

MODELING DOMESTIC VIOLENCE AND PREDICTING ITS GROWTH USING  
DIFFERENTIAL EQUATIONS, A CASE STUDY OF WOMEN AND CHILDREN IN  
TAMALE

BY  
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## **DECLARATION**

I hereby declare that this submission is my own work towards the M Sc. Industrial Mathematics and that, to the best of my knowledge it contains no material previously published by any other person or the material therein has ever been accepted for the award of a degree of this or any other university, except where due acknowledgement has been made in the text.

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## **DEDICATION**

With fondest thoughts to Hajia Zaratu, Adam Rafa and Abdul-Razak Iman for your time,  
sacrifice, and endless love and to the memory of my late father

KNUST



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## **ABSTRACT**

The research work focused on analytic and numerical modeling of Domestic Violence. In the case of the analytic modeling, this thesis discusses a simple continuous model for the spread of Domestic Violence, using Ordinary Differential Equations. A mathematical model is inspired from the spread of Domestic Violence in Tamale Metropolitan in which the interaction of the widespread is likely to be minimized. A modeling technique ASV, similar to the SIR model in Epidemics, is used for formulating the spread of Domestic Violence as a system of Differential Equations. Hence the population of three distinct classes- the Abusive, Susceptible/Unreported Victims, and the Reported Victims, are considered in the model. The system of Differential Equations is analyzed by linearization of nonlinear systems and nondimensionilization, and to predict the behaviour of the spread of the Domestic Violence.

Finally, in the case of the numerical analysis, a general model for the population of Domestic Violence Victims is constructed. The present model shows that the given data is reasonably Logistic. Moreover, this model shows that the population of Domestic Violence Victims is limited. A projected limiting number is given by this model. Some typical mathematical models are introduced such as Exponential model and logistic model. The solutions of those models are analyzed.

## TABLE OF CONTENTS

DECLARATION.....	ii
DEDICATION.....	iii
ACKNOWLEDGEMENT.....	iv
ABSTRACT.....	v
TABLE OF CONTENTS.....	vi
LIST OF TABLES.....	ix
LIST OF FIGURES.....	x
LIST OF ACCRONYMS/ABBREVIATIONS.....	xi
CHAPTER 1, ITRODUCTION.....	1
1.0 OVERVIEW.....	1
1.1 BACKGROUND OF THE STUDY.....	1
1.2 STATEMENT OF THE PROBLEM.....	4
1.3 GENERAL OBJECTIVES OF THE STUDY.....	4
1.4 METHODOLOGY.....	5
1.5 JUSTIFICATION OF THE STUDY.....	5
1.6 LIMITATION OF THE STUDY.....	6
1.7 THESIS ORGANIZATION.....	7
CHAPTER 2, LITERATURE REVIEW.....	8
2.1 OVERVIEW OF DOMESTIC VIOLENCE.....	8
2.1.1 Education.....	14

2.1.2 Witchcraft and Female Genital Mutilation.....	16
2.1.3 Rape, Defilement, Assault and Murder.....	18
2.1.4 Culture and Tradition as Constraints.....	21
2.2 NATURE OF DOMESTIC VIOLENCE.....	22
2.3 CAUSES OF DOMESTIC VIOLENCE.....	25
2.4 EFFECTS OF DOMESTIC VIOLENCE.....	26
2.5 DIFFERENTIAL EQUATION MODELING.....	26
2.5.1 Malthusian Model.....	32
2.5.2 The Simple Epidemic Model (SI Model).....	34
CHAPTER 3, METHODOLOGY .....	36
<b>3.0 INTRODUCTION.....</b>	<b>36</b>
3.1 DATA.....	36
3.2 MODEL DEVELOPMENT.....	36
3.2.1 Malthusian Model.....	36
3.2.2 The Logistic Growth Model.....	38
3.2.3 The Violence Epidemic Model (ASV).....	43
3.2.3.1 Formulated Mathematical Model.....	44
3.2.3.1 Definition of Variables and Parameters.....	44
CHAPTER 4, DATA ANALYSIS, DISCUSSION AND MODELING .....	47
4.1 ANALYSIS OF THE VIOLENCE EPIDEMIC (ASV) MODEL.....	47
4.2 ANALYSIS OF THE LOGISTIC GROWTH MODEL.....	57

4.2.1 Graph of the Data.....	57
4.2.2 Logistic Model for the Data.....	59
4.2.3 Determining the Values of $\beta$ and N.....	62
4.2.4 The Solution for the Logistic Model.....	64
<b>CHAPTER 5, SUMMARY, CONCLUSION AND RECOMMENDATION.....</b>	<b>73</b>
<b>5.1 SUMMARY OF RESULTS.....</b>	<b>73</b>
5.1.1 The Violence Epidemic Model.....	73
5.1.2 The Logistic Growth Model.....	73
<b>5.2 CONCLUSION.....</b>	<b>75</b>
<b>5.3 RECOMMENDATION.....</b>	<b>76</b>
<b>REFERENCES.....</b>	<b>78</b>
<b>APPENDICES.....</b>	<b>84</b>
APPENDIX A: Data collected from DOVVSU.....	84
APPENDIX B: Differential Equation Models and Solutions.....	85

## **LIST OF TABLES**

TABLE 4.1: Data on Domestic Violence Victims in Tamale between 1999-2011.....	58
TABLE 4.2: Data of the ratio ( $a$ ) and Domestic Violence Victims.....	61
TABLE 4.3: Data on Actual cases and Predicted cases of DVV.....	66
TABLE 4.5: Predicted Domestic Violence cases with $N = 1000$ .....	69-70
TABLE 4.6: Predicted Domestic Violence cases with $N = 1257855$ .....	71-72



