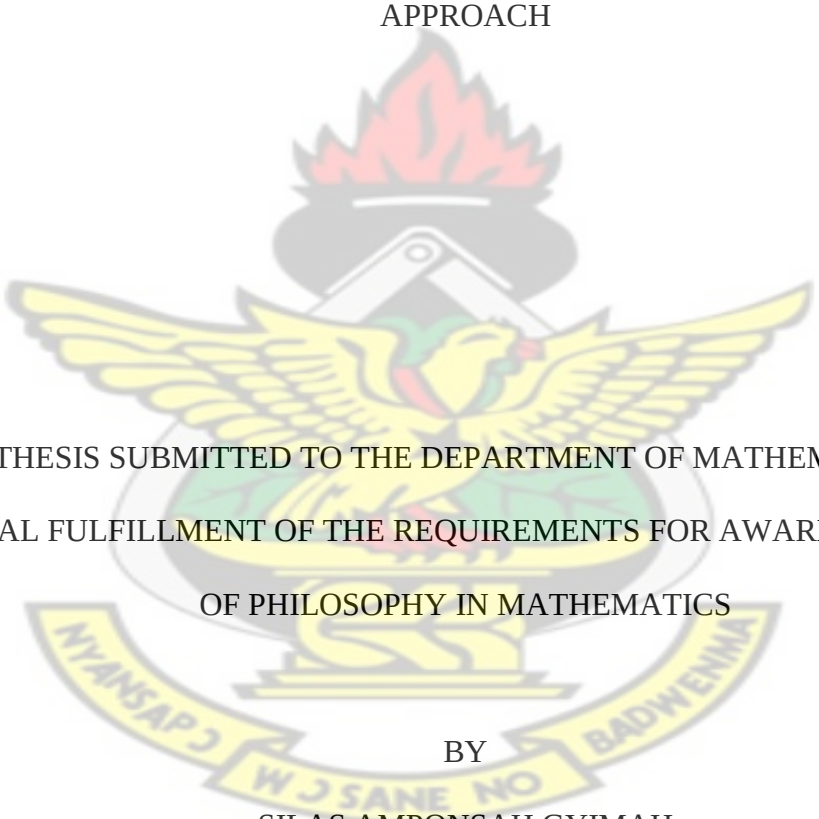
KWAME NKRUMAH UNIVERSITY OF SCIENCE AND TECHNOLOGY

KUMASI

COLLEGE OF PHYSICAL SCIENCE

PRICING CRITICAL ILLNESS USING THE MULTIPLE DECREMENT MODEL APPROACH



A THESIS SUBMITTED TO THE DEPARTMENT OF MATHEMATICS IN PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR AWARD OF MASTER OF PHILOSOPHY IN MATHEMATICS

BY

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DECLARATION

I hereby declare that this submission is my own work towards the award of MPhil (Mathematics) degree and that to the best of my knowledge, it contains no material previously published by another person or material which had been accepted for the award of any other degree of the university, except where due acknowledgement had been made in the text.

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.....	.....	.....
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DEDICATION

To my parents, Mr Joseph Annor Gyimah and Mrs. Dorothy Aidoo, I owe all that I am to your love and nurturing ability.

My Sweet Sister, Leticia-Abigail Agyekum, thanks for bringing so much joy into my life.

All who buy a critical illness insurance policy, I say to you that are doing the right thing.

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ACKNOWLEDGEMENTS

First and foremost I thank the almighty God for His strength and protection for all these years of my being. It is His own diverse ways that has brought me this far.

This dissertation would not have been possible without the guidance and the help of several individuals who in one way or the other contributed their valuable assistance in the preparation and completion of this study.

I will like to acknowledge the input of various persons at different stages of its development.

I will first and foremost like to appreciate one man who has virtually been the engine of this project, Mr Derrick Owusu Asamoah. After almost three years of studying Actuarial Mathematics, my supervisor, Mr Derrick Owusu Asamoah is without question the mentor of this project. His necessary inspiration and motivation for me even in the midst of adversities has brought this me this far. He has proven to me that surely his love for me is undying. I owe all that I know and perhaps will ever know in Actuarial Science to him. His patience, care and encouragement has always spurred me on. Boss, your immense contribution in my life will forever be appreciated.

My heartfelt gratitude again goes to the following people and body for allowing me to use their data for my analysis. By virtue of Critical Illness being a new product in the Ghanaian Market, not all the data for the analysis available, so some of the data used in this project was sorted out from outside the country- The English Life Table (ELT)- Series DS No. 14 Life Tables and also to Azim Dinani, Dave Grimshaw Neil Rob johns, Stephen Somerville, Alasdair Spry and Jerry Staffurth for their Report on Critical Illness dated 14th March, 2000.

ABSTRACT

The introduction of this booklet is about what the project entails (i.e. Pricing Critical Illness Using the Multi-State Model)

To know the amount of money one has to pay for an insurance premium of a condition of a serious illness occurring, the first and critical step is to know the incidence rate and its corresponding premium rate for that particular illness.

Due to the frequent hardships that an individual diagnosed of a particular illness go through in paying for medical bills, there is the need for an insurance that can help reduce these hardships. This happens because some of these diseases sometimes render them incapable of being able to work for a living.

In this project we consider three major diseases (known as the big three) which can cause such major problems in one's life. They are Cancer, Heart Attack and Stroke. Where the incidence rate, and the premium rates for these illnesses are computed for individuals from the age of 20 to age 74. We use the a three state multiple decrement model, where the states are Active (Healthy) for state 1, Critical (Dread Disease) Sufferer for State 2 and Dead for State 3

The Dash and the Grimshaw model was used to accomplish this and it was found out that the premium rate for any the illnesses considered was directly proportional to one's age. It was also realized that the incidence rate of cancer was prevalent amongst females than males.

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INTRODUCTION

1.1 OVERVIEW

The rapid progress made in medical science and clinical examination methods during the recent years and decades means that doctors can diagnose and treat many life-threatening diseases much earlier nowadays. Thanks to such developments, many lives have been saved. However, the financial burden for the people affected and their families can still be extremely painful. Many serious illnesses can only be cured using state-of-the art and highly expensive forms of therapy. Rehabilitation and the sudden changed circumstances caused by serious illness also exact a high price. Health insurance is dominated by the principle of cost reimbursement for medical treatment and rehabilitation. Patients are rarely free to choose their own doctor or form of therapy, with only private insurers offering such an option. Health insurance thus only partially covers the financial consequences of a serious illness and does not normally address the problem of consequential costs for changed circumstances and rehabilitation.

As the traditional life and disability insurance policies generally have a different focus, very few insurers in the past were willing to cover the needs of the seriously ill. However, this gap has been filled over the last twenty years or so, as a new form of cover has been developed and established in many markets which provides for payment in the event of a serious illness occurring. In this project such products are referred to as Critical Illness (CI) insurances.

Critical Illness (Dread Disease) Cover is becoming increasingly important in the developed and developing countries, like ours, as the problems of demographic aging come to the fore. Solutions to these pressures, together with demands from a population that is better educated and more prosperous, are increasingly being found through insurance by the many sectors. This chapter reviews the situation of Critical Illness Pricing. The chapter is divided into sections under the following appropriate headings: Background of the problem (study), Statement of the problem, Purpose of the study, Significance of the study, Objectives of the study and Scope and Limitations of the study.

1.2 BACKGROUND OF THE STUDY

Many firms are interested in pricing Critical Illness conditions. Individuals are more concerned with knowing the amount of money they would be entitled to should they suffer a condition of Critical Illness. The Multi-State Model can be used to accomplish this pricing situation.

Critical Illness (Dread Disease-DD) Insurance or Critical Illness Cover provides an insurance benefits that is payable upon the first occurrence or diagnosis of one of a number of serious medical conditions (Depending on the definitions stipulated in the policy wording) such as

- Cancer
- Heart Attack
- Stroke

- Coronary Artery Bypass Graft
- Multiple Sclerosis
- Kidney Failure
- Major Organ Transplant
- Total and Permanent Disability (TPD)

A Dread Disease contract pays out a lump sum on the diagnosis of a number of specified diseases. Unlike conventional life assurance and Permanent Health Insurance, Dread Disease covers do not meet any specific need in indemnifying the claimant against any loss of earnings or any expenses incurred. Neither does the claimant have to fulfil any criteria for disability-Dread disease covers pay out on diagnosis regardless of the extent of ill-health.

The conditions will be specified in the policy document and can vary considerably from insurer to insurer. It is one of the numbers of insurance products that we have. The insurer is contracted to typically make a lump sum-cash payment if the policyholder is diagnosed with one of the illnesses listed in the insurance policy. The policy may also be structured to pay out regular income.

Studies on the pricing of Critical Illnesses provide a self-examination that enable institutions to measure their effectiveness in meeting the expectations of their clients. Research findings indicate that Critical Illness pricing is one of institutional priorities. (Witney and Hudson, 2001).The pricing policy provides people in the working

environment with real pictures of the key issues relating to the actual benefits that they can enjoy.

The initial step in pricing Critical Illness using the Multiple State model is to develop the model (in the discrete case), obtain a sample of data from the Health Institutions, and with the data test the usefulness of the model and finally use the model to predict the price of the Critical Illness Insurance policy.

1.3 STATEMENT OF THE PROBLEM

Pricing of Critical Illness was necessitated by the notion that clients or buyers of insurance policies have the right and need to fully understand what is taken into consideration before a premium is charged to them, in this case, Critical Illness pricing which in most cases is not fully understood by most clients is an area which needs close attention. Perhaps the existing method of always using insurance terminologies to explain certain terms to ignorant clients no longer seem to be working. A comprehensive survey is therefore needed in relation to all the aspects of the pricing of Critical Illness premiums. The insurance companies in Ghana is contracted to typically make a lump sum cash payment if the policyholder is diagnosed with one of the Critical Illnesses listed in insurance policy. By the above procedure, the insurance companies have noticed the unfairness in making their clients pay for an optimal premium for their insurance. Due to this, some clients always feel cheated whenever their lump sums (or entitlement, benefits) are paid. I have therefore used the multiple decrement approach

for developing the pricing of Critical Illness benefits. I hope the various insurance companies will adopt this method of pricing reported herein for improved pricing.

1.4 OBJECTIVES OF THE STUDY

In this project we give an actuarial perspective to the process of developing an innovative product. Inevitably this goes wider than the purely actuarial function and other disciplines such as marketing and underwriting are considered as part of the process.

This project covers two major aspects of Ghana's Critical Illness business.

a) Development of incidence rates table for Stand-Alone Rider and Acceleration Rider, for the three prevalent Critical Illnesses. .i.e. Cancer, Heart-Attack and Stroke.
b) Development of premium rates for Acceleration rider benefits using the Dash and Grimshaw model.

1.5 SIGNIFICANCE OF THE STUDY

Pricing of premium for the purchase of any insurance forms the basics of any quality insurance in any insurance company, therefore pricing of Critical Illness is no exception. Thus the data from the survey would therefore be of immense importance to the various insurance companies in the country like, GLICO, VLA, SSNIT, SIC, QIC, etc. It will increase the selling of Critical Illness product which will intend increase their net income. At the end of the project, the various insurance companies will have a better and scientific means of pricing and selling the Critical Illness product.

1.6 JUSTIFICATION OF THE STUDY

Current premium rates for the various illnesses vary from insurer to insurer. This is due to a lack of standard premium rate and its corresponding incident rate for the various products sold. Some insurers charge higher premiums which normally results in the low patronage of the product whereas others charge relatively low premium which also results in the company running at a loss. This calls for the need to find a rigorous and accurate formula for costing an acceleration benefit. Thus necessitated the determination of an accurate pricing formula. Pricing critical illness is also the key objective of any insurance industry that sell such a product. The project which considers the various premiums and incidence rates for the age group of twenty to seventy four will serve as a platform for insurers to know which group of people are most likely to suffer one condition than the other. The idea of knowing that one must insure himself/ herself against conditions of not being able to carry out one's normal duties, by virtue of suffering a major illness makes the project very justifiable.

1.7 ORGANIZATION OF THE THESIS

In Chapter 2, we review past and recent studies that have been conducted on critical illness experience and pricing. This section considers: The literature review, the pricing, Stochastic process review and the Markov process review. Chapter 3 also illustrates our methodology together with the research design and the method of analysis. In chapter 4, we describe the source of the data used, develop our model for pricing and later use it for our construction of premium rates. Finally, in chapter 5, we present our data,

construct the incidence rate table and also calculate our premium rates. Summary and recommendations are also considered here.

1.8 SUMMARY

To know an efficient price one's has to pay for a Critical Illness (Dread Disease) insurance, the first and critical step is to know the incidence rate for that disease, the premium rate for the type of product and the person's present age.

1.9 PRODUCT STRUCTURE

The first question to be addressed when considering the introduction of Dread Disease benefit is whether to issue a 'Stand-Alone' product or to attach the cover to an existing contract. In the latter case, one must also decide on which policy (or policies) and whether to offer the Dread Disease benefit in addition to or as an 'Acceleration' of an attaching life cover; these questions are not independent since some combinations of cover may have a particular marketing appeal.

CI insurance usually takes the form of a rider offered in combination with a life policy. The underlying or main policy, for example, be a term or an endowment policy. Stand-alone CI policies are offered in certain markets but have been of only minor importance to date. A basic distinction has to be made between the two types of CI insurance. The first and most common form of cover provides only a prepayment on the sum insured of the underlying policy (acceleration benefit). The second form provides for additional benefits without affecting the life sum insured of the main policy (additional benefit).

With traditional products the benefit under a CI policy is usually only paid out once, after which the insurer's liability ends.

1.9.1 The Acceleration Benefit (Prepayment Benefit)

The Acceleration Benefit or Prepayment product provides a combination of a death benefit and Critical Illness cover. A payment is made when either the policy holder dies, or he or she is diagnosed as having one of the conditions specified in the policy wording. Typically, a proportion of the sum assured is paid when a Critical Illness is diagnosed and the balance is paid on death. This type of Critical Illness product is very popular in the U.K. and Irish markets. The design can be beneficial to insurers, since it reduces some of the uncertainty surrounding the pricing of the product. For example, when a Critical Illness benefit is attached to a whole-life assurance or endowment assurance, a Critical Illness claim would simply be bringing forward the payment that would ultimately have been made on death.

The advantages of offering the Dread Disease as an acceleration of the death benefit are as follows:

i. The cost of the Dread Disease cover is substantially lower than that for an additional benefit, increasing sales potential.

ii. Any uncertainty in the premium rates charged may be reduced for an acceleration benefit where we are concerned only with the addition required to the mortality rate.

The usual form is a rider to a life insurance policy providing for full or partial prepayment of death benefit in the event of a CI claim. The CI sum insured is then paid out as a lump-sum benefit. The amount if CI benefit is given as a percentage of the life sum insured. As soon as the CI benefit has been paid, the sum insured under the main policy is reduced by this amount and at the same time the premiums to be paid decrease accordingly.

1.9.2 The Stand-Alone Benefit (Additional payment)

The Stand-Alone product provides a sum assured if the policyholder is diagnosed with one of the conditions specified in the policy, provided he or she survives for a period (of 28-30 days) after diagnosis. This period is known as the 'Survival Period'. It is important for policyholders to understand that if they die during the survival period, there is no payment made to their beneficiaries under this type of cover.

The advantage to offering an additional benefit (payment) as follows:

i. Dread Disease can be sold in conjunction with contracts that do not contain life cover.
ii. After suffering a Dread Disease the policyholder may be uninsurable or subject to a high rating so that the continuation of all the life cover (where applicable) would be extremely valuable to him.
iii. Premium income is increased.

In the event of a CI claim, an additional benefit is due with the underlying main insurance continuing unaltered. As a rule, the CI benefit is paid as a lump sum.

However it is also possible to arrange payment in three to five instalments. For such cover the insured must survive a short period to time to trigger the claim.

Two problems can arise with an additional benefit if death occurs soon after the incidence of a Dread Disease. Firstly, if there is an attaching life cover, then the payment of both claims may result in unnecessarily large benefits, since there will be insufficient time for the Dread Disease benefit to be used for any of the purposes suggested. Secondly, there could be difficulties in verifying the validity of a Dread Disease claim-for example, although the death certificate could record 'Heart Attack' as the primary cause of death, but it may be clear that this event satisfied the required definition of a Dread Disease. Worldwide, most CI products are still offered with a prepayment benefit in combination with whole life, term or endowment insurances. However, each of the two main forms has its special advantages.

A decision for or against a prepayment or additional payment should be made in consideration of the actual insurance demand. In this context, it should be noted that the inclusion of an additional payment benefit leads to higher premiums than the inclusion of a prepayment cover.

1.10 STANDARDIZATION OF POLICY WORDING

Any disease to be included in a Dread Disease contract should satisfy the following criteria, which reflect marketing, medical and actuarial considerations:

a) It should be perceived by the public as a disease that could afflict them and one that could then leave them in need of a lump sum.
b) It should be capable of clear and precise definition.
c) There should be adequate data for costing
d) It should not allow anti-selection by applicants.

The main diseases included in contracts are Heart Attack, Cancer and Stroke.

As mentioned earlier, it is imperative that the policy document contains precisely worded definitions. During the 1990s, insurers in the U.K. collectively decided to standardize the definitions of the conditions covered in the policies. This led to a Statement of Best Practice being issued by the Association of British insurers (ABI) in 1999, which was updated in 2003 (ABI, 2003). It has proved to be very successful, as policyholders no longer need to examine the illness definitions in detail when comparing different insurers' products. In addition, it means that any inter-office claims experience that is collected is made more valuable, since the definitions used by each insurer are the same. Today insurers in Ghana employ the same words of standardization.

The Statements of Best Practice defines model wordings for seven 'core' medications, thirteen 'additional' medical conditions and nine 'common exclusions'. The conditions are listed below:

Heart Attack, Cancer, Stroke, Coronary Artery Bypass Surgery, Kidney Failure, Major Organ Transplant, and Multiple Sclerosis.

1.11 LIMITATIONS OF THE STUDY

Although every effort to my best of knowledge to minimise errors in this survey was ensured, I must admit that this project is not without fault. Errors occur at various stages of every survey being conducted and so this case was no exception.

