KWAME NKRUMAH UNIVERSITY OF SCIENCE AND TECHNOLOGY

INSTITUTE OF DISTANCE LEARNING

MINIMUM CONNECTION OF PIPE LAYOUT SYSTEM AND OPTIMAL LOCATION OF MAINTENANCE UNIT
A CASE STUDY OF ABUAKWA TOWNSHIP

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BY

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DECLARATION

I hereby declare that this submission is my own work towards the award of the MSc degree and that, to the best of my knowledge, it contains no material previously published by another person nor material which had been accepted for the award of any other degree of the university, except where due acknowledgement had been made in the text.

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ABSTRACT

Network design phase is done before laying the infrastructure. As the water supply industry matures, there is vital interest in good pipe network designs.The study sought to assess the causes of the numerous water supply problems faced by the Abuakwa community.To find the optimal solution to the pipe network system for the supply path at efficient supply,a solution-oriented modelling approach can help to devise efficient automatic planning and optimization schemes for Atwima Nwabiagya District.This thesis discusses how good mathematical models can be obtain for real-world problems using prim's algorithm, which model structures to minimize pipe network systems.The prim's algorithm can also help in easy and less expensive water supply especially Abuakwa in Kumasi from Owabi Dam.The prim's algorithm employed to find the optimal solution was able to reduce the total pipe network from the original network.

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DEDICATION

I dedicate this thesis with all my love and respect to my dear Husband ASP. Philip Bidipume Jagri and my sweet son Benedict Danamor Jagri not forgetting my Dad, Mr. Luke Maanu, and Mum the late Mrs Dorothy Maanu.

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Chapter 1

Introduction

1.1 Background

That is why the quality and quantity of water supplied to a community is crucial in determine their status standard of living and level of development (Joy & Widstrand, 1990). Water is fundamental for life and essential for nearly every human endeavor. The earth is endowed with enormous water resources, approximately 1.4 billion kilometers and of these only about 2.5 percent (i.e. 35,000,000km^3)- (Cunningham & Siago, 2000). The seriousness of it is that only about tiny fraction of the total freshwater are available for use by humankind. Water is necessary for human survivor, agriculture, transportation and economic potential in every society. However, access to freshwater is actually an important part of the economy of a society. In places where water is scarce, irrigation and access to underground aquifers is essential for agriculture. This study intends to find the best network system to solve the problem of water to the consumer in Ghana especially in Kumasi Metropolis. The study is therefore to provide solutions to the stakeholders of Ghana Water Company Limited so that pipe network system will be humanized leading to low cost paving way for others to get water supply.

1.1.1 Different Sources of Water in Ghana

The sources of water that we get are rainwater, oceans, rivers, lakes, streams, ponds and springs which are the national, tube wells, hand-pumps, canals etc. are man-made sources of water. However in Kumasi Metropolis the main source of water for households are the treated water sources supplies from Barekese and Owabi dams.

The largest river in Ghana is the Volta, with two huge hydroelectric dams, has a catchment of 165.700 km^2 within the country. Volta Lake, behind Akosomba Dam, is about 300km long and traverses the center of the country. The other rivers are all in the south and southwest and drain about one third of the country. However, because of heavy rainfall in these areas, the rivers make up about 50% of the internal runoff. The major rivers in the north, such as the Black Volta and the White Volta, and their tributaries are perennial. The rest are dry during the long Harmattan season (Quansah, 2000). Urban communities get most of their water supply from rivers at dams and diversion structures. Most of the surface water has to be treated to meet health standards. Surface-water resources can probably serve all urban needs for the foreseeable future through parallel programs of development and conservation. The rural communities rely on groundwater provided that it is available in sufficient quantities, is reliable throughout the year and does not required treatment. The quality of groundwater is generally good although in some locations the water contains iron and manganese deposits. Groundwater from shallow wells near streams and spring has always accounted for a large of the potable-water supply in rural communities.

Ghana has a tropical climate with a wide variation of rainfall, influenced by the southwest monsoon. Mean rainfall varies from 2,000 mm in the southwest coastal area to about 850mm in the eastern coastal area and 1,000mm in the north (Cornish & Aidoo, 2000). Any site with static water levels of less than 10m is a possible source of groundwater. Two types of wells are common, namely, hand-dug wells and drilled wells (boreholes):

- Hand-dug wells generally have a large diameter and are constructed using simple tools, such as pick axes and shovels. Their depth ranges from 5 to 20 m. They can be lined with concrete cast on site, precast concrete rings rock, or concrete blocks. These are structured to serve about 200 people.

- Boreholes are hand or machine-drilled well. Hand-drilled wells have a small diameter and are sunk using special tools, such as bits or augers. Machine-drilled wells are typically 100-200 mm in diameter and are sunk using relatively sophisticated equipment powered by diesel or electric motors. Machine drilling is suitable for depths of up to 50 mm but the depth depends on the power of the rig and the geological conditions. The number of people that can be served by a drilled well depends on the capacity of the hand pump, but it is typically about 300

For rural communities, a hand-dug well has several advantages. The use of simple hand tools for construction provides employment for local artisans, serves as a focus for community mobilization, and could be opportunity for educating the villages on water-related health and social issues.

Domestic demand includes water for drinking, cooking, laundering and other household functions. Public demand includes water for fire protection, street cleaning and uses in schools, and other public buildings. Commercial and industrial demands include water for stores, offices, hotels, laundries, restaurants and most manufacturing plants. There is usually a wide variation in total water demand among different communities. This variation depends on population, geographical location, climate, the extent of local community industry activity, the cost of water, habits of people and efficiency of the water supply system. Public demand includes water for fire protection, street cleaning and uses in schools and other public buildings. Commercial and industrial demands include water for stores, offices, hotels, laundries, restaurants and most manufacturing plants. There is usually a wide variation in total water demand among different communities. This variation depends on population, geographical location, climate, the extent of local community industry activity, the cost of water, habits of people and efficiency of the water supply system. Water demand is expressed numerically by average daily consumption per capita (per person). In the United State, the average is approximately 100 gallons (380 litres) per capita per day for domestic and public needs. Overall average total demand is about 180 gallons (68 litres) per capita per day, when commercial and industrial water uses are included. The total quantity available may remain constant after some uses, but the quality is degraded so that water is no longer as valuable as it was. The combined total of assumption and degradation accounts for about half of the withdrawal in most industrial countries. The other half of the water we withdraw would still be valuable for further uses if we could protect it from contamination and make it available to potential consumers. The rate at which we are using water now may make it necessary to conscientiously protect, conserve and replenish our water supply. Agriculture sector alone claims about 70 per cent of the total water withdrawal worldwide (Cunningham & Siago, 1990). It is interesting to note that closed to 70 to 90 per cents of the water withdrawn for agriculture never reaches the crops for which it is intended.The peak demands in residential areas usually occur in the morning and early evening (just before and after the workday) and weekends. Water demands in commercial and industrial districts are usually uniform during the working day. The minimum water demands typically occur in the very early or predawn morning hours.

Water Company is an organization which is mandated to build dams to impound water, treat water and distribute to homes. The water company is much concern for transporting water from areas of abundance to areas of shortage. The work of the company includes collection, transmission, treatment, storage and distribution of water for homes, commercial establishments, industry and irrigation as well as for such public needs as fire fighting and street flushing. The evaporative loss from Lake Mead and Lake Powell on the Colorado River is about one 3km per year. That is about 10% of the annual river flow. This amounts to nearly 4,500litres (1,200 gallon) for each person in the United State per year (Cunningham & Siago, 1990). Water-treatment works employ a variety of other treatment processes, which include long period storage, aeration, coagulation, sedimentation, softening and filtration. These processes are used in varying combinations, depending mostly on the characteristics of the water and its intended purposes. Long-period storage usually in the reservoir gives particulates a chance to settle out filtration through beds of fine sand or through crushed anthracite coal trap the suspended matter. Different chemical additives causes particles to coagulate and thus to settle. Aeration mixed air with water either by spraying the water into the air by allowing the water to cascade or by forcing small air bubbles through the water and it is primarily to reduce unpleasant odours and tastes. Softening is the process of removing calcium and magnesium from the water either by chemical precipitation or by ion exchange. After treatment, water is pumped either directly into the distribution system or to an elevated storage location, such as overhead reservoir. For adequate distribution, water systems must operate under pressure. In some cases, the gravity drop of water from its elevated storage provides enough pressure to aid distribution; otherwise; it is supplied by sufficient pressure at a pumping station. Adequate pressures range between 30 and 100 pounds per square inch (2 and 7 kilograms per square centimetre).Materials used in transporting water to homes and industry include pipes of cast iron, steel, concrete, PVC and asbestos cement. Meters record water usage at the site of consumption and charges are levied to help pay for operation and maintenance of the system. As the worlds population continues to grow, dams, aqueducts and reservoirs will still have to be built particularly in the developing countries where basic human needs have not been met. But such projects must be built to higher standards and with more accountability to the local people and their environment. Even in regions where the projects seem warranted, we must find ways to meet demands with fewer resources, minimum ecological disruption and less loses. In many countries, 30 percent more of the domestic water supply never reaches its intended destinations, disappearing from leaky pipes, faulty equipment or poorly maintained distribution systems. In Mexico City, the quantity of water supply system losses is enough to meet the needs of a city the size of Rome. Even in more modern systems, losses of 5% to 7% are common. (Frederick and Pontius (1997)).

1.1.2 History of Water Supply in Ghana

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Ghana Water Company Limited was established on 1st July 1999 following the conversion of Ghana Water and Sewerage Corporation into a state-owned limited liability company under the Statutory Corporations (Conversion to Companies) Act 461 of 1993 as amended by LI 1648. The first public water system in Ghana then Gold Coast was established in Accra just before World War. Other systems were built exclusively for the other urban areas among them the colonial capital of Cape Coast, Winneba and Kumasi in the 1920s. During this period, the supply systems were managed by the Hydraulic Division of Public Works Department. With time the responsibilities of the Hydraulic Division were widened to include the planning and development of water supply systems in other parts of the country. In 1948, the Department of Rural Water Development was established to engage in the development and management of rural water supply through the drilling of boreholes and construction of wells for rural communities.

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After Ghana's independence in 1957, a Water Supply Division with headquarters in Kumasi was set up under the Ministry of Works and Housing with responsibilities for both urban and rural water supplies. During the dry season of 1959 there was severe water shortage in the country. Following this crisis, an agreement was signed between the Government of Ghana and the World Health Organization for a study to be conducted into water sector water sector development of the country. The study focused on technical engineering, establishment of a national water and sewerage authority and financing methods. Furthermore the study recommended the preparation of a Master Plan for water supply and sewerage services in Accra-Tema covering the twenty-year period from 1960 to 1980.

In line with the recommendations of the WHO, the Ghana Water and Sewerage Corporation (GWSC), was established in 1965 under of Parliament (Act 310) as a legal public utility entity. GWSC was to be responsible for:

- Water supply and sanitation in rural as well as urban areas.
- The conduct of research on water and sewerage as well as making of engineering surveys and plans.
- The construction and operation of water and sewerage works,
- The setting of standards and prices and collection of revenues.

GWSC therefore met its operating costs at a level constrained by unavailability or inadequacy of funds. The lack of funds to meet operational costs resulting the poor state of existing infrastructure especially the distribution systems. Before 1957, there were 35 pipe-borne water supply systems in the country. In a bid to promote rapid national development after Ghana's Independence, the government launched a crash programme for urban water expansion and accelerated rural development. As a result by 1984 there were 194 pipe-borne and 2,500 hand pumped borehole systems in the country. By 1984, additional 3000 boreholes had been drilled and fitted with hand pumps. However, by the late 1980s and early 1990, 33% of the water supply systems had deteriorated greatly or completely broken down due to inadequate funding to carry out maintenance and rehabilitation.

1.1.3 Interventions to Improve Efficiency in supply of water

To reverse the decline in supply services, various sector reforms and improvement projects were undertaken in 1970, 1981 and 1988 by the World Bank, IDA, donor countries and other external support agencies such as Austrian Government, Italian Government, Nordic Development Fund and the African Development Bank. Though some gains were derived from these interventions, their general impact on service delivery was very disappointing. Due to the failure of these interventions to achieve the needed results, several efforts were made to improve efficiency within the water supply sector in Ghana especially during the era of the Economic Recovery Programme from 1983 to 1993.

