

MODELLING THE DISTRIBUTION OF BANKNOTES BY BANK OF GHANA AS A TRANSSHIPMENT PROBLEM

By

KNUST
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A Thesis submitted to the Department of Mathematics Institute of
Distance Learning Kwame Nkrumah University of Science and
Technology in partial fulfillment of the requirement for the degree of
Master of Science
Industrial Mathematics

November, 2012

DECLARATION

I hereby declare that this submission is my own work towards the Master of Science degree and that, to the best of my knowledge it contains no material previously published by another person nor material which has been accepted for award of any other degree of the University except where due acknowledgement has been made in the text.

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ABSTRACT

The transportation problem is a special class of the linear programming problem. It deals with the situation in which a commodity is transported from Sources to Destinations. The transshipment problem is an expansion of the transportation problem where intermediate nodes which are also referred to as transshipment nodes are added to account for locations such as warehouses. My main objective is to model Bank of Ghana's transportation of Banknotes as a transshipment problem and also minimize the cost in transporting them. I will formulate the Transshipment problem as a Transportation problem and use the Transportation algorithm to solve it. The Quantitative Method (QM) Software will be used to analyze the data. I conclude that if Bank of Ghana adapts this method, the cost of transporting Banknotes will be minimized.

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DEDICATION

I dedicate this work to my family (ie my wife Mrs. Esther Ofori, my daughter Caryn Fiankobe Ofori and my son Melvin Paa Kwesi Ofori)

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ACKNOWLEDGEMENT

EBENEZER!!! This is how far the Lord has brought us.

First, I would like to thank the Omnipotent God for making it possible for me to be able to finish this work.

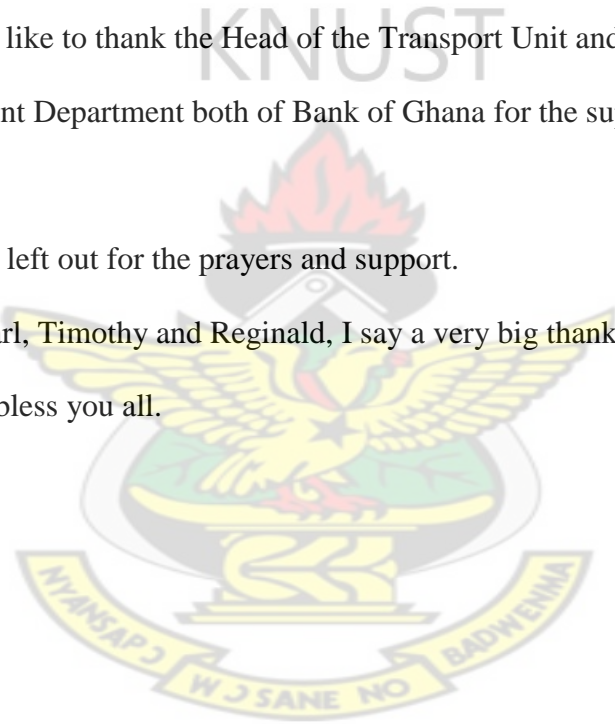
I would also want to thank my supervisor Mr. Kweku Darkwah for the support, guidance, encouragement, and patience given me throughout this work. I am so grateful.

Furthermore I would like to thank the Head of the Transport Unit and the Head of the Currency Management Department both of Bank of Ghana for the support during the collection of Data

My family cannot be left out for the prayers and support.

To my colleagues Carl, Timothy and Reginald, I say a very big thank you.

May the Good Lord bless you all.



CHAPTER 1

INTRODUCTION

Consider a distribution system consisting of multiple retail locations. Demands occur at each retail location, which replenishes its inventory from some central warehouse on a periodic basis. Demands at each retail location are first met from the available inventory at the location. When shortage occurs at one location, the shortage can be covered from available inventory at other retail locations through possible lateral transshipment. The objective is to determine the optimal order quantity of each retail location and the resulting optimal transshipment policy after demands are realized at each period so as to minimize the total expected replenishment costs, inventory holding costs, shortage costs and the transshipment costs among the multiple retail locations during some finite time horizon. Transshipment, when possible, can be used as one effective way to reduce total inventory and increase service level in a distribution system. Essentially, transshipment allows the distribution system to take advantage of the risk pooling effect to deal with uncertain demands at different retail locations. Excess inventory at one retail location can be used to cover shortage at another location. Physically, one can interpret inventory stocking at each individual location as being “pooled” together to meet the demands at any other location within the distribution system. As such, the use of transshipment provides more flexibility in deploying the available inventory in the system to meet uncertain customer demand. Consequently, transshipment can help to reduce the total system inventory and stock-out level at each individual location, at the expense of a higher transportation cost for transshipping the products among the different retail

locations. It is interesting yet unclear as to what kind of system configurations and retailer characteristics would benefit most from using transshipment. One objective of this paper is to address a number of managerial issues regarding the impact of transshipment on reducing the costs of the distribution system under different system configurations and retailer characteristics. Our distribution system with transshipment involves a convoluted decision problem consisting of two basic types of decisions that influence each other throughout the finite time zone. The first type involves the decision for the optimal order quantity at the retail locations during each replenishment cycle. We refer to this decision as the optimal replenishment policy. The second type involves the decision for transshipping the products among the retail locations after demands are observed and shortages occur at different retail locations. We refer to this decision as the optimal transshipment policy. The combined optimal replenishment problem with transshipment and stochastic demand is generally difficult to solve. The problem is complicated even for the single period model consisting of a two-stage decision problem, where the transshipment decisions are considered as a recourse action to cover shortage after the replenishment quantity has been selected and uncertain demands have been realized. For a finite time horizon, the optimal replenishment policy generally depends on the replenishment and transshipment decisions as well as realized demands in earlier periods. On the other hand, the optimal transshipment policy, which entails the decision of how much as well as from which location the transshipments should come from, also depends on the replenishment policy and realized demands in earlier periods.

This thesis presents a mathematical modeling of distribution of banknotes to branches and agencies of bank of Ghana as a transshipment problem, which can provide useful information to aid decision-makers in their supply chain decision making.

In this chapter of the thesis, a historical background of Bank of Ghana would be given; a brief description of the problem statement of the thesis is also presented together with the objectives, the methodology, the justification and the organization of the thesis.

1.1 BACKGROUND OF STUDY

A central bank, reserve bank, or monetary authority is a public institution that manages the nation's currency, money supply and interest rates. Central banks also usually oversee the commercial banking system of their respective countries. In contrast to commercial banks fractional-reserve requirement, a central bank possesses a monopoly on creating limitless credit, and sometimes also on creating the physical national currency, such as notes and coins, which usually serves as the nation's tender. The primary function of a central bank is to manage the nation's money supply (monetary policy), through active duties such as managing interest rates, setting the reserve requirement, and acting as a lender of last resort to the banking sector during times of bank insolvency or financial crisis. Central banks usually also have supervisory powers, intended to prevent commercial banks and other financial institutions from reckless or fraudulent behavior. Central banks in most developed nations are institutionally designed to be independent from political interference.

Examples include the European Central Bank (ECB), the Federal Reserve of the United States, the People's Bank of China and the Bank of Ghana.

The Central Bank of Ghana traces its roots to the Bank of the Gold Coast (BGC), where it was nurtured. As soon as local politicians and economists saw political independence in sight in the mid 1950's the agitation for a central bank was revived. It was argued that a central bank was one institution which would give true meaning to political independence. It may be recalled that way back in 1947 some leading politicians had called for the establishment of a national bank with central bank functions to act as banker to government and to cater for the indigenous sector of the economy. Proposals of the advocates for a central bank were accepted and in early 1955 another Select Committee was set up by the Government to take a new look at the Trevor Report and prepare the grounds for the establishment of a central bank in Ghana. Fortunately, the BGC had already set the stage for central banking: all that was needed was specially trained personnel in central banking and suitable accommodation for the bank to take off. By the end of 1956, all was set for the establishment of the Bank of Ghana. A new and modern five-storey building had been put up on the High Street, adjacent to the Accra Metropolitan Assembly (AMA) to house both the Bank of Ghana and the Ghana Commercial Bank (GCB). On the 4th March 1957, just two days before the declaration of political independence, the Bank of Ghana was formally established by the Bank of Ghana Ordinance (No. 34) of 1957, passed by the British Parliament. Frantic preparations then began to put in place an organisational structure for the new central bank. By the middle of July 1957, all was set for the official commissioning of the new Head Office of the Bank on the High Street. In his opening address at the end of July 1957, the then

Leader of Government Business (Prime Minister) stated with pleasure that the occasion marked the beginning of independent monetary administration in the newly independent Ghana – a cherished dream had at long last become a reality. The Leader of Government Business had put the aspiration of the country in establishing the central bank as follows: “In the modern world a central bank plays a very important and decisive role in the life of a country. It is essential to our own independence that we have a government-owned bank and that the central bank follows a policy designed to secure our economic independence and to further the general development of our country.”

The principal objects of the new central bank, as enshrined in the 1957 Ordinance, were “to issue and redeem Banknotes and coins; to keep and use reserves and to influence the credit situation with a view to maintaining monetary stability in Ghana and the external value of the Ghana pound; and to act as banker and financial adviser to the Government. The opening ceremony paved the way for the Bank to commence formal banking operations on 1st August 1957, when the Banking Department opened for business. The Issue Department did not commence operations until July 1958. The Bank of Ghana has since 1957 undergone various legislative changes. The Bank of Ghana Ordinance (No.34) of 1957 was repealed by the Bank of Ghana Act (1963), Act 182. This Act was subsequently amended by the Bank of Ghana (Amendment Act) 1965, (Act 282). The Bank of Ghana Law, 1992 PNDCL 291 repealed Acts 182 and 282. The current law under which the Bank operates is the Bank of Ghana Act 2002 (Act 612). (www.bog.gov.gh).

1.1.2 BRANCHES

Bank of Ghana, as a Public Institution which serves the whole nation, is suppose to have branches at some specific places in Ghana to be able to serve the Commercial Banks and the Public at large. In view of this the Bank of Ghana has six main branches (Regional Offices). These offices are Kumasi, Sunyani, Takoradi, Hohoe, Tamale, and Sefwi Buako. However Sefwi Buako is referred to as a Currency Office, which only deals with the distribution of money to the commercial Banks in its environs due to the high production of Cocoa. There is a main branch in Accra which is the main head office where the Governor and his two deputies together with all the Head of Departments operates. Of all these branches, Bank of Ghana has two main branches (Accra and Kumasi) which are the two main sources of Banknotes distribution. These two offices are the only branches that have Currency Processing Machines to process Banknotes received from the Commercial Banks for authentication. The other Regional Offices serves as branches for the distribution of Banknotes, because Banknotes are often sent to them from Accra or Kumasi for onward distribution. After authenticating the Banknotes, they are then sent to other Regional Offices, Currency Office or Bank of Ghana Agencies.

