THE OPTIMAL TRANSPORTATION PROBLEM OF ACCRA BREWERY LIMITED

BY

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DECLARATION

I hereby declare that this thesis is the true account of my own research work except for references to other people's work which have been fully acknowledged.

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DEDICATION

I would like to dedicate this thesis to the glory of God and my wife, Mrs. Pearl Mills-Lamptey and my son Gerard Nii Odartey Mills-Lamptey for their prayers, love and support through this course.

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ABSTRACT

In Ghana, there are a number of beverage manufacturing companies with the aim of distributing their products to the entire, country.

The products need to be distributed within the ten regions of the country at a minimum total cost.

Accra Brewery Limited was chosen as a case study with their distributing cost of beverage particularly beer for the past ten years from their archival data cannot be overemphasized. Huge transportation cost in the past shows that there is a need to investigate the cause.This study is aim at formulating a transportation model to minimize the cost of transporting the beverage (beer) within the entire country, (i.e Ghana).

In pursuit of the objectives, secondary data on the transportation cost and number of beer transported for the past six months in the year 2007 i.e, May- October was collected and analyzed for this project.

The company's major distributors transport beer to different retail warehouses within the country. The various production plants and retail warehouse were analyzed.

The objective is to develop a mathematical model to optimize the total transportation cost for the Distribution Department of Accra Brewery Limited. Based on a few assumptions made, a mathematical model was formulated to help in solving the problem. Excel Solver was used to solve the problem.

The result shows that during a normal season the cost of transporting beer for the entire country was 2,272,455.9 Ghana cedis, and 2,996,936.7 Ghana cedis in the lean season. The cost of transporting beer in festive season was 3,439,329.8 Ghana cedis.

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LIST OF ABBREVIATION

ABBREV	WAREHOUSES
C.R.W	Central Region Warehouse
A.R.W	Ashanti Region Warehouse
G.R.W	Greater Accra Region Warehouse
W.R.W	Western Region Warehouse
V.R.W	Volta Region Warehouse
E.R.W	Eastern Region Warehouse
B.R.W	Brong Ahafo Region Warehouse
N.R.W	Northern Region Warehouse
UE.R.W	Upper East Region Warehouse
UW.R.W	Upper West Region Warehouse

ABBREV	SOURCES
C.R.S	Central Region Source
A.R.S	Ashanti Region Source
G.R.S	Greater Accra Region Source
V.R.S	Volta Region Source
B.R.S	Brong Ahafo Region Source
N.R.S	Northern Region Source

CHAPTER ONE

INTRODUCTION

1.0 TRANSPORTATION SYSTEMS AND PROBLEMS FACING THE INDUSTRY IN GHANA

Transport in Ghana is mostly road, rail, air and water. Ghana's transportation and communications networks are centered in the southern regions, especially the areas in which gold, cocoa, and timber are produced and big industries are located. The northern and central areas are connected through a major road system; some areas, however, remain relatively isolated. The huge percentage of the population uses the road network transport system.

The large numbers of users of the roads has constituted most road network problems. Myriads of problems facing the transportation in Ghana cannot be over emphasized. Transportation and communications networks in Ghana have been blamed for impeding the distribution of economic inputs and food as well as the transport of crucial exports. Ministry of Roads and Highways major priority was to repair and maintain the road in Ghana. During the 1980's the government of Ghana used about US$1.5 billion for road and rail rehabilitation.

Most new vehicles are intended for private use rather than for hauling goods and people, a reflection of income disparities. Transportation is difficult in eastern regions, near the coast, and in the vast, underdeveloped northern regions, where vehicles are scarce. At any one time, moreover, a large percentage of intercity buses and Accra city buses are out of service. Third world countries such as Ghana face a lot of problem in the public transportation system due to the fact that the country is encountering major economic growth and improvement in transporting people for economic activities.

. In Ghana, the modern public transportation system started in the late 1800 when the first rail line was constructed for the commercial exploitation of gold, and the first road created from Accra to the Eastern region. Omnibus Services Authority (OSA), State Transport company (STC), City Express Services (CES), and lately Metro Mass Transit (MMT) Ltd are public transport companies owned by the government to facility the mobility of citizens to various destinations. The public transportation companies are also established for the various reasons including one of the social services on government to provide for the people, environmental factors, energy considerations and the promotion of efficient public transportation to increase productivity and economic growth.

The public transport system can be divided in into the formal and informal sectors. The informal public transport system can be categorized into two groups namely taxis and "trotro" for transporting people and small goods whiles the big trucks for transporting huge goods such as cartons of beer etc.

Transportation by rail is also paramount and patronize in Ghana. The percentage of population using the rail transport system is about 18%. In Accra the rail line stretches from Nsawam to Accra Central. Most market women and children use the rail as a means of transportation to various destinations. In the Western region, the rail transportation system is used to transport ~~raw~~ materials from Tarkwa to Bawjiase for exploration of Gold. Mostly this type of transportation is predominantly used in the rural areas.

A lot of challenges are encountered in the rail transportation system and the Ministry of Transportation is to address this situation.

African Development Bank (ADB) conducted the feasibility studies and estimated $5 million for the project. Possible projects at the time included extending a line from Ejisu to Nkoranza and Techiman; a line from Tamale to Bolgatanga and Paga to Burkina Faso; a line from Wenchi, Bole to Wa and Hamile and also to Burkina Faso, and a line to Yendi where there are iron ore deposits Ejisu, Kumasi with extension lines via Mampong, Nkoranza, Tamale, Bolgatanga and Paga was proposed for rehabilitation in March 2007.(Ministry of Transportation Journal).

Water transportation is also predominant in Ghana. It is mostly used in the rural areas in Ghana. The main water bodies known are the Lake volta, the Ankobra river, the Densu river and the Weija river. The challenges on the water transportation systems are many. Most of the lakes have tree stumps and this normally causes accidents on the lakes. In 2002 a ferry hit a tree stump in a stormy weather and sunk in the Volta Lake. Human lives were lost. Most ferries have no life jackets on board in case of any eventualities.Dredging of the Volta Lake in 2004 caused the government of Ghana billions of Ghana Cedis .(Ministry of Transportation Journal).

Accra Brewery Limited (ABL) is no exception and has a lot of challenges in transporting the beverages produced by the company. Accra Brewery Limited was incorporated on April 1, 1975 though it was originally referred to as Overseas Breweries Limited dating back in 1931. The ~~company was~~ provisionally listed on the Ghana Stock Exchange on November 12, 1990 and formally on December 20 2001. ABL has one billion authorized shares of which 249,446,664 are issued making its stated capital GH¢7,332,000.00. It has

Overseas Breweries Limited as it major shareholders with about 62.9% of the shares outstanding.

The company has distributions depot all over the Ghana. Its main business is brewing of beer, malt and aerated soft drink. Accra Brewery Limited is the producer of Club Beer, Castle Milk Stout, Stone Lager, Club Shandy, Vitamalt, Muscatella etc. Due to market forces and inflation over the past ten years the financial statement summary for the year 2002, 2004 and 2005 are as follows. In 2002 the total net sales was 89,780,000 Ghana Cedis, the net profit was 5,064,000 Ghana Cedis, the key ratios net profit margin was 5.64%, currency ratio 1.4400 and the debt/capital ratio was 0.1200. In 2004 the total net sales was 183,909,000 Ghana Cedis, the net profit was 13,645,000 Ghana, the key ratios for net profit margin was 7.42%, currency ratio 1.6600 and debt/capital ratio was 0.0700.The year 2005 indicated that the total net sales was 198,246,000 Ghana Cedis and the net profit was 13,742,000 Ghana Cedis. The key ratio for the net profit margin was 6.93%, the currency ratio was 1.1500 and the debt/capital ratio was 0.1500. The project centered on the distribution of Club Beer all over Ghana. (Cal Brokers Limited)

1.1 MODELLING

The transportation model seeks the determination of a transporting plan for a single commodity from a number of sources (e.g. factories) to a number of destinations (warehouse). The data of ~~the model~~ should include:

(i) The level of supply at each source and the amount of demand at each destination.

(ii) The unit transportation cost of the commodity from each source to each destination.

Since there is only one commodity, a destination can receive its demand from one or more sources. The objective of the model is to determine the amount to be transported from each source to each destination such that the total transportation cost is minimized. The basic assumption of the model is that the transportation cost on a given route is directly proportional to the units transported.

A Model is a symbolic or physical representation of reality. Modelling is an art, because judgments are made when selecting the important features of reality for the problem at hand. Modelling also is a science; however, because data are collected to measure the relationship between decision variables and clarifying objectives imposes a discipline that is useful in itself as the definition implies, a model does not represent precisely what is supposed to represent, that is, it cannot be mistaken for or replace reality. The major component of the modelling system is that there is an opportunity to evaluate the alternatives. These alternatives will be measured in some quantitative form. Some scaling must be done to select the better alternative. These alternatives considered arises from the effects of input variables where inputs enter the model and results in dependent variables. These independent variables are also called input variables. Their origin is from outside the model system and result from external causes. They are differentiated from another quantity called the parameter and it becomes the quantity that is assignable and generally known as a constant. The dependent variable originates from within the modelling ~~system and~~ results from the interaction of input variables and parameters. The modelling system is characterized also by constraints or limitations on the values that the model operates and help to deride the limits within which the model will operate. We solve the model using a technique called Linear Programming (LP). Http://www-neos.msc.anl.gov.neos/

Linear Programming is a mathematical technique designed to aid institutions in allocating scarce resources (energy, labour, capital, etc). For a typical LP model an attempt is made to achieve some objectives (maximizing profit, minimizing cost) in view of limited resources (available labour or capital, service level of available machine time). A linear objective function that is to be optimized (either maximizes or minimizes) subject to linear inequality or equality constraints is the bedrock of an LP. The proportionate relationship of two or more variables is called 'linear' that is a given change in one variable will always cause a resulting proportionate change in another variable. The LP technique uses a systematic method called iterations to find an optimal solution to an LP problem. Each step of the procedure is an attempt to improve on the solution until the best answer is obtained.

However, the "ordinary" LP cannot solve our transportation problem since the LP has some rigid limitations that can restrict its application in transportation Problems. The Transportation technique is a particular form of LP that solves transportation problems.

Linear Programming (LP) is a general computer-based modelling tool for making resource allocation decisions that transcend all aspects of service operations management. The success of an Operation Research (OR) / Management Science (MS) technique is ultimately measured by ~~the spread~~ of its use as a decision making tool. Ever since its introduction in the late 1940s, linear programming (LP) has proven to be one of the most effective operations research tools. Its success stems from its flexibility in describing multitudes of real life situations in the following areas: military, industry, agriculture, transportation, economics, health systems, and even behavioral and social sciences.

The usefulness of LP extends beyond its immediate applications. Indeed, LP should be regarded as an important foundation for the development of other Operation Research techniques including integer, stochastic, network flow and quadratic programming. Linear programming is a deterministic tool meaning that all the model parameters are assumed to be known with certainty. In real life, however, it is rare that one encounters a problem in which true certainty prevails. The LP technique compensates for this "deficiency" by providing systematic post-optimal and parametric analysis that allows the decision maker to test the sensitivity of the "static" optimum solution to discrete or continuous changes in the parameters of the model. In essence, these additional techniques add a dynamic dimension to the property of optimum LP solution. Linear programming should not be viewed as computer programming.

The use of models such as LP springs from a belief that applying the scientific method can enhance the decision-making process. Science study nature and conduct controlled experiments to understand better the phenomena of interest. Decision models are the laboratories of managers who are interested in testing the outcomes of decisions before their actual implementation. In this way, potential disasters may be avoided, and the decision- making process may be improved through a better understanding of the environment. Wayne et al., (2005)

1.1.1 MODEL REVIEW

The art of formulating LP models and interpreting computer output is discussed here. The mathematical details involved in solving an LP model are not discussed. The availability of computer programs to solve such models is extensive. The chapter concludes with a discussion of a particular form of LP called Transportation Technique. We begin by discussing the concept of an optimum solution to a constrained model.

1.1.2 CONSTRAINED OPTIMIZATION PROBLEMS

Each day, we are faced with making decisions in which our potential set of alternatives is restricted by money, time, physical limitations, or some other elements. For example, suppose we wish to buy a car, we can qualify for a Ghana cedis (GH¢10,000) loan, and we want a vehicle with an Environmental Protection Agency rating of at least thirty kilometres (30) per gallon. The set of possible cars is constrained by time, budget, and mileage performance. These constraints are restrictions that reduce the allowable set of solutions to our problem. Thus, constraints actually, help us to make decisions by limiting our search for a solution to cars that meet the stipulated requirements [1].

If economy were our goal, we could measure this by calculating the cost per kilometre for each car that meets our constraints. The car with the lowest cost-per-kilometre value would be considered the optimum solution to our constrained decision problem.