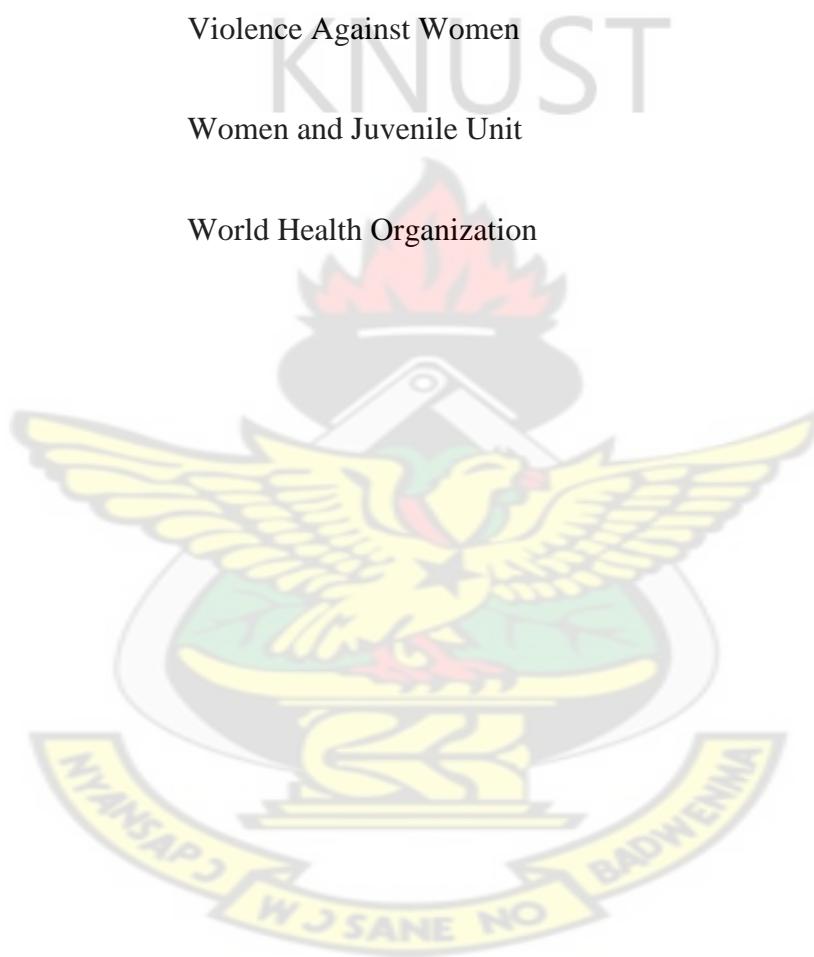
## LIST OF FIGURES

FIGURE 2.1: Simple Epidemic Model (SI Model).....	34
FIGURE 2.2: Simple Epidemic Model (SI Model).....	35
FIGURE 3.1: Violence Epidemic Model (SV Model).....	39
FIGURE 3.2: Violence Epidemic Model (SV Model).....	39
FIGURE 3.3: Phase line diagram.....	40
FIGURE 3.4: The Violence Epidemic Model (ASV Model).....	46
FIGURE 4.1: Graph showing the intersection of Abusive and Susceptible.....	48
FIGURE 4.2: A phase plane diagram of a nodal sink at equilibrium when $\rho \geq 4$ .....	51
FIGURE 4.3: A phase plane diagram of a spiral sink at equilibrium when $\rho < 4$ .....	52
FIGURE 4.4: steady state equilibrium as $t \rightarrow \infty$ , when $\rho \geq 4$ .....	55
FIGURE 4.5: steady state equilibrium in an oscillating manner as $t \rightarrow \infty$ , when $\rho < 4$ .....	56
FIGURE 4.6: A plot of Domestic Violence Data.....	59
FIGURE 4.7: Least Square Approximation Graph.....	62
FIGURE 5.1: Graph of Predicted Values of Domestic Violence.....	74
FIGURE 5.2: Graph of Actual Verses Predicted Values of DVV.....	75

## **ACRONYMS/ ABBREVIATIONS**

ASV	Abusive, Susceptible and Violence Victims
BPFA	Beijing Platform for Action
CHRAJ	Commission on Human Rights and Administrative Justice
DE	Differential Equations
DOVSSU	Domestic Violence and Victim Support Unit
DV	Domestic Violence
DVB	Domestic Violence Bill
FGM	Female Genital Mutilation
FIDA	International Federation of Women Lawyers
GES	Ghana Education Service
GSS	Ghana Statistical Service
KNUST	Kwame Nkrumah University of Science and Technology
NGOs	Non-Governmental Organizations
ODEs	Ordinary Differential Equations
SIR	Susceptible, Infectious and Recovered
UN	United Nation

UN	United Nations
UNCEF	United Nations Children's Fund
UNESCO	United Nations Educational, Scientific and Cultural Organization
UNFPA	United Nation Population Fund
VAW	Violence Against Women
WAJU	Women and Juvenile Unit
WHO	World Health Organization



# **CHAPTER 1**

## **INTRODUCTION**

### **1.0 Overview**

This chapter gives a background of the study, statement of the problem, purpose of the study, objectives, methodology and justification of the study. In addition to these are limitations as well as organization of the study.

