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DEPARTMENT OF INDUSTRIAL MATHEMATICS

MODELLING TRANSPORTATION PROBLEMS IN THE TIMBER INDUSTRY: A CASE OF ASUO BOOSADU TIMBER SAWMILL LIMITED (ABTS), BEREKUM

By

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Declaration

We hereby declare that this submission is our own work towards the award of the Master of Science degree and that, to the best of my knowledge, it contains no material previously published by another person nor material which had been accepted for the award of any other degree of the university, except where due acknowledgement had been made in the text.

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Dedication

This study is dedicated to Almighty God, Owusu-Boahene family and my daughter Richlove

Abstract

The transportation problem is a special class of the linear programming problem. It deals with the situation in which a commodity is transported from source to destinations.

The proposed transportation model of manufacturing good to customers (key distributors) is considered in this research. The data gathered were modelled as a linear programming model of transportation type and represent the transport problem as a tableau and solve it with computer software solves to generate an optional solution.

The main objective is to model the product of ABTS transportation as a transportation problem and minimize the cost of transportation of plywood in the ABTS Company. The quantitative method (QM) software will be used to analyze the data.

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Contents

List of Abbreviation

LP .... Linear programming

TP ....Transportation Problem

ABTS ....AsuoBomosadu Timber And Sawmill Limited

TP ....Transportation problem

BKM ....Berekum (factory)

TM ....Tamale (warehouse)

KS ....Kumasi (warehouse)

MDA .... Millennium Development Goal

GDP .... Gross Domestic Product

C.E.O .... Chief Executive O cer

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CHAPTER 1

INTRODUCTION

1.1 Introduction

Forests are very important in the development of many African countries as they play a key role in most aspects of the socio-economic lives of the people.

Also, considering the signi cant role forests play in mitigating climate change, it is essential to conserve forests by reducing deforestation and increasing forest cover. The United Nations' Millennium Development Goals (Henceforth referred to as MDG) encourages developing countries to meet certain targets aimed at helping them achieve a higher status of development. Forestry plays an important part in Ghana's economy. In the 1980s, timber was the third-largest export commodity after cocoa and gold, accounting for 5-7% of the total gross domestic product (GDP), and the forestry sector employed some 70 000 people. Forests also provide 75% of Ghana's energy requirements(Arsham,1992).

1.1.1 TIMBER USES

Some of the uses of timber are as follows: Air dispensers (eg aquariums),Arti cial limbs, Bakers equipment, Balance, decks and terraces, Boat and ship construction, Cladding, Beehives, Carving and sculpture, Cooperage, Cabinet making, Fencing, Flooring, Furniture's, Glass manufacture log cabins, Musical instruments, Pallets, Paper and paper products, Power poles, Saunas and hot tubs, Sca olding, Shingles, Smocking produce(eg sh and meat), Railways sleepers and Windows.

1.1.2 Timber bene ts (over other construction

materials)

i. Timber is the only 100% renewable resource of construction material ii. Renewable resource allows for the direct employment of hundreds of thousands of people. Thus improving local economy. iii. Timber from managed plantations are Greenhouse Gas Reducing.

iv. Ecologically safe and sound to handle and dispose

v. Natural Variations add esthetic interest.

1.1.3 Some Types of Timbers Found in Ghana

Some of these timber are;

Odum(miliciaexcelsa), Awilemfosamina (albiziaferruginea), teak (tactinagranais), wawa (tropolochitmseleroxylon), watapuo (cola gigantean), potrodom (erythrophleumivorense), kokradua (pericopsiselata), kusia (naucleadiderrichii), mansonai (African black walnut) , ofram (terminaliasuperba) and ceiba.

1.2 Background to the Research

1.2.1 Overview of Forestation in Africa

Forest resources are essential to social and economic activities in Africa; as a result, they are important elements in both poverty reduction and sustainable development strategies for many Sub-Saharan African countries(Reeb and Leavengood,2000). There is therefore the need to protect forests and implement policies and programs that ensure that these forests are sustained for future generations. Also, considering the rise in development activities such as the discovery of oil, increasing activities in mining and the ever growing telecommunications industry on the continent, it is necessary to evaluate or assess policies aimed at sustaining forests so this essential resource is not lost in the future(?).

One of the most important and successful applications of quantitative analysis to solving business problem has been in the physical distribution of products, commonly referred to as transportation problems(Goldfarb and Kai,1986).
Basically, the cost of shipping goods from one location to another is to meet the needs of each arrival area and every shipping location operation within its capacity. In this context, it refers to a planning process that allocates
resources-labour, materials, machines, capital in the best possible (optional) way
so that cost are minimized or pro ts are maximized. In Linear programming (LP), these resources are known as decision variable. The criterion for selecting the best values of the decision variable (eg to maximize pro ts or minimize cost) is known as the objective function.
Limitations on resource availability form what is known as a constraint set(?).

Transportation model is one of those techniques that can help to nd an optimum solution and save the cost in transportation models or problems primarily concerned with the optimal (best possible) way in which a product factories or plants (called supply origins) can be transported to a number of warehouses or customers (called demand destinations)(?). The objective in a transportation problem is to fully satisfy the destination requirements within the operating production capacity constraints at the minimum possible cost. Whenever there is a physical movement of goods from the point of manufacturer to the nal consumers through a variety of channels of distribution (wholesalers, retailers, distributors etc), there is a need to minimize the cost of transportation so as to increase pro t on sales(?).

The transportation problem is a special class of linear programming problem that commodities from source to destinations. The objective of the transportation problem is to determine the shipping schedule that minimizes the total shipping cost while satisfying supply and demand limits. The model assumes that the shipping cost is proportional to the number of units shipped on a given route. In general, the transportation model can be extended to other areas of operation, including, among others, inventory control, employment scheduling and personnel assignment.

The transportation problem received this name because many of its applications involve in determining how to optimally transport goods. The transportation problem deals with the distribution of goods from several points, such as factories often known as sources, to a number of points of demand, such as warehouses, often known as destinations. Each source is able to supply a xed number of units of products, usually called the capacity or availability and each destination has a xed demand, usually known as requirement.

Because of its major application in solving problems which involves several products sources and several destinations of products, this type of problem is frequently called the transportation problem. The classical transportation problem is referred to as a special case of Linear Programming (LP) problem and its model is applied to determine an optional solution for delivering available amount of satis ed demand in which the total transportation cost is minimized. The transportation problem can be described using linear programming mathematical model and usually it appears in a transportation tableau. One possibility to solve the optional problem would be optimization method. The problem is however formulated so that objective function and all constraints are linear and thus the problem can be solved. There is a type of linear programming problem that may be solved using a simpli ed version of the simplex technique called transportation method. The simplex method is an iterative algebraic procedure for solving linear programming problems(Badr,2007).

Transportation theory is the name given to the study of optional transportation and allocation of resources. The model is useful for making strategic decisions involved in selecting optimum transportation routes so as to allocate the production of various plants to several warehouses or distribution centres. The transportation model can also be used in making location decisions. The model helps in locating a new facility, manufacturing a new facility, manufacturing plant or an o ce when two or more of the locations are under consideration.

The total transportation cost, distribution cost or shipping cost and production costs are to be minimized by applying the model. Transportation problem is a particular class of linear programming, which is associated with day-to-day activities in our real life and mainly deals with logistics. It helps in solving problems on distribution and transportation of resources from one place to another. The goods are transported from a set of sources (eg factory) to a set of destinations (eg. warehouse) to meet the speci c requirements.

There is a type of Linear programming problem that may be solved using a simpli ed version of the simplex technique called transportation method.

Because of its major application in solving problems involving several product sources and several destinations of products, this type of problem is frequently called the transportation problem. It gets its name from its application to problems involving transporting products from several sources to several destinations. Although the formation can be used to represent more general assignment and scheduling problems it is also a transportation and distribution problems. The two common objectives of such problems are to;

1. Minimize the cost of shipping in units to destinations

2. Maximize the pro t of shipping in units to destinations

The transportation problem itself was rst formulated by Hitchcock (1941), and was independently treated by Koopmans and Kantorovich. In fact, Monge (1781) formulated it and solved it by geometrical means. Hitchaxic (1941) developed the basic transportation problem; however it could be solved for optimally as answers to complex business problem only in 1951. When George B. Dantizig applied the concept of Linear programming in solving the transportation model, Dantzing (1951) gave the standard Linear Programming (LP) formulation, Transportation problem (TP) and applied the simplex method subject in almost every textbook on operation research and mathematical programming.

Linear programming has been used successfully in solution of problem concerned with the assignment of personnel, distribution and transportation, engineering, banking education, petroleum etc. Furthermore, LP algorithms are used in subroutines for solving more di cult optimization problems. A widely considered quint essential LP algorithm is the simplex Algorithm developed by Dantzig (1947) in response to a challenge to mechanise the Air Force planning process. Linear Programming has been applied extensively in various areas such as transportation, health care and public services etc. The simplex algorithm was the forerunner of many computer programs that are used to solve complex optimization problem (Baynto, 2006). The transportation method has been employed to develop many di erent types of process. From machine shop

scheduling, Mohaglegh (2006) optimized operating room schedules in hospitals (Goldfarb and Kai,1986).The transportation problem is a special kind of the network optimization problem. The transportation models play an important role in logistics and supply chains. The objective is to schedule shipments from sources to destinations so that total transportation cost is minimized. The problem seeks a production and distribution plan that minimizes total transportation cost.

1.3 HISTORY OF THE COMPANY

1.3.1 Pro le of Company

The Asuo Bomosadu Timber and Sawmills Limited (A. B. T. S) is among the oldest timber company in the Brong-Ahafo Region of Ghana. It is a private limited liability company registered on the 16th October, 1980 and was authorized to commence business on October 22, 1980 with its certi cate of incorporation. The name Asuo Bomosadu Timber and Sawmill (A. B. T. S) was coined from a river AsuoKoraa in consultation with Traditional leaders of the Berekum Community. Asuo Bomosadu had a humble beginning with startup capital from the sale of the hardware that the Managing Director was into after he had decided to divert into a Timber Merchant with the sole aim of employing many youths in the community of Berekum and its environs, In 1993, the company got its rst concession from ministry of Lands and Forestry and now the company has ve di erent timbers concession with a total area of about $562.94 sqarekilometers$. The purpose for establishing the company in the rural area included the following;

To be nearest to raw materials

To get the tax rebate for establishing factory in the rural areas

And also to nd employment to people within Berekum and outside

A. B. T. S has two main supportive o ces; Takoradi and Accra in Amasaman. The one in Takoradi headed by Export Manager which deals speci cally in Export shipping documentation and contracts negotiation and Accra o ce deals in general matters of the company.

The company exports about 60% of the total production with the local fallen 40% because of high production cost.

1.3.2 Company Brands

A. B. T. S - Berekum produces and markets many brands of products and these include plywood, T & G, Parquet and boards. The company exports the following products: Parquet, T & G, Plywood and Veneer.

1.3.3 Warehouse

Raw materials, semi- nished goods and nished goods are kept at the warehouse at Berekum (factory). Semi- nished goods and nished goods are kept at the warehouse in Accra (Amasaman).The Takoradi one is a store House for export. The distribution of semi- nished and the nished productsare outsourced to third party contractors, that is, A. B. T. S. operates in 3 party logistics which ensures materials and nished goods are delivered at the right time to the right place in accordance with the planning schedule and at a minimum cost. The few
registered transporters are responsible for loading, o loading and movement of raw materials from the bush to the company warehouse and movement of nished product to the distributors.