Constrained optimization problems are common to service operations. For example, a potential location for a service facility is constrained by the available sites. Schedule telephone operators are constrained by variations in demand for the service and by the personnel policies regarding split shifts.

Linear programming models are a special class of constrained optimization models. In LP, all relationships are expressed as linear functions, and all LP models have the following algebraic form:

Maximize (minimize) $c_1x_1 + c_2x_2 + \dots + c_nx_n$

Subject to $$a_{11}x_1 + a_{12}x_2 + \dots\dots + a_{1n}x_n$$

$$\sum a_{ij} \begin{Bmatrix} \geq \\ \leq \\ = \end{Bmatrix} b_i$$

And non-negativity constraints

$$x_1, x_2, \cdots x_n \geq 0$$

Each constraint is limited to only one of the conditions $\leq$ or = or $\geq$.

1.2 STATEMENT OF THE PROBLEM

The cost of transportation of club beer in Accra Brewery Limited for some years now is very high. This has caused the marginal profit of the company to dwindle. The expenditure of the balance sheet of the company is high on transportation. Transportation Technique is used to determine the optimum transport cost of transporting a product from a number of sources to various distribution centers.

1.3 PURPOSE OF THE STUDY

The cost of transporting products from various sources to various destinations to maximize profit is a major headache for many Distribution/Transport managers of various companies. One school of thought is of the view that the total profit is not maximized since the supply does not meet the demand. Another school of thought is that the unit cost of transporting a product from a source to a destination in not minimized.

This thesis seeks to delve into the transportation problem of Accra Brewery Limited (ABL).

1.4 OBJECTIVE

This thesis aims at developing a mathematical model to ensure an optimum total transportation cost for ABL.

1.4 JUSTIFICATION

- To help improve efficiency in the transport system.
- To help establish a better distribution of products depending on the demands at the various centres so as to maximize profit.
- To achieve an optimum transportation cost for ABL.

1.6 METHODOLOGY

The data was collected from Accra Brewery Limited in Accra. A data format and a questionnaire were designed for the collection of data from the Distribution Manager.
A model was built from the data collected and a computer program was used in solving the transportation Problem. Attached is a copy of the data format and the questionnaire, see Appendix A

CHAPTER TWO

LITERATURE REVIEW

2.0 INTRODUCTION

Real world situations may be encountered with many transportation problems. The demand and supply quantities may vary due to factors that can not be controlled.

Harizan., (2007) studied and presented an algorithm solving the fuzzy transportation using membership functions of fuzzy numbers. The solution yielded optimal compromise solutions.

Schmidt., (1986) investigated the multi-period, multicommodity transportation of fertilizer in the Republic of Indonesia. He developed a model which handles implicitly all commodities but at least two commodities are needed for each sub problem. The main goal of the Republic of Indonesia is to be advance in agriculture and self-sufficiency in rice production for the next ten years and domestic consumption of fertilizer is to increase. The distribution of the fertilizer was paramount and therefore experts in shipping, handling storage, corporate manager, transport economist, mechanical engineer, structural engineer, rail expert, port expert and operation research expert team was setup. The operational research expert was to design a transportation model to optimize the fertilizer flow in Indonesia considering suggestions from the other experts on the project. The fertilizer transportation in Indonesia model shows that it is a multi-period, multi commodity transportation problem. The author explained how the model was and steps needed to reduce and simplify the model without loosing the optimality. After the first step reduction they showed up the multi-period aspect and how to reduce the transportation links and ultimately explained how the multi-commodity aspect was reduced to the two commodities approach. Transportation problem was and continues to be one of the main forces in mathematical disciplines of graph theory, optimization and operation research.

The Propositions and acuendos invenes book has a story the "River Crossing Problem" which was translated as a mathematical problem by Alcuin of York in the 18^{th} Century. The Alcuin's river crossing problem has no impact on mathematics. However, the characteristics of the literature displayed to large extent a large scale real transportation problem. Borndorfer et al.,(1995) used a point of combinations, optimization and operation research to explain the Alcuin's transportation a wolf, a goat and a bunch of cabbage. The story is about a wolf, a goat and a bunch of cabbages. A man had to take a wolf, a goat and a bunch of cabbages across the river. The only boat he could find could only take two of them at a time but he had been ordered to transfer all of these to the other side in good condition. How could this be done? Alcuin solved the problem as I quote "I would take the goat and leave the wolf and the cabbage. Then I would return and take the wolf across. Having put the wolf on the other side I would take the goat back over. Having left that behind, I would take the cabbage across. I would then row across again, and having picked up the goat takes it over once more. By this procedure there would be some healthy rowing but no catastrophe". Our aim is to design a mathematical model to describe a general technique based on geometry (polyhedral theory) and integer progra~~mming to~~ solve Alcuin's transportation problem. The problem was solved using the polyhedral theory. A characterization of the transport property is given. New properties for strong nonatomic probabilities are established. Belili et al., (1999) study the relationship between nondifferentiability of a real function f and the fact that the probability measure λ_f f := (f(x)−1, where f(x) := (x, f(x)) and is the λ_f Lebesgue measure, has the transport property. In their paper, in order to simplify matters, they consider E = R^d with the inner product $\langle .,. \rangle$ and the quadratic cost. They give a characterization of the probabilities which have the transport property, preceded by the necessary definitions; they introduce the notion of strongly nonatomic probabilities and they investigated their properties. Finally, they examine the relationship between the nondifferentiability of a real function f and the fact that the probability measure λ_f f := $\lambda \circ (f^*)^{-1}$, where f(x) := (x, f(x)) and is the λ_f Lebesgue measure, has the transport property. For example,they showed that the probability λ_B , where B is the Brownian motion, isstrongly nonatomic i.e.; on the other hand, if we take f(x) = $\int_0 B(t)dt$, the probability has λ_f the transport property i.e. but it is not strongly nonatomic. Often in supply chain optimization multi-items have to be considered together, due to the dependency of the cost structure or the operational constraints on the total quantities transported and/or replenished. In other cases, the exact composition of the individual items in a single vehicle/batch is important. Other complicating characteristics of multi-item transportation/replenishment problems include capacity limitations of the vehicles/batches, time dependency of demand and cost parameters and the existence of fixed costs per vehicle/batch. Anily et al.,(2004) considered a multi-item lot-sizing problem in which transportation production takes place in mixed vehicles batches of constant capacity. Apart from the usual unit storage and production costs, there is a fixed cost per vehicle batch representing the use of limited capacity resource. In the transportation context, a warehouse ships a number of items by using trucks of a given capacity so as to possibly stock items and satisfy demand forecasts at the retailer. The objective is to find replenishment decisions for all items in order to satisfy the demand for the items over a finite planning horizon, and minimize the sum of fixed costs associated with the vehicles used, variable production costs and storage costs.

Transportation has a fundamental role in the economic development of all countries. It is not just a means to service commuting people, but also to collect products and materials from producers and distribute them to consumers. Transportation has become a significant factor affecting the production costs of commodities. The production of sugar cane in Thailand is no exception. The cost of transporting sugarcane from the farm gate to the mills is quite high, owing to the multiple transport facilities and time-consuming activities involved in the delivery process. The total transportation expenditure was estimated at 5,708 million baht for the crop year 1999-2000. The average cost per transaction incurred by farmers (excluding other labour costs) was in the range of 180-220 baht per ton in 1999. A large portion of this cost comprises truck rental and driver wages.

These two elements together represent a high proportion of the overall production cost. The transportation issue has been over looked in many industrial sectors and in the agricultural sector, in particular. Chetthamrongchai et al., (2001) findings was on a study on the transportation and other relevant costs of sugar cane production. The findings and the subsequent recommendations could be considered for the enhancement of welfare of the sugar cane farmers and the increased efficiency of the industry in general and may also be applied to other agro-based industries facing similar problems. Schrijver., (2002) reviewed two papers that are of historical interest for combinatorial optimization: an article of Tolsto from 1930, in which the transportation problem is studied, and a negative cycle criterion is developed and applied to solve a (for that time) large-scale (10X68) transportation problem to optimality; and an, until recently secret, Rand report of Harris and Ross from 1955, that Ford and Fulkerson mention as motivation to study the maximum flow problem. Dymowa et al., (1993) further development of approach proposed by Chanas and Kuchta for the transportation problem solution in the case of fuzzy coefficients. The direct fuzzy extension of usual simplex method is used to realize the elaborated numerical fuzzy optimization algorithm with fuzzy constraints. It must be emphasized that the fuzzy numerical method proposed is based on the practical embodiment of the pioneer Stefan Chanas idea to consider the fuzzy values in the probabilistic sense. The problem is formulated in the more general form of the distributor's benefit maximization. Srdjevic et al., (1997) continued general discussion introduced in [1] on two methods for solving transportation problems: standard and network linear programming. They put on modeling and computing issues related to both methods. Simple illustrative example is used to demonstrate how transportation problem may be attacked by two related solvers, Simplex and Out-of-kilter. Specific notes are given on knowledge analyst has to be armed with in order to be able to apply the network approach and use network methods and solvers. Goosens et al., (2005) considered the so-called Transportation Problem with Exclusionary Side Constraints (TPESC), which is a generalization of the ordinary transportation problem. They determined the complexity status for each of two special cases of this problem, by proving NP-completeness, and by exhibiting a pseudo-polynomial time algorithm. For the general problem, they showed that it cannot be approximated with a constant performance ratio in polynomial time (unless P=NP). These results settle the complexity status of the TPESC. The balanced relation between supply and demand in transportation problem makes it difficult to use traditional sensitivity analysis methods. Therefore, in the process of changing supply or demand resources, at least one more resource needs to be changed to make the balanced relation possible. Doustdargholia et al., (2009) utilizing the concept of complete differential of changes for sensitivity analysis of right-hand-side parameter in transportation problem, a method is set forth. This method examines simultaneous and related changes of supply and demand without making any change in the basis. The mentioned method utilizes Arasham and Kahn's simplex algorithm to obtain basic inverse matrix. The validity of mentioned method's results are compared to and inspected by the well-known transportation problems in the literature review. Chen et al., (2007) considered fuzzy transportation problems with satisfaction degrees of routes since except of transportation costs about routes, its safety or transportation time etc should be taken into account. Further flexibility of demand and supply quantity should also be taken into account. Moreover the fuzzy goal about total transportation cost is considered in place of minimizing the total transportation cost directly. They considered two criteria. One is to maximize the minimal satisfaction degree with respect to the flexibility and fuzzy goal .The other is to maximize the minimal satisfaction degree among routes used in transportation. But usually there exists no solution that optimizes both objectives at a time.

They seek some non-dominated solutions after defining non-domination. In the framework of transport theory, Brancolini et al., (2000) were interested in the following optimization problem: given the distributions μ+ of working people and μ− of their working places in an urban area, build a transportation network (such as a railway or an underground system) which minimizes a functional depending on the geometry of the network through a particular cost function. The functional is defined as the Wasserstein distance of $\mu+$ from $\mu-$ with respect to a metric which depends on the transportation network. In the late 1940s, Dantzig and his contemporaries were faced with monumental problems that arose in the areas of military logistics, management, shipping, and economics. In 1947 Dantzig invented the simplex method a way to reduce the number of calculations involved in optimization problems. This was the advent of linear programming. Coover., (1985) studied a cost minimization problem associated with Royal Dutch Shell's distribution system in the Chicago area. The solution is made easier by using a program called SIMPMETH, which was developed by Andree Chea. This software was designed as a TI-83 calculator application. Badra., (2007) presented a sensitivity analysis to multi-objective transportation problem. The proposed approach yields to maximum tolerance percentages in multi-objective transportation problem. The weighted sum approach is used to solve the multi-objective transportation problem. The proposed approach allows changing in both the weights and the objective function coefficients and the right hand side constraints simultaneously and independently from their specified values while remaining the same basis optimal. Formulation of both perturbed solution and the corresponding perturbed objective values are presented. An illustrative example is presented to clarify the idea of the proposed approach. Research on expert-novice differences falls into two complementary classes. The first assumes that novice skills are a subset of those of the expert, represented by the same vocabulary of concepts. The second approach emphasizes novices' misconceptions and the different meanings they tend to attribute to concepts. Our evidence, based on observations of problem solving behavior of experts and novices in the area of mathematical programming, reveals both types of differences: while novices are to some extent underdeveloped experts, they also attribute different meanings to concepts. The research suggests that experts' concepts can be characterized as being more differentiated than those of novices, where the differentiation enables experts to categorize problem descriptions accurately into standard archetypes and facilitates attribution of correct meanings to problem features. Orlikowski et al., (1986) results are based on twenty five protocols obtained from experts and novices attempting to structure problem descriptions into mathematical programming models. They have developed a model of knowledge in the LP domain that accommodates a continuum of expertise ranging from that of the expert who has a highly specialized vocabulary of LP concepts to that of a novice whose vocabulary might be limited to high school algebra. They discussed the normative implications of this model for pedagogical strategies employed by instructors, textbooks and intelligent tutoring systems. Wieselquist., (2009) develop a quasidiffusion (QD) method for solving radiation transport problems on unstructured quadrilateral meshes in 2D Cartesian geometry, for example hangingnode meshes from adaptive mesh refinement (AMR) applications or skewed quadrilateral meshes from radiation hydrodynamics with Lagrangian meshing.

