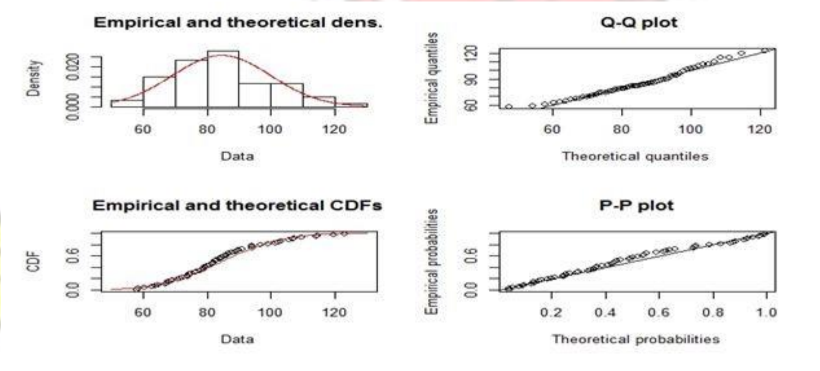


Figure 4.2: Normal fitting of the Gold price

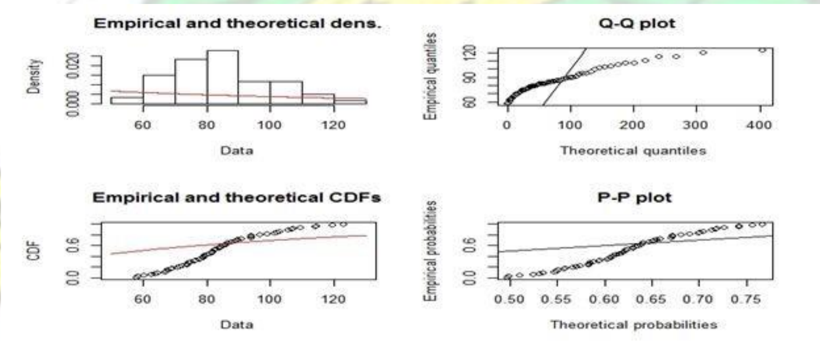


Figure 4.3: Exponential fitting of the Gold price

Figure (4.1) - (4.3) and Figures in Appendix (B2) provide the graphical representation of the Empirical and theoretical probability density function (PDF), Q-Q plots, Empirical and theoretical CDFs and P-P plots. These plots were obtained using R software. For us to achieve the aim of this section, Kolmogorov-Smirnov test and Anderson-Darling test were conducted together with the Q-Q plots to make any decision concerning the distributions. Figure 4.1 - 4.3 and those in Appendix (B2) show the fitting of the Gold prices for the distributions considered in this study. The same experiments were conducted for the Oil and Bonds refer from Appendix B2.

The quantile-quantile (Q-Q) plot is a graph of the observed data values plotted against the theoretical (fitted) distribution quantiles as shown in section 4.10.1. and Appendix B2 . Both axes of this graph are in units of the input data set. The Q-Q plot will be approximately linear if the specified theoretical distribution is the correct model. The middle line is called reference diagonal line, along which the graph points should fall or spread around it.

It is obvious from Figure (4.1) using the Q-Q plot that, most of the data points fell on the reference line when fitted the Gold price as lognormal distribution. We compared Figure (4.1) with other distributions on the same Gold price from Figure (4.2) and (4.3), Lognormal was chosen over Normal and Exponential. This was because the normal Q-Q plot had some extreme values that deviate from the reference line as compared to the lognormal. The Q-Q plot on Exponential has greater fluctuations in the body of the plot. The deviations are wider than those that has been seen from the other graphs of their distributions.

4.10.2 Tabular Representation and Algebraic Assessment of GOF Tests

The Figures 4.4 – 4.6 go to support the tests conducted from the graphical analysis from Figure 4.1 - 4.3 and Appendix (B3), it was shown that the Exponential Distribution is not accurate to be used as a model for the stock prices as shown from Figure 4.3.

Lognormal GOF Test on the Gold Prices					
Kolmogorov-Smirnov					
Sample Size Statistic P-Value	60 0.06015 0.9726				
α	0.2	0.1	0.05	0.02	0.01
Critical Value	0.13573	0.15511	0.17231	0.19267	0.20673
Reject?	No	No	No	No	No
Anderson-Darling					
Sample Size Statistic	60 0.20242				
α	0.2	0.1	0.05	0.02	0.01
Critical Value	1.3749	1.9286	2.5018	3.2892	3.9074
Reject?	No	No	No	No	No

Figure 4.4: Log-normal GOF Test on the Gold Prices

Exponential GOF Test on the Gold Prices					
Kolmogorov-Smirnov					
Sample Size Statistic P-Value Rank	60 0.49798 8.8921E-14 3				
α	0.2	0.1	0.05	0.02	0.01
Critical Value	0.13573	0.15511	0.17231	0.19267	0.20673
Reject?	Yes	Yes	Yes	Yes	Yes
Anderson-Darling					
Sample Size Statistic Rank	60 18.715 3				
α	0.2	0.1	0.05	0.02	0.01
Critical Value	1.3749	1.9286	2.5018	3.2892	3.9074
Reject?	Yes	Yes	Yes	Yes	Yes

Figure 4.5: Exponential GOF Test on the Gold Prices

Normal GOF Test on the Gold Prices

Kolmogorov-Smirnov					
Sample Size Statistic P-Value	60 0.09623 0.60077				
α	0.2	0.1	0.05	0.02	0.01
Critical Value	0.13573	0.15511	0.17231	0.19267	0.20673
Reject?	No	No	No	No	No
Anderson-Darling					
Sample Size Statistic	60 0.57012				
α	0.2	0.1	0.05	0.02	0.01
Critical Value	1.3749	1.9286	2.5018	3.2892	3.9074
Reject?	No	No	No	No	No

Figure 4.6: Normal GOF Test on the Gold Prices

Similar from the figure 4.4 - 4.6, the statistical distributions are subjected to the various algebraic distributions. The rejection or acceptance decision is determined at various levels of significance. For both tests, the null hypothesis that the data comes from the said distribution is rejected if the test statistic obtained is greater than the critical value obtained at the specified level of significance. For the Kolmogorov-Smirnov test, the rejection or otherwise of the null hypothesis can also be made based on the P-value. If the P-value is lesser than the significance level, the null hypothesis is rejected. Otherwise, we fail to reject it.

The various tests are conducted at varied levels of significance. The statistic computed using the distributions fit of the data must be less than the critical value obtained. Based on the KolmogorovSmirnov test, the Log-normal and the Normal distributions are the only distributions that passed the entire test at significance levels from 0.2 to 0.01, that is, an 80% to 99% confidence level. They have test statistic values of (0.06015) and (0.09623) respectively. These values are lesser than the critical values obtained at a respective confidence level. The Exponential statistic values showed greater values than the critical values see Table 4.7 and APPENDIX (B3).

On the basis of the Anderson-Darling test, all the distributions showed a similar result as we obtained from Kolmogorov-Smirnov test.

On account of the two tests the researcher decided on log-normal distribution to be the best between the two distributions tests since it is the one whose deviations from the reference line in the Q-Q plot was not much compared to normal and it had lesser test statistic. As stated earlier that we want to know the likely distributions the selected stocks prices follow, we came to a conclusion that Gold stock prices followed log-normal distribution, Oil followed normal distribution and Bonds followed log-normal which would be relied upon to help 2^{nd} Tier Fund Manager to make his investment decisions under the scenarios.

4.11 Returns of the Data

Before we begin anything we have to estimate the returns of the stocks and report their descriptive statistic. When an investor wants to make investment decision or wants to buy a security, he has some view as to the risk involved and the likely returns the investment will bring. In the market world, security prices vary from one stock to the other, as well as their dividends payment. To make stocks comparable, we express their performance in terms of their returns.

4.11.1 Graph of the Returns

Figure (4.7) reports the behavior of the stocks in the market. We observed from the graph that the Gold returns experienced the largest fluctuations as compared to the returns of the Oil and the Bonds during the period. It is evident that it had the maximum and the minimum fluctuation thus (0.23491) and (-0.26547) respectively from Table (4.6).

