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DEPARTMENT OF MATHEMATICS

PROMETHEE METHODOLOGY TO SELECT LEAST COST TELECOMMUNICATION NETWORK OPERATOR IN THE ASHANTI REGION

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BY

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DECLARATION

I hereby declare that this submission is my own work towards the award of Master of Science in Industrial Mathematics and that, to the best of my knowledge; it contains neither material previously published by another person nor material, which has been accepted for the award of any other degree of the University, except where the acknowledgement has been made in the text.

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DEDICATION

It is great to acknowledge the presence of Almighty Jehovah and to receive His enormous benefits to make this thesis a reality. This thesis is dedicated to my wife Agnes Boateng, Madam Beatrice, Kodua and many friends who encouraged me to come up to this realization.

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Jehovah is my light and my salvation; he has been my father, friend and God. I thank him so much for seeing me through it all even when I felt like given up. I am very grateful to my supervisor Mr. Kwaku Darkwa who made me work hard for my own benefit and for his immense contributions, advice and support he gave me in making this work a success. I say a very big thank you. My sincere gratitude to my dear wife Agnes Boateng for her encouragement, tolerance and patience through this hard period. I render unlimited appreciation to Madam Beatrice and the family for their support. Finally to my coach Mr. John. N. Kodua and all those who contributed in diverse ways towards this thesis I say God richly bless you.

ABSTRACT

The selection of a mobile network operator has become a problem to the subscriber. This is so because the individual is faced with the option to choose so as to reduce cost.

Promethee methods have taken an important place among the existing outranking multiple criteria methods.

In this thesis the Promethee method was used to rank the six (6) mobile network operators based on six (6) given criteria data.

Conclusions and recommendations were given based on the results from the ranking of the six (6) network operators

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CHAPTER ONE

INTRODUCTION

1.1 Background of the study

The Ashanti region is located in south Ghana and the third largest of ten (10) administrative regions in Ghana. (Wikipedia, 2013).It is occupying a total land surface oftwenty four thousand, three hundred and eighty-nine (24389) km square and this represent 10.2 percent of the total land surface of Ghana (Wikipedia 2013). It is the third largest region after Northern (70,384 sq. kms) and BrongAhafo (39,557 sq. kms) regions

The main language spoken is Twi. The scenic and hilly Kumasi is the regional capital, the second largest city in Ghana. It has also served for three centuries as the royal.

The region has a population density of 148.1 persons per square kilometer, the third after Greater Accra and Central Regions. More than half of the region lies within the wet, semi-equatorial forest zone

Due to human activities and bushfires, the forest vegetation of parts of the region, particularly the north-eastern part, has been reduced to savanna. The region has an average annual rainfall of 1270 mm and two rainy seasons. The major rainy season starts in March, with a major pick in May. There is a slight dip in July and a pick in August, tapering off in November. December to February is dry, hot, and dusty

The Ashanti Region is centrally located in the middle belt of Ghana. It lies between longitudes 0.15W and 2.25W, and latitudes 5.50N and 7.46N. The region shares boundaries with four of the ten political regions, Brong-Ahafo in the north, Eastern region in the east, Central region in the south and Western region in the South west.

The region is drained by Lake Bosomtwe, the largest natural lake in the country, and Rivers Offin, Prah, Afram and Owabi. There are other smaller rivers and streams which serve as sources of drinking water for residents of some localities in the region. The political administration of the region is through the local government system .Under this administration system, the region is divided into 27 districts. Each district ,municipal or metropolitan area is administer by a chief executive representing the central government but deriving authority from an assembly headed by a presiding member elected from among the members themselves. (Wikipedia, the free encyclopedia).

1.1.2 Mobile phone

A mobile phone (also known as a cellular phone, cell phone, hand phone, or simply a phone) is a phone that can make and receive telephone calls over a radio link while moving around a wide geographic area. It does so by connecting to a cellular network provided by a mobile phone operator, allowing access to the public telephone network. By contrast, a cordless telephone is used only within the short range of a single, private base station. In addition to telephony, modern mobile phones also support a wide variety of other services such as text messaging, MMS, email, Internet access, short-range wireless communications (infrared, Bluetooth), business applications, gaming, and photography. Mobile phones that offer these and more general computing capabilities are referred to as smart phones. The first hand-held cell phone was demonstrated by John F. Mitchell and Dr. Martin Cooper of Motorola in 1973, using a handset weighing around 4.4 pounds (2 kg). In 1983, the DynaTAC 8000x was the first to be commercially available. From 1983 to 2014, worldwide mobile phone subscriptions grew from zero to over 7 billion (Wikipedia, the free encyclopedia, 2014).

1.1.3 Uses of Mobile Phone

Mobile phones are used for a variety of purposes, including keeping in touch with family members, conducting business, and having access to a telephone in the event of an emergency. Some people carry more than one cell phone for different purposes, such as for business and personal use. Multiple SIM (Subscriber Identity Module) cards may also be used to take advantage of the benefits of different calling plans — a particular plan might provide cheaper local calls, long-distance calls, international calls, or roaming. The mobile phone has also been used in a variety of diverse contexts in society, for example:

- ✓ A study by Motorola found that one in ten cell phone subscribers have a second phone that often is kept secret from other family members. These phones may be used to engage in activities including extramarital affairs or clandestine business dealings.
- ✓ Some organizations assist victims of domestic violence by providing mobile phones for use in emergencies. They are often refurbished phones.
- ✓ The advent of widespread text messaging has resulted in the cell phone novel; the first literary genre to emerge from the cellular age via text messaging to a website that collects the novels as a whole.
- ✓ Mobile telephony also facilitates activism and public journalism being explored by Reuters and Yahoo! and small independent news companies such as Jasmine News in Sri Lanka.

- ✓ The United Nations reported that mobile phones have spread faster than any other technology and can improve the livelihood of the poorest people in developing countries by providing access to information in places where landlines or the Internet are not available, especially in the least developed countries. Use of mobile phones also spawns a wealth of micro-enterprises, by providing work, such as selling airtime on the streets and repairing or refurbishing handsets.
- ✓ In Mali and other African countries, people used to travel from village to village to let friends and relatives know about weddings, births and other events, which are now avoided within mobile phone coverage areas, which is usually greater than land line penetration.
- ✓ The TV industry has recently started using mobile phones to broadcast live TV through mobile phones, advertising, social TV, and mobile TV.86% of Americans use their mobile phone while watching TV.

1.2 Advantages of using mobile phone

The positive effects of using mobile phones are so great that it can never be overlooked at.

Cell phone is the most popular technological product nowadays. (Wikipedia, the free encyclopedia). It's used by almost people of all ages. Cell phone really makes life easier, it is very convenient and helps us contact with people at anywhere over the world in a fastest way. In fact cell phone helps remove loneliness from once life since communicating with friends and family members is always possible. Again cell phone is the most multifunction tool nowadays. It is not only a cell phone but also a calculator, a MP3 player, a handheld computer because people can use it to call to other, calculate, listen to music, arrange the schedule, surf on the

Internet, check email and so on. In addition cell phone helps to reduce unnecessary transport expenses since issues of less importance can best be discussed.

Economic Gains of using Mobile Phones

The best exchange of ideas help reduce the knowledge gap among developed and developing nations, enabling developing countries to increase their standards of living. Information technologies, such as cell phones, can increase efficiencies within a country by enabling the exchange of information among its inhabitants and lowering the cost of acquiring information. Mobile phones are especially important in developing Nations where the needs of separate groups within the population may differ substantially (Unwin, 2009). For example, the poorest individuals in marginalized communities more immediately need information about sources of food and shelter. Producers and consumers, the majority of the population, would instead need information about employment opportunities, prices of goods, education, health, acceptable norms of Behavior and elections. With cell phones, distinct groups can receive the specialized information they need. The use of mobile phones also implies a two-way communication. After individuals receive the information they need, they can communicate it to others. In this manner, cell phones increase the flow of information, as well as its overall availability.

Bedia (1999) suggests that in developing countries, reliable information communication technologies lower the costs of transmitting information, which shifts the information supply curve to the right. The technologies can improve the quality of information by providing up-to-date and complete data. With more abundant and accurate information, people in developing countries will be able to make better and quicker decisions in order to facilitate economic growth and development and reduce poverty.

Moreover, as Unwin writes, "Information and knowledge have always been central to the effective functioning of human societies. They are the means through which societies reproduce themselves, through which understanding is passed on to future generations" (2009). Mobile devices proliferate knowledge, helping individuals in society communicate and establish an intricate network of information. Mobile devices decrease Transaction costs and broadens product markets (Waverman, Meschi, & Fuss, 2005). They also lower search costs, reduce the degree of asymmetric information in markets, and reduce price dispersion (Abraham, 2007). Telecommunications further enhance the spread of information through network effects. As more and more users are linked into an information network, network externalities are generated, providing a benefit to citizens of developing countries like Ghana.

1.1.2. Disadvantages of using mobile phones

It is not a surprise to hear of bad things associated with the use of mobile phone since it use involve emission of microwave radiation.

All of us know that something that can broadcast electro-magnetic wave really has some bad influence to our health, including cell phone. Many researches show that people using cell phone frequently have more risk to get ears and fingers diseases. In addition, electro-magnetic wave from cell phone can cause more diseases like epilepsy, childless male and especially brain cancer. A worrying fact is that a large number of people using cell phone are not considering the bad effects from cell phones.

(a) Harmful Health Effects

According to Dr Vini Khurana, an Australian neurosurgeon, the use of mobile phone has far broader public health ramifications than asbestos and smoking. Having analyzed data from more than 100 different studies, he concluded that most of them did not cover timeframes long enough to measure the potential impact on brain cancer risk. Again, there are studies which reveal the negative impact of cell phones on the health of the young ones. For example, quite recently, Dr Kheifets and researchers in Denmark looked at over 13,000 Danish children born in 1997 and 1998. The children were part of a study called the Danish

National Birth Cohort.

The study discovered that those children who used cell phones and whose mothers had used cell phones during their pregnancy had 80% higher incidence of behavioural issues. These include emotional issues, hyperactivity, inattention and having problems with their peers. Even those children who themselves did not have cell phone exposure except during their mothers' pregnancies had 54% higher incidence of such problems. According to Dr Kheifers these figures are indeed worrying.

(b) Decreased Male Fertility

In 2007, research at the Cleveland Clinic found that men who used cell phones for more than four hours per day had markedly poorer sperm quality than their counterparts with lower cell phone usage.

Recently, the same team delved deeper. In a study published in *Fertility & Sterility*, they obtained sperm samples from 32 men and split them into 2 groups – control and test.

After being placed an inch from a 850-Mhz cell phone which was in 'talk' mode, the sperm in the test group samples had higher levels of harmful free radicals as well as lower levels of protective antioxidants as compared to the control group, i.e. the unexposed sperm. These resulted in a drop in the affected sperm's function, motility as well as overall health.

There was, however, no significant difference in DNA damage between the two groups.

This is a small, laboratory-based study, and more research is definitely still needed in this area. On top of that, sperm in men would actually be further away from cell phones, and also be protected by a few layers of human tissue. However, despite these factors, the findings of this study still offer cause for concern, and men, especially those planning to start a family, may want to keep the cell phones as far away from their reproductive parts as possible.

1.3 Problem statement

Mobile telecommunication has now become an integral part of the Ghanaian society to the extent that to possess a mobile phone is more of a necessity than a privilege. This is so because of the numerous benefits derived from the possession and use of mobile phone.

The passage of the mobile number portability (MNP) has made it possible for every citizen to choose whichever mobile network he/she wishes to join. So the issue of being slave to one mobile network has become a thing of the past. However, because of the many advert on the news, media and many papers, it become more difficult to come out vividly as whether mobile network A or B is the best in terms of cost analysis.

The Government has been firm as far as the liberalization of the telecom market is concerned This has given birth to private sector participation to meet the changing demands of the Ghanaian populates. The outcome of this has exert serious competition among the six mobile telecom operators in the country and Ashanti in this case; namely MTN, TIGO, VODAFONE, KASAPA, AIRTEL and GLO.

The issue is, individuals, institution and groups are face with which of these six telecom operators serve them best in terms 0f cost**.**

1.4 Objectives of the study

The objectives of this thesis are to;

1. Rank the telecommunication network operators in the Ashanti region using promethee method.
2. Select the best network operator with respect to cost.

1.5 Methodology

The introduction of the mobile number portability (M.N P) in Ghana has brought a high level 0f competition among the six network in the region.The network operators are, Globacom (GLO), Airtel, Vodafone, Expresso, Tigo and MTN. All these network operators (N WO) compete for the same market in the region.

The methodology consists of the sample procedure, sample size and how the data is to be analysed.

The six mobile telecommunication operators in Ghana will be used in this case study.

Ashanti region with a large number of customers using these network operators has been selected as the reference area for the study.

The model to be used to calculate f0r the network with the minimum cost will be the PROMETHEE (I and II).

The data to be used will be from both primary and secondary sources.

The study will make use of quantitative data from well designed questionnaire and on the internet.

Resources for the study include KNUST library, college of science library and the internet.

1.6 Justification

The result from the thesis will;

1. Reveal to mobile phone subscribers in the Ashanti region the network with the least cost service and subsequently inform their decisions on the network to subscribe or switch to
2. Compel other network operators to reduce their service rate if possible.
3. Enable governments identify the network with least cost of services in the region and for reword or recommendation.

1.7 Thesis organization

This thesis like all other thesis is organized in to five chapters.

These includes chapter one which is the introduction and contains the background of study, the problem statement, the objectives, methodology and the thesis justification.

Chapter two is the literature review and contains keywords in topics, objectives, and methodology. These include, the problem, method, used together with solution to the problem and conclusions.

The chapter three deals with models formulation, variants and illustrative examples.

The fourth chapter considers study and analysis of data collected and the thesis end with chapter five that contains conclusions and recommendations.

CHAPTER TWO

LITERATURE REVIEW

2.1 Introduction

According to Collins (2003), telecommunication is the technology by which signals and messages are send from one place to another through electronic equipment such as radio, telephones and other devices. One definition of communication is “any act by which one person gives to or receives from another person information about that person's needs, desires, perceptions, knowledge, or affective states”. Communication may be intentional or unintentional, may involve conventional or unconventional signals, may take linguistic or nonlinguistic forms, and may occur through spoken or other modes. Communication requires a sender, a message, and a recipient, although the receiver doesn't have to be present or aware of the sender's intent to communicate at the time of communication; thus communication can occur across vast distances in time and space. Communication requires that the communicating parties share an area of communicative commonality. The communication process is complete once the receiver has understood the message of the sender. There is various categories of telephone available in the system but cell phone (mobile phone) is the common among the lot.

2.1.1 Importance of mobile telecommunications and its impact on Economic Development.

According to (Harald Gruber and Pantelis, 2010), although mobile devices have infiltrated and revolutionized the modern world, the effects of it on society are vast and can be observed through a variety of discipline. The contribution of mobile telecommunications infrastructure to economic growth for low penetration Countries is found to be smaller than for high penetration countries, suggesting increasing Returns from mobile adoption and use. Growth effects are estimated for individual countries and compared. More generally, the annual contribution of mobile telecommunications infrastructure to growth for high income countries is doubles that of low income countries. The increasing returns are also emerging when assessing the impact of mobile telecommunications infrastructure on productivity growth Policy.

(Koski and Kretschmer, 2005), recommends that to further support the diffusion of mobile telecommunications, especially in low income countries the mobile telecommunications industries have to grown rapidly over the last three decades representing one of the most intriguing stories of technology diffusion. Since 2002, Mobile subscribers have exceeded the number of fixed lines globally. The process to achieve what fixed phones have struggled for more than 120 years took less than a fifth of the time for mobile networks. This cross-over time of mobile users have been even shorter for developing countries. At the end of 2009 the number of mobile telecommunications subscribers reached 4.6 billion, which is equivalent to 67 per cent of the world population. This technology is particularly relevant in developing countries, where there are more than twice as many subscriptions (3.2 billion) as in developed countries. The importance of the telecommunications sector becomes also evident by comparing the share of telecommunications revenues in GDP: telecommunications services accounted for an average 4.8% of the total GDP of sub-Saharan Africa compared to 3.1% in the European Union.