The main result ~~of the work~~ is a new low-order quasidiffusion (LOQD) discretization on arbitrary quadrilaterals and a strategy for the efficient iterative solution which uses Krylov methods and incomplete LU factorization (ILU) preconditioning. The LOQD equations are a non-symmetric set of first-order PDEs that in second-order form resembles convection-diffusion with a diffusion tensor, with the difference that the LOQD equations contain extra cross-derivative terms. Our finite volume (FV) discretization of the LOQD equations is compared with three LOQD discretizations from literature. We then present a conservative, short characteristics discretization based on subcell balances (SCSB) that uses polynomial exponential moments to achieve robust behavior in various limits (e.g. small cells and voids) and is second-order accurate in space. A linear representation of the isotropic component of the scattering source based on face-average and cell-average scalar fluxes is also proposed and shown to be effective in some problems. In numerical tests, our QD method with linear scattering source representation shows some advantages compared to other transport methods. We conclude with avenues for future research and note that this QD method may easily be extended to arbitrary meshes in 3D Cartesian geometry.The Transportation Problem is a classic Operations Research problem where the objective is to determine the schedule for transporting goods from source to destination in a way that minimizes the shipping cost while satisfying supply and demand constraints. Although it can be solved as a Linear Programming problem, other methods exist. Linear Programming makes use of the Simplex Method, an algorithm invented to solve a linear program by progressing from one extreme point of the feasible polyhedron to an adjacent one. The algorithm contains tactics like pricing and pivoting. For a Transportation Problem, a simplified version of the regular Simplex Method ~~can be used~~, known as the Transportation Simplex Method. Kumar discussed the functionality of both of these algorithms, and compares their run-time and optimized values with a heuristic method called the Genetic Algorithm. Genetic Algorithms, pioneered by John Holland, are algorithms that use mechanisms similar to those of natural evolution to encourage the survival of the best intermediate solutions. The objective of the study was to find out how these algorithms behave in terms of accuracy and speed when a large-scale problem is being solved. Patel., (2006) dealt with the formulation of Classic Transportation Problem and different methods to solve the classic transportation problem. The auction algorithm is a parallel relaxation method for solving the classical assignment problem. It resembles a competitive bidding process whereby unassigned persons bid simultaneously for objects, thereby raising their prices. Once all bids are in, objects are awarded to the highest bidder. Bertsekas et al., (1989) generalized the auction algorithm to solve linear transportation problems. The idea is to convert the transportation problem into an assignment problem, and then to modify the auction algorithm to exploit the special structure of this problem. Computational results show that this modified version of the auction algorithm is very efficient for certain types of transportation problems.

The transportation problem is a specialized problem in Operations Research. Wermes., (2007) discussed the development of a program designed to solve the transportation problem. The program is used as an educational tool to learn how to solve the transportation problem and can be used as a tool to solve the problem without the educational parts. The logical development of the program includes a discussion of the steps to solve the transportation problem. The first step in the process is the collection of the information about the transportation problem. After collecting the data check to see if the problem is balanced, look for a basic feasible solution, then check to see if it is optimal. A detailed discussion of each step is included, but the focus is on the development of the program. The code and program flow is the central theme of his work. Screen shots from the program are included as well as code and flow to understand the processes of the program and how it relates to the solution of the transportation problem. Recently Althoefer et al., (2004) have shown that a transportation problem is immune against the"more for less"-paradox if and only if the cost matrix $C = (c_{i,j})$ (of dimension $m \times n$) does not contain a bad quadruple. In this note a counter-example with infinite-dimensional supply and demand vectors is given. In the second part they showed that the quadruple-characterization of paradox-immune cost matrices remains valid in the infinite-dimensional case in a slightly weaker form. As a side result a smooth inequality is obtained for the situation where a transportation plan is split in two or more arbitrary subplans. Jana et al., (2004) studied the solution procedure of Multi-objective Fuzzy Linear Programming Problem (MOFLPP) with mixed constraints and its application in solid transportation problem is going to be presented. There are two parts in this paper. In the first part, a Multi-objective Linear Programming Problem MOLPP) with fuzzy coefficients occurring in constraints and objective functions and fuzzy constraint goals, has been considered. Here fuzzy constraint goals and coefficients of objective and constraint functions are characterized by Triangular Fuzzy Numbers (TFNs). Using Zadeh et al., (1970) multicriteria fuzzy decision-making process, the very problem has been converted to a crisp non-linear programming problem. Then it has been solved using fuzzy decisive set method. In other part, a linear multi-objective solid transportation problem with mixed constr~~aint as well~~ as additional restriction in fuzzy environment is considered. In this transportation problem, cost coefficients of objective functions and additional restriction function, the supply, demand and conveyance capacity have been expressed as TFNs. This MOFLPP is solved by fuzzy decisive set method as before. Numerical examples have been provided for two parts to illustrate the solution procedure. The decision-making process is one of the everyday manager's problems. The decision making process is arising with the problem's occurrence, which the manager firstly name, define and formulate and then the manager is observing the factors, that are influencing and limiting the solutions options. By the problems analyzing and solving, there can be used the linear programming methods. Most of stoneware is still being transported by trucks from a place of mining (stone pits) to a place of consumption (concrete factory), where it is used as a basic material component for a production of concrete. The transportation costs, which are not low, have to be included in a final price of the concrete. Therefore, it must be carefully considered what routes to select for individual trucks in order to assure a maximum volume of transported stoneware, and at the same time the shortest possible total kilometre distance gone by all trucks Lampa., (2005). Mikami et al., (2005) solved optimal transportation problem using stochastic optimal control theory. Indeed, for a super linear cost at most quadratic at infinity, we prove Kantorovich duality theorem by a zero noise limit (or vanishing viscosity) argument.. They also obtain a characterization of the support of an optimal measure in Monge-Kantorovich minimization problem (MKP) as a graph. Our key tool is a duality result for a stochastic control problem which naturally extends (MKP).Dahiya et al., (2006) discussed a paradox in ~~fixed charge~~ capacitated transportation problem where the objective function is the sum of two linear fractional functions consisting of variables costs and fixed charges respectively. A paradox arises when the transportation problem admits of an objective function value which is lower than the optimal objective function value, by transporting larger quantities of goods over the same route. A sufficient condition for the existence of a paradox is established. Paradoxical range of flow is obtained for any given flow in which the corresponding objective function value is less than the optimum value of the given transportation problem. Numerical illustration is included in support of theory. Transportation problems appear in many practically highly relevant areas of our daily life. In general, they include the assignment of produced goods to customers and decisions on how and at which times the goods are picked up and delivered. Improvements in solutions often have a direct and substantial impact on costs and on other important factors like customer satisfaction. Because of the many facets and decisions to be made, such transportation problems are often complex combinations of assignment, scheduling, and routing problems. Raidl., (2007) focus on special classes of transportation problems, namely those dealing with multiple visits. Multiple visits problems occur when customers from a fixed set have to be visited repeatedly. The basic form of this type of problems is given by the so-called periodic vehicle routing problem. Further reductions in costs may be achieved by exploiting the possibility of switching from the more frequent vendee managed inventory setup to a vendor managed inventory system. The resulting problem type, known as inventory routing problem, will also be considered in the project. A third important class is given by the periodic full truckload problem, where customers require repeatedly at least one full truckload of the transported unit. For all these problem types, both heuristic and exact algorithms exist. Exact algorithms have the aim to find an optimal solution and to prove its optimality; the run-time, however, often increases dramatically with a problem instance's size, and only small or moderately sized instances can be solved to provable optimality in practice. For larger instances, one usually has to resort to heuristic algorithms that trade optimality for run-time; i.e., these algorithms are designed to obtain good but not necessarily optimal solutions in acceptable time. Two particularly successful categories of methods that traditionally can be distinguished by these aspects are mathematical programming techniques on one side and metaheuristics on the other. To some degree, they can be seen as complementary; therefore it is highly promising to combine concepts from both streams. Nevertheless, most of today's hybrid optimizers of this type, for which the term "matheuristics" has been coined recently, follow rather simple combination schemes, despite a potential for farther-reaching synergies. More work is necessary in order to obtain a better general understanding as well as guidelines indicating under which circumstances which hybridization strategies are most promising. The general aim of the project is to develop and to investigate different hybrids of metaheuristics with integer linear programming methods for solving the indicated classes of transportation problems in a better way than by current state-of-the-art approaches. In more detail, we have the following major goals to which the project's work plan is oriented: (i) boosting the performance of heuristic and of exact algorithms by exploiting hybridization possibilities, (ii) developing hybrid algorithms for bi-objective and stochastic problem formulations. The last two features are important insofar as in applications of periodic routing, situations requiring decisions under uncertainty and/or encompassing more than one single objective are frequently encountered. This project will be the first in which a variety of "matheuristic" solution techniques for real-world transportation problems with periodic visits will be developed and studied. Existing research results clearly indicate that such approaches are highly promising for the considered problem domain. A particularly innovative aspect of our project is the consideration of models that combine bi-objective and stochastic aspects; for such models, very few solution techniques are known at present.

Finally, they expected the findings of this research also to be useful in the future development of solution approaches for other classes of combinatorial optimization problems. For students learning the simplex method of linear programming it is a well beloved occasion to solve the so-called transportation problem by the method of distribution. This method is simple to calculate and easy to follow. The simple way of solution suggests that its correctness may be proven by basic means. Schmidt., (2009) has two main aims. One of them is to present the problem and to solve it by basic means. The other one is the analysis of the so-called array-bases defined for this reason. In case of a transportation problem m stores and n destinations are given, and the goods have to be taken from the stores to the destinations such that the cost of transporting has to be minimal.

The unit costs of the transportation are given by an array. In the solution some routes (elements of the array) are chosen and the number of units to transport there is given. It will be proven that the routes for the optimal transportation compose a basis, and the solution is also achieved by those through the searches. (The basis of an m n × array consists of m+n−1 elements such that they do not span a loop.) In the proof some characteristics ~~of~~ the bases ~~are needed~~, for example that the number of them is finite. To prove this it is enough to give an easily calculated upper bound, the exact value is given in the appendix. As an extra result of this calculation some interesting formulas of combinatorics are also proven. Estimating simultaneous hierarchical logic models is conditional to the availability of suitable algorithms. Powerful mathematical programs are necessary to maximize the associated non-linear, non-convex, log-likelihood function. Even if classical methods (e.g.Newton-Raphson) can be adapted for relatively simple cases, the need of an efficient and robust algorithm is justified to enable practioners to consider a wider class of models. Bierlaire., (1995) analyzed and to adapt to this context methodologies available in the optimization literature. An algorithm is proposed based on two major concepts from non-linear programming: a trust region method, that ensures robustness and global convergence, and a conjugate gradients iteration, that can be used to solve the quadratic sub problems arising in the estimation process described in this paper.

Numerical experiments are finally presented that indicate the power of the proposed algorithm and associate software. Lukač et al., (2005) formulated two new models of the production-transportation problem which can be described as follows. Let us supposed that there are several plants at different locations producing certain number of products and large number of customers of their products. Each plant can operate in several modes characterized by different quantities of products and variable production costs. The customers' demand for each product during the considered time period is known. They considered the problem of finding the production program for each plant as well as the transportation of products to customers for which the sum of the production and transportation costs is minimized given the condition that each customer can satisfy its demand for a given type of product from one plant only. They also formulated the problem as a bilevel mixed-integer programming problem. They solved the models for the available data from a petroleum industry and compare the results. Transportation problem is a particular form of linear programming problem, which is solved by a different technique called transportation Technique. The model of the transportation problem deals with how to transport quantities of single product from a number of factories or production centres, called sources to a number of warehouses or retail shops, called destinations. The usual objective of the model is to minimize the total transportation cost or maximize total profit for supplying the quantities of the product(s) from the sources to the destinations so as to meet the requirements at the destinations. The transportation technique differs from the Simplex method but has some basic similarities, which makes it computationally more efficient. These similarities are as follows:

(i) It is an iterative process;

(ii) It starts with an initial basic feasible solution (BFS),

(iii) At each step of the iteration, a test is made to check whether the total transportation cost can be reduced or total profit can be increased, and

(iv) The optimal solution is reached when no further cost reduction or profit is possible. Hiller et al., (2001)

CHAPTER THREE

METHODOLOGY

3.0 INTRODUCTION

This chapter deals with preliminary analysis of data from Accra Brewery Limited.