During this period, loans and grants were sought from the World Bank and other donors for rehabilitation and expansion programmes, training of personnel and procurement of transport and maintenance equipment. In 1986, subvention for operations and maintenance was withdrawn although funding for development programmes continued. User fees for water supply were increased and subsides on water tariffs were gradually removed for GWSC to achieve self-financing. The government at that time approved a formula for annual tariff adjustments to enable the Corporation generate sufficient funds to cover all annual recurrent costs as well as attain some capacity to undertake development projects. For political reasons, this tariff formula was not applied and over the years irregular tariff increases were always below cost recovery levels resulting in heavy corporation deficit financing and ineffective service delivery.

1.1.4 Water Sector Reforms in Ghana

In 1987, a Five-Year Rehabilitation and Development Plan for the sector were prepared which resulted in the launching of the Water Sector Restructuring Project (WSRP). Multilateral and bilateral donors contributed \$140 million to support the implementation of the WSRP. The WSRP was aimed at reducing unaccounted for water, rationalisation of the workforce, hiring of professionals and training of staff. A strong focus of the WSRP was also on improved management and increased efficiency through organisational change of the water sector. Accordingly, a number of reforms within the Ghanaian water sector were initiated in the early 1990s.As a first step, responsibilities for sanitation and small town water supply were decentralized and moved from Ghana Water and Sewerage Corporation to the District Assemblies in 1993. The Environmental Protection Agency (EPA) was established in 1994 to ensure that water operations would not cause any harm to the environment. The Water Resources Commission (WRC) was founded in 1996 to be in charge of overall regulation and management of water resources utilization. In 1997, the Public Utilities Regulatory Commission (PURC) came into being with the purpose of setting tariffs and quality standards for the operation of public utilities. Community Water and Sanitation Agency (CWSA) was established in 1998 to be responsible for management of rural water supply systems, hygienic education and provision of sanitary facilities. After the establishment of CWSA, 120 water supply systems serving small towns and rural communities were transferred to the District Assemblies and communities to manage under the community-ownership and management scheme. Finally, pursuant to the Statutory Corporation (Conversion to Companies) Act 461 of 1993 as amended by LI 1648, on 1st July 1999, GWSC was converted into a hundred percent state owned limited liability, Ghana Water Company Limited, with the responsibility for urban supply only.

1.1.5 Water Supply in Ghana (Rural and Urban Areas)

The water supply and sanitation sector in Ghana faces a number of challenges, including very limited access to sanitation, intermittent supply, high water losses and low water pressure. Since 1994, the sector has been gradually reformed through the creation of an autonomous regulatory agency, introduction of private sector participation, decentralization of the rural supply to 138 districts and increased community systems. An international company managed all urban water systems since 2006 under a 5 year management contract which expired after achieving only some of its objectives. The reforms also aim at increasing cost recovery and a modernization of the urban utility Ghana Water Company Ltd. (GWCL). Another problem which partly arose from the recent reforms is the existence of a multitude of institutions with overlapping responsibilities. The National Water Policy (NWP), launched at the beginning of 2008, seeks to introduce a comprehensive sector policy.

1.1.6 Urban areas

The Ghana Water Company Ltd. (GWCL) is responsible for providing, distributing, and conserving water for domestic, public, and industrial purposes in 82 urban systems in localities with more than 5,000 inhabitants. Moreover, the company is mandated to establish, operate, and control sewerage systems in Ghana. Local private companies are in charge of meter installation, customer billing, and revenue collection. In the framework of the urban water project, since October 2006 the private operator AVRL supports GWCL under a five year management contact to improve its performance and rehabilitate and extend the infrastructure. Urban sanitation is a responsibility of local governments.

1.1.7 Rural areas

The Company Water and Sanitation Agency (CWSA) are in charge of coordinating and facilitating the implementation of the National Community Water and Sanitation Programme (NCWSP) in rural areas, which is carried out directly by the communities and their District Assemblies. The NCWSP focuses on three main objectives in order to achieve health improvements: safe water supply, hygiene education, and improved sanitation. The CWSA was created in 1994 under the framework of the Ghana decentralization policy and became autonomous in 1998. The institution does not directly construct, operate, and maintain facilities for water supply and sanitation. Instead, its role is to coordinate the work of a number of actors which carry out the services in rural areas, including public sector organizations, local beneficiary communities, private sector organization and NGOs. The CWSA is also expected to ensure that financial support from development partners is effectively used and to ensure hygiene education. The agency operates ten regional offices besides its head office in Accra.

In communities with fewer than 5,000 inhabitants, water supply systems are owned and managed by the respective community on a demand driven basis. According to the NCWSP, these systems do not receive any cross subsidies and 5% of investment costs are paid by District Assemblies. Communities and in rural areas and small towns elect gender-balanced water and sanitation boards consisting of volunteers, in communities with fewer than 5,000 inhabitants, water supply systems are owned and managed by the respective community on a demand-driven basis. According to the NCWSP, these systems do not receive any cross-subsidies and 5% of investment costs are paid by District Assemblies. Communities and rural areas and small towns elect gender-balanced water and sanitation boards consisting of volunteers, including one or two village-based caretakers who received special training in repair and maintenance. Communities can contract private companies or NGOs to provide technical assistance or services.

The share of non-functional supply systems in Ghana is estimated at almost one third with many others operating substantially below designed capacity. Moreover, domestic water supply competes with a rising demand for water by the expanding industry and agriculture sectors. Ghana aims at achieving 85% coverage for water supply and sanitation 2015, which would exceed the Millennium Development Goals' target of 78% (African Economic Outlook 2007 - Ghana Country Note). The World Water Day is celebrated annually on the 22nd of March. Its main purpose is to address issues relation to water resources, their management and the supply of portable water. In Ghana, portable water coverage is very low-about 45% for rural areas and 70% for urban centres. However, the Millennium Development Goals (MDGs) aim at halving the number of people without portable drinking water by 2015. Water, undoubtedly is a universal commodity and very important for the survival of mankind. Its use spans from domestic, industrial, agricultural to power generation. The use and importance of water is felt most when our taps cease to flow and our rivers run dry, we suddenly realize that water indeed is life. Sometimes, this tells us why people struggle to their last breath to get even the worse water-from canals and ponds. It is difficult for one to live without water, especially those of us in the tropical zone. It is anticipated that by 2020, the consumptive demand of water would be 5.3 billion of portable water. (Sam-Okyere.2010).

Over the years, mathematicians have tried to solve network problems using varying algorithms that have proved to be a set of useful concepts for analyzing the interaction between science and mathematics. Creating an environment for the nodes within the network is essential to the mathematical evaluation and furthermore the mathematician belonging to the environment just as the network nodes (Prims, 1957). The primary concern is to minimize the distance between nodes generally so that the resource present is utilized effectively.

Researchers have considered few algorithms in this area as follows; Prim's algorithm for example is an algorithm in graph theory that finds a minimum spanning tree for a connected weighted graph. This particularly was developed by Vojtech Jarnik and later independently by computer scientist Robert Prim and rediscovered by Dijkstra. Therefore it is sometimes DJP algorithm or Jarnik algorithm. There is also the P-median problem which considers the distance minimization in addition to considering the population at the various points. These routines when employed provide stakeholders with varying options in optimally considering variables in real life.

1.2 Problem Statement

Lots of resources are invested into the running of pipe networks therefore ineffective layout of pipe network system raise concerns for government and partners in development in development. A significant fact within the Ghanaian Water supply industry is that only 24 percent or 235 million m^3 of water is distributed for domestic use with two-thirds used for agricultural purposes and 10% withdrawn for industry.

This has led to some districts in the country especially in the southern section of the country getting water once a week or monthly inconsistently. One such district is the Abuakwa Towship within the fastest growing city in Ghana, Kumasi – (Amoateng & Owusu-Adade, 2013), (Cobbinah & Amoako, 2012). Abuakwa is located specifically along the Kumasi-Sunyani and Bibiani trunk roads about 12km from the Central Business District of Kumasi. With a current population of 23, 201 and growing as a peri-urban district within Kumasi, it experiences this water supply problem of inconsistency with access being once a week and sometimes once a month primarily because of leakages, high demand and traditionally costly pipe network supply layout.

1.3 Objectives

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Looking at the problem statement above, the researcher seeks to solve the following:

1. To find the optimal network system of Ghana Water Company Limited at Abuakwa Township using the Prim's algorithm.

2. To find the optimal cost of pipe network to replace the existing one.

3. To locate a maintenance unit using the P-median model

1.4 Methodology

The main purpose of this research is to minimize the pipe network system and find the optimal location of a maintenance unit in Abuakwa Township under the Ghana Water Company Limited. Due to the nature of the problem the researcher uses the method of Prim's algorithm under the shortest path algorithms. The researcher also uses the P-median algorithm to locate a maintenance unit which is based on distance and demand for water supply; this will be the shortest node in the network relative to all other nodes.

The entire research map was obtained from the Geographical Information Service (GIS) of Ghana Water Company Limited and Town and Country Planning of the Ghana Statistical Service, Ashanti Regional Office in Kumasi. The map is used for the pipe network system of Abuakwa Township and the distances they cover within the Township. The algorithms used in this research are presented in their general forms and used to solve the problem based on the data collected. The solution based on the Prim's algorithm is hand calculated and the solution based on the P-median for locating the maintenance unit is computed using Microsoft Excel. The sources of reading materials for research include journals on water management, mathematical journals and presentations related to the research. These sources have been included in the reference section.

1.5 Justification

The thesis will enable management of Ghana Water Company to have an in-depth knowledge on how to minimize pipe network and also find a location point relative to other nodes in the pipe network that will enable optimal distribution of water from source to various destination nodes.

The project also considers minimizing the usage of pipe network based on distance and also considers an algorithm that factors population density or demand dependent. This will therefore provide alternative solutions based on distance only and another based on both distance and population.

1.6 Organization of the study

Chapter one presents the background to the study, problem statement, objectives significance and justification of the study, scope of the study, limitations, data analysis tools and organization of the study. Chapter two lay more emphasis on review of relevant literature. This chapter discusses both the theoretical and empirical background literature. Chapter three discusses the Methodology. Chapter four focuses on analysis of data collection. In chapter five summary, conclusion and recommendations of the study are presented.

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Chapter 2

Literature Review

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This chapter is a review of literature in the area of shortest path algorithms with the aim of finding the optimal network system in order to solve the problem of the Abuakwa pipe layout. The related review will focus on Network and Shortest Path Algorithms. Some forms of the Shortest Path Algorithms to consider will include:

- Prim's Algorithm
- Dijkstra's Algorithm
- P-median model
- Floyd-Warshall Algorithm etc.

2.1 Introduction

For this chapter, the research reviews pertinent literature in the field of shortest path algorithms. The purpose of this study is to find the optimal network system to eradicate the problem of water supply of the residents of Abuakwa Township employing the use of the shortest path algorithm and the P-median approaches. It is in this light that this chapter will constitute mainly a literature review of the water supply network system and shortest path algorithms to support the research.

2.2 Network Systems

According to Xu (2008), the visualization of pipe network has become one of the essential characteristics of urban pipe network management system. But for the distribution of related resources such as data storage, modeling platform etc. and complex structure, the design and implementation of a pipe network visualization is always a challenge. This contribution presents a service view in design and deployment phase to cope with the questions mentioned above.

Many social and biological networks consist of communities/groups of nodes within which connections are dense, but between which connections are sparser. Recently, there has been considerable interest in designing algorithms for detecting community structures in real-world complex networks. (Maini, 2005) proposed an evolving network which exhibits community structure. The network model is based on the inter-community preferential attachment and the inter-community preferential attachment mechanisms. In relation to connecting a pipe network in which water flows we have a structure as such: A water supply system is considered a system of engineered hydrological and hydraulic components which provide water. This comprises of a watershed (i.e. an area of land where surface water from rain and melting snow converges to a single point). The raw untreated water collection point is transfered to a purification facility. The purified water is then stored. A pipe network for distribution of water to consumers is then engineered and connected to sewers.

