

MINIMISING TRANSPORTATION COST OF PETROLEUM PRODUCTS
THROUGH DISTRIBUTION NETWORKS LINKING DEPOTS OPERATED BY
BULK OIL STORAGE AND TRANSPORTATION COMPANY LIMITED (BOST),
GHANA USING TRANSPORTATION MODULE

by

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of

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DECLARATION

I hereby declare that this project is my own work towards the Master of Science degree and that to the best of my knowledge, it contains no material previously published by another person nor material which has been accepted for the award of any other degree of the University, except where due acknowledgement has been made in the text.

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ABSTRACT

The transportation model deals with the distribution of goods from several points of supply (sources) to a number of points of demands (destinations) such that the total transportation cost is minimized. Typical examples include petroleum products distribution system, railways, roadways, telephone system and citywide water system.

In transportation model, a transportation table is setup such that the relevant data are summarized conveniently and concisely while observing restrictions regarding supply and requirements. The basic feasible solution for a balanced problem is then found by applying any of following methods such as Northwest Corner Method, Minimum Cost Method and Vogel's Method. The initial feasible solution is then moved to an optimal feasible solution by using an iterative technique such as Stepping-stone method or Modified Distribution Method.

The data collected from the Marine Services, Volta Lake Transportation Company Limited and Bulk Oil Storage and Transportation Company Limited revealed that pipeline is the cheapest mode of transportation of petroleum products in the country. After analyzing the data on the petroleum products distribution network linking all the five petroleum products storage depots located at Eastern, Greater Accra, Ashanti, Northern and Upper East Regions showed that the bulk transportation cost of products can be reduced by 14.16%. Also, two optimal solutions were obtained.

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DEDICATION

I wish to dedicate this successful work to the Almighty God who has made it possible for me to be at this level. Also, to my family members especially my lovely wife and daughter, management and staff of Bulk Oil Storage and Transportation Company Limited, lecturers and friends by whose support and inspiration this work was successfully completed.

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CHAPTER ONE

INTRODUCTION

1.1 BACKGROUND TO THE STUDY

The Oil industries play a major role in the economies of oil producing countries such that petroleum products have constituted the major source of fuel for driving engines and automobiles as well as for domestic needs. Oil industries have also facilitated the development of infrastructures such as road, telecommunication, industries and other facilities that have brought about better standard of living. Apart from these benefits, this sector has also affected the environment negatively such that lives and valuable properties have been destroyed. The effects are worsened by oil spillage into water bodies and land resulting to the death of man, animals and plants and the pollution of the environment. Their uses have increased the incidence of oil spillage in the process of transportation as well as their discharge in depots and petroleum stations or areas where needed.

Petroleum products have been reported to have adverse effects on plants irrespective of their habits or habitats. Such effects are manifested in reduced and retarded growth, vestigial plant parts, morphological defects, and irregular physiological processes such as chlorosis and in extreme cases, death. Mckendrick (2002) reported that wilting, defoliation and loss of productive cycle as immediate impacts on the vegetation. Furthermore, destruction of normal plant water relation which indirectly affects plant

metabolism with respect to nutrient availability or directly toxic to plant (McCown *et al.*, 1972).

The most urgent problem of man is environmental pollution as clean air is no more a normal feature of the environment. The environment is now loaded with pollutants from agricultural chemicals, radioactive materials, sewage and chemicals in water and solid wastes on land (Little and Martin, 1974). Natural and agricultural sources, energy production and recycling operation, urban and industrial complexes and cars are sources of atmospheric pollution (Seaward and Richardson, 1990, Wegner *et al.*, 1990). The presence of heavy metals such as lead and cadmium in the petroleum products have been indicted in the blockage of conducting vessels of plants thereby interfering in the physiological and biochemical processes necessary for plant development (Jastrow and Koeppe 1980).

Petroleum, when subjected to fractional distillation and refined gives the Dual Purpose Kero (kerosene), Automobile Gas Oil (diesel) and Premium Motor Spirit (petrol). Each of these products has important usage in the environment. Kerosene is widely used in aviation industry as fuel for jet engines as well as in the homes for cooking in stoves and lightening in lanterns. Diesel is used for power generating plants and by heavy trucks and also as illuminant. In the same vein, petrol is used to move automobiles and for light generation by generators.

The exploration, transportation and utilization of these petroleum products pose serious danger to the lives of the flora and fauna in the environment. There is the need therefore to ascertain or monitor the pending risk or potential damage level of these pollutants

from time to time. A good bio-indicator is therefore required. Renzaglia and Vaughn (2000) identified bryophytes as the primary receptors of atmospheric deposition. To this effect, *Barbula lambarenensis*, a common moss has this potential and can be used to monitor the effects of petroleum products.

There is little argument that liquid petroleum (crude oil and the products refined from it) plays a pervasive role in modern society. As recently as the late 1990s, the average price of a barrel of crude oil was less than that of a take-out dinner. Yet a fluctuation of 20 or 30 percent in that price can influence automotive sales, holiday travel decisions, interest rates, stock market trends, and the gross national product of industrialized nations, whether they are net exporters or importers of crude oil. A quick examination of world history over the last century would reveal the fundamental impact access to crude oil has had on the geopolitical landscape. Fortunes are made and lost over it; wars have been fought over it. Yet its sheer magnitude makes understanding the true extent of the role of petroleum in society difficult to grasp. Furthermore, widespread use of any substance will inevitably result in intentional and accidental releases to the environment. The frequency, size, and environmental consequences of such releases play a key role in determining the extent of steps taken to limit their occurrence or the extent and nature of mitigation efforts taken to minimize the damage they cause.

Consequently, the United States and other nations engaged in strategic decision making regarding energy use spend a significant amount of time examining policies affecting the extraction, transportation, and consumption of petroleum. In addition to the geopolitical aspects of energy policymaking, the economic growth spurred by inexpensive fuel costs must be balanced against the environmental consequences associated with widespread

use of petroleum. Petroleum poses a range of environmental risks when released into the environment (whether as catastrophic spills or chronic discharges). In addition to physical impacts of large spills, the toxicity of many of the individual compounds contained in petroleum is significant, and even small releases can kill or damage organisms from the cellular- to the population-level. Compounds such as polycyclic aromatic hydrocarbons (PAH) are known human carcinogens and occur in varying proportions in crude oil and refined products. Making informed decisions about ways to minimize risks to the environment requires an understanding of how releases of petroleum associated with different components of petroleum extraction, transportation, and consumption vary in size, frequency, and environmental impact.

Since the mid 1980s the inflation adjusted price of a barrel of crude oil on NYMEX had been generally under \$25/barrel and was still at this level in September 2003. A series of events led the price to reach over \$60 by August 11, 2005, surpass \$75 in the summer of 2006, fall below \$60/barrel by the early part of 2007, then rise steeply, reaching \$92/barrel by October 2007 and \$99.29/barrel for December futures in New York on November 21, 2007[1. Throughout the beginning of 2008, oil hit several new record highs. On February 29, 2008, oil prices hit an inflation-adjusted all-time peak at \$103.05 per barrel, and reached \$110.20 on March 12, 2008, the sixth record high in seven trading days. The most recent price per barrel maximum of \$135.09 was reached on May 22, 2008.

The effect that rising oil prices have on a market is not directly proportional to the cost of crude oil. For example, while crude oil prices increased 400% from 2003–2008,

United States gasoline prices did not rise by the same factor. This is because the profits of distributors and retailers, production costs (such as refining, transportation), and taxes are all part of the price of auto fuel. However, as the cost of crude oil increases, the crude-oil cost becomes a relatively larger component of the retail price of gasoline, causing any further increases in crude oil prices to have correspondingly larger impact on consumers.

1.2 STATEMENT OF PROBLEM

This research is to investigate the least cost of bulk transportation of petroleum products across the storage depots of Bulk Oil Storage and Transportation Company Limited (BOST) in Ghana strategically located country-wide. The vision of BOST is to be a major quality player in the Government's vision of ensuring that Ghana has a continuous, uninterrupted, reliable and safe supply, effective and efficient distribution of petroleum products at the most competitive prices for its socio-economic development.

The Bulk Oil Storage and Transportation Company Limited was incorporated in December, 1993 as a private limited liability company with the Government of Ghana as the sole shareholder responsible for storing and transporting strategic and operational bulk petroleum products throughout the country in the most efficient way by pipelines, lake, road, rail and sea. Its depots are located at Accra Plains, Kumasi, Takoradi, Bolgatanga, Buipe, Maimi Water and Akosombo in the Greater Accra, Ashanti, Western, Upper East, Northern, Volta and Eastern Regions respectively. Its Accra Plains

Depot is fed with products from a Convention Buoy Mooring (CBM) floated thirty-two kilometers (32 km) offshore, which is operated by Marine Services at Kpong near Tema and Tema Oil Refinery (TOR) through pipelines while the Kumasi depot is fed by bulk road vehicles (BRVs) from Tema Oil Refinery or convention Buoy Mooring (CBM). The depots at Maimi Water, Akosombo, Buipe, Takoradi and Bolgatanga are fed by pipelines and Volta Lake.

The transportation of petroleum products by pipeline is the safest and cheapest followed by lake, which uses river barges, railway and road where tankers or bulk road vehicles are used.

Petroleum products are useful materials derived from crude oil (petroleum) as it is processed in oil refineries. Major products of oil refineries are asphalt, diesel, fuel oil, gasoline, kerosene, liquefied petroleum gas (LPG), lubricating oils, paraffin wax and tar.

At the moment BOST stores premium, gas oil and kerosene at Kumasi Depot and the rest of the depots keep only premium and gas oil.

Although as yet, its upstream oil industry has no crude oil production, Ghana is one of four West African countries with an oil refinery. The Tema refinery operated by the Tema Oil Refinery Corporation (TOR) has an operating capacity of forty-five thousand barrels (45,000 bbl) per day running on crude oil imported from Nigeria. The downstream sector, which is regulated by the National Petroleum Authority (NPA), is key to the economy and Oil-derived products supply seventy per cent (70%) of Ghana's commercial energy needs. Current consumption of petroleum products is in the region of ninety-five thousand tones (950,000 tons) per annum. Increasing power demands by

industry and domestic consumption and a need to reduce the reliance on hydroelectric power is not only fuelling the search for oil and gas but also has set in motion projects relating to the importation of gas via pipeline from Nigeria and Cote d'Ivoire.