1.3.4 Distributors

Finished products are sold to registered distributors, some are sent to Takoradi for export and some are sold to retailers at the company warehouse at Berekum and Accra (Amasaman). The distributors act as whole sales that sell directory to the public.

1.4 PROBLEM STATEMENT

The transportation costing policies of the company under study is not based on results obtained from mathematical modeling but from rule-ofthumb methods.This thesis seek to address the problem of determining the optional transportation schedule that will improve and minimize the total cost of transporting nished products from the company to the various
distributors.

1.5 OBJECTIVES

The aim of this study is to look principally at a speci c type of Linear Programming problem, known as the Transportation Problem. The speci c
objectives are;

1. To model the distribution of A. B. T. S products as a transportation problem
2. To use the mathematical model to derive optimal timber distribution using data of timber supply and demanded by product type and location.
3. To use the LP to minimize the transportation problem in A. B. T. S.

1.6 METHODOLOGY

The propose of this thesis utilize a Linear Programming Technique in solving the transportation Problem of a Sawmill producing company in Berekum with a view to minimizing the total transportation cost and obtaining an optimal schedule bearing in mind the present transportation policy of the company.

This work become necessary because it was discover that the transportation costing policies of the company under study were not based on results obtained from mathematical modeling but from rule-of thumb methods. The data gathered will be modeled as a linear Programming model of transportation type and solve it with Linear Programming (LP) computer software solver to generate an optimal solution.

This LP solver application management scientist is based on a simpli ed version of the simplex technique. The transportation model will be useful for making strategic decisions by the logistics managers in making optimal allocation of the product from the warehouse (source) to the various customers (distribution) at minimum transportation cost.

The Quantitative Method (QM) software will be used to analyses the data

1.7 JUSTIFICATION

The pro t gained as a result of minimizing the transportation cost will enable A. B. T. S to contribute to its continuous projects target and programmes such as:

1. Employment: create employment to people within Berekum and outside
2. Culture and entertainment: Sponsoring festivals in Brong Ahafo Region, such as Techiman Apoo Yam festival.
3. Education: Given scholarship to the brilliant but needy students to higher institution.
4. Environmental support: Planting trees (A orestation) projects around Berekum area.
5. Social responsibility: Construction of two public toilet facilities at Berekum.

1.8 ORGANIZATION OF THE THESIS

Chapter one which introduces the thesis in general contains sub-section like the review of transportation problem, the background for Transportation Problem, the background of company (A. B. T. S), the problem statement, objective, methodology, justi cation and the organization of the thesis.

Chapter two is the literature review, which looks at brie y work done by other researchers on this topic.

Chapter three is about the methodology that is used to analysis available data.

Chapter four provides an over view of the computational platforms for implementation and solution of the model and introduces the real life data sets used in the solution process.

Finally, chapter ve summaries the conclusions with respect to overall aims of the project and proposed recommendation for future research/study. It reports the computational results and provides a comprehensive analysis of the outcome and performance of the proposed solution approaches.

CHAPTER 2

LITERATURE REVIEW

2.1 INTRODUCTION

The Transportation Problem is probably the most important special Linear Programming problem in terms of relative frequency with which it appears in the application and also in the simplicity of the procedure developed for its solution. The Transportation Problem were the earliest class of linear programs discovered to have totally unit modular matrices and integral extreme points resulting in considerable simpli cation of the simplex method.

The study of the Transportation Problems laid the foundation for further theoretical and algorithmic development of the minimal cost network ow problems. This chapter will review previous and relevant research work in study area.

2.2 LITERATURE REVIEW

The Linear programming Theory and technique have been successfully applied to various transportation problems almost since its early beginning. A famous example is given by Dantzig (1951) to adopt his simplex method to solve (Hitchcock's) transportation problem.

The origin of transportation was rst presented by Hitchcock (1941), in a study entitled The Distribution of a product from several sources to numerous Localities . This studywas considered to be the rst important contribution to the solution of transportation problems. Koopmans (1947), presented an independent study, not related to Hitchcock's, and called it "Optimum Utilization of the Transportation System". These two contributions helped in the development of transportation methods which involve a number of shipping sources and a number of destinations. The transportation problem, received this name because many of its applications involve determining how to optimally transport goods.

However, it could be solved as optimally solution for an answer to complex business problem only in 1951, when George B.

The terminology, such as transportation/assignment problems and allocation problems have become a standard in these contexts since then (Adlakha V at el,2006).

In Mathematics and Economics transportation theory is a name given to the study of optimal transportation and allocation of resources.

The problem was formulized by the French mathematician Gaspard Monge in 1781.

In the 1920s A.N Tolstol was one of the rst to study the transportation problem mathematically. In 1930, in the National commissaritat of Transportation of the soviet union he published a paper 'methods of nding the minimal kilometrage in Cargo transportation in space.

Major advances were made in space in Economics the eld during World war 2 by the Soviet/Russian Mathematician and Leonid Kantoronch consequently the problem as it is sometimes known as the monge-Kantorovich transportation problem.

The Transportation Problem was formalized by the French mathematician (Monge, 1781). Major advances were made in the eld during World War II by the Soviet/Russian mathematician and economist Leonid Kantorovich consequently, the problem as it is now stated is sometimes known as the Monge Kantorovich (1942), published a paper on continuous of the problem and later with Gavurian and applied study of the capacitated transportation problem (at el,2008). Many scienti c disciplines have contributed toward analysing problems associated with the transportation problem including operation research, economics, engineering, information science and geography. It is explored extensively in the mathematical programming and engineering literatures. Sometimes referred to as the facility, location and allocation problem can be modelled as a large-scale mixed integer linear programming problem.

Ji and Chu (2002) have discussed Dual-Matrix Approach Method to solve the Transportation Problem as an alternative to the Stepping Stone Method. The approach considers the dual of the Transportation Model instead of the prima land then obtains the optimal solution of the dual using Matrix operations hence it is called dual matrix approach. In this method, the unit transportation cost is generally indicated on the North-East Corner in each cell(Barr and Klingman,1978). However

the dual matrix approach introduced by Ji and chu(2002) does not required that a transportation problem to be
balanced.
This method can be used both for balanced and unbalanced transportation
problem.
This is one of advantage in Ji and Chu (2002) approach over the Stepping Stone method. The dual matrix approach is similar to that of Stepping Stone Method where rst nd an initial feasible solution and then get next improved solution by assessing all non-basic cells until the optimal solution found.
Adlakha and Kowalski (1999, 2006) suggested an alternative solution algorithm for solving certain TP based on the theory of absolute point. Recently Adalkha and Kowalski (2009) presented various rules governing load distribution for alternate optimal solution in Transportation Problem. The load assignment for an alternate optimal solution is left mostly on the decision of the practitioner. They illustrated the structure of alternate solution in a transportation problem using the shadow price.
They also provided a systematic analysis for allocating loads to obtain and
alternate optimal solution(Bertsekas and Tseng,1985).
Many problems like multi-commodity transportation problem with di erent kind of vehicles, multi-stage transportation problem with capacity limit are an extension of the classical transportation problem considering the additional
special condition.
Solving such problems many optimization techniques are used like dynamic programming, linear programming and heuristic approaches etc.
Brezinaet. al. (2010) developed a method for solving multi-stage transportation problem with capacity limit that re ects limits of transported materials quantity. They also developed algorithm to nd optimal solution. Further they discussed e ciency of presented algorithm depends on selection of algorithm used to
obtain the starting solution.

Sudhakar et al proposed zero su x method for nding an optimal solution for transportation problem directly in 2012.

Dantizing, (1963), then uses the simplex method on transportation problem as the primal simplex transportation method. Stringer and Haley have developed a method of solution using a mechanical analogue. May be the rst algorithm to nd an optimal solution for the incapacitated transportation problem was that of Efroymson and Ray. They assumed that each of the unit production cost function has a xed charge form. But they remark that their branch and bound method can be extended to the case in which each of these function is concave and consists of several linear segments. And each unit transportation cost function is linear. Roy and Gelders (1980) solved a real life distribution problem of a liquid bottled product through a 3-stage logistic system; the stages of the system are plant-depot, depot-distributor and distributor -dealer. They modelled the customer allocation, depot location and transportation problem as a 0-1 integer programming model with the objective function of minimization of the eet operating costs, the depot set up costs, and delivery cost subject to supply constraints, demand constraints, truck load capacity constraints and driver hours constraints. The problem was solved optimally by branch-andbound and Langrangian relaxation(Bertsekas and Tseng,1985).Tzerget at el. (1995) solved the problem of how to distribute and transport the imported coal to each of the power plants on time in the required amounts and at the

required 18 quality under conditions of stable and supply with least delay. They formulated LP that minimizes the cost of transportation subject to supply constraints, demand constraints, and handing constraints vessel constraints of the ports. The model was solved to yield optimum results, which is then used as input to a decision support system that help manage the coal allocation voyage scheduling and dynamic eet assignment. Frank sharp et al, developed an algorithm for reaching an optimal solution to the production-transportation problem for the convex case. The algorithm utilizes the decomposition approach. It iterates between a linear programming transportation problem which allocates previously set plant production quantities to various markets and a routine which optimally sets plant production quantities to equate total marginal production cost, including a shadow price representing a relative location cost determined from the transportation problem. Williams applied the decomposition principle of Dantzing and Wolf to the solution of the Hitchcock transportation problem of it. In generalizations, the case in which the cost are piece wise linear convex functions is included (Bertsekas and Tseng,1988).

He decomposed the problem and reduced to a strictly linear program. In addition, he argued that the two problems are the same by a theorem that he called the reduction theorem. The algorithm given by him to solve the problem, is a variation of the simplex method with "generalized pricing operation". It ignores the integer solution property of the transportation problem such that some problems which are not strictly transportation type, and for which the integer solution property may not hold be solved.

Shetty (1959) also formulated an algorithm to solve transportation problem taking nonlinear cost. He considered the case when a convex production cost is included at each supply centre besides the linear transportation cost. Some of the approaches used to solve the concave transportation problem are presented as follows. The branch and bound algorithm approach is based on using a convex approximation to the concave cost function. It is equivalent to the solution of a nite sequence of transportation problems. The algorithm was developed as a particular case of the simpli ed algorithm for minimizing separable concave function over linear polyhedral as Falk and Soland (1969). Soland (1971) presented a branch and bound algorithm to solve concave

separable transportation problem which he called it the "Simpli ed algorithm" in comparison with similar algorithm given by Falk and himself in 1969. The algorithm reduces the problem to a sequence of linear transportation problem with the same constraint set as the original problem.

Caputo et al. presented a methodology for optimally planning long-haul road transport activities through proper aggregation of customer orders in separate full-truckload or less-than truckload shipments in order to minimize total transportation costs. They study demonstrated that, evolutionary computation techniques may be e ective in tactical planning of transportation activities. The model shows that substantial savings on overall transportation cost may be achieved adopting the methodology in a real life scenario. Equi et al. (1996) modelled a combined transportation and scheduling in one problem where a product such as sugar cane, timber or mineral are transported from multi origin supply points to multi destination demand points using carriers such as ships, trains or trucks. They de ned a trip as a full-loaded vehicle travel from one origin to one destination. They solved the model optimally using Langrangian Decomposition.

Saumis et al. (1991) considered a problem of preparing a minimum cost transportation plan by simultaneously solving following two sub-problem:
rst the assignment of units available at a series of origins to satisfy demand at a series of destinations and second, the design of vehicle tours to transport these units when the vehicles have to be brought back to their departure point. The cost minimization mathematical model was constructed, which is converted into a relaxation total distance minimization, then nally decomposed to network problems, a full vehicle problem, and an empty vehicle problem. The problems were solved by tour construction and improvement procedures. This approach allows large problems to be solved quickly and solutions to large test problems have been shown to be 1% or 2% from the optimum. Equi et al. (1996) modelled a combined transportation and scheduling in one problem where a product such as sugar cane, timber or mineral ore is transported from multi origin supply points to multi destination demand point or transshipment points using carries such as ships, trains or trucks(Adlakha V at el,2006).