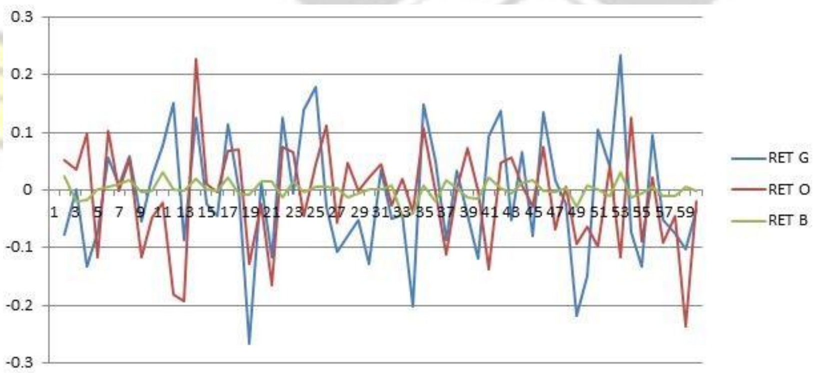


Figure 4.7: Time series graphs of returns of the stock prices

The time series graphs showed that Gold returns were fluctuating strongly during the analysis period, which explicated the high standard deviation, reported in Table (4.6). Oil prices are priced primarily based demand and supply, and are subject to speculation, which causes a lot of fluctuations. Gold returns shows to be more volatile than Oil returns. Bond returns on the other hand, are more stable, with fewer and much smaller fluctuations from Figure (4.7). In fact one could argue that oil, gold and bond returns all exhibit some sort of mean reversion, which can have significant impact on the optimal portfolio choice.

4.11.2 Statistical Features of the Returns

This section provides some descriptive statistics of the returns. Table (4.6) reports the mean, standard deviation, the maximum and the minimum of the returns of the stocks.

Table 4.6: Summary Statistic of the Returns

STATISTICS	GOLD	OIL	BONDS
Mean	-0.00935604	-0.01111	0.000534
Sharpe Ratio	-0.56592686	-0.6897	-3.28714
Volatility	0.10488288	0.08861	0.015048
Maximum	0.234912	0.226218	0.031807
Minimum	-0.26547202	-0.23726	-0.04539

From Table (4.6), we observed that the fund manager of this portfolio over the five-year period earned an average return of 0.05% in Bonds as the highest returns, low volatility of 1.50%, -0.95% in Gold being the moderate returns and -1.11% in Oil being the lowest returns during the period under studied. Any investor using the mean-variance approach can conclude at this point that Bonds is the most attractive security in the market based on these preliminary observations; one would expect the fund manager to allocate a larger proportion of his wealth to Bonds compared to the Gold and Oil. But the problem is what percentage of his wealth should be invested in each of the assets in the stock market. Again it is doubtful whether Bonds will still be attractive if we find the returns correlation or variance-covariance matrix and incorporate into our model (3.61).

4.12 Diversification

When we combine stocks in a portfolio, we reduce some of the risk through diversification. The degree to which the stocks face common risk and how their returns move together are determined by these two Tables 4.7 and 4.8.

Table 4.7: Variance - Covariance Matrix

Stocks	GOLD	OIL	BONDS
GOLD	0.011	0.002587899	0.00065512
OIL	0.002588	0.007851784	-6.32571E-05
BONDS	0.000655	-6.32571E-05	0.000226453

From Table (4.7), we can see that Gold and Oil, Gold and Bonds have positive covariance (0.002588) and (0.000655) respectively, indicating a tendency of the stocks to moving together. Whereas Oil and Bonds have negative covariance (-0.0000633), indicating a tendency of the stocks moving oppositely.

Table 4.8: Correlation Matrix

Stocks	GOLD	OIL	BONDS
GOLD	1	0.27845729	0.41507488
OIL	0.27845729	1	-0.04743903

BONDS	0.41507488	-0.04743903	1

The correlation determines how two variables have linear relationship with each other. The linearity of the stocks has the same interpretation as the covariance. The stocks will have the same sign as their covariance; it ranges between -1 and +1. From the correlation table above, it shows that the negative correlation between the stocks Oil and Bonds will make diversification relatively ease to reduce portfolio risk. Even though those with positive correlations have weak positive correlations which will still lead to a greater diversification in the portfolio risk.

4.13 Optimal Trading Strategy of the Market Data Fitting

The optimal trading strategy will be found for the next period of investment. That is a period ahead.

4.14 Case 1 - Fixed Interest Rate

As stated above, the distribution from which the stock prices follow, based on the observations on the portfolio (GOLD, OIL and BONDS), it was seen that the portfolio came from Log-normal, Normal and Lognormal distributions respectively. From the return analysis it was shown that the portfolio shown mixed correlation (that is positively and negatively correlated stocks).

The researcher incorporated the market data under the said distributions into the optimal Trading Strategy model (3.61), and observed the behavior with;

- GH X_t = 1000 cedis as Wealth
- r_t = 17.19% as fixed interest rate for all the stocks
- λ = -0.5, -5 and -20 as the levels of risk aversion

This analysis was performed in R software as shown in APPENDIX C3

Table 4.9: OTS for Log-normal, Normal and Log-normal Correlated Stock Prices under case 1

Stocks	levels of risk aversion		
	$\lambda = -0.5$	$\lambda = -5$	$\lambda = -20$
LOG-NORMAL (GOLD)	23.41	4.42	0.69
NORMAL (OIL)	28.89	3.15	0.9
LOG-NORMAL (BOND)	47.7	92.43	98.41

From Table (4.9) and Figure (4.8) it can be seen that, the optimal trading strategy (OTS) at all levels of λ consisted of long positions of the portfolio as mentioned in our simulation study. (A) the coefficient of risk aversion, as stated in the previous chapter section (3.9.4), the more (A) becomes greater, the more the fund manager become risk averse and allocate more of his wealth in less risky stocks and less of his wealth into more risky stocks. From the table 4.9 the fund manager invested 98.41% of his wealth into log-normal stock which is Bond at $\lambda = -20$ and 0.9 %

in oil (normal) and 0.69% in Gold (Lognormal). Similar thing was observed in the APPENDIX (A1.a) which is an uncorrelated stock price with fixed interest rates.

4.15 Case 2 - Varying Interest Rate

Now we are considering risk aversion coefficients with varying interest rate for all the stocks. The optimal Trading Strategy model (3.61) was applied to the data under the said distributions of the market data fitting, and observed their dynamics with

- GH X_t = 1000 cedis as Wealth
- r_t = 17.19% as fixed interest rate for all the stocks
- $\lambda = -0.5$, -5 and -20 as the levels of risk aversion

Table 4.10: OTS for Lognormal, Normal and Lognormal Correlated Stock Prices under case 2

Stocks	levels of risk aversion		
	$\lambda = -0.5$	$\lambda = -5$	$\lambda = -20$
LOG-NORMAL (GOLD)	4.14	3.49	3.72
NORMAL (OIL)	3.54	3.31	11.08
LOG-NORMAL (BOND)	92.32	93.20	85.21

From the Table (4.10) and Figure (4.9), one can observed that, a similar observations in (case 1) is depicted here, the investor apportioned a larger percentages in Bonds for all levels of λ while the proportion of Gold and Oil were very small. Table 4.10 showed that the fund manager preferred investing (11.08%) in Oil as (A) increased to 21 instead it should have been lesser than (3.31%) at A = 6. This was as a result of the drift process because it was modeled by Gaussian distribution and the fund manager hoped that Oil price will perform better in the future than Gold stock. It can be seen from Table (4.6) that the Gold price is highly volatile than all the stocks. This kind of scenario was seen from Table (4.4) showing correlated stock prices with varying interest rates, APPENDIX A1.a (showing uncorrelated stock prices with fixed interest rates) and APPENDIX A4.a (showing correlated stock prices with varying interest rates). The results shown that stocks following this kind of combination, the fund manager allocated more of his wealth into one of the log-normal stock prices which Table (4.9) and Table (4.10) have shown.