While the determinants for the diffusion of mobile telecommunications have been extensively studied.

From Helpman (1998), relatively little is known about the impact of this technology at a macroeconomic level. The pervasiveness of the technology in terms of transforming the way economic activity is organized suggests that mobile telecommunications have features of what is referred to as *general purpose technology*.

Bresnahan and Trajtenberg (1995). There is widespread anecdotal evidence about the surge of new companies and business models with worldwide brands linked to the sector (e.g. *Nokia*, *Vodafone*) and the appearance of new modes of communication such as 'personal reachability'. Because of the lower access cost to the user compared to wired telecommunications, linked with the solution of the problem of creditworthiness of customer through prepaid cards, the technology could reach completely new segments of population particularly in developing countries. As revenues from mobile telecommunications account nowadays for a significant percentage of GDP especially in developing countries, mobile telecommunications have also become an important and efficient means for tax collection. Moreover, telecommunications infrastructure has significant network externalities.

In line with the network economics' literature, one of their key characteristics is that the value of the network increases with the usage base.

Economides and Himmelberg (1995), this has frequently been referred to as a direct network externality, with the implication that critical mass effects may occur when certain threshold levels of diffusion occur which can then trigger off additional benefits, such as the availability of new services. Ultimately one would expect increasing returns from the adoption of the technology. The implication suggests that high mobile penetration yields incentives for further investment, very much along the "success breed success" paradigm. As a result, low penetration countries, which typically are developing countries, could have a double disadvantage: they not only have a lower growth impact due to lower mobile diffusion; they also have lower incentives for further development of the mobile network.

Hence, the economic cost in terms of foregone growth is highest in these countries. This paper assesses the impact of mobile telecommunications on growth taking in to account the fact that economic growth is itself a determinant for the diffusion of mobile telecommunications. In a similar setting

(Roller and Waverman 2001) used as simultaneous equations model to measure the returns from fixed telephony on growth. In this paper we introduce a similar simultaneous equation model. Mobile telecommunications diffusion allows for a more accurate estimate of its impact on growth. It also corrects for possible simultaneity biases in estimating the impact of mobile telecommunications on growth. Results show that the contribution of mobile telecommunications infrastructure to economic growth for low mobile penetration countries (or in fact low income countries) is much smaller than for high penetration countries: low income countries forego 0.20 per cent of annual growth due to lack of a mobile telecommunications infrastructure compared to a high income country. This suggests increasing returns from mobile penetration. The increasing returns result is also obtained when the existing model is extended to assess the impact of mobile telecommunications infrastructure on productivity growth.

2.1.2 National Communications Authority (N.C A)

According to Wikipedia (2014) The National Communications Authority (N.C.A) is the government of Ghana agency responsible for licensing of media houses and organizations in

Ghana. The commission was formed by the National Communications Authority Acts of 2008, Acts 769.

The Authority's main objective is to regulate the provision of communications services in Ghana, by setting and enforcing high standards of competence and performance to enable it contribute significantly and fairly to the nation's prosperity through the provision of efficient and competitive services.

2.1.3.1 Mission

According to The National Communications Authority (2014), the main mission of the National Communications Authority is to regulate the communications industry by setting and enforcing high standards of competence and performance to enable it to contribute significantly and fairly to the nation’s prosperity through the provision of efficient and competitive services.

2.1.3.2 Vision

The prime vision of the National Communications Authority (NAC) is to become the most forward-looking and innovative Communications Regulatory Authority in the sub-region; by creating and maintaining an efficient, transparent and business friendly environment to enable Ghana become the premier destination of ICT investment in the sub-region.

2.2 Analytic Hierarchy Process (AHP)

Saaty L. T (1970) has it that the **analytic hierarchy process** (**AHP**) is a structured technique for organizing and analyzing complex decisions, based on mathematics and psychology. It was developed and has been extensively studied and refined since then .It has particular application in group decision making, and is used around the world in a wide variety of decision situations, in fields such as government, business, industry, healthcare, and education.

Rather than prescribing a “correct” decision, the AHP helps decision makers find one that best suits their goal and their understanding of the problem. It provides a comprehensive and rational framework for structuring a decision problem, for representing and quantifying its elements, for relating those elements to overall goals, and for evaluating alternative solutions.

Users of the AHP first decompose their decision problem into a hierarchy of more easily comprehended sub-problems, each of which can be analyzed independently. The elements of the hierarchy can relate to any aspect of the decision problem—tangible or intangible, carefully measured or roughly estimated, well or poorly understood—anything at all that applies to the decision at hand.

2.2.2 Applications of Analytic Hierarchy Process

Saaty L. T (1914) has it that the applications of AHP to complex decision situations have numbered in the thousands, and have produced extensive results in problems involving planning, resource allocation, priority setting, and selection among alternatives. Other areas have included forecasting, total quality management, business process re-engineering, quality function deployment, and the balanced scorecard. Many AHP applications are never reported to the world at large, because they take place at high levels of large organizations where security and privacy considerations prohibit their disclosure. But some uses of AHP are discussed.

Recently these have included:

- Deciding how best to reduce the impact of global climate change (Fondazione Eni Enrico Mattei)
- Quantifying the overall quality of software systems (Microsoft Corporation)
- Selecting university faculty (Bloomsburg University of Pennsylvania)
- Deciding where to locate offshore manufacturing plants (University of Cambridge)
- Assessing risk in operating cross-country petroleum pipelines (American Society of Civil Engineers)
- Deciding how best to manage U.S. watersheds (U.S. Department of Agriculture)

AHP is sometimes used in designing highly specific procedures for particular situations, such as the rating of buildings by historic significance. It was recently applied to a project that uses video footage to assess the condition of highways in Virginia. Highway engineers first used it to determine the optimum scope of the project, then to justify its budget to lawmakers (Zemelo E, 1982).

Stuat (1994) conducted a study on using AHP for decision making in engineering and some challenges faced were; in many industrial engineering applications, the final decision is based purely on the evaluation of a number of alternatives in terms of the number of criteria. This problem may come to be a difficult one when the criteria are expressed in difference units and making the data difficult to be quantified.

2.3 A Brief History of PROMETHEE

The Promethee I (partial ranking) and Promethee II (complete ranking) were developed by J. P. Brans and presented for the first time in 1982 at a conference organized by R Nadeau and M. Landry at the University Laval. In the same year several applications using this methodology were also treated by G, Davignon in the field of health care.

A few years later, J. P Brans and B Marschal developed Promethee III (ranking based on interval and Pomethee IV (continuous case). The same authors proposed in 1988 the visual interactive module GALA which is providing a marvelous graphical representation supporting the PROMETHEE methodology.

Brains J. P and Mareschal B. (1994) further suggested two nice extensions; PROMETHEE V (MCDA including segmentation constraints) and PR0METHEE VI (representation of the human brain). A considerable number of successful applications has been treated by the PROMETHEE methodology in various fields such as Banking, Industrial location, Manpower planning, water resources, Investments, Medicine Chemistry, Healthcare, Tourism, Ethics in OR, Dynamic management and so on. The success of the methodology is basically due to its mathematical properties and to its particular friendliness of use. According to Peter Peeter (1994) the Promethee methods have been found to be very popular in the world of outranking methods. According to users, one of the reasons for this popularity is the existence of the very user-friendly software called PROmCALC-PROmetheeCALCUlation. This software is used by more and more practitioners to handle their multiple criteria problems.

According to (Peter Peeter 2010) the PROMETHEE method has great advantages over other multiple criteria decision making (MCDM) approaches The promethee methodology consist of six outranking methods, these are the PROMETHEE I, PROMETHEE II, PROMETHEE III, PROMETHEE IV and finally PROMETHEE VI (Behzadian et al, 2010) (Brans 1982) came out with the first two; PR0METHEE I and PROMETHEE II which are the partial and complete ranking respectively. PROMETHEE is one of the most resent multiple criteria decision making (MCDM) methods that was proposed by Brans etal (1982), and has successively been applied in many fields. Especially in investment analysis and performance evaluation.

2.3.1 The Promethee Method

Brans (1982) developed the Promethee I (partial ranking) and Promethee II (complete ranking) and presented for the first time in 1982 at a conference organised by R. Nadeau and M. Landry at the Université Laval, Québec, Canada (L'Ingéniérie de la Décision. Elaboration d'instrumentsd'Aideà la Décision).

The same year several applications using this methodology were already treated by G. Davignon in the field of Heath care. A few years later J. P. Brans and B. Mareschal developed Promethee III (ranking based on intervals) and Promethee IV (continuous case). The same authors proposed in 1988 the visual interactive module GAIA which is providing a marvellous graphical representation supporting the Promethee methodology.

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Vincke (1985) and Brans et al. (1986). The scores a_{ij} need not necessarily be normalized or transformed into a common dimensionless scale.

We only assume that, for the sake of simplicity, a higher score value means a better performance. It is also assumed that the weights w_i of the criteria have been determined by an appropriate method (this is not a part of the Promethee methods).

Preference function based outranking method is a special type of MCDM tool that can provide a ranking ordering of the decision options. The Promethee (preference ranking organization method for enrichment evaluation) method was developed by Brans and Vincke in 1985. The Promethee I method can provide the partial ordering of the decision alternatives, whereas, Promethee II method can derive the full ranking of the alternatives.

2.3.2 Applications of Promethee method

Ali, (2011) used the Promethee method combined with multi-objective linear programming (MOLP) to develop a model for outsourcing. The model was built based on two phases. First, with using Promethee, we start to rank the priority of our partners. In the second phase, we assign the products to partner with multi-objective linear programming based on the priorities that was earned from the first phase. Minimizing costs and defect products, maximizing on time delivery and referring demands to better suppliers are the major goals in this article.

The model is improved to solve the problem of a company that periodically purchases different products from different suppliers to fulfill its aggregate demand.

Each of suppliers can provide a few products. However, it was expected that these suppliers have different characteristics.

The supplier evaluation and order allocation plan of the company is a strategic issue. In general, these plans are made for a time period of at least 6-12 months, due to economic and market conditions, because of difficulty in determine the environmental coefficient and related parameters, especially in a medium time horizon

In this study, order allocation methodology focuses on developing the MOLP approach to allocate order quantities to satisfy the aggregate demand of a company considering prices, rejected units rate, on time delivery, and suppliers ranks calculated from the PROMETHEE.

Constanta (2005), adopted the Promethee method together with the AHP to study Water Resources Planning. The projects goal is the rational water resources management of Nestos River in relation to the operation of two recently constructed dams. The management of the water supply system should balance the needs for irrigation, the needs of the Public Electrical Corporation for hydropower generation, as well as environmental requirements given the presence of valuable natural ecosystems in the area. In order to evaluate the projects, the Analytic Hierarchy Process (AHP) and PROMETHEE multi criteria methods are used. The projects evaluation is based on economic, social, environmental and cost criteria. Alternative scenarios on the availability of water resources are also incorporated in the model.

2.4 **Technique for Order of Preference by Similarity to Ideal Solution (TOPSIS)**

(Hwang and Yoon K, 1981) Developed the Technique for Order of Performance by Similarity to Ideal Solution (TOPSIS), they found this to be one of the most classical methods for solving MCDM problem. It is based on the principle that the chosen alternative should have the longest distance from the negative-ideal solution i.e. the solution that maximizes the cost criteria and minimizes the benefits criteria; and the shortest distance from the positive-ideal solution i.e. the solution that maximizes the benefit criteria and minimizes the cost criteria. In classical TOPSIS the rating and weight of the criteria are known precisely. However, under many real situations crisp data are inadequate to model real life situation since human judgments are vague and cannot be estimated with exact numeric values. To resolve the ambiguity frequently arising in information from human judgments fuzzy set theory has been incorporated in many MCDM methods including TOPSIS. In fuzzy TOPSIS, all the ratings and weights are defined by means of linguistic variables. A number of fuzzy TOPSIS methods and applications have been developed in recent years. (Chen and Hwang, 1981) first applied fuzzy numbers to establish fuzzy TOPSIS. (Triantaphyllou and Lin 1996) developed a fuzzy TOPSIS method in which relative closeness for each alternative is evaluated based on fuzzy arithmetic operations. (Liang, 2007) proposed Fuzzy MCDM based on ideal and anti-ideal concepts. He considered triangular fuzzy numbers and defined crisp Euclidean distance between two fuzzy numbers to extend the TOPSIS method to fuzzy GDM situations. (Chu and Lin, 2007) further improved the methodology proposed by Chen and Tsao, and extended the TOPSIS method based on Interval-valued fuzzy sets in decision analysis. (Jahanshahloo et al. 1994) extended the fuzzy TOPSIS method based on alpha level sets with interval arithmetic.Later on, Chen and Lee extended fuzzy TOPSIS based on type-2 fuzzy TOPSIS method in order to provide additional degree of freedom to represent the uncertainties and fuzziness of the real world. Fuzzy TOPSIS has been introduced for various multi-attribute decision-making problems. (Yong 1981) used fuzzy TOPSIS for plant location selection and Chen et al. used fuzzy TOPSIS for supplier selection. (Kahraman et al, 2002) utilized fuzzy TOPSIS for industrial robotic system selection. Wang and Chang, applied fuzzy TOPSIS to help the Air Force Academy in Taiwan choose optimal initial training**.**

(Hwang and Yoon, 2007), Identified TOPSIS to be known as one of the most classical MCDM methods. According to them, TOPSIS is based on the idea that the chosen alternative should have the shortest distance from the Positive Ideal Solution (PIS) and on the other side the farthest distance of the Negative Ideal Solution (NIS). The Positive Ideal Solution maximizes the benefit criteria and minimizes the cost criteria, whereas the Negative Ideal Solution maximizes the cost criteria and minimizes the benefit criteria. In the process of TOPSIS, the performance ratings and the weights of the criteria are given as exact values. (Abo-sinna and Amer, 2005) extend TOPSIS approach to solve multi-objective nonlinear programming problems.

(Jahanshahloo et al, 2005) extends the concept of TOPSIS to develop a methodology for solving multi-criteria decision-making problems with interval data. The steps of TOPSIS model are as follows:

- Calculate the normalized decision matrix.
- Calculate the weighted normalized decision matrix.
- Determine the Positive Ideal Solution and Negative Ideal Solution.

(Weber et al, 1991) Outline the main advantages of using TOPSIS method as follows:-

1. It is simple to use.

2. It takes into account all types of criteria (subjective and objective).
3. It is rational and understandable.
4. The computation processes are straight forward.

2.4.1 Application of TOPSIS

(Abo-Sinna 2004) Appied TOPSIS approach to solve multi-objective dynamics programming (MODP) problems. He shows, that using the fuzzy max–min operator with non-linear membership functions, the obtained solutions are always non-dominated solutions of the original MODP problems. Further extensions of TOPSIS for large scale multi-objective non-linear programming problems with block angular structure was presented by Abo-Sinna et al. (Deng et al.) formulated the inter-company comparison process as a multi-criteria analysis model, and presented an effective approach by modifying TOPSIS for solving such a problem. Chen extended the concept of TOPSIS to develop a methodology for solving multi-person multi-criteria decision-making problems in a fuzzy environment and he defined the fuzzy positive ideal solution (FPIS) and the fuzzy negative ideal solution (FNIS). Baky and Abo-Sinna extended the TOPSIS approach to solve Bi-Level MODM Problems.

Generally, TOPSIS provides a broader principle of compromise for solving multiple criteria decision making problems. It transfers m-objectives (criteria), which are conflicting and non-commensurable, into two objectives (the shortest distance from the PIS and the longest distance from the NIS). They are commensurable and most of time conflicting. Then, the objective problem can be solved by using membership functions of fuzzy set theory to represent the satisfaction level for both criteria and obtain TOPSIS's compromise solution by a second-order compromise operation. And then, the max–min operator is considered to resolve the conflict between both criteria (the shortest distance from the PIS and the longest distance from the NIS).

(Baky, 2004), further extended the concept of TOPSIS to develop a methodology for solving multi-level non-linear multi-objective decision-making (MLN-MODM) problems of maximization-type.

