MANAGING INVENTORY SYSTEMS USING OPTIMIZATION

ERNEST ALEMA-MENSAH (BSc (MATHEMATICS AND COMPUTER SCIENCE), CILT)

A Thesis Submitted to the Department of Mathematics, Kwame Nkrumah University of Science and Technology, Kumasi, in partial fulfillment of the requirement for the degree of Master of Science in Industrial Mathematics, Institute of Distance Learning.

JULY, 2011

DECLARATION

I hereby declare that this submission is my own work towards the MSc and that, to the best of my knowledge, it contains no material(s) previously published by another person(s) nor material(s), which have been accepted for the award of any other degree of the University, except where the acknowledgement has been made in the text.

ERNEST ALEMA-MENSAH	__________	______
(20103243)	**Signature**	**Date**

Certified by:

DR. S. K. AMPONSAH	___________	______
Supervisor	**Signature**	**Date**

Certified by:

DR. S. K. AMPONSAH	___________	______
Head of Department	**Signature**	**Date**

DEDICATION

This thesis is dedicated to my parents Gifty and Ernest Alema-Mensah.

ACKNOWLEDGEMENT

It is with great pleasure that I take this opportunity to recognize those who have played a major role in bringing this significant work to its full realization. It has been satisfying to see all the pieces come together, often in ways much better than I expected.

It is therefore, my pleasure to express my greatest indebtedness to my supervisor, Dr. Francis T. Oduro, who performed the invaluable service of helping me create the overflow and text of the work. His diligence and tenacity are truly appreciated.

I owe a considerable measure of appreciation to Mr. Takashi Araki of Toyota Ghana Company Limited for helping me with information about the logistics of new Toyota vehicles needed for this thesis.

I am also grateful for the useful suggestions and the encouragement I received from Rev. Dr. Lartey, Department of Mathematics, KNUST; Dr. Prempeh, Mathematics Department, KNUST; Dr. S.K. Amponsah, Head of Department of Mathematics, KNUST; Late. Mr. Agyeman, formally of Department of Mathematics, KNUST; Mr. Prince Koree, Department of Mathematics, University of Ghana; Dr. Margaret MacIntye Head of Department of Mathematics, University of Ghana.

To my course mates especially Gilbert Debrah and George Boafo, I say thank for your honesty and friendship. Also to great friends like Rita, Deborah, Blay, Seth, Prosper, Harry, Botwe, Gotty and John. Simple words of acknowledgement and appreciation seem inadequate to my girlfriend, Grace Teikuor Newman. Thank you for understanding intuitively from the beginning that I had to make it. Thanks for your love and support.

ABSTRACT

In managing inventory systems, proper simulation of the inventory or stock levels is very likely to reduce holding cost, losses and improve general customer satisfaction or profit. This thesis aims to investigate simulation optimization of an Inventory Management System. It also discusses Optimal Inventory Policy (cost saving in inventory management and maximizing profit). A mathematical model is used for this purpose. Also a case study adjusts mathematical model to practicality of Toyota Ghana Company Limited business situation.

TABLE OF CONTENT

CHAPTER THREE: METHODOLOGY

CHARPTER ONE

INTRODUCTION

1.0 Background to the study

In recent years, Inventory Management has attracted a great deal of attention from people both in academia and industries. A lot of resources have been devoted into research in the inventory management practices of organizations. Companies with superior forecasting abilities can afford to procure or produce a large fraction of their demand by making use of low production methods and inexpensive logistics services. These companies pay more for faster production and logistics services only when the demand surges or goes up unexpectedly. On the other hand, companies with irregular demands and inferior forecasting abilities have to pay more for using fast production methods to respond to unexpected surges in demand.

The advances in manufacturing technologies, logistics services, and globalization makes it possible for companies to satisfy their customer demands from sources with different prices and lead time. On the hand the ability to provide better forecasting increases as the delivery date approaches. Also the cost increases as the lead time increases.

It is therefore critical to be able to simulate in advance the demand information, lead time in logistics services and to strike a balance between the quality of demand information and the cost of production and logistics services.

1.1.0 Problem Statement

The most common problem in inventory management is to attain optimal inventory levels. Decisions about how many of which products are to be stored in the warehouse, when to place the next order, the quantities to be ordered are some of the problems encountered every day. High level of inventory locks up the capital of any company. Customers on the other hand, lose confidence in the company and look elsewhere if there is no availability. This can reduce the profitability of the company and eventually crumple the company.

The science of balancing the right levels of inventory can be solved by modeling the inventory system into a mathematical model. This model can then be simulated and the result analyzed to reach the best practices in inventory management.

Goods in transit, obsolete stock, dead stock, fast and slow moving stock, back orders are all problems associated with managing inventory systems.

1.1.1 Inventory management problems (Defects, inventory levels, allocation)

Defects in the inventory and incoherent levels of inventory form a common problem in the area of inventory management planning. They affect the optimal operation of the inventory system. These problems are very common occurrence in most inventory systems. In different studies, they have been addressed using analytical models, queuing theory and deterministic programming techniques like integer programming. In order to properly understand the complexity of these problems, simulation models will be used to demonstrate the inventory system. To optimize stock allocation level and resources, a detailed analysis will be in Chapter 3. Another classic problem is the lead time. The objective is to minimize the average lead time and cost of holding high levels of inventory subject to the constraints on the throughput and the budget available.

1.2 Objectives

First of all, An Economic Order Quantity will be modeled. As this simulated Economic Order Quantity (EOQ) model is fully developed and tested, optimal level of inventory can be achieved. This implies cost saving in inventory control and achievement of maximum profit. Also the carrying cost is reduced to the lowest possible value and that money can be invested in other parts of the company.

Secondly, this Economic Order Quantity Model will be analyzed against the current practices of Toyota Ghana Company Limited's ordering policies. Also the availability of stock at all times will be studied to guarantee customer satisfaction and loyalty. The goal is to prevent losing customers, who may turn towards other sources to buy what they need.

Every inventory consists of products stored in order to meet future potential demand. The emphasis is on meeting future demand hence it has the ability to forecast the company's future profit and the direction in which it will go.

Finally, the best policy in Managing Inventory will be determined. The average cycle time will have to be minimized to achieve this goal. Various simulation based stochastic optimization algorithms are presented and an observation of their behavior with respect to the quality of solution and the number of simulation each algorithm requires. This is then compares with the analytical algorithm. The Just-In-Time method is also used to establish the results.

1.3 Methodology

Simulation based optimization of the Economic Order Quantity is an effective tool to model, analyze and optimize any inventory systems. It is useful in forecasting the behavior of systems with both continuous and discrete variables like a typical inventory system. Discrete and continuous systems need to be modeled or designed into complex systems. This complex system or model must be linked with a specific simulation optimization technique that best calculate the output.

In continuous optimization problems, the decision variables are continuous in nature. These problems are solved using techniques such as stochastic approximation methods, response surface search/methodology and the Gradient estimation method.

In discrete optimization problems, the decision variables are discrete in nature. These problems are solved using techniques such as gradient estimation techniques, discrete random and non-random search method, stochastic comparison algorithm, simulated annealing algorithm, stochastic ruler method. There are non-random search methods too, like the branch and bound algorithm and the low dispersion point set method.

1.4 Justification

The inventory system has diverse decision variables that can be considered as continuous like regular orders, demand on the stock, regular supply et cetera. On the other hand, there are discrete variables like special orders that come in at a particular time, theft or accidents that occur without any warning. Based on the kind of information that management or decision makers need to enable them plan properly for their inventory, these discrete and continuous variables always play an important role in determining the results.

1.5 Complexity of an Inventory system

If inventory levels were easily predictable, managing an inventory system would be very easy and simple.

However, factors such as lead time, order times, back orders are very difficult to predict in real life scenario. These complexities of inventory systems arise due to the stochastic nature of the system and constant changes that are made to the stocks. In most businesses with inventory levels, demand is uncertain and hard to forecast. In general as stock life cycles get shorter, the randomness and unpredictability of these demands have become even more complex.

In the designing of an inventory system, many factors will have to be met. For example, there could be an optimal allocation of resources such as stocks at each location so as to maximize the productivity of the system. An important constraint here could be the limited resources that are allocated. Another problem could be planning the operations of the inventory system in such a way that stocks are not kept there over a long period of time. The obvious constraint here would be that the overall stocks should never run out or be more that the acceptable stock level.

1.6.0 Simulation optimization

Simulation modeling is an effective tool to model, analyze and optimize systems. It is particularly useful in predicting the behavior of systems with an inherent stochastic nature, hence the term simulation-based stochastic optimization. Based on the nature of the decision space, such optimization problems could be categorized as continuous or discrete.

1.6.1 Simulation optimization of an Inventory System

Inventory systems could be analyzed as queuing systems, where the stocks being kept in the warehouse or processed is considered as a customer and warehouse is considered as the server. The important characteristic of such systems is their event-based nature. The state of the system changes only at the occurrence of an event such as the arrival or departure of stock, damage in the warehouse, processing of stock, or other warehouse operations. Since the occurrence of such events takes place at different points in time, we generally refer to inventory systems as discrete event inventory systems.

Even though there are analytical models to analyze inventory systems given their natural stochastic nature, it becomes increasingly difficult to adjust them or develop new analytical models to accommodate complex features and enhanced variability in the system. There could be unforeseen delays in the arrival of stocks, theft in the warehouse or sudden increase in stock. When that happens it is important to use simulation-based models with higher flexibility to get a more accurate picture. In inventory systems the decision variables are usually discrete in nature. Therefore the techniques used for optimizing an inventory system are based on simulation-based discrete stochastic optimization methodologies, due to the discrete solution space over which we try to optimize the performance of the system.

1.7 Organization of the thesis

The thesis is organized as follows. Chapter 2 presents a literature review and discuss the Kanban or Just-In-Time (JIT), Economic Order Quantity (EOQ), Inventory Management Problems, Supply Chain Management and Logistics Management. Chapter 3 discusses the methodology used to tackle the inventory management problem; specify the objective function, constraints and the decision variables. The chapter also describes our simulation EOQ model and the set-up of our analysis. It defines the improvements based on which a comparison of behavior of all the simulation-based algorithms. Chapter 4 defines a process of collecting data and analyzing the data on inventory along with the simulation-based algorithms and the analytical algorithm. A case study is also defined at the end of the chapter to demonstrate the implementation of the algorithms. Chapter 5 discusses the results obtained, concludes the thesis, summarizing the results and discussing the contributions, limitations and the future work, concerning the research conducted.

