# PROJECTING LOSS RESERVES USING TAIL FACTOR DEVELOPMENT METHOD

# A CASE STUDY OF STATE INSURANCE COMPANY (MOTOR INSURANCE)

By

DANIEL OKYERE *DWEBENG*,

(B.Sc. Statistics)

A Thesis submitted to the Department of Mathematics,

Kwame Nkrumah University of Science and Technology

in Partial Fulfillment of the Requirements for the Degree of

MASTER OF SCIENCE

INSTITUTE OF DISTANCE LEARNING

March, 2016

# Declaration

I hereby declare that this submission is my own work towards the award of Master of Science (M.Sc.) and that, to the best of my knowledge, it contains no material previously published by another person nor material which has been accepted for the award of any other degree of the University, except where due acknowledgement has been made in the text.

DANIEL OKYERE DWEBENG (PG8732812) ..................... ...............

Student Signature Date

Certified by:

NANA KENA FREMPONG ..................... ...............

Supervisor Signature Date

Prof. S. K. AMPONSAH ..................... ...............

Head, Department of Mathematics Signature Date

# Abstract

In many loss reserve analyses, especially those involving long-tail casualty lines, the loss development triangle may end before all the claims are settled and before the final costs of any year are known. That is, in loss reserving we implicitly assume that there are no claims after the oldest origin year is fully developed. However, it is not appropriate to assume that the oldest origin year is fully settled. This is because it is possible to have incurred but not reported claims after the oldest origin year is fully settled.The study was conducted to project future claims after the run-off triangle is fully run off. Claims data was collected from the State Insurance Company (SIC) Limited motor insurance for a period of six(6) years. The R-software and the spreadsheet were used to analysis claims data using the chain ladder and tail factor techniques. The study revealed that the motor insurance department of SIC would pay extra 3% of claims after estimated outstanding reserves of about 14 million Ghana cedis. It was therefore, recommended that SIC motor insurance department should set aside extra 3% of the estimated reserve to cater for possible incurred but not reported claims in 2015 and beyond.

# Dedication

First, I want to thank God Almighty for the strength and guidance that enabled me to complete this work. Then secondly to my family for the great support they gave me, prayers and sponsorship for this second degree education.

# Acknowledgements

I thank God Almighty for the grace and support to undertake this course and for successful completion. I also thank my supervisor, Nana Kena Frempong, of the the Mathematics Department of the Kwame Nkrumah University of Science and Technology, whose guidance, encouragements and directions made it possible for me to complete this work. God bless you abundantly. I am also grateful to my course mates especially Samuel Boateng and Maxwell Bemere for their assistance during the studies. I am also indebted to Gaston Edem Awashie for his support in this project. I am also appreciative of Regina Boakye, for her love, care encouragement and supports for my studies.

# Table of Contents

# List of Figures

# Chapter 1

# Introduction

## 1.1 Background of the Study

The insurance industry, like State Insurance Company (SIC) LTD, does not sell commodities as such but rather assurances. An insurance policy is an assurance by the insurer to the policyholder to recompense for future claims for an upfront received premium.Consequently, insurers do not know the upfront cost for their service, but rely on past data analysis and judgment to predict a viable price for their contribution. In Non-Life Insurance, e.g. motor insurance; most policies administered for a period of twelve (12) months. Nevertheless, the claims allowance or payment manner can take years. As a result, at all times not even the delivery date of their product is known to insurers. Especially, losses coming from non-life insurance can take a long time to settle and even when the claims are acknowledged, it may take time to establish the extent of the claims settlement cost. Claims can take years to show up. It should come as no shock that the largest item on the liabilities side of an insurer's balance sheet is often the provision or reserves for future claims payments. These reserves can be divided into case reserves (or outstanding claims), which are losses already reported to the insurance company and losses that are incurred but not reported (IBNR) yet.

Historically, reserving was based on deterministic calculations with pen and paper, combined with expert judgement. Since the 1980s, with the arrival of personal computer, spreadsheet software has become very popular for reserving. Spreadsheets not only reduced the calculation time, but allowed actuaries to test different scenarios and the sensitivity of their forecasts. As computers became more powerful, ideas of more sophisticated models started to evolve. Changes in regulatory requirements, have fostered further research and promoted the use of stochastic and statistical techniques. In particular, for many countries, extreme percentiles of reserve deterioration over a fixed time period have to be estimated for the purpose of capital setting. Several methods and models have been developed over the years, to estimate both the level and variability of reserves for insurance claims; Schmidt (2012) or P.D. England & R.J. Verrall (2002) for an overview.

In tradition, many actuaries use the Mack chain-ladder and bootstrap chain-ladder models along with stress testing scenario analysis and expert judgment to estimate ranges of reasonable results; surveys of U.K. actuaries in 2002.

In most analysis of loss reserve , mainly those relating to long tail casualty lines, the loss development triangle may end before all the claims are settled and before the final costs of any year are known. In most times,some actuaries add to the development factors they obtain from available run off triangle with a tail factor that estimates the development beyond the end of the development period for which the link ratio could be estimated.

## 1.2 Background of study area

### 1.2.1 SIC Non-life insurance contracts:

The State Insurance Company has three main contracts. They are usually casualty, property and personal accident insurance contracts. The Casualty insurance contracts defend customers of SIC against the risk of causing harm to third parties as a result of their justifiable activities. The Property insurance policies mainly reimburse SICs customers for damages suffered to their properties or for the value of property lost. The Personal accident insurance contracts mainly recompense the policy holder for injuries suffered. For all these three contracts, premiums are recognised as earned premiums proportionally over the period of coverage. The premium share received on operational contracts that relates to unexpired risks at the statement of financial position date is reported as the unearned premium liability. Furthermore, in statement of financial position position premiums are shown before deduction of commission. Loss and claims expenses are charge to income as incurred, based on the estimated liability for compensation owed to contract holders or third party properties damaged by the contract holders. These include indirect and direct claims settlement costs emanating from events that have occurred up to the statement of financial position date event if they have not yet been reported to the SIC. The State Insurance Company does not discount its liabilities for unpaid claims other than for disability claims. Liabilities for unpaid claims are estimated using the input of assessments for individual cases reported to SIC and the use of statistical analyses for the claims incurred but not reported, and to estimate the expected ultimate cost of more complex claims that may be affected by external factors like court decisions.

### 1.2.2 Management of insurance risk in SIC

Insurance risk or financial risk or both are contracts that the State Insurance Company issues. This section summarises these risks and the way SIC manages them.

(i) *Insurance risk:*

The possibility that an insured event occurs and the uncertainty of the amount of the resulting claim is the risk under any of the insurance contract. This risk is random and therefore unpredictable by the very nature of the insurance contract. For a portfolio of insurance contracts where the probability theory is applied to pricing and provisioning, the major risk that SIC faces under this insurance contract is that the actual claims and benefit payments go beyond the carrying amount of the insurance liabilities. This could happen because of the severity of claims and benefits are greater than estimated. Insurance events are random, the actual number and amount of claims and benefits will differ from year to year from the estimate established using statistical techniques.

(ii) *Sources of uncertainty in the estimation of future claim payments:*

Claims on casualty contracts are to be paid when the insured event happens. SIC is legally responsible for all insured events that occur during the period of the contract even if the loss is discovered after the end of the contract term. Therefore, liability claims are settled over a long period of time and larger variables affect the amount and timing of cash flows from these policies. These mainly relate to the natural risks of the business activities carried out by individual policyholders and the risk management measures accepted by them. The payment made on these policies is the monetary awards settled for injury suffered by employees or members of the public. Such monetary awards are lump-sum payments that are calculated as the present value of the lost earnings and expenses for rehabilitation that the injured party will incur as a result of the accident. The estimated cost of claims includes direct expenses to be incurred in settling claims, net of the expected subrogation (legal right) value and other recoveries. SIC as insurance company uses all realistic steps steps to ensure that it has appropriate information regarding its claims exposures.

However, given the uncertainty in establishing claims provisions, it is likely that the final outcome will prove to be different from the original liability established. The liability for these contracts comprise a provision for IBNR, a provision for reported claims not yet paid and a provision for unexpired risks at the statement of financial position date. The amount of casualty claims is particularly sensitive to the level of court awards and to the development of legal precedent on matters of contract and tort. Casualty contracts are also subject to the emergence of new types of latent claims, but no allowance is included for this at the statement of financial position date.

## 1.3 Problem Statement

Loss reserving is the term used for estimating unknown future payments and the reserve is the money put aside by the insurance company to pay those estimated future obligations, Schewe (2012). Good estimation techniques for reserves are essential since

- Too high reserves are unnecessarily bounded capital. That means the insurance company could have used a fraction of the capital for other investments with higher returns on capital.
- Too low reserves can lead to serious problems for an insurance company. If not enough money has been reserved the insurance company might not be able to pay all obligations and could become insolvent.