2.5 Multi Attribute Utility Theory (MAUT)

The MAUT approach, involving additive utility functions and preference modeling, has been researched and studied in great detail over a period of decades. (Fishburn 1970, Keeney and Raiffa 1976), provide excellent summaries of the background and history of MAUT. The historical development of the MAUT approach can be traced through the work of Debreu (1960), Luce and Tukey (1964), Krantz (1964), Pollak (1967), Keeney (1968), and others.

(Pollak,1967 and Keeney 1968), use a framework sometimes referred to as the Analytica Hierarchy Process (AHP) (Saaty 1980), which supports the ability to assign relative values to qualitative attributes so they can be included in the quantitative multi-attribute scoring of decision options. This is a key aspect of the approach we apply because it allows users to integrate the entire range of attributes into a single analysis. The approach allows attribute weights (smart values that reflect relative importance to users, or values) to be set explicitly and adjusted by users.

According to (Tukey and Krantz, 1964), this method is highly effective in enabling users to conduct their own sensitivity analysis to identify breakeven points and evaluate preference trade-offs between conflicting objectives or choices. According to them, applying this approach involves the following steps, which are listed below:

- Define a utility function that scales the importance or impact of each attribute from 0 to 1
- Define and rate each attribute on a five-point scale scored from best (100) to worst (0) that reflects the range of conditions that might occur as a result of decommissioning
- Scale and normalize attribute scores for a given decommissioning scenario in terms of each attribute's Best-to-Worst range; this will result in a proportion somewhere between 0 and 1 for each attribute
- Weight the attributes to reflect user preferences about the relative importance of each attribute
- Calculate the overall multi-attribute score

CHAPTER THREE

MATHEMATICAL METHODOLOGY

3.0 Introduction

This chapter deals with the details of the PROMETHEE methodology for the multi decision making problem.

3.1 Multi decision Making Problem

Multi-criteria decision-making (MCDM) problems are well known due to their widespread application in the public and private sectors. A typical MCDM problem examines a set of alternatives across a set of decision criteria. Decision makers (DMs) usually evaluate the performance of each alternative with respect to each criterion, and also consider the relative importance of each criterion with regard to the overall goal using qualitative and/or quantitative measures. However, often only limited information is available for DMs on such occasions, and thus it is important for researchers and practitioners to develop more effective methods for decision-making in an uncertain environment. This has motivated the development of various techniques for handling uncertainties in the decision-making processes. For example, the theory of fuzzy sets (FSs) was introduced by Zadeh in 1965 to handle fuzzy decision-making problems in various fields.

An important feature of FSs is that only a single membership value (2[0, 1]) is assigned to each element to identify the degree to which it belongs to a given universal set. However, for some decision-making cases in an uncertain environment, not only the positive assessment (the degree of membership) is of interest, but also the negative one (the degree of non-membership). For example, the assessment of several investment proposals should take into account the positive as well as negative degrees of each evaluation criterion (Fishburn, 1970).

There are a lot of methods used in solving multi criteria decision making problems, the chapter will consider some of these methods and provide sample problems and their solutions.

Let us consider a multi-criteria decision making problem with m criteria and n alternatives. Let $C_1,.., C_m$ *and* $A_1,..,A_n$ denote the criteria and alternatives, respectively. A standard feature of multi-criteria decision making methodology is the decision table as shown in Figure 3.1 below. Each row belongs to a criterion and each column describes the performance of an alternative. The score a_{ij} describes the performance of alternative A_j against criterion C_i. For the sake of simplicity the assumption that a higher score value means a better performance is made since any goal of minimization can be easily transformed into a goal of maximization.

As shown in decision table Figure 3.1, weights $w_1,...,w_m$ are assigned to the criteria. Weight w_i reflects the relative importance of criteria C_i to the decision, and is assumed to be positive. The weights of the criteria are usually determined on subjective basis. They represent the opinion of a single decision maker. Again this may represent the opinions of a group of experts using a group decision technique. The values a_{ij} ,…., a_{in} associated with the alternatives in the decision table are used in the Multi Attribute Utility Theory (MAUT) methods as shown in figure 3.1. This table shows the final ranking values of the alternatives. Usually, higher ranking values means a better performance of the alternatives, so the alternatives with the highest ranking value are the best of the alternatives.

$$x_i \ldots\ldots\ldots\ldots\ldots\ldots\ldots x_n$$

$$A_i \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots A_n$$

$$\begin{array}{cc|cc|}
\cline{3-4}
W_i & C_i & a_{11} & a_{m1} \\
“ & “ & & \\
“ & “ & & \\
“ & “ & & \\
“ & “ & & \\
W_m & C_m & a_{m1} & a_{mn} \\
\cline{3-4}
\end{array}$$

Figure 3.1. Multi decision making model

3.2 Multi Attribute Utility Theory (MAUT)

Choosing among decommissioning alternatives is complicated by the fact that each option involves multiple characteristics, or attributes, that are important to decision makers. Some attributes, such as cost, can more readily be quantified. For others, such as impacts on marine mammals or on the broader regional economy, data gaps and/or inherent uncertainty means they can only be described and evaluated qualitatively. Focusing only or primarily on quantitative attributes can undermine the quality of decisions because qualitative attributes can be as important as, or more important than, those that can be quantified. Attributes discussed in the preceding subsections ranged from those that can be quantified relatively reliably (decommissioning costs) to ones with quantitative but uncertain estimates (biological production) to those that can only be addressed in qualitative terms (impacts on marine mammals).

Because both quantitative and qualitative attributes are important to be considered in decisions about decommissioning, we have included both types of attributes within a single integrated decision framework based on multi-attribute utility theory (MAUT). The MAUT approach, involving additive utility functions and preference modeling, has been researched and studied in great detail over a period of decades (Fishburn 1970)

Keeney and Raiffa (1976) provide excellent summaries of the background and history of MAUT. The historical development of the MAUT approach can be traced through the work of Debreu (1960), Luce and Tukey (1964), Krantz (1964), Pollak (1967), Keeney (1968), and others.

We use a framework sometimes referred to as the Analytica Hierarchy Process (AHP) (Saaty 1980), which supports the ability to assign relative values to qualitative attributes so they can be included in the quantitative multi-attribute scoring of decision options. This is a key aspect of the approach we apply because it allows users to integrate the entire range of attributes into a single analysis. The approach allows attribute weights (smart values that reflect relative importance to users, or values) to be set explicitly and adjusted by users. This permits users to systematically explore the effects on the choice of decommissioning option of different perceptions of the relative value, importance, or weighting of each attribute. This method is highly effective in enabling users to conduct their own sensitivity analysis to identify breakeven points and evaluate preference trade-offs between conflicting objectives or choices.

According to Krantz and Pollak (1964, 1967), applying this approach involves the following steps, which are described below and then applied to illustrative decommissioning scenarios:

1. Define a utility function that scales the importance or impact of each attribute from 0 to 1
2. Define and rate each attribute on a five-point scale scored from best (100) to worst (0) that reflects the range of conditions that might occur as a result of decommissioning

3. Scale and normalize attribute scores for a given decommissioning scenario in terms of each attribute's Best-to-Worst range; this will result in a proportion somewhere between 0 and 1 for each attribute
4. Weight the attributes to reflect user preferences about the relative importance of each attribute
5. Calculate the overall multi-attribute score

3.2.1 MAUT Utility Functions

There are three fundamental utility functions been discussed below:

1) Additive Utility Function has the following properties:

- A Simplified Utility Model
- Ignores interactions among attributes

Mathematically, for a consequence set that has values x_1, x_2, …, x_m on the attributes of *m* objectives, its overall utility is computed as:

$$U(x_1, x_2,\ldots,x_m) = k_1 u_1(x_1) + k_2 u_2(x_2) + k_m u_m(x_m) = \sum_{i=1}^{m} k_i\, u_i(x_i) \qquad (3.1)$$

Where $u_i(x_i)$ is the utility function of the ith attribute $0 \leq U_i(x_i) \leq 1$

K_i is the weight of the *ith* attribute ($k_i + k_2 + \ldots + k_m = 1$)

$0 \leq U(x_1,x_2,..,x_m) \leq 1$

2) Multilinear utility function captures a limited form of interaction, mathematically it is expressed below:

$$U(x,y)= kxUx(x) + k_y u_y(y) + (1-k_x-k_y)\, U_x(x)\, U_y(y) \qquad (3.2)$$

$U_x(x)$ is the utility function of x scaled so that $U_x(x-)=0$ and $U_x(x+)=1$

$U_y(y)$ is the utility function of y scaled $U_y(y-)=0$ and $U_y(y+)=1$

$K_x = U(x+, y-)$ is Not relative weight of U_x

$K_y = U(x-, y+)$ is Not relative weight of U_y

$K_x + k_y \neq 1$

$U(x+, y-) = k_x u_x(x+) + k_y u_y(y-) + (1-k_x-k_y) U_x(x+) U_y(y-) = k_x(1) + k_y(0) + (1-k_x-k_y)(1)(0) = k_x$

$U(x-, y+) = k_x u_x(x-) + k_y u_y(y+) + (1-k_x-k_y) U_x(x-) U_y(y+) = k_x(0) + k_y(1) + (1-k_x-k_y)(1)(0) = k_y$

3) The Multiplicative utility function

Let x and y be two attributes with values $x_1 \leq \ldots \leq x_n$, $n \geq 2$ and $y_1 \leq \ldots \leq x_m$, $m \geq 2$.

$$U(x,y) = (k.u(x,y_1) + 1) . (k.u(x_1,y) + 1) \quad (3.3)$$

Where

$u(x_1,y_1) = 1$, $u(x_n,y_1) > 1$ and $u(x_1,y_m) > 1$

$k=\frac{|u(x_n,y_n)-u(x_n,y_1)-u(x_1,y_m)}{u(x_n,y_1).u(x_m)} > 0$, that is scaling constant

3.2.2 Illustrative example of the Multi Attribute Utility Theory (MAUT)

A buyer wants to buy a car with a long expected life span and a low price. The three alternatives under consideration are: the Camary (a relatively expensive sedan with a reputation for longevity), the BMW (renowned for its reliability), and the Kia Pride (a relatively inexpensive domestic automobile). These three cars are evaluated on both attributes, as in Table 3.2 below:

Table 3.1: Cost and Criteria of three cars

	Alternatives		
Attributes	Camry	BMW	Kia pride
Price($)	17000 (worst)	10000	8000 (best)
Life span (years)	12 (best)	9	6 (worst)

The graph below compares the three cars using a line graph; the horizontal axis gives the price of cars in dollars whiles the vertical axis gives the life span of the cars in years.

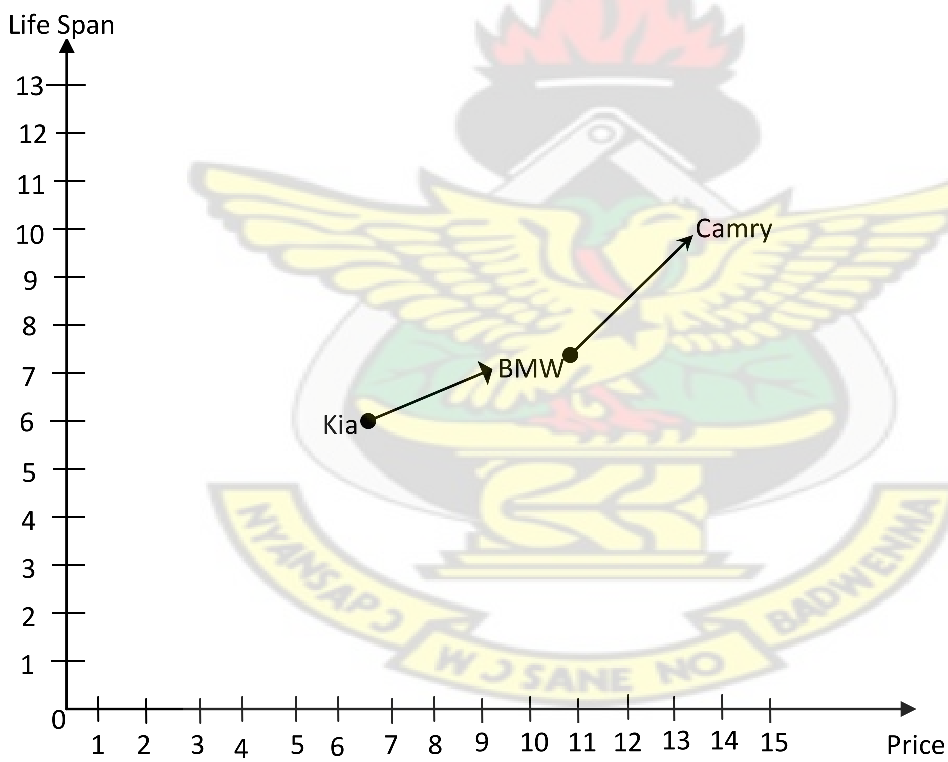


Figure 3.2: Graph of Comparison of three cars.

3.2.3 Additive Utility Function

For a consequence set that has values x_1, x_2, …, x_m on the attributes of m objectives, its overall utility is computed as:

$$U(x_1, x_2, \ldots x_m = k_1 u_1(x_1) + k_2 u_2(x_2) + \ldots + k_m u_m = \sum_{i=1}^{m} k_i u_i(x_i) \qquad (3.4)$$

$U_i(x_i)$- the utility function of the i*th* attribute $0 \leq U_i(x_i) \leq 1$

K_i- the weight of the i*th* attribute ($k_1 + k_2 + \ldots + k_m = 1$)

$0 \leq U_i(x_1, x_2, \ldots, x_m) \leq 1$

Set UPrice(Kia) =UPrice(8000) = 1, UPrice(Camary) = UPrice(17000) = 0

ULife(Camary) = ULife(12) = 1, ULife(Kia) = ULife(6) = 0

$$U(x) = \frac{x - x_1^{-1}}{x^{+} - x_1^{-}} \qquad (3.5)$$

Xi-: is the worst value of attribute *x*i and xi+: is the best value of *x*i

UPrice(Norushi) = UPrice(10000) = (10000 – 17000) / (8000 – 17000) = 0.78

ULife(Norushi) = ULife(9) = (9 – 6) / (12 – 6) = 0.5

Table 3.2: Utility values of cars

	Alternatives		
Attributes	Camary	Bmw	Kia
U_{Price}	0	0.78	1
U_{Life}	1	0.5	0

3.2.4 Weight Assessment (Pricing Out)

Directly specify the ratio of the weights

e.g. $kPrice = 2kLife$

Because $kPrice + kLife = 1$, then $kPrice = 2/3$ and $kLife = 1/3$

$U(Camry) = 2/3 \cdot UPrice(Camry) + 1/3 \cdot ULife(Camry)$

$= 2/3(0) + 1/3(1) = 1/3$

$U(Bmw) = 2/3 \cdot UPrice(Bmw) + 1/3 \cdot ULife(Bmw)$

$= 2/3(0.78) + 1/3(0.5) = 0.69$

$U(Kia) = 2/3 \cdot UPrice(Kia) + 1/3 \cdot ULife(Kia) = 2/3(1) + 1/3(0) = 2/3$

Suppose taking the Kia Pride as the base case. You are indifferent between paying \$8000 for 6 years of life span and paying \$8,600 for 7 years of life span

$U(\$8{,}000, 6\ Years) = U(\$8{,}600, 7\ Years)$

$kPrice \cdot UPrice(8000) + kLife \cdot ULife(6) = kPrice \cdot UPrice(8600) + KLife \cdot ULife(7)$

$UPrice(8600) = (8600-17000)/(8000-17000) = 0.933$, $ULife(7) = (7-6)/(12-6) = 0.167$

$kPrice \cdot 1 + kLife \cdot 0 = kPrice \cdot 0.933 + kLife \cdot 0.167$ □□ $0.067kPrice = 0.167kLife$ (Equation 1)

$kPrice + kLife = 1$ (Equation 2)

Solve Equations (1) and (2) □□ $kPrice = 0.714$, $kLife = 0.286$

$U(Camry) = 0.714 \cdot UPrice(Camry) + 0.286 \cdot ULife(Camry) = 0.286$

$U(Bmw) = 0.714 \cdot UPrice(Bmw) + 0.286 \cdot ULife(Bmw) = 0.7$

$U(Kia) = 0.714 \cdot UPrice(Kia) + 0.286 \cdot ULife(Kia) = 0.714$

From the calculations of the utility functions, $U(Kia) > U(Bmw) > U(Camry)$, thus the best alternative is Kia Pride, followed by Bmw and finally the Camry.

