KWAME NKRUMAH UNIVERSITY OF SCIENCE AND TECHNOLOGY, KUMASI

QUEUING THEORY AND THE MANAGEMENT OF WAITING-TIME: A CASE STUDY OF OUT-PATIENT DEPARTMENT OF KOMFO ANOKYE TEACHING HOSPITAL POLYCLINIC KUMASI

By

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DECLARATION

I hereby declare that this submission is my own work towards the award of the Msc. Industrial Mathematics degree and that, to the best of my knowledge, it contains no material previously published by another person nor material which had been accepted for the award of any other degree of the university, except where due acknowledgement had been made in the text.

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DEDICATION

This work is dedicated to my parents and all my family members especially my wife Adelaide Poku, my children Archimedes Adjei Poku, Amanda Adjei Poku, John Adjei Poku and Ebenezer Adjei Poku.

ACKNOWLEDGMENT

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I acknowledge the valuable time and technical assistance received from management of KATH Polyclinic.

Finally, I am greatly indebted to the Almighty God for his Grace and Mercy in my academic and professional journey.

LIST OF ABBREVIATION

KATH ....Okomfo Anokye Teaching Hospital

SQL .... Structured Query Language

WHO .... World Health Organisation

KNUST .... Kwame Nkrumah University Of Science And Technology

OPD ....Out Patient Department

FCFS ....First Come First Served

FIFO .... First in First Out

LCFS ....Last Come First Served

ABSTRACT

The effect of queuing in relation to the time spent by patients to access medical services is increasingly becoming a major source of concern to health care providers. This is because keeping patients waiting too long could result in inconveniences or at times deaths. Also, providing too much service capacity to operate a system involves excessive cost. But not providing enough service capacity results in excessive waiting time and cost.

In this study, the queuing characteristics at the Komfo Anokye Teaching Hospital (KATH) Polyclinic were analysed using a Multi-server single-phase Model. Data for this study was collected at the outpatient department of the polyclinic in the 3rd Week of March, 2015 between the hours of 8 am to 12 noon through observations, interviews and by administering questionnaire. With the help of three research assistants a stop watch was used to calculate the number of minutes spent by each patient from the Record section, Assessment center and the Consulting rooms. The data gathered were analysed using Excel software as well as using calculator.

The results showed that Monday recorded the highest number of patients in the waiting line at the Assessment Centre while the least number of patients in line was recorded on the same Monday at the Consulting Room. The study also showed that there were only two record centres, one assessment center and three consulting rooms at the polyclinic serving all the patients that arrived at the facility compelling them to join long queues. Patients had to wait on the average of 7.36 minutes in the queue at the assessment center on Mondays and 9.75 minutes in the system before receiving service.

As a result, it is recommended that more doctors should be deployed to the hospital so as to convert the single-channel queuing units in multi-channel queuing units. It is also recommended that more health care centers should be created to take care of all categories of patients in the community.

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Chapter 1

INTRODUCTION

Government and private health providers in Ghana provide medical services to diverse groups of people. Demand of health services has been growing in recent years due to increase of both preventable and communicable diseases. In the absence of enough hospitals, health personnel and well-organise operating system, patients especially in government hospitals spend excessively long times waiting for treatment. As a result the hospitals and clinics are congested leaving patients
dissatisfied.

In order to survive, most hospitals are making efforts to improve their service quality to satisfy their patients. In the out-patient service for example, the main indicator of worth promise for patients is waiting time; patients should be attended to within an acceptable time. Several studies (Cayirli et al., 2008; Kujala et al., 2006; Zhu et al., 2009) suggest that hospital managers and policymakers are becoming more and more concerned with patient waiting time which is a measure of organizational efficiency. The waiting time is particularly important for a hospital, since the customers are patients who are human beings. Waiting for treatment can be frustrating given that time is unproductively spent and according to Katzman (1999) people are impatient and do not want to wait to be attended to. As Bielen and Demoulin (2007) observe, the literature on service quality indicates that waiting experiences are typically negative and have been shown to affect overall satisfaction of consumers with the service generally.

A common situation that occurs in everyday life is that of queuing or waiting in line. Queues are usually seen at bus stops, hospitals and bank counters Sharma (2009). In general, queue is formed when the demand for service exceeds its supply (Kandemir-Caues and Cauas, 2007). Wait time depends on the number of customers in a queue, the number of servers serving line and the amount of service time for each individual customers. In health care institution the effect of queuing in relation to the time spent for patients to access treatment is increasingly becoming a major source of concern to a modern society that is currently expose to great strides in technological advancement and speed (Stakutis and Boyle, 2009).

The organizations that care for persons who are ill and injured vary widely in scope and scale from specialized outpatient clinics to large urban hospitals, teaching hospitals, to regional health care systems. Despite these differences one can view the health care processes that these organizations generate within the context of queuing systems in which patients arrive, wait for service, obtain service, and then depart (Fomundam and Hermann, 2007). From birth to death, we are all part of the health care system. We rely on hospitals to provide preventive care and treat our illnesses, diseases and injuries. In fact, health care is perhaps the stage determinant of people's quality of life and longevity (Hall, 2006).

Health care systems have been challenged in recent years to deliver high quality services with limited resources (Hall et al., 2001). Health care resources are becoming increasingly limited and expensive thereby placing greater emphasis on the efficient utilization of the resources and the corresponding level of service provided to patients. Consequently, one of the most important operational issues in health care delivery involves capacity planning such that the goals of efficient resource utilization and providing high quality service are met using queuing models (Pierskalla and Wilson, 1989). Queuing is a challenge for all health care systems. In the developed world, considerable research has been done on how to improve queuing systems in various hospital settings. This unfortunately has not been the case in developing countries like Ghana.

Queuing theory is a potent mathematical approach to the analysis of waiting lines performance parameters in healthcare delivery systems (Ozcan, 2006). It has increasingly become a common management tool for decision making in the developed world. This vital tool is unfortunately minimally used in most health care systems in African countries including Ghana. Review of extensive literature establishes the use of queuing theory and modelling in improving waiting time in various hospital settings (McQuarrie, 1983; Green, 2006; Siddhartan et al., 1996). It has also been used in reducing cost relating to various aspects of healthcare Keller and Laughhum (1973) and generally improving system performance Murray (2000) in hospital systems.

Application of queuing theory to model hospital settings has been widely published (Ivalis and Millard, 2003; Adele and Barry, 2005; Vasanawala and Desser, 2005). Also, the use of queuing analysis and simulation to enhance performance at various hospital departments has been widely researched Green (2002), Kim et al. (1999) and emergency departments (Green, 2006). In most health care settings, unless an appointment system is in place, the queue discipline is either first-in-first-out or a set of patient classes that have different priorities (as in an emergency department, which treats patients with life-threatening injuries before others). McQuarrie (1983) showed that it is possible to minimize waiting times by giving priority to clients who require shorter service times. Green (2006), also provide models for queue disciplines while Siddhartan et al. (1996) analyzed the effect on patient waiting times when primary care patients use the Emergency Department. They proposed a priority discipline for different categories of patients and then a first-in-first-out discipline for each category. Singh (2006) looked into minimizing total cost incurred and also minimizing the waiting costs by comparing the outputs for two nurses, three nurses and four nurses by evaluating the performance measures for each of the scenarios. In that study, it was found that scenario of three(3) nurses was the optimal solution with optimum trade-off between the two types of cost involved in queuing models. In another study, Obamiro (2003) also applied the queuing theory in a study to determine the optimum number of nurses required in an antenatal clinic to reduce the time spent by pregnant women in the queue and the system. Queuing theory and modelling can thus be said to be useful modern tools for decision making on issues of capacity and resourcing.

Also the number of messengers required to transport patients or specimens in a hospital by assigning costs to the messenger and to the time during which a request is in queue was determined by (Gupta and Ascots, 1971). In a queuing network, a patient may have to go through several nodes and consequently several queues in order to obtain the desired service. Nodes where the ratio of demand to available service capacity is relatively high become bottlenecks. Such bottlenecks increase overall patient waiting times even though other nodes may have low utilization. One of the major elements in improving efficiency in the delivery of health care services is patient flow. Good patient flow means that patient queuing is minimized and poor patient flow means that patients suffer considerable queuing delays (Hall, 2006). Effective resource allocation and capacity planning are determined by patient flow because it informs the demand for health care services (Murray, 2000). Queuing theory provides exact or approximate estimation of performance measures for such systems based upon specific probability assumptions. In a hospital, these assumptions rarely hold, and so results are approximated (Cochran and Bharti, 2006).

1.1 Background of the Study

A Hospital or Medical Center is an institution for health care which is able to provide long-term patient stays. One distinguishes between two types of patientsinpatients and outpatients. Some patients in a hospital come only for a diagnosis and /or therapy and then leave (outpatient), while others are admitted and stay overnight or for several weeks or months (inpatients). Hospitals usually differ from other types of medical facilities by their ability to admit and care for inpatients. Within hospitals, the two types of patients are usually treated in separate systems, and thus can be analyzed separately. In the modern age, a hospital constitutes a combination of several Medical Units specializing in different areas of medicine such as internal medicine, surgery, plastic surgery, and childbirth. In addition to these medical units, the hospital includes some service units such as laboratories, imaging facilities, and Information Technology that provide service to the medical units.

Each Medical Unit is managed autonomously with its own medical staff usually with limited capacity which is a function of the physical space or capacity available. The physical space is usually measured by the number of beds allocated to that Medical Unit and the staffing levels such as doctors, nurses, and general workers. Naturally, capacity restrictions can lead to a situation of system blocking where patients will have to wait for several hours before being attended to.In large medical centers such as Komfo Anokye Teaching Hospital Polyclinic, such blocking systems is common especially on the first, middle and last days of the working week, that is Mondays, Wednesdays, and Fridays.

Time is always a valuable asset for patients seeking treatment at any health care centre either public or private, and even more valuable for patients who are in critical conditions. Doctors and specialists need to maximize their service time since some of them are assigned with administrative work, reading medical reports and keep moving from one department to another. Waiting idly in the waiting room is not a productive situation where patients can spend their waiting time to do other activities that might benefit them rather than sitting for nothing. Whenever the demand for a service exceeds its supply then queues are formed. Long waiting time in any hospital is considered as an indicator of poor quality and needs improvement. Managing waiting lines create a great problem for managers seeking to improve upon quality health care delivery and patient satisfaction. Patients dislike waiting for a long time. For many patients, queuing or waiting in lines is annoying Obamiro (2003) or negative experience (Scotland, 1991). If the waiting and service time is high, patients may leave the queue prematurely and this in turn results in customer dissatisfaction. This would reduce patients demand and eventually reduce revenue and profits gained by hospitals.

Health care is riddled with delays. Almost all of us have waited for days or weeks to get an appointment with a physician or schedule a procedure, and upon arrival we wait some more until being seen. In hospitals, it is not unusual to find patients waiting for beds in hallways, and delays for surgery or diagnostic tests. Delays are the result of a disparity between demand for a service and the capacity available to meet that demand. Usually this mismatch is temporary and due to natural variability in the timing of demands and in the duration of time needed to provide service. The variability and the interaction between the arrival and service processes make the dynamics of service systems very complex. Consequently, it's impossible to predict levels of congestion or to determine how much capacity is needed to achieve some desired level of performance without the help of a queuing model.

Queuing models require very little data and result in relatively simple formulas for predicting various performance measures such as mean delay or probability of waiting more than a given amount of time before being served. This means that they are easier and cheaper to use and can be more readily used to find optimal solutions rather than just estimating the system performance for a given scenario. Timely access has been identified as one of the key elements of health care quality Jestor and Redici (2001) and consequently, decreasing delays has become a focus in many health care institutions. Given the financial constraints that exist in many of these facilities, queuing analysis can be an extremely valuable tool in utilizing resources in the most cost effective way to reduce delays. Some people use the information gathered from queuing theory in order to determine how to best serve customers and so prevent them from waiting in line longer than they have to. The theory allows researchers to analyze several things such as arriving in line, waiting in line, and the time it takes to service customers. This allows them to gather and derive information on a customer's waiting time, the expected amount of customers that will be in a line, the probability of a customer encountering a line, as well as other data. This information is used in order to find ways to reduce lines and wait time.

1.1.1 Profile of KATH Polyclinic

The Komfo Anokye Teaching Hospital (KATH) was established in 1940 to serve European expatriates and few fortunate Africans in the Gold Coast era. In 1945 the Nurses Training College for the training of State Registered Nurses began. This was followed in 1950 with the establishment of the Midwifery Training School for the training of Midwives. In 1975, the hospital became a Teaching hospital for the training of Medical Students by the School of Medical Sciences (SMS) of the Kwame Nkrumah University of Science and Technology (KNUST), Kumasi. KATH offers delivery of health care not only to its catchment area but also the rest of the country with its tertiary referral services. In addition to health care services provided, the hospital offers facilities for teaching/training of all health professionals (nurses, undergraduate medical students, postgraduate resident medical practitioners, pharmacists, laboratory technologists) and also conducts research into various health related issues. Due to its strategic location, inadequate facilities as well as inadequate health personnel the Polyclinic is always flooded with patients compelling them to join long queues to receive medical attention.

1.2 Statement of Problem

It is a goal universally acknowledged that a health care system should treat its patients in a timely manner. However this is often not achieved in practice, particularly in public health care systems that suffer from high patient demand and limited resources (AU-Yeung et al., 2006; Bruin et al., 2007b). Today's health care system operates under severe pressure along with improved medical and health care science and possibly healthier lifestyles, the proportion of people that need medical attention in the population continues to increase.

Furthermore, the long waiting queues have become symbols of the inefficiency of hospitals services all over the world, particularly in publicly funded hospitals (Gauld, 2000). According to him overcrowded outpatient departments, patient care delays and scarce resources are common in large public hospitals. Variability in the length of waiting time has a major impact on a day-to-day hospital operation and capacity requirement Bruin et al. (2007a). Studies have shown that overcrowding, prolonged waiting times and protracted lengths of stay increase the proportion of patients who leave without being seen by a physician (Stock et al., 1994; Fernandes et al., 1997).

Although several measures have been adopted over the past years to ensure quality service, the issues of waiting times delays and cancellations with respect to both in-patient and outpatient flows are still a problem to reckon with. A great deal of research has shown that waiting time is a source of dissatisfaction in patients (Uehira and Kay, 2009; Hart, 1996; Gupta et al., 1993; McKinnon et al., 1998). Hart (1996) argues that waiting to be treated is the one consistent feature of dissatisfaction that has been expressed with outpatient service. There is a scarcity of research on hospital waiting times with very few studies focusing on methods to improve the situation. Different researchers studying different queue systems have come up with different models that best fits the situation being studied. Hence the need of this study which intends to develop a flow model that can be used effectively to solve queue management problems at the outpatient department of KATH Polyclinic.

1.3 Objectives of the Study

The main objective of the study is to explore multi-server exponential queuing system. Based on this exploration, a designed model would be used for determining specified steady-state average quantities such as waiting and service time of patients at KATH Polyclinic. Furthermore, the effects of traffic intensity would be examined on these steady state average quantities given the arrival and the service rates. Specifically, the following system performance characteristics will be measured;

1. The average number of arrivals at the outpatients departments of the polyclinic.
2. The average service time of customers at various sections of the outpatient department.
3. The probability that the facility will be idle.
4. The average time a patient spends waiting for a doctor.

1.4 Justification of the Study

The previous works on the subject matter of this study only identify the need for the application of queuing models to patient waiting problem and its associated costs but clearly not determining the maximum number of servers that can be used in order to minimize total expected costs and achieve optimal patient satisfaction. The findings of this research will assist hospital administrators and management to reduce the waiting time of outpatient in the system, improve on customer service and maximize the utilization of human and material resources. The findings could also be replicated in other service facilities where queuing is a major problem.

1.5 Research Methodology

The study adopts a descriptive and observation case study approach. In this approach, a specific phenomenon is being studied in order to gain understanding of issues that affects a relatively complete organizational unit. Owojori (2002) asserts that case study method is used when a research focuses on a set of issues in a single organization, and how to identify the factors involved in an in-depth study of the organization. The data in a case study approach are obtained largely through a review of written records and by means of interview and questionnaire techniques. The data used in this study satisfied the following conditions. First, historical data of patients at the OPD were used to model patient average arrival and length of stay. Second, some concerned persons in the outpatient department were interviewed to gather some relevant information in validating the historical data. Finally, Questionnaires were distributed to key persons in the selected units and hospitals and the responses were analyzed using modeling and simulation techniques.

1.6 Scope of the Study

The study looked at the outpatient department of KATH Polyclinic Kumasi. It focused on the number of patients who came to the outpatient section for health care delivery. It was targeted at the waiting and service times of patients to see how much time they spend in the outpatient department before they finally receive medical attention. These were done to determine the duration of service period outpatients undergo at the various sections of the hospital so as to design a system capable of ameliorating the situation.

1.7 Limitations of the Study

The study was confined to outpatient department of Komfo Anokye Teaching Hospital Polyclinic. This implies that the results may be peculiar to the chosen case studies and may be of limited generalization except for those health care institutions with similar characteristics in terms of patient flow and resources. Furthermore, the study covered a period of one week (23rd March, 2015 to 27th

March, 2015).

1.8 Organization of the Study

The study is organized into five chapters. Chapter one deals with the background, problem statement, objectives, scope, limitations and research methodology of the study. Chapter two presents the review of related literature under the following sub-headings: The concept of queuing, queue management, factor that influence outpatient management and queuing models analysis. Chapter three discusses the research methodology. Chapter four is devoted to data collection and analysis while Chapter five presents the summary of findings, conclusions and recommendation.

Chapter 2

LITERATURE REVIEW

The study examines the impact of waiting time at the outpatient department of KATH polyclinic. This chapter concerns itself with the contributions of other researchers on the issue of queueing problems in service institutions including hospitals. The literature is reviewed under the following relevant sub-headings:

1. The concept of queueing
2. Queue management
3. Factors That Influence Outpatient Management

2.1 The Concept of Queuing

A queue is a waiting line (like customers waiting at a supermarket checkout counter or patients waiting at the outpatients department and wanting to see a doctor); queuing theory is the mathematical theory of waiting lines. More generally, queuing theory is concerned with the mathematical modeling and analysis of systems that provide service to random demands. A queuing model is an abstract description of such a system. Typically, a queuing model represents

(1). the system's physical configuration, by specifying the number and arrangement of the servers, which provide service to the customers, and

(2). the stochastic (that is, probabilistic or statistical) nature of the demands, by specifying the variability in the arrival process and in the service process. Queuing theory was developed by A.K. Erlang in 1904 to help determine the capacity requirements of the Danish telephone system (Brockmeyer et al., 1948). It has since been applied to a large range of service industries including banks, airlines and telephone call centers Brewton (1989), Stem and Hersh (1980) , as well as emergency systems such as police patrol, fire and ambulances (Kolesar et al., 1975). Queuing models can be very useful in identifying appropriate levels of staff, equipment, and beds as well as in making decisions about resource allocation and the design of new services.

Waiting lines in health care organizations can be found wherever either patients or customers arrive randomly for services, such as walk-in patients and emergency room arrivals. Patients arriving for health care services with appointments are not considered as waiting lines, even if they wait to see their health care provider. Most sorts of health care service systems have the capacity to serve more patients than they are called to over the long term.

Basic structure of queuing model can be separated into input and output queuing system which include queue that must obey a queuing rule and service mechanics Hiller and Lieberman (2005). The simplest queuing model is called single-server single queue model as illustrated in figure 2.1. Single-server model has a single server and a single line of customers (Krasewski and Ritzman, 1998). It is a situation in which customers from a single line are to be served by a single service facility or server, i.e. one after the other. For application of queuing model to any situation we should first describe the input process and the output process (Singh, 2006).

The figure 2.1 below illustrates a view of basic queuing process.

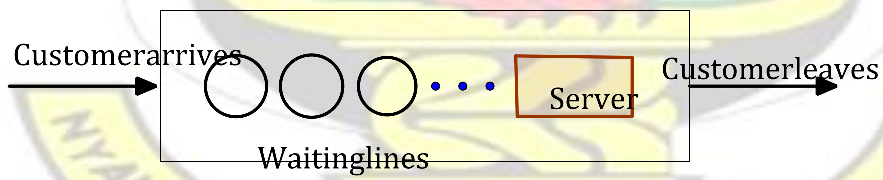


Figure 2.1: A view of Basic Queuing Process

2.1.1 Input and Output Processes

Input process is known as the arrival process. Customers/patients are known as arrivals which are generated one time by an input source randomly from finite or infinite population. These Patients/customers enter the queuing system and join a queue to be served. In the hospital setting, the group of individuals from which arrivals come is referred to as the call-in population. Variations occur in this population's size. Total patient demand requiring services from time to time constitute the size of arrival (Tutunic and Newlands, 2009). At all times, a member of the patients on the queue is selected for service by some rules known as the queue discipline. The required service is then performed for the customer by the service mechanism, after which the customer leaves the queuing system (Hiller and Lieberman, 2005). The output process tells the time that a customer leaves the system after going through all the service mechanisms. In other words, it is final stage of the service delivery from the point of arrival to the point of departure (Ozcan, 2006).