4.16 Graphs of the Trading Strategy under the Cases

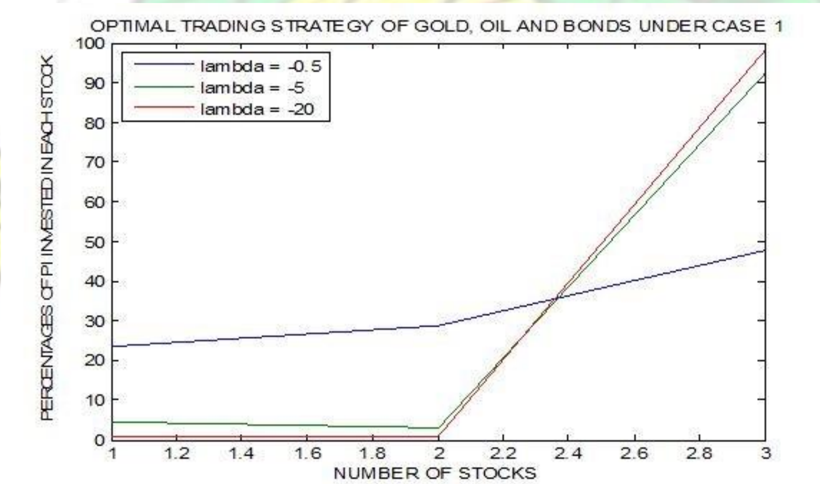


Figure 4.8: Plot of OTS for Lognormal, Normal and Lognormal Correlated Stock Prices under Case 1

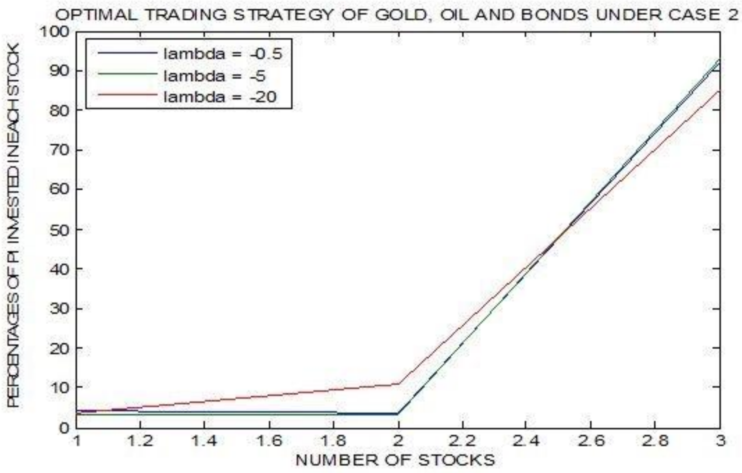


Figure 4.9: Plot of OTS for Lognormal, Normal and Lognormal Correlated Stock Prices under Case 2

CHAPTER 5

CONCLUSIONS AND RECOMMENDATIONS

5.1 Introduction

In this section the conclusions and recommendations are made based on our observations and findings from our study.

5.2 Summary of Results

The objective of this thesis was to solve the portfolio optimization problem of a 2^{nd} Tier pension fund manager in Ghana under Isoelastic utility theory. In line with the objective of the thesis, a lot of observations were found on the behavior of the trading strategies under different scenarios or observations in a risk aversive wealth allocation setting. The study did not permit short selling or borrowing restriction. In finding data for the thesis, two studies were considered; simulation study and market data fitting study.

Monte Carlo simulation was used to generate the stock prices from existing probability distributions; (lognormal, normal and Exponential), with specified covariance structures; (identity covariance structure and compound symmetry), which were decomposed by Cholesky decomposition to enable for the data generation. The simulated data was incorporated into the optimal trading strategy model 3.61 under the scenarios study. The results of the simulation study pointed out that, any possible combination of stocks under the scenario study, log-normally stocks are apportioned with the highest percentages of wealth irrespective of the level of risk aversion. The market data fitting study was carried out on the stocks returns; (Randgold Resources limited (Gold), Murphy Oil Corporation (Oil) and ishares Tips Bonds (Bonds)) which were called Gold, Oil and Bonds. Distribution fitting checks were done on the stocks and observed that two of the stocks were lognormal that is Bond and Gold and the other been normal that is Oil. From Table (4.6), the statistical descriptive of the returns were revealed, which was used to argue about the problem statement outlined. To answer the questions of this thesis the optimal trading strategy was modeled for two investment cases; One where the investor can only invest in stocks with fixed interest rate with varying risk aversion coefficients, and

Two investing in the portfolio with varying interest rate for all the stocks with varying risk aversion coefficients.

The TWO CASES were studied under the simulation and market data scenarios or observations. The results of Case one under market data settings shown that the fund manager should invest (98.41%) of his Wealth into ishares Tips Bonds (Bonds) at $\lambda = -20$, (0.9%) of his Wealth into Murphy Oil Corporation (Oil) and (0.69%) into Randgold Resources limited (Gold) from Table (4.9).

Another observation under the Case two of the market data settings was that, fund manager still allocated higher proportion of Wealth (85.21%) in Lognormal stocks which is ishares Tips Bonds (Bonds), (11.08%) in Normal stock which is Murphy Oil Corporation (Oil), and (3.72%) in Lognormal stock Randgold Resources limited (Gold). Comparing the fixed interest rate and the varying interest rate, it was seen that the both rate followed the same trend but the rate of wealth allocation changes at varying risk aversion.

The simulation study and the market data fitting depicted the same result under the scenarios study. It should be noted that this thesis was performed under too restrictive assumption of the constraint $\lambda < 0$.

5.3 Conclusions

From the objective of the study, this thesis should be seen as a stepping stone in the attempt to determining the optimal trading strategy for a 2^{nd} Tier pension fund manager under Iso-elastic utility, which contributes and extends existing literature on optimal trading strategy of a 2^{nd} Tier Pension Fund Manager in Ghana. Therefore the trading strategy model has been identified for fund managers.

Based on the three distributions considered in this study, any portfolio that had a log-normally distributed stock under the scenarios or observations in this thesis, it is advisable for Fund Managers to invest larger proportion of their wealth in one of the log-normally distributed stocks irrespective of the level of risk aversion coefficient and the interest rate.

Log-normally stocks would yield higher returns hence this improves 2^{nd} Tier Fund Manager's wealth to aid contributors when they go on retirement.

5.4 Recommendations

By comparing the results obtained, the trading strategy from the simulation study and the market data scenarios, therefore the optimal trading strategy model be recommended and adopted for allocating wealth in the stock market in real world situations.

2^{nd} Tier Pension Fund Managers, Actuaries, Financial advisors, Researchers, etc. can use the trading strategy as a model for allocating wealth in Ghana.

It is advisable for Fund Managers to invest larger proportion of their wealth in log-normally stocks irrespective of the level of risk aversion coefficient.

5.5 Further Studies

Further research could be conducted to observe the behavior of the trading strategy by looking at different covariance structures like positively correlated, negatively correlated, or a mixture of negative and positive correlation for fund managers in Ghana and other countries in the world.

REFERENCES

Adjei, E. (1999). Pension schemes in africa: The national experience of ghana. *Social Security Documentation, International Social Security Association*, African Series No. 21.

Agyeman, A. (2011). An assessment of the returns on employee pension fund investments and their impact on future benefit payments: A case study of social security and national insurance trust (ssnit). Master's thesis, Kwame Nkrumah University of Science and Technology. Kumasi, Ghana.

Aitken, W. (1994). *A Problem Solving Approach to Pension Funding and Valuation.* Actex Publication, Inc.

Ameko, M. and Baah, P. (2014). Application of stochastic differential equations in determining the optimal trading strategy for an investor. Technical report, BSc thesis, Kwame Nkrumah University of Science and Technology, Kumasi: Ghana.

Arrow, K. (1970). *Essays in the theory of risk-bearing*. Amsterdam, London: North-Holland.

Ashidam, B. (2011). The problem of the cap 30 pension scheme in ghana. Master's thesis, Kwame Nkrumah University of Science and Technology. Kumasi, Ghana.

Banks, J. (1998). *Handbook of Simulation*. John Wiley, U.S.A.

Banks, J. (2000). *Getting started with Automod.* Vol. II., Chelsford, U.S.A.

Beveridge, W. (1921). Weather and harvest cycles. *Econ. J*, 31:429– 452.

Bianchi, L., J.E.Jarrett, and R.C.Hanumara. (1993). Forecasting incoming calls to telemarketing centers. *The journal of Business Forecasting*, pages 429–452.

Bloomfield, P. (1976). *Fourier Analysis of Time Series An Introduction*. North Carolina University Raleigh: North Carolina.

Bloomfield, P. (2000). *Fourier Analysis of Time Series An Introduction*. Second Edition. North Carolina University Raleigh: North Carolina.

Campbell, J. and Viceira, L. (2002). Strategic asset allocation: portfolio choice for long term investors. Oxford: Clarendon.

Catalan, M. (2004). What do we know and what do we need to know? In *Pension Funds and Corporate Governance in Developing Countries*.