### **1.1 Background of the study**

The provisional results of the 2010 Population and Housing Census shows that total population of Ghana is 24,233,431 (11,801,661 males and 12,421,770 females). The males form 48.7 percent of the population and females constitute 51.7 percent. The total population in Northern Region is 2,468,557 (1,210,702 males and 1,257,855 females, Ghana Statistical Service, 2010). Tamale is a bustling Regional Capital of the Northern Region about 400 miles north of the Atlantic Coast in West Africa. The geographical area of Tamale is about 750 kilometer square (289.58 square miles). The economy of the area is predominantly Agricultural. Most people in Tamale are muslims and they practice the polygamous system of marriage and also have large family size. The dominant tribe is the Dagombas but there are other tribes like Gonja, Mamprusi, Konkomba, Hausa, Dagarti, and Grusi. Globally, in 90 percent of the cases of Domestic Violence, the victim is a woman and 90 percent of the time the abuser is a man. It is also estimated that one out of four women will experience intimate partner violence at some time in their life (Perttu-Rautava, 2002). Violence against women and children continues to be a global epidemic that kills, tortures, and maims – physically, psychologically, sexually and economically. It is one of the most pervasive of human rights violations,

denying women and children equality, security, dignity, self-worth, and their right to enjoy fundamental freedoms. Violence against women and children is present in every country, cutting across boundaries of culture, class, education, income, ethnicity and age. Even though most societies proscribe violence against women, the reality is that violations against women's human rights are often sanctioned under the garb of cultural practices and norms, or through misinterpretation of religious tenets (Manuh, 2002).

Domestic violence has been outlined by Amnesty International as most violent attacks on an individual or group of people or women. It involves physical, sexual and psychological violence in the family including battering, sexual abuse of female children in the household, dowry related violence, marital rape, female genital mutilation and other traditional practices harmful to any member of the household. Domestic violence may be exhibited by any member of the household.

All over the World, governments and non-governmental and international organizations are trying to collaborate to come out with appropriate ways to handle these burning issues. On the international front, (Avoke et al, 1999) cited the United Nations declaration (1998) which enjoined member states to protect the rights of citizens, particularly the vulnerable groups such as women, children, the disabled and disadvantaged. The repercussions of these violent acts and how spouses and their children suffer thereafter appears to be taken for granted. Women all over the World agitated and fought for human dignity, more especially, the dignity of women and Children. Significant among these moves was the Beijing Platform for Action (BPFA, 1998). In Ghana, a Ministry is established to focus on the interest of women and children. The philosophies behind its establishment were borne out of the idea that, women and children are vulnerable and are prone to domestic violence. Another

important unit was also set up in the Police Service that is Women and Juvenile Unit (WAJU) to handle cases concerning women and juveniles in the society and it is now called Domestic Violence Victim Support Unit (DOVVSU). International Federation of Women lawyers (FIDA) is one of the organizations in Ghana which plays advocacy roles in combating all kinds of violence against children and women in society.

Ironically many people in Ghana generally tend to see domestic violence as part of everyday experiences. (Cusack et al, 1999) remarked of a queen mother who was reported to have commented as follows “I hear every day that somebody has beaten the wife but women do not come to report the men. I also keep quite in my house” Such comments underpin the mini-subculture of abuse and violence that exist in many homes, and their subsequent effect on children growing up in those homes. In many communities in Ghana, it is often the case that when defilement, assault, sexual harassment, rape, and battering are reported, little attention or regard is giving to them thereby, reinforcing the perpetuation of violence in many homes. On February 2007, Ghana’s Parliament passed the much-awaited Domestic Violence Bill (DVB), which had been laid before it in 2003 and had been the subject of heated debate. The process leading to the passage of the law involved not only the introduction of new legislation, but also confronting a social system that tolerates various forms of violence against women and children, especially in the context of gender relations and in the domestic sphere. It is against this background that this study is being conducted to ascertain the current state of domestic violence and its growth.

## **1.2 Statement of the Problem**

In 90 percent of the cases of Domestic Violence in Tamale, the victim is a woman and 90 percent of the time the abuser is a man. Violence against women and children continues to be a Tamale Metropolitan epidemic that kills, tortures, and maims – physically, psychologically, sexually and economically. It is one of the most pervasive of human rights violations, denying women and children equality, security, dignity, self-worth, and their right to enjoy fundamental freedoms in the metropolis. Women and children are often in great danger in the place where they should be safest within their families. For many, ‘home’ is where they face a regime of terror and violence at the hands of somebody close to them – somebody they should be able to trust. They are unable to make their own decisions, voice their own opinions or protect themselves and their children for fear of further repercussions. Their human rights are denied and their lives are stolen from them by the ever-present threat of violence. Government and Civil Society Organizations have made some attempts to solve the problem. Domestic violence is still going on unabatedly in the Metropolis. This study seeks to develop a model for Domestic Violence and also consider its stability in the future.

## **1.3 General objective of the study**

The main objective of the study is to develop a Differential Equation Model which will be used to predict the growth of Domestic Violence.

Specifically, the study aims at:

- ❖ Modeling Domestic Violence using Differential Equations.
- ❖ Solving the model by both analytical techniques and numerical techniques.

- ❖ To help test the application of that model since there are so many method and models that could have been used.

## **1.4 Methodology**

In the study Metropolis, the Domestic Violence Victim Support Unit (DOVVSU) of the Ghana police in Tamale was contacted for data on Domestic Violence Against women and children. The data for monthly reported cases of Domestic Violence over a period of 13 years was taken. The KNUST library, Tamale main library and the internet were other sources of vital information to this research. A graph will be plotted to show the trend of the incidence of Domestic Violence. Finally, this research will use Differential Equations in the modeling process (modeling through Differential Equations).