CHAPTER 2

LITERATURE REVIEW

2.0 Introduction

This chapter reviews some of the research work that has been conducted so far in the field of Inventory Management, Economic Order Quantity (EOQ), Just In Time (JIT), Logistics Management, Supply Chain Management and Simulation Optimization.

2.1 Inventory Management

Inventory management deals with decisions regarding supply levels: the correct amount of material and the correct time to reorder. There are many reasons for a company to hold excess inventory; variation in demand and production; poor quality and unreliable suppliers and shippers. However, there are also good reasons to cut down the amount held in inventory: carrying cost, storage space and material handling. Thus an exchange has to be considered between the two situations.

Manufacturers are moving towards lean manufacturing and JIT, so companies are decreasing the amount of inventory being held. Retail stores are also applying the philosophy of JIT to reduce inventories and in turn reduce the associated costs at the store. But determining the exact amount that is needed to cover contingencies changes based on the situation faced by the company. There is one model that is commonly used to determine optimal order size. This is the Economic Order Quantity (EOQ) model. Another model that is also used is one that includes purchasing cost per unit in the total cost equation and aims at offering quantity discounts to customers that order large quantities. This is called the EOQ with Quantity Discounts.

Inventory management is defined as the direction and control of activities with the purpose of getting the right inventory in the right place at the right time in the right quantity in the right form and at the right cost. (Cudjoe, 2010)

Inventory is an important current asset with far reaching financial ramifications which deserves very organizations serious attention to ensure cost savings and optimum utilization of scare resources.

Cudjoe, (2010) explained that the terminology-Inventory was of American origin which was synonymous with stock associated with British authors. Assets in the form of goods, property or services held for sale in the ordinary course of business, in the process of production for sale or to be consumed in the production of goods for sale or in the rendering of services. In order words, inventory may exist in three main forms namely;

--- Finished goods, Work in progress and raw materials.

Cudjoe, (2010) also said that Inventory was held for the following purposes;

i. To enable the organization to achieve economies of scale

ii. To balance supply and demand

iii. To enable speculation activities

iv. To provide protection from uncertainties in demand and order cycle

v. To act as a buffer between critical and interfaces within the channel of distribution.

2.2 Type of Inventory

Inventory can be classified based on the reasons for which they are accumulated. The categories of inventories include cycle stock, in-transit inventories, safety or buffer stock, speculative stock, seasonal stock and dead stock.

Cycle Stock is inventory that results from replenishment of inventory sold or used in production. It is required in order to meet demand under condition of certainty, that is, when the organization can predict demand and replenishment times (lead times).

In-Transit inventories are items that are on the way from one location to another. They may be considered part of cycle stock even though they are not available for sale or shipment until after they arrive at the destination.

Safety or Buffer Stock is held in excess of cycle stock because of uncertainty in demand or lead time. Average inventory at a stock keeping location that experiences demand or lead time variability is equal to half the order quantity plus the safety stock.

Dead Stock refers to items for which no demand has been registered for some specified time.

2.3.0 Economic Order Quantity (EOQ)

Muckstadt *et al*., (2010) discussed that EOQ model was determined by minimizing the total annual cost incurred by the company by virtue of its ordering cost and carrying cost. The expression for total annual cost is:

$$TC = \frac{Q}{2}h + \frac{D}{Q}s$$

where,

$$TC = Total\_annual\_cost$$
$$Q = Order\_quantity$$
$$D = Annual\_demand$$
$$s = Ordering\_cost$$
$$h = Annual\_carrying\_cost\_per\_unit$$

They said that this model was based on the basic assumption that there was a single item, with deterministic demand and lead time, no shortages, and inventory was replenished in batches rather than continuously over a period of time.

Muckstadt *et al*., (2010) also said that the first component of this equation represented the inventory management costs and the second component represents the ordering cost. Differentiating with respect to order quantity, the expression for EOQ was obtained as indicated in the equation below.

$$Q = \sqrt{\frac{2Ds}{h}}$$

where,

$$Q = Economic\_Order\_Quantity$$

The literature in the area of inventory management included different types of inventory models dealing with different real-world constraints. Many of these models are variations of the basic EOQ model where the alterations include the conditions that are encountered in the situation being studied. Despite these new conditions these models still try to determine the optimal order quantity, which is one area where the model developed in this research is different from other models for the EOQ.

Liberatore,(1979) discussed an EOQ model, with a few alterations to the assumptions on the basis of which the traditional EOQ model had been developed. Typically, demand always followed a pattern that could be traced by a probability distribution for analysis. The basic EOQ model, however, assumed that this demand was deterministic to simplify the calculations involved.

The traditional EOQ model also assumed that if the inventory is zero when the order was received then that particular order was lost. This was not the scenario in real life as orders may be backordered and fulfilled when the inventory was available. Liberatore, (1979) considered a more realistic situation for his model and developed an equation for the order size based on stochastic lead times and backlogged demand. The traditional equations of inventory theory with deterministic lead times and no backlogging were special cases of this model.

Kim *et al*., (2003) analyzed the suitability of using the Order Quantity Reorder point (Q, R) model where Q is the order quantity and R is the reorder point, for different situations in production/inventory systems. Kim *et al*., (2003) presented a Production/Inventory (Q, R) model that included the production lead times and the order replenishment lead times explicitly with the inventory costs. Comparisons between this model and the traditional (Q, R) model showed that the optimal order quantity and reorder point were different for each of the models. This indicated that the average inventory and backorders would also be different and in turn, the estimated costs would also be different.

Therefore the value of lead time used in the models made a substantial difference in the costs. If the lead times were fixed then the costs in both the models would be the same. But in an actual manufacturing environment, the lead times were rarely constant and therefore the traditional model could severely overestimate or underestimate the order quantity and the reorder point. The authors also portrayed the impact of setup times on the quantity and they showed that the system stability depended on the order sizes. Kim *et al*., (2003) concluded by presenting the extensions that could be done to make this research more broad.

2.3.1 EOQ with Quantity Discounts

Quantity discounts are price reductions that are offered to the retailer when they place an order that is beyond a certain specific level. It is an incentive to the retailer to buy larger quantities. When quantity discounts are offered the retailer is forces to consider the possible benefit of ordering larger number of items with a lower price per item over the increase in the inventory costs that would be incurred by the retailer (Kim *et al*. 2003). The total quantity discount model can be written below:

$$TC = \left(\frac{Q}{2}\right) \cdot HC + \left(\frac{D}{Q}\right) \cdot S + PD$$

$$where,$$

$$P = unit\_price$$

Including the purchasing cost in the total cost equation does not change the EOQ point but changes the total cost for the retailer since the unit costs for certain ranges are different. There are two cases of this model:

i. Carrying costs are constant: When carrying costs are constant, the EOQ remains, the same for all the curves.

ii. Carrying costs are a percentage of the purchasing cost: When the carrying costs are a percentage of purchasing cost per unit, the EOQ starting with the lowest price range is found. If this EOQ is feasible (i.e. falls in the correct quantity cost range), it is the EOQ for that model. If the EOQ found is infeasible, then the EOQ for the other prices are calculated starting form the next highest one. This procedure is continued until a feasible solution is reached.

There is a large body of research that has dealt with quantity discounts in the case of single supplier-single buyer situations and single supplier-multiple buyer situations. Stevenson (1993) had compiled a paper that reviewed the literature in determining lot sizes using the principle of quantity discounts. Stevenson (1993) categorized the literature based on whether the quantity discounts were all-units or incremental and also categorize from buyer's or the seller's perspective.

This section of the literature focuses on some of the research that has been done regarding the quantity discount model and modifications of the EOQ model is this regard.

Benton *et al*., (1996) proposed an algorithm that determined the EOQ with a demand that had been adjusted to consider the effects of the increased demand in the previous period due to discounted costs. The authors considered the situation where suppliers that had excess inventory sold these by the end of the period at discounted cost. Taking advantage of this situation, when products could be stored for more that a single period, buyers bought larger quantities at discounted prices so that it would decrease their costs for the next period. If the supplier did not consider the effect of such large order quantities, the classic EOQ will be suboptimal. The authors thus suggested a technique that would help suppliers calculate the true order quantity and true profit.

Khouja (2001) presented a heuristic (trial and error, encourage to find out own solution) that determined order quantities for multiple items when incremental quantity discounts and a single resource constraint were given. The results obtained by this heuristic were compared with the results obtained by a combinatorial algorithm, which considered all price levels for all items, used to find the optimal solution for small problems. This combinatorial algorithm assumed that the reorder times for each item are independent. However, when the number of items was large and there were many price breaks, this algorithm could not solve the problem to optimality. This was when the heuristic came into play. This heuristic used the Lagrangian relaxation technique. The heuristic worked well when compared to the optimal algorithm for small problems and hence could be used to solve large problems to optimality.

Guder *et al*., (1994) presented a non-linear procurement model which considered quantity discounts in order to reduce the total procurement cost. This model was developed for a multinational oil company and compared with the technique currently used by the company. The authors used the non-linear programming technique for this model. The model considered all combinations of shipments to all the customers in the cost minimizing function. The constraints included those of supplier capacity, customer demand, price to volume relationship and order requirement. This model was found to be flexible and could adapt to changes in the objective and can consider multiple objectives as well.

Dada *et al*., (1987) studied quantity discounts from a seller's point of view. The authors characterized the rand of order quantities and prices that would lower costs for both the buyer and the seller. Pricing policies that helped with balancing the savings for both the buyer and the seller were developed according to these characteristics.

This principle of offering quantity discounts is similar to the principle discussed in this research but the benefit of ordering large quantities is implicitly included in the model as opposed to explicitly considering the purchasing cost per unit and providing discounted rates to buyers when they order larger quantities. The discount is obtained by the retailer when large quantities are ordered that larger unit's loads are used.