Ideally, every good enumerator would want to minimize the errors to be encountered in a survey and that was what I tried to ensure. Even though proper survey procedures in terms of data organization, motivation and persuasion of our respondent, and good processing of data and interpretation or results were all implemented, there were some errors which were obviously inevitable. Among which are discussed below:

i. The problem of Non-response. There was failure on the part of some respondents to part certain vital information. There was a problem of hesitance and delays of officials to release information for security reasons and lack of time on their part.
ii. The limited time that was available to go through the volume of data that would be collected.
iii. The problem of insufficient resources in terms of books and money to move to and fro from one area to the other for data collection.
iv. As Critical Illness contracts cover a variety of medical conditions, a number of data sources need to be found in order to cost each of these components. However, since Critical Illness is relatively new product, for most countries, very little insurance claims experience is available.

1.12 SCOPE OF THE STUDY

In terms of content, the study considers the pricing of Critical Illness Insurance policy in Ghana and how best it could help the various insurance companies in the country.

CHAPTER 2

LITERATURE REVIEW

2.0 INTRODUCTION

This chapter seeks to review briefly some studies that have been carried out on multi-state models and how to relate it with critical illnesses of which cancer, heart attack and stroke are part of it.

There has been a great deal of research examining the Critical Illness pricing. Multiple State models provide a powerful tool for application in many areas of Actuarial Science, particularly in the Actuarial Assessment of sickness insurance and disability income benefits, and as such feature prominently in this project.

The early history of these models has been described by Seal (1977) and Daw (1979) in some detail and our purpose here is merely to outline the key historical developments in terms of the theory and its practical applications to insurance problems. The problem is the following:

Given two states A and B such that individuals in state A have mutually exclusive probabilities, possibly dependent on the time spent in state A, and the possibility of leaving state A because if (i) death or (ii) passage to state B, then what is the probability of an individual passing to state B and dying there within a given period?

Bernoulli's state A consisted of individuals who had never had smallpox, while state B comprised those who had contracted smallpox and would either die from it, almost immediately, or survive and no longer be suffering from that disease. In solving this problem, Bernoulli started with Edmund Halley's (Breslau) life table and effectively produced the first double decrement life table with one of the related single decrement tables as well as considering the efficacy of inoculation and deriving a mathematical model of the behaviour of smallpox. During the next 50 years, there were a number of contributions from other authors on the subject, including Jean d'Alembert and Jean Trembley.

Lambert (1772) explained how numerical data could be used to study Bernoulli's problem and laid the practical foundations for the double decrement model and life table. He obtained an approximate formula for the rate of mortality and thereby setting down a practical connection between the double decrement model and the underlying single decrement models. Despite this progress by the early 1800s there were two outstanding problems, namely (i) deriving accurate practical formulae for application to numerical data, linking the discrete and continuous cases; and (ii) obtaining exact results in a convenient form (d'Alembert had derived an exact results in terms of an integral that was difficult to evaluate). These problems were attacked successfully and independently by Cournot (1843) and Makeham (1867). They were the first to set down the fundamental relations of multiple decrement models: in modern notation:

$\mu_x^k = (a\mu)_x^k$ for k = 1, 2,…, m

$(a\mu)_x = \sum_{k=1}^{m} \mu_x^k$ From which it follows that

${}_n(ap)_x = \prod_{k=1}^{m} {}_n p_x^k$

Makeham (1867) also contains an analysis of the 'partial' forces of mortality for different causes of death, suggesting an interpretation of his well –known formula for the aggregate force of mortality $\mu_x = A + Bc^x = (\sum_{i=1}^{n} A_i) + (\sum_{i=1}^{m} B_j)c^x$ to represent separate contributions from *m* + *n* causes of death. Makeham went on to use connection between forces of decrement to interpret the prior development of the theory; he demonstrated that the earlier results of Bernoulli and d'Alembert satisfied this addictive law for the forces of decrement and this multiplicative law for the probabilities (or corresponding l_x functions)

In an internal report in 1875 (which was not placed in the public domain) on the invalidity and widows' pension scheme for railway officials, Karup described the properties and use of singe decrement probabilities and forces of decrement in the context of an illness-death model (with no recoveries permitted), i.e. the 'independent or pure' probabilities of mortality and disablement. Hamza (1900) represents an important development by providing a systematic approach to disability benefits in both the continuous and discrete cases. Hamza's paper is significant, setting down a notation which has been widely adopted in the following decades and which forms the basis for the notation we have utilized.

Du Pasquier took a dramatic step forward by providing a rigorous, mathematical discussion of the invalidity or sickness process with the introduction of a three state-death model in which recoveries were permitted. He derived the full differential equations for the transition probabilities and showed that these lead to a second-order differential of Riccati type which he then solved for the case of constant forces of transition. Du Pasquier work is very significant, presenting an early application of Markov Chains, and laying the foundations for modern actuarial applications to disability insurance, long-term care insurance and critical illness.

Despite the interest and importance of these problems to actuaries and the consistent contribution made to the actuarial literature since the mid-nineteenth century, these contributions have essentially been rediscovered and renamed as the Theory of competing risk by Neyman (1950) and Fix and Neyman (1951), and other statistical workers.

2.1 APPLICATION TO INSURANCE

The actuarial contribution to health and sickness insurance is closely linked with the evolution of friendly societies in Ghana and other corresponding institutions in other countries. The society is a mutual association which gives financial assistance to its members in times of sickness and old age. Its operations are based on insurance principles, the benefits being paid from a fund accumulated from the member's regular contributions. The actuarial profession began to be concerned with friendly society finance during the nineteenth century. As their techniques developed, actuaries advised

on the rates of contribution and on the accumulation of funds to meet the future liabilities being promised. The first attempt to produce age-related sickness rate was made by Price (1792) at the request of a House of Commons Committee. The rates were used to produce tables of contributions for given levels of benefit in respect of incapacity for work. Finlaison (1829) presents a classic investigation of annuities which contains sickness rates based on six years' data. Finlaison calculated the present value of a standard set of friendly society benefits at different ages and the equivalent age-dependent contribution rate for all. This work represents the first investigation into sickness rates.

Hubbard (1852) contains the first investigation into the sickness experience with an analysis by sex and occupation. For example, we find the first published analysis of data on length of hospitals stays for each spell of sickness for different types of occupation. In 1855, K.F. Heym published work on the organization of friendly societies, with special reference to Leipzig, in which he advocated the use of premiums which were dependent on age at entry. He also published age related sickness rates and probabilities of being disabled, based on date collected from local patterns of sickness benefits. Heym is regarded by some commentators as the 'creator of invalidity insurance science'.

Watson (1903) provides an account of investigations into the sickness and mortality experience for 1893-97 of the Manchester Unity Friendly Society which had become one the standard works of sickness insurance for almost nine decades. Although

methods have now moved on from Watson's approach, it is still possible to appreciate his clear presentations and analysis of data cross-classified by several factors. Earlier investigations had revealed that occupation was an important variable, but Watson's consideration of the combined effect of occupation and region was new.

The friendly society movement has declined in importance during the twentieth century; however, there is now a resurgence of interest in many countries in long-term disability income insurance (or permanent health insurance), long-term care insurance for the elderly, critical illness protection and other types of related cover offered by life insurance companies. It is with these important modern applications in mind that this project has been carried out.

2.2 PRICING METHODOLOGY

There are two main approaches to calculating premiums rates for Critical Illness products: the traditional approach and the more modern multiple decrement model approach. The former for project objective 1and the later for project objective 2

2.2.1 The Traditional Approach

Muchener Rick and the Munich Re has provided adequate information on the pricing of critical illness. It is possible that the insurer will have insufficient of its own insured life data to be able to produce credible Critical Illness incidence rates. In this situation, the insurer might choose to derive such rates approximately from population-based incidence rates. To derive the insured life incidence rates, it is assumed that the

relationship between insured life and population-based incidence rate is approximately the same as that between insured life and population-based mortality rates. The formulae are set out below, together with a general logical explanation for each result. In each case, the formula for the risk premium is given for a term of 1 year unit sum assured, for a policyholder at age *x*. This project will adopt this method since the product is relatively new in the Ghanaian market.

2.2.1.1 Pricing formula for acceleration product

To calculate the Acceleration product, we use the Dash and Grimshaw model for risk premium which is given by:

Risk Premium $= i_x + q_x(1-k_x)$ where i_x is the incidence rate of Critical Illness, k_x is the proportion of deaths caused by Critical Illness, and q_x is the mortality rate. The i_x term covers the cost of the Critical Illness element of the benefit. The $q_x(1-k_x)$ term covers the cost of the mortality element of the benefit, but only in respect of deaths due to a cause or causes other than Critical Illness.

2.2.1.2 Pricing formula for stand-alone product:

Risk premium$= i_x(1-q_x^i)$ where i_x is the incidence rate of Critical Illness and q_x^i is the proportion of lives who die during the survival period following a Critical Illness. The deduction of $i_x q_x^i$ from the incidence rate of i_x is required since no payment is made on this policy if the policyholder dies during the survival period.

2.3 REVIEW OF STOCHASTIC PROCESS

Cunningham R.J., Herzog T. N and London R.L (2006) in their book, models for Quantifying Risk, has provided a good review on Stochastic processes and Markov Chains. It is a collection of random variables, denoted by $X(t)$, where $t \in [0, \infty)$, with t identified as time. A stochastic process can be thought of as a random variable that evolves with time or "depends" on time. For a continuous process, time can be any positive number, but we will be interested primarily in discrete stochastic processes, for which time will take on only integer values. In the latter case we will denote the process by X_n instead of $X(t)$.

For a specific time, t, $X(t)$ is called the state of the process at time t. An example is the total number of students studying for an exam at any specific time t, t is a random variable which can take on only discrete values like 10, 100, or 1000. Looking at this random variable over all times $t > 0$ creates a stochastic process.

2.4 REVIEW OF MARKOV CHAINS

In defining models that allow the subject to move back and forth among various states, we are going to make some simplifying restrictions and consider only models with

1. Discrete time (meaning that the states are described at times 0, 1, 2, …)
2. a finite number of states in which the subject may be; and
3. history independence (meaning that the probability distribution of the state of the subject at time $n + 1$ may depend on the time n and on the state at time n but does not depend on the states at times prior to n)

Throughout this project, a Markov chain is understood to be a stochastic process in discrete time possessing a certain conditional independence property. The state space may be finite, countably infinite **or** even in more general, let $\{X_n,\ n=0,1,2,...\}$ be a stochastic process with X_n having possible values $\{1,2,3,...\}$. We think of this as a process that depends on time *n* where, at any time *n*, X_n must be equal to one of the integers between 1,2,3,... If $X_n = i$, we say the process is in State i at time n. Given that $X_n = i$, we are interested in the probability that X_{n+1} equals some number j, including possibly $j = i$. For now we assume that whenever the process is in state $\boldsymbol{i}$, the probability that at the next time, it will be in state j remains the same over time. For example, suppose the maximum number of passengers (including the driver) in a car pool is 4, but the actual number of passengers on any given day could be any of 1, 2, 3, or 4. If on Day 17 the number of passengers is $X_{17} = 3$, then we are interested in the probability that the number on Day 18 will be, say, $X_{18} = 2$. This process has 4 possible states since the number of passengers on any given day is at least 1 and at most 4. If we assume that the number of passengers on Day n depends on only on the number on Day $n-1$, then this stochastic process would be an example of a Markov Chain, which we describe below.

For a Markov Chain stochastic process, the probability of being in State j at time $t = n+1$ depends on which state the process is in at time $t = n$. It does not matter which state the process is in at time $t < n$. For this reason, a Markov Chain is sometimes called

a *memoryless* stochastic process. For a Markov Chain, the future development of an evolution is often independent of its development in the past, provided that the present state of the evolution is given. After moving to a new state, we can completely forget where we were in the past. For a Markov Chain, the probability of moving to State *j*, given that we are currently in State i, is called a transitional probability, and is denoted by $Q_n^{(i,j)}$.

Definition (non-homogenous Markov Chain)

M is a **non-homogeneous Markov Chain** when M is an infinite sequence of random variables $M_0, M_1 \ldots$ with the following properties.

1. M_n denotes the **state number** of a subject at time *n*.
2. Each M_n is a discrete-type random variable over r values (usually $1,2, \ldots, r$ but sometimes $0,1,2, \ldots, m$ with $r = m+1$).
3. The ***transition probabilities***

$$Q_n^{(i,j)} = \Pr\left[M_{n+1} = j \middle| M_n = i \text{ and various other previous values of } \mathrm{M_k} \right]$$

$$= \Pr\left[M_{n+1} = j \middle| M_n = i \right]$$

are history independent.

If the transition probabilities $Q_n^{(i,j)}$ -pronounced *"q-sub-n i-to-j"* – do not in fact depend on *n*, then they denoted by $Q_n^{(i,j)}$ and the chain is a ***homogeneous Markov Chain.***

We also note that, history independence implies the important and useful fact that the probability of moving from State #*i* to State #*j* and then to State #*k* is simply the product

of the probability of moving from State #i to State #j with the probability of moving form State *#j* to State *#k* --- that is, successive transitions are independent events.

For a Markov Chain, if we are in State *i* today, then we must be in some state in the next period. Thus the sum of the transition probabilities starting from State *i* must be 1. That is,

$$\sum_{i=1}^{m} Q_n^{(i,j)} = 1$$

for each relevant value of *i*, where *m* is the total number of states. (Although models with infinitely many states are theoretically possible, all models considered in this text will assume a finite number of states). We also note that $Q_n^{(i,j)}$ may or may not be zero. That is we may or may not be able to remain in the same state for the next period. We now develop a way to represent all of the $Q_n^{(i,j)}$ values for all (i, j) at the same time. To do this, we define the *transition probability matrix* Q_n to be (or later to be redefined in terms of Q) as

$$Q_n = \begin{bmatrix} Q_n^{(1,1)} & Q_n^{(1,2)} & \cdots & Q_n^{(1,j)} \\ Q_n^{(2,1)} & Q_n^{(2,2)} & \cdots & Q_n^{(2,j)} \\ \cdots & \cdots & \cdots & \cdots \\ Q_n^{(i,1)} & Q_n^{(i,2)} & \cdots & Q_n^{(i,j)} \end{bmatrix}$$

Thus, for example, if the process is currently in State1, the probability of moving to State 2 in the next period is given by $Q_n^{(1,2)}$, the second entry in the first row of the transition matrix. Because this matrix represents a Markov Chain, the sum of all the elements in any row must be 1.

2.5 PROBABILITIES FOR MULTIPLE STEPS (LONGER TERM PROBABILITIES)

In the previous section, transition probability matrix described the movements possible in one step of the stochastic process. We now consider *n*-step matrices that give the probability values that a process in State *i* will be in State *j* after n transitions. We denote these probabilities by

$$Q_n^{(i,j)} = \Pr\left(X_{m+n} = j \mid X_m = i\right)$$

for $n, i, j \geq o$. To obtain $Q_n^{(i,j)}$, we simply multiply the transition matrix times itself *n* times and select the appropriate entry from the resulting matrix. The resulting matrix is denoted by $(Q_n)^n$ and is given by

$$(Q_n)^n = \frac{Q.Q.Q.....Q}{n \text{ times}}$$

Actuarial notation often uses *q* to denote failure probabilities [such as moving from State #0 (intact) to the different state #1 (failed) in the basic survival model] analogously; it is sometimes convenient to use: $P_n^i = Q_n^{(i,i)}$ as the "success probability" of remaining in State *#i* at the next step.

Even a more convenient way is to place the probabilities $Q_n^{(i,j)}$ in a matrix as shown above.

2.6 CLASSIFICATION OF STATES

Markov Chains are of several different types, based on how easy (or possible) it is to move between states. First we present some definitions.

For a Markov Chain, State *j* is called *accessible* from State *i* if State *j* can be reached from State *i* in a finite number of steps. Note that, by definition, each state is accessible from itself.

State *i* and State *j* are said to *communicate* if State *j* is accessible from State *i* and State *i* is accessible from State *j*. Every state is considered to communicate with itself, because State *i* can always be reached from State *i* in zero steps. Also, if State *i* communicate with State *j*, and State *j* communicates with State *k*, then State *i* communicates with State *k*, so communication is an equivalence relation.

Since communication is an equivalence principle relation, then the sets of communicating states form *equivalence classes* for which every state in the class communicates with every other state in the class. Two states are said to be in the same class in they communicate. A Markov Chain that has only one class is called *irreducible*.

For State *i*, let r_i be the probability that the process eventually returns to State *i*, given that it begins in State *i*. When $r_i = 1$, we say that State *i* is *recurrent*. When $r_i < 1$, then, we say that State *i* is *transient*. When a state is recurrent, then it is certain to be revisited and will be visited infinitely if the process is ever in that state at all. When a state is transient, it is not certain to be visited and will be visited only a finite number of times.

State *i* is called *absorbing*, if $Q_n^{(i,i)} = 1$. That is once in an absorbing state, a Markov Chain remains in that state for all subsequent periods. Since the probability of returning is $r_i = 1,$ then an absorbing state is also a recurrent state. (An example of an absorbing state in an actuarial model is death. Once a person enters the state of death, the process never leaves that state.)

There are two final observations we can make about transient and recurrent states.

The first is that every Markov Chain must have at least one recurrent state, in order that the process can continue for an indefinite length of time. If all states were transient, then each could be visited only a finite number of times, and therefore the process could not continue indefinitely.

The second observation is that if two states communicate, so that they are in the same class, then either both states are transient or both are recurrent. It is not possible for one state to be transient and the other to be recurrent. If State *i* is recurrent, and if States *i* and *j* communicate, then it is always possible to move from State *i* to State *j*. But State *i*, being recurrent, can be visited an indefinite number of times, and, since the process can always move form State *i* to State *j*, then State *j* can be visited an infinite number of times as well. This implies that State *j* is also recurrent. A similar argument shows that if State *i* is transient then State *j* cannot be recurrent, therefore transient as well. Thus we conclude that the characteristic of being either transient or recurrent for a state must apply to all states in the class, so we see that transient and recurrent are class properties.

CHAPTER 3

METHODOLOGY

3.1 OVERVIEW

Under methodology: data collection, population, sample size, sampling survey and the method of analysis will be considered. The project aims at pricing critical illness insurance using the Multiple Decrement model approach.