## **1.5 Justification of the Study**

The problem of Domestic Violence Against women and children is a national issue, and may not be limited to the area in which the study is to be carried out. Several programs have been initiated in the country to help reduce violence against women and children such as the passage of the Domestic Violence Bill (DVB) but based on empirical evidence its prevalence rate is still high. Also, considering the simultaneous loss of live, cost in the medication of victims, loss of productive hours and low performance by women at work and children at school, it requires renewed commitment from the Government, Non-Governmental Organizations and all in sundry to help fight for complete reduction of Domestic Violence Against women and children in Tamale.

The study seeks to develop a Differential Equations Model that will be used to predict the growth of Domestic Violence and will also help on systematic basis to monitor trends and rates in prevalence and its outcomes.

Additionally, the study will help in the following areas of the economy. It will contribute to an increased awareness of men, women and children's understanding and experiences of Domestic Violence. This in the long run will help to improve the general wellbeing of women and children so they can contribute to nation building as women are the majority of the Ghanaian population and the children are the future leaders.

Economically, it will help Government to allocate the appropriate budgetary support to the Ministry of Women and Children Affairs and also encourage interested organizations to help fight this epidemic.

Finally, it is important to contribute to knowledge on Domestic Violence with a view to, among other things, stimulate further research.

## **1.6 Limitation of the Study**

This project will be accomplished successfully notwithstanding constraints encountered. Inadequate logistical arrangement to facilitate collection and storing information constituted a major drawback. Officers of the Domestic Violence Victim Support Unit in Tamale had to search through bulk documents for the needed information. This made it a hard task for me in obtaining the required data. Time allotted to carry out the study wasn't sufficient enough to cover a broader scope like the whole of Northern Region or country wide. One other factor which may account for some possible error is the use of already collected data. This is because the collected data may have been taken under

certain limitations and conditions that are not available to the researcher. With a few exceptions, this review was limited to sources published within the previous 10 years.

### **1.7 Thesis Organization**

Chapter 1 is the introduction which comprises the background to the problem, statement of the problem, objectives of the study, justification of the research, methodology and limitation.

Chapter 2 (Literature Review) basically deals with the review of literature on Domestic violence against women and children and Differential Equation models. Chapter 3 (Methodology) presents the description and mathematical formulation of the problem (the model) with underlying assumptions. The data used in the study and characteristics of the study area, data analysis and results are considered in Chapter 4 (Data Collection, Analysis and Discussion). The summary of results, conclusions and recommendations of the study are presented in Chapter 5.

## **CHAPTER 2**

### **LITERATURE REVIEW**

The literature review was discussed along the following lines

- ❖ Overview of Domestic Violence
- ❖ Nature of domestic violence
- ❖ Causes of domestic violence
- ❖ Effects of domestic violence
- ❖ Differential Equation Models.

#### **2.1 OVERVIEW OF DOMESTIC VIOLENCE**

According to Galles and Straus (1979), "Violence is an act carried out with the intention of perceived intention of physically hurting another person". It can also be defined as the exercise of power in order to impose one's will on a person or to have ones will with a thing. According to Schuler (1992), when this violence comes in the circle of gender relations, it becomes a pervasive and prevalent problem worldwide, touching all aspects of women's lives, from the home to the workplace. The definition of gender-based violence was discussed in detail at the Asia Pacific Forum on Women, Law, and Development (APWLD) in 1990 and participants concluded that the definition of gender-based violence is "any act involving use of force or coercion with intent of perpetuating or promoting hierarchical gender relations". As women who are overwhelmingly victimized because of being women and because of their sex, the term gender-based violence is often used for the violence against women worldwide. Adding gender dimension to that definition amplifies it to include violent acts perpetrated on women because they are women. With this addition, the definition is no longer simple or

obvious. Understanding the phenomenon of gender violence requires an analysis of the patterns of violence directed towards women and the underlying mechanisms that permit the emergence and perpetuation of these patterns.

According to UNESCO (1999), violence against women is one of the crucial social mechanisms by which women are force into a subordinate position compared with men.

Symonides and Volodin (2002), the United Nations General Assembly defines violence against women as any act of gender-based violence that results in, or is likely to result in, physical, sexual or mental harm or suffering to women, including threats of such acts, coercion or arbitrary deprivation of liberty, whether occurring in public or in private life.

Tomaseveski (1993) highlights violence against women as:

Violence against women includes any act, omission or conduct by means of which physical, sexual or mental suffering is inflicted, directly or indirectly through deceit, seduction, threat harassment. Coercion or any other means on any woman with the purpose or effect of intimidating, punishing or humiliating her or of denying her human dignity, sexual self determination, physical, mental or moral integrity or of undermining the security of her person, her self-respect or her personality or of diminishing her physical or mental capacities.

According to Seager (1997), "domestic violence is a leading cause of injury and death to women children and worldwide. It is often ignored or even condoned by the state on the grounds that are a 'private' matter". She further explores domestic violence as a tool of power used to sustain particular sexual, family and household structures and to keep women subordinate to them. Here it is important to notice that common perception of domestic violence is that it is the violence inflicted by a husband or wife. However, according to Yasmeen (1995) a women's rights activist from Pakistan, "for many women

in developing countries like Pakistan, family life involves a complex set of power relations with many different people (men as well as women) who may take part in the abuse of women in the home". This is the case of Pakistan where there are also many women who poison their husbands. There are women (mothers-in-law) who are also the perpetrators of bride burnings to some extent in many parts of the world especially in Pakistan, India and Bangladesh. Therefore, it can be said that it is not only a war between the masculine and the feminine sex, but also a war between family members in which women and children are mostly victims because of their subordinate and inferior position.

