KWAME NKRUMAH UNIVERSITY OF SCIENCE AND TECHNOLOGY

INSTITUTE OF DISTANCE LEARNING

LOCATION OF AN AIR AMBULANCE RESPONSE UNIT IN THE BRONG AHAFO REGION OF GHANA USING THE PLANAR K-CENTRA SINGLE-FACILITY EUCLIDEAN LOCATION ALGORITHM

BY

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CERTIFICATION

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DECLARATION

I hereby declare that this submission is my own work towards the Master of Science and that, to the best of my knowledge, it contains no material previously published by another person nor material which has been accepted for the award of any other degree of the university, except where due acknowledgement has been made in the text.

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DEDICATION

I dedicate this thesis to my wife Charlotte Koranteng and my daughter Maria-GorettiFosuaaAcheampong.

ACKNOWLEDGEMENT

I wonder if I can sufficiently acknowledge the enormous debt of gratitude which I owe in connection with the writing of this thesis. This thesis brought me into contact with several people many of whom are lectures of Kwame Nkrumah University of Science and Technology.

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ABSTRACT

The Planar k-Centra Single-Facility Euclidean Location Algorithm was used to strategically locate an Air Ambulance Response Facility in the BrongAhafo Region of Ghana which will be closest to the k-6 farthest districts. Sixteen(16) rectangular co-ordinates were generated for 16 district capitals as the inputs for the algorithm. Matlab codes were written and used to run the algorithm. The algorithm generated (397.3km, 929.2km) as the plane Cartesian rectangular coordinate of the optimal point with 273.8km as its objective function value which is given by sum of the distances of 6 farthest locations away from the optimal location. The proposed community for the Air Ambulance Response Facility was found to coincide with Tadieso, a village which is about nine (9) kilometres away from Techiman in the BrongAhafo Region of Ghana.

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CHAPTER ONE

INTRODUCTION

1.1 Background to theStudy

1.1.1 Ambulance

An ambulance is a vehicle for transportation of sick or injured people to, from or between places of treatment for an illness or injury and in some instances will also provide out of hospital medical care to the patient. The word is often associated with road going emergency ambulances which perform part of an emergency medical service, administering emergency care to those with acute medical problems (Wikipedia, 17/09/2011).

The term ambulance does however extend to a wide range of vehicles other than those with flashing warning lights and sirens, including a large number of non-urgent ambulances which are for transport of patients without an urgent acute condition and a wide range of vehicles including trucks, vans, bicycles, motorbikes, station wagons, buses, helicopters, fixed-wing aircraft, boats and even hospital ships.

The term ambulance comes from the Latin word 'ambmulare', meaning to walk or move about which is a reference to early medical care where patients were moved by lifting or wheeling. The word originally meant a moving hospital, which follows an army in its movements. During the American Civil War vehicles for conveying the wounded off the field of battle were called ambulance wagons. Field hospitals were still called ambulances during the Franco-Prussian War of 1870 and in the Serbo-

Turkish war of 1876 even though the wagons were first referred to as ambulances about 1854 during the Crimean War. (Wikipedia, 17/09/2011).

There are other types of ambulance, with the most common being the patient transport ambulance. These vehicles are not usually (although there are exceptions) equipped with life-support equipment, and are usually crewed by staff with fewer qualifications than the crew of emergency ambulances. Their purpose is simply to transport patients to, from or between places of treatment. In most countries, these are not equipped with flashing lights or sirens. In some jurisdictions there is a modified form of the ambulance used, that only carries one member of ambulance crew to the scene to provide care, but is not used to transport the patient. Such vehicles are called fly-cars. In these cases a patient who requires transportation to hospital will require a patient-carrying ambulance to attend in addition to the fast responder.

1.1.2 The History of Ambulance

The history of the ambulance begins in ancient times, with the use of carts to transport incurable patients by force. Ambulance was first used for emergency transport in 1487 by the Spanish, and civilian vanants were put into operation during 1830s. Advance in technology throughout the 19th and 20th centuries led to the modern self-powered ambulances.

1.1.3 Types of Ambulance

Ambulances can be grouped into types depending on whether or not they transport patients, and under what conditions. In some cases, ambulances may fulfill more than one function (such as combing emergency ambulance care with patient transport). The functional types of ambulance are: emergency ambulance, patient transport ambulance, response unit and charity ambulance. Other types of ambulance are motorcycle ambulance, water ambulance, ambulance bus and air ambulance on fleet but doing little work.

1.1.4 Air Ambulance

The study will mainly focus on an air ambulance. An air ambulance is an aircraft used for emergency medical assistance in situations where either a traditional ambulance cannot reach the scene easily or quickly enough, or the patient needs to be transported over a distance or terrain that makes air transportation the most practical transport. These and related operations are referred to as Aero medical. Air ambulance crews are supplied with equipment that enables them to provide medical treatment to a critically injured or ill patient. Common equipment for air ambulances includes Ventilators, medication, monitoring unit, CPR equipment and structures. An important distinction should be made between a medically staffed and equipped air ambulance which can provide medical care in flight.

As with many innovations in Emergency Medical Service (EMS), the concept of transporting the injured by aircraft has its origin in the military, and the concept of using aircraft as ambulances is almost as old as powered flight itself. It is often stated that air medical transport likely first occurred in 1870 during the siege of Paris when 160 wounded French Soldiers were transported by hot-air balloon to France but this canard has been definitively disproven. During the First World War air ambulances were tested by various military organizations, and were used regularly for crash rescue by the American Army and Navy within the United States, though none were actually used in combat. Aircrafts were still primitive at the time, with limited capabilities and the effort received mixed reviews. In post-World War 1, several Dehavilland DH-4 were converted to air ambulances for the US Army and Navy.

The exploration of the idea continued, however, and fully organized air ambulance services were used by France and the United Kingdom during the African and Middle Eastern Colonial Wars of the 1920s; over 7,000 casualties were evacuated by the French during this period. By 1936, an organized military air ambulance service was evacuating wounded from the Spanish Civil War for medical treatment in Nazi Germany. The first use of helicopters to evacuate combat casualties was by the United States Army in Burma during World War 2, and the first dedicated use of helicopters by U.S forces occurred during the Korean War, during the period from 1950-1953.

The first Civilian uses of aircraft as ambulances were probably incidental. In Northern Canada, Australia, and in the Scandinavian countries, remote, sparsely populated settlements were often in accessible by road for months at a time, or even year round.

Air ambulances were useful in remote areas, but their usefulness in the developed world was still uncertain. Following the end of the World War II, the first civilian air ambulance in North America was established by the Saskatchewan government in Regina, Saskatchewan, Canada, which had both remote communities and great distances to consider in the provision of health care to its citizens. The Saskatchewan Air Ambulance Service continues to be active as of 2011 (Wikipedia, 17/09/2011).

1.1.5 Facility Location – An Overview

Since the thesis deals with the new facility location, the major elements considered are type of location problems, the nature of the solution space, distance measures and the objective function.

1.1.5.1 Type of Location Problem

A facility location problem can be classified according to the number of facilities involved. As shown in Figure 1.1, a new facility problem could be single or a multiple facility problem.

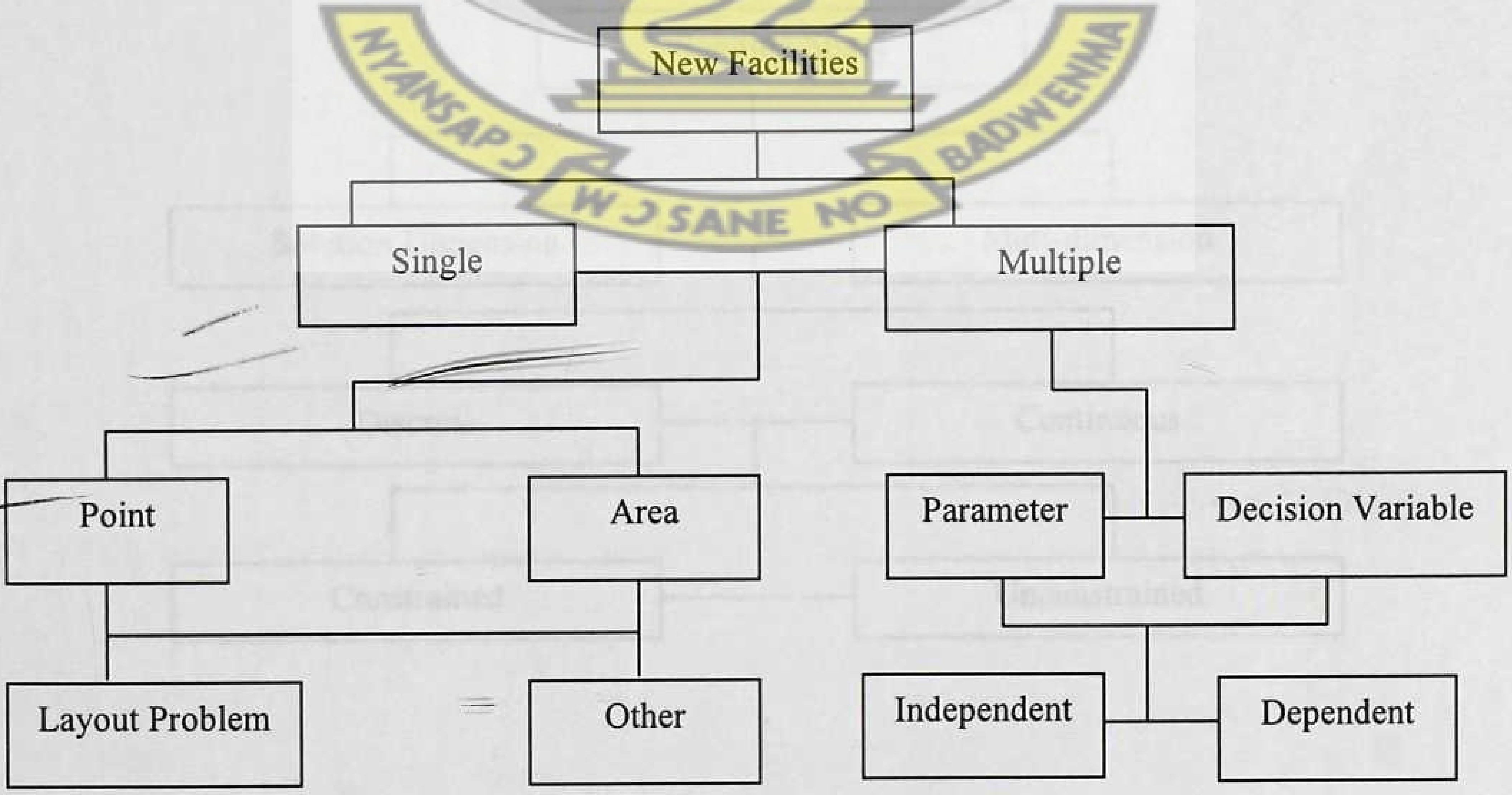


Figure 1.1: Type of Location Problem

A single facility location can be locating a single point or an enclosed area. Multiple facility location problems can be problems defined by certain parameters like length or certain decision variables. The obtained solution can be dependant or independent of other parameters or decision variables. For examples, the new located point could be dependent or independent of the existing customer's location.

1.1.5.2 Nature of Solution Space

The other category of classification would be the solution space for the location problem. As shown in Figure 1.2, some spaces can be one-dimensional such as locating points on a line while others are multidimensional as in locating points in 2-D or 3-D spaces. The solution subspace can be discrete (finite number of possible location) or continuous (infinite number of possible locations). Also, the solution space can be constraint bounded or unconstrained.

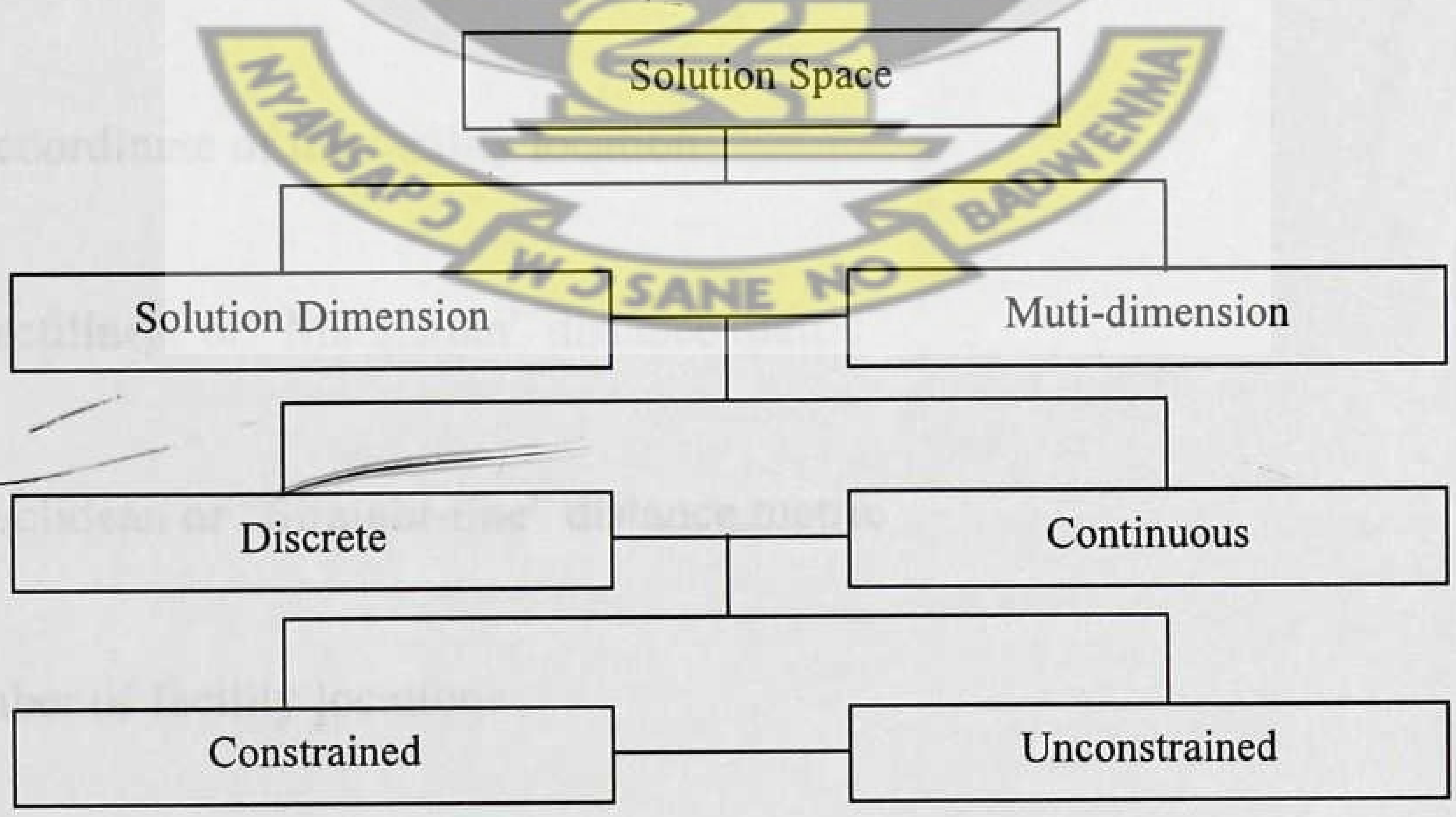


Figure 1.2: Nature of Solution Space

In continuous space, there is no restriction on the locationof the new facility. The new facility could be located anywhere inside the convex hull. In the case of discrete space problems, the new facility has to be located among one of the discrete points.

