

APPLICATION OF GENETIC ALGORITHM TO FIND OPTIMAL WORK SCHEDULING FOR NURSES-A CASE STUDY AT THE ENT DEPARTMENT,KATH

By

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## DETION

I hereby declare that this submission is my own work towards the award of the MSC degree and that, to the best of my knowledge, it contains no material previously published by another person nor material which had been accepted for the award of any other degree of the university, except where due acknowledgement had been made in the text.

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## Dedication

This research paper is lovingly dedicated to my parents Mr.Joseph Maxwell Mensah and Mrs. Agatha Sey who have been a constant source of financial support and inspiration.

I would also like to dedicate this research to Mr. Emmanuel Eghan who has given me the drive and discipline to tackle any task with enthusiasm and determination.

Without their love and support, this project would have not been possible.Am so grateful


#### Abstract

Hospitals need to repeatedly produce duty rosters for its nursing staffs. The good scheduling of nurses has an impact on the quality of health care,the recruitment of nurses, the development of budgets and other nursing functions.


Nurse scheduling is a well known scheduling problem that aims at allocating the required workload to the available staff nurses. This work tends to formulate a model to minimise the shift of nurses using the genetic based algorithm. The results obtained showed that the shift of nurses has been minimised so as to prevent a ward from being overstaffed or understaffed while satisfying the needs of the patients.

The study shows that genetic algorithm based method can find optimal schedules that are feasible and acceptable for hospital nursing units.

## Acknowledgement

A major research project like this is never the work of anyone alone, the contribution of many different people,in their different ways made this possible. I would like to extend my appreciation especially to the following :

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## Contents

Acknowledgement ..... iv
Declaration ..... v
Dedication KNUST ..... v
Acknowledgement ..... v
List of Tables ..... ix
List of Figures ..... x
1 INTRODUCTION ..... 1
1.1 BACKGROUND OF THE STUDY ..... 1
1.1.1 Profile of Komfo Anokye Teaching Hospital (KATH) ..... 3
1.2 STATEMENT OF THE PROBLEM ..... 4
1.3 OBJECTIVES OF THE STUDY ..... 4
1.4 METHODOLOGY ..... 5
1.5 JUSTIFICATION ..... 5
1.6 THESIS ORGANISATION ..... 5
2 LITERATURE REVIEW ..... 7
2.1 INTRODUCTION ..... 7
2.2 SCHEDULING ..... 7
2.2.1 THE ROLE OF SCHEDULING ..... 7
2.2.2 WHY IS SCHEDULING SO IMPORTANT? ..... 8
2.2.3 SCHEDULING DISCIPLINE ..... 8
2.3 BACKGROUND OF SCHEDULING ..... 8
2.4 LITERATURE REVIEW ..... 10
3 METHODOLOGY ..... 13
3.1 INTRODUCTION ..... 13
3.2 GENETIC ALGORITHM ..... 13
3.2.1 WHAT ARE GENETIC ALGORITHM ..... 14
3.2.2 ESSENTIAL COMPONENTS OF GENETIC ALGORITHM ..... 15
3.2.3 ADVANTAGES OF GENETIC ALGORITHM ..... 23
3.2.4 POTENTIAL DRAWBACKS OF GENETIC ALGORITHM ..... 23
3.3 GENETIC REPRESENTATION ..... 23
3.3.1 Summary ..... 25
3.3.2 THE CROSSOVER PROCESS ..... 29
3.3.3 DETERMINING THE CORRECT NUMBER OF GENES FOR A CHROMOSOME ..... 30
3.3.4 MUTATION OPERATION ..... 31
3.4 MODEL FORMULATION ..... 33
4 RESULTS AND ANALYSIS ..... 38
4.1 MODELS FORULATED FOR THE WARDS ..... 38
4.1.1 Model for ward B1 ..... 38
4.1.2 Model for ward B3 ..... 42
4.1.3 Model for ward B2 ..... 46
4.1.4 Model for ward C1A ..... 50
4.1.5 Model for ward C1B ..... 54
4.1.6 Model for ward C2 ..... 58
4.1.7 Summary ..... 62
5 CONCLUSION AND RECOMMENDATION ..... 63
References ..... 65
Appendix A ..... 66
Appendix B ..... 67
KNUST


## List of Tables

3.1 chromosome 1 associated with staff member B ..... 24
3.2 chromosome 2 associated with staff member A ..... 24
3.3 Example of chromosome ..... 25
3.4 This chromosome is allowed ..... 26
3.5 This chromosome is not allowed ..... 26
3.6 Invalid (too few genes) ..... 26
3.7 Invalid (too many genes) ..... 27
3.8 valid chromosome ..... 27
3.9 Chromosome 1 ..... 28
3.10 Chromosome 2 ..... 29
3.11 Chromosome from roster X ..... 29
3.12 Chromosome from roster Y ..... 29
3.13 Chromosome for roster Z ..... 30
3.14 Not full chromosomes ..... 31
3.15 Full chromosomes ..... 31
3.16 WARD B1(13 NURSES)63 SHIFTS ..... 35
3.17 WARD B2(17 NURSES)91 SHIFTS ..... 35
3.18 WARD B3(13 NURSES) 70 SHIFTS ..... 35
3.19 WARD C1A(13 NURSES)61 SHIFTS ..... 36
3.20 WARD C1B(14 NURSES)72 SHIFTS ..... 36
3.21 WARD C2(14 NURSES) 73 SHIFTS ..... 36
3.22 SUMMARY OF THE SEVEN WARDS ..... 37
4.1 WARD B1(13 NURSES)63 SHIFTS ..... 38
4.2 WARD B3(13 NURSES)70 SHIFTS ..... 42
4.3 WARD B2(17 NURSES)91 SHIFTS ..... 46
4.4 WARD C1A(13 NURSES)61 SHIFTS ..... 50
4.5 WARD C1B(14 NURSES)72 SHIFTS ..... 54
4.6 WARD C2(14 NURSES) 73 SHIFTS ..... 58

## List of Figures



## Chapter 1

## INTRODUCTION

This chapter discusses the background of the study,statement of the problem,objectives of the research and methodology.It also discuss the justification of the study and the thesis organisation.

### 1.1 BACKGROUND OF THE STUDY

The Nurse scheduling problem (NSP) problem is all about assignment of shifts and holidays to nurses. A nurse has her/his wishes/restrictions. The problem is described as finding a schedule that both respects the constraints of the nurses and fulfills the objectives of the hospital. Conventionally a nurse can work 3 shifts because nursing is shift work,that is morning,afternoon and evening shift.

In a world where demand for round-the-clock services and higher output has lead to an increase in shift work in a wide variety of hospitals, the ability to devise a good staff schedule can be crucial to the success of an hospital. The task of scheduling staff is a complicated balancing act between the hospital needs and the legal contractual obligations to its employees. Moreover, shift work can have a significant impact on the health and lives of staff, which can in turn affect a persons productivity at work. All such factors should be considered when designing a staff shift schedule, leading to an extremely complex problem for which finding a good solution, within a reasonable time frame, can be difficult. Differences between hospitals, their goals and restrictions, often mean that specific models and algorithms must be developed for each of them.(Ernst and Sier (2004)) This together with the importance of finding the best schedule has lead to much research covering industries including airlines, call centres and hospitals
with a range of solution methods including mathematical programming, heuristic techniques, simulation and artificial intelligence being utilised.

An important area for research and the focus of this paper is the scheduling of nurses. Hospital wards must be staffed 24 hours a day 7 days a week by a limited number of staff, which in itself can be difficult. However, as well as the usual legal and cost constraints, patient care is an important factor. The effects of shift work on the individual are even more important, with patient lives at risk if nurses are made to work undesirable schedules that may affect their performance in some way. With shortages in qualified nurses reported regularly, good schedules are important to provide satisfactory patient care and potentially improve nurse retention.

Many solutions to the nurse scheduling problem have been proposed along with software that has been shown to provide better solutions and/or solutions in significantly quicker time frames than devising them by hand. Problems arise however, if a member of staff is unable to work their assigned shift and alterations have to be made to cover the shortfall. There are limited options for rectifying the problem with the main ones being the production of a completely new schedule, running a ward understaffed, employing temporary staff, or making changes to the schedule by hand. All of these options can be undesirable for a variety of different reasons including cost and the effect on the quality of patient care,hence properly scheduling the nursing staff has a great impact on the quality of health care, the recruitment of nursing personnel, the development of a nursing budgets and various other functions of the nursing service.

Employee scheduling for nurses is generally a challenging and laborious task that must be routinely completed by the head nurse in each nursing unit. An automatic optimal scheduler is needed to evenly and fairly distribute the workload.

The goal of this study is to create a genetic algorithm based scheduling method for creating impartial schedules for specific number of nurses during each period of the day.

### 1.1.1 Profile of Komfo Anokye Teaching Hospital (KATH)

Komfo Anokye Teaching Hospital(KATH)is located in kumasi,the regional capital of Ashanti Region with a total projected population of 4,839,100(2010)The geographical location of the 1200-bed KATH,the road network of the country and the commercial nature of the kumasi make the hospital accessible to all the areas that share boundaries with Ashanti Region and others that are further away.

## Historical background

In the 1940s,there was a hospital located on the hill over-looking Bantama Township designated African and European Hospitals.As the name implied, the African side treated Africans while the European side treated Europeans. By 1952, the need to construct a new hospital to cater for the fast increasing population in kumasi and therefore Ashanti Region arose.The european hospital was therefore transferred for thenew project to begin.In 1954/55 the new hospital complex was completed and named Kumasi Central Hospital which was later changed to the Komfo Anokye Hospital in honour of Komfo Anokye. The hospital became a Teaching hospital in 1975 for the training of medical students in collaboration with the school of medical sciences of the University of Science and Technology,kumasi.

## Vision

To become a medical centre of excellence offering clinical and non-clinical services to the highest quality standards comparable to any international standards.

## Mission

To provide quality services to meet the needs and expectations of all its clients.This will be achieved through well-motivated and committed staff applying best practice and innovation.

### 1.2 STATEMENT OF THE PROBLEM

To utilise labour effectively as possible, it is important to analyse manpower requirement during various times of the day. Since nurse usually work an eight-hour shift,it is possible to schedule their working hours so that a single shift covers two or more peak periods of demand.

The human resource officials objective or problem is to find a feasible schedule so as to minimise the shift of nurses thereby preventing a ward from being understaffed or overstaffed, also a reduction in payroll cost.

Hospitals routinely face the problem of scheduling nurses working hours. An optimal work scheduling model for nurses will address the problem precisely.

### 1.3 OBJECTIVES OF THE STUDY

The main objectives of the study are:

- To formulate a model of work scheduling using genetic algorithm to minimise the shifts of nurses at KATH
- To implement the model with MATLAB using data from KATH


### 1.4 METHODOLOGY

The mathematical method/algorithm used is the genetic algorithm which have been proved to be very efficient in obtaining near optimality solutions for a variety of hard combinational problems including the NRP(Nurse Rostering Problem) A secondary data was obtained from annual and monthly reports of duty rosters of nurses at KATH.The method of solution of the genetic algorithm is to be implemented on the MATLAB software

### 1.5 JUSTIFICATION



Assigning of shifts to staff members in a time period is very essential in the scheduling problem which helps to minimise the number of shifts of nurses during each period of the day. Hospitals faces the problem of scheduling of nurses working hours, but optimal work scheduling using genetic algorithm will solve this problem. The results obtained after the findings will prevent a ward from being overstaffed or understaffed with respect to all available nurses to provide a quality care that patients needs.Economically the cost involve will be minimised while the workload will be evenly distributed between nurses.

