

**Kwame Nkrumah University of Science and Technology, Kumasi**



**Optimal Production Scheduling: A Case Study of Everpure Ghana  
Limited**

By

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## Declaration

I hereby declare that this submission is my own work towards the award of the MSc degree and that, to the best of my knowledge, it contains no material previously published by another person nor material which had been accepted for the award of any other degree of the university, except where due acknowledgment had been made in the text.

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## Dedication

I dedicate this piece of dissertation to my Godfather, Apostle Dr. Kwadwo Safo for his spiritual support.

Also to my supervisor, Prof. SK Amponsah for his time and encouragement, my husband, Mr. Kwaku Asante for his financial support. I say may the almighty God bless you all.

KNUST



## Abstract

The manufacturing industry undoubtedly has immeasurable impact on countries worldwide. It has opened up business opportunities, create jobs, and improved quality of life. It thus improves the socio-economic development of nations. For instance, in 2012, the manufacturing industry contributed \$1.87 trillion to the United States' economy. This was 11.9% of the United States' total GDP (Gross Domestic Product). In the same way, in Ghana, it accounted for 27.6% of GDP in 2011. The pivotal role of the manufacturing industry to the global economy is therefore not in doubt. The main objectives of this study among others were to establish a good production schedule that will satisfy future demand for the manufacturing industry, determine how to optimize a production scheduling using transportation model and establish schedule formulation and analyze using MATLAB software. The study modeled the production problem as a transportation problem which involve determine how to optimally transport goods. In a balanced transportation problem, the supply must be equal to the demand. The initial basic feasible solution is found using the Vogel's Approximation Method and to iterate to optimality, the Modified Distribution Method (MODI) is used. A twelve month data on production capacity at Everpure Ghana Limited is collected for the study. The methods used have proven appropriate in obtaining the optimum schedule. From the result, it is clear that efficient scheduling could reduce production and inventory cost whilst satisfying customer demand.

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# Chapter 1

## INTRODUCTION

### 1.1 Background of the Study

The manufacturing industry undoubtedly has immeasurable impact on countries worldwide. It has opened up business opportunities, created jobs, and improved quality of life. It thus improves the socio-economic development of nations.

The manufacturing industry is the mainstay of most economies. For instance, in 2012, the manufacturing industry contributed \$1.87 trillion to the United States' economy. This was 11.9% of the United States' total GDP (Gross Domestic Product)(National Association of Manufacturers,2013). In the same way, in China, it accounted for 46.8% of GDP in 2010 (Economy Watch, 2013). Finally, in Ghana, it accounted for 27.6% of total GDP in 2011(Ghana Statistical Service, 2012). The pivotal role of the manufacturing industry to the globally economy is therefore not in doubt. The competitive nature of the world market, however, encourages management of industries to bring new products and supply strategies. In their quest to executing these, many challenges emerge: machine-failure, inefficiency and scarce production resources, late and high production and inventory cost, which prevent firms from meeting customers demand, which is a typical problem in dealing with many complex man-made system(Cassandras,1993). This has detrimental effect on productivity and stifles economic growth.

Lots of efforts have directed at improving on the efficiency of the overall manufacturing process: to lower costs, reduce inventory, and improve customer

service overall applicable time horizons. One of such novel interventions is scheduling. It is essentially the competitive process of allocating limited resources in order to meet the demands of customers(Baker, 1974).

Scheduling is a decision-making function that plays a vital role in most manufacturing and service industries. It is applied in procurement and production, in transportation and distribution and in information processes and communication.

In manufacturing the purpose of scheduling is to minimize the production time and cost, by defining what to make, when, with which staff and which equipment. Two essential techniques are involved in the scheduling process: backward and forward scheduling. Backwards scheduling plans the tasks from the due date to determine the start date or any change in the capacity required. Forward scheduling, in contrast, plans the tasks when the date and resources become available to determine the the due date.

Scheduling is of different types, which includes project scheduling, production scheduling, work-force scheduling etc. Therefore, production scheduling is the management and allocation of resources, event and processes to create goods and services. A firm adjust its production scheduling based on the availability of resources, client orders and efficiencies. Production scheduling system rely on human decision-makers and many of them need assistance dealing with the swampy complexities of real-world scheduling(McKay and Wiers, 2004). The goal of production scheduling is to balance client needs with available resources while operating in the most cost-efficient manner. The resources include the raw materials used to produce goods, the availability of machines and the available of workers.

Mostly production schedulers track all resources and find constraints or resources outages that will affect different volume levels of production. Once a scheduler identifies resources constraints, he/she adds additional supplies, machines and personnel to ensure production goals are met and also review client orders based on the time frame requested, client importance and available production capacity. Schedulers also work closely with sales and marketing to meet customers demand and maximize sales. The traditional scheduling approach is however, riddled with challenges. According to Wight (1984), the two main problem in production scheduling are, "priorities" and "capacity".

There are a plethora of measures at improving production scheduling. The primary measures include problem-solving perspective, decision-making perspective and organizational perspective. Each perspective has a peculiar scope, set of assumption and a different approach to improving production scheduling.

The problem-solving perspective views the scheduling as an optimization problem. It is the formulation of scheduling as a combinatorial optimization problem isolated from the manufacturing planning and control system place.

The decision-making perspective is the view that scheduling is a decision that a human must make. Schedulers perform different tasks and use both formal and informal information to accomplish these. Schedulers must address uncertainty, bottlenecks and anticipate the problem that people cause (McKay and Wiers, 2006).

The organizational perspective is a systems-level view that scheduling is part of the complex flow of information and decision-making that forms the manufacturing planning and control system (Herrmann, 2004). Such system

are separated into modules that perform different functions such as aggregate planning and material requirement planning. There are three main goals of production scheduling, these include: the due date and avoiding late completion of jobs, throughput time; the system from the opening of a job order until it is completed, the utilization of work centers (Hurtubise et al., 2004).

### 1.1.1 Classification of Production Scheduling

Production scheduling can be classified according to the following criteria:

Flow patterns

- Flow shop: all the jobs have identical process flow and require the same sequence of operations.
- Job shop: jobs have different process flows and may require significantly different sequence of operations.

Processing mode

- Unit processing: Jobs are processed one by one.
- Batch processing: number of jobs are processed as a batch.

Job release pattern (job release time is the earliest time which processing can start)

- Static: jobs are released to the shop floor at time zero.
- Dynamic: jobs assumed to be released to the shop floor over time.

Work center configuration

- Single machine.
- Identical parallel machine.
- Uniform parallel machine.
- Unrelated parallel machines.

### 1.1.2 Benefits of Production Scheduling

There are some goals and benefits of production scheduling which need to be considered.

- A production schedule can determine whether delivery promises can be met and identify time period available for prevention maintenance.
- Maximize machine and /or worker utilization.
- Minimize setup times.
- Minimize average flow time through the system.
- Better coordination to increase productivity and minimizing operating cost.
- A production schedule can identify resource conflicts, control the release of jobs to shop and ensure that required raw materials are ordered in time.
- A production schedule gives shop floor personnel and explicit statement of what should be done so that supervisors and managers can measure their performance.

## 1.2 Statement of Problem

Production tasks are done within the factory through production scheduling, which is essentially the strategy of allocating equipment, utility and manpower resources over time to execute the processing tasks required to manufacture one or several production in time. The aim of production scheduling is to balance client needs with available resources while operating in the most cost-efficient manner.

Unfortunately, there could be a limitation in the production sequence. Such as, a set of tasks to be performed, and the criteria may involve both

trade-offs between early and late completion of a task, holding inventory for the task and frequent production changeover. Unforeseen events on the shop floor, for instance, machine breakdowns or longer-than-expected processing time may have a great impact on the schedules.

The afore mentioned problems can be remedied using mathematical techniques and heuristic methods to allocate limited resources to the activities that have to be done. The allocation of resources has to be done in such a way that the company optimizes its objectives and achieve its goals. It helps the target firm establishes an efficient production schedule that minimizes both total production and inventory cost while satisfying customer demands.

### 1.3 Objective of the Study

The main objective of this study among others were to :

- establish good production schedule that will satisfy future demands for the manufacturing industry.
- determine how to optimize a production scheduling using transportation model.
- establish schedule formulation and analyze using MATLAB software.

### 1.4 Research Methodology

Production problems have been treated as a linear programming framework by many books. The study will model the production problem as a transportation problem which involve determine how to optimally transport goods. Since the transportation problem is a linear programming problem, it can be solve by the simplex method but because of its special nature it can be solved more easily by special form of the simplex method taking merit of the special structure (Amponsah, 2009). These special algorithm are more efficient for the

transportation problem than the parent simplex method .

The transportation problem concerns with finding the minimum cost distribution of a commodity from a source ( supply:  $K_i$ ) to a receiving center(demand:  $W_j$ ) while satisfying all constraints of productive capacity and demand,  $i = 1, 2, \dots, m$  and  $j = 1, 2, \dots, n$  respectively. The cost of transporting from a source to a destination is directly proportional to the number of units transported. It is important to note that production scheduling has nothing to do with transportation. In a balanced transportation problem, the supply must be equal to the demand, ie

$$\sum_{i=1}^m K_i = \sum_{j=1}^n W_j \quad (1.1)$$

To search for initial basic feasible solution (IBFS) several methods can be used but this work will concentrate on the Vogel's Approximation Method. And to iterate to optimality, the Modified Distribution Method(MODI) will be used. A twelve months data on production capacity and expected demand(in L) at Everpure Ghana Limited would be collected for this study. Regular production cost(in GH¢) and inventory at the onset of the year will be gathered for this study. The data will be formulated and analyzed using appropriate software. The information and references for this work will be taken from books, libraries, the internet etc.

It is very hard to think of a situation where the mathematical algorithms and planning logics are self-setting, self-tuning and self-installing and self-adapting to the situational context of business realities. Unluckily, the role of the human element has been largely ignored and under-researched compared to the effort placed on the mathematical and software aspect(Wight, 1984).



### **1.4.1 Everpure Ghana Limited**

Everpure Ghana Limited is a wholly Ghanaian company established in December, 2010 by Ghanaian professionals from different fields. The company is of partnership and limited liability ownership type. Everpure is based in Tema and has recently setup a factory at Kumasi to enable it meet the ever growing demand for the company's product. Though established not long ago the company has gained a reputation for consistently producing quality products. The sole aim of establishing the company is to provide the best purified water to clients, based on a process that is unique and of the highest standard in water purification technology, utilizing best practices, backed by quality customer service. And also to support on-going Government efforts to promote good and healthy lifestyle habits, by actively encouraging and educating clients on the benefits of drinking quality purified water at all times. The firm has different department of which includes production, account, marketing etc to aid smooth running. Everpure produces a wide range of products to fit different socio-economic sector of the population. The products come in 0.60L, 1.5L and 18.9L bottles, as well as in the 500ml sachet, which is a better value proposition as other competitors sell in 450mL sachet and 50cL bottle, for the same price. Because of the specialized production method for company's ice cubes, drinks can be chilled within 15 minutes, and such chilling stays longer because the ice cubes melts so slowly, giving long-lasting cooling effect to your drinks and refrigerated products like fish, chicken, and other seafood products.