1.1.5.3 Distance Metric

Another form of classification is the distance metric selected which could be rectilinear, Euclidean or some other form of defined metrics (Figure 1.3). The most common distance metric is the l_p distance metric, where l_p is given by Equation 1.

$$l_p = \left(\sum_{i=1}^{n} |x_i - y_i|^p\right)^{1/p} \quad \text{.........................(1.1)}$$

where,

xi = x – coordinate of the facility location

yi = y – coordinate of the facility location

p = 1, Rectilinear or "Manhattan" distance metric

p = 2, Euclidean or "Straight-line" distance metric

n = Number of facility locations

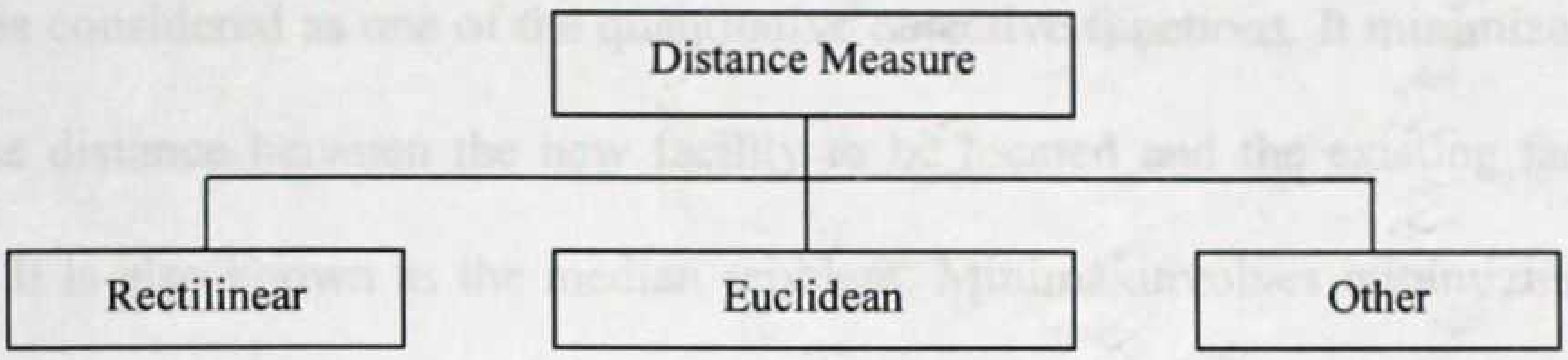


Figure 1.3: Classification of Distance Metric

Other distance metrics include the weight 1 – α norm, round norm, mixed norms, Jaccard, expected, central and mixed gauges.

1.1.5.4 Type of Objective Function

The final category is the classification of objective functions. As shown in figure1.4, a problem can be qualitative (generally facility layout problem) or quantitative (facility location problem). Quantitative objectives could be minimization, maximization or some other function depending on the formulated problem.

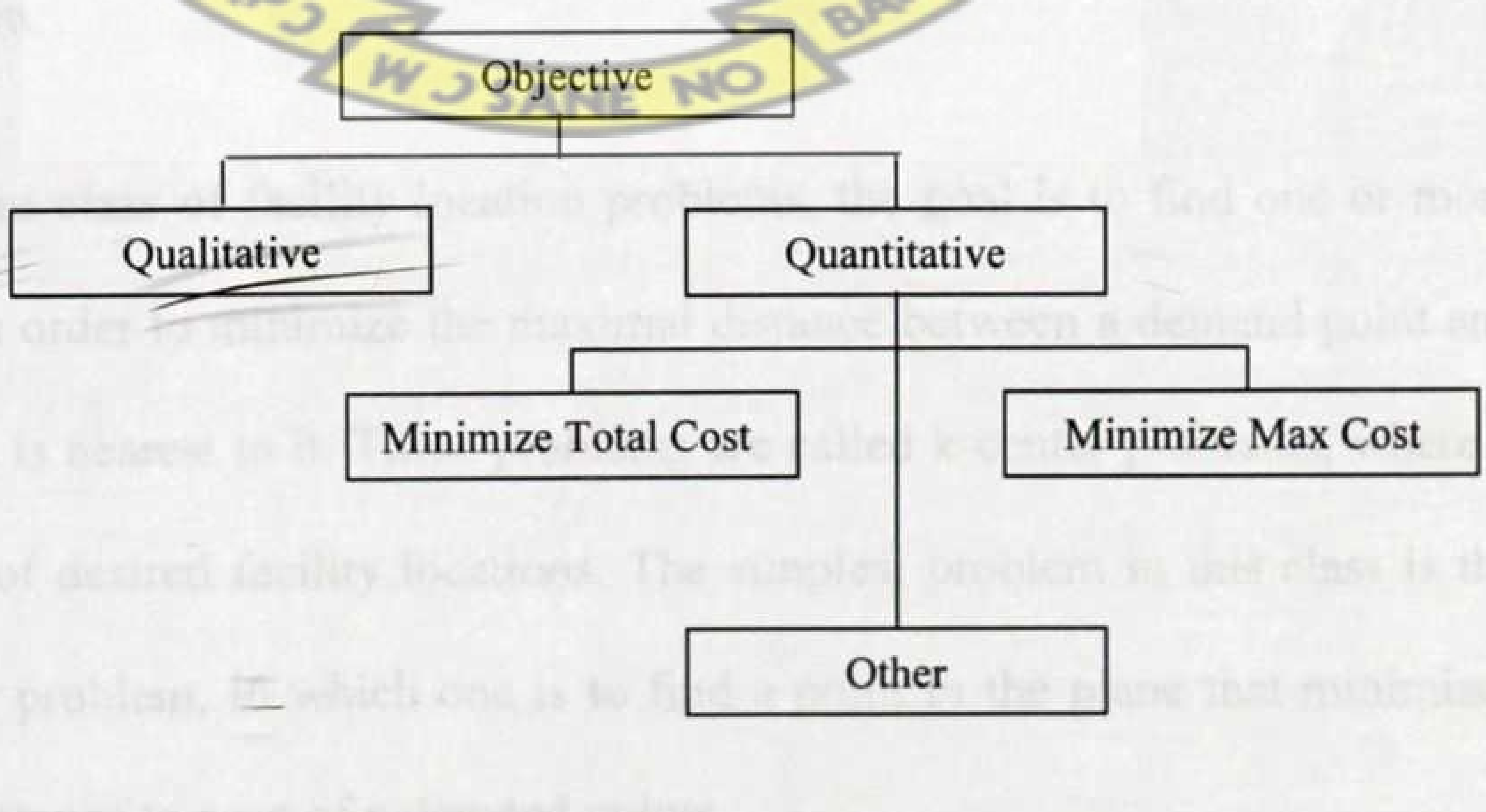


Figure 1.4: Type of Objective Function Involved

Minisum is considered as one of the quantitative objective functions. It minimizes the sum of the distance between the new facility to be located and the existing facility locations. It is also known as the median problem. Minimaxinvolves minimizing the maximum distance between the existing facility location and the new facility to be located. It is also known as the center problem.

Maxisum is maximizing the sum of the distances between the new facility to be located and the existing facility locations. Maximin is maximizing the minimum distance between the existing facilities and the new facility to be located. The maxisum and the maximin objective functions are useful for evaluating problems involving undesirable facilities like nuclear waste sites, garbage dumps etc. a set covering objective function refers to the problem in which a minimal number of new facilities to be located are determined. The constraint involved is that the new facility cannot be further than some pre-specified distance away from the existing facility. The maximal covering problem has the number of new facilities to be located as the problem input, but the distance between the existing facility and the new facilities is to be maximised. These are the frequently used objective functions of a facility location problem.

In one important class of facility location problems, the goal is to find one or more center points in order to minimize the maximal distance between a demand point and the facility that is nearest to it. These problems are called k-center problems, where k is the number of desired facility locations. The simplest problem in this class is the 'basic' l-center problem, in which one is to find a point in the plane that minimizes the maximal distance to a set of n demand points.

In another important class of problems, the goal is to find one or more median points in order to minimise the average distance. (i.e. the sum of the distance) between a demand point and the facility that is nearest to it. These problems are called k-median problems, where k is the number of desired facility locations. When k = 1 the problem is known as the Weber problem.

1.1.6 Planar K-Centra Single-Facility Location Problem

Facility location handles problems with an aim of locating one or more service points so as to satisfy objectives. In the case of this thesis, the optimizing factor is distance. Median and center problems are well known distance dependant location problems with a multitude of applications. The median approach locates a set of service points in order to minimize the average distances from other customer location points. The center approach handles a different type of location problem with the objective of having farthest users as close as possible. An application of center approach is the location of emergency facilities. However, in real life applications, most problem that come up generally require a unique combination of both the discussed objective functions. In the process of locating a new facility, more than one deciding factors are involved. This is due to ~~the fact~~ that typically the cost associated with this decision is very high. Thus approaches are evaluated and compared in order to be implemented.

A median approach provide solution where remote and low-population(less weight) density areas are neglected compared to centrally situated or high-population (more weight) density areas. On the other hand using a center approach may cause a large increase in total travelled distance, thereby causing inefficient use of available space.

The problem is therefore to select the best factors out of the two objectives functions to solve the problem. Similar logic problems are the centdian problem or the k-centra problem. A centdian problem minimizes the convex combination of the center and median objective function. The convex combination is decided depending on the associated weights allocated to the customer points.

The objective of this thesis is to solve the planar k-centra single facilityproblem with Euclidean distances. The problem statement is to locate a single service point that minimises the sum of the distances to the k-farthest demand points out of a set of given points. A *k*-centra problem is a location analysis which encloses attributes from both the center as well as the median approaches. However, this is not a linear combination. In a k-centra problem, a set value of *k* is predefined and is used in the evaluation. If k=2, then the corresponding problem becomes a center problem and when k = n, the corresponding problem becomes a median problem, thus enclosed both approaches. It also gives the decision maker a more flexible tool to make an informed and a calculated decision. A real life example for a k-centra problem could be locating warehouse to satisfy a set number of distant customers. The advantage here over a minimax solution for a similar problem is that the minimax approach generally governed by a single largest distance. A good example here would be the case of an isolated unit ~~located at~~ a significant distance from all other units. In this case, the minimax problem is reduced to the minimization of the distance of the particular unit leaving few of the location decision unsatisfied.

A k-centra problem considers *k* largest distances thus encompassing more of the problem characteristics before deciding on the solution. Another application is when the problem is very large and there are a multiple of existing locations to evaluate, thereby costing a lot of analysis time. The k-centra approach simplifies the solution method by selectively evaluation does not sacrifice too many problem characteristics and give a near optimal solution in a shorter period of time. However, the k-centra method comes with a few disadvantages. Solution optimality is not guaranteed in case of a k-centra method. The iterative nature of the solution method can cause the problem to go in a loop.

The solution procedure used in this thesis is an extension of one of the oldest heuristic methods existing in location science-Weiszfeld's algorithm. Starting out with 2$n$minimax solution (n being the number of demand points), the distance from the radius of the solution would be reduced until k points lie outside the circle. Then, minisum is applied on the problem and it is solved for the k points by considering the smallest possible circle that encompasses $(n - k)$ points. The solution is tested for feasibility, and if the original k points are outside the smallest circle, the solution is optimal. If not, this procedure is repeated again. The solution method used to solve this problem would be an Alternating Weiszfield's Algorithm. Here, Weiszfeld's algorithm would be alternated with reducing radius points until all k points are inside the smallest possible circle (one at a time) and then Weiszfled's is applied one more time. This method is repeated until optimality is obtained.

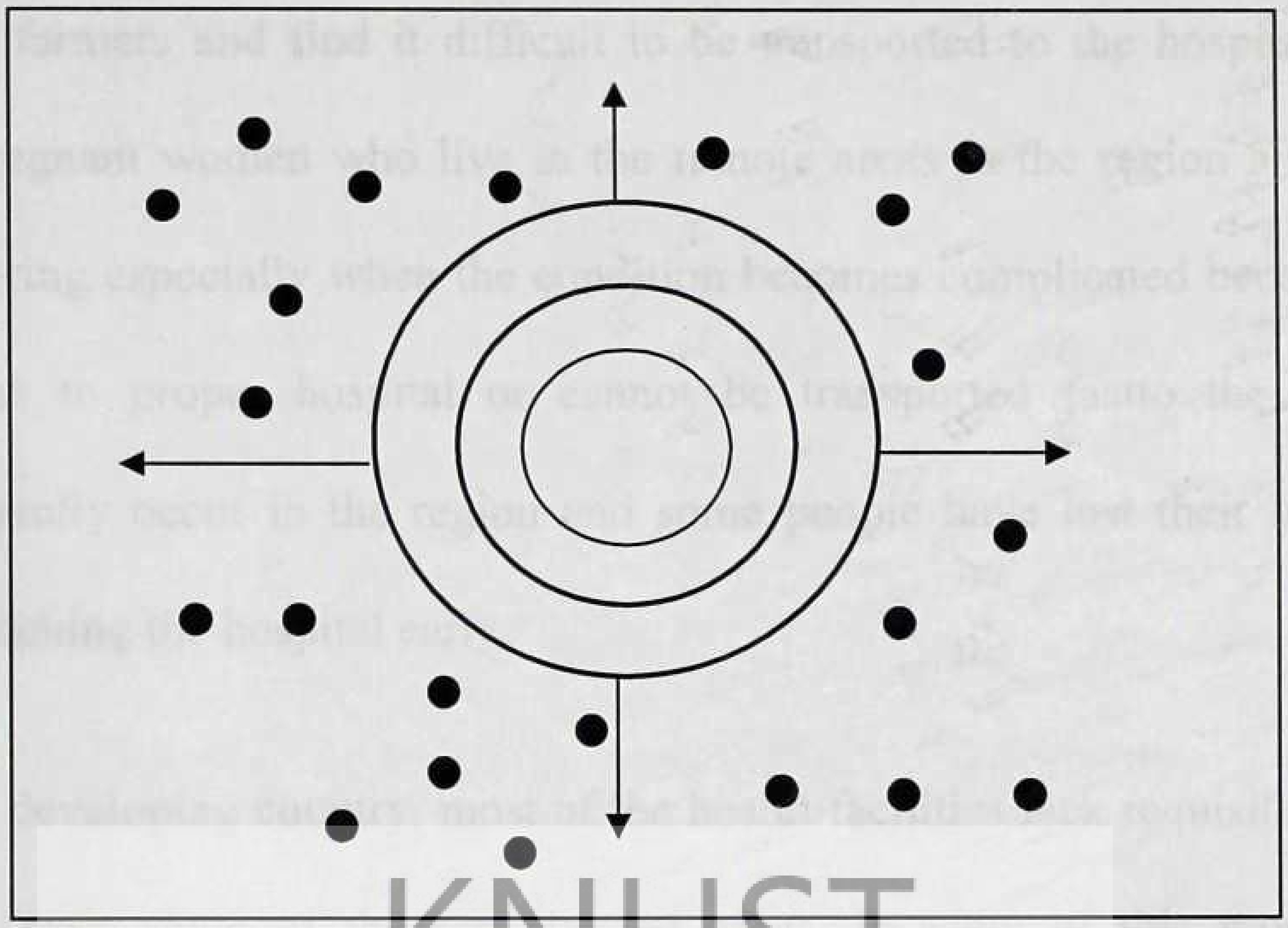


Figure 1.5: A Schematic Example of the Single Facility k-Centra Problem

1.2 Problem Statement

The ambulance existed in most of the developed countries to transport sick or injured people to places of treatment. This function of the ambulance has saved a lot of life especially accident victims and has also reduce maternal mortality in most of the countries in the world.

Ghana as in other developing countries, ambulance exist in only the regional capitals and some few district capitals and serve the purpose of transporting sick or injured people to referred hospitals. However, the people living in the remote areas have no access to ambulance to serve the same purpose and as a result some of the victims die before reaching hospitals in the district capitals since most of the hospitals are sited at the district capitals. Most of the people in Ghana living in the remote areas especially BrongAhafoRegion are famers and sometimes get injured or snake bite in performing their duties as farmers and find it difficult to be transported to the hospitals. Also some of the pregnant women who live in the remote areas in the region also die in cause of delivering especially when the condition becomes complicated because they have no access to proper hospital or cannot be transported fastto the hospital. Accidents generally occur in the region and some people have lost their lives as a result of not reaching the hospital early.