3.3 Promethee Theory

Brans and Mareschal, (1994), had it in their work that in order to take the deviations and the scales of the criteria into account, a preference function is associated to each criterion. For this purpose, a preference function $P_i(A_j,A_k)$ is defined, and this representing the degree of the preference of alternative A_j over A_K for criterion C_i. Consider a degree in normalized form, so that

$0 \leq P_i(A_j,A_k) \leq 1$ and

$P_i(A_j,A_k) = 0$ means no preference or indifference,

$P_i(A_j,A_k) \approx 0$ means weak preference,

$P_i(A_j,A_k) \approx 1$ means strong preference, and

$P_i(A_j,A_k) = 1$ means strict preference.

In most practical cases $p_i(A_j,A_k)$ is function of the deviation $d=a_{ij}-a_{ik}$, i.e. $p_i(A_j,A_k) = p_i(a_{ij}-a_{ik})$, where p_i is a non decreasing function, $p_i(d)=0$ for $d \leq 0$, and $0 \leq p_i(d) \leq 1$ for $d > 0$. A set of six typical preference functions was proposed by Brans and Vincke (1985) and Brans et al. (1986). The simplicity is the main advantage of these preferences functions: no more than two parameters in each case, each having a clear economic significance.

A multi criteria preference index $\pi(A_j,A_k)$ of A_j over A_k can then be defined considering all the criteria:

$$\pi(\text{a,b}) = \sum_{j=1}^{k} (W_j \; P_j(\text{a,b})) \qquad (3.6)$$

This index also takes values between 0 and 1, and represents the global intensity of preference between the pair of alternatives. In order to rank the alternatives, the following precedence flows are defined:

Positive Outranking flow:

$$\emptyset^{+}(A_j)=\frac{1}{n-1}\sum_{k=1}^{n}\pi(A_j,A_k) \tag{3.7}$$

Negative Outranking:

$$\emptyset^{-}(A_j)=\frac{1}{n-1}\sum_{k=1}^{n}\pi(A_k,A_j) \tag{3.8}$$

The positive outranking flow expresses how much each alternative is outranking all the others. The higher $\emptyset^{+}$ (A_j), the better the alternative.$\emptyset^{+}$ (A_j), represents the power of A_j, its outranking character. The greater$\emptyset^{-}$ (A_j), the greater the weakness of A_j. The negative outranking flow expresses how much each alternative is outranked by all the others. The smaller $\emptyset^{-}(A_j)$, the better the alternative.$\emptyset^{-}(A_j)$, represents the weakness of A_j, its outranked character.

3.3.1 The Promethee I partial Ranking

A_j is preferred to A_K when $\emptyset^{+}A_j) \geq \emptyset^{+}(A_j)$, $\emptyset^{-}(A_j) \leq \emptyset^{-}(A_k)$, and at least one of the inequalities holds as a strict inequality. A_j and A_k are indifferent when $\emptyset^{+}(A_j) = \emptyset^{-}(A_k)$, $\emptyset^{-}(A_j) = \emptyset^{-}(A_k)$,

A_j and A_k are incomparable otherwise. In this partial ranking some couples of alternatives are comparable, some others are not. This information can be useful in concrete applications for decision making.

3.3.2 The Promethee II Complete Partial Ranking

If a complete ranking of the alternatives is requested by the decision maker, avoiding any incomparability, the net outranking flow can be considered. This is defined to be

$\emptyset(A_j) = \emptyset^{+}(A_j) - \emptyset^{-}(A_j)$. (3.9)

3.3.3 Preference Functions and their Feature

The preference function $p(d)$ is the function of deviation or difference (d) between values of two evaluated alternatives on the same criterion (perhaps over a set of criterion).

Mathematically, written as $p_i(A_k,A_l) = p_i(d_i(A_k,A_l))$, $i = 1,\ldots n$

The main features of preference functions are

a) Values of the preference functions: these values are within the interval zero to one such that $0 \leq p_i(A_k,A_l) \leq 1$

b) Preference functions are functions that maximize criteria through normalized values such that the higher the value of the function $p(d)$, the preference of A_k to A_l.

c) Most preference functions have one or more of the following parameters p, d, σ. Values of these parameters are always determined by the decision maker and thereby aid in determining the intensity of preference of one alternative over the other on a criterion. The parameter q, indicated along the deviation axis, is the greatest point of deviation (d) between two evaluations, below which the decision maker regards the corresponding alternatives (A_k,A_l) = as indifferent. p which is fixed to the right of the parameter q on the deviation axis measures the lowest point of deviation (d) between two alternatives above which the decision maker expresses strict preference $p_i(A_k,A_l)$ for the first alternative A_k over the second alternative A_l when the deviation d between two evaluations falls between q and p, preference for the alternative A_k over alternative A_l ranges between 0 and 1

The value of a preference function p (d) equals zero when the deviation or difference (d) is below the lower boundary q, in other words, when the value of deviation is less than the value of q: p (d) = 0 if $d \leq q$ (in case however, the value of q is not specified it is regarded as zero, q = 0) So long as the deviation value remains a value in between the threshold q and p, the following conclusions are worth noting:

(i) $p_i(A_k,A_l) = 0$ (3.9a)

implies indifference between A_k and A_l or no preference of A_k over A_l

(ii) $pi(A_k,A_l) \approx 0$ (3.9b)

implies there is a weak preference of A_k over A_l where the symbol " ≈ " denotes a value of $pi(A_k,A_l)$ closed to zero (0)

(iii) $p_i(A_k,A_l) \approx 1$ (3.9c)

implies a strong preference of A_k over A_l where the symbol " ≈ " denotes a value of $p_i(A_k,A_l)$ closed to 1

(iv) $p_i(A_k,A_l) = 1$ (3.9d)

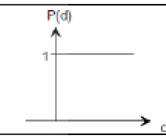
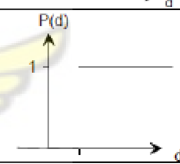
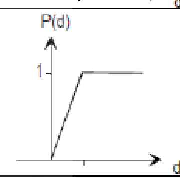
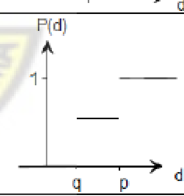
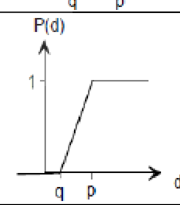
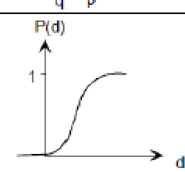
implies a strict preference of A_k over A_l (Brans et al., 1986)

There is also a parameter σ which is regarded as an intermediate value between q and p, therefore the choice of generalized criterion is preceded by the selection of the appropriate parameters.

d) If the upper boundary of deviation p is defined then p (d) = 1 if and only if $d \geq p$ also, there are times the value of p is not explicitly stated and in such cases lim d→∞ p (d) = 1 (Podvezko and Podviezko, 2010)

There exist basically six preference functions in the Promethee method. The 'usual function' is an easy to use preference function and is generally used with quantitative criteria. 'U-shape function’ uses a single indifference threshold and is generally used with qualitative criteria. ‘V-shape function’ uses a single preference threshold and is often used with quantitative criteria. ‘Level function’ is similar to ‘U-shape’, but with an additional preference threshold and it is mostly used with qualitative criteria. ‘Linear function’ is similar to ‘V-shape’, but with an additional indifference threshold and is often used with quantitative criteria. ‘Gaussian function’ is rarely used and is best suited for quantitative criteria, (Bertrand, 2009). The graph of each of the preference function is detailed in Table 3.4.

Table 3.3: Different Preference Function available in Promethee

Type of generalized criteria	Analytical definition	Parameters to define	Shape
Type I. Usual criterion	$H(d)=\begin{cases}0, & d=0;\\ 1, & \lvert d\rvert>0.\end{cases}$	--	
Type II. Quasi-criterion	$H(d)=\begin{cases}0, & \lvert d\rvert\le q;\\ 1, & otherwise.\end{cases}$	p	
Type III. Criterion with linear preference	$H(d)=\begin{cases}\frac{\lvert d\rvert}{p}, & \lvert d\rvert\le p;\\ 1, & \lvert d\rvert>0.\end{cases}$	p	
Type IV. Level-criterion	$H(d)=\begin{cases}1, & \lvert d\rvert\le q;\\ 1/2, & q<\lvert d\rvert\le p;\\ 1, & otherwise.\end{cases}$	q, p	
Type V. Criterion with linear preference and indifference area	$H(d)=\begin{cases}1, & \lvert d\rvert\le q;\\ \frac{\lvert d\rvert-q}{p-q}, & q<\lvert d\rvert\le p\\ 1, & otherwise.\end{cases}$	q, p	
Type VI. Gaussian criterion	$H(d)=1-\exp\{-\frac{d^2}{2\sigma^2}\}$	σ	

1. Usual Criterion or Preference Function

This function is applicable to cases when the decision maker is only interested in the difference between criteria values. Here there is no allocation of importance for the differences between criteria values. The decision maker only has strict preferences for an alternative with the greatest criteria values. In short, their preference judgment is based on the principle that the "more the better". This type of function is boundary free (neither q nor p is defined).

The decision maker's focus is only on the evaluation difference and so $p\ (d) = 1$ and if and only if $d_i(A_k,A_l) = C_i(A_k) - C_i(A_k)$ is positive and $p\ (d) = 0$ if $d_i(A_k,A_l) = C_i(A_k) - C_i(A_k)$ is negative and the value of the difference does not matter (Podvezko and Podviezko, 2010).

For example, one job offer is preferred over another if offered salary is higher without assigning any importance to the difference; it is important if distance to the office is higher or smaller; if one candidate for a job knows more languages than another etc. the usual preference function is defined

$$p(d) = \begin{cases} 0, & x \le 0 \\ 1, & x > 0 \end{cases} \tag{3.10}$$

2. U-shape preference function or the quasi criterion

This differs from the usual preference function by the establishment of the indifference threshold q, this indifference threshold marks the lower boundary of the evaluation difference such that when the difference (d) between the evaluation of two alternatives is below q the decision maker considers the two alternatives indifferent and the preference function $p\ (d) = 0$ since $d \le q$. On the other hand, if the evaluated difference between the two alternatives is above q then there is a strict preference of one alternative over the other and the preference function $p\ (d) = 1$ since d >

q, though the function is u-shape our focus is on the right side of it. Hence, to use the u-shape criterion the decision maker has to determine only the value of q and this has economic signification- the greatest value of deviation between two alternative actions below which the decision maker declares the affected alternatives indifferent. For example a new job will have strict preference p (d) = 1 over another if only the salary difference exceeds 500 Ghana cedis (q= 500) otherwise the difference will be of no value to the employee and p (d) = 0. The same way, a candidate becomes preferable to another if the work experience of that candidate is more than another four years (q= 4) or that candidate correctly answered at least 4 questions more than another and so on. The algebraic definition of the function is:

$$p(d) = \begin{cases} 0, & x \leq 0 \\ 1, & x > 0 \end{cases} \qquad (3.11)$$

3. Level preference function

This function makes use of the indifference and preference threshold, q and s respectively which must therefore be defined simultaneously by the decision maker.

As usual, if the value difference between two evaluated alternatives is below indifference threshold q then the two alternatives concerned are regarded as indifferent and [p (d) =0] by the decision maker. If the difference (d) is above the preference threshold s, the decision maker expresses a strong preference [p (d) =1] of one alternative over another. And if the difference d is between q and s then there is a weak preference of one alternative over another denoted by [p (d) =1/2] as the value of the preference function. The analytical expression is as shown below:

$$\begin{cases} 0, when\ d \leq q \\ 0.5, when\ q < d \leq s \\ 1\ when\ d > s \end{cases} \qquad (3.12)$$

4. Criterion with linear preference function

This has a boundary parameter such that if the evaluation difference d is below s then the preference of the decision maker increases linearly with the difference d, if d is above s then the decision maker will have a strict (constant) preference for one option over another. This function is therefore different from the u-shape function in the interval 0 to s where the link between the point of indifference p (d) = 0 and the point of strict preference of one alternative over another [p(d) = 1] is linear but not a shift. This preference function has only an upper boundary s , a preference threshold above which there is a strict preference for one alternative over another. In effect, the preference threshold s is the lowest value of difference (d) above which the decision maker has strict preference for one of the corresponding alternatives. The analytical expression for the v-shape preference function is as follows:

$$\begin{cases} 0, when\ d \leq 0 \\ d/s, when\ 0 < d \leq s \\ 1\ when\ d > s \end{cases} \tag{3.13}$$

5. Criterion with linear preference and indifference area preference function

This function too has the parameters q and s as defined before and the decision maker has to determine their values. In this case the preference of the decision maker increases but linearly from the point of indifference threshold (q) to the point of strict preference threshold (s), in other words, the preference function increases steadily and linearly from zero to one based on the formula (d-q)/(s-q). The value of this formula suggests the degree of preference of one alternative over another. In view of this, when q = 0 the function turns to v-shape preference function. For example, a job seeker already into another job will be indifferent over the job he is engaged in and a new one if the salary difference of these two jobs is less than 500 cedis (p(d) = 0). ON the other hand, the seeker expresses strict preference for the new job if the salary of the new job over his current job if the salary offer of the new job offer exceeds 1000 cedis ($p(d)$ =1) and there will be a preference of some sort for the new job over his current job if the salary offer of the new one falls within 500 and 1000 cedis.

The preference level is calculated by the formula $p(d)$= (d-500)/(1000-500)= (d-500)/500 The algebraic definition of this function is given as below :

$$\begin{cases} 0, when\ d \leq q \\ d - \frac{q}{s} - q, when\ q < d \leq s \\ 1\ when\ d > s \end{cases} \tag{3.14}$$

6. Gaussian Preference Function

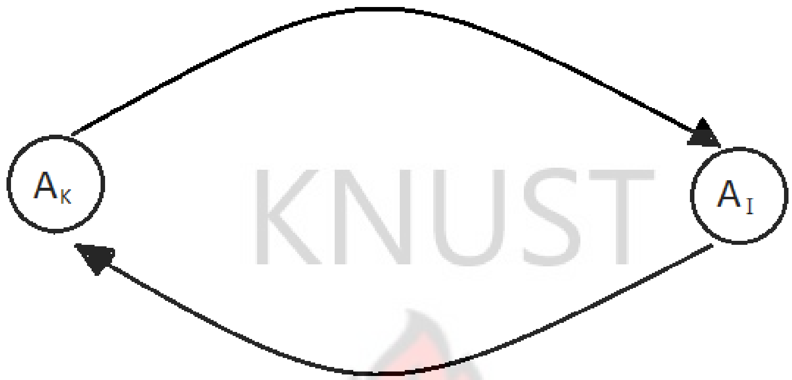
This makes use of statistical data involving random values with normal distribution. The decision maker requires only determining the parameter of standard deviation of the given random values. The function increase most considerably at values of difference close to parameter Preference increases gradually from point zero along with the gradual increase (d). As the difference (d) in criteria values becomes considerably large so does the preference increases towards the preference threshold 1 but never hit on the exact mark. The algebraic definition is presented below:

$$p(d) = \begin{cases} 0,\ when\ d \leq 0 \\ 1 - \exp\left(\frac{-d^2}{2\sigma^2}\right), when\ d > 0 \end{cases} \tag{3.15}$$

3.3.4 Ranking of Alternatives in Promethee

The two indices $\pi(A_k,A_l)$ and $\pi(A_l,A_k)$ connect every pair of alternatives say A_k,A_l to each other . Such a connection or relation is known as outranking relation. Graphically, the relation is often represented by two nodes denoting the two alternatives linked to each other by a corresponding two arcs each for a preference index as presented in Figure 3.1 below:

$\pi(A_k,A_l)$



$\pi(A_I,A_K)$

Figure 3.1: Outranking flow relation

From Figure 3.1, the alternatives A_k and A_l are the nodes. The preference index $\pi(A_k,A_l)$ which links node A_k and $A_l$as indicated by the arrow of the upper arc of Figure 3.1 shows the magnitude of the preference of the alternative A_k over A_l. The preference index $\pi(A_l,A_k)$ on the other hand connects node $A_l$to A_k and is indicated by the arrow of the lower arc of Figure 3.1.