Goal Programming (G. P) model and its variants have been applied to solve large-scale multi criteria decision-making problems. Charnes and cooper (1960) rst used the Goal Programming technique. This solution approach has been extended by Ijiri (1965), Lee (1972) and others. Lee and Moore (1973) used GP model for solving transportation problem with multiple con icting objective. Arthur and Lawrence (1982) designed a GP model for production and shipping patterns in chemical and pharmaceutical industries. Kwak and Schniederjans (1985) applied GP to transportation problem with variable supply and demand requirement. Several other researchers Sharma et al. (1999) have also used the GP model for solving the transportation problem. Veenanet al. proposed a heuristic method for solving transportation problem with mixed constraints which is based on the theory of shadow price. The solution obtained by heuristics method introduced is an initial solution of the transportation problems with constraints. Klingman and Russel (1975) have developed an e cient procedure for solving transportation problems with additional linear constraints. Their method exploits the topological properties of basis trees within a generalized upper bound framework. Swarup (1970) developed a technique, similar to
transportation problem.

Further developed a heuristic, called TOM (Total Opportunity-Cost Method), for obtaining an initial basic feasible solution for the transportation problem. Gass (1990) detailed the practical issues for solving transportation problems and o ered comments on various aspects of transportation problem methodologies along with discussions on the computational results, by the respective researchers(?). Sharma and Sharma (2000) proposed a new heuristic approach for getting good starting solutions for dual based approaches used for solving transportation problems. The transportation criterion is, however, hardly mentioned at all where the transportation problem is treated. Apparently several researchers have discovered the criteria independently from each other. But most papers on the subject refer to the paper by Charnes, Klingman and Szwarc as the initial papers. In Charnes and Klingman, it is called more for-less criteria (MFL), and they wrote: The criteria was rst observed in the early days of linear programming history (by whom no one knows) and has been a part of the Folklore, known to some (eg. A. Charnes and W. W. Cooper), but unknown to the great majority of workers in the eld of linear programming(?).

According to White (1972) the movement of vehicles and goods in a transportation system can be represented as ows through a time-dependent transshipment network. An inductive out-of-kilter type of algorithm is presented which utilizes the basic underlying properties of the dynamic transshipment network to optimize the ow of a homogeneous commodity through the network, given a linear cost function.

According to Chou(2007), transshipment of Cargo means that the direct handling cost per ton will be doubled. This is a severe disadvantage even for unitized cargo. Advocate of a shuttle-service/sea-feeder transportation system seem to believe that this disadvantage will be o set by savings of the very expensive time of big trunk liners. Allahviranloo and Afandizadeh(2008) formulated a model to determine the optimum investment on port development from a national investment prospective.

On the other hand, costs and bene ts are calculated from consumer and investor's view point.

Kutanoglu and Mahajan(2009) considered the time-based service levels. They discussed models to examine cost and service level in this setting within two and three location networks .This allows time-based service levels to be achieved while it is noted that there is noted that there is particular sensitivity to changes in demand. They also considered how suitable stocking levels can be obtained so as to minimize cost. This is achieved in the form of an enumeration algorithm. Mues et el,(2005) stated that the transshipment problems and vehicle Routing problems with Time Windows (VRP TW) are common network ow problem and well-studied. Combinations of both are known as intermodal transportation problems. This concept describes some real world transportation problems more precisely and can lead to better solutions, but they are examined rarely as Mathematical optimization problems.

Ozdeminet el (2006) gave a literature review on transshipment theory but limited it to deterministic cases. A more recent review covering stochastic and some non-linear facility location problems was discussed by Sydney, (2006). In particular, this survey considers some classical problems including the P-median problem.

The transportation criteria is known as Doig's criteria at the London School of Economics, named after Alison Doig who used it in exams, around 1959.

Doig did not publish any paper on it. Since the transportation criteria seems not to be known to the majority of those who are working with the transportation problem, one may be tempted to believe that this phenomenon is only an academic curiosity, which will most probably not occur, in any practical situation. Experiments done by Finke, with randomly generated instances of the transportation problem of size $100x100$ and allowing additional shipments (Post optimal) show that the transportation cost can be reduced considerably by exploiting the criteria properties. More precisely, the average cost reductions achieved are reported to be 18.6% with total additional shipment of 20.5%.In a recent paper, Deinekoet al. developed necessary and su cient conditions for a cost matrix C to be protected against the transportation criteria. These conditions are rather restrictive, supporting the observation by Finke. The existing literature has demonstrated the identifying cases where MFL paradoxical situation exists and also, has provided various methods for nding MFL solution for transportation problems(Dial and Klingman,1979).

Gupta et al. and Arsham obtained the more-for-less solution for the TPs with mixed constraints and by introducing new slack variables. While yielding the best more-for-less solution, their method is tedious since it introduces more variables and requires solving sets of complex equations. The perturbed method was used for solving the TPs with constraints Adlakha et al. proposed a heuristic method for solving TPs with mixed constraints which is based on the theory of shadow price. In the heuristic algorithm for an MFL solution in Adlakha et al. Vogel Approximation Method (VAM) and MODI (Modi ed Distribution) method were used. Arsham developed an approach to post optimality analysis of the TPs through the use of perturbation analysis(?).

Adlakha and Kowalki introduced a theory of absolute points for solving a TP and used these points for search opportunities to ship more for less in TP.

Adlaka et al. developed an algorithm for nding an optimal MFL solution for TPs which builds upon any existing basic feasible solution. Since then, these problems have been studied extensively by many authors and have found application in such diverse elds as geometry, mechanics, statistics, economics, shape recognition, inequalities and meteorology(Dantzig,1951).

Li and Shi (2000) formulated a dynamic transportation model with multiple criteria and multiple constraint levels (DMC2) using the framework of multiple criteria and multiple constraints(MC2) LP. An algorithm is developed to solve such DMC2 transportation problems. In this algorithm, dynamic programming ideology is adopted to nd the optimal sub-policies and optimal policy for a given DMC2 transportation problem.

Then the MC2-simplex method is applied to locate the set of all potential solutions over possible changes of the objective coe cient parameter and the supply and demand parameter for the DMC2 transportation problem(Dantzing,1963).

CHAPTER 3

METHODOLOGY

3.1 INTRODUCTION

This chapter expatiates on the various procedure and methods employed in this project The classical transportation problem can be stated mathematically as follows:

Let a_i denotes quantity of product available at origin i, b_j denotes quantity of product required at destination j, C_{ij} denotes the cost of transporting one unit of product from source/origin i to destination j and x_{ij} denotes the quantity transported from origin i to destination, then problem is to determine the transportation Schedule so as to minimize the total transportation Cost satisfying supply and demand condition.

3.2 TRANSPORTATION PROBLEM

This is a type of linear programming problem that may be solved using a simpli ed version of the simplex technique called transportation method.

Because of its major application in solving problems involving several product sources and several destinations of products, this type of problem is frequently called the transportation problem.

In a transportation problem, we have certain origins, which may represent factories where we produced items and supply a required quantity of the products to a certain number of destinations. This must be done in such a way as to maximize the pro t or minimize the cost. Thus we have the places of production as origins and the places of supply as destinations. Sometimes the origins and destinations are also termed as sources and sinks.

Transportation model is used in the following:

♣ To decide the transportation of new materials from various centres to di erent manufacturing plants. In the case of multi-plant company this is highly useful.

♣ To decide the transportation of nished goods from di erent manufacturing plants to the di erent distribution centres. For a multi-plant-multi-market company this is useful. These two are the uses of transportation model. The objective is minimizing transportation cost.

3.3 MATHEMATICAL FORMULATION

Supposed a company has m warehouses and n retail outlets. A single product is to be shipped from the warehouses to the outlets. Each warehouse has a given level of supply, and each outlet has a given level of demand. We are also given the transportation cost between every pair of warehouse and outlet, and these costs are assumed to be linear. More explicitly, the assumptions are:

♠ The total supply of the products from warehouse $i = a_i$,where $i = 1,2,...,m$

♠ The total Demand of the products at the outlet $j = b_j$, where $j = 1,2,...,n$.

♠ The cost of sending one unit of the product from warehouse i to outlet j is equal to C_{ij} , where $i = 1,2,...,m$ and $j = 1,2,...,n$. The total cost of a shipment is linear in size of shipment.

3.3.1 The Decision Variables

The variables in the Linear Programming (LP) model of the TP will hold the values for the number of units shipped from one source to a destination. The decision variables are: X_{ij} = the size of shipment from warehouse i to outlet j, Where $i = 1,2,...,m$ and $j = 1,2,...,n$. This is a set of $\{m,n\}$ variables.

3.3.2 The Objective Function

The objective function contains costs associated with each of the variables. It is a minimization problem. Consider the shipment from warehouse i to outlet j. For i any j and , the transportation cost per unit C_{ij} and the size of the shipment is X_{ij} .Since we assume that the total cost function is linear, the total cost of this shipment is given by $C_{ij}X_{ij}$ Summing over all i and j now yields the overall transportation cost for all warehouse-outlet combinations. That is, our objective function is:

$$Minimize \sum_{i=1}^{m} \sum_{j=1}^{n} C_{ij} X_{ij}$$

3.3.3 The Constraints

The constraints are the conditions that force supply and demand needs to be satis ed. In a Transportation Problem, there is one constraint for each node. Let a_1 denote a source capacity and b_1 denote destination needs

i) The supply at each source must be used:

$$\sum_{j=1}^{n} X_{ij} = a_i, \quad i = 1, 2, \ldots, m$$

ii) The demand at each destination must be met:

$$\sum_{i=1}^{m} X_{ij} = b_j, \quad j = 1, 2, \ldots, n$$

and iii) Non

negativity:

$$X_ij > 0, \quad \forall \quad i \quad \text{and} \quad j$$

.

The transportation model will then become: Minimizing the transportation cost

$$Minimize Z = \sum_{i=1}^{m}\sum_{j=1}^{n} X_{ij}C_{ij} \tag{3.1}$$

$$\sum_{j=1}^{n} X_{ij} \leq a_i, \quad (i = 1, 2, \ldots, m) \quad \text{(Demand Constraint)} \tag{3.2}$$

$$\sum_{i=1}^{m} X_{ij} \geq b_j, \quad (j = 1,2,\ldots,n) \quad \text{(Supply Constraint)} \tag{3.3}$$

$$X_{ij} \geq 0, \quad (i = 1,2,\ldots,m; j = 1,2,\ldots,n)$$

This is a linear program with $\{m,n\}$ decision variables, functional constraints, and non-negative constraints. m =Number of sources n =Number of destinations a_i = Capacity of i^{th} source (in tons, pounds, litres, etc) b_j = Demand of j^{th}

destination (in tons, pounds, litres, etc.) $C_{ij=}$cost coe cients of material shipping (unit shipping cost) between $i - th$ source and $j - th$ destination (in \$ or as a distance in kilometres, miles, etc.) X_{ij} = amount of material shipped between $i - th$ source and $j - th$ destination (in tons, pounds, litres etc.)

A necessary and su cient condition for the existence of a feasible solution to the transportation problem is that

$$\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j$$

Remark: The set of constraints

$$\sum_{i=1}^{m} X_i = b_j \quad \text{and} \quad \sum_{j=1}^{n} X_{ij} = a_i$$

represents $m + n$ equations in $m \cdot n$ non-negative variables. Each variable X_{ij} appears in exactly two constraints, one is associated with the origin and the other is associated with the destination.