Darko, S. (2012). Constructing optimal stock portfolio with markowitz model. Master's thesis, Kwame Nkrumah University and Science Technology, Kumasi: Ghana.

Davies, B. (1993). Better pensions for all. *Institute for Public Policy Research, London.*

D.J.S (1994-2002). Encyclopedia britannica: Fourier. *Jean-BaptisteJoseph, Baron,Oxford University Press.*

Ekpenyong, E. J., Omekara, C. O., and Ekerete, M. P. (2014). Modeling inflation rates using periodogram and fourier series analysis method. the nigerian case. *International Journal of African and Asian Studies. Nigeria.*

Haberman, S. (1990). State pension provision and social security in the united kingdom. *Actuarial Research Report No.25 City University, London.*

Hebb, T. and Clark, G. (2003). Pension fund corporate engagement in global arena, society for the advancement of socio-economics annual meeting. *France, June.*

Hughes, C. (1995). Four steps for accurate call-centre staffing. *HR Magazine*, pages 87–89.

Inderst, R.and Shaffer, G. (2009). Market power, price discrimination and allocative efficiency in intermediate goods market. *RAND Journal of Economics*, 40(4):658–72.

Ivanova, N. (2010). Optimal features of micro pension schemes. econometrics and management science, erasmus university rotterdam.

Karatzas, I. and Shreve, S. (1988). *Brownian motion and Stochastic Calculus.* Springer-Verlag, New York.

Klungle, R. and J.Maluchnik. (1997-'98). Call center forecasting at aaa michigan. *Journal of Business Forecasting*, page 813.

Knott, C. (1897). *The Spectral Analysis of Time Series*. (Reprint ed.). New York: Academic.

Kwapong, N. (2013). The role of ghana's new tier pension scheme and its effectiveness in providing social security for ghanaians. Technical report, Bachelor thesis, Ashesi University Ghana.

Kyng, T. and Konstandatos, O. (2014). Multivariate monte-carlo simulation and economic valuation of complex financial contracts: An excel based implementation." spreadsheets in education. *eJSiE*, Vol. 7: Iss. 2, Article 5.

Lakner, P. (1998). Optimal trading strategy for an investor: the case of partial information. *Elsevier.*

Lewis, B., R.D.Herbert, and Chivers., W. (2002). The migration of a call centre to a contact centre, paper presented at the first international conference on information technology & applications (icita, 2002). pages 25–28.

Mercer, N. (2007). Commentary on the reconciliation of congnitive and socio cultural accounts of conceptual change. *Educational Psychologist*, pages 42,1,75–78.

Norstad, J. (2011). An introduction to utility theory. Technical report.

Ocone, D. and Karatzas, I. (1990). A generalized clark representation formula, with application to optimal portfolios. stochastic and stochastic. *Reports*, Vol 34:187–220.

Pedersen, U. (2013). Strategic asset allocation commodities' influence on the long-term investor's portfolio choice. Master's thesis, Aarhus University.

Pratt, J. (1964). Risk aversion in the small and in the large. *Econometrics*, 32(1/2):122–136.

Pulley, B. (1981). A general mean-variance approximation to expected utility for short holding periods. *Journal of Financial and Quantitative Analysis*, Volume 16 / Issue 03:361–373.

Sandstrom, M. and Zulumovski, D. (2012). Simulation of finished goods inventory. next step towards factory simulation, packaging logistics. Technical report, Lund University. Sweden.

Schuster, A. (1987). On lunar and solar periodicities of earthquakes. *Proc. Roy. Soc*, 61:455–465.

Sheffrin, S. and Sullivan, A. (2002). *Economics: Principles in Action*.

Pearson Prentice Hall.

Shreve, S. (1997). *Stochastic Calculus and Finance*.

Stewart, B. and W.Dodgson (1879). Preliminary report to the committee on solar physics on a method of detecting the unknown inequalities of a series observation. *Proc. Roy.Soc*, 29:106–122.

Stewart, F. and Yermo, J. (2009). Pensions in africa. oecd working papers on insurance and private pensions,. *The Corporate Guardian (2009). Creating Pensions for Ghana's Informal Sector*, No. 30:1–34.

Stroke, G. (1879). Note on paper by stewart and dodgson. *Poc. Roy. Soc*, 29:122–123.

Thomson, W. (1876). On an instrument for calculating the integral of the product of two given functions. easuring risk aversion: an alternative approach mit press. *Proc. Roy. Soc*, 24:266–268.

Tukey, J. (1949-1950). The sampling theory of power spectrum estimates. 22.

Vanderhei, J., E.J., A., Mehome, J., and J.S, R. (1992). *Pension Planning Pension, Profit Sharing and other Deferred Compensation Plans 7th edition*. Homewood, III. Richard D. Irwin.

Vives, A. (1999). *Pension Funds in Infrastructure Project Finance*.

Von Neumann, J. and Morgenstern, O. (1947). The theory of games and economic behavior.

Ward, A. (2011). Danish pensions invest in giant offshore wind farm.

APPENDIX

5.6 APPENDIX A: Simulation study (Tables of Optimal Trading Strategies)

Table A1.a: Lognormal, Lognormal and Normal Distribution

level of risk = -0.5				
	N=500	N=1000	N=2000	N=10000
stock 1	56.73	57.69	57.59	57.43
stock 2	36.84	35.83	35.93	36.07
stock 3	6.44	6.48	6.48	6.5
level of risk = -5				
	N=500	N=1000	N=2000	N=10000
stock 1	55.91	57.93	57.33	57.34
stock 2	37.38	35.6	36.18	36.16
stock 3	6.71	6.47	6.5	6.5
level of risk = -20				
	N=500	N=1000	N=2000	N=10000
stock 1	56.44	56.75	57.51	57.54
stock 2	36.82	36.68	35.98	35.96
stock 3	6.74	6.57	6.51	6.51

Table A1.b: Lognormal, Lognormal and Exponential Distribution

level of risk = -0.5				
	N=500	N=1000	N=2000	N=10000
stock 1	65.45	67	66.07	65.84
stock 2	34.12	32.53	33.5	33.73
stock 3	0.43	0.46	0.44	0.43
level of risk = -5				
	N=500	N=1000	N=2000	N=10000
stock 1	65.59	66.25	65.37	65.57
stock 2	33.98	33.31	34.21	34
stock 3	0.43	0.45	0.42	0.43
level of risk = -20				
	N=500	N=1000	N=2000	N=10000
stock 1	67.05	65.3	65.45	65.67
stock 2	32.49	34.27	34.12	33.9
stock 3	0.46	0.42	0.42	0.43

Figure 5.1: A1: Tables of Uncorrelated stock returns prices with Fixed Interest Rate

Table A2.a: Lognormal, Normal and Normal Distribution

level of risk = -0.5	N=500	N=1000	N=2000	N=10000
stock 1	30.28	29.77	29.51	29.82
stock 2	32.81	32	32.59	32.74
stock 3	36.92	38.23	37.91	37.44

level of risk = -5	N=500	N=1000	N=2000	N=10000
stock 1	30.35	29.12	29.69	29.67
stock 2	33.28	32.29	33.72	32.86
stock 3	36.37	38.6	36.59	37.46

level of risk = -20	N=500	N=1000	N=2000	N=10000
stock 1	30.17	30.22	28.92	30.18
stock 2	32.21	31.49	32.75	32.57
stock 3	37.62	38.29	38.33	37.24

Table A2.b: Normal, Normal and Exponential Distribution

level of risk = -0.5	N=500	N=1000	N=2000	N=10000
stock 1	30.44	31.59	31.8	31.54
stock 2	48.39	47.77	47.55	48.39
stock 3	21.18	20.64	20.65	20.08

level of risk = -5	N=500	N=1000	N=2000	N=10000
stock 1	32.37	31.17	31.96	31.54
stock 2	47.43	48.94	46.97	48.28
stock 3	20.19	19.88	21.07	20.18

level of risk = -20	N=500	N=1000	N=2000	N=10000
stock 1	30.78	32.52	31.21	31.35
stock 2	50.55	47.58	48.31	48.39
stock 3	18.66	19.9	20.48	20.26

Figure 5.2: A2: Tables of Correlated stock returns prices with Fixed Interest Rate