2.1.2 Queuing System Characteristics

According to Adedayo et al. (2006) and Medhi (2003), queuing phenomenon comprises of the following basic characteristics:

1. Arrival characteristics
2. The queue or the physical line itself
3. The number of servers or service channels
4. Queue discipline
5. Service mechanism
6. The capacity of the system
7. Departure

2.1.2.1 Arrival Characteristics

Arrival pattern describes the behaviour of way customers' arrive. It is specified by the inter-arrival time between any two consecutive arrivals Medhi (2003).

The inter-arrival time may be deterministic or probabilistic in nature. Arrival can occur from unlimited population (infinite) or limited (finite or restricted population) (Adedayo et al., 2006). There are four main descriptor of arrivals as put forth by Davis et al. (2003), the pattern of arrivals may be controllable or uncontrollable); whether the arrival occurs one at a time or in batches/bulk; whether the time between arrivals is constant or follow statistical distribution such as Poisson or exponential and whether the arrival stays in line or leave.

2.1.2.2 Waiting Line or Queue

A waiting line or queue occurs when customers wait before being served because the service facility is temporarily engaged. A queue is characterized by the maximum permissible number of customers that it can contain. Queues are called infinite or finite according to whether the number is infinite or finite Hiller and Lieberman (2001). An infinite queue is one in which for all practical purposes an unlimited number of customers can be held there. When the capacity is small enough that it needs to be taken into account then the queue is called a finite queue (Hillier and Hillier, 2003). Unless specified otherwise, the adopted queuing network model in this study assumes that the queue is an infinite queue.

2.1.2.3 Queue Discipline

The queue discipline refers to the order in which members of the queue are selected for service (Hiller and Lieberman, 2001). Winston and Albright (1997) posit that the usual queue discipline is first come, first served (FCFS or FIFO), where customers are served in order of arrival. In this study KATH uses FCFS queuing discipline. Although sometimes there are other service disciplines: last come, first served (LCFS) which happens sometime in case of emergencies, or servicein-random order and priority rule.Davis et al. (2003) assert that reservations first, emergencies first, highest profit customer first, largest orders first, best customers first, longest waiting time in line, and soonest promised date are other examples of queue discipline. Unless otherwise stated, the queuing model adopted in this study assumes arrival from infinite source with infinite queue and with first in first served (FCFS) queue discipline.

2.1.2.4 Service Mechanism

According to Mosek and Wilson (2001), service mechanism describes how the customer is served. In a single server system each customer is served by exactly one server even though there may be multiple servers. In most cases service times are random and they may vary greatly. Sometimes the service time may be similar for each job or constant. The service mechanism also describes the number of servers. A queuing system may operate with a single server or a number of parallel servers. An arrival who finds more than one free server may choose at random any one of them for receiving service. If he finds all the servers busy he joins a queue common to all servers. The first customer from the common queue goes to the server who becomes free first Medhi (2003).

2.1.2.5 Capacity of the System

A system may have an infinite capacity that is; the queue in front of the server(s) may grow to any length. Furthermore, there may be limitation of space and so when the space is filled to capacity an arrival will not be able to join the system and will be lost to the system. The system is called a delay system or a loss system according to whether the capacity is infinite or finite respectively Medhi (2003).

2.1.2.6 Departure

Once customers are served they depart and may not likely re-enter the system to queue again. It is usually assumed that departing customers do not return into the system immediately Adedayo et al. (2006).

2.1.3 Types of Queuing System

There are four major types of queuing system and different combinations of the same can be adopted for complex networks. Lapin (1981) broadly categorized queuing system structures into the following.

2.1.3.1 Single-Server, Single-Phase System:

This is a situation in which single queue of customers are to be served by a single service facility (server) one after the other. An example is where a nurse practitioner is the server who does all the work as well as attending to patients Singh (2007). Figure 2.2 depicts a single server-single- phase system.

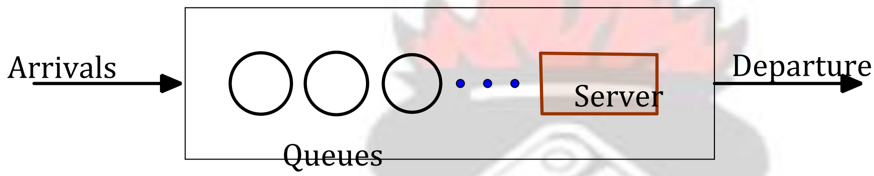


Figure 2.2: Single-Server, Single-Phase System

2.1.3.2 Single-server, Multiple-phases System:

In this situation, there's still a single queue but customers/patients receive more than one kind of service before departing the queuing system as shown in figure 3. For example, at outpatient department, patient first arrive at the records section, get the registration done and then wait in a queue to see a nurse for ancillary services before being seen by the doctor. Patients have to join queue at each phase of the system.

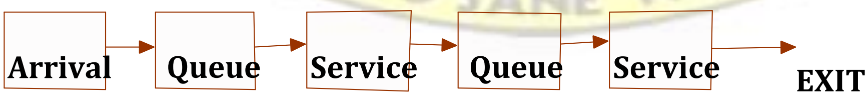


Figure 2.3: Single-Server, Multiple-Phases System

2.1.3.3 Multiple-servers, Single-phase System:

This is a queuing system characterized by a situation where there is more than one service facility (servers) providing identical service but drawn on a single waiting line (Obamiro, 2005). An example is patient waiting to see a doctor at general outpatient department of teaching hospitals as illustrated by figure 2.3.

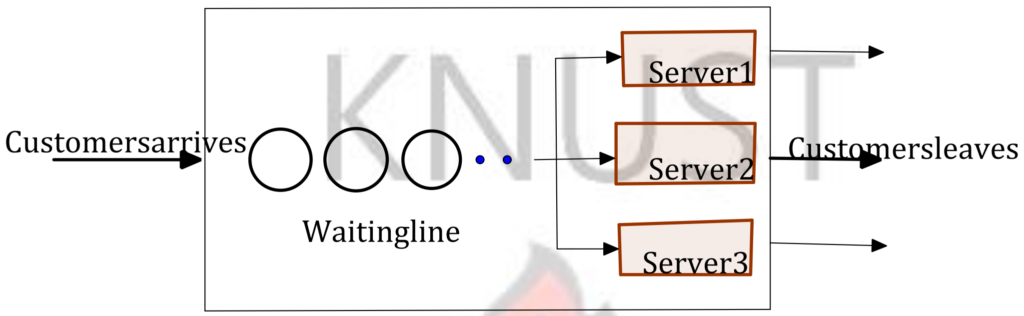


Figure 2.4: Multi Server Single-Phase

2.1.3.4 Multiple servers, Multiple-phases System:

According to Singh (2007), this type of system has numerous queues and a complex network of multiple phases of services involved as can be seen in figure 2.4. This type of service is typically seen in a hospital setting, in a multi-specialty outpatient clinics patient first form the queue for registration, and then he/she is triage for assessment, then for diagnostics, review, treatment, intervention or prescription and finally exits from the system or triage to different provider.

2.2 Queue Management

Queuing management refers to the control of queues and waiting lines Obamiro (2006). He said health systems should have an ability to deliver safe, efficient and smooth services to the patients. Increasing demand of quality and efficacy from highly aware and educated patients have started putting more pressure on the health care managers to respond to unduly delays for receiving health care.

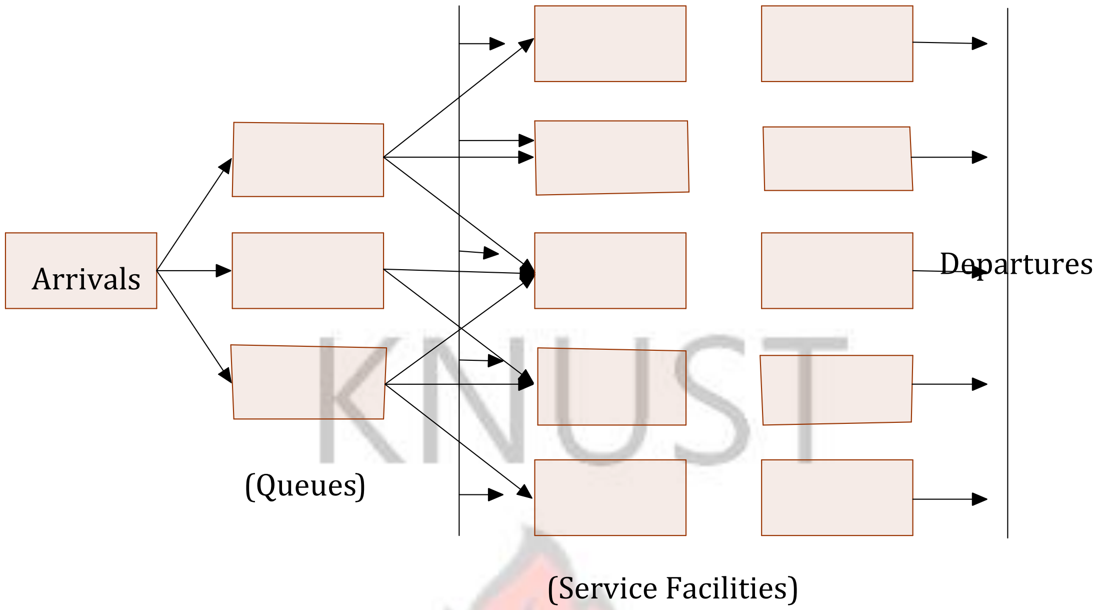


Figure 2.5: Multi Server, Multi-Phase System

Queuing theory is essentially used to manage long waiting time at a health facility. It essentially deals with patient flow through the system, if patient flow is good then patient queuing is minimized, if it is bad then the system may suffer loss of business and patients may suffer considerable queuing delays Parasuraman et al. (2009). They indicated that health care system can be visualized as a complex queuing network in which delays can be reduced through the following ways:

a. Synchronization of work among service stages (e.g., coordination of tests, treatments, discharge processes)

b. Scheduling of resources (e.g., doctors and nurses) to match patterns of arrival

c. Constant system monitoring (e.g., tracking number of patients waiting by location, diagnostic grouping and acuity) linked to immediate actions.

The management of health care facilities such as outpatient clinics is very complex and demanding to manage. The most common objectives of studies on the clinics have included the reduction of patient's time in the system (outpatient clinic), improvement on customer service, better resource utilization, and reduction of operating costs Davis and Vollman (1990). Analysis in such cases involves in depth analysis of the patients arrival and flow structure of the system, manpower characteristics and the scheduling system. Appropriate queuing models are then developed and applied for process modifications, appropriate staffing, scheduling or facility changes. Queuing theory can also be applied to hospital settings, particularly outpatient clinics and surgeries McManus et al. (2004). For example, small surgeries are performed by interns or assisting staff members in a hospital and the complicated ones by the experienced surgeons or a team. The experienced surgeons or team members for support services arrive later during the day. But the interns start their work earlier then the experienced surgeons. Using queuing theory in such a case, we can determine the arrival patterns of patients or the service rate and time and appropriately schedule surgeries for better quality and efficiency.

Queues and waiting lines could be controlled by two techniques namely; the operations management approach and the psychology approach

2.2.1 The Operation management approach

This approach deals with the management of how customers, queues and servers could be coordinated towards the goal of rendering effective service at the least cost. It has a way of reducing the length of the queue which helps to reduce customers waiting time. Increasing productivity by training existing staff or employing more staff is the way to achieve the operation management approach to satisfy customer's demands Katz and Martin (1989).

2.2.2 The Psychology approach

The psychology approach is used to improve upon customer satisfaction in relation to queuing. It plays with the mind of customers by manipulating their perceptions and expectations Katz and Martin (1989). The approach is founded

on the premise that customers see what they actually want to see. That is, they rely on what their minds tell them is true and look down upon the reality on the ground. Customers are satisfied when they see a fast moving queue because their minds tell them that the service providers in that system offer quick service. They however become unsatisfied when they see a long, slow – moving queue. Katz and Martin (1989). Customer's evaluation of service quality is affected not only by the actual waiting time but also, by the perceived waiting time. The act of waiting has great impact on customer's satisfaction. Davis and Vollman (1990). One of the issues in queuing management is not only the actual amount of time the customer has to wait Davis and Heineke (1994). The gap between the customer's perceptions of what happened during the service transaction and the customers' expectations of how the service transaction should have been performed is represented by the SERVQUAL model proposed by Parasuraman et al.
(1985). Mathematically, the model is presented as; Satisfaction (S) = Perception (P) = Expectation (E)

2.2.3 Customer Expectations

Kano (1984) suggested three categories of customer expectations. They are;

2.2.3.1 Satisfiers:

These are the characteristics which customers say they want in a service. The presence of these characteristics when provided leads to the satisfaction of customers. Example is experienced when more nurses are employed to take medical histories of patients. No matter how serious a patient sickness is, he/she is satisfied with this kind of service. They are satisfied because they have in mind that the greater the number of nurses, the lesser their waiting times. Thus customers are pleased when their perceptions of performances are equal to their expectations. That is, Perception (P) = Expectation (E) → Satisfied (S) customers.

We could say from the equation that the satisfaction (S) is zero (0) when customers are satisfied.

2.2.3.2 Dis-satisfiers:

They are the expected characteristics in a product or service and their absence leads to customer dissatisfaction. They are termed as the ”must - be”s” in a product or service and no matter what the quality of that product is they must be present to satisfy customers. Their absence would make customers go elsewhere. A laboratory centre must have a wash room to make patients take samples of say their urine for diagnosis. The absence of this would make customers dissatisfied no matter how good the quality of service provided to them is. Customers are dissatisfied when their perceptions of performance fall below their expectations. Thus, Perceptions (P) < Expectation (E) → Dissatisfied customers. From the equation, Satisfaction (S) is negative ($S < 0$) when customers are dissatisfied.

2.2.3.3 Enchantment or Exciters:

Customers generally do not expect to see these characteristics in a product or service because they are new to them. They are the unexpected qualities or bonuses that customers receive from their service providers. Example is the bonus customers enjoy when they recharge their mobile-phones with top-up cards. They are delighted or excited when their perceptions of performance exceed their expectations. Thus, Perception (P) > Expectation (E) → Delighted or excited customers. From the equation the Satisfaction (S) is positive ($S > 0$) when customers are delighted or excited. To minimize cost and maximize profit, the psychology approach provides a great benefit in that it is less expensive to apply as compared with the operation management approach. Service providers must at all times provide quality services with those characteristics which would make customer satisfied and excited always. Management should not over rely on the psychology approach since it plays with the mind of its customers, though it is good. The reason is when customers get to know the reality on the ground; they might turn away without coming back.

2.3 Factors That Influence Outpatient Management

2.3.1 Appointment System

The term ”appointment” refers to the amount of time the physician actually spends with the patient which may be shorter or longer than the appointment duration Ling et al. (2002) According to Cayirli and Veral (2005) Appointment scheduling can be classified into four broad categories:

2.3.1.1 Static:

With this system all decisions must be made prior to the beginning of a clinic session, which is the most common appointment system in health care.

2.3.1.2 Dynamic:

With this system the schedule of future arrivals are revised continuously over the course of the day based on the current state of the system. This is applicable when patient arrivals to the service center can be regulated dynamically, which generally involves patients already admitted to a hospital or clinic.

2.3.1.3 Single Block Rule:

The most primitive form of outpatient management is single block scheduling. The single block rule assigns all patients to arrive at the same time. The patients are served on a first come first serve basis.

2.3.1.4 Individual Block Rule:

The common form of appointment scheduling is the individual block rule. Here, patients are assigned unique appointment times that are spaced throughout the clinical session.

One of the major elements in improving efficiency in the delivery of health care services is patient flow. From a clinical perspective, patient flow represents the progression of a patient's health status. Patient flow management requires addressing three aspects of an outpatient unit: arrival of patients, service process, and queuing process. Working on the patient's arrival includes controlling its patient panel size, balancing patient volumes across available sessions, and achieving desirable patient arrival pattern within a session (Torres et al., 2004). According to Cote (2001) Patient flow can be described by one of two complementary approaches: clinical or operational. Regardless of approach, all patient flows share four common characteristics: an entrance, an exit and the random nature of the health care elements. He said resource planning, scheduling, and utilization are all affected by patient flows. Quantitative tools, like forecasting and queuing models, can help decision makers assess health care services in light of the patient flows. Queuing performance measures such as time in the system and traffic intensity have direct correspondence to the patient flow characteristics Goldstein et al. (2002).

2.3.2 Queuing System Terminology and Notations

Queuing theory is a mathematical theory with its own standard terminologies and notations. Few of the basic terminology and notations used in queuing theory that are relevant in this study are enumerated below; λ: Average (mean) arrival rate i.e. the rate of arrivals of patients/customers at a system.

μ: Average (mean) service rate i.e. the rate at which customers/patients could be served.

$\frac{1}{\lambda}$: Expected inter-arrival time.

$\frac{1}{\mu}$: Expected inter-service time.

ρ : system utilization factor. i.e. $\rho = \frac{\lambda}{s\mu}$ where s is the number of servers. It represents the fraction of the system's service capacity ($s\mu$) that is being utilized in the average by arriving customers/patients (λ) (Hiller and Lieberman, 2001).

L_q : Average number of customers waiting for service or waiting in the queue. i.e $L_q = \frac{\lambda^2}{\mu(\mu-\lambda)} = \frac{\rho^2}{1-\rho^2}$

L_s : Average number of customers in the system (those waiting and receiving service).ie $L_s = \frac{\lambda}{(\mu-\lambda)}$ *or* $\frac{\rho}{(1-\rho)}$

W_q : Average time customers spent in the queue. i.e $= \frac{\lambda}{\mu-\lambda}$

W_s : Average time customers spent in the system. ie $W_s = \frac{1}{(\mu-\lambda)}$ or $\frac{1}{\lambda(1-\rho)}$

P_o : probability of zero customers in the system. i.e $P_o = 1 - \frac{\lambda}{\mu}$ or $1 - \rho$

P_n : probability of exactly n units or customers in the system. i.e

$P(x = n) : 1 - \frac{\lambda}{\mu}\left(\frac{\lambda}{\mu}\right)^n = (1-\rho)\rho^n$

Probability of more than n units in the system = p^n

Chapter 3

METHODOLOGY

The main objective of the study is to explore multi-server exponential queuing system using modeling techniques. Based on this exploration, a designed model would be used for determining specified steady-state average quantities such as waiting times of patient at the out-patient department given the arrival and the service rates.

This study focuses on the derivation of the steady state probabilities for the M/M/s :(∞ /FCFS) queuing system (infinite capacity of customers). This system has a single queue with two or more servers. It is assumed that both inter-arrival and service time distributions are exponential and only one arrival can occur during a given time interval.

3.1 Derivation of Steady-State Probabilities

In this section, steady-state probabilities for the M/M/I queuing system are derived using the following notation: λ : Average number of arrivals entering the system per unit time.

L : Average number of customers present in the queuing system.

L_q : Average number of customers waiting in line.

L_s : Average number of customers in service.

μ : Average number of customers served per time period.

W : Average time a customer spends in the system.

W_q : Average time a customer spends in line.

W_s : Average time a customer spends in service.

Recall that, the M/M/1 :(∞ FCFS) queuing system has exponential inter-arrival times (we assume the arrival rate per unit time is λ) and a single server with exponential service times (we assume each customer's service time is exponential with rate μ). The M/M/1 :(∞ /FCFS) can be modelled as a birth-death process with the following equations (Winston, 1994).

$$\lambda_j = \lambda \qquad (j = 1, 2, 3,...)$$

$$\mu_j = \begin{cases} 0 & j = 0 \\ \mu & j > 0 \end{cases}$$

Where λ_j is the arrival rate in any state j (where j represents the number of customers in the system) and μ_j is the service rate in any state j.