### 1.6 THESIS ORGANISATION

The study will be organised in this manner

Chapter 1 is the introductory chapter which discusses the background of the study,statement of the problem,the objectives of the study and the methodology.It further discuss the justification of the study and finally the thesis overview of the study
chapter 2 consist of the literature review.In this chapter, various methods em-
ployed under this topic will be defined and explained.The mathematical method used will be addressed.

In chapter 3 the mathematical method employed is the genetic algorithm which will be defined and explained.A model will be formulated for the scheduling of nurses.

Chapter 4 includes the data collection, results and analysis of the findings

Chapter 5 consist of the conclusion and recommendation of the study based on the findings of the research and improvement of scheduling nurses respectively

## Chapter 2

## LITERATURE REVIEW

### 2.1 INTRODUCTION

This chapter defines scheduling and discuss the role of scheduling,importance and the historical background of scheduling. The various methods used by researchers for nurse scheduling problems are discussed

### 2.2 SCHEDULING

It is the allocation of resources to activities over time so that input demands are met in time and cost-effective manner

### 2.2.1 THE ROLE OF SCHEDULING

Scheduling is a decision-making process that is used on a regular basis in many manufacturing and services industries. It deals with the allocation of resources to tasks over given time periods and its goal is to optimize one or more objectives. The resources and tasks in an organization can take many different forms Duty scheduling deals with the arrangement of duty periods for routine workers in an establishment. The group can be nurses ,teachers, telephone operators,sales personnel at a supermarket and so on. In a literature sense, scheduling refers to apportioning duty periods to workers so that they can rotate their time periods for working and also satisfy the minimum duty hours needed Each task may have a certain priority level, an earliest possible starting time and a due date. The objectives can also take many different forms. One objective may be the minimization of the completion time of the last task and another may be the
minimization of the number of tasks completed after their respective due dates. Scheduling, as a decision-making process, plays an important role in most duty rosters, manufacturing and production systems as well as in most information processing environments.

### 2.2.2 WHY IS SCHEDULING SO IMPORTANT?

In order to utilise labour as efficient as possible it is important to analyse manpower requirement during various times of the day. This is especially true in large service organisations in which customer demand is repetitive, but changes significantly during different hours. For example, many more telephone operators are required during the period of noon to 2:00pm than from midnight to 2:00am. Nevertheless, some operator must be on duty during the early morning hours. Since nurses usually work an eight-hour shift, it may be possible to schedule operators working hours so that a single shift covers two or more peak periods of demand. By devising intelligent schedules, the productivity of the operation is increased resulting in smaller staff and reduction in payroll cost

### 2.2.3 SCHEDULING DISCIPLINE

Scheduling discipline are algorithms used for distributing resources among parties with simultaneously and asynchronously request them. The main purpose of scheduling algorithms are to minimise resources starvation and to ensure fairness among the parties utilising the resources.

### 2.3 BACKGROUND OF SCHEDULING

The origin of staff rostering and scheduling can be traced back to 1954, to Edie work on traffic delays at toll booths. Since then staff rostering and scheduling methods have been applied to many fields. Vast amount of literature on personnel scheduling has been made over the years. Ever since staff scheduling was
first applied, many different methods have been used to solve the staff scheduling problem. These are methods like Mathematical programming, Goal programming, Artificial intelligence methods, Heuristics and Metaheuristic scheduling like Tabu search and Genetic algorithms . Even though many different methods have been and are used to solve staff scheduling problems, only three different key approaches are usually used when solving the staff scheduling problem, these are cyclical scheduling, self scheduling and preference scheduling. The self scheduling approach was first documented in 1963 by Jenkinson, however the possibilities of using this approach have increased with the emergence of the computers since it is possible to implement this approach more fairly using computers.

Abernathy et al. (1973)presented a staff planning and scheduling model that had specific application in the nurse-staffing process in acute hospitals in 1973. That model was solved using mathematical (stochastic) programming techniques.

In 1972 Warner and J.Prawda (1972)developed a model to solve the nursing personnel scheduling problem. They presented a mixed-integer quadratic programming formulation to calculate the number of specifically skilled nurses to undertake a number of shifts per day. Warner improved the previous formulation by adding weights or fairness levels and introduced it in 1976.

Burns and Carter (1985)presented a paper in 1985 where they developed lower bounds on the workforce size and then introduced them as an additional constraint in a linear programming model to ensure integer solution.

The same year Bailey and Field (1985)introduced a general mathematical model for the nurse scheduling problem. This model is still today one of the few published models that allow shifts to start at any time during the day. But since the nurse scheduling is becoming more and more complicated and complex this
approach cannot address the current needs of todays hospitals.

Bard et al. (2009)formulated a mixed-integer program for physicians shift scheduling and solved it with the CPLEX optimization software package. Even though physician scheduling and nurse scheduling are similar problems, physician scheduling is more complex than nurse scheduling.In 2010

Asgeirsson (2010)introduced the same problem using a local-search based algorithm to find a solution

### 2.4 LITERATURE REVIEW

Over the past decade, a variety of methods have been used to create work schedules for nurses.

Yih (2010)showed that heuristic methods, such as modified hill climbing, can be used to create work schedules for any type of employee. The modified hill climbing method uses a deterministic greedy search technique to repeatedly assign the most favorable jobs to the most favorable employees.

Azaiez and Al Sharif (2005)used a goal programming approach to creating work schedules for nurses. The goal programming method creates work schedules to meet the expectations of the nurses.

Brucker and Berghe (2010)used an adaptive constructive method to create work schedules for nurses. The adaptive constructive method creates rosters and then uses a greedy local search technique to create work schedules.

Dowsland and Thompson (2000)identified requirements for creating day and night shift work schedules for nurses. Dowsland and Thompson used a three-stage
method to create complex work schedules for nurses. The first stage uses a knapsack technique to create schedules that fulfill human resource requirements. The secondstage uses a tabu search technique to create schedules that fulfill day and night shift work schedule requirements. The third stage divides the day shifts into early and late shifts.

Li and Aickelin (2004)enhanced the three-stage method to create work schedules for nurses. The method uses four scheduling rules and a Bayesian optimization algorithm to create feasible work schedules. They used the method to create work schedules for 52 test cases.


Yaw (2008)used the branch and bound method to determine the least number of nurses required while still satisfying the needs of the patients using integer programming model computations of the matlab software.A model was formulated to minimise the shift of nurses which was implemented on the matlab software.

Chiaramonte2008 used an agent-based method to create work schedules for nurses. The method uses nurse agents that simulate intelligent human behaviors. The nurse agents bid for work shifts and work leaves until all of the agents are satisfied with their work schedules. Prior studies created methods that improve work schedules for nurses. The methods used to create work schedules include local search techniques, heuristic search techniques, stochastic search techniques, or intelligent agents. Most of the optimization techniques can conduct local searches effectively. However, they cannot conduct extensive searches effectively. Metaheuristic algorithms, on the other hand, can conduct extensive searches effectively. Genetic algorithms (GAs), ant colony optimization algorithms (ACOs), and particle swam optimization algorithms (PSOs) have been used to solve many different types of optimization problems. They can be used to find near-optimal solutions to complex optimization problems with nonlinear constraints and multiple goals.

Meta-heuristic algorithms have many advantages over traditional optimization techniques. They can search the optimal solution without gradient information; they can conduct effective population search for a wider solution space; they can use heuristic information for advantageous optimum search; and they are generally controlled or governed by evolution laws or natural intelligence techniques. This study develops a genetic algorithm-based method to create impartial work schedules for nurses.


## Chapter 3

## METHODOLOGY

### 3.1 INTRODUCTION

In this chapter, genetic algorithm will be discussed and explained and the model for scheduling the shifts of nurses will be formulated. The model to determine least number of shifts will be formulated while still satisfying the needs of the patients using genetic algorithm computations of the MATLAB software

### 3.2 GENETIC ALGORITHM

There are many ways of searching the space of all possible rosters to find a suitable one, and many different algorithms may be applied which include tabu search, simulated annealing, mathematical methods of optimisation etc. I however, decided to focus mainly upon a class of algorithms known a genetic algorithm. Their use is popular in the field of timetabling and duty scheduling and they have been used successfully in the past. John Holland is often considered the inventor of Genetic Algorithms. A genetic algorithm is an optimisation algorithm which takes a heuristic approach to optimisation, searching through a search space in a directed manner. A genetic algorithm does not exhaustively consider every possible combination, although it is not guaranteed to find an optimal solution. In a genetic algorithm, we start with an initially random population of solutions. An evolutionary process can be applied to this population. To do this we rank the solutions in terms of their fitness for use, mate the best solutions in some way, and allow the offspring to replace existing members of the population. Possibly some genetic mutations are introduced to simulate the mutation which occurs in biological processes. This process is repeated until some termination conditions
are met. Each iteration is known as generation. We assume that by mating good solutions, at least some of the offspring will contain the best elements from the parents. Genetic algorithms can be applied to any type of problem which requires searching of a space, and have been successfully applied to problems like travelling salesperson and timetabling, where results within $10 \%$ of the optimum can be produced. This flexibility makes genetic algorithms particularly suited to duty scheduling, where constraints and roster structure can vary between wards.

### 3.2.1 WHAT ARE GENETIC ALGORITHM

Genetic algorithms are adaptive heuristics search algorithm based on evolutionary ideas of natural selection and genetics.GA are part of evolutionary computing, a rapidly growing area of artificial intelligence.GA are inspired by Darwin's theory about evolution (survival of the fittest). Genetic algorithm represents an intelligent exploitation of a random search used to solve optimisation problems.GA, although randomized, exploit historical information to direct the search into the region of better performance within the search space. In nature, competition among individuals for scanty resources results in the fittest individuals dominating over the weaker one. The following principles in biological evolution inspired GA.

- Species live in a competitive world
- The continued survival of the species depends on fitness competition and having offspring's who are stronger than or equally as strong as their parents
- The offspring's genetically take the characteristics of their parents
- The offspring's are however unique and there is probability of slight variations in some of their genes, and
- In the competitive environment less fit individuals die off and may not become parents for breeding. If they should breed they have only a small probability for breeding.


### 3.2.2 ESSENTIAL COMPONENTS OF GENETIC ALGORITHM

There are a number of components which make up any genetic algorithm. A genetic algorithm always requires some form of genetic representation of the entity that the genetic algorithm is trying to optimise. Often this genetic representation is based around biological concepts.In general,the components of genetic algorithm are the operators of genetic algorithm

## OPERATORS OF GENETIC ALGORITHM

Genetic operators used in genetic algorithm maintain genetic diversity. Genetic diversity or variation is necessity for the process of evolution. Genetic operators are analogous to those which occur in the natural world:

- Reproduction (or selection)
- Crossover (or recombination)
- Mutation

In addition to these operators, there are some parameters of genetic algorithm. One important parameter is the population size. Population size says how many chromosomes are in a population (in one generation). If there are only few chromosomes, the GA would have a few possibilities to perform crossover and only a small part of search space is explored. If there are many chromosomes, then GA slows down. Research shows that after some limit, it is not useful to increase population size, because it does not help in solving the problem faster. The population size depends on the type of encoding and the problem.