As a result of the deregulation policy currently in place every oil Company is at liberty to bring or import and sell her own products in the country. The transportation of petroleum products from Accra Plains Depot to Kumasi, Buipe, Bolga, maimi water and Akosombo depots by bulk road vehicles (BRVs), pipelines and barges and pipelines respectively is one of the main tasks of BOST, which consumes chunk of its resources. Therefore a significant reduction in transportation cost would ensure its competitiveness in the face of steep competition from other bulk distribution companies(BDC) such as Chase Petroleum Ghana Limited, which is building its depot at Tema, Cirus Oil, which has depots at Tema and Takoradi, Sahara, which has a depot at Tema and so on. These companies haul petroleum products by only pipelines into their depots which impact positively in their operations.

In this work, I consider the transportation problem characterized by a set of depots having different cost of transporting bulk petroleum products per litre and these locations are replenished by different or a combination of two or more modes of transportation.

The bulk transportation of petroleum products from Tema Oil Refinery (TOR) and Conventional Buoy Mooring (CBM) operated by Marine Services are the only sources of finished products for BOST 'S depots country-wide at the moment. The transportation cost of petroleum products forms a huge portion of the prices of petroleum products in

Ghana which has a significant effect on the socio-economic development in Ghana. Therefore appreciable reduction of cost of transporting products will lead to a decrease in prices of goods and services in general.

1.3 OBJECTIVES

The research to reduce the bulk transportation cost of petroleum products in Bulk Oil Storage and Transportation Company Limited will help to make it more competitive and more viable in the face of steep competition from private distribution companies operating at the moment in the country. All the private distribution companies receive finished products by pipelines which is the cheapest mode of transportation. The main objective for undertaking this project is to minimize total transportation cost of petroleum products in Bulk Oil Storage and Transportation Company Limited's operations.

1.4 METHODOLOGY

Transportation problem is a member of a category of linear programming (LP) called network flow problem which deals with the distribution of goods from several points of supply (sources) to a number of points of demand (destination). It involves finding an initial solution, testing the solution to see if it is optimal and developing an improved solution. This process continues until an optimal solution is reached.

The methods that will be used in solving the transportation problem of Bulk Oil Storage and Transportation Company Limited are the least cost, Northwest Corner rule or

Vogel's Method for finding a basic Feasible Solution and then using either the Stepping-stone or Modified Distribution Method (MODI) to obtain the optimal solution.

The source of the relevant data in this work was obtained from Bulk Oil Storage and Transportation Company Limited (BOST), Volta Lake Transportation Company Limited (VLTC), Marine Services and National Petroleum Authority (NPA) of Ghana.

1.5 JUSTIFICATION

The cost of bulk transportation of petroleum products in Ghana is part of petroleum products price build-up or ex-pump price or retail price therefore its reduction will result in reduction of prices of general goods and services. Primary distribution Margin (PDM) in the petroleum products prices build-up is used to pay for cost of transportation of petroleum products in the country to ensure that the ex-pump price is the same throughout the country.

Higher prices of petroleum products will result in hiking of prices of food stuff, goods and services will impact negatively on the economy of Ghana. Therefore a significant reduction in cost of bulk transportation of petroleum products will impact positively on the economy of Ghana.

Repetitive price increases erode the purchasing power of money and other financial assets with fixed values, creating serious economic distortions and uncertainty. Inflation results when actual economic pressures and anticipation of future developments causes

goods and services to exceed the supply available at existing prices, or when an available output is restricted by faltering productivity and marketplace constraints. When the upward trend of prices is gradual and irregular averaging only a few percentage points each year, such creeping inflation may not be considered as a serious economic threat and social progress.

1.6 LIMITATIONS

The Ghana railway which is the cheapest and safest mode of hauling products across the length and breadth of the country apart from pipeline is malfunctioning therefore the relevant data could not be obtained. There was no proper data on the cost of transporting products through all the pipelines operated by the Bulk Oil Storage and Transportation Company Limited except that operated by Marine Services which was used to estimate for that of BOST.

CHAPTER TWO

LITERATURE REVIEW

2.1 INTRODUCTION

This chapter examines research works carried out so far by other people about transportation.

That is, we are going to review the intellectual contributions of earlier researchers in a form of literature on transportation.

2.2 LITERATURE REVIEW

Glover et al., (1978) developed a primal simplex procedure to solve transshipment problems with an arbitrary additional constraint. The procedure incorporates efficient methods for pricing-out the basis, determining certain key vector representations, and implementing the change of basis. These methods exploit the near triangularity of the basis in a manner that takes advantage of computational schemes and list structures used to solve the pure transshipment problem. The authors also developed a fast method for determining near optimal integer solutions. Computational results showed that the near optimum integer solution value is usually within 0.5% of the value of the optimum continuous solution value.

Gong and Yucsan (2006) used simulation optimization by combining an LP/network flow formulation with infinitesimal perturbation analysis to examine the multi-location

transshipment problem with positive replenishment lead times, and demonstrated the computation of the optimal base stock quantities through sample path optimization. From a methodological perspective, the authors deployed an elegant duality-based gradient computation method to improve computational efficiency. In test problems, their algorithm was also able to achieve better objective values than an existing algorithm.

Melachrinoudis et al., (2005) proposed a novel multiple criteria methodology called physical programming (PP). The proposed PP model enables a decision maker to consider multiple criteria (i.e., cost, customer service and intangible benefits) and to express criteria preferences not in a traditional form of weights, but in ranges of different degrees of desirability. The proposed model was tested with real data involving the reconfiguration of an actual company's

distribution network in the United States and Canada.

Topkis (1984) considered complementarity and substitutability among locations for a two-stage transshipment problem with locations being factories, warehouses, and demand centers. A direct generalization of properties known for the transportation problem would be that any two locations of different types are complements and any two locations of the same type are substitutes. Examples showed that these properties need not hold for pairs of locations that include at least one warehouse. An algorithm of Nagelhout and Thompson (1981) for locating warehouses is based on the incorrect supposition that any two warehouses are substitutes, and an example shows that their

algorithm need not generate an optimal solution as claimed. For pairs of locations that do not include a warehouse, complementarity and substitutability properties hold just as in the transportation problem.

Kwak and Schniederjans (1985) presented an application of goal programming as an aid in facility location analysis. Specifically, they illustrated the use of a goal programming model to resolve a site location problem. The results of the study showed that the goal programming model presented could be used to improve the site selection process over existing models. This was accomplished by allowing consideration of substitutable resources that exist in the decision environment.

Weber (2004) studied complementarity in the transshipment problem. In contrast to the typical distinction between complements and substitutes in the operations literature, the distinction here involved the impact of a change in the unit shipping cost for one route on the amount of product shipped via another route. Among other results, he showed that each route must have at least one and in some cases two substitutes. Furthermore, for each factory, which operates at capacity and each customer who receives exactly his given demand for the product, there is at least one route which has at least one complement.

Hoppe and Tardos (2000) gave the first polynomial-time algorithm for the quickest transshipment problem. Their algorithm provided an integral optimum flow. Previously, the quickest transshipment problem could only be solved efficiently in the special case of a single source and single sink.

Herer and Tzur (2001) investigated the strategy of transshipments in a dynamic deterministic demand environment over a finite planning horizon. This is the first time that transshipments are examined in a dynamic or deterministic setting. The authors considered a system of two locations which replenish their stock from a single supplier, and where transshipments between the locations are possible. Their model includes fixed (possibly joint) and variable replenishment costs, fixed and variable transshipment costs, as well as holding costs for each location and transshipment costs between locations. The problem is to determine how much to replenish and how much to transship each period; thus this work can be viewed as a synthesis of transshipment problems in a static stochastic setting and multi-location dynamic deterministic lot sizing problems. The authors provided interesting structural properties of optimal policies which enhanced their understanding of the important issues which motivated transshipments and allowed them to develop an efficient polynomial time algorithm for obtaining the optimal strategy. By exploring the reasons for using transshipments, they enabled practitioners to envision the sources of savings from using this strategy and therefore motivate them to incorporate it into their replenishment strategies.

Herer et al., (2006) considered a supply chain, which consists of several retailers and one supplier. The retailers, who possibly differ in their cost and demand parameters, may be coordinated through replenishment strategies and transshipments, that is, movement of a product among the locations at the same echelon level. They proved that in order to minimize the expected long-run average cost for this system, an optimal replenishment policy is for each retailer to follow an order-up-to S policy. Furthermore, they

demonstrated how the values of the order-up-to quantities can be calculated using a sample-path-based optimization procedure. Given an order-up-to S policy, the authors showed how to determine an optimal transshipment policy, using a linear programming/network flow framework. Such a combined numerical approach allowed them to study complex and large systems.

Belgasmi et al., (2008) investigated the case where locations have a limited storage capacity. The problem was to determine how much to replenish each period to minimize the expected global cost while satisfying storage capacity constraints. They proposed a Real-Coded Genetic Algorithm (RCGA) with a new crossover operator to approximate the optimal solution. They analyzed the impact of different structures of storage capacities on the system behaviour. They found that Transshipments were able to correct the discrepancies between the constrained and the unconstrained locations while ensuring low costs and system-wide inventories. Their genetic algorithm proved its ability to solve instances of the problem with high accuracy.

McLean and Biles (2008) presented a simulation model of the operation of a liner shipping network that considers multiple service routes and schedules. The objective was to evaluate the operational costs and performance associated with liner shipping, as well as the impact of individual service schedules on the overall system. The approach proposed a discrete-event simulation model where shipping activities, container ship operations, and intermodal container movements were considered. The model allows for direct and transshipment operations of container cargo, and the evaluation of fuel

consumption and other logistics metrics. The model was used to evaluate a liner shipping network consisting of four service routes, up to 64 container ships, and up to twenty (20) ports with diverse physical characteristics and cost components. The results showed the contribution of service routes, ports, container ships, and containers to the cost and performance of the system.