Ghana being a developing country, most of the health facilities lack requisite medical equipments to treat some of the complicated cases. In view of this most of the complicated cases are referred to the so called well-equipped hospitals for treatment.

According to the Berekum Holy Family Hospital in the BrongAhafo Region, in 2011, 7% out of 40 referred cases to the Sunyani Regional Hospital die before reaching the hospital. Base on this there is the need for a study that will come out with useful suggestions in order to minimize the situation. This prompted the researcher to locate a single service point(an air ambulance response unit) in the BrongAhafo Region using the Planar K-Centra Single-facility Euclidean Location Algorithm to minimizes the sum of the distances to the k-farthest demand points out of a set of given points.

Due to poor road network in the region and sometimes traffic and speed rumps on the roads, some referred patients from clinics, maternity homes and hospitals die when being referred to regional hospital in Sunyani.

The aim of this thesis is to use the Planner K-Centra Single-Facility Euclidean Location Problem Strategy to locate Air Ambulance Response Unit in the BrongAhafo Region to minimize travel distances of K-6 farthest locations.

1.3 The Objectives of the Study

The objective of the study is to determine the optimal location of an air ambulance response unit by the Planar K-Centra Single Facility Euclidean Algorithm in the BrongAhafo Region of Ghana. Specifically, the study seeks:

1. To minimize the travel distances of transferring patients from the primary hospitals to the Sunyani Regional Hospital in the BrongAhafo Region.

2. To reduce the rate at which patients die when they have been referred to Sunyani Regional hospital.

3. To locate a centre that is not too far from all the centres and also quickly accessible to contribute to curb the situation.

4. To recommend to stakeholders the most appropriate location for the Air Ambulance Response Unit in the BrongAhafo Region.

1.4 Methodology

The problem is to locate an air ambulance response unit in the BrongAhafo Region so as to minimize the travel distances between the new facility and the users using the planar K-Centra Single Facility Euclidean location strategy. The data type is a primary data recorded from the map of BrongAhafo Region. The data was taken from Town and Country Planning Department from BrongAhafo Regional Office. The names of the hospitals in the region were obtained from the regional headquarters of National Health Insurance Authority (NHIA). The data on the number of referred cases was taken from the transport unit of Berekum Holy Family Hospital and Sunyani regional Hospital in the BrongAhafo Region. The data will be in the x-y coordinates. The algorithm used was the Weiszfield's algorithm.

The scale for all the reading will be 1:50000. The software for the work will be the Matlab and the focus will be on locating an air ambulance response unit to minimize the total distance of the K-6 farthest customer locations. The resources for the study include my personal laptop, the internet, Berekum College of Education Library, the Sunyani Municipal Library, the Library of Kwame Nkrumah University of Science and Technology (Kumasi) and Sunyani Polytechnic Library.

1.5 Significance of the Study

The study is significant because of the following reasons:

- It will give an effective coordinates for the computation distances and subsequently a central location for an air ambulance response unit.
- It will reduce the average travel time and distances from all the k-6 farthest costumer locations to the air ambulance response unit.
- It will help to conserve fuel since the algorithm provides average shortest distances for all customer locations.
- It will also help reduce the number of death cases when patients are being referred to the regional hospital due to delay.

1.6 Organization of the Thesis

This thesis is structured as follows;

Chapter one deals with the introduction which comprises the background to the study, the problem statement, the objectives, methodology, significance of the study and the organization of the study.

Chapter two deal with the related literature relevant of the study. Chapter three discusses the methodology used for the study. Chapter four presents the results and the discussion of the study. Chapter five which is the last chapter, deals with conclusion and the recommendations.

CHAPTER TWO

LITERATURE REVIEW

2.0 Introduction

This chapter reviews the literature existing in the area of facility location. Facility location problems date back to the 17th century when Fermat (1643) and Cavalieri, et al. (1647) simultaneously introduced the concept, although this theory is widely contested by location analyst experts. Late in 18th century, Pierre Varignonpresented "The Varignon Frame" which an analog solution to the planar minisum location problem. However, it started gathering more interest when Weber (1909) presented the Planar Euclidean single facility minisum problem in1909. The Weber problem considers the example of locating a warehouse in the best possible location such that the distance travelled between the warehouse and the customer is minimized. Weiszfeld (1937) conceptualised an iterative method to solve the minisum Euclidean problem today referred to as the Weiszfeld's procedure.

Hakimis (1964) introduced a seminal paper on locating one or more points on a network with the objective to minimize the maximum distance. Francis et al. (1983) presented a survey paper in location analysis which defined four classes of location problems and described algorithms to optimize them. They are continuous planar, discrete planar, mixed planar and discrete network problems.

Daskin et al (1998) reviewed various strategic location problems where they emphasized that a good facility location decision is a critical element in the success of any supply chain. They explained median problems, centre problems, covering problems, and other dynamic location problem formulations in the context of a supply chain environment.

Facility location problems span many research fields like operation research, mathematics, statistics, urban planning designing, etc.

Location analysis goes back to the influential book of (the German industrial author Alfred Weber(1909). The research was motivated by observing a warehouse operation and its inefficiencies. Weber considered the single warehouse location problem and evaluated it such that the travel distances for pickups and replenishment were reduced. Other notable work in this field was by Fermat (1643), who solved the location problem for three points constituting a triangle.

Another major concept in the field of location analysis was the concept of competitive location analysis introduced by (Hotelling, 1929). The paper discussed a method to locate a new facility considering already existing competition. The considered facilities were on a straight line. He proposed that the customers generally prefer visiting the closest service facility. He introduced the "Hotelling's Proximity Rule" which can be used to determine the market share captured by each facility. He just considered the distance metric during his analysis. The Hotelling model was extended by Drezner (1993) who introduced the concept of varying attractiveness among competing facilities. He analyzed cost and quality factors in addition to distance metric involved. Huff (1966) proposed the famous "Gravity Model" for estimating the market share captured by competitors. The gravity model states that existing customer locations attract business from a service in direct proportion to the existing locations and in inverse proportion to the distance between the service location and the existing customer locations.

2.1 Pure Location Problem

Apure location problem deals with optimally locating one or more new service facilities to serve the demand at existing customer locations. Selecting different distance measures, and by using different constraints, we obtain different variants of the multi or single facility location problem. Furthermore, we can introduce additional variants by restricting new facility locations to specific sites, and by letting demands be either stochastic or distributed over areas.

2.1.1 Rectilinear Distance Metric Problem

The rectilinear distance location problem is a variant of the classic location problem. Rectilinear distance metric is the axial distance between two corresponding points taken at right angles from each other. Francis (1963) first considered the single facility location problem with rectilinear distances. The paper considered a simple substitution method for solving the proposed problem. Francis (1964) also solved the multi-facility rectilinear location problem with equal demand weights. The objective function is as given by Equation 2.1.

Minimize $f(x) = \sum (i = 1 \ldots m) Wi.d(X, Pi)$ ……………………….(2.1)

Where;

Wi = Weights associated with the locations.

m = Number of existing locations

2.1.2 Euclidean Distance Metric Problem

The difference between Euclidean and rectilinear distances is that where rectilinear distance metric considers right-angle distances, a Euclidean metric considers straight line distances between points. In a plane with a points at (x_1, y_1) and (x_2, y_2), it is given by Equation 2.2.

Minimize $f(x, y) = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$ ……………..(2.2)

This problem is also known as the Weber problem. It is also known as the planar Euclidean minisum distance single facility location problem, which adequately defines its characteristic. The upper and lower bound on f_{euc} is as shown in Equation 2.3.

$\sqrt{f_1(x)^2 + f_2(y)^2} <= f_{euc}(x^*_{euc}, y^*_{euc}) <= f_{euc}(x^*_{rec}, y^*_{rec})$ ……………(2.3)

For collinear single facility location problems with Euclidean distances, a user can always obtain an optimal solution as discussed in the paper by (Rosen et al., 1993). This thesis assumes that the existing facilities are non-collinear. For non collinear location problems, Weiszfield (1937) was the first to propose a fixed-point iterative method that is known as the Weiszfield procedure. Weiszfield'salogarithm iteratively solves for the minisum location to the Weber problem based on the objective function.

Drezner (1985) in his paper conducted some sensitivity analyses for the single facility problem. He studied various variants of the problem with weight restrictions and location restrictions.

2.1.3 Location-Allocation Problems

Since 1963, when the first location-allocation model was formulated by Cooper (1963), there has been extensive research on the field. The simplest location-allocation problem is the Weber problem addressed by (Friedrich, 1929). This paper discussed the steps in locating a machine so as to minimize the sum of the weighted distances from all the raw materials sources. The seminal work in this area was on the p-median problem, initially formulated by (Hakimi, 1964). The median problem was considered on a graph and the objective function was to reduce the average or the sum of the transportation costs from the service facility to the demand locations. It was derived that one of the optimal solutions locates the service facility on one of the nodes of the network.

2.1.4 K – Central Problems

Most of the research in the field of location analysis focuses on the minimization of the average (or total) distance (median function) or the minimization of the maximum distance (center function) to some service facility. Approaches based on the median model are primarily concerned with achieving maximum spatial efficiency. This approach generally provides solutions in which remote and low – population density areas are neglected in favour of providing maximum accessibility to centrally situated and high – population density areas. To avoid this drawback, the center method can be applied which primarily addresses geographical equity issues compromising optimum spatial efficiency. Sometimes locating a facility using the center method doesn't satisfy all the complex underlying constraints a location problem is associated with. These factors led to more research in lieu of obtaining a compromise solution concept.

Halpern (1976) first introduced the cent – dian model as a parametric solution concept based on the bicriteriacenter/median model. Halpern modelled the problem in such a way that a compromise was achieved between median and centre objective functions such that the inherent objective function characteristics of both the problems are considered while solving. The two objective functions considered are total distance minimization and the maximum distance minimization criterions. The goal here was to find an optimal balance between efficiency (least - cost) and equity (worst - case). However, this particular method can sometimes fail to provide a solution to a discrete location problem mostly due to the limitations involved with direct combinations of two different functions.

Hansen et al. (1991) introduced a variation of the cent – dian problem in the generalized center problem, which minimizes the difference between the maximum distance and the average distance. This model can be extended to formulate solutions for multiple facility location problems on a plane as well as on a network. This model can also be applied to discrete location problems.

The k – centra problem concept was formulated by (Slater, 1978). The k – centra model combines both the center as well as the median concepts by minimization of the sum of the *k* largest distances. If *k* = *2* the model reduces to a standard center problem while with *k*=*n* it becomes a standard median problem. This paper concentrated on the discrete single facility location problem on a tree graph.

Peeters (1998) studied the k – centrum model and introduced a full classification of the k – centrum criteria and some solution concepts. He proposed two different variations on the median and the center functions each. The functions considered were upper k – median where the sum of the *k* largest distances are minimized, lower k – median where the sum of the *k* smallest distances are minimized, upper k – center where the *k* largest distances are minimized, and lower k– center where the *k* smallest distance are minimized.

The k–centrum model is generally reserved for unweighted problems. However, significant research has been performed to show that satisfying the above criteria is not always necessary. Recently, Tamir (2001) solved a weighted multiple facility k – centrum problem on paths and tree graphs using simple polynomial time algorithms. In this method, weights are assigned to all the distances from the new location to the existing locations and the distances are scaled accordingly.

Ogryczak and Zawadski(2002) in their paper introduced the conditional median method which is an extension of the k-centrum concept when applied to weighted problems. The paper proposes that a k-centrum problem can be evaluated for optimality by just considering only that specific part of the demand which is in direct proportion to the existing largest distances for a specified portion of the demand.

CHAPTER THREE

METHODOLOGY

3.0 Introduction

This chapter discusses the methodology used to locate an ambulance response unit in the BrongAhafo Region.

3.1 Optimization

Optimization is the process by which the best solution is selected from among possible solutions. Most engineering problems, including those associated with the analysis, design, construction, operation, and maintenance, involve decision making. Usually, there will be a criterion that is to be minimized or maximized while satisfying several social, economical, physical, and technological constraints. There will be a number of parameters that can be varied in the decision making process. As the number of parameters increases, it becomes necessary to use systematic procedures for solving the decision making problems.

3.1.1 Definition of an optimization problem

The formulation of an optimization problem involves the development of a mathematical model for the physical or engineering problem. In practice, several assumptions have to be made to develop a reasonably simple mathematical model that can predict the behavior of the system fairly accurately. The results of optimization will be different with different mathematical model of the same physical system. Hence, it is necessary to have a good mathematical model of the system, so that the results of optimization can be used to improve the performance of the system.

A general optimization problem can be stated in mathematical form as

$$\text{Find } \vec{X} = \begin{Bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{Bmatrix}$$

that minimizes $f(\vec{X})$

subject to

$g_j(\vec{X}) \le 0; \; j = 1, 2, \ldots, m;$

and

$h_k(\vec{X}) = 0; \; k = 1, 2, \ldots, p;$

where x_i ($i = 1, 2, \ldots, n$) are the decision variables, $\vec{X}$ is the vector of decision variables, $f(\vec{X})$ is the merit, or criterion, or objective, function, $g_j(\vec{X})$ is the jth inequality constraint function that is required to be less than or equal to zero, $h_k(\vec{X})$ is the kth equality/ Constraint function that is required to be equal to zero, n is the number of decision variables, m is the number of inequality constraints, and p is the number of equality constraints.

3.1.2 Decision variables

The formulation of an optimization problem begins with the identification of a set of variables that can be varied to change the performance of the system. These are called the decision or design, variables, and their values are freely controlled by the decision maker. A set of numerical values, one for each decision variables, constitutes a solution (acceptable or unacceptable) to the optimization problem.

3.1.3 Objective function

When different solutions are obtained by changing the decision variables, a criterion is needed to judge whether one solution is better than another. This criterion, when expressed in terms of the decision variables, is called the objective, merit, or cost function. The interest of the decision maker is to select suitable values for the decision variables so as to minimize or maximize the objective function.

3.1.4 Inequality constraints

In any decision making problem, there will be limitations or conditions imposed on the decision variables that may include economical, physical, or functional limitations. In many cases, the validity of the mathematical model used for the physical system imposes restrictions on the decision variables. These limitations, or conditions, when expressed in terms of the decision variables, are known as constraint functions. When the constraint function is restricted to have only negative or zero values, the resulting constraint is known as the inequality constraint. On the other hand, if the constraint function is required to be equal to zero, the resulting constraint is called the equality constraint

3.1.5 Feasible Solution

Any set of decision variables that satisfy the constraints of the problem (both inequality and equality constraints) is known as a feasible solution. A feasible solution is an acceptable solution to the decision maker in terms of the constraints, but may or may not minimize the objective function.

3.1.6 Optimum Solution

A feasible solution that minimizes the objective function is known as the optimum solution.