A new method for the direct least cost solution of pipe sizes of a branched pipe network of known geometry consisting of pumping station and network of pipes for known water consumptions incorporating various cost functions is being developed. The method developed gives optimum pumping head and total head loss in the network system its optimum distribution among the various pipes of the overall least cost solution. The variation of the total cost with the total head loss in the system is studied. The method outline herein is simple and can be efficiently used for the least cost design of branched water main systems only, with the help of a desk calculator.

According to Shau, (2005), in the last three decades, a significant number of methods for optimal design of pipe network systems have been developed using the linear programming, non-linear programming, dynamic programming, enumeration techniques, and genetic algorithm.

2.2.1 Shortest Path Problem

The shortest path problem is concerned with finding the shortest path from a specified origin to a specified destination in a given network while minimizing the total cost associated with the path. The shortest path problem is an archetypal combinatorial optimization problem having widespread applications in a variety of settings. The applications of the shortest path problem include vehicle routing in transportation system, traffic routing in communication networks, pipe network system and path planning in robotic systems. Furthermore, the shortest path problem also has numerous variations such as the minimum weight problem, the quickest path problem, the most reliable path problem, and so on.

In graph theory, the shortest path problem of finding a path between two vertices (or nodes) such that the sum of the weights of its constituent edges is minimized. An example is finding the quickest way from one location to another on a road map; in this case, the vertices represent location and edges represent segments of road and are weighted by the time needed to travel that segment.

Formally given a weighted graph (i.e. a set V of vertices, a set E of edges, and a real-valued weight function $f : E \rightarrow R$) and one element v of V, find the path p from v to a v' of V so that

$$\sum_{p=1} f(p)$$

is minimal among all paths connecting v to v'.

According to Mouli, (2010), shortest path problems are among the most studied network flow optimization problems with interesting applications in a wide range of fields.

2.2.2 The Time-Dependent shortest Path Problem

Deterministic, time-dependent shortest path (TDSP) problems have been widely studies for the case of determining a single shortest path (TD-ISP). ? (?) have extended Bellman' principal of optimality for dynamic programming (1958) to this case and Dreyfus (1969) has suggested the use of Dijkstras algorithm (1959) for determining time-dependent shortest paths. Halpern (1977) first noted the limitations of the approach of Dreyfus (1969), and showed that if there exists a $y > \sigma$ such that $y + d_ij(t + y) < d_ij(t)$ is the travel time on arc (i, j) as a function of time t, then the departure from node i must be delayed, or the optimal path might include cycles, Kaufman and Smith (1990) subsequently studies the assumptions under which the existing TDSP algorithms would work, and showed that if the link-delays follow the first-in-first-out (FIFO) rule or consistency assumption, then one could use an expanded static (time-space) network to obtain optimal paths. Orda and Rom (1990), on the other hand, studies various types of waiting-at-nodes scenarios, and proposed algorithms for these different cases. They showed that if waiting is allowed at nodes, then the consistency assumption is not required, and they prescribed an algorithm for identifying optimal waiting times at the source node if waiting is not allowed elsewhere in the network. Furthermore, they demonstrated that for the forbidden waiting case, the paths obtained without the consistency assumption may not be simple, and showed that the continuous-time version of the problem is NP-Hard (1989). Malandraki (1993) analyzed the TDSP problem and extended Halpern's result for the special case of differentiable link delay functions and showed that the consistency assumption would be satisfied by verified by that the first derivative of the link delay function did not exceed negative unity. Frieze et al (1986) analysed the source waiting case in the context of the traffic equilibrium problem. They postulated that route choice is not independent of departure times and they calculated optimal source waiting times based on minimizing the total cost to the user. KoutsopoulosandXu (1994) noted the need for realistic traffic link delay functions in order for TDSP algorithms to be effective and prescribed the use of a link-delay function that gives weight age for both real-time and historic delay components.

Ziliaskopoulos and Mahmassani (1993) provided an efficient solution approach to the problem by discrediting time into time periods and developed a pseudo-polynomial.

2.3 Some Forms of Shortest Path Algorithms

There are several well-known algorithms for solving the shortest path problem, some of which are listed below. However, for the purpose of this work we will only concentrate on a few of them

- Dijkstras algorithm solves the singular-pair, single-source, and single destination shortest path problems.
- Bellman-Ford algorithm solves the single source problem if edge weights may be negative.
- A search algorithm solves for single pair shortest path using heuristics to try to speed up the search.
- Floyd-Warshall algorithm solves all pairs shortest paths.
- Johnsons algorithm solves all pairs shortest paths, and may be faster than Floyd-Warshall on sparse graphs.
- Others include Prim's,Perturbation theory finds, etc.

The following subsections present some forms of shortest path algorithms

2.3.1 Floyd-Warshall Algorithm

The shortest path between the two nodes might not be a direct edge between them, but instead involve a detour through other nodes. The all- pairs shortest path problem requires that we determine shortest path distances between every pair of nodes in the network. The Floyd-Warshall's algorithm obtains a matrix of shortest path distances with $O\{n^3\}$ computations.

The Floyd- Warshall algorithm, also variously know as Floyd's algorithm, the Roy-Floyd algorithm, the Roy-Warshall algorithm, or the WFI algorithm, is an algorithm for efficiently and simultaneously finding the shortest paths (i.e., graph geodesies) between every pair of vertices in a weighted and potentially directed graph (Floyd, 1962).

Hougardy, the Floyd-Waeshall algorithm is a simple and widely used algorithm to compute shortest paths between all pairs of vertices in an edge weighted directed graph. It can also be used to detect the presence of negative cycles. Hougardy, show that For this task many existing implementations of the Floyd-Warshall algorithm will fail because exponentially large numbers can appear during its execution.

Floyd-Warshall algorithm or the WFI algorithm is a graph algorithm for finding shortest paths in a weighted graph (with positive or negative edge weights).It is also for finding transitive closure of a single execution of the algorithm which will find the lengths (summed weights) of the shortest paths between all pairs of vertices through it which does not return details of the paths themselves. The algorithm is an example of dynamic programming. It was published in its currently recognized form by Robert Floyds in 1962. However, it is essentially the same as by Stephen Warshall in 1962 for finding the transitive closure of a graph.

The Floyd-Warshall algorithm compares all possible paths through the graph between each pair of vertices. It able to do this with only $(|V|^3)$ comparisons in a graph. This is remarkable considering that there may be up to $\omega \in (|V|^2)$ edges in the graph, and every combination of edges is tested. It does so by incrementally improving an estimate on the shortest path between two vertices, until the estimate is optimal.

2.3.2 Kruskals Algorithm

Kruskals algorithm is an algorithm in graph theory that finds a minimum spanning tree for connected weight graph. This means it find a subset of the edges that form a tree that includes every vertex, where the total weight of all the edges in the tree is minimized .If the graph is not connected , then it finds a minimum spanning forest (a minimum spanning tree for each connected component). Kruskals algorithm is an example of a greedy algorithm. An algorithm for computing a minimum spanning tree. It maintains a set of partial minimum spanning tree, and repeatedly adds the shortest edge in the graph whose vertices are in different partial minimum spanning trees.

At the termination of the algorithm, the forest has only one component and forms a minimum spanning tree of the graph (Cormen, 2001).

This minimum spanning tree algorithm was first described by Krustal in 1956 in the same paper where he rediscovered Jarnik's algorithm. This algorithm was also rediscovered in by Loberman and Weinberger, but somehow avoided being renamed after him. The basic idea of the Kruskals algorithm is as follows: scan all edges in increasing weight order; if an edge is safe, keep it (i.e. add to the set A).

2.3.3 Prim's Algorithm

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In computer science, prims algorithm is an algorithm that finds a minimum spanning tree for a connected weighted undirected graph. This means it finds a subset of the edges that forms a tree that includes every vertex, where the total weight of all the edges in the tree is minimized. Prisms algorithm is an example of a greedy algorithm. The algorithm was developed by Czech mathematician Vojtech Jarnik and later independently by computer scientist Robert C. Prim in 1957 and rediscovered by Edsger Dijkstra in 1959. Therefore it is also sometimes called the DJP algorithm, the Jarnik algorithm, or the prim-Jarnik algorithm (Prim, 1957).

Gonina, (2007) described parallel implementation of Prims algorithm for finding a minimum spanning tree of a dense graph. Their algorithm uses novel extension of adding multiple vertices per iteration to achieve significant performance improvements on large problems (up to 200,000 vertices). They described several experimental results on large graphs illustrating the advantages of our approach on over a thousand processor.

Gloor, (1993) described a system for visualizing correctness proofs of graph algorithms.

The system has been demonstrated for a greedy algorithm. Prim's algorithm for finding a minimum spanning tree of an directed weighted graph. They believe that their system is particularly appropriate for greedy algorithm, though much of what discuss can guide visualization of proofs in other contexts. While an example is not a proof, our system provides concrete examples to illustrate the operation of algorithm. These examples can be referred to by the user interactively and alternatively with the visualization of the proof where the general case is portrayed abstractly. Martel, (2002), studied the expected performance of Prim's Minimum spanning trees MST algorithm implemented using the ordinary heaps. We show that this implementation runs in linear or almost linear expected time on a wide range of graphs. This helps to explain why Prim's algorithm often beats MST algorithms which have better worst case run times.

2.3.4 Dijkstra's Algorithm

Chang described the reasons about why it is beneficial to combine with graph theory and board game. For bye, it also descants three graph theories Dijsktras, Prims, and Kruskals minimum spanning tree. Then it would describe the information about the board game they choose and how to and the game specifically.

The literature makes it abundantly clear that the procedure commonly known today as Dijkstras Algorithm was discovered in the late 1950s, apparently independently; by a number of analysts. There are strong indications that the algorithm was known in certain circle before the publication of Dijkstras famous paper. It is therefore somewhat surprising that this fact is not manifested today in the official title of the algorithm. Buriol, (2008), described dynamic shortest path algorithms as an update of the shortest paths taking into account a change in an arc weight. This paper describes a new generic technique that allows the reduction of heap sizes used by several dynamic single-destination shortest-path algorithms. The proposed scheme finds the shortest paths using a simultaneous multi-path search method. In contrast with Dijkstras algorithm, several nodes can be determined at one time. Moreover, we partition the network into different groups (networks groups) and find the all-node pairs shortest path in each group using a pipeline operation. Networks can be abstracted, and the shortest paths in very large networks can be found easily. The proposed scheme can decrease calculation time from O (N2) to O (N) using a pipeline operation on DAPDNA-2. Our simultaneous show that the proposed algorithm can be applied to the very large Internet network designs of the future. Pangilinan, (2007), presented an overview of the multi-objective shortest path problem (MSPP) and a review of essential and recent issues regarding the methods to its solution. The paper further explored a multi-objective evolutionary algorithm as applied to the MSPP and describes its behavior in terms of diversity of solutions, computational complexity, and optimality of solutions to the MSPP in polynomial time (based on several network instances) and can be an alternative when other methods are trapped by the tractability problem.

Chang, described the shortest distance between two points as a straight line. But in the real world, if those two points are located at opposite ends of the country, or even in different neighborhoods, it is unlikely you will find a route that enables you to travel from origin to destination via one straight road. You might pull to determine the fastest way to drive somewhere, but these days, you are just as likely to use a Web-based service or a hand held device to help with driving directions. The popularity of mapping applications for mainstream consumer use once again has brought new challenges to the research problem knows as the "shortest-path problem." The shortest-path problem, one of the fundamental quandaries in computing and graph Theory, is intuitive to understand and simple to describe, In mapping terms, it is the problem of find the quickest way to get from one location to another. Expressed more formally, in a graph in which vertices are joined by edges and in which each edge has a value, or cost, it is the problem of finding the lowest-cost path between two vertices. There are already several graph-search algorithms that solve this basic challenge and its variations, so why is shortest path perennially fascinating to computer Scientists?