To solve for the steady-state probability, π_j that j customers will be present in the system, we substitute the above set of equations into $\pi_j = \pi_j C_j$

where $$C_j = \frac{\lambda_0 \lambda_1 ... \lambda_{j-1}}{\mu_1 \mu_2 .... \mu_j}$$

Thus,

$$\pi_1 = \frac{\lambda\pi_0}{\mu}, = \pi_2 = \frac{\lambda^2\pi_0}{\mu^2}, ...\pi_j = \frac{\lambda^j\pi_0}{\mu^j, \ldots} \quad (3.1)$$

Since traffic intensity or utilization rate ρ, is given by $\rho = \frac{\lambda}{\mu}$ equation 3.2 can be written as $\pi_1 = \rho\pi_0$, $\pi_2 = \rho^2\pi_0$, $\pi_3 = \rho^3\pi_0$ ....

Since π_1, π_2 .... are probabilities, it follows that

$$\sum_j^{\infty} \pi = 1 \quad (3.2)$$

Hence, substituting for π_j in equation 3.2 and simplifying, we have

$$\pi_0(1 + \rho + \rho^2 + ...) = 1 \qquad 0 \leq \rho \leq 1 \quad (3.3)$$

Denoting $1 + \rho + \rho^2 + ...$ in equation 3.3 by S, we have $S = 1 + \rho + \rho^2 + \rho^3 + \rho^4 ...$
Multiplying through this equation by ρ yields $\rho S = \rho + \rho^2 + \rho^3 + \rho^4 ...$

This implies that $S - \rho S = 1$

Hence, $S = \frac{1}{(1-\rho)}$

Subsequently, by equation 3.3

$$\pi_0 \left(\frac{1}{1-\rho} \right) = 1$$

Therefore,

$\pi_0 = 1 - \rho$

and $\pi_j = \rho^j(1 - \rho)$; $\quad 0 \leq \rho \leq 1 \quad$ (Since ρ is a probability measure).

Hence for the M/M/1 :(∞ /FCFS), the steady-state probability in any state is given by $\pi_j = \rho^j(1 - \rho)$; where $0 \leq \rho \leq 1$. This result provides the basis for the derivation of steady-state average quantities in the next section.

3.2 Derivation of Steady-State Average Quanti-

ties

The relationship between the average quantities; average number of arrivals entering the system per unit time (λ), average number of customers present in the queuing system (L), average number of customers waiting in line (L_q), average number of customers in service (L_s), average number of customers served per unit time period (μ), average time a customer spends in the system (W), average time a customer spends in line (W_q) and average time a customer spends in service (W_s) are collectively known as Little's Queuing formulae. In the derivation of these quantities, it is first assumed that $\rho < 1$. This assumption ensures that the system will reach a steady-state ;that is $\lambda < \mu$.

3.2.1 Average Number of Customers in the System (L)

The average number of customers L, present in a single server queuing system is given by

$$L = \sum_{j=0}^{\infty} j\pi_j$$

Substituting for π_j, in this equation we have

$$L = \sum_{j=0}^{\infty} j\rho^j(1-\rho) \quad (3.4)$$

$$= (1-\rho)\sum_{j=0}^{\infty} j\rho^j \quad (3.5)$$

Denoting $\sum_{j=0}^{\infty} j\rho^j$ by S^i, we have $S^i = \rho + 2\rho^2 + 3\rho^3 + ...$

Multiplying through this equation by ρ gives ρS^i

$= \rho^2 + 2\rho^3 + 3\rho^4 + ...$

Thus,

$$S^i - \rho S^i = \rho + \rho^2 + \rho^3 + \dots \quad (3.6)$$

$$S^i(1-\rho) = \frac{\rho}{1-\rho} \quad (3.7)$$

$$S^i = \frac{\rho}{1-\rho} \times \frac{1}{1-\rho} \quad (3.8)$$

$$= \frac{\rho}{(1-\rho)^2} \quad (3.9)$$

Therefore, from equation 3.5

$$L = (1-\rho) \times \frac{\rho}{(1-\rho)^2} \quad (3.10)$$

$$= \frac{\rho}{1-\rho} \quad (3.11)$$

Noting that $\rho = \frac{\lambda}{\mu}$, it follows that the average number of customers, L, in a single server queuing system is given by;

$$L = \frac{\lambda}{\mu - \lambda} \quad (3.12)$$

3.2.2 Average Number of Customers in the Queue(L_q)

In some circumstances, we are interested in the expected number of people waiting in line or in the queue (L_q). Note that, if 0 or 1 customer is present in the system, then nobody is waiting in line, but if j people are present (j ≥ 1), there will be j - 1 customers in line. Thus, the average number of customers, L_q, present in a single server queue is given by;

$$L_q = \sum_{j=0}^{\infty} (j-1)\pi_j$$

Simplifying,

$$L_q = \sum_{j=0}^{\infty}(j\pi_j - \pi_j)$$

$$\sum_{j=0}^{\infty} j\pi_j - \sum_{j=0}^{\infty} \pi_j$$

Evaluating the above equation in terms of L and π_0 gives

$L_q = L - (1 - \pi_0)$

$= L - \rho$

Since $L = \frac{\rho}{(1-\rho)}$,it follows that

$$L_q = \frac{\rho}{1-\rho} - \rho \tag{3.13}$$

$$= \frac{\rho^2}{1-\rho} \tag{3.14}$$

Noting that $\rho = \frac{\lambda}{\mu}$, it follows that in a single server queuing system, the average number of customers in a queue, L_q, is given by

$$L_q = \frac{\lambda^2}{\mu(\mu - \lambda)} \tag{3.15}$$

3.2.3 Average Number of Customers in Service (L_s)

Also, of interest is L_S the average number of customers in service. For M/M/1 :(∞ /FCFS) queuing system, there are (0, 1, 2,.., n) customers in service and their respective results are as follows;

$L_s = 0\pi_0 + 1(\pi_1 + \pi_2 + \ldots)$

$= 1 - \pi_0$

$= 1 - (1 - \rho)$

Thus, the number of customers in service, L_S in a single server exponential queuing system is given by

$L_s = \rho$

Since every customer who is present in the system is either in line or in service, it follows that for any queuing system,

$$L = L_q + L_s \text{ or } L = L_q + \frac{\lambda}{\mu} \quad (3.16)$$

3.3 Waiting Times in the System (W, W_q, W_s)

For any queuing system in which a steady-state distribution exists, the following relations hold:

$L = \lambda W$

$L_q = \lambda W_q$

$L_s = \lambda W_s$

These set of equations are called Little's queuing formula and these form the basis for the easy computation of W_q and W_s where the W^{0S} denote the time customers spend in the queue and system respectively.

It has been shown in 3.11 that by definition M/M/1 queuing system is given by;

$$L = \frac{\rho}{1-\rho}$$

Since $L = \lambda W_s$, it follows that

$$W = \frac{L}{\lambda}$$

Substituting for L in this equation gives

$$W = \frac{\rho}{\lambda(1-\rho)}$$

Now, substituting for $\rho = \frac{\lambda}{\mu}$, in the above equation, it follows that

$$W = \frac{1}{\mu - \lambda} \tag{3.17}$$

Which is the average time the customer spends in the system, similar arguments as above give the waiting time in the queue (W_q) as

$$W_q = \frac{\lambda}{\mu(\mu - \lambda)} \tag{3.18}$$

And waiting time in service (W_S) is

$$W_s = \frac{1}{\mu} \tag{3.19}$$

The quantities that have been derived are useful in calculating steadystate quantities (rate variables).

3.4 Effect of Arrival and Service Rates on Steady-State Variables

Recall that the single server exponential queuing system made use of the following assumptions:

1. The system has a single server and a single queue
2. Arrival and service times are exponential but the service times is independent (do not depend on the arrival process).
3. Furthermore, arrivals are defined to be non-overlapping and at most only one arrival can occur at any given instant.

3.5 Effect of Traffic Intensity on Number of Customers in the System

An important parameter in any queuing system is the traffic intensity also called the load or the utilization, defined as the ratio of the mean service time $E(X) = \frac{1}{\pi}$ over the mean inter-arrival time $E(t) = \frac{1}{\lambda}$

$$\rho = \frac{E(X)}{E(t)} \qquad = \frac{\lambda}{\mu}$$

Where λ and μ are the mean inter-arrival and service rate, respectively. Clearly, if $\rho > 1$ or $E(X) > E(t)$, which means that the mean service time is longer than the mean inter-arrival time, then the queue will grow indefinitely long for large t, because patients/customers/packets are arriving faster on average than they could be served. In this case ($\rho > 1$), the queuing system is unstable or will never reach a steady-state. The case where $\rho = 1$ is critical. In practice therefore, mostly situations where $\rho < 1$ are of interest. If $\rho < 1$, a steady state can be reached. These considerations are a direct consequence of the law of conservation of packets in the system.

Traffic intensity ρ is defined as

$$\rho = \frac{\lambda}{\mu} \tag{3.20}$$

Where λ, is the number of arrivals per unit time and μ, is the number of departures per unit time. Traffic intensity is the measure of the congestion of the system. Low traffic intensity means that fewer people are in the system, while high intensity means that more people are in the system. In general, as traffic intensity increases the corresponding queue length increases. In a single-server system, only one customer is served at a time. It follows therefore that, the expected number of customers in the queue is one less than the expected number of customers in the system. That is,

$$
\begin{aligned}
L_q &= L - L_s \\
&= \frac{\rho}{1-\rho} - \rho \\
&= \frac{\rho^2}{1-\rho} \\
&= \rho L
\end{aligned}
$$

As a result, the effect of traffic intensity on number of customers in the queue is the same as the effect on number of customers in the system.

3.6 Effect of Difference in Arrival and Service Rates on the Time a Customer Spends in the Queue

It has been shown in 3.18 that the waiting time in the queue, W_q, is given by

$$W_q = \frac{\lambda}{\mu(\mu-\lambda)}$$

Substituting for $\rho = \frac{\lambda}{\mu}$ in this equation gives

$$W_q = \rho\left(\frac{1}{\mu-\lambda}\right) = \rho W$$

Thus, the waiting time in the queue is directly proportional to the waiting time in the system. As a result, the effect of the difference in arrival and service rates on waiting time in the queue is almost the same as on the waiting time a customer spends in the system.

3.6.1 Illustration

A hospital is exploring the level of staffing needed for a booth at the local clinic where they would test and provide information on diabetes. Previous experience has shown that, on average, every 15 minutes a new person approaches the booth.

A nurse can complete testing and answering questions on average in 12 minutes. If there is a single nurse at the booth, calculate system performance measures including the probability of idle time and of one or two persons waiting in the queue. What happens to the utilization rate if another workstation and nurse are added to the unit?

Solution

Parameters:

Arrival rate: λ = 1(hour) ÷ 15 = 60(minutes) ÷ 15 = 4 persons per hour.

Service rate: μ = 1(hour) ÷ 12 = 60(minutes) ÷ 12 = 5 persons per hour.

Traffic intensity(ρ) is given by:

$\rho = \frac{\lambda}{\mu} = \frac{4}{5} = 0.8$ average persons served at any given time.

The average number of persons in a queue, with a single server is given by:

$L_q = \frac{\lambda^2}{\mu(\mu - \lambda)}$ $L_q = \frac{4^2}{5(5-4)} = \frac{16}{5}$ = 3.2 persons waiting in the queue

The average number of patients in the system is given by:

$L = L_q + \frac{\lambda}{\mu} = 3.2 \div 0.8$ = 4 persons .

The average time a patient spends waiting in the queue is given by

$W_q = \frac{L_q}{\lambda} = \frac{3.2}{4} = 0.1067 = 6.4$ minutes of waiting time in the queue.

The average time the patient spends waiting in queue and in service is

$W = W_q + \frac{1}{\mu} = 6.4 + \frac{60}{5} = 6.4 + 12 = 18.2$ minutes in the system (waiting and service).

Probability of zero, one and two persons in the queue is

$P_0 = 1 - \frac{\lambda}{\mu}$ where $\frac{\lambda}{\mu} = \rho$

$$P_1 = P_0 \left(\frac{\lambda}{\mu}\right)^1$$

$$P_2 = P_0 \left(\frac{\lambda}{\mu}\right)^2$$

$$\vdots$$

$$P_n = P_0 \left(\frac{\lambda}{\mu}\right)^n$$

$$P_n = \left(1 - \frac{\lambda}{\mu}\right) \left(\frac{\lambda}{\mu}\right)^n$$

where n = (0, 1, 2 ...)

$$P_0 = 1 - \frac{\lambda}{\mu} = 1 - \frac{4}{5} = 1 - 0.8 = 0.$$
2 or 20% probability of idle.

$$P_1 = P_0\left(\frac{\lambda}{\mu}\right)^1 = (0.2)\left(\frac{4}{5}\right)^1 = (0.2)(0.8)^1 = (0.2)(0.8) = 0.$$
16 or 16%

$$P_2 = P_0\left(\frac{\lambda}{\mu}\right)^2 = (0.2)\left(\frac{4}{5}\right)^2 = (0.2)(0.8)^2 = (0.2)(0.64) = 0.128 \text{ or } 12.$$
8%

Current system utilization is (s = 1) $\rho = \frac{\lambda}{\mu} = \frac{4}{1 \times 5}$= 80%

System utilization with an additional nurse is (s = 2) $\rho = \frac{\lambda}{\mu} = \frac{4}{2 \times 5}$= 40%

3.7 Birth-and-Death Processes

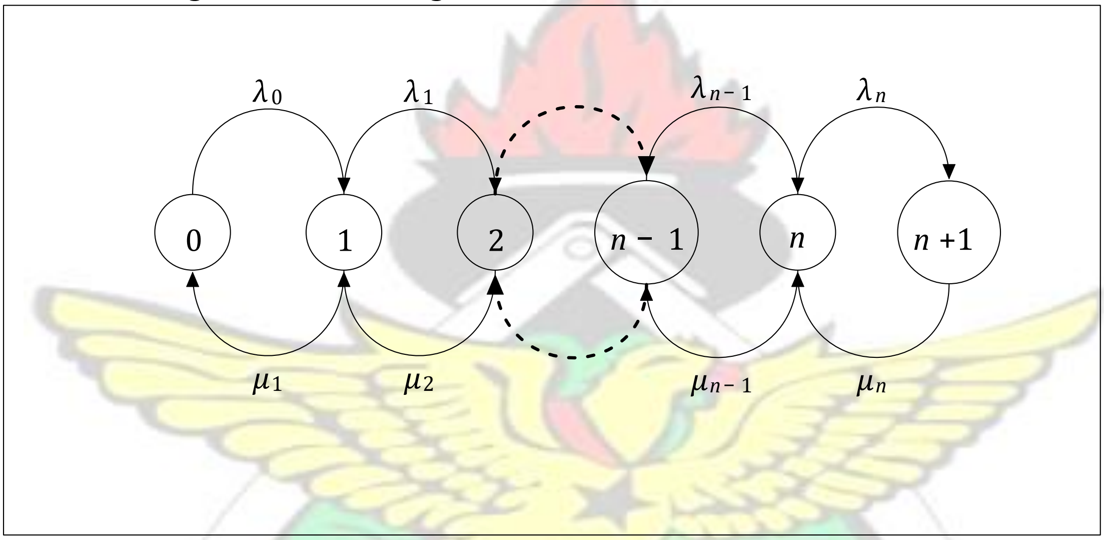
In the context of queuing theory (Hiller and Lieberman, 2005; Carter and Price, 2001), the term birth refers to the arrival of a new customer into the queuing system, and death refers to the departure of a served customer. Only one birth or death may occur at a time: therefore, transitions always occur to the ”next higher” or ”next lower” state. The rates at which births and deaths occur are prescribed precisely by the parameters of the exponential distributions that describe the arrival and service patterns. All the possible transitions can be illustrated in the rate diagram in figure 6. The state of the system at time t ($t \geq 0$), denoted by N (t), is the number of customers in the queuing system at time t. The birth-anddeath process describes probabilistically how N (t) changes as t increases. More precisely, the assumptions of the birth-and-death process are the followings:

Assumption 1: Given N (t) = n, the current probability distribution of the remaining time until next birth (arrival) is exponential with parameter λ_n (n = 0, 1, 2...).

Assumption 2: Given N (t) = n, the current probability distribution of the remaining time until the next death (service completion) is exponential with parameter (n = 1, 2 ...).

Assumption 3: The random variable of assumption 1 (the remaining time until the next birth) and random variable of assumption 2 (the remaining time until the next death) are mutually dependent. Furthermore, an arrival causes a transition from state n into sate n+1, and the completion of a service changes the system's state from n to n-1. No other transitions are considered possible. This birth-and-death process illustration is shown in the figure 3.1 leads directly to the formulas that measure the performance of this queuing system.

Figure 3.1: Rate Diagram for the Birth-and-Death Process



A fundamental flow in the birth-and-death process structure is a reliance on equilibrium between birth and death rates. This assumes the overall population shall remain constant at long run Tutunic and Newlands (2009). The approach is based on the rate-equality principle Medhi (2005) or balanced population model. Rate-Equality Principle states that the rate at which a process enters a state n (≥ 0) equals the rate which the process leaves that state n. In other words, the rate of entering and the rate of leaving a particular state are the same for every state. Rate in = rate out principle Medhi (2005). This principle implies that any state of the system can be expressed by an equation which is called the balance equation for state n (n = 0, 1, 2...), and mean entering rate = mean leaving rate.

One requirement for any steady-state analysis is that parameters of the system remain constant for the entire time period. In particular, the arrival rate must remain constant. Another requirement for steady-state analysis is that the system must be stable. Basically, this means that the servers must serve fast enough to keep up with arrivals; otherwise, the queue could theoretically grow without limit (Winston and Albright, 1997).

3.8 Other Queue System Models

Other queue system models such as single server - finite population model, multiple servers infinite population model (M/M/K/∞) and M/M/S/K model (s = 1) systems with finite waiting space are discussed in detail as a basis for evaluating their performance characteristics which may serve as useful guide for the study.

There are at least, 40 queuing models based on different queue management goals and service conditions (Weber, 2006). Kolker (2009) asserts that development of tractable analytic formulas is possible only if a flow of event in the system is a steady-state Poisson process where the average inter-arrival time assumes a Poisson distribution and service time is assumed to follow an exponential distribution. Based on the steady-state behaviour and performance notations, researchers have developed many different queuing models for different cases. Few among the queuing models used most often are:

3.8.1 M/M/1 Queue with Poisson Input and Exponential Service

An M/M/1 queue represents the queue length in a system having a single server, where arrivals are determined by a Poisson process and job service times have an exponential distribution. The model is the most elementary of queueing models Sturgul (2000). An M/M/1 queue property is the set {0, 1, 2, 3 ...} where the value corresponds to the number of customers in the system, including any currently in service.

3.8.1.1 Assumptions

1. Arrivals occur at rate λ according to a Poisson process and move the process from state i to $i + 1$

2. Service times have an exponential distribution with parameter $\frac{1}{\mu}$ in the M/M/1 queue, where μ is the mean service rate.

3. A single server serves customers one at a time from the front of the queue, according to a first-come, first-served discipline. When the service is complete the customer leaves the queue and the number of customers in the system reduces by one.

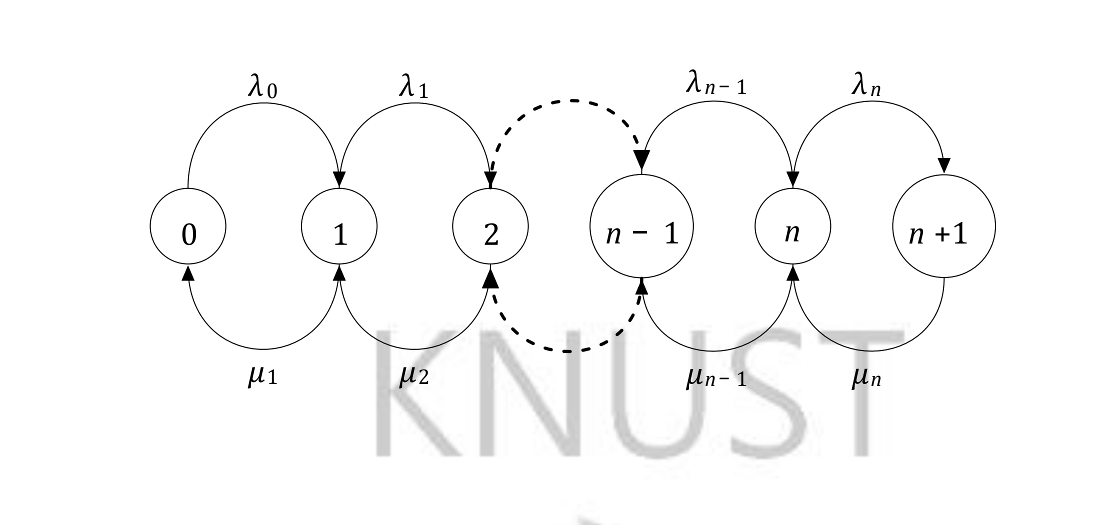
4. The buffer is of infinite size, so there is no limit on the number of customers it can contain.