## **1.5 Justification of work**

The scheduling problem involves a set of tasks to be performed, and the criteria may involve both trade offs between early and late completion of a task, and between holding inventory for the task and frequent production changeovers. This work will present a broad classification for various scheduling problems,



to review important theoretical developments for these problem classes, and to distinguish the presently available theory with the practice of production scheduling. The work will underline problem areas for which there is both a significant variation between theory, practice and for which the practice corresponds closely to the theory. Basically, production scheduling aims at managing and allocating resources, event and processes to create goods and services. This in effect will increase production as well as the country's GDP. In certain industries the viability of a company may depend on the effectiveness of its scheduling systems. Good scheduling often allows an organization to conduct its operations with a minimum of resources while meeting customer demands through the used a good software like the excel solver. Companies use backward and forward scheduling to allocate plant and machinery resources, plan human resources, plan production processes and procure materials.

Finally, this piece will contribute to the existing store of knowledge, especially in Ghana as expressed in articles, academic work, books etc. showing the level of importance attached to the topic under discussion.

## **1.6 Organisation of the Study**

The study consist of five chapters. The first chapter comprises of the introduction which talks about the background of the study, statement of problem, research questions, research methodology, justification and organization of the study. Chapter two is the review of related literature. Chapter three focused on the methodology and the proposed model. Chapter four deals data collection and analysis. Chapter five focuses on summary, conclusion, recommendation and suggestion for further study.

## Chapter 2

### LITERATURE REVIEW

This chapter deals with the review of related literature on production scheduling expressed by different authorities in industrial practices.

The history of production scheduling in manufacturing facilities over the last hundred(100) year was discussed by (Herrman, 2006). This shows how production scheduling has been improved through critical analysis of existing scheduling system and the approaches to be used and its timeless advantage. The tools used to support decision-making in real-word production and others.

Computer software can be helpful, the first chart developed by Henry Gantt(1973) to advance scheduling system, his finding has helped engineers, production schedulers and researchers comprehend the true nature of production scheduling in dynamic manufacturing systems and to encourage them to research on how production scheduling can be improved. In the 1980s, IBM developed the Logistics Management System Fordyce et al. (1992) an innovative scheduling system for semi-conductor manufacturing facilities that was used at six IBM facilities and by some customers(Fordyce, 2006).

Pinedo(2005) and McKay and Wiers(2004) provided practical guidelines on selecting and implementing scheduling software. Meanwhile, information technology is not really the answer. Based on their survey of hundreds of manufacturing facilities LaForge and Craighead (1998) conclude that computer-based scheduling can assist manufacturers improve on time delivery, respond quickly to the orders of customers and create realistic schedules but success demands using finite scheduling techniques and integrating them with other

manufacturing planning system. Finite scheduling uses actual shop floor condition, including capacity constraints and the demanding of orders that have already been released.

According to Vollmann et al.(1988) scheduling is defined as allocating a set of tasks within a given time frame. In production system, this concerns with the allocation of a set of machines to perform a set of task within a certain time frame. To them the result of scheduling is a schedule, which can be explained as a plan with references to the sequence of and time allocated for each item or operation necessary to its completion.

Lee and Liman (1993) consider the scheduling problem of minimizing total completion time with two parallel machines where one of them is always available and the other one is available in time zero up to a fixed point in time where feasible solutions can be obtained.

Baker (1974), makes it known that most work on production scheduling focused on developing an algorithm and method for identifying the optimal sequence of the expected tasks considering either only one machine or multiple machines. The optimal schedule is achieved in these scheduling method base on a certain expected goal such as to minimize the total mean flow of these selected tasks.

According to Pfund and Scott (2006) one of the most difficult manufacturing environments is wafer fabrication facilities. Such facilities represent the most costly and time-consuming aspect of semi-conductor manufacturing process. The outcome of the survey of semi-conductor manufacturers focused on the resent state of the practical and future needs were given after a short introduction to wafer fabrication operations. The review of some current dispatching approaches

were presented. They went ahead to give the overview of recent scheduling approaches.

Lodree and Norman(2006) summarized research relating to scheduling to scheduling personnel with the objective to optimize system performance and personnel well-being were considered. Such topics like job rotation, cross-training, work rest scheduling, task learning, mathematical models and best practices were also discussed.

Considering manufacturing facilities, production schedules declare when certain controllable activities like processing of job by resource should take effect. Production schedules gather activities to increase production and minimize operating cost. Managers can identify resources conflicts, control the release of job to the shop, ensures that raw materials are ordered in time, show whether delivery promises can be fulfilled and identify periods available for preventive maintenance when production schedules are used. The two main problems in production scheduling "priorities" and 'capacity"(Wight,1984). In other words, "what should be done first?" and "who should do it?". These mind blowing questions are answered by Cox et al. (1992), "the actual assignment of starting and/or completion dates to operations or group of operations to show when these must be done if the manufacturing order is to be completed on time". This can be also known as shop scheduling,detailed scheduling, operation scheduling and order scheduling.

Production scheduling systems depend on human decision-makers and many of them need help dealing with the complexities of real-world scheduling(Mckay and Wiers, 2005). Many manufactures have feeble production scheduling system. They produce goods and transport them to their customers through the use of broken collection of independent plans that are mostly ignored, unreliable

information is shared during most of their meetings,expeditors who run from one crisis to another and ad-hoc decision made by persons who cannot see the entire system.

Academic research on production scheduling has presented countless papers on the topic. Pinedo and Chao (1999)lists a number of important surveys on production scheduling. Vieira et al. (2003) presented a framework for rescheduling and Leung(2004) covers both the fundamentals and the most resent advances in a variety of scheduling research topics. Nevertheless, there are difficulties in applying these results because real-world situations mostly do not match the assumptions made by scheduling researchers (Dudek et al., 1992).

The increase number of studies of production scheduling in industrial practice have led to the development of a business process perspective that considers the knowledge management and organization aspects of production scheduling (MacCarthy, 2006).

Frederick W.Taylor's important contribution to production scheduling was his invention of the planning office(describe Taylor, 1911). His separation of planning from execution justified the use of formal scheduling methods, which became critical as manufacturing organizations grew in complexity. It shows the view that production scheduling is a distinct decision-making process in which individuals share information, make plans and react to unexpected events. An interesting feature of the planning office was the bulletin board. There was one in the planning office and another on the shop floor (Thompson, 1917). The bulletin board had space for every workstation in the shop. The board showed for each workstation, the operation that the workstation was currently performing, the orders currently waiting and future order that would eventually need processing. It was an important resource for sharing information about scheduling decision to many people. Many firms implemented versions of Taylor's production planning office, which perform routing, dispatching and scheduling, "the timing of all

operations with a view to insuring their completion when required ”(Mitchell, 1939). The widespread adoption of Taylor’s approach reflects the importance of the organizational perspective of scheduling, a system level view that scheduling is part of the complex flow of information and decision-making that forms the manufacturing planning and control system (Herrmann, 2004; MacCarthy, 2006). The rise of information technology did not eliminate the planning functions defined by Taylor, it simply automated them using over more complex software that is typically divided into modules that perform the different functions more quickly and accurately than Taylor’s clerks could, see (Vollmann and Berry, 1997).

Johnson analyzed the properties of an optimal solution and presented an elegant algorithm that constructs an optimal solution. The published paper (Johnson, 1954) not only analyzed the two-stage flow shop scheduling problem but also considered problems with three or more stages and identified a special case for the three-stage problem.

Jackson(1956) generalized Johnson’s results for a two-machine job shop scheduling problem. Smith(1956) considered some single-machine scheduling problems with due dates. Both of these early, notable works cited Johnson’s paper and used the same type of analysis. Bellman et al. (1954) addressed a slightly simplified version with a different approach while employing Johnson,s results.

Conway et al.(1967) describe Johnson’s paper as a vital influence, as it was ”perhaps the most frequently cited paper in the field of scheduling”. In particular they noted the importance of its proof that the solution algorithm was optimal. Johnson’s paper ”set a wave of research in motion”(Dudek et al., 1992).

Johnson’s paper summarizes the problem-solving perspective, in which scheduling is an optimization problem that must be solved. A good number



of effort has been spent developing methods to generate optimal production schedules, and countless papers discussing this topic have appeared in scholarly journals and articles.

Though, there exists a significant difference between scheduling theory and practice (as discussed by Dudek et al., 1992; Portougal and Robb, 2000) good problem-solving have been used by researchers to improve real-world production scheduling in some settings(as in Zweben and Fox, 1994; Dawande et al., 2004; Newman et al., 2006). This might be that the results of production scheduling theory are applicable in some, but not all production sites (Portougal et al., 2000).

Artificial intelligence (AI) has provided a good foundation for modeling and solving the scheduling problem: artificial intelligence research has significantly provided a successful way in solving complex problems in a number of scientific areas. Particularly artificial intelligence was expected to be able to capturing formerly intangible human decision behavior in scheduling. The potential use of artificial intelligence in scheduling is advocated by comparing operations research and artificial intelligence methods in the context of developing a scheduling system for repairing job shop scheduling. Artificial intelligence techniques, modeling human expertise, turn out to be useful to develop more efficient search strategies than would have been possible with operations research techniques. The applicability of expert systems to job shop scheduling is also investigated by Randhawa et al.(1990). The problem of job shop scheduling is described from two perspectives: industry and academia. Industry has generally focused on pragmatic approaches to job shop scheduling, such as just-in-time(JIT), Manufacturing Resource Planning(MRP), and Optimized Production Technology(OPT).

Academia has attempted to solve the job shop scheduling problem by mathematical approaches or to predict system performance by using simulation.

Randhawa and McDowell stated that these efforts from academia show that mathematical techniques are not suited for solving real-world problems. They also discuss the potential benefits of artificial intelligence techniques because of the limited applicable use of operations research techniques in job shop scheduling. However, from other reports on the applicable use of artificial intelligence in scheduling in practice, it can be concluded that the same problems that triggered the implementation of scheduling techniques from operations research in practice, also arise in the application of artificial intelligence to production scheduling.

A good number of existing expert systems for scheduling and issues that should be taken into account when developing expert systems for job shop scheduling were listed by (Kathawal et al., 1996). The problem solving domain should be well understood, stable and not subject to negotiation. Again, human experts should be available and willing to cooperate; they should fear losing their jobs and obstruct expert systems development. Also, the costs of expert systems, which can become very high, should be evaluated carefully against the potential profits.

In Kanet et al.(1987), the applicability of expert systems to production scheduling is discussed. A state of the art review is given, along with the remark that the area of expert systems in production scheduling is still in its basic level. They indicated that in order to include sole imitating of human scheduling behavior, successful scheduling systems of the future should be able to enumerate more alternatives than a human scheduler and be able to learn from experience. This leads to the observation that artificial intelligence not only inherited problems of operations research, but that some additional pitfalls were introduced as well.