Table A3.a: Normal, Normal and Exponential Distribution

level of risk = -0.5	N=500	N=1000	N=2000	N=10000
stock 1	36.1	37.38	38.51	38.1
stock 2	61.98	60.72	59.6	59.95
stock 3	1.92	1.9	1.89	1.95

level of risk = -5	N=500	N=1000	N=2000	N=10000
stock 1	37.74	38.35	38.87	38.64
stock 2	60.32	59.7	59.23	59.39
stock 3	1.94	1.94	1.89	1.97

level of risk = -20	N=500	N=1000	N=2000	N=10000
stock 1	37.62	39.14	38.55	37.82
stock 2	60.65	58.71	59.45	60.23
stock 3	1.73	2.15	2	1.95

Table A3.b: Normal, Normal and Lognormal Distribution

level of risk = -0.5	N=500	N=1000	N=2000	N=10000
stock 1	13.3	12.91	13.35	12.93
stock 2	32.36	31.79	31.61	31.61
stock 3	54.34	55.3	55.04	55.45

level of risk = -5	N=500	N=1000	N=2000	N=10000
stock 1	13.52	12.89	13.4	12.73
stock 2	31.84	32.19	31.23	31.61
stock 3	54.64	54.92	55.38	55.65

level of risk = -20	N=500	N=1000	N=2000	N=10000
stock 1	12.61	12.63	12.99	12.83
stock 2	30.26	31.99	31.76	30.95
stock 3	57.13	55.38	55.25	56.22

Figure 5.3: A3: Tables of Uncorrelated stock returns prices with Varying Interest Rate for all the stocks

Table A4.a: Lognormal, Normal and Lognormal Distribution

level of risk = -0.5				
	N=500	N=1000	N=2000	N=10000
stock 1	29.83	28.84	28.94	28.84
stock 2	27.09	28.36	28.21	28.62
stock 3	43.08	42.8	42.85	42.54
level of risk = -5				
	N=500	N=1000	N=2000	N=10000
stock 1	28.84	28.62	29.27	28.52
stock 2	28.62	29.24	27.87	29.03
stock 3	42.54	42.15	42.86	42.45
level of risk = -20				
	N=500	N=1000	N=2000	N=10000
stock 1	30.43	29.18	28.75	28.92
stock 2	27.4	29.83	28.87	28.73
stock 3	42.17	40.99	42.38	42.35

Table A4.b: Lognormal, Normal and Normal Distribution

level of risk = -0.5				
	N=500	N=1000	N=2000	N=10000
stock 1	27.32	28.24	28.63	28.7
stock 2	35.94	33.61	32.44	33.91
stock 3	36.74	38.15	38.93	37.39
level of risk = -5				
	N=500	N=1000	N=2000	N=10000
stock 1	29.74	28.74	28.3	28.78
stock 2	32.97	33.86	33.89	33.13
stock 3	37.29	37.4	37.81	38.09
level of risk = -20				
	N=500	N=1000	N=2000	N=10000
stock 1	27.12	29.17	28.61	28.7
stock 2	34.03	33.62	32.78	33.04
stock 3	38.85	37.21	38.61	38.27

Figure 5.4: A3: Tables of Uncorrelated stock returns prices with Varying Interest Rate for all the stocks