1.1.3 AGENCIES

For equal distribution of Banknotes throughout Ghana, Bank of Ghana in an agreement with Ghana Commercial Bank has Agencies within the premises of Ghana Commercial Bank. These branches of Ghana Commercial Banks include the following; Tema, Swedru, Nkawkaw, Cape coast, Koforidua, Wa, and Dunkwa-on-Offin. Staffs who work

at these Agencies are Ghana Commercial Bank Staff who operates on behalf of Bank of Ghana. They receive Deposits from all Commercial Banks in their catchment area.

They also Issue Cash to the Commercial Banks on request. In short, they operate just like Staff of Bank of Ghana Vaults. Daily transactions are being sent to Bank of Ghana who has a Unit in the Issue Department specifically for that, both Soft copies (on-line) and hard copies (through curia service).

1.1.4 MONEY

Money is any object or record that is generally accepted as payment for goods and services and repayment of debts in a given country or socio-economic context. The main functions of money are distinguished as: a medium of exchange; a unit of account; a store of value; and, occasionally in the past, a standard of deferred payment. Any kind of object or secure verifiable record that fulfills these functions can serve as money.

Money originated as commodity money, but nearly all contemporary money systems are based on fiat money. Fiat money is without intrinsic use value as a physical commodity, and derives its value by being declared by a government to be legal tender; that is, it must be accepted as a form of payment within the boundaries of the country, for "all debts, public and private".

The money supply of a country consists of currency (banknotes and coins) and bank money (the balance held in checking accounts and savings accounts). Bank money

usually forms by far the largest part of the money supply (<http://en.wikipedia.org/wiki/Money>).

1.1.5 BANKNOTES

A banknote (often known as a bill, paper money or simply a note) is a kind of negotiable instrument, a promissory note made by a bank payable to the bearer on demand, used as money, and in many jurisdictions is legal tender. In addition to coins, banknotes make up the cash or bearer forms of all modern fiat money. With the exception of non-circulating high-value or precious metal commemorative issues, coins are used for lower valued monetary units, while banknotes are used for higher values.

The banknote was first developed in China during the Tang and Song dynasties, starting in the 7th century. Its roots were in merchant receipts of deposit during the Tang Dynasty (618–907), as merchants and wholesalers desired to avoid the heavy bulk of copper coinage in large commercial transactions. During the Yuan Dynasty, banknotes were adopted by the Mongol Empire. In Europe, the concept of banknotes was first introduced during the 14th century, with proper banknotes appearing in the 17th century (<http://en.wikipedia.org/wiki/Banknote>). Presently Banknotes issued in Ghana, are of the denomination 1,2,5,10,20 and 50 cedis.

1.1.6 COINS

A coin is a piece of hard material that is standardized in weight, is produced in large quantities in order to facilitate trade, and primarily can be used as a legal tender token for commerce in the designated country, region, or territory. Coins are usually metal or a

metallic material and sometimes made of synthetic materials, usually in the shape of a disc, and most often issued by a government. Coins are used as a form of money in transactions of various kinds, from the everyday circulation coins to the storage of large numbers of bullion coins. In the present day, coins and banknotes make up currency, the cash forms of all modern money systems. Coins made for paying bills and general monetized use are usually used for lower-valued units, and banknotes for the higher values; also, in many money systems, the highest value coin made for circulation is worth less than the lowest-value note. In the last hundred years, the face value of circulation coins has usually been higher than the gross value of the metal used in making them; exceptions occurring when inflation causes the metal value to surpass the face value, causing the minting authority to change the composition and the old coins to begin to disappear from circulation (see Gresham's Law.) However, this has generally not been the case throughout the rest of history for circulation coins made of precious metals.

Exceptions to the rule of coin face-value being higher than content value, also occur for some bullion coins made of silver or gold (and, rarely, other metals, such as platinum or palladium), intended for collectors or investors in precious metals. Examples of modern gold collector/investor coins include the American Gold Eagle minted by the United States, the Canadian Gold Maple Leaf minted by Canada, and the Krugerrand, minted by South Africa. The American Gold Eagle has a face value of US\$50, and the Canadian Gold Maple Leaf coins also have nominal (purely symbolic) face values but the Krugerrand does not. (<http://en.wikipedia.org/wiki/Coin>)

Historically, a great number of coinage metals (including alloys) and other materials have been used practically, artistically, and experimentally in the production of coins for circulation, collection, and metal investment, where bullion coins often serve as more convenient stores of assured metal quantity and purity than other bullion.

Coins have long been linked to the concept of money, as reflected by the fact that in some other languages the words "coin" and "currency" are synonymous. Fictional currencies may also bear the name coin as such, an item may be said to be worth 123 coin or 123 coins (<http://en.wikipedia.org/wiki/Coin>). Presently Coins issued in Ghana are of the denomination 1, 5, 10, 20, 50 pesewas and 1 cedi.

1.1.7 THE CEDI

Prior to independence, the issue of currency was the responsibility of the West African Currency Board (WACB). The West African pounds shillings and pence, constituted currency issued by the Board and was in circulation in Ghana until July 1958.

After Independence, the new monetary authority, the Bank of Ghana, issued its own currency in the form of Ghana pounds, shillings and pence on 14th July, 1958. With that issue, the Bank of Ghana formally took over the issue of currency notes and coins from the WACB. (www.bog.gov.gh).

The second issue of currency was in early 1965, when Ghana decided to leave the British colonial monetary system and adopt the widely accepted decimal system. Accordingly,

Cedi notes and Pesewa coins were introduced on the 19th July, 1965 to replace the Ghana pounds, shillings and pence. The cedi was equivalent to eight shillings and four pence (8s 4d) and bore the portrait of the then President, Dr. Kwame Nkrumah. The name “cedi” was derived from the word “sedie” meaning cowrie, shell money which gained popularity and wider circulation in the later part of the 19th Century. The “Pesewa” represented the smallest denomination (quantity) of the gold-dust currency regime. The name was chosen to replace the British Colonial penny. (www.bog.gov.gh).

After the overthrow of the CPP government, the military government decided to replace the existing currency, which bore Nkrumah’s portrait, with one without his portrait. The New Cedi (N¢), as it was called, was introduced on 17th February, 1967 to replace the 1965 cedi at a rate of ¢ 1.20=N¢ 1.00. The N¢ notes remained in circulation until March 1973 when it became simply known as the cedi. (www.bog.gov.gh).

On 9th March, 1979, the Government announced the introduction of new cedi notes to replace the old ones at a discount of 30% for amounts up to ¢5,000 and 50% for amounts in excess of ¢5,000. The old cedis were therefore, demonetized. New denominations issued included ¢1, ¢2, ¢5, ¢10, ¢20 and ¢50. (www.bog.gov.gh).

From 1965 to present, various cedi and pesewa denominations, ranging from ¢1 to ¢5,000 for notes and ½ P to ¢500 for coins, were put into circulation. Currency issued in 1965 comprised ¢1, ¢5, ¢10, ¢50, ¢100, ¢1,000, 5P, 10P, and 20P. Between 1972 and 1994, additional seven different note denominations and eight coin denominations were introduced. These ranged between ¢2 to ¢5,000 for notes and ¢100p to 50,000p (¢500) for coins.

Since 2002, two more notes ¢10000 and ¢20000 have been added to notes in circulation.

(Statistics and Reports Office Issue Department) (www.bog.gov.gh).

In 2007, Bank of Ghana embarked on a re-denominations exercise by eliminating four zeros at the end of every amount (ie ₵10,000 became GH₵1.00). (www.bog.gov.gh).

1.2 PROBLEM STATEMENT

Bank of Ghana for evenly distribution of Banknotes has Branches and Agencies all around the country. It is the Mandate of the Bank to make money available no matter which part of the country one may be. Ghana is a country which has about 80% of its population that uses Banknotes as its main mode of payment. Bank of Ghana sometimes experiences shortages of Banknotes at some Regional Offices and Agencies although Banknotes may be available at its two main sources. Transshipments, the monitored movement of material between locations at the same echelon, provide an effective mechanism for correcting discrepancies between the locations' observed demand and their available inventory. As a result, transshipments lead to cost reductions and improved service without necessarily increasing system-wide inventories.

1.3 OBJECTIVES

This research project proposes to;

1. Model the distribution of Banknotes by Bank of Ghana as a transshipment problem.
2. To find the Optimal Transshipment module by a Transshipment Algorithm.

1.4 METHODOLOGY

Our proposed methodology to our problem would be solved by using the transshipment model with intermediate destinations between the sources and the destinations. The transshipment problem will be converted to a transportation problem and the transportation algorithm will be used to solve it. A Data from Bank of Ghana which is a secondary data for one year period (2011) would be analyzed and the Quantitative Methods (Q. M.) for windows software will be used.

1.5 JUSTIFICATION

The relevance of this research was to come out with a model for the distribution of Banknotes to all Branches and Agencies at an optimal cost and to make Banknotes readily available at all Bank of Ghana Branches and Agencies.

This will help to reduce cost of transshipment of Banknotes from Accra and Kumasi to the various Regional Branches and Agencies as there is no schedule of transporting money until a request is being made by the Branch or Agency in need of money. It will also help to reduce the transportation problems facing Bank of Ghana in distributing Banknotes to the various destinations.

1.6 ORGANIZATION OF THE THESIS

Chapter one presents a historical background study of Bank of Ghana.

In chapter two, related work in the transshipment problem will be discussed.

In chapter three, the transshipment and transportations algorithms by Amponsah and Darkwah (2009) will be introduced and explained.

Chapter four will provide a computational study of the algorithm applied to our transshipment problem for Bank of Ghana.

Chapter five will conclude this thesis with comments and recommendations



CHAPTER 2

LITERATURE REVIEW

The literature on distribution problems has grown enormously during the past years, and there is a clear need to develop a classification scheme for all their variants. The need for such a scheme exists because there are many connections and dependencies among the many variants of these problems and it is conceivable of such a scheme (Psaraftis, 2007).

Lin et al., (2003) addressed a limited form of the two-stage lightering practice for large tankers, first stage at an offshore location farther from the refinery and the second stage at the lightering location closer to the refinery using an event-based approach. The authors assumed single-compartment vessels, did not restrict the number of simultaneous services for a single tanker, did not allow pickups from more than two tankers within one voyage of a lightering vessel, ignored differences in crude densities, and did not allow the freedom to select lightering crudes. In their paper, the authors developed a new continuous-time MILP formulation that addresses all of the above drawbacks. Thus, the authors allowed multi-compartment lightering vessels, restricted the number of simultaneous transfers to two, allowed more than two pickups in one voyage for any lightering vessel, considered the impact of varying crude densities, selected optimally the right lightering crudes, and most importantly used a realistic cost-based scheduling objective. The authors MILP model generated optimal lightering schedule with lightering volumes, sequence, times, and assignments, which minimized the operating costs of lightering vessels, the demurrage costs of tankers as well as the delivery times of crude oil from the lightering location to refinery ports.

Krishnan and Rao (1965) studied a reactive mode of transshipment. The authors considered a model that was similar to the periodic lateral transshipment model which has negligible transshipment times, but aim to minimize cost through transshipments once all demand is known. The author's model provided an optimal solution for a multi-location, multiperiod model. However, this solution can only be determined for networks with either two nonidentical locations or any number of identical locations. For more than two non-identical locations, a LP based heuristic solution procedure is proposed and shown to perform well for a number of scenarios.

