**KWAME NKRUMAH UNIVERSITY OF SCIENCE AND TECHNOLOGY.**

**INSTITUTE OF DISTANCE LEARNING**

# OPTIMAL LOAN PORTFOLIO

**(A CASE STUDY OF AGRICULTURAL DEVELOPMENT BANK, SUNYANI)**



**BY**

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## DECLARATION

I, Prince Kusi hereby declare that except for reference to other people‘s work, which have duly been cited. This submission is my own work towards the Master of Science degree and that, it contains no material neither previously published by another person nor presented elsewhere.



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# DEDICATION

I dedicate this research work to my wife Juliana Agyeiwaa and our unborn baby, my mother Madam Stella Amponsah, my late father Mr. Joseph Nana Kusi who made my education a success. Also to my siblings; Joyce Kusi, Emmanuel Kusi, Samuel, Kusi and Joseph Nana Kusi. I finally dedicate this work to my son Prince Nana Kusi jnr, my dearest friends Bedi (prof), Buka T, Kalasco,   Ebo, Appiah Kubi and Samuel Acheampong.

# ABSTRACT

Many customers in ADB, Sunyani branch take loans for various reasons some being investment in businesses, agriculture or their wards education. Others also take loans to acquire personal properties such as houses and cars. Due to poor allocation of funds by most banks to prospective loan seekers the banks are not able to maximize their profits. In view of this monies that can be used for social services in the community in which they operate go into bad debt. The main aim of this study is to develop Linear Programming (LP) model to help ADB Sunyani branch in the Brong Ahafo Region to allocate their funds to prospective loan seekers in order to maximize profits. To achieve this aim, a secondary data were extracted from the annual reports and financial statements of the bank. Based on these empirical data, LP model was formulated. A computerized software application called LP solver (Lips) based on Revised Simplex Algorithm was used to solve the problem. The results from the model showed that ADB Sunyani branch would be making annual profit of GH¢ 476732.00 on loans alone as against GH¢ 190693.00 in 2011 if they stick to the model. From the study, it was realized that the Scientific method used to develop the proposed model can have a significant increase in the bank's profit margin if put into use.

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# CHAPTER ONE

## 1.0 Introduction

Decision making involves a variety of a course of action among several alternatives. In our everyday life, one has made numerous decisions in a day and many of these decisions must be made in the context of randomness. Banks as well as other firms aim at offering loan facilities to their customers in order to maximize profit. To quantify, understand, and predict the effects of randomness, the mathematical theory of probability and stochastic processes should be developed, (Feldman and Valdez-Flores, 2010). A stochastic process is a random process, which evolves with time. For instance random variables $X_t$, $Y_t$etc depend on time t, where $X_t$ could be the location at time t.

## 1.1 Background to the study

Lending is one of the main activities of banks in Ghana and other parts of the world. This is evidenced by the volume of loans that constitute banks assets and the annual substantial increase in the amount of credit granted to borrowers in the private and public sectors of the economy. According to Comptroller (1998), lending is the principal business for most commercial banks. Loan portfolio is therefore typically the largest asset and the largest source of revenue for banks. In view of the significant contribution of loans to the financial health of banks through interest income earnings, these assets are considered the most valuable assets of banks. A survey in 2006 on the Ghanaian banking sector revealed that loans accounted for about fifty percent(50%) of total bank assets which had increased from forty one point five (41.5%) in 2005 (Appertey and Arkaifie, 2006). In 2007, the figure increased to fifty three percent (53%) of the industry's total assets of GH¢ 7,795.6 million (Infodata Associates, 2009).

The reason why banks give much attention to the lending activity, especially in periods of a stable economic environment, is that a substantial amount of banks income is earned on loans which contribute significantly to the financial performance of banks. A financial report of ADB in 2007, indicated that out of the total interest income of GH¢42,327,367.00 earned in that year, about sixty six point five percent (66.5%) was earned on loans and advances.

The above literature gives ample evidence that healthy loan portfolios are vital assets for banks in view of their positive impact on the performance of banks. Unfortunately, some of these loans usually do not perform and eventually result in bad debts which affect banks earnings on such loans. These bad loans become cost to banks in terms of their implications on the quality of their assets portfolio and profitability. This is because in accordance with banking regulations, banks make provisions for non-performing loans and charge for bad loans which reduce their loan portfolio and income. For example in February, 2009, a Bank of Ghana report revealed that non-performing loans ratio increased from six point four percent (6.4%) in 2007, to seven point seven percent (7.7%) in 2008.

A cursory study of the annual reports and financial statements of banks in Ghana indicate that bad loans are seriously affecting most banks hence necessitating a study into the problem. (ADB, Desk Daily, 2008).

### 1.1.1 Overview of banking system in Ghana

Ghana's financial system is based on a number of banks and non-banking financial institutions, including the Bank of Ghana which as the Central Bank has the responsibility of advising the government on the implementation and control of monetary policies. Their transaction compliesmaking loans with customers and the purchase of investment securities in the market place, (Mensah, 1997).

The Ghanaian financial sector has moved from repressed era to more liberal environment as a response to the international calls of the World Bank and the International Monetary Fund (IMF) several benefits, which are reminiscent of a liberal financial market, have been enjoyed as evidenced by the contribution of the financial liberalization policy to the improved economic growth and development of the Ghanaian economy.Loans are usually granted in a form of contract between the bank and the borrower. The bank will like to know the purpose of the loan and whether the use of the loan will yield results. In this way they become very effective partners in process of economic development. Today modern banks are very useful for the utilization of the resources of the country.

### 1.1.2Loan explained

A loan in terms of small business finance is a sum of money advanced to a business that must be repaid with interest at some point in the future. The lender must bear the risk that the borrower may not repay the loan. The interest rate charge is the price for that risk. A loan is money, classified as debt for temporal use.  Lending organizations might be a commercial bank, credit unions and other lending organization like a thrift institution or an alternative source of loans for businesses. Loans may also be guaranteed by the small business Administration which usually makes them easier to obtain .A loan is made to a small business for many reasons. The loan may be for working capital and purchases of equipment or buildings and land. Interest rates on a loan depend on a verity of factors as do the terms of a business loan. Businesses have to be creditworthy in order to be granted a loan. (www.htt://en.Wikipedia.org/wiki/loan).

Normally, the borrowers pay back the money either in regular installments or partial repayments in an annuity; each installment is the same amount. The loan is generally provided at a cost, called interest on the debt, which provides an incentive for the lender to engage in the loan. In a legal loan, each of these obligations and restrictions is enforced by contract, which can also place the borrower under additional restrictions known as loan covenants. Acting as a provider of loans is one of the principal tasks for financial institutions.

### 1.1.3Organizational profile of ADB



The organizational profile of ADB comprises brief history, ownership and focus, and scope of activities.

#### 1.1.3.1 Brief history of ADB

The ADB was established in 1965 by an Act of Parliament, Act 286 under the name Agricultural Credit and Cooperative Bank to promote and modernize the agricultural sector and other related economic activities through appropriate financial intermediation. The name of the bank was changed to Agricultural Development Bank (ADB) in 1967 by NLC Decree 182. Its functions were further broadened by the passage of the Act of Parliament (Act 352) in 1970.

At this stage the bank's corporate mission centered on the provision of agriculture credit and management projects. However, with the enactment of the Banking Law (PNDC Law 225), the bank considered this event as an opportunity and broadened its corporate mission to include the entire range of financial intermediation without sacrificing its primary function.

#### 1.1.3.2Ownership and focus

The bank is jointly owned by the Government of Ghana and the Central Bank of Ghana with a shareholding structure of fifty two percent (52%) and forty eight (48% ) respectively. Thus, it is pure government bank.

**Vision statement**

The bank's vision is ''Achieving the position of being the largest, prudently managed and the most profitable growth-oriented agricultural development bank in Africa''

**Mission statement**

Its mission is ''ADB is committed to building a strong customer-oriented bank, run by knowledgeable and well-motivated staff, providing profitable financial intermediation and related services for a sustained and diversified agricultural and rural development''.



In line with its primary functions as contained in the mission statement, the bank seeks to finance agricultural sub-sectors such as primary production, agric-business, agro- processing, agro-export and cocoa sub-sector, with short term, medium term and long term loan facilities.

**1.1.3.3 Scope of activities**

The bank's services are grouped into six main areas as outlinedbelow.

**1.1.3.3.1 Development Banking Services**

These include agricultural production and marketing credit, agro-processing financing, cocoa farm maintenance and bean purchases, export development financing and agriculture business financing.

(i) **Corporate banking**

Services include foreign account services, domestic current account service, business credit and international banking.

(ii) **Commercial credit services**

Under this category, credit is extended on commercial basis to individual, groups and societies, sole proprietorships, partnerships, limited liability companies, and other corporate organizations. These services include overdrafts, letters of credit, advance mobilization guarantees, issue of bonds and guarantees.

#### 1.1.3.3.1 Retail and Consumer Banking

Services include current account, salary account, savings and deposits account, personal loans, institutional managed personal loan, local payment service, and Automated Teller Machines (ATM) services.

#### 1.1.3.3.2 International Banking



This provides services such as foreign exchange account, foreign deposit account, travelers' checques, international funds transfer, export finance, export documentation and processing, export advisory service and import financing

#### 1.1.3.3.3Treasury Management Services

This includes cedi account deposits, special call deposits, fixed deposits and Bank of Ghana treasury bills or bonds, and foreign currency deposits.

#### 1.1.3.3.4 Western Union Money Transfer

This service allows the payment of inward remittances with ease, convenience and speed from every city in the world in local currency to the beneficiary in accordance with an agreement with the sender.

#### 1.1.3.3.5 The bank's offices and branch network

The bank has its Head office in Accra with various departments in with its scope of activities as stated above. It has seven Area offices, fifty-two branches and thirteen Agencies and Farm Loan offices which are strategically spread across the ten Regions of Ghana. (ADB, 2008)

### 1.1.4 Kinds of loans in ADB

Normally, ADB offers the following loan facilities:

(i) Mortgage loan

(ii) Personal loans

(iii) Commercial loans (small and medium scales)

(iv) Construction loans

(v) Home improvement

(vi)Young farmers' loan (agric)

(vii) Retail loan



### 1.1.5 Aims of lending ADB loans

The inability of rural borrowers to offer adequate security for loans and the enormousrisks associated with agricultural production, are the typical reasons given for urban-base bias of commercial lending. The Agricultural Development Bank was created to service the rural sector in particular. It eventually began to concentrate on traditional urban-based banking activities.

The aims of ADB are:

(i) to give loans to farmers to expand their farms in other to maximize profit.

(ii) to mobilize resources locked up in the rural areas into the banking systems.

(iii) to stimulate banking habits among rural dwellers to facilitate development .

(iv) to identify viable industries in their respective catchment areas for investment and development.

(v) to strengthen domestic food security.

(vi) to generate foreign exchange savings through cost effective production of import substitutes.

(vii) to generate sustained increases in foreign exchange earnings through rapid expansion of particularly non-traditional agricultural export crops.

(viii) to generate productive employment and poverty reduction and

(ix) to promote profitable value-addition to agricultural produce through investment in agricultural marketing and processing.