5.7 APPENDIX B: Market Data Fitting

5.7.1 APPENDIX B1: Distribution Fitting to Data

Table 5.1: B1.a: Fitted Parametric Distribution to the Gold

Distribution	Parameters
	λ=0.01184
	σ=15.525 μ=84.471 μ=4.4202

Table 5.2: B1.a: Fitted Parametric Distribution to the Oil

Distribution	Parameters
	λ=0.01719
	σ=9.3065 $\mu = 58.172$ μ = 4.0493

Table 5.3: B1.c: Fitted Parametric Distribution to the Bonds

Distribution	Parameters

	λ=0.00871
	σ=4.2233 μ = 114.83 μ = 4.7428

5.8 APPENDIX B2: Graphs of Goodness of Fit Test for all the three Distributions

5.8.1 GOF test of the Gold price for all the three distributions

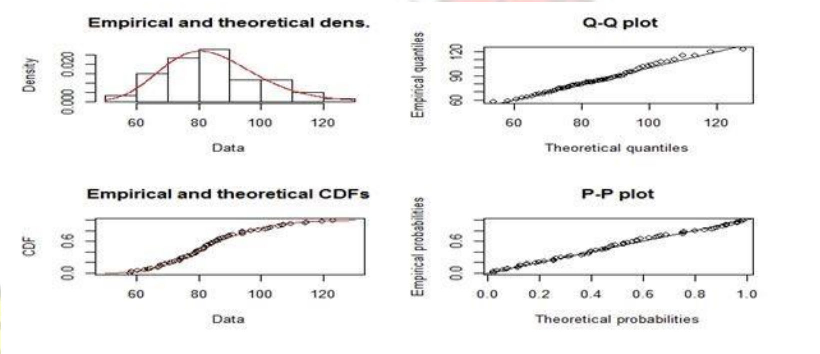


Figure 5.5: B2.a: Lognormal fitting of Gold price

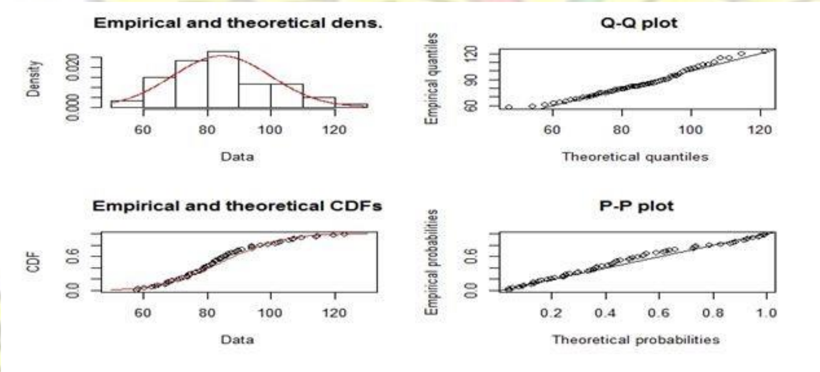


Figure 5.6: B2.b: Normal fitting of Gold price

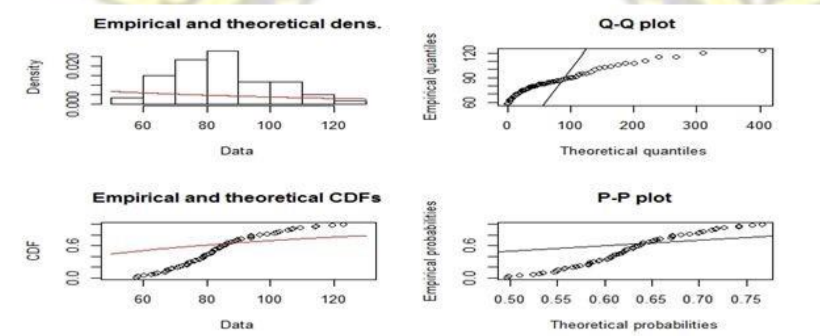


Figure 5.7: B2.c: Exponential fitting of Gold price

5.8.2 GOF test of the Oil price for all the three distributions

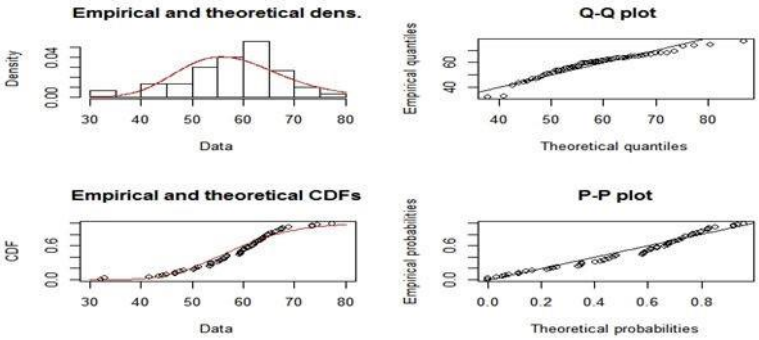


Figure 5.8: B2.a: Lognormal fitting of Oil price

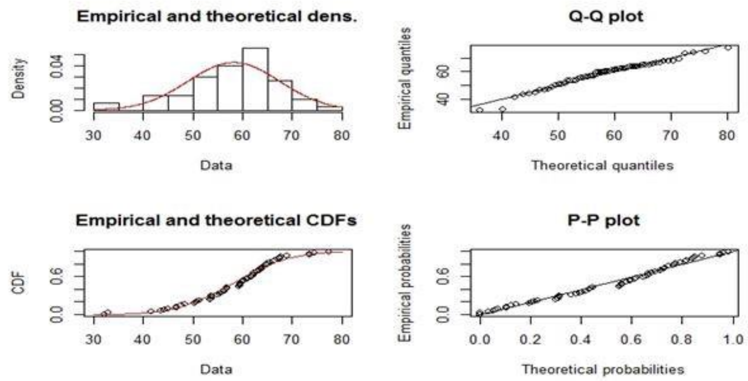


Figure 5.9: B2.b: Normal fitting of Oil price

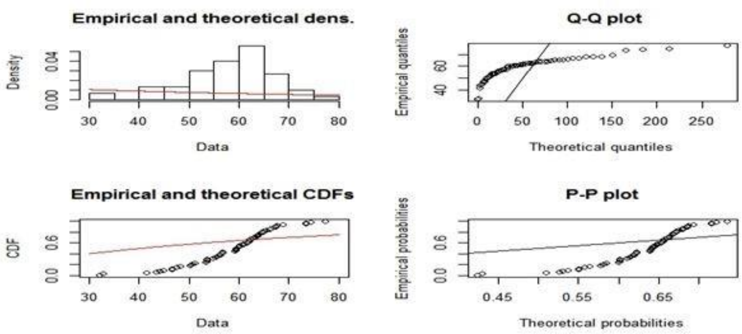


Figure 5.10: B2.c: Exponential fitting of Oil price

5.8.3 GOF test of the Bond price for all the three

distributions

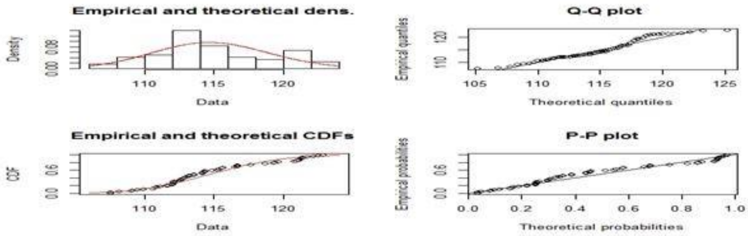


Figure 5.11: B2.a: Lognormal fitting of Bond price

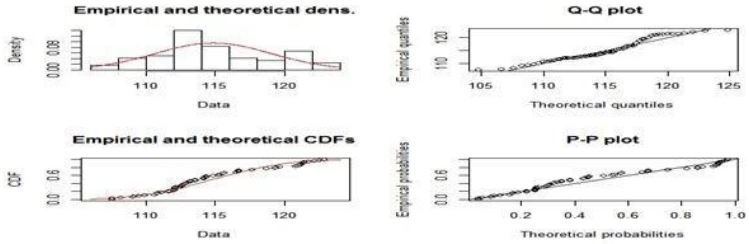


Figure 5.12: B2.b: Normal fitting of Bond price

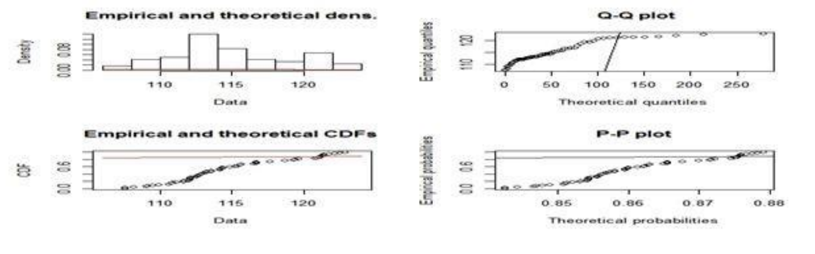


Figure 5.13: B2.c: Exponential fitting of Bond price

5.9 APPENDIX B3: Tabular Representation of GOF tests on all the distributions

5.9.1 B3.a: Tabular Representation of GOF tests on the Gold prices

Lognormal GOF Test on the Gold Prices

Kolmogorov-Smirnov					
Sample Size Statistic P-Value	60 0.06015 0.9726				
α	0.2	0.1	0.05	0.02	0.01
Critical Value	0.13573	0.15511	0.17231	0.19267	0.20673
Reject?	No	No	No	No	No
Anderson-Darling					
Sample Size Statistic	60 0.20242				
α	0.2	0.1	0.05	0.02	0.01
Critical Value	1.3749	1.9286	2.5018	3.2892	3.9074
Reject?	No	No	No	No	No

Figure 5.14: Log-normal GOF Test on the Gold Prices

Exponential GOF Test on the Gold Prices

Kolmogorov-Smirnov					
Sample Size Statistic P-Value Rank	60 0.49798 8.8921E-14 3				
α	0.2	0.1	0.05	0.02	0.01
Critical Value	0.13573	0.15511	0.17231	0.19267	0.20673
Reject?	Yes	Yes	Yes	Yes	Yes
Anderson-Darling					
Sample Size Statistic Rank	60 18.715 3				
α	0.2	0.1	0.05	0.02	0.01
Critical Value	1.3749	1.9286	2.5018	3.2892	3.9074
Reject?	Yes	Yes	Yes	Yes	Yes

Figure 5.15: Exponential GOF Test on the Gold Prices

Normal GOF Test on the Gold Prices

Kolmogorov-Smirnov					
Sample Size Statistic P-Value	60 0.09623 0.60077				
α	0.2	0.1	0.05	0.02	0.01
Critical Value	0.13573	0.15511	0.17231	0.19267	0.20673
Reject?	No	No	No	No	No
Anderson-Darling					
Sample Size Statistic	60 0.57012				
α	0.2	0.1	0.05	0.02	0.01
Critical Value	1.3749	1.9286	2.5018	3.2892	3.9074
Reject?	No	No	No	No	No

Figure 5.16: Normal GOF Test on the Oil Prices

5.9.2 B3.a: Tabular Representation of GOF tests on the Oil

prices

LOGNORMAL GOF TEST ON THE OIL PRICES					
Kolmogorov-Smirnov					
Sample Size Statistic P-Value Rank	60 0.14539 0.1432 2				
α	0.2	0.1	0.05	0.02	0.01
Critical Value	0.13573	0.15511	0.17231	0.19267	0.20673
Reject?	Yes	No	No	No	No
Anderson-Darling					
Sample Size Statistic Rank	60 1.3194 2				
α	0.2	0.1	0.05	0.02	0.01
Critical Value	1.3749	1.9286	2.5018	3.2892	3.9074
Reject?	No	No	No	No	No

Figure 5.17: Log-normal GOF Test on the Gold Prices

EXPONENTIAL GOF TEST ON THE OIL PRICES					
Kolmogorov-Smirnov					
Sample Size Statistic P-Value Rank	60 0.47728 1.0861E-12 3				
α	0.2	0.1	0.05	0.02	0.01
Critical Value	0.13573	0.15511	0.17231	0.19267	0.20673
Reject?	Yes	Yes	Yes	Yes	Yes
Anderson-Darling					
Sample Size Statistic Rank	60 19.729 3				
α	0.2	0.1	0.05	0.02	0.01
Critical Value	1.3749	1.9286	2.5018	3.2892	3.9074
Reject?	Yes	Yes	Yes	Yes	Yes

Figure 5.18: Exponential GOF Test on the Oil Prices

NORMAL GOF TEST ON THE OIL PRICES					
Kolmogorov-Smirnov					
Sample Size Statistic P-Value	60 0.11787 0.34753				
α	0.2	0.1	0.05	0.02	0.01
Critical Value	0.13573	0.15511	0.17231	0.19267	0.20673
Reject?	No	No	No	No	No
Anderson-Darling					
Sample Size Statistic	60 0.60112				
α	0.2	0.1	0.05	0.02	0.01
Critical Value	1.3749	1.9286	2.5018	3.2892	3.9074
Reject?	No	No	No	No	No

Figure 5.19: Normal GOF Test on the Oil Prices

5.9.3 B3.a: Tabular Representation of GOF tests on the Bond prices

LOGNORMAL GOF TEST ON THE BOND PRICES					
Kolmogorov-Smirnov					
Sample Size Statistic P-Value	60 0.12156 0.3122				
α	0.2	0.1	0.05	0.02	0.01
Critical Value	0.13573	0.15511	0.17231	0.19267	0.20673
Reject?	No	No	No	No	No
Anderson-Darling					
Sample Size Statistic	60 1.1891				
α	0.2	0.1	0.05	0.02	0.01
Critical Value	1.3749	1.9286	2.5018	3.2892	3.9074
Reject?	No	No	No	No	No