2.4.0 Kanban/Just-in-Time (JIT) System

Kanban and just-in-time systems have become much more important in manufacturing and logistics operations in recent years.

Kanban, also known as the Toyota Production Systems (TPS), was developed by Toyota Motor Cooperation during the 1950's and 1960's. The philosophy of Kanban is that parts and materials should be supplied at the very moment they are needed in the factory production process. This is the optimal strategy, from both a cost and service perspective. The Kanban system can apply to any manufacturing process involving repetitive operations.

Just-in-time (JIT) systems extend Kanban, linking purchasing, manufacturing and logistics. The primary goal of JIT are to minimize inventories, improve product quality, maximize production efficiency, and provide optimal customer service levels. It is basically a philosophy of doing business.

2.4.1 Definition of JIT

JIT has been defined in several ways including:

As a production strategy, JIT works to reduce manufacturing cost and to improve quality markedly by waste elimination and more effective use of existing company (Amirk *et al*., 1993).

A philosophy based on the principle of getting the right materials to the right place at the right time (Snehemay *et al*., 1993).

A program that seeks to eliminate non value-added activities form any operation with the objectives of producing high quality products (zero defects), high productivity levels, and lower levels of inventory, and developing long term relationships with channel members (Larry *et al*., 1993).

At the heart of JIT system is the notation that waste should be eliminated. This is in direct contrast to the traditional "just-in-case" philosophy where large inventories or safety stocks are held just in case they are needed. In JIT, the ideal lot size or EOQ is one unit, safety stock is considered unnecessary and any inventory must be eliminated.

2.4.2 Toyota Pioneers Kanban and JIT

Perhaps the best know example of Kanban and JIT systems is the approach developed by Toyota Motor Cooperation. The Company identified problems in supply and product quality through reduction of inventories, which forced problems into the open. Safety stocks were no longer available to overcome supplier delays and faulty components, thus forcing Toyota to eliminate "hidden" production and supply problems.

The same type of procedure has been applied to many companies in the world. The advantage to the system becomes very evident when we see that raw materials can be reduced by 75% with

JIT implementation (Sohal *et al*., 2003). Not every component can be handled by the Kanban or JIT approaches, but the systems work very well for items that are constantly on demand.

2.4.3 Benefits of JIT

According to Ibid, (2003), any companies have successfully adopted the JIT approach. Companies that dealt in metal products, automobile manufacturing, electronics, food and beverages had implemented JIT and realized a number of benefits, including:

i. Improvement in productivity and greater control between various production stages.
ii. Diminished raw material, Work in progress and finished goods inventory.
iii. Reduction in manufacturing time cycle.
iv. Improvement in inventory turnover rates

In general, JIT produced benefits for firms in the following major areas (Francis *et al.* 1990):

i. Improved inventory turns
ii. Better customer service
iii. Decrease warehouse space
iv. Improve response time
v. Reduced distribution cost
vi. Lower transportation cost
vii. Improved quality of supplier products

viii. Reduced number of transportation carriers and suppliers

Examples of multinational companies that have achieved success through JIT include Rank Xerox Manufacturing (Holland), Ford Motor Company, Brunswick, Cummings Engineering, General Motors, Textro, Whirlpool, Sony etc.

2.4.4 Problems Associated with JIT

While JIT offers a number or benefits it may not suitable for all firms. It has some inherent problems which fall into three categories:

I. Production scheduling (Plant)

When leveling of the production schedule is necessary due to uneven demand, companies will require higher levels of inventory. Items can be produced during slack periods even though they may not be demanded until a later time. Finished goods inventory has a higher value because of its form utility; hence, there is a greater financial risk resulting from product obsolescence, damage or loss.

However, higher levels of inventory, coupled with a uniform production schedule can be more advantageous that a fluctuating schedule with less inventory. In additions when stock outs cost are great because of production slowdowns or shutdowns, JIT may not be optimal system. JIT reduces inventory levels to the point where there is little if any safety stock, and parts shortages can adversely affect production operations.

II. Supplier production Schedules

Success of JIT system depends on supplier's ability to provide parts in accordance with the company's production schedule. Smaller, more frequent orders can result in higher ordering costs and must be taken into account when calculating any cost savings due to reduced inventory levels. When a large number of small lot quantities are produced, suppliers incur higher production and setup costs. Generally, suppliers will incur higher costs, unless they are able to achieve the benefits associated with implementing similar system with their suppliers.

III. Supplier location

As distances between the companies and its supplier's increases, delivery times may become more erratic and less predictable. Shipping cost increases as less truck movement are made. Transit time variability can cause inventory stock outs that disrupt production scheduling; when this is combined with higher delivery costs on a per unit basis, total costs may be greater that the savings in inventory carrying cost.

Other problem areas that can become obstacles in JIT, especially in implementation, are (Louis-Guist, 1993):

i. Organizational resistance to change
ii. Lack of systems support
iii. Inability to define service levels
iv. Lack of planning
v. Shift of inventory to suppliers

2.4.5 Implication of JIT

JIT has numerous implications for logistics operations

First, proper implementation of JIT requires that the firm fully integrate all logistics. Many trade-offs are required, but without the coordination provided by integrated logistics management, JIT systems cannot be fully implemented.

Second, transportation becomes an even more vital component of logistics under a JIT system. In such an environment, the demands placed on the firm's transportation network are significant and include a need for shorter, more consistent transit times; more sophisticated communications; the use of fewer carriers with long term relationships; a need for efficiently designed transportation and materials handling equipment; and better decision-making strategies relative to when private, common, or contract carriage should be used.

Third, warehousing assumes an expanded role as it assumes the role of consolidation facility instead of a storage facility. Since many products come into the products come into the manufacturing operation at shorter intervals, less space is required for storage, but there must be an increased capability for handling and consolidating items. Different forms of materials handling equipment may be needed to facilitate the movement of many products in smaller quantities. The location decision for warehouses serving inbound materials needs may change because suppliers are often located closer to the manufacturing facility in a JIT system.

JIT systems are usually combined with other systems that plan and control material flow into, within, and out of the organization.

2.5 Supply Chain Management (SCM)

Many theorists have given the definitions for the term supply chain management. One of them that can describe the term supply chain management really well and it seems to cover all related activities is that;

According to Basu *et al*., (2008) Supply chain management was a set of approaches utilized to efficiently integrate suppliers, manufacturers, warehouses and stores, so that merchandise was produced and distributed at the right quantities, to the right locations, and at the right time, in order to minimize system-wide costs while satisfying service level requirements.

Coyle *et al*., (2003) discussed that as the definition implied; supply chain management had been developed for customers who played the most important role in businesses. Especially in the globalization era, customers, ever more demanding and powerful than before, were seeking for products and services with higher criteria. In order to meet customers' requirements and satisfactions, companies had to be proactive against globalized markets which could be changed and influenced by several factors. With an increase of use of technology like internet, some claim that there was no more geography in business nowadays. Offshore production, collaboration between international companies, and openness of the global market were the significance of the global environment. Supply chain management could therefore be labeled as global supply chain management in today's environment.

Supply Chain Management evolved soon after lean manufacturing and Just-in-Time system were implemented in the 1970's. This was after manufacturers realized the impact carrying excess inventory and work in progress had on the quality of the products and lead time. Excess inventory along the manufacturing line leads to congestion and consequently affects the quality of the products. Once the quality is affected, the rework rate increases and hence lead time increase. Carrying smaller inventories required fostering a better relationship with the suppliers so that the manufacturers could expect a better response time from the suppliers. This led to development of supplier partnerships. The manufacturers also realized that close relationships with the customers helped the manufacture of products that conformed to customer's needs and helped the manufacturers decide on their next product line based on what the customer wanted. Thus customer partnerships were promoted. These new dimensions in the manufacturing chain led to supply chain management.

Hence supply chain management originated in the later parts of 1980. Since that time a lot of researchers have studied this management concept extensively. The literature present in the field ranges from the different definitions coined to explain and categorize SCM, to the different principles and algorithms needed to apply it to the manufacturing and distribution industries.

Harland, (1996) stated that SCM was the technique of managing business practices and relationships within and outside an organization including all the suppliers and the customers. Scott *et al.*, (1991) defined SCM as material management for the products until they reach the end of their life in the supply chain (i.e. until they reach the customer).

The definition of New *et al.*, (1995) emphasized the importance of the transportation and logistics function of SCM.

Tan *et al.*, (2001) emphasized that SCM literatures spanned different aspects of manufacturing, but as they developed their summary, they detected two distinct perspectives that were more prominent than the others: the purchasing and supply perspective and transportation and logistics perspective. The purchasing and supply perspective refers to integration and standardization of the suppliers for a manufacturing company to make the purchasing function more effective (Farmer, 1997). The transportation and logistics perspective refers to the area of integration of the transportation providers with the manufacturing company to make their transportation and distribution function more effective.

Fredendall *et al*. (1997) presented a comprehensive view of the supply chain and the reasons for it being a focus of research for the last ten years. They also discuss reasons for the change in the operating policy of the manufacturing environment. Lead time and customer satisfaction gained importance as the traditional policy of “make as much as possible to fulfill any amount of demand to gain profit” took a back seat.

The authors also described different aspects of the supply chain such as management basics, performance measures, purchasing and distribution. Fredendall *et al*. (1997) explained the logistics cost analysis as shown in details below:

$$
\begin{aligned}
& C_l = C_t + C_w + C_o + C_{lq} + C_i \\
& where, \\
& C_l = Total\_Logistics\_Cost \\
& C_t = Transportation\_Cost \\
& C_w = Warehou\sin g\_Cost \\
& C_o = Order\_\Pr ocess\sin g\_Cost \\
& C_{iq} = Lot\_Quantity\_Cost \\
& C_i = Inventory\_Carrying\_Cost
\end{aligned}
$$

The author pointed out that most analysts treated the order processing costs as constant and only took into consideration the inventory holding costs and the transportation costs in the process of minimizing the total cost. However, this assumption did not lead to optimal solutions.

Each part of the logistics cost had to be considered during the optimization process and the order processing costs (per unit) depended on the size of the order being filled. This principle was used in developing an optimization methodology.