3.1.7 Types of Optimization Problems

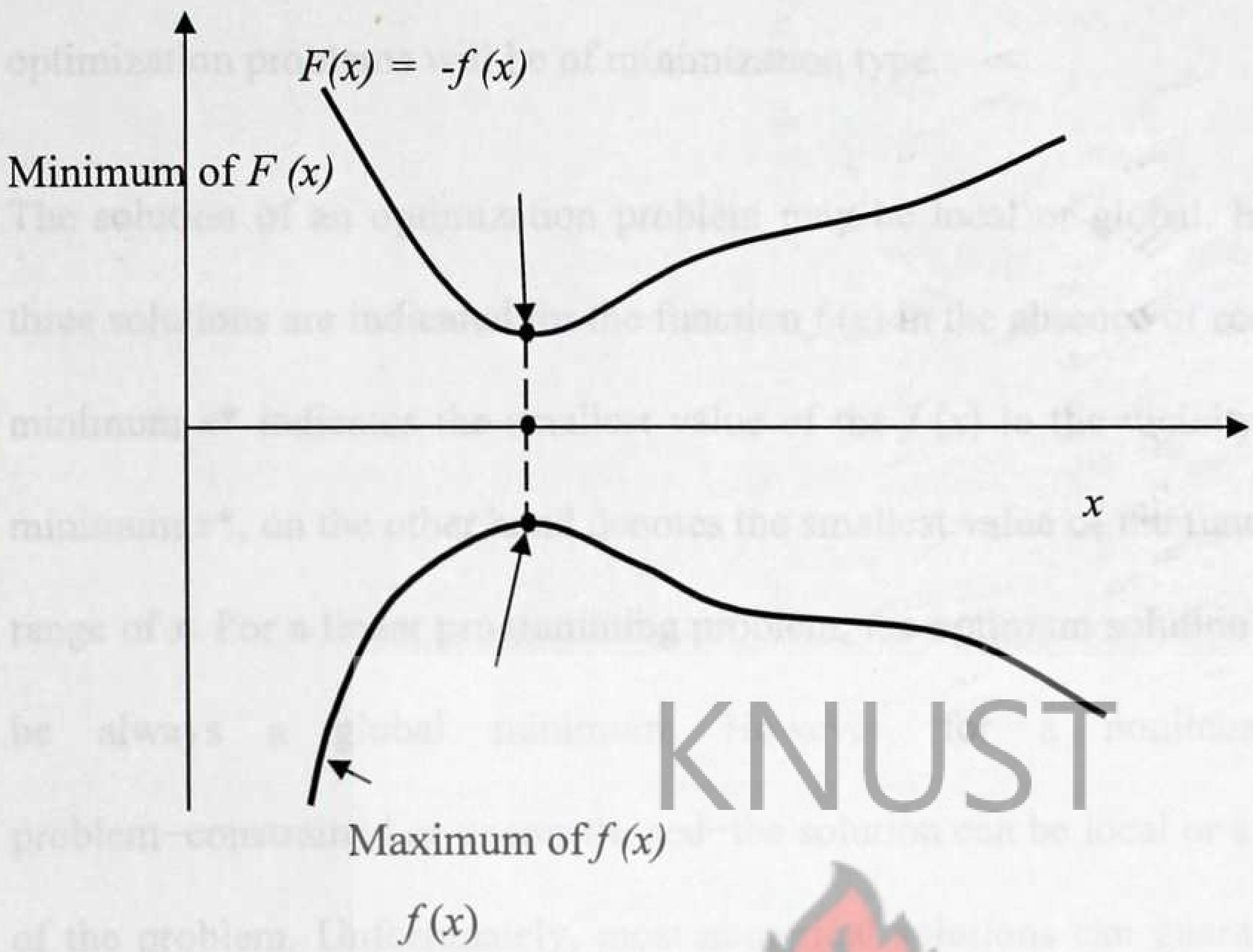
Optimization problem are also known as mathematical programming problems. In some practical situations, the constraints may be absent, and the problem reduces to

$$\text{Find } \vec{X} = \begin{Bmatrix} x_1 \\ x_2 \\ \cdot \\ \cdot \\ \cdot \\ x_n \end{Bmatrix}$$

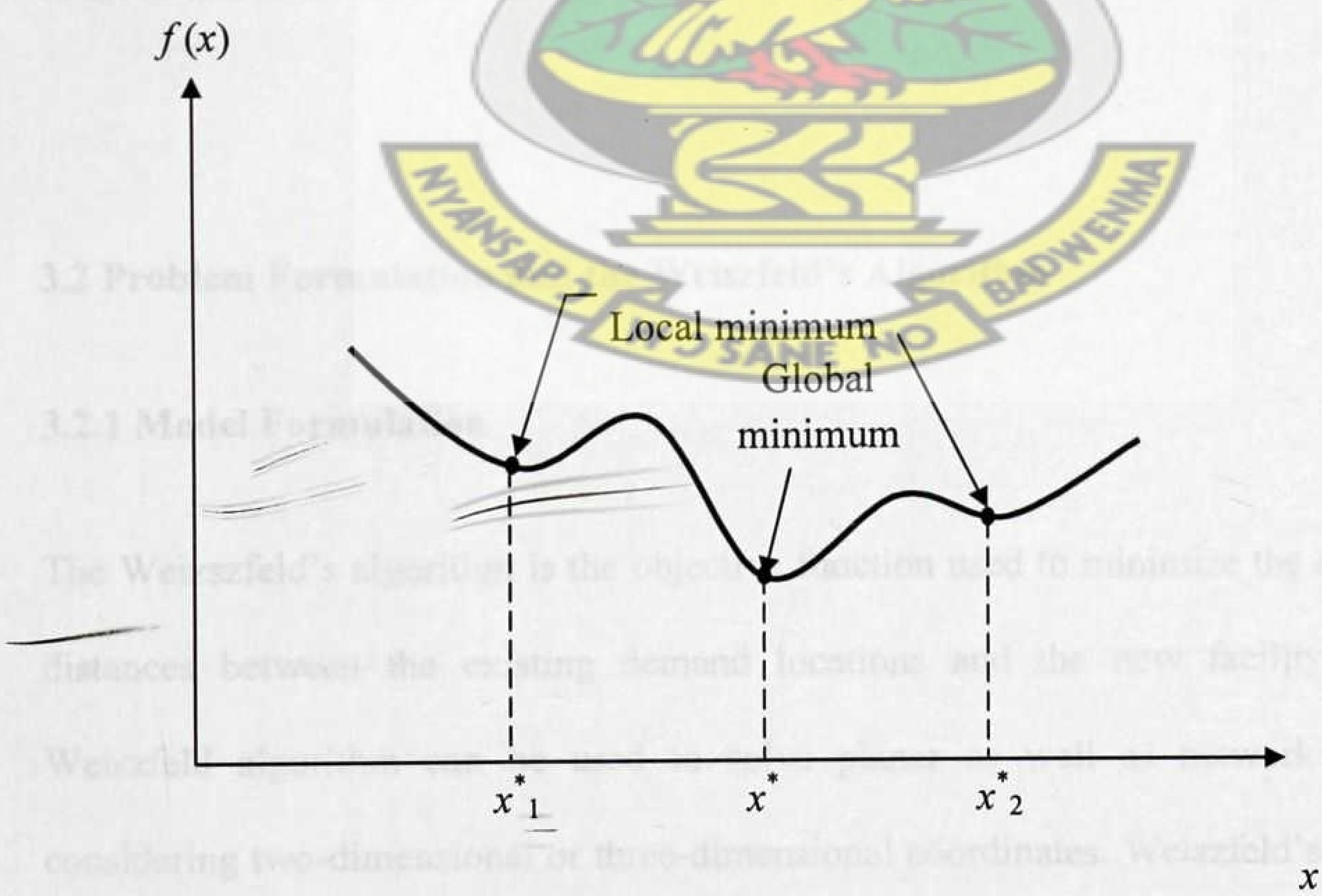
that minimizes $f(\vec{X})$

The problem is known as an unconstrained optimization problem. Depending on the nature of functions, optimization problems can be classified as linear and nonlinear programming problems. A linear programming problem is one in which all the functions involved, namely, $f(\vec{X})$, $g_j(\vec{X})$, and $h_k(\vec{X})$ are linear in terms of the design variables. A nonlinear programming problem is one in which at least one of the functions is nonlinear in terms of the decision variables.

Some practical problems requires the maximization a function $f(\vec{X})$ instead of minimization. However, the maximum of a function $f(\vec{X})$ can be found by seeking the minimum of the negative of the same function. This situation is illustrated below for the case of a function of a single variable. In this figure, $x = x^*$ correspond to the maximum of the function $f(x)$. The same solution, $x = x^*$, also corresponds to



F(x) = -f (x)
Minimum of F (x)
x
Maximum of f (x)
f (x)



f (x)
Local minimum
Global
minimum
x*1
x*
x*2
x

the minimum of the function F(x) = - $f(x)$. Hence, without loss of generality, all optimization problems will be of minimization type.

The solution of an optimization problem may be local or global. In diagram above three solutions are indicated for the function $f(x)$ in the absence of constraints. A local minimum x^* indicates the smallest value of the $f(x)$ in the vicinity of x^*. A global minimum x^*, on the other hand denotes the smallest value of the function in the entire range of x. For a linear programming problem, the optimum solution can be proved to be always a global minimum. However, for a nonlinear programming problem−constrained or unconstrained−the solution can be local or a global minimum of the problem. Unfortunately, most numerical solutions can guarantee only a local minimum of the problem. In spite of this limitation, optimization methods are quite useful in solving several practical (and complex) decision-making problems.There are several methods of solving location problem which comes under optimization and one of the methods is Weiszfeld Algorithm.

3.2 Problem Formulation and the Weiszfeld's Algorithm

3.2.1 Model Formulation

The Weizszfeld's algorithm is the objective function used to minimize the sum of the distances between the existing demand locations and the new facility location. Weiszfeld algorithm can be used to solve planar as well as network problems considering two-dimensional or three-dimensional coordinates. Weiszfeld's algorithm iteratively solves for the minisum location to the Weber problem based on its objective function as shown in equation 3.1.

Minimize $f(x, y) = \sum wi\sqrt{(x - a_1)^2 + (y - b_1)^2}$......................................3.1

Where,

w_i = weight's associated with the existing locations

x = x coordinate of the starting solution and later obtained by successive iterations.

y = y coordinate of the starting solution and later obtained by successive iterations.

a_i = x coordinate of existing locations b_i = y coordinate of existing locations.

Equation 3.1 is the sum of the weighted distance.

3.2.2 The Weiszfeld's Algorithm

The steps followed in Weiszfeld's Algorithm to solve a planar Euclidean location problem are as listed below:

Step1: Input initial coordinates (x,y).

Step 2: Solve for every γ_i as per Equation 3.2.

$$\gamma_i = \frac{w_i}{\sqrt{(x - a_i)^2 + (y - b_i)^2}} \qquad \text{............(3.2)}$$

Where, x= x-coordinates of starting point for the iterative algorithm.

y= y-coordinates of starting point for the iterative algorithm.

Step 3: sum for every γ_i for $\Gamma(x,y)$ shown by Equation 3.3.

$$\Gamma(x,y) = \sum_{i=1}^{m} \gamma_i (x,y) \quad \text{.................(3.3)}$$

Step 4: determine all $\lambda_i = \gamma_I/\Gamma$ as shown in Equation 3.4.

$$\lambda_i(x,y) = \frac{\gamma_i (x,y)}{\Gamma(x,y)} \quad \text{................(3.4)}$$

Step 5: Weiszfeld's (x) = WFx= $\sum$ (i = 1…m) $\lambda_i . a_i$ ....(3.5)

Weiszfeld's (y) = WFy= $\sum$ (i = 1…m) $\lambda_i . b_i$ ....(3.6)

Step 6: Determine the objective function value by summing the individual objective function values for all i as per Equation 3.7.

$$f_{euc} = \sum w_i \sqrt{(WFx - a_i)^2 + (WFy - b_i)^2} \quad \text{................(3.7)}$$

Step 7: Repeat until stopping conditions are met that is $|f_{eucn+1} - f_{eucn}| \leq 0$ 3.8

Considering a random starting solution, the algorithm is evaluated to obtain a second set of trial values (WFx, WFy). The obtained coordinates are substituted to obtain the third set of trial values and so on. By reiterating the Weisfeld's values for x, y, we can find the Euclidean solution close to or equal to the optimal solution. The stopping conditions are met when the difference between subsequent objective function values obtained is less than or equal to zero.

3.2.3. Solved Example for Weiszfeld's Algorithm

The objective of the Weiszfeld's algorithm is to find the minisum solution to a location problem. The example problem considered here is a non – weighted planar problem with coordinates listed in Table 3.1.

Column one is the points, column two is the x-coordinates of the existing locations and column three is the y-coordinates of the existing locations.

Table 3.1: Weiszfeld's Example Coordinates

Points	a_i	b_i
P1	2	3
P2	0	8
P3	9	3
P4	6	2
P5	7	2
P6	1	5

Considering the initial solution as (x,y) = (3,2). This value could be selected as any value depending on the user's discretion. To reduce the number of iterations required to solve the object function, the starting point based on visual judgment.$(x,y) = (3,2)$

By substituting the value of (x,y) in Equation 3.2 we get the following $\gamma_i$values:

$$\gamma i\,(x, y) = \frac{1}{\sqrt{(3-2)^2 + (2-3)^2}} = 0.707$$

$\gamma_2\,(x,y) = 0.149$

$\gamma_3\,(x,y) = 0.164$

$\gamma_4\,(x,y) = 0.333$

$\gamma_5\,(x,y) = 0.25$

$\gamma_6\,(x,y) = 0.277$

The above γvalues are summed up to obtain the Γ value.

$\Gamma(x,y) = (0.707+0.149+0.164+0.333+0.25+0.277) = 1.881$

Using Γ and the corresponding γ value, λ values were calculated for each of the existing facility locations as in Equation 3.4.

$$\lambda_1\,(x,y) = \frac{0.707}{1.881} = 0.375$$

$\lambda_2\,(x,y) = 0.0792$

$\lambda_3\,(x,y) = 0.0873$

$\lambda_4\,(x,y) = 0.177$

$\lambda_5\,(x,y) = 0.132$

λ_6 (x,y) = 0.147

Using the obtained λ values, Weiszfeld coordinates (WFx, WFy) are calculated using the Equations 3.5 and 3.6.

$$WF(\mathrm{x}) = \sum_{i=1}^{m} \lambda_i * a_i$$

$= (0.375*2) + (0.0792*0) + (0.0873*9) + (0.177*6) + (0.132*7) + (0.147*1)$

= 3.14

KNUST

$$\mathrm{WF}(y) = \sum_{i=1}^{m} \lambda_i * b_i$$

$= (0.375*3) + (0.0792*8) + (0.177*2) + (0.132*2) + (0.147*5)$

= 3.201

By using the trial solution coordinates, calculate the individual function values for all *i*. The sum of the obtained individual function values would give the objective function value for trial solution 2 as shown in Equation 3.7.

$f_{eucPi} = \sqrt{(\mathrm{WF(x)} - a_i)^2 + (\mathrm{WF(y)} - b_i)^2}$

$f_{eucP1} = \sqrt{(3.14 - 2)^2 + (3.201 - 3)^2} = 1.157$

$f_{eucP2} = 5.734$

$f_{euP3} = 5.863$

$f_{euP4} = 3.101$

$f_{euP5} = 4.042$

$f_{euP6} = 2.795$

$$f_n = \sum f_{euP1} = 22.695 \qquad \text{3.9}$$

The procedure described above is repeated until stopping conditions are achieved. Stopping condition is defined as;

$$\left| f_{eucn+1} - f_{eucn} \leq 0 \right| \qquad \text{3.10}$$

3.3. Application of Weiszfeld Algorithm to k – Centra Problem

The objective of this thesis is to solve a single facility, unweighted, planar k – centra problem with an aim to locate an ambulance response unit in the BrongAhafo Region to minimize the distance to the k – farthest customer points among several existing points. The input required for the problem includes the coordinates for the existing customer locations as well as *k* value. This *k* value depends on the type of problem, the type of coverage required, the existing customer locations on the plane or the discretion of the user.

The solution methodology used in this thesis is an iterative heuristic method utilizing Weiszfeld's algorithm. The solution steps are listed below:

- Step 1: Find the minimax solution for all the existing customer locations. This minimax solution is used as starting point for following iterations.
- Step 2: The next step is to find the distances of all existing locations from the minimax solution. The top *k* distances are considered to find the *k* farthest locations points.

- Step 3: the next step is to use the minisum approach to evaluate the obtained *k* farthest points. Weiszfeld's algorithm is applied to solve the minisum aspect. After the first iteration, a new location point is obtained.

- Step 4: the step is to check whether the same *k* points still are the points lyingfarthest away.

  - If so, the solution obtained is the k – centra solution.

  - If not, reiterate by considering the new k-largest distances and the corresponding existing points.

  - Stopping conditions are met when you end up with the same k-farthest points you started with.