3.3.4 BALANCE TRANSPORT

The transportation algorithm is based on the assumption that the model is balanced, meaning that the total demand equals the total supply.

$$\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j$$

3.3.5 UNBALANCED TRANSPORTATION PROBLEM

$$\sum_{i=1}^{m} a_i \neq \sum_{j=1}^{n} b_j$$

The transportation problem is known as an unbalanced transportation problem.

There are existence of two cases :

Case(1):

$$\sum_{i=1}^{m} a_i > \sum_{j=1}^{n} b_j$$

Introduce a dummy destination in the transportation table. The cost of transporting to this destination is all set equal to zero. The requirement at this destination is assumed to be equal to

$$\sum_{i=1}^{m} a_i - \sum_{j=1}^{n} b_j$$

Case (2):

$$\sum_{i=1}^{m} a_i < \sum_{j=1}^{n} b_j$$

Introduce a dummy origin in the transportation table, the costs associated with this origin ia a set equal to zero. The availability is

$$\sum_{j=1}^{n} b_j - \sum_{i=1}^{m} a_i = 0$$

According to Jay Heizer and Barry Render, "Transportation modelling is an iterative procedure for solving problems that involve minimizing the cost of shipping products from a series of sources to a series of destinations". Transportation modelling nds the least-cost means of shipping supplies from
several origins to several destinations.

To use the transportation model, the following information must be considered:

1. The origin points and the capacity or supply per period at each.
2. The destination points and demand per period at each.
3. The cost of shipping one unit from each origin to each destination

Supposed a company has m warehouses and n retail outlets. A single product is to be shipped from the warehouses to the outlets. Each warehouse has a given level of supply, and each outlet has a given level of demand. We are also given the transportation cost between every pair of warehouse and outlet, and these costs are assumed to be linear. More explicitly, the assumptions are:

- The total supply of the products from warehouse $i = a_i$, where $i = 1,2,\ldots,m$
- The total Demand of the products at the outlet $j = b_j$, where $j = 1,2,\ldots,n$.

- The cost of sending one unit of the product from warehouse i to outlet j is equal to C_{ij} , where $i = 1,2,...,m$ and $j = 1,2,...,n$. The total cost of a shipment is linear in size of shipment.

3.4 TRANSPORTATION TABLEAU

The transportation problem can be described using linear programming mathematical model and usually it appears in a transportation tableau. The model of a transportation problem can be represented in a concise tabular form with all the relevant parameters. The transportation tableau (A typical TP is represented in standard matrix form), where supply availability (a_i) at each source is shown in the far right column and the destination requirements (b_j) are shown in the bottom row. Each cell represents one route. The unit shipping cost (C_{ij}) is shown in the upper right corner of the cell, the amount of shipped material is shown in the centre of the cell. The transportation tableau implicitly expresses the supply and demand constraints and the shipping cost between each demand and supply point.

To Destination→ ↓From Source↓	D_1	D_2	...D_j...	D_n	Source Supply
S_1	c_{11} x_{11}	c_{12} x_{12}		c_{1n} x_{1n}	a_1
S_2	c_{21} x_{21}	c_{22} x_{22}		c_{2n} x_{2n}	a_2
...S_i...			c_{ij} x_{ij}		...a_i...
S_m	c_{m1} x_{m1}	c_{m2} x_{m2}		c_{mn} x_{mn}	a_m
Destination Requirements	b_1	b_2	...b_j...	b_m	$\sum a$ $\sum b_j$

Figure 3.1: THE TRANSPORTATION TABLEAU

3.5 NETWORK REPRESENTATION OF TRANSPORTATION PROBLEM

Graphically, transportation problem is often visualized as a network with m source nodes, n sink nodes, and a set of $m.n$ "directed arcs" This is depicted in Fig 1.

In the diagram there are $S_1,\ldots,S_n$ sources and $D_1,\ldots,D_n$ Destination. The arrows show ows of output from source to destination .Each destination is linked to each source by an arrow. The number $C_1,\ldots,C_n$ above each arrow represents the cost of transporting on that route.

Problems with the above structure arise in many applications. For example, the

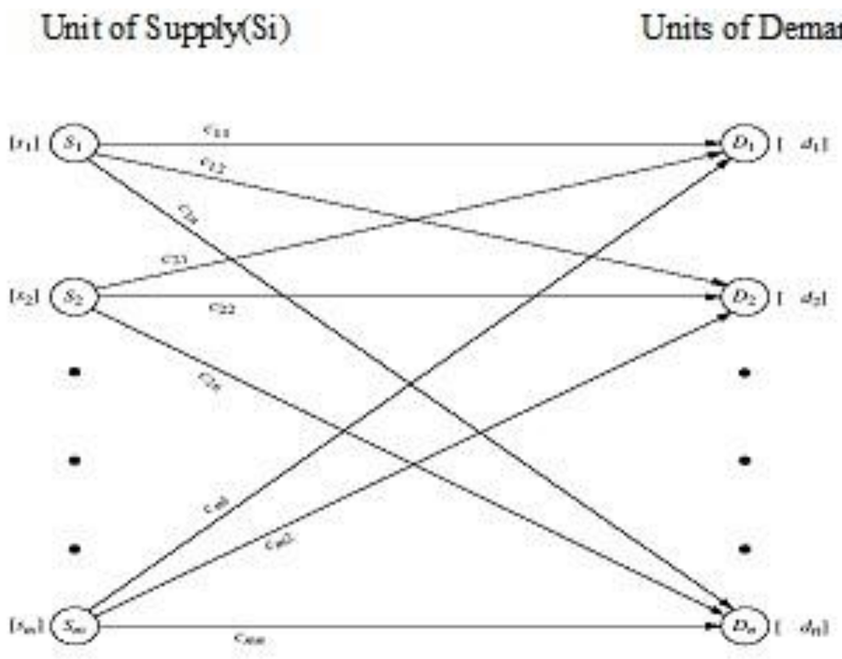

Figure 3.2: Network representation of the transportation problem sources could represent warehouses and the sinks could represent retail.

Transportation modelling methods

Based on transportation theory, after all needed data is arranged in tabular form (transportation matrix); The next step of the technique is to establish an initial feasible solution to the problem. The transportation method consists of the following three steps.

1. Obtaining an initial solution, that is to say making an initial assignment in such a way that a basic feasible solution is obtained.
2. Ascertaining whether it is optimal or not, by determining opportunity costs associated with the empty cells, and if the solution is not optimal.
3. Revising the solution until an optimal solution is obtained

Methods for Obtaining Basic Feasible Solution for Transportation Problem.

There are three di erent methods to obtain the initial basic solution of a transportation problem. These are Northwest-Corner Rule, Lowest cost entry and Vogel's approximation methods.

1. Feasible Solution (F.S.)

   A set of non-negative allocations $X_{ij} > 0$ which satis es the row and column restrictions is known as feasible solution.

2. The Initial Basic Feasible Solution (BFS)

   Let us consider a T.P involving m origins and n destinations.
   Since the sum of origin capacities equals the sum of destination requirements, a feasible solution always exists. Any feasible solution satisfying $m+n$˘1 of the $m+n$ constraints is a redundant one and hence can be deleted. This also means that a feasible solution to a T.P can have at the most only $m+$ n˘1 strictly positive component, otherwise the solution will degenerate.
   It is always possible to assign an initial feasible solution to a T.P. in such a manner that the rim requirements are satis ed. This can be achieved either by inspection or by following some simple rules. We begin by imagining that the transportation table is blank i.e. initially all $X_{ij} = 0$.

DEGENCY IN TRANSPORTATION PROBLEM

Degeneracy exists in a transportation problem when the number of lled cells is less than the number of rows plus the number of columns minus one $(m+n-1)$. Degeneracy may be observed either during the initial allocation when the rst entry in a row or column satis es both the row and column requirements or during the Steppingstone method application, when the added and subtracted values are equal.

Transportation with m-origins and n-destinations can have $m + n - 1$ positive basic variables, otherwise the basic solution degenerates. So whenever the number of basic cells is less than $m + n - 1$, the transportation problem is degenerate.

To resolve the degeneracy, the positive variables are augmented by as many zero-valued variables as is necessary to complete $m + n\breve{}1$ basic variable.

Cell: It is a small compartment in the transportation tableau. Circuit: A circuit is a sequence of cells (in the balanced transportation tableau) such that

i) It starts and ends with the same cell.

ii) Each cell in the sequence can be connected to the next member by a horizontal or vertical line in the tableau.

Allocation: The number of units of items transported from a source to a destination which is recorded in a cell in the transportation tableau.

Basic Variables: The variables in a basic solution whose values are obtained as the simultaneous solution of the system of equations that comprise the functional constraints.

Loop: In a transportation table, an ordered set of four or more cells is said to form a loop if :

i) Any two adjacent cells in the ordered set lie in the same row or in the same column.

ii) Any three or more adjacent cells in the ordered set do not lie in the same row or in the same column.

Optimal Solution: A feasible solution (not necessarily basic) is said to be Optimal if minimizes the total transportation cost

Test of Optimality

In the previous section we have learnt how to obtain an initial basic feasible solution. Obtained solutions may be optimal or maybe not, so it is essential for us to test the Optimality.

3.5.1 Algorithm for Optimality test

In order to test the optimality we should follow the procedure as given below:

Step 1:

Start with a basic feasible solution consisting of $m + n - 1$ allocations in independent positions.

Step 2:

Determine a set of $m+n$ numbers $u_i (i = 1,2,\ldots,m)$ and $v_j (j = 1,2,\ldots,n)$ such that for each occupied cells (r,s) $c_{r,s} = u_r + v_s$.

Step 3:

Calculate cell evaluations (unit cost di erence) di,j for each empty cell (i, j) by using the formula $d_{i,j} = c_{i,j} - (u_i + v_j)$.

Step 4:

Examine the matrix of cell evaluation di,j for negative entries and conclude that

i) If all $di,j > 0$ =) Solution is optimal and unique.

ii) If all di,j_0 =) Solution is optimal and alternate solution also exists.

iii) If at least one $di,j < 0$ =) Solution is not optimal.

iv) If it is so, further improvement is required by repeating the above process. See step 5and onwards.

Step 5:

i) See the most negative cell in the matrix $d_{i,j}$.

ii) Allocate q to this empty cell in the nal allocation table. Subtract and add the iii) Amount of this allocation to other cornes of the loop in order to restore feasibility.

iv) The value of q, in general is obtained by equating to zero the minimum of the

v) Allocations containing $-q(not + q)$ only at the corners of the closed loop.

vi) Substitute the value of q and nd a fresh allocation table.

Step 6: Again, apply the above test for optimality till we nd all $d_{i,j} > 0$.

