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KUMASI

SOLUTION TO BUS SCHEDULING PROBLEM

A CASE STUDY OF METRO MASS TRANSIT LIMITED - KUMASI

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DECLARATION

This thesis is the result of research work conducted by **GRACE AJAO (PG3007509)** of the Industrial Mathematics Programme in the Department of Mathematics (Institute of Distance Learning), Kwame Nkrumah University of Science and Technology, under the supervision of Dr. E. Osei- Frimpong.

I further certify that this thesis has not been submitted in any form to this University or elsewhere for the award of a degree or diploma. Other works which served as sources of information have been duly acknowledged by reference to the authors.

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ABSTRACT

The problem of bus scheduling involves determining the optimal route structure, span of service, and service frequencies for the transit agency and assigning vehicles to the routes. This involves considering cycle times, number of vehicles, timed transfers, layover time and locations, recovery time, and any difference in weekday and weekend services (Wren and Rousseau, 1995).

The problem is common due to its direct application to problems arising in industry for example, vehicle routing and vehicle assignment and also for their contribution to the solution methods for integer programming problems. Several exact algorithms based on branch and bound, dynamic programming and heuristics have been proposed to solve the Scheduling Problems.

This study formulates and solves the scheduling problem of the Metro Mass Transit unit in the Ashanti region with respect to how scheduling should be managed for the intra city, inter urban/rural urban and intercity routes. The use of integer linear programming, specifically the branch and bound method is incorporated in this study. The objective is to find an optimal bus schedule for the routes plied by Metro Mass Transit buses.

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DEDICATION

This work is dedicated to the Almighty God for his grace.

I also dedicate it in honour of my late father, **Reuben Ajao** and my mother, **Agnes Boateng** for the numerous sacrifices they made for me to reach this stage in life. Their encouragements and prayers have never been in vain. My success is yours' and the crown is also for you. May God richly bless and keep you, Dad wherever you are.

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CHAPTER ONE

INTRODUCTION

Public transportation planning covers a very wide research area. From the design of networks to the scheduling of vehicles and rostering of crews, the process of generating a public transportation system has been approached from many sides.

In Ghana, road transport is easily accessible and affordable yet not enough to satisfy all customers at a given time, hence the long queues at most bus stops in urban areas and even rural areas where few vehicles operate. Though buses are able to convey a large number of the passengers at a time, the issue of efficiency matters a lot to the passenger.

One major problem of transportation is the nature of our roads. Drivers have to choose a suitable route whenever there is traffic jam or poor road condition.

In order to ensure that both passengers and transport owners are fairly served, transport companies may engage in intensive planning so as to consider all the potential problems and find solutions to them.

An urban public transit system planning involves making decisions on number of buses, routes, and passenger demand rate as well as time table for the transports. Confronted by traffic congestion, urban parking problems and increasing pollution, car drivers might consider switching to public transit if they had an affordable and good quality system at their disposal. It is the duty and goal of transit agencies to provide such conditions, by adequately adjusting their systems, so as to maximize the quality of service to users while minimizing the costs.

1.1 BACKGROUND OF THE STUDY

The bus transit system has become an essential part of our daily life, especially in large developed urban cities. It has been a supporting industry for the nation to meet the goals of improving mobility, protecting the environment and saving energy (Qiao, 2008).

Considering the ever increasing demand for public transport, most bus operators and their customers encounter inconvenient situations which may sometimes provide unsatisfactory services. In practice, the bus operators would consider their desire to provide a good overall level of service as well as maximizing their profit. Therefore they do not decide the routes directly from the customers' demand. Most buses also do not have a fixed arrival and departure time, causing passengers to be stranded at bus stations. Also drivers may be indisposed when they are supposed to be at post. This situation has offered the opportunity for investigating one of the solutions to the discussed problem.

The Metro Mass Transit Limited (M.M.T.) is a local bus company which is identified to explore the practices of scheduling. Established in October 2003, the MMT, held by both the government of Ghana and private investors had a mission to provide an efficient urban mass transport system in Ghana through the use of buses (www.metromass.com). The company's aim was to operate an effective and affordable transport system in an economical sustainable way in Ghana and with this, was able to implement three bus service systems.

Currently, these three bus services are run by the company, namely;

- Bus Rapid Transit System - designed only for the congested roads in Ghana; these are presently the main corridors of Accra and Kumasi.

- Urban Service - operates in any greater urban area connecting central bus terminals with city outskirts and provides upon that medium-distance transportation to villages in the surrounding of a regional capital.
- Rural Bus services - This long distance rural bus service of MMT operate mainly on rough roads. Because of the long journey, the rural service has a low but solid frequency.

The company's bus depot in Kumasi has eight DAF buses, forty-six VDL Neoplan City buses, thirty-two VDL Commuter buses and fourteen VDL Jonckheere buses that operate on six intercity routes; twelve inter urban / rural urban routes and thirteen intra city routes.

1.1.1 ROAD TRANSPORT

Transportation involves the conveyance of people and goods from one place (source) to another place (destination) by means of air, land, sea, among others. Transport on roads can be roughly grouped into two categories: transportation of goods and transportation of people (Wikimedia foundation, 2010). For instance, when transporting goods, factors such as type of goods, weight and volume of individual shipment, and the coverage distance are considered. This helps to identify the appropriate vehicle to use.

While a van or pickup truck may be used for short distances, light and small shipments, a truck is believed to be more appropriate for large shipments. However, people are transported on roads either in individual cars or automobiles or in mass transit/public transport by bus.

Road transport in Ghana may be grouped into four main segments, namely urban, express, rural-urban and rural services. The demand for urban passenger transport is mainly by residents commuting to work, school and other economic, social and leisure activities (www.mrt.gov.gh).

1.1.2 HISTORY OF ROAD TRANSPORT

Modern roads tend to follow the structure established by previous roads, as it was the case for the modern European road network (especially in Italy, France and Britain) that follows the structure established by the Roman road network centuries before (Jean-Paul and Slack, 1998).

Before wheeled vehicles emerged, trails were used to move from one hunting territory to another. The need for the construction of better roads arose as a result of the emergence of wheeled vehicles in order to support additional weight. The first major road system was established by The Roman Empire from 300 BC and onwards, mainly for economic, military and administrative reasons. It relied on solid road engineering methods, including the laying of foundations and the construction of bridges (Jean-Paul and Slack, 1998).

Following the fall of the Roman Empire in the 5th century, integrated road transportation fell out of favor as most roads were locally constructed and maintained. Because of the lack of maintenance of many road segments, land transport became a very hazardous activity. It is not until the creation of modern nation-states in the 17th century that national road transportation systems were formally established by John Loudon McAdam, (1756-1836).

Road development accelerated in the first half of the 20th century. By the 1920s, the first all-weather transcontinental highway, the Lincoln Highway, spanned over 5,300 km between New York and San Francisco. All road transport modes have limited potential to achieve economies of scale. This is due to size and weight constraints imposed by governments and also by the technical and economic limits of engines. In most jurisdictions, trucks and buses have specific weight and length restrictions which are imposed for safety reasons.

In addition, there are serious limits on the traction capacities of cars, buses and trucks because of the considerable increases in energy consumption that accompany increase in the vehicle weight. For these reasons the carrying capacities of individual road vehicles are limited (Jean-Paul and Slack, 1998).

1.1.3 ADVANTAGES OF ROAD TRANSPORT

Road transport has significant advantages over the other transports. Some of the advantages are:

- Drivers are not restricted in making route choices once a network of roads is provided.
- The capital cost of vehicles is relatively small. For instance, the cost of an aeroplane is more expensive than a car. It is obvious that very few wealthy people can own a train or a ship.
- The speed of vehicles is relatively high, but the major constraint is speed limits imposed by government.
- Road transport provides door to door service for both passengers and freight.

1.1.4 SCHEDULING

In our everyday activities, one may encounter situations where too many tasks or events have to be undertaken simultaneously. In such situation it is quite difficult to know which tasks to implement first and at what time. It is therefore prudent to assign specific resource to tasks so as to achieve optimal result. Identifying a sequence of tasks to be performed and subsequently allocating optimal specific resources to the tasks is termed as scheduling.

According to Belletti *et al*.(1985) and Hagberg (1985) the objective of scheduling is to achieve trade-offs between conflicting goals which minimize costs and time such as waiting time, process time and inventory cost .

1.1.5 APPLICATION OF SCHEDULING

Scheduling is applied in many real life situations such as transportation, library, and hospital staff among others. Some of the applications of scheduling have been briefly explained below:

Airline Scheduling

Airline scheduling involves the planning of various destinations, assigning duties to crew, determining fares and so on. Unlike road transport, airlines do not encounter any traffic jams or bad roads, rather one of their major problems in unfavourable climatic conditions.

One objective of the airline scheduling problem is to decide for each flight, how to develop the route to follow and cancellation, if any, according to company priorities. The problem restrictions are due to capacities of departures and arrivals in airports and the number of flights that may fly within a sector, at a given time.

Vehicle Scheduling

The vehicle scheduling problem, arising in public transport bus companies, addresses the task of assigning buses to cover a given set of timetabled trips taking into consideration the various depots, vehicle types and other factors. This problem which could also be called the routing problem is such that each bus is allocated to a route to serve and a schedule is set for each vehicle starting and returning to the depot after serving a sequence of trips (Joubert, 2007).

Usually, the goal of bus scheduling is to find the minimum number of buses needed to meet the level of service required. Developing a convenient bus system depends heavily on an efficient and fast scheduling process of the bus system (Qiao, 2008).

Crew Scheduling

This scheduling process involves assigning tasks to bus drivers. This process is both labor intensive and time consuming since it includes collecting the data, and scheduling vehicles and crews. It takes the schedulers a significant amount of time to come up with a new schedule and it is impossible to change the current schedule on a short notice.

1.2 PROBLEM STATEMENT

Metro Mass Transit Limited (MMT) is a bus company whose service is highly patronized by people in and outside the Kumasi metropolis. Most people prefer the services of MMT because of relatively cheaper fares compared to other transport services.

Although a fixed number of buses are assigned to the various routes, it is the aim of the company to satisfy all customers as and when they arrive. However, it is observed that there are seasons when buses on particular routes are either highly patronized or less patronized. In the former situation, some passengers may have to wait long hours until bus returns from trip, whereas in the latter situation buses have to wait for long hours until they are full.

When this happens the Company may not be able to realize its expected daily revenue from the affected buses. Passengers on the other hand may be frustrated as a result. There is therefore the need to optimize the number of buses operating in a day in order to provide efficient and effective service to customers.

1.3 OBJECTIVES

The main objective of the study is to develop a bus schedule for Metro Mass Transit Limited, specifically:

i) To formulate an Integer Programming approach to schedule buses to routes.

ii) To find an optimal bus schedule for the inter city, inter urban/rural urban and intra city routes.

1.4 METHODOLOGY

This study will employ the following methods:

a) *Data collection and survey:* Data on bus types, capacity, various routes and number of buses will be obtained from the statistics department of the company through questioning and observation.

b) *Integer programming method:* The Branch and Bound algorithm which best solves scheduling problems, will be employed. This algorithm applies the Simplex method to solve for feasible solution to determining the minimum number of buses to operate on a route whilst ensuring minimum number of daily buses in operation.

c) *Computer programming:* The problem will be solved using Quantitative Manager for windows.

d) Related literature will be obtained through search on the internet. Other references will be made at the KNUST library and the Mathematics Department's library.