This is illustrated in the work of Randhawa et al.(1990), who indicate that a prerequisite for developing an expert system for production scheduling is the availability of expert knowledge. Unfortunately, this knowledge is dispersed among operators, foremen, supervisors, schedulers, and so on (Patten, 1968). They visualized tackling this problem by simulating the job shop and training experts through simulations.

Fox (1983) initiated a research on intelligent scheduling for solving production problems considering real world constraints. In the research, constraints were used for guiding the direction of search to identify the feasible and the optimal schedules. Ever since, many researches on constraint-based scheduling have been carried out(Zweben and Fox, 1994). The methodologies of intelligent scheduling are classified into two main categories: constructive approach and repair-approach. The constructive approach achieves a complete schedule gradually from a partial schedule using constraints as guidance (Fox et al., 1989). The repair-based method on the other hand, start with a complete schedule and modifies it through iterations towards the optimal solution (Zweben et al., 1992). Both approaches aim at identifying the optimal schedules considering the demanding constraints through iterative search process. Most advanced computing techniques including genetic algorithm(Goldberg, 1989), tabu-search (Glover, 1989), can also be employed to improve the efficiency of scheduling while maintaining the quality of the schedule created. The review of the main contributions to the area of deterministic scheduling problems, with emphasis on the classical models was presented by (Lawler et al., 1993) and (Hoogeveen et al., 1997). Most of the references are on theoretical work, and with respect to setup times, the only references are on sequence-independent batch setup times for the single-machine scheduling problem. Several works have been published for the single-machine scheduling problems with sequence-independent batch setup times where different performance measures were taking into account.

Bruno and Downey(1978), Mason and Anderson(1991), Monma and Potts(1989), Williams and Wirth(1996), Gupta(1988) and Zdrzalka(1992) discussed single machine problems. A computer simulation model for a limited machine job shop scheduling problem with sequence-dependent setup times was presented by(Kim and Bobrowski, 1994). They study the influence of setup times and due date's information in priority rules performance for job-shop problem with setup times.

Ovacik and Uzsoy(1994) presented a family of rolling horizon heuristics to minimize the maximum lateness on a single machine in the presence of sequence-dependent setup times. They also presented a survey on the work done on this scheduling problem.

Laguna(1997) presents a heuristic procedure to a realistic production and inventory control problem with sequence-dependent setup times. The heuristic is based on a simple short-term tabu search coordinated with a linear programming and traveling salesperson solvers to guide the search.

Ríos-Mercado and Bard(1997) present a branch-and-bound enumeration method scheme for the make span minimization of the flow-shop scheduling problem with sequence-dependent setup times.

Furthermore, Production planning and scheduling for the two-stage parallel flow shop problem is an intricate procedure. Caie et al.(1980) modeled an injection molding production planning problem as a mixed binary integer programming problem, with the objective function defined as the sum of setup cost and holding costs and overtime costs over the planning horizon.

Van Wassenhove and De Bodt(1983) then described a case study of injection molding. Using machine mold compatibility, the problem domain is break down into five subproblems. And each subproblem is considered as a single

machine problem, which is solved by heuristic procedures of Lambrecht and Vandervcken(1979), and Dixon and Silver(1980), both of which are modified versions of a heuristic proposed by Eisenhut(1975). All of them did not consider any shortage or backorder costs.

The three industrial scheduling problems in manufacturing systems were discussed by Kusiak (1992). The first problem is the single machine scheduling problem with sequence-dependent setup times and precedence constraints. A mixed integer formulation was proposed. The second problem is a machine cell scheduling problem. A new dispatching rule was then developed to minimize the total tool setup time. The third problem is concerned with scheduling laser cutting operations. An integer programming formulation was proposed.

Nam and Logendran(1995) analyzed some switching rules for aggregate production planning problems. Depending on the net amount of a product to be produced, the rules specify whether the production rate should be high, normal, or medium. The rules give some simple, practical approaches to the managers for decision making, but they are applicable only to single-product problems.

Kalpic et al.(1995) described a multi-period, multi-criteria production planning problem in a thermoplastic factory. Two objectives, such as, financial contribution and duration of the longest resource engagement were considered. The model was formulated and solved as a linear programming problem, with proper weights given to the objectives. A goal programming variation was also applied and the two methods were compared. A production planning and scheduling model for injection molding of PVC pipe fittings was proposed by (Nagarur et al., 1997). The aim was to minimize the total costs of production, inventory and shortages. A goal programming method is used to generate the solution.

Golovin(1997) proposed a linear programming based production scheduling model with an objective function of minimizing the total cost, including cost of setups, holding cost of inventory, production cost and cost of additional resources (overtime).

Westenberger and Kallrath(1994) formulated a typical but generic scheduling problem with the objective to push the development of algorithms for scheduling problems in process industry. The proposal was to establish a working group to develop standardized benchmark problems for planning and scheduling in the chemical industry initiated many research projects and activities. Their problem has been understood as a typical scheduling problem occurring in process industry including the major characteristics of a real batch production process(involving multi-product facilities, multi-stage production, combined divergent and convergent product flows, variable batch sizes, non-preemptive processes, shared intermediates, alternative recipes, flexible proportions of output products, blending processes, sequence and usage dependent cleaning operations, finite intermediate storage, cyclic material flows, re-usage of carrier substances, and no-wait production for certain types of products) to encourage researchers and engineers to test their algorithms and software tools by applying them to this test.

In transportation systems, crew scheduling and integrated vehicle are regular research areas. Benders decomposition scheme to solve aircraft routing and crew scheduling problems was proposed by (Cordeau et al., 2001). They made use of a set partitioning formulation for both the aircraft routing and the crew scheduling. With the first scheme, the primal sub-problem involves only crew scheduling variables and the master problem involves only aircraft routing variable. Both problems relaxation are solved by column generation. Integer solutions are found by a three-phase method, adding progressively the integrity constraints.

Recently, Mercier et al. (2005) have improved the hardness of the proposed model. Their method reverses the Benders decomposition proposed in(2001) by considering the crew scheduling problem as the master problem.

Haase and Friberg (1999) proposed a method to solve bus and driver scheduling problems. The problem was formulated as a set partitioning problem with additional constraints in which a column represents either a schedule for a crew or for a vehicle. Additional constraints were introduced to connect both schedule types. A branch-and-price-and-cut algorithm is proposed in which column generation is performed to generate both vehicle and crew schedules. The method was improved in(2001) by a set partitioning formulation only for the driver scheduling problem that incorporates side constraints for the bus itineraries. These side constraints ensure that a feasible vehicle schedule can be derived afterwards in polynomial time. Moreover, the inclusion of vehicle costs in this extended crew scheduling formulation ensures the overall optimality of the proposed two phase crew-first, vehicle-second approach.

A method to solve bus and driver scheduling problems on individual bus lines was proposed by(Freling et al., 2003). The proposal deal with a formulation that mixes the set partitioning formulation for crew scheduling and the assignment formulation for the vehicle scheduling problem. They compute lower bound and feasible solutions by combining Lagrangian relaxation and column generation. The constraints involving the current columns are relaxed in a Lagrangian way. The obtained Lagrangian dual problem is a single-depot vehicle scheduling problem(SDVSP). Once the Lagrangian relaxation is solved a new set of columns with negative reduced costs is generated. The method is iterated until the gap between the so-computed lower bound and an estimated lower bound is small enough. Feasible solutions are generated from the last feasible SDVSP and the current set of columns.

Moreover, specific employee scheduling problems in production scheduling are often tackled by considering if the job schedule is fixed. Valls et al.(1996) consider a fixed schedule in a multi-machine environment and consider the problem of finding the minimal number of workers. The problem is formulated as a restricted vertex coloring problem and a branch and bound algorithm is presented. A large part of work involving both job scheduling and employee timetabling aims at keeping the number of required employees at each time period under a threshold without considering the regulation constraints of employee schedules nor the individual preferences and skills of employees.

Daniel and Mazzolla (1994) analyze a flow-shop problem in which the duration of an operation depends on the selected mode to process an operation. Each mode defines a number of resources(workers) needed during the processing of the operation. The scheduling horizon is discretized in periods and at each time period, the number of workers cannot exceed a fixed number. Heuristic and Optimal approaches are proposed.

Bailey et al. (1995) and Alfare and Bailey (1997) propose an integrated model and a heuristic for project task and manpower scheduling where the aim is to find a trade-off between labor cost and daily overhead project cost. The labor cost therefore depends on the number of employed workers at each time period. The project duration determines the daily overhead cost. There are no machine constraints and the labor restrictions consist in setting a maximal number of workers per period.

Faaland and Schmitt (1993) proposed an assembly shop with multiple workstations. Each task must be performed on a given workstation by a worker. There are production and late-delivery costs on one hand and labor cost linked to the total number of employees on the other hand. The authors then study



the benefits of cross-training which allows employees to have requisite skills for several work-centers. A heuristic based on a priority rule and on the shifting bottleneck procedure is proposed.

Daniel et al. (2004) studied a general problem and extend the model proposed in(1994) to an individual representation of employees in a flow-shop environment. Where each employee has the requisite skills for only a subset of machines and can be assigned to a single machine at each period. The time period of a job operation depends on the number of employees assigned to its machine during its processing. Employees assigned to an operation are required during all its processing time. No schedule regulations are considered except unavailability periods. A branch and bound method is developed and the benefit of the level of worker flexibility for make span minimization is designed.

There has been a general model for integrating production scheduling and employee timetabling, based on the concepts of load center, configuration, employee assignment and sequence. This model allows one to represent the simultaneous work of an employee on several machines. Moreover, the computation method of the job durations performed simultaneously by the same operator is not provided. The authors provide two examples of integrated resolution in a flow-shop context. In the first example, they propose a dynamic programming algorithm to find a feasible path in the configuration graph with a fixed number of equivalent operators and a fixed sequence of jobs. Whist in the second example they propose a heuristic and a lower bound of the make span in a flow-shop where the timetabling problem is reduced to the assignment of an employee to each machine, the duration of the jobs depending of the employee performance and skills.

More so, Drezet and Billaut (2005) proposed a project scheduling problem

with human resources and time-dependent activities requirements. Employees have different skills and the main legal constraints dictated by the workforce legislation have to be considered. The model was quite general and only human resources are considered since the considered context is not a production scheduling problem where machines are critical resources. A tabu search method was proposed as well as proactive scheduling techniques to deal with the uncertainty of the problem.

The economic lot scheduling problem has been studied by different researchers for over 40 years and more than 100 papers have been published in a varied journals. The earliest contributions to this problem include (Roger, 1958) and Hanssmann(1962). Independent solution(IS) is an approach for obtaining a lower bound on the minimum average cost by taking each product in isolation and calculating economic production quantities. This ignores the capacity issue of the sharing of the machine by several products.