Despite the above laudable aims of ADB, some customers still abuse the loan facility. One form of abuse in the granting of loans involves granting a loan in order to put the borrower in a position that one can gain advantage over. A situation where the lender charges excessive interest is also considered as one of the abuses. Abuses can also take place in the form of the customer abusing the lender by not repaying the loan or with intent to defraud the lender. In different time periods and cultures the acceptable interest rate has variedly from no interest at all to unlimited interest rates. Credit card companies in some countries have been accused by consumer organizations of lending at usurious interest rates and making money out of frivolous extra charges.

**1.1.6 Terms and duration of A D B loans**

Bank loans come in three broad categories they are short-term, medium-term and long-term.Short term loans offer payday loans online and at our locations throughout Illinois. Short term loans specialize in providing personal cash loans to clients. They usually have a maximum term of one year; however, the bank may agree to extend them for several years.Medium term loan is a type of loan that is paid off in monthly installments over two to seven years, sometimes longer. This type of loan is suitable for capital expenses. It has the following benefits:

(i) Interest is linked to prime

(ii) The amount of the loan, the interest rate and your repayment plan depend on how much collateral you have, and the value of the assets you want to buy.

(iii)Medium term loan has the following types;

(iv)Term loan**:** It is monetary loan that is repaid in regular payments over a set period of time. Term loans usually last between one and ten years, but may last as long as thirty years in some cases.

(v) Monthly payment business loan: It is a variation of term loan, structured with monthly payments. The borrower may negotiate lower payments for the first two years.

(vi)Equipment leasing: Equipment leasing is an alternative means of financing an asset for a business. A key difference between leasing company retains the title to the equipmentduring the term of the lease. With equipment leasing we have the same process except that the leasing company owns the equipment.

Another term used in loans is long term loan .This is a type of loan that has an extended time period for repayment usually lasting between three and thirty years. Car loans and home mortgages are examples of long –term loans

Long-term loans are collateralized by a business's assets and typically require quarterly or monthly payments derived from profits or cash flow. These loans usually carry wording that limits the amount of additional financial commitments the business may take on (including other debts but also dividends or principals' salaries) and they sometimes require that a certain amount of profit be set-aside to, pay the loan. (www.agricbank.com/)

## 1.2Statement of the problem

Agricultural development banks record marginal profits with some running at a loss. Loan is granted in a form of contract between the bank and the borrower. Due to poor allocation and loans disbursement, the banks are not able to optimize profits from the loans.Monies that could have been used to offer social services like building of schools, hospitals etc in the community in which they operate goes into “Bad Debts”.

As a result, the ADB is advocating for a device or a model that will enable the bank to solve the problem of loan disbursement optimally for both long term and short term basis in other to maximize their profit. It is from the above observations that this research work is skewed at developing a linear programming model with the specific purpose of providing anoptimal solution of banks problem with a case study of A.D.B in Sunyani.

## 1.3 Objectives of the study

Most banks operating in Ghana today are faced with the complex problems of how to manage their loan portfolios in such a manner that the goals of the bank are best achieved. The main objectives of this study are:

(i) to develop linear programming (LP) model of disbursing funds allocated for loans effectively and efficiently in order to optimize profit margin of ADB, Sunyani.

(ii) to help decision makers at the bank to formulate prudent and effective loan policies.

(iii) to make recommendations that can address the issue of portfolio in the banking industry.

## 1.4Justification

In today's word money is scarce commodity for a lot of people. No matter how hard one's work there is always a need for more money. Hence banks and financial institutions have taken an advantage of this current need for money to satisfy various needs. The money given out also serves a useful purpose because the loan portfolio of bank which comprises the principal and the interest gained on the loan serves as major asset of bank.

If lending authorities loan limitations are not revised when circumstances change, a Bank could be operating within guidelines that are too restrictive or too lenient. If guidelines do not comply with current laws and rules, lending decisions may not reflect best practices or regulatory requirements. A loan policy that does not anticipate risks can lead to asset quality problems and poor earnings. The bank might run at a loss or even collapse if they are not able to retrieve all the loans they give out. Due to this, a more scientific approach must be employed by banks to ensure adequate, effective and efficient distribution of funds they have available for loans to ensure constant growth of these banks. When banks run efficiently they are able to allocate a larger amount of its funds for social services in the community in which they operate. The proposed model is going to help banks to efficiently distribute the funds they have available for loan in order to maximize their profit. The proposed model will also help decision makers at the Bank to formulate prudent and effective loan policies. This makes this study justifiable and worthwhile.

## 1.5 Methodology

The data for this study will consist of secondary data collected from ADB; Sunyani branch in the BrongAhafo from annual report, journals and dailies. In order for the bank to maximize profit, the proposed model will be based strictly on the Bank's loan policy and its previous history on loan disbursement. The model will be solved using revised Simplex Algorithm.

The computerized software application program called LP solver (lips) based on the revised Simplex Algorithm will be usedto facilitate the solution of the linear programming model developed. The LP solver is considered the best option for the project because the spreadsheet offers a very convenient data entry and editing features which allows for a greater understanding of how to construct linear programs. The method will also be selected due to the fact that it is a popular program used by the operational researchers.



The model will be analyzed by the use of LP solver (lips).

### 1.6 Significance of the study

This project seeks to access the performance of the loan portfolio of ADB on the community,especially its customers. It is hoped that the model designed in the course of this study based on empirical evidence, will go a long way in providing useful planning and decision tool for the banks operating in Ghana. The study will contribute significantly to the development of the banking industry which plays a pivotal role in the development of the economy. This is because the study seeks to identify problems of loan disbursement in ADB and recommend some measures that can solve these problems.

The study will also play a significant role of engineering further research into other aspects of the topic under consideration or other related topics in the banking sector. This would provide various solutions to some of the problems in banking institutions.

The findings will also enable management of banking institutions come out with pragmatic policies for loan portfolio management aimed at improving the quality of their loan portfolios.

### 1.7 Limitation of the study

The research only focuses on ADB and for that matter it will not give general impression of other banks in Ghana. The loan disbursement practices in financial institutions in Ghana seem to differ from bank to bank. Different banks have their own disbursement policies and problems. The model will not measure the portfolio risk but only the returns (profit).

Where the problem consists of inflicting multiple objectives, the LP technique cannot provide a solution. Also, LP problem cannot be used efficiently for large scale problems, the computational difficulties are enormous, even when the large digital computer is available. The research will focus only on linear constraints and so other constraints such as quadratic and exponential will not be considered.

### 1.8 Organization of the study

The remaining chapters will be structured as follows;

Chapter two looks at review of related literature, which covers application of linear programming to portfolio and risk associated with loans. Chapter three presents the research methodology used for the study.Chapter four is devoted for data collection and analysis. Chapter five which is the last chapter of the study presents the conclusions and recommendations of the study.

### 1.9 Summary

In this introductory chapter, weconsidered background of loan portfolio, a brief history of ADB and some of their social responsibilities, statement of the problem, main objectives of the study, Justification, Methodology, Significance, Limitation and Organization of the study. The objectives of the work were also presented. In the next chapter, we shall review some literature in the area of linear programming and some loan portfolio theories.

## CHAPTER TWO

## LITERATURE REVIEW

### 2.0 Introduction

In this chapter of the study, other people's works, journals of various fields of research on linear programming programs and portfolio theory will be considered. One of the most popular methods to choose optimal investment portfolio is the method that measures the value of risk proposed by Markowitz. This model analysis different measure of risk such as conditional value risk, variance and standard deviation. Different measures of risk is focusing on the different properties of distribution of rate of return for example, the variance measures the dispersion of rate of return and the value-at-risk.

The optimal portfolio selected on the bases of proposed models will be compared according to level of risk and profitability.

### 2.1 Linear Programming Model

Linear programming theory and technique have been successfullyapplied to various transportation problems almost since its early beginning. A famous example is given by Dantzig to adapt his simplex method to solve Hitchcock`s transportation problem. The terminology, such as transportation/assignment problems, and have become a standard in these contexts since then. Linear programming methods were first used to study origin-Destination distributions in 1970s.

### 2.2 Linear Programming in Financial Management

The use of linear and other types of mathematical programming techniques has received coverage in the extensive banking literature,( Chambers and Chames ,1961), as well as (Cohen and Hammer 1967;1972), developed a series of sophisticated linear programming models for managing the balance sheet of larger banks, while,(Waterman and Gee, 1963) and (Fortson and Dince,1977) proposed less elegant formulations which were better suited for the small to medium-sized bank. Several programming models have also been proposed for managing a bank's investment security portfolio, including those by, (Booth, 1972).

## 2.3 Linear programming for bank portfolio management

Various portfolio theories have been propounded for the management of bank funds. In 1961, Ronald I Robinson reserved the proposed four priorities of the use of banks funds. These include primary reserves, (or protective investment), loans and advances (customer creditdemand) and investment account (open market investment for income) in descending order of priority. His assessment has been fully supported in other works by (Sheng-Yi and Yong,1988).

A bank has to place primary reserves at the top of the priority in order to comply with the minimum legal requirement, to meet any immediate withdrawal demand by depositors and to provide a means of clearing cheques and credit obligations among banks.

Secondary reserves include cash items from banks, treasury bills and other short-term securities. Bank should have to satisfy customers' loan demand before allocating the balance of the funds in the investment market.

Loans and investment are in fact complementary. According to Robinson, (1961) investment should be tailored to the strength, seasonality and character of loan demand. He reiterated that banks that experience sharp seasonal fluctuations in loan demand need to maintain more liquidity in their investment programmed. Moreover, during a boom when loan demand is high and credit-worthy customers are available, banks should allocate more funds to loans and fewer funds to investment, and vice versa during recession when loan demand is low. According to Robinson, (1961) the crucial banking problem is to resolve the conflict between safety and profitability in the employment of bank funds. The conflict is essentially the problem between liquidity and the size of the earning assets. Robinson suggested that where there is a conflict between safety and profitability, it is better to err on the side of safety.



The best practice is identifying procedures that can bring out the optimal mixture of management of banks funds. According to Tobin (1965) portfolio theory can be applied to bank portfolio management in that a bank would maximize the rates of return of its portfolio of assets, subject to the expected degree of risk and liquidity. Chambers and Charnes (1961) applied linear programming analysis on the consolidated balance sheets of commercial banks in Singapore for the period 1978-1983. The results show that large banks do not try to maximize the returns of their portfolios, subject to legal, policy, bounding and total assets constraints, which denote riskiness and liquidity of the portfolio of assets. In a direct way, banks conform to the portfolio choice theory; they have to balance yield and liquidity against security. They pointed out that although the computer cannot replace a manager, linear programming can serve as a useful guide.

**2.4 Modern Portfolio Theory**

According to Bodie et al., (2009), the concept of investment diversification is an old one and existed long before modern finance theory. It was, however, not until 1952 that Harry Markowitz published a formal model of portfolio selection based on diversification principles. This work contributed to Markowitz receiving the Nobel Prize in Economics in 1990. His model can be regarded a first step in portfolio management, which is the identification of the efficient set of portfolios or the efficient frontier of risky assets.