2.6.0 Logistics Management

Logistics Management is defined as the process of planning, implementing and controlling the efficient flow and storage of goods, services and related information from point of origin to point of consumption for the purpose of conforming to customer requirements.

The following are some of the key activities required to facilitate the flow of a product from point of origin to point of consumption, they include;

i. Customer service
ii. Demand forecasting
iii. Inventory Management
iv. Logistics Communication
v. Materials Handling
vi. Transportation
vii. Warehousing

2.6.1 Push System

Push system is referred when raw materials are stored before production and products are produced to stock before orders are placed. The action is stimulated by demand estimation or demand forecast. Products and information flow the same way, from seller to buyer. Communication carried out in the supply chain of this approach can be either interactive or non-interactive since customers or buyers do not always response to messages sent by producer or sellers. For example, there is no direct feedback from customers after message in advertisement was sent by vendors through media channels. Push system, typical and traditional, is still widely utilized by many firms in different industries.

2.6.2 Pull System

Pull system, on the other hand, is used in response to confirmed orders. Products are produced after or at production planning stage. Therefore, stock does not contain finished goods, but semi-finished materials. Customers send their requirements and place orders to producers or sellers. The requested product is pulled through the delivery channel. Communication carried out in pull system is usually interactive. Pull model is also widely used inside the same firm, for instance, a department sends an internal order to the other department to manufacturer an item that is needed in their work process.

Pull system includes just-in-time (JIT) which is an inventory strategy to improve to improve business‟ inventory turnover by bringing inventory to a minimum. JIT strategy considers inventory as waste, its emphasis therefore is ensure that supplies are delivered at when and to where they are needed.

2.7.0 Inventory Control

Inventory control is challenging in business. Managing inventory control can directly affect business performance. The reason for having inventories or stocks is to buffer against demand and supply. Having too much inventory on hand means high holding cost, and having too little leads to a rise in ordering cost. Therefore, inventory management should be well planned in order to achieve the lowest possible total cost.

Even though inventory is considered as a negative impact in business since large proportion of total expenses is generated here, but having inventory is still a must for many kinds of business. Managing and controlling inventory are compulsory practices for firms that seek for profitability. The goals for controlling inventory are minimizing the total cost and maximizing service level by balancing demand and supply. There are several approaches involved in managing inventory. Businesses are characterized by two distinguished systems, push and pull. JIT is a pull system while EOQ (Economic Order Quantity) includes elements of push strategies in proactive manner.

When it comes to hospital pharmacy, being proactive is the most crucial qualification. Generally, order or demand is not confirmed beforehand since number of patients is really difficult to predict. However, it is predictable in some cases, for instance, diabetic and HIV patients who must regularly get treatments and constantly require particular medicines. Hence, push system is mostly used in hospital pharmacy and some other healthcare facilities since drugs must be available when they are needed.

Medicinal products are really unique compared to the other commodities since they deal with illness and life saving. It is common for warehouse managers to try to reduce inventory level and minimize the total cost. Sometimes, this leads to falling in service level. Inventory management in hospital is handled differently compared to some other organizations in healthcare industry since hospitals do not seek for a big margin from drug sales. Inventory in hospitals should be therefore managed a bit differently. Service level should be the first priority, then minimizing costs and losses.

2.7.1 Justification for Having Inventory

- Economies of scale can be obtained by purchasing large volumes which allows cost reduction of per unit fixed cost. Also, transportation can get economies of scale through utilization by moving larger volume of products.

- Balancing supply and demand is another important reason for having inventory. If supply is seasonal, inventory can help meet demand when materials or products are not available. Vice versa, if there is an occurrence of seasonal demand, firms must accumulate inventory in advance to meet demand in the future.

- Specialization can bring economies of scale to manufacturers by long production run. Instead of producing a variety of products, each plant can product a product and ship to customers or other warehouse.

- Protection from uncertainties is a primary reason for holding inventory. Having stock on hand can reduce risk of shortage or stockout situation which might lead to lost sales and lack of reliability. Customer can possibly buy products from competitors instead.

2.7.2 Inventory Costs

Inventory is associated with three major costs as follows.

- Ordering Cost covers all costs occurring during the ordering processes of one order regardless of volume or quantity ordered. It includes costs and time spent on requesting for quotations, entering purchase order, approving order, checking received order, invoicing, making payment and reviewing order report. Ordering cost might not be a big component of the total cost for firms but considering time spent on one order and management efforts, ordering cost should be therefore properly reduced. Not only can saving in ordering cost bring the total cost down, some other costs such as wages (less staff required) is also cut.

- Holding Cost is divided into three categories namely risk costs, storage costs and finance costs. Risk costs include deterioration, obsolescence, damage and theft. Storage costs are associated with renting, building, racking, special storage such as refrigeration, and handling costs. Finally, finance costs include interest on money invested in inventories and insurance.

- Stockout Cost is the cost of not having products available or enough when they are demanded by customers. It might be difficult to calculate this cost. For example, stockout cost causes losing in sales both current and future since customers might turn to competitors. Stockout cost in some organization such as hospital might be greater than the other types of organization.

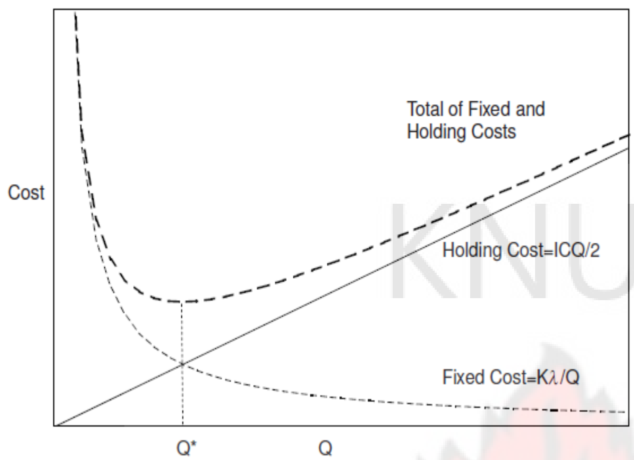


Figure 1: Cost Trade-offs

Figure 1 illustrates the general relationship of holding cost and ordering cost. The tendency of holding and ordering costs are generally opposed to each other. Different sizes of order have a significant impact to the total cost. The lowest total cost can be obtained at the most suitable order quantity which can be determined by balancing holding and ordering costs. However, the other types of costs can also affect the total cost depending on different logistic processes such as, transportation, shortage, or in-transit inventory carrying costs.

CHAPTER 3

METHODOLOGY

3.0 Introduction

The Economic Order Quantity model is widely used based on its simple nature. Simple and limiting modeling assumptions usually go together, and the EOQ model is not an exception.

Nevertheless, the presence of these modeling assumptions does not mean that the model cannot be used in practice. There are several instances where the EOQ model will produce good results. For example, the models have been successfully working in automotive, pharmaceutical and retail sectors of the economy for many years. Another benefit is that the model usually gives the optimal solution in closed form. This gives way to gain more insights about the behavior of the inventory system.

The closed-form solution is also easy to compute as compared to, for example, an iterative method of computation.

In this chapter, the development of models for a single-stage system in which inventory of a single item; in this case Toyota Vehicle will be managed. The purpose of these models is to determine how much to order or order quantity and when to place the order or reorder point. The common factor across these models is the assumption that demand occurs continuously at a constant and known rate. First of all the simple model in which all demand is satisfied on time is considered. Secondly a model in which some of the demand could be backordered is studied. Thirdly the EOQ model is considered again; but this time, the unit purchasing cost depends on the order size. The final section briefly discuss how to manage many item types when constraints exist that link the lot size decisions across items (Muckstadt *et al*. 2010)

3.1 EOQ Model Development

To begin with, various assumptions underlying the EOQ model are discussed. This discussion is also used to present the notation:

i. Demand arrives continuously at a constant and known rate of λ units per year. Arrival of demand at a continuous rate implies that the optimal order quantity may be non-integer. The fractional nature of the optimal order quantity is not a significant problem so long as the order quantity is not very small; in practice, one simply rounds off the order quantity. Similarly, the assumption that demand arrives at a constant and known rate is rarely satisfied in practice. However, the model produces good results where demand is relatively stable over time.

ii. Whenever an order is placed, a fixed cost K is incurred. Each unit of inventory costs $\$I$ to stock per year per US$ invested in inventory. Therefore, if a unit's purchasing cost is C, it will cost $I \cdot C$ to stock one unit of that item for a year.

iii. The order arrives τ years after the placement of the order. It is assumed that τ is deterministic and known.

iv. All the model parameters are unchanging over time.

v. The length of the planning horizon is infinite.

vi. All the demand is satisfied on time.

The goal is to determine the order quantity and the reorder interval. In view of the fact that all the parameters are stationary over time, the order quantity, denoted by Q, also remains stationary. The reorder interval is related to when an order should be placed, since a reorder interval is equal to the time between two successive periods at which an order is placed and is called the cycle length. A cycle is the time between the placing of two successive orders. The question of when to place an order has a simple answer in this model. Since demand occurs at a deterministic and fixed rate and the order once placed arrives exactly τ years later, we would want the order to arrive exactly when the last unit is being sold. Thus the order should be placed τ years before the depletion of inventory.

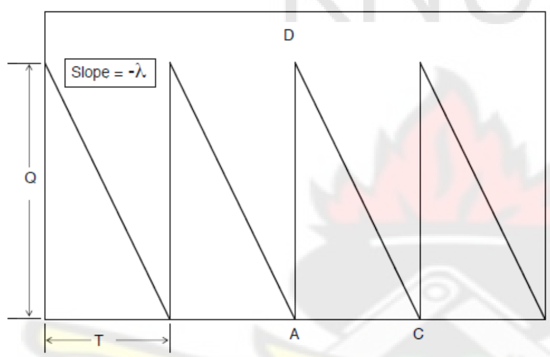


Figure 2: Inventory/ Time Graph

The first step in the development of the model is the construction of cost expressions.

Since total demand per year is λ, the total purchasing cost for one year is $C\lambda$.