Claims on casualty contracts in SIC are payable if the insured incident happens or if the loss is discovered after the end of the policy period. Consequently, liability claims are settled over a long period of time and larger variables affect the amount and timing of cash flows from these polices. The deterministic chain ladder technique is based on the assumption that payments from each accident year will develop in the same way. Most reserve methodology supposed that there are no claims payment after the oldest accident year is fully developed, Boor, (2006).

However, most claims are not fully settled after the run off triangle is fully developed.

## 1.4 Objectives of the study

The main objectives of this project are

1. to explore the development patterns of the run-off triangle.

2. to estimate the unobserved claims using the chain ladder method.

3. to use the tail factor development method to project the not settled claims after the oldest accident year is fully developed.

## 1.5 Methodology

Often in claims reserving problems, the claims settlement process goes beyond the latest development period available in the observed claims development triangle. This means that there is still an unobserved part of the insurance claims for which one needs to build claims reserves. In such situations, claims reserving actuaries apply tail development factors to the last column of the claims development triangle which account for the settlement that goes beyond this latest development period. The claim development triangle is a unique way of arranging the annual loss evaluation for several past policy periods.

Typically, one has only limited information for the estimation of such tail development factors. Therefore, various techniques are applied to estimate these tail factor development factors. Most of these estimation methods are informal methods that do not fit into any stochastic modelling framework.

Typically, one has only limited information for the estimation of such tail development factors. Therefore, various techniques are applied to estimate these tail factor development factors. Most of these estimation methods are informal methods that do not fit into any stochastic modelling framework.

Run-Off triangles which are mainly used by actuaries in general insurance, the spread sheets and R software would be used to forecast future claims and amounts. Also the Chain Ladder method which estimates the age-to-age development factors on claims arising in each year would be applied to estimate the expected outstanding claim reserve for SIC. The Run-Off triangle is used in general insurance to forecast future claims number and amounts whiles the chain ladder is used primarily to estimates the development of cumulative claim payments. The chain ladder methods estimates IBNR loss estimates using the run-off triangle.

Finally, the ratio analysis of the loss development factors to the development year would be plotted on a log scale to give the expected claim percentage and amounts of claims to be reserved after all the development years are fully run-off.

## 1.6 Justification

Although the Deterministic Chain Ladder (CL) and the Bornhuetter-Ferguson(B-F) methods have become a certain standard or benchmark in claims reserving, it was not satisfactory because then it could not estimate expected claims beyond development years.

Also Claim payments are inevitable in any insurance company in the world. General insurers need to be able to estimate the ultimate cost of claims for several reasons. This will enable insurers set up reserves to cover their liabilities for future payments in respect of events that have occurred and even beyond fully run-off development periods.

## 1.7 Limitation

First of all, the researcher could not have access to a lot of the claims data from the website of SIC. It was only six years claims data which were placed there on their website. Persistence attendance and calls to their head office could not give me more information. Information in this thesis is based on the audited balance sheet of SIC from the year 2009 to 2014. Finally, the study was also limited to SIC due to unavailability of obtaining claim data from other insurance companies.

## 1.8 Thesis organization

The study is in five chapters. Chapter one considers the introduction of the study, SICs background and its management of risk, the problem statement and objectives of the study. It also considers the justification for the study, the methodology, thesis organization and limitations. Chapter two covers the review of available literature that is relevant to the study. Chapter three is devoted for the research methodology. Chapter four focuses on the data analysis, which involves the analysis of the incremental and cumulative claims from the run-off triangle, calculating the age-to-age link ratios from the chain ladder, estimation of total outstanding claims and ends with using the tail-factor method to project the expected reserve after all the development years are fully run-off. Chapter five looks at the discussion, conclusion and recommendations of the study.

# Chapter 2

# Literature Review

## 2.1 Introduction

This chapter provides a review of some previous study on loss reserves methods, economic value of loss reserves and some methods used in estimating tail developing factor.

## 2.2 The challenge of reserving

Most insurance model depends on the exact measurement of reserves and risk. In solving the measurement of reserve solvency, professional actuarist need to be able to measure the exact loss reserve because it is a regulatory requirement. One major tasks for an actuary is to estimate the proper amount of reserves to be set aside to meet future liabilities of current in-force business.

In this research we propose the tail factor development method in estimating loss reserves even when all the accident years are fully run off. It is straightforward to use and apply data which is almost always available from the reserve-setting procedure.

## 2.3 Review of variability in claim reserves

About 30 years ago, many researchers have made significant contributions to the study of the variability of reserving methods. The CAS working party paper (2005) presented a comprehensive review that brought all of the important historical research simultaneously together. The CAS working party (2005) ultimately concluded, ”There is no clear preferred method within the actuarial community.” Professional Actuaries need to select one of these several methods that are considered appropriate for specific situation. When it comes to the final decision, judgement still overrules.

The general linear model techniques were used by Taylor and McGuire (2004) to attain an alternative method in cases where the chain ladder method performs inadequately. Verrall (2004) also utilized Bayesian models within the framework of generalized linear models that led to posterior predictive distributions of quantities of interest.

There is a conformity with some models for claims reserving in non-life insurance that assume, explicitly or implicitly, that the proportion of claim payments, payable in the $j$−th development period, is the same for all periods of origin, Taylor (1986).

The second moment of estimates of outstanding claims were introduced by Taylor and Ashe (1983). Verrall (1991) derived an unbiased estimates of total outstanding claims as well as the standard errors of these estimates. Mack (1993) used a distribution-free formula to calculate the standard errors of chain ladder reserve estimates. England and Verrall (2002) presented analytic and bootstrap estimates of prediction errors in claims reserving.

De Alba (2002) gave a Bayesian approach to obtain a predictive distribution of the total reserves.

England and Verrall (2002) also researched and concluded that ”there is little in the actuarial literature which considers the predictive distribution of reserve outcomes. To date the focus has been on estimating variability using prediction errors. It is complex to obtain analytically these distributions which take into account both the process variability and the estimation variability. The models allow the actuary to provide not only point estimates of the required reserves, but also some measures of dispersion such as the variance, as well as the complete distribution for the reserves. This will make it feasible to compute other risk measures, e.g. the VaR.

## 2.4 Review of loss reserving methods

The most important responsibility of the insurer is to ensure they have adequate capital to pay outstanding losses. A lot of work has been done on methods to evaluate and set these loss reserves. Mack (2008) developed a model for the B-F method on the basis of the B-F reserve formula which was stochastic. Also a model was developed from the formula for the prediction error of the B-F reserve estimate. Meanwhile, Mack (1993) had already published a method for the prediction error of the Chain Ladder method. This formula resolved the question of significant differences to the other method and measured the inconsistency of true reserves for business which made the chain Ladder acceptable. The new developed model introduced a vital advice on how to calculate the parameters for the B-F reserve method. This resulted that the appropriate B-F development pattern is different from the CLM pattern. The B-F method was made a standard method for reserving which is fully independent from the Chain Ladder Method. The research also came with the other parameter required for the B-F reserve which is the well-known initial estimate for the ultimate claim amount. The stochastic model identified what was meant by initial estimate for the ultimate amount. Sequentially, to use the formula for the predictor error Macks research recommended professional actuaries to access their ambiguity about sets of parameter about the development pattern and initial ultimate claims estimate.

Schmidt (2006) presented a paper on some important methods in models of loss reserving which was based on run-off triangle. He resulted with the usage of the run off triangle in loss reserving can be acceptable only under the assumption that the development of losses of every accident year follows a development pattern which is common to all accident year. The theory was seen as a primitive stochastic model of loss reserving. He noticed that a development pattern turns out be a unifying force in the comparison of the methods which to a large extent can be summarized under the B-F method. His research also showed that the loss development method and the Chain Ladder as well as the Cape-Cod method and the additive method can be viewed as special cases of the general B-F method. The research further vindicated these methods by general principle of statistical inference applied to suitable and more sophisticated stochastic models. It showed that credibility prediction and Gauss-Markov prediction as well as maximum

likelihood estimation can contribute in a substantial way to the understanding of various methods of loss reserving. Shapland
(2003) explored the meaning of reasonable loss reserves. His research emphasized the need for models to take into consideration the various risks involved along with ”reasonable” assumptions. His work pointed out that reasonableness is subject to many factors, such as culture, guidelines and availability of information, as such the paper concludes that more explicit input is needed on what should be considered reasonable in the actuarial industry.