In a system with two echelons there are several ways in which stockouts can be satisfied through emergency stock movements. Lateral transshipments are one possibility but there could be situations where it is beneficial also to perform emergency shipments from the central warehouse. Wee and Dada (2005) considered this problem with five different combinations of transshipments, emergency shipments and no movements at all and devises a method for deciding which setup is optimal under a given model description. The author's research allows the structure of the emergency stock movements to be established.

Dong and Rudi (2004) examined a different aspect by looking at the benefits of lateral transshipments for a manufacturer that supplies a number of retailers. The authors compared the case where the manufacturer is the price leader to the case of exogenous prices. For exogenous prices, it was found that retailers benefited more when demand across the network was uncorrelated. For the endogenous price case, modeled as a

Stackelberg game, the manufacturer exploits his leadership to increase his benefits, leaving retailers worse off if they use transshipments. These results were restricted to demand that follows a normal distribution.

In a more retail case study based approach, Bendoly (2004) studied a model with internet and store based customers. The authors utilized lateral transshipment ideas to show how partial pooling of goods can improve a system's performance. The examined model considered a modern retail environment where stores are operated alongside internet channels and is an example of the practical uses of lateral transshipments.

Cross docking is a logistic technique which seeks to reduce costs related to inventory holding, order picking, transportation as well as the delivery time. Most of the existing studies in the area are interested in the dock assignment problem and the design of the cross dock transportation networks. Little attention has been given to the transshipment operations inside a cross docking platform. Larbi et al. (2003) studied the transshipment scheduling problem in a simple cross dock with a single strip door and single stack door. The authors proposed a graph based model for the problem. The shortest path in the graph gives the schedule which minimizes the total cost of transshipment operations.

Deniz et al. (2009) studied transshipment problem of a company in the apparel industry with multiple Sub-contractors and customers, and a transshipment depot in between. Unlike a typical transshipment problem that considers only the total cost of transportation, the authors model also considered the supplier lead times and the customer

due dates in the system and can be used for both supplier selection and timely distribution planning. The authors proposed their model can also be adapted easily by other companies in the industry.

Tagaras (1989) used the fill rate and the probability of no-stock out to reflect the level of service. For an identical demand structure, balancing the fill rate is equivalent to starting with identical beginning inventory at each location. In the authors study, while analyzing the effect of risk pooling in a setting with one central warehouse and three stocking locations, the author compared random allocation with a 'risk balancing' transshipment policy. In risk balancing, transshipment quantities are determined so as to equalize the probability of a stockout in the following period, and for an identical demand structure, risk of stockout will be balanced if each location starts with the same inventory.

Kut (2006) studied a distribution system consisting of multiple retail locations with transshipment operations among the retailers. Due to the difficulty in computing the optimal solution imposed by the transshipment operations and in estimating shortage cost from a practical perspective, the authors proposed a robust optimization framework for analyzing the impact of transshipment operations on such a distribution system. The authors demonstrated that their proposed robust optimization framework is analytically tractable and is computationally efficient for analyzing even large-scale distribution systems. From a numerical study using this robust optimization framework, the authors addressed a number of managerial issues regarding the impact of transshipment on reducing the costs of the distribution system under different system configurations and

retailer characteristics. The authors considered two system configurations, line and circle, and studied how inventory holding cost, transshipment cost, and demand size and variability affect the effectiveness of transshipment operations for the cases of both homogeneous and non-homogeneous retailers. The results obtained from the robust optimization framework helped to evaluate the potential benefits when investing in transshipment operations.

In situations where a seller has surplus stock and another seller is stocked out, it may be desirable to transfer surplus stock from the former to the latter. Krishnan and Rao (1965) studied the transshipment problems with multiple retail locations with identical cost structure, and examined how the possibility of such transshipments between two independent locations affects the optimal inventory orders at each location. If each location aims to maximize its own profits—the authors called this local decision making—their inventory choices will not, in general, maximize joint profits. The authors found transshipment prices which induce the locations to choose inventory levels consistent with joint-profit maximization. The authors showed that the optimal stocking quantities satisfy the equal fractile property.

Tagaras (1989) presented a model which deals with the analysis of two-location periodic review inventory systems with non-negligible replenishment lead times. Emergency transshipments were used in these systems as a recourse action to reduce the occurrence of shortages. A class of partial pooling policies is proposed for the control of transshipments. The cost performance of this class of policies was shown to be inferior to that of complete pooling. An approximate model and a heuristic algorithm were

introduced to compute near-optimal stocking policy solutions. Comparisons with simulation results verified the satisfactory performance of the approximate model and algorithm. Numerical sensitivity analysis provided additional insight into the nature of optimal transshipment behavior. The author's model also allowed for a service constraint on the minimum acceptable fill rates.

Supply chain designs are constrained by the cost-service trade-off. Cost minimization typically leads to physically efficient or lean supply chains at the expense of customer responsiveness or agility. Recently, the concept of leagility has been introduced. Research on leagility, defined as the capability of concurrently deploying the lean and agile paradigms, hinges heavily on the identification of the decoupling point, which, in turn, is enabled by postponement. Postponement strategies, however, present a cross-functional challenge for implementation. As a tactical solution to achieve leagility without postponement, Yale et al.(2002), studied transshipments problem, which represented a common practice in multi-location inventory systems involving monitored movement of stock between locations at the same echelon level of the supply chain. Through a series of models, the authors established how transshipments can be used to enhance both agility and leanness.

Deniz et al. (2006) considered coordination among stocking locations through replenishment strategies that explicitly take into account lateral transshipments, i.e., transfer of a product among locations at the same echelon level. The basic contribution of our research is the incorporation of supply capacity into the traditional emergency transshipment model. The authors formulated the capacitated production case as a

network flow problem embedded in a stochastic optimization problem. The authors developed a solution procedure based on infinitesimal perturbation analysis (IPA) to solve the stochastic optimization problem numerically. The authors analyzed the impact on system behavior and on stocking locations' performance when the supplier may fail to fulfill all the replenishment orders and the unmet demand is lost. The authors found that depending on the production capacity, system behavior can vary drastically. Moreover, in a production-inventory system, the authors found evidence that either capacity flexibility (i.e., extra production) or transshipment flexibility is required to maintain a certain level of service.

Taragas and Cohen (1993) studied two-location transshipment model which allow for positive replenishment lead-times. With positive replenishment lead-times, it might be beneficial to hold back stock for future demands, and so it is not necessarily optimal to always transship from the other location (complete pooling) when shortages occur. However, their numerical results showed that complete pooling generally dominates partial pooling.

Herer and Rashit (1999) studied the two-location transshipment problem to include fixed and joint replenishment costs and the multiperiod case and defined a set of assumptions that lead to "complete pooling." Complete pooling means that if one location has excess stock while another location is short, the number of units transshipped will be the minimum of the excess and the shortage. The authors showed that no transshipments will occur if both locations are short or if both have excess stock, and derived several

properties regarding the structure of the corresponding optimal replenishment and transshipment policies.

Herer and Tzur (2001) investigated the strategy of transshipments in a dynamic deterministic demand environment over a finite planning horizon. The authors considered a system of two locations which replenish their stock from a single supplier, and where transshipments between the locations are possible. The authors model included fixed (possibly joint) and variable replenishment costs, fixed and variable transshipment costs, as well as holding costs for each location and transshipment costs between locations. The problem was to determine how much to replenish and how much to transship each period; thus the study can be viewed as a synthesis of transshipment problems in a static stochastic setting and multilocation dynamic deterministic lot sizing problems. The authors provided interesting structural properties of optimal policies which enhance the understanding of the important issues which motivate transshipments and allowed the development of an efficient polynomial time algorithm for obtaining the optimal strategy. By exploring the reasons for using transshipments, the model enabled practitioners to envision the sources of savings from using this strategy and therefore motivated them to incorporate it into their replenishment strategies. With this model the authors were able to minimize the total replenishment, holding and transshipment costs over a finite horizon.

Dong and Rudi (2004) studied how transshipments affect manufacturers and retailers, considering both exogenous and endogenous wholesale prices. For a distribution system where a single manufacturer sells to multiple identical-cost retailers, the authors

considered both the manufacturer being a price taker and the manufacturer being a price setter in a single-period setup under multivariate normal demand distribution. In the case of the manufacturer being a price taker, the authors provided several analytical results regarding the effects of key parameters on order quantities and profits. In the case of the manufacturer being a price setter, the authors characterized the Stackelberg game that arises, and provided several insights into how the game dynamics are affected by transshipments. Specifically, the authors found that risk pooling makes retailers' order quantities less sensitive to the wholesale price set by the manufacturer; hence, in general, the manufacturer benefits from retailers' transshipments by charging a higher wholesale price, while retailers are often worse off. The author's model captures the effect of demand correlation and the effect of the number of retailers throughout, and it illustrates the findings by a numerical example. The authors also provided an interactive Web page for numerical experiments.

Roberto et al. (2009) considered the stochastic capacitated transshipment problem for freight transportation where an optimal location of the transshipment facilities, which minimizes total cost, must be found. The total cost is given by the sum of the total fixed cost plus the expected minimum total flow cost, when the total throughput costs of the facilities are random variables with unknown probability distribution. By applying the asymptotic approximation method derived from the extreme value theory, a deterministic nonlinear model, which belongs to a wide class of Entropy maximizing models, is then obtained. The computational results showed a very good performance of this deterministic model when compared with stochastic one.

Mangal and Chandna (2007) examined the antecedents of retailer - retailer partnership and to explore its impact on the supply chain performance. The authors considered coordination among stocking locations through replenishment strategies that take explicitly into consideration transshipments, transfer of a product among locations at the same echelon level. A continuous review inventory system was adopted, in which lateral transshipments are allowed. In general, if a demand occurs at a location and there is no stock on hand, the demand is assumed to be backordered or lost. Lateral transshipments serve as an emergency supply in case of stock out and the rule for lateral transshipments is to always transship when there is a shortage at one location and stock on hand at the other. The aim is to explore the role of lateral transshipment to control inventory and associated cost within supply chain and, from this, to develop an exploratory framework that assists understanding in the area. A simple and intuitive model was presented that enables us to characterize optimal inventory and transshipment policies for 'n' locations. The study was based on a case study of a bi-wheeler company in India by using its data and to strengthen its supply chain. The results obtained enabled the managers to overcome the uncertainties of demand and lead-time resulting into customer satisfaction and cost reduction.

Banu and Sunderesh (2005) studied a single-item two-echelon inventory system where the items can be stored in each of N stocking locations is optimized using simulation. The aim of this study was to minimize the total inventory, backorder, and transshipments costs, based on the replenishment and transshipment quantities. In this study,

transshipments which are the transfer of products among locations at the same echelon level and transportation capacities which are the transshipment quantities between stocking locations, were also considered. Here, the transportation capacities among the stocking locations are bounded due to transportation media or the locations' transshipment policy. Assuming stochastic demand, the system is modeled based on different cases of transshipment capacities and costs. To find out the optimum levels of the transshipment quantities among stocking locations and the replenishment quantities, the simulation model of the problem was developed using ARENA 10.0 and then optimized using the Opt Quest tool in this software.