3.6 Solution for a transportation problem

3.6.1 Flow Chart Solution For the transportation

Summary description of the Flow chart

1. First the problem is formulated as transportation matrix.

2. Check weather is a balance transportation model?

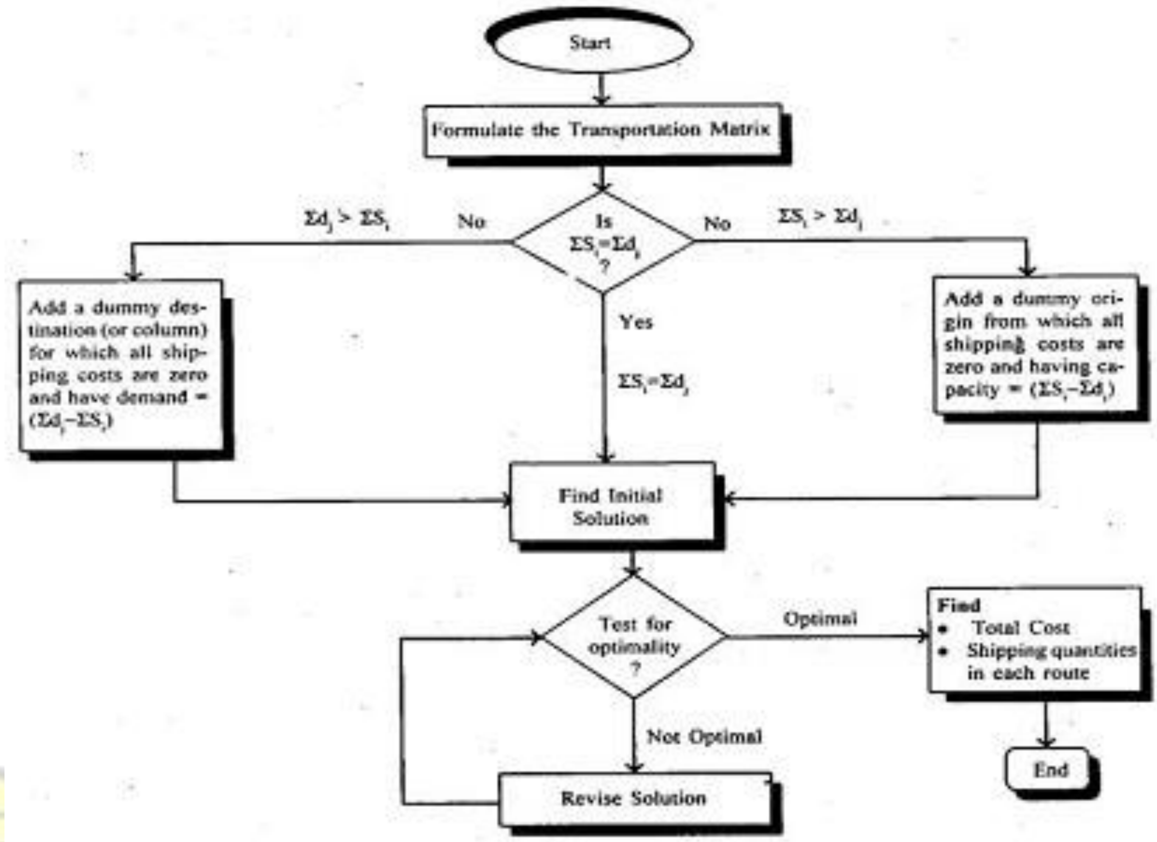

Figure 3.3: The ow chart showing the transportation problem approach

3. If not balance add a dummy to either the supply or the demand to balance the transportation model.

4. Find the initial solution of the transportation problem.

5. Check whether the solution is optimized?
   If the solution is not optimize Go to 4.

6. When optimal solution is obtained

7. We compute the total transportation cost and also shipped the respective quantity demand to its route.

3.6.2 Solution Algorithm For the transportation Problem

Transportation models do not start at the origin where all decision values are zero; they must instead be given an initial feasible solution.

The solution algorithm to a transpiration problem can be summarized into following steps:

Step 1: Formulate the problem and set up in the matrix form.

The formulation of transportation problem is similar to LP problem formulation. Here the objective function is the total transportation cost and the constraints are the supply and demand available at each source and destination, respectively.

Methods for Obtaining Basic Feasible Solution for Transportation Problem.

There are three di erent methods to obtain the initial basic solution of a transportation problem.

Step 2. Obtain an initial basic feasible solution. This initial basic solution can be obtained by using any of the following methods:

i) North West Corner Rule ii) Matrix

Minimum(Least Cost) Method iii) Vogel

Approximation Method

The solution obtained by any of the above methods must ful l the following conditions:

i) The solution must be feasible, i.e., it must satisfy all the supply and demand constraints. This is called RIM CONDITION.

ii) The number of positive allocation must be equal to m + n 1, where, m is number of rows and n is number of columns.

The solution that satis es the above mentioned conditions are called a non-degenerate basic feasible solution.

Step 3. Test the initial solution for optimality.
Using any of the following methods can test the optimality of obtained initial basic solution:

(i Stepping Stone Method

(ii Modi ed Distribution Method (MODI)

If the solution is optimal then stop, otherwise, determine a new improved solution.

Step 4: Updating the solution Repeat Step 3 until the optimal solution is arrived at.

3.6.3 FINDING INITIAL BASIC FEASIBLE SOLUTION OF BALANCED TRANSPORTATION PROBLEMS

Northwest Corner Method (NWC)

The North West corner rule is a method for computing a basic feasible solution of a transportation problem where the basic variables are selected from the North West corner (i.e., top left corner). The method starts at the northwest-corner cell (route) The major advantage of the north west corner rule method is that it is very simple and easy to apply. Its major disadvantage, however, is that it is not sensitive to costs and consequently yields poor initial solutions The Northwest Corner Method Summary of Steps.

1. Allocate as much as possible to the cell in the upper left-hand corner, subject to the supply and demand conditions.
2. Allocate as much as possible to the next adjacent feasible cell.
3. Repeat step 2 until all rim requirements are met

ILLUSTRATIVE EXAMPLE 1 ON TRANSPORTATION PROBLEM

In this tableau the decision variable X_{ij}, represent the number of tons of wheat transported from each grain elevator, $i(where i = 1,2,3)$,to each mill, $j(where j = A,B,C)$. The objective function represents the total transportation cost for each route. Each term in the objective function re ects the cost of the tonnage transported for one route. The problem is to determine how many tons of wheat of transport from each grain elevator to each mill on monthly basis in order to minimize the total cost of transportation.

ILLUSTRATIVE EXAMPLE 1

METHOD OF SOLUTIONS TO BALANCE PROBLEM USING NORTH WEST CORNER METHOD.

- In the northwest corner method the largest possible allocation is made to the cell in the upper left-hand corner of the tableau, followed by allocations to adjacent feasible cells.

To / From	A	B	C	Supply
1	6 150	8	10	150
2	7 50	11 100	11 25	175
3	4	5	12 275	275
Demand	200	100	300	600

Figure 3.4: A Balance transportation Problem

To / From	A	B	C	Supply
1	6 150	8	10	150
2	7 50	11 100	11 25	175
3	4	5	12 275	275
Demand	200	100	300	600

Figure 3.5: THE INITIAL NORTH WEST CORNER SOLUTION

The Initial NW Corner Solution This transportation tableau has:

The total supply= $200 + 100 + 300 = 600 units$ The

total supply= $150 + 175 + 275 = 600 units$

Hence the tableau is balance

We rst allocate as much as possible to cell 1A(northwest corner).this amount is 150 tons, since that is the maximum that can be supplied by grain 1 , even though 200 tons are demanded by mill A . This initial allocation, in this initial allocation is shown in Table 2. We next allocate to cell adjacent to cell 1A, in this case either cell 2A or cell 1B. However, cell 1B no longer represents a feasible allocation, because the total

tonnage of wheat available at source 1 (i.e. 150tons) has been allocated. Thus, cell 2A represents the only feasible alternative, and as much as possible is allocated to this cell. The amount allocated at 2A can be either 175 tons, the supply available from source 2, or 50 tons, the amount now demanded at destination A. Because 50 tons is the most constrained amount, it is allocated to cell 2A. As shown in table 2. The third allocation is made in the same way as the second allocation. The only feasible cell adjacent to cell 2A is cell 2B. The most that can be allocated is either 100 tons(the amount demanded at mill B) or 125 tons(175 tons minus the 50 tons allocated to cell2A).the smaller(most constrained) amount, 100 tons, is allocated to cell 2B, as shown in Table 2. The fourth allocation is 25 tons to cell 2C, and the fth allocation is 275 tons to cell
3C, both of which are shown in Table 2.1

Testing for Optimality

The allocations made by the method is BFS since $(m+n-1) = 3+3-1 = 5$, which equals the number of allocations made.

Since the number of occupied cell 5 is equal $(3 + 3 - 1)$,

The condition is satis ed

The initial solution is complete when all rim requirements are satis ed.

The starting solution (consisting of 4 basic variables) is

$X2A = 50 tons$,

$X2B = 100 tons$,

$X2C = 25 tons$

$X3C = 275 tons$

Transportation cost is computed by evaluating the objective function:

$Z = \$6X_{1A} + 8X_{1B} + 10X_{1C} + 7X_{2A} + 11X_{2B} + 11X_{2C} + 4X_{3A} + 5X_{3B} + 12X_{3C} = 6(150) + 8(0) + 10(0) + 7(50) + 11(100) + 11(25) + 4(0) + 5(0)+!2(275)$

$$= \$5,925$$

3.6.4 The Minimum Cell Cost (Least cost) Method

Matrix minimum method is a method for computing a basic feasible solution of a transportation problem where the basic variables are chosen according to the unit cost of transportation.

The minimum-cost method nds a better starting solution by concentrating on the cheapest routes. The method starts by assigning as much as possible to the cell with the smallest unit cost. Next, the satis ed row or column is crossed out and the amounts of supply and demand are adjusted accordingly. If both a row and a column are satis ed simultaneously, only one is crossed out, the same as in the northwest corner method. Next, look for the uncrossed-out cell with the smallest unit cost and repeat the process until exactly one row or column is left uncrossed out.

Steps

1. Identify the box having minimum unit transportation cost (C_{ij}).
2. If there are two or more minimum costs, select the row and the column corresponding to the lower numbered row.
3. If they appear in the same row, select the lower numbered column.
4. Choose the value of the corresponding X_{ij} as much as possible subject to the capacity and requirement constraints.
5. If demand is satis ed, delete the column.
6. If supply is exhausted, delete the row.
7. Repeat steps 1-6 until all restrictions are satis ed.

In the minimum cell cost method as much as possible is allocated to the cell with the minimum cost followed by allocation to the feasible cell with minimum cost

APPLICATION OF LEAST COST METHODS TO ILLUSTRATIVE

EXAMPLE 1 OF BALANCED TRANSPORTATION PROLEM

Table 3.4 The starting solution using Minimum Cell Method.

In the minimum cell cost method as much as possible is allocated to the cell with the minimum cost followed by allocation to the feasible cell with minimum cost.

From \ To	A	B	C	Supply
1	6	8	10	150
2	7	11	11	175
3	4 200	5	12	275
Demand	200	100	300	600

Figure 3.6: The Second Minimum Cell Cost Allocation

The complete initial minimum cell cost solution; total cost = $4,550.

The minimum cell cost method will provide a solution with a lower cost than the northwest corner solution because it considers cost in the allocation process.

The Minimum Cell Cost Method Summary of Steps

From \ To	A	B	C	Supply
1	6	8	10	150
2	7	11	11	175
3	4 200	5 75	12	275
Demand	200	100	300	600

Figure 3.7: The starting solution using Minimum Cell Method

1. Allocate as much as possible to the feasible cell with the minimum transportation cost, and adjust the rim requirements.
2. Repeat step 1 until all rim requirements have been met

Vogel's Approximation Method (VAM)

VAM is an improved version of the least-cost method that generally, but not always, produces better starting solutions. VAM is based upon the concept of minimizing opportunity (or penalty) costs. The opportunity cost for a given supply row or demand column is de ned as the di erence between the lowest cost and the next lowest cost alternative.