Bomberger (1966) suggested a tight lower bound and this has been rediscovered in several different ways by several researchers((Dobson, 1987), (Moon and Christy, 1998)). The idea is to compute economic production quantities under a constraint on the capacity of a machine. The capacity constraint is that enough time must be made available for set-ups. The problem can be formulated as a non-linear program and easily solved via a line search algorithm. Nonetheless, the adjustment constraint, stating that no two items can be scheduled to produce at the same time, is ignored. Thus, the value of the non-linear program results in a lower bound on the minimum average cost. Several research on economic lot scheduling problem(ELSP) focused on cyclic schedules. More so, almost all researchers have limited their attention to cyclic schedules that satisfy the zero switch rule(ZSR). The rule states that a production run for any particular product can be started only if its physical inventory is zero. Several examples



to the optimality of this rule have been found in (Maxwell (1964), Delporte and Thomas (1978)) but are rare. The objective value obtained from this approach serves as the upper bound on the general ELSP.

Therefore, Jones and Inman (1989) and Gallego(1990) proved that this approach works well under certain situations. There are two other heuristics approaches for the ELSP: the basic period approach and the time-varying lot sizes approach. The basic period approach, in addition to the ZSR, requires that every item must be produced at equally spaced intervals that are multiples of a basic period. Most heuristic algorithms that follow this approach first select the frequency i.e number of production runs per cycle with which each product is to be produced, and then search for a feasible schedule that implements these frequencies (Doll and Whybark, 1973). The time-varying lot sizes approach, which relaxes the restriction of equally spaced production runs, was initiated by Maxwell(1964) and Delporte and Thomas(1978). Furthermore, Dobson (1992) made it known that any production sequence i.e the order in which the products are produced in a cycle can be converted into a feasible production schedule in which the quantities and timing of production lots are not necessarily equal provided that, beyond the time needed for production, there is sometime available for setups. He also developed a heuristic to generate production frequencies and a reasonable production sequence. Optimal schedules can be obtained by combining Dobson's heuristic with Zipkin's(1991) algorithm which finds the production run times and machine idle times for each product for a given production sequence.

Moreover, Gallego and Roundy(1992) extended the time-varying lot sizes approach to the ELSP which allows backorders. Gallego and Shaw(1997) showed that the ELSP is strongly NP-hard under the time-varying lot sizes approach with or without the ZSR restriction, by giving theoretical justification to the

development of heuristics.

Silver (1993) in his review, stated that, if quantitative models are to be more useful as aids for managerial decision-making, they must represent more realistic problem formulations, particularly permitting some of the usual (givens) to be treated as decision variables. Givens can therefore be explained as the parameters which have been treated as fixed or given, for instance, setup time, setup cost, production rate, defective rate, etc. Silver (1995) listed a wide variety of possible improvements to undertake in manufacturing operations, such as set-up time or cost reduction, higher quality level, controllable production rates, lead time reduction. There is a rapid growing literature on modeling the effects of changing the givens in manufacturing decisions. In the domain of changing the givens, a variety of change on the ELSP have been developed (Silver et al., 1998).

A modification was made which allows production rates to be decision variables(Allen, 1990). Allen then developed a graphical method for the rates and cycle times for a two-product problem. The following researches also showed that production rate reduction was more profitable for underutilized facilities: Silver(1990), Moon et al.(1991), Gallego (1993), Khouza et al. (1998), and Moon and Christy(1998). Another group, Silver(1995) and Viswanathan and Goyal (1997) considered the situation in which a family of products follows a cyclic schedule, but there is a limit on shelf life. The cycle length and production rate are altered to ensure a feasible schedule.

Again, Gallego and Moon(1992) examined a multiple product factory that employs a cyclic schedule to minimize holding and set-up costs. Set-up times can be reduced, at the expense of set-up costs, by externalizing internal set-up operations, they further showed that dramatic savings are possible for highly

utilized facilities. Gallego and Moon (1995) developed an ELSP with the assumptions that set-up times can be reduced by a one-time investment. Hwang et al. (1993) developed an ELSP in which both set-up reduction and quality improvement can be achieved through investment. Moon et al.(1998) recently applied the stabilization period concept, in which yield rates gradually increase during the period, to the ELSP.

Boukas and Haurie (1990) present a model for a continuous-time stochastic flow control for production scheduling with preventive maintenance. In this treatment, the failure and preventive maintenance rates depend on the operational age of the workstation, which is defined as the time since the last repair or preventive maintenance. A control variable is used to determine when to perform preventive maintenance in order to avoid failures in an optimal way.



## Chapter 3

### METHODOLOGY

#### 3.1 INTRODUCTION

In this chapter, we shall put forward an algorithm for solving production problems that would be modeled as a transportation problem. The transportation problem seeks the determination of a shipping plan of a single commodity from a number of source ( $m$ , say) to a number of determinations ( $n$ , say) at a minimum cost while satisfying the demands of the various destinations. However, some of its important applications like the production problem/scheduling problems absolutely have nothing to do with transportation. The problem of balancing costs of regular and/or overtime production and inventory storage to minimize the total cost of meeting given sales requirements can be treated as a transportation problem.

#### 3.2 The Model and Optimization Task

The production problem shall be modeled as a balanced transportation problem by noting the time period during which production occurs at source  $S_1, S_2 \dots, S_m$  and the periods in which units of the commodity will be shipped to the warehouses/destinations. The capacity of the source  $S_i$  in a given period is  $a_i$  and the demand at the warehouse  $W_j$  is  $b_j$ .

The production cost per unit during time period 'i' and the storage cost per unit from time period 'j' is denoted by  $C_{ij}$ . The main challenge is to find a production schedule, which will meet all demands at a minimum cost while satisfying all constraints of production capacity and demands. To find a solution, let  $X_{ij}$

denote the amount of commodity to be produced per period from source  $S_i$  to destination  $W_j$ ,  $i = 1, 2, \dots, m$  and  $j = 1, 2, \dots, n$ . Then  $X_{ij} \geq 0$  for all 'i' and 'j'. For each 'i', the total amount of commodity produced at source  $S_i$  is

$$\sum_{j=1}^n X_{ij} \quad (3.1)$$

Considering a set of 'm' supply points from which a unit of the product is produced. Since supply point  $K_i$  can supply at most  $a_i$  units in any given period, we have

$$\sum_{j=1}^n X_{ij} \leq a_i, i = 1, 2, \dots, m \quad (3.2)$$

Also considering a set of 'n' demanded points to which the product is transported. If demand points  $W_j$  must receive  $b_j$  units of the transported products. We have

$$\sum_{i=1}^m X_{ij} \geq b_j, j = 1, 2, \dots, n \quad (3.3)$$

Since units produced cannot be transported prior to being produced,  $C_{ij}$  is excessively large for  $i > j$  to force the equivalent  $X_{ij}$  to be zero or if transporting is impossible between a given source and destination, a large cost of M is entered. Therefore, the total cost of production is given as;

$$\sum_{i=1}^m \sum_{j=1}^n C_{ij} X_{ij} \quad (3.4)$$

Hence the general formulation of a production is given as:

$$\text{Minimize } Z = \sum_{i=1}^m \sum_{j=1}^n C_{ij} X_{ij} \quad (3.5)$$

$$\text{Subject to } \sum_{j=1}^n X_{ij} \leq a_i, i = 1, 2, \dots, m \quad (3.6)$$

$$\sum_{i=1}^m X_{ij} \leq b_j, j = 1, 2, \dots, n \quad (3.7)$$

$$X_{ij} \geq 0, i = 1, 2, \dots, n, j = 1, 2, \dots, m \quad (3.8)$$

The non-negative condition  $X_{ij} \geq 0$  is included single negative value for any  $X_{ij}$  have no physical meaning.

Since the production problem is a linear programming problem, it can be solved by the Simplex method but because of its special nature, it can be solved more easily by special forms of the Simplex methods, which are more efficient for the production problem than the main Simplex method.

### 3.3 The Balanced Problem

For a production problem to be balanced, the supply and demand constraints must be equal, thus

$$\sum_{i=1}^m a_i = \sum_{j=1}^n b_j. \quad (3.9)$$

Therefore, if

$$\sum_{i=1}^m a_i \geq \sum_{j=1}^n b_j \quad (3.10)$$

then the production is said to be unbalanced. It is indicated that, a special algorithm works well for the balanced production. This cannot be a hindrance since the unbalanced problem can be modeled as an equivalent balance problem to which the special method can be applied.

All the constraints must be tried up in a balanced production problem. The available product will not be sufficient to satisfy all demands, if any supply

constraints are not tied up. The balanced production problem may therefore be written as:

$$\text{Minimize } Z = \sum_{i=1}^m \sum_{j=1}^n C_{ij} X_{ij} \quad (3.11)$$

$$\text{Subject to } \sum_{j=1}^n X_{ij} = a_i, i = 1, 2, \dots, m \quad (\text{Supply constraints}) \quad (3.12)$$

$$\sum_{i=1}^m X_{ij} = b_j, j = 1, 2, \dots, n \quad (\text{Demand constraints}) \quad (3.13)$$

Finding a basic feasible solution from a balanced production problem is easy. It is necessary to formulate a production problem as a balanced production problem. The balanced production problem can be identified clearly by the supply, demand and production cost, therefore the relevant data can be represented by the table 3.1. Since this is a minimization problem, the numbers in the upper right corner in each cell is then the unit cost. The quantities produced are shown on the right hand side of each row while demand is shown along the columns.

Note that:

- i. The coefficient of each variable  $X_{ij}$  in each constraint is either 1 or 0.
- i. The constant on the Right Hand Side(RHS) of each constraint is a positive integer.
- i. Each variable  $X_{ij}$  appears only twice in the constraints.

Again, it can be observed that any Linear Programming problem with the above properties satisfies the following: if the problem has a feasible solution in which all the variables are integers. It is on this property that the modification of the Simplex method that provides efficient solution for the algorithm is based.

We observe that the (m+n) conditions:



Table 3.1: The Transportation Tableau

Warehouse							Supply
		$W_1$	$W_2$	$W_3$	.....	$W_n$	
$S_1$		$c_{11}$	$c_{12}$	$c_{13}$		$c_{1n}$	$a_1$
	$X_{11}$		$X_{12}$	$X_{13}$	.....	$X_{1n}$	
$S_2$		$c_{21}$	$c_{22}$	$c_{23}$		$c_{2n}$	$a_2$
	$X_{21}$		$X_{22}$	$X_{23}$	.....	$X_{2n}$	
$S_3$		$c_{31}$	$c_{32}$	$c_{33}$		$c_{3n}$	$a_3$
	$X_{31}$		$X_{32}$	$X_{33}$	.....	$X_{3n}$	
$\vdots$					.....		$\vdots$
$S_m$		$c_{m1}$	$c_{m2}$	$c_{m3}$		$c_{mn}$	$a_m$
	$X_{m1}$		$X_{m2}$	$X_{m3}$	.....	$X_{mn}$	
Demand		$b_1$	$b_2$	$b_3$	.....	$b_n$	

$$\sum_{i=1}^m X_{ij} = b_j, 1 \leq j \leq n \quad (3.14)$$

$$\sum_{j=1}^n X_{ij} = a_i, 1 \leq i \leq m \quad (3.15)$$

are independent since

$$\sum_{i=1}^m b_j = \sum_{j=1}^n \sum_{i=1}^m X_{ij} = \sum_{j=1}^n a_i \quad (3.16)$$

Therefore, the effective number of constraints on the balanced production problem is  $(m+n-1)$ . We expect a basic feasible solution of the balanced production problem to have  $(m+n-1)$  non-negative entries.