Anderson et al., (2008) presented a simulation model of a barge transportation system for petroleum delivery within an inland waterway. The simulation was employed as an evaluation model within a decision support system, which also included a criterion model, represented as a decision maker's utility function, and an optimization procedure which employed scatter search. Variance reduction techniques were also employed in order to improve the accuracy of the estimates of the performance measures associated with the system. The main purpose of the system was to determine values for important inventory policy variables.

Al-Nory and Brodsky (2008) proposed and developed a framework and an extensible library of simulation modeling components for strategic sourcing and transportation. The components included items, suppliers, volume-discount schedules, aggregators, procurement rules, and less-than-truck-load delivery. Service models are classes in the Java programming language extended with decision variables, assertions, and business objective constructs. The optimization semantics of the framework was based on finding an instantiation of real values into the decision variables in the service object constructor that satisfies all the assertions and leads to the optimal business objective. The

optimization was not done by repeated simulation runs, but rather by automatic compilation of the simulation model in Java into a mathematical programming model in AMPL and solving it using an external solver.

Ekren and Heragu (2008) studied a single-item two-echelon inventory system where the items could be stored in each of N stocking locations is optimized using simulation. The aim of the study was to minimize the total inventory, backorder, and transshipments costs, based on the replenishment and transshipment quantities. Transshipments which are the transfer of products among locations at the same echelon level and transportation capacities, which are the transshipment quantities between stocking locations, were also considered. Here, the transportation capacities among the stocking locations are bounded due to transportation media or the locations' transshipment policy. Assuming stochastic demand, the system is modeled based on different cases of transshipment capacities and costs. To find out the optimum levels of the transshipment quantities among stocking locations and the replenishment quantities, the simulation model of the problem is developed using ARENA 10.0 and then optimized using the OptQuest tool in this software.

Rossetti, et al.,(2008) presented an object oriented framework that facilitates modeling inventory systems whose policy updating is driven by forecast estimates. In an inventory system, the forecast estimates and the forecast error measures are used to set the inventory policy. A simulation approach can address questions regarding the choice of the forecasting technique and the frequency of updating the policy, especially in non-

stationary demand scenarios. This paper discusses how the framework can be used to develop simulation models through which these questions can be addressed. In addition, two examples illustrate how to use the framework and how to analyze supply chains with forecast based policy updating.

Ogryczak et al., (1989) described the results of research, development and implementation of the Dynamic Interactive Network Analysis System (DINAS), which enables the solution of various multi-objective transshipment problems with facility location. DINAS utilizes an extension of the classical reference-point approach to handling multiple objectives. In this approach, the decision-maker (DM) forms his requirements in terms of aspiration and reservation levels, i.e., he specifies acceptable and required values for given objectives. For providing DINAS with solutions to single-objective problems, a special TRANSLOC solver was developed. It is based on the branch-and-bound scheme with a pioneering implementation of the simplex special ordered network (SON) algorithm with implicit representation of the VUB and SUB (variable and simple upper bound, respectively) constraints.

Moore et al., (1978) presented the formulation of a goal programming model for analysis of the transshipment problem, where multiple conflicting objectives must be considered. Included were the general G. P. model for the transshipment problem, and a representative application of goal programming to such a problem. Analysis and interpretation of the G.P. solution to the problem was presented.

Scott et al., (2006) compared two genetic algorithms tailored to solve this problem based on permutation and bin-packing inspired encodings. Results were presented and analysed in order to evaluate the validity and robustness of the two approaches. As part of the analysis, bounds were calculated to determine how well both GAs perform in absolute terms as well as relative to each other. Of the two GA there was one clear winner, although it was not the one that would have been indicated by previous research. Whilst the winning GA is able to generate significant savings in practice, compared to the optimum there remains room for further improvement.

Reyes (2006) provided a numerical example for a three-person game in a non-cooperative environment, a selected cooperative environment, and a fully cooperative environment. The well-known Shapely Value concept from cooperative game theory was used as an approach to solve the transshipment problem and showed “how” the pooling coalitions should be formed. The appurtenance of the Shapley Value is verified to ensure that a stable solution exists.

Staniec and Cyrus (1987) examined two categories of solution algorithms for the large-scale multi-commodity transshipment problem (MCTP): resource direction and price direction. In the former category they constructed RDLB, a new algorithm which uses a simplified projection method in the subgradient capacity reallocations and conjugate subgradient directions with approximate line search to provide better termination conditions in the Lagrangean lower-bounding iteration. In the latter category, they developed DDC, a dual decomposition, and they introduced RSD (P) and RSD (A), new

non-linear decomposition algorithms for the MCTP based on penalty transformations of the original problem and using restricted simplicial decomposition.

Huang and Greys (2009) studied a newsvendor game with transshipments, in which n retailers face a stochastic demand for an identical product. Before the demand is realized, each retailer independently orders her initial inventory. After the demand is realized, the retailers select an optimal transshipment pattern and ship residual inventories in order to meet residual demands. Unsold inventories are salvaged at the end of the period. We compare two methods for distribution of residual profit – transshipment prices (TP) and dual allocations (DA) -- that were previously analyzed in literature. Transshipment prices are selected a priori, before the demand is known, while dual allocations, which are obtained by calculating the dual prices for the transshipment problem, are calculated ex post, after observing the true demand. We first study the conditions for the existence of the Nash equilibria under DA, and then compare the performance of the two(2) methods and show that neither allocation method dominates the other. Their analysis suggests that DA may yield higher efficiency among "more asymmetric" retailers, while TP works better with retailers who are "more alike", but the difference in profits does not seem significant. They also link expected dual prices to TP, and use those results to develop heuristics for TP with more than two symmetric retailers. For general instances with more than two asymmetric retailers, they proposed a TP agreement which uses a neutral central depot to coordinate the transshipments (TPND). While DA in general outperformed TPND in their numerical simulations, its ease of implementation makes TPND an attractive alternative to DA when the efficiency losses are not significant (e.g., high critical fractiles or lower demand variances).

Hindi (1996) showed that the heuristic solutions obtained are effective, in that they are extremely close to the best known solutions. The computational efficiency makes it possible to solve realistically large problem instances routinely on a personal computer; in particular, the solution procedure is most effective, in terms of solution quality, for larger problem instances.

Shakeel(2007) analysed a transshipment problem whose objective function is fractional under stochastic environment where the demand considered is uncertain. The objective here is to maximize the net expected revenue per unit transportation cost, i.e., the total expected revenue minus the transportation and transshipments cost. The stochastic transshipment problem is converted to an equivalent deterministic transportation problem for which an algorithm is developed and numerically illustrated.

Perng and Ho (2007) discussed how to apply the concept of the extra supply chain cost to managing an operational problem in a transshipment center. They set four scenarios. Scenario 1 is: goods will be sent to the transshipment center until there is no free space. Scenario 2 is: goods will be sent to the transshipment center and a temporary space. Scenario 3 is: Scenario 1 without free space reorganization. Scenario 4 is: Scenario 2 without free space reorganization. In all the four scenarios, they created an extra supply chain cost equation to validate the results. The extra supply chain cost consists of renting temporary space, transportation, reorganization and returned goods. They used empirical data gathered from some transshipment centers in central Taiwan to simulate our research problem. In conclusion they showed that the larger the transshipment center,

the lower the extra cost. The extra cost will be minimized if the transshipment center returns the superfluous goods directly because of high space and reorganization cost. If the transshipment center is full: first the return of the goods should be considered; secondly, the renting of temporary space should be considered, with the reorganization of space being the final consideration.

Özdemir et al., (2006) considered coordination among stocking locations through replenishment strategies that explicitly take into account lateral transshipments, i.e., transfer of a product among locations at the same echelon level. The basic contribution of our research is the incorporation of supply capacity into the traditional emergency transshipment model. The authors formulated the capacitated production case as a network flow problem embedded in a stochastic optimization problem. They developed a solution procedure based on infinitesimal perturbation analysis (IPA) to solve the stochastic optimization problem numerically. They analyzed the impact on system behavior and on stocking locations' performance when the supplier may fail to fulfill all the replenishment orders and the unmet demand is lost. The authors found that depending on the production capacity, system behavior can vary drastically. Moreover, in a production-inventory system, they found evidence that either capacity flexibility (i.e., extra production) or transshipment flexibility is required to maintain a certain level of service.

Benes (1992) investigated the set of all finite measures on the product space with given difference of marginals. Extremal points of this set are characterized and constructed. Sets of uniqueness are studied in the relation to marginal problem. In the optimization

problem the support of the optimal measure is described for a class of cost functions. In an example the optimal value is reached by an unbounded sequence of measures.

Bradley et al., (1977) gave a complete description of the design, implementation and use of a family of very fast and efficient large scale minimum cost (primal simplex) network programs. The class of capacitated transshipment problems solved is the most general of the minimum cost network flow models, which include the capacitated and uncapacitated transportation problems and the classical assignment problem; these formulations are used for a large number of diverse applications to determine how (or at what rate) a good should flow through the arcs of a network to minimize total shipment costs. The presentation tailors the unified mathematical framework of linear programming to networks with special emphasis on data structures which are not only useful for basis representation, basis manipulation, and pricing mechanisms, but which also seem to be fundamental in general mathematical programming. A review of pertinent optimization literature accompanies computational testing of the most promising ideas. Tuning experiments for the network system, GNET, are reported along with important extensions such as exploitation of special problem structure, element generation techniques, postoptimality analysis, operation with problem generators and external problem files, and a simple noncycling pivot selection procedure which guarantees finiteness for the algorithm.

Bhaumik and Kataria (2006) described a large watch manufacturer in India and a proposed system to reduce its cost and simultaneously improve its service level by planning lateral transshipments. Many authors have studied the multi-location inventory

situations, under different demand and cost characteristics, and almost all of them have developed push systems developing optimum or good replenishment/transshipment policies. Yet pull systems may offer simpler and more practical joint replenishment/transshipment policies, particularly when practical considerations as faced by real companies are studied. The process map including the inputs and the outputs for the replenishment system is described, as well as the feedback mechanism to revise the system parameters.