Berquist and Sherman (1977) and Wiser, Cockley, and Gardner (2001) explored on the standard approaches used to obtain a point estimate for loss reserves. They provided excellent descriptions of the standard approaches used. Estimating loss reserve became an issue in the past and many researchers have also addressed this issue in numerous papers. Mack (1993) presented the chain-ladder estimates and ways to calculate the variance of the estimate. Murphy (1994) offered other variations of the chain-ladder method in a regression setting. There have been a lot of contributions and discussion to loss reserve estimates on the strengths and weaknesses of various evaluation models used by various writers. These works typically dealt with nominal undiscounted value of loss reserves in line with constitutional reserve obligation. Traditionally some methods rely on historical inflation to produce the nominal reserves. Outstanding losses are exposed to inflation until they are finally claimed. If inflation rate during the period is high, loss severity will increase significantly generating large loss reserves. Also, after periods of low inflation, loss severity will be proposed to increase more slowly, leading to lower loss reserves. Since inflation and interest rates are correlated, an insurer with an effective Asset Liability Management (ALM) strategy for dealing with interest rate risk can reduce some of the impact of changing inflation. There have been reserving techniques that attempt to isolate the inflationary component from the other effects, such as those proposed by Butsic (1981), Richards (1981), and Taylor (1977). Butsic studied the performance of inflation upon incurred losses and loss reserves, as well as the effect of inflation on investment income. For both increases and decreases in inflation, these components are found to vary proportionally. According to Butsic, as competitive pricing is hooked on a mixture of both claim costs and investment income, insurers are to a large extent unaffected by unanticipated changes in inflation. Richards provides a basic technique to evaluate the impact of inflation on loss reserves by factoring out inflation from historical loss data.

## 2.5 Economic value of loss reserves

The cost of the underwriters liabilities is generally supposed to be independent of the underwriters underlying the property or assets. The two bodies, The Audit and Actuarial Reporting and Valuation (2006) and Institute of Actuaries of Australia Professional Standard 300 (2007) proposed that loss reserves be discounted by present observable marketbased rates. The rates were based on future obligations or a replicating portfolio of low-risk securities. Their proposal mentioned that appropriate allowance can be made for future claim escalation from inflation and superimposed inflation (e.g. social or legal costs), but no clear methodology was provided as to how inflation should be account for. The work recalled that though there has been much discussion on the meaning of fair or economic value within the world, little attention has been given so far to the impact of economic value on loss reserve. Discounting expected future cash flow by their appropriate discount rate dterminses the financial value of an insurers liabilities. Butsic (1988) and D'Arcy(1987) discounted reserves using an interest rate risk-adjusted that reflects the risk inbuilt in the outstanding reserve. Actuarial Standard of Practice No. 20 addressed the issues in determining discounted loss reserves by actuaries. They set the Standard which suggests that possible discount factors could be the risk-free interest rate or the discount rate used in asset estimation.

## 2.6 Deterministic claims reserving in short term insurance contract

In recent years there has been increased reliance on actuaries to sign off on the claims reserves held by insurance companies. Currently the guidelines set require that a signed statement of actuarial opinion be included with annual statement submitted to the national insurance commission. Other regulators also make use of actuaries to help determine whether reserves are in accordance with regulations and legislations. Accounting firms use both in-house and consulting actuaries to determine whether reserves held by their insurance company client make fair provision for claim-related liabilities. This increased reliance is based on the understanding that actuarial analysis provides an independent, scientific evaluation of contingent liabilities (Blum and Otto, (1998)). One concept that appears both in actuarial literature and in regulations pertaining to claims reserves is the concept of a ”best estimate”.

## 2.7 Some methods used in estimating tail development factors

### 2.7.1 Hertig’s model

Hertig’s model (1985) analysis showed that in the presence of tail development, the outstanding loss liabilities may substantially be underestimated compared to the Paid Incurred Chain (PIC) tail development factor models .Only the PIC tail development factor model gives similar reserves. This comes from the fact that the incurred development factors still give a downward trend to incurred losses in development periods which contradicts some model assumptions. Including tail development factors also gave a higher prediction uncertainty compared to Hertig’s model (1985) without tail development factors. This finding was in line with the ones in Verrall and Wthrich (2012) which showed that prediction uncertainty needs a careful evaluation in the presence of tail development. Paid Incurred Chain reserving model from Merz and Wthrich (2010) was modified so that it allows for the incorporation of tail development factors. These tail development factors were estimated considering claims incurred-paid ratios in an appropriate way. This extended the ad hoc methods used in practice and because analysis were perform in a mathematically consistent way which obtained formulas for the prediction uncertainty. These were obtained analytically for the conditional MSEP and were obtained numerically for other uncertainty measures using Monte Carlo simulations (because they worked in a Bayesian setup). The PIC highlighted the need to incorporate tail development factors in the presence of tail development, since otherwise both the outstanding loss liabilities and the prediction uncertainty were underestimated.

### 2.7.2 The Bondy type methods

Early 1980 Martin Bondy published The Bondy Type methods. This method was introduced at a time where development factors decreased from link factors to link factors. Bondy promoted a process of simply working out the last link factor for use as the tail factor. Afterwards, many differences of this method which are all based on the tail factor on the last available link factors have been developed. This method used the last link factor that could be calculated from the run-off triangle, that is, the link factor of the last stage of the development present in the run-off triangle. The repeating of the last link factor approach would possible look crude and difficult for long-tailed lines, where link factors decrease slowly. The critism in the long-tail lines must usually be acceptable. The Bondy method has a lot of merits which are extremely simple to estimates and easy to comprehend. Also, the method entails reasonable assumptions. However, a major merit is that they tend to greatly underestimate long-tailed lines.

### 2.7.3 The algebraic method

This Algebriac method is used after a certain stage when incurred loss development stops, that is, the link factors are near to one. Because of non-existence of repeat development, reported incurred losses are a good forecaster of the ultimate claim for the oldest years without additional tail factor development. A tail factor that is suitable for paid incurred loss development can then be estimated as the ratio of the case incurred losses to-date for the oldest accident year in the run-off triangle divided by the paid losses to-date for the same accident year. For this method, the paid and incurred development tests will result exactly into the same ultimate losses for that oldest year. This method depends on some assumptions. One of the assumption is that the paid loss and incurred loss development estimates of incurred loss would estimate the same quantity. Therefore the ultimate loss estimate of the two should be equal. The second assumption is that the incurred loss estimate for the oldest year is accurate. The last assumption is that the other development years would show the same development in the tail as the oldest year. This algebraic method can be made general to where case incurred losses are still showing development factors near the tail. In that case, the implied paid loss tail factor will be equal to incurred loss development ultimate loss estimate for the oldest year divided by paid losses to date for the oldest year.

### 2.7.4 The Sherman-Boor method

This method was developed by Richard Sherman, and later by Joseph Boor in the course of analyzing very long tail workers compensation data during the 1987-1989 periods. Although it was originally published some time ago as an adjunct to other tail factor methods, it has only recently received much attention. Thus, a comparatively small percentage of practicing actuaries are aware of it. It was developed largely to provide an alternative to the use of fitted curves and their heavy reliance on theoretical assumptions.

The significant strengths of this method are that it requires only the data already in the triangles. The weakness is that it can be distorted if the adequacy of the ending case reserve has changed significantly over time. This method can be a reasonable approach in predicting tail factors without reliance on extensive assumptions, but it needs to be focused on data mature enough so that the overwhelming majority of claims have been reported.

### 2.7.5 Curve-fitting method

The strategy for developing tail factors is to develop some relationship between the link ratios at various development ages or, some similar quantity such as incremental paid by development age, and use that relationship as an assumption to fit a curve to the link ratios. Projected link ratios in the development ages covered by the tail factor can then be generated. All those projected link ratios can then be multiplied together to provide an estimate of the tail factor. The methods below represent only those methods where curvefitting is the primary source of the tail factor. There are several methods (e.g., Mueller's method) that involve curve-fitting but are not solely curve-fitting type methods.

### 2.7.6 Corro's method

Daniel R. Corro published this method in his 2003 research paper titled ”Annuity Densities with Application to Tail Development.” The paper considers the task of modeling ”pension” claims whose durations may vary, but whose payment pattern is uniform and flat. The aggregate payout pattern is derived from the duration density and can be applied to calculating tail development factors. The tail factor notation is the same as in Corro's original research paper. The following assumptions were made. All payments on all claims are of the same amount.

# Chapter 3

# Methodology

## 3.1 Introduction

This section examines some definitions and concepts of loss reserves and development factor of the run-off triangle. It also shows how a set of development factors can be used to project the future development of delay triangle. Chapter three further gives an overview of the data structure used for typical reserving exercise and discuss classical deterministic chain ladder reserving method. It further introduces the concept of the tail factor method.

## 3.2 Some basic concepts and definitions of the delay triangle.

Loss reserving or outstanding claims reserves refer to the calculation of the required reserves for a tranche of general insurance business. Typically, the claims reserves represent the money which should be held by the insurer so as to be able to meet all future claims arising from policies currently in force and policies written in the past.

There are different ways used in estimating reserves in general insurance from those used in life insurance, pensions and health insurance. This is because general insurance contracts or policies are usually of a much shorter period. A lot of contracts of general insurance are underwritten for a period of one year. Most general insurance contracts are written for a period of one year. Normally only one payment of premium is at the beginning of the contract in exchange for coverage over the year.

Reserves are calculated differently from contracts of a longer duration with multiple premium payments since there are no future premiums to consider in this case. The reserves are calculated by forecasting future losses from past losses.

The following actuarial methods, Chain Ladder Method and the Bornhuetter-Ferguson Method are popularly used in claim reserving.