## 3.4 Finding an Initial Basic Feasible Solution (IBFS)

There are three methods that can be used to find the initial basic feasible solution for a balanced transportation problem. These are:

- i. The Northwest corner Methods
- i. The Least Cost Methods and
- i. The Vogel's Approximation Methods (VAM)

The result obtained under each of the three methods may not be optimal.

### 3.4.1 The North West Corner Rule

Considering this method, the entry in the upper left hand corner (Northwest corner) of the transportation balance tableau must be chosen, (i.e. the shipment from source 1 to warehouse 1). This must be used to supply as much of the demand at  $W_1$  as possible. The shipment should be recorded with a circle in the cell. If the supply at  $S_1$  is not used up by the allocation, the remaining supply should be used to fill the remaining demands at  $W_2, W_3, \dots$  in that order until supply at  $S_1$  is used up, then all shipments in circles in the appropriate cells recorded. When one supply is used up, the next supply follows and start filling the demands beginning with the first warehouse in that row, where there is still a demand unfilled, recording in circle numbers all allocations.

In few cases degeneration situation arises and the solution is not a Basic Feasible Solution (BFS) because it has fewer than  $(m+n-1)$  cell in the solution. This usually happens because at some point during the allocation when a supply is used up, there is no cell with unfulfilled demand in the column. In non-degeneration cases, until the end, whenever a supply is used up, there is always an unfulfilled demand in the column.

In the case of degeneracy, the Northwest corner method still yield a BFS, if it is modified as follows: after obtaining a solution, which is not basic, some empty cells should be chosen and the solution with circled zero(s) in them should be added to produce a BFS, i.e.,

- The total number of cells with allocations should be  $(m+n-1)$ .
- There should be no circuit among the cells of the solution.

The Northwest corner method does not utilize production cost per unit, so it can yield an initial basic feasible solution that has a very high production cost.

### 3.4.2 The Least Cost Method

Determining an optimal solution may require several pivots or iterations. The least cost method uses the production costs in an effort to produce a basic feasible solution that has a lower total cost. Fewer iterations will then be required to find optimal solution for the problem. With this method, the variable with the least unit cost ( $C_{ij}$ ) is found, then assign the largest possible value ( $X_{ij}$ ), the  $(a_i \ b_j)$ . Satisfied row or column is crossed out.

The next least weight cost is identified and assigned to its cell, without violating any of the supply or demand constraints. This procedure is continued until all row and columns have been added. If there are more than one per unit cost, break them arbitrary.

### 3.4.3 Vogel's Approximation Method

Vogel's deals with penalties, the column penalties should be computed for each column by identifying the least unit cost and the next least unit cost in that column and take either positive difference. In the same way, the row penalties for each row must be computed as the positive difference between the least unit cost and the next least unit cost in that row. The method is a variant of the minimum cost method and based on the idea that if for some reasons the allocation cannot

be made to the least unit cost cell via row or column, then, it is made to the next least cost cell in that row or column and the appropriate penalty paid for not being able to make the best allocation. The cell for which the value of the row and column penalties is greatest must be looked for. Then allocate as much as possible cell the row supply and column demand will allow. This shows either a supply is used up or a demand is satisfied. In either case cross out the row of the used up supply or the column of the satisfied demand. New row and column penalties for the remaining rows and columns must be calculated for and the process repeated until a BFS is found.

The least cost method provides a BFS, which is closer to optimal and performs better than the Northwest corner method. The least cost method may lead to an allocation with fewer than  $(m+n-1)$  non-empty cells even in the non-degenerate case unlike the Northwest corner method. To get the right number of cells in the solution, enough zero entries are added to empty cells, avoiding the generation of circuits among the cells in the solution.

NB: All the three methods discussed above can be used to obtain a Basic Feasible Solution but the Vogel's Approximation Method would be used for this work since it yields the best BFS as compared to the other two methods.

### 3.5 Improving the solution to optimality

To solve the transportation problem, a basic feasible solution is found by any of the methods discussed earlier, then the optimal solution can be found by the following two methods.

- i. The steppingstone Method
- i. The Modified Distribution Method (MODI)

### 3.5.1 The Steppingstone Method

This method requires an initial basic feasible solution, which is then improved to optimality. Assuming there is a basic feasible solution of such problem consisting of non-negative allocation in  $(m+n-1)$  cells. The cells which are not in the basic feasible solution are called unoccupied cells. And it must be noted that for each unoccupied cell, there is a unique circuit beginning and ending in that cell consisting of that unoccupied cell and other cells all of which are occupied in such a way that each row or column in the tableau either contains two or none of the cells of the circuit. To use this method make sure there is no loop.

#### Test for Optimality

Testing for the current basic feasible solution for optimality, each of the unoccupied cells must be taken in turns and one(1) unit allocation is placed in it. This can be introduced by just the plus(+) sign knowing the special circuit containing this cell, the signs '-' and '+' are placed alternately until all the cells of the circuit are covered. Having known the unit cost of each cell, the total change in cost produced by the allocation of one unit in the empty cell is computed and the corresponding placements in the other cells of the circuit. The improvement index of the unoccupied cell is the change in cost. If the improvement index of each unoccupied cell in the given basic feasible solution is non-negative, then the current basic feasible solution is optimal since every reallocation increases the cost. If there is at least one unoccupied cell with a negative improvement index then a reallocation to produce a new basic feasible solution decreases cost and in effect the current basic feasible solution is not optimal. Therefore, the current basic feasible solution is optimal only if each unoccupied cell has a non-negative improvement index and so on.

## Improvement to Optimality

If there are one or more unoccupied cell in a given basic feasible solution which has a negative improvement index, then the basic feasible solution is not optimal. Therefore, to improve on such solution, the unoccupied cell with the most negative improvement index of assuming  $N$  is found, using the circuit that was used in the calculation of its improvement index, the least allocation in the cells of the circuit with the sign '-' found. This must be noted as smallest allocation 'z'. 'K' is subtracted from the allocation in all the cells in the circuit with the '-' sign, then it is added to all the allocations in the cells in the circuit with the '+' sign. This in effect will satisfy the constraints on demand and supply in the transportation tableau. Since the cell which carried the allocation 'k' now has zero allocation, it is then deleted from the solution and is replaced by the cell in the circuit which was originally unoccupied and now has an allocation 'k'. The result of each reallocation is new basic feasible solution. The cost of the new basic feasible solution is  $N$  less than the cost of the previous basic feasible solution. The new basic feasible solution is then tested for optimality and the whole procedure is repeated until an optimal solution is realised.

### 3.5.2 The Modified Distribution Method (MODI)

Take into account the balance production problem below:

If a basic feasible solution is obtained, then  $(m+n-1)$  cells are occupied.

#### Test for Optimality

A new index  $u_i$ , and column index  $v_j$  are computed for each occupied cell  $(i,j)$  of the transportation tableau, such that  $C_{ij} = U_i - V_j$ .

Since there are  $(m+n-1)$  occupied cells, it follows that there are  $(m+n-1)$  of these equations. There are  $(m+n)$  row and column indices altogether, and it also follows that by prescribing any arbitrary value for one of them, like  $U=0$ , then equations for the remaining  $(m+n-1)$  unknown  $U_i, V_j$  are solved. With all the

Table 3.2: The Transportation Tableau

Warehouse							Supply
		$W_1$	$W_2$	$W_3$	.....	$W_n$	
$S_1$		$c_{11}$	$c_{12}$	$c_{13}$		$c_{1n}$	$a_1$
	$X_{11}$		$X_{12}$	$X_{13}$	.....	$X_{1n}$	
$S_2$		$c_{21}$	$c_{22}$	$c_{23}$		$c_{2n}$	$a_2$
	$X_{21}$		$X_{22}$	$X_{23}$	.....	$X_{2n}$	
$S_3$		$c_{31}$	$c_{32}$	$c_{33}$		$c_{3n}$	$a_3$
	$X_{31}$		$X_{32}$	$X_{33}$	.....	$X_{3n}$	
$\vdots$					.....		$\vdots$
$S_m$		$c_{m1}$	$c_{m2}$	$c_{m3}$		$c_{mn}$	$a_m$
	$X_{m1}$		$X_{m2}$	$X_{m3}$	.....	$X_{mn}$	
Demand		$b_1$	$b_2$	$b_3$	.....	$b_n$	

$U_i, V_j$  known, each unoccupied cell must be computed such that the evaluation factor  $e_{st} = C_{st} - U_s - V_t$ .

The evaluation factors are the relative cost factors corresponding to the non-basic variables when the simplex method is applied to the transportation problem. The current basic feasible solution is then applied if and only if  $e_{st} \geq 0$  for all unoccupied cells (s,t), since the production problem is a minimization problem. if there are unoccupied cell with negative evaluation factors, then the current basic feasible solution is not optimal and needs to be improved.

### Improvement to Optimality

To improve the current non-negative basic feasible solution, find the unoccupied cell with the most negative evaluation factor then construct its circuit and adjust the values of the allocation in the cells of the circuit in exactly the same way as was done in the steppingstone method. This will then yield a new basic feasible solution. With a new basic feasible solution available, the whole process

is repeated until optimality is obtained.

NB: The fact that the circuit is not constructed for every unoccupied cell makes the Modified Distribution Method more efficient than the Steppingstone Method. The MODI method is currently the most efficient method of solving the transportation problem.

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## Chapter 4

### DATA COLLECTION AND ANALYSIS

#### 4.1 INTRODUCTION

This chapter will focus on computational procedure, data analysis and finding an optimal schedule. The production problem entails the manufacturing of a single product, which can be stored or transported.

#### 4.2 Computational Procedure and Data Analysis

The production plan for the financial year January-December 2013 is shown in Table1. The cost per box of the 0.60L bottle water is GH¢10.50 and with a unit cost of GH¢0.5 per a bottle of 0.60L water.

Table 4.1: Production Plan for the firm -2013(in GH¢)

Month	Demand	Produce
January	9200	8584
February	7200	5052
March	6000	9658
April	12000	9477
May	13052	14308
June	10000	9955
July	8000	9952
August	11250	9731
September	10314	8984
October	11800	11660
November	12050	9148
December	10113	11066

The firm decides on how much to be produced based on the demand taking into account the resources availability. Production is carried out throughout the day and goods produce cannot be allocated prior to being produced. Goods produced

in a particular month are allocated to the demands in the month ahead. Due to this there will be a holding cost. This is a cost of carrying one unit of inventory cost due to the possibility of spoilage, theft or disuse. The most vital component of holding is the opportunity cost incurred by binding up capital with inventory. Production takes place on regular bases for each month. Each of these month is a source, since it can be used to satisfy demands, the firm incurred total production cost of GH¢61719.3 for producing 117575 of 0.60L bottle water. To ensure that no goods are used to meet demand during a month prior to their production, a cost is assigned to a cell to meet demand for a current or an earlier month. The production tableau was modeled as a transportation problem in order to minimize the total cost of production whilst satisfying demand.