Figure 5.20: Log-normal GOF Test on the Bond Prices

EXPONENTIAL GOF TEST ON THE BOND PRICES					
Kolmogorov-Smirnov					
Sample Size Statistic P-Value Rank	60 0.60793 0 3				
α	0.2	0.1	0.05	0.02	0.01
Critical Value	0.13573	0.15511	0.17231	0.19267	0.20673
Reject?	Yes	Yes	Yes	Yes	Yes
Anderson-Darling					
Sample Size Statistic Rank	60 25.608 3				
α	0.2	0.1	0.05	0.02	0.01
Critical Value	1.3749	1.9286	2.5018	3.2892	3.9074
Reject?	Yes	Yes	Yes	Yes	Yes

Figure 5.21: Exponential GOF Test on the Bond Prices

NORMAL GOF TEST ON THE BOND PRICES					
Kolmogorov-Smirnov					
Sample Size Statistic P-Value Rank	60 0.12805 0.2562 2				
α	0.2	0.1	0.05	0.02	0.01
Critical Value	0.13573	0.15511	0.17231	0.19267	0.20673
Reject?	No	No	No	No	No
Anderson-Darling					
Sample Size Statistic Rank	60 1.2796 2				
α	0.2	0.1	0.05	0.02	0.01
Critical Value	1.3749	1.9286	2.5018	3.2892	3.9074
Reject?	No	No	No	No	No

Figure 5.22: Normal GOF Test on the Bond Prices

5.10 APPENDIX C: R code to determining the trading strategy for an investor based on formula (3.61)

5.10.1 APPENDIX C1: This chunk of codes is to help generate correlated combination of the distributions CHOLESKY DECOMPOSITION OF STOCK PRICES

##This chunk of codes is to help generate corrlated combination of the distributions CHOLESKY DECOMPOSITION

OF STOCK PRICES

#N is the number of observations to be worked with

#n is the number of stocks to be understudied

#sigma is the monthly volatility of the historical stock prices

#mu is the expected return of the historical stock prices

#cs is the covariance structure

#m is the correlation matix

#z is the distribution of the simulation #x becomes

the simulated stock prices

```
corrcomb <- function(N,n,mu,sigma,compsym){ z=matrix(NA,nrow=n,ncol=N)
```

```
for(i in 1:n){ if(i == 1){ z[i,] =
100*rlnorm(N, mu, sigma) }else if(i ==
2){ z[i,]=100*rnorm(N, mu, sigma)
}else z[i,]=100*rexp(N, mu)
   } compsym <- function(n){ m<-
matrix(c(1.0,0.5,0.5,0.5,1.0,0.5,0.5,0.5,1), nrow=n) cs =
0.01983*m cs
   } c=chol(compsym(n))
   c z x=t(c)%*%z x t(x)
}

##TRY
sp=corrcomb(60,3,0.562457,0.2345953) colnames(sp) <-
c("stock 1", "stock 2", "stock 3") sp

#we compute the returns, mean(muR) and the volality matrix(volmat)

returns=diff(log(sp))
muR=colMeans(returns, na.rm =TRUE)
muR volmat=var(returns, na.rm = TRUE)
volmat cor(returns)

#Here we find the interest rate R(t) using the fourier transform
# k is the number of observations of the historical data
```

```
# t is the number of oscilliations from the historical data
# w is the angular frequency measured in radians
# f is the frequency
# A and B are the fourier coefficients(constants)
# ubar is the mean of the historical data
# Rt is the interest rate
fourR<- function(k,n,t)
{
f <- k/t # where k is the number of rate length w <- (2*pi)/f
Rates<-read.csv("C:/Users/APPLE/SkyDrive/Documents/ERICUS
PROJECT/Rates.csv")
Rates
Rates$Cos <- with(Rates, Rates$Rate*cos(w*Rates$Time)) Rates$Sin <-
with(Rates, Rates$Rate*sin(w*Rates$Time))
Ubar <- (1/k)*sum(Rates$Rate)
A  <- (2/k)*sum(Rates$Cos)
B  <- (2/k)*sum(Rates$Sin)
# n = NROW(Rates$Time)
R= rep(NA,n) for (i in 1:n){
R[i] <- Ubar + A*cos(w*Rates$Time[i]) + B*sin(w*Rates$Time[i])
   } return(R[i])
}
Rt <- fourR(55,3,10)
```

```
Rt

# Trading Strategy Function for an Investor
# n is the number of stocks/ securities we want to invest in
# W is the amount we want to distribute in the stocks
# l is the risk aversion parameter of the investor
# mu is the drift
# volmat is the volatility matrix
# Rt is the interest rate from Fourier series

TradStra <- function(W, n, l)
{ beta <- matrix(runif(n*n, 0, 1), nrow = n) alpha <- diag(runif(n, 0, 1))
delta <- matrix(runif(n, 0, 1), nrow = n) mu=muR+alpha*(delta-
muR)+beta*matrix(rnorm(n,mean=0,sd=1), nrow=n)
TS=(1/(1-l))*solve(volmat%*%t(volmat))%*%(mu-Rt*matrix(rep(1,n)
,nrow=n)) prop =
1/sum(TS)
    TS=prop*TS
    TS=TS*W
    sum = 0 for(i in
    1:n)
    { value = TS[i];
        if(value > 0)
```

```
        {
            sum = sum + value
        }
    } if(sum <= W)
    { return(TS)
    }else
    { return(matrix(c(mat.or.vec(n,1))))
    }
}

#simulated Trading Strategy for larger numbers in percenatges form

simulatedTS <- function(T,W,n,l){

    totalRequired <- T; counter <- 1;

    PI = matrix(data = NA, nrow = n, ncol = totalRequired)

    while(counter <= totalRequired){
        strategy = TradStra(W, n, l) zeroVector =
        matrix(c(mat.or.vec(n,1))) if(all(strategy == zeroVector)){
        # Bad strategy next
        }else{ # Good strategy PI[,counter] =
            strategy counter <- counter + 1
```

```
        }
    }
    PI <- matrix(c(rowMeans(PI)), nrow = n, ncol = 1) total <-
    sum(PI) percPI <- ((PI/total)*100) PI <- round(percPI,2)
    plot(PI,type="o",col="blue") return(PI)
}

### TRY THIS

simulatedTS(500,1000,3,l=-0.5) simulatedTS(1000,1000,3,l=-0.5)
simulatedTS(2000,1000,3,l=-0.5) simulatedTS(10000,1000,3,l=-0.5)
simulatedTS(500,1000,3,l=-5) simulatedTS(1000,1000,3,l=-5)
simulatedTS(2000,1000,3,l=-5) simulatedTS(10000,1000,3,l=-5)

simulatedTS(500,1000,3,l=-20) simulatedTS(1000,1000,3,l=-20)
simulatedTS(2000,1000,3,l=-20) simulatedTS(10000,1000,3,l=-20)
```

5.10.2 APPENDIX C2: This chunk of codes is to help generate independent CHOLESKY DECOMPOSITION OF STOCK PRICES

```
# this chunk of codes is to help generate independent
CHOLESKY DECOMPOSITION OF STOCK PRICES
# N is the number of observations to be worked with
```

```
# n is the number of stocks to be understudied
# sigma is the monthly volatility of the historical stock prices
# mu is the expected return of the historical stock prices
# cs is the covariance structure
# m is the correlation matix
# z is the distribution of the simulation
# x becomes the simulated stock prices
uncorrcomb <- function(N,n,mu,sigma,identitycs){ z =
   matrix(NA,nrow=n,ncol=N) for(i in 1:n){ if(i == 1){ z[i,] =
   100*rlnorm(N, mu, sigma)
      }else if(i == 2){ z[i,] = 100*rnorm(N, mu,
         sigma)
      }else z[i,] = 100*rlnorm(N, mu, sigma)
   } identitycs <- function(n){ m <-
   diag(n) cs = 0.01983*m cs
   } c=chol(identitycs(n)) c z
   x=t(c)%*%z x t(x)
}

##TRY
spx=uncorrcomb(60,3,0.56246,0.23460) colnames(spx) <-
c("stock 1", "stock 2", "stock 3")
#spx
```