The Fourth Conference of Women (1995) has defined violence against women as a physical act of aggression of one individual or group against another or others. Violence against Women (VAW) is any act of gender-based violence which results in, physical, sexual or arbitrary deprivation of liberty in public or private life and violation of human rights of women in violation of human rights of women in situations of armed conflicts (Conference on Women, Beijing, 1995 Country Report).

Liz Kelly (1998), Surviving Sexual Polity has defined violence as "any physical, visual, verbal or sexual act that is experienced by the woman or girl at the time or later as a threat, invasion or assault that has the effect of hurting her or degrading her and/or takes away her ability to contest an intimate contact".

Dr Joanne Liddle (2000), modified this definition as "any physical, visual, verbal or sexual act that is experienced by the person at the time or later as a threat, invasion or assault, that has the effect of hurting or disregarding or removing the ability to control one's own behaviour or an interaction, whether this be within the workplace, the home, on the streets or in any other area of the community".

A resolution adopted by the United Nations General Assembly in 1994 states that Violence Against Women (VAW) is any form of gender based violence that results in or is likely to result in physical, sexual or psychological harm or suffering to women including threats of such acts, coercion or arbitrary deprivation of liberty whether occurring in public or private life. In line with global concerns regarding VAW, November 25 of every year has been earmarked as the United Nations Day against VAW.

According to Galtung (1990), violence can be cultural, structural, and direct. These types of violence affect, intersect, and influence one another. Direct violence is an event, structural violence is a process with ups and downs, and cultural violence is an invariant permanence. According to Stark et al (1979) have defined structural violence as “the confiscation of someone’s rights through the use of ideas”. Applied to arranged marriages without women’s consent, agents of structural violence will argue that it is in the woman’s interest since women lack knowledge, are incompetent in making decisions, and men know better. Illiteracy among many women and some men, and the tribal codified customary law strengthen the use of structural violence, supporting Galtung’s argument that structural violence will become a social pattern that people will learn and easily transmit. Stark et al (1979) have defined direct violence as “the confiscation of one’s right or interest through the use of physical violence”. Galtung (1996) has added psychological and verbal abuse to direct violence because consistent abuse may leave behind trauma. Honor killing is an example of direct violence in Kurdistan. It takes place when a woman has committed adultery in a pre-marriage relationship, when a woman insists in marrying a person of her own choice, or refuses an arranged marriage and also wife beating. This is very common and takes place for trivial

reasons, such as not preparing food for her husband in time. Wife beating sometimes leads women to self-immolation. Galtung (1996) held cultural violence to be the core, the intermediate, and facilitators for all types of violence. He stated that many aspect of culture, such as religion and ideology, language and arts, empirical science and formal science can be used to justify or legitimize direct or structural violence. He meant that violence might be invisible to people, and people may not recognize it as violence because it has became part of their life. He pointed out that the culture may "change the moral color of an act from red/wrong to green/right or at least to yellow/accepted". According to Johnson (1996), they concluded that women were "four times as likely to experience the most serious and potentially lethal violence, such as threats, assault with a gun or knife, choking or sexual assault, three times more likely to report suffering a physical injury and twice as likely to report chronic ongoing assaults". Edelson and Tolman, (1992) put it at 30 -60%. That is, "30 -60% of children whose mothers had experienced abuse were themselves likely to be abused". According to Campbell and Soeken, (1999); Women's Rights International, (1998); Human Rights Watch, (1992), Gender-based violence in various forms including rape, domestic violence, "honour" killings and trafficking in women exacts a heavy toll on mental and physical health of the victims. Increasingly, gender-based violence is recognized as a major public health concern and a serious violation of basic human rights. According to a report by the United Nations Population Fund (1999), Gender-based violence constitutes a life-long threat for hundreds of millions of women and children around the world. At least one in every three women has been beaten, coerced into sex, or abused in some other way, usually by an intimate partner or family member. According to Davidson et al., (1996); Bergman and Brismar, (1991), in Africa, domestic violence, rape and other sexual abuse

and female genital mutilation are of special concern. Studies suggest that domestic violence is widespread in most societies and is a frequent cause of suicides among women. According to Saunders and Kindy, (1993); Burnam et al., (1988), rape and other forms of sexual violence are increasing. It is unfortunate that many rapes go unreported because of the stigma and trauma associated with them and the lack of sympathetic treatment from legal systems. It has been found that sexual abuse can lead to a wide variety of unhealthy consequences, including behavioral and psychological problems, sexual dysfunction, relationship problems, low self-esteem, depression, thoughts of suicide, alcohol and substance abuse, and sexual risk-taking.

According to Fergusson et al., (1997); Boyer and Fine, (1992), physical and sexual abuse also increases a woman's risk for a number of common gynecological disorders, including chronic pelvic pain. Furthermore, women who are abused or afraid to raise the issue of family planning with their partners are at risk of repeated unwanted pregnancies. Women who are sexually abused in childhood also are at greater risk of being physically or sexually abused as adults. At least 130 million women have been forced to undergo female genital mutilation or cutting, another 2 million are at risk each year from this degrading and dangerous practice according to United Nations Population Fund (1999).