## REPRODUCTION OR SELECTION

Reproduction is usually the first operator applied on population. From the population, the chromosomes are selected to be the parents to crossover and produce
offspring's.

## HOW TO SELECT THESE CHROMOSOMES

According to Darwin's evolution theory- survival of the fittest, the best one should survive and create new offspring's. The reproduction operator is also called the selection operators. Selection means extract a subset of genes from an existing population, according to any definition of quality. Every gene has a meaning, so one can derive from a gene a kind of quality measures called the fitness function. Following this quality (fitness value), selection can be performed. Fitness function quantifies the optimality of a solution (chromosome) so that a particular solution may be ranked against all the other solutions. The function depicts the closeness of a given solution to the desired results. Fitness function therefore is a measure associated with the collective objective functions of the optimisation problem. The measure indicates the fitness of a particular chromosome representation of a particular individual or solution. Many reproduction operators exist and they all essentially do same thing. They pick from current population the strings of above average and insert this multiple copies in the mating pool in a probabilistic manner. The most commonly used methods of selecting chromosomes for parents to crossover are roulette wheel selection, rank selection, steady state selection, tournament selection etc.

## Roulette wheel selection (fitness proportionate selection)

Roulette wheel selection, also known as fitness proportionate selection is a genetic operator used for selecting potentially useful solutions for recombination. In fitness-proportionate selection: The chance of an individual being selected is proportional to its fitness, greater or less than its competitor's fitness. Conceptually, this can be thought as a game of roulette.

## fig 1. roulette wheel shows 8 individual with fitness



Roulette wheel simulates 8 individuals with fitness value F marked at its circumference, example the 5th individual has a higher fitness than others, so the wheel would choose the 5 th individual more than other individuals. The fitness of the individual is calculated as the wheel is spun $\mathrm{n}=8$ time, each time selecting an instance, of the string, chosen by the wheel pointer.Probability of the ith string is

$$
\begin{equation*}
F_{i}=\sum_{j=1}^{n} F_{j} \tag{3.1}
\end{equation*}
$$

where n is the number of individuals, called the population size, Pi is the probability of the ith string being selected; Fi is the fitness of the ith string in the population. Because the circumference of the wheel is marked according to string fitness, the roulette wheel mechanism is expected to make

$$
\begin{equation*}
F \div \text { mean } \tag{3.2}
\end{equation*}
$$

of F copies of the ith strings.

## Tournament selection

Two chromosomes are chosen at random, the one with the higher fitness is selected. The process is repeated until the required number of chromosomes is obtained.

## Random selection

Chromosomes may be selected randomly irrespective of the fitness

## CROSSOVER

Crossover is a genetic operator that combines (mate) two chromosomes (parents) to produce a new chromosome (offspring). The idea behind crossover is that the new chromosome may be better than both of the parents if it takes the best characteristics from each of the parents. Crossover occurs during evolution
according to a user-definable crossover probability. Crossover selects genes from parent chromosome and creates a new offspring's. The crossover operators are of many types;

- One- point crossover
- Two-point crossover
- Uniform crossover
- Arithmetic crossover and
- Heuristics crossover


The operator is selected based on the way chromosomes are selected.

## One- point crossover

One-point crossover operator randomly selects one crossover point and then copies everything before this point from the first parent and then everything after the crossover point copy from the second parent. The crossover would then look as shown below.


Consider the two parents selected for crossover

| parent 1 | 11011 | 11000011110 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| parent 2 | 11011 | 00100111010 |

NOTE: the first vertical line is the chosen crossover point
Interchanging the parent chromosomes after the crossover points-the offspring's produced are;

| offspring's 1 | 11011 | 00100110110 |
| :---: | :--- | :--- |
| offspring's 2 | 11011 | 11000011110 |

## Two- point crossover

Two-point crossover operator randomly selects two crossover points within a chromosome then interchanges the two parent chromosomes between these parents to produce two new offspring's.

Consider two parents selected for crossover

| parent 1 | 11011 | 0010011 | 0110 |
| :--- | :--- | :--- | :--- | :--- | :--- |
| parent 2 | 111011 | 1100001 | 1110 |

Interchanging the parent chromosome between the crossover points
The offspring's produced are

| offspring's | 1 | 1 | 0 | 1 | 1 | 0 | 0 | 1 | 0 | 0 | 1 | 1 | 1 | 1 | 1 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |

## Heuristics

Heuristics crossover operator uses the fitness value of the two parent chromosomes to determine the direction of the search. The offspring's are created according to the equations:

Offspring $1=$ Best parent $+\mathrm{r} *$ (Best parent-Worst parent)
Offspring 2= Best parent
where r is a random number between 0 and 1 . It is possible that offspring will not be feasible. It can happen if r is chosen such that one or more of its genes fall outside of the allowable lower and upper bounds. For this reason, heuristics crossover has a user defined parameter $n$ for the number of times to try and find $r$ that result in a feasible chromosome. If the feasible chromosome is not produced after n tries, the worst parent is returned as offspring 1.

## MUTATION

After a crossover is performed, mutation takes place. Mutation is a genetic operator used to maintain genetic diversity from one generation of a population of chromosome to the next generation. Mutation occurs during evolution according to a user-definable mutation probability set to a very fairly low value, say 0.01 a good first choice.

Mutation alters one or more gene values in a chromosome from its initial state. This can result in entirely new gene values being added to the gene pool. With a new gene value, the genetic algorithm may be able to arrive at a better solution than was previously possible.

Mutation is an important part of the genetic search, helps to prevent the population from stagnating at any local optima. Mutation is intended to prevent the search falling into a local optimum of the state spaces.

The mutation operators are of many types;

- flip bit
- Boundary
- Non-uniform
- Uniform
- Gaussian

The operators are selected based on the way chromosomes are encoded.

## Flip Bit

The mutation operator simply inverts the value of the chosen gene, which is 0 goes to 1 and 1 goes to 0 . This mutation operator can only be used for binary genes.

Consider the two original offspring's selected for mutation

| Original offspring's 1 | 1101111000011110 |
| :---: | :--- | :--- |
| Original offspring's 2 | 1101100100110110 |

Invert the value of the chosen genes as 0 to 1 and 1 to 0 . The mutated offspring's produced are

| Mutated offspring's 1 | 1100111000011110 |
| :--- | :--- |
| Mutated offspring's 2 | 1101101100110100 |

## Boundary

The mutation operator replaces the value of the chosen gene with either the upper or lower bound for that gene (chosen randomly). The mutation operator can only be used for integer and float genes.

## Non-Uniform

The mutation operator increases the probability such that the amount of the mutation will be close to 0 as the generation number increases. This mutation operator prevents the population from stagnating in the early stages of the evolution then allows the genetic algorithm to fine tune the solution in the later stages of evolution. This mutation can only be used for integer and float genes.

## Uniform

The mutation operator replaces the value of the chosen gene with a uniform random value selected between the user-specified upper and lower bounds for that gene. This mutation operator can only be used for integer and float genes.

## Gaussian

The mutation operator adds a unit Gaussian distributed random value to the chosen gene. The new gene value is clipped if it falls outside of the user-specified
lower or upper bounds for that gene. This mutation operator can only be used for integer and float genes

### 3.2.3 ADVANTAGES OF GENETIC ALGORITHM

- It solves problem with multiple solutions and it's very easy to understand
- Genetic algorithms are easily transferred to existing simulations and models
- Multi-point search
- Resistant to becoming stuck on a roster which is sub-optimal


### 3.2.4 POTENTIAL DRAWBACKS OF GENETIC ALGORITHM

- Certain optimisation problem (variant problems) cannot be solved by means of genetic algorithm. This occurs due to poorly known fitness function which generates bad chromosome blocks in spite of the fact that only good chromosomes blocks crossover.
- There is no absolute assurance that a genetic algorithm will find a global optimism. It happens very often when the populations have a lot of subjects.
- Genetic algorithm applications in control which are performed in real time are limited because of random solution and convergence gradient method. This unfortunate genetic algorithm property limits the genetic algorithm use in real time application
- It's computational intensive


### 3.3 GENETIC REPRESENTATION

A "gene" is defined to be a pair of a day a shift. Example (Monday, Early) is a gene and (Tuesday, Late) is a gene. A set of genes constitutes a chromosome.

Each staff member has an associated chromosome. Each chromosome represents the shift assigned to that staff member.

Table 3.1: chromosome 1 associated with staff member B

| GENE |  | PRESENT |
| :---: | :---: | :---: |
| MON | EARLY | X |
| MON | LATE |  |
| MON | NIGHT |  |
| TUE | EARLY |  |
| TUE | LATE | X |
| TUE | NIGHT |  |

Table 3.2: chromosome 2 associated with staff member A

| GENE |  | PRESENT |
| :---: | :---: | :---: |
| MON | EARLY |  |
| MON | LATE |  |
| MON | NIGHT | X |
| TUE | EARLY |  |
| TUE | LATE |  |
| TUE | NIGHT |  |

Table 1.1 and 1.2 illustrates two chromosomes. Chromosome 2 contains the gene (Monday night) for example, yet chromosome 1 does not. This indicates that staff A works night shift on Monday, yet staff B does not. Chromosome 2 has no gene of the form (Tuesday, X ) where X is a shift, this represents that staff B has a day off on Tuesday. In this example staff A works early shift on Monday and late shift on Tuesday

A staff member can only work a certain number of hours per week, so the chromosome can only contain a finite number of genes. The number of hours worked in a week is a property of the staff member. The total duration of all shifts in all genes present must be no more than the correct number of genes for a chromo-
some.

To prevent a staff member working too few hours in one week, a chromosome must contain sufficient genes to ensure the total duration of all shifts in all genes is greater than a lower bound, specific to each staff member.

### 3.3.1 Summary

Some staff members cannot work certain days of the week, therefore another property of a staff member is the days they may work (the valid days). All genes in a chromosome must have a day in the "valid days" set of the associated staff member. For example, the staff member associated with the chromosome B in table 1.1 can work only on Monday and Wednesday.

Table 3.3: Example of chromosome

| GENE |  | PRESENT |
| :---: | :---: | :---: |
| MON | EARLY |  |
| MON | LATE |  |
| MON | NIGHT | X |
| TUE | EARLY |  |
| TUE | LATE |  |
| TUE | NIGHT | X |

10

A final property of a staff member is the shift that they may work (the valid shifts). No chromosome may contain a gene with a shift which is not in the "valid shifts" for the associated staff member. For example the staff member associated with the chromosome in table 2 only works night shifts.