3.2 RESEARCH DESIGN

The importance of methodology to every research cannot be overstated if the validity and reliability of the results are to be attained. It is also to ensure the replication and generalization of the research results (Baume, 2006). Research design is the overall plan for collecting data in order to answer the research question. It also involves specific data analysis techniques or methods the researcher intends to use. This study will adopt the survey research method since though the focus will be on people suffering from such a disease, data will be collected from the various insurance companies in Ghana and if not available, then from other parts where the product is highly patronised. The Quantitative and Qualitative technique involved in surveys will also be used. A cross sectional survey method will be used to collect information from the sample, thus all relevant data will be catered for.

3.3 DATA COLLECTION

Data for the study will be collected by the secondary data collection method. Since the product is new in the Ghanaian market and it is now gradually gaining grounds. The

secondary data will be collected from the Hospital Episode Statistics-HES and the Office of National Statistic-ONS.

3.4 POPULATION

Fraenkel and Wallen (1993) point out that, "Population is the group of interest to the researcher, the group to whom the researcher would like to generalize the results of the study." Jankowicz (1995) points out that in order to draw a sample; you have to know how many people are in the population, and how this total is made up from people falling into various subgroups in which you might be interested. The population for this study consists of people who by virtue of their diagnosis has been found to be suffering from one of the medical conditions specified by the insurance company and the entire Ghanaian society; since there is a possibility/risk of everyone suffering from one of the diseases specified in the policy.

3.5 THE SAMPLE SIZE

Fraenkel and Wallen (1993) refer to a "sample" in a research study as any group from which information is obtained. Jankowicz (1995) defines sampling as the choice of the number of people. The sample provides data from which to draw conclusions about some larger group, the population, whom these people represent. Therefore, not all the members of the study population will be surveyed. Also it is considered economically feasible to use a part of the population. This enables the research to be conducted within the limited time frame.

Professor N. Nuamah (2001) mentions two types of sampling techniques used in various research studies. These are probability and non-probability sampling. He points out that in probability sampling, each and every unit within the population is given equal chance of being selected. Thus, simple random sampling (probability technique) is the main method that will be used in selecting the subjects.

In the sampling survey, a preliminary survey will be carried out in the study field and this will help in the choice of the sampling method and the sample size. In a study which has a large number of people suffering from these medical conditions, we choose a sample size which will form the units of enquiry. The study area will be clustered into units with each cluster consisting of some specified unit, out of this; a specified number will be selected from each cluster areas to give a sample size of the specified executives.

3.6 METHOD OF ANALYSIS

As mentioned earlier, Quantitative and Qualitative techniques of data analysis such as sample averages and statistical tables will be employed. Excel, a powerful spread sheet program for analysing data will be employed in the analysis of using the discrete case multiple state model to price critical illness.

3.7 MULTIPLE STATE MODELS FOR LIFE AND OTHER CONTINGENCIES THE DISCRETE TIME APPROACH

Multi-State transition models are probability models that describe the random movements of a subject among various states. Often the subject is a person, but it could just as well be a piece of machinery or a loan contract in whose survival or failure you are interested in.

In this section, we generalize discrete cash flow models to situations where in each period an entity (life, machine, etc.) can move from the current state to several other states, and perhaps even return to the current state after leaving it. The three-factor method for calculating an Actuarial Present Value (APV) which are *the amount due at time, let say k, the probability that this payment is made, and a discount factor for k periods*, remains the same, but now the probability factor will come from a multi-state transition model-known as Markov chain which was described earlier and will be revisited later.

3.7.1 Two State Model

In **single life theory** where an individual is in two possible states:

- state 1-living
- state 2-dead

A discrete life annuity due of 1 per year paid to or by life (x) will result in a series of payments equal to 1 at the start of each period while life (x) remains is state 1. A

discrete while life insurance of 1 on life (x) will result in a one-time payment of 1 at the end of the period during which life (x) makes a transition from state 1 to state 2 (i.e. dies).

3.7.2 Three State Model

a) Let us first of all suppose that we have a double decrement model for insured lives where decrement (1) is non-accidental death, and decrement (2) is accidental death. We might look at this situation as a 3-state model:

- state 1-(x) is surviving
- state 2-(x) had died non-accidentally (natural death)
- state 3-(x) had died accidentally

Secondly, suppose that we have a fully discrete plan of life insurance issued on (x) where the benefit is 1 if death is non-accidental or 2 if the death is accidental. Premiums would be paid as an annuity due while (x) remains in state 1. A one-time benefit payment is made at the end of the period when (x) makes the transition from state 1 to either state 2 or state 3. The payment is 1 if the transition is to state 2 or it is 2 if the transition is to state 3.

3.7.3 Four State Model

Case 1

Consider a discrete, multiple-life insurance issued on a pair of lives (x) and (y). Suppose that level annual premiums are paid while both lives are surviving, and the benefit is

paid at the end of the period in which the second death occurs. We might look at this situation as a 4-state model:

- state 1-both (x) and (y) are surviving
- state 2-(x) is surviving, but (y) is deceased
- state 3-(y) is surviving, but (x) is deceased
- state 4-both (x) and (y) are deceased

A series of premiums would be paid at the start of each period while the status remains in state 1. Premiums would cease when the status moves from state 1 to state 2, state 3 or perhaps even state 4 (if both lives dies in the same period).If the movement is to state 2 or 3, then the benefit is paid at the end of the period when the transition to state 4 eventually occurs.

In each of these three models, the possible transitions at the end of each period are very limited. In the single life model there is a one-time transition from state 1 to state 2. In the multiple decrement model there is only one transition that ever occurs. It is either from state 1 to state 2 or from state 1 to state 3. In the multiple life model there are possibly two transitions of state: one after the first death from state 1 to either state 2, state 3, or state 4, and then usually a second transition to state 4 after the second death. If both deaths occur in the same period, then there is only one transition.

Case 2

In multi-state models, an individual can move back and forth between a number of states in an unlimited number of times. A good example might be a multiple state model that involves disability. We might consider a 4-state model:

- state 1-living and able
- state 2-temporarily disabled
- state 3-permanently disabled
- state 4-dead

If the life is currently in state 1 it could be in any of the 4 states at the end of this period. The same is true of a life in state 2, since an individual can recover from temporary disability. However, if a life is currently is state 3, it can only be in state 3 or 4 at the end of this period. A life in state 4 will remain is state 4 forever. Because something like a temporary disability can last a matter of weeks or months, one time period might be a week or a month.

A simple two-state example; the up-down model

For this model, as the name implies, the states are called "up" (state 1) and "down" (state 2). We might model the operation over time of some system that produces cash flow. It could be the networks of an internet server, a power plant, or a printing press. The state "up" indicates that the system is functioning as intended and providing a stream of income to the operator. When the system goes down (makes transition from state 1, up, to state 2, down) a repair expense will be incurred. Each period that the

system is down, there will also be a cost in terms of lost income. The cost of going down is like a discrete insurance-the cost of repair is incurred at the end of the period in which the system makes a transition from state 1 to state 2. The cost of being down is like an annuity due-a cost is incurred at the beginning of each period for as long as the system remains in state 2. The first step in constructing the model is to specify probabilities for the end-of –period state, given the beginning –of-period state.

3.8 MULTIPLE STATE MODELS FOR LIFE CONTINGENCIES-THE TIME DISCRETE APPROACH-GENERAL CASE

The evolution of an insured risk can be viewed as a sequence of events which determine the cash flows of premiums and benefits when insurances of the person are concerned, examples include disablement, recovery, death, marriage, birth of a child, **onset of a particular illness (critical illness)**, etc.

We assume that the evolution of a risk can be described in terms of the presence of the risk itself, at every point of time, in a certain **state** belonging to a specified set of states, or **state space**. Furthermore, we assume that the aforementioned events correspond to **transitions** from one state to another state.

The graph Fig. 3.1 illustrates a set of four states, numbered 1 to 4 (**nodes** of the graph), and a set of possible 'direct' transitions between states, denoted by pairs such as (1, 2), (2, 1), (1, 3), etc. (**arcs**). 'Indirect' transitions can be represented by the sequences of arcs: for example, a transition from 2 to 3 can be represented by the (2, 1), (1, 3).

For instance, let us suppose that state 1 is the state at policy issue. Possible paths of the insured risk are as follows

1 (until the policy term);

1 – 2 – 4;

1 – 2 – 1 – 3 – 4

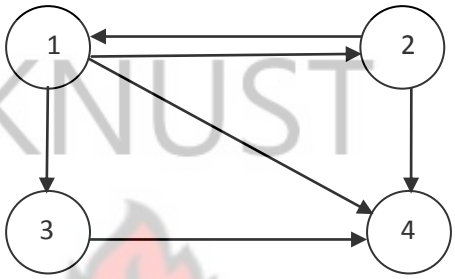


Figure 3.1: Set of states and set of transitions

From a merely intuitive point of view, it appears that:

- states 1 and 2 are **transient states**: it is possible to leave and to re-enter these states;
- state 3 is a **strictly transient state**: it is not possible to enter this state once it has been left;
- State 4 is an **absorbing** state: it is not possible to leave this state once it has been entered.

A rigorous definition of these types of states will be given in terms of transitional probabilities. Formally, we denote by λ the state space. We assume that λ is a finite set. Then denoting the states by integral numbers, we have:

$$\lambda = \{1,\ 2,\ 3,\ \ldots\ ,\ N\}$$

The set of directional transitions is denoted by τ. In general, τ is a subset of the set of pairs *(i, j)*

$$\tau \subseteq \{(i, j) | i \neq j; i, j \in \lambda\}$$

If state 1 is the initial state at time 0, it is assumed that all the states $j \in \lambda$ can be reached from state 1 by direct or indirect transitions. The pair (λ, τ) is **called a multiple state model**. We also note that multiple state model (λ, τ) simply describes the 'uncertainty', i.e. the 'possibilities' pertaining to insured risk, as far as its evolution is concerned.

Let us suppose that we are at policy at issue, i.e. at time 0. The time unit is one year. Let $S(t)$ denote the random state occupied by the risk at time t, $t = \{0,1, 2, 3, \ldots, n\}$. Of course, $S(0)$ is a given state; we can assume for example $S(0) = 1$. $\{S(t); t = 0,1,2,3,\ldots\}$ is time-discrete stochastic process, with values in the finite set λ. The variable t is often called **seniority**. It represents the duration of the policy; when a single life is concerned, whose age at policy issue is *x*, $x + t$ represents the attained age. Any possible realization $\{s(t)\}$ of the process is called a sample path; thus, $s(t)$ is a function of the discrete positive variable *t*, with values in λ.

3.9 BENEFITS AND PREMIUMS

As the next step in describing the insurance contract, benefits and premiums must be introduced. First, let us consider Fig 3.2. The items in Fig 3.2 are as follows: *s*(*t*) a sample path of the stochastic process $\{S(t)\}$; $P_1(t)$ a discrete premium, at a rate $P_1(t)$,

paid by the insured while the risk is in state 1; $b_2(t)$ a discrete annuity, at a rate $b_2(t)$, paid by the insurer while the risk is in state 2;

$c_{13}(t_3)$ a lump sum paid by the insurer at time t_3 because a transition from state 1 to state 3 occurs;

$c_{34}(t_5)$ a lump sum paid by the insurer at time t_5 because a transition from state 3 to state 4 occurs;

$d_3(t_4)$ a lump sum paid by the insurer at fixed time t_4 because the risk is in state 3.

Thus from the point of view of the insurer, $P_1(t)$ determines an inflow, whilst the bs, cs and the ds correspond to outflows, i.e. benefit

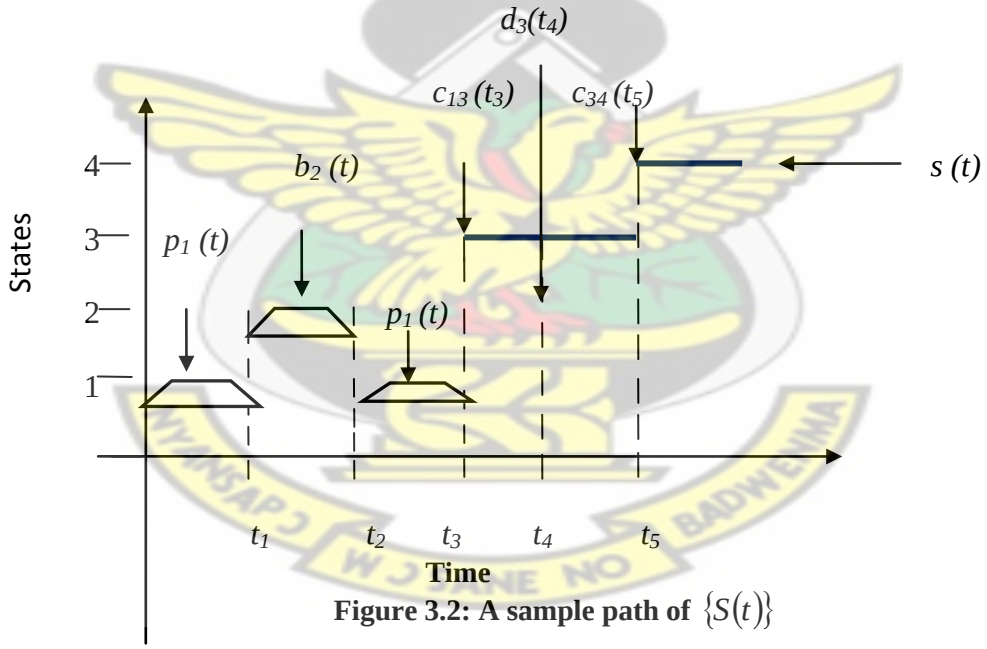


Figure 3.2: A sample path of $\{S(t)\}$

Benefits and premiums

Discrete-time premiums of the following types can be usefully considered:

- A premium $\pi_i(t)$ at some fixed time t if $s(t)=i$; for instance, as we assume $s(0)=1$, $\pi_1(0)$ can be represent the (initial) single premium (in this case, all other premium functions are identically equal to zero)

3.10 GENERALIZED DEFINITION OF DISCRETE-TIME MARKOV CHAIN

Consider a discrete-time stochastic process $\{S(t);t=0,1,2,3,...\}$, with a finite state space λ, we say that $\{S(t);t=0,1,2,3,...\}$ is a discrete-time Markov chain if, for any n and each finite set of integer times $(0\le)\ t_0<...<t_{n-1}<t_n<u$ and corresponding set of states $i_0,...i_{n-1},i_n,j$ in λ , then

$$\Pr\{S(t_0)=i_0\wedge...\wedge S(t_{n-1})=i_{n-1}\wedge S(t_n)=i_n\wedge S(u)\}>0$$

The following property (the so-called Markov property) is satisfied:

$$\Pr\{S(u)=j\,|\,S(t_0)=i_0\wedge...S(t_{n-1})=i_{n-1}\wedge S(t_n)=i_n\}$$

$$=\Pr\{S(u)=j\,|\,S(t_n)=i_n\} \qquad (3.10.1)$$

Thus for the discrete-time case, it is assumed that the conditional probability only depends on the 'most recent' information $S(t_n)=i_n$ and is independent of the path before t_n.

To assess the above probability, for example the following set of hypotheses might be assumed:

- No more than two transitions within the unit period, i.e. the year;
- Uniform distribution of first transition time within the year;
- The probability that the second transition occurs within the second half of the year is equal to one half of the probability that a transition of the same type occurs within the year.

The conditional probabilities $\Pr\{S(u)=j \mid S(t)=i\}$ are called transitional probabilities, also in a time discrete context. They are usually denoted by:

$$P_{ij}(t,u) = \Pr\{S(u)=j \mid S(t)=i\}$$

These probabilities satisfy the probability conditions

$$0 < P_{ij}(t,u) \le 1 \quad \text{for all } (i,j); \qquad 0 \le t \le u$$

$$\sum_{j\in\lambda} P_{ij}(t,u) = 1 \quad \text{for all } i; \qquad 0 \le t \le u$$

and the **Chapman-Kolmogorov equation**:

where t, w, u are integer times ($(t \le w \le u)$.

The proof of the **Chapman-Kolmogorov equation** is as shown below:

From equation (3.10.1) $\Pr\{S(u)=j \mid S(t_n)=i_n\}$ we have

$$P_{ij}(t,u) = \Pr\{S(u)=j \mid S(t)=i\}$$

$$= \sum_{k\in\lambda} \Pr\{S(u)=j \wedge S(w)=k \mid S(t)=i\}$$

$$= \sum_{k\in\lambda} \Pr\left\{S(w)=k \middle| S(t)=i\right\} \Pr\left\{S(u)=j \middle| S(t)=i \wedge S(w)=k\right\}$$

$$= \sum_{k\in\lambda} \Pr\left\{S(w)=k \middle| S(t)=i\right\} \Pr\left\{S(u)=j \middle| S(w)=k\right\}$$

$$= \sum_{k\in\lambda} P_{ik}(t,w) P_{kj}(w,u) \qquad \text{(Hence the proof)}$$

The occupancy probabilities satisfy the following relation:

$$P_{\underline{ii}}(t,u) = P_{\underline{ii}}(t,w) P_{\underline{ii}}(w,u) \quad (t \le w \le u) \qquad (3.10.2)$$

The relation (3.10.2) can be proved by using the Markov property as defined by the equation

$$\Pr\left\{S(u)=j \middle| S(\tau)=i\right\}$$

The proof is shown below:

$$P_{\underline{ii}}(t,u) = \Pr\left\{S(z)=i \text{ for all } z\in[t,u] \middle| S(t)=i\right\}$$

$$= \Pr\left\{S(z)=i \text{ for all } z\in[t,w] \wedge (S(z)=i \text{ for all } z\in[w,u]) \middle| S(t)=i\right\}$$

$$= \Pr\left\{(S(z)=i \text{ for all } z\in[t,w]) \middle| S(t)=i\right\}$$

$$\text{x } \Pr\left\{(S(z)=i \text{ for all } z\in[w,u]) \middle| S(z)=i \text{ for all } z\in[t,w]\right\}$$

$$= \Pr\left\{(S(z)=i \text{ for all } z\in[t,w]) \middle| S(t)=i\right\}$$

$$x \Pr\left\{(S(z)=i \text{ for all } z\in[w,u]) \middle| S(w)=i\right\}$$

$$P_{\underline{ii}}(t,u) = P_{\underline{ii}}(t,w) P_{\underline{ii}}(w,u) \qquad \text{(Hence the proof)}$$

Transition probabilities and occupancy probabilities can be used as a tool to formally label the states of a Markov process. In particular

- State *i* is an **absorbing state** if $P_{\underline{ii}}(t,u)=1 \quad (0\le t\le u)$
- State *i* is a **transient state** if $P_{\underline{ii}}(t,+\infty)=0 \quad (t\ge 0)$
- State i is a strictly transient if $P_{\underline{ii}}(t,u)=P_{\underline{ii}}(u,t)<1 \quad (0\le t\le u)$

Thanks to the Chapman-Kolmogorov equations, the following recursion in particular holds:

$$P_{\underline{ii}}(t,u)=\sum_{k\in\lambda}P_{\underline{ik}}(t,t+1)\,P_{\underline{kj}}(t+1,u)$$

Hence all probabilities $P_{\underline{ii}}(t,u)$ can be derived from the set of one-year transition probabilities

$$P_{ij}\left[z\right]=P_{ij}(t,t+1); \qquad z=0, 1, 2, ...$$

In particular, when a time-homogeneous process is concerned, we simply have:

$$P_{ij}\left[z\right]=P_{ij} \qquad z=0, 1, 2, ...$$

We shall denote with

$$P_{\underline{ii}}(t,u)$$

the probability of remaining in state *i* up to time *u*, given that the risk is in state in at time t. If we assume (as is rather usual in actuarial practice) that no more than one transition can occur during one year (part from the possible death of the insured), we simply have:

$$P_{\underline{ii}}(t,u)=\prod_{h=t}^{u-1}P_{ii}(h,h+1)=\prod_{h=t}^{u-1}P_{ii}[h]$$

EXAMPLES

In this section we present several examples which illustrate states, transitions and benefits pertaining to insurances of the person.