Actually, the work began in 1900 when the French mathematician, Louis Bachelier, studied financial markets. Based on his studies, Bachelier argued that prices will go up or down with equal probability and that their volatility is measurable. The so-called bell curve was born, whereby the distribution of price movements is thought to be bell-shaped with very large changes assumed to be extremely rare. It was Markowitz who took the first step in applying Bachelier's ideas (Mandelbrot, 2004).



### 2.5 Markowitz's portfolio theory

In the 1950's the investment community talked about risk but there was no measurable specification for the term. However, investors were eager to quantify their risk variable.

Markowitz showed that the variance of the rate of return was an important measure of risk under a reasonable set of assumptions and came forward with the formulas for computing the variance of the portfolio. The use of this formula revealed the importance of diversifying to reduce risk and also provided guidance on how to diversify effectively (Reilly, 1989).

When Markowitz first published his ideas of portfolio selection in 1952 he rejected the notion that investors should maximize discounted returns and choose their portfolio accordingly. Markowitz's view was that this rule failed to imply diversification, no matter how the anticipated returns were formed. The rule he rejected implied that the investor should place all of his or her funds in the security with the greatest discounted value. He also rejected the law of large numbers in portfolios made up of securities, objecting to the claim that it would result in both maximum expected returns and minimum variance, and pointing out that returns from securities are too intercorrelated for all variance to be eliminated with diversification. Markowitz also pointed out that a portfolio with maximum expected returns is not necessarily the one with the minimum variance. Hence, that there is a rate at which the investor can gain expected returns by accepting more variance, or reduce variance by giving up expected returns. Building on these observations he presented the 'expected returns-variance of returns' rule (Markowitz, 1952).

Markowitz's idea was that investors should hold mean- variance efficient portfolios. While not an entirely new concept, mean-variance optimization was not a widely used strategy at the time. Most investment managers were focusing their efforts on identifying securities with high expected returns (Chan, Karceski, andLakonishok, 1999).The authors formally presented his view that although investors want to maximize returns on securities they also want to minimize uncertainty, or risk. These are conflicting objectives which must be balanced against each other when the investor makes his or her decision. Markowitz asserts that investors should base their portfolio decisions only on expected returns, i.e. the measure of potential rewards in any portfolio, and standard deviation, the measure of risk. The investorshould estimate the expected returns and standard deviation of each portfolio and then choose the best one on the grounds of the relative magnitudes of these two parameters (Sharpe, Alexander and Bailey, 1999). As previously mentioned, Markowitz rejected the expected returns rule on the grounds that it neither acknowledged nor accounted for the need for diversification, contrary to his expected return-variance of return rule. In addition, he concluded that the expected return-variance of return rule not only revealed the benefits of diversification but that it pointed towards the right type of diversification for the right reason. It is not enough to diversify by simply increasing the number of securities held. If, for example, most of the firms in the portfolio are within the same industry they are more likely to do poorly at the same time than firms in separate industries.

In the same way it is not enough to make variance small to invest in large number of securities. It should be avoided to invest in securities with high covariance among themselves and it is obvious that firms in different industries have lower covariance than firms within the same industry Markowitz, (1952), simply put, Markowitz concluded that by mixing stocks that flip tail and those that flip heads you can lower the risk of your overall portfolio. If you spread your investments across unrelated stocks you will maximize your potential profit whether the economy is slowing down or growing. If you then add more and more stock in different combinations you have what Markowitz called an efficient portfolio. An efficient portfolio is the portfolio which gives the highest profit with the least risk. The aim of Markowitz's methods is to construct that kind of portfolio (Mandelbrot, 2004).

Until Markowitz, (1952), suggested this approach to portfolio analysis no full and specific basis existed to justify diversification in portfolio selection. Also the concept of risk had rarely been defined in a thorough manner in portfolio analysis before Markowitz's writings, let alone treated analytically. With his approach these issues, diversification and risk, got a specified framework and a workable algorithm for employing that framework for practical problems was provided. Markowitz did not, however, suggest a preferred technique for security analysis or a suitable method for portfolio selection. He concentrated on providing a general structure for the whole process and providing an algorithm for performing the task of portfolio analysis (Sharpe W. F., Portfolio Analysis, 1967). Markowitz created a theory of portfolio choice in the uncertain future. He quantified the difference between the risk that was taken on individual assets and the aggregated risk of the portfolio. He showed that the portfolio risk came from covariance of the assets which made up the portfolio. The marginal contribution of a security to the portfolio return variance is therefore measured by the covariance between the return of the security and the return of the portfolio but not by the variance of the security itself. In his writings, Markowitz argues that the risk of a portfolio is less than the risk of each asset in the portfolio taken individually and provides quantitative evidence of the merits of diversification (Amenc andSourd, 2003).

In his model of portfolio management, Markowitz indentified the efficient set of portfolios, or the efficient 'frontier of risky assets'. The principal idea behind the frontier set of risky portfolios is that the investor should only be interested in the portfolio which gives the highest expected return for any given risk level. Alsothe frontier is a set of portfolios that minimizes the variance for any target expected return (Bodie, Kane, and Marcus, 2009).

With his work, Markowitz introduced a parametric optimization model that was both sufficiently general to be applicable to a significant range of practical situations and simple enough to be usable for theoretical analysis. Nevertheless, the subject is so complicated that Markowitz's work in the 1950's probably raised more questions than it answered. Indeed, it spurred a tremendous amount of related research (Steinbach, 2001).

### 2.6 Risk measures in loan portfolio management

Given that a bank wants to find the optimal way of funding its Loan portfolio without taking on too much risk, a good risk measure is essential. The risk measure must be easy for management to interpret and suitable to act as a part of an optimization problem.

#### 2.6.1 Conditional Value at Risk

A popular and widely used risk measure is Value at Risk (VaR). VaR is a measure that is defined as the lowest amount $\zeta$ such that with probability $\alpha$ loss will not exceed $\zeta$ during a specified time period. For example, if you choose the probability level $\alpha$ to be 0.95 and the time period to be one week, VaR states the maximum loss that you can expect over a one- week period with 95% certainty.

Value at Risk (VaR) has a role in the approach, but the emphasis is on Conditional Value at Risk (CVaR) which is known also as Mean Excess Loss, Mean Shortfall, or Tail VaR. Conditional Value at Risk (CVaR) is defined as the conditional expected loss above VaR. by definition with respect to a specific probability $\beta$, the loss will not exceed $\alpha$, whereas the $\beta$- CVaR is the conditional expectations of losses above the amount $\alpha$. Three values of $\beta$ are commonly considered: 0.90, 0.95, and 0.99. The definitions ensure that the $\beta$-CVaR is never more than the $\beta$-CVaR , so portfolios with low CVaR must have low VaR as well.

Rockafellar and Uryasev (1999) and Uryasev (2000) showed that VaR has undesirable mathematical features. For instance, VaR has a lack of subadditivity, resulting in the fact that the sum of the VaR of two different portfolios can be greater than the VaR of the combination of the two portfolios. Under most circumstances, the portfolios are not perfectly correlated and therefore the VaR of the combination of the two portfolios should not be greater than the sum of the individual portfolios' risk measures. In addition, Rockafellar and Uryasev (1999) clarify that it is problematic to optimize a problem where VaR is used as a risk measure. Difficulties arise, for example, from the fact that VaR then will be non-convex. Convexity is a key property in optimization since it assures that a local optimum is also a global optimum. The major drawback of using VaR as a risk measure is the fact that tail events are not considered. In other words, great losses that might be devastating for a company are not taken into account by using VaR as the risk measure of choice. For more information on the difficulties regarding VaR as a risk measure in an optimization problem, we refer to Rockafellar and Uryasev (1999).

According to Uryasev (2000), VaR can be restricted by constraining CVaR because of the fact that CVaR always will be greater than VaR. This means that portfolios with low CVaR also will have low VaR.Uryasev (2000) also shows how to optimize a problem with the CVaR risk measure as a constraint while calculating VaR at the same time. Since the Division has expressed that they want to know the VaR of the combined portfolio, this is a very useful feature of the CVaR approach presented by Rockafellar and Uryasev (1999) and Uryasev (2000).

### 2.7 Probability of loss on loan portfolio

According to Klaus Rheinberger and Martin Summer in their credit risk portfolio models, three parameters drive loan losses: The probability of default by individual obligors (PD), the Loss Given Default (LGD) and the exposure at default (EAD). While the standard credit risk models focus on modeling the PD for a given LGD, a growing recent literature has looked closer into the issue of explaining LGD and of exploring the consequences of dependencies between PD and LGD. Most of the papers on the issue of dependency between PD and LGD have been written for US data and usually find strong correlations between these two variables. The first papers investigating the consequences of these dependencies for credit portfolio risk analysis were Frye (2000a) and Frye (2000b) using a credit risk model suggested by Finger and Gordy (1999; 2000). The authors used a different credit risk model in the tradition of actuarial portfolio loss models and focus directly on two risk factors: an aggregate PD and an aggregate API as well as their dependence. The authors used this approach because their interest was to investigate the implications of some stylized facts on asset prices and credit risk that have frequently been found in the macroeconomic literature for the risk of collateralized loan portfolios. The authors also believe that the credit risk model we use gives us maximal flexibility with assumptions about the distribution of systematic risk factors. There are a variety of models that try to capture the dependence between PD and LGD. These models are developed in the papers of Jarrow (2001), Jokivuolle and Peura (2003), Carey and Gordy (2003), Hu and Perraudin (2002), Bakshi et al. (2001), G¨urtler and Heithecker (2005) and Altman et al., (2004). Most of these papers look at bond data but some also cover loans. There is a literature that looks in some detail into the determinants of LGD. (Acharya et al., 2003) investigated defaulted bonds, Duellmann and Trapp (2004) look into recoveries of US corporate credit exposures, Grunert and Weber (2005) investigated recoveries of German bank loans and (Schuermann,2004) summarizes existing knowledge about recoveries. While these papers show a nuanced picture of the determinants of recoveries that consist of many microeconomic and legal features such as the industry sector in which exposures are held or the seniority of a claim all papers find that macroeconomic conditions play a key role.

**2.8. Boom and Bust Cycles**

The close relationship between macroeconomic cycles and boom and bust cycles in bank lending and asset prices has been described as a stylized fact by several authors dealing with financial stability. Two recent examples are Borio(2002) and Goodhart et al., (2004). Borio (2002) provided evidence about the cyclical co-movements between credit, asset prices and the macro-economy. (Goodhart et al., 2004) analyze this dependency in the context of banking system liberalization and banking regulation during the last two decades. While these authors focus mainly on the past two decades, (Bordo et al., 2001) point out that financial accelerator mechanisms and boom and bust cycles in a long term perspective were the rule rather than the exception.

When banking systems were liberalized after the break down of the Bretton Woods System in the early 1970s banks were suddenly confronted with volatile exchange rates and interest rates, tighter margins and increased competition from financial markets.

Due to this disintermediation process, banks often lost their biggest and safest borrowers in industry to the capital market. As consequence banks began to increase lending to smaller and also riskier borrowers such as small and medium sized enterprises and persons. Banks also increasingly engaged in mortgage lending to households. Since such a larger and more dispersed pool of borrowers makes information acquisitions and monitoring more costly (compared to a small pool of large industry customers), the weight of collateralized lending increased. Goodhart et al., (2004) pointed out that this increasing weight of collateral as basis for bank lending automatically accentuates financial accelerator mechanisms described in the literature by Bernanke and Gilchrist, Kyotaki and Moore (1999;1997) .