5. Arrivals are served on a first-in, first-out (FIFO) basis, and every arrival waits to be served, regardless of the length of the line or queue.

3.8.1.2 Rate In = Rate Out Principle

For any state of the system n (n = 0, 1, 2, ...), mean entering rate = mean leaving rate. The equation expressing this principle is called the **balance equation** for state **n**. After constructing the balance equations for all the states in terms of the unknown P_n probabilities, we can solve this system of equations (plus any equation stating that the probabilities must sum to 1) to find these probabilities. Using rate-equality principle,the first equation for the type of system is determined:

Figure 3.2: Rate Diagram for the Birth-and-Death Process



$\lambda_0 P_0 = \mu_1 P_1$

To understand the above relationship, consider state 0. When in state 0, the process can leave this state only by an arrival. Since the arrival rate is λ_0 and the proportion of the time that the process is in state 0 is given by P_0, it follows that the rate at which the process leave state 0 is $\lambda_0 P_0$. On the other hand, state 0 can only be reached from state 1 via a departure. That is, if there is a single customer in the system and he completes service, then the system becomes empty. Since the service rate is μ_1 and the proportion of the time that the system has exactly one customer is P_1, it follows that the rate at which the process enters is $\mu_1 P_1$, the balance equations using this principle for any n can now be written as:
State rate at which the process leaves = rate which it enters Thus,

$\lambda_0 P_0 = \mu_1 P_1$

For every other state there are two possible transitions both into and out of the state. Therefore, each side of the balance equations for these states represents the sum of the mean rates for the two transitions involved. Otherwise, the reasoning is just the same as for state 0. These balance equations are summarized in table

3.1

Table 3.1: Balance equations for the birth-and-death process

State	Rate In = Rate Out
0	$\mu_1 P_1 = \lambda_0 P_0$
1	$\lambda_0 P_0 + \mu_2 P_2 = (\lambda_1 + \mu_1) P_1$
2	$\lambda_1 P_1 + \mu_3 P_3 = (\lambda_2 + \mu_2) P_2$
...	...
n-1	$\lambda_{n-2} P_{n-2} + \mu_n P_n = (\lambda_{n-1} + \mu_{n-1}) P_{n-1}$
n	$\lambda_{n-1} P_{n-1} + \mu_{n+1} P_{n+1} = (\lambda_n + \mu_n) P_n$
...	...

Applying this procedure yields the following results:

state:

$$
\begin{array}{lll}
0: & P_1 = \dfrac{\lambda_0}{\mu_1} P_0 & \\
1: & P_2 = \dfrac{\lambda_1}{\mu_2} P_1 + \dfrac{1}{\mu_2}(\mu_1 P_1 - \lambda_0 P_0) & = \dfrac{\lambda_1}{\mu_2} P_1 = \dfrac{\lambda_1 \lambda_0}{\mu_2 \mu_1} P_0 \\
2: & P_3 = \dfrac{\lambda_2}{\mu_3} P_2 + \dfrac{1}{\mu_3}(\mu_2 P_2 - \lambda_1 P_1) & = \dfrac{\lambda_2}{\mu_3} P_1 = \dfrac{\lambda_2 \lambda_1 \lambda_0}{\mu_3 \mu_2 \mu_1} P_0 \\
\vdots & \vdots & \vdots \\
n-1: & P_n = \dfrac{\lambda_{n-1}}{\mu_n} P_{n-1} + \dfrac{1}{\mu_n}(\mu_{n-1} P_{n-1} - \lambda_{n-2} P_{n-2}) & = \dfrac{\lambda_{n-1}}{\mu_n} P_{n-1} = \dfrac{\lambda_{n-1} \lambda_{n-2} \ldots \lambda_0}{\mu_n \mu_{n-1} \ldots \mu_1} P \\
n: & P_{n+1} = \dfrac{\lambda_n}{\mu_{n+1}} P_n + \dfrac{1}{\mu_{n+1}}(\mu_n P_n - \lambda_{n-1} P_{n-1}) & = \dfrac{\lambda_n}{\mu_{n+1}} P_n = \dfrac{\lambda_n \lambda_{n+1} \ldots \lambda_0}{\mu_{n+1} \mu_n \ldots \mu_1} P_0 \\
\vdots & \vdots &
\end{array}
$$

To simply notation, let

$C_n = \dfrac{\lambda_{n-1} \lambda_{n-2} \ldots \lambda_0}{\mu_n \mu_{n-1} \ldots \mu_1}$, for n = 1, 2 ...

and then define $C_n = 1$ for n = 0. This, study-state probabilities are

$P_n = C_n P_n$ for n = 0, 1, 2 ...

The requirement that

$$\sum_{n=0}^{\infty} P_n = 1$$

implies that

$$\left(\sum_{n=0}^{\infty} C_n\right) P_0 = 1$$

So that

$$P_0 = \left(\sum_{n=0}^{\infty} C_n\right)^{-1}$$

3.8.1.3 Summary of M/M/1 Queue Model Formulas

$$\rho = \frac{\lambda}{\mu} \qquad P_n = \rho^n(1-\rho) \qquad L = \frac{\lambda}{\mu - \lambda}$$

$$L_q = \frac{\lambda^2}{\mu(\mu-\lambda)} \qquad W = \frac{1}{\mu-\lambda} \qquad W_q = \frac{\lambda}{\mu(\mu-\lambda)}$$

$$P(n > N) = \left(\frac{\lambda}{\mu}\right)^{N+1}$$

3.8.2 Multiple-Server Model with Poisson Input and Exponential Service M/M/S (S>1)

The operating characteristics for the multiple-server model are based on the same assumptions as the single-server model-Poisson arrival rate, exponential service times infinite calling population and queue length and FIFO queue discipline. Also, recall that in the single-server model m > 1; however, in the multiple-server model,sm > 1, where s is the number of servers and m the number of customers.

For this case the service rate of the system is given by:

$cu \qquad \eta \geq c$

$\eta\mu \qquad \eta < c$

Thus, a multiple-server model is equivalent to a single-server system with service rate varying with η

$\lambda\eta = \lambda$ & $\mu\eta = \eta\mu$ $\eta < c$

Using the equality rate principle we have the following balance equations,

$-(\lambda_\eta + \mu_\eta)P_\eta + \mu_{\eta+1} + P_{\eta+1} + \lambda_{\eta-1}P_{\eta-1} = 0$

$-\lambda_0 P_0 + \mu_1 P_1 = 0$

$$P_1 = \frac{\lambda_0}{\mu_1}P_0$$

$$-(\lambda_1 + \mu_1)P_1 + \mu_2 P_2 + \lambda_0 P_0 = 0$$

$$P_2 = \frac{(\lambda_1 + \mu_1)P_1 - \lambda_0 P_0}{\mu_2}$$

$$P_2 = \frac{(\lambda_1 + \mu_1)P_1 - \mu_1}{\mu_2}$$

$$P_2 = \frac{\lambda_1 P_1}{\mu_2} = \frac{\lambda_1 \lambda_0}{\mu_1 \mu_2}P_0$$

$$P_2 = \frac{\lambda_0 \lambda_1}{\mu_1 \mu_2}P_0$$

$$P_\eta = \frac{\lambda_0 \lambda_1 .....\lambda_\eta}{\mu_1 \mu_2 .....\mu_\eta}P_0 \qquad \eta \geq 1$$

But

X

$P_\eta = 1$

Therefore,

$$P_\eta = \frac{\lambda^n}{\mu(2\mu)(3\mu).....(\eta\mu)}P_0 = \frac{\lambda^n}{\eta!\mu^n}P_0$$

$\eta \geq c$

$$P_n = \frac{\lambda^n}{\mu(2\mu)(3\mu).....(c-1)\mu(c\mu)(c\mu)(c\mu).....(c\mu)}P_0$$

$$P_n = \frac{\lambda^n}{c!c^{n-c}\mu^n}$$

if $\rho = \dfrac{\lambda}{\mu}$

$$\begin{cases} \dfrac{\rho^n}{n!}P_0 & 0 \le n \le c \\ \dfrac{\rho^n}{c^{n-c}c!}P_0 & n > c \end{cases}$$

To solve for P_0 we note that $\sum_{n=0}^{\infty} P_n = 1$. Hence,

$$\sum_{n=0}^{c-1}\frac{\rho^n}{n!}P_0 + \sum_{n=c}^{\infty}\frac{\rho}{c^{n-c}c!}P_0 = 1$$

$$P_0 = \frac{1}{\sum_{n=0}^{c-1}\frac{\rho^n}{n!} + \sum_{n=c}^{\infty}\frac{\rho^n}{c^{n-c}c!}}$$

$$P_0 = \frac{1}{\sum_{n=0}^{c-1}\frac{\rho^n}{n!} + \frac{\rho^c}{c!}\frac{1}{1-\frac{\rho}{c}}}$$

To solve for L_q

$$L_q = \sum_{n=0}^{c-1} 0P_n + \sum_{n=c}^{\infty}(n-c)P_n$$

$(k = n - c)$

$$L_q = \sum_{k=0}^{\infty} kP_{k+c} + \sum_{n=0}^{\infty} nP_{n+c}$$

To simplify,

$$L_q = \sum_{n=0}^{\infty} n\frac{\rho^{n+c}}{c^{n-c}c!}P_0$$

$$L_q = \frac{\rho^c}{c!}P_0\sum_{n=0}^{\infty} n\left(\frac{\rho}{c}\right)^n$$

$$L_q = \frac{\rho^c}{c!}P_0\frac{\rho}{c}\sum_{n=0}^{\infty} n\left(\frac{\rho}{c}\right)^{n-1}$$

$$L_q = \frac{\rho^c}{c!} P_0 \frac{\rho}{c} \sum_{n=0}^{\infty} \frac{d}{d\left(\frac{\rho}{c}\right)} \left(\frac{\rho}{c}\right)^n$$

$$L_q = \frac{\rho^c}{c!} P_0 \frac{\rho}{c} \frac{d}{d(\frac{\rho}{c})} \sum_{n=0}^{\infty} \left(\frac{\rho}{c}\right)^n$$

$$L_q = \frac{\rho^c}{c!} P_0 \frac{\rho}{c} \frac{d}{d(\frac{\rho}{c})} \left(\frac{1}{1-\frac{\rho}{c}}\right)$$

$$L_q = \frac{\rho^{c+1}}{c!c} \frac{1}{\left(1-\frac{\rho}{c}\right)^2} P_0$$

3.8.2.1 Summary of Multiple-Server M/M/S (S > 1) Formulas

$$P_0 = \frac{1}{\sum_{n=0}^{c-1} \frac{\rho^n}{n!} + \frac{\rho^c}{c!} \frac{1}{1-\frac{\rho}{c}}}$$

$$\begin{cases} \frac{\rho^n}{n!} P_0 & \frac{\rho}{c} < 1 \\ \frac{\rho^n}{c^{n-c} c!} P_0 & \end{cases}$$

$$L_q = \frac{\rho^{c+1}}{c!c} \frac{1}{\left(1-\frac{\rho}{c}\right)^2} P_0$$

3.8.3 Finite Source Model of the M/M/S Model

This model is similar to single channel Poisson arrival and service exponential rate except that the queue is finite,that is, when the total number of customers/patients in the system reaches the allowable limit, all arrivals balk or leave.

Let m = maximum number allowed in the system. The balance equations are obtained as follows;

$-\rho P_0 + P_1 = 0$ n = 0

$-(1+\rho)P_n + P_{n+1} + P_{n-1} = 0 \qquad 0 < n < m$

$-P_m + \rho P_{m-1} = 0 \qquad n = m$

Where n = 0 $P_1 = \rho P_0$

Where n = 1 $-(1 + \rho)P_1 + P_2 + \rho P_0 = 0$

$P_2 = (1 + \rho)P_1 - \rho P_0$

$= (1 + \rho)P_1 - P_0$

$P_2 = \rho P_1 = \rho 2P_0$

Where $n = m - 1$ $P_{m-1} + P_{m-1}.\rho = \rho_{m-1}P_0$

Where $n = m$ $P_m = \rho P_{m-1} = \rho^m P_0$

But

$$\sum_{n=0}^{m} P_n = 1$$

$P_0 + \rho P_0 + \rho 2P_0 + \ldots + \rho_{m-1}P_0 + \rho_m P_0 = 1$

$P_0[1 + \rho + \rho^2 + \ldots \rho_N] = 1$

$$P = \frac{1}{\sum_{n=0}^{m} \rho^n}$$

Where $\sum_{n=0}^{m} \rho^n$ is finite geometric series with sum

$$\frac{1 - \rho^{m+1}}{1 - \rho}$$ if ρ 6= 1

Therefore;

$$P_0 = \frac{1 - \rho}{1 = \rho^{m+1}}$$

Now;

$P_1 = \rho P_0$

$P_2 = \rho 2P_0$

...

$P_m = \rho_m P_0$

Summary

This chapter so far has explored several queueing models with much emphasis on multi-server single queue model which is basis for this study. In the next chapter, performance parameters such as the arrival time and service system capacities of KATH polyclinic are evaluated.

Chapter 4

DATA ANALYSIS

This chapter looks at the analysis of the data collected during the period of the study. Calculation of various performance parameters discussed in chapter three are presented in this chapter. Tables would be used to show the data collected and the findings used to answer the research objectives that were formulated in Chapter one.

4.1 Sources of Data Collection

For the purpose of obtaining the required data for this study, both primary and secondary data sources were utilized.

4.1.1 Primary Data

Primary data were collected by (i) obtaining raw data from the hospitals' database, (ii) observing the patient flow at the outpatient department and the entire hospital (iii) interviewing concerned participants from different units of the hospital to capture comprehensive information needed to construct patient flow, (iv) structured questionnaire was equally used to obtain data on factors which affect outpatient department efficiency in terms of patient flow. This method of data collection is considered important to validate the historical data and highlight the other factors affecting the efficiency of health delivery system of the Polyclinic. The semi-closed ended questionnaire was designed to obtain the opinions of health professionals and administrators about suitability of queuing model to address problems confronting the various units. These concerned individuals include; (a) Medical record officers, (b) administrative staff, (c) doctors and other nurses. These individuals were consulted because of the complexity of a hospital as well as the domain expertise required to define the model.

4.1.2 Secondary Data

Patient data were collected from the database of the outpatient department. Several administrative data files were consulted to obtain data on dates and sources of referrals, admission or diversion. Also, important data such as number of facilities available at the facility were collected and subsequently validated by interviewing the departmental staff. The patient flow was quantified using this data. Existing data were also obtained from the medical records office reports. The average number patient arrivals per day were computed from the secondary data.

With the help of 3 research assistances a stop watch was used to calculate the number of minutes spent by each patient from the records section where patients collected their folders through to the last section which is the pharmacy where they finally leave. Data were collected from Monday to Friday from the hours of 8:00 am to 12:00 noon in March 2015. The number of patients who arrived from each section was taken.

4.2 Distribution of Respondents

The study was conducted at the Outpatient department of KATH Polyclinic. The Polyclinic has three separate units comprising of two records unit, one assessment center and three consulting rooms. Observations were made during the last week of the month of March 2015 from Monday to Friday between the hours of 8:00 am to 12:00 noon concerning the number of patients that arrived and attended to at the three centers are shown in table 4.1.

Table 4.1: **Number and Time of Patients arrival on Monday at the Records Section**

Number of Patients	1	2	3	4	5	6	7	8	9	10
Arrival rate in Mins.	3	5	8	9	12	18	22	35	28	30
Number of,Patients	11	12	13	14	15	16	17	18	19	20
Arrival rate in Mins.	31	36	41	44	47	49	51	54	55	59

4.3 Analysis of Data Collected on Monday at the Records Section

On Monday twenty patients arrived at the records section within one hour. The data gathered is shown in table 4.1.

4.4 Calculation of Mean Arrival Time

The inter-arrival time is the amount of time between the arrival of one patient and the arrival of the next patient. It is calculated for each customer after the first and is often averaged to get the mean inter-arrival time, represented by lambda (λ).

(5 - 3) + (8 - 5) + (9 - 8) +(12 - 9)+ (18 - 12) + (22 - 18) + (25 - 22) + (28 - 25) + (30 - 28) + (31 - 30) + (36 - 31) + (41 - 36) + (44 - 41) + (47 - 44) + (49 - 47) + (51 - 49) + (54 - 51) + (55 - 54) + (59 - 55)

Inter Arrival time = 2 + 3 + 1 + 3 + 6 + 4 + 3 + 3 + 2 + 1 + 5 + 5 + 3 + 3 + 2 +2 + 3 + 1 + 4 = 56 minutes

1. The mean arrival rate $\lambda = \frac{56}{20}$= 2.8 minutes

4.5 Calculation of Performance Parameters on Monday at the Records Section

The mean service rate is the amount of time between the service of one patient and the service of the next patient. It is often averaged to get the mean service rate, represented by Mu, (μ).

Table 4.2: **Number and Time Patients were served at the Records Section on Monday**

No. Of patients	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
Service Time in minutes	6	12	17	22	28	31	39	46	55	61	68	77	84	92	99	108

(12 - 6) + (17 - 12) + (22 - 17) + (28 - 22) + (31 - 28) + (39 - 31) + (46 - 39)

+ (55 - 46) + (61 - 55) + (68 - 61) + (77 - 68) + (84 - 77) + (92 - 84) + (99 92) +

(108 - 99)

Inter service rate = 6 + 5 + 5 + 6 + 3 + 8 + 7 + 9 + 6 + 7 + 9 + 7 + 8 + 7

+ 9 = 102 minutes

2. Mean service rate $\mu = \frac{102}{16}$ = 6.38 minutes.
3. Number of servers k = 2
4. Mean combined rate of all servers = $k\mu$ = 2(6.38) = 12.76
5. Utilization factor of the entire system = $\rho = \frac{\lambda}{k\mu} = \frac{2.8}{12.76}$ = 0.22 or 22%
6. The probability that there are no patients in the system (all servers are idle)

is

$$P_0 = \left| \left[\sum_{n=0}^{k-1} \frac{1}{n!} \left(\frac{\lambda}{\mu} \right)^n \right] + \frac{1}{K!} \left(\frac{\lambda}{\mu} \right)^k \left(\frac{K\mu}{K(\mu - \lambda)} \right) \right|^{-1}$$

$$= \left| 1 + \frac{2.8}{6.38} + \frac{1}{2} \left(\frac{2.8}{6.38} \right)^2 \frac{2(6.38)}{2(6.38 - 2.8)} \right|^{-1}$$

$$= \left[1 + \frac{2.8}{6.38} + \frac{1}{2}(0.19) \left(\frac{12.76}{7.16} \right) \right]^{-1} = 0.62$$

62% Probability of zero patients in the system

(7). The expected number of patients in the waiting line

$$L_q = \frac{\left(\frac{\lambda}{\mu} \right)^k}{k!(1 - P)^2} (P_0)$$

$$= \frac{\left(\frac{2.8}{6.38}\right)^2}{2!(1-0.22)^2}(0.62)$$

$$= \frac{\left(\frac{2.8}{6.38}\right)^2}{2(1-0.22)^2}(0.62) = 0.098$$

(8). The expected number of patients in the system,

$$L_s = L_q + \frac{\lambda}{\mu}$$

$$= 0.098 + \frac{2.8}{6.38} = 0.54$$

(9). The expected waiting time in the queue

$$W_q = \frac{L_q}{\lambda}$$

$$W_q = \frac{0.098}{2.8} = 0.035 \text{ minutes}$$

= 0.035 minutes average time a patient spends in the queue waiting

(10). The expected time a patient spends in the system,

$$W_s = W_q + \frac{1}{\lambda}$$

$$= 0.035 + \frac{1}{2.8} = 0.392 \text{ minutes}$$

= 0.392 minutes average time a patient spends in the system

4.6 Analysis of Data Collected on Monday at the Assessment Centre

Sixteen (16) patients arrived at the assessment centre within a period of fifty (50) minutes. The data gathered is shown in table 4.3.