Now given the set of possible alternatives in A, each alternative $A_k \epsilon A$ faces (n-1) other alternatives in A, where n connotes the number of alternatives in A. The PROMETHEE method

sums up all preference indices that are in favour of the alternative A_k, to get what is referred to as positive outranking flow:

$$\emptyset^+(A_j)=\frac{1}{n-1}\sum_{k=1}^{n}\pi(A_j,A_k) \qquad (3.16)$$

It sums up all indices which are not in favor of A_k to be the negative outranking flow:

$$\emptyset^{-}(A_j)=\frac{1}{n-1}\sum_{k=1}^{n}\pi(A_k,A_j) \qquad (3.17)$$

Therefore, the positive outranking flow shows how the alternative A_k is outranking all else in A overall criteria. It is called the power of A_k over the strength of the outranking character A_k. On the other hand, the negative outranking flow indicates how an alternative A_k being outranked by all other alternatives in A. This measure represents the weakness of the outranked character A_k. The higher the positive outranking flow and the lower the negative flow the better the alternative A_k..

In graphical representation, the positive outranking flow is represented by Figure 3.2

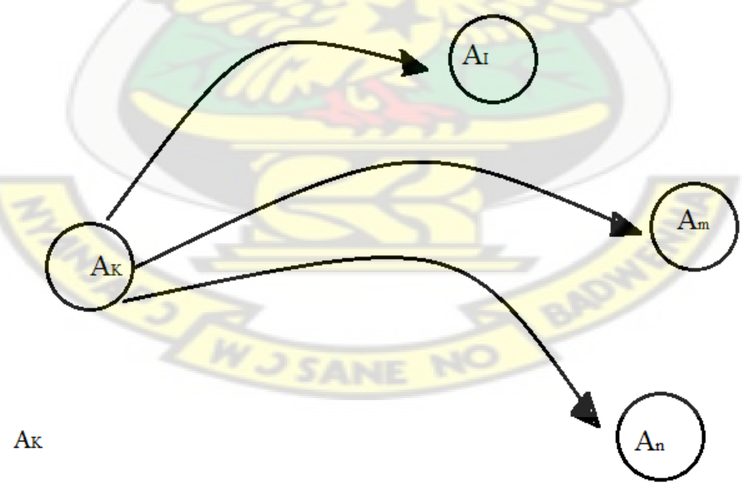


Figure 3.2: Positive outranking flow

From Figure 3.2 the arrows directed at nodes A_l, A_m, A_n from node A_k show how the alternative A_k outranks all other alternatives. These directed arrows from A_k are called the positive outranking flow (leaving flows) denoted by $\emptyset^+$ (A_k). The negative outranking flow$\emptyset^-$ (A_k) is graphically represented by Figure 3.3. In Figure 3.3, the arrows from nodes A_l, A_m, A_n directed at node A_k are called the negative outranking (entering flows) and they show how the alternative A_k is outranked by the other alternatives.

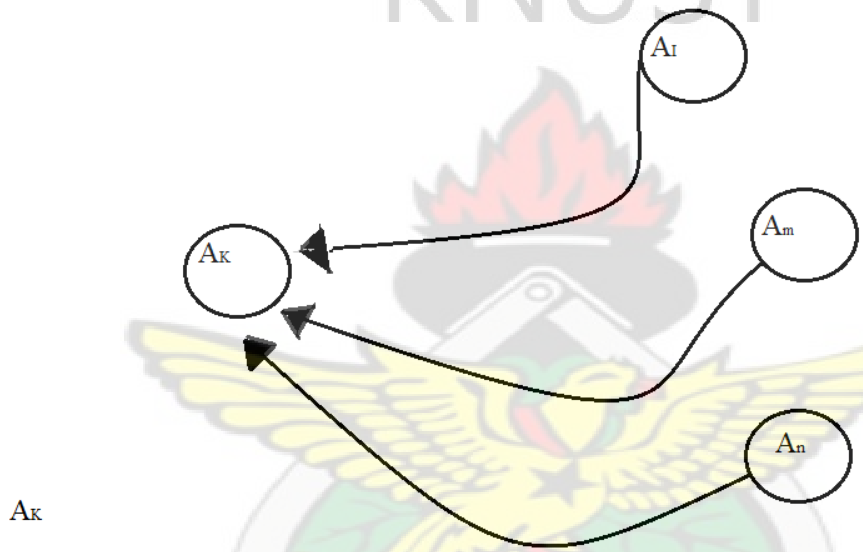


Figure 3.3: Negative Outranking flow

The net flow, denoted by $\emptyset$ (A_k), is the difference between the positive flow and the negative flow. Essentially, the net flow is used for PROMETHEE II (complete ranking). The ranking of a finite set of alternatives under PROMETHEE methodology may involve two ranking processes which are namely:

(i) The partial ranking process

(ii) The complete ranking process

3.3.5 Promethee I - The Partial Ranking Method

The partial ranking (PROMETHEE I) establishes the outranking relation existing between various alternatives via the leaving $\emptyset^{+}$ (A_k) and the entering $\emptyset^{-}$ (A_k) flows on node A_k.

The possible outcomes may be denoted by P, I and R, where P, often placed between two alternatives as, $A_k PA_l$, signifies the preference of the alternative A_k over A_l.$AkiA_l$ signifies the indifference between alternatives A_k and A_i and $A_k RA_l$ signifies the incomparability of the two alternatives A_k and A_l over all criteria.

The first column in Table 3.4 represents the preference relation which indicates the three possible outcomes when alternatives are compared pair wise. The possible outcomes are

(i) $A_k PA_l$ means A_k is preferred to A_l

(ii) $A_k IAl$ means A_k is indifferent to A_l

(iii) $A_k RA_l$ means A_k is incomparable A_l

The second column of Table 3.4 labelled cases, give the condition under which a given pair wise comparison of alternatives can be regarded as preference (P), indifference (I) or incomparable (R). The third column is the graphical representation column which shows how one alternative A_k is preferred to A_l by means of a directed arrow from A_k to A_l($A_k \rightarrow A_l$). However, indifference or incomparable relations are shown by means of a dash (-).These three cases are identified using the following preorders as shown in Table 3.4 below:

Table 3.4: Outranking relations for Partial Promethee Method

Preference Relation	Cases	Graphical Representation
$A_K \mathrm{P} A_I$	$\emptyset^+(\boldsymbol{A_K}) > \emptyset^+\boldsymbol{A_I}$ **and** $\emptyset^-(\boldsymbol{A_K}) < \emptyset^-(\boldsymbol{A_I})$ $\emptyset^+(\boldsymbol{A_K}) > \emptyset^+\boldsymbol{A_I}$ **and** $\emptyset^-(\boldsymbol{A_K}) = \emptyset^-(\boldsymbol{A_I})$ $\emptyset^+(\boldsymbol{A_K}) = \emptyset^+(\boldsymbol{A_I})$ **and** $\emptyset^-(\boldsymbol{A_K}) < \emptyset^-(\boldsymbol{A_I})$	$A_K \blacktriangleright A_I$
$A_K \mathrm{I} A_I$	$\emptyset^+(\boldsymbol{A_K}) = \emptyset^+(\boldsymbol{A_I})$ **and** $\emptyset^-(\boldsymbol{A_K}) = \emptyset^-(\boldsymbol{A_I})$	-
$A_K \mathrm{R} A_I$	$\emptyset^+(\boldsymbol{A_K}) > \emptyset^+(\boldsymbol{A_I})$ **and** $\emptyset^-(\boldsymbol{A_K}) > \emptyset^-(\boldsymbol{A_I})$ $\emptyset^+(\boldsymbol{A_I}) > \emptyset^+(\boldsymbol{A_K})$ **and** $\emptyset^-(\boldsymbol{A_K}) > \emptyset^-(\boldsymbol{A_I})$	-

The conclusion from the above table is that:

(i) $A_k PA_l$ implies a higher power of alternative A_k is matched to a lower weakness of A_k, in relation to A_l. In such a consistency the alternative A_k is automatically preferred to A_l.

(ii) $A_k IA_l$ implies the respective leaving flows and entering flows are the same.

(iii) $A_k RA_l$ implies a higher power of the alternative A_k is associated to a lower weakness of A_l. This type of situation arises when out of a set of criteria, alternative A_k is better than A_l on some, and conversely, the alternative A_l is better than A_k on other criteria. When the flows experience such an inconsistency the alternatives therein are declared incomparable. Over here, PROMETHEE I does not decide which alternative is better than the other. The choice is left to the decision maker to make, based on his or her perception, priorities, knowledge, experience etc.

This is the reason why PROMETHEE I is regarded a partial preorder ranking method. It only compares alternatives that are comparable (i.e. only those under P and I) and thus makes the whole ranking incomplete.

The partial ranking can be represented graphically using the leaving and the entering flows. Decision to be made according to this ranking is done by considering the alternative with the highest number of leaving flows. This indicates the alternative most preferred in the comparison to other alternatives.

3.3.6 Promethee II - Complete Ranking

At this stage it is the PROMETHEE II (preorder complete ranking) method which completes the whole ranking process, establishing a relation that links all alternatives be they comparable or incomparable and placing them in their right perspective in a hierarchy from best to worst. If after partial ranking of PROMETHEE I some alternatives are found to be incomparable then we apply PROMETHEE II (the complete ranking) method to finish the ranking process for an optimal decision to be made.

It makes use of only the parameter P and I (preference and indifference respectively). This approach makes use of what is called the net outranking flow, the higher the net flow, the better the alternative.

The alternative A_k is preferable to A_l if and only if $\emptyset\ (A_k) > \emptyset A_I$

(i) The alternative A_k is indifferent to the alternative A_l if and only if $(A_k) = (A_l)$

Tab 3.5: Two existing relations between alternatives in complete ranking

Preference Relation	Cases	Graphical Representation
A_KPA_I	$\emptyset\ (\boldsymbol{A_K}) > \emptyset(\boldsymbol{A_I})$	$A_K \blacktriangleright A_I$
A_KIA_I	$\emptyset\ (\boldsymbol{A_K}) = \emptyset(\boldsymbol{A_I})$	

3.3.7 Illustrative example of Promethee Method

In the selection of an appropriate house the table below gives the cost and location of each house.

The weight assign to cost is 0.75 and that assign to location is 0.25.

Table 3.6a. Cost and location of three houses

	House A	House B	House C
Cost	60	25	30
Location	70	50	60

Table 3.6b. Deviation di(A_K, A_I), on the minimization of cost criterion C_1

Min cost (C)	A=60	B=25	C=30
A=60	0	-35	-30
B=25	35	0	5
C=30	30	-5	O

Table 3.6c.Deviation di(A_K, A_I) on the maximization of location criterion C_2

Max location (C)	A=70	B=50	C=60
A=70	0	20	10
B=50	-20	0	-10
C=60	-10	-10	0

3.3.8 Preference Evaluation

In this Illustrative example the Quasi-criterion preference function is used and is expressed as shown mathematically.

$$P(d) = D_i(A_K, A_I) = \begin{cases} 0, & d \leq 0 \\ 1, & d > 0 \end{cases} \qquad (3.18)$$

Table 3.6 devalues of p_1 (A_k, A_I) 0n the minimization criterion on C_1

C1	I=1	I=2	I=3
K=1	0	0	0
K=2	1	0	1
K=3	1	0	0

Table 3.6e; values of p_2 ($A_{k,}$ A_I) 0n the maximization criterion on C_2

C1	I=1	I=2	I=3
K=1	0	1	1
K=2	0	0	0
K=3	0	1	0

3.3.9 Preference Index

Table 3.6f values of $\pi_1(A_K, A_I)$ on the minimization criterion C_1

C1	A	B	C
A	0	0	0
B	O.75	0	0.75
C	0.75	0	0

Table 3.6g values of $\pi_2(A_K, A_I)$ on the maximization criterion C_2

C1	A	B	C
A	0	0.25	0.25
B	O	0	0
C	0	0.25	0

3.4. Aggregated preference index

The aggregated preference index is derived by summing all the individual preference index and the result is displayed in table below,

Table 3.6h: Aggregated preference index

$\pi(A_K,A_I)$	House A	House B	House C
House A	0	0.25	0.25
House B	0.75	0	0.75
House C	0.75	0.25	0

3.4.1 Calculation of positive (Leaving) and Negative (Entering) flow values

Using the preference functions and weight of the criteria, every action is compared with each other. From this pair wise comparison, table information can be extracted in order to rank all actions. This is done by computing three different flows.

Positive outranking flow;

$$\emptyset^{+}(A_j)=\frac{1}{n-1}\sum_{k=1}^{n}\pi(A_j\,,A_k) \qquad (3.19)$$

Negative outranking flow;

$$\emptyset^{-}(A_j)=\frac{1}{n-1}\sum_{k=1}^{n}\pi(A_k,A_j) \qquad (3.20)$$

Net flow;

$\emptyset=\emptyset^{+}-\emptyset^{-}$

Table 3.6i Calculation of preference flow

$\pi(A_K, A_I)$	House A	House B	House C	$\emptyset^+$
House A	0	0.25	0.25	0.25
House B	0.75	0	0.75	0.75
House C	0.57	0.25	0	0.50
$\emptyset^-$	0.75	0.25	0.50	
$\emptyset$	-0.50	0.50	0.0	

3.4.2 Promethee 1 Partial ranking of Houses

Pomethee 1 uses partial ranking of the actions includes only preferences that are confirmed by both entering and leaving flow. Incomparability arises when both flows give opposite information because the actions have quite different profiles and are thus difficult to compare.

Table 3.7: Partial Promethee ranking.

Ranking	Promethee
1	House B
2	House C
3	House A

3.4.3 Promethee II complete ranking of Houses

The Promethee II uses the net flow to rank completely all the actions from the best to the worst. In this case no incomparabilities are possible. In this example both the partial and complete promethee ranking yielded the same results as shown in table 3.8. House *B* has the highest net flow followed by House *C* and House *A*, that is $B>C>A$.

3.5 Analytic Hierarchy Process (AHP) Theory

According to Geoff (2004), considering n elements to be compared, $C_1 \ldots C_n$ and denote the relative 'weight' (or priority or significance) of C_i with respect to C_j by a_{ij} and form a square matrix $A = (a_{ij})$ of order n with the constraints that $a_{ij} = 1/a_{ji}$, for $i \neq j$, and $a_{ii}= 1$, all i.

Such a matrix is said to be a reciprocal matrix. The weights are consistent if they are transitive, that is $a_{ik} = a_{ij}a_{jk}$ for all *i*, *j*, and *k*. Such a matrix might exist if the a_{ij} are calculated from exactly measured data.

Then find a vector ω of order *n* such that $A_\omega = \lambda_\omega$. For such a matrix, ω is said to be an Eigen vector (of order n) and λ is an eigen value. For a consistent matrix, $\lambda = n$.

For matrices involving human judgment, the condition $a_{ik} = a_{ij}a_{jk}$ does not hold as human judgments are inconsistent to a greater or lesser degree. In such a case the ω vector satisfies the

Equation $A_\omega = \lambda max_\omega$ and $\lambda max \geq n$. The difference, if any, between λmax and n is an indication of the inconsistency of the judgments. If $\lambda max= n$ then the judgments have turned out to be consistent. Finally, a Consistency Index can be calculated from $(\lambda max\text{-}n)/(n\text{-}1)$. That needs to be assessed against judgments made completely at random and (Saaty, 1980) has calculated large samples of random matrices of increasing order and the Consistency Indices of those matrices. A true Consistency Ratio (CR) is calculated by dividing the Consistency Index for the set of judgments by the Index for the corresponding random matrix. (Saaty, 1980) suggests that if that ratio exceeds 0.1 the set of judgments may be too inconsistent to be reliable. In practice, CRs of more than 0.1 sometimes have to be accepted. A CR value of 0 means that the judgments are perfectly consistent.

(Anagnostopolous, 2005) determined that hierarchy evaluation is based on pair wise comparisons. The decision-maker compares two alternatives A_i and A_j using a criterion and assigns a numerical value to their relative weight. The result of the comparison is expressed in a fundamental scale of values ranging from 1 (A_i, A_j contribute equally to the objective) to 9 (the evidence favoring A_i over A_j) the highest possible order of affirmation. Given that the *n* elements of a level are evaluated in pairs using an element of the immediately higher level, an *n*x*n* comparison matrix is obtained.