Mabel et al. (2006) developed an analytical framework for studying a two-echelon distribution system consisting of one central warehouse and multiple retail locations with transshipment operations among the retailers. The authors' framework can be used to model very general distribution systems and analyze the impact of transshipment under different system configurations. The authors demonstrated that their proposed analytical framework was analytically tractable and computationally efficient for analyzing even large-scale distribution systems. From a numerical study using the authors framework, they addressed a number of managerial issues regarding the impact of transshipment on reducing the costs of the distribution system under different system configurations and retailer characteristics. The managerial insights obtained from the authors analysis was able to evaluate the potential benefits by investing in transshipment operations.

The transportation problem has offered two mathematical facets: (1) as a specialized type of linear programming problem, (2) as a method of representation of some combinatorial problems. Orden (1956) developed a third aspect of the mathematical properties of the transportation problem. It was shown that the same mathematical framework can be extended beyond pair-wise connections, to the determination of optimum linked paths over a series of points. This extension although viewed here as a linear programming problem, takes advantage of the combinatorial aspect of the transportation problem, and applications may arise which, like the assignment problem, appear to be combinatorial problems, but which can be solved by linear programming.

A dynamic network consists of a graph with capacities and transit times on its edges. The quickest transshipment problem is defined by a dynamic network with several sources and sinks; each source has a specified supply and each sink has a specified demand. The problem is to send exactly the right amount of flow out of each source and into each sink in the minimum overall time. Variations of the quickest transshipment problem have been studied extensively; the special case of the problem with a single sink is commonly used to model building evacuation. Similar dynamic network flow problems have numerous other applications; in some of these, the capacities are small integers and it is important to find integral flows. There are no polynomial-time algorithms known for most of these problems. Hoppe and Tardos (1997) presented the first polynomial-time algorithm for the quickest transshipment problem. The author's algorithm provides an integral optimum flow. Previously, the quickest transshipment problem could only be solved efficiently in the special case of a single source and single sink.

Transshipments, monitored movements of material at the same echelon of a supply chain, represent an effective pooling mechanism. With a single exception, research on transshipments overlooks replenishment lead times. The only approach for two-location inventory systems with non-negligible lead times could not be generalized to a multi-location setting, and the proposed heuristic method cannot guarantee to provide optimal solutions. Gong and Yucesan (2006) studied a model that uses simulation optimization by combining an LP/network flow formulation with infinitesimal perturbation analysis to examine the multi-location transshipment problem with positive replenishment lead times, and demonstrates the computation of the optimal base stock quantities through sample path optimization. From a methodological perspective, the authors deployed an elegant duality-based gradient computation method to improve computational efficiency. In test problems, the author's algorithm was also able to achieve better objective values than an existing algorithm.

Glover et al. (2005) developed a primal simplex procedure to solve transshipment problems with an arbitrary additional constraint. The procedure incorporates efficient methods for pricing-out the basis, determining certain key vector representations, and implementing the change of basis. These methods exploit the near triangularity of the basis in a manner that takes advantage of computational schemes and list structures used to solve the pure transshipment problem. The authors implemented these results in a computer code, I/O PNETS-I. Computational results (necessarily limited) confirm that this code is significantly faster than APEX-III on some large problems. The authors also developed a fast method for determining near optimal integer solutions. Computational

results showed that the near optimum integer solution value was usually within 0.5% of the value of the optimum continuous solution value.

A transshipment problem with demands that exceed network capacity can be solved by sending flow in several waves. How can this be done using the minimum number of iterations? This is the question tackled in the quickest transshipment problem. Hoppe and Tardos (1997) described the only known polynomial time algorithm that finds an integral solution to this problem. The author's algorithm repeatedly minimizes sub-modular functions using the ellipsoid method, and is therefore not at all practical. Fleischer presented an algorithm that finds a fully integral quickest transshipment with a polynomial number of maximum flow computations. When there is only one sink, the quickest transshipment problem is significantly easier. For this case, the authors showed how the algorithm can be sped up to return an integral solution using $O(k)$ maximum flow computations, where k is the number of sources.

Ozdemir et al. (2003) studied a supply chain model, which consists of N retailers and one supplier. The retailers may be coordinated through replenishment strategies and lateral transshipments, that is, movement of a product among the locations at the same echelon level. Transshipment quantities may be limited, however, due to the physical constraints of the transportation media or due to the reluctance of retailers to completely pool their stock with other retailers. The authors introduced a stochastic approximation algorithm to compute the order-up-to quantities using a sample-path-based optimization procedure. Given an order-up-to S policy, the authors determined an optimal transshipment policy, using an LP/network flow framework. Such a numerical approach allows the authors to study systems with arbitrary complexity.

The decentralized transshipment problem is a two-stage decision making problem where the companies first choose their individual production levels in anticipation of random demands and after demand realizations they pool residuals via transshipment. The coordination will be achieved if at optimality all the decision variables, i.e. production levels and transshipment patterns, in the decentralized system are the same as those of centralized system. Hezarkhani and Kubiak (2009) studied a model with the coordination via transshipment prices. The authors proposed a procedure for deriving the transshipment prices based on the coordinating allocation rule introduced by Anupindi et al. (2006). With the transshipment prices being set, the companies are free to match their residuals based on their individual preferences. The authors drew upon the concept of pair-wise stability to capture the dynamics of corresponding matching process. As the main result of the study, the authors showed that with the derived transshipment prices, the optimum transshipment patterns are always pair-wise stable, i.e. there are no pairs of companies that can be jointly better off by unilaterally deviating from the optimum transshipment patterns.

Rosa et al. (2001) studied the Arc Routing and Scheduling Problem with Transshipment (ARPT), a particular Arc Routing Problem whose applications arise in garbage collection. In the ARPT, the demand is collected by specially equipped vehicles, taken to a transfer station, shredded or compacted and, finally, transported to a dump site by means of high-capacity trucks. A lower bound, based on a relaxation of an integer linear formulation of the problem, was developed for the ARPT. A tailored Tabu Search heuristic was also devised. Computational results on a set of benchmark instances were reported which proved to be efficient as compared with existing methods.

Asmuth et al., (1979) studied a multi-commodity transshipment problem where the prices at each location are an affine function of the supplies and demands at that location and the shipping costs are an affine function of the quantities shipped. A system of prices, supplies, demands, and shipments is defined to be equilibrium, if there is a balance in the shipments, supplies, and demands of goods at each location, if local prices do not exceed the cost of importing, and if shipments are price efficient. The authors used Lemke's algorithm to compute the equilibrium.

Perincherry and Kikuchi (1990) presented a transshipment problem in which the projected demand and supply at different locations on different days are known in fuzzy quantities. The formulation of the model follows that of fuzzy linear programming in that the solution is a shipment schedule which satisfies the objective at a 'reasonable cost'. Priorities for satisfying requirements at demand points and supply points on selected days were incorporated by multiplying corresponding weights to h , the level of satisfaction. The authors provided several examples for their formulation.

Dahan (2009) studied a model which considered two retailers between which transshipments can take place at the end of the period. The retailers differ in cost and demand distributions, operate in a single period, and cooperate to minimize joint costs. The authors work differs from previous analyses as it considered the possibility that customers are not always willing to wait for transshipments. Instead, only some customers are willing to wait and return to the retailer for transshipments. The objective of the study was to find the replenishment levels and transshipment quantities that minimize the total expected system cost. The authors considered two cases - a partially

deterministic case, and a fully stochastic case. In the partially deterministic case, the number of returning customers was a known fraction of those that could not be satisfied off-the-shelf. The fully stochastic case treated the number of returning customers as a random variable whose probability density function is known and whose expected value was a fraction of the customers that could not be satisfied off-the-shelf. In the partially deterministic case, the authors showed that the transshipment decision has a form similar to complete pooling. They proved that the objective function was convex in the replenishment levels, and suggested numerical methods for finding the optimal replenishment levels.

Topkis (1984) developed a complement and substitution principles applicable to settings in transshipment dual stage problems such as those encountered in factories and warehouses. Direct examination of the basic property of this transportation problem suggested that two locations of a similar nature would be reasonable substitutes. Such elements may not apply to location pairs where there were one or more warehouses. Where no warehouse was present, complement and substitution principles are functional. Model illustrations of factory warehouses and demand centre locations were highlighted in the author's results.

Huang and Greys (2008) studied a newsvendor game with transshipments, in which n retailers face a stochastic demand for an identical product. Before the demand was realized, each retailer independently orders her initial inventory. After the demand was realized, the retailers selected an optimal transshipment pattern and ship residual inventories to meet residual demands. Unsold inventories were salvaged at the end of the

period. The authors compared two methods for distribution of residual profit—transshipment prices (TPs) and dual allocations (DAs)—that were previously analyzed in literature. TPs are selected before the demand is known, and DAs, which were obtained by calculating the dual prices for the transshipment problem, were calculated after observing the true demand. The authors first studied the conditions for the existence of the Nash equilibrium under DA and then compared the performance of the two methods and showed that neither allocation method dominates the other. The author's analysis suggested that DAs may yield higher efficiency among “more asymmetric” retailers, whereas TPs worked better with retailers that were “more alike,” but the difference in profits does not seem significant. The authors also linked expected dual prices to TPs and used those results to develop heuristics for TPs with more than two symmetric retailers. For general instances with more than two asymmetric retailers, the authors proposed a TP agreement that uses a neutral central depot to coordinate the transshipments (TPND). Although DAs in general outperform TPND in our numerical simulations, its ease of implementation makes TPND an attractive alternative to DAs when the efficiency losses are not significant (e.g., high critical fractiles or lower demand variances).

Lateral transshipments in multi-echelon stochastic inventory systems imply that locations at the same echelon of a supply chain share inventories in some way, in order to deal with local uncertainties in demands. While the structure of a transshipment policy will depend on many important factors, a commonly observed phenomenon at the retail level, called “customer switching”, may be of some significance. Under such a phenomenon, a customer, who cannot obtain a desired product at a specific location, may visit one or

more other retail locations in search of the item. Liao (2010) studied the inventory replenishment and transshipment decisions in the presence of such stochastic "customer switching" behavior, for two firms which were either under centralized control, or operate independently. The first model adopted in this study considered two retailers that sell the same product to retail customers. After demand was realized, transshipments occur if only one location has insufficient inventory. Under this circumstance, a random fraction of the unfulfilled demand from the stocked out firm (which was referred to as the "shortage firm") may switch to the other firm with surplus inventory (which was referred to as the "surplus firm"). We examine the impact of such customer switching behavior on the firms' inventory decisions, and found out that the firm with surplus inventory is willing to (1) transship the entire quantity requested ("complete pooling policy"), (2) transship a portion of the amount requested ("inventory keeping policy"), or (3) transship nothing ("no-shipping policy") to the shortage firm. The authors demonstrated that a unique pair of optimal order quantities exists if the two firms are centrally coordinated.