Example 3.10.1

Consider a temporary assurance, with a constant sum assured c and discrete premium at a constant rate p, let n denote the term of the policy. The graph describing states and possible transitions is depicted in **Fig 3 .10.1**.

$$c_{12}(t) = c \qquad t = 0,1,2,3,\ldots,n-1$$

$$p_1(t) = \begin{cases} p & \text{if } t=0,1,2,\ldots,n-1 \\ 0 & \text{if } t \geq n \end{cases}$$

Figure 3.3: A two-state model

It is understood that the functions which do not appear in the definition of benefits and premiums (for example, $p_2(t)$ must be considered identically equal to zero. In the case of a single premium, on the other hand, we have $p_1(t) = 0$ *and* $\pi_1(0) = \pi$

Example 3.10.2

Consider an endowment assurance, with c as sum assured in the case of death and in the case of survival to maturity as well. The graph is still given by **Fig. 3.3** and we have (for the case of a discrete premium):

$$c_{12}(t) = c \qquad t = 0,1,2,3,\ldots,n-1$$

$$d_1(n) = c$$

$$p_1(t) = \begin{cases} p & \text{if } t=0,1,2,\ldots,n-1 \\ 0 & \text{if } t \geq n \end{cases}$$

Example 3.10.3

Consider a **deferred annuity**. Premiums are assumed to be paid discretely at a rate p over $[0, m)$ when the contract stays in state 1 (i.e. the insured is alive). The benefit is a discrete annuity at a rate *b* after *m* when the contract stays in state 1, i.e. until the death of the insured. Thus, benefit and premium functions are as follows:

$$b_1(t) = \begin{cases} 0 & \text{if } t=0,1,2,\ldots,m-1 \\ b & \text{if } t \geq m \end{cases}$$

$$p_1(t) = \begin{cases} p & \text{if } t=0,1,2,\ldots,m-1 \\ 0 & \text{if } t \geq m \end{cases}$$

Example 3.10.4

As the next step in building up more complex models, consider a **temporary assurance with a rider benefit in the case of accidental death.** In this case, we have to distinguish between death due to an accident and death due to other causes. The graphical representation is given by

Fig. 3.10.2

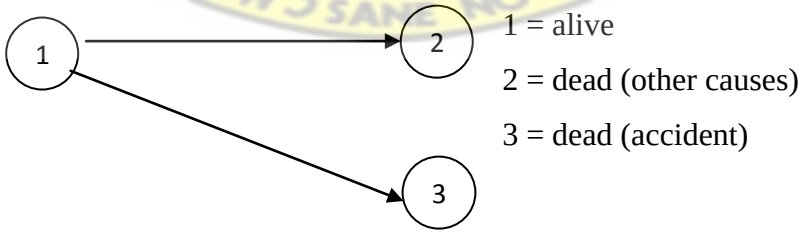


Figure 3.4: A three-state model with two causes of 'decrement'.

The benefit and premium functions are specified as follows:

$$c_{12}(t) = c \qquad t = 0,1,2,3,...,n-1$$

$$c_{13}(t) = c' \qquad t = 0,1,2,3,...,n-1$$

$$p_1(t) = \begin{cases} p & \text{if } t=0,1,2,...,n-1 \\ 0 & \text{if } t \geq n \end{cases}$$

Where $c^1 > c$, and $c^1 - c$ is the amount of the supplementary benefit in the case of accidental death.

Example 3.10.5

Consider now an n-year insurance contract just providing a lump sum benefit in the case of permanent and total disability. Also in this case a three-state model is required ,and the states are 'active', 'disabled' and 'dead', must be considered. It is important to stress that, since permanent and total disability only is involved, the label' active' concerns any insured who is alive and non-permanently (or non-totally) disabled. The model is presented in Fig. 3.10.3. Let c denote the sum assured. Premiums are assumed to be paid continuously at a rate p over when the insured stays in state 1 (i.e. when the insured is active).

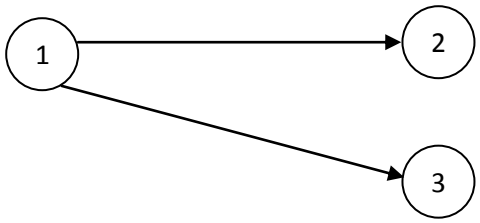
The benefit and premium functions are as follows:

$$c_{12}(t) = c \qquad t = 0,1,2,3,...,n-1$$

$$p_1(t) = \begin{cases} p & \text{if } t=0,1,2,...,n-1 \\ 0 & \text{if } t \geq n \end{cases}$$

The above-described model is very simple but rather unrealistic. It is more realistic to assume that the lump sum benefit will be paid out after a qualification period, which is required by the insurer in order to ascertain the permanence of the disability; the length

of the qualification period would be chosen in such a way that recovery would be practically impossible after that period.



1 = active
2 = disabled (permanent disability)
3 = dead

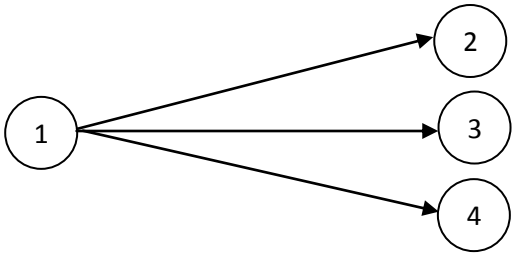
Figure 3.5: A further three-state model with two causes of 'decrement'

Example 3.10.6

Examples 3.10.4 and 3.10.5 can be generalized to include more than two causes of 'decrement'. This leads to the widely discussed **multiple decrement model** For example, we may be interested in the benefits provided to members within an occupational pension scheme should they retire, die or leave the scheme. A possible model is presented in Figure 3.5. Note that in this simplified model it is assumed that there is no impact if death occurring after retirement or withdrawal.

Example 3.10.7

A more complicated structure can be used to represent mortality due to a certain disease. In this case, we have the following



1 = active membership
2 = retired
3=withdrawn from membership
4 = dead

Figure 3.6: A four-state model with three causes of 'decrement'

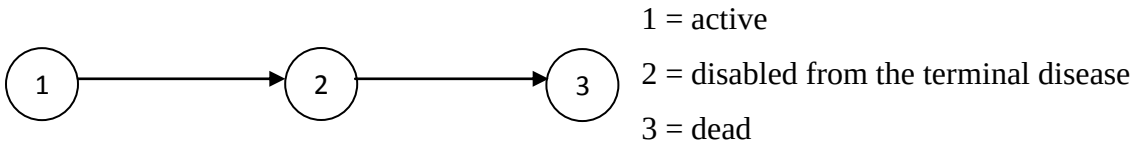


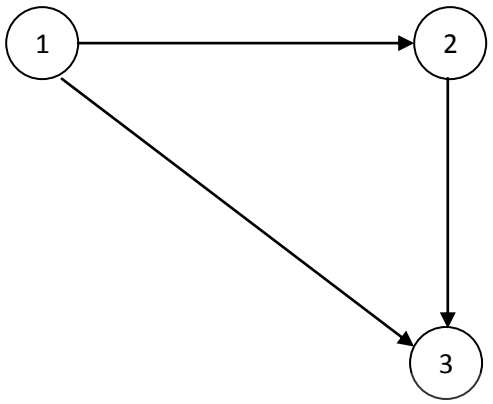
Figure 3.7: A Further three –state model

three states; 'active', 'disabled from the terminal disease', 'dead'. If the probability of death among those not suffering from the disease is deemed to be sufficiently small for it, it will to be ignored in the model, we can consider only the transitions depicted in **Figure 3.10.5**

Example 3.10.8

A more complicated structure than Example 3.10.5 is needed in order to represent an annuity benefit in the case of permanent and total disability. In this case, the death of the disabled insured must be considered, and then transition 2 – 3 must be added to the three-state model. The resulting graph is depicted in Fig. 3.10.5.

Let n denote the policy term; assume that the annuity is payable if the disability inception time belongs to the interval $[0, n-1)$. Let r denote the stopping time (from policy issue) of the annuity payment, r ≥ n; for example, if x is the entry age and ξ is the retirement age, then we can assume $r = \xi - x$. The annuity benefit is assumed to be paid continuously at a rate b. Premiums are assumed to be paid continuously at a rate p over $[0, n-1]$ when the risk stays in state 1 (i.e. the insured is active).



1 = active

2 = disabled (permanent disability)

3 = dead

Figure 3.8: A three-state model with two causes of 'decrement' and with a second-order decrement

Hence, the benefit and premium functions are as follows:

$$b_1(t) = \begin{cases} 0 & \text{if } t=0,1,2,\ldots,r-1 \\ b & \text{if } t \geq r \end{cases}$$

$$p_1(t) = \begin{cases} p & \text{if } t=0,1,2,\ldots,n-1 \\ 0 & \text{if } t \geq n \end{cases}$$

Remark

Note that transitions 1 – 2 and 1 – 3, in Example 3.10.4, correspond to two' causes of decrements', according to the traditional actuarial terminology (or 'competing risks', according to the statistical terminology); actually, from a collective point of view, transitions 1 – 2 and 1 – 3 correspond to decrements in the number of lives belonging to the 'group of lives'. An analogous interpretation holds with reference to examples 3.10.5 and 3.10.6. As regards example 3.10.8, transition 2 – 3 corresponds to a 'second-order cause of decrement', in the sense that it denotes a decremental factor pertaining to a group (the disabled lives) which, in its turn, has originated by decrement from the 'initial' group (the active lives).

Example 3.10.9

Let us generalize Example 3.10.8, considering an annuity benefit in the case of total disability: thus, the permanent character of the disability is not required. Hence, we have to consider the possibility of recovery, and then the transition 2 – 1 must be added to the three-state model depicted in Fig. 3.10.6**.** The resulting model is illustrated by Fig. 3.10.7. It we assumed that the policy conditions are as described in example 3.10.6 (as far as policy term, stopping time and premium payment are concerned). The benefit and premium functions are the same as in Example 3.10.8

Remark

The models presented in examples 3.10.1 to 3.10.8 contain only strictly transient states and absorbing states, whilst the model discussed in example 3.10.9 contains (non-strictly)transient states (states 1 and 2) and an absorbing state (state 3)

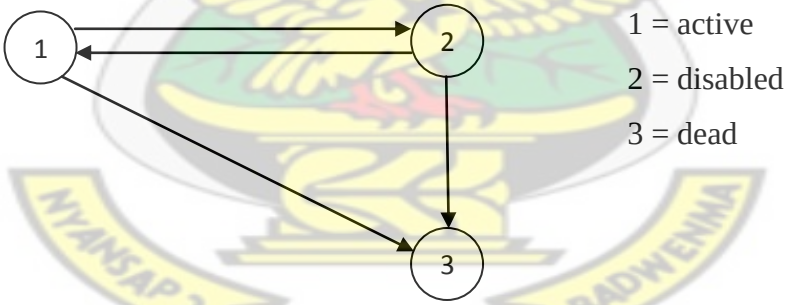


Figure 3.9: A more generalized three-state model relating to disability benefits

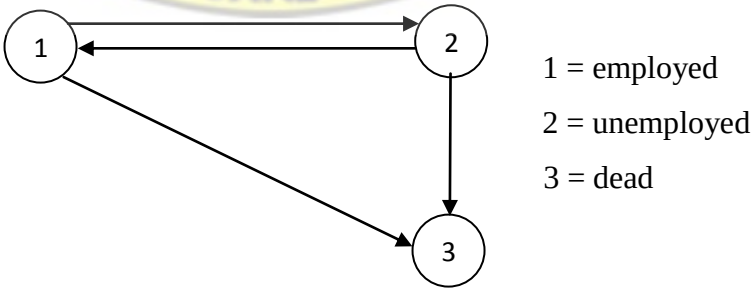


Figure 3.10: A three-state model relating to an annuity benefit in the case of unemployment

Example 3.10.10

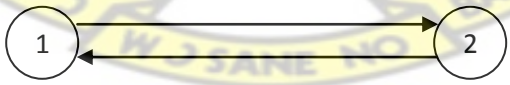
Example 3.10.9 and the model depicted in Fig. 3.10.7 can also be adapted to refer to an annuity benefit in the case of unemployment (as we can see in Fig. 3.10.8).

Example 3.10.10a

A simplified version of the model depicted in Figure 3.10.8 is widely used for practical calculations in respect of annuity benefits paid in the case of unemployment. The model is illustrated in Figure 3.10.9, corresponding to Figure 3.10.8 but with state 3 omitted completely. This adaptation may be made because the age range covered by such insurance contracts is characterized by low probabilities of death relative to the probabilities of moving from state 1 to state 2 or from state 2 to state 1, or because the financial effects of death may be small relative to that of unemployment.

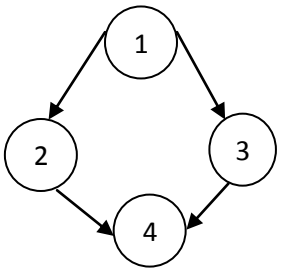
Example 3.10.10b

States and transitions can be used as a starting point also to represent insurance contracts with last-survivor benefits. Let us consider a **reversionary annuity (i.e. a widow's pension).**



1 = employed
2 = unemployed

Figure 3.10.9. A simplified two-state model relating to an annuity benefit.



1 = *(x)* alive, *(y)* alive
2 = *(x)* dead, *(y)* alive
3 = *(x)* alive, *(y)* dead
4 = *(x)* dead, *(y)* dead

Figure 3.11: A four State model

Suppose two lives are involved, say (x) and (y), and then we have four states to be considered. We also assume that (x) and (y) cannot die simultaneously. States and transitions are illustrated in Fig. 3.10.10

The policy provides a continuous life annuity to (y) at a rate b after the death of (x) and vice versa. Premiums are assumed to be paid continuously at a rate p over $[0, n-1]$ while both (x) and (y) are alive. Hence, the benefit and premium functions are as follows:

$$b_2(t) = b \qquad (t = 0,1,2,\ldots)$$

$$p_1(t) = \begin{cases} p & \text{if } t=0,1,2,\ldots,n-1 \\ 0 & \text{if } t \geq n \end{cases}$$

3.11 MULTIPLE DECREMENT MODEL APPLICATION TO CRITICAL ILLNESS COVER

TYPES OF BENEFITS: STAND ALONE BENEFIT RIDER (ADDITIONAL PAYEMENT) AND ACCELERATION (PREPAYEMENT) BENEFITS

As mentioned earlier, a Dread Disease (DD, or 'Critical Illness') policy provides the policyholder with a lump sum in case dread disease, i.e. when he/she is diagnosed as having an illness included in a set of diseases specified by the policy conditions. The most commonly covered diseases are heart attack, coronary artery bypass graft, total and permanent disability, kidney failure, cancer and stroke.

As the first modern DD policies appeared in South Africa in about 1983. The policy was simple, providing a rider benefit for a temporary or endowment assurance in the event that the insured was diagnosed as having one of the set of specified conditions.

Since then, critical illness policies have appeared in a number of countries including the UK and Ireland, Australia, Japan, Israel, Korea and Taiwan, and the scope of the policies has widened. Benefits are now available on an individual and group basis. Individual policies comprise two principal types: (a) **rider benefits** for a basic life, or (b) **stand-alone** cover. Type (a) itself takes either of two main forms: it may provide an **acceleration** of all or part of the basic life cover or it may be an **additional** benefit. An important point to note is that the benefit is paid on diagnosis of a specified condition, rather than on disablement. Thus, unlike other types of policy, critical illness differs in its objectives in that it does not meet any specified need, nor does it indemnify the policy against any specific financial loss (for example, loss of earnings or reimbursement of medical or other expenses incurred)

3.11.1 Pricing

For quite some time, there have been claims analyses of CI portfolios. However, the base data are not so comprehensive that incidence rates for insured lives could be obtained. An initial indication of the different CI risks of insured on one side and the overall population on the other can currently only be derived, if at all, by drawing a comparison between the claims cases observed and the claims figures expected on the basis of population statistics. As long as CI incidence rate cannot be deduced from the claims experience directly, they will still essentially be determined in accordance with the system described below.

Incidence rates are calculated in several steps which basically have to be taken separately for each illness to be covered.

3.11.2 Population rates

Population statistics on the incidence of each CI are taken as a basis. These statistics should be broken down by sex and age or at least by age group. The CI definition underlying the statistics should of course conform to that of the policy.