Similarly, the number of orders placed per year is equal to λ / Q. Therefore, the total annual average cost of placing orders is $K\lambda / Q$. The derivation of the total holding cost per year is a bit more involved. By first computing the average inventory per cycle it is started. Since each cycle is identical to any other cycle, the average inventory per year is the same as the average inventory per cycle. The holding cost is equal to the average inventory per year times the cost of holding one unit of inventory for one year.

The average inventory per cycle is equal to:

$$\frac{Area\_of\_triangle\_ADC}{Length\_of\_the\_cycle} = \frac{\frac{1}{2}QT}{T} = \frac{Q}{2}$$

The annual cost of holding inventory = $\dfrac{ICQ}{2}$

Adding the three types of costs together, the following objective function is obtained, which is to be minimized over Q:

$$\min\ Z_{Q\geq 0} = C\lambda + \frac{K\lambda}{Q} + \frac{ICQ}{2}$$

Prior to computing the optimal value of Q, what the optimal solution should look like is examined. First, the higher the value of the fixed cost K, the fewer the number of orders that should be placed every year. This means that the quantity ordered per order will be high. Second, if the holding cost rate is high, placing orders more frequently is economical since inventory will on average be lower. A higher frequency of order placement leads to lower amounts ordered per order. Therefore, our intuition tells us that the optimal order quantity should increase as the fixed ordering cost increases and decrease as the holding cost rate increases.

To compute the optimal order quantity, the first derivative of $Z(Q)$ with respect to Q is taken and set it equal to zero:

$$\frac{dZ}{dQ} = 0 - K\frac{\lambda}{Q^2} + \frac{IC}{2} = 0$$

or

$$Q^* = \sqrt{\frac{2K\lambda}{IC}}$$

Where Q^* is the optimal order quantity.

Note that the derivative of the purchasing cost $C\lambda$ is zero since it is independent of Q.

To gain more insights, additional properties of the optimal solution possesses are explored.

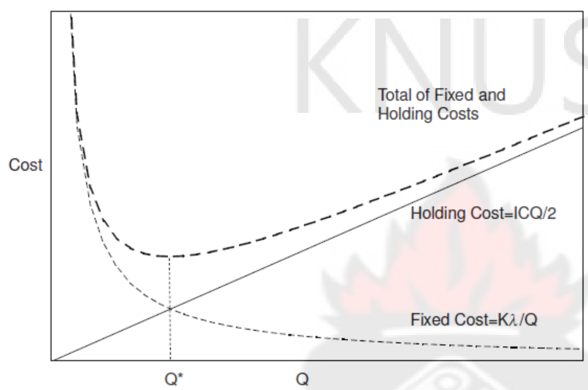


Figure 3: Cost/Inventory Graph

The figure shows the plot of the average annual fixed order cost $K\lambda\ /Q$ and the annual holding cost $ICQ/2$ as functions of Q. The aver age annual fixed order cost decreases as Q increases because fewer orders are placed. On the other hand, the average annual holding cost increases as Q increases since units remain in inventory longer. Thus the order quantity affects the two types of costs in opposite ways. The annual fixed ordering cost is minimized by making Q as large as possible, but the holding cost is minimized by having Q as small as possible. The two curves intersect at $Q=Q_1$

By definitionof Q_1

$$K\frac{\lambda}{Q_1} = \frac{ICQ_1}{2} \Rightarrow Q_1 = \sqrt{\frac{2K\lambda}{IC}}$$

and, in this case, $Q1 = Q^*$.

The exact balance of the holding and setup costs yields the optimal order quantity. In other words, the optimal solution is the best compromise between the two types of costs. (Inventory models are based on finding the best compromise between opposing costs.) Since the annual holding cost $ICQ^*/2$ and the fixed cost $K\lambda /Q^*$ are equal in the optimal solution, the optimal average annual total cost is equal to:

$$
\begin{aligned}
Z(Q^*) &= C\lambda + \frac{ICQ^*}{2} + K\frac{\lambda}{Q^*} \\
&= C\lambda + 2K\frac{\lambda}{Q^*} \\
&= C\lambda + 2K\frac{\lambda}{\sqrt{\frac{2\lambda K}{IC}}} \\
&= C\lambda + \sqrt{2\lambda KIC}
\end{aligned}
$$

3.2 Robustness of the EOQ Model

In the real world, it is often difficult to estimate the model parameters accurately. The cost and demand parameter values used in models are at best an approximation to their actual values. The policy computed using the approximated parameters, henceforth referred to as approximated policy and cannot be optimal. The optimal policy cannot be computed without knowing the true values of the model's parameters. Clearly, if another policy is used, the realized cost will be greater than the cost of the true optimal policy.

An upper bound is now derived on the realized average annual cost of using the approximate policy relative to the optimal cost. Suppose the actual order quantity is denoted by Q^*_a . This is the answer we would get if we could use the true cost and demand parameters.

Let, true fixed cost and holding cost rate be denoted by K_a and I_a, respectively.

It is assumed that the purchasing cost C and the demand rate λ have been estimated accurately. The estimates of the fixed cost and holding cost rate are denoted by K and I, respectively. The estimated order quantity is denoted by Q^*.

Let,

$$\frac{Q^*}{Q_a^*} = \alpha$$

$$Q^* = \alpha Q_a^*$$

Thus,

$$Q_a^* = \sqrt{\frac{2\lambda K_a}{I_a C}}$$

$$Q^* = \sqrt{\frac{2\lambda K}{IC}} = \alpha \sqrt{\frac{2\lambda K_a}{I_a C}}$$

$$\Rightarrow \alpha \sqrt{\left(\frac{K}{I}\right)\left(\frac{I_a}{K_a}\right)}$$

Since the purchasing cost Cλ is not influenced by the order quantity, it is not included in the comparison of costs.

The true average annual operating cost (the sum of the holding and order placement costs) is equal to

$$Z(Q_a^*) = \sqrt{(2K_a \lambda I_a C)}$$

The actual incurred average annual cost (sum of the holding and order placement costs) corresponding to the estimated order quantity Q^* is equal to:

$$Z(Q^*) = \frac{K_a \lambda}{Q^*} + \frac{I_a C Q^*}{2}$$

$$Z(Q^*) = \frac{K_a \lambda}{\alpha Q_a^*} + \frac{I_a C(\alpha Q_a^*)}{2}$$

$$= \frac{K_a \lambda}{\alpha \sqrt{\frac{2\lambda K_a}{I_a C}}} + \frac{I_a C \alpha \sqrt{\frac{2\lambda K_a}{I_a C}}}{2}$$

$$= \frac{1}{2}\left(\alpha + \frac{1}{\alpha}\right)\sqrt{2 K_a \lambda I_a C}$$

$$= \frac{1}{2}\left(\alpha + \frac{1}{\alpha}\right) Z(Q^*)$$

Therefore, if the estimated order quantity is a times the optimal order quantity, the average annual cost corresponding to the estimated order quantity is

$\frac{1}{2}\left(\alpha + \frac{1}{\alpha}\right)$ times the optimal cost.

For example, if α = 2 (or 1/2), that is, the estimated order quantity is 100% greater (or 50% lower) than the optimal order quantity, then the cost corresponding to the estimated order quantity is 1.25 times the optimal cost.

Similarly, when α = 3 (or 1/3), the cost corresponding to the estimated order quantity is approximately 1.67 times the optimal cost.

Two observations can be made. First, and importantly, even for significant inaccuracies in the order quantity, the cost increase is modest. As showed, the cost increase is only 25% for a 100% increase in the estimated order quantity from the optimal order quantity. The moderate effect of inaccuracies in the cost parameters on the actual incurred average annual cost is very profound.

Second, the cost increase is symmetric around α = 1 in a multiplicative sense. That is, the cost increase is the same for

$$Q^* / Q_a^* = \alpha$$

or

$$Q^* / Q_a^* = \frac{1}{\alpha}$$

How to estimate α? Clearly, if α could be precisely estimated, then it could compute Q_a^* precisely as well and there would be no need to use the estimated order quantity. Since it cannot ascertain its value with certainty, perhaps it can estimate upper and lower bounds for α. These bounds can give bounds on the cost of using the estimated order quantity relative to the optimal cost.

3.3 Reorder Point and Reorder Interval

In the EOQ model, the demand rate and lead time are known with certainty. Therefore, an order is placed such that the inventory arrives exactly when it is needed. This means that if the inventory is going to be depleted at time t and the lead time is τ, then an order should be placed at time $t - \tau$. If an order is placed before time $t - \tau$, then the order will arrive before time t. Clearly holding costs can be eliminated by having the order arrive at time t. On the other hand, delaying the placement of an order so that it arrives after time t is not permissible since a backorder will occur.

How to determine the reorder point in terms of the inventory remaining on the shelf? There are two cases depending upon whether the lead time is less than or greater than the reorder interval, that is, whether $\tau \leq T$ or $\tau > T$.

Since the on-hand inventory at the time an order arrives is zero, the inventory at time $t-\tau$ should be equal to the total demand realized during the time interval $(t - \tau, t]$, which is equal to $\lambda\tau$.

Therefore, the reorder point when $\tau \leq T$ is equal to:

$$r^* = \lambda\tau$$

In other words, whenever the inventory drops to the level $\lambda\tau$, an order must be placed. Observe that r^* does not depend on the optimal order quantity.

On the other hand, when $\tau > T$, the reorder point is equal to

$$r^* = \lambda\tau_1$$

where τ_1 is the remainder when τ is divided by T.

That is,

$$\tau = mT + \tau_1,$$

Where m is a positive integer.

The time between the placements of two successive orders, T, is equal to the time between the receipts of two successive order deliveries, since the lead time is a known constant. Since orders are received when the inventory level is zero, the quantity received, Q^*, is consumed entirely at the demand rate λ by the time the next order is received.

Therefore, if the optimal reorder interval is denoted by T^*,

$$Q^* = \lambda T^*$$

Hence,

$$T^* = \frac{Q^*}{\lambda}$$

$$= \sqrt{\frac{2K}{\lambda IC}}$$

3.4 EOQ Model with Backordering Allowed

Here one of the assumptions made is about satisfying all demand on time. Now some of the demand is to be allowed to be backordered, but there will be a cost penalty incurred. The rest of the modeling assumptions remain unaltered.