1.5 SIGNIFICANCE OF THE STUDY

The inadequate number of public transport buses in Ghana, coupled with the nature of some of our road networks poses a lot of problems to passengers. In most bus stations, passengers have to wait in queues for very long hours during certain periods of the day before they get to their destinations.

Meanwhile some buses are less utilized due to the routes they ply and the demand for buses during certain periods. Although the operations of Metro Mass Transit Limited (MMT) buses are mostly based on passenger demand, the solution of the bus scheduling problem by the use of integer programming method will assist MMT and other Transport Service Providers to provide a better schedule for the buses in order to benefit both the customers and the companies involved.

1.6 ORGANIZATION OF THE THESIS

Chapter one gives an introduction on the research work by giving a brief background study to the problem under study. It also states the main objectives of the work.

Chapter two discusses the various literature and research works which make use of integer programming and are relevant to this work.

Chapter three deals with the methodology used for the entire study whereas chapter four discusses the results of the analysis and the model used.

Findings, conclusions and recommendations are discussed in Chapter five.

CHAPTER TWO

LITERATURE REVIEW

This chapter reviews the existing literature about scheduling of vehicles for public transportation systems.

Salzborn (1972) proposed a mathematical model for the bus scheduling problem. For a given passenger arrival rate, the problem was to determinate the bus departure rate as a function of time. The primary objective of the study was to minimize the number of buses and a secondary criterion was the minimization of the passenger waiting time. The results showed that during the peak period loads represented actual passengers, but at off-peak times the actual loading was much lower. Thus, it was often desirable to reduce the number of buses that was in operation during off-peak periods.

Park (2005) applied Genetic algorithms and simulation in optimizing bus schedules in an urban transit network. Based on the mode of arrival of buses, two different algorithms were considered. First, a simple Genetic algorithm combined with problem with specific operators was used to determine optimized headways in a case where buses arrive following a deterministic process. On the other hand, a simulation based Genetic algorithm was used to optimize both headways and slack times in cases where buses arrive stochastically. Problem of specific genetic algorithm operators include coordinated headway generator, crossover and mutation.

Salzborn (1980) investigated some rules for scheduling a bus system consisting of an inter-town route linking a string of interchanges each of which was the center of a set of feeder routes. He presented the requirements for the inter-town route and feeder route scheduling under pre-determined parameters.

Hall (1985) developed a model for scheduling vehicle arrivals at transportation terminals where vehicles were randomly delayed en route and evaluated the optimal slack time when vehicles were delayed according to an exponential probability distribution. The results showed that coordinating arrivals with departures was most important when the headway was large relative to average vehicle delay.

White (1972) studied the class of dynamic transshipment problems. These are transportation problems that are characterized by the movement of vehicles and goods from location to location over time. Such movements can be represented by a network. The author states that if no directed cycles exist in this network, then an inductive algorithm can be used to optimize the flow of a homogeneous commodity for a linear cost function. The inductive algorithm employs dynamic programming within an out-of-kilter framework. This algorithm can be modified to handle networks in which there are directed cycles.

The problem was formulated as an asymmetric traveling salesman problem. Chen and Kallsen (1988) considered a school bus routing and scheduling problem. The routing aspect of the problem is concerned with the determination of a stop-to-stop route to be traversed to each school by each bus. The scheduling aspect is concerned with the determination of times at all bus stops for each bus. The objective is to minimize the number of buses required in operation, fleet travel time, and to balance the bus loads. The authors developed an expert system approach which was programmed in Turbo PROLOG for use on an IBM/XT and was applied to rural county school system in Alabama.

Yan (1988) presented a heuristic method for scheduling of trucks from many warehouses to many delivery points subject to constraints on truck capacity, traveling time, and loading and unloading time. He considered the truck scheduling problem faced by STARLINK, a warehousing and Distribution Company based in Hong Kong. This heuristic method was used to build a complete schedule. In each step of the method, two things can happen, a delivery point may be inserted into the set of partial routes, or a delivery point may be moved in the partial solution to another position in the set of routes.

Ferland and Fortin (1989) investigated the problem of scheduling vehicles with sliding time windows. They used a heuristic approach to tackle this problem. The approach was based on the identification of pairs of tasks offering good opportunity costs for reducing the overall cost, and searching for ways to modify the starting times in order to permit them to be linked. This method was first developed for the vehicle scheduling with time windows problem, and then modified to deal with the sliding time windows.

Balakrishnan (1993) described three heuristics for designing an efficient (cost effective) route for the vehicle routing problem with soft time windows. Appropriate penalties are incurred for violated time windows. Upper limits are imposed on the penalty and the waiting time permitted at any customer location. A number of assumptions were considered:

- the fleet considered is homogenous and stationed at a single depot and
- the penalty is assumed to be a linear function of the amount of time window violation

The author concluded that the results obtained from a number of benchmark problems showed that by permitting violations of certain customer time window constraints, it could be possible to considerably reduce both the number of vehicles required and/or the total route distances while controlling both customer penalties and waiting times.

Baaj and Mahmassani (1995) described a hybrid route generation heuristic algorithm for network route design. The route generation algorithm (RGA) determines a set of routes that correspond to different trade-offs between user and operative costs. It starts by determining initial set of skeletons and expands them to form transit routes. This process of expansion continues until a pre-specified minimum percentage of total demand can be satisfied. Once a set of routes is generated, the routes are analyzed taking into account the assignment of demand to the transit network.

Gao, Sun and Shan (2003) proposed a continuous equilibrium network design model where the attention was mainly on setting optimal transit line frequencies. A bi-level programming technique with an upper-level problem and a lower-level problem has been used for this transit network design problem. In the upper-level problem the objective function is to minimize the total deterrence of the transit system and cost caused by frequency setting. The lower-level model is a transit equilibrium assignment model that is used to describe the path alternative activities to transit users. A heuristic solution based on sensitivity analysis is designed to solve this model to obtain optimal frequencies that optimize the systems performance.

Haghani and Banihashemi (2002) proposed a heuristics approach for solving a large-scale vehicle-scheduling problem with route time constraints. In this work a new formulation for multi depot vehicle scheduling (MDVS) and multi depot vehicle scheduling problem with route time constraints (MDVSRTC) have been proposed.

To solve a medium size MDVSRTC problem it provides an exact and heuristic solution procedure that cannot solve a real time problem.

Hence for a real-world application, they have proposed a solution procedure that reduces the size of a large-scale problem by decreasing the number of trips and by decreasing the number of variables. It is shown that the solution obtained from proposed strategy has decreased the number of vehicles required and also the operating costs.

Wren and Wren (1995) proposed a genetic algorithm for solving a public transport driver-scheduling problem. The genetic algorithm proposed in this work uses a new crossover operator. It is shown that the algorithm would produce more efficient results than the presented existing method in terms of quality of result and time taken to obtain the schedule.

Beasley and Cao (1996) presented an algorithm for crew scheduling problem based upon the lagrangean relaxation of a linear integer-programming problem, together with a sub-gradient optimization and tree search procedure. The problem faced by many schedulers, is how to set departure times in the transition segments between adjacent time periods. Using the common average headway rule may result in over-crowding.

Palma and Lindsey (2001) analyzed optimal timetables for a given number of vehicles on a single transit line. In this method it is assumed that the preferred travel times and unit schedule delay costs vary from person to person. Here two models are considered: line model and the circle model.

In the line model, preferred travel times of the individuals are distributed over a segment of the day and rescheduling trips between days is impossible. But in the circle model the preferred travel times are distributed round the clock and rescheduling trips between days is possible. The analysis for both the models proceeds in two steps. The first step is to determine for an arbitrary timetable of vehicles, which individuals will travel on which vehicles. The second step is to determine the timetable that minimizes total schedule delay costs given the behavior of individuals identified in step one. This model can be best applied to create timetables when the objective of the transit planners is to reduce the riders delay costs.

Yan and Chen (2002) developed a solution algorithm to produce timetables and bus routes/schedules for inter-city bus carriers. Urban bus and inter-city bus operations differ in terms of their scheduling practices and demand arrival patterns, mainly due to the fixed time schedule set for the latter, and the rough service frequency set for the former.

In this research they developed a model for inter-city carriers with given passenger trip demand, bus fleet size and related cost data. The model demanded the optimal management of both bus and passenger movements in the network through the systematic manipulation of direct bus trips, multi-stop bus trips, and passenger transfer operations, utilize data as the projected passenger trip demand, the available fleet size, the bus operating speed, the station turn-around time, the passenger trip ticket fare, and the related cost data.

Mathematically, the model is formulated as a mixed integer multiple commodity network flow problem. This method of constructing timetables can be better applied when the objective is to maximize the system profits.

In a follow-up study Ceder (2002), three different procedures are proposed and analyzed for better matching the passenger demand with a given timetable while attempting to minimize the number of departures. Procedure 1 produces departure times with evenly spaced headways while considering smooth transition between adjacent hours. Procedure 2 determines departure times such that, in average sense, vehicles will carry on even loads (equal to desired occupancy) at hourly maximum load points.

Procedure 3 derives the departure times such that, in average sense, the on-board passenger load will not exceed desired occupancy, and will be equal to desired occupancy at each individual vehicle max load point. All these three procedures are applicable to situations when the transit planners want to have balanced loads on all the buses to prevent over crowding.

According to Ceder et al. (2001), the importance of transfers in public transport service is motivated by several operational considerations. In a large public transport network, all the origins and destinations are not connected by a single route and have a number of transfer points. In this case passengers who want to travel between different routes have to change routes at transfer points. If there are a large numbers of such transfer points a "perfect" timetable can be achieved only if the waiting time between the transfer points is minimized.

In a research to create timetables for off period operations by incorporating the waiting times at each transfer point, they defined simultaneous arrivals as the arrival of two buses at the transfer node at the same time. However, their solution seemed to be applicable to only peak period. For non-peak period transit operation, the frequency of buses is generally low and the system is characterized with a certain waiting time.

Unfortunately, transfers involve certain inconveniences connected with discomfort of boarding a new vehicle (necessity of passenger orientation and walking between vehicles on feeder and receiving lines), negative perception of waiting for arrival of a vehicle and existence of some delay during a trip. The elimination of these inconveniences by schedule synchronization to provide an attractive service level with easy access and transfer possibilities is continuously a challenging problem in timetable construction.

Bookbinder and Désilets (1992) proposed transfer optimization in a transit network to minimize the overall inconvenience to passengers. Bus trips are scheduled to depart from their terminal so as to minimize some objective function measuring that inconvenience. A mean disutility function is defined here which is used to evaluate the inconvenience under random bus travel times of a transfer connection. This disutility function $g(w)$ is some function of waiting time, which gives the desirability of a waiting time w, as perceived by the user.

To obtain a heuristic solution, an iterative improvement procedure is used. This procedure starts with an initial solution and looks for improvements by changing the departure times for each route from a set of possible starting times, until no further improvement can be obtained.