A comparison matrix is consistent if and only if $a_{ij} * a_{jk} = a_{ik}$ for all *i*, *j*, *k*. AHP measures the inconsistency of judgments by calculating the consistency index CI of the matrix, as

$$CI=(\frac{n_{max} - n)}{(n-1)} \qquad (3.21)$$

The consistency index CI is in turn divided by the average random consistency index RI to obtain the consistency ratio defined as

$$CR = CI / RI \qquad (3.22)$$

The RI index is a constant value for an *n*x*n* matrix, which has resulted from a computer simulation of *n*x*n* matrices with random values from the 1-9 scale and for which $a_{ij} = 1/a_{ji}$. If CR is less than 5% for a 3x3 matrix, 9% for a 4x4 matrix, and 10% for larger matrices, then the matrix is consistent.

$$RI=\frac{1.98(n-2)}{n} \tag{3.23}$$

Once its values are defined, a comparison matrix is normalized and the local priority (the relative dominance) of the matrix elements with respect to the higher level criterion is calculated.

The overall priority of the current level elements is calculated by adding the products of their local priorities by the priority of the corresponding criterion of the immediately higher level. Next, the overall priority of a current level element is used to calculate the local priorities of the immediately lower level which use it as a criterion, and so on, till the lowest level of the hierarchy is reached. The priorities of the lowest level elements (alternatives) provide the relative contribution of the elements in achieving the overall goal.

3.5.1 Illustrative example of AHP Method

Considering making a choice among three hospitals based on some selected criteria as shown in the table 3.3 below.

Table 3.8 Choosing a Hospital Problem

HOSP/CRITERIA	A	B	C	Weight
Location	12.9	27.7	59.4	17%
reputation	54.5	27.3	18.2	83%

The ranking of each hospital is based on computing the composite weight as follows;

U(A)=(12.9×0.17) + (54.5× 0.83)=47.428

U(B)=(27.7×0.17) +(27.3× 0.83)=27.368

U(C)=(59.4× 0.17) +(18.2× 0.83)=25.204

3.5.2 Calculating Consistency in Hospital Selection Problem

Considering n criteria, this establishes n x n matrix A. The criteria in row (i = 1,2,…n) is ranked relative to every other criterion. AHP uses a discrete scale 1-9 in which a_{ij} = 1 signifies i and j are of equal importance, a_{ij} =5 signifies i is strongly more important than j and a_{ij} = 9 indicates that i is extra ordinary important than j. All other intermediate values between 1 and 9 are interpreted correspondingly. Consistency in judgment requires that $a_{ij} = k$ automatically implies that $a_{ji} = 1/k$, also all diagonal elements in a_{ii} of A must be equal to 1 because they rank a criterion against itself. Using the scale 1-9 in Table 3.2, the pair wise comparison matrix A can be derived.

$$A_L=\begin{pmatrix} 1 & 3 & 1/5 \\ 2 & 1 & 1/2 \\ 5 & 2 & 1 \end{pmatrix}$$

Summing all the column elements C1=8, C2=3.5 and C3=1.7, we thus divide all column elements by their respective column sum value to form the matrix:

$$N_L=\begin{pmatrix} 1/8 & 0.5/3.5 & 0.2/1.7 \\ 2/8 & 1/3.5 & 0.5/1.7 \\ 5/8 & 2/3.5 & 1/1.7 \end{pmatrix}$$

$$N_L=\begin{pmatrix}0.125 & 0.143 & 0.118\\ 0.25 & 0.286 & 0.294\\ 0.625 & 0.571 & 0.588\end{pmatrix}$$

Since NL does not have identical columns the matrix is inconsistent. Thus compute the following:

$\Sigma R1/3= (0.125+0.143+0.118)/3=0.129$

$\Sigma R2/3= (0.25+0.286+0.294)/3=0.277$

$\Sigma R3/3= (0.625+0.571+0.588)/3=0.594$

$$A_L=\begin{pmatrix}0.125 & 0.143 & 0.118\\ 0.25 & 0.286 & 0.294\\ 0.625 & 0.571 & 0.588\end{pmatrix}\times\begin{pmatrix}0.129\\ 0.277\\ 0.594\end{pmatrix}$$

$$A_L=\begin{pmatrix}0.3863\\ 0.832\\ 1.793\end{pmatrix}1$$

$N_{max}=(0.3863+ \quad 0.832+ \quad 1.793)=3.0113$

Consistency index CI is given as:

$CI=(n_{max} \quad - \quad n)/(n \quad - \quad 1)$

$$CI=\left({(3.0113-3)}/{3-1}\right)=0.0565$$

The random consistency index is expressed below;

$$RI=\frac{1.98(n-2)}{n}$$

$RI=\frac{1.98(3-2)}{3}=0.66$

Finally, compute the consistency ratio as below:

$CR=\frac{CI}{CR}=\frac{0.0565}{0.66}=0.0856$

Since CR<0.1 the level of inconsistency in AL acceptable.

We conclude based , on the computations Hospital A has the highest composite weight (47.428) and thus is the number one Hospital, followed by Hospital B (27.368) and Hospital C (25.204) respectively.

3.6 Technique for Order of Preference by Similarity to Ideal Solution (TOPSIS)

The Technique for Order of Preference by Similarity to Ideal Solution (TOPSIS) is a multi-criteria decision analysis method, which was originally developed by Hwang and Yoon in 1981' with further developments by Yoon in 1987, Hwang, Lai and Liu in 1993. TOPSIS is based on the concept that the chosen alternative should have the shortest geometric distance from the positive ideal solution and the longest geometric distance from the negative ideal solution. It is a method of compensatory aggregation that compares a set of alternatives by identifying weights for each criterion, normalizing scores for each criterion and calculating the geometric distance between each alternative and the ideal alternative, which is the best score in each criterion. An assumption of TOPSIS is that the criteria are monotonically increasing or decreasing. Normalization is usually required as the parameters or criteria are often of incongruous dimensions in multi-criteria problems. Compensatory methods such as TOPSIS allow trade-offs between criteria, where a poor result in one criterion can be negated by a good result in another criterion. This provides a more realistic form of modeling than non-compensatory methods, which include or exclude alternative solutions based on hard cut-offs.

3.6.1 TOPSIS method

The TOPSIS process is carried out as follows:

Step 1

Create an evaluation matrix consisting of m alternatives and n criteria, with the intersection of each alternative and criteria given as x_{ij}, we therefore have a matrix $(x_{ij})_{mxn}$.

Step 2

The matrix $(x_{ij})_{m\times n}$ is then normalised to form the matrix

R=(r_{ij}) m$\times$ n, using the normalisation method

$r_{ij}=x_{ij}/P^{MAX}(v_j)$,i=1,2,….m,j=1,2,…,n,where $P^{MAX}(V_j)$ is the maximum possible value of the indicator v_j,j=1,2,…,n.

Step 3

Calculate the weighted normalised decision matrix

T=(t_{ij})m×n=$(w_j r_{ij})$m× n,i=1,2,…,m

$w_j = W_j / \sum_{j=1}^{n} wj$,j=1,2,…,n so that $\sum_{j}^{=1n} w_j$ =1, and w_j is the 0riginal weight given to the indicator v_j ,j=1,2,…,n.

Step 4

Determined the worst alternative (A_w) and the best alternative (A_b);

A_w ={[max(t_{ij} |i=1,2,…m)|j∈ J_-],[min(t_{ij} |i=1,2,…,m]|j∈ $J_{+]\}}$ ≡{t_{wj} |j=1,2,…,n }.

A_B={[min(t_{ij} |i=1,2,…,m)|j∈ J_-], [max(t_{ij} |i=1,2,…,m)|j∈ J_+]}≡{t_{bj} |j=1,2,…,n},

Where,

J_+={j=1,2,…,n|j associated with the criteria having a positive impact, and

J_-={j=1,2,…,n|j associated with the criteria having a negative impact

Step 5

Calculate the L2-distance between the target alternative i and the worst condition A_W

$$d_{iw} = \sqrt{\sum_{j=1}^{n} (t_{ij\,-t_{twj}}\ )^2},\text{i=1,2,…,m} \qquad (3.24)$$

and the distance between the alternative i and the best condition A_b

$$d_{ib} = \sqrt{\sum_{j=1}^{n} (t_{ij\,-t_{bj}}\ )^2}.\backslash,\text{i=1,2,…,m}$$

Where d_{wi} and d_{ib} are L2-norm distances from the target alternative I to the worst and best condition respectively.

Step 6

Calculate the similarity to the worst condition:

s_{iw} =d_{ib}|(d_{iw}+d_{ib}),0≤ s_{iw} ≤ 1,i=1,2,…m. (3.25)

s_{iw} =1 if and only if the alternative solution has the worst condition; and

s_{iw} =0 if and only if the alternative solution has the best condition.

Step 7

Rank the alternatives according tos_{iw} (i=1,2,…m)

Normalizations

Two methods of normalization that have been used to deal with incongruous criteria dimensions are linear normalization and vector normalization.

Linear normalisation can be calculated as in Step 2 of the TOPSIS process above. Vector normalisation was incorporated with the original development of the TOPSIS method and is calculated using the following formula:

r_{ij} =$\frac{x_{ij}}{\sqrt{\sum_{i=1}^{m} x_{ij}{}^{2}}}$,i=1,2,…m,j=1,2,…,n (3.26)

3.6.2 Illustrative example of TOPSIS

Consider a decision matrix in students to select the best teacher out of four (4) teachers based on seven (7) criteria using Topsis.

Evaluation of four teachers performances based on the following seven (7) criteria: subject knowledge (C1), method of teaching (C2), communication skill (C3), accessibility (C4), discipline & behavior (C5), power of explanation (C6) and attitude(C7). The proposed model yields the ranking of the four teachers based on evaluating their performances.

Step -1: Construct normalized decision matrix.

Normalize scores or data as follows:

$$r_{ij} = \frac{x_{ij}}{(\sum x^2_{ij})^{1/2}} \text{for } i=1,\ldots m{:}j=1,\ldots n \tag{3.27}$$

TABLE 3.9: NORMALIZED DECISION MATRIX

Weight	**0.30953**	**0.20661**	**0.10786**	**0.06532**	**0.10106**	**0.16433**	**0.04529**
Teacher	C_1	C_2	C_3	C_4	C_5	C_6	C_7
T_1	0.065828	0.060948	0.061911	0.051743	0.04864	0.148883	0.03949
T_2	0.117698	0.014797	0.146781	0.151129	0.101391	0.014741	0.01323
T_3	0.025548	0.228318	0.018001	0.065029	0.063892	0.03567	0.12721
T_4	0.057852	0.02361	0.054706	0.016581	0.043376	0.092338	0.108631
SUM	**0.26692**	**0.32767**	**0.2814**	**0.28448**	**0.2573**	**0.29163**	**0.28856**
SQ.ROOT	**0.51665**	**0.57243**	**0.53047**	**0.53337**	**0.50725**	**0.54003**	**0.53718**

Weight	**0.30952**	**0.20661**	**0.10786**	**0.06532**	**0.10106**	**0.16433**	**0.04529**
Teacher	**C_1**	**C_2**	**C_3**	**C_4**	**C_5**	**C_6**	**C_7**
T_1	0.496604	0.43128	0.469055	0.42648	0.434788	0.714505	0.369933
T_2	0.664033	0.212502	0.722227	0.728864	0.627741	0.224825	0.214124
T_3	**0.309371**	**0.834737**	**0.252921**	**0.478107**	**0.498315**	**0.349731**	**0.66396**
T_4	**0.465548**	**0.268429**	**0.440916**	**0,241426**	**0.410588**	**0.562694**	**0.613561**

Step 2: Construct the weighted normalized decision matrix. Multiply each column of the normalized decision matrix by its associated weight.

An element of the new matrix is:

$v_{ij} = w_j . r_{ij}$ (3.28)

TABLE 3.10: WEIGHTED NORMALIZED DECISION MATRIX

Weight	**0.30952**	**0.20661**	**0.10786**	**0.06532**	**0.10106**	**0.16433**	**0.04529**
Teacher	**C_1**	**C_2**	**C_3**	**C_4**	**C_5**	**C_6**	**C_7**
T_1	0.153709	0.089107	0.050592	0.027858	0.04394	0.117415	0.016754
T_2	0.205531	0.043905	0.077899	0.047609	0.06344	0.036946	0.009698
T_3	0.095757	0.172465	0.02728	0.03123	0.05036	0.057471	0.030071
T_4	0.144096	0.05546	0.047557	0.01577	0.041494	0.092468	0.027788

Step 3: Determine the ideal and negative ideal solutions.

Ideal solution:-

A* = {v1* , …, vn*},where

$v_j^* = \{\max(v_{ij})$ if j∈J; $\min(v_{ij})$ if j∈ $J^{|}\}$

= {0.205531, 0.172465, 0.077899, .047609, 0.06344, 0.117415, 0.030071}

Negative ideal solution:-

A' = {v1' , …, vn' }, where

vj' = { min(v_{ij}) if j∈J: max (v_{ij}) if j∈J}

= {0.095757, 0.043905, 0.02728, 0.01577, 0.041494, 0.036946, 0.009698}

Step .4: Calculate the separation measures for each alternative**.**

The separation from the ideal alternative is:

Si * =$\left\lfloor \sum_j (v_j^* - v_{ij})^2 \right\rfloor^{½}$I;j=1,…m

TABLE 3.11: SEPARATION MEASURE FROM IDEAL ALTERNATIVE

Teacher	**C_1**	**C_2**	**C_3**	**C_4**	**C_5**	**C_6**	**C_7**	**SUM**	**$S_I^{\uparrow}$**
T_1	0.002686	0.006949	0.000746	0.00039	0.00038	0	0.000178	0.011329	0.106438
T_2	0	0.016528	0	0	0	0.006475	0.000415	0.023418	0.153029
T_3	0.01205	0	0.002562	0.000268	0.000171	0.003593	0	0.018644	0.136543
T_4	0.003774	0.01	0.000921	0.001014	0.000482	0.000622	0.000005	0.020508	0.143206

Similarly, the separation from the negative ideal alternative is:

S'i ==$\left\lfloor \sum_j (v_j^{'} - v_{ij})^2 \right\rfloor^{½}$I; $j = 1, \ldots m$ (3.29)

TABLE 3.12: SEPARATION MEASURE FROM NEGATIVE IDEAL ALTERNATIVE

Teacher	**C_1**	**C_2**	**C_3**	**C_4**	**C_5**	**C_6**	**C_7**	**SUM**	**$S_I^{'}$**
T_1	0.003358	0.002043	0.000543	0.000146	0.00006	0.006475	0.0005	0.012621	0.112343
T_2	0.01205	0	0.002562	0.001014	0.000482	0	0	0.O1608	0.126917
T_3	0	0.016528	0	0.000239	0.000079	0.000421	0.000415	0.017682	0.132974
T_4	0.002337	0.000134	0.000411	0	0	0.003083	0.000327	0.006292	0.079322

Step 5: Calculate the relative closeness to the ideal solution Ci* and the corresponding rank of the candidate.

$$C_i^{*}=\frac{S'}{S_i^{*+}+S_i'} \text{ ,} 0<C_i^{*}<1 \qquad (3.30)$$

TABLE 3.12: RELATIVE CLOSENESS AND RANK OF TEACHERS

Teacher	Result	Rank
T_1	0.1513495	1
T_2	0.453362	3
T_3	0.493379	2
T_4	0.356459	4

The overall ranking for each teacher is presented

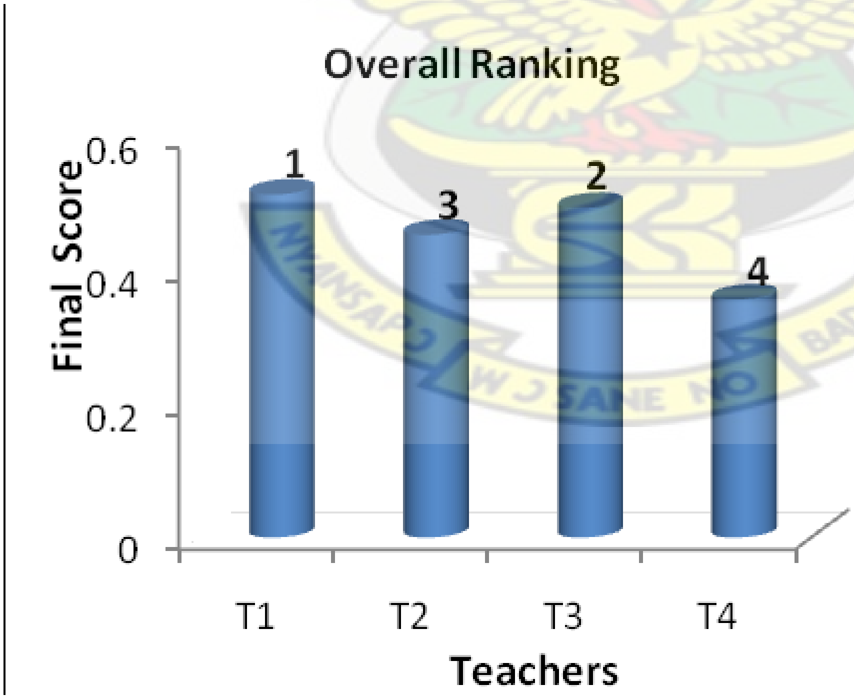


Figure 3.6 Overall Ranking of Teachers

CONCLUSION

The conclusion is that, the teacher, T1 is best in his performance and followed by teacher T3 and teacher T2. The overall performance of the teacher T4 is not good enough with respect to different criteria among all other teachers. It is notable that the subject knowledge of the teacher T2 is better than the teachers T1 & T3 and T4 is also better than T3. That means it can be also concluded that in spite of having sufficient knowledge of a teacher about his subject he/she may not be the best faculty member in his department.