3.11.3 "First-ever" adjustment

Many people have two or more of the diseases specified in the policy. CI's however, only cover the first occurrences after the commencement of the policy and cease after that even. On the other hand, people who have already had an infection prior to insurance inception should be prevented from taking out a CI policy on the basis of the medical risk assessment. This means that the actuarial CI calculation only has to consider the actual "first" ("first ever in a lifetime") infection of a person.

3.11.4 Overlap with other conditions

Often one and some health impairment causes the occurrence of several CIs one after the other. As the CI cover usually ceases after the first claim, such overlap effects must be considered in the calculation of actuarial bases.

3.11.5 Mortality after critical illness

In pricing acceleration benefits, the probability of the insured dying following a CI is used to calculate the overlap of CI with death. By considering this overlap, the incidence rates for the underlying life insurance could be reduced, as for all insured who have received a CI benefit no death benefit of that amount becomes due in the event of

death at a later date. Instead of changing the actuarial bases of the main insurance however, the overlap is usually considered by granting a discount on the CI incidence rates.

The following graph is intended to further illustrate the situation.

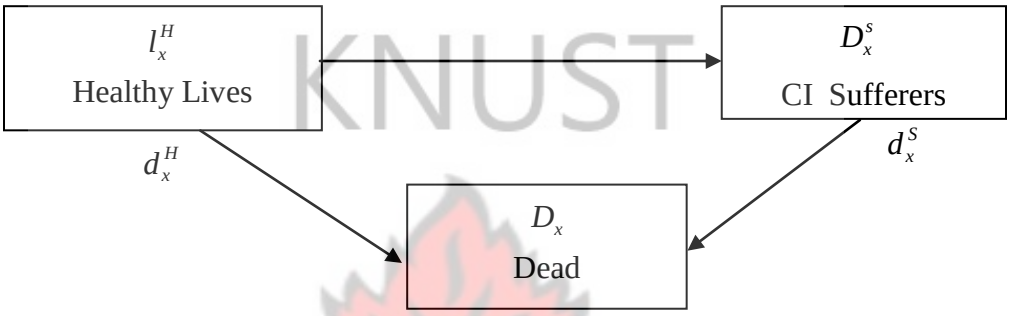


Figure 3.12: (Three State model for deriving the incidence rate for acceleration product)

The overlap corresponds exactly to the number d_x^S of deaths among CI sufferers. However, there are hardly any reliable statistics on the mortality of CI sufferers. This is why an approximation model was developed to help determine the cost of an acceleration benefit. On the basis of this model, the incidence rate for an acceleration product is approximately given as follows:

Extra rate of CI^{AAC} the mortality $= i_x - k_x, q_x$ (Which represents the Dash and Grimshaw model)

i_x being the CI incidence rate without survival period (i.e. all deaths due to a CI must be considered) and k_x being the portion of deaths caused by a CI. These k_x can be deduced from cause-of-death statistics.

3.11.6 Trend

The population statistics used will be based on historical data. However, in order for them to be utilized for current actuarial bases, they must be investigated for any improvement or deterioration. For example, a trend in disease frequencies made visible with the help of a time-series must be incorporated.

In view of the above items, it becomes clear that careful deriving of actuarial bases represents a task that should not be underestimated. In this context, it will be a problem to find adequate statistical material for actuarially using these items.

3.11.7 Smoker/Non-smokers

The incidence of several CIs is strongly related to the smoking habits of the insured. For example, about 90% of all lung cancers are related to smoking, but also the risk of heart attack or stroke is about twice as high for smokers as for non-smokers. These ratios are different for males and females and dependent on age.

Many markets have therefore established different base tables for smokers and non-smokers, implying the need to estimate the effect of smoking on the incidence rates. These considerations will of course be equally relevant for insurers providing CI cover on the basis of aggregate premium rates.

3.11.8 Substandard Risks

Loadings are required if applicants are classified as substandard CI risks due to poor health conditions, hazardous professions or other relevant factors. In this contest, it may well be commensurate with a risk to fix loadings that are different from those for the death or disability risk. In the event of especially serious impairments, a CI cover cannot be granted.

3.11.9 Reserves

For CI policies running for longer than one year, reserves for future claims payments must be set up. As a rule, such reserves are calculated for each policy in accordance with formulae corresponding to those for the mortality reserves. In addition to such individual actuarial reserves, further reserves are recommended to cover claim fluctuations and any deterioration tendencies of the CI risk. If business with long-term premium guarantees is concerned, even more emphasis must of course be put on the long-term performance of contracts. Careful observation of actual claims paid and subsequent adjustments of the reserve are indispensable.

3.11.10 Premium Guarantee

The future claims experience may change with new diagnostic procedures and improved surgical methods. For example, obligatory cancer examinations for females could increase the CI claims frequency owing to earlier detection, whereas the total mortality might decrease. Likewise, changes in lifestyle could have a significant influence. Future CI claims experience is therefore subject to a high degree of uncertainty.

Medical progress, changes in conditions and habits of living will continue to make the CI risk change constantly. Therefore, it is recommended not to guarantee premium rates on a long-term basis. If experience shows that the pricing bases are inadequate for a particular market, it will certainly be helpful to be able to adjust the premium rates, even for business in force. For the same reason, cover of the CI risk should be granted very restrictively in return for a single premium, as in such cases adjustments on the premium or benefit side cannot be implemented. The question of premium guarantees must also be seen in the context of the safety loadings in the actuarial bases. For long-term guarantees, substantially higher safety loadings are required than for short –term covers such as yearly renewable term insurances.

3.11.11 Derivation of Approximate Risk Premium Formula for an Acceleration Product

The intuitive approach for costing an acceleration benefit is to build up a multiple decrement model for the population of insured lives denoted by l_x

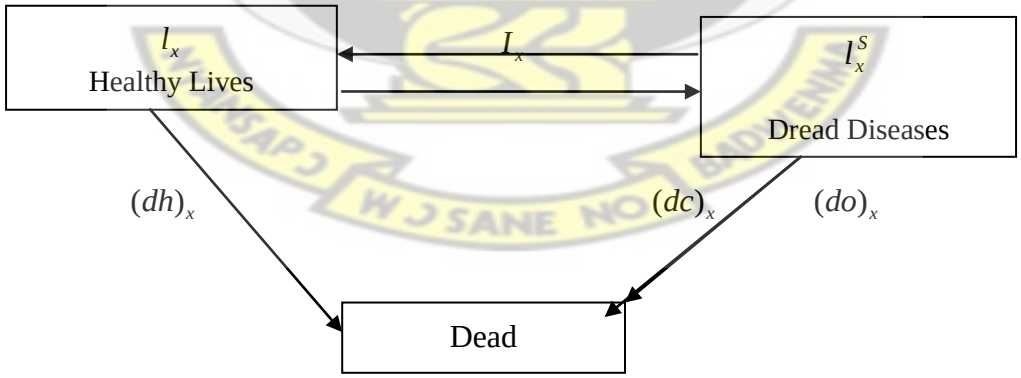


Figure 3.13: Multiple Decrement model to price CI benefits

For the purpose of this project, we follow the approach described in Dash and Grimshaw to derive the approximate risk premium rate for the Acceleration product.

The approach is an approximate, in that it uses a discrete time representation of the continuous model. The cover for this product is for 1 year and the benefit involves a unit sum assured.

The discrete time model is shown in the figure 3.11.1, where I_x is the number of incidences of CI between age x and $x+1$, and $(do)_x$ is number of deaths due to a cause other than CI amongst lives suffering from critical illness between age x and $x+1$.

The decrements are summarised below:

I_x = number of incidences of Critical Illness (Dread Disease) from the population of healthy lives.

$(dh)_x$ = number of deaths among healthy lives.

$(dc)_x$ = number of deaths from the population of Critical Illness (Dread Disease) sufferers due to Critical Illness.

$(do)_x$ = number of deaths from the population of Critical Illness (Dread Disease) sufferers due to other causes.

It can be seen that the three states are healthy, CI sufferer, and dead. The number of CI incidents, I_x, is the aggregate of the number of incidents for each CI covered. Also, the number for deaths amongst CI sufferers needs to be subdivided between those where deaths was caused by CI and those where death was attributable to another cause.

The total number of claims between age x and $x+1$ on an acceleration of CI policy or the total claims on a Critical Illness acceleration product per unit sum assured is given by

$$I_x + (dh)_x$$

(3.11.1)

We note that our definition above includes 'sudden deaths'. So for example a life that dies instantly from his first heart attack will be counted in both I_x and $(dc)_x$. This means that all Critical Illness deaths are counted $(dc)_x$. We therefore note that:

$$(dc)_x = k_x d_x$$

where $d_x = q_x l_x$ and $q_x =$ mortality rate of insurance population l_x.

k_x is the proportion of deaths due to Critical Illness in the population l_x between ages x and $x+1$

d_x is the number of deaths in the population between ages x and $x+1$

Further, considering the deaths due to causes other than Critical Illness

$$(dh)_x + (do)_x = (1-k_x)d_x$$

Hence the total claim cost can be re-expressed as

$$I_x + (dh)_x = I_x + (1-k_x)d_x - (do)_x$$

As noted earlier, we have reliable data for I_x and k_x. Further, d_x follows from the assumption of insured lives mortality used in costing the life cover. To complete our costing we need values for $(do)_x$ for which we do not have any data which can be used directly. Thus we have to deduce acceptable values for $(do)_x$ and this is the point at

which the existing methods for costing Critical Illness have varied. We now consider three different approaches and the merits for each.

3.11.11.1 Mortality of Critical Illness (Dread Disease) sufferers

Here we make use of the data that is available on the mortality experience of those who have suffered a Critical Illness (Dread Disease). This gives us figures for the sum of the mortality rates:

$$\frac{(dc)x}{l_x^S} + \frac{(do)x}{l_x^s}$$

Now since

$$(dh)_x = d_x - (dc)_x - (do)_x$$

we have all the elements required to build a multiple decrement table from which we can extract the required rates directly.

The difficulty with this approach is that the data is insufficient to have confidence in the rates. Because of this difficulty, we shall consider alternative methods that do not directly use the mortality after a Critical Illness.

3.11.11.2 Proportions of Deaths

Suppose we define a factor f_x as the proportion of deaths among Dread Disease sufferers attributable to Dread Diseases in the year age of age x to $x+1$

$$f_x = \frac{(dc)_x}{(do)_x + (dc)_x} = \frac{k_x d_x}{(do)_x + k_x d_x}$$

Thus $(do)_x$ is found if we can arrive at suitable values for f_x. It has been suggested elsewhere that f_x is relatively constant between .80 and .85 for many ages.

Unfortunately we have been unable to locate any data to validate this. However, extending the method explained in the following section suggests that f is almost unity. This highlights the danger in relying on assumptions to which the premium rates will be sensitive since the higher value for f_x results in significantly higher premiums.

3.11.11.3 Extra Mortality

A different approach is to compare the mortality of Critical Illness (Dread Disease) sufferers from causes other than Critical Disease with the mortality of healthy lives. Suppose the latter exceeds the former by an extra mortality of *m*:

then we have,

$$\frac{(do)_x}{l_x^s} = \frac{(dh)_x}{l_x - l_x^S}(1+m)$$

We also have

$$(dh)_x + (do)_x = (1-k_x)d_x$$

and eliminating $(do)_x$ from these equations gives

$$\frac{(dh)_x}{l_x - l_x^S}(l_x + ml_x^S) = (1-k_x)d_x$$

Thus if we know m and l_x^S then we know $(dh)_x$, and the first equation gives $(do)_x$, completion the values needed. If we continue the algebra then we can find the net premium rate for a Dread Disease acceleration benefit:

$$\frac{I_x + (dh)_x}{l_x - l_x^S}$$

$$= \frac{I_x}{l_x - l_x^S} + \frac{(1-k_x)q_x l_x}{l_x + ml_x^S}$$

$$= i_x + \frac{(1-k_x)q_x}{1 + ml_x^S / l_x}$$

where i_x is the incidence rate of Critical Illness.

If m =0, then the rate can be expressed simply as:

$$i_x + (1-k_x)q_x$$

So that the extra premium over the mortality rate is:

$$i_x - k_x q_x$$

This method is the most robust of those considered and we shall now proceed to produce premium rates on this basis. This is what the project seeks to establish.

A simplified model of the above derivation is illustrated below:

The total number of claims between age x and $x+1$ on an acceleration Critical Illness policy is given by

$$I_x + (dh)_x \qquad (3.1)$$

The difficulty is then to find suitable data to calculate these quantities and, in particular, $(dh)_x$. We can find a good approximation to the above equation by making suitable assumptions as follows.

d_x is the number of deaths in the population between age x and $x+1$

k_x is the proportion of deaths in the population between age x and $x+1$

which are due to CI

k_x should exclude deaths that occur immediately following the onset of CI, since such lives will not enter the population of CI sufferers. Then

$$(dc)_x = k_x d_x \qquad (3.2)$$

and since $d_x = (dh)_x + (dc)_x + (do)_x$, it follows that the number of deaths not due to CI is

$$(dh)_x + (do)_x = (1-k_x)d_x \qquad (3.3)$$

If now we assume that the probability of CI sufferers from causes other than CI is the same as the mortality of healthy lives, then

$$\frac{(dh)_x l_x}{l_x - (Ic)_x} = \frac{(dh)_x}{l_x - (Ic)_x} \qquad (3.4)$$

where l_x is the number of lives aged x exact and $(Ic)_x$ is the number of lives aged x exact suffering from a CI.

From Equation (3.3) and Equation (3.4), we can eliminate $(do)_x$ to

$$\frac{(dh)_x l_x}{l_x - (Ic)_x} = (1-k_x)d_x \qquad (3.5)$$

Now Equation (3.1) is an expression for the number of claims between age x and $x+1$ on an acceleration CI policy. To convert this into an incidence rate, we must divide by the healthy population to age x (i.e., divide by $l_x - (Ic)_x$).

Hence, the incidence rate for the acceleration product is

$$\frac{l_x}{l_x - (Ic)_x} + \frac{(dh)_x}{l_x - (Ic)_x} \qquad (3.6)$$

From (3.5), this can be rewritten as

$$i_x + (1-k_x)q_x \qquad (3.7)$$

where the incidence rate $i_x = I_x / (l_x - (Ic)_x)$ and the mortality rate is $q_x = d_x / l_x$.

Hence, Equation (3.7) represents a simple expression for the risk premium rate for an acceleration CI product. This above model is what shall be used in this project.

It must be stated that the formulas derived here illustrates premium rates, and this is what the project seeks to establish.

CHAPTER 4

RESULTS: DATA PRESENTATION AND ANALYSIS

4.1 Construction of the Base Table

Introduction

There is currently no standard table which relates to Critical Illness experience in Ghana. Our aim in this section of the project is to develop a set of population incidence rates for Critical Illness which can be used as a Base Table for benchmarking the experience under Ghana policies and as a reference point for pricing and reserving. The base table is developed using the population of Ghana data and where not available population of England (a country developed in pricing and selling Critical Illness), as this provides a large data source in order to derive a smooth set of rates. It is my hope that the Base Table will nevertheless provide a useful standard against which the insured lives experience can be compared. Furthermore, the development of the Base Table from first principles should give a good understanding of the relative impact of the various risks covered.

The Critical Illnesses covered in the Base Table are the core illnesses which virtually all the offices include within the terms of their Critical Illnesses policies. These are Cancer, Heart Attack and Stroke.

In deriving the incidence rates for each condition we have attempted to match the recently introduced Association of British Insurers (ABI) definitions. The base table has

been developed on an aggregate basis with no attempt to differentiate between Smokers and Non-Smokers.

4.2 Data Sources

For each illness, the full list of data sources used in deriving the incidence rates is given in the section relating to that illness. The following provides some background information on the primary data sources used. These are all published by the office of Population Censuses and Surveys (OPCS). A great deal of work has been done by Azim Dinani, Dave Grimshaw, Neil Rob johns, Stephen Somerville, Alasdair Spry and Jerry Stafford in their project, Critical Review. (Report of the Critical Illness Healthcare Study Group). For the purpose of this project, we will make an extension to calculating the Acceleration premium rates and the extra cost for the acceleration product for sufferers between the ages of twenty and seventy four. The project seeks to make an extension of their work by proving the Dash and Grimshaw model used (as shown earlier) using the multiple state model as described in chapter three. As Critical Illness is very new in the Ghanaian market, we resorted to use a data from the Ghanaian Ghana Health Service for the limited ages and then smooth it for the other age values.

4.3 General Approach

Incidence rates applicable to both Stand-Alone and Accelerated Critical Illness policies have been calculated for the Base Table. In doing this, a standardised approach using the steps detailed below has been followed, wherever possible. For certain illnesses, it has been necessary to vary this approach. The precise approach adopted for each illness

is documented in the section relating to that illness and the detailed derivation of the rates are shown in tables at the end of each section.

4.4 Adjustment For Unreported Cases

An appropriate adjustment needs to be made for unreported cases, which depend on the Critical Illness concerned and the data sources used. Unreported cases can arise from three sources:-

- where a patient dies suddenly following a Critical Illness, before being admitted to hospital;
- where the patient is admitted to a private hospital; and
- where the patient is not hospitalised at all.

Within the Base Table we have only adjusted for sudden deaths- we have no data to ascertain the significance of the other omissions.

4.5 Calculation of Crude Incidence Rates

Crude incidence rates can be calculated by dividing the adjusted number of incidences by the relevant population.

4.6 Overlap with Other Illnesses

Where there is a strong correlation between two Critical Illness conditions, it is necessary to adjust the incidence rates for one or both of these, to eliminate the double counting which would otherwise exist.

4.7 Prevalence Adjustment

The crude population incidence rates have been calculated by using the total population as the denominator. For the purpose of calculating a first time incidence rate, the denominator needs to be reduced by the prevalence of lives who have already suffered from that particular Critical Illness. The prevalence of the population can be estimated using the patients consulting rate reported in Morbidity Statistics from General Practice (MSGP).

If C_x is the consultation rate for a particular Critical Illness at age x, an approximation of the required adjustment to the incidence rates is to divide through by the factor $1 - C_x$.

4.8 Smoothing

The next step is to smooth the resulting incidence rates. Where individual age data is available, it has been smoothed by using a 5 year moving averages. In other cases, the smoothing methods are covered under the individual sections for the specific illnesses.