Topkis (1984) developed complement and substitution principles applicable to settings in transshipment dual stage problems such as those encountered in factories and warehouses. Direct examination of the basic property of this transportation problem suggests that two locations of a similar nature would be reasonable substitutes. Such elements may not apply to location pairs where there is one or more warehouses. Where no warehouse is present, complement and substitution principles are functional. Model illustrations of factory warehouses and demand center locations are highlighted.

Sarker and Namatame (2004) developed three evolutionary algorithms for solving a transshipment problem. The authors investigated the effect of population sizes on the quality of solutions to be obtained, the computational time to be required and the size of search spaces of the problems under consideration. They also used a well-known conventional optimization package to compare the quality of solutions. The numerical results are analysed and the interesting findings are discussed.

Iakovou (2001) presented the development of a strategic multi-objective network flow model, allowing for risk analysis and routing, with multiple commodities, modalities and origin-destination pairs. The development of an interactive solution methodology is also presented followed by its implementation via a World Wide Web-based software package. Government agencies will find the model helpful in determining how regulations should be set to derive desirable routing schemes. Shippers will also find the model useful in optimizing their logistics costs.

Boamah (2009) used optimisation techniques in solving the transportation problem of two large marketing firms. The problem involves finding the cheapest way of transporting or shipping the firms' commodity from their sources of supply to their destinations. The identity of each of the firms is not disclosed since this was the condition for the supply of data. A special purpose algorithm that is more efficient and intuitive than the Simplex Method is discussed and used to solve the transportation problem for each one of the firms separately and the transportation problem for the two firms put together and the results compared. It turned out on the average that the minimum cost of the combined transportation problem was less by over fifteen million cedis per month as compared to the sum of the minimum costs for each of the firms.

Bazlamacci and Hindi (1996) developed further the adjacent extreme point search method to enhance its overall computational efficiency. The enhanced search algorithm is then imbedded in a tabu search scheme which proved capable of finding better solutions, by a wide margin in some instances. Another tabu search scheme, somewhat inferior in terms

of solution quality but computationally more efficient, is also developed to provide an alternative solution vehicle for larger networks. Results of extensive computational testing are included.

Koo et al., (2006) looked at a refinery supply chain simulation and attempt to optimize the refinery operating policies and capacity investments by employing a genetic algorithm. The refinery supply chain is complex with multiple, distributed, and disparate entities which operate their functions based on certain policies. Policy and investment decisions have significant impact on the refinery bottom line. To optimize them, they developed a simple simulation-optimization framework by combining the refinery supply chain simulator called Integrated Refinery In Silico (IRIS) and genetic algorithm. Results indicate that the proposed framework works well for optimization of supply chain policy and investment decisions.

CHAPTER THREE

METHODOLOGY

3.1 INTRODUCTION

Transportation problem is a category of linear programming (LP) techniques called network flow problem which deals with the distribution of goods from several points of supply (sources) to a number of points of demand (destinations). Usually, we have a given capacity of goods at each source and a given requirements for the goods at each destination. The objective of this project is to schedule shipments from Tema Oil Refinery and Conventional Buoy Mooring (sources) at Tema and Kpong respectively to BOST's depots (destinations) strategically located nationwide so that the total transportation cost is minimized. Although Linear Programming can be used to solve this type of problems, more efficient special-purpose algorithms have been developed for the transportation application. As in the simplex algorithm, they involve finding an initial solution, testing this solution to see if it is optimal, and developing an improved solution. This process continues until an optimal solution is reached. Unlike the simplex method, the transportation method is fairly simple in terms of computation.

The streamlined versions of the simplex method are important for two reasons:

1. Their computation times are generally 100 times faster than the simplex algorithm.

2. They require less computer memory (and hence can permit larger problems to be solved).

3.2 THE TRANSPORTATION PROBLEM

A commodity is being produced at the sources S_1, S_2, \dots, S_n and shipped to the depots or warehouses W_1, W_2, \dots, W_n . The cost of transportation of a unit of the commodity from S_i to warehouse W_j is C_{ij} . The capacity of the source S_i in a given period is a_i and the demand at the warehouse W_j is b_j .

The transportation problem is to transport petroleum products from Accra Plains Depot (APD), Tema Oil Refinery (TOR) and Conversional Buoy Mooring (CBM) at Kpone to various depots strategically located nationwide at minimum cost while satisfying the constraints of productive capacity and demands.

To solve this, we let x_{ij} denote the amount of the products transported per period from source S_i to depot W_j , $i = 1, 2, \dots, m$, $j = 1, 2, \dots, n$, then $x \geq 0$ for all i and j .

Hence the transportation problem can be modeled by the following linear programming problem.

If x_{ij} = quantity transported from i to j .

Minimize $\sum \sum c_{ij} x_{ij}$

Subject to $\sum x_{ij} \leq a_i \forall i$

$$\sum x_{ij} \geq b_j \quad \forall j$$

$$x_{ij} \geq 0$$

Where a_i and b_j are demand and supply quantities.

Since the transportation problem is a linear programming problem, it can be solved by the simplex method. But because of its special nature, to be discussed soon, it can be solved more easily by special forms of the simplex method taking advantage of the special structure. These special methods are more efficient for the transportation problem than the parent simplex method.

3.3 THE BALANCED PROBLEM

3.3.1 POSSIBILITIES

If $\sum a_i < \sum b_j$ then the total availability is more than what is required. Thus, the requirements at each b_j will be met.

If $\sum a_i > \sum b_j$, then the total availability is less than what is required, and so the requirements at each b_j will not be met.

If $\sum a_i = \sum b_j$, then the total availability will be fully utilized at b_j . When we have this condition we say the problem is a balanced transportation problem.

A balanced transportation problem occurs when total supply is equal to total demand which does not happen often in real- life. Therefore with a balanced transportation

problem with m supply points and n demand points, the cells corresponding to a set of $m + n - 1$ variables contain no loop if and only if the $m + n - 1$ variables yield a basic solution.

From equations (i) and (ii), we always have $\sum_{i=1}^m a_i \geq \sum_{i=1}^m \sum_{j=1}^n x_{ij} \geq \sum_{j=1}^n b_j$.

In the case for which $\sum_{i=1}^m a_i = \sum_{j=1}^n b_j$ the problem is said to be balanced while in the case $\sum_{i=1}^m a_i > \sum_{j=1}^n b_j$, it is said to be unbalanced. The special algorithm works for the balanced problem. This is no restriction since the unbalanced problem can always be converted to an equivalent balanced problem to which the special method may be applied.

3.3.2 REMARKS

We note that if a problem is balanced

Then $a_i = \sum_{j=1}^n x_{ij}$ for each i

and $b_j = \sum_{i=1}^m x_{ij}$ for each j

For if there exists $i = i_0$ such that $a_{i_0} > \sum_{j=1}^n x_{i_0 j}$,

then $\sum_{i=1}^m a_i > \sum_{i=1}^m \sum_{j=1}^n x_{ij} > \sum_{j=1}^n b_j$

and the problem is not balanced. Hence for each i , we have $\sum_{j=1}^n x_{ij} = a_i$

Similarly, $b_j = \sum_{i=1}^m x_{ij}$ for each j .

Hence the balanced transportation problem may be written as

If $\sum a_i = \sum b_j$ the problem becomes

$$\text{Min } \sum \sum c_{ij} x_{ij}$$

$$\text{Subject to } \sum x_{ij} = a_i \quad \forall i$$

$$\sum x_{ij} = b_j \quad \forall j$$

$$x_{ij} \geq 0$$

This balanced problem may therefore be represented in a tabular form as shown in Table 1.

TABLE 1

	W_1	W_2	
W_n			
	c_{11}	c_{12}	c_{1n}
	c_{21}	c_{22}	c_{2n}
	c_{1m}	c_{2m}	c_{mn}
	b_1	b_2	b_n

There is a row for each source and a column for each storage depot. The supplies and the demands of these are shown for the right and below respectively of the rows and columns of table 1. The unit costs are shown in the upper right hand corners of the cells.

- (i) We observe from table 1 that
 - (a) The coefficient of each variable x_{ij} in each constraint is either 1 or 0.
 - (b) The constant on the right hand side of each constraint is an integer
 - (c) The coefficient matrix A has a certain pattern of 1's and 0's.

It can be shown that any linear programming problem with these properties has the following properties:

If the problem has a feasible solution then there exist feasible solutions in which all the variables are integers. A solution is called a basic feasible solution if it involves $(m + n - 1)$ cells with non-negative allocation (circled). There are no circuits among the cells in the solution. It is this property on which the modification of the simplex method that provides efficient solution algorithms is based.

- (ii) We note that the $m + n$ conditions

$$\sum_{i=1}^m x_{ij} = b_j, 1 \leq j \leq n,$$

$$\sum_{j=1}^n x_{ij} = a_i, 1 \leq i \leq m$$

are not independent since $\sum_{i=1}^m b_j = \sum_{j=1}^n \sum_{i=1}^m x_{ij} = \sum_{i=1}^m a_i$

Thus the effective number of constraints on the balanced transportation problem is $m + n - 1$. Hence we expect a basic feasible solution of the balanced transportation problem to have $m + n - 1$ non-negative entries.

3.4 METHODS OF SOLUTION

The methods of solution to be discussed are variants of the simplex method. They are the stepping-stone method and the Modified Distribution Method (MODI) initiated by Dantzig(

Since these methods are variants of the simplex method they require initial basic feasible solutions to start with. These initial basic feasible solutions may be obtained by the Northwest corner method, the least cost method or Vogel's Approximation Method (VAM) of which we will discuss the Northwest Corner Method in the next section.

3.4.1 SEARCH FOR INITIAL BASIC FEASIBLE SOLUTION

3.4.2 THE NORTHWEST CORNER RULE

This systematic procedure requires that we start in the upper left-hand cell (or northwest corner) of the table and allocate units to shipping routes as follows;

- (i) Exhaust the supply (factory capacity) at each row before moving down to the next row.

- (ii) Exhaust the (demand) requirements of each column before moving to the right of the next column.
- (iii) Check that all supply and demands are met.