CHAPTER FOUR

COLLECTION OF DATA, ANALYSIS AND RESULTS

4.1 Data Collection

The data to be used for this study was obtained through interviews of workers of the various network operators and from the website of the National Communications Authority (NCA) in Ghana. The data that is quantitative was on the performance of six (6) telecommunication network operators in the Ashanti region.

The data obtained was from National Communications Authority (NCA) website of February 2014 and that stand for the tariffs of six (6) telecommunication networks in that particular period within the year.

4.2 Components of Data

The six (6) mobile telecommunication network operators considered are; MTN, TIGO, VODAFONE, EXPRESSO, AIRTEL and GLO GHANA and these communication network operators are identifies with the letters A_1, A_2, A_3, A_4, A_5 and A_6 respectively.

4.2.1 Alternatives

These six (6) telecommunication networks are defined as the set of alternatives.

A= $\{A_j\} = \{A_{1,}\ A_{2,,}\ A_3,, A_{4\ ,}\ A_5,, A_6\}$

The data collected is displayed in table 4.1 below;

Table 4.1 TARIFFS OF MOBILE TELEPHONE OPERATORS (PREPAID) AS AT FEBUARY, 2014

All rates are quoted in Ghana Cedis and the billing rate is per minute

	MTN	Tigo	Vodafone	Airtel	Glo Mobile	Expresso	Industry Average
On Net	0.105	0.036	0.144	0.0999	0.14	0.0954	0.1034
Other Local Networks	0.13	0.108	0.144	0.0999	0.14	0.1494	0.1286
UK	0.44	0.354	0.3	0.36	0.44	0.3601	0.3757
USA	0.1	0.132	0.13	0.144	0.11	0.22	0.1393
Canada	0.1	0.132	0.13	0.144	0.11	0.22	0.1393
Italy	0.144	0.354	0.45	0.44	0.275	0.3601	0.3372
Nigeria	0.192	0.24	0.3	0.2	0.165	0.2118	0.2181
South Africa	0.44	0.354	0.88	0.39	0.275	0.3601	0.4499
Germany	0.44	0.54	0.45	0.44	0.275	0.3601	0.4175
China	0.1	0.132	0.13	0.144	0.11	0.22	0.1393
UAE	0.44	0.354	0.45	0.39	0.44	0.3601	0.4057
SMS-On Net	0.045	0.0403	0.04	0.04	0.04	0.0424	0.0413
SMS-Other Networks	0.055	0.0477	0.05	0.044	0.04	0.0438	0.0468
MMS	0.18	0.1	0.19	0.19	0.18	0.1625	0.1625
Data/MB	0.09	0.2	0.2	0.09	0.05	0.05	0.1133

The first column, labeled tariffs is the column for the tariffs of the six (6) telecommunication network providers. The second column, on net indicates when your call originates on your home operator's network and terminates to another using the home network, the third column, other local networks (off-net). 'Off-Net' applies when the call is made on a different network, e.g. whilst you are roaming, or if you are using your home network and make a call to a number that resides with a different network provider. The fourth column, international calls (IDD), International direct dialing (IDD) or international subscriber dialing (ISD) is the process of an international telephone call being placed by the caller (the subscriber) rather than by an operator. The fifth column, SMS ON-NET On-Net' is used when your message originates on your home operator's network and terminates to another s using the home network. The six column SMS OTHER NETWORKS (OFF-NET), (C_5), 'Off-Net' applies when the message is made on a different network, e.g. whilst you are roaming, or if you are using your home network and send a message to a number that resides with a different network provider. The seventh column

4.2.2 Criteria (C_i)

The criteria identified by National Communications Authority for measuring tariffs includes

(i) On net (C_1)

(ii) Other local networks (OFF-NET) (C_2)

(iii) International (IDD) calls (USA),(C_3)

(iv) SMS on net(C_4)

(v) SMS (other network),(C_5)

(vi) Data/MB (C_6)

ON NET; (C_1)

'On-Net' is used when your call originates on your home operator's network and terminates to another s using the home network.

OTHER LOCAL NETWORKS (C_2)

'Off-Net' applies when the call is made on a different network, e.g. whilst you are roaming, or if you are using your home network and make a call to a number that resides with a different network provider.

INTERNATIONAL DIRECT DIALING (IDD) CALLS (USA),(C_3)

International direct dialing (IDD) or international subscriber dialing (ISD) is the process of an international telephone call being placed by the caller (the subscriber) rather than by an operator.

SMS ON NET, (C_4)

On-Net' is used when your message originates on your home operator's network and terminates to another s using the home network.

SMS OTHER NETWORKS (OFF- NET), (C_5)

'Off-Net' applies when the message is made on a different network, e.g. whilst you are roaming, or if you are using your home network and send a message to a number that resides with a different network provider.

DATA/ MB, (C_6)

In telecommunications, data transfer rate is the average number of bits (bitrate), characters or symbols (baudrate), or blocks (packet rate) per unit time passing between equipment in a data transmission system. Most commonly, measurements of data transfer rate are reported in multiples of the unit *bits per second* or occasionally in *bytes per second*, but in this case it is the cost of data per megabyte.

4.2.3 Weight of a Criterion

Weight of the criteria w_i, for i=1,…6 are taken by (NCA) to be the same. Thus, each criteria was taken to be $^1/_6$ ie 0.16667 according to NCA, S measure. Summing all together for the six (6) criteria given 1 as expected. This implies that all the criteria were of equal value of significance.

4.2.4 The Decision Table

The decision table showing the tariffs of each of the six (6) network in the Ashanti region as at February 2014.The first column labeled criteria is the column for the six (6) criteria ($C_1 \ldots C_6$).

The second column indicates type of criteria which in this case all the criteria are minimizing criterion. The third column is the alternatives, is a 6×6 matrix in which each of the six (6) rows represent C_1, C_2, C_3, C_4, C_5 and C_6 respectively whiles each of the six (6) column represent each of the alternatives, A_1, A_2, A_3, A_4, A_5 and A_6.The entries of the matrix x_{ij} where I = 1, 2, 3, 4, 5 and 6; j = 1, 2, 3, 4, 5 and 6 are the scores of the various alternatives under each criterion for all the criteria.

Table 4.2: the decision table showing the tariffs of each of the six (6) networks in the Ashanti region as at February 2014.

Table 4.2: Decision Table displaying the performance of network providers

ALTERNATIVES							
CRITERIA	**TYPE OF CRITERIA**	**A_1**	**A_2**	**A_3**	**A_4**	**A_5**	**A_6**
C_1	Min	0.105	0.036	0.144	0.0999	0.14	0.0954
C_2	Min	0.13	0.108	0.144	0.0999	0.14	0.1494
C_3	Min	0.1	0.132	0.13	0.144	0.11	0.22
C_4	Min	0.045	0.043	0.04	0.04	0.04	0.0424
C_5	Min	0.055	0.0477	0.05	0.044	0.04	0.0438
C_6	Min	0.09	0.2	0.2	0.09	0.05	0.05

4.3.1 Multiple Criteria Optimization

These six (6) multiple criteria optimization has to do with the evaluation and ranking of the six (6) alternatives on the six (6) criteria concurrently. In this, ranking is done by taking into account all the six (6) criteria at the same time. In this case one of the multiple criteria approaches to be used here is the PROMETHEE method which is found to be one of the best multiple criteria decision methods. The PROMETHEE method involves the following steps:

1. **The preference Function**: The data was sampled from a continuous set and as such the Gaussian preference function is used since the preference function to be chosen is the priority

of the decision maker. The Gaussian preference function is mostly chosen in PROMETHEE methodology for evaluating criteria on continuous data (Villot, 2009).

The Gaussian criterion Function is defined by;

$$P\ (d) = D_i\ (A_K, A_I) = \begin{cases} 0,\ d \leq 0 \\ 1 - e^{-\frac{d^2}{2\sigma^2}}, d > 0 \end{cases} \tag{4.1}$$

As we use this function, the parameter to be define is the standard deviation ð .

The standard deviation is calculated using the decision matrix of table 4.2 with the formula below;

$$\delta_i^2 = \sum_{j=1}^{n} \left(\frac{X_{ij} - \mu^2}{n-1}\right), i = 1, \ldots 6 \tag{4.2}$$

$$\mu_{i=} \frac{1}{n} \sum_{j=1}^{n} x_{ij} \tag{4.3}$$

Where µ is the mean of the data

The standard deviation (δ^2) and mean (μ_i) for each of the criterion C_i are displayed in Table 4.3.

Table 4.3: The mean and standard deviation of the six (6) criteria.

Criteria	**Mean(µ)**	**Standard deviation (δ)**
C_1	0.1034	0.0232
C_2	0.1286	0.0202
C_3	0.1393	0.0426
C_4	0.0417	0.0021
C_5	0.0468	0.0053
C_6	0.1133	0.0695

2. **Calculation of the Deviations**

We calculate the deviations, by $d_i(A_K, A_i)$ through the normal pair wise comparison of the alternatives, A_K, $A_I \in A$ for each criterion over all the criteria. It it is noted that the deviations are given by,

$\{x_{ik} - x_{il}\}$-maximization (4.3a)

$\{-(x_{ik} - x_{il})\}$-minimization (4.3b)

Where x_{ik} and x_{il} correspond to two (2) alternatives on a criterion as provided in the decision matrix of Table 4.2

Table 4.4 presents all possible deviations $d_i(A_K, A_I)$ from the pair wise comparism of all the alternatives on each criterion.

Table 4.4a: Deviations d_1 (A_K, A_I) on the minimizing criterion C_1

Min C_1	**A_1**	**A_2**	**A_3**	**A_4**	**A_5**	**A_6**
A_1	0	-0.069	0.039	-0.0051	0.035	-0.0096
A_2	0.069	0	0.108	0.0639	0.104	0.0594
A_3	-0.039	-0.108	0	-0.0441	-0.004	-0.0486
A_4	0.0051	-0.0639	0.0441	0	0.0401	-0.0045
A_5	-0.035	-0.104	0.004	-0.0401	0	-0.0446
A_6	0.0096	-0.0594	0.0486	0.0045	0.0446	0

Table 4.4b; Deviations d2 (Ak, Ai) on minimizing criterion C2

Min C_2	**A_1**	**A_2**	**A_3**	**A_4**	**A_5**	**A_6**
A_1	0	-0.022	0.014	-0.0301	0.01	0.0194
A_2	0.022	0	0.036	-0.0081	0.032	0.0414
A_3	-0.014	-0.036	0	-0.0441	-0.004	0.0054
A_4	0.0301	0.0081	0.0441	0	0.0401	0.0495
A_5	-0.01	-0.032	0.004	-0.0401	0	0.0094
A_6	-0.0194	-0.0414	-0.0054	-0.0495	-0.0094	0

Table 4.4c: Deviations d3 (A_k, A_i) on minimizing criterion on C_3

MinC_3	**A_1**	**A_2**	**A_3**	**A_4**	**A_5**	**A_6**
A_1	0	0.032	0.03	0.044	0.01	0.12
A_2	-0.032	0	-0.002	0.012	-0.022	0.088
A_3	-0.03	0.002	0	0.014	-0.02	0.09
A_4	-0.044	-0.012	-0.014	0	-0.034	0.076
A_5	-0.01	0.022	0.02	0.034	0	0.11
A_6	-0.12	-0.088	-0.09	-0.076	-0.11	0

Table 4.4d: Deviations d4 (A_K, Ai) on the minimizing criterion C_4

MinC_4	**A_1**	**A_2**	**A_3**	**A_4**	**A_5**	**A_6**
A_1	0	-0.002	-0.005	-0.005	-0.005	-0.0026
A_2	0.002	0	-0.003	-0.003	-0.003	-0.0006
A_3	0.005	0.003	0	0	0	0.0024
A_4	0.005	0.003	0	0	0	0.0024
A_5	0.005	0.003	0	0	0	0.0024
A_6	0.0026	0.0006	-0.0024	-0.0024	-0.0024	0

Table 4.4e: Deviations d5 (A_k, A_i) on the minimizing criterion C_5

MinC_5	**A_1**	**A_2**	**A_3**	**A_4**	**A_5**	**A_6**
A_1	0	-0.0073	-0.005	-0.011	-0.015	-0.0112
A_2	0.0073	0	0.0023	-0.0037	-0.0077	-0.0039
A_3	0.005	-0.0023	0	-0.006	-0.01	-0.0062
A_4	0.011	0.0037	0.006	0	-0.004	-0.0002
A_5	0.015	0.0077	0.01	0.004	0	0.0038
A_6	0.0112	0.0039	0.0062	0.0002	-0.0038	0

4.4f: Deviations d6 (A_k, A_i) on the minimizing criterion C_6

MinC_6	**A_1**	**A_2**	**A_3**	**A_4**	**A_5**	**A_6**
A_1	0	0.11	0.11	0	-0.04	-0.04
A_2	-0.11	0	0	-0.11	-0.15	-0.15
A_3	-0.11	0	0	-0.11	-0.15	-0.15
A_4	0	0.11	0.11	0	-0.04	-0.04
A_5	0.04	0.15	0.15	0.04	0	0
A_6	0.04	0.15	0.15	0.04	0	0

Step Two: Preference Evaluation

After calculating the deviations $d_i(A_K, A_I)$,the evaluation $p_I(A_K, A_I)$ that measures the intensity of the decision makers preference of A_K over A_I is computed. This is done by using the formula; (d)

$$= D_i\,(A_K, A_I) = \begin{cases} 0, \; d \leq 0 \\ 1 - e^{-\frac{d^2}{2\sigma^2}}, d > 0 \end{cases} \qquad (4.5)$$

$d = d_i(A_K, A_I)$

$\partial^2 = \partial^2{}_i$

Table 4.5 present the summary of the values of $p_I(A_K, A_I)$ for each criterion $C_I \in C$

Table 4.5a : values of $P_1(A_k, A_i)$.For criterion C_1

C_1	I=1	I=2	I=3	I=4	I=5	I=6
K=1	0	0	0.7566	0	0.6795	0
K=2	0.988	0	1	0.9933	1	0.9623
K=3	0	0	0	0	0	0
K=4	0.0239	0	o.8358	0	0.7755	0
K=5	0	0	0.0148	0	0	0
K=6	0.0821	0	0.8885	0.0186	0.8424	0