The procedure is illustrated with a flowchart as shown in figure 3.1. However there are chances that the algorithm may go in a loop. This happens when the solution comes up with a different set of *k* points and this keeps repeating. This leads to an infeasible solution. Generally it is observed that the infeasibility is due to a single point getting displaced from the initial set of *k* points. The only solution possible here is a semi-optimal solution which neglects the coordinates of the rogue point. This point is better explained in the later part of the thesis where a MATLAB code for the discussed k- centra approach is provided.

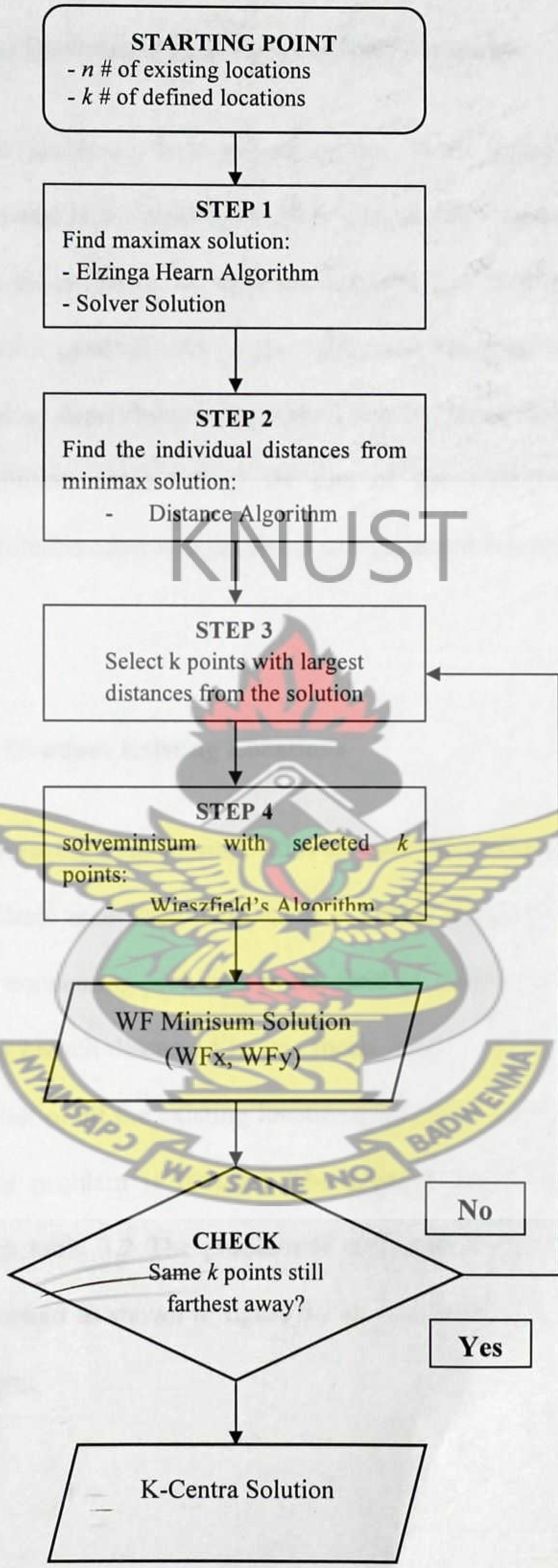


Figure 3.1: Flowchart showing k-centra Solution Approach

3.4. Solved Examples Illustrating Real Life Location Scenarios

Three small location problems were solved in this thesis using the k – centra methodology as discussed in the previous section. The analogy used was real life area planning problems in cities. There are numerous examples of planned and unplanned city layouts. Older cities generally fall in the unplanned category while newer cities are methodically planned depending on population density, topographical features and business sectors. Solution methodology for one of the considered examples is discussed in detail while the other two problems are explained briefly.

3.4.1. Problem with Random Existing Locations

This is most common layout seen in most cities all over the world. A single – facility planar location problem associated with such a layout presents one of the most complex and time – consuming problems in the field of location analysis. However, using the k- centra approach discussed in this thesis, only a selective few points are randomly evaluated out of all the existing locations. Thus the computing time as well as complexity of the problem is reduced. The problem considered is a 15 point problem as shown in table 3.2 The problem is evaluated for different values of k (k=1to 15). The flowchart as shown in figure 3.1 is used as the solution guideline for evaluating the problem.

Table 3.2: Problem of Random Cities

Points	P1	P2	P3	P4	P5	P6	P7	P8	P9	P10	P11	P12	P13	P14	P15
a_i	2	2	5	3.5	7	9.5	10.5	11.5	9.5	7	5	2	5.5	8.5	11
b_i	2	4	2	4	4	4.5	2	5.5	8	6.5	6.5	9	8.5	9	9.5

The graphical representation is shown in Figure 3.2. As the graph suggests, the existing points are randomly located on a planar layout. The aim is to locate a single service facility in the plane so as to minimize the sum of the distance to the k farthest points. The data required for this problem includes the coordinates for existing customer locations and a value for k. k value could depend on the nature of the problem or it could be based on logical reasoning and demand flow.

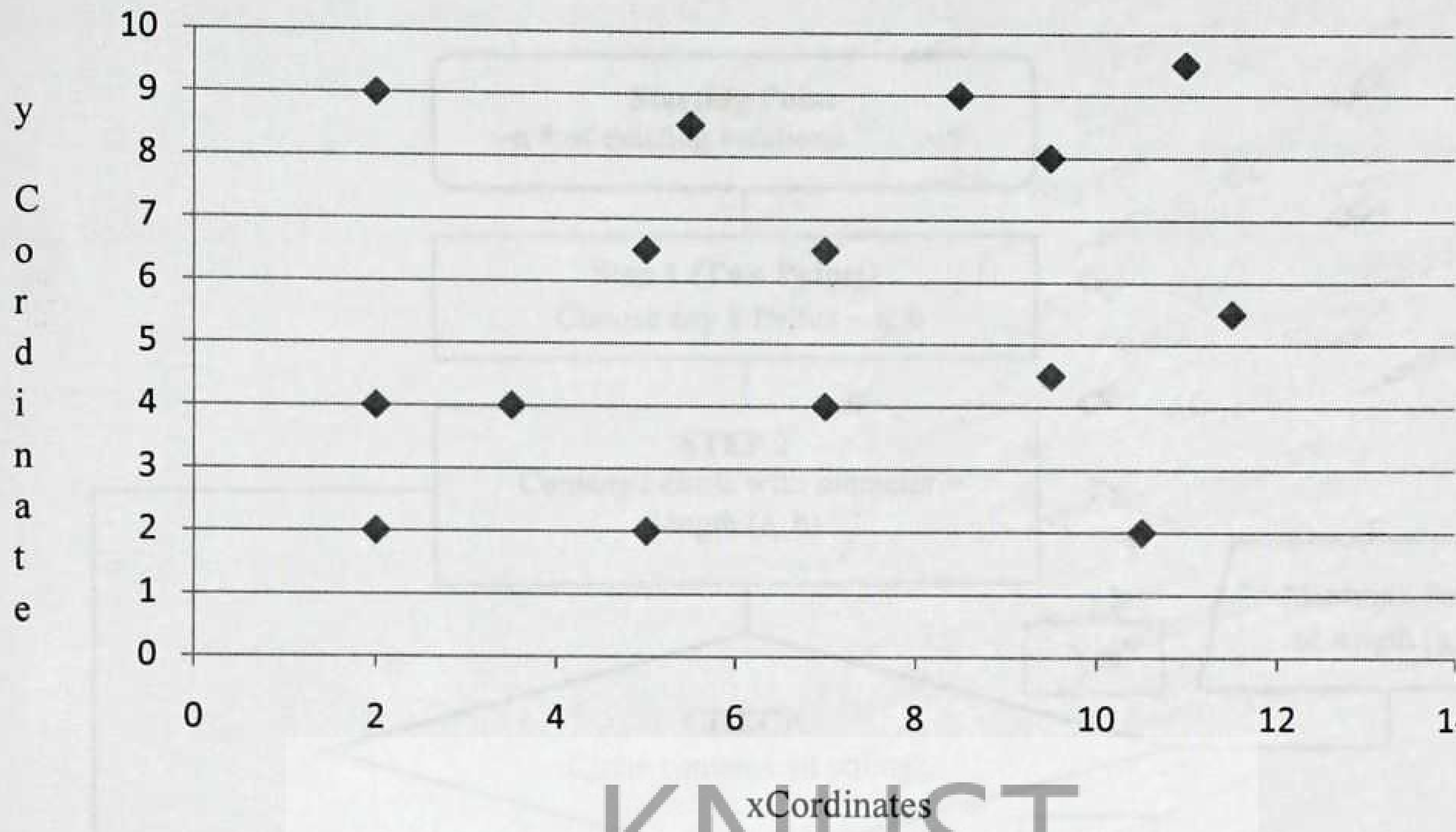


Figure 3.2: Graphical Representation of Problem of Random cities layout

The first step in the discussed methodology is obtaining the minimax solution considering Euclidean metric. Two different methods were used to obtain the minimax solution. The first method is a graphical solution based on the Elzinga-Hearn algorithm. The Elzinga –Hearn algorithm is a graphical method for solving the minimax objective function for a location problem. The method funds is the smallest circle covering method where the objective is to find the smallest possible circle which encompasses all the existing locations in the plane. The method is shown in the flowchart below.

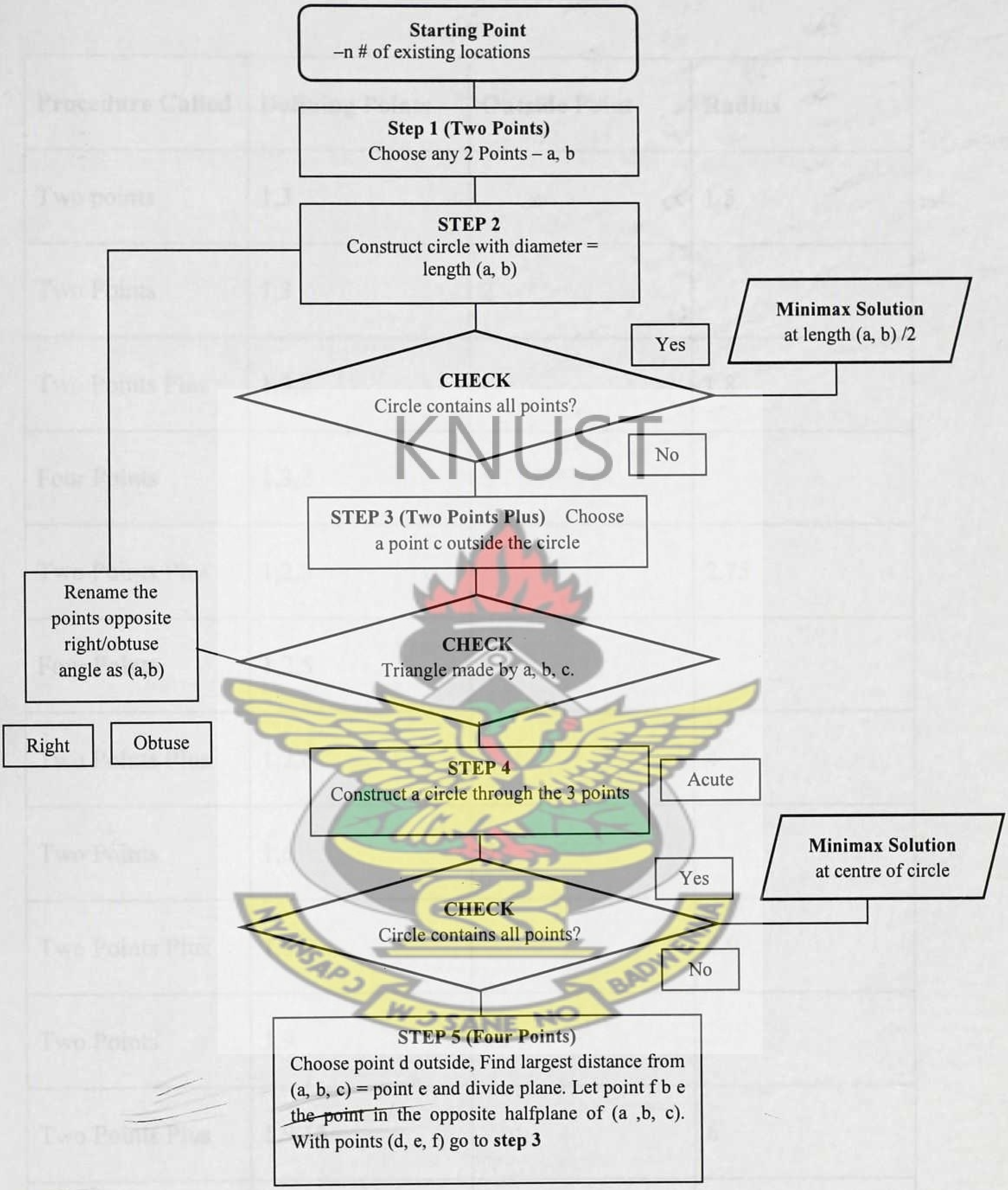


Figure 3.3: Flowchart for Elzinga – Hearn Algorithm

Table 3.3.Elzinga – Hearn Algorithm

Procedure Called	Defining Points	Outside Point	Radius
Two points	1,3		1.5
Two Points	1,3	2	
Two Points Plus	1,3,2		1.8
Four Points	1,3,2	5	
Two Points Plus	1,2,5		2.75
Four Points	1,2,5	6	
Two Points Plus	1,2,6		4
Two Points	1,6	9	
Two Points Plus	1,6,9		4.9
Two Points	1,9	15	
Two Points Plus	1,9,15		6
Two Points	1,15	None	

The minimax solution is obtained at $x = 6.5$ and $y = 5.75$. Excel solver was also used to obtain the minimax solution and verify the results obtained by using the Elzinga – Hearn algorithm. The next step is to find the distance between the calculated minimax solution and all the other existing location points. This provides the starting point for the k – centra algorithm. Distance algorithm is used for this computation. The distances are sorted in descending order as shown in Table 3.4. The top distances are considered and the problem is evaluated for just the obtained k farthest points.

Table 3.4: Distance Algorithm

Points	Distance
P1	5.857687
P15	5.857687
P12	5.550901
P7	5.482928
P8	5.006246
P2	4.828302
P3	4.038874
P14	3.816084
P9	3.75
P4	3.473111
P6	3.25
P13	2.926175
P5	1.820027
P11	1.677051
P10	0.901388

Considering k=6 for this illustration, points P1, P15, P12, P7, P8 and P2 are the points farthest away from the minimax solution (6.5,5.75). Considering the minimax solution as the starting solution, Weiszfeld's algorithm is used to solve the minisum objective function. The Weiszfeld's function is shown in Equation 3.10.

$$\textit{Minimize } f(x,y) = \sum w_i \sqrt{(x - a_i)^2 + (y - b_i)^2} \qquad \text{..................(3.1)}$$

The next step is to solve for everyγ_i as shown in Equation 15. x and y are the starting solution coordinates.

$$\gamma_i = \frac{1}{\sqrt{(x - a_i)^2 + (y - b_i)^2}} \qquad \text{................(3.2)}$$

The γ_i values obtained are shown in Table 3.5

Table 3.5: **γ_i Values**

γ1	0.17072
γ12	0.18015
γ2	0.20711
γ8	0.19975
γ7	0.18238
γ15	0.17072

The next step is to obtain the $\Gamma(x,y)$ values by summing for every γ_i. The value of $\Gamma(x,y)$ comes out to be 1.1108 in this case. The $\Gamma(x,y)$ value calculation is shown in Equation 3.3.

$$\Gamma(x,y) = \sum_{i=1}^{m} \gamma_i(x,y) \qquad \ldots\ldots\ldots\ldots\ldots(3.3)$$

The next step to determine all $\lambda_i$values as shown in Equation 3.4 and the values obtained are listed in Table 3.6.

$$\lambda_i(x,y) = \frac{\gamma_i(x,y)}{\Gamma(x,y)} \qquad \ldots\ldots\ldots\ldots\ldots(3.4)$$

Table 3.6: λ_i values

λ_1	0.15368
λ_{12}	0.16218
λ_2	0.18645
λ_8	0.17982
λ_7	0.16419
λ_{15}	0.15368

The final step is to find the Weiszfeld's values for x and y and the objective function value. The formulae are listed in Equations 3.5, 3.6, and 3.7. The WF values are presented in Table 3.7.