To prevent staff members working more than one shifts per day, we need the constraints that genes present in a chromosome cannot share days, this is illustrated in table 1.4 and 1.5

Table 3.4: This chromosome is allowed

| GENE |  | PRESENT |
| :---: | :---: | :---: |
| MON | EARLY |  |
| MON | LATE |  |
| MON | NIGHT | X |

Table 3.5: This chromosome is not allowed

| GENE |  | PRESENT |
| :---: | :---: | :---: |
| MON | EARLY | X |
| MON | LATE |  |
| MON | NIGHT | X |

To summarize, a chromosome will be invalid if:

- Genes present within a chromosome share days
- Genes have shifts not in the "valid days" for the associated staff member
- Genes have days not in the "valid days" for the associated staff member
- There are too many genes in a chromosome to represent a full week for the associated staff member

There are not enough genes in a chromosome to represent a full week for the associated staff as illustrated in the table 1.6

Table 3.6: Invalid (too few genes)

| GENE |  | PRESENT |
| :---: | :---: | :--- |
| MON | EARLY |  |
| MON | LATE |  |
| MON | NIGHT |  |
| TUE | EARLY |  |
| TUE | LATE |  |
| TUE | NIGHT |  |

There are too many genes in a chromosome to represent a full week for the associated staff member as illustrated in table 1.7

Table 1.7 as illustrated below shows a chromosome that is valid.

Table 3.7: Invalid (too many genes)

| GENE |  | PRESENT |
| :---: | :---: | :---: |
| MON | EARLY | X |
| MON | LATE | X |
| MON | NIGHT | X |
| TUE | EARLY | X |
| TUE | LATE | X |
| TUE | NIGHT | X |

Table 3.8: valid chromosome

| GENE |  | PRESENT |
| :---: | :---: | :---: |
| MON | EARLY |  |
| MON | LATE |  |
| MON | NIGHT | X |
| TUE | EARLY |  |
| TUE | LATE |  |
| TUE | NIGHT | X |

Invalid chromosomes are not permissible and no rosters should ever be created with invalid chromosomes. This genetic representation now enforces 5 constraints:

- Any staff member cannot work two shifts in one day
- Any staff member cannot work more hours than they are contractually obliged to
- Any staff member cannot work significantly few hours than they are obliged to
- Any staff member may not be assigned to shifts which they do not work
- Any staff member may not be assigned to shifts on days which they do not work

For every genetic representation, there is an equivalent tabular representation, but not all tabular representations can be represented genetically.

EXAMPLE

|  | MONDAY | TUESDAY |
| :--- | :---: | ---: |
| Staff member 1 | EARLY | LATE |
| Staff member 2 | EARLY | EARLY |

Translating a genetic representation to a tabular representation is a process of iterating over a chromosome. Each gene corresponds to a different cell in the table. Likewise to translate a tabular representation into a genetic representation, a chromosome is created for each staff member and a gene is created for each populated cell in the table. From the example two chromosomes are created which is illustrated below.

Table 3.9: Chromosome 1

| GENE |  | PRESENT |
| :---: | :---: | :---: |
| MON | EARLY | X |
| MON | LATE |  |
| MON | NIGHT |  |
| TUE | EARLY |  |
| TUE | LATE | X |
| TUE | NIGHT |  |

Translating staff member 2 into chromosome, this is what we obtain

Table 3.10: Chromosome 2

| GENE |  | PRESENT |
| :---: | :---: | :---: |
| MON | EARLY | X |
| MON | LATE |  |
| MON | NIGHT |  |
| TUE | EARLY | X |
| TUE | LATE |  |
| TUE | NIGHT |  |

### 3.3.2 THE CROSSOVER PROCESS

Conceptually, the crossover process is illustrated in table below. The crossover process is well explained in the tables

Table 3.11: Chromosome from roster X

| GENE |  | PRESENT |
| :---: | :---: | :---: |
| MON | EARLY | X |
| MON | LATE |  |
| MON | NIGHT |  |
| TUE | EARLY |  |
| TUE | LATE | X |
| TUE | NIGHT |  |

Table 3.12: Chromosome from roster Y

| GENE |  | PRESENT |
| :---: | :---: | :---: |
| MON | EARLY | X |
| MON | LATE |  |
| MON | NIGHT |  |
| TUE | EARLY | X |
| TUE | LATE |  |
| TUE | NIGHT |  |

When crossover process occurs, chromosomes from both roster X and Y , parents share a common assignment that is, early shift on Monday to obtain chromosome for roster Z.

Table 3.13: Chromosome for roster Z

| GENE |  | PRESENT |
| :---: | :---: | :---: |
| MON | EARLY | X |
| MON | LATE |  |
| MON | NIGHT |  |
| TUE | EARLY | $?$ |
| TUE | LATE | $?$ |
| TUE | NIGHT | $?$ |

Common genes are passed unto child in chromosome for roster Z. Remainder of genetic representation randomly generated, or randomly chosen from a parent. The crossover operator acts upon the genetic representation of two rosters X and Y to produce a new roster Z. For each chromosome the operator takes genes present in both x and Y and places them in the corresponding chromosome Z. It then randomly fills the chromosome in Z until they are full. The crossover operator works by iteratively adding genes into a chromosome until that chromosome becomes valid. Sometimes the operator can create a sequence of genes which can never be part of a valid chromosome. For example, certain combination of shifts with unusual durations can prevent the target number of hours in a week being reached. In this case all the work must be discarded and the crossover operator must start again. This event is detected if more than a threshold number of loops iterations have occurred. This threshold is directly proportional to the number of genes which can exist in the chromosome.

### 3.3.3 DETERMINING THE CORRECT NUMBER OF GENES FOR A CHROMOSOME

A chromosome is defined as being "full" when it contains sufficient genes to represent a working week for its associated staff member. Determining whether a chromosome is full has become an increasingly complex task, over the course of the development of the project. The main difficulty arises from the fact that
the presence of certain shifts in the genes of a chromosome can cause a reduction in the number of genes required to fill the chromosome. A good example of this is Night Shifts. Generally if a person is working one or more night shifts in a week, they work one less shift in total. In other words the presence of night shifts reduces the number of allowed working days by 1 .

## EXAMPLE OF FULL AND UNDER FILLED CHROMOSOMES

Table 3.14: Not full chromosomes

| GENE |  | PRESENT |
| :---: | :---: | :---: |
| MON | EARLY | X |
| MON | LATE |  |
| MON | NIGHT |  |
| TUE | EARLY |  |
| TUE | LATE | X |
| TUE | NIGHT |  |

Table 3.15: Full chromosomes

| GENE |  | PRESENT |
| :---: | :---: | :---: |
| MON | EARLY |  |
| MON | LATE |  |
| MON | NIGHT | X |
| TUE | EARLY |  |
| TUE | LATE |  |
| TUE | NIGHT | X |

### 3.3.4 MUTATION OPERATION

The mutation operators act upon the tabular representation of a roster, applying random modifications, which in turn cause the genetic representation to be altered. This is because it is more intuitive to apply this mutation operation to a table. Commonly a mutation operator takes one of the entities under consideration (i.e. rosters), and places a random sequence into its genetic representation. Again, this may or may not be based around biological processes. In fact the mutation operator does not need to act on the genetic representation, and could act on the entity itself (i.e. modify the roster directly), while this is not the classical
method by which mutation is performed. The idea is to introduce some random variation into the population, which may lead to improvements and prevents the algorithm getting stuck.

## SOME FORMS OF MUTATION OPERATIONS

Consider the table below which will be used to explain some forms of mutation processes in genetic algorithm.

|  | MONDAY | TUESDAY |
| :--- | :---: | ---: |
| Staff member 1 | EARLY | LATE |
| Staff member 2 | EARLY | EARLY |

- swap two shifts around for an existing staff member to obtain a mutated staff member.

|  | MONDAY | TUESDAY |
| :--- | :---: | ---: |
| Staff member 1 | LATE | EARLY |
| Staff member 2 | EARLY | EARLY |

- Choose a random day and two staff members. Swap the assignment for the staff members around. The two staff members must be working on shifts which are allowable for both staff members until it is valid. It is available to swap a day off for one staff member with a shift assignment for another staff member.

|  | MONDAY | TUESDAY |
| :--- | :---: | ---: |
| Staff member 1 | LATE | EARLY |
| Staff member 2 | EARLY | LATE |

### 3.4 MODEL FORMULATION

## The model

The human resource department of KATH has a shift pattern for the staff members including nurses. There is a three shift periods (8:00-14:00, 14:00-20:00, 20:00-8:00) denoted by morning, afternoon and night respectively for each day of every week. Nurses must be on duty all the time and the minimum number of nurses required for each of the shifts period for the nine wards under the director in charge of ENT department and the summary is also given. In the case of KATH the union of nurses governing body has acceptable shifts order for nurses and it is as follows:

- Each nurse is assigned to work either in the morning shift or afternoon shift or the night shift and once nurse has been assigned to a shift, he or she must remain on the same shift everyday that they work.
- Each nurse works four consecutive days during any seven day period and takes three consecutive days off.


## Decision variable

The union agreement is such that any nurse can only start their four consecutive work days on one of the seven days (Monday to Sunday) and in one of the three shifts( morning, afternoon, night). Let Monday $=1$, Tuesday $=2$ and so on. Morning $=$ shift 1 , Afternoon=shift 2 and Night $=3$ Xij is the number of nurses starting their four consecutive work days on day $\mathrm{i}(1,2,3,4,5,6,7)$ and shift $\mathrm{j}(1,2,3)$

## Objective function

The human resource official's objective is simply to find a feasible schedule so as to reduce the size of the staff members.

Hence the objective function is given as;

$$
\begin{equation*}
\text { Minimize } Z=\sum_{i=1}^{7} \sum_{j=1}^{3} X_{i} j \tag{3.3}
\end{equation*}
$$

Subject to the constraints

$$
\begin{align*}
& \operatorname{Monday}\left(X_{1} j+X_{7} j+X_{6} j+X_{5} j \geq Y_{1} j\right)  \tag{3.4}\\
& \text { Tuesday }\left(X_{2} j+X_{1} j+X_{7} j+X_{6} j \geq Y_{2} j\right)  \tag{3.5}\\
& \text { Wednesday }\left(X_{3} j+X_{2} j+X_{1} j+X_{7} j \geq Y_{3} j\right)  \tag{3.6}\\
& \text { Thursday }\left(X_{4} j+X_{3} j+X_{2} j+X_{1} j \geq Y_{4} j\right)  \tag{3.7}\\
& \text { Friday }\left(X_{5} j+X_{4} j+X_{3} j+X_{2} j \geq Y_{5} j\right)  \tag{3.8}\\
& \text { Saturday }\left(X_{6} j+X_{5} j+X_{4} j+X_{3} j \geq Y_{6} j\right)  \tag{3.9}\\
& \text { Sunday }\left(X_{7} j+X_{6} j+X_{5} j+X_{4} j \geq Y_{7} j\right)  \tag{3.10}\\
& \qquad X_{i} j \geq 0 \tag{3.11}
\end{align*}
$$

A nurse working on Tuesday ( day 2), works shift j on day 2 whose shift started either on Tuesday (day 2, X2J) or on Monday (day 1, X1J) or on Sunday (day $7, \mathrm{X} 7 \mathrm{~J}$ ) or on Saturday ( day $6, \mathrm{X} 6 \mathrm{j}$ ). Hence the sum of these variables is the total number of nurses on day 2 in shift j and this must be at least the minimum number required Y2J

## Data collected

Based on the general model formed, data was obtained from wards under the director of ENT department of Komfo Anokye Teaching Hospital(KATH),Kumasi(Ghana).The data covers a period of seven months that is from June to December 2013. The data below represents the duty roster or time tabling of nurses at duty at any shift period in the seven days of the week.