4.9 Stand Alone-Rates

The Stand-Alone rates calculated for the Base Table assumes that a survival period of 28 days applies. That is, the claimant needs to survive 28 days before any benefit is payable. The Stand-Alone rates are calculated by multiplying the smoothed incidence rates, calculated above, by the probability of survival during this 28 day period, which depend on the Critical Illness concerned.

4.10 Accelerated Rates

To calculate the Accelerated incidence rates, the Dash and Grimshaw model, developed in chapter three has been used. The values of k_x are calculated using the Office of Population Census and Statistics (OPCS). The q_x are taken from Life Table 15 (LT15). These were compared to crude population mortality rates derived from OPCS by cause for 1992-1993, the central year of the Base table.

4.11 Construction of incident Rate, the Acceleration Premium Rate and the Extra Cost for Accelerated Rate

In this section we derive the Incidence rates and its corresponding Stand-Alone Premium and Acceleration Rider Premium for the three most prevalent illnesses among Ghanaians in the Critical Illness industry. They are:

1. Cancer
2. Heart Attack and
3. Stroke.

4.11.1 Cancer

Definition

The ABI model definition for Cancer is:

A malignant tumour characterised by the uncontrolled growth and spread of malignant cells and invasion of tissue. This include leukaemia (other than chronic lymphocytic leukaemia), but excludes non-invasive cancers tumours in the presence of any human immune-deficiency virus and any skin cancer other than malignant melanoma. The term

cancer includes leukaemia and Hodgkin's disease but there are others which are excluded by the insurer.

For computation of incidence rates, exclusions and trends, a great deal of work has been done by the aforementioned people in their exclusive research work. (Report of the Critical Illness Healthcare Study Group) and our work is to make an extension to Premiums and Benefits.

The Data Sources for all Diseases will be taken from:

- The Cancer Registration Office
- Hospital Episode Statistics (HES)
- Office of Population Census and Statistics (OPCS)-Mortality Statistics

The data presented in this project have a 0% adjustment for overlap with other CI's.

Table 4.1: Critical Illness Incidence Rates-Cancer(males)

Age	HES Data- No. of Sufferers	Population *1000's*	Prevalence Rate Adjustment	Adjusted Crude Rate (i_x)	28 day Mortality Rate *(ELT 15)* (q^i_x)	Stand Alone Rate $i_x(1-q^i_x)$	LT 15 Population q_x (per 10,000)	Proportions of Death from Cancer (k_x)	Extra cost for Accelerated $(i_x-k_x*q_x)$	Accelerated Rate $i_x+q_x(1-k_x)$
20	603	334.96	0.18%	2.16	0.00006	**2.16**	8.4267	0.0878	**1.42**	**2.16**
21	765	357.45	0.21%	2.31	0.00007	**2.31**	8.6371	0.0876	**1.55**	**2.31**
22	738	375.82	0.20%	2.46	0.00007	**2.46**	8.8479	0.0881	**1.68**	**2.46**
23	754	375.00	0.20%	2.67	0.00007	**2.67**	8.8558	0.0904	**1.87**	**2.67**
24	773	387.17	0.20%	2.87	0.00007	**2.87**	8.8636	0.0935	**2.04**	**2.87**
25	825	390.34	0.21%	3.08	0.00007	**3.08**	8.5656	0.0970	**2.25**	**3.08**
26	850	402.53	0.21%	3.29	0.00006	**3.29**	8.4709	0.1007	**2.43**	**3.29**
27	934	411.42	0.23%	3.49	0.00007	**3.49**	8.4780	0.1049	**2.60**	**3.49**
28	908	419.52	0.22%	3.71	0.00007	**3.71**	8.6897	0.1101	**2.75**	**3.71**
29	914	419.46	0.22%	3.93	0.00007	**3.93**	8.9019	0.1158	**2.89**	**3.93**
30	1009	409.81	0.25%	4.14	0.00007	**4.14**	9.1147	0.1217	**3.03**	**4.14**
31	1235	399.12	0.31%	4.36	0.00007	**4.36**	9.3280	0.1277	**3.17**	**4.36**
32	1019	389.39	0.26%	4.58	0.00007	**4.58**	9.7471	0.1340	**3.27**	**4.58**
33	1004	370.49	0.27%	4.94	0.00008	**4.94**	9.9620	0.1406	**3.54**	**4.94**
34	1126	358.25	0.32%	5.30	0.00008	**5.30**	10.5888	0.1475	**3.74**	**5.30**
35	969	350.00	0.28%	5.66	0.00009	**5.66**	11.6291	0.1544	**3.86**	**5.66**
36	1090	338.26	0.32%	6.02	0.00010	**6.02**	12.7760	0.1623	**3.95**	**6.02**
37	1262	325.30	0.39%	6.39	0.00011	**6.39**	13.7208	0.1719	**4.03**	**6.39**
38	1097	315.45	0.35%	7.21	0.00011	**7.21**	14.9793	0.1850	**4.44**	**7.21**

39	1196	320.34	0.37%	8.05	0.00012	**8.05**	16.0364	0.1998	**4.84**	**8.05**
40	1397	318.62	0.44%	8.88	0.00013	**8.88**	17.2021	0.2156	**5.17**	**8.88**
41	1749	311.99	0.56%	9.72	0.00014	**9.72**	18.5812	0.2310	**5.43**	**9.72**
42	1944	318.96	0.61%	10.56	0.00015	**10.56**	20.1758	0.2459	**5.60**	**10.56**
43	1906	327.52	0.59%	12.03	0.00017	**12.03**	21.8839	0.2595	**6.35**	**12.03**
44	2250	339.37	0.67%	13.51	0.00018	**13.51**	24.0206	0.2725	**6.96**	**13.51**
45	2386	362.17	0.66%	14.98	0.00020	**14.98**	26.6957	0.2852	**7.37**	**14.98**
46	3155	397.14	0.80%	16.47	0.00023	**16.47**	29.7063	0.2973	**7.64**	**16.47**
47	3105	306.90	1.02%	17.99	0.00025	**17.99**	33.1638	0.3081	**7.77**	**17.99**
48	3059	302.27	1.02%	21.20	0.00029	**21.19**	37.1826	0.3162	**9.44**	**21.20**
49	3241	304.44	1.08%	24.43	0.00032	**24.42**	41.4564	0.3231	**11.04**	**24.43**
50	3402	288.22	1.19%	27.68	0.00036	**27.67**	46.4200	0.3293	**12.39**	**27.68**
51	3627	263.16	1.40%	30.96	0.00040	**30.95**	51.8778	0.3351	**13.58**	**30.96**
52	3712	241.88	1.56%	34.24	0.00044	**34.22**	57.7395	0.3401	**14.60**	**34.24**
53	4009	256.27	1.59%	38.74	0.00049	**38.72**	64.2391	0.3436	**16.67**	**38.74**
54	4544	257.84	1.79%	43.32	0.00055	**43.30**	71.4029	0.3463	**18.59**	**43.32**
55	4779	255.84	1.90%	47.87	0.00061	**47.84**	79.7001	0.3486	**20.09**	**47.88**
56	5157	250.26	2.10%	52.47	0.00068	**52.43**	89.0706	0.3506	**21.24**	**52.48**
57	5855	244.78	2.45%	57.18	0.00076	**57.14**	99.4603	0.3522	**22.15**	**57.19**
58	5978	239.71	2.56%	65.22	0.00085	**65.16**	111.1587	0.3528	**26.00**	**65.23**
59	6054	230.49	2.70%	73.30	0.00095	**73.23**	124.2526	0.3529	**29.45**	**73.31**
60	6351	228.25	2.86%	81.42	0.00107	**81.33**	139.1932	0.3528	**32.32**	**81.43**
61	7401	233.43	3.27%	89.78	0.00120	**89.67**	156.0106	0.3525	**34.80**	**89.79**

62	7857	234.01	3.47%	98.01	0.00134	**97.88**	174.8779	0.3517	**36.50**	**98.02**
63	8281	228.02	3.77%	110.51	0.00151	**110.34**	196.4910	0.3528	**41.70**	**110.52**
64	8883	220.83	4.19%	123.39	0.00169	**123.18**	219.9171	0.3529	**46.64**	**123.40**
						0.00				
65	9302	215.61	4.51%	135.93	0.00188	**135.67**	244.6622	0.3528	**51.23**	**135.95**
66	9502	214.24	4.64%	148.43	0.00208	**148.12**	271.0949	0.3525	**55.29**	**148.45**
67	10231	213.63	5.03%	161.36	0.00230	**160.99**	299.6439	0.3517	**59.49**	**161.38**
68	10345	204.58	5.33%	174.83	0.00253	**174.39**	329.1736	0.3502	**64.76**	**174.85**
69	10620	199.28	5.63%	188.39	0.00276	**187.87**	360.2085	0.3483	**70.32**	**188.41**
70	10985	193.52	6.02%	202.18	0.00301	**201.57**	392.9175	0.3462	**76.19**	**202.21**
71	11234	196.09	6.08%	215.41	0.00331	**214.70**	431.1650	0.3436	**80.26**	**215.44**
72	11668	195.56	6.34%	229.09	0.00364	**228.26**	474.5692	0.3400	**83.86**	**229.12**
73	12236	195.24	6.69%	243.57	0.00400	**242.60**	521.7056	0.3344	**87.99**	**243.60**
74	8672	123.55	7.55%	259.39	0.00437	**258.26**	569.7189	0.3278	**94.06**	**259.43**

Table 4.2: Critical Illness Incidence Rates-Cancer(females)

Age	HES Data-No. of Sufferers	Population *1000's*	Prevalence Rate Adjustment	Adjusted Crude Rate (i_x)	28 day Mortality Rate *(ELT 15)* (q^i_x)	Stand Alone Rate $i_x(1-q^i_x)$	LT 15 Population q_x (per 10,000)	Proportions of Death from Cancer (k_x)	Extra cost for Accelerated $(i_x-k_x*q_x)$	Accelerated Rate $i_x+q_x(1-k_x)$
20	408	316.68	0.13%	2.10	0.00002	**2.10**	3.1327	0.1520	**1.62**	**2.10**
21	382	337.82	0.11%	2.27	0.00002	**2.27**	3.2347	0.1624	**1.75**	**2.27**
22	600	358.72	0.17%	2.45	0.00002	**2.45**	3.2358	0.1735	**1.89**	**2.45**
23	507	355.66	0.14%	2.81	0.00003	**2.81**	3.3380	0.1860	**2.19**	**2.81**
24	628	371.38	0.17%	3.17	0.00002	**3.17**	3.2379	0.1991	**2.52**	**3.17**
25	592	375.35	0.16%	3.52	0.00003	**3.52**	3.4414	0.2125	**2.79**	
26	701	387.10	0.18%	3.88	0.00003	**3.88**	3.4426	0.2269	**3.10**	**3.88**
27	803	395.73	0.20%	4.24	0.00003	**4.24**	3.5451	0.2433	**3.38**	**4.24**
28	881	402.87	0.22%	5.00	0.00003	**5.00**	3.8503	0.2634	**3.98**	**5.00**
29	1039	401.82	0.26%	5.75	0.00003	**5.75**	3.9531	0.2855	**4.62**	**5.75**
30	1251	394.34	0.32%	6.51	0.00003	**6.51**	4.3603	0.3085	**5.17**	**6.51**
31	1270	385.16	0.33%	7.27	0.00004	**7.27**	4.6665	0.3312	**5.72**	**7.27**
32	1295	374.28	0.35%	8.02	0.00004	**8.02**	5.1762	0.3531	**6.20**	**8.02**
33	1277	357.70	0.36%	9.19	0.00004	**9.19**	5.7882	0.3735	**7.03**	**9.19**
34	1496	351.75	0.43%	10.37	0.00005	**10.37**	6.1979	0.3932	**7.93**	**10.37**
35	1605	344.10	0.47%	11.55	0.00005	**11.55**	6.9134	0.4126	**8.69**	**11.55**
36	1717	333.81	0.52%	12.72	0.00006	**12.72**	7.5287	0.4315	**9.47**	**12.72**

37	1645	321.73	0.51%	13.89	0.00006	**13.89**	8.2470	0.4497	**10.18**	**13.89**
38	2211	312.23	0.71%	15.52	0.00007	**15.52**	8.9671	0.4662	**11.34**	**15.52**
39	2136	318.12	0.68%	17.12	0.00008	**17.12**	9.7911	0.4819	**12.40**	**17.12**
										0.00
40	2254	316.65	0.72%	18.73	0.00008	**18.73**	10.7195	0.4971	**13.40**	**18.73**
41	2383	311.87	0.77%	20.34	0.00009	**20.34**	11.6508	0.5119	**14.38**	**20.34**
42	2599	316.82	0.83%	21.96	0.00010	**21.96**	12.8923	0.5257	**15.18**	**21.96**
43	3280	327.42	1.01%	24.59	0.00011	**24.59**	14.1383	0.5373	**16.99**	**24.59**
44	3788	338.39	1.13%	27.21	0.00012	**27.21**	15.7999	0.5480	**18.56**	**27.21**
45	4224	362.00	1.18%	29.82	0.00014	**29.82**	17.7773	0.5581	**19.90**	**29.82**
46	4890	393.71	1.26%	32.45	0.00015	**32.45**	19.7649	0.5671	**21.24**	**32.45**
47	4641	306.96	1.54%	35.14	0.00017	**35.13**	21.8669	0.5737	**22.60**	**35.14**
48	4409	303.03	1.48%	38.46	0.00019	**38.45**	24.1890	0.5756	**24.54**	**38.46**
49	4845	304.01	1.62%	41.86	0.00020	**41.85**	26.6310	0.5751	**26.55**	**41.86**
50	4915	287.83	1.74%	45.26	0.00023	**45.25**	29.4035	0.5734	**28.40**	**45.26**
51	5025	262.71	1.95%	48.71	0.00023	**48.70**	32.5122	0.5710	**30.15**	**48.71**
52	4579	242.40	1.93%	52.05	0.00027	**52.04**	35.7546	0.5671	**31.78**	**52.05**
53	5246	256.64	2.09%	55.37	0.00030	**55.35**	39.0305	0.5604	**33.49**	**55.37**
54	5346	259.76	2.10%	58.60	0.00033	**58.58**	42.7648	0.5523	**34.98**	**58.60**
55	5586	257.23	2.22%	61.90	0.00037	**61.88**	47.6029	0.5434	**36.03**	**61.90**
56	5732	251.32	2.33%	65.21	0.00041	**65.18**	53.0388	0.5342	**36.87**	**65.21**
57	5997	247.67	2.48%	68.54	0.00045	**68.51**	59.1988	0.5243	**37.50**	**68.54**
58	4529	241.58	1.91%	73.04	0.00051	**73.00**	55.0009	0.5131	**39.18**	**73.04**
59	4339	234.65	1.88%	77.91	0.00057	**77.87**	73.9058	0.5012	**40.86**	**77.91**

60	4679	235.06	2.03%	82.91	0.00064	**82.86**	82.9591	0.4890	**42.34**	**82.91**
61	5056	240.45	2.15%	87.90	0.00071	**87.84**	92.2272	0.4763	**43.97**	**87.90**
62	5546	247.05	2.30%	92.93	0.00078	**92.86**	101.5178	0.4630	**45.93**	**92.94**
63	5668	246.93	2.35%	96.10	0.00087	**96.02**	112.8709	0.4481	**45.52**	**96.11**
64	5905	240.99	2.51%	99.38	0.00097	**99.28**	126.6296	0.4324	**44.62**	**99.39**
65	6247	237.03	2.71%	102.70	0.00107	**102.59**	139.8505	0.4164	**44.46**	**102.71**
66	6392	238.22	2.76%	105.88	0.00117	**105.76**	152.3144	0.4006	**44.87**	**105.89**
67	6682	239.70	2.87%	109.13	0.00129	**108.99**	167.5594	0.3851	**44.60**	**109.14**
68	6950	237.43	3.02%	114.26	0.00141	**114.10**	184.3656	0.3705	**45.95**	**114.27**
69	6708	237.42	2.91%	119.10	0.00155	**118.92**	201.7962	0.3563	**47.20**	**119.11**
										0.00
70	7015	237.78	3.04%	124.22	0.00168	**124.01**	218.9571	0.3423	**49.28**	**124.23**
71	7020	244.70	2.95%	129.08	0.00184	**128.84**	239.8394	0.3280	**50.41**	**129.10**
72	7619	253.39	3.10%	134.24	0.00207	**133.96**	269.3109	0.3133	**49.85**	**134.26**
73	8663	259.69	3.45%	139.17	0.00231	**138.85**	301.3986	0.2978	**49.42**	**139.19**
74	5928	171.14	3.59%	143.84	0.00252	**143.48**	328.3922	0.2818	**51.31**	**143.86**

DISCUSSION FOR THE CASES OF CANCER RECORDED

The definition of the diseases and the data sources used is stated above. Generally incidence of cancer recorded amongst females is more than that of males. The stand-alone rates are computed using the figures obtained in column five and column eight. The accelerated rates are computed by using the elements in column five, seven and eight from the data. The stand-alone rate and its corresponding accelerated rates computed from the table are what is used to determine the office premium and its corresponding chargeable premium to the insured. For example suppose a male individual currently at age 45, wants to buy a critical illness product of cancer with a benefit of 1500 to be enjoyed at the end of one year. Then office premium to be paid by this client for a stand-alone product is given by $\frac{14.98}{1000}\times 1500 = 22.47$. Suppose that this individual is to pay 25 in loadings, then the chargeable premium is $22.47 + 25 = 47.47$. This is the amount of money to be paid to in order to enjoy the benefit 1500. For a female also at the same age, the office premium is given by $\frac{29.82}{1000}\times 1500 = 47.73$. With a loading of also 25, the chargeable premium is given by $47.73 + 25 = 69.73$. Premiums for other ages can be computed in the same way and basically all other quantities increases as the age also increases.

4.11.2 Heart Attack

The ABI model definition for Heart Attack is:

The death of a portion of the heart muscle as a result of inadequate blood supply as evidenced by an episode of typical chest pain, new electrocardiograph changes and by

the elevation of cardiac enzymes. The evidence must be consistent with the diagnosis of heart attack.