Borio (2002) described a stylized pattern of such an accelerator mechanism or a financial cycle as we have observed it repeatedly in the past. The buildup of imbalances that trigger a crisis usually starts with booming economic conditions. This boom is accompanied by a climate of overly optimistic risk assessment, the gradual weakening of financing and credit constraints and hiking asset prices (in particular property and real estate prices). In this climate financial and real imbalances are building up. At some point an essentially unpredictable trigger like an asset price drop or the interruption of an investment boom causes a sudden run down of financial buffers and once these buffers are exhausted and the contraction exceeds a certain threshold a full scale financial crisis occurs.

**2.9 Data on Asset Prices and Default**

Since loan quality and asset prices both depend on the general macroeconomic conditions, these variables tend to be highly correlated. For instance, in the terminology of quantitative risk management, probabilities of default (credit risk) of individual borrowers are high at the same time when asset prices (market risk) are depressed. The distinction between market and credit risk, which has been common standard in the regulatory and supervisory community, has frequently been criticized by economists in the past for instance (Hellwig, 1995). Academic research on quantitative risk management as well as risk management practitioners are currently undertaking substantial efforts to include these dependencies into their risk models and the integration of credit and market risk is an active field of research.

Following the work of Borio et al., (1994) the Bank for International Settlements (BIS) has constructed an aggregate API for several of the major industrial countries (Arthur, 2004). The aggregate application programming interface (API) of the BIS is a geometric weighted average of equity, commercial and residential real estate, the most important asset classes used in collateralized bank lending. The weights represent estimates of the shares of those assets in the total private sector wealth. While the aggregate API provides only a broad brush perspective on the risk of collateral values – and therefore LGD in a collateralized loan portfolio – we think that these data provide an excellent starting point to explore the order of magnitude by which credit risk measures are underestimated when collateral values and thus recovery rates are taken to be fixed or independent from credit risk. According to (Crouhy et al., 2000), this assumption is currently used in most of the standard portfolio credit risk models used in the banking industry.

**2.10 Stochastic programming**

Stochastic programming is an approach intended for finding optimal solutions to problems including random variables such as interest and exchange rates. It is an approach that can be used for practical decision making under uncertainty. The solutionshould be derived with respect to the problem's objective function (the Division's preference functional), which will encompass a utility function, and given constraints regarding for example the amount of risk that the Division deems acceptable.

### 2.10.1 Expected utility theory

Gollier (2001) states that "Before addressing any decision problem under uncertainty, it is necessary to build a preference functional that evaluates the level of satisfaction of the decision maker who bears risk. If such a functional is obtained, decision problems can be solved by looking for the decision that maximizes the decision maker's level of satisfaction." The maximization of a decision maker's satisfaction is one of the fundamental tenets of expected utility theory (EUT). According to Mongin (1997), EUT "stated that the decision maker chooses between risky or uncertain prospects by comparing the expected utility values, i.e., the weighted sums obtained by adding the utility values of outcomes multiplied by their respective probabilities." By choosing the prospect with the greatest expected utility value, the decision maker maximizes his level of satisfaction.

Utility values are calculated with a utility function. (Gustafsson and Salo, 2004) explained the distinction between a preference functional and a utility function. They assert that a preference functional is $U[X]= [u(X)]$ where $u$ is the investor's utility function and $X$ is an act that can result in many different outcomes. To each outcome, we can apply $u$ and calculate the decision maker's utility associated with the outcome.

Kall and Wallace, (1994) explained that attitudes towards risk can be characterized by utility functions. A utility function can be regarded as a function that describes one's happiness or utility from a certain wealth. The function is used in order to determine if one outcome is better or more preferable than another. We could e.g. choose to participate in a game where we can win or lose a certain amount of money with given probabilities and level of initial wealth. Given that we participate in the game, with a utility function we can quantify our satisfaction (utility) with each of the possible outcomes. With a preference functional, we can quantify our expected satisfaction from participating in the game. We can also choose not to participate in the game. A preference functional can then determine which of the two alternatives, to participate or not, is preferable.

From the perspective of an investor, the distinction between a preference functional and a utility function can be explained as follows. The preference functional lets the investor compare risky portfolios and rank them according to the degree of his satisfaction with the portfolios. In essence, the investor can compare portfolios head-to-head and decide which one is best. The utility function lets the investor compare different outcomes, given that he has already selected a portfolio. By changing the utility function, the investor can adapt it so that it mirrors his tolerance for losses.

Kall and Wallace, (1994) show an example of a game where we can win or lose $\delta w$with equal probabilities (50%). The initial wealth is $w0$ and it costs nothing to participate in the game. After the outcome of the game is known, we will have a wealth of ($w0 + \delta w$or $w0 - \delta w$)depending on whether we win or lose. If we choose to maximize the expected value of the total wealth, we would consider the decision maker to be risk-neutral, meaning that the decision maker would accept to participate in a fair game. A fair game is one where the expected payoff is zero, as the case is for the game mentioned above.

### 2.11 Summary

In this chapter, other research works done by some scholars' in connection with Linear Programming Problems were reviewed. In the next chapter, we shall put forward the methodology use in solving Linear Programming Problems.

# CHAPTER THREE

# METHODOLOGY

## 3.0 Introduction

This Chapter takes a look at relevant methodology, use in solving Linear Programming Problem. We shall discuss linear programming with particular emphasis on the Simplex method and revised Simplex method.

## 3.1 Linear Programming



Linear Programming is a method for determining a way to achieve the best outcome (such as maximum profit or lowest cost) in a given mathematical model for some list of requirements represented as linear relationships. Linear programming is a specific case of mathematical programming (mathematical optimization).

More formally, linear programming is a technique for the optimization of a linear objective function, subject to linear equality and linear inequality constraints. It's feasible region is a convex polyhedron, which is a set defined as the intersection of finitely many half spaces, each of which is defined by a linear inequality. Its objective function is a real-valued affine function defined on this polyhedron. A linear programming algorithm finds a point in the polyhedron where this function has the smallest (or largest) value if such point exists.

Linear programming can be applied to various fields of study. It is used most extensively in business and economics, but can also be utilized for some engineering problems. Industries that use linear programming models include transportation, energy, telecommunications, and manufacturing. It has proved useful in modeling diverse types of problems in planning, routing, scheduling, assignment, and design.

### 3.1.1 Types of LP problems

A linear Programming is the problem of optimizing linear objective in the decision variables $x_1$, $x_{2, \ldots,} x_n$subject to linearly or infinitely constraints on the x's.

LP has so many forms, some of which include:

(i) Integer programming

(ii) Quadratic programming

(iii) Exponential Programming

(iv) Logarithmic programming etc.

We shall focus on integer programming and Quadratic programming.

#### 3.1.1.1Integer programming problem

A mixed-integer program is the minimization or maximization of a linear function subject to linear constraints. More explicitly, a mixed-integer program with n variables and m constraints has the form:

$$Maximize \quad \sum_{j=1}^{n} c_j \mathrm{x}_\mathrm{j} (3.1)$$

$$Subjectto; \sum_{j=1}^{n} a_{ij} x_i = bi \; (3.2)$$

$$\mathrm{x}_\mathrm{j} \geq 0 \qquad (\mathrm{j}=1, 2\ldots \mathrm{n})$$

$x_j$integer (for some or all$j = 1, 2, \ldots, n$).

This problem is called the (linear) integer-programming problem. It is said to be a mixedinteger programwhen some, but not all, variables are restricted to be integer, and is called a *pure* integer program when all decision variables are integers.

#### 3.1.1.1.1 Branch and Bound

The most widely used method for solving integer programs is branch and bound. Subproblems are created by restricting the range of the integer variables. For binary variables, there are only two possible restrictions: setting the variable to 0, or setting the variable to 1. More generally, a variable with lower bound l and upper bound u will be divided into two problems with ranges l to q and q+1 to u respectively. Lower bounds are provided by the linear-programming relaxation to the problem: keep the objective function and all constraints, but relax the integrality restrictions to derive a linear program. If the optimal solution to a relaxed problem is (coincidentally) integral, it is an optimal solution to the sub problem, and the value can be used to terminate searches of subproblems whose lower bound is higher.



#### 3.1.1.2 Quadratic Programming

The general quadratic program can be written as

Minimize $f(x) = cx+\frac{1}{2}X^{T}QX$, (3.3)

Subject to $AX \leq b$, (3.4)

$x \geq 0$



wherecis an *n*-dimensional row vector describing the coefficients of the linear terms in the objective function, and Qis an (*n* ´ *n*) symmetricmatrix describing the coefficients of the quadratic terms. If a constantterm exists it is dropped from the model. As in linear programming, thedecision variables are denoted by the *n*-dimensional column vector x, and

the constraints are defined by an (*m* ´ *n*) Amatrix and an *m*-dimensionalcolumn vector bof right-hand-side coefficients. We assume that afeasible solution exists and that the constraint region is bounded. When the objective function $f$(x) is strictly convex for all feasiblepoints the problem has a unique local minimum which is also the globalminimum. A sufficient condition to guarantee strictly convexity is for Qto be positive definite.

#### 3.1.1.2.1 Karush-Kuhn-Tucker Conditions

TheLagrangian function for the quadratic program is;

$L(x, \mu) = cx + \frac{1}{2} X^T QX + \mu\,(Ax - b)$, Where μ is an $m$-dimensional row vector. The Karush-Kuhn-Tucker conditions for a local minimum are given as follows.

$\frac{\partial L}{\partial xj} \geq 0$, j=1,…,n $\quad\quad c + x^T Q + \mu A \geq 0$ (3.5)

$\frac{\partial L}{\partial \mu i} \leq 0$, i=1,…m $\quad\quad Ax - b \leq 0$ (3.6)

$x_j \frac{\partial L}{\partial xj} = 0$, j=1,…,n $x^T(c^T + Qx + A^T\mu) = 0$ (3.7)

$\mu_i g_i(\mathbf{x}) = 0$, $i = 1, \ldots, m$ $\quad\quad \mu(Ax - b) = 0$ (3.8)

$x_j \geq 0$, $j = 1, \ldots, n$ $x \geq 0$ (3.9)

$\mu_i \geq 0$, $i = 1, \ldots, m$ $\mu \geq 0$ (3.10)

To put (3.5) to (3.10) into a more manageable form we introducenonnegative surplus variables y $\in \mathcal{R}^n$ to the inequalities in (3.5) andnonnegative slack variables v $\in \mathcal{R}^m$ to the inequalities in (3.6) to obtainthe equations

$c^T + Qx + A^T\mu^T - y = 0$ and $Ax - b + v = 0$.

The KKT conditions can now be written with the constants moved to the right-hand side.

$Qx + A^T\mu^T - y = -c^T$ (3.11)

$Ax + v = b$ (3.12)

$x \geq 0,\ \mu \geq 0,\ y \geq 0,\ v \geq 0$ (3.13)

$y^T x = 0$, $\mu v = 0$ (3.14)

The first two expressions are linear equalities, the third restricts all the variables to be nonnegative, and the fourth prescribes complementary slackness.