Schwartz (1968) considered the problem of determining the routing and timing of movements of barges and towboats to fulfill agreed upon freight movements at minimum fleet expense. The problem was modeled as a linear discrete programming problem. A solution of the model provided the numbers of barges and towboats of each size needed to render the service.

Conley et al. (1968) presented a linear programming formulation for the transport of a homogeneous product from an overseas port through United States ports to over 400 inland destinations. The fleet considered was composed of 50 ships of six types to be assigned to routes between a group of overseas ports and up to seven United States ports. The objective was to minimize the total cost of transporting the product. The linear programming solution indicated that fewer ports should be used.

Stochastic aspects of ship scheduling were addressed by Koenigsberg and Lam (1976). In their model, they studied queueing aspects in a small system of liquid gas tankers operating in closed routes between a small number of terminals. For any particular system, the model could provide the expected number of ships in each stage, the expected number waiting in each stage, and most importantly the expected waiting time in port. Exponential service time distributions were used; however, a series of parallel simulation computations were used to analyze the impact of other distributions.

Schechter (1976) considered a ship routing problem which resembles the vehicle routing problem. The cargoes were collected from various ports to a central transshipment point. The author employed a heuristic that was reverse to that of Clark and Wright (1964), in which the model initially commenced with one ship making all the pickups and added ships until a solution was obtained. The total distance traveled by the ships was minimized.

Ronen (1982) developed basic models for the determination of the optimal speed of one ship for three kinds of legs: income generating leg, positioning (empty) leg, and a leg for which income is related to the speed. The results of this model were applicable to tramp and industrial operators.

Boykin and Levary (1985) developed a simulation based interactive decision support system that could be used for scheduling one chemical tanker. This system was used to evaluate different voyage itineraries, including various steaming speeds.

Miller (1987) investigated the problem of fleet scheduling and inventory resupply encountered by an international chemical company. The company had a fleet of small ocean-going tankers to transport bulk fluid to warehouses worldwide. The author developed an interactive computer model, which was successfully employed to deal with daily scheduling concerns as well as longer range planning problems. A network flow model and a mixed-integer programming model were used to analyze the underlying decision problem.

Perakis and Papadakis (1989) considered the two dimensional minimal time routing problem for a ship traveling from a single origin to a number of ordered destination points. They emphasized that knowing the departure time beforehand could ease the problem tremendously. They derived an optimal bound for the optimal state evolution which significantly reduced the dimensionality of the problem. Finally, they presented numerical examples to validate their methodologies.

Cline et al. (1992) considered the problem of routing and scheduling of buoy maintenance by the United States Cost Guard. They used a best-schedule heuristic for solving a large class of real-world routing and scheduling problems to approach the buoy routing and scheduling problem. For a routing and scheduling problem, the objective is to minimize the cost of the distance traveled as well as the cost of being either early or late at each destination.

This model was used to determine the optimum arrival times for the locations (the best schedule) for any given set of locations to be traversed (that is, for any route), and then it employed this information to locate an optimum route.

Schardy and Wadsworth (1991) developed a computerized system to evaluate (among other things) fuel consumption of naval combat ships. Good estimates of fuel consumption can be utilized to obtain a proper scheduling of resupply activities. This computerized system had been examined and implemented in US naval fleet exercises.

Newell (1971) analyzed the dispatching policies for a transit route which a given number of vehicles might be dispatched at any times and the arrival rate of passengers was a given smooth function of time, typically having one or more peaks. He showed that if the capacity of vehicles was sufficiently large to serve all waiting passengers and the number of vehicles was large, then the optimal flow rate of vehicles and the number of passengers served per vehicle, both varied with time approximately as the square root of the arrival rate of passengers. If the vehicles had limited capacity, their dispatch schedule would be distorted so that certain vehicles were dispatched as soon as they were full.

Abkowitz et al. (1986) proposed headway control strategies as methods for correcting transit service irregularities and reducing passenger wait times at stops and addressed a particular strategy which could be implemented on high frequency route (headways under 10-12 minutes), in which buses were held at a control stop to a threshold headway. They developed an algorithm which yielded the optimal control stop location and optimal threshold headway with respect to a system wait function. They concluded that the headway variation did not increase linearly along a route and that the location of the optimal control stop and threshold value was sensitive to the passenger boarding profile.

Özekici (1987) formulated an analytic model for analyzing and exploiting the relation between the arrival and service processes, with emphasis on the impact of this relation on average waiting times.

The results showed that, when a timetable of the scheduled services is available, the arrival pattern of passengers was not stationary and passengers chose an optimal time to arrive at the bus stop based on the information they had about the timetable and their observation on the service performance.

Banks (1990) studied multi-route transit systems to determine net-benefit maximizing headways, which were subjected to constraints on vehicle capacity, subsidy, and fleet size. Conditions of optimality were derived for the unconstrained case and the various constrained cases.

The relation between optimality conditions based on the assumption of fixed demand and those based on the assumption of variable demand was expressed with terms incorporating the elasticity of demand with respect to frequency of service. The results showed that the magnitude of discrepancies between the true conditions of optimality and their fixed-demand approximations depended on the elasticity of demand and on the distribution of ridership and cycle times among the various routes of the system.

Lee and Schonfeld (1991) developed a numerical model for optimizing slack times for simple systems with transfers between one bus route and one rail line which could work with arrival distributions. Some analytic results were derived for empirical discrete and Gumbel distributions of bus arrival times. Relations between the optimal slack times and headways, transfer volumes, passenger times values, bus operating cost, and standard deviations of bus and train arrivals were also developed numerically using normally distributed arrivals. The results provided some guidelines on desirable slack times and showed that schedule coordination between the two routes was not worth attempting when standard deviations of arrivals exceeded certain levels.

CHAPTER THREE

METHODOLOGY

There are several methods for solving the scheduling problem. One of such methods (specifically the branch and bound) will be used to solve the MMT bus scheduling problem. Before solving the problem, it is important to discuss the linear programming, integer programming problems and solution methods such as dynamic programming and branch and bound in detail.

3.1 LINEAR PROGRAMMING

Linear Programming (LP) is a Mathematical program which involves the maximization or minimization of a linear function subject to linear constraints. It could also be viewed as a quantitative technique for selecting an optimum plan, and is efficient for finding the best solution to a problem that contains many decision variables. Linear Programming is used to solve many problems in vehicle scheduling, job assignment as well as transportation problem.

In solving linear programming problem the desired objective is to maximize some function such as profit, or to minimize some function such as costs. Determination of the optimum objective is usually subject to various constraints or restrictions on possible alternatives. These constraints describe availabilities, limitations, and relationships of resources to alternatives.

3.1.1 DEFINITIONS

Before one can formulate a good linear programming problem, the various components of the model should be identified.

Minimize $c^T x$ (objective function)

Subject to $Ax \geq b$ (constraint)

$x_j \geq 0$, $j \in J$ (non-negativity constraint)

Decision variables: These are variables that represent a number of related quantifiable decisions to be made and their respective values are to be determined. For instance, in the model above, the decision variable is x and its value is to be determined.

Objective function: This is a function that can be expressed as a mathematical function of some decision variables. In the preceding model, $c^T x$ is the objective function and is expressed as $c_1x_1 + c_2x_2 + c_3x_3 + \ldots + c_nx_n$, where c is the coefficient of the decision variables.

Constraints: They are mathematical expressions for the restrictions assigned to decision variables by means of inequalities or equations. They are usually equated to constants at the right-hand side. That is, $Ax \geq b$.

Parameters: These consist of the coefficients of the objective function and constraints as well as constants at the right-hand side of the constraints. The parameters in the model are c, A, and b.

3.1.2 CONVEX SET

Definitions:

1. A *Line Segment* defined by vectors x and y is the set of points of the form

   $\mu x + (1-\mu) y$ for $\mu \in [0, 1]$.

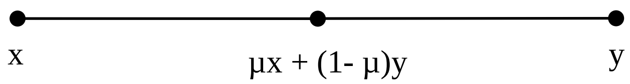

Figure 3.1: A line segment

2. A point on the line segment for which $0 \leq \mu \leq 1$ is called *Interior Point* of the line segment.

3. A subset C of R^n is said to be *Convex* if for any two elements x, y in C, the line segment [x,y] is contained in C. Thus x and y in C imply $\mu x + (1-\mu)y \epsilon C \ for\ all\ 0 \leq \mu \leq 1$ if C is convex.

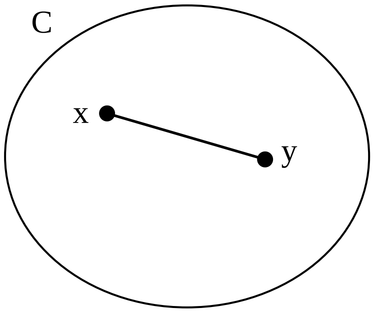

Figure 3.2: Convex set

Sets in R^n are convex if they contain no "hole", "indentation" or "protrusion" and are non-convex otherwise (Amponsah, 2007).

4. A point *u* in a non – empty convex set C is said to be an *Extreme Point* of C if it is not an interior point of any line segment in C. Thus if $u = \alpha x + (1-\alpha)\ y$ for x, y in C and $0 \leq \alpha \leq 1$, then $x = y = u$. For example in a triangle with vertices say EFG the only extreme points are the vertices E, F and G.

Some examples of convex sets are:

i. The intersection of any family of convex sets in R^n.
ii. A closed half-space or open half-space in R^n

3.2 INTEGER PROGRAMMING

This is a special case of linear programming in which all variables are required to take on integer values only (Weisstein, 2010). Whereas some solution methods yield continuous solution, integer programming solution methods implement a systematic procedure that restricts continuous solutions sequentially until an integer solution is attained.

3.2.1 TYPES OF INTEGER PROGRAMMING

Integer Programming (IP) problems could be classified as either Binary, Mixed or Pure integer programming, depending on the nature of values the decision variables are required to take.

Pure Integer Programming: An Integer Programming problem is said to be pure when all decision variables are required to be integers.

Binary Integer Programming: One area of application of integer programming is problems involving a number of interrelated "yes-or-no decisions". For example, there are only two choices to be made when deciding on locating a facility in a particular site. Such decisions could be represented by decision variables that are restricted to just two values, say 0 and 1(binary variables). Thus binary integer programming (BIP) problems (or 0–1 integer programming problems) are IP problems that contain only binary variables.

Example: Minimize $z = \sum^{n}_{j=1} c_j x_j$

Subject to: $\sum^{n}_{k=1} a_{ij} x_j \leq b_i \; (i = 1, 2, 3 ... m)$

x_j = binary (0 or 1)

Mixed Integer Programming (MIP): Another type of integer programming is the Mixed Integer Programming problem in which some, but not all decision variables are restricted to be integers. A Mixed Integer Programming problem results when some of the variables are real-valued (can take on fractional values) and some are integer–valued (Chinneck, 2004). MIP can be used to create a model to schedule the production of products in order to reduce cost.