Table 4.3: **Number and Time of patients that arrived on Monday at the Assessment Centre**

Number of Patients	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16

Service Time In Minutes	8	10	13	15	21	24	26	29	31	33	34	37	39	41	45	49

4.7 Calculation of Performance Parameters on Monday at the Assessment Centre

(10 - 8) + (13 - 10) + (15 - 13) + (21 - 15) + (24 - 21) + (26 - 24) + (29 - 26) + (31 - 29)

(33 - 31) + (34 - 33) + (37 - 34) + (39 - 37) + (41 - 39) + (45 - 41) + (49 - 45) Inter arrival time = 2 + 3 + 2 + 6 + 3 + 2 + 3 + 2 + 2 + 1 + 3 + 2 + 2 + 4 + 4 = 41

1. Mean arrival rate $\lambda = \frac{41}{16}$= 2.56 minutes

Table 4.4: **Number and Time of patients who were served on Monday at the Assessment Centre**

Number of Patients	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Service Time In Minutes	2	4	5	7	9	11	15	18	19	22	26	29	33	39	42

(4-2)+(5-4)+(7-5)+(9-7)+(11-9)+(15-11)+(18-15)+(19-18)+(22-19)+(26-22)+(29-26)+ (33-29)+ (39-33) + (42-39)

Inter mean service rate =2 + 1 + 2 + 2 + 2 + 4 + 3 + 1 + 3 + 4 + 3 + 4 + 6 + 3 = 40 minutes

2. Mean service rate $\mu = \frac{40}{15}$= 2.67 minutes.
3. Number of servers = 1
4. Mean combined rate of all servers = $k\mu$ = 1(2.67) = 2.67
5. Utilization factor of the entire system = $\rho = \frac{\lambda}{k\mu} = \frac{2.56}{2.67}$ = 0.96 or 96%
6. The probability that there are no patients in the system (all servers are idle) is

$$P_0 = \left| \left[\sum_{n=0}^{k-1} \frac{1}{n!} \left(\frac{\lambda}{\mu} \right)^n \right] + \frac{1}{K!} \left(\frac{\lambda}{\mu} \right)^k \left(\frac{K\mu}{K(\mu - \lambda)} \right) \right|^{-1}$$

$$= \left| \left[\sum_{n=0}^{k-1} \frac{1}{1!} \left(\frac{2.56}{2.67} \right)^0 \right] + \frac{1}{1!} \left(\frac{2.56}{2.67} \right)^1 \left(\frac{2.67}{1(2.67 - 2.56)} \right) \right|^{-1}$$

$$= \left[1 + 1 \left(\frac{2.56}{2.67} \right) \left(\frac{2.67}{0.11} \right) \right]^{-1} \quad = \quad 0.04$$

4% Probability of zero patients in the system

(7). The expected number of patients in the waiting line

$$L_q = \frac{\left(\frac{\lambda}{\mu} \right)^k}{k!(1-P)^2} (P_0)$$

$$= \frac{\left(\frac{2.56}{2.67} \right)^2}{1!(1-0.22)^2} (0.62)$$

$$= \frac{\left(\frac{2.8}{2.67} \right)^1}{1(1-0.96)^2} (0.04)$$

$$= \frac{09588}{0.0016} (0.04) = 23.97$$

(8). The expected number of patients in the system,

$$L_s = L_q + \frac{\lambda}{\mu}$$

$$= 23.97 + \frac{2.56}{2.67} = 24.93$$

(9). The expected waiting time in the queue

$$W_q = \frac{L_q}{\lambda}$$

$$W_q = \frac{23.97}{2.56} = 9.$$ 36 minutes

= 9.36 minutes average time a patient spends in the queue waiting

(10). The expected time a patient spends in the system,

$$W_s = W_q + \frac{1}{\lambda}$$

$= 9.36 + \frac{1}{2.56} = 9.$75 minutes

= 9.75 minutes average time a patient spends in the system

4.8 Analysis of Data Collected on Monday at the Consulting Rooms

Ten (10) patients arrived at the consultation section within a period of thirty (30) minutes. The number of patients and the time they reported at the consultation centre is shown in table 4.5

Table 4.5: **Number and Time of patients that arrived on Monday at the Consulting Rooms**

Number of Patients	1	2	3	4	5	6	7	8	9	10
Arrival Rate In Minutes	5	7	10	12	14	17	21	24	29	33

(7 - 5) + (10 - 7) + (12 - 10) + (14 - 12) + (17 - 14) + (21 - 17) + (24 - 21) + (29-24) + (33 - 29)

Inter arrival rate = 2 + 3 + 2 + 2 + 3 + 4 + 3 + 5 + 4 = 28 minutes

1. Mean arrival rate $\lambda = \frac{28}{10}$= 2.8 minutes per patient

Table 4.6: **Number and Time Patients were served at the Consulting Rooms on Monday**

Number Of Patients	Service Time In Minutes		
	Consulting Room 1	Consulting Room 2	Consulting Room 3
1	3	3	5
2	9	10	15
3	16	19	21
4	23	23	25
5	29	28	32
6	36	34	35
7	42	44	45
8	50	51	52
9	59	60	61

10	68	67	74

4.9 Calculation of Performance Parameters on Monday at the Consulting Rooms

Inter service rate (consulting room 1) = (9 - 3)+(16 - 9)+(23 - 16)+(29 - 23)+(36 - 29)+(42 - 36)+ (50 - 42)+ (59 - 50) + (68 - 59)

= 6 + 7 + 7 + 6 + 7 + 6 + 8 + 9 + 9 = 65 minutes

Mean service rate (consulting room 1) = $\mu = \frac{65}{10}$= 6.5 minutes.

Inter service rate (consulting room 2) = (10 - 3)+(19 - 10)+(23 - 19)+(28 - 23)+(34 - 28)+(44 - 34)+ (51 - 44)+ (60 - 51) + (67 - 60) 7 + 9 + 4 + 5 + 6 + 1 0 + 7 + 9 + 7 = 64 minutes

Mean service rate (consulting room 2) = $\mu = \frac{64}{10}$= 6.4 minutes.

inter service rate (consulting room 3) = (15 - 5)+(21 - 15)+(25 - 21)+(32 - 25)+(35 - 32)+(45 - 35)+ (52 - 45)+ (61 - 52) + (74 - 61)

10 + 6 + 4 + 7 + 3 + 1 0 + 7 + 9 + 13 = 69 minutes

Mean service rate (consulting room 3) = $\mu = \frac{69}{10}$= 6.9 minutes.

2. Mean service rate of the 3 consulting rooms = $\frac{(6.5 + 6.4 + 6.9)}{3} = \frac{19.8}{3} = 6.6$
3. Number of servers k = 3
4. Mean combined rate of all servers = $k\mu$ = 3(6.6) = 19.8
5. Utilization factor of the entire system = $\rho = \frac{\lambda}{k\mu} = \frac{2.8}{19.8}$ = 0.14 or 14 %
6. The probability that there are no patients in the system (all servers are idle) is

$$P_0 = \left| \left[\sum_{n=0}^{k-1} \frac{1}{n!} \left(\frac{\lambda}{\mu}\right)^n \right] + \frac{1}{K!} \left(\frac{\lambda}{\mu}\right)^k \left(\frac{K\mu}{K(\mu - \lambda)}\right) \right|^{-1}$$

$$= \left| 1 + \left(\frac{2.8}{6.6}\right) + \left(\frac{2.8}{6.6}\right)^2 + \frac{1}{3}\left(\frac{2.8}{6.6}\right)^3 \frac{3(6.6)}{3(6.6 - 2.8)} \right|^{-1}$$

$$= \left| 1 + \left(\frac{2.8}{6.6}\right) + \left(\frac{196}{1089}\right) + \frac{1}{3}\left(\frac{2744}{35937}\right)\left(\frac{19.8}{11.4}\right) \right|^{-1} = 0.61$$

Hence, the Probability of zero patients in the system = 61%

(7). The expected number of patients in the waiting line

$$L_q = \frac{\left(\frac{\lambda}{\mu}\right)^k}{k!(1-\rho)^2}(P_0)$$

$$= \frac{\left(\frac{2.8}{6.6}\right)^3}{3!(1-0.14)^2}(0.61)$$

$$= \left(\frac{0.076}{4.44}\right)(0.61) = 0.010$$

(8). The expected number of patients in the system,

$$L_s = L_q + \frac{\lambda}{\mu}$$

$$= 0.010 + \frac{2.8}{6.6} = 0.43$$

(9). The expected waiting time in the queue

$$W_q = \frac{L_q}{\lambda}$$

$W_q = \frac{0.010}{2.8} = 0.0036$ minutes

= 0.0036 minutes average time a patient spends in the queue waiting

(10). The expected time a patient spends in the system,

$$W_s = W_q + \frac{1}{\lambda}$$

$= 0.0036 + \frac{1}{2.8} = 0.36$ minutes

= 0.36 minutes average time a patient spends in the system

Table 4.7: **Summary of Performance Parameters at All the Various Sections on Monday**

Performance Rate	Records Section	Assessment Center	Consulting Room
Mean arrival Rate λ	2.8 mins / pnt	2.56 mins/pnt	2.8 mins/pnt
Mean service rate μ	6.38 mins/pnt	2.67 mins/pnt	6.6 mins/pnt

Mean combined rate of all servers kμ	12.76 minutes	2.67 minutes	19.8 minutes
Utilization factor of the entire system *rho*	22%	96%	14%
Probability of zero patients in the system P_0	62%	4%	61%
Average number of patients in the queue L_q	0.10	24	0.010
Average number of patients in the system,L_s	0.54	25	0.43
Average waiting time in the queue,W_q	0.0346 minutes	9.74 minutes	0.0036 minutes
Average time a patient spends in the system W_s	0.392 minutes	10.13 minutes	0.36 minutes

It can be seen in from table 4.7 that the assessment center was the busiest of all the sections. Its utilization factor is 96%. The next busiest section is the records sections having a utilization rate of 22%. The least busy section is the consulting rooms with a utilization rate of 14%. The Table also shows that the assessment centre has more patients waiting in the queue and the system than that of the consulting rooms and the record section. Also, the number of minutes a person spends before his or her biological data is taken from the assessment centre is far more than that of the records section and the consulting rooms. On the average, a patient spends about 9.36 minutes in the queue and 9.75 minutes in the system.

4.10 Analysis of Data Collected On Tuesday at the Records Section

Sixteen (16) patients arrived at the Records section within an hour. The number of patients and time they arrived is shown in table 4.8.

Table 4.8: **Number and Time of Patients that Arrived on Tuesday at the Records Section**

Number of Patients	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16

Arrival Rate In Minutes	2	6	10	16	21	25	29	32	34	38	41	46	49	52	56	60

Inter arrival rate = (6 - 2) + (10 - 6) + (16 - 10) + (21 - 16) + (25 - 21) + (29 - 25) + (32 - 29) + (34 - 32) + (38 - 34) + (41 - 38) + (46 - 41) + (49 - 46) + (52 - 49) + (56 - 52) + (60 - 56)

4 + 4 + 6 + 5 + 4 + 4 + 3 + 2 + 4 + 3 + 5 + 3 + 3 + 4 + 4 = 58

1. Mean arrival rate $\lambda = \frac{58}{16}$= 3.63 minutes per patient

Table 4.9: **Number and Time patients were served at the Records Section on Tuesday**

Number of Patients	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
Service Time In Minutes	5	11	15	22	27	35	40	47	52	59	65	71	78	83	89	95

4.11 Calculation of Performance Parameters on Tuesday at the Records Section

(11 - 5) + (15 - 11) + (22 - 15) + (27 - 22) + (35 - 27) +(40 - 35) + (47 - 40) + (52 - 47) + (59 - 52) + (65 - 59) + (71-65) + (78 - 71) + (83 - 78) + (89 - 83) + (95 - 89)

Inter service rate = 6 + 4 + 7 + 5 + 8 + 5 + 7 + 5 + 7 + 6 + 6 + 7 + 5 + 6 + 6 = 90

2. Mean service rate = $\mu = \frac{90}{16} = 5.$ 63 minutes per patients
3. Number of servers = 2
4. Mean combined rate of all servers $k\mu$ = 2(5.56) = 11.25 minutes
5. Utilization factor of the entire system = $\rho = \frac{\lambda}{k\mu} = \frac{3.63}{11.28}$ = 0.32 or 32%
6. The probability that there are no patients in the system (all servers are idle) is

$$P_0 = \left| \left[\sum_{n=0}^{k-1} \frac{1}{n!} \left(\frac{\lambda}{\mu} \right)^n \right] + \frac{1}{K!} \left(\frac{\lambda}{\mu} \right)^k \left(\frac{K\mu}{K(\mu - \lambda)} \right) \right|^{-1}$$

$$= \left|1 + \frac{3.63}{5.63} + \frac{1}{2}\left(\frac{3.63}{5.63}\right)^2 \frac{2(5.63)}{2(5.63-3.63)}\right|^{-1}$$

$$= \left[1 + \frac{3.63}{5.63} + \frac{1}{2}(0.42)\left(\frac{11.26}{4}\right)\right]^{-1} \quad = \quad 0.45$$

45% Probability of zero patients in the system

(7). The expected number of patients in the waiting line

$$L_q = \frac{\left(\frac{\lambda}{\mu}\right)^k}{k!(1-P)^2}(P_0)$$

$$= \frac{\left(\frac{3.63}{5.63}\right)^2}{2!(1-0.32)^2}(0.45)$$

$$= \frac{\left(\frac{3.63}{5.63}\right)^2}{2(1-0.32)^2}(0.45) \quad = 0.21$$

(8). The expected number of patients in the system,

$$L_s = L_q + \frac{\lambda}{\mu}$$

$$= 0.21 + \frac{3.63}{5.63} = 0.85$$

(9). The expected waiting time in the queue

$$W_q = \frac{L_q}{\lambda}$$

$$W_q = \frac{0.21}{3.63} = 0.06 \text{ minutes}$$

= 0.06 minutes average time a patient spends in the queue waiting

(10). The expected time a patient spends in the system,

$$W_s = W_q + \frac{1}{\lambda}$$

$$= 0.06 + \frac{1}{3.63} = 0.34 \text{ minutes}$$

= 0.34 minutes average time a patient spends in the system

4.12 Analysis of Data Collected on Tuesday at the Assessment Centre

Sixteen (16) patients arrived at the assessment centre for their biological data to be taken within a period of eighty (80) minutes. The number of patient and time they arrived is shown in table 4.10.

(9 - 4) + (15 - 9) + (19 - 15) + (26 - 19) + (31 - 26) + (37 - 31) + (42 - 37) +

(47 - 42) + (51 - 47) + (55 - 51) + (59 - 55) + (65 - 59) + (70-65) + (76-70) + Table 4.10: **Number and Time of patients that arrived on Tuesday at the Assessment Centre**

Number of Patients	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
Service Time In Minutes	4	9	15	19	26	31	37	42	47	51	55	59	65	70	76	79

(79-76)

Inter arrival rate = 5 + 6 + 4 + 7 + 5 + 6 + 5 + 5 + 4 + 4 + 4+ 6 + 5 + 6 +

3 = 75

1. The mean arrival rate $\lambda = \frac{75}{16}$= 4.69 minutes per patients

Table 4.11: **Number and Time of patients were served on Tuesday at the Assessment Centre**

Number of Patients	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
Service Time In Minutes	7	11	18	26	34	40	47	54	59	66	70	76	81	88	94	103

4.13 Calculation of Performance Parameters on Tuesday at the Assessment Center

(11 - 7) + (18 - 11) + (26 - 18) + (34 - 26) + (40 - 34) + (47 - 40) + (54 - 47)

+ (59 - 54) + (66 - 59) + (70 - 66) + (76 - 70) + (81 - 76) + (88 - 81) + (94 88) + (103 - 94)

Inter mean service rate = 4 + 7 + 8 + 8 + 6 + 7 + 7 + 5 + 7 + 4 + 6 + 5 + 7 + 6 + 9 = 96

2. Mean service rate $\mu = \frac{96}{16}$= 6.00 minutes per patients.
3. Number of servers = 1
3. Mean combined rate of all servers = $k\mu$ = 1(6.00) = 6.00 mins
4. Utilization factor of the entire system = $\rho = \frac{\lambda}{k\mu} = \frac{4.69}{6.00}$ = 0.78 or 78%
5. The probability that there are no patients in the system (all servers are idle) is

$$P_0 = \left| \left[\sum_{n=0}^{k-1} \frac{1}{n!} \left(\frac{\lambda}{\mu}\right)^n \right] + \frac{1}{K!} \left(\frac{\lambda}{\mu}\right)^k \left(\frac{K\mu}{K(\mu - \lambda)}\right) \right|^{-1}$$

$$= \left| \left[\sum_{n=0}^{k-1} \frac{1}{1!} \left(\frac{4.69}{6.00}\right)^0 \right] + \frac{1}{1!} \left(\frac{4.69}{6.00}\right)^1 \left(\frac{6.00}{1(6.00 - 4.69)}\right) \right|^{-1}$$

$$= \left[1 + 1\left(\frac{4.69}{6.00}\right)\left(\frac{6.00}{1.31}\right)\right]^{-1} = 0.22$$

22% Probability of zero patients in the system

(7). The expected number of patients in the waiting line

$$L_q = \frac{\left(\frac{\lambda}{\mu}\right)^k}{k!(1 - P)^2}(P_0)$$

$$= \frac{\left(\frac{0.69}{6.00}\right)^1}{1!(1 - 0.78)^2}(0.22)$$

$$= \frac{\left(\frac{4.69}{6.00}\right)^1}{1(1 - 0.78)^2}(0.22)$$

$$= \frac{0.782}{0.0484}(0.22) = 3.43$$

(8). The expected number of patients in the system,

$$L_s = L_q + \frac{\lambda}{\mu}$$

$$= 3.43 + \frac{4.69}{6.00} = 4.21$$

(9). The expected waiting time in the queue

$$W_q = \frac{L_q}{\lambda}$$

$$W_q = \frac{3.43}{4.69} = 0.73 \text{ minutes}$$

= 0.73 minutes average time a patient spends in the queue waiting

(10). The expected time a patient spends in the system,

$$W_s = W_q + \frac{1}{\lambda}$$

$$= 0.73 + \frac{1}{4.69} = 0.94 \text{ minutes}$$

= 0.94 minutes average time a patient spends in the system

4.14 Analysis of Data Collected on Tuesday at the Consultation Centres

Sixteen (16) patients reported at the consultation centers within a period of seventy-five minutes. Table 4.12 shows the number of patients and the time they took to arrive at the centre.

Table 4.12: **Number and Time Patients arrived on Tuesday at the Consulting Rooms**

Number of Patients	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
Arrival Rate In Minutes	4	7	11	15	18	22	27	31	36	42	47	53	61	66	69	75

(7-4)+(11-7)+(15-11)+(18-15)+(22-18)+(27-22)+(31-27)+(36-31)+(42-36)+(47-

42)+ (53-47)+ (61-53) + (66-61) + (69-66) + (75-69) Inter arrival rates: 3 + 4

+ 4 + 3 + 4 + 5 + 4 + 5 + 6 + 5 + 6 + 8 + 5 + 3 + 6 = 71 minutes

1. Mean arrival rate $\lambda = \frac{71}{16}$= 4.44 minutes per patient

Table 4.13: **Number and Time Patients were served at the Consulting Rooms on Tuesday**

Number Of Patients	Service Time In Minutes		
	Consulting Room 1	Consulting Room 2	Consulting Room 3
1	5	3	5
2	15	10	15
3	21	19	21
4	25	23	25
5	29	28	32
6	33	34	35
7	37	44	45
8	41	51	52
9	47	60	61
10	52	67	74
11	55	69	78
12	60	74	85
13	67	79	90
14	70	84	93
15	76	87	97
16	81	91	101

4.15 Calculation of Performance Parameters on Tuesday at the Consulting Rooms

Inter service rate (consulting room 1) = (15 - 5) + (21 - 15) + (25 - 21) + (29 - 25) + (33 - 29) + (37 - 33) + (41 - 37) + (47 - 41) + (52 - 47) + (55 - 52) + (60 - 55) + (67 - 60) + (70 - 67) + (76 - 70) + (81 - 76)

10 + 6 + 4 + 4 + 4 + 4 + 4 + 6 + 5 + 3 + 5 + 7 + 3 + 6 + 5 = 76 minutes

Mean service rate (consulting room 1)= $\mu = \frac{76}{16}$= 4.75 minutes per patient

Inter service rate (consulting room 2) = (10 - 3) + (19 - 10) + (23 - 19) + (28 - 23) + (34 - 28) + (44 - 34) + (51 - 44) + (60 - 51) + (67 - 60) + (69 - 67) + (74 - 69) + (79 - 74) + (84 - 79) + (87 - 84) + (91 - 87)

7 + 9 + 4 + 5 + 6 + 1 0 + 7 + 9 + 7 + 2 + 5 + 5 + 5 + 3 + 4 = 88

Mean service rate (consulting room 2) = $\mu = \frac{88}{16}$= 5.50 minutes per patient

Inter service rate (consulting room 3) = (15 - 5) + (21 - 15) + (25 - 21) + (32 - 25) + (35 - 32) + (45 - 35) +(52 - 45) + (61 - 52) + (74 - 61) + (78 - 74) + (85 - 78) + (90 - 85) + (93 - 90) + (97 - 93) + (101 - 97)

10 + 6 + 4 + 7 + 3 + 10 + 7 + 9 + 13 +4 + 7 + 5 + 3 + 4 + 4 = 96 minutes

Mean service rate (consulting room 3) = $\mu = \frac{96}{16}$= 6.00 minutes per patient.