This method is preferred over the methods discussed above because it generally yields, an optimum, or close to optimum, starting solutions. Consequently, if we use the initial solution obtained by VAM and proceed to solve for the optimum solution, the amount of time required to arrive at the optimum solution is greatly reduced. The steps involved in determining an initial solution using VAM are as follows: The steps involved in determining an initial solution using VAM are as follows:

Step 1: Write the given transportation problem in tabular form (if not given). Step 2: Compute the di erence between the minimum cost and the next minimum cost corresponding to each row and each column which is known as
penalty cost.

Step 3: Choose the maximum di erence or highest penalty cost. Suppose it corresponds to the ith row. Choose the cell with minimum cost in the ith row. Again if the maximum corresponds to a column, choose the cell with the minimum cost in this column.

Step 4: Suppose it is the $(i,j)^{th}$ cell. Allocate $min(a_i,b_j)$ to this cell. If the min (a_i,b_j) = a_i, then the availability of the i^{th} origin is exhausted and demand at the j^{th} destination remains as $b_j - a_i$ and the i^{th} row is deleted from the table. Again if $min(a_i,b_j) = b_j$, then demand at the j^{th} destination is ful lled and the availability at the i^{th} origin remains to be $a_i - b_j$ and the j^{th} column is deleted from the table.

Step 5: Repeat steps 2,3,4 with the remaining table until all origins are exhausted and all demands are ful lled.

- Method is based on the concept of penalty cost or regret.
- A penalty cost is the di erence between the largest and the next largest cell cost in a row (or column).
- In VAM the rst step is to develop a penalty cost for each source and destination.
- Penalty cost is calculated by subtracting the minimum cell cost from the next higher cell cost in each row and column.

Vogel's Approximation Method (VAM) Summary of Steps

1. Determine the penalty cost for each row and column.
2. Select the row or column with the highest penalty cost.
3. Allocate as much as possible to the feasible cell with the lowest transportation cost in the row or column with the highest penalty cost.
4. Repeat steps 1,2, and 3 until all rim requirements have been met

APPLICATION OF VOLGEL'S APPROXIMATION METHOD TO ILLUSTRATIVE EXAMPLE 1 ON BALANCE TRANSFORMATION PROBLEM.

From \ To	A	B	C	Supply	
1	6	8	10	150	2
2	7	11	11	175	4
3	4	5	12	275	1
Demand	200	100	300	600	
	2	3	1		

Figure 3.8: The VAM Penalty Costs

- VAM allocates as much as possible to the minimum cost cell in the row or column with the largest penalty cost.

After each VAM cell allocation, all row and column penalty costs are recomputed.

From \ To	A	B	C	Supply	
1	6	8	10	150	2
2	7 175	11	11	175	
3	4	5	12	275	1
Demand	200	100	300	600	
	2	3	2		

Figure 3.9: The Initial VAM Allocation

From \ To	A	B	C	Supply
1	6	8	10	150
2	7 175	11	11	175
3	4	5 100	12	275
Demand	200	100	300	600
	2		2	

Figure 3.10: The Second VAM Allocation

Recomputed penalty costs after the third allocation.

Table 3.10: The Initial VAM Solution

- The initial VAM solution; total cost = $5,125

From \ To	A	B	C	Supply
1	6	8	10 150	150
2	7 175	11	11	175
3	4 25	5 100	12 150	275
Demand	200	100	300	600

Figure 3.11: The Third VAM Allocation

- VAM and minimum cell cost methods both provide better initial solutions than does the northwest corner method

From \ To	A	B	C	Supply
1	6	8	10 150	150
2	7 175	11	11	175
3	4 25	5 100	12 150	275
Demand	200	100	300	600

Figure 3.12: The Third VAM Allocation

3.6.5 METHODS FOR SOLVING TRANSPORTATION PROBLEMS TO OPTIMALITY

3.6.6 AN OPTIMAL SOLUTION

To obtain an optimal solution by making successive improvements to initial basic feasible solution until no further decrease in the transportation cost is possible. An optimal solution is one where there is no other set of transportation routes that will further reduce the total transportation cost. Thus, we have to evaluate each unoccupied cell in the transportation table in terms of an opportunity of reducing total transportation cost. An unoccupied cell with the largest negative opportunity cost is selected to include in the new set of transportation routes (allocations). This value indicates the per unit cost reduction that can be achieved by raising the shipment allocation in the unoccupied cell from its present level of zero. This is also known as an incoming cell (or variable). The outgoing cell (or variable) in the current solution is the occupied cell (basic variable) in the unique closed path (loop) whose allocation will become zero rst as more units are allocated to the unoccupied cell with largest negative opportunity cost. That is, the current solution cannot be improved further. This is the optimal solution.

The widely used methods for nding an optimal solution are:

- Stepping stone method (not to be done).
- Modi ed Distribution (MODI) method.

They di er in their mechanics, but will give exactly the same results and use the same testing strategy.

5. To develop the improved solution, if it is not optimal. Once the improved solution has been obtained, the next step is to go back to 3.

Note: Although the transportation problem can be solved using the regular simplex method, its special properties provide a more convenient method for solving this type

of problems. This method is based on the same theory of simplex method. It makes use, however, of some short-cuts which provide a less burdensome computational scheme. There is one di erence between the two methods. The simplex method performs the operations on a simplex table. The transportation method performs the same operations on a transportation
table.

APPLICATION OF STEPPING STONE METHOD TO ILLUSTRATIVE EXAMPLE 1 ON BALANCE TRANSFORMATION
PROBLEM

3.6.7 The Stepping-Stone Solution Method

- Once an initial solution is derived, the problem must be solved using either the stepping-stone method or the modi ed distribution method (MODI).
- The initial solution used as a starting point in this problem is the minimum cell cost method solution because it had the minimum total cost of the three methods used

The stepping-stone method determines if there is a cell with no allocation that would reduce cost if used.

From \ To	A	B	C	Supply
1	6	8 25	10 125	150
2	7	11	11 175	175
3	4 200	5 75	12	275
Demand	200	100	300	600

Figure 3.13: The Minimum Cell Cost Solution

From \ To	A	B	C	Supply	
1	6	8 25	10 125	150	151
2	7	11	11 175	175	
3	4 200	5 75	12	275	
Demand	200	100	300	600	

Figure 3.14: The Allocation of One Ton from Cell 1A

- Must subtract one ton from another allocation along that row.

- A requirement of this solution method is that units can only be added to and subtracted from cells that already have allocations, thus one ton must be added to a cell as shown.

An empty cell that will reduce cost is a potential entering variable. • To evaluate the cost reduction potential of an empty cell, a closed path connecting used cells to the empty cells is identi ed.

To / From	A	B	C	Supply
1	+1 [6]	−1 [8] 25	[10] 125	150
2	[7]	[11]	[11] 175	175
3	[4] 200	[5] 75	[12]	275
Demand	200	100	300	600

99

Figure 3.15: The Subtraction of One Ton from Cell 1B

To / From	A	B	C	Supply
1	+1 [6]	−1 [8] 25	[10] 125	150
2	[7]	[11]	[11] 175	175
3	−1 [4] 200	+1 [5] 75	[12]	275
Demand	200	100	300	600

Figure 3.16: The Addition of One Ton to Cell 3B and the Subtraction of One Ton from Cell 3A

The remaining stepping-stone paths and resulting computations for cells 2B and 3C

- After all empty cells are evaluated, the one with the greatest cost reduction potential is the entering variable.
- A tie can be broken arbitrarily.

From \ To	A	B	C	Supply
1	6	8 25	10 125	150
2	7	11	11 175	175
3	4 200	5 75	12	275
Demand	200	100	300	600

2A → 2C → 1C → 1B → 3B → 3A
+ \$7 − 11 + 10 − 8 + 5 − 4 = −\$1

Figure 3.17: The Stepping-Stone Path for Cell 2A

From \ To	A	B	C	Supply
1	6	8 25	10 125	150
2	7	11	11 175	175
3	4 200	5 75	12	275
Demand	200	100	300	600

2B → 2C → 1C → 1B
+ \$11 − 11 + 10 − 8 = +\$2

Figure 3.18: The Stepping-Stone Path for Cell 2B

- When reallocating units to the entering variable (cell), the amount is the minimum amount subtracted on the stepping-stone path.
- At each iteration one variable enters and one leaves (just as in the simplex method).

Check to see if the solution is optimal.

Continuing check for optimality.

To / From	A	B	C	Supply
1	(6)	+ (8) 25	− (10) 125	150
2	(7)	(11)	(11) 175	175
3	(4) 200	− (5) 75	+ (12)	275
Demand	200	100	300	600

3C → 1C → 1B → 3B
+ \$12 − 10 + 8 − 5 = +\$5

Figure 3.19: The Stepping-Stone Path for Cell 3C

To / From	A	B	C	Supply
1	+ (6)	− (8) 25	(10) 125	150
2	(7)	(11)	(11) 175	175
3	− (4) 200	+ (5) 75	(12)	275
Demand	200	100	300	600

Figure 3.20: The Stepping-Stone Path for Cell 1A

- The stepping-stone process is repeated until none of the empty cells will reduce costs (i.e., an optimal solution).
- In example, evaluation of four paths indicates no cost reductions; therefore Table 16 solution is optimal.
- Solution and total minimum cost:

$X_{1A} = 25 tons,$

To / From	A	B	C	Supply
1	(6) 25	(8)	(10) 125	150
2	(7)	(11)	(11) 175	175
3	(4) 175	(5) 100	(12)	275
Demand	200	100	300	600

Figure 3.21: The Second Iteration of the Stepping-Stone Method

To / From	A	B	C	Supply
1	− (6) 25	(8)	+ (10) 125	150
2	+ (7)	(11)	− (11) 175	175
3	(4) 175	(5) 100	(12)	275
Demand	200	100	300	600

2A → 2C → 1C → 1A
+ \$7 − 11 + 10 − 6 = \$0

Figure 3.22: The Stepping-Stone Path for Cell 2A

$X_{2C} = 175 tons, X_{3A} = 175 tons, X_{1C} = 125 tons, X_{3B} = 100 tons$

$$Z = \$6(25) + 8(0) + 10(125) + 7(0) + 11(0) + 11(175) + 4(175) + 5(100) + 12(0)$$

$$= \$4{,}525$$

1. A multiple optimal solution occurs when an empty cell has a cost change of zero and all other empty cells are positive.

2. An alternate optimal solution is determined by allocating to the empty cell with a zero cost change.

To / From	A	B	C	Supply
1	− [6] 25	+ [8]	[10] 125	150
2	[7]	[11]	[11] 175	175
3	+ [4] 175	− [5] 100	[12]	275
Demand	200	100	300	600

1B → 3B → 3A → 1A
+ \$8 − 5 + 4 − 6 = +\$1

Figure 3.23: The Stepping-Stone Path for Cell 1B

To / From	A	B	C	Supply
1	− [6] 25	[8]	+ [10] 125	150
2	[7]	+ [11]	− [11] 175	175
3	+ [4] 175	− [5] 100	[12]	275
Demand	200	100	300	600

2B → 3B → 3A → 1A → 1C → 2C
+ \$11 − 5 + 4 − 6 + 10 − 11 = +\$3

Figure 3.24: The Stepping-Stone Path for Cell 2B

3. Alternate optimal total minimum cost also equals \$4,525

The Stepping-Stone Solution Method Summary

1. Determine the stepping-stone paths and cost changes for each empty cell in the tableau.

To / From	A	B	C	Supply
1	+ 6 25	8	− 10 125	150
2	7	11	11 175	175
3	− 4 175	5 100	+ 12	275
Demand	200	100	300	600

3C → 3A → 1A → 1C
+ \$12 − 4 + 6 − 10 = +\$4

Figure 3.25: The Stepping-Stone Path for Cell 3C

To / From	A	B	C	Supply
1	6	8	10 150	150
2	7 25	11	11 150	175
3	4 175	5 100	12	275
Demand	200	100	300	600

Figure 3.26: The Alternative Optimal Solution

2. Allocate as much as possible to the empty cell with the greatest net decrease in cost.

3. Repeat steps 1 and 2 until all empty cells have positive cost changes that indicate an optimal solution.

3.6.8 The Modi ed Distribution Method (MODI)

MODI is a modi ed version of the stepping-stone method in which mathematical equations replace the stepping-stone paths.