3.2.2 THE SIMPLEX METHOD

Developed by George Dantzig in 1947 and first published in 1951, the simplex method is an iterative procedure that provides a structured method for moving from one basic feasible solution to another, always maintaining or improving the objective function until the optimal solution is obtained. The iteration process helps to reduce the distance (mathematically and graphically) from the objective function. The method can be used to solve any linear programming problem provided it is in canonical form.

3.2.3 Initial Basic Feasible Solution

A basic feasible solution is a solution with non – negative basic variables ($n \geq 0$). Given a set of m equations in n variables ($n > m$) the basic solution is obtained by setting (n-m) variables equal to zero and solving the resulting system of m equations in m variables. Thus, the *number of non-basic variables* equals the total number of variables *minus* the number of functional constraints (n-m). The remaining m variables are referred to as the basic variables. The *number of basic variables* equals the number of functional constraints (equations) (Amponsah, 2007).

A solution is said to be feasible when all the constraints in a given model are satisfied. A non-degenerate basic feasible solution is a basic solution with $m > 0$. An optimal solution is a feasible solution that has the most favorable value of the objective function where the most favorable value is the largest value for a maximization problem, and the smallest value for a minimization problem.

3.2.4 THE INITIAL SIMPLEX TABLEAU

The simplex tableau is made up of rows and columns with the components defined as follows:

The top (objective row) contains the coefficients of the objective function for variables *j*, represented by c*j*. The next row (variable row) contains the variables of the problem, including slack variables. The problem row (also known as the a_{ij} row) contains coefficient of variable *j* in constraint *i*, with one row for each constraint. Variables which are not in a constraint are assigned zero coefficients.

During each iteration, new problem rows are computed. The right side column of the table b_i represents the right-handside values for constraints *i*. The left-handside column (c_B) contains the coefficients of the basic variables. The values in the z_j row are calculated by multiplying the elements in the c_B column by the corresponding elements in the columns of the matrix and summing them. It is observed that all the z_j values in the initial tableau are zeros but may change in subsequent iterations.

The last row, known as the index row (c_j-z_j) indicates the net contribution per unit of the j^{th} variable. It contains values which indicate whether an optimum solution has been reached. The values are determined by subtracting the appropriate *zj* value from the corresponding objective function coefficient *cj* for that column. The table below represents the format of the initial simplex tableau.

Table 3.1: Format of the Initial Simplex Tableau

	c_j	c_1	c_2	…	c_n	0	0	…	0	
c_B	*Basic Variable*	x_1	x_2	…	x_n	s_1	s_2	…	s_m	*Solution*
0	s_1	a_{11}	a_{12}	…	a_{1n}	1	0	…	0	b_1
0	s_2	a_{21}	a_{22}	…	a_{2n}	0	1	…	0	b_2
.	.	.	.		.	.	.	…	.	.
.	.	.	.		.	.	.	…	.	.
.	.	.	.		.	.	.	…	.	.
0	s_m	a_{m1}	a_{m2}		a_{mn}	0	0		1	b_m
	Zj	0	0	…	0	0	0	…	0	0
	cj-zj	c_1-0	c_2-0	…	c_n-0	0	0	…	0	

In the initial tableau, the coefficients of the basic variables form an identity matrix at the constraint column.

Thus, the basis matrix *B*, formed by *Si's* is given by;

$$\begin{array}{c} \begin{matrix} s_1 & s_2 & s_3 & s_4 \end{matrix} \\ \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \end{array} = I$$

Figure 3.3: Identity matrix formed by basic variable coefficients

3.2.5 MAJOR STEPS IN THE SIMPLEX ALGORITHM

The major steps in the Simplex algorithm are as follows:

Step 1

Given the problem formulation with *m* equations in *n* variables, select the set of *m* variables that yields an initial basic feasible solution.

Maximize z = $c_1x_1 + c_2x_2 + c_3x_3 + \ldots + c_nx_n + 0s_1 + 0s_2 + \ldots + 0s_m$

Subject to $a_{11}x_1 + a_{12}x_2 + \ldots + a_{1n}x_n + s_1 \leq b_1$

$a_{21}x_2 + a_{22}x_2 + \ldots + a_{2n}x_n + s_2 \leq b_2$

. . .

. . .

. . .

$a_{m1}x_1 + a_{m2}x_2 + \ldots + a_{mn}x_n + s_m \leq b_m$

$x_1, x_2, x_3 \ldots x_n, s_1, s_2, \ldots s_m \geq 0$

Step 2

Analyse the objective function to see if there is a non – basic variable that is equal to zero in the initial basic feasible solution, but that would improve the value of the objective function if made positive.

If no such variable can be found, the current basic feasible solution is optimal, and the simplex algorithm stops.

If however, such a variable can be found, the simplex algorithm continues.

Step 3

Using the non – basic variable selected in step 2, determine how large it can become before one of the m variables in the current basic feasible solution becomes zero. The current solution can be improved by replacing a variable in the basis with a current non-basic variable.

The variable entry criterion is based on the values of the cj-zj row. At this stage it is expected that the iteration terminates if all the *cj-zj* values are zeros and negatives (for minimization problem) and zeros and positives (for maximization problem).

Suppose the initial tableau below represents a minimization problem, then the variable corresponding to the most positive (largest) *cj-zj* value is selected for entry (Amponsah, 2007).

Table 3.2: Non-basic variable entering the basis

	c_j	**50**	40	0	0	0	
c_B	*Basic Variable*	x_1	x_2	s_1	s_2	s_3	*Solution*
0	s_1	3	5	1	0	0	150
0	s_2	0	1	0	1	0	20
0	s_3	8	5	0	0	1	300
	Zj	0	0	0	0	0	0
	cj-zj	**[50]**	40	0	0	0	

It is observed that 50 is the largest *cj-zj* value and its corresponding variable is x_1, therefore x_1 enters the basis.

The variable leaving the basis is determined by:

1) dividing the right hand side values by the coefficients of x_1 in the constraint column respectively. That is; b_1/ a_{11}, $b_2/ a_{21}, \ldots, b_m / a_{m1}$.
2) selecting the minimum ratio. If $b_1/ a_{11} < b_2/ a_{21} < \ldots < b_m / a_{m1}$ then select b_1/ a_{11}.
3) removing the basic variable corresponding to the minimum ratio. Remove s_1 since it corresponds to b_1/ a_{11}. For example, in Table 3.3, s_3 is removed since it corresponds to b_3/ a_3.

Table 3.3: Removal of basic variable

	c_j	50	40	0	0	0		
c_B	*Basic Variable*	x_1	x_2	s_1	s_2	s_3	*Solution*	*Ratio*
0	s_1	(3)	5	1	0	0	150	150/3=50
0	s_2	0	1	0	1	0	20	-
0	$\mathbf{s_3}$	(8)	5	0	0	1	300	300/8=**37.5**
	Zj	0	0	0	0	0	0	
	cj-zj	50	40	0	0	0		

We note that, a_{ij} values that are zeros or negatives are ignored when computing the ratios. Comparing the two remaining ratios, it is observed that the minimum is 37.5 and it corresponds to s_3 in the basis, therefore s_3 and its coefficient are removed and replaced by x_1 and its coefficient.

Step 4

Perform Gauss-Jordan elimination procedure on the rows to solve the system of equations in the constraints in terms of the new basic variables.

The first step is to locate the pivot element as the number that represents the intersection of the pivot row and the pivot column.

In table 3.4, it is observed that the arrowed number (8) is chosen as the pivot element.

Table 3.4: Selection of the pivot element

	c_j	50	40	0	0	0		
c_B	Basic Variable	x_1	x_2	s_1	s_2	s_3	Solution	Ratio
0	s_1	3	5	1	0	0	150	150/3=50
0	s_2	0	1	0	1	0	20	-
0	s_3	**(8)**	5	0	0	1	300	300/8=**37.5**
	Zj	0	0	0	0	0	0	
	$cj\text{-}zj$	50	40	0	0	0		

The following are the row elements of Table 3.4.

R1: {3, 5, 1, 0, 0, 150}

R2: {0, 1, 0, 1, 0, 20}

R3: {**8**, 5, 0, 0, 1, 300}

The next step is to apply row reduction method these elements to obtain new elements as shown in Table 3.5.

Table 3.5: Row reduction method

	Cj	50	40	0	0	0	
c_B	Basic Variable	x_1	x_2	s_1	s_2	s_3	Solution
0	s_1	**0**	25/8	1	0	-3/8	75/2
0	s_2	**0**	1	0	1	0	20
50	x_1	**1**	5/8	0	0	1/8	75/2
	Zj	50	125/4	0	0	25/4	1875
	$cj\text{-}zj$	0	[35/4]	0	0	-25/4	

In Table 3.4, eight (8) is chosen as the pivot element and is reduced to one (1) as shown in Table 3.5, with its corresponding column and row elements reduced.

The new row elements in Table 3.5 are obtained as follows;

R1* = R1 – 3(R3*)

= {3, 5, 1, 0, 0, 150} – {3 × (1, 5/8, 0, 0, 1/8, 75/2)}

= {**0**, 25/8, 1, 0, -3/8, 75/2}

R2* = R2

= {**0**, 1, 0, 1, 0, and 20}

R3* = 1/8 (R3)

=1/8 × {8, 5, 0, 0, 1, 300} = {**1**, 5/8, 0, 0, 1/8, 75/2}

Given that a feasible solution exists and that the optimal value of the objective function is finite, the simplex algorithm as outlined in the preceding steps will lead to an optimal solution in a finite number of iterations.

Table 3.6: The final tableau showing optimal solution

	cj	50	40	0	0	0	
c_B	*Basic Variable*	x_1	x_2	s_1	s_2	s_3	Solution
40	x_2	0	1	8/25	0	-3/25	**12**
0	s_2	0	0	-8/25	1	3/25	**8**
50	x_1	1	0	-1/5	0	1/5	**30**
	Zj	50	40	14/5	0	26/5	**1980**
	cj-zj	**0**	**0**	**-14/5**	**0**	**-26/5**	

In the final tableau, all the c_j-z_j values are now zeros and negatives which means that the optimal solution has been obtained with **(x_1, x_2, s_1, s_2, s_3)=(30, 12, 0, 8, 0)** and

z =1980.

3.2.6 SOME COMPLICATIONS IN APPLYING THE SIMPLEX METHOD

In applying the simplex method to solve an LP problem, there are some complications that may be encountered. Some of those complications are discussed as follows:

Tie for Entering Variables

In the application of the simplex method to solve LP problem, we may encounter a situation in which there is a tie between two or more entering variables. This occurs when two or more variables have the same $cj–zj$ values. When this happens any of the variables are selected arbitrarily for entry into the basis.

Tie for Leaving Variables – Degeneracy

Another situation that can be encountered during solution process is degeneracy. Since the leaving variable is determined by minimum ratio the variable corresponding to the minimum non-negative ratio is selected. However, when there is more than one variable having the same minimum non-negative ratios, one of them is selected arbitrarily to leave the basis. When this happens the pivoting process drives that variable to zero. Other variables may be driven to zero but will remain in the basis. Thus a solution with one or more basic variables equal to zero is known as a degenerate solution. An example is found in Table 3.5 above.