The table below is the data for ward B1, there are 13 nurses with 61 shifts in the ward

Table 3.16: WARD B1(13 NURSES)63 SHIFTS

|  | Mon | Tue | Wed | Thu | Fri | Sat | sun |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Morning | 3 | 2 | 2 | 2 | 3 | 2 | 2 |
| Afternoon | 5 | 5 | 5 | 4 | 4 | 5 | 5 |
| Night | 2 | 2 | 2 | 2 | 2 | 2 | 2 |

The table below is the data for ward B2, there are 17 nurses with 91 shifts in the ward

Table 3.17: WARD B2(17 NURSES) 91 SHIFTS

|  | Mon | Tue | Wed | Thu | Fri | Sat | sun |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Morning | 5 | 5 | 3 | 4 | 4 | 4 | 3 |
| Afternoon | 8 | 7 | 6 | 6 | 6 | 5 | 4 |
| Night | 3 | 3 | 3 | 3 | 3 | 3 | 3 |

The table below is the data for ward B3, there are 13 nurses with 70 shifts in the ward

Table 3.18: WARD B3(13 NURSES) 70 SHIFTS

|  | Mon | Tue | Wed | Thu | Fri | Sat | sun |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Morning | 3 | 2 | 2 | 4 | 2 | 2 | 3 |
| Afternoon | 7 | 6 | 6 | 6 | 5 | 4 | 4 |
| Night | 2 | 2 | 2 | 2 | 2 | 2 | 2 |

The table below is the data for ward C 1 A , there are 13 nurses with 61 shifts.

Table 3.19: WARD C1A(13 NURSES)61 SHIFTS

|  | Mon | Tue | Wed | Thu | Fri | Sat | sun |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Morning | 3 | 3 | 2 | 3 | 2 | 2 | 2 |
| Afternoon | 6 | 6 | 5 | 4 | 4 | 3 | 5 |
| Night | 2 | 1 | 1 | 2 | 2 | 1 | 2 |

The table below is the data for ward C1B, there are 14 nurses with 72 shifts in the ward

Table 3.20: WARD C1B(14 NURSES) 72 SHIFTS

|  | Mon | Tue | Wed | Thu | Fri | Sat | sun |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Morning | 3 | 2 | 3 | 3 | 3 | 3 | 3 |
| Afternoon | 7 | 7 | 7 | 7 | 5 | 5 | 5 |
| Night | 2 | 2 | 2 | 2 | 2 | 2 | 2 |

The table below is the data for ward C2,there are 14 nurses with 73 shifts.

Table 3.21: WARD C2(14 NURSES) 73 SHIFTS

|  | Mon | Tue | Wed | Thu | Fri | Sat | sun |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Morning | 3 | 3 | 2 | 3 | 3 | 3 | 3 |
| Afternoon | 6 | 6 | 5 | 5 | 6 | 5 | 6 |
| Night | 2 | 2 | 2 | 2 | 2 | 2 | 2 |

The table below is the data for all the seven ward. The total number of nures in the wards are 84 with a total of 409 shifts.

## KNUST

Table 3.22: SUMMARY OF THE SEVEN WARDS

|  | Mon | Tue | Wed | Thu | Fri | Sat | sun |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Morning | 20 | 17 | 14 | 19 | 16 | 15 | 15 |
| Afternoon | 39 | 37 | 34 | 32 | 26 | 27 | 28 |
| Night | 13 | 12 | 12 | 13 | 13 | 12 | 13 |

## Chapter 4

## RESULTS AND ANALYSIS

In this chapter the minimum number of shifts will be determined and the results obtained will be analysed.

### 4.1 MODELS FORULATED FOR THE WARDS

The models for each ward will be formulated and they are as follows;

### 4.1.1 Model for ward B1

Table 4.1: WARD B1(13 NURSES)63 SHIFTS

|  | Mon | Tue | Wed | Thu | Fri | Sat | sun |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Morning | 3 | 2 | 2 | 2 | 3 | 2 | 2 |
| Afternoon | 5 | 5 | 5 | 4 | 4 | 5 | 5 |
| Night | 2 | 2 | 2 | 2 | 2 | 2 | 2 |

From the table of ward B1, there are 13 nurses and the shifts that occur in this ward is 63 .The idea is to minimise the number of shifts so as to reduce the workload. Hence the model for ward B1 is as follows;

$$
\begin{equation*}
\text { Minimize } Z=\sum_{i=1}^{7} \sum_{j=1}^{3} X_{i} j \tag{4.1}
\end{equation*}
$$

Subject to the constraints

$$
\begin{align*}
& \operatorname{Monday}\left(X_{1} j+X_{7} j+X_{6} j+X_{5} j \geq 3\right)  \tag{4.2}\\
& \operatorname{Monday}\left(X_{1} j+X_{7} j+X_{6} j+X_{5} j \geq 5\right)  \tag{4.3}\\
& \text { Monday }\left(X_{1} j+X_{7} j+X_{6} j+X_{5} j \geq 2\right) \tag{4.4}
\end{align*}
$$

$$
\begin{align*}
& \text { Tuesday }\left(X_{2} j+X_{1} j+X_{7} j+X_{6} j \geq 2\right)  \tag{4.5}\\
& \text { Tuesday }\left(X_{2} j+X_{1} j+X_{7} j+X_{6} j \geq 5\right)  \tag{4.6}\\
& \text { Tuesday }\left(X_{2} j+X_{1} j+X_{7} j+X_{6} j \geq 2\right)  \tag{4.7}\\
& \text { Wednesday }\left(X_{3} j+X_{2} j+X_{1} j+X_{7} j \geq 2\right)  \tag{4.8}\\
& \text { Wednesday }\left(X_{3} j+X_{2} j+X_{1} j+X_{7} j \geq 5\right)  \tag{4.9}\\
& \text { Wednesday }\left(X_{3} j+X_{2} j+X_{1} j+X_{7} j \geq 2\right)  \tag{4.10}\\
& \text { Thursday }\left(X_{4} j+X_{3} j+X_{2} j+X_{1} j \geq 2\right)  \tag{4.11}\\
& \text { Thursday }\left(X_{4} j+X_{3} j+X_{2} j+X_{1} j \geq 4\right)  \tag{4.12}\\
& \text { Thursday }\left(X_{4} j+X_{3} j+X_{2} j+X_{1} j \geq 2\right)  \tag{4.13}\\
& \text { Friday }\left(X_{5} j+X_{4} j+X_{3} j+X_{2} j \geq 3\right)  \tag{4.14}\\
& \text { Friday }\left(X_{5} j+X_{4} j+X_{3} j+X_{2} j \geq 4\right)  \tag{4.15}\\
& \text { Sunday }\left(X_{7} j+X_{6} j+X_{5} j+X_{4} j \geq 2\right)  \tag{4.16}\\
& \text { Friday }\left(X_{5} j+X_{4} j+X_{3} j+X_{2} j \geq 2\right)  \tag{4.17}\\
& \text { Sunday }\left(X_{7} j+X_{6} j+X_{5} j+X_{4} j \geq 5\right)  \tag{4.18}\\
& \text { Sunday }\left(X_{7} j+X_{6} j+X_{5} j+X_{4} j \geq 2\right) \tag{4.19}
\end{align*}
$$

where,for all variables

$$
\begin{equation*}
X_{i} j \geq 2 \tag{4.23}
\end{equation*}
$$

This means that for any shift period, there should be at least 2 nurses in the ward.

The upper limit on the total number of nurses at ward B1 is the number of shifts within that ward which is 63

Hence

$$
\begin{equation*}
\sum_{i=1}^{7} \sum_{j=1}^{3} X_{i} j \leq 63 \tag{4.24}
\end{equation*}
$$

Again,since there are 13 nurses in the ward and each nurse is to work for at least 4 continuous days, it means the number of shifts should not be less than 52 .

Hence

$$
\begin{equation*}
\sum_{i=1}^{7} \sum_{j=1} X_{i} j \geq 52 \tag{4.25}
\end{equation*}
$$

By the use of mathematical programming software (MATLAB)which has the syntax;
\% X = ga(FITNESSFCN,NVARS,A,b,[],[],lb,ub,NONLCON,INTCON,options) The results are obtained in a tabular form as:

| Days | Shift period | Solution | Number of nurses |
| :--- | :--- | :--- | :--- |
| Monday | 1 | 2 | 2 |
| Monday | 2 | 2 | 2 |
| Monday | 3 | 3 | 3 |
| Tuesday | 1 | 5 | 5 |
| Tuesday | 2 | 2 | 2 |
| Tuesday | 3 | 2 | 2 |
| Wednesday | 1 | 2 | 2 |
| Wednesday | 2 | 4 | 4 |
| Wednesday | 3 | 2 | 2 |
| Thursday | 1 | 2 | 2 |
| Thursday | 2 | 5 | 5 |
| Thursday | 3 | 2 | 2 |
| Friday | 1 | 2 | 2 |
| Friday | 2 | 2 | 2 |
| Friday | 3 | 3 | 3 |
| Saturday | 1 | 2 | 2 |
| Saturday | 2 | 2 | 2 |
| Saturday | 3 | 2 | 2 |
| Sunday | 1 | 3 | 2 |
| Sunday | 2 | 2 | 3 |
| Sunday | 3 | 2 | 52 |
| TOTAL |  | 2 |  |
|  | 2 | 2 |  |
|  | 2 | 2 |  |
|  | 2 | 2 |  |
|  | 2 | 2 |  |

From the results obtained, the number of shifts for the 13 nurses in the ward B1 has been reduced from 63 to 52, which implies that the shifts has been minimised, indicating that the 13 nurses are entitled to 52 shifts periods in which each nurse is to work for only four continuous days while satisfying the needs of the patients. The shift that has been minimised indicating that either some of the
nurses were doing overtime or the timetable has been drawn arbitrary.There has been a reduction of 11 shifts which implies that the amount spent on the entire shifts will be reduced.From the results obtained there is a peak periods, that is 8:00am to $4: 00 \mathrm{pm}$ where majority of nurses are needed to attend to patients.