(Barry Render et al., Quantitative Analysis for Management, 9th Edition, p.396)

That is, choose the entry in the upper left hand corner (Northwest corner) of the transportation tableaux, i.e. the shipment from source one (1) to warehouse one (1). Use this to supply as much of the demand at W_1 as possible. Record the shipment with a circled number in the cell. If the supply at S_1 is not used up by the allocation in (b) use the remaining supply to fill the remaining demands at $W_2, W_3 \dots$ in that order until supply at S_1 is used up record all shipments in circles in the appropriate cells. When one supply is used up, go to the next supply and start filling the demands beginning with the first warehouse in that row, where there is still a demand unfilled, recording in circled numbers all allocations.

3.4.3 CIRCUIT

A circuit is made up of cells of the tableau of the balanced transportation problem is a sequence of cells such that it starts and ends with the same cell. Each cell in the sequence can be connected to the next member of the sequence by a horizontal or vertical line in the tableau.

The optimal solution to the balanced transportation problem is achieved by some BFS.

3.4.4 DEGENERACY

In certain cases, called Degenerate, the solution obtained by this method is not a basic feasible solution because it has fewer than $m + n - 1$ cells in the solution. This happens because at some point during the allocation when a supply is used up there is no cell with unfulfilled demand in the column. In the non-degenerate case, until the end, whenever a supply is used up there is always an unfulfilled demand in the column. Even in the case of degeneracy, the Northwest corner Rule still yield a basic feasible solution if it is modified as follows. Having obtained a solution which is not a basic feasible solution (BFS) choose some empty cells and add the solution with circled zeros in them, so to produce a basic feasible solution (BFS), that is

- (i) The total number of cells with allocations should be $m + n - 1$.
- (ii) There should be no circuit among the cells of the solution.

3.5 THE LEAST COST METHOD

The least cost method identifies the least unit cost in the transportation tableaux and allocates as much as possible to its cell without violating any of the supply or demand constraints. The satisfied row or column is then deleted (crossed out). The next least weight cost is identified and as much as possible is allocated to its cell, without violating any of the supply or demand constraints. The satisfied row or column is deleted (crossed out). This procedure is continued until all rows and columns have been deleted.

Applying the procedure to the balanced transportation problem on table 2, we obtained the allocations shown in table 3.

TABLE 2

				Supply
6	5	7	9	40
3	2	4	1	40
7	3	9	5	25

TABLE 3

				Supply
6 30	5	7 10	9	40
3	2 20	4	1 20	40
7	3	9 25	5 0	25
30	20	35	20	

The total initial cost is $= 6 \times 30 + 7 \times 10 + 2 \times 20 + 1 \times 20 + 9 \times 25 = 535$

The number of cells in the solution is 5. Adding a '0' allocation in cell (3,4) produces a basic feasible solution (BFS), since there is no circuit among the cells.

3.6 THE VOGEL'S APPROXIMATION METHOD (VAM)

We first compute column penalties for each column by identifying the least unit cost and the next least unit cost in that column and taking positive difference. This is the column penalty for the column. In a similar way we may compute the row penalty for each row as the positive difference between the least unit cost and the next least unit cost in that row. Column penalties are shown below in table 5 the columns and row penalties are shown to the right of each row. The method is a variant of the least cost method and is based on the idea that if for some reason the allocation cannot be made to be the least unit cost cell via row or column then it is made to the next least cost cell in that row or column and the appropriate penalty paid for not being able to make the best allocation. We illustrated the method with the example on table 4.

TABLE 4

					Supply	Row penalties
	6	5	7	9	40	1
	3	2	4	1	40	1
	7	3	9	5	25	2
Demand	30	20	35	20		
Column penalties	3	1	3	4		

The row and column penalties have been calculated as shown on table 4. The next step is to find the cell for which the value of the row and column penalties is greatest. Allocate as much to this cell as the row supply or column demand will allow. This implies that either a supply exhausted or a demand is satisfied. In either case delete the row of the exhausted supply or the column of the satisfied demand. Calculate row and column penalties for the remaining rows and columns and repeat the process until a basic feasible solution is found.

3.6.1 REMARKS

Vogel's Approximation method provides a BFS which is close to optimal or is optimal and thus performs better than the Northwest Corner Rule or the Least Cost Method.

Unlike the Northwest Corner Rule, Vogel's Approximation method may lead to an allocation with fewer than $m + n - 1$ non-empty cells even in the non-degenerate case.

To obtain the right number of cells in the solution we add enough zero entries to empty cells, avoiding the generation of circuits among the cells in the solution.

3.7.1 IMPROVING THE SOLUTION TO OPTIMAL

The minimum additional cost per unit that will be incurred by not being able to allocate to the least cost is the penalty. The solutions obtained from the three methods discussed earlier are feasible but not optimal. We improve them to optimal by employing the following methods:

3.7.2 THE STEPPING-STONE METHOD

The stepping –stone method, being a variant of the simplex method, requires an initial basic feasible solution which is then improved to optimality. Such an initial basic feasible solution may be obtained by either the modified distribution method or stepping stone method.

Consider the balance transportation problem shown on table 5 below:

TABLE 5

6	5	7	9	40
3	2	4	1	40
7	3	9	5	25
30	20	35	20	

Suppose that we have a basic feasible solution of this problem consisting of non-negative allocations in $(m + n - 1)$ cells. Let us call the cells which are not in the basic feasible solution unoccupied cells. It can be shown that for each unoccupied cell, there is a unique circuit beginning and ending in that cell, consisting of that unoccupied cell and other cells all of which are occupied such that each row or column in the tableau either contains two or none of the cells of the circuit.

3.7.3 TEST FOR OPTIMALITY

To test the current basic feasible solution for optimality, take each of the unoccupied cells in turns and place one (1) unit allocation in it. This is indicated by just the sign+. Following the unique circuit containing this cell as described above place alternately the signs – and + until all the cells of the circuit are covered. Knowing the unit cost of each cell, we compute the total change in cost produced by allocation of one (1) unit in the empty cell and the corresponding placements in the other cells of the circuit. This change in cost is called improvement index of the unoccupied cell. If the improvement index of each unoccupied cell in the given basic feasible solution is non-negative then the current basic feasible solution is optimal since every reallocation increases the cost. If there is at least one unoccupied cell with a negative improvement index then a reallocation to produce a new basic feasible solution with a lower cost is possible and so the current basic feasible solution is not optimal.

Thus the current basic feasible solution is optimal if and only if each unoccupied cell has a non-negative improvement index.

3.7.4 IMPROVEMENT TO OPTIMALITY

As we have just seen, if there exist at least one unoccupied cell in a given basic feasible solution which has a negative improvement index then the basic feasible solution is not optimal. To improve this solution, we find the unoccupied cell with the most negative improvement index say, N. Using the circuit that was used in the calculation of its

improvement index find the smallest allocation in the cells of the circuit with the sign ‘-’ call this smallest allocation m .

Subtract m from the allocations in all the cells in the circuit with the sign ‘-’ add to all the allocations in the cells in the circuit with the sign ‘+’ this has the effect of satisfying the constraints on demand and supply in the transportation tableau. Since the cell which carried the allocation m now has a zero allocation, it is deleted from the solution and is replaced by the cell in the circuit which was originally unoccupied and now has an allocation m , the result of each reallocation is new basic feasible solution. The new basic feasible solution is tested for optimality and the whole procedure repeated until an optimal solution is attained.

Consider the following balance transportation problem:

TABLE 6

C_{11}	C_{12}		C_{1n}
C_{21}	C_{22}		C_{2n}
C_{m1}	C_{m2}		C_{mn}
b_1	b_2		b_n

The initial basic feasible solution obtained by Vogel’s Approximation method is shown circle above. The associated cost is 515.

3.7.5 THE MODIFIED DISTRIBUTION METHOD (MODI)

This method allows us to calculate improvement indices quickly for each unoccupied cell without drawing all the closed paths. It often provides considerable savings over stepping-stone method for solving transportation problems. If there is a negative improvement index indicating an improvement can be made, then only one stepping-stone path must be found. This is used to determine what changes should be made to obtain the improved solution. It is applied by first calculating a value, R_i , for each row and K_j for each column in the transportation table.

In general, we let

C_{ij} = cost in square ij (cost of shipping from source i to destination j)

R_i = value assigned to row i

K_j = value assigned column j

3.7.5. 1 THE STEPS IN THE MODI METHOD TO TEST UNUSED CELLS

1.0 Calculate the values for each row and column for only those squares that are currently used or occupied by setting $R_i + K_i = C_{ij}$

2.0 Write equations for all those squares that are used or occupied and set, $R_1 = 0$.

3.0 Solve the system of equations for all R and k values.

4.0 Compute the improvement index for each unused square by the formula

$$\text{Improvement index } (I_{ij}) = C_{ij} - R_i - K_j$$

5.0 The most negative index is selected and proceed to solve the problem using the stepping- stone method. Consider the balanced transportation problem on table 8.

TABLE 8

c_{11}	c_{12}		c_{1n}
c_{21}	c_{22}		c_{2n}
c_{m1}	c_{m2}		c_{mn}
b_1	b_2		b_n

Let an initial basic feasible solution be available. Then $(m + n - 1)$ cells are occupied.

3.7.5.2 TEST FOR OPTIMALITY

For each occupied cell (i, j) of the transportation tableau, compute a row index u_i and a column index v_j such that $c_{ij} = u_i + v_j$. Since there are $((m + n - 1))$ occupied cells, it follows that there are $m + n - 1$ of these equations. Since there are $(m + n)$ row and column indices altogether, it follows that by prescribing an arbitrary value for one of them, say $u_i = 0$, we then solve the equations for the remaining $(m + n - 1)$ unknowns u_i, v_j . With all the u_i, v_j known, we compute for each unoccupied cell such that the evaluation factor

$e_{st} = c_{st} - u_s - v_t$. It can be shown that the evaluation factors are the relative cost factors corresponding to the non-basic variables when the simplex method is applied to the transportation problem. Hence the current basic feasible solution is optimal if and

only if $e_{st} \geq 0$ for all unoccupied cells (s, t), since the transportation problem is a minimization problem. If there are unoccupied cells with negative evaluation factors, then current basic feasible solution is not optimal and needs to be improved.