Herer et al. (2006) considered coordination among stocking locations through replenishment strategies that take explicitly into consideration transshipments, that is, transfer of a product among locations at the same echelon level. The authors incorporated transportation capacity such that transshipment quantities between stocking locations are bounded due to transportation media or the location's transshipment policy. The authors modeled different cases of transshipment capacity as a capacitated network flow problem embedded in a stochastic optimization problem. Under the assumption of instantaneous transshipments, the authors developed a solution procedure based on infinitesimal perturbation analysis to solve the stochastic optimization problem, where the objective

was to find the policy that minimizes the expected total cost of inventory, shortage, and transshipments. Such a numerical approach provides the flexibility to solve complex problems. Investigating two problem settings, the authors showed the impact of transshipment capacity between stocking locations on system behavior. The authors found out that transportation capacity constraints do not only increase total cost, but also modify the inventory distribution throughout the network.

Zhaowei et al. (2009) studied a new type of transshipment problem, the flows through the cross dock are constrained by fixed transportation schedules and any cargos delayed at the last moment of the time horizon of the problem will incur relative high inventory penalty cost. The problem is known to be NP-complete in the strong sense. The authors therefore focused on developing efficient heuristics. Based on the problem structure, the authors proposed a Genetic Algorithm to solve the problem efficiently. Computational experiments under different scenarios showed that GA outperforms CPLEX solver.

Herer and Tzur (1998) investigated the strategy of transshipments in a dynamic deterministic demand environment over a finite planning horizon. The authors considered a system of two locations which replenished their stock from a single supplier, and where transshipments between the locations are possible. The authors model included fixed and variable replenishment costs, fixed and variable transshipment costs, as well as holding costs for each location and transshipment costs between locations. The problem was to determine how much to replenish and how much to transship in each period. The authors provided interesting structural properties of optimal policies which enhanced the

understanding of the important issues which motivate transshipments and allowed the development of an efficient algorithm for obtaining the optimal strategy. By exploring the reasons for using transshipments, the model enabled practitioners to envision the sources of savings from using this strategy and therefore motivated them to incorporate it into their replenishment strategies

Belgasmı et al. (2008) studied a multi-location inventory system where inventory choices at each location are centrally coordinated. Lateral transshipments are allowed as recourse actions within the same echelon in the inventory system to reduce costs and improve service level. However, this transshipment process usually causes undesirable lead times. The authors proposed a multi-objective model of the multi-location transshipment problem which addressed optimizing three conflicting objectives: (1) minimizing the aggregate expected cost, (2) maximizing the expected fill rate, and (3) minimizing the expected transshipment lead times. The authors applied an evolutionary multi-objective optimization approach using the strength Pareto evolutionary algorithm (SPEA2), to approximate the optimal Pareto front. Simulation with a wide choice of model parameters showed the different trades-off between the conflicting objectives.

Transshipments, monitored movements of material at the same echelon of a supply chain, represent an effective pooling mechanism. Earlier papers dealing with transshipments either do not incorporate replenishment lead times into their analysis, or only provide a heuristic algorithm where optimality cannot be guaranteed beyond settings with two locations. Gong and Yucasan (2010) presented a method that uses infinitesimal perturbation analysis by combining with a stochastic approximation method to examine

the multi-location transshipment problem with positive replenishment lead times. It demonstrates the computation of optimal base stock quantities through sample path optimization. From a methodological perspective, this study deploys a duality-based gradient computation method to improve computational efficiency. From an application perspective, it solves transshipment problems with non-negligible replenishment lead times.

One of the most important problems in supply chain management is the distribution network design problem system which involves locating production plants and distribution warehouses, and determining the best strategy for distributing the product from the plants to the warehouses and from the warehouses to the customers. Vahidreza et al., (2009) studied a model which allows for multiple levels of capacities available to the warehouses and plants. The authors developed a mixed integer programming model for the problem and solved it by a heuristic procedure which contains 2 sub-procedures. The authors used harmony-search meta-heuristic as the main procedure and linear programming to solve a transshipment problem as a subroutine at any iteration of the main procedure.

Glover et al. (1974) presented a primal simplex procedure to solve transshipment problems with an arbitrary additional constraint. The procedure incorporates efficient methods for pricing-out the basis, determining representations, and implementing the change of basis. These methods exploit the near triangularity of the basis in order to take full advantage of the computational schemes and list structures used in solving the pure transshipment problem. Also reported was the development of a computer code, I/O

PNETS-I for solving large scale singularly constrained transshipment problems. This code has demonstrated its efficiency over a wide range of problems and has succeeded in solving a singularly constrained transshipment problem with 3000 nodes and 12,000 variables in less than 5 minutes on a CDC 6600. Additionally, a fast method for determining near optimal integer solutions is also developed. Computational results showed that the near optimum integer solution value is usually within a half of one percent of the value of the optimum continuous solution value.

Cheng and Karimi (2002) addressed a special case of the general chemical transshipment problem, namely the tanker lightering problem. When tankers are fully loaded with crude oil, they may not be able to enter the shallow channels or refinery ports due to the draft limitation. Under such circumstances, it is necessary to transfer some part of the crude oil from the tanker to lightering vessels in order to make the tanker “lighter”. After such transshipment operation, the tanker can travel to the refinery port, which it previously cannot. And, the lightering vessels also travel to the refinery port to deliver the lightered crude oil. With tanker lightering operation, large tankers can also deliver crude oil to shallow-draught refinery ports. Furthermore, it helps to reduce the demurrage costs of tankers as well as inventory holding costs (Chajakis, 2000) at the refinery. During congested time, tankers could spend days awaiting lightering service. Since the demurrage costs of tankers are extremely high, effective scheduling of lightering operation is crucial for minimizing the system cost by reducing the waiting times of tankers and increasing the utilization of lightering vessels.

Chajakis (1997) considered a scheduling problem faced by a shipping company that provides lightering services to multiple refineries clustered in a region. The company

operates a fleet of multi-compartment lightering vessels with a mix of different configurations such as numbers of compartments, sizes, speeds, heating equipment, and so on. When a tanker arrives at the lightering location, one lightering vessel pumps off crude oil from one side of the tanker. Therefore, at most two lightering services can take place simultaneously for a tanker, one at each side of the tanker. And, these multi-compartment lightering vessels can pick up multiple types of crude from the same/different tankers during a voyage. After enough crude oil has been offloaded, the tanker leaves the lightering system and travels to its designated refinery port. However, lightering vessels travel to the refinery ports, deliver the crude oils, and then return to the lightering location to continue their service. In other words, the lightering vessels make multiple voyages among the refinery ports and lightering location in order to service multiple tankers. Furthermore, the authors considered a two-stage lightering practice for large tankers, first stage at an offshore location farther from the refinery and the second stage at the lightering location closer to the refinery.

Gilbert et al. (1997) examined a multiperiod capacity transshipment model with upgrading. There are multiple product types, corresponding to multiple classes of demand, and the firm purchases capacity of each product before the first period. Within each period, after demand arrives, products are allocated to customers. Customers who arrive to find that their product has been depleted can be upgraded by at most one level. The authors showed that the optimal allocation policy is a simple two-step algorithm: First, they used any available capacity to satisfy same-class demand, and then upgrade customers until capacity reaches a protection limit, so that in the second step the higher-level capacity is rationed. The authors showed that these results hold both when all

capacity is salvaged at the end of the last demand period as well as when capacity can be replenished (in the latter case, an order up to policy is optimal for replenishment). Although finding the optimal protection limits was computationally intensive, the authors described bounds for the optimal protection limits that take little effort to compute and can be used to effectively solve large problems.

Mues et al. (2005) stated that the transshipment Problems and Vehicle Routing Problems with Time Windows (VRPTW) are common network flow problems and well studied. Combinations of both are known as intermodal transportation problems. This concept describes some real world transportation problems more precisely and can lead to better solutions, but they are examined rarely as mathematical optimization problems.

According to White (1972), the movement of vehicles and goods in a transportation system can be represented as flows through a time-dependent transshipment network. An inductive out-of-kilter type of algorithm was presented which utilizes the basic underlying properties of the dynamic transshipment network to optimize the flow of a homogeneous commodity through the network, given a linear cost function.

Banerjee et al. (2003) examined the effects of transshipments in different operating conditions one of which is based on the concept of inventory balancing via transshipments. Under their redistribution policy, the beginning inventory at each location is equalized. Bertrand and Bookbinder (1998) also use the balancing of the beginning inventory as a redistribution policy for identical retailers.

Hsu and Bassok (1999) considered a single period problem with one input resulting in a random yield of multiple, downward substitutable products. They showed how the network structure of the problem can be used to devise an efficient algorithm.

McGillivray and Silver (1978) considered a case where products had identical costs and there is a fixed probability that a customer demand for a stocked-out product can be substituted by another available product.

Bertrand and Bookbinder (1998) extend the complete pooling policy problem, whereby the amount transshipped from location i to location j is the minimum between the excess at location i and the shortage at location j : to a system whose stock keeping locations have non-identical costs. In particular, the authors considered a warehouse following a periodic order up to S policy based on the system stock. Once the warehouse receives a shipment, it is entirely allocated to the retailers, who experience independent and (not necessarily identically distributed) normal demand. Prior to a new replenishment (order by the warehouse), system stock is redistributed—in a preventive transshipment mode—among the retailers to minimize the expected holding, backorder and transshipment costs. In the case with identical retailers, the authors analytically showed that redistribution reduces the variance of the net inventory prior to a new order. For the case with non-identical retailers, a one-parameter-at-a-time simulation experiment showed that higher values of the length of the replenishment cycle, the number of retailers, holding costs, lead times (LTs) from the warehouse to the retailers, coupled with low values for transshipment costs, supplier LTs, and shortage penalties, favor a redistribution policy.

CHAPTER 3

METHODOLOGY

3.0 INTRODUCTION

In this chapter we shall discuss the transportation and the transshipment problems and their solution procedures.

3.1.1 THE TRANSPORTATION PROBLEM

The transportation problem seeks to find the determination of a shipping plan of a single commodity from a number of sources, (say, m), to a number of destinations, (say, n), at a minimum total cost, while satisfying the demand at all destination.

The standard scenario where a transportation problem arises is that of sending units of a product across a network of highways that connect a given set of cities. Each city is considered either as a "source," (supply point) or a "sink," (demand point). Each source has a given supply, each sink has a given demand, and each highway that connects a source-sink pair has a given transportation cost per unit of shipment. This can be visualized in the form of a network, as depicted in Figure 3.1.