According to Donkor and Boateng, (2004), psychological violence, though difficult to detect and diagnose, includes denial of sex, false accusations, and humiliation before others, refusal to talk to partners, isolation, husbands staying away from home, discrimination and favoritism between wives. Emotional and psychological violence is capable of killing, and causing psychiatric disorders to, women. Emotional violence also includes showing disrespect for one's partner, blackmail, verbal abuse and making people feel bad about their own selves. Threatening behaviour such as insults, threats of

sacking the woman from the home, as well as threats to injure or kill a woman or child, threats to stop paying school fees of children, generate fear and a sense of insecurity. In fact, having others make decisions for them, supervise and monitor them and treating them in a manner similar to that of children erodes women's sense of worth, their visibility and their confidence. The foregoing discussion represents the dismal picture violence and discrimination against women and girls paints in any society wherever it is found. In Ghana, more than 2,500 women suffered various forms of abuse between January and September 2004 alone and of this number, 837 were sexually assaulted, 130 were abducted, and 1,358 were battered. The statistics are rather disturbing because the figures are likely to be higher since many cases of VAW go unreported. Violence against women and children is also a developmental issue because they form the majority of the country's population is at stake and impinges on developmental efforts. In what follows, a modest attempt is made at highlighting the incidence of violence and discrimination against females in the Ghanaian context especially in Tamale Metropolis. In particular, attention is focused on education, witchcraft and female genital mutilation, rape, defilement, assault and murder.

### **2.1.1 Education**

Education is a fundamental human right and Ghana is signatory to Article 26 of the Universal Declaration of Human Rights (1948), which stipulates that elementary education shall be free and compulsory, and that the higher levels will be equally available to all on the basis of merit. The intrinsic human value of education is its ability to add meaning and value to all people's lives without discrimination. However, most recent available evidence suggests that in Ghana 28 percent of women have no education

as opposed to 18 percent of men (Ghana Statistical Service and Noguchi Memorial Institute for Medical Research, 2004), implying that in general males have more education than females. The picture is even grimmer when higher levels of education are considered. Also, school dropout rates in 1997 were about 20 percent for boys and 30 percent for girls at the primary level, and 15 percent for boys and 21 percent for girls at the junior secondary level (Ministry of Education, 1997). According to Tamakloe, (2004), there is little or no discrimination against female children in education in Ghana, but girls and women frequently drop out of school due to societal or economic pressures. The Government actively campaigns for girls' education and in 1997 established a girls' education unit within the basic education division of the Ghana Educational Service. The low female enrolment is attributable to the fact that many girls marry early or become pregnant. Although the percentages of girls enrolled in school have been increasing steadily over the years, participation is still low; only about one quarter of university students are females. Women's rights groups are active in educational campaigns and in programs to provide vocational training, legal aid, and other support to women. According to Tamakloe (2004), The Government also is active in educational programs and the President and First Lady, Mr. and Mrs. J.A. Kufuor, are among the most outspoken advocates of women's rights. However, gender gap in educational attainments in favour of males is common knowledge.

### **2.1.2 Witchcraft and Female Genital Mutilation**

According to Africa Recovery (2004), Ghanaian women, especially in rural areas, remain subject to burdensome labour conditions and traditional male dominance. Traditional practices and social norms often deny women their statutory entitlements to

inheritances and property, a legally registered marriage (and with it, certain legal rights), and the maintenance and custody of children. According to Niriti (2001), although the Constitution prohibits slavery, it exists on a limited scale. Trokosi, a traditional practice found among the Ewe ethnic group and in part of the Volta Region, is an especially severe abuse and a flagrant violation of children's and women's rights. It is a system in which a young girl, usually under the age of 10, is made a slave to a fetish shrine for offences allegedly committed by a member of the girl's family. In rare instances, boys are offered. The belief is that, if someone in that family has committed a crime, such as stealing, members of the family may begin to die in large numbers unless a young girl is given to the local fetish shrine to atone for the offense. The girl becomes the property of the fetish priest, must work on the priest's farm, and perform other labors for him. Because they are the sexual property of the priests, most Trokosi slaves have children by them. Although the girls' families must provide for their needs such as food, most are unable to do so. There are at least 2,510 girls and women bound to various shrines in the Trokosi system, a figure that does not include the slaves' children. Even if released, generally without skills or hope of marriage, a Trokosi woman has continued obligations to the shrine for the duration of her life. When the fetish slave dies, the family is expected to replace her with another young girl for the fetish shrine.

According to Niriti (2001), in 1998 the Parliament passed a comprehensive legislation to protect women and children's rights, a legislation that amended the 1960 Criminal Code to provide additional protection for women and children. The legislation added new definitions of sexual offences and strengthened punishments for others. The provisions of the bill ban the practice of "customary servitude" (known as Trokosi), protect women accused of witchcraft, double the mandatory sentence for rape, raise the age of criminal

responsibility from 7 years to 12, criminalize indecent assault and forced marriages, and raise punishments for defilement, incest, and prostitution involving children. Human rights activists believe that the goal of eradicating the Trokosi practice is achievable with the new law. Non-governmental organizations (NGOs), such as International Needs, and government agencies, like the Commission on Human Rights and Administrative Justice (CHRAJ), have been campaigning against Trokosi for several years and are familiar with the locations of the fetish shrines and the numbers of women and children enslaved. The CHRAJ and International Needs have had some success in approaching village authorities and fetish priests at over 116 of the major and minor shrines, winning the release of 2,190 Trokosi slaves to date and retraining them for new professions. The organizations continue to work for additional releases. Rural women can also be punished with banishment by traditional village authorities for teenage pregnancy or suspected witchcraft. Women accused of witchcraft have been sent to penal villages in the Northern Region by traditional authorities, such as a shaman and the Gambaga witch camp. Although the women face no formal legal sanction if they leave, most fear that they would be beaten to death if caught outside the penal villages. Forced labour also occurs at the camps for women accused of witchcraft. However, the Trokosi practice is yet to be eradicated in Ghana. The challenge lies not only in persuading custodians of the witches' homes to abolish the practice, but also in educating the community so the women will be allowed to return safely to their homes. Female Genital Mutilation (FGM) refers to the removal of all or part of the clitoris and other genitalia. Those who perform the more extreme form, undergone this procedure. A Ministry of Health survey conducted between 1995 and 1998 found that FGM is practiced among nearly all the northern sector ethnic groups. Officials at all levels have spoken out against the practice,