we compute the returns, mean(muR) and the volality matrix(volmat)

```
returns=diff(log(spx))
muR=colMeans(returns, na.rm =TRUE)
muR volmat=var(returns, na.rm = TRUE)
volmat cor(returns)
#Here we find the interest rate R(t) using the fourier transform

# k is the number of observations of the historical data
# t is the number of oscilliations from the historical data
# w is the angular frequency measured in radians
# f is the frequency
# A and B are the fourier coefficients(constants)
# ubar is the mean of the historical data
# Rt is the interest rate
fourR<- function(k,n,t)
{ f<-k/t #where k is the number of rate length, t is the
number of peaks w <- (2*pi)/f
Rates<-read.csv("C:/Users/APPLE/SkyDrive/Documents/ERICUS
PROJECT/Rates.csv")
Rates
Rates$Cos <- with(Rates, Rates$Rate*cos(w*Rates$Time)) Rates$Sin <-
with(Rates, Rates$Rate*sin(w*Rates$Time))
```

```
Ubar <- (1/k)*sum(Rates$Rate)

A  <- (2/k)*sum(Rates$Cos)

B  <- (2/k)*sum(Rates$Sin)

# n = NROW(Rates$Time)

R= rep(NA,n) for (i in 1:n){

R[i]<-Ubar+A*cos(w*Rates$Time[i])+B*sin(w*Rates$Time[i])

    } return(R)

}

Rt <- fourR(55,3,10)

Rt

# Trading Strategy Function for an Investor

# n is the number of stocks/ securities we want to invest in

# W is the amount we want to distribute in the stocks

# l is the risk parameter of the investor

# mu is the drift

# volmat is the volatility matrix

# Rt is the interest rate from Fourier series


TradStra <- function(W, n, l)

{ beta <- matrix(runif(n*n, 0, 1), nrow = n) alpha <- diag(runif(n, 0, 1)) delta

<- matrix(runif(n, 0, 1), nrow = n) mu=muR+alpha%*%(delta-

muR)+beta%*%matrix(rnorm(n,mean=0,sd=1)

,nrow= n)
```

```
TS=(1/(1-l))*solve(volmat%*%t(volmat))%*%(mu-Rt*matrix(rep(1, n)
,nrow = n)) prop =
    1/sum(TS)
    TS=prop*TS
    TS=TS*W
    sum = 0 for(i in
    1:n)
    { value = TS[i];
        if(value > 0)
        {
            sum = sum + value
        }
    } if(sum <= W)
    { return(TS)
    }else
    { return(matrix(c(mat.or.vec(n,1))))
    }
}
```

##simulated Trading Strategy for larger numbers in percenatges form

```
simulatedTS <- function(T,W,n,l){
```

```
totalRequired <- T; counter <- 1;

PI = matrix(data = NA, nrow = n, ncol = totalRequired) while(counter <=
totalRequired){ strategy = TradStra(W, n, l) zeroVector =
matrix(c(mat.or.vec(n,1))) if(all(strategy == zeroVector)){ # Bad strategy
next
  }else{ # Good strategy PI[,counter] =
    strategy counter <- counter + 1
  }
}
PI <- matrix(c(rowMeans(PI)), nrow = n, ncol = 1) total <-
sum(PI) percPI <- ((PI/total)*100) PI <- round(percPI,2)
plot(PI,type="o",col="blue") return(PI)
}

### TRY THIS

simulatedTS(500,1000,3,l=-0.5) simulatedTS(1000,1000,3,l=-0.5)
simulatedTS(2000,1000,3,l=-0.5) simulatedTS(10000,1000,3,l=-0.5)
simulatedTS(500,1000,3,l=-5) simulatedTS(1000,1000,3,l=-5)
simulatedTS(2000,1000,3,l=-5) simulatedTS(10000,1000,3,l=-5)

simulatedTS(500,1000,3,l=-20) simulatedTS(1000,1000,3,l=-20)
simulatedTS(2000,1000,3,l=-20) simulatedTS(10000,1000,3,l=-20)
```

5.10.3 APPENDIX C3: This chunk of codes is to help model a real STOCK PRICES FROM YAHOO FINANCE

```
# this chunck of codes is to help model a real STOCK PRICES FROM
YAHOO FINANCE

PRICES<-read.csv("C:/Users/APPLE/SkyDrive/Documents/ERICUS
PROJECT/PRICES.csv")
PRICES

# Fitting of the distribution by maximum likelihood parameters
#PRICES$GOLD
s1 <- fitdist(PRICES$GOLD,"lnorm") s1
plotdist(PRICES$GOLD,"lnorm", para=list(meanlog=s1
$estimate[1],sdlog=s1$estimate[2]))

s2 <- fitdist(PRICES$OIL.MUR.,"norm") s2
plotdist(PRICES$OIL.MUR.,"norm", para=list(mean=s2
$estimate[1], sd=s2$estimate[2]))

s3 <- fitdist(PRICES$BONDS.TIP.,"lnorm") s3
plotdist(PRICES$BONDS.TIP.,"lnorm", para=list(meanlog=s3
$estimate[1],sdlog=s3$estimate[2]))
```

we compute the returns, mean(muR) and the volality matrix(volmat)

returns = matrix(c(diff(log(PRICES$GOLD)),diff(log(PRICES

$OIL.MUR.)),diff(log(PRICES$BONDS.TIP.))), ncol = 3) returns muR =

colMeans(returns, na.rm = TRUE)

muR volmat = var(returns, na.rm = TRUE)

volmat corr = cor(returns) corr

#Here we find the interest rate R(t) using the fourier transform

k is the number of observations of the historical data # t is the

number of oscilliations from the historical data

w is the angular frequency measured in radians

f is the frequency

A and B are the fourier coefficients(constants)

ubar is the mean of the historical data

Rt is the interest rate

fourR<- function(k,n,t)

{

f<-k/t # where k is the number of rate length, t is the number of peaks w <-

(2*pi)/f Rates<-read.csv("C:/Users/APPLE/SkyDrive/Documents/ERICUS

```
PROJECT/Rates.csv")
Rates
Rates$Cos<-with(Rates, Rates$Rate*cos(w*Rates$Time)) Rates$Sin<-
with(Rates, Rates$Rate*sin(w*Rates$Time))
Ubar<-(1/k)*sum(Rates$Rate)
A<-(2/k)*sum(Rates$Cos)
B <- (2/k)*sum(Rates$Sin)
   # n = NROW(Rates$Time)
   R= rep(NA,n) for (i in 1:n){
R[i] <- Ubar + A*cos(w*Rates$Time[i]) + B*sin(w*Rates$Time[i])
   } return(R[i])
}
Rt <- fourR(55,3,10)
Rt


# Trading Strategy Function


# n is the number of stocks/ securities we want to invest in
# W is the amount we want to distribute in the stocks
# l is the risk parameter of the investor
# mu is the drift
# volmat is the volatility matrix
# Rt is the interest rate from Fourier series
```

```
TradStra <- function(W, n, l)
{ beta<-matrix(runif(n*n, 0, 1), nrow = n) alpha<-diag(runif(n, 0, 1)) delta<-
matrix(runif(n, 0, 1), nrow = n) mu=muR+alpha%*%(delta-
muR)+beta%*%matrix(rnorm(n,mean=0,sd=1)
,nrow= n)
TS=(1/(1-l))*solve(volmat%*%t(volmat))%*%(mu-Rt*matrix(rep(1,n)
,nrow=n)) prop =
    1/sum(TS)
    TS = prop*TS TS
    = TS*W sum = 0
    for(i in 1:n)
    { value = TS[i];
        if(value > 0)
        {
            sum = sum + value
        }
    } if(sum <= W)
    { return(TS)
    }else
    { return(matrix(c(mat.or.vec(n,1))))
    }
}
```

```
##simulated Trading Strategy for larger numbers in percenatges form

simulatedTS <- function(T,W,n,l){

    totalRequired <- T; counter <- 1;

PI=matrix(data = NA, nrow = n, ncol = totalRequired)

    while(counter <= totalRequired){ strategy = TradStra(W, n, l)
        zeroVector = matrix(c(mat.or.vec(n,1))) if(all(strategy ==
        zeroVector)){ # Bad strategy next
        }else{ # Good strategy PI[,counter] =
            strategy counter <- counter + 1
        }
    }
PI<-matrix(c(rowMeans(PI)),nrow=n,ncol=1) total
   <- sum(PI) percPI <- ((PI/total)*100) PI <-
   round(percPI,2) plot(PI,type="o",col="blue")
   return(PI)
}

simulatedTS(T=1, W=1000, n=3, l=-0.5) simulatedTS(T=1,
W=1000, n=3,l=-5) simulatedTS(T=1, W=1000, n=3,l=-20)
```