Goldberg (2001), Principal researcher at Microsoft Research Silicon Valley said there are many reasons why researchers keep studying the shortest-path problem. "Shortest path is an optimization problem thats relevant to a wide range of applications, such as network routing, gaming, and circuit design and mapping", Goldberg says. "The industry comes up with new applications all the time, creating different parameters for the problem. Technology with more speed and capacity allows us to solve bigger problems, so the scope of the shortest-path problem itself has become more ambitious. And now there are Web-based services, where computing time must be minimized so that we can respond to queries in real time."

2.3.5 The P-Median Model

The p-median problem is, with no doubt one of the most studied facility location models. Basically, the p-median problem seeks the location of a given number of facilities so as to minimize some measure of transportation costs, such as distance or travel time. Therefore, demand is assigned to the closest facility.

The p-median problem is widely used in both public and private sector location decisions. Its uses include the original practical case suggested by Hakimi, which is locating a number of switching centers on a telephone network, as well as a large number of other applications, both geographical and non-geographical. Among the first, it is worth mentioning the location of public facilities so that the distance the public must travel to them is minimized; facilities such as schools and hospitals are typical examples.

The model is called the p-median because the median vertex of a network or graph for which the sum of the lengths of the shortest paths to all other vertices is the smallest. Locating a school on the median vertex of a network in which edges or arcs represent roads and each node represents a child, minimizes the total distance that children have to walk to go to that school. Or, if each node represents a customer, and a maintenance center housing a vehicle has to be located on some vertex of the network, the median vertex will be the location that minimizes the total distance traveled by the vehicle, if all customers have to be served, one at a time. On a network, finding the median vertex solves a problem similar to that posed by Fermat on a plane, consisting of finding the location of the point on a plane which minimizes the sum of its distances to three points whose location is known.

Weber, in the early twentieth century generalized this problem by adding weights to them, which could represent amount of demand or population aggregated at the point. If a facility is located at this weighted median, it will satisfy the demand of the three points with the minimal transportation cost. Later, the Weber problem was generalized to include more than three demand points, and to locate more than one facility. The 3 version with multiple facilities became known as the Multi-Weber problem. In the twentieth century, Cooper (1963-4) provided heuristics solution for it.

In its current form of usage it must be stated that Hakimi did not formulate the p-median as an integer programing problem. This was first done by ReVellle Swain (1970) who, not being familiar with the results of Hakimi, assumed node, only location of what they called central facilities. This formulation opened a new line in the search of solution procedures for the p-median problem.

The p-median can be formally stated in words as: 'Given the location of n points that house known amounts of demand, designate p of these points as facilities and allocate each demand to a facility, in such a way as to minimize the total weighted distance between demands and facilities.

This problem can be solved after modeling using different methods. Total enumeration is always an alternative, although its complexity makes this method useless when the problem grows. The first methods that were proposed for solving the p-median were heuristic. Louis Hakimi was one of the first researchers addressing the problem on a network. In his 1964 paper, the best location of a facility was sought, considering that all demand must be attended. Similarly to the problem on a plane, the demand is distributed over the region of interest. In the network version of the problem, demand is located only on vertices or nodes, each of them having a weight representing th total amount of a point on an edge of the network, distinction that does not exist when the problem lies on the plane. Hakimi proved, however, that there is always an optimal solution at a node. The problem consists of finding this optimal location, in such a way that the sum of the distances between the facility and each demand node, weighted by the amount of demand, is minimum. Because of this minimization of a sum of terms, the problem has also been called "minsum" problem. Hakimi was able to generalize his main result (node solution) to the case of multiple facilities. Now, the problem consists of finding the locations of p facilities, in such a way that the sum of the weighted distances between each demand node and its closest facility is the least. He called this problem the p-median. Note that the presence of more than one facility introduces an additional level of difficulty, since the solution must now answer to two questions; where to locate the p facilities – the "location" problem; and what demand node is assigned to which facility – the "allocation" problem. In the Hakimi (1965) version, the allocation problem is defined as assignment of demand nodes to their closest facilities. However, the location of multiple facilities allows different possibilities, including allocation of a demand to more than one facility, which could be optimal if facilities have a limited capacity, or if customers located at demand nodes can choose different facilities in different opportunities in real life.

Chapter 3

Methodology

KNUST

3.1 Introduction

Water Supply in Abuakwa Township is a problem connecting to various areas like homes, schools, churches, organizations and industries. The main problem involves the insufficient water supply to the residents in Abuakwa Township. This has made people in the township to practice illegal connection to the main service lines. Majority of the township members consider it as generating of water from the main service lines is costly therefore it is easy for them to connect to any lines anyhow without any cost involve. Shortest path problems are the most fundamental and most commonly encountered problem in the study of transportation and distribution in water companies. For the purpose of this research, work we are interested in determine the shortest, economical and easiest way of Water Company to plan on how to supply water to Abuakwa Township.

3.2 Minimum Connector Problem

The Minimum Connector Problem (MCP) for networks has been understood well for many years and applied in a variety of situations (Du & Pardalos, 1993). Given a number of nodes, or vertices, we seek to find the optimum set of edges (an edge is a link between two nodes) that fully connects the node set in question. (A network is connected if a path exists between each pair of nodes.) To this end, a cost matrix is applied to the nodes requiring interconnection with the cost element between each node pair - i.e. the cost of that edge - reflecting the expenditure, distance, difficulty, etc. involved in joining the two. Finding the Minimum Spanning Tree (MST) for the cost matrix will then result in the optimal solution across the nodes i.e. the minimum cost set of connecting edges.

An example is given in Figure 3.1. Here, cost has been taken to be the Euclidean distance between node pairs and the MST minimizes the total edge length in the connected solution. It should be noted that the degree of each node (the number of edges in the solution adjacent to that node called the valency in some texts) differs from node to node. Terminal nodes (1, 3, 5, 9 & 10 in Figure 1) have degree one. A node of higher degree (2, 4, 6, 7 & 8) acts as some form of connector or relay between/among the two, or more, nodes to which it is connected

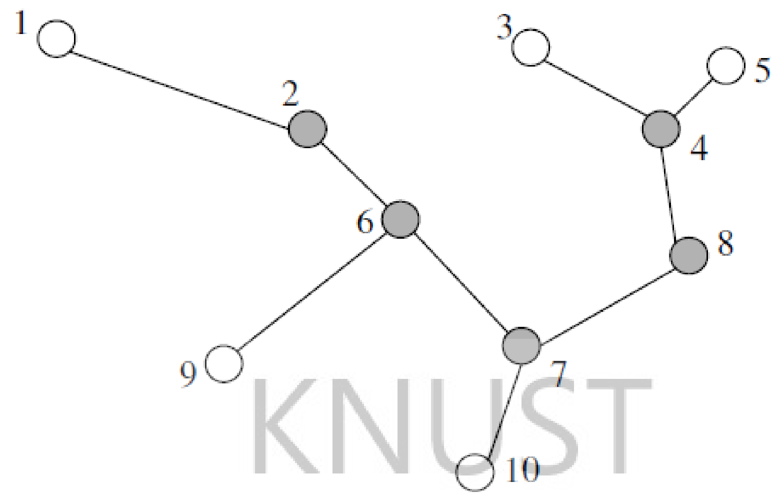

Figure 3.1: Minimal Spanning Tree Example

3.3 Prim's Algorithm

In computer science, Prim's algorithm is a greedy algorithm that finds a minimum spanning tree for a connected weighted undirected graph. This means it finds a subset of the edges that forms a tree that includes every vertex, where the total weight of the edges in the tree is minimized. The algorithm was developed in 1930 by Czech mathematician Vojtech Jarnik and later independently by computer scientist Robert C. Prim in 1957 and rediscovered by Edsger Dijkstra in 1959.

It works as follows:

- Create a tree containing a single vertex, chosen arbitrarily from the graph
- Create a set containing all the edges in the graph
- Loop until every edge is then connected to vertices in the tree.

- Remove from the set an edge with minimum weight that connects a vertex in the tree with a vertex not in the tree.

- Add that edge to the tree like Kruskal's algorithm, Jarnik's algorithm as described idea of Jarnik's algorithm is similar to that of Dijkstra's algorithm for finding shortest path in a given graph.

The Jarniks algorithm has the property that the edges in the set A always form a single tree. We begin with some vertex v in a given graph $G = (V, E)$, defining the initial set of a vertices A. Then, in each iteration, we choose a minimum weight edge (u, v), connecting a vertex v in the set A to the vertex u outside of set A. Then vertex u is brought in to A. This process is repeated until a spanning tree is formed. Like Kruskal's algorithm, here too, the important fact about MSTs is that we always choose the smallest - weight edge joining a vertex inside set A to the one outside the set A. The implication of this fact is that it adds only edges that are safe for A; therefore when the Jarniks algorithm terminates, the edges in set A form a minimum spanning tree.

The implementation of the Prim's algorithm can be demonstrated as follows Consider a five node distance network provided in Figure 3.2. We can construct a distance matrix from one node by considering the distance given from one node to the next. Therefore we obtain a square matrix from one node to the next. In the case where a particular distance is unavailable from the network diagram, ∞ is placed in that distance.

We choose a starting point say D. Delete the row D after selecting the smallest entry in column D. This is C. It implies DC is the smallest edge joining C to the other vertices

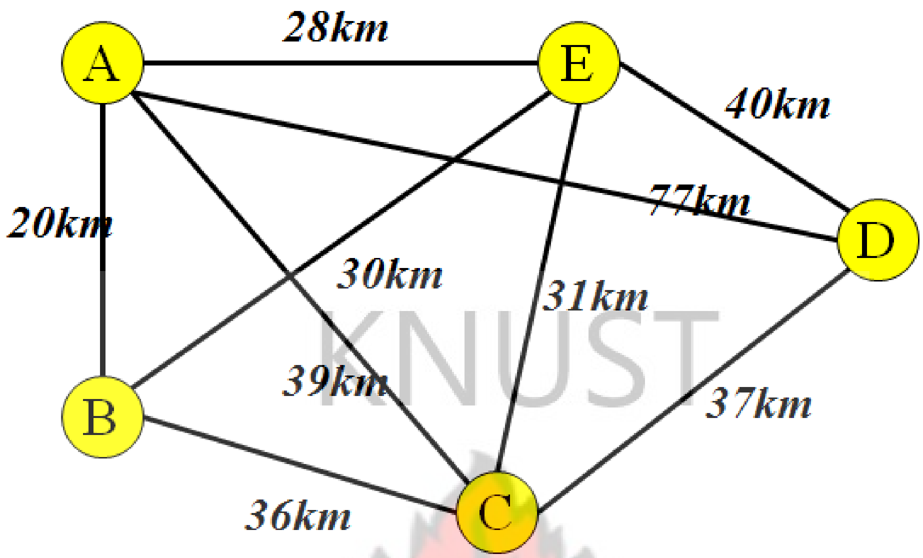

Figure 3.2: Distance network for five node problem

Table 3.1: Distance-Matrix for Prim's schedule

Site/Node	D	E	C	A	B
D	∞	40	37	77	∞
E	40	∞	31	28	30
C	37	31	∞	39	36
A	77	28	39	∞	20
B	∞	30	36	20	∞

put edge DC into the solution. Then row C will be deleted as given in the Table below:

Table 3.2: Distance-Matrix for Prim's schedule after deleting C & D

Site/Node	**D**	E	**C**	A	B
E	40	∞	31	28	30
A	77	28	39	∞	20
B	∞	30	36	20	∞

CE is the smallest edge D and C to any other vertex. Put edge EC into the solution. Delete row E and look for the smallest entry in columns D, C and E. AE is now the smallest edge joining D, C and E to the other vertices. Put EA into the solution. Delete row A. And then we follow the same routine by looking for the smallest entry in the columns D, C, E and A. AB is the smallest edge joining D, C, E and A to the other vertices. Put edge BA into the solution and delete row B. The resulting spanning tree is given in Figure 3.3.

We have now connected all the vertices into the spanning tree. The length is

$$(37 + 31 + 28 + 20 = 116km)$$

. This is the algorithm suitable to the minimum connector of this project together with the P-median which will also be considered.