### Scheduling Formulation

The formulation considers the unit cost of production,  $C_{ij}$ , the supply at  $a_i$  at source  $S_i$  and the demand  $b_j$  at destination for  $i \in (1, 2, \dots, 13)$  and  $j \in (1, 2, \dots, 12)$ . The problem is:

$$\text{Minimize } Z = \sum_{i=1}^{13} \sum_{j=1}^{12} C_{ij} X_{ij} \quad (4.1)$$

$$\text{Subject to } \sum_{j=1}^{12} X_{ij} \leq a_i, i = 1, 2, \dots, 13 \quad (4.2)$$

$$\sum_{i=1}^{13} X_{ij} \leq b_j, j = 1, 2, \dots, 12 \quad (4.3)$$

The aim is to determine the amount of  $X_{ij}$  allocated from source  $i$  to a destination  $j$ , such that the production cost

$$\sum_{i=1}^{13} \sum_{j=1}^{12} C_{ij} X_{ij} \quad (4.4)$$

is minimized. Thus, minimized

$$Z = \sum_{i=1}^{13} \sum_{j=1}^{12} C_{ij} X_{ij} \quad (4.5)$$

Subject to the following supply constraints

$$x_{11} + x_{12} + x_{13} + x_{14} + x_{15} + x_{16} + x_{17} + x_{18} + x_{19} + x_{1,10} + x_{1,11} + x_{1,12} \leq 8584$$

$$x_{21} + x_{22} + x_{23} + x_{24} + x_{25} + x_{26} + x_{27} + x_{28} + x_{29} + x_{2,10} + x_{2,11} + x_{2,12} \leq 5052$$

$$x_{31} + x_{32} + x_{33} + x_{34} + x_{35} + x_{36} + x_{37} + x_{38} + x_{39} + x_{3,10} + x_{3,11} + x_{3,12} \leq 9658$$

$$x_{41} + x_{42} + x_{43} + x_{44} + x_{45} + x_{46} + x_{47} + x_{48} + x_{49} + x_{4,10} + x_{4,11} + x_{4,12} \leq 9477$$

$$x_{51} + x_{52} + x_{53} + x_{54} + x_{55} + x_{56} + x_{57} + x_{58} + x_{59} + x_{5,10} + x_{5,11} + x_{5,12} \leq 14308$$

$$x_{61} + x_{62} + x_{63} + x_{64} + x_{65} + x_{66} + x_{67} + x_{68} + x_{69} + x_{6,10} + x_{6,11} + x_{6,12} \leq 9955$$

$$x_{71} + x_{72} + x_{73} + x_{74} + x_{75} + x_{76} + x_{77} + x_{78} + x_{79} + x_{7,10} + x_{7,11} + x_{7,12} \leq 9952$$

$$x_{81} + x_{82} + x_{83} + x_{84} + x_{85} + x_{86} + x_{87} + x_{88} + x_{89} + x_{8,10} + x_{8,11} + x_{8,12} \leq 9731$$

$$x_{91} + x_{92} + x_{93} + x_{94} + x_{95} + x_{96} + x_{97} + x_{98} + x_{99} + x_{9,10} + x_{9,11} + x_{9,12} \leq 8984$$

$$x_{10,1} + x_{10,2} + x_{10,3} + x_{10,4} + x_{10,5} + x_{10,6} + x_{10,7} + x_{10,8} + x_{10,9} + x_{10,10} + x_{10,11} + x_{10,12} \leq 11660$$

$$x_{11,1} + x_{11,2} + x_{11,3} + x_{11,4} + x_{11,5} + x_{11,6} + x_{11,7} + x_{11,8} + x_{11,9} + x_{11,10} + x_{11,11} + x_{11,12} \leq 9148$$

$$x_{12,1} + x_{12,2} + x_{12,3} + x_{12,4} + x_{12,5} + x_{12,6} + x_{12,7} + x_{12,8} + x_{12,9} + x_{12,10} + x_{12,11} + x_{12,12} \leq 11066$$

$$x_{13,1} + x_{13,2} + x_{13,3} + x_{13,4} + x_{13,5} + x_{13,6} + x_{13,7} + x_{13,8} + x_{13,9} + x_{13,10} + x_{13,11} + x_{13,12} \leq 3404$$

and the following demand constraints

$$\begin{aligned}
x_{11} + x_{21} + x_{31} + x_{41} + x_{51} + x_{61} + x_{71} + x_{81} + x_{91} + x_{10,1} + x_{11,1} + x_{12,1} &\leq 9200 \\
x_{12} + x_{22} + x_{32} + x_{42} + x_{52} + x_{62} + x_{72} + x_{82} + x_{92} + x_{10,2} + x_{11,2} + x_{12,2} &\leq 7200 \\
x_{13} + x_{23} + x_{33} + x_{43} + x_{53} + x_{63} + x_{73} + x_{83} + x_{93} + x_{10,3} + x_{11,3} + x_{12,3} &\leq 6000 \\
x_{14} + x_{24} + x_{34} + x_{44} + x_{54} + x_{64} + x_{74} + x_{84} + x_{94} + x_{10,4} + x_{11,4} + x_{12,4} &\leq 12000 \\
x_{15} + x_{25} + x_{35} + x_{45} + x_{55} + x_{65} + x_{75} + x_{85} + x_{95} + x_{10,5} + x_{11,5} + x_{12,5} &\leq 13052 \\
x_{16} + x_{26} + x_{36} + x_{46} + x_{56} + x_{66} + x_{76} + x_{86} + x_{96} + x_{10,6} + x_{11,6} + x_{12,6} &\leq 10000 \\
x_{17} + x_{27} + x_{37} + x_{47} + x_{57} + x_{67} + x_{77} + x_{87} + x_{97} + x_{10,7} + x_{11,7} + x_{12,7} &\leq 8000 \\
x_{18} + x_{28} + x_{38} + x_{48} + x_{58} + x_{68} + x_{78} + x_{88} + x_{98} + x_{10,8} + x_{11,8} + x_{12,8} &\leq 11250 \\
x_{19} + x_{29} + x_{39} + x_{49} + x_{59} + x_{69} + x_{79} + x_{89} + x_{99} + x_{10,9} + x_{11,9} + x_{12,9} &\leq 10314 \\
x_{1,10} + x_{2,10} + x_{3,10} + x_{4,10} + x_{5,10} + x_{6,10} + x_{7,10} + x_{8,10} + x_{9,10} + x_{10,10} + x_{11,10} + x_{12,10} &\leq 11800 \\
x_{1,11} + x_{2,11} + x_{3,11} + x_{4,11} + x_{5,11} + x_{6,11} + x_{7,11} + x_{8,11} + x_{9,11} + x_{10,11} + x_{11,11} + x_{12,11} &\leq 42050 \\
x_{1,12} + x_{2,12} + x_{3,12} + x_{4,12} + x_{5,12} + x_{6,12} + x_{7,12} + x_{8,12} + x_{9,12} + x_{10,12} + x_{11,12} + x_{12,12} &\leq 10113
\end{aligned}$$

The solution for the scheduling formulation will be found using MATLAB software. The MATLAB administers the Vogel and the MODI to solve the production scheduling formulation.

### Using MATLAB to obtain the BFS and the optimal solution

MATLAB is a windows package which can be used to obtain the optimal solution to a production scheduling problem. The machine specification for this work is a Lenovo230 with 500HDD and a processor i5 and 2.6Ghz, it has 4GB external and 8GB internal memory, OS Ubuntu 13.04LTS and a software version MATLAB2013-64bit. Before using the MATLAB, an initial table is created. This is presented in Table 4.2.

Each cell in Table 4.2 contains the cost per unit of the product plus the storage cost but in this study, the storage cost is zero since production is strictly based on order from the customer. For example, in  $C_{11}$  the cost is 0.5 whereas in the second cell  $C_{12}$  the cost is 0.6 (i.e  $0.5+0.1 = 0.6$ ), thus, a holding cost of 0.6 incurred since the firm couldn't supply all demands in the month of January

and have to carry it to the following month. Again in cell  $C_{21}$  the cost is 0.55 and in  $C_{22}$  the cost is 0.5, this indicated that there is a backlogging of 0.05, since the production in the month of February is used to satisfy the demand in January.

For the solution to the production problem to actualize, the total demand should be equal to the total supply. The total demand according to Table 4.2 is 120979 and the total supply is 117575 examining these values it can be perceived that total demand exceeds total supply, hence, the production problem is not balanced. To make the production problem balanced, dummy supply are created whose capacity is exactly the excess of demand over supply, such that the unit cost from source to every ware house is '0' which in effect do not have any effect on the allocations. The dummy supply of 3404 (i.e 120979-117575) is created to balance the production problem with a cost per unit of zero.

The IBFS and the optimal solution to the problem are given in Table 4.3 and 4.4. The IBFS gives the initial allocations of production resources necessary to meet a given demand. Each occupied cell contains the individual allocation for each of the period during the financial year. A cell with no allocation is termed as an unoccupied cell or an empty cell.

The solution in Table 4.4 shows the optimal solution. It gives the allocations which minimize the total cost of production. This is so because from MODI, if all the factors  $C_{ij}$  calculated for the empty cells are positive or zero, then the solution is optimal. The optimal solution gave the final total cost of production and is thus :

$$0.5(6436 + 5052 + 6000 + 8342 + 11917 + 7609 + 5654 + 6952 + 7535 + 10351 + 9148 + 10113) + 0.6 (2148 + 3658 + 1135 + 2391 + 2346 + 4298 + 2779 + 1449 + 1309) + 0.55 (953) = 60986.45$$

The  $S_i$  with  $i = 1, 2, \dots, 13$  shows the monthly supplies and the  $B_j$  with  $j =$

Table 4.2: Initial table the MATLAB software uses to generate results

	Jan	Feb	Mar	Apr	May	Ju	Jul	Aug	Sep	Oct	Nov	Dec	Supply
Jan	0.5	0.6	0.7	0.8	0.9	1	1.1	1.2	1.3	1.4	1.5	1.6	8584
Feb	0.55	0.5	0.6	0.7	0.8	0.9	1	1.1	1.2	1.3	1.4	1.5	5052
Mar	0.6	0.55	0.5	0.6	0.7	0.8	0.9	1	1.1	1.2	1.3	1.4	9658
Apr	0.65	0.6	0.55	0.5	0.6	0.7	0.8	0.9	1	1.1	1.2	1.3	9477
May	0.7	0.65	0.6	0.55	0.5	0.6	0.7	0.8	0.9	1	1.1	1.2	14308
Jun	0.75	0.7	0.65	0.55	0.5	0.6	0.7	0.8	0.8	0.9	1	1.1	9955
Jul	0.8	0.75	0.7	0.65	0.6	0.55	0.5	0.6	0.7	0.8	0.9	1	9952
Aug	0.85	0.8	0.75	0.7	0.65	0.6	0.55	0.5	0.6	0.7	0.8	0.9	9731
Sep	0.9	0.85	0.8	0.75	0.7	0.65	0.6	0.55	0.5	0.6	0.7	0.8	8984
Oct	0.95	0.9	0.85	0.8	0.75	0.7	0.65	0.6	0.55	0.5	0.6	0.7	11660
Nov	1	0.95	0.9	0.85	0.8	0.75	0.7	0.65	0.6	0.55	0.5	0.6	9148
Dec	1.05	1	0.95	0.9	0.85	0.8	0.75	0.7	0.65	0.6	0.55	0.5	11066
Dum	0	0	0	0	0	0	0	0	0	0	0	0	3404
Dem	9200	7200	6000	12000	13052	10000	8000	11250	10314	11800	12050	10113	120979

1,2,...,12 also represents the monthly demands. The allocation  $C_{11}$  of the optimum production schedule above shows that the firm should supply 6436 instead of the actual supply 8584, to hold some for the next month. Likewise,  $C_{11,11}$  shows the firm should supply all the 9148 medium bottle water in the month of November. The allocations continue till the end of the year. Dummy supply are only created to balance the production problem, so all their allocations do not do not count.