**3.1.1.2.2 Optimal solution**

The simplex algorithm can be used to solve (3.11) to (3.14) by treating the complementary slackness conditions (3.14) implicitly with a restricted basis entry rule. The procedure for setting up the linear programming model follows.

(i) Let the structural constraints be Equations (3.11) and (3.12) defined by theKKT conditions.

(ii) If any of the right-hand-side values are negative, multiply the corresponding equation by –1.

(iii) Add an artificial variable to each equation.

(iv) Let the objective function be the sum of the artificial variable

(v) Put the resultant problem into simplex form.

The goal is to find the solution to the linear program that minimizes the sum of the artificial variables with the additional requirement that the complementarily slackness conditions be satisfied at each iteration. If the sum is zero, the solution will satisfy (3.11) to (3.14).

To accommodate (3.14), the rule for selecting the entering variable must be modified with the following relationships in mind.$x_j$and $y_j$are complementary for $j = 1. . .n$ while

m$i$and $vi$ are complementary for $i = 1, . . . , m$.

The entering variable will be the one whose reduced cost is most negative provided that its complementary variable is not in the basis or would leave the basis on the same iteration. At the conclusion of the algorithm, thevector **x** defines the optimal solution and the vector $\mu$defines the optimaldual variables.

This approach has been shown to work well when the objective function is positive definite, and requires computational effort comparableto a linear programming problem with $m + n$ constraints, where $m$ is thenumber of constraints and $n$ is the number of variables in the QP. Positivesemi-definite forms of the objective function, though, can presentcomputational difficulties. Van De Panne (1975) presents an extensivediscussion of the conditions that will yield a global optimum even when$f$(x) is not positive definite. The simplest practical approach to overcomeany difficulties caused by semi-definiteness is to add a small constant toeach of the diagonal elements of Qin such a way that the modified Qmatrix becomes positive definite. Although the resultant solution will notbe exact, the difference will be insignificant if the alterations are keptsmall.

### 3.2 Forms of Linear Programming Problems

A linear programming may take one of the following forms:

(i) Matrix form

(ii) General form and

(iii)Standard form

#### 3.2.1 A Linear Programming in the Matrix Form

Linear programs are problems that can be expressed in canonical form as:

Maximize $c^T x$, (3.15)

Subject to $Ax \leq b$, (3.16)

Where x represents the vector of variables (to be determined), c and b are vectors of (known) coefficients and A is a (known) matrix of coefficients. The expression to be maximized or minimized is called the objective function ($c^T x$in this case).The inequalities are the constraints which specify a convex poly-tope over which the objective function is to be optimized.

A Linear programming model may simply be presented in the matrix vector form as;

Maximize (Minimize) $c^T x$

Subject to: $Ax \leq b, x \geq 0$

### 3.2.2 A Linear Program in the General Form

A linear programming in the general form may be presented as;

Maximize (Minimize) $\sum_{j=1}^{a} C_i X_i$ (3.17)



Subject to; $\sum_{i=1}^{n} x_{i \leq b, 1 \leq i \leq p}$ ( 3.18)

$$\sum_{j=1}^{n} a_{ij \leq} b_b, \; p+1 \leq i \leq k \quad (3.19)$$

$$\sum_{j=1}^{n} a_{ij} x_i \leq b, k+1 \leq i \leq m \quad (3.20)$$



### 3.2.3 Linear Program in the Standard Form

Standard form is the usual and most intuitive form of describing a linear programming problem.

It consists of the following four parts:

(i) A linear function to be maximized

e.g., Maximize: $c_1 x_1 + c_2 x_2$

(ii) Problem constraints of the following g form

e.g.,

$$a_{11} x_1 + a_{12} x_2 \leq b_1$$

$$a_{21} x_1 + a_{22} x_2 \leq b_2$$

$$a_{31} x_1 + a_{32} x_2 \leq b_3$$

(i) Non-negative variables

e.g.

$x_1 \geq 0, x_2 \geq 0$

(ii) Non-negative right side constant

(iii) $b_2 \geq 0$

Other forms, such as minimization problems, problems with constraints on alternative forms, as well as problems involving negative variables can always be rewritten into an equivalent problem in standard form.

**3.3 Solution Techniques for LP**

LP problems can be solved using the following methods;

(i) Simplex Method

(ii) Revised Simplex Method,

(iii) Graphical Method

(iv) Carmaker Method

(v) Matlabetc

For this research work, the researcher will talk about the use of Simplex Method and the Revised Simplex Method.

### 3.3.1 Simplex Method

The Simplex method is the name given to the solution algorithm for solving linear programming problems developed by George Dantzig in 1947. A simplex is an $n$ -dimensional convex figure that has exactly ($n$+ 1) extreme point. For example, a simplex in two dimensions is a triangle, and in three dimensions is a tetrahedron. The Simplex method refers to the idea of moving from one extreme point to another on the convex set that is formed by the constraint set and non-negativity conditions of the linear programming problem.

The solution algorithm is an iterative procedure having fixed computational rules that leads to a solution to the problem in a finite number of steps (i.e., converges to an answer). The Simplex method is algebraic in nature and is based upon the Gauss-Jordan elimination procedure.

The principle underlying the Simplex method involves the use of the algorithm which is made up of two phase, where each phase involves a special sequence of number of elementary row operations known as pivoting. A pivot operation consist of finite number of $m$ elementary row operations which replace a given system of linear equations by an equivalent system in which a specified decision variables appears in only one of the system and has a unit coefficient.

The algorithm has two phases, the first phase of the algorithm, is finding an initial basic feasible solution (BFS) to the original problem and the second phase, consists of finding an optimal solution to the problem which begins from the initial basic feasible solution.

#### 3.3.1.1 Algorithm for Simplex Method

A basic feasible solution to the system of $m$ linear constraint equations and $n$ variables is required as a starting point for the Simplex method. From this starting point, the simplex successively generates better basic feasible solutions to the system of linear equations. We proceed to develop a tabular approach for the simplex algorithm. The purpose of the tableau form is to provide an initial basic feasible solution that is required to get simplex method started. It must be noted that basic variables appear once and have coefficient of positive one.

### 3.3.1.2 Simplex method with mixed constraints

Some Linear Programming problem may consists of a mixture of ≤, =, and ≥ sign in the constraints and wish to maximized or minimized the objective function. Such mixture of signs in the constraints is referred to as mixed constraints.

The following procedure is followed when dealing with problem with mixed constraints.

Step1:

Ensuring that the objective function is to be maximized. If it is to be minimized then we convert it into a problem of maximization by

Max W = -Min (-Z)

Step2:

For each constraint involving 'greater or equal to' we convert to 'less than or equal to' that is, constraints of the form $a_{21}\, x_1 + a_{22}x_2 + \ldots + a_{2n}x_n \geq b_2$ is multiplied by negative one to obtain $-a_{21}\, x_1 - a_{22}\, x_2 - \ldots - a_{2n}x_n \leq -\, b_2$

Step 3:

Replace constraints

$a_{21}\, x_1 + a_{22}\, x_2 + \ldots + a_{2n}x_n = b_2$ by

$a_{21}\, x_1 + a_{22}\, x_2 + \ldots + a_{2n}x_n \leq b_2$ and

$a_{21}\, x_1 + a_{22}\, x_2 + \ldots a_{2n}x_n \geq b_2$ where the latter is written as

$-a_{21}\, x_1 - a_{22}\, x_2 - \ldots - a_{2n}x_n \leq -\, b_2$

Step 4:

Form the initial simplex tableau

Step 5:

If there exist no negative entry appearing on the right hand side column of the initial tableau, proceed to obtain the optimum basic feasible solution.

Step 6:

If there exist a negative entry on the Right Hand Side column of the initial tableau;

(i) identify the most negative at the Right Hand Side , this row is the pivot row

(ii) select the most negative entry in the pivoting row to the left of the Right Hand Side. This entry is the pivot element.

(iii) reduce the pivot element to 1 and the other entries on the pivot column to 0 using elementary row operation

Step 7:

Repeat step 6 as long as there is a negative entry on the Right Hand Side column. When no negative entry exists on the Right Hand Side column, except in the last row, we proceed to find the optimal solution.

### 3.3.2 Types of Simplex Method Solutions

The Simplex method will always terminate in a finite number of steps with an indication thata unique optimal solution has been obtained or that one of three special cases has occurred.

These special cases are:

(i) Alternative optimal solutions

(ii) Unbounded solutions

(iii) Infeasible solutions

### 3.3.2.1 Alternative Optimal Solutions

The Simplex method provides a clear indication of the presence of alternative or multiple, optimal solutions upon its termination. These alternative optimal solutions can be recognized by considering the ($c_j$-$z_j$) row. Assume that we are maximizing and remember that when all ($c_j$-$z_j$) values are all negative; we know that an optimal solution has been obtained. Now, the presence of an alternative optimal solution will be indicated by the fact that for some variable not in the basis, the corresponding ($c_j$-$z_j$) value will equal zero.

Thus, this variable can be entered into the basis; the appropriate variable can be removed from the basis, and the value of the objective function will not change. In this manner, the various alternative optimal solutions can be determined.

### 3.3.2.2 Unbounded Solutions

In the case of an unbounded solution, the Simplex method will terminate with the indication that the entering basic variable can do so only if it is allowed to assume a value of infinity.

Specifically, for a maximization problem we will encounter a simplex tableau having a non basic variable whose ($c_j$-$z_j$) row value is strictly greater than zero. And for this same variable all of the a. elements in its column will be zero or negative value (i.e. every coefficient in the pivot column will be either negative or zero). Thus, in performing the ratio test for the variable removal criterion, it will be possible only to form ratios having negative numbers or zeros as denominators. Negative numbers in the denominators cannot be considered since this will result in the introduction of a basic variable at a negative level. Zeros in the denominator will produce a ratio having an undefined value and would indicate that the entering basic variable should be increased indefinitely (i.e. infinitely) without any of the current basic variables being driven from the basis.

Therefore, if we have an unbounded solution, none of the current basic variables can be driven from solution by the introduction of a new basic variable, even if that new basic variable assumes an infinitely large value.

Generally, arriving at an unbounded solution indicates that the problem was originally misformulated within the constraint set and needs reformulation.

**3.3.2.3 Infeasible Solution**

An indication that no feasible solution is possible will be given by the fact that at least one of the artificial variables, which should be driven to zero by the Simplex method will be present as a positive basic variable in the solution that appears to be optimal. For example, assuming one wish to solve a maximization problem in which artificial variables are required. Then, at some iteration one achieve a solution in which all the ($c_j - z_j$) values are zero or negative, but this has one or more artificial variables as positive basic variables.

When an infeasible solution is indicated the management science analyst should carefully reconsider the construction of the model, because the model is either improperly formulated or two or more of the constraints are incompatible.

Reformulation of the model is mandatory for cases in which the no feasible solution condition is indicated. Consider the following linear programming problem as an example of Simplex algorithm.