Table 4.5b: Values of $p_2(A_k, A_i)$, For criteria C_2

	I=1	I=2	I=3	I=4	I=5	I=6
K=1	0	0	0.2135	0	0.1153	0.3695
K=2	0.4474	0	0.7957	0	0.7149	0.8776
K=3	0	0	0	0	0	0.0351
K=4	0.6705	0.0772	0.9077	0	0.8606	0.9503
K=5	0	0	0.0194	0	0	0.049
K=6	0	0	0	0	0	0

Table 4.5c: Values of $P_3(A_k, A_i)$, for criterion C_3

	I=1	**I=2**	**I=3**	**I=4**	**J=5**	**I=6**
K=1	0	0.2458	0.2196	0.4134	0.0272	0.9811
K=2	0	0	0	0.0389	0	0.8816
K=3	0	0.OO11	0	0.0527	0	0.8927
K=4	0	0	0	0	0	0.7964
K=5	0	0.1248	0.1044	0.2728	0	0.9643
K=6	0	0	0	0	0	0

Table 4.5d: Values of p_4 (A_k, A_i), for criterion C_4

	I=1	**I=2**	**I=3**	**I=4**	**I=5**	**I=6**
K=1	0	0	0	0	0	0
K=2	0.3646	0	0	0	0	0
K=3	0.9413	0.6396	0	0	0	0.4795
K=4	0.9413	0.6396	0	0	0	0.4795
K=5	0.9413	0.6396	0	0	0	0.4795
K=6	0.5353	0.04	0	0	0	0

Table 4.5e: Values of P_5 (A_k, A_i),for criterion C_5

	I=1	**I=2**	**I=3**	**I=4**	**I=5**	**I=6**
K=1	0	0	0	0	0	0
K=2	0.6127	0	0.0899	0	0	0
K=3	0.3592	0	0	0	0	0
K=4	0.884	0.2163	0.4731	0	0	0
K=5	0.9818	0.6519	0.8314	0.2478	0	0.2267
K=6	0.8928	0.2372	0.4955	0.0007	0	0

Table 4.5f: Values of P_6 (A_k, A_i), for criterion C_6

	I=1	**I=2**	**I=3**	**I=4**	**I=5**	**I=6**
K=1	**0**	0,7142	0.7142	0	0	0
K=2	0	0	0	0	0	0
K=3	0	0	0	0	0	0
K=4	0	0.7142	0.7142	0	0	0
K=5	0.1526	0.9026	0.9026	0.1526	0	0
K=6	0.1526	0.9026	0.9026	0.1526	0	0

Step Four: Aggregate Preference Index

The result that will be used in further analysis is obtained by the evaluation of the aggregate preference index. The aggregate preference index is evaluated by π(A_K, AI) and is mathematically expressed below;

$\pi(A_K,A_I) = \sum_{I=1}^{n} W_i \, P_i(A_k,A_i) \;\forall\; A_K,A_I \in A.$ (4.6)

With k=1, 2, 3,4,5 and 6 and $w_i=(\frac{1}{6})$ ≈0.16667 is the weight of each criterion. The values of $\pi(A_K, A_I)$ for all the six (6) alternatives in A are shown in Table 4.6

Table 4.6: Aggregated Preference Indices $\pi(A_K, A_I)$

	I=1	I=2	I=3	I=4	I=5	I=6
K=1	0	0.1600	0.3173	0.0689	0.1358	0.2251
K=2	0.4021	0	0.3143	0.1720	0.2858	0.4536
K=3	0.2168	0.1068	0	0.0088	0	0.2345
K=4	0.4199	0.2745	0.4885	0	0.2727	0.3710
K=5	0.3459	0.3865	0.3121	0.1122	0	0.2866
K=6	0.2771	0.1966	0.3811	0.0286	0.1404	0

Step five: Partial Ranking

Through the aggregated Preference indices we could make the following analysis:

We obtain the partial ranking of our finite set of alternatives through the equations,

Positive Outranking flow:

$$\emptyset^{+}(A_j)=\frac{1}{n-1}\sum_{k=1}^{n}\pi(A_j, A_k) \qquad (4.7)$$

Negative Outranking flow:

$$\emptyset^{-}(A_j)=\frac{1}{n-1}\sum_{k=1}^{n}\pi(A_k, A_j) \qquad (4.8)$$

Table 4.7 present the values of the positive $e\ \emptyset^{+}(A_j)$ and negative $\emptyset^{-}(A_j)$ outranking flows for the six (6) alternatives

Table 4.7: Values of the positive and negative flow

Aj	**$\emptyset^{+}(A_j)$**	**$\emptyset^{-}(A_j)$**
A1	0.18166	0.33236
A2	0.32556	0.33236
A3	0.11338	0.25194
A4	0.36532	0.20858
A5	0.28866	0.1246
A6	0.20476	0.05542

The following conditions are very necessary when using the partial ranking:

1. A_k is preferred to A_I if and only if one of the following three conditions is satisfied

   i. $\emptyset^{+}(A_k) > A\emptyset^{+}(A_I)$ and $\emptyset^{-}(_K) < \emptyset^{-}(A_I)$

   ii. $\emptyset^{+}(A_k) > \emptyset^{+}(A_I)$ and $\emptyset^{-}(A_K) = \emptyset^{-}(A_I)$

   iii. $\emptyset^{+}(A_k) = \emptyset^{+}(A_I)$ and $\emptyset^{-}(A_K) < \emptyset^{-}(A_I)$

The Table 4.8a,is a table of 6x6 matrix and the entries denoted by the dash (-) represents no preference between any pair of alternatives while entries with the value one (1) indicate preference of alternative A_K over A_I.

Table 4.8a; Preference table for the six (6) alternatives

	A_1	**A_2**	**A_3**	**A_4**	**A_5**	**A_6**
A_1	-	-	-	-	-	-
A_2	1	-	-	-	-	-
A_3	-	-	-	-	-	-
A_4	1	1	1	-	-	-
A_5	1	-	1	-	-	-
A_6	1	-	1	-	-	-

2. Indifference: indifference exists between any pair of the six (6) alternatives if and only if the condition below is satisfied:

:$\emptyset^{+}(A_k)= A\emptyset^{+}(A_I)$ and $\emptyset^{-}(_K)=\emptyset^{-}(A_I)$

There exists no indifference in this case

3. Incomparability: two of the alternatives are incomparable if and only if

4. :$\emptyset^{+}(A_k)> A\emptyset^{+}(A_I)$ and $\emptyset^{-}(_K)> \emptyset^{-}(A_I)$

Table 4.8b: present the incomparability between pairs of alternatives in this study, entry 1 means A_K is incomparable to A_I

Table 4.8: Incomparability between pairs of alternatives

	A_1	**A_2**	**A_3**	**A_4**	**A_5**	**A_6**
A_1	-	-	-	-	-	-
A_2	-	-	-	-	-	-
A_3	-	-	-	-	1	1
A_4	-	-	-	-	-	-
A_5	-	-	-	-	-	1
A_6	-	-	-	-	-	-

Table 4.8c: the incidence table for alternatives

	A_1	**A_2**	**A_3**	**A_4**	**A_5**	**A_6**
A_1	-	-	-	-	-	-
A_2	1	-	-	-	-	-
A_3	-	-	-	-	-	-
A_4	1	1	1	-	-	-
A_5	1	-	1	-	-	1
A_6	1	-	1	-	-	-

From the incidence table, the row with the highest number of one's (1) is the row with the highest number of directed arcs and the corresponding alternative in the row is the best alternatives.

Based on table 4.8c, the graph of partial ranking is displayed in figure 4.1 below;

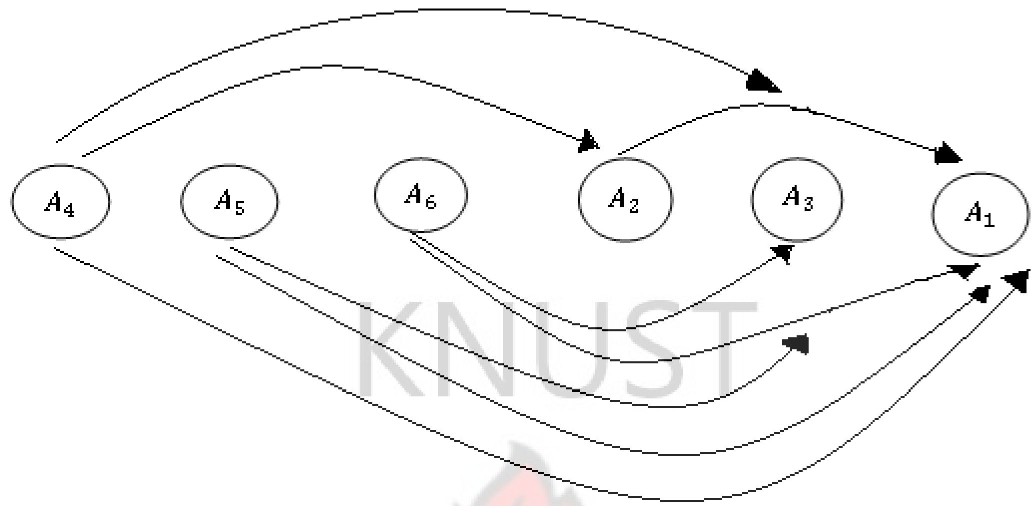


Figure 4.1: Graph of partial ranking

It is realized from figure 4.1that there are no connections between A_4, A_5 and A_6thus three alternatives are incomparable, hence we proceed to use the complete ranking method.

Step Six: Complete Ranking

The complete Ranking deals with the analyzing of the pairs of alternatives by the use of their net flows ($\emptyset(A_j)$).The net flow is mathematically calculated by the use of the equation below:

$\emptyset(A_j)=\emptyset^{+}(A_j)-\emptyset^{-}(A_j)$

Table 4.9 summarized the values of all the net flow for the six alternatives

Aj	$\boldsymbol{\emptyset^{+}(A_j)}$	$\boldsymbol{\emptyset^{-}(A_j)}$	$\boldsymbol{\emptyset(A_j)}$
A1	0.18166	0.33236	-0.15070
A2	0.32556	0.33236	-0.0068
A3	0.11338	0.25194	-0.1387
A4	0.36532	0.20858	0.1567
A5	0.28866	0.1246	0.1641
A6	0.20476	0.05542	0.1493

1. Preference exists between a pair of alternatives (A_K, A_I) if and only

   $\emptyset^{+}(A_K) \neq \emptyset^{+}(A_I)$

   Taking the alternative (A_K, A_I), alternative A_K is preferred to alternative A_I if and only if $\emptyset^{+}(A_K) > \emptyset^{+}(A_I)$, otherwise A_K is not preferred to alternative A_I

   The Table 4.10 shows that the entries denoted by the dash (-) indicates no preference between any pair of alternatives, whiles entries with the value one (1) show preference of alternative A_K over A_I.

Table 4.10: Pair wise comparison of net flow

	A_1	**A_2**	**A_3**	**A_4**	**A_5**	**A_6**
A_1	-	-	-	-	-	-
A_2	1	-	1	-	-	-
A_3	1	-	-	-	-	-
A_4	1	1	1	-	-	1
A_5	1	1	1	1	-	1
A_6	1	1	1	-	-	-

2. Indifference exists between two alternatives if and only if

$\emptyset^{+}(A_K) = \emptyset^{+}(A_I)$

In our case, there exists no indifference

The graph of the complete ranking based on Table 4.3 is shown in Figure4.2

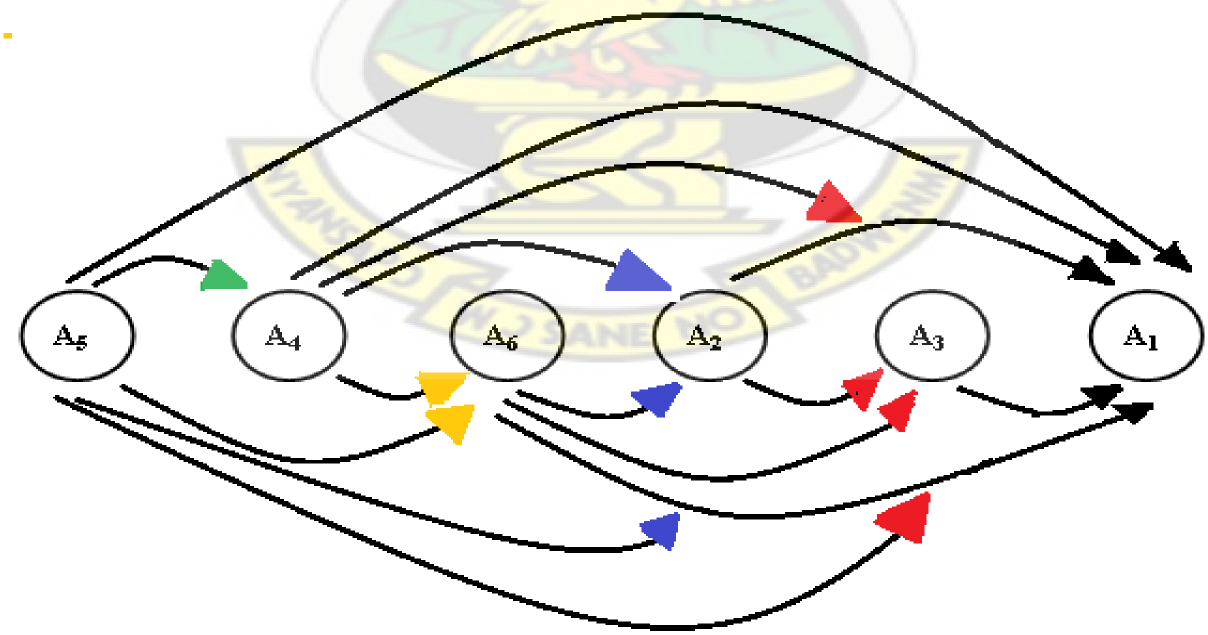


Figure 4.2: The graph of complete ranking

The ranking is done considering the number of directed arcs that is recorded by each alternative such that the best alternative A_5 is the one with the highest number of directed arcs and the alternative A_I without directed arc becomes the worst alternatives.

From Table 4.11, Column one, contains all alternatives (A_1…, A_6), the entries in the second column 0…,5 denotes the number of arcs coming from the nodes (A1…, A6) in figure 4.2. The third column is the position of alternatives based on the number of arcs.

Table 4.11: Ranking of Six alternatives using Promethee

Alternatives	Number of directed arcs	Ranking position
A_1	0	6th
A_2	2	4th
A_3	1	5th
A_4	4	2nd
A_5	5	1st
A_6	3	3rd

From the above table, it can be observed that $A_5 > A_4 > A_6 > A_2 > A_3 > A_1$ where ">" means "is better than"

Hence, the alternative A_5 is the best alternatives and the alternative A_1 is the worst alternative.

4.4 Discussion

The results obtained ranked all carriers from best to worst using the complete ranking in the Promethee method since there was incomparability in the system the partial raking could not be used to rank alternatives. Again the partial ranking could not produce reliable result.

CHAPTER FIVE

CONCLUSIONS AND RECOMMENDATIONS

5.1 Conclusion

It is established from the multi criteria ranking of the six (6) mobile networks telecommunication operators using the Promethee method, the conclusions arrived at were as follows;

1. The Promethee method conveniently ranked all the six network operators from the best to the worst in terms of cost
2. From the result obtained, incomparability existed when using the partial ranking in Promethee method, so the Promethee complete ranking with no occurrence of incomparability.
3. With partial ranking the total number of nodes entering and leaving is not equal, but with complete ranking the total numbers of nodes entering and leaving are the same throughout.
4. From table 4.11, it can be seen that the best alternative is $A_{5,}$ follow by $A_{4,}$ A_6, A_2 ,A_3 and A_1 as the worst alternative.

5.2 Recommendations.

The following were the recommendations emanating from the research.

1. The Promethee methodology should be used to solve any multi criteria decision making problem with a very high level of accuracy.

2. Network operators need expert in Promethee method to help them make better decision in areas of multi criteria decision making.
3. Further study should be done in this area using other methods such as TOPSIS, AHP and others.
2. National Communications Authority (NCA) should use Promethee method periodically to rank network operators so as to ensure competition among it members.
3. This ranking could change based on the review of tariffs by network operators and as such could be done periodically.

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