## DISCUSSION

The optimum production schedule presented in Table 4.4 gives the amount of the product to be allocated to satisfy demand during each period of the financial year. The allocations in the MODI are supply giving to a particular demand and have been done with objective of minimizing cost.

Table 4.3: Basic Feasible Solution (BFS) to the production scheduling obtain by the Vogel's Approximation Method

5796	2788	0	0	0	0	0	0	0	0	0	0
0	4412	640	0	0	0	0	0	0	0	0	0
0	0	5360	4298	0	0	0	0	0	0	0	0
0	0	0	7702	1775	0	0	0	0	0	0	0
0	0	0	0	11277	3031	0	0	0	0	0	0
0	0	0	0	0	6969	2986	0	0	0	0	0
0	0	0	0	0	0	5014	4938	0	0	0	0
0	0	0	0	0	0	0	6312	3419	0	0	0
0	0	0	0	0	0	0	0	6895	2089	0	0
0	0	0	0	0	0	0	0	0	9711	1949	0
0	0	0	0	0	0	0	0	0	0	9148	0
0	0	0	0	0	0	0	0	0	0	953	10113
3404	0	0	0	0	0	0	0	0	0	0	0

Table 4.4: Optimal solution to the production scheduling obtain by the MODI Method

6436	2148	0	0	0	0	0	0	0	0	0	0
0	5052	0	0	0	0	0	0	0	0	0	0
0	0	6000	3658	0	0	0	0	0	0	0	0
0	0	0	8342	1135	0	0	0	0	0	0	0
0	0	0	0	11917	2391	0	0	0	0	0	0
0	0	0	0	0	7609	2346	0	0	0	0	0
0	0	0	0	0	0	5654	4298	0	0	0	0
0	0	0	0	0	0	0	6952	2779	0	0	0
0	0	0	0	0	0	0	0	7535	1449	0	0
0	0	0	0	0	0	0	0	0	10351	1309	0
0	0	0	0	0	0	0	0	0	0	9148	0
0	0	0	0	0	0	0	0	0	0	953	10113
2764	0	0	0	0	0	0	0	0	0	640	0

The optimal solution gives the allocation that minimizes the total cost of production. For the firm to make profits or minimize cost, it has to allocate 6436 to meet part of the demand in the same month. For the month of February, the firm fulfill all the supply, there wasn't any holding in that month.

The firm has a production of 8984 boxes of products in the month of September and 11660 in the month of October. However, the optimum schedule revealed that the firm should produce only 7535 in the month of September and 10351 in the month of October. The allocations in the dummy column are not



Table 4.5: Summary of the optimum schedule generated by the MATLAB software

From(Supply)	To(Demand)	Allocation	Cost per unit	Production cost
S1	D1	6436	0.5	3218
S1	D2	2148	0.6	1288.8
S2	D2	5052	0.5	2526
S3	D3	6000	0.5	3000
S3	D4	3658	0.6	2194.8
S4	D4	8342	0.5	4171
S4	D5	1135	0.6	681
S5	D5	11917	0.5	5958.5
S5	D6	2391	0.6	1434.6
S6	D6	7609	0.5	3804.5
S6	D7	2346	0.6	1407.6
S7	D7	5654	0.5	2827
S7	D8	4298	0.6	2578.8
S8	D8	6952	0.5	3476
S8	D9	2779	0.6	1667.4
S9	D9	7535	0.5	3767.5
S9	D10	1449	0.6	869.4
S10	D10	10351	0.5	5175.5
S11	D11	9148	0.5	4574
S12	D11	953	0.55	524.15
S12	D12	10113	0.5	5056.5
S13	Dummy	2764	0	0
s13	dummy	640	0	0
Optimal cost				60986.45

taken into consideration.

## Chapter 5

### CONCLUSION AND RECOMMENDATION

#### 5.1 Conclusion

The application of the model revealed how the monthly allocations should be done in order to reduce the production cost. Also it showed which month the stocks available should be allocated to so that they do not pile up unnecessarily. The modeling of the production problem as a balanced transportation problem and its specialized methods of solution such as the Least Cost Method and the Vogel's Approximation Method, which are modification of the parent simplex algorithm have proven appropriate in obtaining the optimum schedule.

The company is able to produce using regular working period. This means that overtime or subcontracting is not necessary in reducing the cost of production. The production of the firm would have yielded a total production cost GH¢61719.3, but the optimum production schedule gave a final total production cost of GH¢60986.45. This result is vital because the decrease of 733 (i.e 61719.3-60986.45) in the total cost of production is important. Moreover, the optimal solution demonstrated how the reduction will be achieved. From the result it's clear that efficient scheduling could reduce production and inventory cost whilst satisfying customer demands. The demand and supply at each level were determine using MATLAB.

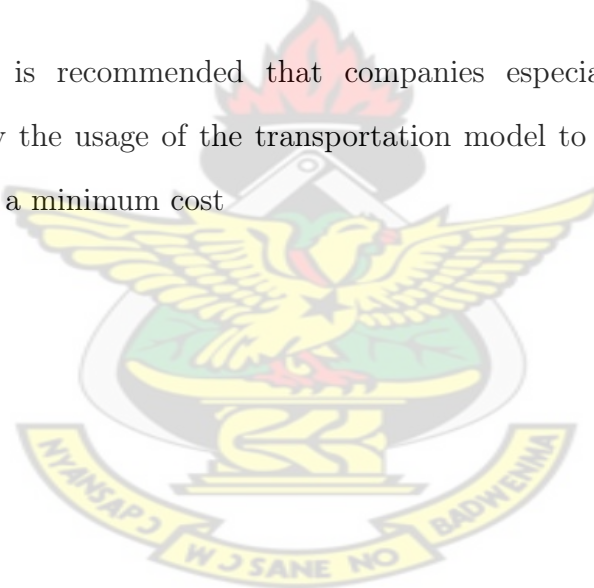
Computer-based scheduling could help manufacturers to easily attend to customers' orders, improve on-time delivery and create realistic schedules. This attest to the fact that computerized scheduling tools outperform older manual scheduling tools. The analysis also recommends that production scheduling and control can facilitate the production processes in a number of ways. A

production scheduling can result in optimum utilization of capacity. Firms with the help of production scheduling could schedule their production capacities such as employees and machinery do not remain idle but be fully utilized.

A good production scheduling ensures quality in terms of processes, products and packaging. It is shown that production scheduling is of immense importance to every production firm in terms of capacity utilization, inventory control and improving the company's response to time and quality. Again, effective production scheduling contributes to time, quality and cost parameters of a company's success.

## 5.2 Recommendation

Therefore, it is recommended that companies especially, production firms should employ the usage of the transportation model to achieve optimum level production at a minimum cost



## REFERENCES

- Alfare, H. and Bailey, J. (1997). Integrated project task and manpower scheduling. *IIE Transaction*, 29:711–717.
- Allen, S. J. (1990). Prod rate planning for two products sharing a simple process facility : a real world cast study. *Production and Inventory Management*, 31:24–29.
- Amponsah, S. K. (2009). *Lecture note, Optimization Technique 1, institute of Distance Learning, Department of Mathematics, KNUST, Kumasi, Ghana.*
- Bailey, J., Alfare, H., and Lin, W. (1995). Optimization and heuristic models to integrate project task and manpower scheduling. *Computer and Industrial Engineerings*, 29:473–476.
- Baker, K. R. (1974). *Introduction to Sequencing and Scheduling*. Wiley, New York.
- Bellman, Richard, and Gross, O. (1954). "some combinational problems arising in the theory of multi-stage process",. *Journal of the society for industrial and Applied Mathematics*, 2(3):175–183.
- Bomberger, E. (1966). A dynamic programming approach to a lot size scheduling problem. *Management Science*, 12:778–784.
- Boukas, E. K. and Haurie, A. (1990). Manufacture flow control and preventive maintenance:. *IEEE Trans. Autom Control*, 35:1024–1031.
- Cordeau, J. F., Stojkovi'c, G., Soumis, F., and Desrosiers, J. (2001). Bender decomposition for simultaneous aircraft routing and crew scheduling. *Transportation Science*, 35:375–388.

- Cox, J. F., Blackstone, J. H., and Spencer, M. S. (1992). *APICS Dictionary, American Production and Inventory Control Society, Falls Church, Virginia.*
- Daniel, R. L. and Mazzolla, J. B. (1994). *Flow shop scheduling with resource flexibility Operations Research.*
- Daniel, R. L., Mazzolla, J. B., and Shi, D. (2004). Flow shop scheduling with partial resource flexibility. *Management Science*, 50:658–669.
- Delporte, C. and Thomas, L. (1978). Lot sizing and sequencing for products on one facility. *Management Science*, 23:1070–1079.
- Dobson, G. (1987). The economic lot scheduling problem: achieving feasibility using time varying lot sizes. *Operations Research*, 35:764–771.
- Dobson, G. (1992). The cyclic lot scheduling problem with sequence-dependent set-ups. *Operation Research*, 40:736–749.
- Doll, L. and Whybark, C. (1973). An iterative procedure for the single machine, multiproduct lot scheduling problem. *Management S*, 20:50–55.
- Drezet, L. E. and Billaut, J. C. (2005). *Tabu search algorithms for a predictive and a reactive project scheduling problem. In 6th Metaheuristics International Conference, Vienna.*
- Dudek, R. A., Panwalkar, S. S., and Smith, M. L. (1992). "the lesson of flowshop scheduling reasearch". *Operation Research*, 40:7–13.
- Faaland, B. and Schmitt, T. (1993). Cost-based scheduling of worker and equipment in a fabrication and assembly shop. *Operation Research*, 41:253–268.
- Fordyce, K., Dunki-Jacobs, R., Gerad, B., Sell, R., and Sullivan, G. (1992). Logistics management system: An advanced decision support system for the fourth tier dispatch or short-interval scheduling,. 1:70–86.