The Chain Ladder Method uses data in a two dimensional array representing occurrence and development of claims. The upper left of this matrix contains known values (in the past) which are used to estimate the remaining figures (i.e. arising in the future).

The Bornhuetter-Ferguson Method is a Bayesian technique. This means that it incorporates both an independently derived prior estimate of ultimate expected losses as well as estimates generated.

## 3.3 The origins of run-off triangles.

Run-off triangles also normally known as delay triangles usually arise in non-life insurance where it may take some time after a loss until the extent of the claims which have to be paid are known. It is necessary that the claims are ascribed to the year in which the contract/policy was written. The insurer needs to know how much it is accountable to pay in claims so that it can estimate how much surplus it has made. However, it may be many years before it knows the exact claims totals. There are many causes for the delays in the claim totals being finalized. The delay may occur before or between notification of the claim and final settlement. It is certain that though the insurance company does not know the accurate amount for total claims each year, it must make an effort to estimate that amount with as much confidence and accuracy as possible.

### 3.3.1 Types of reserves

Insurance companies need to be able to estimate the ultimate cost of claims for a number of reasons. For example, they need to know the full cost of paying claims in order to set future premium rates. They also need to set up reserves in their accounts to make sure that they have sufficient assets to cover their liabilities.

After the claim event has occur (e.g. a policyholder has been involved in a motor accident), the policyholder will report the incident to the insurer. In due time the insurer will make any payments required (e.g. paying for the repairs vehicle, compensation to an injured person). There may be several payments made under a single claim. When the insurer considers that no further payments will be required for this claim, the claim file will be closed. A general insurer will need to set up reserves to cover its liabilities for future payments in respect of accidents that have already occurred. These reserves will relate to claims at different stages in the settlement process.

Claim event occur → claim reported → claim payments made → claim file close

Figure 3.1: Normal steps involved in settling a general insurance claim are shown in the diagram

In particular, reserves will be required for outstanding reported claims and IBNR (Incurred But Not Reported) claims.

1. An IBNR claims reserve is required in respect of claims that have been incurred but not reported.

2. An outstanding reported claims reserve is required in respect of claims that have been reported, but have not been closed.

### 3.3.2 The evolving nature of claims and reserve

. In non-life general insurance claims are due to substantial damage or theft are reported and settled quickly. However, in some areas of general insurance, there can be some delay between the time of a claim event, and determination of the actual amount the company will have to pay in settlement. When there is an incident leading to a claim, it may not be reported for some time. Also a claim may be reported reasonably soon after an incident, but extensive amount of time may pass before the actual extent of the damage is known. In the case of an accident the incident may be quickly reported, but it may be some time before it is determined actually who is liable and of what extent. In some situations, one might have to wait for the outcome of legal action before damages can be properly ascertained.

The insurer needs to know on a regular basis how much it should be allocating in reserves in order to handle claims emanating from incidents that have already occurred, but for which it does not yet know the full extent of its liability. Claims arising from incidents that have already occurred but which have not been reported to the insurer is termed Incurred But Not Reported, IBNR claims, and a reserve set aside for these claims is called an IBRN reserve. Claims that have been reported but for which a final settlement has not been determined are called outstanding claims. Some insurance companies may make interim payments on a claim, thus a claim remains open and outstanding until it has been settled and closed. Incurred claims are those which have been already paid, or which are outstanding. Reserves for those claims that have been reported, but where a final payment has not been paid, are called reserves.

In respect of claims that occur (or originate) in any given accounting or financial year, ultimate losses at any point in time may be estimated as the sum of paid losses, case reserves and IBNR reserves. Estimated incurred claims or losses are the case reserves plus paid claims, while total reserves are the IBNR reserves plus the case reserves. In practice, companies usually set aside combined reserves for claims originating in several different years.

Claim reserving is a challenging task in general insurance. One should never take too lightly the knowledge and insight that an experienced claims actuary makes use of in establishing case reserves and estimating ultimate losses. However, mathematical models and techniques can also be very useful, giving the added advantage of laying a basis for simulation.

One of the most frequently used techniques for estimating reserves is the chain ladder method. In this method, one looks at how claims arising from different accident (or cohort) years have developed over subsequent development years, and then use relevant ratios (eg. Development factors or grossing-up factors) to predict how future claims from these years will develop. There are many ways in which one might define a development factor for use in projecting into the future. It is a ratio (normally greater than 1) based on a given data which will be used as a multiplier to estimate the progression into the future between consecutive or possibly many years. Also the grossing-up factors can be used to project into the future. It is usually a proportion (< 1) representing that part of the ultimate estimated cumulative losses which have been incurred or paid to date.

## 3.4 Presentation of claims data

There are several ways of presenting claims data which emphasizes different aspects of the data. Claims data will be presented as a triangle, which is the most commonly used method.

The year in which the incident happens and the insurer was on risk is called the accident year. The number of years until a payment a payment is made is called the delay, or development period/year. The claims data are divided up by the accident year and the development year.

In some types of insurance it might be relevant to look at development of claims by month or quarter, but the principles are unchanged. Also data may be presented cumulatively, or on an individual year basis. In this thesis, claim data will be referenced by accident year and development year.

We consider a portfolio of risks and we assume that each claim of the portfolio is settled either in the accident year or in the following $n$ development years. The portfolio may be modelled either by incremental losses or cumulative losses in the following n development years.

### 3.4.1 Portfolio modelled by incremental losses

To model a portfolio by incremental losses, we consider a family of random variables $\{\mu_{ik}\}$,$quadi,k \in 0,1,2,\cdots,n$ and we interpret the random variable $\{\mu_{ik}\}$ as the loss of accident year $i$ which is settled with delay of $k$ years and hence in the development year $k$ and in the calendar year $i+k$. We refer to $\mu_{i,k}$ as the incremental loss of accident year $i$ and development $k$ as shown in Table (3.1) below.

We assume that the incremental loss $\mu_{i,k}$ are observable for calendar years $i+k \leq n$ and that they are non-observable for calendar years $i + k \geq n + 1$. The observable incremental losses are represented by the following run-off triangle.

Table 3.1: Incremental incurred claims data

| Accident year | Development year | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | $0$ | $1$ | ............ | $k$ | ............ | $n-i$ | ............ | $n-1$ | $n$ |
| $0$ | $\mu_{0,0}$ | $\mu_{0,1}$ | | $\mu_{0,k}$ | ............ | $\mu_{0,n-i}$ | ............ | $\mu_{0,n-1}$ | $\mu_{0,n}$ |
| $1$ | $\mu_{1,0}$ | $\mu_{1,1}$ | ............ | $\mu_{1,k}$ | ............ | $\mu_{1,n-i}$ | | $\mu_{i,1,n-1}$ | |
| $\vdots$ | $\vdots$ | $\vdots$ | | $\vdots$ | | $\vdots$ | | | |
| $\vdots$ | $\vdots$ | $\vdots$ | | $\vdots$ | | $\vdots$ | | | |
| $i$ | $\mu_{i,0}$ | $\mu_{i,1}$ | ............ | $\mu_{i,k}$ | ............ | $\mu_{i,n-i}$ | | | |
| $\vdots$ | $\vdots$ | $\vdots$ | | $\vdots$ | | | | | |
| $\vdots$ | $\vdots$ | $\vdots$ | | $\vdots$ | | | | | |
| $\vdots$ | $\vdots$ | $\vdots$ | | $\vdots$ | | | | | |
| $\vdots$ | $\vdots$ | $\vdots$ | | $\vdots$ | | | | | |
| $n-k$ | $\mu_{n-k,0}$ | $\mu_{n-k,1}$ | ............ | $\mu_{n-k,k}$ | | | | | |
| $\vdots$ | $\vdots$ | $\vdots$ | | | | | | | |
| $\vdots$ | $\vdots$ | $\vdots$ | | | | | | | |
| $n-1$ | $\mu_{n-1,0}$ | $\mu_{i,n-1,1}$ | | | | | | | |
| $n$ | $\mu_{i,0}$ | | | | | | | | |

The problem is to predict the non-observable incremental losses.

### 3.4.2 Portfolio model by cumulative losses

To model a portfolio by cumulative losses, we consider a family of random variables $\{\alpha_{ik}\}$,*quadi,k* $\in 0,1,2,\cdots,n$ and we interpret the random variable $\{\alpha_{ik}\}$ as the loss of accident year $i$ which is settled with delay of at most $k$ years and hence not later than in the development year $k$. we refer to $\{\alpha_{ik}\}$ as the cumulative loss of accident year $i$ and development $k$, to $\{\alpha_{i,n-i}\}$ as a cumulative loss of the present calendar year $n$ and to $\{\alpha_{in}\}$ as an ultimate (cumulative) loss. We assume that the cumulative loss $\{\alpha_{ik}\}$ are observable for calendar years $i+k \leq n$ and that they are non-observable for calendar years $i+k \geq n+1$.

The observable cumulative losses are represented by the following run-off triangle.

The problem is to predict the non-observable cumulative losses.