As a result, the cost function now consists of four components:

i. Purchasing cost
ii. Fixed cost of order placement
iii. Inventory holding cost
iv. Backlog penalty cost.

Each order cycle is comprised of two sub-cycles. The first sub-cycle (ADC) is T_1 years long and is characterized by positive on-hand inventory which decreases at the demand rate of λ. The second sub-cycle (CEF) is T_2 years long, during which demand is backordered. Hence there is no on-hand inventory during this time period. Since no demand is satisfied in this latter period, the backlog increases at the demand rate λ. The total length of a cycle is $T = T_1+T_2$.

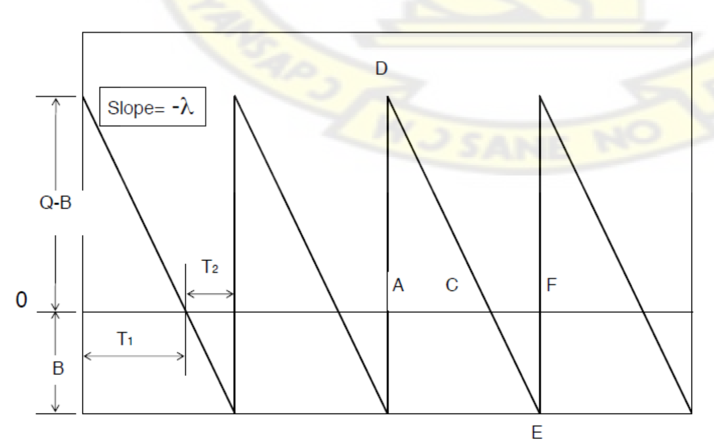


Figure 4: Inventory/ Time Graph with backorder

There are two decisions to be made:

i. How much to order whenever an order is placed

ii. How large the maximum backlog level should be in each cycle.

The order quantity is denoted by Q as before, and we use B to denote the maximum amount of backlog allowed. When an order arrives, all the backordered demand is satisfied immediately. Thus, the remaining $Q-B$ units of on-hand inventory satisfy demand in the first subcycle.

Since this on-hand inventory decreases at rate λ and becomes zero in T_1 years,

$$Q - B = \lambda T_1$$

In the second sub-cycle, the number of backorders increases from 0 to B at rate λ over a period of length T_2 years.

Thus,

$$B = \lambda T_2$$

and

$$Q = \lambda(T_1 + T_2)$$
$$= \lambda T$$

The cost expressions for the purchasing cost and annual fixed ordering cost remain the same as for the EOQ model and are equal to $C\lambda$ and $K\lambda /Q$, respectively. The expression for the average annual holding cost is different. First compute the average inventory per cycle and then multiply the result by the holding cost IC to get the annual holding cost. Average inventory per cycle is equal to:

$$\frac{Area\_of\_triangle\_ADC}{T} = \frac{\frac{(Q-B)T_1}{2}}{T}$$

Next substitute for $T1$ and T. This results in an expression which is a function only of Q and B, the decision variables.

$$Average\_Inventory\_per\_Cycle = \frac{\frac{(Q-B)^2}{\lambda}}{2\frac{Q}{\lambda}}$$

$$= \frac{(Q-B)^2}{2Q}$$

The computation of the average annual backordering cost is similar. Let π be the cost of backordering a unit for one year. The first step is to compute the average number of backorders per cycle. Since all cycles are alike, this means that the average number of outstanding backorders per year is the same as the average per cycle. To get the average backorder cost per year, multiply the average backorder quantities per year by the backorder cost rate. The average number of backorders per cycle is equal to the area of triangle CEF in Figure 4 divided by the length of the cycle T:

$$\frac{Area\_of\_tiangle\_CEF}{T}$$

$$= \frac{\frac{BT_2}{2}}{T}$$

$$= \frac{\frac{B^2}{2\lambda}}{\frac{Q}{\lambda}} = \frac{B^2}{2Q}$$

In the last equality, the relationships $T_2=B/\lambda$ and $T=Q/\lambda$ are used. Therefore, the average annual backorder cost is equal to $\pi \frac{B^2}{2Q}$

Now a combination of all the cost components and an expression of the average annual cost of managing inventory as:

$$Z(B,Q) = C\lambda + K\frac{\lambda}{Q} + IC\frac{(Q-B)^2}{2Q} + \pi\frac{B^2}{2Q}$$

Before obtaining the optimal solution, it is anticipated what properties to expect the optimal solution to possess. As before, if the fixed order cost *K* increases, fewer orders will be placed, which will increase the order quantity. An increase in the holding cost rate should drive the order quantity to lower values. The effect of the backorder cost on the maximum possible number of units backordered should be as follows: the higher the backorder cost, the lower the maximum desirable number of backorders.

To obtain the optimal solution, take the first partial derivatives of $Z(B,Q)$ above with respect to Q and B and set them equal to zero. This yields two simultaneous equations in Q and B:

$$\frac{\partial Z(B,Q)}{\partial Q} = -K\frac{\lambda}{Q^2} + IC\frac{(Q-B)}{Q} - IC\frac{(Q-B)^2}{2Q^2} - \pi\frac{B^2}{2Q^2}$$

$$\frac{\partial Z(B,Q)}{\partial B} = -IC\frac{(Q-B)}{Q} + \pi\frac{B}{Q}$$

$$let, \frac{\partial Z(B,Q)}{\partial B} = 0$$

Hence,

$$B = Q\frac{IC}{IC+\pi}$$

Substituting $B = Q\frac{IC}{IC+\pi}$ in equation $\frac{\partial Z(B,Q)}{\partial Q}$

$$\frac{\partial Z(B,Q)}{\partial Q} = -K\frac{\lambda}{Q^2} + IC\frac{(Q-(Q\frac{IC}{IC+\pi}))}{Q} - IC\frac{(Q-(Q\frac{IC}{IC+\pi}))^2}{2Q^2} - \pi\frac{(Q\frac{IC}{IC+\pi})^2}{2Q^2}$$

$$= -K\frac{\lambda}{Q^2} + IC\frac{\pi}{IC+\pi} - IC\frac{\pi}{2(IC+\pi)}$$

$$= -K\frac{\lambda}{Q^2} + IC\frac{\pi}{2(IC+\pi)}$$

$$\Rightarrow Q^* = \sqrt{\frac{2K\lambda(IC+\pi)}{IC\pi}}$$

$$= \sqrt{\frac{(IC+\pi)}{\pi}}\sqrt{\frac{2K\lambda}{IC}}$$

$$= Q_E\sqrt{\frac{IC+\pi}{\pi}}$$

Where Q_E is the optimal solution to the EOQ model.

Also,

$$B^* = Q^*\frac{IC}{IC+\pi} = \sqrt{\frac{2K\lambda IC}{(IC+\pi)\pi}}$$

$$T^* = \frac{Q^*}{\lambda} = \sqrt{\frac{2K(IC+\pi)}{\lambda IC\pi}}$$

Several observations can now be made.

(i). The pre-derivation intuition holds. As K increases, Q^* increases, implying a decrease in the number of orders placed per year. As the holding cost rate I increases, $\frac{IC+\pi}{IC} = 1 + \frac{\pi}{IC}$ decreases, and Q^* decreases. Finally, as π increases, the denominator of B^* increases then B^* decreases.

(ii). The maximum number of backorders per cycle, B^*, cannot be more than the order quantity per cycle Q^* since $\frac{B^*}{Q^*} = \frac{IC}{IC+\pi} \leq 1$

(iii). Q^* is never smaller than Q_E, the optimal order quantity for the EOQ model without backordering. Immediately after the arrival of an order, part of the order is used to fulfill the backordered demand. This saves the holding cost on that part of the received order and allows the placement of a bigger order. As π increases, backordering becomes more expensive and B^* *d*ecreases. As a result, the component of Q^* that is used to satisfy the backlog decreases and Q^* comes closer to Q_E.

(iv). As the holding cost rate I increases, $\frac{IC}{IC+\pi}$ increases and B^* increases.

To improve the understanding even further, a comparison between the fixed cost to the sum of the holding and backordering costs at $Q = Q^*$ and $B = B^*$

From,

$$B^* = Q^* \frac{IC}{IC+\pi}$$

which means that:

$$\frac{Q^* - B^*}{Q^*} = 1 - \frac{B^*}{Q^*}$$

$$\Rightarrow$$

$$1 - \frac{IC}{IC+\pi} = \frac{\pi}{IC+\pi}$$

Now, the holding cost at Q = Q^* and B = B^* is equal to

$$\frac{IC(Q^* - B^*)^2}{2Q^*} = \frac{ICQ^*}{2}\left(\frac{Q^* - B^*}{Q^*}\right)^2 = \frac{ICQ^*}{2}\left(\frac{\pi}{IC+\pi}\right)^2$$

Similarly, the backordering cost at Q = Q^* and B = B^* is equal to:

$$\frac{\pi B^{*2}}{2Q^*} = \frac{\pi}{2}\left(\frac{IC}{IC+\pi}\right)^2 Q^*$$

Therefore, the sum of the holding and backordering cost is equal to

$$\frac{Q^*}{2}\left(\frac{\pi^2 IC + (IC)^2\pi}{(IC+\pi)^2}\right) = \frac{Q^*}{2}\left(\frac{IC\pi}{IC+\pi}\right)$$

$$Q^* = \sqrt{\frac{\lambda KIC\pi}{2(IC+\pi)}}$$

On the other hand, the fixed cost of order placement at Q =Q^* is equal to

$$\frac{\lambda K}{Q^*} = \sqrt{\frac{\lambda KIC\pi}{2(IC+\pi)}}$$

Where having substituted for Q^*.

The expressions for the average annual fixed cost and sum of the holding and backordering cost are equal. Note that in an optimal solution the costs are balance.