### 4.1.2 Model for ward B3

Table 4.2: WARD B3(13 NURSES) 70 SHIFTS

|  | Mon | Tue | Wed | Thu | Fri | Sat | sun |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Morning | 3 | 2 | 2 | 4 | 2 | 2 | 3 |
| Afternoon | 7 | 6 | 6 | 6 | 5 | 4 | 4 |
| Night | 2 | 2 | 2 | 2 | 2 | 2 | 2 |

From the table of ward B3, there are 13 nurses and the shifts that occur in this ward is 70 .The idea is to minimise the number of shifts so as to reduce the workload. Hence the model for ward B3 is as follows;

$$
\begin{equation*}
\text { Minimize } Z=\sum_{i=1}^{7} \sum_{j=1}^{3} X_{i} j \tag{4.26}
\end{equation*}
$$

Subject to the constraints

$$
\begin{align*}
& \text { Monday }\left(X_{1} j+X_{7} j+X_{6} j+X_{5} j \geq 3\right)  \tag{4.27}\\
& \text { Monday }\left(X_{1} j+X_{7} j+X_{6} j+X_{5} j \geq 7\right)  \tag{4.28}\\
& \text { Monday }\left(X_{1} j+X_{7} j+X_{6} j+X_{5} j \geq 2\right)  \tag{4.29}\\
& \text { Tuesday }\left(X_{2} j+X_{1} j+X_{7} j+X_{6} j \geq 2\right)  \tag{4.30}\\
& \text { Tuesday }\left(X_{2} j+X_{1} j+X_{7} j+X_{6} j \geq 6\right)  \tag{4.31}\\
& \text { Tuesday }\left(X_{2} j+X_{1} j+X_{7} j+X_{6} j \geq 2\right)  \tag{4.32}\\
& \text { Wednesday }\left(X_{3} j+X_{2} j+X_{1} j+X_{7} j \geq 2\right)  \tag{4.33}\\
& \text { Wednesday }\left(X_{3} j+X_{2} j+X_{1} j+X_{7} j \geq 6\right) \tag{4.34}
\end{align*}
$$

$$
\begin{align*}
& \text { Wednesday }\left(X_{3} j+X_{2} j+X_{1} j+X_{7} j \geq 2\right)  \tag{4.35}\\
& \text { Thursday }\left(X_{4} j+X_{3} j+X_{2} j+X_{1} j \geq 4\right)  \tag{4.36}\\
& \text { Thursday }\left(X_{4} j+X_{3} j+X_{2} j+X_{1} j \geq 6\right)  \tag{4.37}\\
& \text { Thursday }\left(X_{4} j+X_{3} j+X_{2} j+X_{1} j \geq 2\right)  \tag{4.38}\\
& \text { Friday }\left(X_{5} j+X_{4} j+X_{3} j+X_{2} j \geq 2\right)  \tag{4.39}\\
& \text { Friday }\left(X_{5} j+X_{4} j+X_{3} j+X_{2} j \geq 5\right)  \tag{4.40}\\
& \text { Friday }\left(X_{5} j+X_{4} j+X_{3} j+X_{2} j \geq 2\right)  \tag{4.41}\\
& \text { Saturday }\left(X_{6} j+X_{5} j+X_{4} j+X_{3} j \geq 2\right)  \tag{4.42}\\
& \text { Saturday }\left(X_{6} j+X_{5} j+X_{4} j+X_{3} j \geq 4\right)  \tag{4.43}\\
& \text { Saturday }\left(X_{6} j+X_{5} j+X_{4} j+X_{3} j \geq 2\right)  \tag{4.44}\\
& \text { Sunday }\left(X_{7} j+X_{6} j+X_{5} j+X_{4} j \geq 3\right)  \tag{4.45}\\
& \text { Sunday }\left(X_{7} j+X_{6} j+X_{5} j+X_{4} j \geq 4\right)  \tag{4.46}\\
& \text { Sunday }\left(X_{7} j+X_{6} j+X_{5} j+X_{4} j \geq 2\right) \tag{4.47}
\end{align*}
$$

where,for all variables

$$
\begin{equation*}
X_{i} j \geq 2 \tag{4.48}
\end{equation*}
$$

This means that for any shift period, there should be at least 2 nurses in the ward.

The upper limit on the total number of nurses at ward B3 is the number of shifts within that ward which is 70

Hence

$$
\begin{equation*}
\sum_{i=1}^{7} \sum_{j=1}^{3} X_{i} j \leq 70 \tag{4.49}
\end{equation*}
$$

Again,since there are 13 nurses in the ward and each nurse is to work for at least 4 continuous days, it means the number of shifts should not be less than 56 .

Hence

$$
\begin{equation*}
\sum_{i=1}^{7} \sum_{j=1}^{3} X_{i} j \geq 56 \tag{4.50}
\end{equation*}
$$

By the use of mathematical programming software (MATLAB)which has the syntax;
\% X = ga(FITNESSFCN,NVARS,A,b,[],[],lb,ub,NONLCON,INTCON,options) The results are obtained in a tabular form as:

| Days | Shift period | Solution | Number of nurses |
| :---: | :---: | :---: | :---: |
| Monday | 1 | 2 | 2 |
| Monday | 2 | 2 | 2 |
| Monday | 3 | 2 | 2 |
| Tuesday | 1 | 5 | 5 |
| Tuesday | 2 | 3 | 3 |
| Tuesday | 3 | 4 | 4 |
| Wednesday | 1 | 2 | 2 |
| Wednesday | 2 | 2 | 2 |
| Wednesday | 3 | 2 | 2 |
| Thursday | 1 |  |  |
| Thursday | 2 | 2 | 2 |
| Thursday | 3 | 3 | 3 |
| Friday | 1 |  | 2 |
| Friday | 2 |  | 2 |
| Friday | 3 | 2 | 2 |
| Saturday | 1 |  |  |
| Saturday | 2 |  | 5 |
| Saturday | 3 | 2 |  |
| Sunday | 1 | 2 SANIE |  |
| Sunday | 2 | 2 | 2 |
| Sunday | 3 | 2 | 2 |
| TOTAL |  |  | 56 |

From the results obtained, the number of shifts for the 13 nurses in the ward B3 has been reduced from 70 to 56 , which implies that the shifts has been minimised, indicating that the 13 nurses are entitled to 56 shifts periods in which each nurse is to work for only four continuous days while satisfying the needs of the patients. The shift that has been minimised indicates that either some of the
nurse were doing overtime or the timetable has been drawn arbitrary.There has been a reduction of 14 shifts which implies that the amount spent on the entire shifts will be reduced.From the results obtained there is a peak periods, that is 8:00am to $4: 00 \mathrm{pm}$ where majority of nurses are needed to attend to patients.

### 4.1.3 Model for ward B2

Table 4.3: WARD B2(17 NURSES)91 SHIFTS

|  | Mon | Tue | Wed | Thu | Fri | Sat | sun |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Morning | 5 | 5 | 3 | 4 | 4 | 4 | 3 |
| Afternoon | 8 | 7 | 6 | 6 | 6 | 5 | 4 |
| Night | 3 | 3 | 3 | 3 | 3 | 3 | 3 |

From the table of ward B2, there are 17 nurses and the shifts that occur in this ward is 91 .The idea is to minimise the number of shifts so as to reduce the workload.Hence the model for ward B2 is as follows;

$$
\begin{equation*}
\text { Minimize } Z=\sum_{i=1}^{7} \sum_{j=1}^{3} X_{i} j \tag{4.51}
\end{equation*}
$$

Subject to the constraints

$$
\begin{align*}
& \text { Monday }\left(X_{1} j+X_{7} j+X_{6} j+X_{5} j \geq 5\right)  \tag{4.52}\\
& \text { Monday }\left(X_{1} j+X_{7} j+X_{6} j+X_{5} j \geq 8\right)  \tag{4.53}\\
& \text { Monday }\left(X_{1} j+X_{7} j+X_{6} j+X_{5} j \geq 3\right)  \tag{4.54}\\
& \text { Tuesday }\left(X_{2} j+X_{1} j+X_{7} j+X_{6} j \geq 5\right)  \tag{4.55}\\
& \text { Tuesday }\left(X_{2} j+X_{1} j+X_{7} j+X_{6} j \geq 7\right)  \tag{4.56}\\
& \text { Tuesday }\left(X_{2} j+X_{1} j+X_{7} j+X_{6} j \geq 3\right)  \tag{4.57}\\
& \text { Wednesday }\left(X_{3} j+X_{2} j+X_{1} j+X_{7} j \geq 3\right)  \tag{4.58}\\
& \text { Wednesday }\left(X_{3} j+X_{2} j+X_{1} j+X_{7} j \geq 6\right) \tag{4.59}
\end{align*}
$$

$$
\begin{align*}
& \text { Wednesday }\left(X_{3} j+X_{2} j+X_{1} j+X_{7} j \geq 3\right)  \tag{4.60}\\
& \text { Thursday }\left(X_{4} j+X_{3} j+X_{2} j+X_{1} j \geq 4\right)  \tag{4.61}\\
& \text { Thursday }\left(X_{4} j+X_{3} j+X_{2} j+X_{1} j \geq 6\right)  \tag{4.62}\\
& \text { Thursday }\left(X_{4} j+X_{3} j+X_{2} j+X_{1} j \geq 3\right)  \tag{4.63}\\
& \text { Friday }\left(X_{5} j+X_{4} j+X_{3} j+X_{2} j \geq 4\right)  \tag{4.64}\\
& \text { Friday }\left(X_{5} j+X_{4} j+X_{3} j+X_{2} j \geq 6\right)  \tag{4.65}\\
& \text { Friday }\left(X_{5} j+X_{4} j+X_{3} j+X_{2} j \geq 3\right)  \tag{4.66}\\
& \text { Saturday }\left(X_{6} j+X_{5} j+X_{4} j+X_{3} j \geq 4\right)  \tag{4.67}\\
& \text { Saturday }\left(X_{6} j+X_{5} j+X_{4} j+X_{3} j \geq 5\right)  \tag{4.68}\\
& \text { Saturday }\left(X_{6} j+X_{5} j+X_{4} j+X_{3} j \geq 3\right)  \tag{4.69}\\
& \text { Sunday }\left(X_{7} j+X_{6} j+X_{5} j+X_{4} j \geq 3\right)  \tag{4.70}\\
& \text { Sunday }\left(X_{7} j+X_{6} j+X_{5} j+X_{4} j \geq 4\right)  \tag{4.71}\\
& \text { Sunday }\left(X_{7} j+X_{6} j+X_{5} j+X_{4} j \geq 3\right) \tag{4.72}
\end{align*}
$$

where,for all variables

$$
\begin{equation*}
X_{i} j \geq 2 \tag{4.73}
\end{equation*}
$$

This means that for any shift period, there should be at least 2 nurses in the ward.

The upper limit on the total number of nurses at ward B2 is the number of shifts within that ward which is 91

Hence

$$
\begin{equation*}
\sum_{i=1}^{7} \sum_{j=1}^{3} X_{i} j \leq 91 \tag{4.74}
\end{equation*}
$$

Again,since there are 17 nurses in the ward and each nurse is to work for at least 4 continuous days, it means the number of shifts should not be less than 68 .