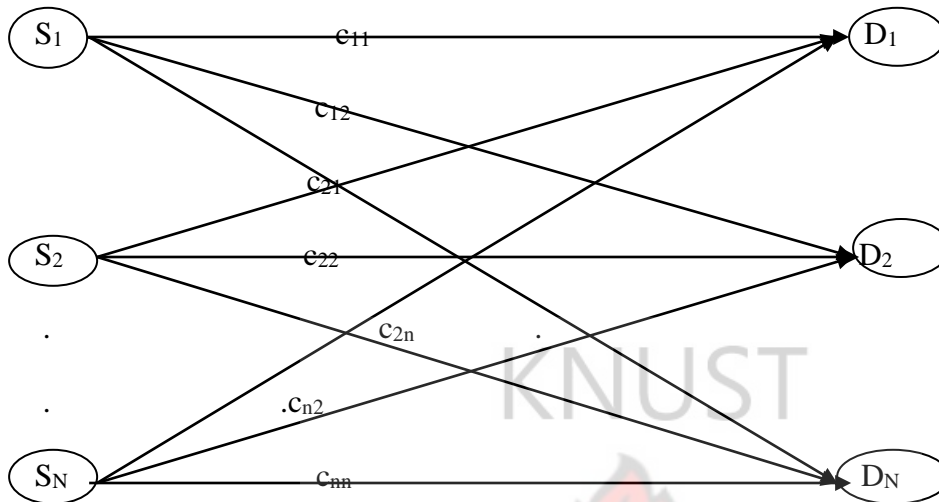


Figure 3.1 Shipment from sources to sinks

Given such a network, the problem of interest is to determine an optimal transportation scheme that minimizes the total cost of shipments, subject to supply and demand constraints. Problems with this structure arise in many real-life situations. The transportation problem is a linear programming problem, which can be solved by the regular simplex method but due to its special structure a technique called the transportation technique is used to solve the transportation problem. It got its name from its application to problems involving transporting products from several sources to several destinations, although the formation can be used to represent more general assignment and scheduling problems as well as transportation and distribution problems.

The two common objectives of such problems are either to:

- minimize the total transportation cost of shipping a single commodity from m sources to n destinations, or
- maximize the profit of shipping from m sources to n destinations.

3.1.2 CHARACTERISTICS OF A TRANSPORTATION PROBLEM

- Objective function is to reduce the transportation cost to the minimum.
- Maximum quantity available at the sources is limited. This is a constraint.
- Maximum quantity required at the destination is specified. This cannot be exceeded. This is another constraint.
- Transportation cost is specified for each movement.
- Sum of the products available from all sources is equal to sum of the products distributed at various destinations

Maximum quantity available at the source, maximum quantity required at the destination and the cost of transportation, all refer to a single product.

3.2 MATHEMATICAL FORMULATION

Let the cost of transporting one unit of goods from i^{th} origin to j^{th} destination be C_{ij} , $i=1,2, \dots, m$, $j=1,2, \dots, n$. If $x_{ij} \geq 0$ is the amount of goods to be transported from i^{th} origin to j^{th} destination, then the problem is to determine x_{ij} so as to

$$\text{Minimize } Z = \sum_{i=1}^m \sum_{j=1}^n X_{ij} C_{ij}$$

subject to the constraint

$$\sum_{j=1}^n X_{ij} = a_i, (i = 1, 2, \dots, m)$$

$$\sum_{i=1}^m X_{ij} = b_j, (j = 1, 2, \dots, n)$$

and $x_{ij} \geq 0$, for all i and j , where a_i and b_j are demand and supply availabilities.

3.2.1 FEASIBLE SOLUTION

A set of non-negative allocations, X which satisfies the row and column restrictions is known as feasible solution. A feasible solution to an m -origin and n -destination problem is said to be Basic Feasible Solution (BFS) if the number of positive allocations are $(m+n-1)$.

3.2.2 NON – DEGENERATE BASIC FEASIBLE SOLUTION

A basic feasible solution of an $(m \times n)$ transportation problem is said to be non-Degenerate if it has following two properties: (a) Initial basic feasible solution must contain exactly $(m+n-1)$ number of individual allocations.

(b) These allocations must be in independent positions. Independent positions of a set of allocations mean that it is always impossible to form any closed loop through these allocations. See fig 3.2 below.

0			0
	0	0	
	0	0	

Closed loop (ie Non – independent position)

0			0
0		0	
0	0		

Independent position

Fig 3.2 Figure showing Non-independent and depended Loop

Definition (Loop)

Given a transportation table, an ordered set of four or more cells is said to form a loop if:

- Any two adjacent cells in the ordered set lie in the same row or in the same column.
- Any three or more adjacent cells in the ordered set do not lie in the same row or in the same column.

3.2.3 DEGENERATE BASIC FEASIBLE SOLUTION

A basic feasible solution that contains less than $(m + n - 1)$ non – negative allocations is said to be degenerate basic feasible solutions.

3.2.4 DEGENERACY IN TRANSPORTATION PROBLEM

Transportation with m -origins and n -destinations can have $(m+n-1)$ positive basis variables or allocation, otherwise the basic solution degenerates. So whenever the number of basic cells or occupied cells is less than $(m + n-1)$, the transportation problem is degenerate.

3.2.5 HOW TO RESOLVE DEGENERACY IN TRANSPORTATION PROBLEM

To resolve the degeneracy, the positive variables are augmented by as many zero-valued variables as is necessary to complete $(m + n - 1)$ basic variables.

3.3 BALANCED TRANSPORTATION PROBLEM

If total supply equals total demand, the problem is said to be a balanced transportation problem: that is

$$\sum_{i=1}^m a_i = \sum_{j=1}^n b_j$$

3.3.1 UNBALANCED TRANSPORTATION PROBLEM

The transportation problem is known as an unbalanced transportation problem for the following two cases.

Case (1).

Here, $\sum_{i=1}^m a_i > \sum_{j=1}^n b_j$

In solving this, we first balance it by introducing a dummy destination in the transportation table. The cost of transporting to this destination is set equal to zero. The requirement at this destination is assumed to be equal to

$$b_{n+1} = \sum_{i=1}^m a_i - \sum_{j=1}^n b_j$$

Case (2) .

Here, $\sum_{i=1}^m a_i < \sum_{j=1}^n b_j$

To solve this, we first balanced it by introducing a dummy origin in the transportation table. The costs associated in transporting the dummy source are set equal to zero. The availability is

$$a_{m+1} = \sum_{j=1}^n b_j - \sum_{i=1}^m a_i$$

3.4 THE TRANSPORTATION TABLEAU

Table 3.4 The Transportation Table

	D₁	D₂	D₃	D₄	SUPPLY
S₁	C ₁₁ X ₁₁	C ₁₂ X ₁₂	C _{1N} X _{1N}	S ₁ = a ₁
S₂	C ₂₁ X ₂₁	C ₂₂ X ₂₂	C _{2N} X _{2N}	S ₂ = a ₂
.....
S_N	C _{M1} X _{M1}	C _{M2} X _{M2}	C _{MN} X _{MN}	S _N = a _m
DEMAND	d ₁ . b ₁	d ₂ . b ₂	d _n . b _n	

3.5 METHODS OF FINDING INITIAL BASIC FEASIBLE SOLUTION FOR A BALANCED TRANSPORTATION PROBLEM

The three basic methods are:

- The Northwest Corner Method
- The Least Cost Method
- The Vogel's Approximation Method

3.5.1 NORTHWEST-CORNER METHOD

The steps below are used in the Northwest- Corner method

Step (1) The first assignment is made in the cell occupying the upper most left-hand (North West) corner of the transportation table. The maximum feasible amount is allocated there, i.e.; $x_{11} = \min(a_1, b_1)$.

Step (2) If $b_1 > a_1$, the capacity of origin S_1 is exhausted but the requirement at D_1 is not satisfied. So move down to the second row, and make the second allocation: $x_{21} = \min(a_2, b_1 - x_{11})$ in the cell $(2,1)$. If $a_1 > b_1$, allocate $x_{12} = \min(a_1 - x_{11}, b_2)$ in the cell $(1,2)$.

Step (3) Continue this until all the requirements and supplies are satisfied.

3.5.2 LEAST-COST METHOD

The least cost method uses shipping costs in order to come up with a basic feasible solution that has a lower cost.

Step (1) Find the decision variable with the smallest shipping cost x_{ij} .

Step (2) Assign x_{ij} its largest possible value, which is the minimum of s_i and d_j .

Step (3) As in the Northwest Corner Method cross out row i and column j and reduce the supply or demand of the non crossed-out row or column by the value of x_{ij} ,

Step (4) Choose the cell with the minimum cost of shipping from the cells that do not lie in a crossed-out row or column and repeat the procedure.

3.5.3 VOGEL'S APPROXIMATION METHOD (VAM)

Step 1 For each row of the transportation table, identify the smallest and the next to-smallest costs. Determine the difference between them for each row. Display them alongside the transportation table by enclosing them in parenthesis against the respective rows. Similarly compute the differences for each column.

Step 2 Identify the row or column with the largest difference among all the rows and columns. If a tie occurs, use any arbitrary tie breaking choice. Let the greatest difference correspond to i^{th} row (j^{th} column) and the minimum cost be C_{ij} . Allocate a maximum feasible amount $x_{ij} = \min(a_i, b_j)$ in the $(i, j)^{\text{th}}$ cell, and cross off the i^{th} row or j^{th} column.

Step 3. Re compute the column and row differences for the reduced transportation table and go to step 2. Repeat the procedure until all the rim requirements are satisfied.

3.6 OPTIMAL SOLUTION

A feasible solution (not necessarily basic) is said to be optimal if it minimizes the total transportation cost.

3.6.1 THEOREM FOR TESTING OPTIMALITY

If we have a BFS consisting of $m + n - 1$ independent positive allocations and a set of arbitrary number u_i and v_j ($i = 1, 2, \dots, m$; $j = 1, 2, \dots, n$) such that $c_{rs} = u_r + v_s$ for all occupied cells (r, s) then the evaluation d_{ij} corresponding to each empty cell (i, j) is given by

$$d_{ij} = c_{ij} - (u_i + v_j)$$

If all $d_{ij} > 0$, then the solution is optimal and unique.

If all $d_{ij} \geq 0$, (ie at least one $d_{ij} = 0$), solution is optimal and alternate solution also exist.

If at least one $d_{ij} < 0$, the solution is not optimal.

3.6.2 SOLUTION TO OPTIMALITY

As mentioned above, the solution method for transportation problems is a streamlined version of the Simplex algorithm. As such, the solution method also has two phases. In the first phase, the aim is to construct an initial basic feasible solution; and in the second phase, to iterate to an optimal solution. For optimality, we need a method, like the simplex method, to check and obtain the optimal solution. The two methods used are:

- a. Stepping-stone method
- b. Modified distributed method (MODI)

3.6.3 STEPPING STONE

1. Select an unused square to be evaluated.
2. Beginning at this square, trace a closed path back to the original square via squares that are currently being used (only horizontal or vertical moves allowed).

You can only change directions at occupied cells.

3. Beginning with a plus (+) sign at the unused square, place alternative minus (-) signs and plus signs on each corner square of the closed path just traced.
4. Calculate an improvement index, I_{ij} by adding together the unit cost figures found in each square containing a plus sign and then subtracting the unit costs in each square containing a minus sign.

5. Repeat steps 1 to 4 until an improvement index has been calculated for all unused squares.

If all indices computed are greater than or equal to zero, an optimal solution has been reached

If not, it is possible to improve the current solution and decrease total shipping costs.

3.6.4 MODIFIED DISTRIBUTED METHOD (MODI)

It is a modified version of the stepping stone method

MODI determines if a tableau is the optimal, tells which non-basic variable should be firstly considered as an entry variable, and makes use of stepping-stone to get its answer of next iteration

Procedure (MODI)

Step 1: let u_i , v_j , c_{ij} variables represent rows, columns variables, and cost in the transportation tableau, respectively

Step 2:(a) Form a set of equations that are used to represent all basic variables x_{ij} as $u_i + v_j = c_{ij}$

(b) Solve variables u_i , v_j by assign one variable = 0

Step3: (a) Form a set of equations use to represent non-basic variable (or empty cell) as

$$\text{Such } c_{ij} - u_i - v_j = k_{ij}$$

(b) Evaluate variables v_j by using step 2b information

Step 4: Select the cell that has the most negative value in step 3b

Step 5: Use stepping-stone method to allocate resource to cell in Step 4

Step 6: Repeat the above steps until all cells in step 3a has no negative Value.