The slack variables form the initial solution mix. The initial solution assumes that all available hours are unused i.e. the slack variables take the largest possible values. Variables in the solution mix are called basic variables. Each basic variable has a column consisting of all zeroes (0's) except for a single one. All variables not in the solution mix take the value zero.

The Simplex method uses a four step process (based on the Gauss Jordan method for solving a system of linear equations) to go from one tableau or vertex to the next. In this process, a basic variable in the solution mix is replaced by another variable previously not in the solution mix. The value of the replaced variable is set to zero (0).

### 3.3.3 The Revised Simplex Method

The original Simplex method is a straight forward algebraic procedure. However, this way ofexecuting the algorithm (in either algebraic or tabular form) is not the most efficient computational procedure for computers because it computes and stores many numbers that are not needed at the current iteration and that may not even become relevant for decision making at subsequent iterations.

The only pieces of information relevant at each iteration are:

(i)The coefficients of the non basic variables.

(ii) The coefficients of the entering basic variable in the other equations.

(iii)The right –hand sides of the equations.

It would be very useful to have a procedure that could obtain this information efficiently without computing and storing the other coefficients. These considerations motivated the development of the revised simplex method. This method was designed to accomplish exactly the same things as the original simplex method, but in a way that is more efficient for execution on computer. Thus, it is a streamlined version of the original procedure. It computes and stores only the information that is currently needed, and it carries along the essential data in a more compact form.

The revised simplex method explicitly uses matrix manipulations, so it necessary to describe the problem in matrix notation. Using matrices, our standard form for the general linear programming model becomes

Maximize $Z = c\,x$,

Subject to

$Ax \leq b$ and $x \geq 0$,

Where c is the row vector

$C = [C_1, C_2 \ldots C_n]$

x, b, are the column vectors such that

$$x = \begin{pmatrix} x_1 \\ x_2 \\ . \\ x_n \end{pmatrix} \qquad b = \begin{pmatrix} b_1 \\ b_2 \\ . \\ b_m \end{pmatrix}$$

and A is the matrix

$$A = \begin{pmatrix} a_1 & a_{12} \ldots\ldots a_{1n} \\ a_{21} & a_{22} \quad a_{2n} \\ a_{m1} & a_{m2} \ldots\ldots a_{mn} \end{pmatrix}$$

To obtain the augmented form of the problem, introduce the column vector of slack variables.

$$\begin{pmatrix} x_{n+1} \\ x_{n+2} \\ . \\ x_{n+m} \end{pmatrix}$$ so that the constraints become

$$[A \;\; I]\begin{bmatrix} x \\ xs \end{bmatrix} = b \quad \text{and} \quad \begin{bmatrix} x \\ xs \end{bmatrix} \geq 0$$ where I is the m x m identify matrix

#### 3.3.3.1 Solving For a Basic Feasible Solution

Recall that the approach of the Simplex method is to obtain a sequence of improving Basic Feasible solutions until on optimal solution is reached. One of the key features of the revisedSimplex method involves the way in which it solves for each new Basic Feasible solution after identifying its basic and non-basic. Given the variables, the resulting basic solution is the solution ofthe m equations.

$$[A \;\; I] \begin{bmatrix} X \\ Xs \end{bmatrix} = b,$$ in which the non-basic variables from the n + m elements of

$\begin{bmatrix} x \\ xs \end{bmatrix}$ are set equal to zero. Eliminating these n variables by equating them to zero leaves a set of m equations in m unknowns (the basic variables). This set of equations can be denoted by

$B\ x_B = b,$

Where the vector of basic variables

$$x_B = \begin{pmatrix} x_{B1} \\ x_{B2} \\ \vdots \\ x_{Bm} \end{pmatrix}$$

is obtained by eliminating the non basic variables from $\begin{pmatrix} x \\ xs \end{pmatrix}$ ,and the basis matrix

$$B = \begin{pmatrix} B_{11} & B_{12} \ldots\ldots & B_{1m} \\ B_{21} & B_{22} & B_{2m} \\ \ldots & \ldots & \ldots \\ B_{m1} & B_{m2} & B_{mm} \end{pmatrix}$$

is obtained by eliminating the columns corresponding to coefficients of non basic variables from [AI]. (in addition, the elements of $x_B$ and, therefore, the columns of B may be placed in a different order when the Simplex method is executed). The Simplex method introduces only basic variables

such that B is nonsingular, so that B always will exist. Therefore, to solve $B x_B = b$, both sides are premultipliedby$B^{-1}$

$B^{-1} B x_B = B^{-1}b$

Since $B^{-1} B = I$, the desired solution for the basic variables is $x_B = B^{-1} b$. Let $C_B$ be vector whoseelements are the objective function coefficients (including Zeros for slack variables) for the corresponding elements of $x_B$. The value of the objective function for this basic solution is then

$Z = C_B x_B = C_B B^{-1} b$

**3.3.3.2 Revised Simplex Algorithm**

Original simplex method calculates and stores all numbers in the tableau – many are not needed.

Revised Simplex Method (more efficient for computing)

Used in all commercial available package. (e. g. IBM MPSX, CDC APEX III)

$$\text{Max} \quad Z = c\,x$$

$$\text{Subject to; } Ax \leq b$$

$$X \geq 0$$

Initially constraints become (standard form):

$$[A \quad I] \begin{pmatrix} x \\ X_s \end{pmatrix} = \begin{pmatrix} b \end{pmatrix}$$



Xs = slack variables Basis matrix: columns relating to basic variables

$$B = \begin{pmatrix} B_{11} & \ldots\ldots\ldots\ldots & B_{1m} \\ .. & . \; . \; . \; . & .. \\ & & \\ B_{m1} & \ldots\ldots\ldots\ldots & B_{mm} \end{pmatrix}$$

(Initially B = I.)

Basic variable values: $x_B = \begin{pmatrix} x_{B1} \\ .. \\ x_{Bm} \end{pmatrix}$

At any iteration non – basic variables = 0

$$B\,x_B = b$$

Therefore $x_B = B^{-1} b$, $B^{-1} \rightarrow$ inverse matrix. At any iteration, given the original vector and the inverse matrix,$x_B$ (current R.H.S.) can be calculated.

$Z = C_B\, x_B$ where $C_B =$ objective coefficient of basic variables.

#### 3.3.3.3 Steps in the Revised Simplex Method.

1. Determine entering variable, $X_J$ with associated vector $P_J$.

   (i) Determine the coefficient of the basic variable, $c_B$

   (ii) Compute $Y = c_B B^{-1}$

   (iii) Compute $z_j - c_j = YP_j - c_j$ for all non – basic variables.

   (iv) Choose largest negative value (maximization) if none, stop.

2. Determine leaving variable, Xr, with associated vector $P_r$.

   (i) Compute $x_B = B^{-1}b$ (current R.H.S.)

   (ii) Compute current constraint coefficients of entering variable:

   $a^j = B^{-1} P_j$

   Xris associated with

   $\Theta = \text{Min } \{(x_B)_k / a^j k, a^j_k > 0\}$

   (Minimum ratio rule)

3. Determine next basis i.e. calculate $B^{-1}$

   Go to step 1.

### 3.4 Duality

Corresponding to any given linear programming problem called the primal problem, is another linear programming problem called the Dual Problem. Since a given linear programming problem can be stated in several forms (standard form, canonical form, etc), it follows that the forms of the dual problem will depend on the form of the primal problem.

A fundamental of the primal dual-relationship is that the optimal solution to either the primal or the dual problem also provides optimal solution to the other.

A maximization problems with all the less-than or equal to constraint and the non-negative requirement for the decision variables is said to be in canonical form as in example 3.3 used below. If the dual problem has optimal solution, then the primal also has an optimal solution and vice versa. The values of the optimal solution to the dual and primal are equal.

These are rules for converting the primal problem in any form into its dual.

Table 3.1: Converting of primal problem to dual form.

| PRIMAL PROBLEM | DUAL PROBLEM |
|---|---|
| Maximization | Minimization |
| Coefficient of objective function | Right hand sides of constraint |
| Cofficient of $i^{th}$constraint | Cofficient of $i^{th}$variable |
| $i^{th}$constraint is an inequality of the form $\leq$ | $i^{th}$ variable satisfies $\geq 0$ |
| $i^{th}$constraint is an equality | $i^{th}$variable is unrestricted |
| $i^{th}$variable is unrestricted | $i^{th}$constraint is an equality |
| $i^{th}$ variable satisfies $\geq 0$ | $i^{th}$constraint is an inequality of the form $\geq$ |
| Number of variable | Number of Constraint |
| Number of Constraint | Number of variable |

### 3.5 Degeneracy

A linear program is said to be degenerate if one or more basic variables have a value zero. This occurs whenever there is a tie in the minimum ratio prior to reaching the optimal solution. This may result in cycling, that is the procedure could possibly alternate between the same set of non optimal basic feasible solutions and never reach the optimal solution.

In order to overcome this problem, the following steps may be used to break the tie between the key row tie,

(i) Select the rows where the ties are found for determining the key row.

(ii) Find the coefficient of the slack variable and divide each coefficient by the coefficients in the key column in order to break the tie. If the ratios at this stage do not break the tie, find the similar ratios for the coefficient of the decision variables.

(iii) Compare the resulting ratio column by column.

(iv)Select the row which has the smallest ratio and this now becomes the key row.

### 3.6 Sensitivity analysis

Suppose that you have just completed a linear programming solution which has a major impact. How much will the result change if your basic data is slightly wrong? Will that have a minor impact on your result? Will it give a completely different outcome, or change the outcome only slightly?

These are the kind of questions addressed by sensitivity analysis. It allows us to observe the effect of changes in the parameters in the LP problem on the optimal solution. It is also useful when the values of the problem parameters are not known. Formally, the question is this; is my optimum solution sensitive to a small change in one of the original problem coefficient. This sort of examination of impact of the input data on output results is very crucial. The procedure and algorithm of mathematical programming are important, but the problems that really appear in practice are usually associated with data: getting it all, and getting accurate data. What is required in sensitivity analysis is which data has significant impact on your results.

There are several ways to approach sensitivity analysis. If your model is small enough to solve quite quickly, you can simply change the initial data and solve the model again to see what results you get. At the extreme, if your model is very large and takes a long time to solve, you can apply formal methods of classical sensitivity analysis. The classical methods rely on the relationships between the initial tableau and any later tableau to quickly update the optimum solution when changes are made to the coefficient of the original tableau. Finally on the state of sensitivity analysis, we are typically limited to analyzing the impact of changing only one coefficient at a time. There are few accepted techniques for changing several coefficients at once.

### 3.6.1Change Objective Function Coefficient

A change of the coefficients of the objective function does not affect the values of the variable directly. So as we change the values of the objective function coefficients we should ensure that the optimality conditions are not violated. The range of values over which an objective function coefficient may vary without any change in the optimal solution is known as the range by those coefficient values that maintain ($c_j - z_j \leq 0$). The computation for the range of optimality can be categorized can be categorized into two; that for the basic and also for the non-basic variable.