Step 1: Under this method we construct penalties for rows and columns by subtracting the least value of row / column from the next least value.

Step 2: We select the highest penalty constructed for both row and column. Enter that row / column and select the minimum cost and allocate
$min(ai,bj)$

Step 3: Delete the row or column or both if the rim availability / requirements is met.

Step 4: We repeat steps 1 to 2 to till all allocations are over.

Step 5: For allocation all form equation $u_i + v_j = c_j$ set one of the dual variable u_i/v_j to zero and solve for others.

Step 6: Use this value to nd $D_{ij} = c_{ij} - u_i - v_j$ of all $D'_{ij}s$, then it is the optimal solution.

APPLICATION OF MODIFIED DISTRIBUTION METHOD TO ILLUSTRATIVE EXAMPLE 1 ON BALANCE TRANSFORMATION
PROBLEM

In the table, the extra left-hand column with the u_i symbols and the extra top row with the v_j symbols represent values that must be computed.
Computed for all cells with allocations:
$u_i + v_j = c_{ij}$ = unit transportation cost for cell *ij.*

Formulas for cells containing allocations: $X_{1B} : u_1 + v_B = 8$ $X_{1C} : u_1 + v_C = 10$

$X_{2C} : u_2 + v_C = 11$ $X_{3A} : u_3 + v_A = 4$ $X_{3B} : u_3 + v_B = 5$

	v_j	$v_A =$	$v_B =$	$v_C =$	
u_i	To / From	A	B	C	Supply
$u_1 =$	1	6	8 25	10 125	150
$u_2 =$	2	7	11	11 175	175
$u_3 =$	3	4 200	5 75	12	275
	Demand	200	100	300	600

Figure 3.27: The Minimum Cell Cost Initial Solution

	v_j	$v_A = 7$	$v_B = 8$	$v_C = 10$	
u_i	To / From	A	B	C	Supply
$u_1 = 0$	1	6	8 25	10 125	150
$u_2 = 1$	2	7	11	11 175	175
$u_3 = -3$	3	4 200	5 75	12	275
	Demand	200	100	300	600

Figure 3.28: The Initial Solution with All $u_i and v_j$ Values

Table 3.27: The Initial Solution with All u_i and v_j Values

- Five equations with 6 unknowns therefore let $u_1 = 0$ and solve to obtain:

  $v_B = 8, v_C = 10, u_2 = 1, u_3 = -3, v_A = 7$

- Each MODI allocation replicates the stepping-stone allocation.

- Use following to evaluate all empty cells:

$c_{ij} - u_i - v_j = k_{ij}$

Where k_{ij} equals the cost increase or decrease that would occur by allocating to a cell.

For the empty cells in Table 26:

x_{1A} : $k_{1A} = c_{1A} - u_1 - v_A = 6 - 0 - 7 = -1$ x_{2A} : $k_{2A} = c_{2A}$

$- u_2 - v_A = 7 - 1 - 7 = -1$ x_{2B} : $k_2B = c2B - u2 - vB$

$= 11 - 1 - 8 = +2$ x_{3C} : $k_{3C} = c_{3C} - u_3 - v_C = 12 - (-3)$

$- 10 = +5$

After each allocation to an empty cell, the u_i and v_j values must be recomputed

	v_j	$v_A =$	$v_B =$	$v_C =$	
u_i	From \ To	A	B	C	Supply
$u_1 =$	1	6 25	8	10 125	150
$u_2 =$	2	7	11	11 175	175
$u_3 =$	3	4 175	5 100	12	275
	Demand	200	100	300	600

Figure 3.29: The Second Iteration of the MODI Solution Method

The Second Iteration of the MODI Solution Method

Re-computing u_i and v_j values: x_{1A} : $u_1 + v_A = 6, v_A = 6$ x_{1C} : $u_1 +$

$v_C = 10, v_C = 10$ x_{2C} : $u_2 + v_C = 11, u_2 = 1$ x_{3A} : $u_3 + v_A = 4, u_3 = -2$

x_{3B} : $u_3 + v_B = 5, v_B = 7$

u_i	v_j / From \ To	$v_A = 6$ A	$v_B = 7$ B	$v_C = 10$ C	Supply
$u_1 = 0$	1	[6] 25	[8]	[10] 125	150
$u_2 = 1$	2	[7]	[11]	[11] 175	175
$u_3 = -2$	3	[4] 175	[5] 100	[12]	275
	Demand	200	100	300	600

Figure 3.30: The New u_i and v_j Values for the Second Iteration

The New u_i and v_j Values for the Second Iteration

- Cost changes for the empty cells, $c_{ij} - u_i - v_j = k_{ij}$;

  x_{1B} : $k_{1B} = c_{1B} - u_1 - v_B = 8 - 0 - 7 = +1$ x_{2A} : $k_{2A} = c_{2A}$

  $- u_2 - v_A = 7 - 1 - 6 = 0$ x_{2B} : $k_{2B} = c_{2B} - u_2 - v_B = 11$

  $- 1 - 7 = +3$ x_{3C} : $k_{2B} = c_{2B} - u_3 - v_C = 12 - (-2) - 10$

  $= +4$

- Since none of the values are negative, solution obtained is optimal.
- Cell 2A with a zero cost change indicates a multiple optimal solution. The Modi

  ed Distribution Method (MODI) Summary of Steps

1. Develop an initial solution.
2. Compute the u_i and v_j values for each row and column.

3. Compute the cost change, k_{ij}, for each empty cell.
4. Allocate as much as possible to the empty cell that will result in the greatest net decrease in cost (most negative k_{ij})
5. Repeat steps 2 through 4 until all k_{ij} values are positive or zero.

3.7 Sensitivity Analysis of TP

This involves the development of understanding how the information in the nal tableau can be given managerial interpretations. This will be done by examining the application of sensitivity analysis to the linear programming problems. To analyse sensitivity in linear programming, after obtaining the optimal solution, one of the right-hand-side values or coe cients of objective function are changed, then, the changes in optimal solution and optimal value are examined.

The balanced relation between supply and demand in transportation problem makes it di cult to use traditional sensitivity analysis methods. Therefore, in the process of changing supply or demand resources, at least one more resource needs to be changed to make the balanced relation possible.

In this study, utilizing the concept of complete di erential of changes for sensitivity analysis of right-hand-side parameter in transportation problem, a method is set forth. This method examines simultaneous and related changes of supply and demand without making any change in the basis. The mentioned method utilizes Arasham and Kahn's simplex algorithm to obtain basic inverse matrix

CHAPTER 4

DATA COLLECTION AND ANALYSIS

4.1 Introduction

ABTS is among the oldest timber company in Brong Ahafo Region of Ghana. It produces a lot of timber product such as plywoods, T & G, parquet and boards. The company export products such as parquet, T & G, plywood and Veneer. For the purpose of this study the $12mm$ plywood product was concerned from the production site at Berekum through other two source station at Tamale and Kumasi to the twelve (12) key distributors.

4.1.1 DATA COLLECTION

The required data includes. A list of all products, sources, demand for each product by customer, the distance from each source to the various destinations, the full truck transportation cost. The study concerned the supply of 12mm plywood product from the the main sources Berekum (BKM),store at the two warehouses Tamale (TM) and Kumasi (KS) to 12 key distributors. The Takoradi harbor(TD) one is for exportation. The company was able to produced 1596500 $12mm$ plywood a year 133042 average a month and 4374 daily. The company
runs three (3) shifts to meet the demand of their customers. The rst shift start from 6:00am to 2;00pm, the second start from 2:00pm to 10:00pm and the third shift is from 10:00pm to 6:00am.

4.1.2 DATA SOURCE

The data used for analysis was collected from the Chief Executive O cer (CEO) of ABTS at Berekum. The data includes the transportation cost per full truck of 1500 $(169.8456tons)$ pieces of $12mm$ plywood from the three source to the
various key distributors.

SOURCE	HAMEG	ISAKA	MIKE	TD	ERICUS	MERCY	ILANI	ODAME	ABORA	ODK	DOMAIN	ANGEL	CAPACITY
BKM	530	113	409	415	387	11	346	369	402	184	404	170	677500
TM	177	260	618	512	302	283	564	592	267	538	590	197	437000
KSI	542	119	251	258	443	123	194	231	251	166	214	180	46200
DEMAND	214801	105615	194991	588866	26503	77132	92549	80023	26413	98757	20869	69981	

Table 4.1: DATA FOR DISTANCE FROM SOURCES TO DESTINATIONS (KM)

The table above which was distance from sources to destination (various customers) was later converted to cost of full truck load (1500 12mm plywood, (169.8456 tons) of shipment from source to destination.

January 2013 to December 2013, the transportation cost is shown in table below (Table 4.2) This data indicate the transportation matrix showing supply (capacity), demand and the unit cost per tone (full truck).

SOURCE	HAMEG	ISAKA	MIKE	TD	ERICUS	MERCY	ILANI	ODAME	ABORA	ODK	DOMAIN	ANGEL	SUPPLY
BKM	1943.8	414.4	1500.0	152210	1419.3	40.3	1268.9	1353.4	1474.3	674.3	1481.7	623.5	697500
TM	649.1	953.5	2266.5	1877.8	1107.6	1037.9	2068.5	2171.1	979.2	1973.1	2165.8	722.5	437000
KSI	1987.8	436.4	920.5	946.2	1624.7	457.1	711.5	847.2	920.5	608.8	784.3	660.1	462000
DEMAND	214801	105615	194991	588866	26563	99132	92549	80023	26413	98757	20869	69985	

Table 4.2: THE MATIX REPRESENTATION OF THE PROBLEM

4.1.3 FORMULATION PROBLEM

Let y1= source at BKM y2

= source at TM y3 =

source at KS

Xij = the unit shipped in tons from source i to distribution centred i = 1, 2, 3 and j = 1, 2, 3 ... 12

Using the shipping cost table 4.2 the annual transportation cost of Ghana cedis is written as

Minimize

$1943.8X11 + 414.1X12 + 1500X13 + 1522X14 + 1419.3X15 + 40.3X16 + 1268.9X17 + 1353.4X18 + 1474.3X19 + 674.8X110 + 1481.7X111 + 623.5X12 + 649.1X21 + 953.5X22 + 2266.5X23 + 1877.8X24 + 1107.6X25 + 1037.9X26 + 2068.5X27 + 2171.1X28 + 979.2X29 + 1973.1X210 + 2165.8X211946.2X212 + 1987.8X31 + 436.4X32 + 920.5X33 + 946.2X34 + 1624.7X35 + 451.1X36 + 711.5X37 + 847.2X38 + 92015X39 + 608.8X310 + 784.3X311 + 660.1X312$

Subject to

$X11+X12+X13+X14+X15+X16+X17+X18+X19+X110+X111+X112 = 69750$ $X21 + X22 + X23 + X24 + X25 + X26 + X27 + X28 + X29 + X210 + X211 + X212 = 43700$$X31 + X32 + X33 + X34 + X35 + X36 + X37 + X38 + X39 + X310 + X311 + X312 = 46200$

$X11 + X21 + X31 = 214801$

$X12 + X22 + X32 = 105615$

$X13 + X23 + X33 = 194991$

$X14 + X24 + X34 = 588866$

$X15 + X25 + X35 = 26503$

$X16 + X26 + X36 = 77132$

$X17 + X27 + X37 = 92549$

$X18 + X28 + X38 = 80023$

$X19 + X29 + X39 = 26413$

$X110 + X210 + X312 = 98757$

$X111 + X211 + X311 = 20869$

$X112 + X212 + X312 = 69981$

4.1.4 OPTIMAL SOLUTION

January 2013 to December 2013

Using the management science 6.0 for linear programming module the optimal solution obtained is displayed below.