3.7 THE TRANSSHIPMENT PROBLEM

The transshipment problem is an extension of the framework of the transportation problem. The extension is in allowing the presence of a set of transshipment points that can serve as intermediate stops for shipments, possibly with a net gain or loss in units. Any given transshipment problem can be converted into an equivalent transportation problem. Hence, the procedure for solving the transportation problems can be applied to the solution of transshipment problem as well.

A transshipment problem consists of finding the cheapest way of shipping goods through a network of routes so that all given demands at all points of the network is satisfied.

Given:

- a network of routes as a graph
- a set of nodes which act as sources (supplies)
- a set of nodes which act as sinks (demands)
- the amount of supply and demand at each node
- the cost of each transport route (edge)

The transshipment problem is similar to the transportation problem except that in the transshipment problem it is possible to ship both into and out of the same junction node (point). It is an extension of the transportation problem in which intermediate nodes, referred to as transshipment nodes are added to source as well as sink nodes to account for locations such as junctions. In this more general type of distribution problem, shipments may be made between any pair of the three general types of nodes: origin

nodes, transshipment nodes and destination nodes. For example (i) transshipment problems permits shipments of goods from origins to transshipment nodes and on to destinations, (ii) From one origin to another origin, (iii) From one transshipment location to another, (iv) from one destination location to another and (v) directly from origins to destinations.

3.7.1 THE MODEL

The general linear programming model of a transshipment problem is

$$\text{Min } \sum_{\text{all arcs}} C_{ij} X_{ij}$$

Subject to

$$\sum_{\text{arc out}} X_{ij} - \sum_{\text{arc in}} X_{ij} = S_i \quad \text{nodes Origin } i$$

$$\sum_{\text{arc out}} X_{ij} - \sum_{\text{arc in}} X_{ij} = 0 \quad \text{Transshipment nodes}$$

$$\sum_{\text{arc in}} X_{ij} - \sum_{\text{arc out}} X_{ij} = D_j \quad \text{demand nodes } j$$

Where

x_{ij} = amount of units shipped from node i to node j

C_{ij} = cost per unit of shipping from node i to node j

S_i = supply at origin node i

D_j = demand at origin node j

The following steps describe how the optimal solution to a transshipment problem can be found by solving a transportation problem.

Step1: Balancing the given problem: Balancing means checking whether sum of availability constraints must be equal to sum of requirement constraints. That is $\sum b_i = \sum d_j$. Once they are equal, go to Step two. If not by opening a Dummy row or Dummy

column balance the problem. The cost coefficients of dummy cells are zero. If $\sum b_i$ is greater than $\sum d_j$, then open a dummy column, whose requirement constraint is equal to $\sum b_i - \sum d_j$ and the cost coefficient of the cells are zeros.

In case if $\sum d_j$ is greater than $\sum b_i$, open a dummy row, whose availability constraint will be equal to $\sum d_j - \sum b_i$ and cost coefficient of the cells are zeros.

Step2: Transshipment occurs in the network because the entire supply amount of goods could conceivably pass through any node of the network before ultimately reaching their destination nodes. In this regard each node of the network with both input and output arcs acts as both a source and a destination and is referred to as a transshipment node. The remaining nodes are either pure supply nodes or pure demand nodes.

In converting the transshipment model into a regular transportation model, a row in the tableau will be needed for each supply point and transshipment point. The amounts of supply and demand at the different nodes are computed as

Supply at a pure supply node = Original supply

Demand at a pure demand node = Original demand

Supply at a transshipment node = Original Supply + Buffer amount

Demand at a transshipment node = Original demand + Buffer amount

The buffer amount should be sufficiently large to allow all of the original supply(or demand) units to pass through any of the transshipment nodes.

Assuming B is the desired buffer amount, then

$B = \text{Total Supply (or Demand)}$, thus, sum of supply or sum of demand.

Using the buffer and the unit shipping costs given in the network, we construct the equivalent regular transportation model.

Step 3: Solve the transportation table of step 2 by the transportation technique.

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CHAPTER 4

DATA COLLECTION AND ANALYSIS

4.1 INTRODUCTION

This chapter deals with data collection, data analysis and discussion, the discussion of transshipment of Banknotes from Bank of Ghana to the other branch offices. The data was obtained from Bank of Ghana Head office Accra; the cost of transporting Banknotes involves fuel consumption of vehicle and cost of labour. The main sources of Banknotes are Accra and Kumasi, the warehouses or junctions are Tarkoradi, Sefwi Buako, Sunyani, Hohoe and Tamale, and the final destinations are Cape Coast, Agona Swedru, Koforidua, Nkawkaw, Dunkwa-On-Offin and Wa.

Accra and Kumasi are the only branches that have currency processing machines to process Banknotes received from the Commercial Banks for authentication.

The other regional offices serve as branches for the distribution of Banknotes.

Table 4.1 below shows the Distances between the Locations in Kilometers. Full table can be seen in Appendix (I)

Table 4. 1 Table of Data Showing Distances Between Locations (KM)

	ACCRA	KUMASI	HOHOE	----	-----	-----	DK. OFF	S. BUAKO
ACCRA	-	270	225				270	391
KUMASI	270	-	385	----	----	----	95.7	140
HOHOE	225	385	-	-----	-----	----	457	525
SUNYANI	400	130	509	-----	-----	----	212	137
TAKORADI	218	242	432	-----	-----	----	166	266
TAMALE	658	388	392	-----	-----	----	471	456
TEMA	30.5	270	201	-----	-----	----	174	410
K'DUA	85	194	190	-----	-----	----	266	335
C-COAST	144	221	362	-----	-----	----	140	260
NKAWKAW	145	105	280	-----	-----	----	177	245
AG. SWE	72.6	220	272	-----	-----	----	197	348
WA	740	470	693	-----	-----	----	537	488
DK. OFF	270	95.7	457	-----	-----	----	-	123
S. BUAKU	391	140	525	-----	-----	----	123	-----

Figure 4.1 below represents data of Bank of Ghana. All the destinations serve as pure destinations. All junctions serve as both sources and junctions. The two main sources, Accra and Kumasi, serve as pure sources and destinations as well.

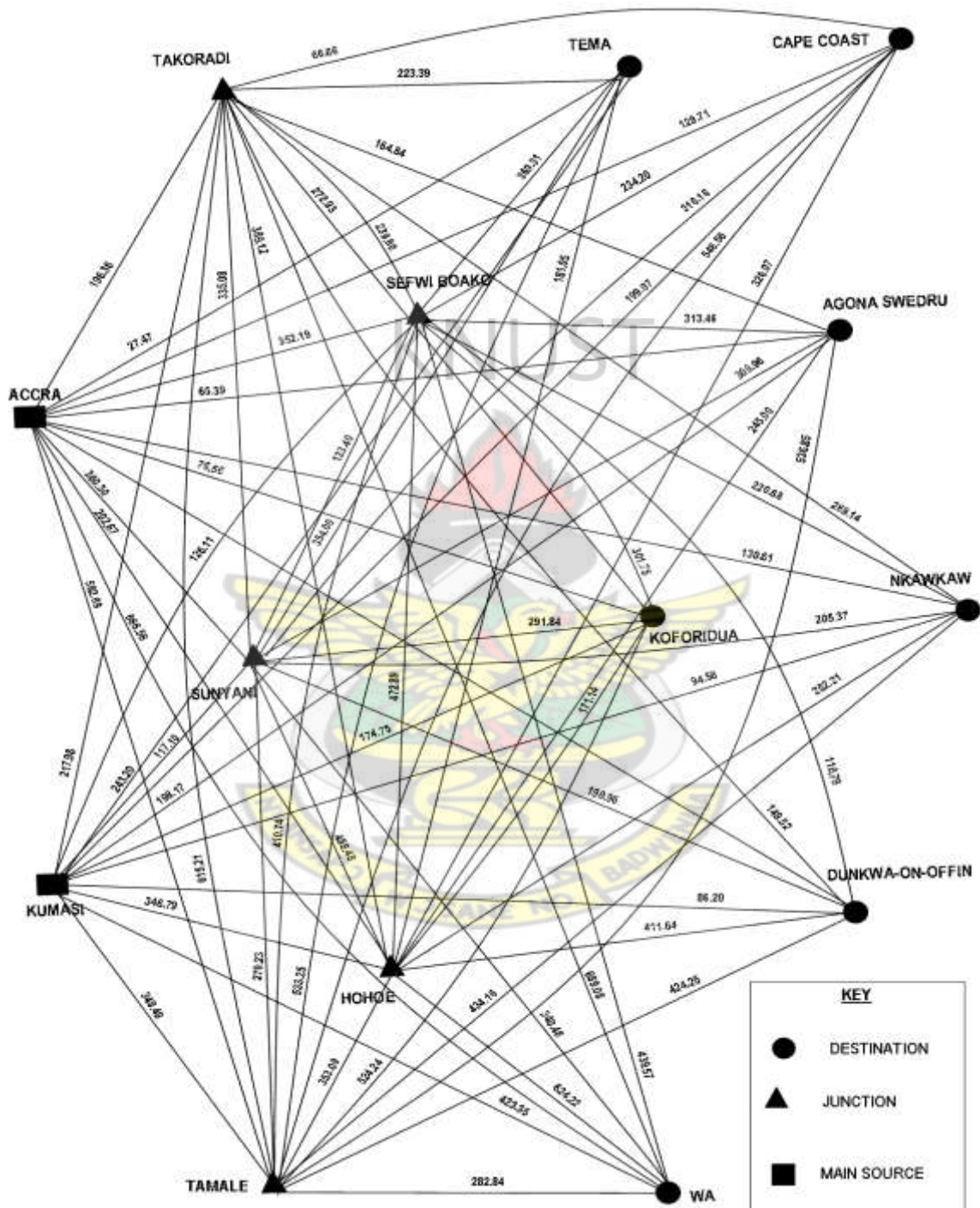


FIGURE 4. 1 This network shows the distances between locations

Table 4.2 below shows the fuel consumption in litres on every movement made. This information was gathered from the Transport Unit of the General Services Department of Bank of Ghana. Full table can be seen on Appendix (II)

Table 4.2 Table of Data Showing the Fuel Consumption (in liters) on every movement made.