2. Mean service rate of the 3 consulting rooms = $\frac{(4.75 + 5.50 + 6.00)}{3} = \frac{16.28}{3} =$ 5.42 minutes per patient

3. Number of servers k = 3

4. Mean combined rate of all servers = $k\mu$ = 3(5.42) = 16.26 minutes

5. Utilization factor of the entire system = $\rho = \frac{\lambda}{k\mu} = \frac{4.44}{16.26}$ = 0.27 or 27 %

6. The probability that there are no patients in the system (all servers are idle) is

$$P_0 = \left| \left[\sum_{n=0}^{k-1} \frac{1}{n!} \left(\frac{\lambda}{\mu} \right)^n \right] + \frac{1}{K!} \left(\frac{\lambda}{\mu} \right)^k \left(\frac{K\mu}{K(\mu - \lambda)} \right) \right|^{-1}$$

$$= \left| 1 + \left(\frac{4.44}{5.42} \right) + \left(\frac{4.44}{5.42} \right)^2 + \frac{1}{3} \left(\frac{4.44}{5.42} \right)^3 \frac{3(5.42)}{3(5.42 - 4.44)} \right|^{-1}$$

$$= \left| 1 + \left(\frac{4.44}{5.42} \right) + (0.67) + \frac{1}{3}(0.55) \left(\frac{16.26}{2.94} \right) \right|^{-1} = 0.29$$

Hence, the Probability of zero patients in the system = 29%

(7). The expected number of patients in the waiting line

$$L_q = \frac{\left(\frac{\lambda}{\mu} \right)^k}{k!(1-\rho)^2} (P_0)$$

$$= \frac{\left(\frac{4.44}{5.42} \right)^3}{3!(1-0.27)^2} (0.29)$$

$$= \left(\frac{0.55}{3.20} \right) (0.29) = 0.05$$

(8). The expected number of patients in the system,

$$L_s = L_q + \frac{\lambda}{\mu}$$

$$= 0.05 + \frac{4.44}{5.42} = 0.87$$

(9). The expected waiting time in the queue

$$W_q = \frac{L_q}{\lambda}$$

$$W_q = \frac{0.05}{4.44} = 0.011 \text{ minutes}$$

= 0.01 minutes average time a patient spends in the queue waiting

(10). The expected time a patient spends in the system,

$$W_s = W_q + \frac{1}{\lambda}$$

$$= 0.011 + \frac{1}{4.44} = 0.24 \text{ minutes}$$

= 0.24 minutes average time a patient spends in the system

Table 4.14: **Summary of Performance Parameters at All the Various Sections on Tuesday**

Performance Rate	Records Section	Assessment Center	Consulting Room
Mean arrival Rate λ	3.63 mins / pnt	4.69 mins / pnt	4.44 mins / pnt
Mean service rate μ	5.63 mins/patient	6.00 mins/pnt	5.42 mins/pnt
Mean combined rate of all servers kμ	11.25 minutes	6 minutes	16.26 minutes
Utilization factor of the entire system *rho*	32%	78%	27%
Probability of zero patients in the system P_0	45%	22%	29%
Average number of patients in the queue L_q	0.21	3.43	0.05
Average number of patients in the system, L_s	0.85	4.21	0.87
Average waiting time in the queue, W_q	0.06 minutes	0.73 minutes	0.01 minutes
Average time a patient spends in the system W_s	0.34 minutes	0.94 minutes	0.24 minutes

Analysis in table 4.14 shows that the assessment center was the busiest of all the sections. Its utilization factor is 78% with just a mean combined rate of 6 minutes per patient. The next busiest section is the records sections having a utilization rate of 32% and a mean combined rate of 11.25 minutes per patient. The least busy section is the consulting rooms with a utilization rate of 27%. The Table also shows that the assessment centre has more patients waiting in the queue and the system than that of the consulting rooms and the record section. Also, the number of minutes a person spends before his or her biological data is taken from the assessment centre is far more than that of the records section and the consulting rooms. On the average, a patient spends about 0.73 minutes in the queue at the assessment centre and 0.94 minutes in the system. The least number of patients in the queue was the consulting rooms while assessment center recorded the highest number of patients in the queue and in the services representing 3 and 4 patients respectively.

4.16 Analysis of Data Collected On Wednesday at the Records Section

Sixteen (16) patients reported at the records section on Wednesday within a period of seventy-one minutes. Table 4.15 shows the breakdown of the number of patients and the time they arrived at the centre.

Table 4.15: **Number and Time of Patients that Arrived on Wednesday at the Records Section**

Number of Patients	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
Arrival Rate In Minutes	3	8	10	14	17	21	26	30	35	41	46	52	59	63	68	71

(8 - 3) + (10 - 8) + (14 - 10) + (17 - 14) + (21 - 17) + (26 - 21) + (30 - 26) + (35 - 30) + (41 -35) + (46 - 41) + (52 - 46) + (59 - 52) + (63 - 59) + (68 - 63) + (71 - 68)

Inter arrival rate = 5 + 2 + 4 + 3 + 4 + 5 + 4 + 5 + 6+ 5 + 6 + 7 + 4 + 5+

3 = 68 minutes

The mean arrival rate $\lambda = \frac{68}{16}$= 4.25 minutes per patient

Table 4.16: **Number and Time Patients were served on Wednesday at the Records Section**

Number of Patients	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
Arrival Rate In Minutes	5	9	12	16	21	24	29	33	37	42	49	53	61	67	73	79

(9 - 5) + (12 - 9) + (16 - 12) + (21 - 16) + (24 - 21) + (29 - 24) + (33 - 29) + (37 - 33) + (42 - 37) + (49 - 42) + (53 - 49) + (61 - 53) + (67 - 61) + (73 - 67) + (79 - 73)

Inter service rate = 4 + 3 + 4 + 5 + 3 + 5 + 4 + 4 + 5 + 7 + 4 + 8 + 6 + 6 + 6 = 74 minutes

2. Therefore the mean service rate $\mu = \frac{74}{16}$= 4.63 minutes per patients.

4.17 Calculation of Performance Parameters on Wednesday at the Records Section

3. Number of servers = 2
4. Mean combined rate of all servers $k\mu$ = 2(4.63) = 9.26 minutes
5. Utilization factor of the entire system = $\rho = \frac{\lambda}{k\mu} = \frac{4.25}{2(4.63)}$ = 0.46 or 46%
6. The probability that there are no patients in the system (all servers are idle) is

$$P_0 \quad = \left| \left[\sum_{n=0}^{k-1} \frac{1}{n!} \left(\frac{\lambda}{\mu} \right)^n \right] + \frac{1}{K!} \left(\frac{\lambda}{\mu} \right)^k \left(\frac{K\mu}{K(\mu - \lambda)} \right) \right|^{-1}$$

$$= \left| 1 + \frac{4.25}{4.63} + \frac{1}{2} \left(\frac{4.25}{4.63} \right)^2 \frac{2(4.63)}{2(4.63 - 4.25)} \right|^{-1}$$

$$= \left[1 + \frac{4.25}{4.63} + \frac{1}{2}(0.84) \left(\frac{9.26}{0.76} \right) \right]^{-1} \quad = \quad 0.14$$

14% Probability of zero patients in the system

(7). The expected number of patients in the waiting line

$$L_q = \frac{\left(\frac{\lambda}{\mu} \right)^k}{k!(1 - P)^2} (P_0)$$

$$= \frac{\left(\frac{4.25}{4.63} \right)^2}{2!(1 - 0.32)^2} (0.45)$$

$$= \frac{\left(\frac{3.63}{5.63} \right)^2}{2(1 - 0.46)^2} (0.14) = 0.20$$

(8). The expected number of patients in the system,

$$L_s = L_q + \frac{\lambda}{\mu}$$

$$= 0.20 + \frac{4.25}{4.63} = 1.12$$

(9). The expected waiting time in the queue

$$W_q = \frac{L_q}{\lambda}$$

$$W_q = \frac{0.20}{4.25} = 0.05 \text{ minutes}$$

= 0.05 minutes average time a patient spends in the queue waiting

(10). The expected time a patient spends in the system,

$$W_s = W_q + \frac{1}{\lambda}$$

$= 0.05 + \frac{1}{4.25} = 0.$29 hours

= 0.29 minutes average time a patient spends in the system

4.18 Analysis of Data Collected On Wednesday at the Assessment Centre

Table 4.17: **Number and Time of patients that arrived on Wednesday at the Assessment Centre**

Number of Patients	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
Service Time In Minutes	7	11	16	22	28	32	37	41	46	50	55	59	66	69	74	80

Within a period of eighty (80) minutes sixteen (16) patients arrived at the assessment center for their biological data to be taken. Table 4.17 shows the breakdown of the number of patients and the time they took to arrive at the facility.

(11 - 7) + (16 - 11) + (22 - 16) + (28 - 22) + (32 - 28) + (37 - 32) + (41 - 37) + (46 - 41) + (50 - 46) + (55 - 50) + (59 - 55) + (66 - 59) + (69 - 66) + (74 69) + (80 - 74)

Inter arrival rate = 4 + 5 + 6 + 6 + 4 + 5 + 4 + 5 + 4 + 5 + 4 + 7 + 3 + 5 + 6 = 73 minutes

1. The mean arrival rate = $\lambda = \frac{73}{16}$ = 4.56 minutes per patient

Table 4.18: **Number and Time of patients were served on Wednesday at the Assessment Centre**

Number of Patients	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
Service Time	9	15	21	29	35	41	47	52	59	66	70	76	81	89	94	97

In Minutes																

(15 - 9) + (21 - 15) + (29 - 21) + (35 - 29) + (41 - 35) + (47 - 41) + (52 - 47) + (59 - 52) + (66 - 59) + (70 - 66) + (76 - 70) + (81 - 76) + (89 - 81) + (94 - 89) + (97 - 94)

Inter service rate = 6 + 6 + 8 + 6 + 6 + 6 + 5 + 7 + 7 + 4 + 6 + 5 + 8 + 5 + 3 = 88 minutes 2. The mean $\mu = \frac{88}{16}$ service rate =

= 5.50 minutes per patient.

4.19 Calculation of Performance Parameters on Wednesday at the Assessment Centre

3. Number of servers = 1
4. Mean combined rate of all servers = $k\mu$ = 1(5.50) = 5.50 minutes
5. Utilization factor of the entire system = $\rho = \frac{\lambda}{k\mu} = \frac{4.56}{5.50}$ = 0.83 or 83%
6. The probability that there are no patients in the system (all servers are idle) is

$$P_0 = \left| \left[\sum_{n=0}^{k-1} \frac{1}{n!} \left(\frac{\lambda}{\mu}\right)^n \right] + \frac{1}{K!} \left(\frac{\lambda}{\mu}\right)^k \left(\frac{K\mu}{K(\mu - \lambda)}\right) \right|^{-1}$$

$$= \left| \left[\sum_{n=0}^{k-1} \frac{1}{1!} \left(\frac{4.56}{5.50}\right)^0 \right] + \frac{1}{1!} \left(\frac{4.56}{5.50}\right)^1 \left(\frac{5.50}{1(5.50 - 4.56)}\right) \right|^{-1}$$

$$= \left[1 + 1 \left(\frac{4.56}{5.50}\right) \left(\frac{5.50}{0.94}\right) \right]^{-1} = 0.17$$

Thus 17% Probability of zero patients in the system

(7). The expected number of patients in the waiting line

$$L_q = \frac{\left(\frac{\lambda}{\mu}\right)^k}{k!(1 - P)^2} (P_0)$$

$$= \frac{\left(\frac{4.56}{5.50}\right)^1}{1!(1-0.83)^2}(0.17)$$

$$= \frac{\left(\frac{4.56}{5.50}\right)}{1(1-0.78)^2}(0.17)$$

$$= \frac{0.829}{0.0289}(0.17) = 4.70$$

(8). The expected number of patients in the system,

$$L_s = L_q + \frac{\lambda}{\mu}$$

$$= 4.70 + \frac{4.56}{5.50} = 5.53$$

(9). The expected waiting time in the queue

$$W_q = \frac{L_q}{\lambda}$$

$$W_q = \frac{4.70}{4.56} = 1.03 \text{ minutes}$$

= 1.03 minutes average time a patient spends in the queue waiting

(10). The expected time a patient spends in the system,

$$W_s = W_q + \frac{1}{\lambda}$$

$$= 1.03 + \frac{1}{4.56} = 1.25 \text{ minutes}$$

= 1.25 minutes average time a patient spends in the system

4.20 Analysis of Data Collected on Wednesday at the Consulting Rooms

Sixteen (16) patients arrived at the consultation bay within a period of eighty-one (81) minutes. Table 4.19 shows the number of patients and their arrival rate.

(14-11)+(19-14)+(23-19)+(26-23)+(30-26)+(34-30)+(39-34)+(42-39)+(49-42)+(53-

49)+ (58-53) + (62-58) + (67-62) + (74-67) + (81-74)

Inter arrival rate = 3 + 5 + 4 + 3 + 4 + 4 + 5 + 3 + 7 + 4 + 5 + 4 + 5 + 7 +

7 = 70 minutes

Table 4.19: **Number and Time Patients arrived on Wednesday at the Consulting Rooms**

Number of Patients	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
Arrival Rate In Minutes	11	14	19	23	26	30	34	39	42	49	53	58	62	67	74	81

1. Mean arrival rate $\lambda = \frac{70}{16}$= 4.38 minutes per patient

Table 4.20: **Number and Time Patients were served at the Consulting Rooms on Wednesday**

Number Of Patients	Service Time In Minutes		
	Consulting Room 1	Consulting Room 2	Consulting Room 3
1	12	11	13
2	18	16	19
3	27	22	25
4	29	29	31
5	34	33	36
6	39	38	40
7	45	47	45
8	49	51	52
9	51	60	61
10	55	66	73
11	61	69	78
12	66	71	83
13	71	78	89
14	77	85	92
15	83	89	98
16	90	95	102

4.21 Calculation of Performance Parameters on Wednesday at the Consulting Rooms

Inter service rate (consulting room 1) = (18- 12) + (27 - 18) + (29 - 27) + (34 - 29) + (39 - 34) + (45 - 39) + (49 - 45) + (51 - 49) + (55 - 51) + (61 - 55) + (66 - 61) + (71 - 66) + (77 - 71) + (83 - 77) + (90 - 83)

⇒ 6 + 9 + 2 + 5 + 5 + 6 + 4 + 2 + 4 + 6 + 5 + 5 + 6 + 6 + 7 = 78 minutes

Mean service rate (consulting room 1)= $\mu = \frac{78}{16}$= 4.88 minutes per patient

Inter service rate (consulting room 2) =(16-11)+(22-16)+(29-22)+(33-29)+(38-33)+(47-38)+ (51 - 47) + (60 - 51) + (66 - 60) + (69 - 66) + (71 - 69) + (78 - 71) + (85 - 78) + (89 - 85) + (95 - 89) ⇒ 5 + 6 + 7 + 4 + 5 + 9 + 4 + 9 + 6 + 3 + 2 + 7 + 7 + 4 + 6 = 84 minutes Mean service rate (consulting room 2) $= \mu = \frac{84}{16}$ = 5.25 minutes per patient

Inter service rate (consulting room 3) = (19 - 13) + (25 - 19) + (31 - 25) + (36 - 31) + (40 - 36) + (45 - 40) + (52 - 45) + (61 - 52) + (73 - 61) + (78 - 73) + (83 - 78) + (89 - 83) + (92 - 89)+ (98 - 92) + (102 - 98)

⇒ 6 + 6 + 6 + 5 + 4 + 5 + 7 + 9 + 1 2 + 5 + 5 + 6 + 3 + 6 + 4 = 89 minutes

Mean service rate (consulting room 3) = $\mu = \frac{89}{16}$= 5.56 minutes per patient.

2. Mean service rate of the 3 consulting rooms = $\frac{(4.88 + 5.25 + 5.56)}{3} = \frac{15.69}{3} =$ 5.23

3. Number of servers k = 3

4. Mean combined rate of all servers = $k\mu$ = 3(5.23) = 15.69 minutes

5. Utilization factor of the entire system = $\rho = \frac{\lambda}{k\mu} = \frac{4.38}{15.69}$ = 0.28 or 28 %

6. The probability that there are no patients in the system (all servers are idle) is

$$P_0 = \left|\left[\sum_{n=0}^{k-1} \frac{1}{n!}\left(\frac{\lambda}{\mu}\right)^n\right] + \frac{1}{K!}\left(\frac{\lambda}{\mu}\right)^k \left(\frac{K\mu}{K(\mu-\lambda)}\right)\right|^{-1}$$

$$= \left|1 + \left(\frac{4.38}{5.23}\right) + \left(\frac{4.38}{5.23}\right)^2 + \frac{1}{3}\left(\frac{4.38}{5.23}\right)^3 \frac{3(5.23)}{3(5.23-4.38)}\right|^{-1}$$

$$= \left|1 + \left(\frac{4.38}{5.23}\right) + (0.70) + \frac{1}{3}(0.59)\left(\frac{15.69}{2.55}\right)\right|^{-1} = 0.27$$

Hence, the Probability of zero patients in the system = 27%

(7). The expected number of patients in the waiting line

$$L_q = \frac{\left(\frac{\lambda}{\mu}\right)^k}{k!(1-\rho)^2}(P_0)$$

$$= \frac{\left(\frac{4.38}{5.23}\right)^3}{3!(1-0.28)^2}(0.27)$$

$$\Rightarrow \left(\frac{0.59}{3.11}\right)(0.27) = 0.05$$

(8). The expected number of patients in the system,

$$L_s = L_q + \frac{\lambda}{\mu}$$

$$= 0.05 + \frac{4.38}{5.23} = 0.89$$

(9). The expected waiting time in the queue

$$W_q = \frac{L_q}{\lambda}$$

$W_q = \frac{0.05}{4.38} = 0.011$ minutes

= 0.01 minutes average time a patient spends in the queue waiting

(10). The expected time a patient spends in the system,

$$W_s = W_q + \frac{1}{\lambda}$$

$= 0.011 + \frac{1}{4.38} = 0.24$ minutes

= 0.24 minutes average time a patient spends in the system

It can be seen in from table 21 that the assessment center was the busiest section. Its utilization factor is 83% with the least combined rate of 5.50 minutes per patient. This is followed by the records sections with a utilization rate of 46%

Table 4.21: **Summary of Performance Parameters at All the Various Sections on Thursday**

Performance Rate	Records Section	Assessment Center	Consulting Room
Mean arrival Rate λ	4.25 mins / pnt	4.56 mins/pnt	4.38 mins /pnts
Mean service rate μ	4.63 mins/pnt	5.50 mins/pnt	5.56 mins/pnt
Mean combined rate of all servers kμ	9.26 minutes	5.50 minutes	15.69 minutes
Utilization factor of the entire system *rho*	46%	83%	28%
Probability of zero patients in the system P_0	14%	17%	27%
Average number of patients in the queue L_q	0.20	5	0.05
Average number of patients in the system,L_s	1.12	6	0.89
Average waiting time in the queue,W_q	0.05 minutes	1.03 minutes	0.01 minutes
Average time a patient spends in the system W_s	1.03 minutes	1.25 minutes	0.24 minutes

and a mean combined rate of 9.26 minutes per patient. The consulting rooms with 28% utilization rate were less busy as compared with the records section and the assessment centre. The Table also shows that the assessment center recorded the highest number of patients both in the queue and in service representing 5 and 6 respectively. Also, the number of minutes a person spends before his or her biological data is taken from the assessment centre is far more than that of the records section and the consulting rooms. On the average, a patient spends about 1.03 minutes in the queue and 1.25 minutes in the system.

4.22 Analysis of Data Collected On Thursday at the Records Section

On Thursday, a total of sixteen (16) patients arrived at the records section within a period of fifty-seven (57) minutes as shown in table 4.22 below.