Weiszfeld's (x) = WFx= $\sum (i = 1 \text{ to } m)\ \lambda_i.a_i$ ………… (3.5)

Weiszfeld's (x) = WFy = $\sum (i = 1 \text{ to } m)\ \lambda_i.b_i$ …………(3.6)

Objective function value: $f_{euc} = \sum w_i \sqrt{(x-a_i)^2 + (y-b_i)^2}$ ……….. (3.7)

Table 3.7: WF values

WF(x)	WF(y)
6.4876	5.29013

The objective function values are shown in Table 3.8.

Table 3.8: Objective Function Values

f_{eucP1}	5.5641
f_{eucP12}	5.8221
f_{eucP2}	4.6688
f_{eucP8}	5.0174
f_{eucP7}	5.1893
f_{eucP15}	6.1717
f_{euc}	32.4334

Using the distance algorithm to find the distance from the obtained minisum solution to all existing locations, it is observed that the same six points are farthest away. In this case, this solution is feasible and optimal for $k = 6$. Thus the k-centra solution for this problem lies at $x = 6.4876$ and $y = 5.29013$.

CHAPTER 4

DATA ANALYSIS AND MODELLING

4.1. Study Area (BrongAhafo Region)

The BrongAhafo Region is located in mid-western Ghana, between the Ashanti Region and the Northern Region. Its capital is Sunyani. The region was created in 1958 out of the then Western Ashanti. The region is the second largest region of Ghana in terms of land mass with a territorial size of 39,557,08sq.kms. There are twenty-two (22) districts in the region.

The region is boarded on the north by the Northern Region, Ashanti and Western on the South, Eastern and Volta on the Southeast and east respectively and the republic of La Cote d'ivoire to the West.

According to the 2000 population censes, the BrongAhafo region has a population of about 1824,827 with an average growth rate of 3.1% and an economically active part of 45% representing the 15-65 age bracket.

The region has a tropical climate with high temperatures of between 23% and 39% enjoying, however maximum rainfall of 45° in the northern parts to 65° in the south of the region.

The region can boast of first class roads linking its major towns to the regional capital, Sunyani and the rest of the country.

The region has one secondary hospital which is the regional hospital in Sunyani, 24 primary hospitals according to National Health Insurance Authority (NHIA) accredited facilities – July 2009 to July 2011.

Sunyani has airport, which connects the region by air to Kumasi, Accra and Takoradi. Air transportation is however irregular and is therefore unreliable. Lake BrongAhafo has three in-land lake ports on its portion of the Volta Lake. These are Yeji, New Buipe and Yapei, which can all be reached from Akosombo using the Yapei Queen among others.

4.2 Data Collection

The Government Agency that was contacted for the primary data and other important information for this thesis was the Regional Town and Country Planning Department (Sunyani) where the map of the BrongAhafo Region was obtained.The names of the hospitals in the region was also obtained from the regional headquarters of National Health Insurance Authority (NHIA) and the data on the number of referred cases was taken from the transport unit of Berekum Holy family Hospital and Sunyani Regional Hospital.

Table 4.1: Table showing Sources of Data

Data	Source
Map of Brong-Ahafo Region	Regional Town and Country Planning Department
Names of Hospitals in the Region	National Health Insurance Authority (NHIA)
Referred Cases	Berekum Holy Family Hospital and Sunyani Regional Hospital

4.3 Map of BrongAhafo Region with its District capitals

The BrongAhafo Regional map which shows the district capitals was obtained from Regional Town and Country Planning Department which was used for the thesis.

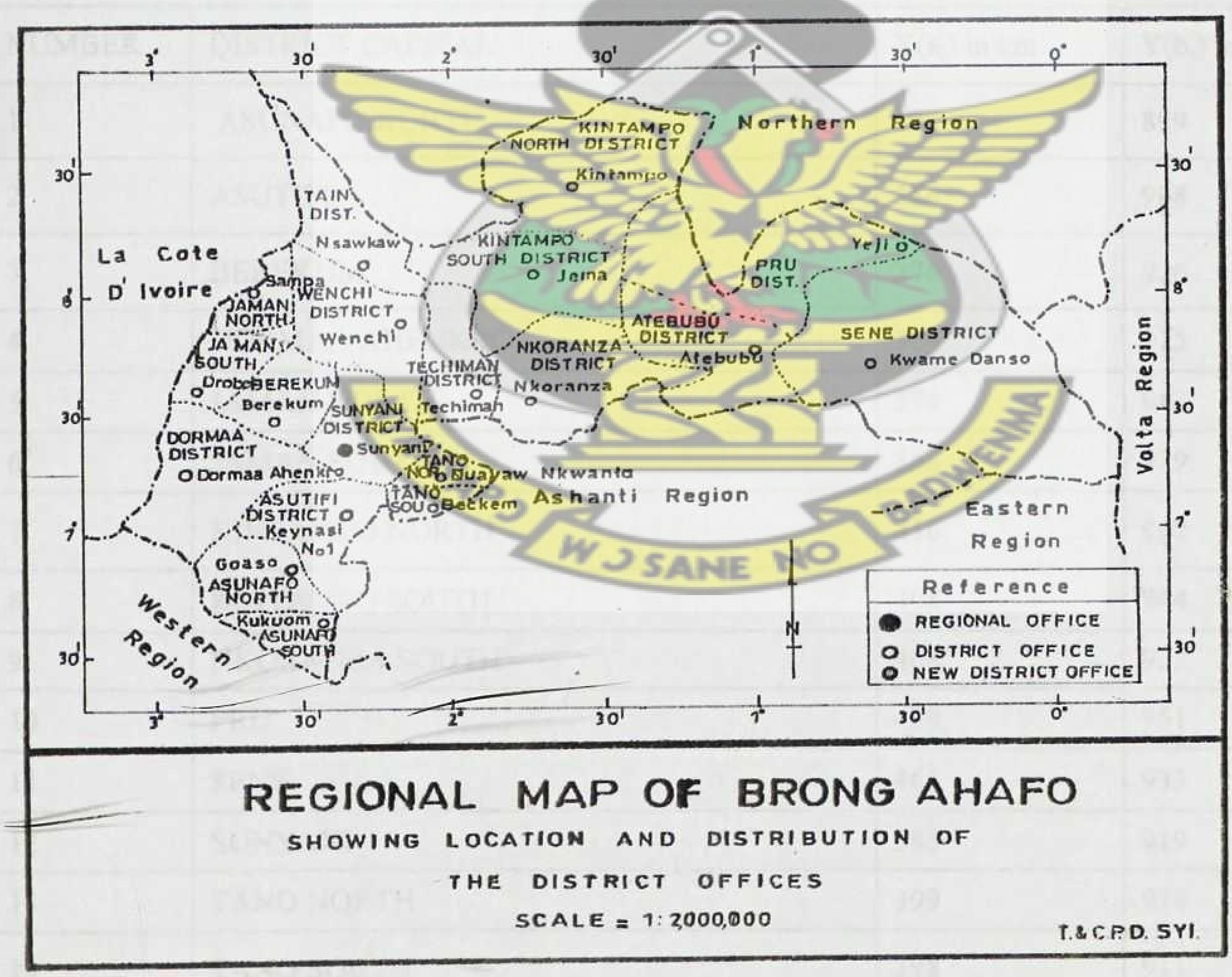


Figure 4.1: shows the BrongAhafo Regional map with its district capitals.

4.4 Data Processing

The rectangular co-ordinates of the district capitals were obtained with the help of a grid sheet since the co-ordinates could not be found on the internet (Google map and Microsoft Encarta).The district capitals were coded as numbers from 1 to 16. The co-ordinates were converted from hundred thousand to hundreds. For example, the co-ordinate for Asunafo North becomes (379, 899) instead of (379000, 899000).

Table 4.2 shows a list of 16 district capitals with their number codes.

Table 4.2; Table Showing number codes for district capitals and their various x and y co-ordinates

NUMBER	DISTRICT CAPITAL	$X(a_i)$ in km	$Y(b_i)$ in km
1	ASUNAFO NORTH	379	899
2	ASUTIFI	386	908
3	BEREKUM	376	926
4	DORMAA AHENKRO	364	915
5	JAMAN NORTH	374	943
6	JAMAN SOUTH	366	929
7	KINTAMPO NORTH	416	962
8	KINTAMPO SOUTH	408	944
9	NKORANZA SOUTH	408	927
10	PRU	459	951
11	SENE	465	933
12	SUNYANI	385	919
13	TANO NORTH	399	914
14	TANO SOUTH	398	911
15	TECHIMAN	403	928
16	WENCHI	393	937

4.5 Model Formulation

$f_{euc} = \sum w_i\sqrt{(WFx - x)^2 + (WFy - y)^2}$ Where ,

x values of the existing location are in column 3 of Table 4.1.

y values of the existing location are in column 4 of Table 4.1.

$W_i=1$ $i=1, 2, 3,\ldots,16$

4.5.1 The Planar k-centra Algorithm

The sixteen rectangular co-ordinates were used as the inputs for the Planar k-Centra Single-Facility Euclidean Location ProblemAlgorithm coded in Mathlab.The steps for the algorithm are shown below;

Step1: Input initial coordinates (x,y), the minimax.

Step 2: Solve for every γ_i as per Equation 4.1.

$$\gamma_i = \frac{w_i}{\sqrt{(x - a_i)^2 + (y - b_i)^2}} \quad \ldots\ldots\ldots\ldots\ldots(4.1)$$

Where,

x=x-coordinates of starting point for the iterative algorithm

y=y-coordinates of starting point for the iterative algorithm

Step 3: sum for every γ_i for $\Gamma(x,y)$ shown by Equation 4.2.

$$\Gamma_1(x,y) = \sum_{i=1}^{m} \gamma_i\,(x,y) \qquad \ldots\ldots\ldots\ldots\ldots(4.2)$$

Step 4: determine all $\lambda_i = \gamma_i/\Gamma$ as shown in Equation 4.3

$$\lambda_i(x,y) = \frac{\gamma_i\,(x,y)}{\Gamma(x,y)} \qquad \ldots\ldots\ldots\ldots\ldots(4.3)$$

Step 5: Weiszfeld's (x) = WFx= $\sum\,(i = 1 \ldots m)\ \lambda_i . a_i$ ……………(4.4)

Weiszfeld's (y) = WFy = $\sum\,(i = 1 \ldots m)\ \lambda_i . b_i$ ……………(4.5)

Step 6: Determine the objective function value by summing the individual objective function values for all i as per Equation 4.6.

$$f_{euc} = \sum w_i \sqrt{(\mathrm{WFx} - a_i)^2 + (\mathrm{WFy} - b_i)^2} \qquad \ldots\ldots\ldots(4.6)$$

Step 7:Repeat until stopping conditions are met that is $\left| f_{eucn+1} - f_{eucn} \right| \leq 0$

Considering a random starting solution, the algorithm is evaluated to obtain a second set of trial values (WFx, WFy). The obtained coordinates are substituted to obtain the third set of trial values and so on. By reiterating the Weisfeld's values for x, y, we can find the Euclidean solution close to or equal to the optimal solution. The stopping conditions are met when the difference between subsequent objective function values obtained is less than or equal to zero.

4.6 Computational Procedure

Matlab program software (Siddarth, 2005) was used for the coding of the Planar k-Centra Single-Facility Euclidean Location Problem algorithm.

The codes were developed and ran on the Intel(R) Pentium(R) Dual CPU T2370, 32 BG Operating system, 1014 MB RAM, 1.73 GHZ speed, with Windows Vista laptop computer. The code runs successfully on the windows vista.

The number of iterations was 100and 4 test runs were carried out.

4.7 Results

The minimax location was (414.0km, 927.0km). The algorithm gave one location as the optimal point. That is (397.3km, 929.2km) and with 273.8km as the objective function value (The sum of the distances of 6 farthest locations away from the optimal location).

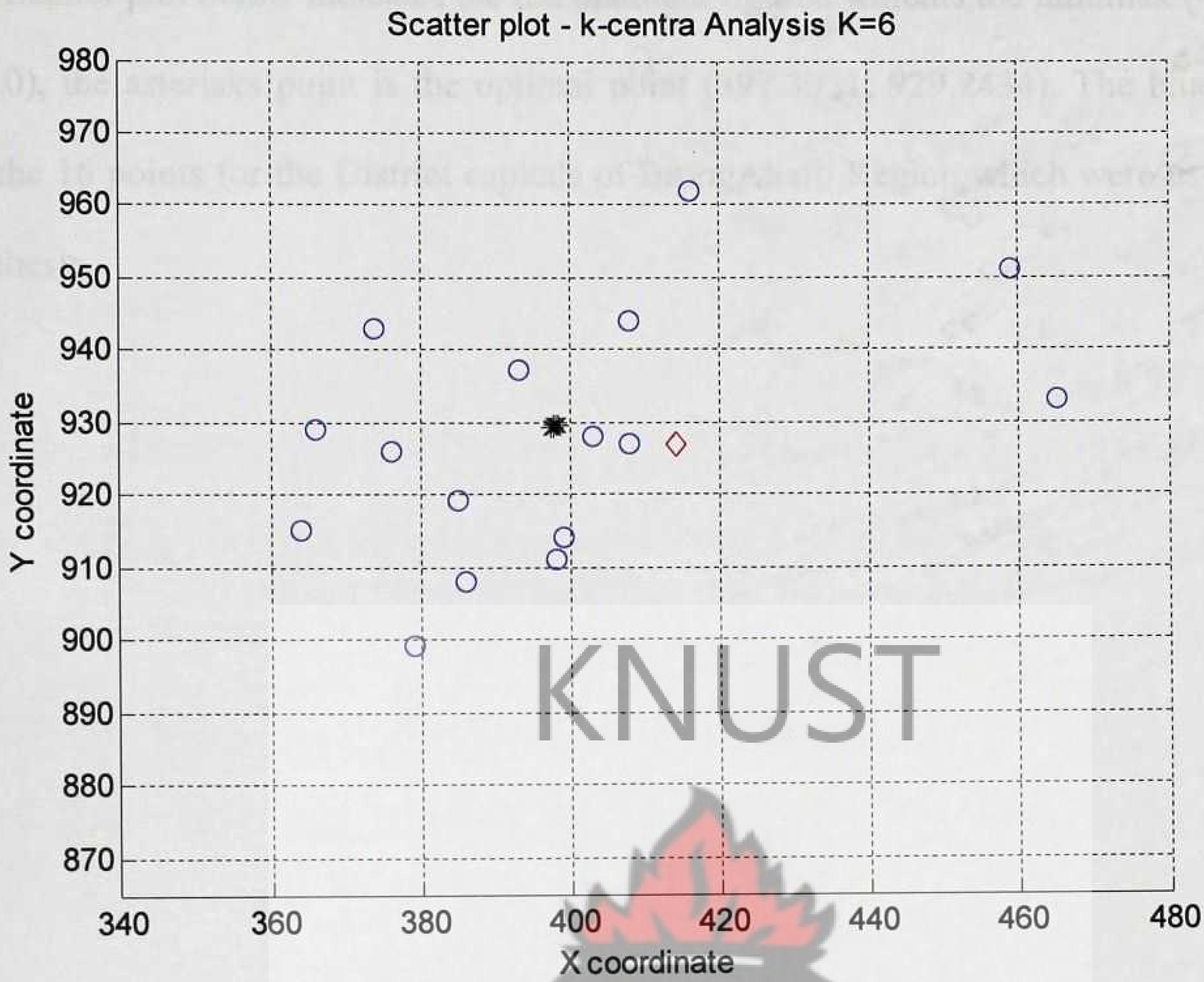


Figure 4.2: The Figure is showing the Sixteen (16) Scatter Points, the Diamond Figured Point is the Minimax and the Asterisk is the Optimal Point

4.8 Discussion of Results

This work is similar to the facility located by Fernandez, et al. (2009) in the city of Murcia in South-East Spain.