Non positive right – hand side values

In case one or more values at the right hand side of a constraint are non - positive, selecting a slack variable to the basis may result in a negative initial basic variable. The best way of solving such complication is to multiply both sides of the constraint by negative (-) and reverse the inequality sign. The new inequality may be changed to equality by adding slack or artificial variables.

Unconstrained variables

One important point about the simplex method is that all the decision variables must be positive at every iteration. However, in a situation where one or more of the variables are unconstrained in sign (that is, either positive or negative), we may express each of the unconstrained variables as the difference of two positive variables and replace the unconstrained variables.

For example, for x_j (unconstrained) we define the new variable $x_j' \geq 0$ and $x_j'' \geq 0$ and let $x_j = x_j' - x_j''$. This means that x_j is replaced with $x_j' - x_j''$ in the objective function and the constraints of the model.

3.2.7 TERMINATION OF THE SIMPLEX METHOD

The simplex method will always terminate in a finite number of steps with an indication that a unique optimal solution has been obtained or that one of the following special cases has occurred. These special cases are;

Alternate optimal solution

All basic variables have zero *cj-zj* values. If there is a variable whose *cj-zj* is 0 but is not a basic variable, then that 0 can be treated as the most positive and use that variable as the entering variable. When this happens the basic variables will change but the total *zj* will remain the same.

Infeasible solution

In finding the optimal solution by the simplex method infeasibility occurs when at the optimal solution the right-hand side values are all zeros and negatives. The indication that no feasible solution is possible will be given by the fact that at least one of the artificial variables will be positive at the basis, instead of zero.

Unbounded solution

If at any iteration no departing variable can be found corresponding to entering variable, the value of the objective function can be increased indefinitely, i.e., the solution is unbounded.

An unbounded solution occurs in a situation such as solving a maximization problem, where a simplex tableau contains a non-basic variable x_j with c_j-z_j row value strictly greater than zero (i.e c_j-z_j > 0) and all of the a_{ij} elements in its column being zeros or negatives. When this happens, the ratio test for the variable removal criterion will indicate that the denominators of the ratios are negatives and zeros resulting in only zeros and negative ratios. In such situation the simplex method will only terminate with the indication that the entering basic variable is allowed to assume a value of infinity.

Multiple (infinite) solutions

If in the final tableau, one of the non-basic variables has a coefficient 0 in the z-row; it indicates that an alternative solution exists. This non-basic variable can be incorporated in the basis to obtain another optimal solution. Once two such optimal solutions are obtained, infinite number of optimal solutions can be obtained by taking a weighted sum of the two optimal solutions.

3.3 DYNAMIC PROGRAMMING

Dynamic programming is an extension of the basic linear programming technique and involves breaking the problem into a set of smaller problems and then reassembling the results of the analysis. It is a useful mathematical technique for making a sequence of interrelated decisions i.e. decisions that must be made in sequence and that influence future decisions in the sequence.

In contrast to linear programming, there is no standard mathematical formulation of dynamic programming problem. Rather, dynamic programming is a general type of approach to problem solving, and the particular equations used must be developed to fit each situation. It starts with a small portion of the original problem and finds the optimal solution for this smaller problem. It then gradually enlarges the problem, finding the current optimal solution from the preceding one, until the original problem is solved in its entirety.

The procedure involves partial optimization of a portion of the sequence and then connection of the optimized portion to the next in line until the entire sequence is optimized. Thus, the final result is the sum of the result of the immediate decision plus the optimal result from all future decisions.

Dynamic programming can be categorized into two main approaches:

- Deterministic problems, where the state at the next stage is completely determined by the state and policy decision at the current stage.
- Probabilistic problems where the state at the next stage is not completely determined by the state and policy decision at the current stage. Rather, there is a probability distribution for what the next state will be. However, this probability distribution still is completely determined by the state and policy decision at the current stage.

3.4 BRANCH AND BOUND METHOD

The branch and bound method is a basic technique for solving integer and discrete programming problems where a larger problem is divided into smaller sub problems that can be solved independently. Described as a systematic refinement of the feasible space that will lead to a feasible optimal integer solution, the method is based on the observation that the enumeration of integer solutions has a tree structure (Al-Salamah, 2008). The main idea in branch and bound is to grow the tree in stages, ensuring that only the most promising nodes are grown at any stage.

3.4.1 DEFINITIONS IN THE BRANCH AND BOUND METHOD

As we discuss the Branch and Bound method we must first know the meanings of some of the terms that may be encountered in the discussion. Some of the terms are defined in the context of the Branch and Bound method and hence may have a different meaning elsewhere.

Node: A node is any partial or complete solution.

Leaf (leaf node): It is a complete solution in which all of the variable values are known. The leaf nodes have objective function values, which are actual values rather than estimates.

Bud (bud node): It is a feasible or infeasible partial solution.

Bounding function: It is the method of estimating the best value of the objective function that is obtained by growing a bud node further. Only bud nodes have associated bounding function values. The bounding function must underestimate the actual best achievable objective function value for a minimization problem and overestimate the best achievable objective function value for a maximizing problem.

Incumbent: This is the best complete feasible solution found so far. There may not be an incumbent when the solution process begins. In that case, the first complete feasible solution found during the solution process becomes the first incumbent.

Branching: This involves the division of a LP feasible region into two sub regions. The term could be likened to what happens when a bud node is selected for further growth and the next generation of children of that node is created. For each possible value of the next variable one child node is created.

Pruning or fathoming: Is a term applied in the branch and bound method to mean the process of cutting off and permanently discarding sub problems and all their potential children which could never be either feasible or optimal from the branch and bound tree. The subproblems that cannot contain the optimal solution are discarded, thereby decreasing the size of the solution space.

Bounding: The bounding comes in when the bound on the best value attained by growing a node is estimated.

3.4.2 STRUCTURE OF THE BRANCH AND BOUND ALGORITHM

The structure of the Branch and Bound algorithm was first developed by Dakin (1965) based on a pioneering branch and bound algorithm by Land and Doig (1960). This algorithm is a class of exact algorithms for various optimization problems, especially integer programming problems and combinatorial optimization problems.

The Branch and Bound algorithm is represented in a tree like structure with the various nodes and sub nodes as shown in figure 3.2.

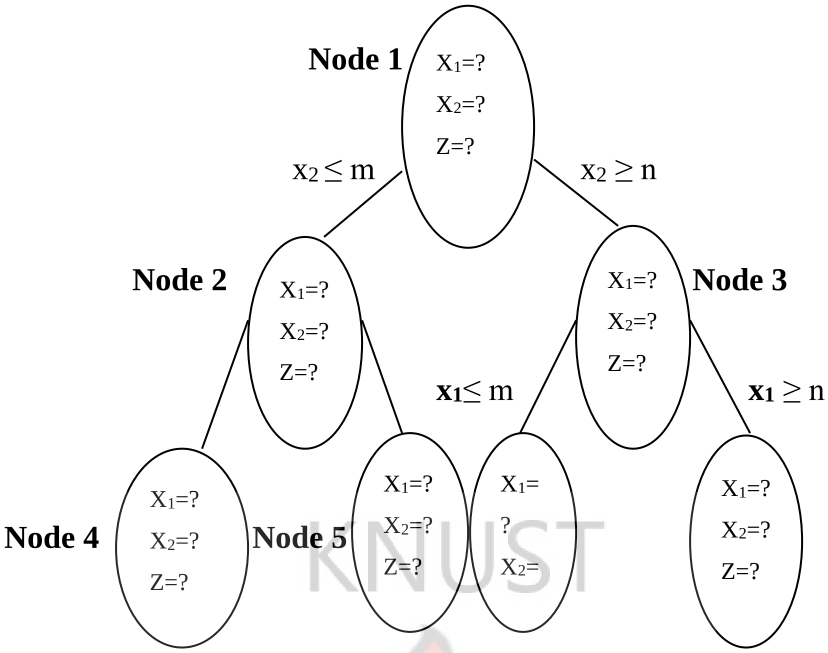

Figure 3.4: Formulation of nodes and sub nodes

3.4.3 STEPS IN THE BRANCH AND BOUND METHOD

Given the Integer Programming problem;

Maximize $z = \sum^{n}_{k=1} c_k x_k$ (1)

Subject to: $\sum^{n}_{k=1} a_{ik} x_k \leq b_i$ $(i = 1, 2, 3... m)$ (2)

$x_k \geq 0$ $(k = 1, 2... n)$ (3)

x_k *integer (for some or all k=1, 2… n)* (4)

The branch and bound solution method is outlined as follows

Step 1: Relaxation

Relax the integrality condition x_k, integer (for some or all $k=1, 2\ldots n$). The LP obtained by deleting the constraints $x \in Z^n$ (or $x \in \{0, 1\}^n$) is called the LP relaxation. The relaxed LP obtained represents the first node in the branch and bound tree.

The optimal value of the objective function is the initial upper bound of the objective function value. Stop when the relaxed LP is infeasible (IP problem is infeasible).

Step 2: Finding optimal solution

To guarantee that we have reached optimality, compare the upper bound values for any currently defined nodes. If the solution at the node with the highest upper bound value is integer, stop the solution is optimal.

When the incumbent solution's objective function value is better than or equal to the bounding function value associated with all of the bud nodes, stop the process.

This means that none of the bud nodes could possibly develop into a better solution than the complete feasible solution.

In other words when there are no remaining sub-nodes, the current *incumbent* becomes the optimal solution and so we stop the process. Otherwise the solution continues until optimality is attained. If the optimal solution yields values that are not all integers but better than the incumbent, then divide this sub problem further and repeat.

For each sub problem, an upper bound on the objective value is calculated which must be greater than or equal to the optimal solution. Upper bounds for a sub problem can be obtained by optimizing the sub problem's LP relaxation.

Step 3: Branching (node expansion)

Among the integer-restricted variables that have a non-integer value in the optimal solution for the LP relaxation of the sub-problem, select the non-integer variable. Let x_j be this variable and x_j* its value. Branch from the node or the subproblem to create two new subproblems by adding the respective constraints $x_j \leq [x_j*]$ and $x_j \geq [x_j*] + 1$

The node selection policy governs how to choose the next bud node for expansion.

There are three popular policies for node selection:

- *Best-first:* choose the bud node that has the best value of the bounding function anywhere on the branch and bound tree. If we are minimizing, this means choosing the bud node with the smallest value of the bounding function; if maximizing choose the bud node with the largest value of the bounding function.
- *Breadth-first:* expand bud nodes in the same order in which they were created.
- *Depth-first:* choose only from among the set of bud nodes just created. Choose the bud node with the best value of the bounding function. Depth-first node selection takes you one step deeper into the branch and bound tree at each iteration, so it reaches the leaf nodes quickly.

Step 4: Bounding

To obtain the bound for each new subproblem, the simplex method (or the dual simplex method when re-optimizing) is applied to its LP relaxation and the value of Z is used for the resulting optimal solution.