3.5 The Optimal Cost

Since the optimal average annual cost is equal to the purchasing cost plus two times the average annual fixed cost,

$$Z(B^*, Q^*) = C\lambda + K\frac{\lambda}{Q^*} + IC\frac{(Q^* - B^*)^2}{2Q^*} + \pi\frac{(B^*)^2}{2Q^*}$$

$$= C\lambda + 2K\frac{\lambda}{Q^*}$$

$$= C\lambda + \sqrt{\frac{2\lambda KIC\pi}{(IC+\pi)}}$$

3.6 Quantity Discount Model

Here, inventory management is studied when the unit purchasing cost decreases with the order quantity *Q*. In other words, a discount is given by the seller if the buyer purchases a large number of units. The objective is to determine the optimal ordering policy for the buyer in the presence of such incentives.

The discussion is based on all units discounts quantity discounts.

Quantity Purchased at a time	Unit price($)
0-10	60000
11-25	58000
26-50	55000
51 and above	50000

Table 1: Table of Quantity Purchased against Price

From the illustration of the price breakdown above, it can clearly be seen the quantity ordered at any point in time influences the price of the inventory.

Such discounts are offered by suppliers or manufacturers to make the customers purchase more per order. Large orders result in high inventory holding cost because an average unit spends a longer time in the system before it gets sold. Thus the seller's price discount subsidizes the buyer's inventory holding cost. A typical example of company that practices this mode of ordering is Shoprite Chain of Supermarkets across Africa which uses this strategy to negotiate good bargains for its clients.

3.7 All Units Discount

The purchasing cost now depends on the value of Q:

$$Z_j(Q) = C_j\lambda + \frac{IC_jQ}{2} + K\frac{\lambda}{Q}$$

where,

$$q_j \le Q \le q_{j+1}$$

Hence,

$$Q_j^* = \sqrt{\frac{2\lambda K}{IC_j}}$$

3.8 Lot Sizing When Constraints Exist

In the preceding portions of this chapter, the focus has been on determining the optimal ordering policy for a single item. In many if not most real-world situations, decisions are not made for each item independently. There may be limitations on space to store items in warehouses; there may be constraints on the number of orders that can be received per year; there may be monetary limitations on the value of inventories that are stocked.

Each of these situations requires stocking decisions to be made jointly among the many items managed at a location.

Holding costs are often set to limit the amount of space or investment consumed as a result of the lot sizing decisions. Rather than assuming a holding cost rate is used to calculate the lot sizes, suppose a constraint is placed on the average amount of money invested in inventory. Thus a budget constraint is imposed that limits investment across items.

Let,

Q_i be the procurement lot size for item i

C_i the per-unit purchasing cost for item i, $i = 1, \ldots, n$,

where n is the number of items being managed.

The sum measures the average amount invested in inventory over time.

$$\sum_{i=1}^{n} C_i \frac{Q_i}{2}$$

Let b be the maximum amount that can be invested in inventory on average.

Furthermore, suppose the goal is to minimize the average annual total fixed procurement cost over all item types while following the budget constraint.

Let λ_i and K_i represent the average annual demand rate and fixed order cost for item i, respectively.

$\lambda iKi/Qi$ measures the average annual fixed order cost incurred for item i given Qi is the lot size for item i.

Define

$$F_i\,(Q_i) = \lambda_i K_i / Q_i.$$

This procurement problem can be stated as follows:

$$\min \sum_{i=1}^{n} \frac{\lambda_i K_i}{Q_i} = \sum_{i=1}^{n} F_i(Q_i)$$

$$subject\_to$$

$$\sum_{i=1}^{n} C_i \frac{Q_i}{2} \leq b,$$
$$Q_i \geq 0$$

CHAPTER 4

DATA COLLECTION AND ANALYSIS

4.0 Introduction

Globally Toyota has become the most preferred model and has been making tremendous sales over the years. Some of the key factors that are contributing to Toyota's success include commitment to customer satisfaction, quality and continuous improvement of its supply chain systems.

The backbone of this superior supply chain is an efficient Inventory Management system.

For the purposes of this thesis, the relationship between Toyota Motor Cooperation (TMC) which is the manufacturer and Toyota Ghana Company Limited (TGCL) the distributor is examined through the supply chain processes. TMC manufacture stocks approximately based on the confirmed orders that are expected. TMC fulfills demands of hundreds of customers or dealership across the world. TMC receives orders from TGCL. Given the huge size of its Supply Chain Management, it is deemed impossible to coordinate all the inventory management of the national and regional distribution centers. Consequently, each national and regional distribution center manages the inventory on its own regardless of the policies at TMC.

For the purpose of this study, three main models of Toyota vehicles will be considered. They are the Prado, Fortuner and the Corolla:

		PRADO	FORTUNER	COROLLA
ENGINE		**2.7Litre Petrol**	5L-E	16V 1.8-Litre
Type		4-Cylinder, In-line, Twin Cam (16 Valve), VVT-i	4-Cylinder, In-line 8-valve,OHC	4-cyl. In-line Twin Cam Dual VVT-i
Piston displacement	Cc	2694	2986	1798
Max output (SAE net)	kW/rpm	120/5200	70/4000	100/6000
Max torque (SAE net)	Nm/rpm	246/3800	197/2200	175/4400
Fuel system		EFI	EFI	EFI
DIMENSIONS & WEIGHT				
Gross Vehicle Weight	Kg	2990	2510	1785
Kerb weight	Kg	2265	1900	1320
Overall Length	Mm	4930	4695	4540
Overall Width	Mm	1885	1840	1760
Overall Height	Mm	1890	1795	1470
Wheelbase	Mm	2790	2750	1320
Tread Front/Rear	Mm	1585	1540	1535
Ground Clearance	Mm	220	220	150

Table 2: Table of Models against Specifications

4.1 Data Expected and Notation

Demand arrives continuously at a constant rate (λ) per year.

Total Demand rate (λ) per year, therefore the optimal order quantity may not be an integer.

Implying that whenever an order is placed, a Fixed Cost (K) is incurred

Order Quantity (Q)

Minimum Order Quantity (s)

Maximum Order Quantity (S)

Optimal Order Quantity (Q^*)

Unit of Inventory Holding Cost rate (I)

Unit of Purchase Cost (C)

Unit of Holding Cost (IC) per year

Lead Time or Order Arrival Time (τ) per year

Total Purchasing Cost ($C\lambda$) per year

Number of Orders placed (λ/Q) per year

Total Average Cost of placing order ($K\lambda/Q$) per year

Holding Cost per year = Average Inventory per year * Holding Cost per unit per year

Holding Cost of Inventory ($ICQ/2$) per year

Time between placing two successive orders (T) = Time between receiving two successive orders (T)

4.2 Preliminary Analysis of Data

Toyota Corolla 1.6 Automatic is considered as a specific car model. The demand for this vehicle is almost steady throughout the year at a rate of 2 units per week. The purchasing cost of the Corolla paid by TGCL to TMC is $23500 per unit. In addition, TGCL spends on average $2500 per unit on transportation, import duties and clearing charges. A breakdown of the different types of costs is as follows:

1. The TGCL calculates its interest rate to be 25% per year.

2. The cost of maintaining the warehouse and its depreciation is $100000 per year, which is not dependent of the amount of inventory stored there. In addition, the costs of pilferage and misplacement of inventory are estimated to be around 3% of average inventory stocked which is covered under insurance.

3. The annual cost of Order Management System and other Operational/HR Cost is $500000 and not dependent on how often sales are made or orders are placed.

4. The cost of invoice preparation, handling charges, time, etc. is estimated to be $10000 per order.

5. The cost of unloading every order that arrives is estimated to be $120000 per order.

To determine the optimal order quantity; the first task is to determine the cost parameters.

The holding cost rate I is equal to the interest rate (.25) plus the cost rate pilferage and misplacement of inventory or loses (.03).

Therefore, unit of Inventory Holding Cost rate (I) = 25% + 3% = 0.28

This rate applies to the value of the inventory when it arrives at TGCL.

This value includes not only the purchasing cost ($23500) but also the value added through transportation, duties and clearing ($2500).

Therefore, the value or Unit of Purchase Cost (C) = $23500+$2500 = $26000.

Finally, the fixed cost of order placement includes all costs that depend on the order frequency. Thus, it includes the order receiving cost ($120000) and the cost of invoice preparation, etc. ($10000), but not the cost of the order management system.

Therefore, Fixed Cost (K) = $130000

Demand arrives continuously at a constant rate (λ) per year. = 2 units per week =104

Lead Time or Order Arrival Time (τ) per year = 0.25 = 3 months =12 weeks

Total Purchasing Cost ($C\lambda$) per year = 2704000

Holding Cost per year = Average Inventory per year * Holding Cost per unit per year

Substituting to get the optimal order quantity:

$$Q^* = \sqrt{\frac{2K\lambda}{IC}}$$

The following is obtained:

$$Q^* = \sqrt{\frac{2(130000)104}{(.28)26000}}$$

$$Q^* = \sqrt{\frac{27040000}{7280}}$$

$$Q^* = \sqrt{3714.285714}$$

$$Q^* = 60.94494002 \approx 61$$

The data from Toyota Ghana Company Limited illustrate the computation of the optimal order quantity for Toyota Fortuner.

Demand arrives continuously at a constant rate (λ) per year = 10 units per week =520

Implying that whenever an order is placed, a Fixed Cost (K) is incurred = 60000

Order Quantity (Q) = not known

Optimal Order Quantity (Q^*) = not known yet

Unit of Inventory Holding Cost rate (I) = 20% = 0.20

Unit of Purchase Cost (C) = 37000

Unit of Holding Cost (IC) per year = 7400

Lead Time or Order Arrival Time (τ) per year = 0.25 = 3 months =12 weeks

Total Purchasing Cost ($C\lambda$) per year = 19240000

Number of Orders placed (λ/Q) per year = not known

Total Average Cost of placing order ($K\lambda/Q$) per year = not known

Holding Cost per year = Average Inventory per year * Holding Cost per unit per year

Holding Cost of Inventory (ICQ/2) per year = not known

Substituting these values in:

$$Q^* = \sqrt{\frac{2K\lambda}{IC}}$$

The following is obtained:

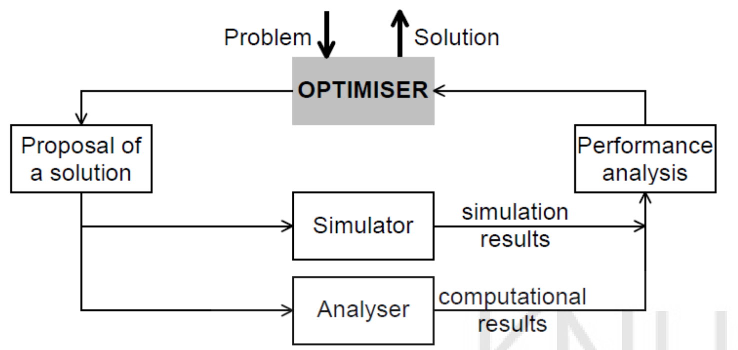
$$Q^* = \sqrt{\frac{2(60000)520}{(.20)37000}}$$

$$Q^* = \sqrt{\frac{62400000}{7400}}$$

$$Q^* = 91.82827 \approx 92$$

This means that in a year the Optimal Order Quantity of Fortuner is approximately 92 units.