3.7.5.3 IMPROVEMENT TO OPTIMALITY

To improve the current non-optimal basic feasible solution we find the unoccupied cell with the most negative evaluation factor, construct its circuit and adjust the values of the allocation in the cells of the circuit in exactly the same way as was done in the stepping-stone method. This yields a new basic feasible solution. With a new basic feasible solution available, the whole process is repeated until optimality is attained.

3.7.5.4 REMARK

The fact that the circuit is not constructed for every unoccupied cell makes the Modified Distribution Method more efficient than the Steppingstone Method. In fact the MODI Method is currently the most efficient method of solving the transportation problem.

3.8 THE UNBALANCED TRANSPORTATION PROBLEM

This situation occurs quite often in real-life problems where total demand is not equal to total supply. This type of transportation problems is handled by introducing dummy sources or dummy destinations. In the event that total supply is greater than total demand, a dummy destination (warehouse), with demand equal to the surplus, is created.

If total demand is greater than total supply, we introduce a dummy source (factory) with a supply equal to the excess of demand over supply. In either case, transportation cost of coefficients of zero are assigned to each dummy location or route because no transportation will actually be made from a dummy factory or to a dummy warehouse. Any units assigned to a dummy destination represent excess capacity, and units assigned to a dummy source represent unmet demand.

Consider the following transportation problem;

Minimise $\sum_{i=1}^m \sum_{j=1}^n c_{ij} x_{ij}$,

Subject to $\sum_{j=1}^n x_{ij} \leq a_i, i = 1, 2, \dots, m$

$$\sum_{i=1}^m x_{ij} \geq b_j, j = 1, 2, \dots, n$$

$$x_{ij} \geq 0, 1 \leq i \leq m, 1 \leq j \leq n.$$

The problem is unbalanced, then $\sum_{i=1}^m a_i \neq \sum_{j=1}^n b_j$. The unbalanced problem is converted to a balanced problem as follows;

3.8.1 IF THE TOTAL SUPPLY EXCEEDS THE TOTAL DEMAND

If the total supply exceeds the total demand, we create a fictitious warehouse or depot W_F whose demand is precisely the excess of supply over demand and such that the unit cost of each source to the fictitious warehouse or depot W_F is zero. This gives the following balanced problem:

3.8.2 IF TOTAL DEMAND EXCEEDS TOTAL SUPPLY

If the total demand exceeds total supply, create a fictitious source S_F whose capacity is precisely the excess of demand over supply and such that the unit cost from the source to every warehouse is zero (0). This produces a balanced transportation problem of the type shown on table 9:

TABLE 9

W_1	W_2		W_n	
c_{11}	c_{12}		c_{1n}	0
c_{21}	c_{22}		c_{2n}	0
c_{n1}	c_{n2}		c_{nn}	0
b_1	b_2			$b_n = \sum_i a_i -$
$\sum_j b_j$				

If the total demand exceeds total supply, create a fictitious source S_F whose capacity is precisely the excess of demand over supply and such that the unit cost from source to every fictitious warehouse W_F is zero (0). This produces a balanced transportation problem of the type shown on table 11.

TABLE 11

W_1	W_2		W_n	
c_{11}	c_{12}		c_{1n}	a_1
c_{21}	c_{22}		c_{2n}	a_2
c_{n1}	c_{n2}		c_{nn}	$\sum_i a_i - \sum_j b_j$
0	0		0	
b_1	b_2		b_n	

CHAPTER FOUR

DATA ANALYSIS

4.1 INTRODUCTION

The Bulk Oil Storage and Transportation Company Limited (BOST) strategic storage depots are fed with products from Tema Oil Refinery (TOR) and Marine Services at Tema and Kpong respectively in the Greater Accra Region of Ghana.

The associated supply of each source and demand of each operational depot is given in the table 7. The cost of sending one (1) litre of petroleum products from a source to an operational depot depends on the distance and mode of transportation. The supply and demand quantities of products are on daily basis and the transportation cost is in Ghana cedis.

TABLE 12: TRANSPORTATION TABLE FOR BULK OIL STORAGE AND

SOURCE	Accra Plains Depot	Kumasi Depot	Buipe Depot	Bolgatanga Depot	Maimi Water Depot	Supply (Litres)
Tema Oil Refinery	0.001554	0.06182	0.061916	0.119867	0.012656	5,184,000
Conventional Buoy Mooring	0.007105	0.068925	0.069021	0.12547	0.018207	
Demand(Litres)	7,000,000	2,835,000	1,500,000	1,000,000	500,000	

TRANSPORTATION COMPANY LIMITED (BOST)

4.2 CALCULATIONS

4.2.1 THE INITIAL BASIC FEASIBLE SOLUTION

Applying the northwest Corner rule to the transportation table 1 above, the resultant solution is as shown on the table 13 below;

TABLE 13

SOURCE	Accra Plains Depot	Kumasi Depot	Buipe Depot	Bolgatanga Depot	Maimi Water Depot	Supply (Litres)
Tema Oil Refinery	0.001554 5,184,000	0.06182	0.061916	0.119867	0.012656	5,184,000
Conventional Buoy Mooring	0.007105 1,816,000	0.068925 2,835,000	0.069021 1,500,000	0.12547 849,000	0.018207	7,000,000
Shortage	0	0	0	0 151,000	0 500,000	651,000
Demand(Litres)	7,000,000	2,835,000	1,500,000	1,000,000	500,000	

The initial total transportation cost = $0.001554 \times 5,184,000 + 0.007105 \times 1,816,000 + 0.068925 \times 2,835,000 + 0.069021 \times 1,500,000 + 0.12547 \times 849,000$

$$= GH\text{¢}426,417.00$$

4.3 TEST FOR OPTIMALITY

We next apply the Modified Distribution Method (MODI) to improve the initial basic feasible solution obtained above.

Let R_1, R_2 and R_3 be the row values and K_1, K_2, K_3, K_4 and K_5 be the column values of the initial basic feasible solution of the transportation table.

Calculating the row and column values using the occupied cells, we have

$$R_1 + K_1 = 0.001554 - - - - - \text{(i)}$$

$$R_2 + K_1 = 0.007105 - - - - - \text{(ii)}$$

$$R_2 + K_2 = 0.068925 - - - - - \text{(iii)}$$

$$R_2 + K_3 = 0.069021 - - - - - \text{(iv)}$$

$$R_2 + K_4 = 0.12547 - - - - - \text{(v)}$$

$$R_3 + K_4 = 0 - - - - - \text{(vi)}$$

$$R_3 + K_5 = 0 - - - - - \text{(vii)}$$

$$\text{Let } R_1 = 0$$

Solving for $K_i, i = 1, \dots, 5$ and $R_j, j = 1, \dots, 3$, we have

From equation (i), $K_1 = 0.001554$

From equation (ii), $R_2 + K_1 = 0.00710$

$$R_2 = 0.007105 - 0.001554, R_2 = 0.005551$$

From equation (iii), $R_2 + K_2 = 0.068925$

$$K_2 = 0.068925 - 0.005551 = 0.063374$$

Substituting 0.005551 for R_2 in equation (iv)

$$R_2 + K_3 = 0.069021 - - - - - (iv), K_3 = 0.069021 - 0.005551 = 0.06347$$

Replacing R_2 with 0.005551 in equation (v),

$$R_2 + K_4 = 0.12547 - - - - - (v),$$

$$K_4 = 0.12547 - 0.005551 = 0.119919$$

Substituting 0.119919 for K_4 in equation (vi),

$$R_3 = -0.119919, \text{ Similarly, } K_5 = 0.119919$$

4.4 IMPROVEMENT INDEX

An improvement index of an unoccupied cell is obtained by subtracting the row and column values of the cell in question from its unit transportation cost. This is expressed mathematically as;

$$I_{ij} = C_{ij} - R_i - K_j$$

Where C_{ij} = unit transportation cost of an unoccupied cell

R_i = Row value

K_j = column value and

I_{ij} = Improvement index

At $(i, j) = (1, 2)$, $I_{12} = C_{12} - R_1 - K_2 - \dots - (i)$

Substituting the values of R_1 , C_{12} and K_2 into equation (i),

$$I_{12} = C_{12} - R_1 - K_2 - \dots - (i)$$

$$I_{12} = 0.06182 - 0 - 0.063374 = -GH\text{¢}0.001554$$

Since the improvement index for the unoccupied cell is negative, the unallocated cell at (1, 2) needs to be allocated a quantity of petroleum products from the Tema Oil Refinery in order to minimize the total transportation cost of products of Bulk Oil Storage and Transportation Company Limited (BOST) in Ghana.

Similarly,

$$I_{13} = C_{13} - R_1 - K_3 = 0.061916 - 0 - 0.06347 = -GH\text{¢}0.001554$$

Cell (1, 3), also needs allocation of products to minimize the total transportation cost.

$$I_{14} = C_{14} - R_1 - K_4 = 0.119867 - 0 - 0.119919 = -GH\text{¢}0.000052$$

The cell located at (1,4) needs allocation of products.

$$I_{15} = C_{15} - R_1 - K_5 = 0.012656 - 0 - 0.119919 = -GH\text{¢}0.107263$$

Cell (1,5) needs to be allocated with a quantity of product from Tema Oil Refinery to minimize the total transportation cost.

$$I_{25} = C_{25} - R_2 - K_5 = 0.018207 - 0.005551 - 0.119919 = -GH\text{¢}0.018207$$

$$I_{31} = C_{31} - R_3 - K_1 = 0 - (-0.119919) - 0.001554 = GH\text{¢}0.118365$$

Since the improvement index for cell (3, 1) is positive, allocation of quantity of products from the CBM for Accra Plains Depot will lead to an increase of transportation by GH¢0.118365 per litre. Therefore no quantity from the CBM will be sent to Accra Plains Depot in order to minimize the total transportation cost.

$$\text{Similarly, } I_{32} = C_{32} - R_3 - K_2 = 0 - (-0.119919) - 0.063374 = GH\text{¢}0.056545$$

There should not be any allocation for cell (3, 2) from the source.

$$I_{33} = C_{33} - R_3 - K_3 = 0 - (-0.119919) - 0.06347 = GH\text{¢}0.056449$$

No allocation of products is required here.