and local NGOs are making some inroads through their educational campaigns to encourage abandonment of FGM and to retrain practitioners. Members of the legal community advocate legislation to close loopholes in the FGM law, including extending culpability to family members and others who aid in carrying out FGM and to Ghanaians who commit the crime outside the country's borders. They contend that any person who conceals information about an instance of FGM would be liable, while FGM would be banned no matter how medically safe the procedure is made--dispelling a belief by some that FGM is acceptable as long as the girls' health is protected. These efforts are laudable. But FGM is still practiced, although in secret, in some parts of Ghana, particularly in the north.

### **2.1.3 Rape, Defilement, Assault and Murder**

According to Donkor and Boateng (2004), after he was elected for his first four-year term in December 2000, Ghanaian President J.A. Kufuor appointed two women to oversee two new ministries created specifically to act on behalf of the country's women and children, The Ministry of Women and Children's Affairs and the Ministry of Education's Girl-Child Education Unit. President Kufuor also established the Women's Endowment Fund to assist women entrepreneurs and affirmed the need for the Women and Juvenile Unit of the country's police service, which was created in 1998 to address an increase in cases of abuse against women and children. The northern sector ethnic groups are those ethnic groups predominant in the Northern, Upper East, and Upper West Regions of Ghana. They include Guan, Mole-Dagbani, Grussi, Gruma, Hausa, and Dagarti. However, both print and electronic media suggest that violence against women is increasing. More than 30 women have been murdered over the last five years by what

authorities describe as a serial killer or gang, and no one has been convicted in connection with the slayings. At least seven women were killed in the course of 2002 by their husbands or companions over alleged infidelities. It is on record that the Women and Children's Affairs Minister, Mrs. Gladys Asmah, condemned the killings, describing a dangerous, emerging culture in the country in which men lash out violently against women, not over alleged transgressions, but to control women's sexuality and sexual behavior. Galvanized by Asmah's remarks, hundreds of women took to the streets in the capital of Accra on April 6, 2003 to protest the killings.

According to data gathered by the International Federation of Women Lawyers (FIDA, 2003), a total of 95 percent of the victims of domestic violence are women. These abuses usually go unreported and seldom come before the courts. However, 1998 legislation doubled the mandatory sentence for rape. In late 1998, the police administration established a "women and juvenile unit" to handle cases involving domestic violence, child abuse, and juvenile offences. The unit works closely with the Department of Social Welfare, FIDA, and the Legal Aid Board.

According to Tamakloe, (2004), the media increasingly report cases of assault and rape. In late 1998, a series of "mysterious murders of women began to occur in the Mateheko area of Accra. Three of the 20 murders reportedly involved husbands' suspicion of their wives' infidelity. The men subsequently were arrested but not convicted. On several occasions, women of Accra demonstrated in a concerted effort to attract attention to violence against women. The White Ribbon Campaign is used to commemorate the serial killing of close to 40 women between 1998 and 2000 in Ghana. The campaign is used to encourage men and boys to publicly take a stand against gender based violence by wearing a white ribbon on the shirt during the 16 days of advocacy and sensitization.

These laudable efforts seem to be a tip of the iceberg as they reach only a small fraction of the relevant population subgroup. Partly because of superiority complex among their male counterparts, the abuse of women in Ghana continues unabated. They think women cannot think on their own. Some Ghanaian men even think women are part of their property and so men should decide what women should do. While more women are reporting domestic violence to the Domestic Violence Victim Support Unit of the police force, many of them continue to take the abuse, intimidated by the stigma and embarrassment heaped on victims and the long delay between reporting and the resolution of a case in the courts. Unfortunately, many people do not even know that an agency exists with the police specifically set up for addressing women and children issues. The Domestic Violence Victim Support Unit has branches in the 10 regions of Ghana, and one outpost per region is not enough to address the crimes reported to officials. Women may not know what options are available to them when they are abused. In addition, there is so much societal pressure on these victims that they refuse to bring the perpetrators to the sanction table. Most Ghanaian women prefer not for their husbands and family members to be jailed, but rather an order to stop them from abusing them.

#### **2.1.4 Culture and Tradition as Constraints**

According to Kyei-Boateng, (2004); Campbell and Soeken, (1999); Bawah et al., (1999); Bergman and Brismar, (1991), justification for violence stems from gender norms - distorted views about the roles and responsibilities of men and women in relationships. Worldwide, studies have shown a consistent pattern of events that trigger violent responses. These include: not obeying the husband, talking back, refusing sex,

not having food ready on time, failing to care for the children or home, questioning the man about money or girlfriends or going somewhere without his permission. The power of tradition also prevents local officials from enforcing reforms.