The Data Sources for all Diseases will be taken from:

- The Heart Foundation
- Hospital Episode Statistics (HES)-The Ghana Health Service
- Office of Population Census and Statistics (OPCS)-Mortality Statistics
- Morbidity Statistics for General Practice Data

For computation of Incidence rates, exclusions and trends, a great deal of work has been done by the aforementioned people in their exclusive research work. (Report of the Critical Illness Healthcare Study Group) and our work is to make an extension to Premiums and Benefits. The data presented in this project have a 0% adjustment for overlap with other CIs.

Table 4.3: Critical Illness Incidence Rates-Heart Attack(males)

Age	Population (1000's)	Number of Heart Attacks (ICD 410)	Prevalence Rate Adjustment	Smoothed Adjusted Crude Rate (i_x)	28 Day Mortality Rate (ELT 15)(q^i_x)	Stand Alone Rate $i_x(1-q^i_x)$	LT 15 Population q_x (per 10,000)	Proportions of Deaths from Heart Attack (k_x)	Extra cost for Accelerated ($i_x-k_x*q_x$)	Accelerated Rate $i_x+q_x(1-k_x)$
20	334.96	0	0.00%	0.09	15%	**0.08**	8.4267	0.0041	**0.05**	**0.09**
21	357.45	3	0.00%	0.14	15%	**0.12**	8.6371	0.0055	**0.10**	**0.14**
22	375.82	7	0.00%	0.16	15%	**0.14**	8.8479	0.0068	**0.10**	**0.16**
23	375.00	13	0.00%	0.23	15%	**0.20**	8.5538	0.0081	**0.16**	**0.23**
24	387.17	5	0.00%	0.31	15%	**0.26**	8.8636	0.0092	**0.22**	**0.31**
										0.00
25	390.34	12	0.00%	0.31	15%	**0.26**	8.5656	0.0104	**0.22**	**0.31**
26	402.53	17	0.00%	0.37	15%	**0.31**	8.4709	0.0118	**0.27**	**0.37**
27	411.42	9	0.01%	0.45	15%	**0.38**	8.4780	0.0135	**0.33**	**0.45**
28	419.52	26	0.01%	0.58	15%	**0.49**	8.6897	0.0160	**0.44**	**0.58**
29	419.46	20	0.01%	0.75	15%	**0.64**	8.9019	0.0188	**0.58**	**0.75**
30	409.81	37	0.02%	1.09	15%	**0.93**	9.1147	0.0218	**0.89**	**1.09**
31	399.12	48	0.03%	1.43	15%	**1.22**	9.3280	0.0254	**1.20**	**1.43**
32	389.39	68	0.04%	1.87	15%	**1.59**	9.7471	0.0303	**1.58**	**1.87**
33	370.49	81	0.05%	2.37	15%	**2.01**	9.9620	0.0378	**1.99**	**2.37**
34	358.25	89	0.06%	3.11	15%	**2.64**	10.5888	0.0465	**2.62**	**3.11**
										0.00
35	350.00	110	0.08%	3.79	15%	**3.22**	11.6291	0.0558	**3.14**	**3.79**
36	338.26	155	0.09%	4.85	15%	**4.12**	12.7760	0.0650	**4.02**	**4.85**
37	325.30	159	0.11%	5.95	15%	**5.06**	13.7208	0.0740	**4.94**	**5.95**

38	315.45	225	0.13%	7.32	15%	**6.22**	14.9793	0.0825	**6.09**	**7.32**
39	320.34	250	0.15%	8.79	15%	**7.47**	16.0364	0.0908	**7.33**	**8.79**
40	318.62	314	0.18%	10.48	16%	**8.65**	17.2021	0.0990	**8.78**	**10.48**
41	311.99	377	0.21%	11.92	16%	**10.01**	18.5812	0.1072	**9.93**	**11.92**
42	318.96	438	0.24%	14.12	16%	**11.86**	20.1758	0.1155	**11.79**	**14.12**
43	327.52	492	0.27%	15.57	17%	**12.92**	21.8839	0.1240	**12.86**	**15.57**
44	339.37	675	0.30%	17.04	17%	**14.14**	24.0206	0.1327	**13.85**	**17.04**
45	362.17	671	0.34%	19.32	17%	**16.04**	26.6957	0.1413	**15.55**	**19.32**
46	397.14	846	0.37%	21.40	17%	**17.76**	29.7063	0.1498	**16.95**	**21.40**
47	306.90	847	0.41%	23.11	18%	**18.95**	33.1638	0.1578	**17.88**	**23.11**
48	202.27	858	0.46%	25.60	18%	**20.99**	37.1826	0.1648	**19.47**	**25.60**
49	304.44	977	0.50%	28.42	18%	**23.30**	41.4564	0.1714	**21.32**	**28.42**
50	288.22	1021	0.55%	30.84	18%	**25.29**	46.4200	0.1777	**22.59**	**30.84**
51	263.16	1082	0.59%	33.46	18%	**27.44**	51.8778	0.1839	**23.92**	**33.46**
52	241.88	1111	0.64%	36.52	18%	**29.95**	57.7395	0.1896	**25.57**	**36.52**
53	256.27	1227	0.69%	40.52	18%	**33.23**	64.2391	0.1945	**28.02**	**40.53**
54	257.84	1372	0.75%	44.09	19%	**35.71**	71.4029	0.1990	**29.88**	**44.10**
55	255.84	1576	0.80%	47.84	19%	**38.75**	79.7001	0.2033	**31.63**	**47.85**
56	250.26	1594	0.85%	51.79	19%	**41.95**	89.0706	0.2071	**33.34**	**51.80**
57	244.78	1670	0.91%	56.55	20%	**45.24**	99.4603	0.2098	**35.34**	**56.56**
58	239.71	1675	0.97%	60.27	21%	**47.61**	111.1587	0.2106	**36.86**	**60.28**
59	230.49	1854	1.02%	63.94	21%	**50.51**	124.2526	0.2104	**37.80**	**63.95**

60	223.25	1858	1.08%	67.51	22%	**52.66**	139.1932	0.2096	**38.33**	**67.52**
61	233.43	1940	1.14%	71.53	22%	**55.79**	156.0106	0.2088	**38.96**	**71.54**
62	234.01	2025	1.19%	75.10	23%	**57.83**	174.8779	0.2078	**38.76**	**75.11**
63	228.02	2089	1.25%	78.80	23%	**60.68**	196.4910	0.2064	**38.23**	**78.82**
64	220.83	2190	1.31%	83.07	23%	**63.96**	219.9171	0.2049	**38.01**	**83.09**
65	215.61	2169	1.36%	86.90	24%	**66.04**	244.6622	0.2032	**37.18**	**86.92**
66	214.24	2262	1.42%	90.80	24%	**69.01**	271.0949	0.2017	**36.13**	**90.82**
67	213.63	2294	1.47%	97.45	24%	**74.06**	299.6439	0.2002	**33.72**	**97.47**
68	204.58	2306	1.52%	97.45	24%	**74.06**	239.1736	0.1989	**31.97**	**97.47**
69	199.28	2281	1.57%	100.18	24%	**76.14**	360.2085	0.1978	**28.92**	**100.21**
70	193.52	2360	1.62%	104.33	24%	**79.29**	392.9175	0.1968	**27.02**	**104.36**
71	196.09	2373	1.66%	108.07	24%	**82.13**	431.1650	0.1954	**23.81**	**108.10**
72	196.56	2563	1.70%	114.45	24%	**86.98**	474.5692	0.1935	**22.61**	**114.49**
73	195.24	2615	1.74%	121.04	24%	**91.99**	521.7056	0.1905	**21.65**	**121.08**
74	123.55	1865	1.78%	125.28	24%	**95.21**	569.7189	0.1870	**18.76**	**125.33**

Table 4.4: Critical Illness Incidence Rates-Heart Attack(females)

Age	Population *(1000's)*	Number of Heart Attacks *(ICD 410)*	Prevalence Rate Adjustment	Smoothed Adjusted Crude Rate (i_x)	28 Day Mortality Rate *(ELT 15)* (q^i_x)	Stand Alone Rate $i_x(1-q^i_x)$	LT 15 Population q_x (per 10,000	Proportions of Deaths from Heart Attack (k_x)	Extra cost for Accelerated $(i_x-k_x*q_x)$	Accelerated Rate $i_x+q_x(1-k_x)$
20	316.68	0	0.00%	0.02	21%	**0.02**	3.1327	0.0010	**0.02**	**0.02**
21	337.82	0	0.01%	0.02	21%	**0.02**	3.2347	0.0015	**0.02**	**0.02**
22	358.72	2	0.02%	0.03	21%	**0.02**	3.2358	0.0023	**0.02**	**0.03**
23	355.66	0	0.03%	0.04	21%	**0.03**	3.3380	0.0036	**0.03**	**0.04**
24	371.38	3	0.04%	0.05	21%	**0.04**	3.2379	0.0051	**0.03**	**0.05**
25	375.35	1	0.05%	0.06	21%	**0.05**	3.4414	0.0068	**0.04**	**0.06**
26	387.10	2	0.05%	0.08	21%	**0.06**	3.4426	0.0085	**0.06**	**0.08**
27	395.73	4	0.05%	0.09	21%	**0.07**	3.5451	0.0102	**0.06**	**0.09**
28	402.86	4	0.05%	0.13	21%	**0.10**	3.8503	0.0119	**0.08**	**0.13**
29	401.82	5	0.05%	0.21	21%	**0.17**	3.9531	0.0136	**0.16**	**0.21**
30	394.34	7	0.04%	0.26	21%	**0.21**	4.3603	0.0153	**0.20**	**0.26**
31	385.16	15	0.04%	0.37	21%	**0.29**	4.6665	0.0168	**0.29**	**0.37**
32	374.28	12	0.03%	0.46	21%	**0.36**	5.1762	0.0179	**0.37**	**0.46**
33	357.70	20	0.03%	0.60	21%	**0.47**	5.7882	0.0183	**0.50**	**0.60**
34	351.75	18	0.02%	0.70	21%	**0.55**	6.1979	0.0184	**0.58**	**0.70**
35	344.10	27	0.02%	0.80	21%	**0.63**	6.9134	0.0182	**0.67**	**0.80**
36	333.31	26	0.01%	0.88	21%	**0.70**	7.5287	0.0186	**0.74**	**0.88**

37	321.73	25	0.01%	1.05	21%	**0.83**	8.2470	0.0199	**0.89**	**1.05**
38	312.23	29	0.00%	1.21	21%	**0.96**	8.9671	0.0231	**1.00**	**1.21**
39	318.12	41	0.00%	1.33	21%	**1.05**	9.7911	0.0272	**1.07**	**1.33**
40	316.65	49	0.00%	1.58	21%	**1.25**	10.7195	0.0318	**1.24**	**1.58**
41	311.87	44	0.00%	1.83	21%	**1.45**	11.6508	0.0360	**1.42**	**1.83**
42	316.82	64	0.00%	2.20	21%	**1.74**	12.8923	0.0393	**1.69**	**2.20**
43	327.42	74	0.01%	2.36	21%	**1.86**	14.1383	0.0410	**1.78**	**2.36**
44	338.39	110	0.01%	2.75	21%	**2.17**	15.7999	0.0418	**2.08**	**2.75**
45	362.00	91	0.02%	3.28	21%	**2.59**	17.7773	0.0422	**2.53**	**3.28**
46	393.71	143	0.03%	3.64	21%	**2.88**	19.7649	0.0428	**2.80**	**3.64**
47	306.96	156	0.04%	4.16	21%	**3.29**	21.8669	0.0440	**3.19**	**4.16**
48	303.03	137	0.05%	4.96	21%	**3.92**	24.1890	0.0463	**3.84**	**4.96**
49	304.01	199	0.07%	5.53	21%	**4.37**	26.6310	0.0491	**4.22**	**5.53**
50	287.83	214	0.09%	6.01	21%	**4.75**	29.4035	0.0522	**4.48**	**6.01**
51	262.71	197	0.10%	6.95	21%	**5.49**	32.5122	0.0557	**5.15**	**6.95**
52	242.40	209	0.13%	7.93	21%	**6.26**	35.7546	0.0599	**5.79**	**7.93**
53	256.64	276	0.15%	9.18	21%	**7.25**	39.0305	0.0657	**6.62**	**9.18**
54	259.76	338	0.17%	10.30	21%	**8.14**	42.7648	0.0722	**7.21**	**10.30**
55	257.23	397	0.20%	11.83	21%	**9.35**	47.6029	0.0792	**8.06**	**11.83**
56	251.32	364	0.23%	13.41	21%	**10.59**	53.0388	0.0860	**8.85**	**13.41**
57	247.67	445	0.26%	15.26	21%	**12.06**	59.1988	0.0925	**9.79**	**15.27**
58	241.58	491	0.30%	16.84	21%	**13.30**	66.0009	0.0985	**10.34**	**16.85**
59	234.65	568	0.33%	19.08	21%	**15.07**	73.9058	0.1042	**11.38**	**19.09**

60	235.06	586	0.37%	21.41	21%	**16.91**	82.9591	0.1097	**12.31**	**21.42**
61	240.45	674	0.41%	23.31	21%	**18.41**	92.2272	0.1152	**12.68**	**23.32**
62	247.05	791	0.45%	25.17	21%	**19.88**	101.5178	0.1206	**12.93**	**25.18**
63	246.93	784	0.49%	28.14	21%	**22.23**	112.8709	0.1256	**13.95**	**28.15**
64	240.99	853	0.53%	30.77	21%	**24.31**	126.6296	0.1306	**14.23**	**30.78**
65	237.03	1014	0.57%	33.20	21%	**26.23**	139.8505	0.1355	**14.26**	**33.21**
66	238.22	1045	0.62%	36.01	21%	**28.45**	142.3144	0.1401	**14.67**	**36.02**
67	239.70	1117	0.66%	38.87	21%	**30.71**	167.5594	0.1445	**14.66**	**38.88**
68	237.43	1155	0.70%	41.34	21%	**32.66**	184.3656	0.1482	**14.02**	**41.36**
69	237.42	1248	0.75%	44.20	21%	**34.92**	201.7962	0.1515	**13.64**	**44.22**
70	237.78	1368	0.79%	47.23	21%	**37.31**	218.9571	0.1546	**13.37**	**47.25**
71	244.70	1493	0.83%	50.87	21%	**40.19**	239.8394	0.1575	**13.10**	**50.89**
72	253.39	1640	0.88%	55.69	21%	**44.00**	269.3109	0.1598	**12.65**	**55.71**
73	259.69	1829	0.92%	59.50	21%	**47.01**	301.3986	0.1610	**10.98**	**59.53**
74	171.14	1397	0.96%	63.89	21%	**50.47**	328.3922	0.1617	**10.80**	**63.92**

DISCUSSION FOR THE CASES OF HEART ATTACK RECORDED

The definition of the diseases and the data sources used is stated above. Generally incidence of Heart Attack recorded, unlike cancer, in males is more than that of females .The stand-alone rates are computed using the figures obtained in column five and column eight. The accelerated rates are computed by using the elements in column five, seven and eight from the data. The stand-alone rate and its corresponding accelerated rates computed from the table are what is used to determine the office premium and its corresponding chargeable premium to the insured. For example suppose a male individual currently at age 45, wants to buy a critical illness product of Heart Attack with a benefit of 1500 to be enjoyed at the end of one year. Then the office premium to be paid by this client for a stand-alone product is given by $\frac{16.04}{1000}\times 1500 = 24.06$. Suppose that this individual is to pay 25 in loadings, then the chargeable premium is $24.06 + 25 = 49.06$. This is the amount of money to be paid to in order to enjoy the benefit 1500. For a female also at the same age, the office premium is given by $\frac{2.59}{1000}\times 1500 = 3.885$. With a loading of also 25, the chargeable premium is given by $3.885 + 25 = 28.89$.. Premiums for other ages can be computed in the same way and basically all other quantities increases as the age also increases. It can be concluded that the premium to be paid for a critical illness product of cancer is more than heart attack among males and less among females. The higher premiums to be paid in males more than females for heart attack product may be attributed to higher smoking habits in males more than females.

4.11.3 Stroke

The ABI model for Stroke is:-

A cerebrovascular incident resulting in permanent neurological damage. Transient ischaemic attacks are specifically excluded.

The Data Sources for all Diseases will be taken from:

- Hospital Episode Statistics (HES)-The Ghana Health Service
- Office of Population Census and Statistics (OPCS)-Mortality Statistics
- Morbidity Statistics for General Practice Data

For computation of Incidence rates, exclusions and trends, a great deal of work has been done by the aforementioned people in their exclusive research work. (Report of the Critical Illness Healthcare Study Group) and our work is to make an extension to Premiums and Benefits.