(5 - 2) + (9 - 5) + (11 - 9) + (15 - 11) + (19 - 15) + (21 - 19) + (27 - 21) + (32 Table 4.22: **Number and Time of Patients that Arrived on Thursday at the Records Section**

Number of Patients	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
Arrival Rate In Minutes	2	5	9	11	15	19	21	27	32	36	39	42	46	51	55	57

- 27) + (36 - 32) + (39 - 36) + (42 - 39) + (46 - 42) + (51 - 46) + (55 - 51) + (57 - 55)

Inter arrival rate = 3 + 4 + 2 + 4 + 4 + 2 + 6 + 5 + 4 + 3 + 3 + 4 + 5 + 4 + 2 = 55 minutes

The mean arrival rate $\lambda = \frac{55}{16}$= 3.44 minutes per patient

(9 - 5) + (15 - 9) + (21 - 15) + (27 - 21) + (34 - 27) + (38 - 34) + (41 - 38) +

Table 4.23: **Number and Time Patients were served on Thursday at the Records Section**

Number of Patients	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
Arrival Rate In Minutes	5	9	15	21	27	34	38	41	48	55	63	69	71	77	81	86

(48 - 41) + (55 - 48) + (63 - 55) + (69 - 63) + (71 - 69) + (77 - 71) + (81 - 77) + (86 - 81)

Inter service rate = 4 + 6 + 6 + 6 + 7 + 4 + 3 + 7 + 7 + 8 + 6 + 2 + 6 + 4 + 5 = 81 minutes

Therefore the mean service rate $\mu = \frac{81}{16}$= 5.06 minutes per patients.

4.23 Calculation of Performance Parameters on Thursday at the Records Section

3. Number of servers = 2

4. Mean combined rate of all servers $k\mu$ = 2(5.06) = 10.13 minutes

5. Utilization factor of the entire system = $\rho = \frac{\lambda}{k\mu} = \frac{3.44}{2(5.06)}$ = 0.34 or 34%

6. The probability that there are no patients in the system (all servers are idle) is

$$P_0 = \left| \left[\sum_{n=0}^{k-1} \frac{1}{n!} \left(\frac{\lambda}{\mu} \right)^n \right] + \frac{1}{K!} \left(\frac{\lambda}{\mu} \right)^k \left(\frac{K\mu}{K(\mu - \lambda)} \right) \right|^{-1}$$

$$= \left| 1 + \frac{3.44}{5.06} + \frac{1}{2} \left(\frac{3.44}{5.06} \right)^2 \frac{2(5.06)}{2(5.06 - 3.44)} \right|^{-1}$$

$$= \left[1 + \frac{3.44}{5.06} + \frac{1}{2}(0.46) \left(\frac{10.12}{3.24} \right) \right]^{-1} = 0.42$$

42% Probability of zero patients in the system

(7). The expected number of patients in the waiting line

$$L_q = \frac{\left(\frac{\lambda}{\mu} \right)^k}{k!(1-P)^2}(P_0)$$

$$= \frac{\left(\frac{3.44}{5.06} \right)^2}{2!(1-0.34)^2}(0.42)$$

$$= \frac{\left(\frac{3.63}{5.63} \right)^2}{2(1-0.34)^2}(0.42) = 0.22$$

(8). The expected number of patients in the system,

$$L_s = L_q + \frac{\lambda}{\mu}$$

$$= 0.22 + \frac{3.44}{5.06} = 0.90$$

(9). The expected waiting time in the queue

$$W_q = \frac{L_q}{\lambda}$$

$$W_q = \frac{0.22}{3.44} = 0.06 \text{ minutes}$$

= 0.06 minutes average time a patient spends in the queue waiting

(10). The expected time a patient spends in the system,

$$W_s = W_q + \frac{1}{\lambda}$$

$$= 0.06 + \frac{1}{3.44} = 0.35 \text{ minutes}$$

= 0.35 minutes average time a patient spends in the system

4.24 Analysis of Data Collected On Thursday at the Assessment Centre

On Thursday, a total of sixteen (16) patients arrived at the assessment centre for their biological data to be taken. Table 24 shows the number of patients that arrived within a period of ninety-seven (97) minutes

Table 4.24: **Number and Time of patients that arrived on Wednesday at the Assessment Centre**

Number of Patients	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
Service Time In Minutes	7	13	17	23	29	36	40	43	50	57	65	70	75	83	88	97

(13 - 7) + (17 - 13) + (23 - 17) + (29 - 23) + (36 - 29) + (40 - 36) + (43 - 40) + (50 - 43) + (57 - 50) + (65 - 57) + (70 - 65) + (75 - 70) + (83 - 75) + (88 - 83) + (97 - 88)

Inter arrival rate = 6 + 4 + 6 + 6 + 7 + 4 + 3 + 7 + 7 + 8 + 5 + 5 + 8 + 5 + 9 = 90 minutes

1. The mean arrival rate = $\lambda = \frac{90}{16}$ = 5.63 minutes per patient

(11 - 7) + (18 - 11) + (26 - 18) + (34 - 26) + (40 - 34) + (47 - 40) + (54 - 47) Table 4.25: **Number and Time of patients were served on Thursday at the Assessment Centre**

Number of Patients	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
Service Time In Minutes	7	11	18	26	34	40	47	54	59	66	70	76	81	88	94	105

+ (59 - 54) + (66 - 59) + (70 - 66) + (76 - 70) + (81 - 76) + (88 - 81) + (94 88) + (105 - 94)

Inter mean service rate = 4 + 7 + 8 + 8 + 6 + 7 + 7 + 5 + 7 + 4 + 6 + 5 + 7 + 6 + 11 = 98

2. The mean service rate = $\mu = \frac{98}{16}$ = 6.13 minutes per patient.

4.25 Calculation of Performance Parameters on Thursday at the Assessment Centre

3. Number of servers = 1

4. Mean combined rate of all servers = $k\mu$ = 1(6.13) = 6.13 minutes 5. Utilization factor of the entire system = $\rho = \frac{\lambda}{k\mu} = \frac{5.63}{6.13}$ = 0.92 or 92%

6. The probability that there are no patients in the system (all servers are idle) is

$$P_0 = \left|\left[\sum_{n=0}^{k-1} \frac{1}{n!}\left(\frac{\lambda}{\mu}\right)^n\right] + \frac{1}{K!}\left(\frac{\lambda}{\mu}\right)^k \left(\frac{K\mu}{K(\mu-\lambda)}\right)\right|^{-1}$$

$$= \left|\left[\sum_{n=0}^{k-1} \frac{1}{1!}\left(\frac{5.63}{6.13}\right)^0\right] + \frac{1}{1!}\left(\frac{5.63}{6.13}\right)^1 \left(\frac{6.13}{1(6.13-5.63)}\right)\right|^{-1}$$

$$= \left[1 + 1\left(\frac{5.63}{6.13}\right)\left(\frac{6.13}{0.50}\right)\right]^{-1} \quad = \quad 0.08$$

Thus 8% Probability of zero patients in the system

(7). The expected number of patients in the waiting line

$$L_q = \frac{\left(\frac{\lambda}{\mu}\right)^k}{k!(1-P)^2}(P_0)$$

$$L_q = \frac{\left(\frac{5.63}{6.13}\right)^1}{1!(1-0.92)^2}(0.08)$$

$$= \frac{\left(\frac{5.63}{6.13}\right)}{1(1-0.92)^2}(0.08)$$

$$= \frac{0.9184}{0.0064}(0.08) = 11.48$$

(8). The expected number of patients in the system,

$$L_s = L_q + \frac{\lambda}{\mu}$$

$$L_s = 11.48 + \frac{5.63}{6.13} = 12.40$$

(9). The expected waiting time in the queue

$$W_q = \frac{L_q}{\lambda}$$

$$W_q = \frac{11.48}{5.63} = 2.04 \text{ minutes}$$

= 2.04 minutes average time a patient spends in the queue waiting

(10). The expected time a patient spends in the system,

$$W_s = W_q + \frac{1}{\lambda}$$

$$= 2.04 + \frac{1}{5.63} = 2.22 \text{ minutes}$$

= 2.22 minutes average time a patient spends in the system

4.26 Analysis of Data Collected On Thursday at the Consulting Rooms

Within a period of seventy-two (72) minutes a total of sixteen (16) patients arrived at the consultation bay for treatment. Table 26 shows the breakdown of the number of patients that arrived together with their arrival rate.

Table 4.26: **Number and Time Patients arrived on Thursday at the Consulting Rooms**

Number of Patients	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
Arrival Rate In Minutes	10	13	17	21	25	28	33	37	41	47	50	54	59	63	69	72

(13 - 10) + (17 - 13) + (21 - 17) + (25 - 21) + (28 - 25) + (33 - 28) + (37 - 33) + (41 - 37) + (47 - 41) + (50 - 47) + (54 - 50) + (59 - 54) + (63 - 59) + (69 63) + (72 - 69)

Inter arrival rate = 3 + 4 + 4 + 4 + 3 + 5 + 4 + 4 + 6 + 3 + 4 + 5 + 4 + 6 + 3 = 62 minutes

1. Mean arrival rate $\lambda = \frac{62}{16}$= 3.88 minutes per patient

Inter service rate (consulting room 1) = (19 - 13) + (25 - 19) + (31 - 25) + (36 - 31) + (40 - 36) + (45 - 40) + (52 - 45) + (61 - 52) + (73 - 61) + (78 - 73) + (83 - 78) + (89 - 83) + (92 - 89) + (98 - 92) + (102 - 98)

6 + 6 + 6 + 5 + 4 + 5 + 7 + 9 + 12 + 5 + 5 + 6 + 3 + 6 + 4 = 89 minutes

Mean service rate (consulting room 1)= $\mu = \frac{89}{16}$= 5.56 minutes per patient

Inter service rate (consulting room 2) = (16 - 11) + (22 - 16) + (29 - 22) + (33 - 29) + (38 - 33) + (47 - 38) + (51 - 47) + (60 - 51) + (66 - 60) + (69 - 66) + (71 - 69) + (78 - 71) + (85 - 78) + (89 - 85) + (95 - 89)

Table 4.27: **Number and Time Patients were served at the Consulting Rooms on Thursday**

Number Of Patients	Service Time In Minutes		
	Consulting Room 1	Consulting Room 2	Consulting Room 3
1	13	11	12
2	19	16	18
3	25	22	27
4	31	29	29
5	36	33	34
6	40	38	39
7	45	47	45
8	52	51	49
9	61	60	51
10	73	66	55
11	78	69	61
12	83	71	66
13	89	78	71
14	92	85	77
15	98	89	83
16	102	95	90

5 + 6 + 7 + 4 + 5 + 9 + 4 + 9 + 6 + 3 + 2 + 7 + 7 + 4 + 6 = 84 minutes

Mean service rate (consulting room 2) = $\mu = \frac{84}{16}$= 5.25 minutes per patient

Inter service rate (consulting room 3) = (18 - 12) + (27 - 18) + (29 - 27) + (34 - 29) + (39 - 34) + (45 - 39) + (49 - 45) + (51 - 49) + (55 - 51) + (61 - 55) + (66 - 61) + (71 - 66) + (77 - 71)+ (83 - 77) + (90 - 83)

= 6 + 9 + 2 + 5 + 5 + 6 + 4 + 2 + 4 + 6 + 5 + 5 + 6 + 6 + 7 = 78 minutes

Mean service rate (consulting room 3) = $\mu = \frac{78}{16}$= 4.88 minutes per patient.

2. Mean service rate of the 3 consulting rooms = $\frac{(5.56 + 5.25 + 4.88)}{3} = \frac{15.69}{3} =$ 5.23

4.27 Calculation of Performance Parameters on Thursday at the Consulting Rooms

3. Number of servers k = 3
4. Mean combined rate of all servers = $k\mu$ = 3(5.23) = 15.69 minutes
5. Utilization factor of the entire system = $\rho = \frac{\lambda}{k\mu} = \frac{3.88}{15.69}$ = 0.25 or 25 %

6. The probability that there are no patients in the system (all servers are idle) is

$$P_0 \quad = \left| \left[\sum_{n=0}^{k-1} \frac{1}{n!} \left(\frac{\lambda}{\mu}\right)^n \right] + \frac{1}{K!} \left(\frac{\lambda}{\mu}\right)^k \left(\frac{K\mu}{K(\mu-\lambda)}\right) \right|^{-1}$$

$$= \left| 1 + \left(\frac{3.88}{5.23}\right) + \left(\frac{3.88}{5.23}\right)^2 + \frac{1}{3!}\left(\frac{3.88}{5.23}\right)^3 \frac{3(5.23)}{3(5.23-3.88)} \right|^{-1}$$

$$= \left| 1 + \left(\frac{3.88}{5.23}\right) + (0.55) + \frac{1}{3!}(0.41)\left(\frac{15.69}{4.05}\right) \right|^{-1} = 0.39$$

Hence, the Probability of zero patients in the system = 35%

(7). The expected number of patients in the waiting line

$$L_q = \frac{\left(\frac{\lambda}{\mu}\right)^k}{k!(1-\rho)^2}(P_0)$$

$$= \frac{\left(\frac{3.88}{5.23}\right)^3}{3!(1-0.25)^2}(0.39)$$

$$= \left(\frac{0.41}{3.38}\right)(0.39) = 0.05$$

(8). The expected number of patients in the system,

$$L_s = L_q + \frac{\lambda}{\mu}$$

$$= 0.05 + \frac{3.88}{5.23} = 0.79$$

(9). The expected waiting time in the queue

$$W_q = \frac{L_q}{\lambda}$$

$W_q = \frac{0.04}{3.88} = 0.$01 minutes

= 0.01 minutes average time a patient spends in the queue waiting

(10). The expected time a patient spends in the system,

$$W_s = W_q + \frac{1}{\lambda}$$

$= 0.01 + \frac{1}{3.88} = 0.27$ minutes

= 0.27 minutes average time a patient spends in the system

Table 4.28: **Summary of Performance Parameters at All the Various Sections on Thursday**

Performance Rate	Records Section	Assessment Center	Consulting Room
Mean arrival Rate λ	3.44 mins / patient	5.63 mins/patient	3.88 mins/patients
Mean service rate μ	5.06 mins/patient	6.13 mins/patient	5.23 mins/patient
Mean combined rate of all servers kμ	10.13 minutes	6.13 minutes	15.69 minutes
Utilization factor of the entire system *rho*	34%	92%	25%
Probability of zero patients in the system P_0	42%	8%	39%
Average number of patients in the queue L_q	0.22	11.48	0.05
Average number of patients in the system,L_s	0.90	12.40	0.79
Average waiting time in the queue,W_q	0.06 minutes	2.04 minutes	0.01 minutes
Average time a patient spends in the system W_s	0.35 minutes	2.22 minutes	0.27 minutes

It can be seen from table 4.28 that the assessment center was the busiest section. Its utilization factor is 92% with the least combined rate of 6.13 minutes per patient. This is followed by the records sections with a utilization rate of 34% and a mean combined rate of 10.13 minutes per patient. The consulting rooms with 25% utilization rate were less busy as compared with the records section and the assessment centre. The Table also shows that the assessment center recorded the highest number of patients both in the queue and in service representing 11 and 12 respectively. Also, the number of minutes a person spends before his or her biological data is taken from the assessment centre is far more than that the records section and the consulting rooms. On the average, a patient spends about 2.04 minutes in the queue and 2.22 minutes in the system.

4.28 Analysis of Data Collected On Friday at the Records Section

Sixteen (16) patients arrived on Friday at the records section within a period of one hour as shown in table 4.29 below.

Table 4.29: **Number and Time of Patients that Arrived on Friday at the Records Section**

Number of Patients	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
Arrival Rate In Minutes	2	5	8	12	14	17	21	25	29	33	36	40	45	49	55	62

(5 - 2) + (8 - 5) + (12 - 8) + (14 - 12) + (17 - 14) + (21 - 17) + (25 - 21) + (29 - 25) + (33 - 29) + (36 - 33) + (40 - 36) + (45 - 40) + (49 - 45) + (55 - 49) + (62 - 55)

Inter arrival rate = 3 + 3 + 4 + 2 + 3 + 4 + 4 + 4 + 4 + 3 + 4 + 5 + 4 + 6 + 7 = 60 minutes

The mean arrival rate $\lambda = \frac{60}{16}$= 3.75 minutes per patient

(10 - 5) + (16 - 10) + (21 - 16) + (27 - 21) + (33 - 27) + (39 - 33) + (44 - 39) + (50 - 44) + (55 - 50) + (61 - 55) + (67 - 61) + (71 - 67) + (77 - 71) + (81 - 77) + (86 - 81)

Inter service rate = 5 + 6 + 5 + 6 + 6 + 6 + 5 + 6 + 5 + 6 + 6 + 4 + 6 + 4 Table 4.30: **Number and Time Patients were served on Friday at the Records Section**

Number of Patients	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
Arrival Rate In Minutes	5	10	16	21	27	33	39	44	50	55	61	67	71	77	81	86

+ 5 = 81minutes

Therefore the mean service rate $\mu = \frac{81}{16}$= 5.06 minutes per patients.

4.29 Calculation of Performance Parameters on Friday at the Records Section

3. Number of servers = 2
4. Mean combined rate of all servers $k\mu$ = 2(5.06) = 10.12 minutes
5. Utilization factor of the entire system = $\rho = \frac{\lambda}{k\mu} = \frac{3.75}{2(5.06)}$ = 0.37 or 37%
6. The probability that there are no patients in the system (all servers are idle) is

$$P_0 = \left|\left[\sum_{n=0}^{k-1} \frac{1}{n!}\left(\frac{\lambda}{\mu}\right)^n\right] + \frac{1}{K!}\left(\frac{\lambda}{\mu}\right)^k \left(\frac{K\mu}{K(\mu-\lambda)}\right)\right|^{-1}$$

$$= \left|1 + \frac{3.75}{5.06} + \frac{1}{2}\left(\frac{3.75}{5.06}\right)^2 \frac{2(5.06)}{2(5.06-3.75)}\right|^{-1}$$

$$= \left[1 + \frac{3.75}{5.06} + \frac{1}{2}(0.55)\left(\frac{10.12}{2.62}\right)\right]^{-1} = 0.36$$

36% Probability of zero patients in the system

(7). The expected number of patients in the waiting line

$$L_q = \frac{\left(\frac{\lambda}{\mu}\right)^k}{k!(1-P)^2}(P_0)$$

$$= \frac{\left(\frac{3.75}{5.06}\right)^2}{2!(1-0.37)^2}(0.36)$$

$$= \frac{\left(\frac{3.75}{5.06}\right)^2}{2(1-0.37)^2}(0.36) = 0.25$$

(8). The expected number of patients in the system,

$$L_s = L_q + \frac{\lambda}{\mu}$$

$$= 0.25 + \frac{3.75}{5.06} = 0.99$$

(9). The expected waiting time in the queue

$$W_q = \frac{L_q}{\lambda}$$

$$W_q = \frac{0.25}{3.75} = 0.07 \text{ minutes}$$

= 0.07 minutes average time a patient spends in the queue waiting

(10). The expected time a patient spends in the system,

$$W_s = W_q + \frac{1}{\lambda}$$

$$= 0.07 + \frac{1}{3.75} = 0.34 \text{ minutes}$$

= 0.34 minutes average time a patient spends in the system

4.30 Analysis of Data Collected On Friday at the Assessment Centre

Within a period of fifty-four minutes sixteen (16) patients reported at the assessment centre as shown in table 4.31 below

Table 4.31: **Number and Time of patients that arrived on Friday at the Assessment Centre**

Number of Patients	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
Service Time In Minutes	2	4	6	10	12	15	19	22	26	31	35	39	42	47	50	54

(4 - 2) + (6 - 4) + (10 - 6) + (12 - 10) + (15 - 12) + (19 - 15) + (22 - 19) + (26 - 22) + (31 - 26) + (35 - 31) + (39 - 35) + (42 - 39) + (47 - 42) + (50 - 47) + (54 - 50)

Inter arrival rate = 2 + 2 + 4 + 2 + 3 + 4 + 3 + 4 + 5 + 4 + 4 + 3 + 5 + 3 +

4 = 52 minutes

1. The mean arrival rate = $\lambda = \frac{52}{16}$ = 3.25 minutes per patient

Table 4.32: **Number and Time of patients were served on Friday at the Assessment Centre**

Number of Patients	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
Service Time In Minutes	3	7	12	16	21	26	33	37	44	47	52	57	65	70	76	80

(7 - 3) + (12 - 7) + (16 - 12) + (21 - 16) + (26 - 21) + (33 - 26) + (37 - 33) +

(44 - 37) + (47 - 44) + (52 - 47) + (57 - 52) + (65 - 57) + (70 - 65) + (76 - 70) + (80

- 76)

Inter service rate = 4 + 5 + 4 + 5 + 5 + 7 + 4 + 7 + 3 + 5 + 5 + 8 + 5 + 6

+ 4 = 77 minutes

2. The mean service rate = $\mu = \frac{77}{16}$ = 4.81 minutes per patient.

4.31 Calculation of Performance Parameters on Thursday at the Assessment Centre

3. Number of servers = 1

4. Mean combined rate of all servers = $k\mu$ = 1(4.81) = 4.81 minutes 5. Utilization factor of the entire system = $\rho = \frac{\lambda}{k\mu} = \frac{3.25}{8.41}$= 0.68 or 68%

6. The probability that there are no patients in the system (all servers are idle) is

$$P_0 = \left|\left[\sum_{n=0}^{k-1}\frac{1}{n!}\left(\frac{\lambda}{\mu}\right)^n\right]+\frac{1}{K!}\left(\frac{\lambda}{\mu}\right)^k\left(\frac{K\mu}{K(\mu-\lambda)}\right)\right|^{-1}$$

$$= \left|\left[\sum_{n=0}^{k-1}\frac{1}{1!}\left(\frac{3.25}{4.81}\right)^0\right]+\frac{1}{1!}\left(\frac{3.25}{4.81}\right)^1\left(\frac{4.81}{1(4.81-3.25)}\right)\right|^{-1}$$

$$= \left[1+1\left(\frac{3.25}{4.81}\right)\left(\frac{4.81}{1.56}\right)\right]^{-1} = 0.32$$

Thus 32% Probability of zero patients in the system

(7). The expected number of patients in the waiting line

$$L_q = \frac{\left(\frac{\lambda}{\mu}\right)^k}{k!(1-P)^2}(P_0)$$

$$L_q = \frac{\left(\frac{3.25}{4.81}\right)^1}{1!(1-0.68)^2}(0.32)$$

$$= \frac{\left(\frac{3.25}{4.81}\right)}{1(1-0.68)^2}(0.32)$$

$$= \frac{0.676}{0.1024}(0.32) = 2.11$$

(8). The expected number of patients in the system,

$$L_s = L_q + \frac{\lambda}{\mu}$$

$$L_s = 2.11 + \frac{3.25}{4.81} = 2.79$$

(9). The expected waiting time in the queue

$$W_q = \frac{L_q}{\lambda}$$

$W_q = \frac{2.18}{3.25} = 0.$65 minutes

= 0.65 minutes average time a patient spends in the queue waiting

(10). The expected time a patient spends in the system,

$$W_s = W_q + \frac{1}{\lambda}$$

$= 0.65 + \frac{1}{3.25} = 0.$96 minutes

= 0.96 minutes average time a patient spends in the system.