3.4.4 SOME FATHOMING RULES

A subproblem could be fathomed (dismissed from further consideration) if either of the following conditions is encountered;

1. The bud node bounding function (optimistic estimator) value is worse than the objective function value for the incumbent. That is, the bound on the subproblem $\leq Z^*$, where Z^* is the current optimal solution (value of Z for the current incumbent).
2. Its LP relaxation has no feasible solutions (If there is no incumbent). When a feasible solution (i.e., no fractional variables remaining) is found, all sub

problems whose upper bounds are lower than this solution's objective value can be discarded. The best known feasible solution represents a lower bound for all sub problems, and only sub problems with an upper bound greater than the global lower bound have to be considered.

3. The optimal solution for its LP relaxation (values for the integer restricted variables) is integer. If this solution is better than the incumbent, it becomes the new incumbent and step one is reapplied to all unfathomed subproblems with the new larger Z^*.

3.5 FLOWCHART OF THE BRANCH AND BOUND ALGORITHM

The stages in the Branch and Bound solution method are represented on a flowchart as shown in figure 3.5 below

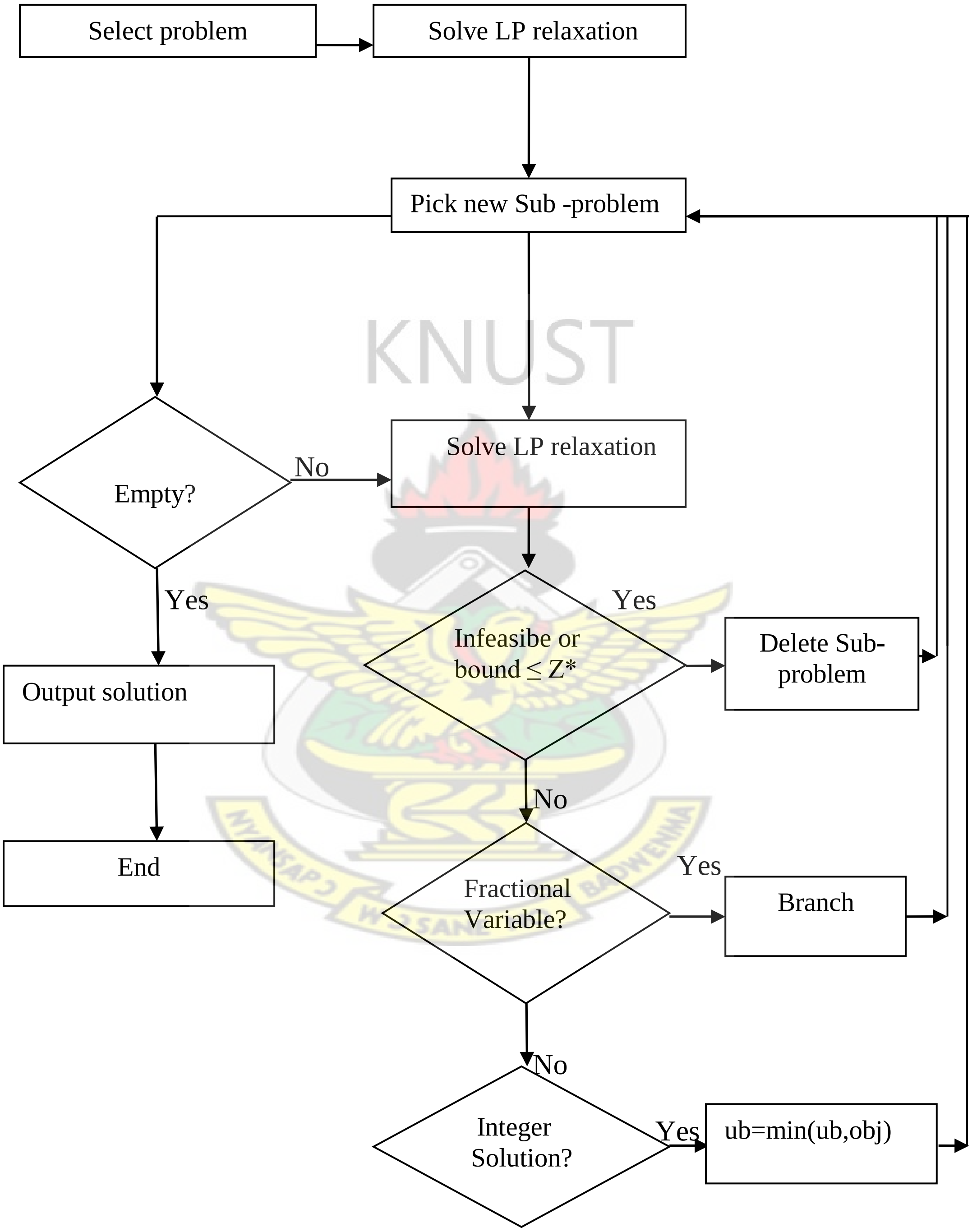

Figure 3.5: Flowchart of the Branch and Bound Algorithm

CHAPTER FOUR

MODEL APPLICATION AND DISCUSSION OF RESULTS

This chapter tackles the modeling of the scheduling problem of the Metro Mass Transit (MMT) section of the Ashanti Region. An integer programming method is applied to develop a schedule system for the company's buses. The computer program will solve and display a new schedule for buses plying each of the three types of routes namely;

i) the inter city routes

ii) the inter urban/rural urban routes

iii) the intra city routes

With one-hundred (100) buses at the depot, twenty-two (22) are allocated for operation on all the inter city with four (4) buses expected to be off duty everyday. Fifty (50) buses are allocated for inter urban / rural urban routes with five (5) buses expected to be off duty everyday. Twenty-eight (28) buses are allocated intra city routes with three (3) expected to be off duty everyday.

4.1 ASSUMPTIONS

The following assumptions were made for the modeling of data for an optimal scheduling of the number of buses needed for operation.

i) There must be at least one bus on each route, whether intercity, inter urban/rural urban or intra city routes.

ii) Each bus is assigned to only one route and cannot be plying more than a single route.

iii) The number of buses on a route should meet the passenger demand on that route.

4.2 DEFINITION OF VARIABLES FOR THE THREE MAJOR ROUTES COMBINED

In modelling the scheduling problem of MMT, the following decision variables are defined as follows:

1) Let X_i $(i = 1, 2, 3, 4, 5)$ be the number of buses needed for the various assignments

   X_1 : inter city routes

   X_2 : inter urban/rural urban routes

   X_3 : intra city routes

   X_4 : off road/inspection

   X_5 : maintenance

2) Let (a, b, c, d and e) represent the respective number of buses available for the various duties

   a (inter city routes) : (22- 4) =18

   b (inter urban/rural urban routes) : (50-5) = 45

   c (intra city routes) : (28-3) = 25

   d (off road/inspection) : 6

   e (maintenance) : 6

3) Let Z be the total number buses operating on all the routes.

4.2.1 MATHEMATICAL FORMULATION FOR THE THREE MAJOR ROUTES COMBINED

Using the variables defined above the linear programming formulation for the three types of routes combined is:

Minimize $Z = X_1+X_2+X_3+X_4+X_5$

Subject to

$X_1 \geq a$, for the inter city route

$X_2 \geq b$, for inter urban/rural urban route

$X_3 \geq c$, for the intra city route

$X_4 \leq d$, off road/inspection

$X_5 \leq e$, maintenance

$X_1, X_2, X_3, X_4, X_5 \geq 1$ and integer

Assigning respective values to the original formulation, the new model becomes:

Minimize $Z = X_1+X_2+X_3+X_4+X_5$

Subject to

$X_1+X_2+X_3+X_4+X_5 \leq 100$

$X_1 \geq 18$

$X_2 \geq 45$

$X_3 \geq 25$

$X_4 \leq 6$

$X_5 \leq 6$

$X_1, X_2, X_3, X_4, X_5 \geq 1$ and integer

The iteration results displayed at appendix yields the optimal results for the scheduling problem involving the three major routes as shown in Table 4.1.

4.2.2 THE SOLUTION TABLEAU FOR THE THREE MAJOR ROUTES COMBINED

Table 4.1: Optimal Schedule of buses for the three major routes combined

Optimal Scheduling of Buses							
	X_1	X_2	X_3	X_4	X_5		RHS
Minimize	1.	1.	1.	1.	1.		
Solution->	19.	45.	27.	0.	0.	Optimal Z->	91.

Inter city route	:	19
Inter urban/rural urban route	:	45
Intra city route	:	27
Off road	:	0
Maintenance	:	0

4.3 DEFINITION OF VARIABLES FOR THE INTER CITY ROUTES

The main aim of the research is to develop a proper schedule for the buses plying the individual inter city routes.

The formulation of the model for the inter city routes transportation of MMT unit is as follows:

Let:

1) X_i (i =1, 2, 3, 4, 5, 6, 7) be the number of buses assigned for the inter city routes.

   X_1: Aflao route

   X_2: Hohoe route

   X_3: Yeji route

   X_4: Sefwi - Juaboso route

   X_5: Sefwi - Asanwinso route

   X_6: Sefwi - Wiawso route

   X_7: off road/maintenance

2) (a, b, c, d, e, f, g) be the number of buses needed for the inter city routes.

a: Aflao	:	2
b: Hohoe	:	2
c: Yeji	:	1
d: Sefwi – Juaboso	:	2

e: Sefwi – Asanwinso : 1

f: Sefwi – Wiawso : 10

g: for maintenance : 4

3) *Z* is the total number of buses available for assignment at the depot.

4.3.1 FORMULATION OF THE MODEL FOR THE INTER CITY ROUTES

The objective function and constraints of the model are given written as:

Minimize $Z = X_1+X_2+X_3+X_4+X_5+X_6+X_7$

Subject to

$X_1+X_2+X_3+X_4+X_5+X_6+X_7 \leq 22$

$X_1 \geq a$, for Aflao route

$X_2 \geq b$, for Hohoe route

$X_3 \geq c$, for Yeji route

$X_4 \geq d$, for Sefwi - Juaboso route

$X_5 \geq e$, for Sefwi - Asanwinso route

$X_6 \geq f$, for Sefwi - Wiawso route

$X_7 \leq g$, for Maintenance

$X_1, X_2, X_3, X_4, X_5, X_6, X_7 \geq 1$ and integers

After applying the data, the model becomes;

Minimize $Z = X_1+X_2+X_3+X_4+X_5+X_6+X_7$

Subject to

$X_1+X_2+X_3+X_4+X_5+X_6+X_7 \leq 22$

$X_1 \geq 2$

$X_2 \geq 2$

$X_3 \geq 1$

$X_4 \geq 2$

$X_5 \geq 1$

$X_6 \geq 10$

$X_7 \leq 4$

$X_1, X_2, X_3, X_4, X_5, X_6, X_7 \geq 1$ and integers

4.3.2 THE SOLUTION TABLEAU FOR INTER CITY ROUTES PROBLEM

The intercity routes allocation problem gives an optimal scheduling as shown in Table 4.2 below:

Table 4.2: Optimal Scheduling of buses for Inter City Routes

Optimal Scheduling of Buses									
	X1	X2	X3	X4	X5	X6	X7		RHS
Minimize	1.	1.	1.	1.	1.	1.	1.		
Solution->	2.	2.	1.	2.	2.	10.	0.	Optimal Z->	19.