The data presented in this project have a 7% adjustment for overlap with other CI's

Table 4.5: Critical Illness Incidence Rates-Stroke(males)

Age	Population *(1000's)*	Number of Strokes	Prevalence Rate Adjustment	Smoothed Adjusted Crude Rate (i_x)	28 Day Mortality Rate *(ELT 15)* (q^i_x)	Stand Alone Rate $i_x(1-q^i_x)$	LT 15 Population q_x *(per 10,000)*	Proportions of Deaths from Strokes (k_x)	Extra cost for Accelerated $(i_x-k_x*q_x)$	Accelerated Rate $i_x+q_x(1-k_x)$
20	334.96	38	0.02%	0.82	0.0444	**0.78**	8.4267	0.0102	**0.73**	**0.82**
21	357.45	32	0.02%	0.95	0.0467	**0.91**	8.6371	0.0113	**0.85**	**0.95**
22	375.82	44	0.02%	1.12	0.0489	**1.07**	8.8479	0.0125	**1.01**	**1.12**
23	375.00	43	0.02%	1.03	0.0511	**0.98**	8.8558	0.0137	**0.91**	**1.03**
24	387.17	54	0.02%	1.17	0.0533	**1.11**	8.8636	0.0149	**1.04**	**1.17**
25	390.34	25	0.02%	1.19	0.0556	**1.12**	8.5656	0.0160	**1.05**	**1.19**
26	402.53	65	0.02%	1.17	0.0578	**1.10**	8.4709	0.0172	**1.03**	**1.17**
27	411.42	53	0.02%	1.20	0.0600	**1.13**	8.4780	0.0186	**1.05**	**1.20**
28	419.52	44	0.02%	1.45	0.0622	**1.36**	8.6897	0.0202	**1.28**	**1.45**
29	419.46	65	0.02%	1.45	0.0644	**1.36**	8.9019	0.0219	**1.25**	**1.45**
30	409.81	79	0.02%	1.63	0.0667	**1.52**	9.1147	0.0237	**1.42**	**1.63**
31	399.12	63	0.02%	1.89	0.0689	**1.76**	9.3280	0.0255	**1.65**	**1.89**
32	389.39	87	0.02%	1.95	0.0711	**1.81**	9.7471	0.0273	**1.68**	**1.95**
33	370.49	87	0.02%	2.22	0.0733	**2.06**	9.9620	0.0290	**1.93**	**2.22**
34	358.25	67	0.02%	2.50	0.0756	**2.31**	10.5888	0.0307	**2.17**	**2.50**
35	350.00	115	0.02%	2.72	0.0778	**2.51**	11.6291	0.0324	**2.35**	**2.72**
36	338.26	102	0.02%	2.91	0.0800	**2.68**	12.7760	0.0339	**2.47**	**2.91**
37	325.30	110	0.02%	3.27	0.0822	**3.00**	13.7208	0.0350	**2.79**	**3.27**

38	315.45	104	0.02%	3.68	0.0844	**3.37**	14.9793	0.0354	**3.15**	**3.68**
39	320.37	119	0.02%	4.29	0.0867	**3.92**	16.0364	0.0354	**3.72**	**4.29**
40	318.62	171	0.02%	4.93	0.0889	**4.49**	17.2021	0.0353	**4.33**	**4.93**
41	311.99	192	0.04%	5.56	0.0911	**5.05**	18.5812	0.0353	**4.90**	**5.56**
42	318.96	212	0.07%	6.13	0.0933	**5.56**	20.1758	0.0357	**5.41**	**6.13**
43	327.52	212	0.13%	6.22	0.0956	**5.63**	21.8839	0.0370	**5.41**	**6.22**
44	339.37	225	0.25%	6.45	0.0978	**5.82**	24.0206	0.0387	**5.52**	**6.45**
45	362.17	263	0.47%	6.63	0.1000	**5.97**	26.6957	0.0406	**5.54**	**6.63**
46	397.14	363	0.47%	7.12	0.1022	**6.39**	29.7063	0.0424	**5.87**	**7.12**
47	306.90	288	0.47%	8.09	0.1044	**7.25**	33.1638	0.0438	**6.63**	**8.09**
48	302.27	339	0.47%	9.35	0.1067	**8.35**	37.1826	0.0446	**7.69**	**9.35**
49	304.44	438	0.47%	10.67	0.1089	**9.51**	41.4564	0.0451	**8.80**	**10.67**
50	288.22	440	0.47%	12.13	0.1111	**10.78**	46.4200	0.0454	**10.02**	**12.13**
51	263.16	462	0.47%	13.18	0.1133	**11.69**	51.8778	0.0457	**10.81**	**13.18**
52	341.88	451	0.47%	14.10	0.1156	**12.47**	57.7395	0.0460	**11.45**	**14.11**
53	256.27	459	0.47%	15.72	0.1178	**13.87**	64.2391	0.0462	**12.75**	**15.73**
54	257.84	522	0.47%	17.11	0.1200	**15.06**	71.4029	0.0464	**13.80**	**17.12**
55	255.84	654	0.47%	18.77	0.1222	**16.48**	79.7001	0.0466	**15.05**	**18.78**
56	250.26	660	0.47%	21.14	0.1244	**18.51**	89.0706	0.0469	**16.96**	**21.15**
57	244.78	714	0.47%	23.51	0.1267	**20.53**	99.4603	0.0474	**18.80**	**23.52**
58	239.71	791	0.47%	25.97	0.1289	**22.62**	111.1587	0.0482	**20.61**	**25.98**
59	230.49	814	0.47%	28.50	0.1311	**24.76**	124.2526	0.0492	**22.39**	**28.51**

60	228.25	939	0.47%	31.07	0.1333	**26.93**	139.1932	0.0504	**24.06**	**31.08**
61	233.43	991	0.59%	34.17	0.1356	**29.54**	156.0106	0.0515	**26.13**	**34.18**
62	234.01	1061	0.74%	37.64	0.1378	**32.45**	174.8779	0.0527	**28.42**	**37.66**
63	228.02	1196	0.93%	38.97	0.1400	**33.51**	196.4910	0.0540	**28.35**	**38.99**
64	220.83	1258	1.16%	39.86	0.1422	**34.19**	219.9171	0.0554	**27.68**	**39.88**
65	215.61	1411	1.46%	41.43	0.1444	**35.45**	244.6622	0.0568	**27.53**	**41.45**
66	214.24	1363	1.46%	43.03	0.1467	**36.72**	271.0949	0.0584	**27.21**	**43.06**
67	213.63	1563	1.46%	44.60	0.1489	**37.96**	299.6439	0.0603	**26.53**	**44.63**
68	204.58	1698	1.46%	47.90	0.1511	**40.66**	329.1736	0.0629	**27.19**	**47.93**
69	199.28	1771	1.46%	51.90	0.1533	**43.94**	360.2085	0.0659	**28.17**	**51.93**
70	193.52	1802	1.46%	56.62	0.1566	**47.75**	392.9175	0.0690	**29.49**	**56.66**
71	196.09	1903	1.63%	61.32	0.1578	**51.64**	431.1650	0.0722	**30.19**	**61.36**
72	195.56	2196	1.83%	68.27	0.1600	**57.35**	474.5692	0.0754	**32.47**	**68.31**
73	195.27	2377	2.05%	76.89	0.1622	**64.42**	521.7056	0.0787	**35.82**	**76.94**
74	123.55	1802	2.30%	83.94	0.1644	**70.14**	569.7189	0.0821	**37.19**	**83.99**

Table 4.6: Critical Illness Incidence Rates-Stroke(females)

Age	Population *(1000's)*	Number of Strokes	Prevalence Rate Adjustment	Smoothed Adjusted Crude Rate (i_x)	28 Day Mortality Rate *(ELT 15)* (q^i_x)	Stand Alone Rate $i_x(1-q^i_x)$	LT 15 Population q_x *(per 10,000)*	Proportions of Deaths from Stroke (k_x)	Extra cost for Accelerated $(i_x-k_x*q_x)$	Accelerated Rate $i_x+q_x(1-k_x)$
20	316.68	33	0.02%	1.02	0.0444	**0.97**	3.1327	0.0228	**0.95**	**1.02**
21	337.82	46	0.02%	1.33	0.0467	**1.27**	3.2347	0.0266	**1.06**	**1.33**
22	358.72	42	0.02%	1.15	0.0489	**1.09**	3.2358	0.0295	**1.10**	**1.15**
23	355.66	53	0.02%	1.46	0.0511	**1.39**	3.3380	0.0305	**1.16**	**1.46**
24	371.38	39	0.02%	1.03	0.0533	**0.98**	3.2379	0.0306	**1.23**	**1.03**
25	375.35	52	0.02%	1.36	0.0556	**1.28**	2.4414	0.0302	**1.38**	**1.36**
26	387.35	65	0.02%	1.64	0.0578	**1.55**	3.4426	0.0303	**1.54**	**1.64**
27	395.73	79	0.02%	1.95	0.0600	**1.83**	3.5451	0.0314	**1.58**	**1.95**
28	402.86	93	0.02%	2.26	0.0622	**2.12**	3.8503	0.0344	**1.80**	**2.26**
29	401.82	51	0.02%	1.24	0.0644	**1.16**	3.9531	0.0384	**1.82**	**1.24**
30	394.34	103	0.02%	2.56	0.0667	**2.39**	4.3603	0.0429	**1.81**	**2.56**
31	385.16	72	0.02%	1.83	0.0689	**1.70**	4.6665	0.0471	**1.76**	**1.83**
32	374.28	81	0.02%	2.12	0.0711	**1.97**	5.1762	0.0508	**2.01**	**2.12**
33	347.70	78	0.02%	2.14	0.0733	**1.98**	5.7882	0.0534	**1.98**	**2.14**
34	351.75	98	0.02%	2.73	0.0756	**2.52**	6.1979	0.0554	**2.23**	**2.73**
35	344.10	93	0.02%	2.65	0.0778	**2.44**	6.9134	0.0572	**2.48**	**2.65**
36	333.81	110	0.02%	3.23	0.0800	**2.97**	7.5287	0.0586	**2.84**	**3.23**

37	321.73	120	0.02%	3.65	0.0822	**3.35**	8.2470	0.0594	**3.08**	**3.65**
38	312.23	132	0.02%	4.14	0.0844	**3.79**	8.9671	0.0589	**3.42**	**4.14**
39	318.12	136	0.02%	4.19	0.0867	**3.83**	9.7911	0.0577	**3.75**	**4.19**
						0.00				
40	316.65	147	0.02%	4.55	0.0889	**4.15**	10.7195	0.0562	**4.13**	**4.55**
41	311.87	161	0.03%	5.06	0.0911	**4.60**	11.6508	0.0549	**4.54**	**5.06**
42	316.82	185	0.05%	5.72	0.0933	**5.19**	12.8923	0.0539	**4.70**	**5.72**
43	327.42	213	0.09%	6.37	0.0956	**5.76**	14.1383	0.0535	**4.64**	**6.37**
44	338.39	183	0.15%	5.30	0.0978	**4.78**	15.7999	0.0535	**4.75**	**5.30**
45	362.00	208	0.25%	4.51	0.1000	**4.06**	17.7773	0.0536	**4.94**	**4.51**
46	393.71	304	0.25%	6.06	0.1022	**5.44**	19.7649	0.0537	**5.40**	**6.06**
47	306.96	283	0.25%	7.24	0.1044	**6.48**	21.8669	0.0538	**5.71**	**7.24**
48	303.03	354	0.25%	9.17	0.1067	**8.19**	24.1890	0.0538	**6.19**	**9.17**
49	304.01	289	0.25%	7.46	0.1089	**6.65**	26.6310	0.0539	**6.85**	**7.46**
50	287.83	275	0.25%	7.50	0.1111	**6.67**	29.4035	0.0539	**7.42**	**7.50**
51	262.71	336	0.25%	10.04	0.1133	**8.90**	32.5122	0.0540	**7.17**	**10.04**
52	242.40	335	0.25%	10.85	0.1156	**9.60**	35.7546	0.0544	**8.01**	**10.85**
53	256.64	287	0.25%	8.78	0.1178	**7.75**	399.0305	0.0552	**8.95**	**8.82**
54	259.76	416	0.25%	12.57	0.1200	**11.06**	42.7648	0.0562	**9.36**	**12.57**
						0.00				
55	257.23	434	0.25%	13.25	0.1222	**11.63**	47.6029	0.0573	**9.69**	**13.25**
56	251.32	427	0.25%	13.34	0.1244	**11.68**	53.0388	0.0585	**10.58**	**13.34**
57	247.67	447	0.25%	14.17	0.1267	**12.37**	59.1988	0.0598	**11.23**	**14.18**
58	241.58	464	0.25%	15.08	0.1289	**13.14**	66.0009	0.0613	**11.78**	**15.09**
59	234.65	539	0.25%	18.03	0.1311	**15.67**	73.9058	0.0629	**13.03**	**18.04**

60	235.06	554	0.25%	18.50	0.1333	**16.03**	82.9591	0.0645	**13.88**	**18.51**
61	240.45	692	0.33%	22.61	0.1356	**19.54**	92.2272	0.0661	**15.09**	**22.62**
62	247.05	688	0.42%	21.90	0.1378	**18.88**	101.5178	0.0678	**16.57**	**21.91**
63	246.93	781	0.55%	24.91	0.1400	**21.42**	112.8709	0.0697	**16.64**	**24.92**
64	240.99	897	0.72%	29.36	0.1422	**25.19**	126.6296	0.0716	**16.16**	**29.37**
65	237.03	949	0.94%	23.74	0.1444	**20.31**	139.8505	0.0735	**16.16**	**23.75**
66	238.22	1053	0.94%	26.21	0.1467	**22.36**	152.3144	0.0758	**15.92**	**26.22**
67	239.70	1132	0.94%	28.00	0.1489	**23.83**	167.5594	0.0786	**14.98**	**28.02**
68	237.43	1202	0.94%	30.02	0.1511	**25.48**	184.3656	0.0827	**15.67**	**30.04**
69	237.42	1314	0.94%	32.82	0.1533	**27.79**	201.7962	0.0874	**16.31**	**32.84**
70	237.78	1507	0.94%	37.58	0.1556	**31.73**	218.9571	0.0924	**17.04**	**37.60**
71	244.70	1704	1.10%	41.36	0.1578	**34.83**	239.8394	0.0975	**18.14**	**41.38**
72	253.39	1900	1.30%	44.62	0.1600	**37.48**	269.3109	0.1028	**19.68**	**44.64**
73	259.69	2232	1.52%	51.26	0.1622	**42.95**	301.3986	0.1085	**19.66**	**51.29**
74	171.14	1776	1.79%	62.06	0.1644	**51.86**	328.3922	0.1144	**18.97**	**62.09**

DISCUSSION FOR THE CASES OF STROKE RECORDED

The definition of the diseases and the data sources used is stated above. Generally incidence of stroke between males and females keeps alternating with regards to the cases recorded, which in one is more than the other. The stand-alone rates are computed using the figures obtained in column five and column eight. The accelerated rates are computed by using the elements in column five, seven and eight from the data. The stand-alone rate and its corresponding accelerated rates computed from the table are what is used to determine the office premium and its corresponding chargeable premium to the insured. Like with other discussions, suppose a male individual currently at age 45, wants to buy a critical illness product of Stroke, with a benefit of 1500 to be enjoyed at the end of one year. Then, the office premium to be paid by this client for a stand-alone product is given by $\frac{5.97}{1000}\times 1500 = 8.955$. Suppose that this individual is to pay 25 in loadings, then the chargeable premium is $8.955 + 25 = 33.96$. This is the amount of money to be paid to in order to enjoy the benefit of 1500. For a female also at the same age, the office premium is given by $\frac{4.06}{1000}\times 1500 = 6.09$. With a loading of also 25, the chargeable premium is given by $6.09 + 25 = 31.09..$ Premiums for other ages can be computed in the same way and basically all other quantities increases as the age also increases.

CHAPTER 5

DISCUSSION, CONCLUSION AND RECOMMENDATIONS

5.1 DISCUSSION

From the data presented in chapter 4, it can be seen that the issues of diseases increase as an individual's age increases. It also means, in general that, the rate at which one can suffer a condition of critical illness is directly proportion to his age. Premium rates also increase with age, which means that the higher one's age the higher one pays for a Critical Illness insurance. This is due to a higher risk of suffering at old age. The actual premiums can be calculated by multiplying the net premium rate calculated in chapter four by the amount of benefit to be paid. For example, suppose that a male individual currently at age thirty want to buy a critical illness product for cancer with a unit sum assured for a 1-year term stand -alone product, then the actual premium can be determined by multiplying $\frac{4.14}{1000}$ or 0.00414 by 1. Other benefits to be enjoyed can be found by multiplying the benefit by the rates for the year concerned. It must be emphasised that this represents the Net (Pure) premium for that year. That is the premium required just to meet claims in respect of those who die during the year. Adjustments or Loadings will have to be made to arrive at the actual premium chargeable. The major loading is to cover the expenses of the life office. These would include:

Salaries of employees, commission paid to the sellers of policies, costs of office buildings used, computer administration and regulatory costs and medical fees during

underwriting. It is here that chargeable premiums vary slightly from insurer to insurer. Suppose that in our example, 0.05 are to be pain for loading expenses, then the chargeable premium to this client will be 0.05414 for a unit sum assured.

Premiums are often calculated on a yearly basis, although in practice most premiums are pain monthly. The monthly premium cannot just be one twelfth of the yearly premium as this will upset the calculations, which assume that the whole premium will be available for investment at the start of the year. Thus if premiums are to be paid more frequently, the life office will impose a frequency loading. i.e. charge slightly more in total for not having all the premium in one go. An example would be a 4% loading, so that if the annual premium was Gh¢300, the monthly premium would be Gh¢26, i.e. $\frac{300}{12} \times 1.04 = 26$.

This represents the official amount of money to be paid at the start of the policy. Generally, almost all the other quantities from the data in chapter four as mentioned earlier, increases as the age also increases and this makes it imperative that it is better to buy a Critical Illness policy at an earlier age than a later age.

5.2 CONCLUSION

Critical Illness insurance is a relatively new product in the health insurance market. The first policies preyed on people's fears of being struck by certain terminal diseases, such as cancer. The early take up rates of CI policies were mixed internationally. Since the early days of the product's existence, the insurance industry has redesigned and

remarketed CI policies to much greater effect. CI is now a significant product in most well-established health insurance markets. The on-going issues that need to be considered are those of policy design. Medical advances in the diagnosis and treatment of conditions result in the need to review the policy documents on a regular basis. In particular, the definitions of the conditions covered are crucial. Failure to keep pace with medical science can have a significantly detrimental effect on the profitability on this line of business. Other countries could benefit from following the approach to definitions that has been adopted in this project. For Heat Attack, the extra cost for accelerated started decreasing from age 61 which was 38.96 to 18.76 at age 74 contrary to all the other critical illnesses. It can be observed that the accelerated rates are greater than the stand alone rates. As the age of a person increases, the differences also between the two products rate also increases.

5.3 RECOMMENDATIONS

The following recommendations have been made for the consideration of the various insurance companies in the country and to the general public:

1. For the insurance companies, it is important they keep pace with advances in medicine, start and develop a way of selling (marketing) the product to their clients and also used the standardised policy wordings so that that they do not run at a lost.
2. To the general public, it is very important that we buy this insurance product because as we do not know on most occasion when we will contract a particular

disease, I recommend that individual from the age thirty-five to sixty-five specifically have no option but to buy this product.

3. Further Studies can be carried out on the experience rates in claims among other diseases specified in the policy wording, the application to the model developed on the premiums and reserves and also Critical Illness marketing can also be further studied.
4. I finally recommend that a further research will be carried out (at a later date) to study the transition probabilities from each of the three states and the Net premium rate for Stand alone and Accelerated Rider Benefits.

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APPENDICES

SAMPLE TABLES