Data collection and a statistical method are used to gather information from a population under study. Population is the total elementary elements under the case study. Sample is subset of elements selected from the total population. Data needed to be analyzed are purposeful for the Linear Programming therefore a personal interview was conducted with Accra Brewery Limited distribution Manager.

3.1 DATA COLLECTION

A face-to-face interview was conducted with the Accra Brewery Limited distribution Manager at ABL premises. Data collected was in raw form therefore an appropriate *STATISTIC* was chosen to analyze the data. Data received was from May to October 2007 on the distribution and transportation cost from six sources to ten destinations. These are all on regional bases and in the appendix B.

Data warehouses and sources are on regional bases. A chart on regional warehouses and sources abbreviations in attached list of abbreviations.

3.2 STATISTICS

Statistics is a body of methods and theory that is applied to numerical evidence when making decisions in the face of uncertainty. A statistic $\hat{\theta}$ is said to be unbiased estimator of the population parameter θ. Inferences about the population parameters will be based on the sample statistic and to make decisions based on these inferences. Based on the data collected point estimator and interval estimation was chosen.

3.2.1 POINT ESTIMATOR

Mean $(\bar{X}) = \frac{1}{n}\sum_{i=1}^{n} X_i$ x are elements in the sample

Standard Deviation $\sigma_X = \frac{1}{n-1}\sum_{i=1}^{n}(X_i - \bar{X})^2$

3.2.2 INTERVAL ESTIMATION

Confidence interval at 95% was chosen. The sample size is small i.e n<30 therefore a t-statistic was used.

$$\bar{X} - t_{\alpha/2,n-1}\frac{s}{\sqrt{n}} < \mu < \bar{X} + t_{\alpha/2,n-1}\frac{s}{\sqrt{n}}$$

Where s= sample standard deviation

n= sample size

The value from t-distribution table cutting $\alpha/2$ of n-1 degrees of freedom.

At 95% level of significance , $t_{0.025,5}$=2.571

The averages of the raw data is shown below

SOURCE	C.R.W	A.R.W	G.R.W	W.R.W	V.R.W	E.R.W	B.R.W	N.R.W	UE.R.W	UW.R.W	TOTAL SUPPLY
C.R.S	1.5	10	15	8	25	25	30	40	55	60	52562
A.R.S	10	2	20	18	30	35	15	45	75	80	54833
G.R.S	15	20	2.2	15	25	25	50	70	90	95	168292
V.R.S	25	30	25	35	1.8	30	50	65	75	100	26678
B.R.S	30	15	50	35	50	45	1.9	55	65	75	18295
N.R.S	40	45	70	70	65	65	55	1.4	35	50	25526
TOTAL DEMAND	33995	50744	141341	26678	25420	15339	16513	15995	11196	8965	

(Averages data)

SOURCE	C.R.W	A.R.W	G.R.W	W.R.W	V.R.W	E.R.W	B.R.W	N.R.W	UE.R.W	UW.R.W	TOTAL SUPPLY
C.R.S	1.5	10	15	8	25	25	30	40	55	60	37762
A.R.S	10	2	20	18	30	35	15	45	75	80	44895
G.R.S	15	20	2.2	15	25	25	50	70	90	95	151281
V.R.S	25	30	25	35	1.8	30	50	65	75	100	24080
B.R.S	30	15	50	35	50	45	1.9	55	65	75	16675
N.R.S	40	45	70	70	65	65	55	1.4	35	50	16244
TOTAL DEMAND	25695	38314	107741	18584	20965	9104	14861	46934	4990	3749	

(Lean Season data)

SOURCE	C.R.W	A.R.W	G.R.W	W.R.W	V.R.W	E.R.W	B.R.W	N.R.W	UE.R.W	UW.R.W	TOTAL SUPPLY
C.R.S	1.5	10	15	8	25	25	30	40	55	60	75695
A.R.S	10	2	20	18	30	35	15	45	75	80	73105
G.R.S	15	20	2.2	15	25	25	50	70	90	95	193638
V.R.S	25	30	25	35	1.8	30	50	65	75	100	37609
B.R.S	30	15	50	35	50	45	1.9	55	65	75	28278
N.R.S	40	45	70	70	65	65	55	1.4	35	50	43146
TOTAL DEMAND	42245	63173	174941	34767	29875	21573	18165	35149	17403	14180	

(During Festivities Data)

3.4 TRANSPORTATION MODEL

The data of a transportation model include:

(i) Level of supply at each source (a_i) and the amount of demand at each destination (d_j),

(ii) The transportation cost per unit of the product from each source to each destination (C_{ij}).

The network for transportation model with *m* sources of supply and *n* destinations of demand is illustrated in the figure 1.1

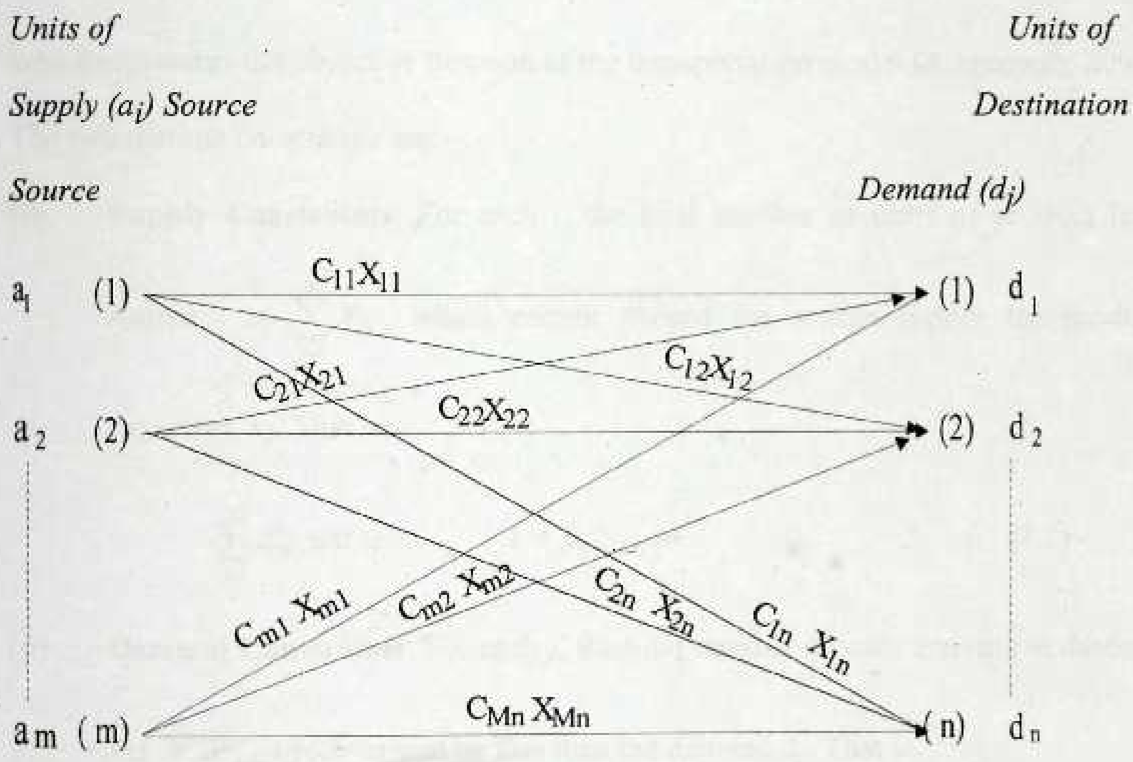


Figure 1.1

3.5 CONSTRUCTION OF TRANSPORTATION MODEL

Suppose certain unit of the product is available at the *m* sources and is to be transported to the *n* destinations. Let a_i denote the quantity available for supply at source *i*, b_j, denote the demand at destination j and C_{ij}, the cost for transporting a unit of the product from source i to destination j. If the number of units to be transported from source i to destination j is represented by the double-subcripted variables, X_{ij}, then $X_{ij} \geq 0$, for each i and j and from a particular source i to the n destinations the cost is given by

$$\sum_{j=1}^{n} C_{ij} X_{ij} \ , \quad i = 1, 2, \ldots m.$$

The total cost for transporting the product from all the m sources is given by

$$Z = \sum_{i=1}^{m} \sum_{j=1}^{n} C_{ij} X_{ij} \qquad (2.1)$$

which represents the objective function of the transportation model.(Amponsah; 2007)

The two distinct constraints are:

(i) **Supply Constraints**: For each i, the total number of units of product leaving source i is $\sum_{j=1}^{n} X_{ij}$, which cannot exceed the source supply (or production capacity), a_i. That is,

$$\sum_{j=1}^{n} X_{ij} \leq a_i \ , \qquad i = 1, 2, \ldots m \qquad (2.2)$$

(ii) **Demand Constraints**: For each j, the total number of units arriving at destination j is $\sum_{i=1}^{m} X_{ij}$, which cannot be less than the demand d_j. That is,

$$\sum_{i=1}^{m} X_{ij} \geq d_j, \quad j = 1, 2, \ldots n \qquad (2.3)$$

Now the transportation model becomes:

Minimize the total transportation cost,

$$Z = \sum_{i=1}^{m} \sum_{j=1}^{n} C_{ij} X_{ij}$$

subject to the supply and demand constraints,

$$\sum_{j=1}^{n} X_{ij} \le a_i \qquad i = 1, 2, \ldots m$$

$$\sum_{i=1}^{m} X_{ij} \ge d_j \qquad j = 1, 2, \ldots n \qquad (2.4)$$

where $X_{ij} \ge 0$, *for all i and j.*

proof

$$\sum_{j=1}^{n} x_{ij} \le a_i \qquad \text{for } i = 1, 2 \ldots \text{m} \qquad (1)$$

$$\sum_{i=1}^{m} x_{ij} \ge d_j \qquad \text{for j} = 1,\ 2 \ldots \text{n} \qquad (2)$$

Summing (1) over i, we obtain

$$\sum_{i=1}^{m} \sum_{j=1}^{n} x_{ij} \le \sum_{i=1}^{m} a_i \qquad (3)$$

Also summing (2) over j we obtain

$$\sum_{j=1}^{n} \sum_{i=1}^{m} x_{ij} \ge \sum_{j=1}^{n} d_j \qquad (4)$$

Which follows from (3) and (4) that $\sum_{i=1}^{m} a_i \ge \sum_{j=1}^{n} d_j$

When the total supply equal the total demand, the resulting formulation is called a BALANCED TRANSPORTATION MODEL.

It differs from the model above only in the fact that all constraints are equations;

Thus:

$$\sum_{j=1}^{n} x_{ij} = a_i \qquad i = 1,\ 2 \ldots m$$

$$\sum_{i=1}^{m} x_{ij} = d_j \qquad j = 1,\ 2 \ldots n$$

PROOF

If $\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} d_j$ the problem is said to balanced

Thus for a balanced problem with

$$\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} d_j$$

$$\sum_{j=1}^{n} x_j = a_i \qquad i = 1,\ 2 \ldots m$$

$$\sum_{i=1}^{m} x_{ij} = d_j \qquad j = 1, 2, \ldots n$$

Suppose that for some k we have

$$\sum_{j=1}^{n} x_{kj} < a_k$$

Then $\sum_{j=1}^{n} d_j \leq \sum_{i=1}^{m} \sum_{j=1}^{m} x_{ij} < \sum_{i=1}^{m} a_i$

This is contradiction and therefore $a_i = \sum_{j=1}^{n} x_{ij}$

Also suppose that for some i $d_i < \sum_{i=1}^{m} x_{ij}$ then $\sum_{j=1}^{n} d_i < \sum_{j=1}^{n} \sum_{i=1}^{m} x_{ij}$

$\leq \sum_{i=1}^{m} a_i$ this is contradiction and therefore $d_j = \sum_{i=1}^{m} x_{ij}$

3.6 GENERAL DESCRIPTION OF A TRANSPORTATION PROBLEM

- A set of m supply points from which a good is shipped. Supply point i can supply at most S_i
- A set of n demand points to which the good is shipped. Demand point j must receive at least d_i units of the shipped good
- Each unit produced at supply point I and shipped to demand point j incurs a variable cost C_{ij}

X_{ij} = number of units shipped from supply point I to demand point j

$$\text{Min} \sum_{i=1}^{i=m} \sum_{j=1}^{j=n} C_{ij} X_{ij}$$

$$\text{Subject to} \sum_{j=1}^{j=n} X_{ij} \leq S_i \left(i = 1,2,.......m\right)$$

$$\sum_{i=1}^{i=m} X_{ij} \geq d_j \left(j = 1,2,.......n\right)$$

$$X_{ij} \geq 0 \left(i = 1,2,m; j = 1,2,....n\right)$$

3.7 BALANCED TRANSPORTATION MODEL AND TRANSPORTATION TABLEAU

(a) Balanced Transportation Model: When the total supply equals the total demand, that is $\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} d_j$, the resulting transportation model is said to be balanced, otherwise, the model is unbalanced. An unbalanced transportation model can always be converted to an equivalent balanced model.