The new allocation of the buses from Kumasi to the individual inter city routes is;

Aflao route : 2

Hohoe route : 2

Yeji route : 1

Sefwi Juaboso route : 2

Sefwi Asanwinso route : 2

Sefwi Wiawso route : 10

Off road/maintenance : 0

4.4 DEFINITION OF VARIABLES FOR INTER URBAN/RURAL URBAN ROUTES

The formulation for the inter- urban/rural urban routes of MMT has the following definition of variables:

1) Let X_i , (i =1, 2, 3,…13) be the number of buses needed for the individual inter urban routes assignment;

X_1: Sefwi Bekwai route

X_2: Techiman route

X_3: Dunkwa Offin route

X_4: Bibiani route

X_5: New Edubiase route

X_6: Bomfa Ofoasi route

X_7: Bomfa Anyinasi route

X_8: Bomfa Anumso route

X_9: Bomfa Brofoyedu route

X_{10}: Kumawu Woraso route

X_{11}: Obuasi route

X_{12}: Kyekyewere route

X_{13}: off road/maintenance under the inter urban/rural urban routes

2) Let (*a, b, c, d,…,m*) be the number of buses available for rural urban routes

a (Sefwi Bekwai)	:	1
b (Techiman)	:	6
c (Dunkwa Offin)	:	8
d (Bibiani)	:	7
e (New Edubiase)	:	1

f (Bomfa Ofoasi) : 1

g (Bomfa Anyinasi) : 1

h (Bomfa Anumso) : 1

i (Bomfa Brofoyedu) : 1

j (Kumawu Woraso) : 1

k (Obuasi) : 16

l (Kyekyewere) : 1

m (off road/maintenance) : 5

3) Let Z be the total number of buses for the inter urban route

4.4.1 FORMULATION OF THE MODEL FOR THE INTER URBAN/RURAL URBAN ROUTES

Minimize $Z = X_1+X_2+X_3+X_4+X_5+X_6+X_7+X_8+X_9+X_{10} +X_{11}+X_{12}+X_{13}$

Subject to

$X_1+X_2+X_3+X_4+X_5+X_6+X_7+X_8+X_9+X_{10} +X_{11}+X_{12}+X_{13} \leq 50$

$X_1 \geq a$, Sefwi Bekwai

$X_2 \geq b$, Techiman

$X_3 \geq c$, Dunkwa Offin

$X_4 \geq d$, Bibiani

$X_5 \geq e$, New Edubiase

$X_6 \geq f$, Bomfa Ofoasi

$X_7 \geq g$, Bomfa Anyinasi

$X_8 \geq h$, Bomfa Anumso

$X_9 \geq i$, Bomfa Brofoyedu

$X_{10} \geq j$, Kumawu Woraso

$X_{11} \geq k$, Obuasi

$X_{12} \geq l$, Kyekyewere

$X_{13} \leq m$, off road/maintenance

$X_1, X_2, X_3 \ldots X_{13} \geq 1$ and integers

After applying the data, the model becomes;

Minimize $Z = X_1+X_2+X_3+X_4+X_5+X_6+X_7+X_8+X_9+X_{10}+X_{11}+X_{12}+X_{13}$

Subject to

$X_1+X_2+X_3+X_4+X_5+X_6+X_7+X_8+X_9+X_{10}+X_{11}+X_{12}+X_{13} \leq 50$

$X_1 \geq 1$

$X_2 \geq 6$

$X_3 \geq 8$

$X_4 \geq 7$

$X_5 \geq 1$

$X_6 \geq 1$

$X_7 \geq 1$

$X_8 \geq 1$

$X_9 \geq 1$

$X_{10} \geq 1$

$X_{11} \geq 16$

$X_{12} \geq 1$

$X_{13} \leq 5$

$X_1, X_2, X_{3\ldots,} X_{13} \geq 1$ and integers

4.4.2 THE SOLUTION TABLEAU FOR INTER URBAN/RURAL URBAN ROUTES

Table 4.3: Optimal Scheduling for Urban/Rural Urban Routes

Optimal Scheduling of Buses															
	X1	X2	X3	X4	X5	X6	X7	X8	X9	X10	X11	X12	X13		RHS
Minimize	1.	1.	1.	1.	1.	1.	1.	1.	1.	1.	1.	1.	1.		
Solution->	1.	6.	8.	7.	1.	1.	1.	1.	1.	1.	16.	1.	0.	Optimal Z->	45.

Kumasi – Sefwi Bekwai route : 1

Kumasi -Techiman route : 6

Kumasi – Dunkwa on Offin route : 8

Kumasi - Bibiani route : 7

Kumasi - New Edubiase route : 1

Kumasi – Bomfa Ofoasi route : 1

Kumasi –Bomfa Anyinasi route : 1

Kumasi – Bomfa Anumso route : 1

Kumasi – Bomfa Brofoyedru : 1

Kumasi – Kumawu Woraso route : 1

Kumasi - Obuasi route : 16

Kumasi – Kyekyewere : 1

Maintenance/ off- road : 0

4.5 DEFINITION OF VARIABLES FOR THE INTRA CITY ROUTES

1) Let X_i, (i =1, 2, 3,…14) be the number of buses needed for the individual intra city routes assignment;

X_1: Bronwire route

X_2: Mamponteng route

X_3: Buoho route

X_4: Kromoase route

X_5: Abuakwa route

X_6: Asuofua route

X_7: Kronum route

X_8: Ejisu route

X_9: Bekwai route

X_{10}: Kuntanasi route

X_{11}: Tetrem route

X_{12}: Esuowin route

X_{13}: Effiduase route

X_{14}: off road/maintenance

2) Let (a, b, c, d,…,n) be the number of buses available for rural urban routes

a (Bonwire)	:	3
b (Mamponteng)	:	3
c (Buoho)	:	2
d (Kromoase)	:	2
e (Abuakwa)	:	3
f (Asuofua)	:	3
g (Kronum)	:	1
h (Ejisu)	:	2
i (Bekwai)	:	2
j (Kuntanasi)	:	1
k (Tetrem)	:	1
l (Esuowin)	:	1

m (Effiduase) : 1

n (maintenance) : 3

3) Let Z be the total number of buses for the intra city routes

4.5.1 FORMULATION OF MODEL FOR INTRA CITY ROUTES

Minimize $Z = X_1 + X_2 + X_3 + X_4 + X_5 + X_6 + X_7 + X_8 + X_9 + X_{10} + X_{11} + X_{12} + X_{13} + X_{14}$

Subject to

$X_1+X_2+X_3+X_4+X_5+X_6+X_7+X_8+X_9+X_{10}+X_{11}+X_{12}+X_{13}+X_{14} \le 28$

$X_1 \ge a$, for Bonwire

$X_2 \ge b$, for Mamponteng

$X_3 \ge c$, for Buoho

$X_4 \ge d$, for Kromoase

$X_5 \ge e$, for Abuakwa

$X_6 \ge f$, for Asuofua

$X_7 \ge g$, for Kronum

$X_8 \ge h$, for Ejisu

$X_9 \ge i$, for Bekwai

$X_{10} \ge j$, for Kuntanasi

$X_{11} \ge k$, for Tetrem

$X_{12} \ge l$, for Esuowin

$X_{13} \ge m$, for Effiduase

$X_{14} \le n$, for maintenance

Introducing the data yields the following model;

Minimize $Z = X_1+X_2+X_3+X_4+X_5+X_6+X_7+X_8+X_9+X_{10}+X_{11}+X_{12}+X_{13}+X_{14}$

Subject to

$X_1+X_2+X_3+X_4+X_5+ X_6+X_7+X_8+X_9+X_{10} +X_{11}+X_{12}+X_{13}+X_{14} \leq 28$

$X_1 \geq 3$

$X_2 \geq 3$

$X_3 \geq 2$

$X_4 \geq 2$

$X_5 \geq 3$

$X_6 \geq 3$

$X_7 \geq 1$

$X_8 \geq 2$

$X_9 \geq 2$

$X_{10} \geq 1$

$X_{11} \geq 1$

$X_{12} \geq 1$

$X_{13} \geq 1$

$X_{14} \leq 3$

$X_1, X_2, X_3 \ldots X_{14} \geq 1$ and integers

4.5.2 THE SOLUTION TABLEAU FOR INTRA CITY ROUTES PROBLEM

Table 4.4: Optimal Schedule of Intra City Routes

Optimal Scheduling Solution																
	X1	X2	X3	X4	X5	X6	X7	X8	X9	X10	X11	X12	X13	X14		RHS
Minimize	1.	1.	1.	1.	1.	1.	1.	1.	1.	1.	1.	1.	1.	1.		
Solution->	3.	3.	2.	2.	3.	3.	2.	2.	2.	1.	1.	2.	1.	0.	Optimal Z->	27.

After the computation the optimal schedule of the buses is as follows:

Kumasi-Bonwire	:	3
Kumasi-Mamponteng	:	3
Kumasi-Buoho	:	2
Kumasi-Kromoase	:	2
Kumasi-Abuakwa	:	3
Kumasi-Asuofua	:	3
Kumasi-Kronum	:	2
Kumasi-Ejisu	:	2
Kumasi-Bekwai	:	2
Kumasi-Kuntanasi	:	1
Kumasi-Tetrem	:	1
Kumasi-Esuowin	:	2
Kumasi-Effiduase	:	1

4.6 DISCUSSION

The computational results in Table 4.2 show that the total number of buses on assignment has been decreased from one-hundred (100) to ninety-one (91). This reduction in number of buses results from the removal of all the twelve (12) buses on maintenance and off road and adding three more buses to the eighty-eight (88) buses that were strictly assigned to the three major routes.

Due to the business activities that go on in some of the towns, demand for the buses may increase during market seasons. Therefore only one bus assigned to a route such as Sefwi-Asanwinso would be inadequate for such seasons, hence an addition of one bus making two buses. There total number of buses has thus reduced from twenty-two (22) to nineteen (19).

According to results in Table 4.3 it is observed that the schedule of buses for the individual inter urban routes however remains the same as what was provided in Appendix A (Table *a* (2)) but no bus is kept for maintenance and off road. This means the fifty (50) buses have reduced to forty-five (45).

The result for the intra city route schedule shows that the number of buses has reduced from 28 to 27 but there is also an increase in the number of buses along the Kronum and Effiduase routes. These two additional buses come from the buses for maintenance.

Since the results obtained for all the routes indicate that no bus should be left for maintenance and off road, the nine (9) buses remaining after re-scheduling should support the routes which are identified to experience unexpected shortage.

Finally, comparing the way buses are assigned to routes at MMT and the computational results obtained, it could be deduced that the Branch and Bound method is the best alternative method for scheduling the buses.

CHAPTER FIVE

CONCLUSION AND RECOMMENDATION

This chapter discusses the conclusion of the study as well as the needed recommendation for an improvement of the general delivery of service of MMT-Kumasi depot as far as bus scheduling is concerned.

5.1 CONCLUSION

The MMT-Kumasi depot's bus scheduling problem for the three major routes combined as well as the individual routes is formulated as an integer programming problem. The study concludes with an optimal number of 91 buses; a minimum of nineteen (19) buses are needed to operate on the intercity routes, 45 buses to operate on the interurban/rural urban routes and twenty seven (27) buses for the intra city routes.