Modelling a portfolio by incremental losses is equivalent to modelling a portfolio by cumulative losses:

Table 3.2: Cumulative incurred claims data

| Accident year | Development year(cumulative claim payments) | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | 0 | 1 | ........ | $k$ | ........ | $n-i$ | ........ | $n-1$ | $n$ |
| 0 | $\alpha_{0,0}$ | $\alpha_{0,1}$ | | $\alpha_{0,k}$ | ........ | $\alpha_{0,n-i}$ | ........ | $\alpha_{0,n-1}$ | $\alpha_{0,n}$ |
| 1 | $\alpha_{1,0}$ | $\alpha_{1,1}$ | ........ | $\alpha_{1,k}$ | ........ | $\alpha_{1,n-i}$ | | $\alpha_{1,n-1}$ | |
| ⋮ | ⋮ | ⋮ | | ⋮ | | ⋮ | | | |
| $i$ | $\alpha_{i,0}$ | $\alpha_{i,1}$ | ........ | $\alpha_{i,k}$ | ........ | $\alpha_{i,n-i}$ | | | |
| ⋮ | ⋮ | ⋮ | | ⋮ | | | | | |
| $n-k$ | $\alpha_{n-k,0}$ | $\alpha_{n-k,1}$ | ........ | $\alpha_{n-k,k}$ | | | | | |
| ⋮ | ⋮ | ⋮ | | | | | | | |
| $n-1$ | $\alpha_{n-1,0}$ | $\alpha_{n-1,1}$ | | | | | | | |
| $n$ | $\alpha_{n,0}$ | | | | | | | | |

- The cumulative losses are obtained from the incremental losses by letting

$$\alpha_{i,k} = \sum_{l=0}^{k} \mu_{i,l} \tag{3.1}$$

Then the non-observable cumulative losses satisfy

$$\alpha_{i,k} = \alpha_{i,n-i} + \sum_{l=n-i+1}^{k} \mu_{i,l} \tag{3.2}$$

From table (3.2), each row in the triangle represents an accident year which defines a cohort of claims (e.g. Accident year cohort). This is because a lot of statistical theory in general insurance was developed in relation to motor insurance, the term accident year.

The accident year 0 row includes all claims relating to accidents that have occurred during the accident year 0 calendar year.

In practice most general insurers use an accounting year starting on 1st January, so the rows really represent calendar years.

The columns represent development years, which show how the cohorts of claims relating to a particular accident year develop over time. Column 0 represents the year in which the accident occurred. Column 1 represents the year after the accident after the accident occurred, etc.

Each entry in the table (3.2) can be defined by its accident year (row) and its development year (column). For example, the random variable $\alpha_{i,k}$ is for accident year i and development year $k$. This includes payment made in year 0, 1 up to year $k$ since it is a cumulative table.

The random variables are figures which are cumulative and represent total amounts paid by the end of each development year. They have been compiled after the end of $n^{th}$ accident year. For the nth accident year, only payments with delay 0 have been reported. For the $(n-1)^{th}$ accident year, payments with delay 0 and delay 1 have been reported, and so on.

Particular calendar years are represented by the diagonals in the triangle. The long diagonal includes all payments made during the most recent calendar year shown in table

(3.2), (as well as past calendar years). Also the upper left corner represents known past payments, while the lower right corner represents unknown future payments as shown in equation (3.2). This research seeks to look at the method used for estimating these unknown figures to complete the lower right triangle. For each accident year, the difference between the figure in the extreme right hand column and the total amount paid so far will give the estimate of the amount we need to hold currently to meet future liabilities arising.

Various alternative tabulations could have been used.

1. The cohorts could be defined by reporting year (all claims reported in year X) or by written year (all claims from policies written in year X).

2. The accident years might be the company's financial years or might be accident quarters or accident months.

3. The entries in the table might show numbers of claims or estimated ultimate cost or related expenses.

### 3.4.3 Estimating future claims

The main purpose of this study is to decide the amounts yet to be paid in respect of the given accident year. This can be done for $n^{th}$ year by looking at previous accident years. If the cumulative payments increase in a similar way it is possible to estimate the reserve figure. This amount is obtained by assuming that the $n^{th}$ accident year is similar to the $0^{th}$ accident year in the pattern of making payments and estimating the cumulative payments at the end of the development year *n* by:

$$\alpha_{1,0} \times \frac{\alpha_{0,1}}{\alpha_{0,0}} = \text{estimate of amount yet to be paid} \tag{3.3}$$

This is not necessarily the best estimate but it is possible to fill in the lower triangle in table (3.2) by comparing present figures with past experience. The estimate amount computed in equation (3.3) above includes what we have already paid out. The actual estimate of the amount yet to be paid using the above method would be

$$\alpha_{1,0} \times \frac{\alpha_{0,1}}{\alpha_{0,0}} - \alpha_{1,0} \tag{3.4}$$

## 3.5 Projections using development factors

### 3.5.1 Run-off patterns

The basic assumptions made in estimating claims concerns the run-off pattern. The simplest assumption is that payments will emerge in a similar way in each accident year. The proportionate increases in the known cumulative payments from one development year to the next can then be used to calculate the expected cumulative payments for future development years.

However, there are a number of choices as to which such ratio should be used to project future claims. The ratios that are used to project future claims are known as development factors or link ratios. Example is shown in equation (3.5) below

$$\frac{\alpha_{0,k}}{\alpha_{0,1}} \qquad (3.5)$$

A development factor may describe the ratio between cumulative claim amounts in consecutive years or between years over a longer period. For each accident year from 0 to $n - 1$ in table (3.2), there is a different ratio for the increase in cumulative payments from development year 0 to development year 1. It is not clear which is the correct one to use when projecting forward for accident year $n$. For a conservative estimate of cumulative payments, it might be best to take the largest ratio. However, some sort of average of the ratios would seem more appropriate. It is possible to use a simple arithmetic average. The disadvantage of this is that it does not take into account the years in which more claims occur. Thus, the greater the amount of claims, the more confidence you can have in the ratio. Assuming a large number of claims would lead to a more predictable average, not a small number of very large claims, which would probably have the opposite effect. This suggests using a weighted average and the usual choice of weight are the cumulative claims values. This is shown as below

$$\text{development factors} = \frac{P_{\text{ratio}} \times \text{weight}}{\qquad} \qquad (3.6)$$

$P_{weight}$

This method of estimating the ratios which describe the run-off pattern is called the chain ladder method.

### 3.5.2 The Chain-Ladder method demonstration

This section explains how to carry out the calculation of completing the run-off triangle using the chain ladder method. This method of calculating development ratios is demonstrated using the table (3.2) above. The ratio in accident year 1 is calculated as follows

$$\frac{\alpha_{1,1}}{\alpha_{1,0}} \tag{3.7}$$

The ratios for other accident years would be calculated in a similar way. The development factor can be calculated using the cumulative claims in the development years 0 and 1; thus

$$\frac{\alpha_{1,0} + \alpha_{1,1} + \cdots + \alpha_{i,1} + \cdots + \alpha_{n-k,1} + \cdots + \alpha_{n-1,1}}{\alpha_{0,0} + \alpha_{1,0} + \cdots + \alpha_{i,0} + \cdots + \alpha_{n-k,0} + \cdots + \alpha_{n-1,0}} \tag{3.8}$$

In other words from equation (3.8), the development factor is the sum of the figures in column one divided by the sum of the corresponding figures from column 0.

The name given to the method presumably arises from the ladder-like operations which are chained over the development years. The development factors for the chain ladder technique would be found for each development year by adding the appropriate number of terms. The development factors would normally be one point estimate. After the development factors have been estimated for each development year, it would be now possible to project forward each accident year.

For accident year *n*, the projection of cumulative claim for the next year would be obtained by equation (3.9)

$$\left(\frac{\alpha_{1,0}+\alpha_{1,1}+\cdots+\alpha_{i,1}+\cdots+\alpha_{n-k,1}+\cdots+\alpha_{n-1,1}}{\alpha_{0,0}+\alpha_{1,0}+\cdots+\alpha_{i,0}+\cdots+\alpha_{n-k,0}+\cdots+\alpha_{n-1,0}}\right)\times\alpha_{n,0} \tag{3.9}$$

Each estimation would follow on from the previous one, by repeated multiplying by the same factors and the preceding development factor. No projection can be done for the first accident year because it is not possible to project beyond the highest development year. That is, claims from accident year 0 are ”fully run-off”, therefore we can expect to make no further payments on these claims. The reserve that needs to be held at the end of the $n^{\text{th}}$ year is the sum over all accident years for which a projection has been made of the difference between the cumulative payment at the end of development year ($n-1$) and the last known entry in the development triangle for that accident year. Numerical examples would be demonstrated for clarity of this method in the next chapter.

### 3.5.3 Assumptions underlying the chain ladder method

The chain ladder technique is based on the assumption that payments from each accident year will develop in the same way. This means, the same development factors are used to project outstanding claims for each accident year. Changes in the rate at which claims emerge can only be incorporated by ”hand adjustment” of the development factors.