The balanced transportation model of equation (3.4) is

Minimize $$Z = \sum_{i=1}^{m} \sum_{j=1}^{n} C_{ij} X_{ij}$$

Subject to: $$\sum_{j=1}^{n} X_{ij} = a_i\,, \qquad i = 1, 2, \ldots m$$

$$\sum_{j=1}^{n} X_{ij} = d_j \qquad j = 1, 2, \ldots n$$

where $X_{ij} \geq 0$, for all i and j.

If the transportation model is unbalanced it is balanced by either creating a fictitious (or dummy) source or destination to absorb the difference. When the total supply exceed the total demand $\left(\sum_{i=1}^{m} a_i > \sum_{j=1}^{n} d_j \right)$, the problem is solved by creating a dummy destination whose demand is exactly the excess. Since shipments to the dummy demand points are not real, they are assigned a cost of zero.In the case where the total demand exceeds the supply $\left(\sum_{j=1}^{n} d_j > \sum_{i=1}^{m} a_i\right)$, a dummy source is created to transport the difference. The unit cost of transporting from a dummy source or to a dummy destination is taken to be zero.

REDUNDANCY IN THE CONSTRAINTS

The constraints are

$$\sum_{j=1}^{n} x_{ij} = a_i \qquad i = 1, 2 \ldots\ldots m \qquad (2a)$$

$$\sum_{i=1}^{m} x_{ij} = d_j \qquad j = 1, 2 \ldots\ldots n \qquad (3a)$$

THEOREM

There is exactly one redundant equality constraint in (2a) and (3a). When any one of the constraints in (2a) and (3a) is dropped, the remaining is a linearly independent system of constraint

If a transportation problem has a total supply that is strictly less than total demand the problem has no feasible solution. There is no doubt that in such a case one or more of the demand will be left unmet. Generally, in such situations a penalty cost is often associated with unmet demand and as one can guess this time the total penalty cost is desired to be minimum.

(b) **Transportation Tableau:** The transportation model is represented in a more compact tabular format called Transportation Tableau. It is a form of matrix where the rows represent the sources and the columns, the destinations. The unit costs C_{ij} are shown in the upper left hand corner of the matrix cells *(i, j)*.

The transportation tableau for *m* sources and *n* destinations is shown in the figure 1.2 below.

Destination j:

Source i:	1	2		n	Supply (a_i)
1	C_{11}	C_{12}		C_{1n}	a_1
2	C_{21}	C_{22}		C_{2n}	a_2
m	C_{m1}	C_{m2}		C_{mn}	a_m
Demand (d_j):	d_1	d_2		d_n	

3.8 SOLUTION OF TRANSPORTATION MODEL

The solution of transportation model is characterized by three stages:

(i) Obtaining an initial basic feasible solution.

(ii) Checking an optimality criteria that indicates whether or not a termination condition has been met.

(iii) Developing a procedure to improve the current solution if a termination condition has not been met.

The requirement for application of a transportation technique is that the model must be balanced. This results in one dependent equation and *(m+n-1)* independent equations. A starting BFS of the balanced model therefore involves *(m+n-1)* variables.

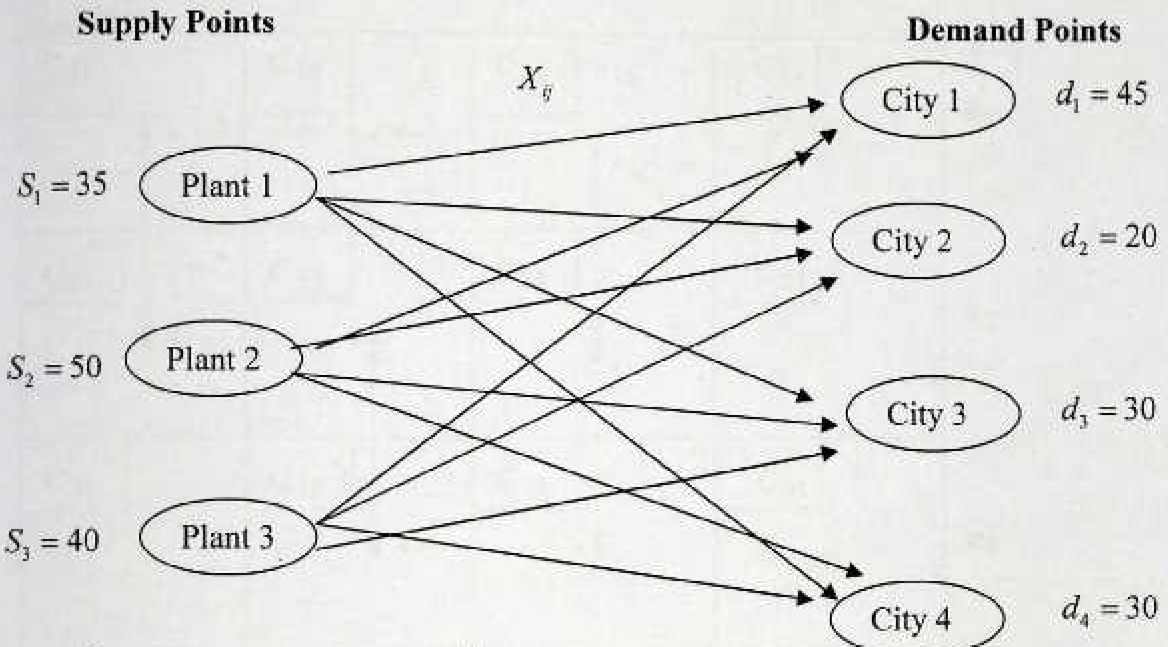
3.9 GRAPHICAL REPRESENTATION OF A TRANSPORTATION PROBLEM FOR BALANCED AND UNBALANCED SITUATIONS

Milamgeo enterprise has 3 elect power plants that supply the needs of 4 cities. Each power plant can supply the following numbers of kilowatt-hours(kwh) of electricity: plant 1-35 million; plant 2 – 50 million; plant 3- 40 million (see Table 1). The peak power demands in these cities which occur at the same time (2pm), are as follows (in kwh): city 1- 45 million; city 2- 20 million; city 3- 30 million; city 4 – 30 million. The cost of sending 1 million kwh of electricity from plant to city depend on the distance the electricity must travel.(Cooray; 2007)

TABLE 1

FROM	CITY 1	CITY 2	CITY 3	CITY 4	TOTAL SUPPLY
PLANT 1	8	6	10	9	**35**
PLANT 2	9	12	13	7	**50**
PLANT 3	14	9	16	5	**40**
TOTAL DEMAND	**45**	**20**	**30**	**30**	

GRAPHICAL REPRESENTATION



$\sum_{i=1}^{i=3} S_i = 125$ This is equal to $\sum_{j=1}^{j=4} d_j = 125$

Decision Variables: X_{ij} # of (million)kwh produced at plant i and sent to city j.

Constraints: Supply (Capacity) constraints and Demand constraints

3.10 DEFINITIONS

(a) Basic Feasible Solution (BFS): A solution is said to be *Basic Feasible Solution (BFS)* if:

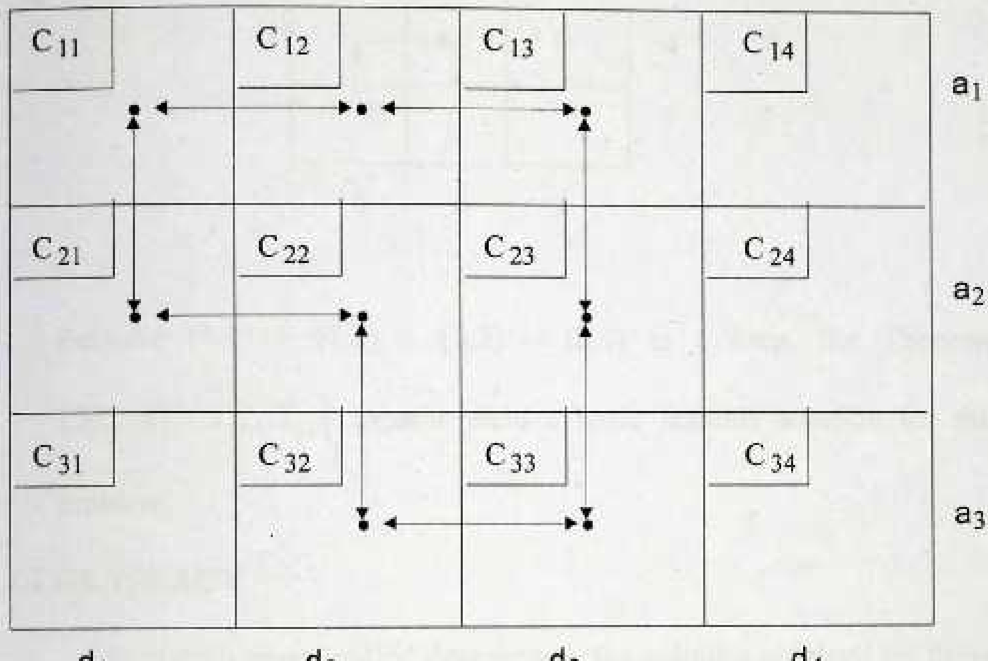
(i) it involves *(m+n-1)* non-negative allocations, and

(ii) there is no circuit among the allocated cells.

(b) Circuit: A *circuit* is made up of sequence of cells of transportation tableau such that:

(i) it starts and ends with the same cell, and

(ii) each cell in the sequence is connected to the next member in the sequence by a horizontal or vertical line as shown in the tableau below.



The required circuit is:

$(1, 1) \rightarrow (1,2) \rightarrow (1,3) \rightarrow (2,3) \rightarrow (3,3) \rightarrow (3,2) \rightarrow (2,2) \rightarrow (2,1) \rightarrow (1,1).$

(c) **LOOP** An ordered sequence of at least 4 different cells is called a loop if:

i. any 2 consecutive cells lie in either the same row or same column

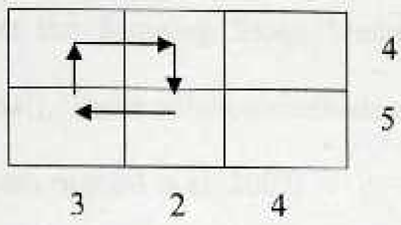
ii. no 3 consecutive cells lie in the same row or column

iii. the last cell in the sequence has a row or column in common with first cell in the sequence.

In a balanced transportation problem with m supply points and n demand points, the cells corresponding to a set of $(m + n - 1)$ variables contain no loop iff the $(m + n - 1)$ variables yield a basic solution.

This follows from the fact that a set of $(m + n - 1)$ cells contains no loop iff the $(m + n - 1)$ columns corresponding to these cells are linearly independent.

Loop (1,1) – (1,2) – (2,2) – (2,1)



Because (1,1) – (1,2) – (2,2) – (2,1) is a loop, the Theorem tells us that $\{X_{11}, X_{12}, X_{22}, X_{21}\}$ **cannot** yield a basic feasible solution for this transportation problem.

DEGENERACY

In certain cases, called degeneracy, the solution obtained by these methods is not a Basic Feasible Solution because it has fewer than (m+n-1) circled numbers (allocations) in the solution. This happens because at some point during the allocation when a supply is used up in a cell there is no cell with unfulfilled demand in the column of that cell. Then it is necessary to make one or more zero allocations to some cells to bring up the number of allocations to (m+n-1). This means that for cost calculation purposes, one or more cells with zero allocations are treated as occupied. The zero allocation is chosen in such a way that:

(i) the total number of cells with allocations is (m+n-1) and

(ii) there is not circuit among the cells of the solution.

3.11 SOLUTION TECHNIQUES OF TRANSPORTATION MODEL

Transportation techniques are variants of the Simplex method and so they require initial BFS to start with. The initial BFS may be obtained by the North-West

Corner Rule, Least Cost Method or the Vogel's Approximation Method. The other methods, which have been devised to improve the starting solution to optimality are the Stepping Stone Method (SSM) and Modified Distribution method (MODI). These solution methods as classified into two and are discussed as follows.(Leavengood et al; 2007)

3.12 INITIAL BASIC FEASIBLE SOLUTION (BFS)

(i) The North-West Corner Rule (NWCR): The method starts by making the maximum allocation allowable by the supply and demand constraints to cell (1, 1), the north-west corner or the tableau. The satisfied row or column is then crossed out row or column is zero (0). If a row and a column are satisfied simultaneously, either one may be crossed out. This condition guarantees locating zero basic variables; if any, automatically. After adjusting the amount of supply and demand for all uncrossed out rows and columns, the maximum feasible amount is allocated to the first uncrossed out cell in the new column or row. The process is completed which exactly one row or column remains uncrossed out. In certain cases, the solution obtained by the method is degenerate. This happens because whenever a supply is used up there is always an unfulfilled demand in the column.