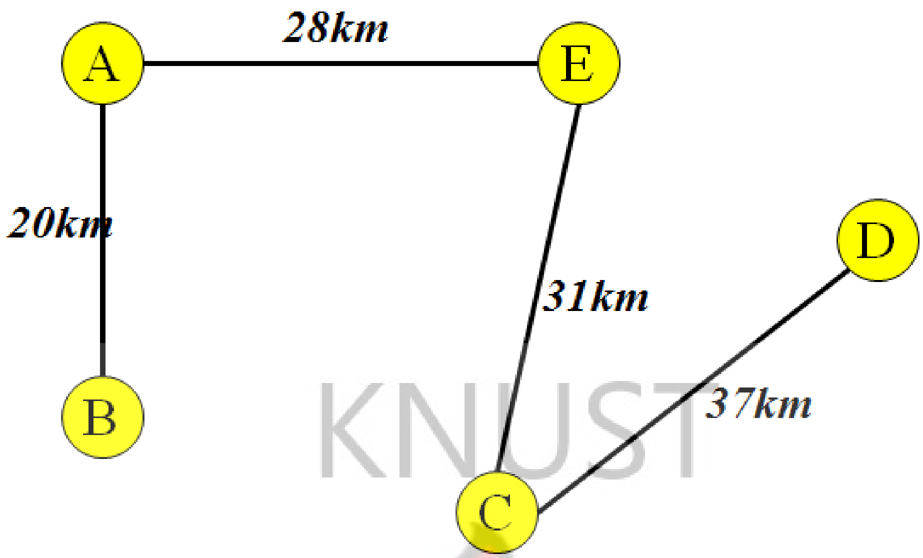

Figure 3.3: Prim's solution for five node problem

3.4 Single-Source Paths, Non-Negative Weight (Dijkstra's Algorithm)

Dijkstra's algorithm is called the single-source shortest path. It is also known as the single source shortest path problem. It computes length of the shortest path from the source to each of the remaining vertices in the graph. The single source shortest path problem can be described as follows:

Let $G = [V, E]$ be a directed weighted graph with V having the set of vertices. The special vertex S in V, where S is the source and let for any edge e in E, Edge Cost (e) be the length of edge e. All the weights in the graph should be non-negative.

The Algorithm

Let the node at which we are starting be called the initial node. Let the distance of node Y be the distance from the initial node to Y. Dijkstra's algorithm will assign some initial distance values and will try to improve them step by step. Set it to zero for our initial node and to infinity for all other nodes. Mark all nodes unvisited. Set the initial node as current. Create a set of the unvisited nodes called the unvisited set consisting of all the nodes except the initial node. For the current node, consider all of its unvisited neighbors and calculate their tentative distances. For example, if the current node A is marked with a tentative distance of 6, and the connecting it with a neighbor B, has length 2, then the distance to B (through A) will be $6 + 2 = 8$. If this distance is less than the previously recorded tentative distance of B, then overwrite that distance. Even though a neighbor has been examined, it is not marked as visited at this time, and it remains in the unvisited set.

When we are done considering all of the neighbors of the current node, mark the current node as visited and remove it from the unvisited set. A visited node will never be checked again; its distance recorded now is final and minimal. If the destination node has been marked visited (when planning a route between two specific nodes) or if the smallest tentative distance among the nodes in the unvisited set is infinity (when planning a complete traversal), then stop. The algorithm has finished. Set the unvisited node marked with the smallest tentative distance as the next current node and go back to step 3.

We can now express Dijkstras algorithm as a set of steps.

1. Assign the permanent label O to the starting vertex.

2. Assign temporary labels to all the vertices that are connected directly to the most recently permanent labeled vertex.

3. Choose the vertex with the smallest temporary label and assign a permanent label to that vertex.

4. Repeat steps 2 and 3 until all vertices have permanent labels.

5. Find the shortest path by tracing back through the network. Note: Recording the order in which we assign permanent labels to the vertices is an essential part of the algorithm.

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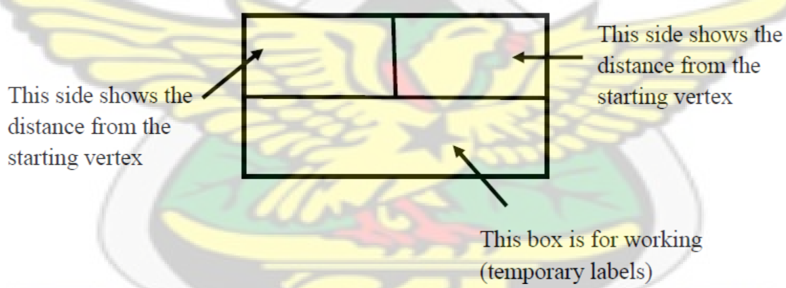
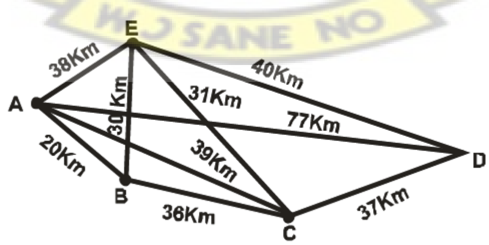
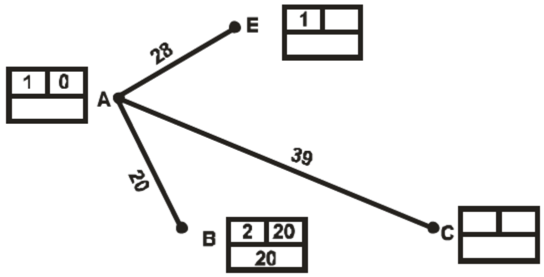
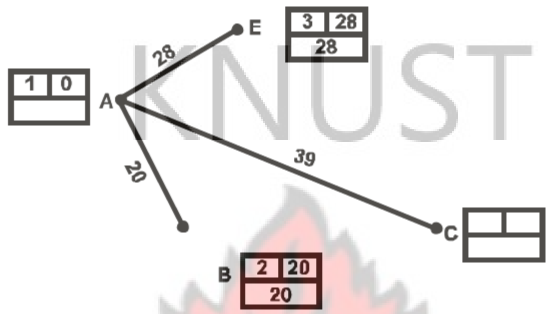


Figure 3.4: Computational Result holder for Dijkstra's algorithm

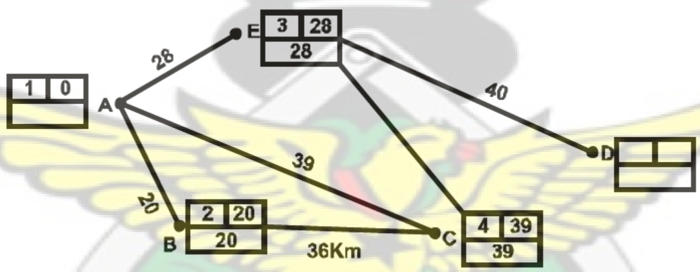




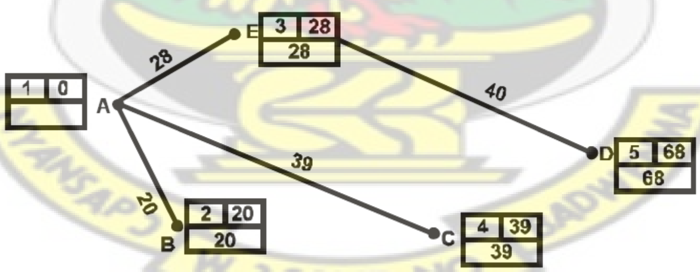
1 | 0
A
28
E
1
20
B
2 | 20
20
39
C



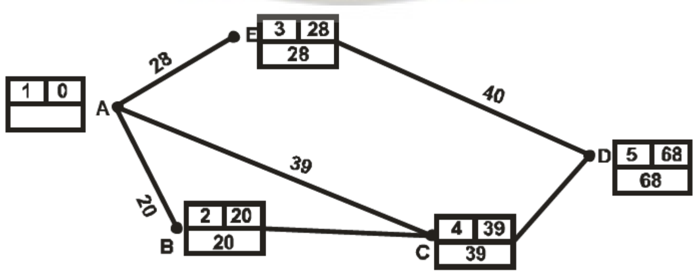
1 | 0
A
28
E
3 | 28
28
20
39
C
B
2 | 20
20



1 | 0
A
28
E
3 | 28
28
40
D
39
20
B
2 | 20
20
36Km
C
4 | 39
39



1 | 0
A
28
E
3 | 28
28
40
D
5 | 68
68
39
20
B
2 | 20
20
C
4 | 39
39



1 | 0
A
28
E
3 | 28
28
40
D
5 | 68
68
39
20
B
2 | 20
20
C
4 | 39
39

Now all the vertices have permanent labels and we can see the shortest distance from A to D is 68.

We find the shortest path by working backward from D, i.e.

$$D \to E \to A$$

Hence the shortest path from A to D is $A \to E \to D$ with length 68.

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3.5 All-Pairs Shortest Path Problem

The all pairs shortest path problem (APSP) finds of the shortest path for all source-destination pairs in a (positively) weight belongs to the most fundamental problems in graph theory. The problem occurs in many algorithms in communication, network, and circuit.

The all-pairs shortest path problem can be considered the mother of all routing problems. Its aim is to compute the shortest path from each vertex v to every other u. Using standard single-source algorithms, you can expect to get naive implementation of $0(n^3)$ if you use Dijkstra for example i.e. running a $0(n^3)$ process n times likewise, if you use the Bellman. Ford-Moore algorithm on a dense graph, it will take about $0(n^4)$, but handle negative arc-lengths too. Storing all the paths explicitly can be very memory expensive indeed, as you need one spanning tree for each vertex. This is often impractical in terms of memory consumption, so these are usually considered as all-pairs shortest distance problems, which aim to find just the distance from each to each node to another. The result of this operation is an $n \times n$ matrix, which estimated distances to the each node. This has many problems when the matrix gets too big, as the algorithm will scale very poorly.

3.5.1 The Floyd-Warshall Algorithm

The Floyd -Warshall algorithm obtains a matrix of shortest path distances within $0\{n^3\}$ Computations. The algorithm is based on inductive arguments developed by an application of a dynamic programming technique.

Let $d^k(i,j)$ represent the length of the shortest path from node i to j subject to the condition that this path uses the nodes $1, 2, 3, \ldots, k-1$ as internal nodes clearly $d^{n+1}(i,j)$ represent the actual shortest path distance from i and j. The algorithm first computes $d^i(i,j)$ for all node pairs i and j. Using $d^1(i,j)$, it then computes $d^2(i,j)$ for all pairs of nodes i and j. It repeats the process until it obtains $d^{n+1}(i,j)$ for all node pairs i and j and then terminates. Given $d^k(i,j)$, the algorithm computes

$$d^{k+1}(i,j) = min\{d^k(i,k), d^1(i,k)\}$$

. The Floyd-Warshall algorithm remains of interest because it handles negative weight edges correctly.

The Algorithm

The Floyd-Warshall algorithm is also use to find the shortest distances between all pairs of nodes. The symbol as means that there is no direct connection between these vertices. The matrix below represents the distance (in Km) of direct direction between five towns.

Site/Node	A	B	C	D	E
A	–	20	∞	77	28
B	20	–	36	∞	30
C	∞	36	–	37	31
D	77	∞	37	–	40
E	28	30	31	40	–

Table 3.3: Distance between all pairs of nodes

The algorithm can be stated as follows:

1. Choose a starting vertex
2. Join this vertex to the nearest vertex directly connected to it
3. Join the nearest vertex, not already in the solution to any vertex in the solution, provided it does not form a cycle
4. Find the shortest path by tracing through the network
5. Repeat until all the vertices have been included

From The Node A

1. From A to B (the direct distance from A to B is 20). Since there is no other direct connection between A and any other node less than 20. So we retain the value 20 as the minimum distance between A and B.

2. A to C: (The direct distance is ∞).

$$\left.\begin{array}{lll} A \to B \to C & = 20 + 36 = & 56 \\ A \to D \to C & = 77 + 37 = & 56 \\ A \to E \to C & = 38 + 31 = & 69 \end{array}\right\} \text{Min}(56{,}114{,}69) = 56$$

3. A to D (The direct distance is 77)

$$\left.\begin{array}{lll} A \to B \to D & = 20 + \infty = & \infty \\ A \to E \to D & = 38 + 40 = & 78 \end{array}\right\} \text{Min}(\infty{,}78) = 78$$

This value is not less than 77. So we retain 77

4. A to E (The direct distance is 38). Only A to B is less than 38.

The distance from A to B =20. So we evaluate the distance

$$A \to B \to E = 20 + 30 = 50$$

So we retain 38 in the cell AE and EA.