The algorithm gave one location as the optimal point. That is (397.3km, 929.2km) with273.8km as the respective objective function value (The sum of the distances of 6 farthest locations away from the optimal location).

The Scatter plot below indicates the red diamond figured whichis the minimax (414.0, 927.0), the asterisks point is the optimal point (397.3051, 929.2434). The blue dots are the 16 points for the District capitals of BrongAhafo Region which were used for the thesis.

Since BrongAhafo Region was used as the case study, the researcher recommends the point (397.3km, 929.2km)which is located at Tadiesoto the regional office, contractors, urban and feeder roads and other developers (Stakeholders) thatTadiesois one of the most closest to the six (6) farthest districts in the region.

REFERENCES

Cooper, L. (1963) "Location-allocation problems" Operation Research, 11: 331-343.

Cavalieri, T. M. (1647) Exercitiosexta.In CAVALIELI, "Exercitationesgeometricae."Bologna, 1647.

Daskin, M. S. and Owen, S. H. (1998) "Two new location of covering problems: the partial covering p-center problem and the partial set covering." Geographical Analysis, 1: 50-100.

Drezner, Z. (1993) "recent result in competitive facility location." Studies in Locational Analysis, 4: 97-103.

Drezner, Z. (1985) "Sensitivity analysis of the optimal of a facility." Naval Research Logistics Quarterly, 32: 209-224.

Fermat, P. D. (1643) Methodus de maximisetminimis. In Tannery, H. (Ed), "Fermat-Oeuvres I.", Paris, 1896, supplement, Paris.

Fernandez, J., Pelegrin, B., Plastria, F. and Boglark, T. (2009) "Planar Location and design of a new facility with inner and outer competition of interval hexicographical – Like Station Procedure." Computer and Operations Research, 1: Pp 15-30.

Francis, R. L. ,McGinnis, L. F. and White, J. A. (1983) "Locational analysis," European Journal of Operational Research, 12: 220-252.

Francis, R. L. (1963) "A note on the optimum location of new machines in existing plant layout." The Journal of Industrial Engineering, 14: 57-59.

Francis, R. L. (1964) "On the location of multiple new facilities with respect to existing facilities." The Journal of Industrial Engineering, 15: 106-107.

Friedrich, F. (1929) "Alfred Weber's theory of the location of industries."Chicago University Press, Chicago, Illinois.

Francis, R. L. ,McGinnis, L. F. and White, J. A. (1983) "Locational analysis," European Journal of Operational Research, 12: 220-252.

Hakimis, S. L. (1964) "Optimum location of switching centres and absolute centre and medians of a graph." Operational Research, 12: 450-459.

Hotelling, H. (1929) "Stability in competition." Economic Journal, 39: 41-57.

Huff, D. L. (1966) "A programmed solution for approximating an optimum retail location." Land Economics, 42: 292-303.

Helpern, J. (1976) "The location of cent-dian convex combination on an undirected tree." Journal of Regional Science, 16: 237-245.

Hansen, P., Labbe, M. and Thisse, J. F. (1991) "An overview of Representative Problems in Location Research." RAIRO RechercheOperationelle, 25: 73-86.

Ogryczak, W. and Zawadzki, M. (2002) "Conditional Median: A Parametric Solution Concept for Location Problems." Annals of Operations Research, 110: 167-181.

Peeter's, P. H. (1998) "Some New Algorithms for Location Problems on Networks." European Journal of Operational Research, 109: 299-309.

Rosen, J. B. and Xue, G. L. (1993) "On the convergence of a hyperboloid approximation procedure for the perturbed Euclidean multifacility location problem." Operations Research, 41: 1164-1171.

Slatter, P. J. (1978) "Stracture of the k-centra in a tree." Proceedings of the 9th Southeast Conference on Combinatorics, Graph Theory and Computing, 663-670.

Siddarth, R. K., Bonilla, C. and Hale, T. S.(2008) "The Planar k-centra Location Problem". The open Industrial and Manufacturing Engineering Journal'.1: 42-49.

Siddarth, R. K. (2005) "Planar K-Centra Single Facility Euclidean Problem". The open Industrial and manufacturing Engineering Journal'1: 1-133.

Tamir, A. (2001) "the K-centrum Multi-Facility Location Problem." Discrete Applied Mathematics, 109: 293-307.

Weber, A. (1909) "Ueber den standort der industrien.ErsterTeil. ReineTeorie der standorte.MiteinemmathematischenAnhang von G. PICK."Verlag. Tubingen, Germany.

Weizfeld, E. (1937) "Sur le point pour la somme des distances de n point donnesest minimum." Tohoku Mathematical journal, 43: 355-386.

APPENDICES

APPENDIX A-MATLAB METHODOLOGY

1. MATLAB is a high-level computer language recognize worldwide in the mathematical community for its computational prowess in performing various complex mathematical tasks. It is superior to various traditional solving methods as well as programming languages like C, or C++ for handling computationally intensive projects. The MATLAB package is available with many software add-ons to solve a multitude of problems in various application areas. It is an interactive tool with options for 2-D or 3-D visualization of input or output data. MATLAB can be integrated with various other software applications to suit every computational need.

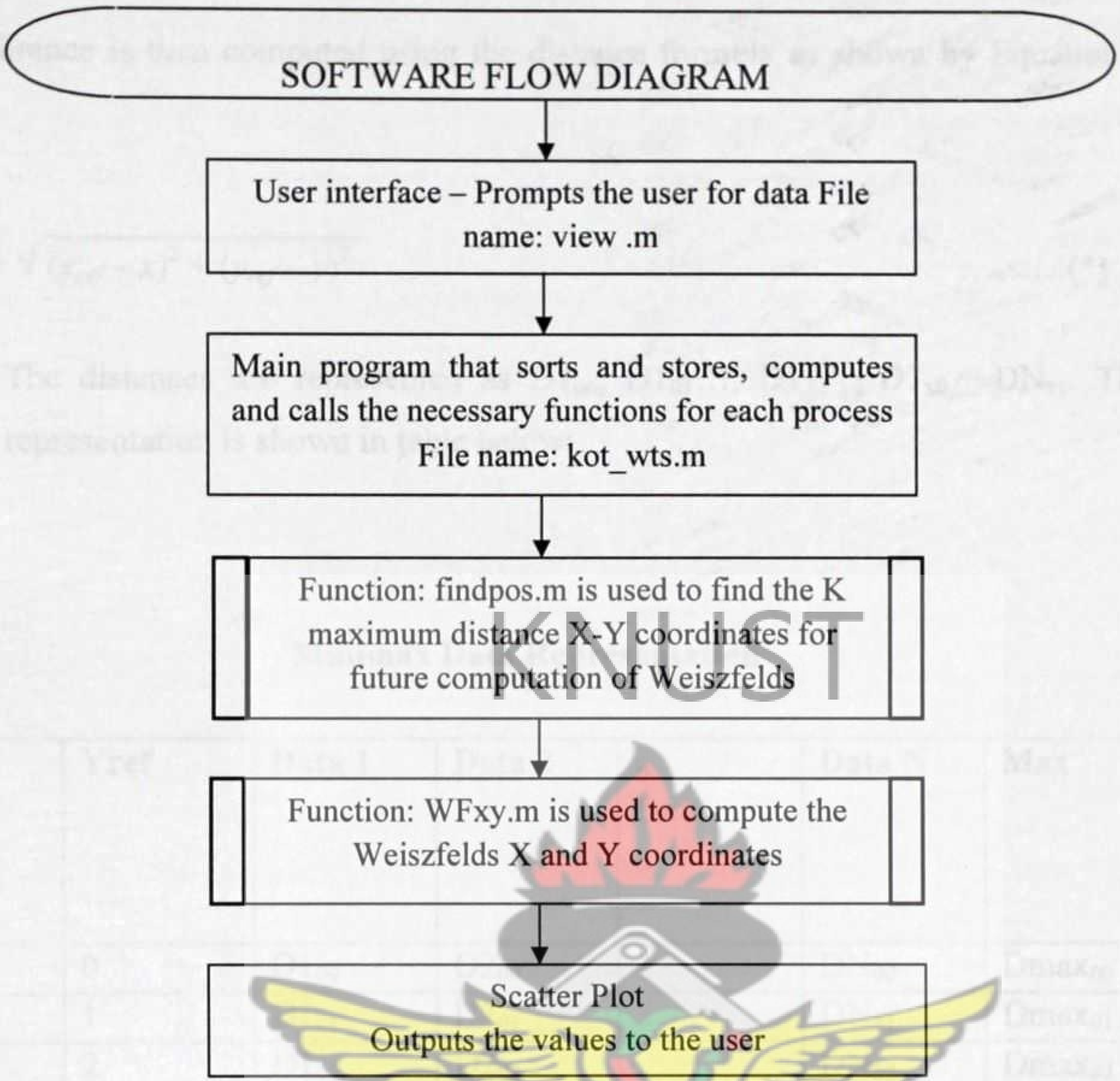
MATLAB Application to the k-Centra Problem

The advantage of using MATLAB to solve a location problem over traditional manual methods is better accuracy as well as increased computational speed. The manual method for evaluating k-centra problems is a very tedious one involving much iteration to get a more accurate solution. The other advantage is you can massage the code to suit your computational needs. The code discussed later in this appendix c has input options which are user-defined like the number of existing locations, *value* of *k,* number of iterations, search space boundaries, and search space increment. The code has been programmed to output the coordinate solution in a graphical representation. You can add a lot more features like inputting coordinates data directly from other applications, 3-D visualization, report generation, etc.

The same logic used for manually solving a k-centra problem is used to code the MATLAB program.

Data Flow Logic and MATLAB Functions

The k-centra example problems involved were solved in MATLAB 7.5.0(R2007B), in the following manner. The user interface is a basic MATLAB program that prompts the user to input various data point coordinates and other parameters. User input includes the number of existing locations (N) and respective coordinates entered one by one (x and y separately), maximum range of x and y, search space increment, k-term, and number of iterations. The added advantage is that this can be modified easily as per the user's requirement and future upgrades. This program then calls the main program which stores the user inputted data, saves and computes the various distances and k-centra parameters by calling the respective functions.



Software Flow Diagram

APPENDIX B-MINIMAX LOGIC

The minimax function forms an important part of the logic flow for this program. The minimax function is called upon after the "View" function and uses user input as its starting variables. Data used for the function includes "N" number of existing locations and corresponding coordinates. Consider the N data points to be Data 1, Data 2…….Data N. The user reference is initiated as Xref and Yref and is started at (0, 0) and is incremented on the basis of user input. The maximum range threshold is also set by the user (Xmax and Ymax). The distance or range for each data point from this reference is then computed using the distance formula as shown by Equation * below;

$$DN_{xy} = \sqrt{(x_{ref} - x)^2 + (y_{ref} - y)^2} \qquad \text{.........(*)}$$

The distances are represented as $D1_{oo}$, $D1_{01}$..., $D1_{20}$..., $D1_{x0}$..., DN_{xy}. The tabular representation is shown in table below;

Minimax Data Representation

Xref	**Yref**	**Data 1**	**Data 2**	**.. Data ..**	**Data N**	**Max**
0	0	$D1_{00}$	$D2_{00}$	..	DN_{00}	$Dmax_{00}$
0	1	$D1_{01}$	$D2_{01}$	..	DN_{01}	$Dmax_{01}$
0	2	$D1_{02}$	$D2_{02}$	..	DN_{02}	$Dmax_{02}$
..	..	..	..	..	..	
0	Ymax=y	$D1_{0y}$	$D2_{0y}$	..	DN_{0y}	$Dmax_{0y}$
1	0	$D1_{10}$	$D2_{10}$	..	DN_{10}	$Dmax_{10}$
1	1	$D1_{11}$	$D2_{11}$	..	DN_{11}	$Dmax_{11}$
1	2	$D1_{12}$	$D2_{12}$	..	DN_{12}	$Dmax_{12}$
..	..	..	..	..	..	
2	0	$D1_{20}$	$D2_{20}$	..	DN_{20}	$Dmax_{20}$
2	1	$D1_{21}$	$D2_{21}$	..	DN_{21}	$Dmax_{21}$
..	..	..	..	..	..	
..	..	..	..	..	..	
Xmax = x	Ymax = y	$D1_{xy}$	$D2_{xy}$	..	DN_{xy}	$Dmax_{xy}$

After each increment, and progressively stepping through the search grid, the program computes the maximum of each of these distances and is represented in the program as $Dmax_{00}$..., $Dmax_{xy}$shown in table below

Maximum Distance Matrix

$Dmax_{00}$	$Dmax_{01}$	…….	$Dmax_{oy}$
$Dmax_{10}$	$Dmax_{11}$	…….	$Dmax_{1y}$
$Dmax_{20}$	$Dmax_{21}$	…….	$Dmax_{2y}$
……..	…….	…….	…….
$Dmax_{x0}$	$Dmax_{x1}$	…….	$Dmax_{xy}$

The minimum values from each of the values are calculated and forms an intermediate step in the calculation. The matrix is shown in table;

Intermediate Distance Matrix

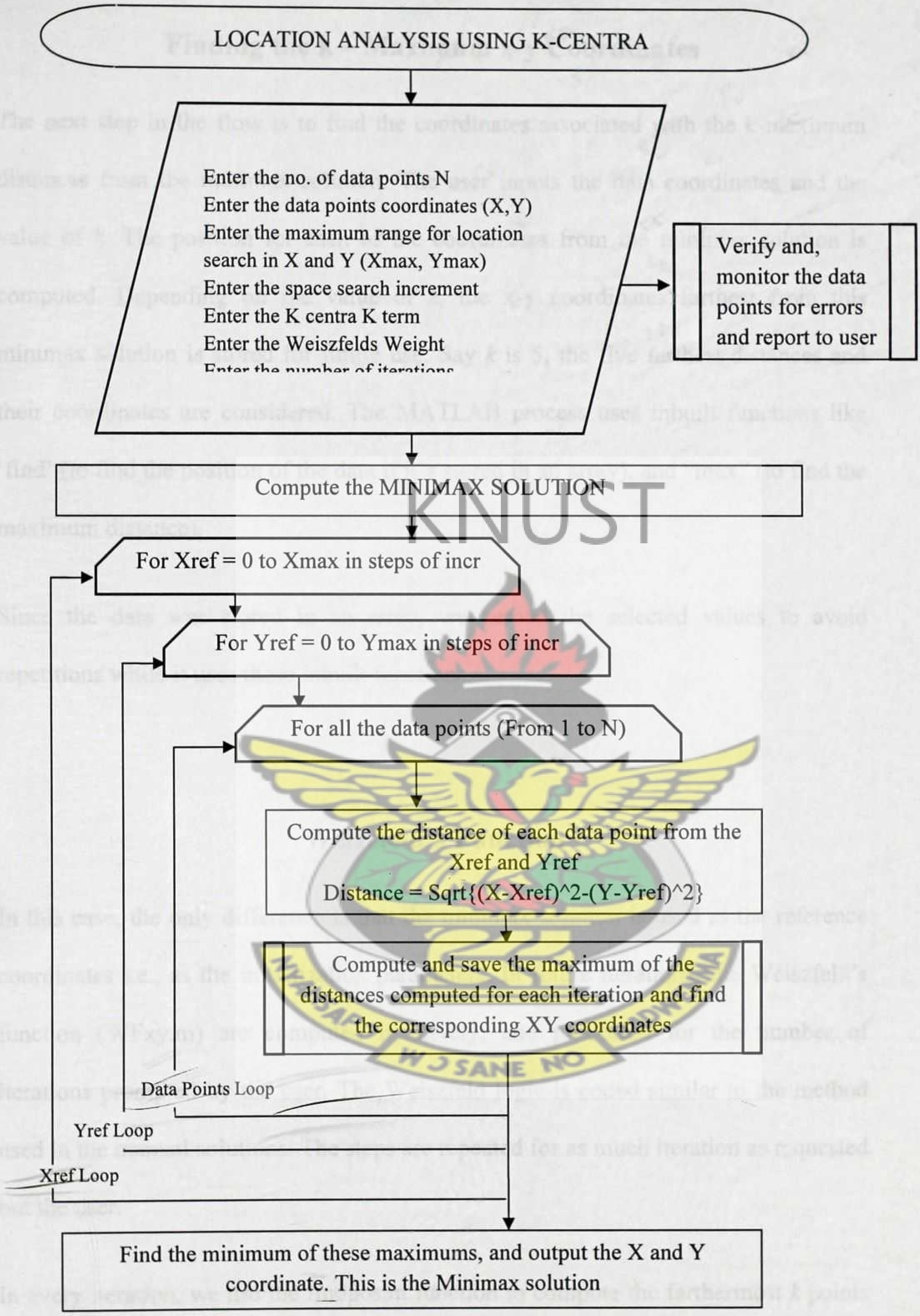
Min of $Dmax_{00}$: $Dmax_{x0}$	Min of $Dmax_{01}$: $Dmax_{x0}$	……..	Min of $Dmax_{0y}$: $Dmax_{xy}$

The next step is to find the minimum of these intermediate minimums. This is the minimax solution distance. We can trace the (Xref, Yref) coordinate where this occurs as the maximum distances matrix directly represents the search grid of (Xref, Yref) coordinates as shown in Table 14 below, and the user data points (D1, D2….DN) will also lie inside this grid.