OPTIMAL SOLUTION

Objective function value = 1477628051.300

VARIABLE	VALUE	REDUCED COST
X_{11}	0	1650.0
X_{12}	105615.0	0
X_{13}	0	3.70
X_{14}	296206	0
X_{15}	0	667.5
X_{16}	77132.0	0
X_{17}	92549.0	0
X_{18}	80023.0	0
X_{19}	0	850.9
X_{110}	98757.0	0
X_{111}	0	121.6
X_{12}	0	256.8
X_{21}	214801.0	0
X_{22}	0	183.3
X_{23}	0	414.4
X_{24}	142083	0
X_{25}	26503.0	0
X_{26}	0	661.8
X_{27}	0	1443.8
X_{28}	0	461.9

X_{29}	26413.0	0
X_{210}	0	942.5
X_{211}	0	447.9
X_{212}	69981.0	0
X_{31}	0	2270.3
X_{32}	0	597.8
X_{33}	194991.0	0
X_{34}	109519	0
X_{35}	0	1448.7
X_{36}	0	986.6
X_{37}	0	1018.4
X_{38}	0	69.6
X_{39}	0	872.9
X_{310}	0	509.8
X_{311}	20869.0	0
X_{312}	0	869.2

Table 4.3: OPTIMAL SOLUTIONS

CONSTRAINTS	SLACK/SURPLUS	DUAL PRICE
1	0.00	354.8
2	0.00	1.0
3	0.00	930.6
4	0.00	648.1
5	0.00	769.2
6	0.00	1851.1
7	0.00	1876.8
8	0.00	1106.6
9	0.00	395.1
10	0.00	623.7
11	0.00	1708.2
12	0.00	978.2
13	0.00	1029.6
14	0.00	1714.9
15	0.00	721.5

Table 4.4: Sensitivity Report

4.1.5 SENSITIVITY REPORT

4.1.6 OBJECTIVE COEFFICIENT RANGES

4.1.7 RIGHT HAND SIDE RANGES

4.1.8 TRANSPORTATION OUTPUT TABLE

VARIABLE	LOWER LIMIT	CURRENT VALUE	UPPER LIMIT
X_{11}	293.300	1943.800	
X_{12}		414.400	697.700
X_{13}	1496.300	1500.00	
X_{14}	1452.400	1522.000	1525.700
X_{15}	751.800	1419.300	
X_{16}		40.300	702.100
X_{17}		1268.900	1287.300
X_{18}		1353.400	1423.000
X_{19}	623.400	1474.300	
X_{110}		674.800	1184.600
X_{111}	1360.100	1481.700	
X_{12}	366.700	623.500	
X_{21}		649.100	2299.600
X_{22}	770.200	953.500	
X_{23}	1852.100	2266.500	
X_{24}	1621.000	1877.800	2061.100
X_{25}		1107.600	1775.100
X_{26}	396.100	1057.900	
X_{27}	624.700	2068.500	
X_{28}	1709.200	2191.100	
X_{29}		979.200	1830.00
X_{210}	1030.600	1973.100	
X_{211}	1715.900	2163.800	
X_{212}		722.500	979.300

X_{31}	282.500	1987.800	
X_{32}	161.400	436.400	
X_{33}		920.500	924.200
X_{34}	942.500	946.200	1015.000
X_{35}	176.000	1624.700	
X_{36}	535.500	451.100	
X_{37}	306.900	711.500	
X_{38}	777.600	847.200	
X_{39}	47.600	920.500	
X_{310}	99.000	608.800	
X_{311}		784.300	905
X_{312}	209.100	660.100	

Table 4.5: Sensitivity Report 2

Constraints	Lower Limit	Current Value	Upper Limit
1	697500	697500	796802
2	437000	437000	
3	462000	462000	561302
4	0	214801	214801
5	6313	105615	105615
6	95689	194991	194991
7	489564	588866	588866
8	0	26503	26503
9	0	77132	77132
10	0	92549	92549
11	0	80023	80023
12	0	26413	26413
13	0	98757	98757
14	0	20869	20869
15	0	69981	69981

Table 4.6: Sensitivity Report 3

SOURCE	DISTRIBUTOR DESTINATION	FULL TRUCK PERTON	COST PER FULL TRUCK LOAD	TOTAL COST GHC
BKM	HAMEG	0	1943.8	0
BKM	ISAKA	105615	414.4	43766856
BKM	MIKE	0	1500	0
BKM	TDA	296206	1522	450826702.1
BKM	ERICUS	0	1419.8	0
BKM	MERCY	77132	40.3	3108419.6
BKM	ILANI	92549	1268.9	117435426.1
BKM	ODAME	80023	1353.4	108303128.2
BKM	ABORA	0	1474.3	0
BKM	ODK	98757	674.8	66641223.6
BKM	DOMAIN	0	1481.7	0
BKM	ANGEL	0	623.3	0
TM	HAMEG	214801	649.1	139427329.1
TM	ISAKA	0	953.5	0
TM	MIKE	0	2266.5	0
TM	TD	142083	1877.8	266804671
TM	ERICUS	26503	1100.6	29169201
TM	MERCY	0	1037.9	0
TM	ILANI	0	2068.5	0
TM	ODAME	0	2171.1	0
TM	ABORA	26413	979.2	25863609.6
TM	ODK	0	1973.1	0
TM	DOMAIN	0	2165.8	0
TM	ANGEL	69981	722.5	50561272.5
KS	HAMEG	0	1987.8	0
KS	ISAKA	0	436.4	0
KS	MIKE	194991	920.5	179489215.5
KS	TDH	109519	946.2	103625142.7
KS	ERICUS	0	1624.7	0
KS	MERCY	0	457.1	0
KS	ILEANI	0	711.5	0
KS	ODAME	0	847.2	0
KS	ABORA	0	920.5	0
KS	ODK	0	608.8	0
KS	DOMAIN	20869	784.3	16367556
KS	ANGEL	0	660.1	0

Table 4.7: TRANSPORTATION OUTPUT TABLE

4.1.9 COMPUTATIONAL PROCEDURE

The management scientist 6.0 soft ware are packaged was used to solve this transportation problem. The management scientist software is mathematical tool solver for optimization and mathematical programming in operation research. The management science model used is based on simpli ed version of the simplex technique called the Transportation Simplex Method. The transportation simplex method is a special version of simplex method used to
solve transportation problems.

It was run on Intel (R) Core (TM) Duo CPU machine with 4.0 GB of RAM. The data gathered (Table 4.2) were used in running the management scientist program, produced the same output for the trials.

4.1.10 RESULTS AND DISCUSSION

The Transportation Problem (TP) was model as Linear Programming and solved using the LP module of management Science application programme to obtain
the optimal solution.

The computer solution shows that the minimum total cost is 1477628051.300 in Ghana cedis as shown in optimal solution.

The values for the decision variables show the optimal amount of goods to be shipped over each route.

The distribution manager should observe the following distribution list if he/she want to optimize the distribution. Ship 105615 tons of 12mm plywood from source BKM to distributor ISAKA.

Ship 243424 tons of 12mm plywood from source BKM to distributor TDH for export.

Ship 77132 tons of 12mm plywood from source BKM to distributor MERCY.

Ship 92549 tons of 12mm plywood from source BKM to distributor ILANI

Ship 80023 tons of 12mm plywood from source BK to distributor ODAME.

Ship 98757 tons of 12mm plywood from source BK to distributor ODK Ship 214801 tons of 12mm plywood from source TM to HAMEG

Ship 142083 tons of 12mm plywood to a distributor TD for export

Ship 26503 tons of 12mm plywood from source TM to distributor ANGEL ship 69981 tons of 12mm plywood from source TM to distributor ANGEL

Ship 194991 tons of 12mm plywood from source KS to distributor MIKE Ship 109519 tons of 12mm plywood from source KS to destination TDH for export.

Ship 208649 tons of 12mm plywood from source KS to distributor DOMAIN.

4.1.11 THE SENSITIVITY ANALYSIS

The Sensitivity Analysis on the optimal solution obtained. The dual price column indicates there could be improvement of the optimal value of the objective function per unit decrease in the Right Hand Side of the constraint. This non zero dual price of 354.800 of constraint one (1) (ie capacity or source 1) and dual price of 930.600 for constraint three (3) (ie capacity or source 3)(?).

This indicate that decrease in the constraint 1 and 3 constraints per unit ton will decrease the optimal objective function value by 354.800 and 930.600 respectively.

The constraint one will decrease the objective function value from 1477628051.300 $(1477628051.300 - 354.30)$ to 1477627697. With all other co-e cient in the problem will remain the same.

The constraint 3 will also decrease the objective function value from 1477628061.300 $(1477628051 - 930.600)$ to 1477627120 with all other co-e cient

in the problem remaining the same.

CHAPTER 5

CONCLUSION AND RECOMMENDATION

5.1 Conclusion

The transportation problem was formulated as a Linear Programming model and solved with the standard LP solvers the Management Science module to obtain the optimal solution.

The optimal solution provided by the computer package (Scienti c Management Science 6.O) provided the valuable information such as sensitivity analysis that would enable ABTS Company to make optimal decisions on transportation planning and distributions of goods.

The study recorded total minimizes transportation cost during the period of January 2013 to December 2013 nancial period. The CEO of ABTS Company recorded the transportation cost of 1567789341.00 during the 2013 nancial year; but after application of the mathematical model the transportation cost reduce to GHC 14776280.30 which represent 6.0% reduction in total transportation cost. This will enable the ABTS to continue to o er their social and cooperative responsibilities they o er to the Brong Ahafo Region and to the people of Ghana if they adopt the proposed transportation model.

5.1.1 RECOMMENDATION

The scienti c transportation problem model for the company ABTS transportation problem using the existing data for the 2013nancial year gave better results.The ABTS Management may bene t from the proposed transportation problem model for their transportation problem planning if they could adopt.

If ABTS managers are to employed the proposed transportation model it will help them to e ciently plan out its transportation scheduled at a minimum cost. There are other mathematical programs that can assist in construction of transportation problem such as. Algebraic model and Quantitative Analysis modeling system.

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Appendix

SOURCE

BKM	Berekum
TM	Tamale
KS	Kumasi

DISTIBUTORS	DISTINATION
Hamea	Bolgatanga
Isaka	Techiman
Mike	Acra
TD	Takoradi
Ericus	Wa
Mercy	Sunyani
ilani	Koforidua
odame	Cape Coast
Abora	Winneba
ODK	SafwiWiaso
Doman	Nsawam
Angel	Kintampo

CONVERSIONS

Full truck load of 12mm plywood = 1500 pieces

Volume of one 12mm plywood

= 122 x 0.012 x2.44 = 0.0357

= 1500 x 0.0357 = 535.5

= 535.5 x Constant   gure of 3.17

= 169.8486 Tons

= full truck load of 1500 pieces of 12mm plywood is 169.84 Tons