The most negative improvement index value is -0.107263 which was calculated at the cells (1, 5) and (2,5) have to be allocated with products first before the rest. The resultant transportation tableau is as shown on table 14.

TABLE 14: SECOND TRANSPORTATION TABLEAU

SOURCE	Accra Plains Depot	Kumasi Depot	Buipe Depot	Bolgatanga Depot	Maimi Water Depot	Supply (Litres)
Tema Oil Refinery	0.001554 4684000	0.06182	0.061916	0.119867	0.012656 500000	5,184,000
Conventional Buoy Mooring	0.007105 2316000	0.068925 2835000	0.069021 1,500,000	0.12547 349,000	0.018207	7,000,000
Shortage	0	0	0	0 651,000	0	651,000
Demand(Litres)	7,000,000	2,835,000	1,500,000	1,000,000	500,000	

Total Transportation cost = $0.001554 \times 4684000 + 0.007105 \times 2316000 +$
 $0.068925 \times 2835000 + 0.069021 \times 1500000 + 0.12547 \times 349000 + 0.012656 \times$
 $500000 = GH\text{¢}366,457.$

4.4.1 TEST FOR OPTIMALITY

Let R_1, R_2 and R_3 be the new rows of table 4.2.0 and that of the columns are $K_1, K_2, K_3,$
 K_4 and K_5 . Calculating the row and column values of the occupied cells, we have

$$R_1 + K_1 = 0.001554 \text{ --- (i) , } R_2 + K_1 = 0.007105 \text{ --- (ii)}$$

$$R_2 + K_2 = 0.068925 - - - - - (iii)$$

$$R_2 + K_3 = 0.069021 - - - - - (iv)$$

$$R_2 + K_4 = 0.12547 - - - - - (v)$$

$$R_3 + K_4 = 0 - - - - - (vi)$$

$$R_1 + K_5 = 0.012656 - - - - - (vii)$$

$$\text{Let } R_1 = 0$$

$$\text{From equations (i) and (vii), } K_1 = 0.001554 \text{ and } K_5 = 0.012656$$

$$\text{Substituting } 0.001554 \text{ for } K_1 \text{ in equation (ii), } R_2 = 0.007105 - 0.001554 = 0.005551.$$

$$\text{Substituting } 0.005551 \text{ for } R_2 \text{ in equation (iii), } K_2 = 0.068925 - 0.005551 = 0.063374.$$

$$\text{Substituting } 0.005551 \text{ for } R_2 \text{ in equation (iv), } K_3 = 0.069021 - 0.005551 = 0.06347$$

$$\text{Substituting } 0.005551 \text{ for } R_2 \text{ in equation (v), } K_4 = 0.12547 - 0.00551 = 0.119919$$

$$\text{Substituting } 0.119919 \text{ for } K_4 \text{ in equation (v), } R_3 = -0.119919.$$

The improvement index for an unoccupied cell is given by

$$I_{ij} = C_{ij} - R_i - K_j, I_{12} = C_{12} - R_1 - K_2 = 0.06182 - 0 - 0.063374 = -0.001554.$$

Cell (1,2) needs to be allocated with products from Tema Oil Refinery.

$I_{13} = C_{13} - R_1 - K_3 = 0.061916 - 0. - 0.06347 = -0.001554$. Cell (1, 3) needs to be allocated with products from Tema Oil Refinery. $I_{14} = C_{14} - R_1 - K_4 = 0.119867 - 0 - 0.119919 = -0.000052$. Cell (1, 4) needs to be allocated with products from Tema Oil Refinery.

$$I_{25} = C_{25} - R_2 - K_5 = 0.018207 - 0.005551 - 0.012656 = 0$$

There is no need to allocate products to cell (2, 5). The resultant transportation tableau is as shown on table 15.

TABLE 15: THIRD TRANSPORTATION TABLEAU

SOURCE	Accra Plains Depot	Kumasi Depot	Buipe Depot	Bolgatanga Depot	Maimi Water Depot	Supply (Litres)
Tema Oil Refinery	0.001554 349000	0.06182 2835000	0.061916 1500000	0.119867 349,000	0.012656 500000	5,184,000
Conventional Buoy Mooring	0.007105 6651000	0.068925	0.069021	0.12547	0.018207	7,000,000
Shortage	0	0	0	0	0	651,000
Demand(Litres)	7,000,000	2,835,000	1,500,000	1,000,000	500,000	

Total transportation cost= $0.001554 \times 349000 + 0.06182 \times 2835000 + 0.012656 \times 500000 + 0.007105 \times 6651000 + 0.061916 \times 1500000 + 0.12547 \times 349000 =$
GH¢366048.

Let R_1, R_2 and R_3 be new the row values and K_1, K_2, K_3, K_4 and K_5 be the new column values.

$$R_1 + K_1 = 0.001554 - - - - - (i)$$

$$R_1 + K_2 = 0.06182 \dots \dots \dots (ii)$$

$$R_1 + K_3 = 0.061916 - - - - - (iii)$$

$$R_1 + K_5 = 0.012656 - - - - - (iv)$$

$$R_2 + K_1 = 0.007105 - - - - - (v)$$

$$R_2 + K_4 = 0.12547 - - - - - (vi)$$

$$R_3 + K_4 = 0 - - - - - (vii)$$

Let $R_1 = 0, K_1 = 0.001554, K_2 = 0.06182, K_3 = 0.061916, K_5 = 0.012656$

Substituting 0.001554 for K_1 in equation (v),

$$R_2 + 0.001554 = 0.00710$$

$$R_2 = 0.007105 - 0.001554 = 0.005551$$

Substituting 0.005551 for R_2 in equation (vi),

$$R_2 + K_4 = 0.12547, R_2 + K_4 = 0.12547$$

$$0.005551 + K_4 = 0.12547$$

$$K_4 = 0.12547 - 0.005551 = 0.119919$$

Substituting 0.119919 for K_4 in equation (vii),

$$R_3 + 0.119919 = 0$$

$$R_3 = -0.119919$$

4.4.2 CALCULATING IMPROVEMENT INDEX

$$I_{14} = 0.119867 - 0 - 0.119919 = -0.000052.$$

This cell needs to be allocated a quantity of product from the Tema Oil Refinery in order to minimize the total transportation cost.

$$I_{22} = 0.068925 - 0.005551 - 0.06182 = 0.001554.$$

This implies that Kumasi depot does not to be allocated with product from conversional Buoy Mooring in order to minimize the total cost of transportation.

$$I_{23} = 0.069021 - 0.005551 - 0.061916 = 0.001554.$$

Similarly, Buie Depot does not need to be allocated with product from the conversional Buoy Mooring in order to minimize the total transportation cost of BOST.

$$I_{25} = 0.018207 - 0.00551 - 0.012656 = 0$$

The Maimi Water Depot does not need product from the conversional Buoy Mooring in order to minimize the total transportation cost.

TABLE 16: THIRD TRANSPORTATION TABLEAU

SOURCE	Accra Plains Depot	Kumasi Depot	Buipe Depot	Bolgatanga Depot	Maimi Water Depot	Supply (Litres)
Tema Oil Refinery	0.001554 0	0.06182 2835000	0.061916 1500000	0.119867 349000	0.012656 500000	5,184,000
Conventional Buoy Mooring	0.007105 7000000	0.068925	0.069021	0.12547	0.018207	7,000,000
Shortage	0	0	0	0 651,000	0	651,000
Demand(Litres)	7,000,000	2,835,000	1,500,000	1,000,000	500,000	

The total transportation cost = $0.06182 \times 2835000 + 0.061916 \times 1500000 +$

$$0.119867 \times 349000 + 0.012656 \times 500000 + 0.007105 \times 7000000$$

$$= \text{GH¢}366,030.00$$

$$\text{Savings} = \text{GH¢}426417 - \text{GH¢}366030 = \text{GH¢}60387 \text{ per day}$$

The results obtained imply that in order to minimize total transportation cost of petroleum products through Bulk Oil Storage and Transportation Company Limited's operations by making sure that Accra Plains Depot takes its products from only the Conventional Buoy Mooring (CBM) at Kpong and the Kumasi, Buipe, Bolga and Maimi Water depots are fed with petroleum products from the Tema Oil Refinery.

Therefore the percentage decrease in transportation cost of petroleum products through

$$\text{BOST's operations} = \frac{60387}{426417} \% = \mathbf{14.16\%}.$$

Alternative solutions for Bulk Oil Storage and Transportation Company Limited's transportation operations are as shown below;

4.5 USING EXCEL QM ANALYSIS.

TABLE 17

Optimal cost = \$366,030.30	Destination 1	Destination 2	Destination 3	Destination 4	Destination 5
Source 1	500000	2835000	1500000	349000	
Source 2	6500000				500000
Dummy				651000	

	Destination 1	Destination 2	Destination 3	Destination 4	Destination 5
Source 1	500000	2835000	1500000	349000	[0]
Source 2	6500000	[.001554]	[.001554]	[5.20004E-05]	500000
Dummy	[.118313]	[.058047]	[.057951]	651000	[.107211]

TABLE 18

From	To	Shipment	Cost per unit	Shipment cost
Source 1	Destination 1	500000	0.001554	777
Source 1	Destination 2	2835000	0.06182	175259.7
Source 1	Destination 3	1500000	0.061916	92874
Source 1	Destination 4	349000	0.119867	41833.58
Source 2	Destination 1	6500000	0.007105	46182.5
Source 2	Destination 5	500000	0.018207	9103.5
Dummy	Destination 4	651000	0	0

This implies that the total transportation cost of BOST could be minimized by feeding Accra Plains Depot with five hundred thousand litres (500,000litres) of petroleum products from Tema Oil Refinery and six million and five hundred thousand litres (6,500,000litres) from conventional Buoy Mooring (CBM) located at Kpong near Tema daily. Also, the rest of the depots should take their products from the Tema Oil Refinery.

$$\text{Percentage decrease in transportation cost} = \frac{60387}{426417} \% = \mathbf{14.16\%}$$

The following chapter gives the details of the results and relevant recommendations in connection with the research.