4.3 Inventory Analysis Formulation Worksheet in Excel



INVENTORY VARIABLE FORMULATION WORKSHEET				
				INSERT VALUE
K	=	Fixed Cost (K) Per Year	=	
I	=	Unit of Inventory Holding Cost rate (I)	=	
λ	=	Demand rate (λ) per year	=	
C	=	Purchase Cost © per Unit	=	
EOQ			=	=SQRT((2*E4*E8)/((E10)*E6))
Optimum Number of Orders Per Year			=	= SQRT((E4*E6)/(2*E8))
Optimum Number of Cost Per Order			=	=SQRT((2*E4*E8)/E6)

Demand arrives continuously at a constant rate (λ) per year = 10 units per week =520
Implying that whenever an order is placed, a Fixed Cost (K) is incurred = 60000
Unit of Inventory Holding Cost rate (I) = 20% = 0.20
Unit of Purchase Cost (C) = 37000

Order Quantity (Q) = not known

Optimal Order Quantity (Q^*) = not known

Number of Orders placed (λ/Q) per year = not known

Total Average Cost of placing order (Kλ/Q) per year = not known

Holding Cost per year = Average Inventory per year * Holding Cost per unit per year

Holding Cost of Inventory (ICQ/2) per year = not known

Next, it is determined whether or not the optimal EOQ solution matches the intuition.

If the fixed cost K increases, the numerator of $Q^* = \sqrt{\dfrac{2K\lambda}{IC}}$ increases and the optimal order quantity $Q*$ increases. Similarly, as the holding cost rate I increases, the denominator of $Q^* = \sqrt{\dfrac{2K\lambda}{IC}}$ increases and the optimal order quantity Q^* decreases. Clearly, the solution fulfills expectations.

$$Z(Q^*) = C\lambda + \frac{ICQ^*}{2} + K\frac{\lambda}{Q^*}$$

$$Z(61) = 26000(104) + \frac{0.28(26000)(61)}{2} + 130000\frac{104}{61}$$

$$Z(61) = 2704000 + 2222040 + 221639.3443$$

$$Z(61) = 5147679.3443 \approx 5147679$$

The optimal cost is:

$$C\lambda + \sqrt{2\lambda KIC}$$

$$= 270400 + \sqrt{2(130000)(0.28)(104)(26000)}$$

$$= 270400 + 2662074.98$$

$$= 5366074.98$$

4.4 Robustness of the EOQ Model

With the Toyota Fortuner the fixed cost of order placement is estimated to be \$60000 and the holding cost rate is estimated to be 20%

To calculate the alternative policy and the cost difference between employing this policy and the optimal policy. Recall that the average annual cost incurred when following the optimal policy is \$5147679.3443

To compute the alternative policy, substitute the estimated parameter values into $Q^* = \sqrt{\frac{2K\lambda}{IC}}$

$$Q^* = \sqrt{\frac{2(60000)520}{(.20)37000}}$$

The realized average annual cost it this policy is used is:

The optimal cost is:

$$C\lambda + \sqrt{2\lambda KIC}$$

$$= 270400 + \sqrt{2(130000)(0.28)(104)(26000)}$$

$$= 270400 + 2662074.98$$

$$= 5366074.98$$

Thus the cost difference between the alternative and optimal policies is

\$5366074.98 −\$5147679.344 = **\$218395.6357**

The optimal ordering policy could save Toyota Ghana Company Limited **\$218,395** per every order.

Note the cost of implementing the alternative policy is calculated using the actual cost parameters.

Still remaining on the Fortuner TGCL is confident that the actual fixed cost is at most 120% but no less than 80% of the estimated fixed cost.

Similarly, TGCL is sure that the actual holding cost rate is at most 110% but no less than 90% of the estimated holding cost rate.

4.5 Reorder Point and Reorder interval

The lead time is equal to 2 weeks. In this case, the reorder interval T^* is equal to

$$T^* = \frac{Q^*}{\lambda}$$

Since

Lead Time or Order Arrival Time (τ) per year = 0.25 = 3 months =12 weeks, which is less than the reorder interval.

The reorder point r^* is equal to

$$r^* = \lambda\tau$$

$$r^* = 520 * 0.25$$

$$r^* = 130$$

CHAPTER 5

SUMMARY, CONCLUSION AND RECOMMENDATIONS

5.0 Practicality of the situation

From the outcome of this analysis of the mathematical optimal policy, the ordering cost and holding cost are almost equal. However, this could occur for every 14 weeks of ordering, which practically could mean 3 or 4 months. In terms of periodical pattern, 1 month is practically appealing because of the shipping pattern of the manufacturer and could disrupt or defy normal shipping schedule. This monthly pattern is now compared with the optimal situation of 3 months.

5.1 Summary

On the other hand, the annual Ordering cost is more than the annual holding cost. This difference shows that the situation is far from optimal because for optimal policy, these costs should be equal or the same. Compared with the monthly policy, the quarterly approach or optimal ordering policy could save Toyota Ghana Company Limited **$218,395** per every order. In effect, the optimal policy edges the monthly policy by implying that saving could be made if the ordering is planned properly.

Also the difference between the monthly policy and the optimal policy is not a problem because it is both practical, convenient and ensures availability of stocks.

Therefore, evident benefits in the efficient policy brought forward by the monthly orders are that;

i. It is practicable when compared with the mathematically optimal one.

ii. It saves the company thousands of dollars.

iii. Availability of stocks at very point in time.

iv. There is warehouse space that could be used for other purposes.

5.2 Conclusion

There could be cases where the optimal solution could be implemented as derived directly form mathematical optimization. In the case study used, it was a learning curve to understand theory in relation to actual industry practice. From the case study it becomes evident that optimality from a mathematical approach serves as a guide to efficiency in real life, and it does not necessarily lead to a conclusion that could be implemented. It is concluded; however, that theory be pursued as it enhances better practice, while practice encourages improvement and advancement of existing theories. Practice could also trigger research that could lead to a new theory.

5.3 Recommendations

1. Economic Order Quantity Model can be used as a tool for analyzing the effectiveness of managing any Inventory System against other economic indicators.

2. We recommend that Toyota Ghana Company Limited continues with the tight Inventory Policy in trying to reduce high Inventory Levels since it was effective.

3. We also recommend a study be carried out to research the effectiveness of some other Mathematical Models like the Economic Order Frequency (EOF) Model in the management of Inventory.

REFERENCES

Mentzer, John 2001, Supply Chan Management, Sage Publications Inc.

Muckstadt, J.A. and Sapra, A. 2010, "Principles of Inventory Management: When You Are Down to Four, Order More" Springer Series Operations Research and Financial Engineering

Harland, C. M. 1996 "Supply Chain Management; relationships, chains and networks" British Academy of Management, 7

Scott and Westbrook 1991, "New Strategic Tools for Supply Chain Management", International Journal of Physical Distribution and Logistics Management

New and Payne 1995, "Research frameworks in Logistics: three models, seven dinners and a survey, International Journal of Physical Distribution and Logistics Management

Council of Logistics Management, 2001 extracted from "What its All About"

Tan, Choon Kean, 2001, "A framework of supply chain management literature" European Journal of Purchasing and Supply Management

Farmer, 1997, "Purchasing myopia-revisited", European Journal of Purchasing and Supply Management

Fisher 1997 "What is the right supply chain for your product?" Harvard Business Review, 75

Fredendall and Hill, 2001 Basics of Supply Chain Management, St. Lucie Press

Frazelle, 2001, World Class Warehousing and Material Handling, The McGraw Hill Companies

Liberatore, 1979, "The EOQ Model under Stochastic Lead Time", Operations Research, Vol. 27

Kim, Joon-Seok and Benjaafar, "Is the (Q,R) Model appropriate for Production-Inventory Systems?" University of Minnesota

Stevenson, 1993, Production/Operations Management, Forth Edition, Richard D Irwin Incorporated.

Benton and Park, 1996, “A classification of literature on determining the lot size under quantity discounts”, European Journal of Operational Research.

Khouja, 2001, “The effect of large order quantities on expected profit in the single-period model”, International Journal of Production Economics

Guder et al, 1994, “Capacitated Multiple Item Ordering with Incremental Quantity Discounts”, The Journal of Operational Research Society

Amrik, S.S., Ramsay Liz, and Samson Danny, 1993 “JIT Manufacturing: Industry Analysis and a Methodology for Implementation”, International Journal of Physical Distributions and Logistics Management

Snehemany B and Golhar D. Y., 1993, “EDI Implementation: A Comparative study of JIT and Non-JIT Manufacturing firms,” International Journal of Physical Distribution and Logistics Management, 23

Giunipero L.C and Law W.K., 1990 “Organizational Support for JIT Implementation,” the International Journal of Logistics Management, 1

Sohal et al., 2003 “JIT Manufacturing”

Ibid, 2003 “JIT Defined”, International Journal of Logistics management

Quinn, F. J. et al., 1990, “Why US Companies are Embracing JIT”, Traffic Management, 29

Dada, Maqbool and Srikanth, K.N., 1987, “Pricing Policies for Quantity Discounts”, Management Science, Vol 33