It is observed that, out of the optimal number of buses (19) available for the intercity routes, only a bus is needed for Kumasi-Yeji route whereas 2 buses are needed for each of the routes namely; Kumasi - Sefwi Asanwinso, Kumasi -Hohoe, Kumasi -Sefwi Juaboso, and Kumasi – Aflao. However, 10 buses are needed for the Kumasi-Sefwi Wiawso route. This result may be due to the high demand for buses on that route.

The computational result for the inter urban/rural urban routes reveals that, for optimality, 16 buses should be assigned to the Kumasi-Obuasi route, 8 buses to the Kumasi-Dunkwa on Offin route, 7 buses to the Kumasi-Bibiani route and 6 buses to the Kumasi-Techiman route with a bus each allocated to the rest of the routes in the inter urban/rural urban category.

The study also concludes with an optimal bus schedule of 3 buses each for the Bonwire, Mamponteng, Abuakwa, and Asuofua routes. The Buoho, Kromoase, Kronum, Ejisu, Bekwai, and Esuowin routes in the intra city category are allocated 2 buses each whereas a bus each is assigned to Kuntanasi, Tetrem and Effiduase routes.

The study shows that, urban bus and inter-city bus operations differ in terms of their scheduling practices and demand arrival patterns, mainly due to the seemingly fixed time schedule set for the latter, and the rough service frequency set for the former as indicated by Yan and Chen (2002). But the model as proposed in this research only demanded the optimal management of buses and not considering passenger movements.

5.2 RECOMMENDATION

This work considers only one depot, specifically Kumasi depot. It is therefore recommended that further research work should be carried out on other depots of MMT using all available routes as basis for a multi depot bus scheduling approach. While the focus of our research is on assigning buses to distinct routes, we provide an overview on bus assignment to multiple routes for future research.

Another line of future research concerning this thesis is the investigation of the applicability of the solution alternatives to the bi-level formulation including transfer and bus capacity constraints; in particular, the special structure of the bi-level formulation may be exploited in order to improve the efficiency of the solution methods.

Finally, since the bus operations at MMT is basically demand based, the management in the long term has to devise a computerised system to collect data on the daily demand for their services, so as to generate proper scheduling procedure to reduce inconsistencies in bus assignment.

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APPENDICES

APPENDIX A

Table a (1): The intercity routes and their number of allocated buses

NO.	INTER CITY ROUTES From	To	NUMBER OF BUSES
1	Kumasi	Aflao	2
2	Kumasi	Hohoe	2
3	Kumasi	Yeji	1
4	Kumasi	Sefwi Juaboso	2
5	Kumasi	Sefwi Asanwinso	1
6	Kumasi	Sefwi Wiawso	10

Table a (2): The inter urban routes and their number of allocated buses

NO.	INTER URBAN / RURAL URBAN ROUTE		NUMBER OF BUSES
1	Kumasi	Sefwi Bekwai	1
2	Kumasi	Techiman	6
3	Kumasi	Dunkwa Offin	8
4	Kumasi	Bibiani	7
5	Kumasi	New Edubiase	1
6	Kumasi	Bomfa Ofoase	1
7	Kumasi	Bomfa Anyinasi	1
8	Kumasi	Bomfa Anumso	1
9	Kumasi	Bomfa Brofoyedu	1
10	Kumasi	Kumawu Woraso	1
11	Kumasi	Obuasi	16
12	Kumasi	Kyekyewere	1

Table a(3): The intra city routes and their number of allocated buses

NO.	INTRA CITY ROUTES		NUMBER OF BUSES
	From	To	
1	Kumasi	Bonwire	3
2	Kumasi	Mamponteng	3
3	Kumasi	Buoho	2
4	Kumasi	Kromoase	2
5	Kumasi	Abuakwa	3
6	Kumasi	Asuofua	3
7	Kumasi	Kronum	1
8	Kumasi	Ejisu Oduom	2
9	Kumasi	Bekwai	2
10	Kumasi	Kuntanasi	1
11	Kumasi	Tetrem	1
12	Kumasi	Esuowin	1
13	Kumasi	Effiduase	1

APPENDIX B

Table b(1): Bus types and their capacities

BUS TYPES	NO. OF BUSES	BUS CAPACITY
DAF	8	63
VDL Neoplan City		
(1st generation)	17	37
(2nd generation)	29	47
VDL Commuter	32	63
VDL Jonckeere	14	62

Table b(2): Time(coverage and departure) and Current fares per distance for the routes of MMT buses

	NO.	ROUTE	ONE WAY DISTANCE (KM)	COVERAGE TIME/HR	FARES (GH ¢)	FIRST DEPARTURE
INTER CITY ROUTES	1	Aflao	455	8	11.00	9pm
	2	Hohoe	397	4	8.00	8am
	3	Yeji	226	3	4.20	8am
	4	Sefwi Juaboso	225	3	4.00	5am
	5	Sefwi Asanwinso	197	3	3.50	5am
	6	Sefwi Wiawso	155	2.5	3.00	5am
INTER URBAN/RURAL URBAN ROUTES	1	Sefwi Bekwai	132	2	2.50	5am
	2	Techiman	120	2	2.50	5am
	3	Dunkwa Offin	96	2	2.00	5am
	4	Bibiani	93	1.4	1.50	5am
	5	New Edubiase	90	2	1.50	5am
	6	Bomfa Ofoase	76	2	1.70	5am
	7	Bomfa Anyinasi	69	2	1.20	5am
	8	Bomfa Anumso	67	2	1.20	5am
	9	Bomfa Brofoyedu	67	2	1.30	5am
	10	Kumawu Woraso	61	2	1.00	5am
	11	Obuasi	61	1.4	1.30	5am
	12	Kyekyewere	53	1	1.00	5am

All the routes provided start from Kumasi.

APPENDIX C

Table c(1): Initial tableau for the scheduling of Buses for the three Major Routes

Optimal Scheduling of Buses							
	X1	X2	X3	X4	X5		RHS
Minimize	1.	1.	1.	1.	1.		
Constraint 1	1.	1.	1.	1.	1.	<=	100.
Constraint 2	1.	0.	0.	0.	0.	>=	18.
Constraint 3	0.	1.	0.	0.	0.	>=	45.
Constraint 4	0.	0.	1.	0.	0.	>=	25.
Constraint 5	0.	0.	0.	1.	0.	<=	6.
Constraint 6	0.	0.	0.	0.	1.	<=	6.
Constraint 7	1.	1.	1.	-1.	-1.	>=	0.
Constraint 8	-1.	0.	1.	0.	0.	>=	0.

Table c(2): Initial tableau for the scheduling of Buses for Inter City Route

Initial Scheduling of Buses									
	X1	X2	X3	X4	X5	X6	X7		RHS
Minimize	1.	1.	1.	1.	1.	1.	1.		
Constraint 1	1.	1.	1.	1.	1.	1.	1.	<=	22.
Constraint 2	1.	0.	0.	0.	0.	0.	0.	>=	2.
Constraint 3	0.	1.	0.	0.	0.	0.	0.	>=	2.
Constraint 4	0.	0.	1.	0.	0.	0.	0.	>=	1.
Constraint 5	0.	0.	0.	1.	0.	0.	0.	>=	2.
Constraint 6	0.	0.	0.	0.	1.	0.	0.	>=	1.
Constraint 7	0.	0.	0.	0.	0.	1.	0.	>=	10.

Constraint 8	0.	0.	0.	0.	0.	0.	1.	<=	4
Constraint 9	1.	1.	1.	1.	1.	1.	-1.	>=	0.

Table c(3): Initial Tableau for scheduling of Buses for Inter Urban/Rural Urban Routes

Initial Scheduling Solution															
	X1	X2	X3	X4	X5	X6	X7	X8	X9	X10	X11	X12	X13		RHS
Minimize	1.	1.	1.	1.	1.	1.	1.	1.	1.	1.	1.	1.	1.		
Constraint 1	1.	1.	1.	1.	1.	1.	1.	1.	1.	1.	1.	1.	1.	<=	50.
Constraint 2	1.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	>=	1.
Constraint 3	0.	1.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	>=	6.
Constraint 4	0.	0.	1.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	>=	8.
Constraint 5	0.	0.	0.	1.	0.	0.	0.	0.	0.	0.	0.	0.	0.	>=	7.
Constraint 6	0.	0.	0.	0.	1.	0.	0.	0.	0.	0.	0.	0.	0.	>=	1.
Constraint 7	0.	0.	0.	0.	0.	1.	0.	0.	0.	0.	0.	0.	0.	>=	1.
Constraint 8	0.	0.	0.	0.	0.	0.	1.	0.	0.	0.	0.	0.	0.	>=	1.
Constraint 9	0.	0.	0.	0.	0.	0.	0.	1.	0.	0.	0.	0.	0.	>=	1.
Constraint 10	0.	0.	0.	0.	0.	0.	0.	0.	1.	0.	0.	0.	0.	>=	1.
Constraint 11	0.	0.	0.	0.	0.	0.	0.	0.	0.	1.	0.	0.	0.	>=	1.
Constraint 12	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	1.	0.	0.	>=	16.
Constraint 13	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	1.	0.	>=	1.
Constraint 14	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	1.	<=	5.
Constraint 15	1.	1.	1.	1.	1.	1.	1.	1.	1.	1.	1.	1.	-1.	>=	0.

Table c(4): Initial Tableau for scheduling of Buses for Intra City Routes

Initial Scheduling Solution																
	X1	X2	X3	X4	X5	X6	X7	X8	X9	X10	X11	X12	X13	X14		RHS
Minimize	1.	1.	1.	1.	1.	1.	1.	1.	1.	1.	1.	1.	1.	1.		
Const 1	1.	1.	1.	1.	1.	1.	1.	1.	1.	1.	1.	1.	1.	1.	<=	28.
Const 2	1.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	>=	3.
Const 3	0.	1.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	>=	3.
Const 4	0.	0.	1.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	>=	2.
Const 5	0.	0.	0.	1.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	>=	2.
Const 6	0.	0.	0.	0.	1.	0.	0.	0.	0.	0.	0.	0.	0.	0.	>=	3.
Const 7	0.	0.	0.	0.	0.	1.	0.	0.	0.	0.	0.	0.	0.	0.	>=	3.
Const 8	0.	0.	0.	0.	0.	0.	1.	0.	0.	0.	0.	0.	0.	0.	>=	1.
Const 9	0.	0.	0.	0.	0.	0.	0.	1.	0.	0.	0.	0.	0.	0.	>=	2.
Const 10	0.	0.	0.	0.	0.	0.	0.	0.	1.	0.	0.	0.	0.	0.	>=	2.
Const 11	0.	0.	0.	0.	0.	0.	0.	0.	0.	1.	0.	0.	0.	0.	>=	1.
Const 12	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	1.	0.	0.	0.	>=	1.
Const 13	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	1.	0.	0.	>=	1.
Const 14	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	1.	0.	>=	1.
Const 15	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	1.	<=	3.
Const 16	1.	1.	1.	1.	1.	1.	1.	1.	1.	1.	1.	1.	1.	-1.	>=	0.