Least Cost Method (LCM) The Least Cost Method identified the least unit cost in the transportation tableau ~~and~~ allocated as much as possible to the associated cell without violating any of the supply or demand constraints. The satisfied row or column in then deleted. The next least unit cost is identified and as much as possible is allocated its cell without violating any of the supply or demand constraints. At this point also, the satisfied row or column is deleted. This procedure is continued until all rows and columns have been deleted. This method performs better than the North-West Corner Rule.

Vogel's Approximation Method (VAM): It provides a BFS which is optimal or close to it and moreover, performs better than the Least Cost method and North-West Corner Rule. The basic idea of VAM is to avoid shipments that have high cost. This is achieved by computing column penalties by identifying the least unit cost and the next least unit cost in that column and taking their positive difference. In a similar way row penalties are computed by taking the positive difference between the least unit cost and the next least unit cost in a row. This method is a variant of the Least Cost method and is based on the idea that if for some reason, the allocation cannot be made to the least unit cost cell in a row or column then it is made to the next least unit cost cell in that row or column and the appropriate penalty is paid for not being able to make the best allocation. Column penalties are shown below columns and row penalties to the right of each row of the transportation tableau.

3.13 OPTIMAL SOLUTION METHODS

The method for obtaining the solution to transportation model is the Stepping-Stone method (SSM) and Modified Distribution method (MODI).

3.14 STEPPINGSTONE ~~METHOD~~

The Steppingstone Method, being a variant of the Simplex method, requires an initial basic feasible solution which it then improves to optimality. Such an initial basic feasible solution may be obtained by the use of the Northwest corner rule, the Least cost method or the Vogel's approximation method.

Let us consider the balanced transportation problem shown below:

	W_1	W_2	...	W_n	
S_1	C_{11}	C_{12}	...	C_{1n}	a_1
S_2	C_{21}	C_{22}	...	C_{2n}	a_2
	⋮	⋮	...		⋮
S_m	C_{m1}	C_{m2}	...	C_{mn}	a_m
		b_2		b_n	

Suppose that we have an initial basic feasible solution of this problem consisting of non-negative allocations in (m+n-1) cells. Let us call the cells which are not in the basic feasible solution unoccupied cells.

It can be shown that for each unoccupied cell, there is a unique circuit beginning and ending in that cell, consisting of that unoccupied cell and other cells all of which are occupied such that each row or column in the tableau either contains two or more of the cells of the circuit.

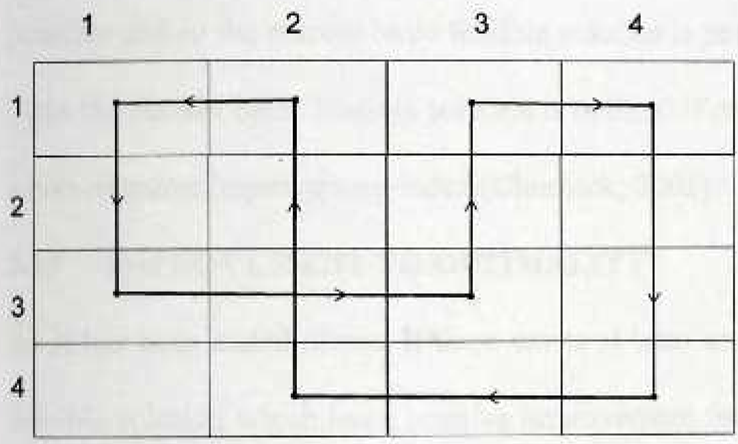
3.15 CIRCUIT

A circuit made up of cells of the tableau of a balanced transportation problem is a sequence of cells such that:

(a) it starts and ends with the same cell

(b) each cell in the sequence can be connected to the next member of the sequence by a horizontal and vertical line in the tableau.

An example is as shown below:



3.16 TEST FOR OPTIMALITY

To test the current basic feasible solution for optimality, we take each of the unoccupied cells in turn and place one unit allocation in it. This is indicated by just the sign (+). Following the unique circuit containing this cell as described above, we place alternatively the sign (-) and (+) until all the cells of the circuit are covered.

Knowing the unit cost of the each cell, we compute the total change in cost produced by the allocation of one unit in the unoccupied cell and the corresponding placements in the other unoccupied cells. This total cost explained above is known as the improvement index of that unoccupied cell under consideration.

If the improvement index of each unoccupied cell in the basic feasible solution is non-negative then the current basic feasible solution is optimal since every re-allocation increases the cost. If there is at least one unoccupied cell with a negative improvement index then a re-allocation to produce a new basic feasible solution with lower cost is possible and so the current basic feasible solution is not optimal.

Thus the current basic feasible solution is optimal if and only if each unoccupied cell has a non-negative improvement index.(Chinneck; 2001)

3.17 IMPROVEMENT TO OPTIMALITY

As it has been stated above, if there exists at least one unoccupied cell in a given basic feasible solution which has a positive improvement index then the basic feasible solution is not optimal.

(1) To improve this solution, we find the unoccupied cell with the most positive improvement index N say. Using the circuit that was used in the calculation of its improvement index, we find the smallest allocation in the cells of the circuit with the sign '-'. Call this smallest allocation m. We then subtract m from the allocation in all the cells of the circuit with the sign '-' and add m to all ~~the~~ allocations ~~in the cells~~ in the circuit with the sign '+'. This has the effect of satisfying the constraints on demand and supply in the transportation tableau. Since the cell which carried the allocation m now has a zero allocation, it is deleted from the solution and is replaced by the cell in the

circuit which was originally unoccupied and now has allocation m. The result of this reallocation is a new basic feasible solution. The cost of this new basic feasible solution is less than the cost of the previous basic feasible solution.

This new basic feasible solution is tested for optimality, and if each unoccupied cell has a non-negative improvement index then the current feasible solution is optimal, .The whole process is repeated until an optimal solution is obtained.

The Stepping Stone Method is summarized as follows:

(i) Select an empty cell

(ii) Identify a circuit beginning and ending in that cell

(iii) Insert alternating signs in the corner cells of the closed circuit starting with a positive (+) sign in the empty cell

(iv) Determine the sum of the cost of positive cells and negative cells.

(v) If the difference of the sum is positive, do not move into empty cell but if it is negative move maximum number of units to empty cell without violating constraints and alternatively subtract the same number of units from cells with positive sign.

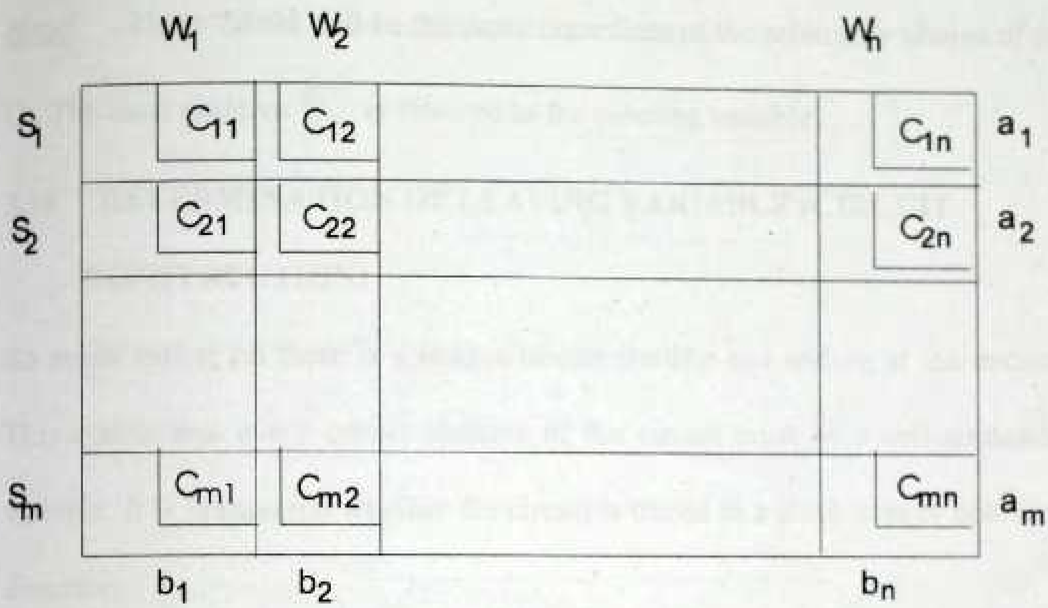
(vi) If it is zero, an alternate solution exists.

(vii) Repeat the procedure until all empty cells are evaluated.

3.18 MODI~~FI~~ED DISTRIB~~UTION~~ METHOD (MODI)

(~~METHOD~~ OF MULTIPLIES)

Consider the balanced transportation problem below:



Let an initial basic feasible solution be available, obtained by the use of the Northwest-corner rule, the Least Cost method or the Vogel's Approximation method.

Then (m+n-1) cells are occupied. In the method of multipliers we associate the multipliers U_i and V_j with row *i* and column *j* of the transportation tableau.

For each basic variable $x_{i,j}$ in the current solution, the multipliers U_i and V_j must satisfy the following equation:

$$U_i + V_j = C_{ij} \quad \text{for each basic variable } x_{ij}$$

These equations yield (m+n-1) equations (because there are only m+n-1 basic variables) in (m+n) unknowns. Values of the multipliers can be determined from these equations by assuming an arbitrarily value for any one of the multipliers (usually U is set equal to zero) and then solving the (m+n-1) equations in the remaining (m+n-1) unknown multipliers. One this is done, the evaluation index of each unoccupied cell is given by

$\overline{C_{pq}} = U_p + V_q - C_{pq}$ for each non basic variable X_{pq}.

Note: These values will be the same regardless of the arbitrarily choice of the value of U_1.The most positive $\overline{C_{pq}}$ is selected as the entering variable.

3.19 DETERMINATION OF LEAVING VARIABLE (CIRCUIT CONSTRUCTION)

As noted earlier on there is a unique circuit starting and ending at the unoccupied cell. This means that every corner element of the circuit must be a cell containing a basic variable. It is immaterial whether the circuit is traced in a clockwise or counter clockwise direction.

It can be noted that for a given basic solution, only one unique circuit can be constructed for each nonbasic variable.

The process is summarized by plus (+) and minus (-) signs in the appropriate corners. The change will keep the supply and demand restriction satisfied.

The leaving variable is selected from among the corner variables of the circuit that will decrease when the entering variable increases above zero level. The leaving variable is then selected as the one having the smallest value, since it will be the first to reach zero value and any further decrease will cause it to negative.

Then the new basic solution is now checked for optimality by computing the new multipliers (i.e. The whole process is repeated until optimality is obtained).

3.20 INTEGER PROPERTY

If all the a_i and b_j are positive integers, then every basic solution is an integer vector. ~~Hence, if~~ all a_i, b_j are positive integers, and if the transportation problem is feasible, it has an optimum solution (x_{ij}) that is an integer vector. (Murty; 2004).

The solution procedure of MODI is as given in the following steps:

(i) Set up the initial balanced transportation tableau

(ii) Obtain the initial BFS using the NWCR, LCM or VAM

(iii) Split the allocated unit costs, C_{ij} into row and column costs, U_i and V_j and compute them using the formula,

$$C_{ij} = U_i + V_j$$

by first, preferably, putting $U_i = 0$

(iv) Compute the shadow costs, $(U_p + V_q)$ associated with the unoccupied or unallocated cells, *(p, q)* and compare with the real costs, C_{pq}. That is,

$$e_{pq} = \underset{\textit{(real cost)}}{C_{pq}} - \underset{\textit{(shadow cost)}}{(U_p + V_q)}$$

If e_{pq} is negative, the cost of transportation can be reduced and if it is positive the cost of transportation can be increased.

(v) Check if all e_{pq} values are positive – no further cost reduction is possible and the solution is optimal, otherwise, the BFS is not optimal and we proceed to step (vi).

(vi) Improving of optimality of BFS: To improve the current BFS, find the most negative e_{pq} ~~value and~~ assign (+) sign to the associated cell.

Form a circuit from this (+) sign cell connecting the allocated cells, assigning alternatively (+) and (-) signs to the cells in the circuit so as to keep the row supplies and column demands.

(vii) Adjust the allocations by adding the smallest allocation with (-) sign to all allocations with (+) signs and subtracting from allocations with (-) signs. This gives a new BFS which is also tested for optimality by proceeding to step (iii).

The shadow cost is the transportation cost for not using a route. It is compared with the real transportation cost to check whether a change of allocation is desirable.