	ACCRA	KUMASI	HOHOE	----	-----	-----	DK. OFF	S. BUAKO
ACCRA	-	160	133.34				160	231.71
KUMASI	160	-	228.15	----	-----	-----	56.71	82.96
HOHOE	133.34	228.15	-	-----	-----	-----	270.82	311.12
SUNYANI	237.04	77.04	301.63	-----	-----	-----	125.63	81.19
TAKORADI	129.19	143.41	256	-----	-----	-----	98.37	157.63
TAMALE	389.93	229.93	232.30	-----	-----	-----	279.11	270.23
TEMA	18.07	160	119.11	-----	-----	-----	103.11	242.97
K'DUA	50.37	114.96	112.59	-----	-----	-----	157.63	198.52
C-COAST	85.33	130.96	214.52	-----	-----	-----	82.96	154.08
NKAWKAW	85.93	62.22	165.93	-----	-----	-----	104.89	145.19
AG. SWE	43.02	130.37	161.19	-----	-----	-----	116.74	206.22
WA	438.52	278.52	410.67	-----	-----	-----	318.23	289.19
DK. OFF	160	56.71	270.82	-----	-----	-----	-	72.89
S. BUAKU	231.71	82.96	311.12	-----	-----	-----	72.89	-----

The shipping costs are obtained by multiplying the costs per litre (GH¢1.52 as at when Data was collected in 2011) by the fuel consumption of Bullion travels. This is shown in Table 4.3 below. Full table can be seen at Appendix (III)

Table 4.3 Table of Data Showing the Amount Spent on Every Specie (GH¢)

	ACCRA	KUMASI	HOHOE	----	-----	-----	DK. OFF	S. BUAKO
ACCRA	-	243.20	202.67				243.20	352.19
KUMASI	243.20	-	346.79	----	-----	-----	86.20	126.11
HOHOE	202.67	346.79	-	-----	-----	-----	411.64	472.89
SUNYANI	360.30	117.10	458.48	-----	-----	-----	190.96	123.40
TAKORADI	196.36	217.98	389.12	-----	-----	-----	149.52	239.60
TAMALE	592.69	349.49	353.09	-----	-----	-----	424.25	410.74
TEMA	27.47	243.20	181.05	-----	-----	-----	156.73	2369.31
K'DUA	76.56	174.75	171.14	-----	-----	-----	239.60	301.75
C-COAST	129.71	199.07	326.07	-----	-----	-----	126.11	234.20
NKAWKAW	130.61	94.58	252.21	-----	-----	-----	159.43	220.68
AG. SWE	65.39	198.17	245.00	-----	-----	-----	177.45	313.46
WA	666.56	423.35	624.22	-----	-----	-----	483.70	439.57
DK. OFF	243.20	86.20	411.64	-----	-----	-----	-	110.79
S. BUAKU	352.19	126.11	472.89	-----	-----	-----	110.79	-----

Table 4.4 is the cost matrix, supply availability at each source is at the far right column and the warehouse demands are shown in the bottom row.

Full table is shown in Appendix (IV)

Total supply = 450 movements

Total demand = 435 movements

Dummy = 15 movements

One (1) movement = 360 trays of Banknotes

One (1) tray = 10,000 pieces of Banknotes

SOURCES:

PURE SOURCES = S1-ACCRA, S2-KUMASI,

JUNCTIONS = S3-HOHOE, S4-SUNYANI, S5-TAKORADI, S6-TAMALE,

S7-S BUAKO

Table 4.4 Table of the cost matrix showing the shipping cost at the locations.

	D1	D2	D3	---	----	D11	D12	DUMMY	SUPPLY
S1	202.67	360.30	196.36	---	----	243.20	352.19	0	150
S2	346.79	117.10	217.98	---	----	86.20	126.11	0	100
S3	-	458.48	389.12	---	----	411.64	472.89	0	30
S4	458.48	-	335.08	---	---	190.96	123.40	0	35
S5	389.12	335.08	-	---	---	149.52	239.60	0	45
S6	353.09	270.23	615.21	---	---	424.25	410.74	0	40
S7	472.89	123.40	239.60	---	---	110.79	-	0	50
DEMAND	35	45	50	---	---	25	60	15	450

DESTINATIONS:

JUNCTIONS = D1-HOHOE, D2-SUNYANI, D3-TAKORADI, D4-TAMALE,
D12-S BUAKO

PURE DESTINATIONS = D5-TEMA, D6-KOFORIDUA, D7-CAPE COAST,
D8-NKAWKAW, D9-AG SWEDRU, D10-WA, D11-DUNKWAW ON OFFIN.

4.2 MODEL FORMULATION

Let x_{ij} be the number of units of Notes shipped from source i to warehouse j for the Banknotes.

The objective is to minimize

$$\text{Cost} = \sum_i^m \sum_j^n c_{ij} x_{ij}$$

Then the supply constrains are

$$\sum_{j=1}^n x_{ij} \leq S_i \quad i = 1, 2, 3, \dots, m \quad \dots \dots \dots (1)$$

The demand constrains are

$$\sum_{i=1}^m x_{ij} \geq d_j \quad j = 1, 2, 3, \dots, n \quad \dots \dots \dots (2)$$

The transshipment constrains are

$$\sum_{j=1}^n x_{ij} - \sum_{i=1}^m x_{ij} = 0 \quad \dots \dots \dots (3)$$

And the non-negativity $x_{ij} \geq 0$, for all i, j

C_{ij} 's are the unit shipping cost and can be obtained in Table 4.4 and Appendix IV

S_i 's are the supply availability at the various sources and can also be seen in Table 4.4 and Appendix IV

d_j 's are the demand at the various destination also to be seen in Table 4.4 and Appendix IV

4.3 COMPUTATIONAL PROCEDURE

The computer used for the computation was Toshiba Intel with 250GB as Hard disk size and 2GB DDR2 RAM size. The operating system that runs the machine is Microsoft Windows 7. Quantitative Method (Q. M. 32) for windows software was used to analyse the data to find solution to the problem. It has a capacity of 760 bytes and approximately 4.00 KB, on disc.

The Q. M. 32 for windows start with an initial starting method (The Vogel's Approximation method) to solve the problem, if an optimal solution is not provided, it then applies a method (Modified distribution method – MODI) that would compute it to optimality to get an optimal solution.

4.4 RESULTS

Table 4.5 below is the result of the data analysed.

The final optimal transshipment cost was GH¢30,719.80.

Column 2 is the sources of the transshipment material, column 3 is the destinations, column 4 the shipment (the number of units shipped), column 5 is the unit cost per shipment, and column 6 is the total shipment cost.

Table 4.5 RESULTS ANALYSIS

	Shipment List				
No.	From	To	Shipment	Cost per unit	Shipment cost
1	ACCRA	HOHOE	5	202.67	1013.35
2	ACCRA	TADI	5	196.36	981.8
3	ACCRA	TEMA	35	27.47	961.45
4	ACCRA	K'DUAH	30	76.56	2296.8
5	ACCRA	C-COAST	25	129.71	3242.75
6	ACCRA	N'KAW	10	130.61	1306.1
7	ACCRA	AG.SWE.	25	65.39	1634.75
8	ACCRA	DUMMY	15	0	0
9	KUMASI	SUNYANI	10	117.1	1171
10	KUMASI	TAMALE	5	349.49	1747.45
11	KUMASI	N'KAW	25	94.58	2364.5
12	KUMASI	WA	25	423.35	10583.75
13	KUMASI	DK.OFF	25	86.2	2155
14	KUMASI	S.BUAKU	10	126.11	1261.1
15	HOHOE	HOHOE	30	0	0
16	SUNYANI	SUNYANI	35	0	0
17	TAKORADI	TADI	45	0	0
18	TAMALE	TAMALE	40	0	0
19	S.BUAKU	S.BUAKU	50	0	0

From column 4 of the Results table above, the maximum shipment is 50 movements, from S Buako to S Buako destination. The next maximum shipment is also from Takoradi to Takoradi destination, which is 45 movements.

The minimum shipments were 5 movements, from Accra to Hohoe destination, Accra to Takoradi destination, and Kumasi to Tamale destination. The next minimums were 10 movements, from Accra to Nkawkaw Destination, from Kumasi to Sunyani Destination and from Kumasi to S Buako Destination.

The optimal transshipment cost for the study is GH¢30,719.80.

4.5 DISCUSSION

From the result in Table 4.5, it could be seen that shipping Banknotes from source to junctions then to the final destination was costly than shipping straight from source to the final destination.

From row 3, 35 movements were made from Accra to Tema, at a cost per unit of GH¢27.47 with a total cost of GH¢961.45. From row 4, 30 movements were made from Accra to Koforidua, at a cost per unit of GH¢76.56, with a total unit cost of GH¢2,296.80, whereas from row 5, 25 movements were made from Accra to Cape Coast at a unit cost of GH¢129.71, with a total of GH¢3,242.75.

It was also realized that the highest unit cost per shipment from sources to the final destination was from Kumasi to Wa, which is 25 movements, at a maximum cost per unit of GH¢423.35, with a total cost of GH¢10,583.75.

CHAPTER 5

CONCLUSIONS AND RECOMMENDATION

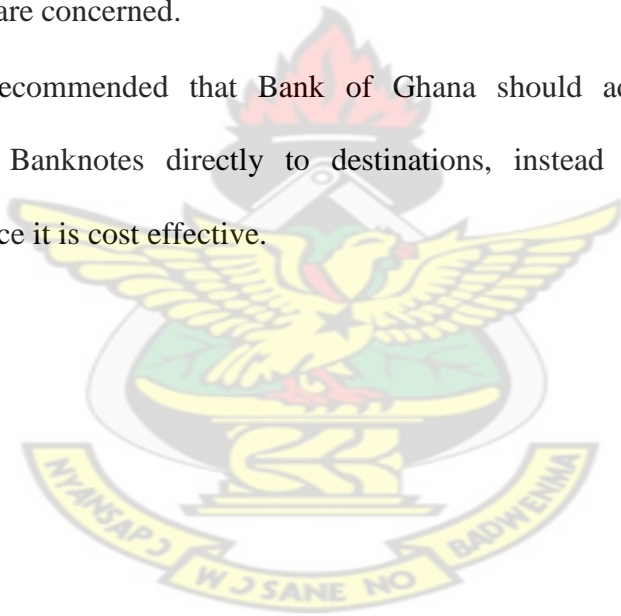
5.1 CONCLUSIONS

Results of data collected from Bank of Ghana consisting of cost of transporting Banknotes from the Sources to the destinations and the number of units being transported to each junction and destination were analysed by using Quantitative Method (QM 32). The minimum cost for the period under study was around GH¢30,719.80 used by BoG.

Banknotes shipment was modelled as a transshipment problem which was converted to a transportation problem, and subsequently solved using QM 32 for windows. Because some supply points and demand locations were on different locations, the results made it clear that it is better to transport more Banknotes within the same locality for a less cost transportation. Based on the findings and analysis of the collated data, it was also realized that it was less costly to transport Banknotes from sources to the branches directly than through the intermediary points or junctions from sources to the destinations.

5.2 RECOMMENDATION

- This study was conducted for only one year and results of this study provide some scope for further studies.
- It could be of interest to use data on the weekly basis. This will provide a more comprehensive view point about the cost of transporting the Banknotes.
- QM for windows software is recommended to supply chain management and the transport officer since it will help them to locate the shortest possible route that will lead to cost effective so far as transporting Banknotes to the final destinations are concerned.
- It is also recommended that Bank of Ghana should adopt the system of transporting Banknotes directly to destinations, instead of passing through junctions since it is cost effective.



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