4.32 Analysis of Data Collected On Friday at the Consulting Rooms

Sixteen (16) patients arrived at the consultation bay within a period of ninety-one (91) minutes as shown in table 4.33 below.

Table 4.33: **Number and Time Patients arrived on Friday at the Consulting Rooms**

Number of Patients	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
Arrival Rate In Minutes	5	11	16	20	26	32	37	44	47	55	61	69	74	80	86	91

(11 - 5) + (16 - 11) + (20 - 16) + (26 - 20) + (32 - 26) + (37 - 32) + (44 - 37) + (47 - 44) + (55 - 47) + (61 - 55) + (69 - 61) + (74 - 69) + (80 - 74) + (86 80) + (91 - 86)

Inter arrival rate = 6 + 5 + 4 + 6 + 6 + 5 + 7 + 3 + 8 + 6 + 8 + 5 + 6 + 6 + 5 = 86 minutes

1. Mean arrival rate $\lambda = \frac{86}{16}$= 5.38 minutes per patient

Table 4.34: **Number and Time Patients were served at the Consulting Rooms on Thursday**

Number Of Patients	Service Time In Minutes		
	Consulting Room 1	Consulting Room 2	Consulting Room 3
1	5	3	3
2	15	9	10

3	21	16	19
4	25	23	23
5	32	29	28
6	35	36	34
7	45	42	44
8	52	50	51
9	61	59	60
10	74	68	67
11	77	72	73
12	83	79	76
13	87	84	79
14	93	87	82
15	97	94	87
16	101	97	92

4.33 Calculation of Performance Parameters on Thursday at the Consulting Rooms

Inter service rate (consulting room 1) = (15 - 5) + (21 - 15) + (25 - 21) + (32 - 25) + (35 - 32) + (40 - 35) + (52 - 45) + (61 - 52) + (74 - 61) + (77 - 74) + (83 - 77) + (87 - 83) + (93 - 87) + (97 - 93) + (101 - 97)

⇒ 10 + 6 + 4 + 7 + 3 +5 + 7+ 9 + 1 3 +3 + 6 + 4 + 6+ 4 + 4 = 91 minutes Mean service rate (consulting room 1)= $\mu = \frac{91}{16}$= 6.69 minutes per patient

Inter service rate (consulting room 2) = (9 - 3) + (16 - 9) + (23 - 16) + (29 - 23) + (36 - 29) + (42 - 36) + (50 - 42) + (59 - 50) + (68 - 59) + (72 - 68) + (79 - 72) + (84 - 79) + (87 - 84) + (94 - 87) + (97 - 94)

⇒ 6 + 1 0 + 7 + 6 + 7 + 6 + 8 + 9 + 9 + 4 + 7 + 5 +3 + 7 + 3 = 97 minutes

Mean service rate (consulting room 2) = $\mu = \frac{97}{16}$= 6.06 minutes per patient

Inter service rate (consulting room 3) = (10 - 3) + (19 - 10) + (23 - 19) + (28 - 23) + (34 - 28) + (44 - 34) + (51 - 44) + (60 - 51) + (67 - 60) + (73 - 67) + (76 - 73) + (79 - 76) + (82 - 79) + (87 - 82) + (92 - 87)

⇒ 7 + 9 + 4 + 5 + 6 + 10 + 7 +9 + 7 + 6 + 3 + 3 + 3 + 5 + 5 = 89 minutes

Mean service rate (consulting room 3) $\mu = \frac{89}{16}$ = 5.56 minutes per patient minutes per patient

2. Mean service rate of the 3 consulting rooms = $\frac{(6.69 + 6.06 + 5.56)}{3} = \frac{18.31}{3} = 6.$10 minutes per patient
3. Number of servers k = 3
4. Mean combined rate of all servers = $k\mu$ = 3(6.10) = 18.30 minutes
5. Utilization factor of the entire system = $\rho = \frac{\lambda}{k\mu} = \frac{5.38}{18.30}$ = 0.29 or 29 %
6. The probability that there are no patients in the system (all servers are idle) is

$$P_0 = \left| \left[\sum_{n=0}^{k-1} \frac{1}{n!} \left(\frac{\lambda}{\mu} \right)^n \right] + \frac{1}{K!} \left(\frac{\lambda}{\mu} \right)^k \left(\frac{K\mu}{K(\mu - \lambda)} \right) \right|^{-1}$$

$$= \left| 1 + \left(\frac{5.38}{6.10} \right) + \left(\frac{5.38}{6.10} \right)^2 + \frac{1}{3} \left(\frac{5.38}{6.10} \right)^3 \frac{3(6.10)}{3(6.10 - 5.38)} \right|^{-1}$$

$$= \left| 1 + \left(\frac{5.38}{6.10} \right) + (0.78) + \frac{1}{3}(0.69) \left(\frac{18.30}{2.16} \right) \right|^{-1} = 0.22$$

Hence, the Probability of zero patients in the system = 22%

(7). The expected number of patients in the waiting line

$$L_q = \frac{\left(\frac{\lambda}{\mu} \right)^k}{k!(1-\rho)^2} (P_0)$$

$$= \frac{\left(\frac{5.38}{6.10} \right)^3}{3!(1-0.29)^2} (0.22)$$

$$= \left(\frac{0.69}{3.02} \right) (0.22) = 0.05$$

(8). The expected number of patients in the system,

$$L_s = L_q + \frac{\lambda}{\mu}$$

$$= 0.05 + \frac{5.38}{6.10} = 0.93$$

(9). The expected waiting time in the queue

$$W_q = \frac{L_q}{\lambda}$$

$$W_q = \frac{0.05}{5.38} = 0.01 \text{ minutes}$$

= 0.01 minutes average time a patient spends in the queue waiting

(10). The expected time a patient spends in the system,

$$W_s = W_q + \frac{1}{\lambda}$$

$$= 0.01 + \frac{1}{5.38} = 0.20 \text{ minutes}$$

= 0.20 minutes average time a patient spends in the system

It can be seen from table 4.35 that the assessment center was the busiest section. Its utilization factor is 68% with the least combined rate of 4.81 minutes per patient. This is followed by the records sections with a utilization rate of 37%

Table 4.35: **Summary of Performance Parameters at All the Various Sections on Thursday**

Performance Rate	Records Section	Assessment Center	Consulting Room
Mean arrival Rate λ	3.75 mins / patient	3.25 mins/patient	5.38 mins/patients
Mean service rate μ	5.06 mins/patient	4.81 mins/patient	6.10 mins/patient
Mean combined rate of all servers kμ	10.12 minutes	4.81 minutes	18.30 minutes
Utilization factor of the entire system *rho*	37%	68%	29%
Probability of zero patients in the system P_0	36%	32%	22%
Average number of patients in the queue L_q	0.25	2	0.05
Average number of patients in the system,L_s	0.99	3	0.93
Average waiting time in the queue,W_q	0.07 minutes	0.65 minutes	0.01 minutes

Average time a patient spends in the system W_s	0.34 minutes	0.96 minutes	0.20 minutes

and a mean combined rate of 10.12 minutes per patient. The consulting rooms with 29% utilization rate were less busy as compared with the records section and the assessment centre. The Table also shows that the assessment center recorded the highest number of patients both in the queue and in service representing 2 and 3 respectively. Also, the number of minutes a person spends before his or her biological data is taken from the assessment centre is far more than that the records section and the consulting rooms. On the average, a patient spends about

0.65 minutes in the queue and 0.96 minutes in the system.

4.34 Summary of Findings

The queuing characteristics at the outpatient department of KATH Polyclinic were analyzed using a Multi-server-single channel queuing Model.

It can be seen from table 4.36 that the Assessment Centre recorded the highest number of patient arrivals (5.63 minutes per patient) on Thursday at the outpatient department of KATH Polyclinic whiles the least arrival rate of 2.56 minutes per patient was recorded on Monday At the Assessment Centre.

Table 4.36: **The Mean Arrival Rates of Patients at the Various Sections**

Days	Records Section	Assessment Center	Consulting Rooms
Mondays	2.8 mins/patient	2.56 mins/patient	2.8 mins/patient
Tuesdays	3.63 mins/patient	4.69 mins/patient	4.44mins/patient
Wednesdays	4.25 mins/patient	4.56 mins/patient	4.38 mins/patient
Thursday	3.44 mins/patient	5.63 mins/patient	3.88 mins/patient
Fridays	3.75 mins/patient	3.25 mins/patient	5.38 mins/patient

Table 4.37: **The Mean Service Rate of Servers at the Various Sections**

Days	Records Section	Assessment Center	Consulting Rooms
Mondays	6.38 mins/patient	2.67 mins/patient	6.6 mins/patient

Tuesdays	5.63 mins/patient	6.00 mins/patient	5.42 mins/patient
Wednesdays	4.63 mins/patient	5.50 mins/patient	5.56 mins/patient
Thursday	5.06 mins/patient	6.13 mins/patient	5.23 mins/patient
Fridays	5.06 mins/patient	4.81 mins/patient	6.10 mins/patient

It can be seen from table 4.37 that the Consulting Rooms recorded the highest mean service rate of 6.6 minutes per patient on Monday while the least mean service of 2.67 minutes per patients was recorded at the assessment center also on Monday.

It is clear from table 4.38 that the consulting rooms had the highest mean combined rate all the servers (19.6 minutes per patient) on Monday, while the least mean combined rate of 2.67 minutes per patient was recorded at the assessment also on Monday.

Table 4.38: **The Combined Mean Rate of the Servers at the Various Sections**

Days	Records Section	Assessment Center	Consulting Rooms
Mondays	12.76 minutes	2.67 minutes	19.6 minutes
Tuesdays	11.25 minutes	6.00 minutes	16.26 minutes
Wednesdays	9.26 minutes	5.50 minutes	15.69 minutes
Thursday	10.13 minutes	6.13 minutes	15.69 minutes
Fridays	10.12 minutes	4.81 minutes	18.30 minutes

The analysis of the mean combined rate of the servers at all the three sections show that the consulting rooms were the busiest of all with a mean combine rate of 85.54 minutes per patient for the period of the study. This was followed by the records section 53.52 minutes per patient and lastly 25.11 minutes per patient for the assessment centre.

The mean arrival rate of patients on all the days was approximately 5 patients per minute which was less than the mean service rate of patients at all the centers. Even though the arrival rate was marginally lower than the mean service rate this resulted in progressive build up of queues at all the centers due to the fact that the utilization factor in all the cases were above 50%. What this means is that a waiting line would be formed which would increase indefinitely pushing extra load on the limited facilities available. The average number of

patients in the queue was higher at the assessment centers on all the days. The average number of patients in the queue and in the system on Monday was 24 and 25 respectively. Even though patients had to wait on the average 9.74 minutes in the queue they had to spend on the average 21 minutes in the system to received
attention.

The consulting rooms recorded the lowest number of patients in the queue and in the system. The lowest number of patients in queue at the consulting rooms was recorded on Thursday, while the highest was recorded on Friday.

Chapter 5

CONCLUSION AND RECOMMENDATIONS

5.1 Conclusion

Objectives of the study were to determine the average number of arrivals at the outpatients departments of the polyclinic, the average service time of patients at various sections of the outpatient department, the average time a patient spends waiting for services at the various sections, the average number of patients present at the outpatient department of the clinic as well as the utilization factor of the entire hospital system.

1. The results show that the Assessment Centre recorded the highestnumber of patient arrivals on Thursday while the least arrival rate of patients was recorded on Monday at the Assessment Centre.

2. The study also showed that there was only one assessment centre atthe polyclinic serving all the patients that arrived at the facility compelling them to join long queues for their biological data to be taken. Patients had to wait for a long time in the queue at the assessment centre before receiving service.

3. It was also found from the study that the facilities at the polyclinicwere inadequate compelling patient to join long queues. It was revealed

that there were only two records centres, one assessment centre and three consulting rooms. This tends to put the doctors and nurses as well as ancillary staff under stress and hence compels them to dispose of patients without in-depth investigation or treatment, which often leads to patient's dissatisfaction.

5.2 Recommendations

Queue theory is a useful statistical technique for solving peculiar problems. Its applications in the organization are indispensable. The queuing problems encountered at KATH Polyclinic are similar to what is encountered in other government hospitals across the country. Excessive waste of time in the hospitals or health centers may lead to patients' health complications and in some cases eventual death which can be avoided. As a result, it is recommended that:

1. More doctors should be deployed to the hospital so as to convert thesingle-channel queuing units to multi-channel queuing units.
2. It is also recommended that more health care centers should be createdto take care of all categories of patients in the community.
3. Again, more paramedical officers should be deployed to these centers.This will take care of patients' preliminary tests or service before they see the doctors. This will reduce the service time spent by the doctors in attending to patients and hence the service efficiency.

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Appendix A

Figure 5.1: Questions For Participations
A.png

(1) What is your current position/title ....

(2) Can you identify predictable periods of peak days for various services in the outpatient department? Yes ☐ No ☐

(3) If yes, what are the peak days periods?

(i) Mondays ☐

(ii) Tuesdays ☐

(iii) Wednesdays ☐

(iv) Thursdays ☐

(v) Fridays ☐

(4) Do you forecast demand and use the results for planning? Yes ☐ No ☐

If no, why..

....

(5) Are there any approaches to alleviate outpatient overcrowding that you have tried and found particularly unsuccessful? Yes ☐ No ☐

(6) If yes, which of the following approaches will you adopt?

(i) Increase staff strength ☐

(ii) Cut patient admissions ☐

(iii) Use first come first serve and turn away late comers ☐

(iv) Create more care centers ☐

(v) None ☐

Figure 5.2: Questions For Participations
A1.png

(7) In the event of a major disaster, what kinds of immediate actions would be most important to make significant to surge capacity?

(i) Cancel surgeries ☐

(ii) Early discharge/transfer ☐

(iii) Use of non-clinical space for overflow ☐

(8) What sorts of public policy initiatives would you consider most important to support or improve outpatient delivery?

(i) …………………………………………………………………………………………………

(ii) …………………………………………………………………………………………………

(iii) …………………………………………………………………………………………………

Thank you again for you participation.

Appendix B

Figure 5.3: Format For Recording The Operating Characteristics At The Various Sections

B.png

Performance Parameters	Records Section	Assessment Center	Consulting Rooms
Mean arrival rate λ			
Mean service rate μ			
Mean combined rate of all servers $k\mu$			
Utilization factor of the entire system ρ			
Probability of zero patients in the system P_0			
Average number of patients in the queue L_q			
Average number of patients in the system, L_s			
Average number of patients in the system, L_s			
Average waiting time in the queue W_q			
Average time a patient spends in the system W_s			

Format for Recording the Mean Arrival Rates of Patients

Days	Records Section	Assessment Center	Consulting Rooms
Mondays			
Wednesdays			
Fridays			

Format for Recording the Mean Service Rate of Servers at the Various Sections

Mean service rates	Mondays	Wednesdays	Wednesdays
Records section			
Assessment centre			
Consulting rooms			

Format for Recording Arrival of Patients at the Various Centres

Arrival of patients	Mondays	Wednesdays	Wednesdays
	Tally	Tally	Tally
Records section			
Assessment centre			
Consulting rooms			

Figure 5.4:
B1.png

Utilization Factor Rate	Records Section	Assessment Center	Consulting Rooms
Mondays			
Wednesdays			
Fridays			

Appendix C

Figure 5.5: KEY FORMULAS

<u>Key Formulas</u>

1. F10: = F$5^E10 (copied down)
2. G10: = E10*G9 (copied down)
3. H10: = H9 + (F10/G10) (copied down)
4. F5: = B5/B6
5. F6: = INDEX (G9:G109, B7 +1)
6. B10: = 1/B5
7. B11: = 1/B6
8. B12: = B7*B6
9. B15: = B5/B12
10. B16: = (INDEX (H9:H109, B7) + (((F5^B7)/F6)*((1)/ (1 – B15))))^(-1)
11. B17: = B5*B19
12. B18: = (B16*(F5^B7)*B15)/(INDEX(G9:G109, B7 + 1)*(1 – B15)^2)
13. B19: = B20 + (1/B6)
14. B20: = B18/B5
15. B24: = IF (B23 <= B7, ((F5^B23)*B16)/ INDEX (G9:G109, B23+1), ((F5^B23)*B16) / (INDEX (G9:G109, B7+2)*(B7^(B23-B7))))

Figure 5.6: Spreadsheets For Multi-Server Operating Characteristics

	A	B	C	D	E	F	G	H
1	**Queuing Analysis: Multiple servers**							
2								
3	**Inputs**				**Working Calculations, mainly for Po calculation**			
4	Time Unit	hour						
5	Arrival Rate (Lambda)	5	Patients / Mins		Lambda/mu	0.714285714		
6	Service Rate per server (mu)	7	Patients / Mins		s!	2		
7	Number of Servers (s)	2	servers					
8					**n**	**(λ/μ)^n**	**n!**	**Sum**
9	**Immediate Calculations**				0	1	1	1
10	Average Time between arrivals	0.200000	Minutes		1	0.714285714	1	1.714286
11	Average service time per server	0.142857	Minutes		2	0.510204082	2	1.969388
12	Combined service rate (s*mu)	14	Patients / Mins		3	0.364431487	6	2.030126
13					4	0.260308205	24	2.040973
14	**Performance Measures**				5	0.185934432	120	2.042522
15	Rho (average server utilization)	0.357143			6	0.132810309	720	2.042706
16	Po (Probability that the system is empty	0.473684			7	0.094864506	5040	2.042725
17	Ls (average number in the system)	0.818713	Patients		8	0.067760362	40320	2.042727

18	Lq (averag number waiting in the queue)	0.104428	Patients		9	0.048400258	362880	2.042727
19	Ws (average time in the system	0.163743	Minutes		10	0.034571613	3628800	2.042727
20	Wq (average time in the queue	0.020886	Minutes		11	0.024694009	39916800	2.042727
21					12	0.017638578	479001600	2.042727
22	**Probability of a specific number of customers in the system**				13	0.012598984	6227020800	2.042727
23	Number	5			14	0.008999275	87178291200	2.042727
24	Probabilty	0.001835			15	0.006428053	1.30767E+12	2.042727
25					16	0.004591467	2.09228E+13	2.042727
26					17	0.003279619	3.55687E+14	2.042727
27					18	0.002342585	6.40237E+15	2.042727
28					19	0.001673275	1.21645E+17	2.042727
29					20	0.001195196	2.4329E+18	2.042727
30					99	3.41448E-15	2.40857E+20	2.042727
31					100	2.43891E-15	2.40857E+22	2.042727