Now From The Node B

1. From B to A (the direct distance from B to A is 20). Since there is no other direct connection between B and any other node less than 20. So we retain the value 20 as the minimum in the cells BA and AB.

2. B to C: (The direct distance is 36).

$$\left.\begin{array}{lll} B \to A \to C & = 20 + \infty = & \infty \end{array}\right\}$$

This is not less than 36 in BC and CB.

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3. B to D (The direct distance is ∞)

$$\left.\begin{aligned} B \to A \to D &= 20 + 77 = 97 \\ A \to E \to D &= 30 + 40 = 70 \end{aligned}\right\} \text{Min(97,70)} = 70$$

So we replace ∞ from B to D and D to B by 70

4. B to E (The direct distance is 30). No other distance in that direction is less than 30 so this will be retained.

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Now From The Node C

1. From C to A (the direct distance from ∞).

$$\left.\begin{aligned} C \to B \to A &= 36 + 20 = 56 \\ C \to D \to A &= 37 + \infty = \infty \\ C \to E \to A &= 31 + 38 = 69 \end{aligned}\right\} \text{Min}(56,\infty,69) = 56$$

So we replace the value in the cell CA by 56

2. C to B: (The direct distance is 36). Only C to E has the value less than 36 so we evaluate.

$$C \to E \to B = 31 + 30 = 61$$

We retain the CB with the value of 36 since the computed value is not less than 36.

3. C to D (The direct distance is 37)

$$\left.\begin{aligned} C \to B \to D &= 36 + \infty = \infty \\ C \to E \to D &= 31 + 40 = 71 \end{aligned}\right\} \text{Min}(\infty,71) = 71$$

Since 71 is not less than 37, we retain 37 in the cells CD and DC.

4. C to E (The direct distance is 31). No other value beginning with C is less than 31, so we retain 31 in the cells CE and EC.

Now From The Node D

1. From D to A (the direct distance from 77).

$$\left.\begin{aligned} D \to C \to A &= 37 + \infty = \infty \\ D \to E \to A &= 40 + 38 = 78 \end{aligned}\right\}$$

So we retain 77

2. D to B: (The direct distance is ∞).

$$\left.\begin{aligned} D \to A \to B &= 77 + 20 = 97 \\ D \to C \to B &= 40 + 30 = 70 \end{aligned}\right\} \text{Min(97,70)} = 7$$

So we replace ∞ in the cell DB by 70

3. D to C (The direct distance is 37) No value in the cell beginning with D is less than 37. So we retain the value 37.

4. C to E (The direct distance is 40).

$$D \to C \to E = 37 + 31 = 68$$

Since 68 is not less than 40, we therefore retain 40.

Hence the shortest distance between all pairs of nodes is as shown in the table below

Table 3.4: The Shortest Distance between all pairs of nodes

Site/Node	A	B	C	D	E
A	–	20	56	77	28
B	20	–	36	70	30
C	56	36	–	37	31
D	77	70	37	–	40
E	28	30	31	40	–

3.6 The P-Median Problem

The p-median problem is defined as an optimization problem that is well known in Operations Research and has been extensively applied to facility location - (Jalali, 2012). This particular technique is very useful in problems where we are interested in finding the location of p facilities to serve demand nodes so that the transportation cost is minimized. The transportation cost is given by the product of the demand node and the distance indicated between the demand node and the facility that served the demand node.

There are no capacity constraints at the facilities and therefore it is optimal to satisfy the demand at a demand node from a single facility. An optimal solution can be found by restricting the search to the demand nodes.

The inputs of this optimization process are therefore

- h_i = demand at node i

- d_{ij} = distance between demand node i and site j

- p = number of facilities

Decision Variables are defined as follows:

$$X_j = \begin{cases} 1 & \text{if a facility is located at site } j \\ 0 & \text{otherwise} \end{cases}$$

$$Y_{ij} = \begin{cases} 1 & \text{if demand at } i \text{ is served by a facility at site } j \\ 0 & \text{otherwise} \end{cases}$$

The outline of the p-median optimization process is therefore given as:

$$\begin{aligned} \text{Min} \quad & \sum_i \sum_j h_i d_{ij} Y_{ij} \\ \text{Subject to} \quad \sum_j Y_{ij} &= 1 \quad \text{for all } i \\ \sum_j X_j &= p \\ Y_{ij} &\leq X_j \quad \text{for all } i, j \\ X_j, Y_{ij} &\in \{0, 1\} \quad \text{for all } i, j \end{aligned}$$

A demonstration of the P-Median is given as follows; Suppose that in addition to the distance network given in Figure 3.1, there was also information on the population present at each node as follows;

The distance between the towns can be calculated and summarized in the table below as

The weights w_1, w_2, w_3, w_4 and w_5 represent the total weighted distances for the five considered nodes.

Table 3.5: Population at individual nodes

Site/Node	A	B	C	D	E
Population	90	85	100	112	120

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Table 3.6: Distance from Source to Destination Within Network

		Site/Node	1	2	3	4	5
	Site/Node	Town	A	B	C	D	E
90	1	A	–	20	59	96	28
85	2	B	20	–	79	116	48
100	3	C	59	79	–	37	31
112	4	D	96	116	37	–	68
120	5	E	28	48	31	68	–

$$
\begin{aligned}
w_i &= \sum_{i,j}^{n} h_i d_{ij} y_i \\
d_{ij} &= \text{the distances between nodes} \\
h_i &= \text{the population of the five nodes} \\
n &= \text{number of nodes}
\end{aligned}
$$

The weights are calculated and the minimum of them is selected to locate the facility. The distance between a node and itself is zero; for example from A to A is equal to zero or – as indicated in Table 3.4. The total weighted distance w_1 of site 1 (A) is calculated as follows using the information from the network diagram in figure 3.5:

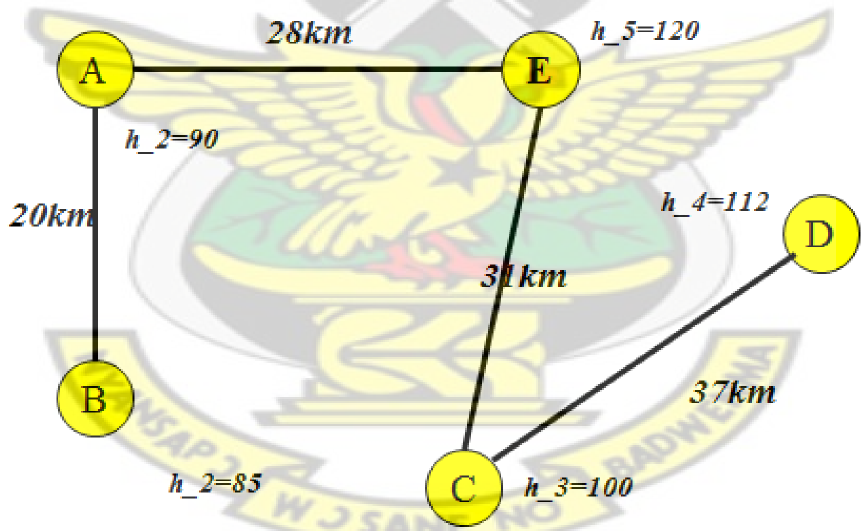

Figure 3.5: Network diagram for citing a facility at site A

$$
\begin{aligned}
w_i &= \sum_{i,j}^{n} h_i d_{ij} y_i \\
&= d_{11}h_1 + d_{12}h_2 + d_{13}h_3 + d_{14}h_4 + d_{15}h_5 \\
&= (0 * 90) + (20 * 85) + (59 * 100) + (96 * 112) + (28 * 120) \\
&= 21712
\end{aligned}
$$

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Therefore w_i which is site A is selected, it means that all the demand from other nodes will be accessing the facility at town A and that will involved a total distance of 21712km.

The total weighted distance (w_2) of site 2 (B) is calculated as follows using Figure 3.6.

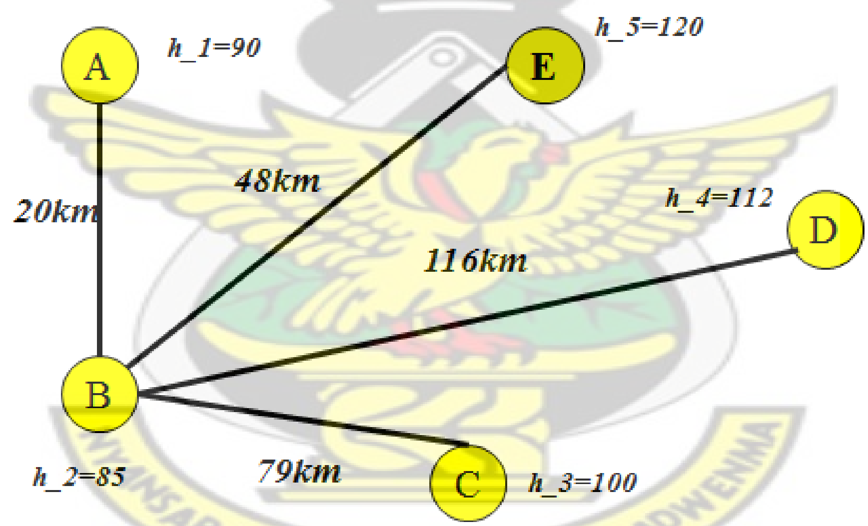

Figure 3.6: Network diagram for citing a facility at site B

$$
\begin{aligned}
w_i &= \sum_{i,j}^{n} h_i d_{ij} y_i \\
&= d_{11}h_1 + d_{12}h_2 + d_{13}h_3 + d_{14}h_4 + d_{15}h_5 \\
&= (20 * 90) + (0 * 85) + (79 * 100) + (116 * 112) + (48 * 120) \\
&= 28452
\end{aligned}
$$

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Therefore, if w_2 which is site B is selected, it means that all the demand from other nodes will access the facility at town B which will total a distance of 28452km.

The total weighted distance (w_3) of site 3(C) is calculated as follows using Figure 3.7.

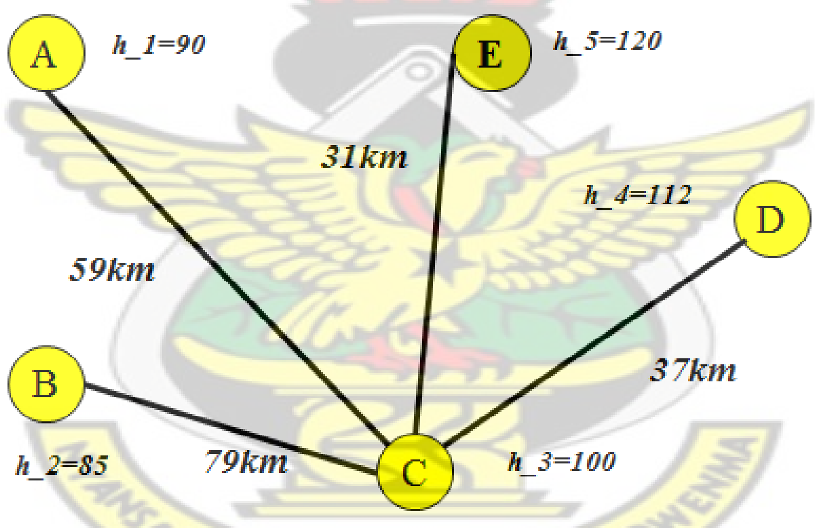

Figure 3.7: Network diagram for citing a facility at site C

$$
\begin{aligned}
w_i &= \sum_{i,j}^{n} h_i d_{ij} y_i \\
&= d_{11}h_1 + d_{12}h_2 + d_{13}h_3 + d_{14}h_4 + d_{15}h_5 \\
&= (59 * 90) + (79 * 85) + (0 * 100) + (37 * 112) + (31 * 120) \\
&= 19889
\end{aligned}
$$

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Therefore, if w_3 which is site C is selected, it means that all the demand from other nodes will access the facility at town C which will total a distance of 19889km.