CHAPTER FOUR

FURTHER ANALYSIS

4.0 INTRODUCTION

In chapter three, a number of observation were made through exploratory analysis of the data. It was noted that while the means, the standard deviations and 95% confidence interval was achieved. However, these observations do not give sufficient basis for a meaningful comparison of transporting unit lager beer within the regions. To address this issue, the data is subjected to further analysis using the North West Corner rule, Least Cost Method and Vogel approximation method for the initial basic solution for the data. And for optimality even though Stepping-Stone method (SSM) and Modified Distribution method (MODI) can be used, I used the Linear optimization module integrated in Microsoft Excel.

4.1 AVERAGE DISTRIBUTIONS

With the averages distribution data above, using a computer program for north-west method, least cost method and vogel approximation method. Find the Initial Basic solution distribution.

Objective function =Min $\sum_{i=1}^{i=m}\sum_{j=1}^{j=n} C_{ij}X_{ij}$

Subject to $\sum_{j=1}^{j=n} X_{ij} \le S_i \,(i=1,2,.......m)$

$$\sum_{i=1}^{i=m} X_{ij} \ge d_j \,(j=1,2,.......n)$$

$$X_{ij} \ge 0 \,(i=1,2,m; j=1,2,....n)$$

SOURCE	C.R.W	A.R.W	G.R.W	W.R.W	V.R.W	E.R.W	B.R.W	N.R.W	UE.R.W	UW.R.W	TOTAL SUPPLY
C.R.S	1.5 **33995**	10	15	8 **18567**	25	25	30	40	55	60	52562
A.R.S	10	2 **50744**	20	18	30	35	15	45	75	80 **4089**	54833
G.R.S	15	20	2.2 **141341**	15 **8111**	25	25 **15339**	50	70	90	95 **3501**	168292
V.R.S	25	30	25	35	1.8 **25420**	30	50	65	75	100 **1258**	26678
B.R.S	30	15	50	35	50	45	1.9 **16513**	55	65 **1665**	75 **117**	18295
N.R.S	40	45	70	70	65	65	55	1.4 **15995**	35 **9531**	50	25526
TOTAL DEMAND	33995	50744	141341	26678	25420	15339	16513	15995	11196	8965	

Average data distribution (Initial Basic Feasible Solution)

4.2 INITIAL BASIC FEASIBLE SOLUTION RESULTS INTERPRETATION

The initial basic feasible solution result shows that, the total transportation cost was 2,450,930.4 Ghana cedis. The Central region capacity with capacity 52,562 supplied only the Central region warehouse and the Western region warehouse with 33,995 and 18,567 respectively. The Ashanti region source also with capacity 54,833 supplied the Ashanti region warehouse and Upper West region warehouse with 50,744 and 4,089. Greater Accra region source with capacity 168,292 supplied the Greater Accra region warehouse with 141,341, the Western region warehouse with 811, the Eastern region warehouse with 15,339 and the Upper West region warehouse with 3,501. The Volta region source with capacity of 26,678 also supplied both Volta region warehouse and Upper West region warehouse with 25,420 and 1,258 respectively. Brong Ahafo region sourcewith capacity 18,295 supplied the Brong Ahafo region warehouse with 16,513, the Upper East region warehouse with 1,665 and the Upper West region warehouse 117. The Northern region source with capacity 25,526 also supplied both the Northern region warehouse and Upper East region warehouse with 15,995 and 9,531 respectively. The above description shows how the allocation was done by the Least Cost Method and because optimum cost was not achieved there is a need to improve on the results. The results below show the optimum allocation achieved by the Microsoft Excel solver.

SOURCE	C.R.W	A.R.W	G.R.W	W.R.W	V.R.W	E.R.W	B.R.W	N.R.W	UE.R.W	UW.R.W	TOTAL SUPPLY
C.R.S	1.5 **29906**	10	15	8 **13808**	25	25	30	40	55	60 **8848**	52562
A.R.S	10 **4089**	2 **50744**	20	18	30	35	15	45	75	80	54833
G.R.S	15	20	2.2 **141341**	15 **12870**	25	25 **14081**	50	70	90	95	168292
V.R.S	25	30	25	35	1.8 **25420**	30 **1258**	50	65	75	100	26678
B.R.S	30	15	50	35	50	45	1.9 **16513**	55	65 **1665**	75 **117**	18295
N.R.S	40	45	70	70	65	65	55	1.4 **15995**	35 **9531**	50	25526
TOTAL DEMAND	33995	50744	141341	26678	25420	15339	16513	15995	11196	8965	

Optimum(Solution)

4.3 OPTIMUM SOLUTION RESULTS INTERPRETATION

The improved solution result shows that, the total transportation cost was 2,272,454.9 Ghana cedis. The Central region capacity with capacity 52,562 supplied only the Central region warehouse, the Western region warehouse and Upper West region warehouse with 29906, 13,808 and 8,848 respectively. The Ashanti region source also with capacity 54,833 supplied the Central region warehouse and Ashanti region warehouse with 4,089 and 50,744. Greater Accra region source with capacity 168,292 supplied the Greater Accra region warehouse with 141,341, the Western region warehouse with 12,870, and the Eastern region warehouse with 14,081. The Volta region source with capacity of 26,678 also supplied both Volta region warehouse and Eastern region warehouse with 25,420 and 1,258 respectively. Brong Ahafo region source with capacity 18,295 supplied the Brong Ahafo region warehouse with 16,513, the Upper East region warehouse with 1,665 and the Upper West region warehouse 117. The Northern region source with capacity 25,526 also supplied both the Northern region warehouse and Upper East region warehouse with 15,995 and 9,531 respectively.

The lean season data distribution is captured in the table below.

SOURCE	C.R.W	A.R.W	G.R.W	W.R.W	V.R.W	E.R.W	B.R.W	N.R.W	UE.R.W	UW.R.W	TOTAL SUPPLY
C.R.S	1.5 **25695**	10	15	8 **12067**	25	25	30	40	55	60	37762
A.R.S	10	2 **38314**	20	18	30	35	15	45	75	80	44895
G.R.S	15	20	2.2 **107741**	15 **6517**	25	25 **9104**	50	70 19180	90 4990	95 **3749**	151281
V.R.S	25	30	25	35	1.8 **20965**	30	50	65 **3115**	75	100	24080
B.R.S	30	15	50	35	50	45	1.9 **14861**	55 **1814**	65	75	16675
N.R.S	40	45	70	70	65	65	55	1.4 **16244**	35	50	16244
TOTAL DEMAND	25695	38314	107741	18584	20965	9104	14861	46934	4990	3749	

Lean Season distribution data (Initial Basic Feasible Solution)

4.4 INITIAL BASIC FEASIBLE SOLUTION RESULTS INTERPRETATION FOR LEAN SEASON DATA

The initial basic feasible solution result shows that, the total transportation cost was 3,609,051.20 Ghana cedis. The Central region capacity with capacity 37,762 supplied only the Central region warehouse and the Western region warehouse with 25,695 and 12,067 respectively. The Ashanti region source also with capacity 44,895 supplied the Ashanti region warehouse and Northern region warehouse with 38,314 and 6,581. Greater Accra region source with capacity 151,281 supplied the Greater Accra region warehouse with 107,741, the Western region warehouse with 6,517, the Eastern region warehouse with 9,104 and the Northern region warehouse with 19,180, Upper East region with 4,990 and Upper West region with 3,749. The Volta region source with capacity of 24,080 also supplied both Volta region warehouse and Northern region warehouse with 20,965 and 3115 respectively. Brong Ahafo region source with capacity 16,675 supplied the Brong Ahafo region warehouse with 14,861 and the Northern region warehouse with 1,814. The Northern region source with capacity of 16,244 supplied the northern region warehouse with 16,244.

The optimum solution for the lean data is as shown below.

SOURCE	C.R.W	A.R.W	G.R.W	W.R.W	V.R.W	E.R.W	B.R.W	N.R.W	UE.R.W	UW.R.W	TOTAL SUPPLY
C.R.S	1.5 **6728**	10	15	8 **12067**	25	25	30	40 **24109**	55 **3176**	60 **3749**	37762
A.R.S	10	2 **38314**	20	18	30	35	15	45 **6581**	75	80	44895
G.R.S	15 **18967**	20	2.2 **107741**	15 **18584**	25	25 **5989**	50	70	90	95	151281
V.R.S	25	30	25	35	1.8 **20965**	30 **3115**	50	65	75	100	24080
B.R.S	30	15	50	35	50	45	1.9 **14861**	55	65 **1814**	75	16675
N.R.S	40	45	70	70	65	65	55	1.4 **16244**	35	50	16244
TOTAL DEMAND	25695	38314	107741	18584	20965	9104	14861	46934	4990	3749	

Optimal solution for lean season

4.5 OPTIMAL SOLUTION RESULTS INTERPRETATION FOR LEAN SEASON DATA

The optimal solution result shows that, the total transportation cost was 2,996,939.7 Ghana cedis. The Central region capacity with capacity 37,762 supplied only the Central region warehouse, Western region warehouse, Northern, Upper East and Upper West warehouse with 6,728, 12,067, 24,109, 3,176, 3,749 respectively. The Ashanti region source also with capacity 44,895 supplied the Ashanti region warehouse and Northern region warehouse with 38,314 and 6,581. Greater Accra region source with capacity 151,281 supplied the Greater Accra region warehouse with 107,741,the Central region with 18,967, the Western region warehouse with 18,584, the Eastern region warehouse with 5989. The Volta region source with capacity of 24,080 also supplied both Volta region warehouse and Eastern region warehouse with 20,965 and 3115 respectively. Brong Ahafo region source with capacity 16,675 supplied the Brong Ahafo region warehouse with 14,861 and the Upper East region warehouse with 1,814. The Northern region source with capacity of 16,244 supplied the northern region warehouse with 16,244.

During festivities such as Easter, Christmas etc, the production of drink by the Accra Brewery limited increases. The initial basic feasible solution of the festivities data is as shown below.

SOURCE	C.R.W	A.R.W	G.R.W	W.R.W	V.R.W	E.R.W	B.R.W	N.R.W	UE.R.W	UW.R.W	TOTAL SUPPLY
C.R.S	1.5 **42245**	10	15	8 **33450**	25	25	30	40	55	60	75695
A.R.S	10	2 **63173**	20	18	30	35	15	45	75	80 **9932**	73105
G.R.S	15	20	2.2 **174941**	15 **1317**	25	25 **17380**	50	70	90	95	193638
V.R.S	25	30	25	35	1.8 **29875**	30 **4193**	50	65	75	100 **3541**	37609
B.R.S	30	15	50	35	50	45	1.9 **18165**	55	65 **9406**	75 **707**	28278
N.R.S	40	45	70	70	65	65	55	1.4 **35149**	35 **7997**	50	43146
TOTAL DEMAND	42245	63173	174941	34767	29875	21573	18165	35149	17403	14180	

Festivities Season(Initial Basic Feasible Solution)

4.5 INITIAL BASIC FEASIBLE SOLUTION RESULTS INTERPRETATION FOR FESTIVE SEASON DATA

The initial basic feasible solution result shows that, the total transportation cost was 3,652,695.80 Ghana cedis. The Central region capacity with capacity 75,695 supplied only the Central region warehouse and the Western region warehouse with 42,245 and 33,450 respectively. The Ashanti region source also with capacity 73105 supplied the Ashanti region warehouse and Upper West region warehouse with 63,173 and 9,932. Greater Accra region source with capacity 193,638 supplied the Greater Accra region warehouse with 174941, the Western region warehouse with 1,317, the Eastern region warehouse with 17,380. The Volta region source with capacity of 37,609 also supplied both Volta region warehouse, Eastern region warehouse and Upper West region warehouse with 29,875, 4,193 and 3,541 respectively. Brong Ahafo region source with capacity 28,278 supplied the Brong Ahafo region warehouse with 18,165 and the Upper East region warehouse with 9,406 and Upper West region warehouse with 707. The Northern region source with capacity of 43,146 supplied the Northern region warehouse with 35,149 and Upper East region warehouse with 7,997.