The next assumption made when the chain ladder technique is used concerns inflation. It is assumed that weighted average past inflation will be repeated in the future. This is because claims inflation is one of the influences swept up within the projection factors.

## 3.6 Tail factor estimation

The tail factor is the ratio of the estimated ultimate loss to cumulative loss at some point in development time. Often in some claims reserving problems, the claims settlement process goes beyond the latest development period available in the observed claims development triangle. This means that there is still an unobserved part of the insurance claims for which one needs to build claims reserves. In such situations, claims reserving actuaries apply tail development factors to the last column of the claims development triangle which account for the settlement that goes beyond this latest development period. Typically, one has only limited information for the estimation of such tail development factors. Therefore, various techniques are applied to estimate these tail development factors.

Most of these estimation methods are ad hoc methods that do not fit into any stochastic modeling framework. Popular estimation techniques, for example, fit parametric curves to the data using the right-hand corner of the claims development triangle (Mack (1999); Boor (2006); Verrall and Wuthrich (2012)). In practice, one often does a simultaneous study of claims payments and claims incurred data, i.e., incurred-paid ratios are used to determine tail development factors (Boor (2006)).

### 3.6.1 Estimating the tail factor method

Let tail factors $\hat{f}_{\infty}^{(n)}$ for $n = 1,2,\cdots,N$ These ultimate tail factors are needed for settled development triangles since there may be still claims payments after development year $J$ which will not be recognized in the claims development triangle. The proposed rule depends on the estimated CL development factors $\hat{f}_j^{(n)} (j = 0,1,\cdots,J-1)$ and $n = 1,2,\cdots,N$ and

their transformation

$$\hat{y}_j^n = \ln\left(\hat{f}_j^{(n)} - 1\right) \tag{3.10}$$

for $j = 0,\cdots,J-1$ and $n = 1,\cdots,N$. The tail factors are projected by extrapolating the values $\hat{y}_j^n (j = 0,\cdots,J-1$ and $n = 1,\cdots,N)$ through a straight line $\hat{y}_j^n = \hat{a}^n + \hat{b}^n j$ for $j = J, J+1,\cdots$ and $n = 1,\cdots,N$

After the extrapolation the calculated values $\hat{y}_j^n (j = J, J+1,\cdots$ and $n = 1,\cdots,N)$ have to be multiplied. The tail factor is then given through

$$\hat{f}_{\infty}^{(n)} = \prod_{j=1}^{\infty} \left(\exp(\hat{y}_j^n) + 1\right) \tag{3.11}$$

### 3.6.2 Stable Method For Determining the tail factors using the Age to Age Development Factors

This method use the last age to age factor when determining the tail factor. It relies on run tests to determine which age to age factors to use in the estimation.

The summary of the Stable Method is shown below.

1. Calculate the logarithm of each age to age factor.
2. Estimate the decay of the logarithm from each period to the next.
3. Use run tests to ascertain changes in rate of decay.
4. Reduce the data until run test does not reject the null of no change in rate of decay.
5. Select median of remaining decays.
6. Use median decay with each of the remaining age to age factors to estimate a tail factor.
7. Select the median of these tail factor estimates.
8. Calculate tail factor exp(log).

Let $F_k$ = Development factors from age i to age i+1

Let $F_t$ = Tail factors to be determined

$$L_k = \ln F_k \tag{3.12}$$

$$D_k = \frac{L_{(k+1)}}{L_k} \tag{3.13}$$

Let j denote the smallest number for which a run test on the ordered set of decay factors $D_k : k > j$ does not reject the hypothesis of randomness.

$\hat{D} = median D_k$

$$T_k = L_k * \frac{\hat{D}^{t-k}}{1-\hat{D}} \tag{3.14}$$

$\hat{T} = median T_k$

$$F_t = \exp(\hat{T}) \tag{3.15}$$

## 3.7 Some concluding thoughts

One of the first steps in putting together an actuarial analysis is the creation of loss triangles. If an organization requires this type of analysis, a good way to reduce the hours billed is to produce the triangles for the actuary. Loss triangles are a valuable tool to determine if your claims administrator has changed their reserving practice. Consistency is the key to successfully estimating development. If the order of magnitude of development factors suddenly changes, then you should immediately review the reserving process. If, in fact, the process has changed, then estimated outstanding liabilities may prove to be erroneous which may affect your balance sheet.

An important benefit of a paid loss triangle is that development factors can also be used as payout percentages. Teamed with an expected discount rate, a present value analysis of expected cash-flows of outstanding liabilities is easily created.

As a wrap-up to this chapter, the development of a loss triangle is the first step in a larger analysis. Whether the purpose is to project losses, estimate liabilities or to benchmark with the competition, the key to a more valuable analysis will be consistent and accurate data.

# Chapter 4

# Results and Discussions

## 4.1 Introduction

This chapter deals with the analysis of the run-off triangle. Data from the annual audited financial statements for SIC insurance company from accident year 2009 to 2014 of the motor service department were employed. These data were obtained from the company's website. This section is structured to give an overview of the data structure used for a typical reserving exercise and introduces the example dataset used throughout this chapter.

We then discuss the classical deterministic chain-ladder reserving method and introduce the concept of a tail factor.

## 4.2 Presentation of claims data

### 4.2.1 Data required for using run-off triangle methods

A claim event is an event that gives rise to a claim against an insurer by a policyholder. The ultimate cost to the insurer of a claim event, including the benefit payments and claims handling expenses, is called the ultimate gross claim loss. The ultimate net claim loss allows for the deduction of any reinsurance recoveries and other recoveries.

For any claim event there may be a delay between the occurrence of the event and the date on which the claim is reported to the insurer (reporting delay) and another delay between the reporting date and the date on which the claim loss is finally settled (settlement delay). Any amount paid or expense incurred in respect of a claim event is called a claim loss settlement amount or a claim loss settled.

In this thesis the use of run-off triangles is demonstrated through a worked example using claims loss data from SIC insurance company a portfolio motor insurance business. The company wishes to determine the technical provisions required for claims that have already been reported but not yet fully settled. The company has data for all claims that have been reported in the past. For purpose of this study we consider only the claims benefit payments received from their audited financial statement.

There are several ways of presenting claims data, which emphasize different aspect of the data. In this research, they will be presented as a triangle which is the most commonly used method in non-life insurance. The year in which the incident happened and the insurer was on risk is called the accident year. The number of years until a payment is made is called the delay or development period. The claim data are divided up by the accident year and the development year . The figures in the table below are in units of GHS 1000.

### 4.2.2 The development of claims losses settled

Typically, claims losses settled for each claims accident year are not paid on one date but rather over a number of years (or time periods). The companys data for the claims loss settlement amounts might be expanded to show the years in which the amounts were settled as in Table 4.1 .

Table 4.1: Incremental claims loss settlement data (in 000) presented as a run-off triangle

| Accident year | Development Years | | | | | |
|---|---|---|---|---|---|---|
| | 1 | 2 | 3 | 4 | 5 | 6 |
| 2009 | 5738 | 1706 | 1279 | 853 | 427 | 180 |
| 2010 | 3277 | 3852 | 855 | 428 | 345 | ? |
| 2011 | 4349 | 2075 | 1037 | 937 | ? | ? |
| 2012 | 14930 | 257 | 191 | ? | ? | ? |
| 2013 | 15128 | 1227 | ? | ? | ? | ? |
| 2014 | 12236 | ? | ? | ? | ? | ? |

The development year for a claims settlement amount reflects how long after the claims occurrence year the amount was settled. An amount settled during the claims accident year is considered to be settled in development year 1, an amount settled in the following year is settled in development year 2, and so on. From the data above, the largest claim settlement was observed in development year 1 for accident year 2013.

The data shown in each of the cells in Table 4.1 represents the incremental claims losses settled in the development year (column) for the given claims accident year (row). For any cell in the table, the value shown represents the incremental claims loss amount that was settled in calendar year (claims accident year + claims development year). Each diagonal set of data represents the amounts settled in a single calendar year. For example, the long diagonal (12236, 1227, 191, 937, 345,180) includes all payment made during the most recent calendar year ie 2014 (as well as past calendar years).

Finally, it can be seen that the upper left corner represents known past payments(observed data) while the lower right corner represent time periods in the future for which we wish to estimate the expected claims settlement amounts.

### 4.2.3 Cumulative claims losses settled

The data in Table 4.2 can be presented as cumulative claims losses settled. For each claims accident year the incremental claims loss settled for a particular development year is the amount settled in that development year. The cumulative claims losses settled is the total amount settled up to that development year, i.e. it is the sum of the incremental claims losses settled up to that date. The cumulative claims losses settled for the below data are presented in Table 4.2 .