The total weighted distance (w_4) of site 4(D) is calculated as follows using Figure 3.8.

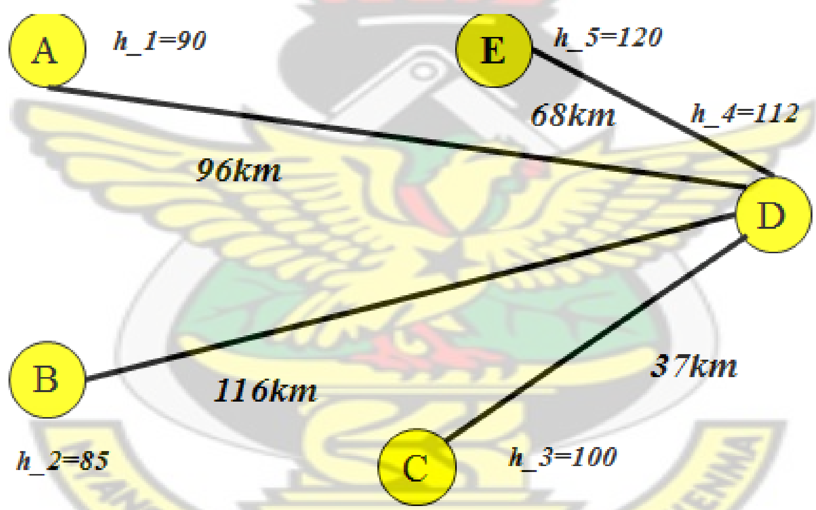

Figure 3.8: Network diagram for citing a facility at site D

$$
\begin{aligned}
w_i &= \sum_{i,j}^{n} h_i d_{ij} y_i \\
&= d_{11}h_1 + d_{12}h_2 + d_{13}h_3 + d_{14}h_4 + d_{15}h_5 \\
&= (96*90) + (116*85) + (37*100) + (0*112) + (68*120) \\
&= 30360
\end{aligned}
$$

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Therefore, if w_4 which is site D is selected, it means that all the demand from other nodes will access the facility at town D which will total a distance of 30360km.

The total weighted distance (w_4) of site 3(E) is calculated as follows using Figure 3.9.

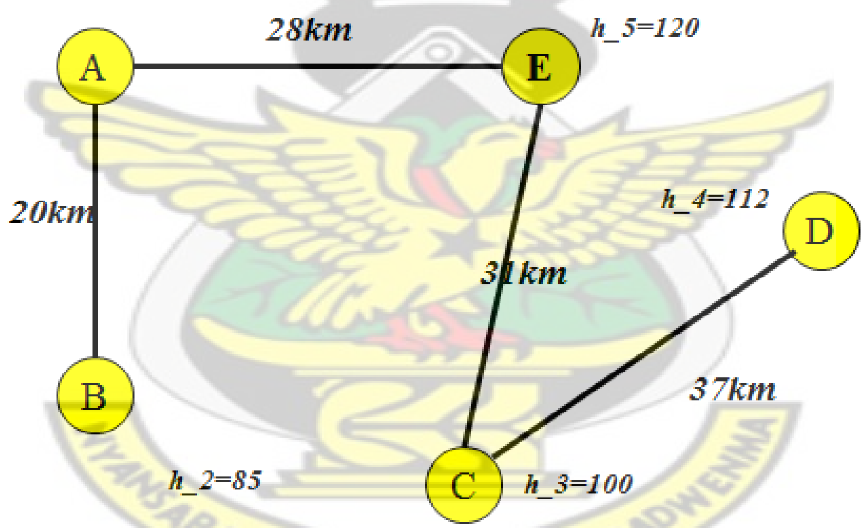

Figure 3.9: Network diagram for citing a facility at site E

$$
\begin{aligned}
w_i &= \sum_{i,j}^{n} h_i d_{ij} y_i \\
&= d_{11}h_1 + d_{12}h_2 + d_{13}h_3 + d_{14}h_4 + d_{15}h_5 \\
&= (28*90) + (48*85) + (31*100) + (68*112) + (0*120) \\
&= 17316
\end{aligned}
$$

Therefore, if w_5 which is site E is selected, it means that all the demand from other nodes will access the facility at town E which will total a distance of 19889km.

The minimum of the weighted distance above is 17316km which corresponds to site 5. Therefore town E is the best location for the facility.

Chapter 4

Data Collection, Analysis and Results

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4.1 Introduction

In this chapter, the data collected on the pipe network will be presented and analyzed. The Prim's algorithm will be used in formulating the problem based on the network derived and this will also be used in locating the optimal structure of the pipe network by finding the shortest distance within the network. The P-median will also be implemented and used in locating a maintenance facility within the Abuakwa Township pipe network. The related costs for the two algorithms are discussed in relation to the optimal solutions provided.

4.2 Data Collection

The main information and data needed for the research was obtained from the Geographical Information Services of the Ghana Water Company Limited (GIS of GWCL) and Town and Country Planning of the Ghana Statistical Service. Figure 4.1 shows the pipe network with the respective distance from one node to the other for the data collected on the North West District situated in the Abuakwa Township.

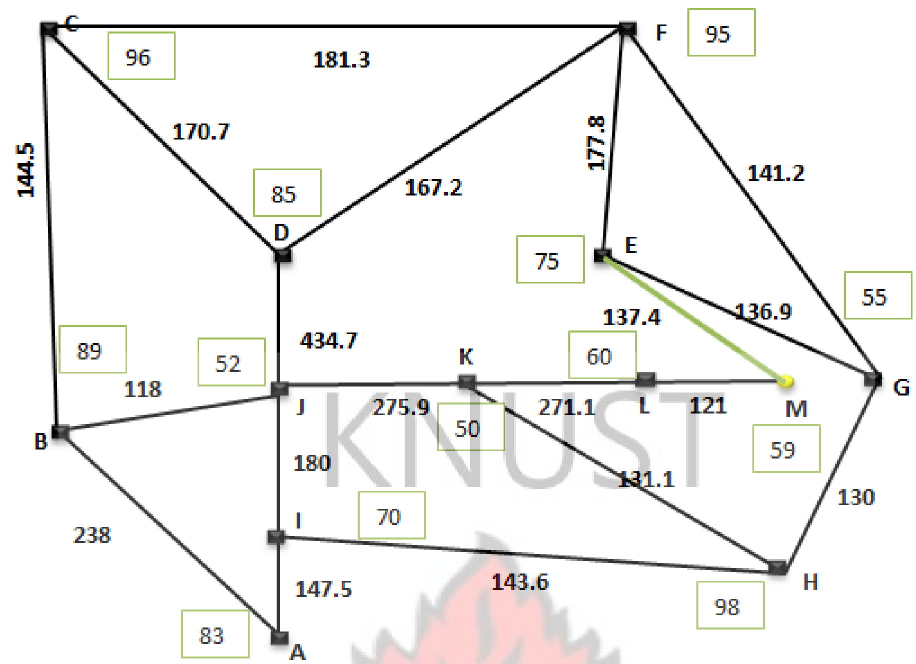

Figure 4.1: Abuakwa Township Network: Source: Ghana Statistical Service

The distances covered by each pipe network are indicated in meters on the network from one point to the other. The pipe network as shown in Figure 4.1 is named Kaana and is located just beside the main road leading to Sunyani. It is made up of thirteen (13) sub stations in the Abuakwa Township. It takes it source from Owabi head works. The thirteen substations are lettered from A–M as indicated in the figure. This data will be used in the problem formulation and computational procedure for both models that are discussed in this research. Table 4.1 is a Distance matrix table for the Abuakwa Township network.

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Table 4.1: Distance from Source to Destination for Abuakwa Township Network

	A	B	C	D	E	F	G	H	I	J	K	L	M
A	∞	238.0	∞	∞	∞	∞	∞	∞	147.5	∞	∞	∞	∞
B	238.0	∞	144.5	∞	∞	∞	∞	∞	118.0	∞	∞	∞	∞
C	∞	144.5	∞	170.7	∞	181.3	∞	∞	∞	∞	∞	∞	∞
D	∞	∞	170.7	∞	167.2	∞	∞	∞	∞	434.7	∞	∞	∞
E	∞	∞	∞	∞	∞	177.8	136.9	∞	∞	∞	∞	∞	137.4
F	∞	∞	181.3	167.2	177.8	∞	141.2	∞	∞	∞	∞	∞	∞
G	∞	∞	∞	∞	136.9	141.2	∞	130.0	∞	∞	∞	∞	∞
H	∞	∞	∞	∞	∞	∞	130.0	∞	143.6	∞	131.3	∞	∞
I	147.5	118.0	∞	∞	∞	∞	∞	413.6	∞	180.0	∞	∞	∞
J	∞	∞	∞	434.7	∞	∞	∞	∞	180.0	∞	275.9	∞	∞
K	∞	∞	∞	∞	∞	∞	∞	131.3	∞	275.9	∞	271.1	∞
L	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	271.1	∞	121.0
M	∞	∞	∞	∞	137.4	∞	∞	∞	∞	∞	∞	121.0	∞

4.3 Prim's Algorithm

The given problem is solved based on the Prim's algorithm and the ensuing computational procedure is demonstrated in this section.

4.3.1 Problem Formulation and Computational Procedure

Given a connected graph the minimum spanning tree problem is to find the set of edges that connect all nodes without a cycle and give the total minimum edge connection within the network. Prim's algorithm finds a minimum spanning tree for connected weighted graph. This means that if it finds a subset of the edges that forms a tree in it includes every vertex, where the total weight of all the edges in the tree is minimized. If a graph is not connected, then it will only find a minimum spanning tree for one of the connected components. The algorithm was discovered in 1930 by Mathematician Vojtech and later independently by computer scientist Robert Prim in 1957 and rediscovered by Dijkstra in 1959. It works as follows; create a tree containing a single vertex, chosen arbitrarily from the graph. Create a set containing all the edges in the graph. Loop until every edge in the set connects two vertices in the tree.

Prim's algorithm works from a starting point and builds up the spanning tree step by step, connecting edges into the existing solution. It can be applied directly to the distance matrix, as well as to the network itself. So it is more suitable for using a computer if the network is large.

The Prim's algorithm can be stated as follows:

- Step 1: Choose a starting vertex
- Step 2: Join this vertex to the nearest directly connected node to it
- Step 3: Join the next nearest vertex, not already in the solution to any vertex in the solution provided it does not form a cycle
- Step 4: Repeat until all vertices have been included

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4.3.2 Results for Prim's Algorithm

The research uses a manual form of calculation in this section. Using the Prim's algorithm described in the previous section and the network data from the Abuakwa Township as shown in Figure 4.1 and further summarized in the Distance Matrix in Table 4.1, a starting point A is chosen. The distances using the Prim's algorithm is summarized in Table 4.2.

Table 4.2: Distance Matrix from Source to Destination for Prim's Algorithm

From		To	Distance (m)	From		To	Distance (m)
A	⇒	I	147.5	F	⇒	G	141.5
I	⇒	J	180	G	⇒	H	130
J	⇒	B	118	H	⇒	K	131.3
B	⇒	C	144.5	K	⇒	L	271.1
C	⇒	D	170.7	L	⇒	M	121
D	⇒	F	167.2	M	⇒	E	137.4

The tree network is

$$A \Rightarrow I \Rightarrow J \Rightarrow B \Rightarrow C \Rightarrow D \Rightarrow F \Rightarrow G \Rightarrow H \Rightarrow K \Rightarrow L \Rightarrow M \Rightarrow E$$

The total distance within the network is therefore calculated by adding all distances in the resulting network.

$$\begin{aligned} \text{Total Distance} &= 147.5 + 180 + 118 + 144.5 + 170.7 + \cdots + 121 + 137.4 \\ &= 1859.9m \end{aligned}$$

The optimal pipe layout distance is therefore 1859.9m. This represents the minimum distance collectively from point A as calculated from Table 4.1. The resulting Network diagram for the Prim's algorithm for Abuakwa Township is given by Figure 4.2.