SOURCE	C.R.W	A.R.W	G.R.W	W.R.W	V.R.W	E.R.W	B.R.W	N.R.W	UE.R.W	UW.R.W	TOTAL SUPPLY
C.R.S	1.5 **32313**	10	15	8 **29909**	25	25	30	40	55	60 **13473**	75695
A.R.S	10 **9932**	2 **63173**	20	18	30	35	15	45	75	80	73105
G.R.S	15	20	2.2 **174941**	15 **4858**	25	25 **13839**	50	70	90	95	193638
V.R.S	25	30	25	35	1.8 **29875**	30 **4193**	50	65	75	100 **3541**	37609
B.R.S	30	15	50	35	50	45	1.9 **18165**	55	65 **9406**	75 **707**	28278
N.R.S	40	45	70	70	65	65	55	1.4 **35149**	35 **7997**	50	43146
TOTAL DEMAND	42245	63173	174941	34767	29875	21573	18165	35149	17403	14180	

4.7 OPTIMAL SOLUTION RESULTS INTERPRETATION FOR FESTIVE SEASON DATA

The optimal solution result shows that, the total transportation cost was 3,439,329.80 Ghana cedis. The Central region capacity with capacity 75,695 supplied only the Central region warehouse and the Western region warehouse with 32,313 and 29,909 respectively. The Ashanti region source also with capacity 73105 supplied the Ashanti region warehouse and Central region warehouse with 63,173 and 9,932. Greater Accra region source with capacity 193,638 supplied the Greater Accra region warehouse with 174941, the Western region warehouse with 4,858, the Eastern region warehouse with 13,839. The Volta region source with capacity of 37,609 also supplied both Volta region warehouse, Eastern region warehouse with 29,875, 7,734 respectively. Brong Ahafo region source with capacity 28,278 supplied the Brong Ahafo region warehouse with 18,165 and the Upper East region warehouse with 9,406 and Upper West region warehouse with 707. The Northern region source with capacity of 43,146 supplied the Northern region warehouse with 35,149 and Upper East region warehouse with 7,997

4.8 SOLVING THE TRANSPORTATION PROBLEM USING THE EXCEL SOLVER

4.8.1 DECISION VARIABLES

The decision variables are the number of units of a single product to ship from the capacities to ~~the~~ warehouses. ~~Initial values~~ of these variables are all ones.

4.8.2 THE CONSTRAINTS

1. The amount shipped from each capacity cannot exceed the available supply.
2. The amount shipped to each warehouse must meet or exceed demand there.

3. All amount shipped must be non-negative.

The total cost of shipping in each warehouse is computed by multiplying the amount shipped by them per unit cost and summing. Total shipping cost is to be minimized.

4.8.3 PROBLEM SPECIFICATION

Target cell

Goal is to minimize the total shipping cost.

Changing Cell

Amount to be shipped from each capacity to each warehouse.

Constraints

Total shipped (from source) must be less than or equal to supply at capacity.

Total shipped to warehouse must be greater than or equal to demand at warehouse.

Number to ship must be greater than or equal to zero.

Algebraic Formulation

Let x_{ij} be the number of units of products shipped from capacity i to warehouse j. Then

the supply constraints are $\sum_{j=1}^{n} x_{ij} \leq \text{available } i,\ i = 1,2,\ldots,n$.

The demand constraints are $\sum_{i=1}^{m} x_{ij} \geq demand\ j,\ j = 1,2,\ldots,m$

And the non-negativity $x_{ij} \geq 0,\ \text{all } i, j$

The objective is to minimize $\text{Cost} = \sum_{j}^{m} \sum_{i}^{n} a_j x_{ij}$

Solver Parameters dialog

To define this problem for excel solver the cell containing the decision variables, the constraints, and the objective must be specified. This is done by choosing the solver command from the tool menu which causes the solver dialog to appear. The target cell is the cell containing the objective function.

Solver option dialog

Selecting the "options" button in the solver parameters dialog brings out a solver option dialog box. The current solver version does not determine automatically if the problem is linear or nonlinear. To inform the solver that the problem is LP, select "assume linear model" box. This causes simplex solver to be used.

Solving the simplex model

Select "show iteration result" box. Click 'ok' in the solver option dialog then click 'solve' on the solver parameter dialog. This causes the simplex solver to stop after each iteration. Because an initial feasible basic is not provided, the simplex method begins with an infeasible solution and proceeds to reduce the sum of infeasibilities. Observe this by pressing 'continue' after each iteration. The simplex solver continuous and finds a solution which is optimal.

Sensitivity Analysis

The most important information is the 'shadow price' column in the 'constraints' section. These shadow prices (also ca~~lled dual~~ variables or Lagrange multiplies) are equal to the change in the optimal objective value if the right hand side of the constraints increases by one unit with all other right hand side value remaining the same. Hence the first *n* multiplies show the effect of increasing the supplies at the capacity. Because the supply in say capacity x are all not used, its shadow price is zero. Increasing the supply in say capacity y by one unit improves the objective. The other shadow prices show the effect of increasing the demand. The 'Allowable Increase' is the amount the right hand side can increase before the shadow price changes and similarly for 'Allowable Decrease'. Beyond these range some shipments that are now zero becomes positive whiles some positive ones becomes zero. The 'Adjustable Cell' contains the sensitivity information on changes in the objective coefficients. The reduce cost are the quantities $\overline{c}_j$ discussed. These are all non-negatives as they must be in an optimal solution. If the $\overline{c}_j$ for say capacity x to warehouse y is zero, it indicates the problem has multiple optima (because the optimal solution is non-degenerate i.e. all basic variables are positive). Http://www.solver.com, 2007

CHAPTER FIVE

SUMMARY, CONCLUSIONS AND RECOMMENDATIONS

5.1 INTRODUCTION

In this chapter the summary of the results of this investigation, details of with have been discussed in Chapters 3 and 4, are provided. Conclusions are also drawn for tends observed from the findings and the relevant recommendations made. The purpose of this chapter is also to determine whether the results obtained could be used to achieve the main objective of the study, which is to minimize the transportation cost.

5.2 SUMMARY AND CONCLUSIONS

Results of data in the four categories namely averages, lean and festive seasons and the sensitivity analysis of the results from excel solver shows that the minimum cost for the period under study was around 2,272,455.9 Ghana cedis on the average range of transporting the within the ten regions of Ghana used by the Accra Brewery Limited (ABL) per month.

During the lean season the total cost of transportation was about 2,996,939.7 Ghana cedis and that of the festive season was 3,439,329.8 Ghana cedis. The results show that the difference between the lean season and the festive season is not much. Because supply points and demand location were on regional bases, the results made it clear that it is better to transport more cartons of beer within the same region or regional transportation cost that is less costly. Based on my findings and data analysis of the collated data, I recommend that more cartons of beer should be transported within the same region than to other regions. Moreover more cartons of beer should be transported to inter-regional bases where cost of transportation is very minimal and also it is better to produce more during the festive season. This will help Accra Brewery Limited marginal profit to go high since during the lean season the cost of transporting much lesser cartons of beer still make the company to pay more.

5.3 RECOMMENDATIONS

This research was conducted for only six months. The results of this research provide some scope for further studies. It could be of interest to provide data on the quarterly or monthly basis. However data as available in the records require a lot of cleaning and therefore a further study on this basis would be time consuming and expensive. It is therefore recommended that the distribution department of Accra Brewery Limited and management and any other stake holders would take up this proposal into consideration. This will provide a more comprehensive view point about the cost of transporting the beverage.

Despite a good work done, many lapses were encountered in the data collection. To avoid this it is suggested that the data collection for the computation should be taken by competent enumerators who understand the use to which the data will be put.

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DATA FORMAT

PRODUCT: A

THE NUMBER OF CARTONS OF PRODUCT DISTRIBUTED OVER VARIOUS DEPOTS FROM JANUARY TO DECEMBER IN 2007.

DEPOTS	MONTHS / CARTONS DISTRIBUTED											
	JAN	FEB	MARCH	APRIL	MAY	JUNE	JULY	AUG	SEPT	OCT	NOV	DEC
ACCRA												
KUMASI												
CAPE COAST												
KOFORIDUA												
TAMALE												
WA												
SUNYANI												
HO												
TAKORADI												
BOLGATANTA												

HOW MANY PLANTS / SUPPLY POINT DO YOU HAVE?

MAJOR DISTRIBUTION CENTERS?

Maximum of six distributions centers in the whole country.

AVERAGE CAPACITY OF EACH PLANT / SUPPLY POINT PER MONTH?

AVERAGE MONTHLY DEMAND AT THE DISTRIBUTION CENTER.

THE TRANSPORTATION COST PER CARTON (ROUNDED TO CEDIS) FROM A SOURCE / SUPPLY POINT TO A DISTRIBUTION CENTER.

	Major distribution centers (in thousands)																								
Plant																									
P1																									
P2																									
P3																									
P4																									
P5																									
P6																									
P7																									
P8																									
P9																									
P10																									

	Major distribution centers (in thousands)																								
Plant																									
P1																									
P2																									
P3																									
P4																									
P5																									
P6																									
P7																									
P8																									
P9																									
P10																									

APPENDIX B

ACCRA BREWERY TRANSPORTATION DISTRIBUTION FOR THE PERIOD MAY-OCTOBER 2007 ON LAGER BEER

					MAY						
Sources	C.R.W	A.R.W	G.R.W	W.R.W	V.R.W	E.R.W	B.R.W	N.R.W	UE.R.W	UW.R.W	TOTAL SUPPLY
C.R.S	1.5	10	15	8	25	25	30	40	55	60	33591
A.R.S	10	2	20	18	30	35	15	45	75	80	60392
G.R.S	15	20	2.2	15	25	25	50	70	90	95	156531
V.R.S	25	30	25	35	1.8	30	50	65	75	100	25391
B.R.S	30	15	50	35	50	45	1.9	55	65	75	25391
N.R.S	40	45	70	70	65	65	55	1.4	35	50	10900
TOT DEMAND	43957	34871	80003	38354	18757	6831	14790	53166	3166	18301	
					JUNE						
Sources	C.R.W	A.R.W	G.R.W	W.R.W	V.R.W	E.R.W	B.R.W	N.R.W	UE.R.W	UW.R.W	TOTAL

											SUPPLY
C.R.S	1.5	10	15	8	25	25	30	40	55	60	49861
A.R.S	10	2	20	18	30	35	15	45	75	80	68750
G.R.S	15	20	2.2	15	25	25	50	70	90	95	170536
V.R.S	25	30	25	35	1.8	30	50	65	75	100	28895
B.R.S	30	15	50	35	50	45	1.9	55	65	75	17260
N.R.S	40	45	70	70	65	65	55	1.4	35	50	38499
TOT DEMAND	24900	68750	153036	24961	28890	17500	17265	10350	20114	8035	

JULY

Sources	C.R.W	A.R.W	G.R.W	W.R.W	V.R.W	E.R.W	B.R.W	N.R.W	UE.R.W	UW.R.W	TOTAL SUPPLY
C.R.S	1.5	10	15	8	25	25	30	40	55	60	47900
A.R.S	10	2	20	18	30	35	15	45	75	80	47000
G.R.S	15	20	2.2	15	25	25	50	70	90	95	162200
V.R.S	25	30	25	35	1.8	30	50	65	75	100	22815

B.R.S	30	15	50	35	50	45	1.9	55	65	75	16566
N.R.S	40	45	70	70	65	65	55	1.4	35	50	23840
TOT DEMAND	28200	47900	147015	19726	21900	15200	16540	7140	10700	6000	

AUGUST

Sources	C.R.W	A.R.W	G.R.W	W.R.W	V.R.W	E.R.W	B.R.W	N.R.W	UE.R.W	UW.R.W	TOTAL SUPPLY
C.R.S	1.5	10	15	8	25	25	30	40	55	60	53310
A.R.S	10	2	20	18	30	35	15	45	75	80	48956
G.R.S	15	20	2.2	15	25	25	50	70	90	95	170837
V.R.S	25	30	25	35	1.8	30	50	65	75	100	26579
B.R.S	30	15	50	35	50	45	1.9	55	65	75	14485
N.R.S	40	45	70	70	65	65	55	1.4	35	50	25830
TOT DEMAND	32720	49054	152695	20492	26579	18142	14485	9052	7090	9688	339997

					SEPTEMBER						
Sources	C.R.W	A.R.W	G.R.W	W.R.W	V.R.W	E.R.W	B.R.W	N.R.W	UE.R.W	UW.R.W	TOTAL SUPPLY
C.R.S	1.5	10	15	8	25	25	30	40	55	60	53579
A.R.S	10	2	20	18	30	35	15	45	75	80	44495
G.R.S	15	20	2.2	15	25	25	50	70	90	95	152008
V.R.S	25	30	25	35	1.8	30	50	65	75	100	26688
B.R.S	30	15	50	35	50	45	1.9	55	65	75	17610
N.R.S	40	45	70	70	65	65	55	1.4	35	50	26240
TOT DEMAND	31074	44486	141343	22505	26697	10671	17604	7340	14900	4000	320620
					OCTOBER						
Sources	C.R.W	A.R.W	G.R.W	W.R.W	V.R.W	E.R.W	B.R.W	N.R.W	UE.R.W	UW.R.W	TOTAL SUPPLY
C.R.S	1.5	10	15	8	25	25	30	40	55	60	77130

A.R.S		10	2	20	18	30	35	15	45	75	80	59403
G.R.S		15	20	2.2	15	25	25	50	70	90	95	197640
V.R.S		25	30	25	35	1.8	30	50	65	75	100	29698
B.R.S		30	15	50	35	50	45	1.9	55	65	75	18438
N.R.S		40	45	70	70	65	65	55	1.4	35	50	27849
TOT DEMAND		43117	59400	173953	34016	29695	23687	18396	8924	11207	7763	410158