Table 4.2: Cumulative claims loss settlement data (in 000) presented as a run-off triangle.

| Accident year | Development Years | | | | | |
|---|---|---|---|---|---|---|
| | 1 | 2 | 3 | 4 | 5 | 6 |
| 2009 | 5738 | 7444 | 8723 | 9576 | 10003 | 10183 |
| 2010 | 3277 | 7129 | 7984 | 8412 | 8757 | ? |
| 2011 | 4349 | 6424 | 7461 | 8398 | ? | ? |
| 2012 | 14930 | 15187 | 15378 | ? | ? | ? |
| 2013 | 15128 | 16355 | ? | ? | ? | ? |
| 2014 | 12236 | ? | ? | ? | ? | ? |

From Table 4.2 the values along the long diagonal line equal to the total amounts settled to date for each claims accident year.

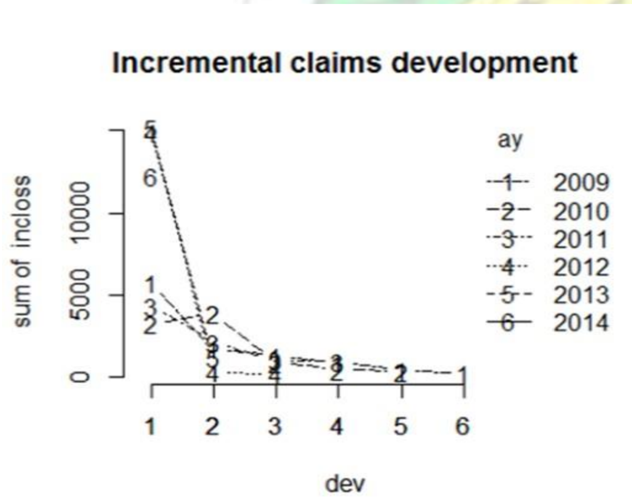
In order to estimate the outstanding loss reserve we will consider the triangular representation of cumulative incurred claims given in the Table 4.2. These incurred claims were extracted from SIC annual audited financial statements.

Each row in the triangle represents an accident year which defines a cohort of claims. The 2009 row includes all claims relating to accidents that occurred during the 2009 calendar year. The columns represent development years which shows how the cohort of claims relating to a particular accident year developed over time. Column 1 represents the year in which the accident occurred. Column 2 also represents the year after the accident occurred and so on.

Each entry in the Table 4.2 is defined by its accident year (row) and its development year (column). The claim figure GHS 8723 is for accident year 2009, development year 2. This figure includes payment made in 2009, 2010 and 2011, since this is a cumulative table. The claim figures given are cumulative and represent total amounts paid by the end of each development year. They were compiled after the end of the 2014 accident year. For the 2014 accident year, only payments with delay 1 have been reported. For the 2013 accident year, payments with delay 1 and delay 2 have been reported and it continues in the same pattern.

To estimate the total outstanding reserve, we will look at a method to help in estimating the unknown figures to complete the lower right triangle. For each accident year the difference between the figure in the extreme right corner column and the total paid so far will give the estimate of the amount we need to hold currently to meet future liabilities arising.

### 4.2.4 Exploring run-off patterns of incremental and cumulative development pattern



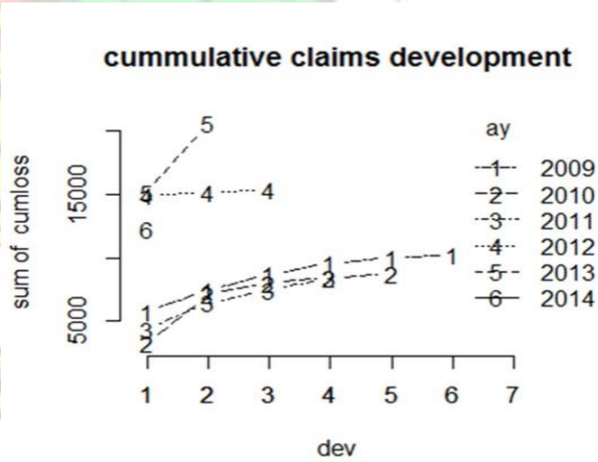



Figure 4.1: Plot of Incremental and cumulative claims development by accident year by using base graphics.

Figures 4.1 present the incremental and cumulative claims development by accident year. The triangle appears to be fairly well behaved. The first three years follows almost the same developmental pattern of which SIC can use to predict other years. But unfortunately the last three years, 2012, 2013 and 2014, appear to be slightly higher than years 2009 to 2011, which did not follow that pattern. The values in 2013 are higher in comparison to the later years, for example, the book changed over the years. The last payment of 180 for the 2009 accident year stands out a bit as well.

Other claims information can provide valuable insight into the reserving process too, such as claims numbers, transition timings between different claims settlement stages and earning patterns.

## 4.3 The Chain-Ladder method

The basic chain-ladder algorithm has the implicit assumption that each accident period has its own unique level and that development factors are independent of the accident periods; or equivalently, there is a constant payment pattern. As the chain-ladder method is a deterministic algorithm and does not regard the observations as realizations of random variables but absolute values, the forecast of the most recent accident periods can be quite unstable. Using the cumulative claims table in Table 4.2, the development factors or the age to age link ratios are obtained.

The Chain-Ladder algorithm is used, as a first step, to estimate the age-to-age link ratios $f_k$ as we expect no further development after year 6.

.

Table 4.3: The age-to-age link ratios $f_k$.

| Accident Year | 2009 | 2010 | 2011 | 2012 | 2013 | 2014 |
|---|---|---|---|---|---|---|
| $f_k$ | 1.21 | 1.093 | 1.092 | 1.043 | 1.018 | 1.00 |

These factors $f_k$ are then applied to the latest cumulative payment in each row to produce stepwise forecasts for future payment years.

.

Table 4.4: Fully run-off triangle showing future payments (in 000).

| Accident year | Development Years | | | | | |
|---|---|---|---|---|---|---|
| | 1 | 2 | 3 | 4 | 5 | 6 |
| 2009 | 5738 | 7444 | 8723 | 9576 | 10003 | 10183 |
| 2010 | 3277 | 7129 | 7984 | 8412 | 8757 | **8915** |
| 2011 | 4349 | 6424 | 7461 | 8398 | **8758** | **8916** |
| 2012 | 14930 | 15187 | 15378 | **16789** | **17510** | **17824** |
| 2013 | 15128 | 16355 | **17875** | **19515** | **20353** | **20719** |
| 2014 | 12236 | **14805** | **16181** | **17667** | **18424** | **18755** |
| $f_k$ | 1.210 | 1.093 | 1.092 | 1.043 | 1.018 | 1.00 |

Comparing table Table 4.2 and table Table 4.4 it can be realized that the unobserved claims have been estimated to produce the full run off triangle. The aim of reserving is to forecast the future claims development in the bottom right corner of the triangle and potential further development beyond accident year 6. From the fully run off triangle in Figure 4.4, it can be realized that accident year 2013 yielded the highest claims in all the development years with the highest estimated claim in development year 6. Also accident year 2010 with development year 6 had the lowest estimated unobserved cumulative claim to be settled in the year 2015

The last column of Figure 4.4 contains the forecast ultimate loss as shown in Table 4.5 .

Table 4.5: Forecast of ultimate loss claims (in 000).

| Accident Year | 2014 | 2015 | 2016 | 2017 | 2018 | 2019 |
|---|---|---|---|---|---|---|
| Ultimate Loss | 10183 | 8914 | 8916 | 17825 | 20719 | 18755 |

The cumulative products of the age-to-age development ratios provide the loss development factors for the latest cumulative paid claims for each row to ultimate:

Table 4.6: Cumulative products of the age-to-age development ratios or factors.

| Accident Year | 2009 | 2010 | 2011 | 2012 | 2013 | 2014 |
|---|---|---|---|---|---|---|
| $f_k$ | 1.21 | 1.093 | 1.09 | 1.04 | 1.02 | 1.00 |
| CUM ATA FACTORS(F) | 1.541 | 1.274 | 1.156 | 1.061 | 1.020 | 1.00 |

The inverse of the cumulative age-to-age development factor estimates the proportion of claims developed to date for each accident year, often also called the gross up factors or growth curve: The figure below gives the growth curve.