- Fox, M. S. (1983). *Constraint -directed search: a case study of job-shop scheduling*, *PhD*.
- Fox, M. S., Sadeh, N., and Baycan, C. (1989). Constrained heuristic search: Proceedings of international joint conference on artificial intelligence, mendo park, c a. pages 309–316.
- Freling, R., Huisman, D., and Wagelmanss, A. (2003). Models and algorithms for integration of vehicle and crew scheduling. *Journal of Scheduling*, 6(1):63–85.
- Gallego, G. (1993). Reduced production rate in the economic lot scheduling problem. *International Journal of Production Research*, 31:1035–1046.
- Gallego, G. and Moon, I. (1995). Strategic investment to reduce setup time in the economic lot scheduling problem,. *Naval Research Logistics*, 42:773–790.
- Glover, F. (1989). Tabu- search part 1,. *ORSA Journal on computing*, 1:190–206.
- Goldberg, D. E. (1989). *Genetic Algorithms in Search, Optimization and Machine Learning*,. Addition -Westly, Reading MA,.
- Haase, K. and Friberg, C. (1999). An exact algorithm for the vehicle and crew scheduling problem . in wilson, n. ed.: Computer-aided trait schedulig. *Lecture notes in Economics and Mathematical Systems*, Springer, Berling, 471:63–80.
- Herrmann, J. W. (2004). information flow and decision -making in production schedulin,2004. *industrial Engineering Research Conference,Houston*, pages pg 15–19.
- Hwang, H., Kim, D., and Kim, Y. (1993). Multiproduct economic lot for size models with investment cost for setup reduction and quality improvement. *International Journal of Production Research*, 31:691–703.
- Khouza, M., Michalewicz, Z., and Wilmot, M. (1998). The use of genetic algorithms to solve the economic lot size scheduling problem. *European of Operational Research*, 110:509–524.

- Kusiak, A. (1992). *Intelligent design and manufacturing* Willey, New York.
- LaForge, R. L. and Craighead, C. W. (1998). *Manufacturing scheduling and supply chain integration: A survey of Current Practice*, American Production and Inventory Control Society, Falls Church, Virginia.
- Lee, C. Y. and Liman, S. D. (1993). *Capacitated Two-parallel Machines Scheduling to Minimize Sum of Job Completion Times-Discrete Appl. Math;* 41,. Number 41.
- Maxwell, W. (1964). The scheduling of economics lot sizes. *Naval Research Logistics Quarterly*, 11:264–124.
- McKay, K. N. and Wiers, V. C. S. (2005). The human factor in planning and scheduling, in handbook of production scheduling, herrmann j. w ,ed.,springer new york.
- McKay, N. K. and Wiers, C. V. (2004). *Practical production control: a survival Guide for planners and schedulers*,. J R Publishing ,Boca Raton, Florida, 2004,co-published with APICS.
- McKay, N. K. and Wiers, C. V. (2006). "the human factor in planning and scheduling,"in handbook of production scheduling , jw herrmann,ed, springer , new york.
- Mercier, A., Cordeaus, J. F., and Soumis, F. (2005). A computational study of benders decomposition for the integrated aircraft routing and crew scheduling problem. *Computers Operations Research*, 32:1451–1476.
- Mitchell, W. N. (1939). *Organizational and Management of Production*. McGraw - Hill Book company , New York.
- Moon, D. and Christy, D. (1998). Determination of optimal production rates on a single facility with dependent mold lifespan. *International Journal of Production Economics*, 54:29–40.



- Pfund, M. E. and Scott, J. M. (2006). *Semi-conductor Manufacturing Scheduling and Dispatching. Chap 9, Handbook of Production Scheduling.*
- Pinedo, M. and Chao, X. (1999). *Operations scheduling with Application in Manufacturing and services*,. Irwin McGraw Hill, Boston,.
- Roger, J. (1958). A computational approach to the economic lot scheduling problem. *Management Science*, 4:264–291.
- Silver, E. (1993). Modelling in support of continuous improvement toward achieving world class operations,. *In perspectives in Operation Management*, pages 23–44.
- Silver, E. (1995). Dealing with shelf life constraint in cyclic scheduling by adjusting both cycle time and production rate. *International Journal of Production Research*, 33:623–629.
- Silver, E., Pyke, D., and Peterson, R. (1998). *Inventory Management and Production Planning and Schedulig, 3rd edition* ,New York: Willey.
- Vieira, G. E., Herrmann, J. W., and Lin, E. (2003). Rescheduling manufacturing system: a frame work of strategies, policies and method. *Journal of Scheduling*, 6(1):35–56.
- Viswanathan, S. and Goyal, S. K. (1997). Optimal cyclic time and production rate in a family production contex with shelf life considerations. *International Journal of Production Research*, 33:1703–1711.
- Vollmann, T. E. and Berry, W. L. (1997). *Manufacturin planning and control system, fourth edition*,. Irwin / McGraw-Hill, New York.
- Wight, O. (1984). Production and inventory management in the computer age, van nostrand reinhold company,inc, new york,.

Zweben, M., Davis, E., Daun, B., Drascher, E., Deale, M., and Eskay, M. (1992). Learning to improve constrain- based scheduling,. *Artificial intelligence*, 58:271–296.

Zweben, M. and Fox, M. S. (1994). *Intelligent Scheduling*, Kaufmann(Morgan), Los Altos, CA.

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## APPENDIX A

### 5.3 COMPUTATIONAL DATA

Table 5.1: CAPACITY DATA OF THE FIRM

Month	Demand	Produce
January	9200	8584
February	7200	5052
March	6000	9658
April	12000	9477
May	13052	14308
June	10000	9955
July	8000	9952
August	11250	9731
September	10314	8984
October	11800	11660
November	12050	9148
December	10113	11066



## APPENDIX B

Table 5.2: SUMMARY RESULT FROM MATLAB

From(Supply)	To(Demand)	Allocation	Cost per unit	Production cost
S1	D1	6436	0.5	3218
S1	D2	2148	0.6	1288.8
S2	D2	5052	0.5	2526
S3	D3	6000	0.5	3000
S3	D4	3658	0.6	2194.8
S4	D4	8342	0.5	4171
S4	D5	1135	0.6	681
S5	D5	11917	0.5	5958.5
S5	D6	2391	0.6	1434.6
S6	D6	7609	0.5	3804.5
S6	D7	2346	0.6	1407.6
S7	D7	5654	0.5	2827
S7	D8	4298	0.6	2578.8
S8	D8	6952	0.5	3476
S8	D9	2779	0.6	1667.4
S9	D9	7535	0.5	3767.5
S9	D10	1449	0.6	869.4
S10	D10	10351	0.5	5175.5
S11	D11	9148	0.5	4574
S12	D11	953	0.55	524.15
S12	D12	10113	0.5	5056.5
S13	Dummy	2764	0	0
s13	dummy	640	0	0
Optimal cost				60986.45

## APPENDIX C

Table 5.3: INITIAL TABLE THE MATLAB USES TO GENERATE RESULT

	Jan	Feb	Mar	Apr	May	Ju	Jul	Aug	Sep	Oct	Nov	Dec	Supply
Jan	0.5	0.6	0.7	0.8	0.9	1	1.1	1.2	1.3	1.4	1.5	1.6	8584
Feb	0.55	0.5	0.6	0.7	0.8	0.9	1	1.1	1.2	1.3	1.4	1.5	5052
Mar	0.6	0.55	0.5	0.6	0.7	0.8	0.9	1	1.1	1.2	1.3	1.4	9658
Apr	0.65	0.6	0.55	0.5	0.6	0.7	0.8	0.9	1	1.1	1.2	1.3	9477
May	0.7	0.65	0.6	0.55	0.5	0.6	0.7	0.8	0.9	1	1.1	1.2	14308
Jun	0.75	0.7	0.65	0.55	0.5	0.6	0.7	0.8	0.8	0.9	1	1.1	9955
Jul	0.8	0.75	0.7	0.65	0.6	0.55	0.5	0.6	0.7	0.8	0.9	1	9952
Aug	0.85	0.8	0.75	0.7	0.65	0.6	0.55	0.5	0.6	0.7	0.8	0.9	9731
Sep	0.9	0.85	0.8	0.75	0.7	0.65	0.6	0.55	0.5	0.6	0.7	0.8	8984
Oct	0.95	0.9	0.85	0.8	0.75	0.7	0.65	0.6	0.55	0.5	0.6	0.7	11660
Nov	1	0.95	0.9	0.85	0.8	0.75	0.7	0.65	0.6	0.55	0.5	0.6	9148
Dec	1.05	1	0.95	0.9	0.85	0.8	0.75	0.7	0.65	0.6	0.55	0.5	11066
Dum	0	0	0	0	0	0	0	0	0	0	0	0	3404
Dem	9200	7200	6000	12000	13052	10000	8000	11250	10314	11800	12050	10113	120979

## APPENDIX D

Table 5.4: BASIC FEASIBLE SOLUTION (IBFS) OBTAIN BY THE VOGEL'S APPROXIMATION METHOD

5796	2788	0	0	0	0	0	0	0	0	0	0
0	4412	640	0	0	0	0	0	0	0	0	0
0	0	5360	4298	0	0	0	0	0	0	0	0
0	0	0	7702	1775	0	0	0	0	0	0	0
0	0	0	0	11277	3031	0	0	0	0	0	0
0	0	0	0	0	6969	2986	0	0	0	0	0
0	0	0	0	0	0	5014	4938	0	0	0	0
0	0	0	0	0	0	0	6312	3419	0	0	0
0	0	0	0	0	0	0	0	6895	2089	0	0
0	0	0	0	0	0	0	0	0	9711	1949	0
0	0	0	0	0	0	0	0	0	0	9148	0
0	0	0	0	0	0	0	0	0	0	953	10113
3404	0	0	0	0	0	0	0	0	0	0	0



## APPENDIX E

Table 5.5: OPTIMAL SOLUTION OBTAIN BY THE MODIFIED DISTRIBUTION(MODI) METHOD

6436	2148	0	0	0	0	0	0	0	0	0	0
0	5052	0	0	0	0	0	0	0	0	0	0
0	0	6000	3658	0	0	0	0	0	0	0	0
0	0	0	8342	1135	0	0	0	0	0	0	0
0	0	0	0	11917	2391	0	0	0	0	0	0
0	0	0	0	0	7609	2346	0	0	0	0	0
0	0	0	0	0	0	5654	4298	0	0	0	0
0	0	0	0	0	0	0	6952	2779	0	0	0
0	0	0	0	0	0	0	0	7535	1449	0	0
0	0	0	0	0	0	0	0	0	10351	1309	0
0	0	0	0	0	0	0	0	0	0	9148	0
0	0	0	0	0	0	0	0	0	0	953	10113
2764	0	0	0	0	0	0	0	0	0	640	0