Hence

$$
\begin{equation*}
\sum_{i=1}^{7} \sum_{j=1}^{3} X_{i} j \geq 68 \tag{4.75}
\end{equation*}
$$

By the use of mathematical programming software (MATLAB)which has the syntax;
\% X = ga(FITNESSFCN,NVARS,A,b,[],[],lb,ub,NONLCON,INTCON,options) The results are obtained in a tabular form as:

| Days | Shift period | Solution | Number of nurses |
| :---: | :---: | :---: | :---: |
| Monday | 1 | 3 | 3 |
| Monday | 2 | 2 | 2 |
| Monday | 3 | 3 | 3 |
| Tuesday | 1 | 4 | 4 |
| Tuesday | 2 | 3 | 3 |
| Tuesday | 3 | 3 | 3 |
| Wednesday | 1 | 4 | 4 |
| Wednesday | 2 | 4 | 4 |
| Wednesday | 3 | 2 | $2 \square$ |
| Thursday | 1 | 2 | 2 |
| Thursday | 2 | 3 | 3 |
| Thursday | 3 | 5 | 5 |
| Friday | 1 |  |  |
| Friday | 2 |  | 4 |
| Friday | 3 |  | 2 |
| Saturday | 1 |  |  |
| Saturday |  |  | 3 |
| Saturday | 3 | 3 | 3 |
| Sunday | 1 | 4 SAANE | 4 |
| Sunday | 2 | 2 | 2 |
| Sunday | 3 | 4 | 4 |
| TOTAL |  |  | 68 |

From the results obtained, the number of shifts for the 17 nurses in the ward B2 has been reduced from 91 to 68 , which implies that the shifts has been minimised, indicating that the 17 nurses are entitled to 68 shifts periods genetically, in which each nurse is to work for only four continuous days while satisfying the needs of the patients.The shift that has been minimised indicates that either
some of the nurse were doing overtime or the timetable has been drawn arbitrary. There has been a reduction of 23 shifts which implies that the amount spent on the entire shifts will be reduced.From the results obtained there is a peak periods, that is $8: 00 \mathrm{am}$ to $4: 00 \mathrm{pm}$ where majority of nurses are needed to attend to patients.

### 4.1.4 Model for ward C1A

Table 4.4: WARD C1A(13 NURSES)61 SHIFTS

|  | Mon | Tue | Wed | Thu | Fri | Sat | sun |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Morning | 3 | 3 | 2 | 3 | 2 | 2 | 2 |
| Afternoon | 6 | 6 | 5 | 4 | 4 | 3 | 5 |
| Night | 2 | 1 | 1 | 2 | 2 | 1 | 2 |

From the table of ward C1A, there are 13 nurses and the shifts that occur in this ward is 61 .The idea is to minimise the number of shifts so as to reduce the workload.Hence the model for ward C1A is as follows;

$$
\begin{equation*}
\text { Minimize } Z=\sum_{i=1}^{7} \sum_{j=1}^{3} X_{i} j \tag{4.76}
\end{equation*}
$$

Subject to the constraints

$$
\begin{align*}
& \text { Monday }\left(X_{1} j+X_{7} j+X_{6} j+X_{5} j \geq 3\right)  \tag{4.77}\\
& \text { Monday }\left(X_{1} j+X_{7} j+X_{6} j+X_{5} j \geq 6\right)  \tag{4.78}\\
& \text { Monday }\left(X_{1} j+X_{7} j+X_{6} j+X_{5} j \geq 2\right)  \tag{4.79}\\
& \text { Tuesday }\left(X_{2} j+X_{1} j+X_{7} j+X_{6} j \geq 3\right)  \tag{4.80}\\
& \text { Tuesday }\left(X_{2} j+X_{1} j+X_{7} j+X_{6} j \geq 6\right)  \tag{4.81}\\
& \text { Tuesday }\left(X_{2} j+X_{1} j+X_{7} j+X_{6} j \geq 1\right)  \tag{4.82}\\
& \text { Wednesday }\left(X_{3} j+X_{2} j+X_{1} j+X_{7} j \geq 2\right) \tag{4.83}
\end{align*}
$$

$$
\begin{align*}
& \text { Wednesday }\left(X_{3} j+X_{2} j+X_{1} j+X_{7} j \geq 5\right)  \tag{4.84}\\
& \text { Wednesday }\left(X_{3} j+X_{2} j+X_{1} j+X_{7} j \geq 1\right)  \tag{4.85}\\
& \text { Thursday }\left(X_{4} j+X_{3} j+X_{2} j+X_{1} j \geq 3\right)  \tag{4.86}\\
& \text { Thursday }\left(X_{4} j+X_{3} j+X_{2} j+X_{1} j \geq 4\right)  \tag{4.87}\\
& \text { Thursday }\left(X_{4} j+X_{3} j+X_{2} j+X_{1} j \geq 2\right)  \tag{4.88}\\
& \text { Friday }\left(X_{5} j+X_{4} j+X_{3} j+X_{2} j \geq 2\right)  \tag{4.89}\\
& \text { Friday }\left(X_{5} j+X_{4} j+X_{3} j+X_{2} j \geq 4\right)  \tag{4.90}\\
& \text { Friday }\left(X_{5} j+X_{4} j+X_{3} j+X_{2} j \geq 2\right)  \tag{4.91}\\
& \text { Saturday }\left(X_{6} j+X_{5} j+X_{4} j+X_{3} j \geq 2\right)  \tag{4.92}\\
& \text { Saturday }\left(X_{6} j+X_{5} j+X_{4} j+X_{3} j \geq 3\right)  \tag{4.93}\\
& \quad \text { Saturday }\left(X_{6} j+X_{5} j+X_{4} j+X_{3} j \geq 1\right)  \tag{4.94}\\
& \text { Sunday }\left(X_{7} j+X_{6} j+X_{5} j+X_{4} j \geq 2\right)  \tag{4.95}\\
& \text { Sun }  \tag{4.96}\\
& \text { Sunday }\left(X_{7} j+X_{6} j+X_{5} j+X_{4} j \geq 5\right)  \tag{4.97}\\
& \text { Sunday }\left(X_{7} j+X_{6} j+X_{5} j+X_{4} j \geq 2\right)
\end{align*}
$$

where,for all variables

$$
\begin{equation*}
X_{i} j \geq 2 \tag{4.98}
\end{equation*}
$$

This means that for any shift period, there should be at least 2 nurses in the ward.

The upper limit on the total number of nurses at ward C1A is the number of shifts within that ward which is 61

Hence

$$
\begin{equation*}
\sum_{i=1}^{7} \sum_{j=1}^{3} X_{i} j \leq 61 \tag{4.99}
\end{equation*}
$$

Again,since there are 13 nurses in the ward and each nurse is to work for at least 4 continuous days, it means the number of shifts should not be less than 52 .

Hence

$$
\begin{equation*}
\sum_{i=1}^{7} \sum_{j=1}^{3} X_{i} j \geq 52 \tag{4.100}
\end{equation*}
$$

By the use of mathematical programming software (MATLAB)which has the syntax;

$\% \mathrm{X}=\mathrm{ga}($ FITNESSFCN,NVARS,A,b,[],[],lb,ub,NONLCON,INTCON,options) The results are obtained in a tabular form as:

| Days | Shift period | Solution | Number of nurses |
| :--- | :--- | :--- | :--- |
| Monday | 1 | 3 | 3 |
| Monday | 2 | 2 | 2 |
| Monday | 3 | 3 | 3 |
| Tuesday | 1 | 3 | 3 |
| Tuesday | 2 | 2 | 2 |
| Tuesday | 3 | 3 | 3 |
| Wednesday | 1 | 2 | 2 |
| Wednesday | 2 | 2 | 2 |
| Wednesday | 3 | 2 | 2 |
| Thursday | 1 | 5 | 5 |
| Thursday | 2 | 2 | 2 |
| Thursday | 3 | 3 | 3 |
| Friday | 1 | 2 | 2 |
| Friday | 2 | 3 | 3 |
| Friday | 3 | 3 | 3 |
| Saturday | 1 | 2 | 2 |
| Saturday | 2 | 2 | 2 |
| Saturday | 3 | 2 | 2 |
| Sunday | 1 | 2 | 2 |
| Sunday | 2 | 2 | 2 |
| Sunday | 3 | 2 | 52 |
| TOTAL |  | 2 |  |
|  | 2 | 2 |  |
|  | 2 | 2 |  |
|  | 2 | 2 |  |
|  | 2 | 2 |  |

From the results obtained, the number of shifts for the 13 nurses in the ward C1A has been reduced from 61 to 52 , which implies that the shifts has been minimised, indicating that the 13 nurses are entitled to 52 shifts periods genetically, in which each nurse is to work for only four continuous days while satisfying the needs of the patients. The shift that has been minimised indicates that either
some of the nurse were doing overtime or the timetable has been drawn arbitrary. There has been a reduction of 9 shifts which implies that the amount spent on the entire shifts will be reduced.From the results obtained there is a peak periods, that is $8: 00 \mathrm{am}$ to $4: 00 \mathrm{pm}$ where majority of nurses are needed to attend to patients.

### 4.1.5 Model for ward C1B

Table 4.5: WARD C1B(14 NURSES) 72 SHIFTS

|  | Mon | Tue | Wed | Thu | Fri | Sat | sun |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Morning | 3 | 2 | 3 | 3 | 3 | 3 | 3 |
| Afternoon | 7 | 7 | 7 | 7 | 5 | 5 | 5 |
| Night | 2 | 2 | 2 | 2 | 2 | 2 | 2 |

From the table of ward C1B, there are 14 nurses and the shifts that occur in this ward is 72 .The idea is to minimise the number of shifts so as to reduce the workload.Hence the model for ward C1B is as follows;

$$
\begin{equation*}
\text { Minimize } Z=\sum_{i=1}^{7} \sum_{j=1}^{3} X_{i} j \tag{4.101}
\end{equation*}
$$

Subject to the constraints

$$
\begin{align*}
& \text { Monday }\left(X_{1} j+X_{7} j+X_{6} j+X_{5} j \geq 3\right)  \tag{4.102}\\
& \text { Monday }\left(X_{1} j+X_{7} j+X_{6} j+X_{5} j \geq 7\right)  \tag{4.103}\\
& \text { Monday }\left(X_{1} j+X_{7} j+X_{6} j+X_{5} j \geq 2\right)  \tag{4.104}\\
& \text { Tuesday }\left(X_{2} j+X_{1} j+X_{7} j+X_{6} j \geq 2\right)  \tag{4.105}\\
& \text { Tuesday }\left(X_{2} j+X_{1} j+X_{7} j+X_{6} j \geq 7\right)  \tag{4.106}\\
& \text { Tuesday }\left(X_{2} j+X_{1} j+X_{7} j+X_{6} j \geq 2\right)  \tag{4.107}\\
& \text { Wednesday }\left(X_{3} j+X_{2} j+X_{1} j+X_{7} j \geq 3\right) \tag{4.108}
\end{align*}
$$

$$
\begin{align*}
& \text { Wednesday }\left(X_{3} j+X_{2} j+X_{1} j+X_{7} j \geq 7\right)  \tag{4.109}\\
& \text { Wednesday }\left(X_{3} j+X_{2} j+X_{1} j+X_{7} j \geq 2\right)  \tag{4.110}\\
& \text { Thursday }\left(X_{4} j+X_{3} j+X_{2} j+X_{1} j \geq 3\right)  \tag{4.111}\\
& \text { Thursday }\left(X_{4} j+X_{3} j+X_{2} j+X_{1} j \geq 7\right)  \tag{4.112}\\
& \text { Thursday }\left(X_{4} j+X_{3} j+X_{2} j+X_{1} j \geq 2\right)  \tag{4.113}\\
& \text { Friday }\left(X_{5} j+X_{4} j+X_{3} j+X_{2} j \geq 2\right)  \tag{4.114}\\
& \text { Friday }\left(X_{5} j+X_{4} j+X_{3} j+X_{2} j \geq 5\right)  \tag{4.115}\\
& \text { Friday }\left(X_{5} j+X_{4} j+X_{3} j+X_{2} j \geq 2\right)  \tag{4.116}\\
& \text { Saturday }\left(X_{6} j+X_{5} j+X_{4} j+X_{3} j \geq 2\right)  \tag{4.117}\\
& \text { Saturday }\left(X_{6} j+X_{5} j+X_{4} j+X_{3} j \geq 5\right)  \tag{4.118}\\
& \quad \text { Saturday }\left(X_{6} j+X_{5} j+X_{4} j+X_{3} j \geq 2\right)  \tag{4.119}\\
& \text { Sunday }\left(X_{7} j+X_{6} j+X_{5} j+X_{4} j \geq 2\right)  \tag{4.120}\\
& \text { Sunday }\left(X_{7} j+X_{6} j+X_{5} j+X_{4} j \geq 4\right)  \tag{4.121}\\
& \text { Sunday }\left(X_{7} j+X_{6} j+X_{5} j+X_{4} j \geq 2\right) \tag{4.122}
\end{align*}
$$

where,for all variables

$$
\begin{equation*}
X_{i} j \geq 2 \tag{4.123}
\end{equation*}
$$

This means that for any shift period, there should be at least 2 nurses in the ward.