Table 4.7: Inverse of loss development factor estimates showing development pattern.

| Accident Year | 2009 | 2010 | 2011 | 2012 | 2013 | 2014 |
|---|---|---|---|---|---|---|
| Inverse of LDF/Gross up factors | 0.65 | 0.79 | 0.860 | 0.940 | 0.98 | 1.00 |

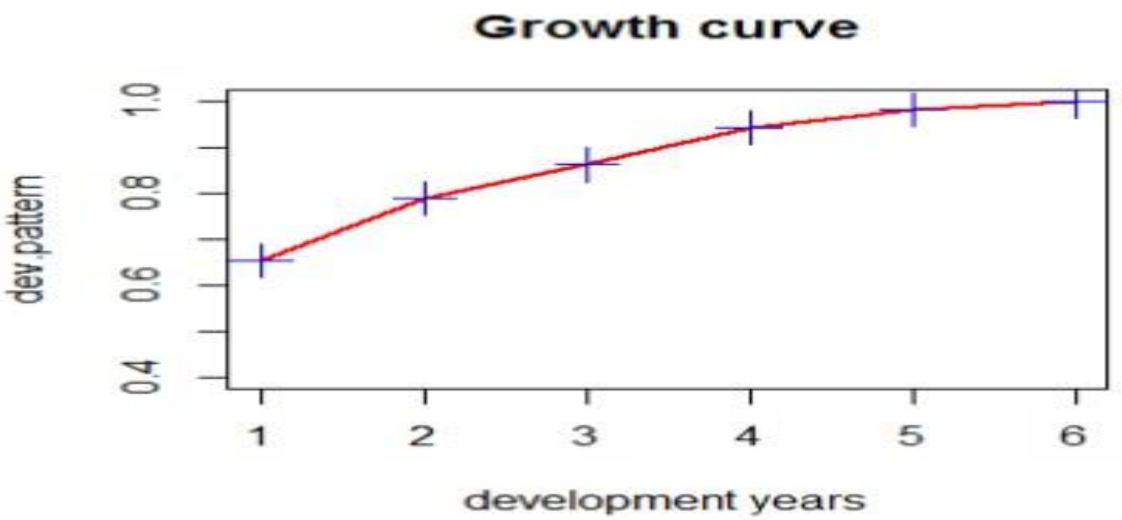



Figure 4.2: the growth curve of development pattern verses development years.

The total estimated outstanding loss reserve with this method is by summing the difference between the ultimate paid and the latest paid (leading diagonal of the cumulative table).

Table 4.8: The total estimated outstanding loss reserve (in 000) for each future accident year.

| Accident Year | 2014 | 2015 | 2016 | 2017 | 2018 | 2019 |
|---|---|---|---|---|---|---|
| Ultimate Loss | 10183 | 8914 | 8916 | 17825 | 20719 | 18755 |
| Latest Loss(diagonals) | 10183 | 8757 | 8398 | 15378 | 16355 | 12236 |
| Outstanding loss | 0 | 158 | 518 | 2447 | 4364 | 6519 |

Therefore, the total outstanding loss reserve with this Chain-Ladder method is estimated to be GHS 14,006,000.00

### 4.3.1 Tail factors method

In the previous method we implicitly assumed that there are no claims payments after six (6) years, or in other words, when the oldest accident year is fully developed. However, often it is not suitable to assume that the oldest accident year is fully settled. A typical approach to overcome this shortcoming is to extrapolate the development ratios, assuming a linear model of the log development ratios minus one, which reflects the incremental changes on the previous cumulative payments as shown in the figure below:

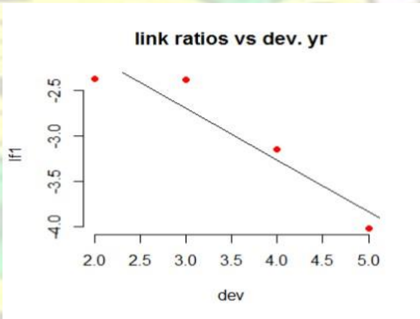



Figure 4.3: Plot of the loss development factors 1 on a log-scale against development year.

### 4.3.2 Stable method for determining tail factor

Using the followig eqautions 3.12 to 3.15 in chapter three above, the tail factor was estimated as shown in the Table below. .

Table 4.9: Estimating the tail factor using the Stable method.

| Cum ATA FACTORS(F) | LN (F) | D | T |
|---|---|---|---|
| 1.529 | 0.425 | 0.570 | 0.023 |
| 1.274 | 0.242 | 0.600 | 0.027 |
| 1.156 | 0.145 | 0.406 | 0.033 |
| 1.061 | 0.059 | 0.336 | 0.027 |
| 1.020 | 0.020 | | 0.019 |
| | | | |
| | | | |
| MEDIAN D = 0.488 | | MEDIAN T = 0.0268 | |
| | | | |
| TAIL FACTOR | | | 1.02721 |

The ratio analysis and the stable method presented above suggest that we can expect another 3% claims development after year 6 and therefore SIC Motor insurance should consider increasing reserve to GHS 14,426,000.00

# Chapter 5

# Summary of Findings, Conclusion and Recommendation

## 5.1 Introduction

This final chapter of the thesis is based on findings from the literature reviews, analysis of claims data from SIC and the tail development method. It thus draws with the conclusions from the study an make the following recommendations

## 5.2 Discussions

In practice, there are many methods used to estimate Ultimate Losses in actuarial profession. There are expected loss ratio method, Loss Development (Chain Ladder), BornhuetterFerguson method, Generalized Cape Cod and many, many others available.

Each Method is based on Idealized Model of Reality. These models have implicit assumptions and these assumptions are rarely satisfied. There is no single method or formula that will work to get accurate result. Therefore, actuary use multiple methods and use judgement in selecting values that will give expected results. Moreover, some actuary also test these assumptions by reviewing it diagnostics and possibly use additional methods to adjust data.

From the analysis of both the incremental and cumulative claim data, it was realized that there is no particular developmental pattern that claims data follow. Claim data might follow a particular pattern to a point in the development year. Although the run-off triangle was not able to get a pattern for the claims data, the the gross up factors or growth curve was able to develop a pattern for the claim data.

The Deterministic Chain Ladder method was used to estimate the total outstanding loss reserves for SIC to be GHS 14,006,000.

As stated before, tail factors have a highly leveraged impact on loss development since they form a portion of the loss development of all accident years analyzed. Further, the tail loss development reflected development occurring after the last development period in the reserving data triangle to be 3% more of the total outstanding loss reserve.

## 5.3 Conclusion

Based on the objectives and the analysis of the data, the following conclusions were made.

- Using the run-off triangle, the pattern of incremental and cumulative claims might be misleading in estimating outstanding reserves.

- The deterministic chain ladder method was used to estimate the outstanding claims for SIC to be GHS 14006000

- Further more, the tail factor development method was used to project the incurred but not settled claims after the run-off triangle was fully developed to be 3% of the outstanding reserves.

## 5.4 Recommendation

Based on the analysis and conclusion, the following recommendations are worth considering

- The methods used in this research are deterministic and therefore the stochastic method can be considered in future studies
- Based on this project SIC should make that 3% (as shown in Table 4.9) difference available this year

# Reference

Boor, J., Winter, (2006). Estimating Tail Development Factors: What to do When the Triangle Runs Out, *Casualty Actuarial Society Forum*, pp. 345390.

CAS Tail Factor Working Party, Fall, (2013). The Estimation of Loss Development Tail Factors: A Summary Report, *Casualty Actuarial Society Forum*.

England, P.D. and Verrall, R.J. (2002). Stochastic claims reserving in general insurance. British Actuarial Journal, Volume 8, pp 443-518.

England P.D., Verrall, R.J. and Wuthrich M V (2012). Bayesian Over-dispersed Poisson model and the Bornhuetter and Ferguson claims reserving method. Annals of Actuarial Science 6(2),258-281.

Gile, Bradford S. 1993. Estimation of Long-tailed Unpaid Losses from Paid Loss Development Using Trended Generalized Bondy Development Actuarial Research Clearing House 3 pp 433-447.

Gogol, D. (1993), Using Expected Loss Ratios in Reserving, Insurance: Mathematics and Economics 12, pp. 297299.

Happ, S., and M.V. Wthrich,(2013) Paid-Incurred Chain Reserving Method with Dependence Modeling, ASTIN Bulletin 43, pp. 120.

Hayne, Roger M.1985. An Estimate of Statistical Variation in Development Factor Methods, PCAS LXXII,pp 25-43.

Hertig, J. (1985). A statistical approach to the IBNR-reserves in marine reinsurance. ASTIN Bulletin, 15, pp 171-183.

Mack, T. (1999), The standard error of chain ladder reserve estimates: Recursive calculation and inclusion of a tail factor, ASTIN Bulletin 29(2), P266.-361.

Mack, T. and Venter, G. (2000). A comparison of stochastic models that reproduce chainladder reserve estimates. pp 101-107.

Mack, T.(1999), The Standard Error of Chain Ladder Reserve Estimates: Recursive Calculation and Inclusion of a Tail Factor, ASTIN Bulletin 29, pp. 361366.

Merz, M., and M. V. Wthrich, Paid-Incurred Chain Claims Reserving Method, *Insurance: Mathematics and Economics* 46, 2010, pp. 568579.

Sherman, Richard E. and Diss, Gordon F., 'Estimating the Workers Compensation Tail, *Casuals Actuarial Sode, Forum*, Winter 2004 pp 207-282.

Sherman, Richard E., 'Extrapolating, Smoothing and Interpolating Development Factors, Proceedings of the Casualty Actuarial Society D, 1984 Vol: LX XI Page(s): 122-155.

Taylor, G. C. (1986). Claims Reserving in Non-Life Insurance.

Verrall, R. J. (1989). A State Space Representation of the Chain Ladder Linear Model. Journal of the Institute of Actuaries, Vol. 116, Part 3,pp 589-609.

Weke, P. G. O. (1992). Statistical Models for Estimating IBNR Claims Reserves,M.Sc. Dissertation, The City University, London.