4.6 SENSITIVITY ANALYSIS

OPTIMAL TABLEAU FOR BOST

The optimal value is GH¢366,030.00

TABLE 16 $K_j = 0.001554 \quad 0.06182 \quad 0.061916 \quad 0.119919 \quad 0.012656$

R_i	SOURCE	Accra Plains Depot	Kumasi Depot	Buipe Depot	Bolgatanga Depot	Maimi Water Depot	Supply (Litres)
0	Tema Oil Refinery	0.001554 0	0.06182 2835000	0.061916 1500000	0.119867 349000	0.012656 500000	5,184,000
0.005551	Conventional Buoy Mooring	0.007105 7000000	0.068925	0.069021	0.12547	0.018207	7,000,000
-0.119919	Shortage	0	0	0	0 651,000	0	651,000
	Demand(Litres)	7,000,000	2,835,000	1,500,000	1,000,000	500,000	

4.6.1 CHANGING THE OBJECTIVE FUNCTION COEFFICIENT OF A NONBASIC VARIABLE

The right- hand side of the optimal tableau of table 16 will remain unchanged when the objective function coefficient of a nonbasic variable is changed. Thus the current basis will still be feasible.

For what values of the cost of shipping one litre of petroleum product from Tema Oil Refinery to Accra Plains Depot will the current basis remain optimal? Suppose we change C_{11} from 0.001554 to $0.001554 + \Delta$. For what values of Δ will the current basis remain optimal?

$$C_{11} = R_1 + K_1 - C_{11} = 0 + 0.001554 - (0.001554 + \Delta)$$

$$C_{11} = 0 - \Delta \leq 0$$

Thus, the current basis remains optimal for $\Delta \geq 0$.

4.6.2 CHANGING THE OBJECTIVE FUNCTION COEFFICIENT OF A BASIC VARIABLE

Since we are changing $C_{BV}B^{-1}$, the coefficient of each nonbasic variable of table 16 row 0 may change, and to determine whether the current basis remains optimal, we must find the new R_i 's and K_j 's and use these values to price out all nonpositive. This idea is illustrated by determining for the BOST transportation problem the range of values of

the cost of hauling one litre of petroleum product from Tema Oil Refinery to Kumasi Depot for which the current basis remains optimal.

Suppose we change C_{14} from 0.119867 to $0.119867 + \Delta$. Then the equation $C_{14} = 0$ changes from $R_1 + K_4 = 0.119867$ to $R_1 + K_4 = 0.119867 + \Delta$. Thus, to find the R_i 's and K_j 's, the following equations need to be solved:

$$R_1 = 0 \dots\dots\dots (1), \quad R_1 + K_2 = 0.06182 \dots\dots\dots (2)$$

$$R_1 + K_3 = 0.061916 \dots\dots\dots (3), \quad R_1 + K_4 = 0.119867 + \Delta \dots\dots\dots (4)$$

$$R_1 + K_5 = 0.012656 \dots\dots\dots (5) \quad R_2 + K_1 = 0.007105 \dots\dots\dots (6)$$

$$R_3 + K_4 = 0 \dots\dots\dots (7)$$

Solving these equations, $R_1 = 0, K_2 = 0.06182, K_3 = 0.061916, K_4 = 0.119876 + \Delta,$

$K_5 = 0.012656$ and $R_3 = -0.119919$.

To price out each nonbasic variable, then the current basis will remain optimal as long as each nonbasic variable has a nonpositive coefficient in row 0.

$$C_{11} = R_1 + K_1 - C_{11} = K_1 - 0.001554$$

$$C_{22} = R_2 + K_2 - C_{22} = R_2 + K_2 - 0.068925 = -0.001554$$

$$C_{23} = R_2 + K_3 - 0.069021 = -0.001554$$

$$C_{24} = R_2 + K_4 - 0.12547 = -0.005617 + \Delta \leq 0 \quad \text{For } \Delta \leq 0.005617.$$

Thus, the current basis remains optimal for $\Delta \leq 0.005617$.

4.6.3 INCREASING BOTH SUPPLY s_i AND DEMAND d_i BY Δ

If the current basis remains optimal,

$$\text{New } Z - \text{value} = \text{old } z - \text{value} + \Delta u_i + \Delta v_j$$

For example, if the Tema Oil Refinery's supply and Kumasi Depot's demand are increased by Δ units $\text{new cost} = 366030 + 1 \times 0 + \Delta \times 0.001554 = \text{GH¢}366030.00 + 0.001554\Delta$.

(1) If C_{ij} is a basic variable in the optimal solution, increase C_{ij} by Δ .

(2) If C_{ij} is a nonbasic variable in the optimal solution, find the loop involving C_{ij} and some of the basic variables. Find the odd cell in the loop that is in row i . Increase the value of this odd cell by Δ and go around the loop, alternatively increasing and then decreasing current basic variables in the loop by Δ .

Suppose s_1 and d_2 are increased by Δ . Since x_{12} is a basic variable in the optimal solution, the new optimal z-value is $366030 + \Delta \times R_1 + \Delta \times K_2 = \text{GH¢}366030.00 + 0.06182\Delta$

Suppose s_1 and d_1 are increased by Δ . Since C_{11} is a nonbasic variable in the current optimal solution, a loop involving C_{11} and some of the basic variables should be found.

CHAPTER FIVE

FINDINGS, CONCLUSION AND RECOMMENDATIONS

5.1 INTRODUCTION

The bulk transportation cost of petroleum products which is factored into ex-pump price or retail price is basically to ensure that fuel is sold at the same price everywhere in the country. The concept of ‘same-price-everywhere’ for fuel ensures fair and equitable distribution for all in that a customer in Bolgatanga pays the same price for fuel as one at Atibie-Kwahu.

The prices of petroleum products have a significant effect on inflation and the overall macroeconomic stability of the country which depends on a sound overall government policy framework which does not itself contribute to economic fluctuations. Inflation is costly in a social justice sense, because it arbitrarily redistributes wealth among different groups of people in a society. Not only does inflation blunt the link between effort and reward, it typically hits hardest at those who least can afford. Inflation is also costly because it obscures the relative price signals that must come through clearly if the economy is to adapt to change and make the most of opportunities for growth.

This is the final chapter of the work to investigate ways of minimizing bulk transportation cost of petroleum products through Bulk Oil Storage and Transportation Company Limited operations which covered the required results are presented in the subsequent sections.

5.2 FINDINGS

This research work which seeks to minimize the bulk transportation cost of petroleum products through the operations of Bulk Oil Storage and Transportation Company Limited discovered that six thousand, three hundred and eighty-seven Ghana cedis per day (GH¢60,387) could be saved through the Bulk Oil Storage and Transportation Company Limited's operations nationwide. This will lead to ease in pressure on prices of the general goods and services in the country as well as rate of inflation.

It was discovered that in order to reduce the bulk transportation cost of petroleum products in the company five hundred thousand litres (500,000litres) of products need to be shipped from Tema Oil Refinery to Accra Plains Depot. Also, Kumasi Depot needs to take all its products from Tema Oil Refinery through contracted bulk transportation companies which use their dedicated bulk road vehicles (BRVs) to haul BOST products. Accra Plains Depot needs to take six million, five hundred thousand litres (6,500,000 litres) of its products from conventional Buoy Mooring (CBM) through a pipeline of thirty-two meters (32m). Buipe and Bolgatanga Depots have to take all their products from Conventional Buoy Mooring (CBM) through pipelines and the river barges operated by Volta Lake Transportation Company Limited.

Furthermore, all the products required to feed the Maimi Water depot should be taken from the Conventional Buoy Mooring operated by Marine Services located at Kpong near Tema.

In order to satisfy the demand of all the depots, the Tema Oil Refinery or the Conventional Buoy Mooring capacity needs to be expanded by at least six hundred and fifty-one thousand litres (651,000litres).

5.3 CONCLUSION

This research was set up to investigate the possibility of reducing the bulk transportation cost of petroleum products using Bulk Oil Storage and Transportation Company Limited's storage depots strategically located at Accra Plains, Kumasi, Buipe, Bolgatanga and Maimi Water and the aim has been achieved.

The company (BOST) can save fourteen million, four hundred and ninety-two thousand eight hundred and eighty Ghana cedis (GHS14,492,880.00) annually from transportation of products.

The results obtained in chapter 4 indicate that Kumasi depot should take its product from the Tema Oil Refinery (TOR) if bulk vehicle roads (BRVs) are used in transporting products to the depot. The Transportation of petroleum products by road is the most unsafe and expensive mode of transporting products.

The Accra Plains Depot (APD) which distributes products to Greater Accra Region, parts of Eastern, Central, and Volta Regions should take its entire products from the Conventional Buoy Mooring (CBM) operated by Marine Services at Kpong while the Maimi water Depot which feeds parts of Easter and Volta regions should take its products from the Tema Oil Refinery through the pipeline.

A combination of a seventy-seven kilometer pipeline from Tema Oil Refinery to Akosombo and four hundred and twelve and a half kilometers distance of the Volta Lake from Akosombo to Buipe operated by Volta Lake Transportation Company Limited should be used to distribute products to Buipe Depot to minimize the bulk transportation cost of petroleum products.

However, the Bolgatanga Depot should be fed with products from a combination of seventy-seven kilometres pipeline from Tema Oil Refinery to Akosombo, four hundred and twelve and a half kilometers distance of the Volta Lake from Akosombo to Buipe and two hundred and sixty-one kilometers pipeline from Buipe to Bolgatanga.

The research shows that the bulk transportation cost of petroleum products can be reduced by 14.16 % daily if and only if the recommended routes are used strictly for the purposes indicated.

5.4 RECOMMENDATIONS

In view of the results and findings emanated from the work carried out to minimize the bulk transportation cost of petroleum products through the Bulk Oil Storage and Transportation operations in Ghana, the following are recommended;

- 1.0 The Tema Oil Refinery or Conventional Buoy Mooring should be expanded by at least 651000 litres to meet the requirement of products in the country.
- 2.0 The Bulk Oil Storage and Transportation Company Limited should avoid the usage of bulk road vehicles in hauling its products.

3.0 Further studies should be carried out on the entire petroleum products distribution network in order to minimize prices of petroleum products in Ghana. The entire network was not covered due to unavailability of the required data.

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