The upper limit on the total number of nurses at ward C1B is the number of shifts within that ward which is 72

Hence

$$
\begin{equation*}
\sum_{i=1}^{7} \sum_{j=1}^{3} X_{i} j \leq 72 \tag{4.124}
\end{equation*}
$$

Again,since there are 14 nurses in the ward and each nurse is to work for at least 4 continuous days, it means the number of shifts should not be less than 56 .

Hence

$$
\begin{equation*}
\sum_{i=1}^{7} \sum_{j=1}^{3} X_{i} j \geq 56 \tag{4.125}
\end{equation*}
$$

By the use of mathematical programming software (MATLAB)which has the syntax;

\% X $=$ ga(FITNESSFCN,NVARS,A,b,[],[],lb,ub,NONLCON,INTCON,options)
The results are obtained in a tabular form as:

| Days | Shift period | Solution | Number of nurses |
| :---: | :---: | :---: | :---: |
| Monday | 1 | 2 | 2 |
| Monday | 2 | 5 | 5 |
| Monday | 3 | 2 | 2 |
| Tuesday | 1 | 2 | 2 |
| Tuesday | 2 | 2 | 2 |
| Tuesday | 3 | 2 | 2 |
| Wednesday | 1 | 2 | 2 |
| Wednesday | 2 | 5 | 5 |
| Wednesday | 3 | 2 | $2 \square$ |
| Thursday | 1 |  |  |
| Thursday | 2 | 3 | 3 |
| Thursday | 3 | 5 | 5 |
| Friday | 1 |  | 2 |
| Friday | 2 |  | 2 |
| Friday | 3 |  | 3 |
| Saturday | 1 |  |  |
| Saturday | $2=$ |  | 2 |
| Saturday | 3 | 2 | 2 |
| Sunday | 1 | 2 SANIE | 2 |
| Sunday | 2 | 5 | 5 |
| Sunday | 3 | 2 | 2 |
| TOTAL |  |  | 56 |

From the results obtained, the number of shifts for the 14 nurses in the ward C1B has been reduced from 72 to 56 , which implies that the shifts has been minimised, indicating that the 14 nurses are entitled to 52 shifts periods genetically, in which each nurse is to work for only four continuous days while satisfying the needs of the patients.The shift that has been minimised indicates that either
some of the nurse were doing overtime or the timetable has been drawn arbitrary. There has been a reduction of 16 shifts which implies that the amount spent on the entire shifts will be reduced.From the results obtained there is a peak periods, that is $8: 00 \mathrm{am}$ to $4: 00 \mathrm{pm}$ where majority of nurses are needed to attend to patients.

### 4.1.6 Model for ward C2

Table 4.6: WARD C2(14 NURSES) 73 SHIFTS

|  | Mon | Tue | Wed | Thu | Fri | Sat | sun |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Morning | 3 | 3 | 2 | 3 | 3 | 3 | 3 |
| Afternoon | 6 | 6 | 5 | 5 | 6 | 5 | 6 |
| Night | 2 | 2 | 2 | 2 | 2 | 2 | 2 |

From the table of ward C2, there are 14 nurses and the shifts that occur in this ward is 73 .The idea is to minimise the number of shifts so as to reduce the workload. Hence the model for ward C2 is as follows;

$$
\begin{equation*}
\text { Minimize } Z=\sum_{i=1}^{7} \sum_{j=1}^{3} X_{i} j \tag{4.126}
\end{equation*}
$$

Subject to the constraints

$$
\begin{align*}
& \text { Monday }\left(X_{1} j+X_{7} j+X_{6} j+X_{5} j \geq 3\right)  \tag{4.127}\\
& \text { Monday }\left(X_{1} j+X_{7} j+X_{6} j+X_{5} j \geq 6\right)  \tag{4.128}\\
& \text { Monday }\left(X_{1} j+X_{7} j+X_{6} j+X_{5} j \geq 2\right)  \tag{4.129}\\
& \text { Tuesday }\left(X_{2} j+X_{1} j+X_{7} j+X_{6} j \geq 3\right)  \tag{4.130}\\
& \text { Tuesday }\left(X_{2} j+X_{1} j+X_{7} j+X_{6} j \geq 6\right)  \tag{4.131}\\
& \text { Tuesday }\left(X_{2} j+X_{1} j+X_{7} j+X_{6} j \geq 2\right)  \tag{4.132}\\
& \text { Wednesday }\left(X_{3} j+X_{2} j+X_{1} j+X_{7} j \geq 2\right) \tag{4.133}
\end{align*}
$$

$$
\begin{align*}
& \text { Wednesday }\left(X_{3} j+X_{2} j+X_{1} j+X_{7} j \geq 5\right)  \tag{4.134}\\
& \text { Wednesday }\left(X_{3} j+X_{2} j+X_{1} j+X_{7} j \geq 2\right)  \tag{4.135}\\
& \text { Thursday }\left(X_{4} j+X_{3} j+X_{2} j+X_{1} j \geq 3\right)  \tag{4.136}\\
& \text { Thursday }\left(X_{4} j+X_{3} j+X_{2} j+X_{1} j \geq 5\right)  \tag{4.137}\\
& \text { Thursday }\left(X_{4} j+X_{3} j+X_{2} j+X_{1} j \geq 2\right)  \tag{4.138}\\
& \text { Friday }\left(X_{5} j+X_{4} j+X_{3} j+X_{2} j \geq 3\right)  \tag{4.139}\\
& \text { Friday }\left(X_{5} j+X_{4} j+X_{3} j+X_{2} j \geq 6\right)  \tag{4.140}\\
& \text { Friday }\left(X_{5} j+X_{4} j+X_{3} j+X_{2} j \geq 2\right)  \tag{4.141}\\
& \text { Saturday }\left(X_{6} j+X_{5} j+X_{4} j+X_{3} j \geq 3\right)  \tag{4.142}\\
& \text { Saturday }\left(X_{6} j+X_{5} j+X_{4} j+X_{3} j \geq 5\right)  \tag{4.143}\\
& \quad \text { Saturday }\left(X_{6} j+X_{5} j+X_{4} j+X_{3} j \geq 2\right)  \tag{4.144}\\
& \text { Sunday }\left(X_{7} j+X_{6} j+X_{5} j+X_{4} j \geq 3\right)  \tag{4.145}\\
& \text { Sunday }\left(X_{7} j+X_{6} j+X_{5} j+X_{4} j \geq 6\right)  \tag{4.146}\\
& \text { Sunday }\left(X_{7} j+X_{6} j+X_{5} j+X_{4} j \geq 2\right) \tag{4.147}
\end{align*}
$$

where,for all variables

$$
\begin{equation*}
X_{i} j \geq 2 \tag{4.148}
\end{equation*}
$$

This means that for any shift period, there should be at least 2 nurses in the ward.

The upper limit on the total number of nurses at ward C 2 is the number of shifts within that ward which is 73

Hence

$$
\begin{equation*}
\sum_{i=1}^{7} \sum_{j=1}^{3} X_{i} j \leq 73 \tag{4.149}
\end{equation*}
$$

Again,since there are 14 nurses in the ward and each nurse is to work for at least 4 continuous days, it means the number of shifts should not be less than 56 .

Hence

$$
\begin{equation*}
\sum_{i=1}^{7} \sum_{j=1}^{3} X_{i} j \geq 56 \tag{4.150}
\end{equation*}
$$

By the use of mathematical programming software (MATLAB)which has the syntax;

\% X $=$ ga(FITNESSFCN,NVARS,A,b,[],[],lb,ub,NONLCON,INTCON,options)
The results are obtained in a tabular form as:

| Days | Shift period | Solution | Number of nurses |
| :---: | :---: | :---: | :---: |
| Monday | 1 | 3 | 3 |
| Monday | 2 | 4 | 4 |
| Monday | 3 | 2 | 2 |
| Tuesday | 1 | 2 | 2 |
| Tuesday | 2 | 2 | 2 |
| Tuesday | 3 | 2 | 2 |
| Wednesday | 1 | 3 | 3 |
| Wednesday | 2 | 4 | 4 |
| Wednesday | 3 | 2 | 2 |
| Thursday | 1 |  |  |
| Thursday | 2 | 4 | 4 |
| Thursday | 3 | 2 | 2 |
| Friday | 1 |  | 2 |
| Friday | 2 |  | 3 |
| Friday | 3 |  | 4 |
| Saturday | 1 |  |  |
| Saturday | $2=$ |  | 4 |
| Saturday | 3 | 3 | 3 |
| Sunday | 1 | 3 SANE | 3 |
| Sunday | 2 | 2 | 2 |
| Sunday | 3 | 2 | 2 |
| TOTAL |  |  | 56 |

From the results obtained, the number of shifts for the 14 nurses in the ward C 2 has been reduced from 73 to 56 , which implies that the shifts has been minimised, indicating that the 14 nurses are entitled to 56 shifts periods genetically, in which each nurse is to work for only four continuous days while satisfying the needs of the patients. The shift that has been minimised indicates that either
some of the nurses were doing overtime or the timetable has been drawn arbitrary. There has been a reduction of 17 shifts which implies that the amount spent on the entire shifts will be reduced.From the results obtained there is a peak periods, that is $8: 00 \mathrm{am}$ to $4: 00 \mathrm{pm}$ where majority of nurses are needed to attend to patients.

### 4.1.7 Summary

In general, the results obtained suggest that the number of shifts has been reduced or minimised genetically in all wards of the hospital.


## Chapter 5

## CONCLUSION AND RECOMMENDATION

Hospitals must create impartial work schedules for nurses.Creating good impartial schedules helps to maintain an efficient and pleasant working environment. Manual scheduling is difficult and time consuming. As a result, manual scheduling can lead to dissatisfaction and complaints.

This study creates genetic-algorithm-based method for creating impartial work schedules for nurses. The method can be used to create stable impartial workloads that are based upon hospital rules and government regulations. The study shows that the genetic algorithm-based method can find optimal or near-optimal work schedules that are feasible and acceptable for hospital nursing units.

Its therefore recommended that further studies on other optimisation methods can be used to help minimise the shifts of nurses.Also, the genetic algorithm method can be used to minimise the shifts of nurses at other hospitals when its needed.

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## Appendix A

This is the


## Appendix B