Search Grid Coordinates

00	01	02	…….	0y
10	11	12	…….	1y
20	21	22	…….	2y
….	….	…..	…….	…..
x 0	x 1	x 2	…….	Xy

Thus the minimax solution, its coordinates and its distance from other existing locations can be calculated. The flow chart for the minimax function is provided in figure below. The function code is provided in this appendix.



Minimax Function Flowchart

Finding the k – Maximum x-y Coordinates

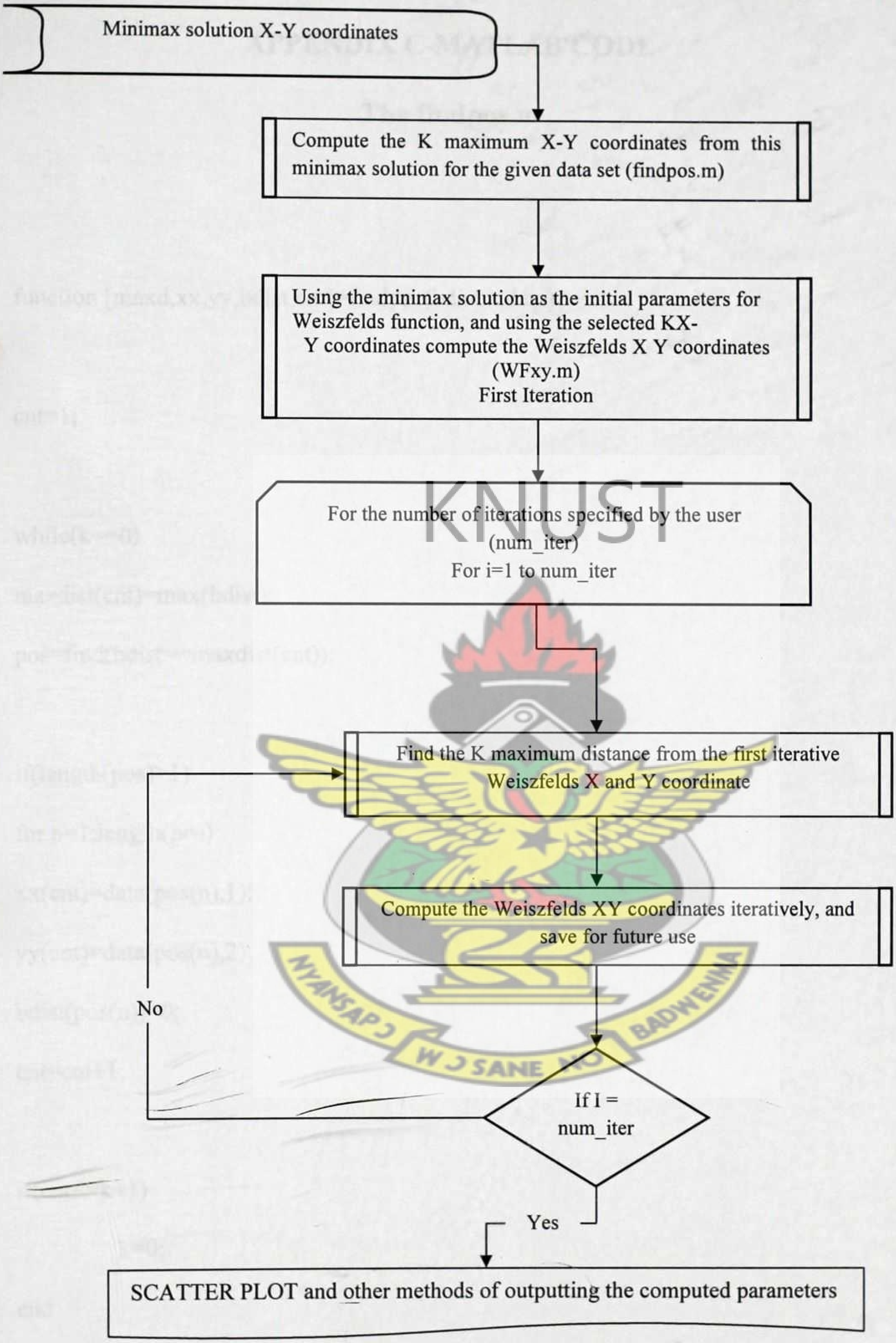
The next step in the flow is to find the coordinates associated with the k-maximum distances from the minimax solution. The user inputs the data coordinates and the value of *k.* The position for each of the coordinates from the minimax solution is computed. Depending on the value of *k*, the x-y coordinates farthest from this minimax solution is stored for future use. Say *k* is 5, the five farthest distances and their coordinates are considered. The MATLAB process uses inbuilt functions like 'find' (to find the position of the data if it's stored in an array), and "max" (to find the maximum distance).

Since the data was stored in an array, we zeroed the selected values to avoid repetitions while it uses these inbuilt functions.

Weiszfeld's Function

In this case, the only difference is that the minimax solution is used as the reference coordinates i.e., as the initialization parameters. In future iterations, the Weiszfeld's function (WFxy.m) are computed iteratively, and processed for the number of iterations prompted by the user. The Weiszfeld logic is coded similar to the method used in the manual solutions. The steps are repeated for as much iteration as requested but the user.

In every iteration, we use the findpos.m function to compute the farthermost *k* points for each iteratively computed WFxy. The flowchart is given below;



Weiszfeld's Function Flowchart

APPENDIX C-MATLAB CODE

The findpos.m

```
function [maxd,xx,yy,bdist,cnt]=findpos(bdist,k,data)

cnt=1;

while(k~=0)

maxdist(cnt)=max(bdist);

pos=find(bdist==maxdist(cnt));

if(length(pos)>1)

for n=1:length(pos)

xx(cnt)=data(pos(n),1);

yy(cnt)=data(pos(n),2);

bdist(pos(n))=0;

cnt=cnt+1;

if(cnt==k+1)

        k=0;

end

end

clearpos
```

```
else

for n=1:length(pos)

xx(cnt)=data(pos(n),1);

yy(cnt)=data(pos(n),2);

bdist(pos(n))=0;

cnt=cnt+1;

if(cnt==k+1)

        k=0;

end

end

% %

clearpos

end

maxd=0;

end
```

The WFxy.m

```
function [WFx,WFy]=WFxy(kpts,bestx,besty,K,Wt)

for g=1:1:K
   gamma(g)=Wt/sqrt((kpts(g,1)-bestx)^2+(kpts(g,2)-besty)^2);
end

tau=sum(gamma);

for l=1:1:K
lamda(l)=gamma(l)/tau;
lam_x(l)=lamda(l)*kpts(l,1);
lam_y(l)=lamda(l)*kpts(l,2);
end

WFx=sum(lam_x);
WFy=sum(lam_y);
```

The codes for the Planer k-centra

```
clearall;

closeall;

clc;


no=input('Number of data points ');


for i=1:no

fprintf('Data Point%d\n',i);

data(i,1)=input('X ');

data(i,2)=input('Y ');

%W(i)=input('W ');

fprintf('\n');

end


xmax=input('Max Range of X ');

ymax=input('Max Range of Y ');

incr=input('Search-Space Increment ');

K=input('K Centra K term ');

Wt=input('Weizfelds Weight ');

num_iter=input('Number of Iterations ');


% kot_wts


x=1;
```

```
y=1;

forxref=0:incr:xmax
foryref=0:incr:ymax
for i=1:length(data)
dist(i)=sqrt((data(i,1)-xref)^2+(data(i,2)-yref)^2);
end
maxdist(x,y)=max(dist);
        y=y+1;
end
if(y>ymax+1)
        x=x+1;
        y=1;
end
end


xcols=[0:incr:xmax];
ycols=[0:incr:ymax];

intmin=min(maxdist);
minimax=min(intmin);

[row,col]=find(maxdist==minimax);

bestX=xcols(row)
bestY=ycols(col)
```

```
for i=1:1:length(data)

bestdist(i)=sqrt((data(i,1)-bestX)^2+(data(i,2)-bestY)^2);

end


  [maxd,xx,yy,bdist,cnt]=findpos(bestdist,K,data);


kpts=[xx' yy'];

  [WFx,WFy]=WFxy(kpts,bestX,bestY,K,Wt);

for z=1:1:num_iter

cleari

for i=1:1:length(data)

newdist(i)=sqrt((data(i,1)-WFx)^2+(data(i,2)-WFy)^2);

end

[maxd_new,xx_new,yy_new,bdist_new,cnt_new]=findpos(newdist,K,data);

kpts_new=[xx_new',yy_new'];

        [WFx,WFy]=WFxy(kpts_new,WFx,WFy,K,Wt);

WFXY(z,:)=[WFx,WFy];

for l=1:1:K

feucp(l)=Wt*sqrt((kpts_new(l,1)-WFx)^2+(kpts_new(l,2)-WFy)^2);

feuc=sum(feucp)

end
```

```
clearnewdist;

end

  WFXY

scatter(data(:,1),data(:,2))
holdon
grid
xlabel('X coordinate')
ylabel('Y coordinate')
axis([340 480 865 980])
title('Scatter plot - k-centra Analysis K=6')
holdon
scatter(bestX,bestY,48,'rd')
holdon
 scatter(WFXY(:,1),WFXY(:,2),48,'k*')
holdon

%function [maxd,xx,yy,bdist,cnt]=findpos(bdist,k,data)

cnt=1;
k=0;
%while(k~=0);
maxdist(cnt)=max(bdist);
pos=find(bdist==maxdist(cnt));
```

```
if(length(pos)>1)

for n=1:length(pos)

xx(cnt)=data(pos(n),1);

yy(cnt)=data(pos(n),2);

bdist(pos(n))=0;

cnt=cnt+1;


if(cnt==k+1)

                k=0;

end

end

clearpos

else

xx(cnt)=data(pos,1);

yy(cnt)=data(pos,2);

bdist(pos)=0;

cnt=cnt+1;

if(cnt==k+1)

             k=0;

end


%code modification here

for n=1:length(pos)

xx(cnt)=data(pos(n),1);

yy(cnt)=data(pos(n),2);
```

```
bdist(pos(n))=0;

cnt=cnt+1;


if(cnt==k+1)

                k=0;

end

end


clearpos

end

maxd=0;

%end

%function [WFx,WFy]=WFxy(kpts,bestX,bestY,K,Wt);

    gamma(g)=Wt/sqrt((kpts(g,1)-bestX)^2+(kpts(g,2)-bestY)^2);

%end

tau=sum(gamma);


for l=1:K

lamda(l)=gamma(l)/tau;

lam_X(l)=lamda(l)*kpts(l,1);

lam_Y(l)=lamda(l)*kpts(l,2);

end


WFx=sum(lam_X);

WFy=sum(lam_Y);
```

APPENDIX D-MATLAB SOLUTION FOR THE CASE STUDY DATA

(100 ITERATIONS)

NO.	X km	Y km	f km		NO.	X km	Y km	f km
1	398.5	929.5	273.8		2	398.2	929.7	273.8
3	398.2	929.6	273.8		4	397.9	929.6	273.8
5	397.8	929.5	273.8		6	397.8	929.5	273.8
7	397.6	929.4	273.8		8	397.6	929.4	273.8
9	397.5	929.4	273.8		10	397.5	929.3	273.8
11	397.5	929.3	273.8		12	397.4	929.3	273.8
13	397.4	929.3	273.8		14	397.4	929.3	273.8
15	397.4	292.3	273.8		16	397.4	929.3	273.8
17	397.4	929.3	273.8		18	397.3	929.3	273.8
19	397.3	929.3	273.8		20	397.3	929.3	273.8
21	397.3	929.3	273.8		22	397.3	929.3	273.8
23	397.3	929.3	273.8		24	397.3	929.3	273.8
25	397.3	929.2	273.8		26	397.3	929.2	273.8
27	397.3	929.2	273.8		28	397.3	929.2	273.8
29	397.3	929.2	273.8		30	397.3	929.2	273.8
31	397.3	929.2	273.8		32	397.3	929.2	273.8
33	397.3	929.2	273.8		34	397.3	929.2	273.8
35	397.3	929.2	273.8		36	397.3	929.2	273.8
37	397.3	929.2	273.8		38	397.3	929.2	273.8
39	397.3	929.2	273.8		40	397.3	929.2	273.8
41	397.3	929.2	273.8		42	397.3	929.2	273.8
43	397.3	929.2	273.8		44	397.3	929.2	273.8
45	397.3	929.2	273.8		46	397.3	929.2	273.8
47	397.3	929.2	273.8		48	397.3	929.2	273.8
49	397.3	929.2	273.8		50	397.3	929.2	273.8
51	397.3	929.2	273.8		52	397.3	929.2	273.8
53	397.3	929.2	273.8		54	397.3	929.2	273.8
55	397.3	929.2	273.8		56	397.3	929.2	273.8
57	397.3	929.2	273.8		58	397.3	929.2	273.8
59	397.3	929.2	273.8		60	397.3	929.2	273.8
61	397.3	929.2	273.8		62	397.3	929.2	273.8
63	397.3	929.2	273.8		64	397.3	929.2	273.8
65	397.3	929.2	273.8		66	397.3	929.2	273.8
67	397.3	929.2	273.8		68	397.3	929.2	273.8
69	397.3	929.2	273.8		70	397.3	929.2	273.8
71	397.3	929.2	273.8		72	397.3	929.2	273.8

73	397.3	929.2	273.8	74	397.3	929.2	273.8
75	397.3	929.2	273.8	76	397.3	929.2	273.8
77	397.3	929.2	273.8	78	397.3	929.2	273.8
79	397.3	929.2	273.8	80	397.3	929.2	273.8
81	397.3	929.2	273.8	82	397.3	929.2	273.8
83	397.3	929.2	273.8	84	397.3	929.2	273.8
85	397.3	929.2	273.8	86	397.3	929.2	273.8
87	397.3	929.2	273.8	88	397.3	929.2	273.8
89	397.3	929.2	273.8	90	397.3	929.2	273.8
91	397.3	929.2	273.8	92	397.3	929.2	273.8
93	397.3	929.2	273.8	94	397.3	929.2	273.8
95	397.3	929.2	273.8	96	397.3	929.2	273.8
97	397.3	929.2	273.8	98	397.3	929.2	273.8
99	397.3	929.2	273.8	100	397.3	929.2	273.8