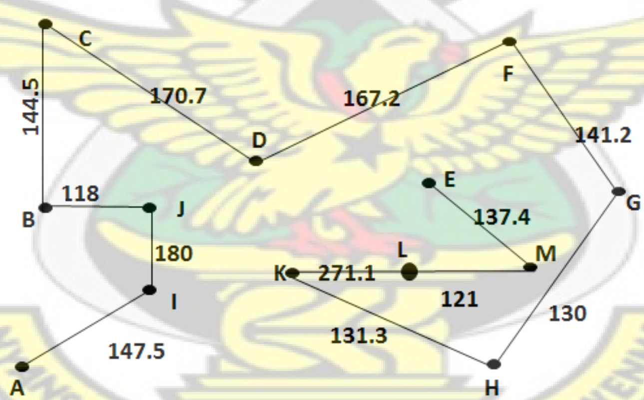

Figure 4.2: The Solution network (Zone 1): Edges in meters not drawn to scale

The minimum distance computed is used in calculating the cost as follows: the pipes used by the research are major pipes that connect the sub stations. The pipes are of

(15) centimeters in diameter. The price at the market for six (6) meters long fifteen (15) centimeters diameter pipe at the time of research (March 2013) was GH 203.00. By the implementation of the Prim's algorithm we arrive at an optimized total distance. The new total distance is now 1859.9 meters and if we consider the price of each six(6) meters long pipe which is GH 203.00 we can then calculate the total cost as:

$$\begin{aligned}
\text{Total Cost} &= \frac{\text{Total Distance of Network}}{\text{Six (6) Meters Pipe Long}} \times GH203.00 \\
&= \frac{1859.9}{6} \times GH203.00 = GH62926.6
\end{aligned}$$

The Average distance by considering all 13 nodes is computed as a ratio of the Total Distance to the number of nodes. This is given by $\frac{\text{Total Distance}}{13} = \frac{1859.9}{13} = 143.06m$

4.4 The P-Median Model

This section is a presentation of the algorithm, problem formulation, computational procedure and results of the Abuakwa pipe layout based on the p-median. The distance matrix and the population at the various nodes as shown in Figure 4.1 is used in this section.

4.4.1 Problem Formulation and Computational Procedure

The p-median is employed when the objective is minimizing the weighted distance in a network problem. This methodology can also be employed, as is the case in this research, to find the minimum weighted distance to locate the substation. So the appropriate objective then is to find the minimum of the calculated weighted distance for all the potential sites.

The p-median involves placing p facilities so that the total user cost or distance to travel to one of those facilities is minimized. The model can be represented mathematically as follows.

- h_i = demand at node i
- d_{ij} = distance between demand node i and site j
- p = number of facilities

Decision Variables are defined as follows:

$$X_j = \begin{cases} 1 & \text{if a facility is located at site } j \\ 0 & \text{otherwise} \end{cases}$$

$$Y_{ij} = \begin{cases} 1 & \text{if demand at } i \text{ is served by a facility at site } j \\ 0 & \text{otherwise} \end{cases}$$

The outline of the p-median optimization process is therefore given as:

$$\begin{aligned} \text{Min} \quad & \sum_i \sum_j h_i d_{ij} Y_{ij} \\ \text{Subject to} \quad \sum_j Y_{ij} &= 1 \quad \text{for all } i \\ \sum_j X_j &= p \\ Y_{ij} &\leq X_j \quad \text{for all } i, j \\ X_j, Y_{ij} &\in \{0, 1\} \quad \text{for all } i, j \end{aligned}$$

Given the sites (locations) the distance and individual should cover to have access to the facility at a selected site as well as the number of people commuting from the various sites to the selected site $w_i = \sum_{i,j}^{n} h_i d_{ij} y_i$ can be used to find the weighted distance and select the minimum of them as the optimal site. The distance between the various towns is put in a table form to form a matrix. The populations h of the various locations are noted. The algorithm can be stated as follows:

- Step 1: Choose a starting vertex
- Step 2: Join this vertex to the next vertex, not already in the solution
- Step 3: Multiply the distance of the next vertex by the population of it
- Step 4: Repeat it with all the other vertices until all the vertices have been included
- Step 5: Sum all the products up
- Step 6: Find the minimum value

4.4.2 Results for P-Median

Using the spanning tree in Figure 4.3 with the demand specified at the various nodes, the weighted distances can be calculated for each node considered as a point at which the maintenance unit should be cited.

The weighted distances which will be used in deciding where to cite the maintenance unit is calculated using $w_i = \sum_{i,j}^{n} h_i d_{ij} y_i$ where h_i is the demand/population specified at each node, d_{ij} represents the distances between nodes and n is the number nodes within the network. The calculation of citing the maintenance unit at node A is given by

$$\begin{aligned} w_A &= \sum_{ij}^{13} h_i d_{ij} y_i \\ &= (0 * 83) + (147.5 * 70) + (180 * 52) + \cdots + (121 * 59) + (137.4 * 75) \\ &= 135,241.0m \end{aligned}$$

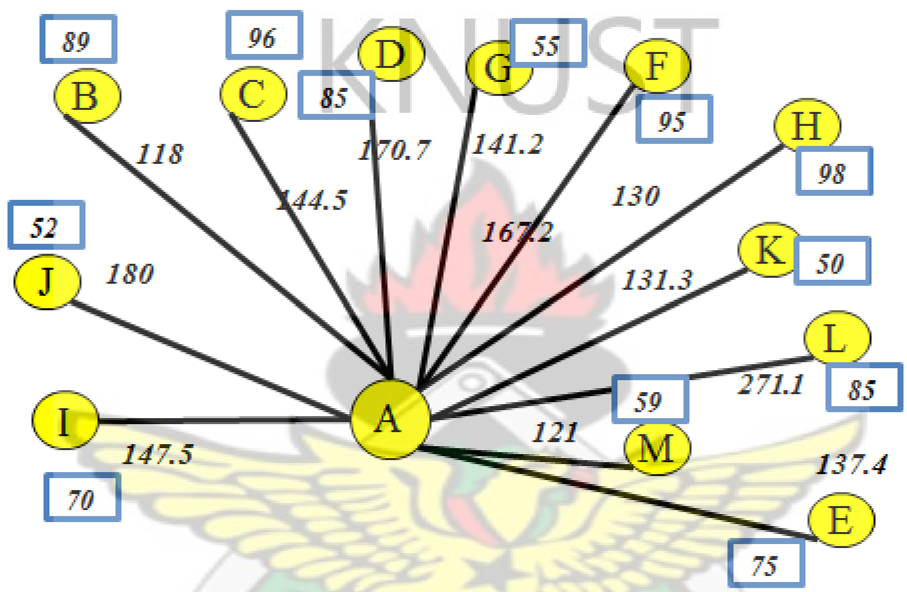

Figure 4.3: Network Diagram for Citing Maintenance Unit at A

This means that although the distance from A to other points is 1859.9 as computed using the spanning tree, the weighted distance is 135,241.0.

Similarly, the calculation of citing the maintenance unit at node I is given by

$$\begin{aligned} w_I &= \sum_{ij}^{13} h_i d_{ij} y_i \\ &= (147.5 * 83) + (180 * 52) + \cdots + (121 * 59) + (137.4 * 75) \\ &= 143928.5m \end{aligned}$$

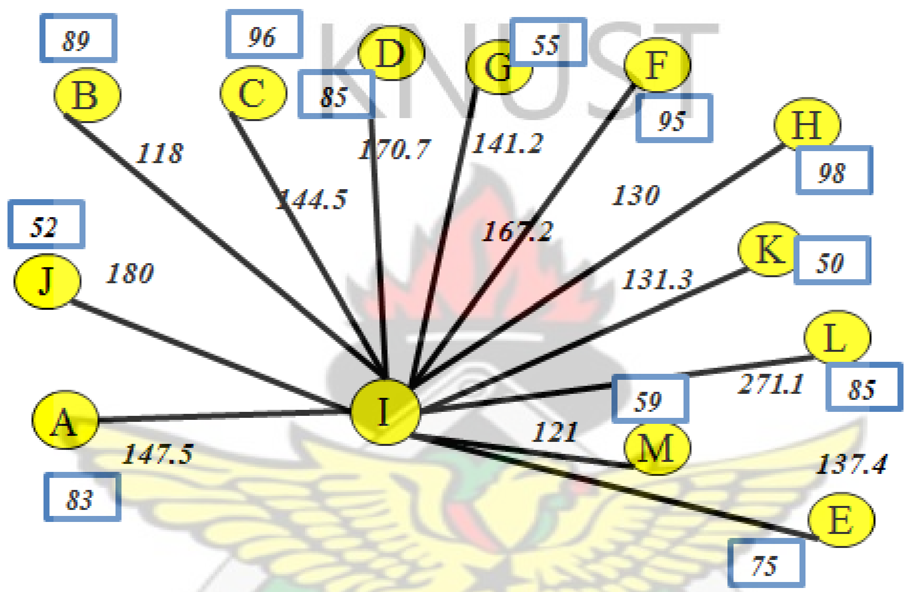

Figure 4.4: Network Diagram for Citing Maintenance Unit at I

This means that although the weighted distance from I to other points is 143,928.5m. The weighted distances for all the other nodes were calculated and have been summarized in Table 4.3

Table 4.3: Weighted Distance For Individual Nodes in Abuakwa Township Network

Number	Node	Weighted Distance (m)	Number	Node	Weighted Distance (m)
1	A	135,241.0	8	H	189,277.0
2	B	193,321.0	9	I	143,928.5
3	C	145,665.5	10	J	250,335.0
4	D	162,125.0	11	K	150,086.0
5	E	301,007.0	12	L	200,411.0
6	F	193,322.5	13	M	253,125.0
7	G	126,256.0			

From the extract given in Table 4.3, the minimized distance based on the population from the point G is 126,256.0. In contrast to the Prim's which is solely based on distance, this solution is based on the demand and distance and so will be appropriate to cite a maintenance facility at cite/node G that will service the demand from all the nodes. The average distance is calculated from the weighted distances as follows:

$$\begin{aligned} \text{Average Distance} &= \frac{\text{Weighted Distance}}{\text{Total Population}} \\ &= \frac{126,256.0}{992} = 127.27m \end{aligned}$$

4.5 Discussion of Results

The research has by way of data provided given solutions using the Prim's algorithm which considers the minimum spanning tree within a network based on distance between nodes. the minimum spanning tree for Abuakwa Township from the data given starting from node A is

$$A \Rightarrow I \Rightarrow J \Rightarrow B \Rightarrow C \Rightarrow D \Rightarrow F \Rightarrow G \Rightarrow H \Rightarrow K \Rightarrow L \Rightarrow M \Rightarrow E$$

The total length of this spanning tree is 1859.9m and the cost is $GH62926.6$. The average distance within this spanning tree is 143.06m.

By implementing the P-median the optimal weighted distance which is based on both distance and demand/population at the various nodes using the data given for the Abuakwah Township is 126,256.0m from the point G. It will therefore be optimal to locate a maintenance unit at G which will serve the demand from all other nodes within the network. The overall average distance computed is 127.27m.

Chapter 5

Conclusion And Recommendations

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5.1 Conclusion

The objectives stated by the research were met as follows:

1. By using the Prim's algorithm the total spanning tree value was 1859.9m representing the optimal distance within the spanning tree

2. The cost of the total spanning tree was computed and is given as $GH62926.6$

3. The citing of the maintenance unit facility using the P-median was found to be at node G and the weighted distance 126,256.0. This will be the optimal node from which other nodes in the network will be served.

4. The average distance by using the Prim's algorithm is 143.06m for the spanning tree and the average weighted distance using the P-median is 127.27m for locating the maintenance unit.

5.2 Recommendations

In view of the optimal pipe network system which has a weighted distance of 1859.9m with the cost of $GH62926.6$ and the citing of the maintenance unit at the substation G, the research recommends the following;

1. The GCWL should adapt the alternate solutions given in this research; the spanning tree network based on distance only and the solution by the P-median if there is a need to cite a maintenance unit within the Abuakwah Township since these are optimized.

2. The GCWL should sponsor and facilitate workshops that seeks to find more optimized routes so as to reduce cost depending on the objectives for each project

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