### 3.6.2 Changing a Right Hand Side Constraint

Right hand side constraints normally represent a limitation on the resources, and are likely to change in practice as business conditions change. An overall procedure for examining proposed changes to the right hand side of constraint is to check whether the proposed changes is within the allowable range of the right hand side of the constraint. So an optimal tableau will continue satisfying the optimal conditions regardless of the altered values of the right hand side coefficients. The change in value of the objective function per unit increase in the constraints right hand side value is known as shadow price. When Simplex method is used to solve LP problem, the values of the shadow price are found in the $Z_j$ of the final Simplex tableau.

### 3.7 Summary

In this chapter, some methods use in solving LP problems were discussed. In the next chapter, we shall solve a real life problem, using Sunyani ADB as a case study.

# CHAPTER FOUR

## DATA COLLECTION AND ANALYSIS

### 4.0 Introduction

In this chapter, we shall formulate and analyze the data collected from the bank. Implementation, solution and findings from the model shall be presented.

### 4.1 Sources and data collection

The data is secondary data extracted from the published annual reports and financial statements of the bank. This category of data was mainly in quantitative form. Saunders et al, (2007) quote Stewart and Kamins (1993) as stating that secondary data are likely to be of higher-quality than could be obtained by collecting empirical data.

A banking institution, ADB, is in the process of formulating a loan policy involving a total of GH¢ 2,119,562.00 for the year 2013.Being a full-service facility, the bank is obligated to grant loans to different clients. Table 4.1 provides the type of loans, the interest rate charged by the bank, and the probability of bad debt as estimated from past experience.

**Table 4.1: Loans available to the ADB, Sunyani.**

| Type of loan | Interest rate | Probability of bad debt |
|---|---|---|
| Commercial | 0.27 | 0.01 |
| Construction | 0.30 | 0.04 |
| Home improvement | 0.28 | 0.035 |
| Mortgage | 0.30 | 0.05 |
| Personal(salary) | 0.28 | 0.015 |
| Retail | 0.30 | 0.03 |
| Young farmers ( agric) | 0.25 | 0.075 |

Bad debts are assumed unrecoverable and for that matter produce no interest revenue. For policy reasons, there are limits on how ADB allocates its funds. The competition with other banking institutions in the Municipality requires that the bank:

(i) Allocates at least 20 percent of the total funds to construction loan and commercial loan.

(ii) To assist agriculture production in the region, agriculture loans must be at least 30 percent of total funds.

(iii) The sum of home improvement and mortgage loans should be at least 15 percent of the total funds.

(iv) Mortgage loans should be at least 15 percent of personal, construction and agric loans.

(v) Construction, mortgage and personal loans must be at least 30 percent of the total funds.

(vi) The total ratio of bad debt on all loans must not exceed 0.04.

## 4.2 Proposed Loan Model for ADB,Sunyani

Base on the empirical data, we formulate the proposed model for ADB, Sunyani.

**Decision variables**

The variables of the model are defined as follows;

$X_1$= amount for Commercial loans

$X_2$= amount for Construction loans

$X_3$= amount for home improvement loans

$X_4$= amount for Mortgage loans

$X_5$= personal loans (salary)

$X_6$= Retail loans

$X_7$= Young farmers (agric) loans

**Objective function**

The objective function of the ADB is to maximize its net returns, Z which comprise the difference between the revenue from interest and lost funds due to bad debts for each amount of loan disburses are shown in Table 4.2.



**Table 4.2**

| Loan amount | Amount of bad debts (pixi) | Amount contributing to profit (1- pi)xi |
|---|---|---|
| $x_1$ | $0.01x_1$ | $0.99x_1$ |
| $x_2$ | $0.4x_2$ | $0.96x_2$ |
| $x_3$ | $0.035x_3$ | $0.965x_3$ |
| $x_4$ | $0.05x_4$ | $0.95x_4$ |
| $x_5$ | $0.015x_5$ | $0.985x_5$ |
| $x_6$ | $0.03x_6$ | $0.97x_6$ |
| $x_7$ | $0.075x_7$ | $0.925x_7$ |

Profit on loan is given by;

$Z = \beta_1 (1-p_1) x_1 + \beta_2(1-p_2)x_2 + \beta_3 (1-p_3)x_3 +\beta_4 (1-p_4) x_4 + \ldots - (p_1x_1 +p_2x_2 +p_3x_3 + \ldots)$ , where $p_i >0$

The above can be written as;

$$\text{Maximize} Z = \sum_{i=1}^{n} \beta i(1 - pi)xi - \sum_{i=1}^{n} p_i x_i, \text{where;}$$

Z is the optimal solution,

βi is the coefficients of objective function, ( i.e. interest rate)

Xi is the various loan items

(1 – Pi) xi is the amount contributing to profit and

Pi is the probability of bad debt

$$\text{Subject to: } \sum_{i=1}^{n} x_i \leq wiA_i$$

$$x_i,\ isaninteger$$

Where $i = 1, 2, 3, \dots, 7$ and *Ai* is the amount and *wi* is the percentage impose on the loan allocated to various loan items.



### 4.3 Implementation of the model

Using the empirical data, we implement the proposed model.

$$\sum_{i=1}^{7} \beta i(1 - pi)xi - \sum_{i=1}^{7} p_i x_i$$

$$\text{Subject to: } \sum_{i=1}^{7} x_i \leq wiA_i$$

#### 4.3.1 Basic Assumptions of the formulation of the above LP

A subtle assumption in the formulation above is that all loans are issued at approximately the same time. This assumption allows us to ignore the differences in the time values of the funds allocated to the different loans. All variables are restricted to nonnegative values (i.e., their numerical value will be $\geq 0$). Also Non – integer values of decision variables are accepted. This is referred to as the assumption of divisibility(Amponsah, 2007)

Maximize $Z = 0.27(0.99x_1) + 0.3(0.96x_2) + 0.28(0.965x_3) + 0.3(0.95x_4) + 0.2875(0.985x_5) + 0.3(0.97x_6) + 0.25(0.925x_7) - 0.01x_1 - 0.04x_2 - 0.035x_3 - 0.05x_4 - 0.015x_5 - 0.03x_6 - 0.075x_7$ this simplifies to;

$Z = 0.2573x_1 + 0.248x_2 + 0.2352x_3 + 0.235x_4 + 0.2608x_5 + 0.261x_6 + 0.1563x_7$

The program has eight constraints.



1. Limit on total funds available($x_1$, $x_2$, $x_3$, $x_4$, $x_5$, $x_6$, and $x_7$)

The total funds available isGH¢2,119,562.00

$x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 \leq 2119562$

2. Construction and commercial loans

$x_1 + x_2 \geq 0.2 \times 2{,}119{,}562$

$x_1 + x_2 \geq 423912.4$

3. Limit on young farmers

$x_7 \geq 0.3 \times 2{,}119{,}562$

$x_7 \geq 635868.6$

4. Limit on home improvement and mortgage loans

$x_3 + x_4 \geq 0.15 \times 2{,}119{,}562$

$x_3 + x_4 \geq 317934.3$

5. Limit on mortgage compared to personal, commercial, and agric

$x_4 \geq 0.15\ (x_1 + x_5 + x_7)$

$0.15\ x_1 - x_4 + 0.15\ x_5 + 0.15\ x_7 \geq 0$

6. Limit on construction, mortgage and personal loans

$x_2 + x_4 + x_5 \geq 0.30(2119562)$

$x_2 + x_4 + x_5 \geq 635868.6$



7. Limit on bad debts

$$\frac{0.01x_1 + 0.04x_2 + 0.035x_3 + 0.05x_4 + 0.015x_5 + 0.03x_6 + 0.075x_7 \leq 0.04}{x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7}$$

$- 0.03x_1 - 0.005x_3 + 0.01x_4 - 0.025x5 - 0.01x_6 + 0.035x_7 \leq 0$

8. Non negativity

$x_1 \geq 0,\ x_2 \geq 0, x_3 \geq 0,\ x+_4\ x \geq 0,\ x_5 \geq 0,\ x_6 \geq 0,\ x_7 \geq 0.$

That is;

Maximize $Z = 0.2573x_1 + 0.248x_2 + 0.2352x_3 + 0.235x_4 + 0.2608x_5 + 0.261x_6 + 0.1563x_7$

Subject to;

$x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 \leq 2119562$

$x_1 + x_2 + 0x_3 + 0x_4 + 0x_5 + 0x_6 + 0x_7 \geq 423912.4$

$0x_1 + 0x_2 + 0x_3 + 0x_4 + 0x_5 + 0x_6 + x_7 \geq 635868.8$

$0x_1 + 0x_2 + x_3 + x_4 + 0x_5 + 0x_6 + 0x_7 \geq 317934.3$

$0.15x_1 + 0x_2 + 0x_3 - x_4 + 0.15x_5 + 0x_6 + 0.15x_7 \geq 0$

$- 0.03x_1 - 0x_2 - 0.005x_3 + 0.01x_4 - 0.01x_6 + 0.035x_7 \leq 0$

$x_i \geq 0,\ i = 1, 2, 3 \ldots 7$

**4.4 Solution of the LP model**

We use the LP software to solve the linear systems as shown below;

Writing the linear system in matrix form, we have,

Z = **[**0.2573 0.248 0.2352 0.235 0.2608 0.261 0.1563**]**

$$P = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0.15 & 0 & 0 & -1 & 0.15 & 0 & 0.15 \\ 0 & 1 & 0 & 1 & 1 & 0 & 0 \\ -0.03 & 0 & -0.005 & 0.01 & -0.025 & -0.01 & 0.035 \end{bmatrix} \qquad A = \begin{bmatrix} 2119562 \\ 423912.4 \\ 635868.6 \\ 317934.4 \\ 0 \\ 635868.6 \\ 0 \end{bmatrix}$$

That is, PX = A

**Table 4.3: Results variables**

| Decision variable | Solution | Objective cost | Total contribution | Reduced cost | Dual price |
|---|---|---|---|---|---|
| Commercial loan ($x_1$) | 423912 | 0.2573 | 109072.5576 | 0 | 0.261 |
| Construction loan ($x_2$) | 0 | 0.248 | 0 | 0.0091 | - 0.0037 |
| Home improvement ($x_3$) | 317934 | 0.2352 | 74778.0768 | 0 | -0.1047 |
| Mortgage loan ($x_4$) | 0 | 0.235 | 0 | 0 | -0.0258 |
| Personal loan ($x_5$) | 635869 | 0.2608 | 165834.6352 | 0 | 0 |
| Retail loan ($x_6$) | 105978 | 0.261 | 27660.258 | 0 | -0.0002 |
| Young farmers ($x_7$) | 635869 | 0.1563 | 99386.3247 | 0 | 0 |

Optimal value (Z) = 476731.8523

**4.5 Explanation of the solution**

The results were found after nine iterations. The optimal solution or value was found to be 476732.Table 4.3 depicts variables (column 1), the optimal value of the variables (column 2), objective cost or the objective function coefficients (column 3), total contribution (column 4), reduced cost (column 4), and the dual price (column 5).The variables show that funds for the loans should be allocated for personal, retail and young farmers loans with the amounts indicated in the table above.

The reduced cost for $x_1$, $x_2$, $x_3$, $x_4$, $x_5$, $x_6$ and $x_7$ are 0, 0.0091, 0, 0, 0, 0, and 0 respectively. The dual price for constraint (1) is 0.261, constraint (2) is – 0.0037, constraint (3) is -0.1047, constraint (4) is -0.0258,and constraint (6) is -0.0002.These are non zero because they correspond to the active constraints at the optimum, hence their slack variables are non basic (0), so the dual can be non zero.

### 4.6 Findings and analysis

The analysis revealed that, the commercial loan contributed GH¢ 109073.00, construction and mortgage loans did not contribute to the optimal solution, home improvement loan contributed GH¢ 74778.00, personal loan contributedGH¢ 165835.00, retail loan also contributedGH¢ 27660.00 and young farmers contributedGH¢ 99386.00. The objective function value is Z = 476732.As shown in Table 4.3, ADB, Sunyani branch should allocate GH¢ 423912.00 to commercial loans, GH¢ 317934.00 to home improvement loans, GH¢ 635869.00 to personal loans, GH¢105978.00 to retail loans and GH¢ 635869.00 to young farmers' loans and should not allocate funds to construction loans and mortgage loans since they do not contribute anything to the profit margin to ADB, Sunyani branch.

The analysis further indicates that personal loans contributed thirty four point seven nine percent (34.79%) which is the highest, followed by commercial loans which contributed twenty two point eight eight percent (22.88%), young farmers' contributed twenty point eight five percent (20.85%), home improvement loans also contributed fifteen point six nine percent (15.69%) and retail loans contributed five point eight percent (5.80%).

Refer to appendix A, B, C, D, E, F,G,H,I,J,K,L and M for tables showing the solution list, LP results and the iterations as displayed by the LP solver.

### 4.7 Summary

In this chapter, the proposed model formulated was implemented and the results were discussed. The next chapter which happens to be the last chapter of this project presents the conclusions, summary of the whole work, and recommendations of the study.

## CHAPTER FIVE

## SUMMARY, CONCLUSIONS AND RECOMMENDATIONS

### 5.0 Introduction

This chapter, which is the last chapter of the project presents the summary, discussion and conclusions drawn from the study and make recommendations to help ADB in order to optimize the profit margin.

### 5.1Summary



The main aim of this project is to develop linear programming (LP) model of disbursing funds allocated for loans effectively and efficiently in order to optimize profit margin of ADB, Sunyani.

To achieve this aim secondary data were collected from the bank which was used to formulate and implement the model. The model which was based strictly on the bank's loan policy was solved using LP solver (lips).The optimal solution was found to be 476732 in which personal loans contributed thirty four point seven nine percent (34.79%) which is the highest, followed by commercial loans which contributed twenty two point eight eightpercent (22.88%), young farmers' contributed twenty point eight five percent (20.85%), home improvement loans also contributed fifteen point six nine percent (15.69%) and retail loans contributed five point eight percent (5.80%). Both construction and mortgage loans did not contribute to optimal solution due to ineffective monitoring of loans and poor credit appraisal were some of the major causes of the loan problems.

### 5.2 Discussions and Conclusions

The results from the study show that most banks in the country do not have any scientific pedagogy to give out loans. Due to this, most banks are unable to optimize their profits, which intern affects their socio economic contributions in the areas in which they operate.

In Table 4.3 we could see that the solution value for construction and mortgage loans are all zeros which means that they do not contribute to the bank's profit in terms of loans allocated to them. The authorities of the bank should not allocate any funds to them since it will not yield any results. Commercial, home improvement, personal, retail and agric loans each respectively contributed GH¢ 109073.00, GH¢74778.00, GH¢ 165834.00, GH¢ 27660.00, and GH¢ 99386.00

It was detected that, the bank was able to achieve maximum profit due to the unit or objective cost of each decision variable. Due to this the bank should allocate funds to areas which have low probability of bad debt and interest rates which largely affect the coefficient of the objective function. The findings show that personal loans contributed so much to the bank in terms profit margin due to its low interest rate and low probability of bad debt. The policy makers and for that matter the loan officers at Sunyani ADB should allocate more funds to personal loans since it contributed thirty four point seven nine percent (34.79%) significantly to the bank's profit.

A model which has been proposed will help ADBs disburse their funds available for loans more effectively, profitably and efficiently. The results from the model show that, if ADB adopts the model they would be making an annual profit of GH¢ 476732.00on loans alone as against GH¢ 190693.00 in 2011if they are to stick to the model. Hence we conclude that the scientific method used to develop the proposed model can have a drastic increase in the profit margin of the bank if put into use.

### 5.3 Recommendations

It has emerged from the conclusion that the use of scientific methods to give out loans helps banks to avoid giving out loans that do not yield any profit there by allocating funds to areas they are sure to get good returns. Hence we recommend that ADB should adopt this model in their allocation of funds for loans.

Secondly, it is recommended that managers of banks and other financial institutions be educated to use scientific methods such as the use of mathematical models to help them disburse funds of the banks and institutions more efficiently and profitably.

Lastly, it is recommended that apart from loan disbursement, banks and other financial institutions should adopt the use of mathematical tools and methods in most of the businesses they conduct to improve efficiency in their work.

## REFERENCES


1. Acharya,U. (2005). Does *industry-wide distress affect defaulted firms*– evidence from creditor recoveries, London School of Business Working paper.
2. Agricultural Development Bank, (2008). Desk Diary, 2008.
3. Agricultural Development Bank, (2003 - 2007), Annual Reports and Financial Statements.
4. Altman, E.I.(1993).Corporate Financial distress and bankruptcy, 2nd. Ed, Wiley, New York.
5. Amponsah, S.K. (2007). *Optimization Technique lecture notes*, Institute of Distance Learning. KNUST, Kumasi, Ghana.
6. Booth, G. G. (1972). Programming Bank Portfolios under Uncertainty, *Journal of Bank Research* Vol. 3, No. 4 (winter), pp. 28-40.
7. Bordo, M.D. andMurshid, A.P. (2001). Are Financial Crises Becoming More Contagious? Chapter 14 in international financial contagion (eds) stijnclaessens and Kristin I. Forbes. Boston: Kluwer Academic publishers' pp 367 – 403.
8. Borio, C. N. K and Prowse, S. (1994). Exploring aggregate asset price fluctuations across countries: measurement, determinants and monetary policy implications, *BIS Economic Papers*,No 40, Basel, April.
9. Carey M., Gordy, M.(2003). Systematic risk in recoveries on defaulted debt, unpublished working paper presented at the 2001 Financial Management Association Meetings, Toronto, and 20th October. Cited in Altman, A. Resti, and A. Soroni.
10. Chambers andChames(1961). Intertemporal Analysis and optimization of bank portfolios, Management Science II,July 1961.

11. Cohen and Hamer, (1967:1972). Linear programming and optimal Bank AssetManagement Decision, *Journal of Finance.* Vol. 22, No. 2 (May pp. 147 - 165).

12. Comptroller. (1998). Handbook on Loan Portfolio management.

13. Crouhy, M., Galai, D. and Mark, R. (1994). A comparative analysis of current credit risk models. *Journal of Banking*.pp 59 -117.

14. Dantzig, G. (1947).*Optimal Solution of a Dynamic Model with substitution*,Economica, University of California,23(3): 295-302.

15. Dince, R. R. and Fortson, J. C. (1977). An Application of Goal Programming to the Management of a Country Bank, *Journal of Bank Research.*Vol. 7, No. 4 (winter), pp. 311-319.

16. Feldman, R. M.and Valdez-Flores, C. (2010). *Applied Probability and Stochastic Processes*, Springer, ISBN: 978-3-642-0505-5.

17. Frye. (2000). Collateral Damage Detected, Federal Reserve Bank of Chicago,Emerging Issues Series, October, 1-14.

18. Good hart, C.A. E. (2004).The Bank of England, 1970- 2000, ch. 17. R. Michie (ed.), *The British Government and the city of London in the Twentieth Century*, Cambridge, Cambridge University Press, 340 – 712.

19. Gollier, C. (2001*).The Economics of Risk and Time*. Cambridge, MA; London, England: The MIT Press.ISBN 0-262-07215-7.

*20.* Gordy, M. (2000). A comparative anatomy of credit risk models. *Journal of Banking and Finance*24, 119-149.

21. Grunert, J.,Nordern, L.,Weber, M. (2005). The role of non-Financial factors in internal Credit Ratings. *Journal of Banking and Finance*, 29, 509 – 531.

22. Gurtler, M., Heithecker, D. (2005). Systematic credit cycle risk of financial collaterals: modeling and evidence, working paper, TU Braunschweig.

23. Gustafsson, J.,Salo, A. (2004). *Valuing Risky Projects with Contingent Portfolio Programming*. Helsinki University of Technology.

24. HuY. T.,Perraudin, W. (2002). *The dependence of recovery rates and defaults*,Birkbech College.

25. Infodata Associates. (2006). Banking Survey, Business and Financial Times,June 16, 2006, pp. 1 – 3.

26. Jokivuolle, E. and Peura, S. (2001). *Regulatory capital volatility*. Risk, May.

27. Kall, P. and Wallace, S.W. (1994). *Stochastic Programming*. New York: John Wiley and Sons. ISBN 0- 471-95108-0.

28. K. Duellmann and M. Trapp. *Systematic risk in recovery rates*. An empirical analysis of US corporate credit exposures. Deutsche Bundesbank Discussion paper 02/ 04/ 2004.

29. Markowitz, H.M. (1952). *Portfolio Selection.* Journal of Finance.Vol. 7, pp. 77-91.

30. Martin, H. (1995). Systemic Aspects of Risk Management in Banking and Finance, Swiss*Journal of Economics and Statistics* (SJES), vol. 131(IV), pages 723-737, December.

31. Mensah, S. (2001). Securities markets and Investments: A Ghanaian Primer SEM Financial Training Centre, University of Michigan.

32. Mongin, P. (1997). Expected Utility Theory, Handbook o f Economic Methodology(J.Davis, W.Hands, and U.Maki, eds. London).

33. Rheinberger., K. and Summer, M. ( 2009). Does Adding up of Economic Capital for market and credit Risk amount to conservative Risk Assessment.?*Journal of Risk*, 1(4): 33 – 53.

34. Rockafellar, R. T. and Uryasev, S. (2002). Conditional Value-at-Risk for General Loss Distributions. *Journal of Banking and Finance*, 26(7), 1443-1471.

35. Robinson,R. I. (1961). The Management of Bank funds, N Y,McGraw-Hill 1991.

36. Tobin, J.(1965). A Dynamic Aggregative Model. *Journal of Political Economy*, University of Chicago Press, vol.63, p. 103.

37. Uryasev, S. and Rockafellar, R. T. (2000). Optimization of Conditional Value-at-Risk.*Journal of Risk*,2(3), 21-41.

38. Van De Panne, (1975). Methods for Linear and Quadratic Programming and its Applications. Amazon.com. p. 521

39. Waterman, R. H. and Gee, R. E. (1981). An Application of Linear Programming to Bank Financial. *The Institute of Management Sciences*. Vol. 11, No. 5, pp. 77-83.

40. Yong,W.Y. and Sheng-Yi, L. (1988).Asia Pacific Journal of Management.Vol. 5 No.2January 1988.

41. www.agricbank.com (Access Date: June 17, 2009)

42. www. htt://en.wikipedia.org/wiki/loan (Access Date: April 15, 2009)