According to Bawah et al., (1999), many cultures condone or at least tolerate a certain amount of violence against women. The right of a husband to beat or physically intimidate his wife is a deeply held conviction in many societies. Even women often view a certain amount of physical abuse as justified under certain conditions. In a study in Ghana, 51 percent of women and 43 percent of men agreed that a husband is justified in beating his wife when she uses a family planning method without his knowledge.

According to Ezeh (1993), when asked what happens if a woman practices family planning without her husband's consent, men interviewed in Ghana gave such replies as, "It is fitting enough to beat her for not consulting you earlier before going ahead to practice family planning," and "It is not good for you to keep such a woman since she did so without first consulting with you". Although the economy of Ghana is predominantly depending on the export of agricultural product, some more modern sectors have also developed in urban areas. Life, however, still remains basically rural. In rural areas women remain subject to burdensome labour conditions and traditional male dominance, which represent a violation of their constitutional rights. In many traditional African settings, the society gives a go ahead and absolute control to everything in the matrimonial home to the man. The women then recoil and concentrate on their duties as women. The case is even much worse in the rural areas where many of these women assist their husbands on the farm. Apart from the tremendous output such as cooking on the farm, collecting firewood and foodstuffs for dinner, there is the tendency to calculate the cooking on the farm as part of her usual duties of cooking in

the house. She therefore earns little or nothing at all as compensation or recognition from her output on the farm. So many wives of male cocoa farmers do not own even a hectare of the cocoa plants as their own. By custom, the man is expected to give her money to buy what she needs. But this does not really happen most of the times? Who will not get angry if someone even if close keeps demanding help and money each day from her/him? So husbands who may face financial difficulty or emotional stress extend their bad feelings to their children and wives in particular.

## **2.2 NATURE OF DOMESTIC VIOLENCE**

According to Cusack et al (1999), most forms of violence inflicted upon women and children in Ghana by persons known to them or in some cases by strangers are considered a “private matter” and given no name or value. In everyday life, we see our relationships in a variety of circumstances. Violence is a forceful way of getting something from someone. It is also living with someone and being forced to do things you do not understand and things that hurt. Violence refers to those actions that have a harmful physical (intentional infliction of pain on an individual) or mental effect (forced to do things that cause displeasure) on the individual. Physical violence can be perceived from five areas. They are;

- ❖ Cruel punishment
- ❖ Forced labour
- ❖ Beatings
- ❖ Assault with weapon and
- ❖ Death

Most at times, children and women are targeted for these acts. Cruel punishment come in the forms of starving children or food rationing, inserting pepper and ginger into anus of children, putting children's fingers into hot oil and the like.

Common among forced labour is the situation in which parents or guardians send their children to relatives and other people to be made a housemaid or houseboy for a little remuneration from their masters. These children are forced to work for long hours at work described as "hard" with little time off, all in exchange for food and shelter. Teachers use students to work for them unofficially without the students consent, and these requests are accompanied by unspoken threat that school work will suffer if not adhered to the request. Beating can be described as physical assault with various degree of physical injury, descriptions of which is severity (stab wound) if weapons are used. Beating can range from isolated slapping using a hand to the regular use of fists, feet and weapons in attacks that targeted any part of the body. Another form of violence may be psychological which is a behaviour that is directed at an emotional level, has emotional impact such as inability to sleep or eat (restlessness). It includes threatening behaviour, verbal abuse and infantilisation of women. Certain traditional practices are violent in nature. Female circumcision is one form of such traditional practices which many women underwent as teenagers or children, some even as infants (Amnesty International, 1995). Other examples include widowhood rites which sometimes has element of violence in its administration and a type of enslavement of female called "Trokosi".

Nolan (1998) categorized a broad range of violent acts and abuse children are likely to fall victim to, and these include;

- ❖ Emotional deprivation of love and physical contact

- ❖ Verbal abuse, that involves constantly being shouted at
- ❖ Exposure to violent aggressive scenes at home
- ❖ Physical neglect
- ❖ Deprivation of food, warmth and shelter
- ❖ Lack of adult protection, bullying, sexual assault and emotional neglect

According to World Health Organization (1996; 2005), the types of Violence Against women and children is classified according to the life cycle of Violence. These are;

- ❖ Pre-birth/infant: sex-selective abortion, infanticide or neglect (health care, nutrition)
- ❖ Childhood: child abuse involving malnutrition, FGC/FGM, excessive discipline, child sex abuse (including prostitution or pornography), violence against girls in schools, child marriage, trafficking for sex, child labour and begging.
- ❖ Adolescence: forced prostitution, trafficking, forced early marriage, psychological abuse and rape.
- ❖ Reproductive age: honor killing, dowry crimes and bride –price abuse, intimate partner violence, sexual assault by a non-partner, sex trafficking, violence against domestic workers and sexual harassment and intimidation.
- ❖ Elderly years: elder/widow abuse.

According to Lehtonen (1999), the types of violence against women and are as follows:

- ❖ Physical violence: pushing, burning, slapping, strangling, kicking and biting.
- ❖ Psychological violence: verbal abuse, isolation, vandalizing property, jealousy and stalking.

- ❖ Sexual violence: forced to have sex, sexual harassment, forcing to take part in pornography, forcing to prostitute, using objects, urinating on someone and rape.
- ❖ Economical violence: forbidding working and getting money of own, using one's money without permission and controlling one's use of money.
- ❖ Spiritual violence: forcing to follow spiritual beliefs against will and not letting practice religion.
- ❖ Latent violence: living in constant fear and behave in a way that pleases the abuser.

### **2.3 CAUSES OF DOMESTIC VIOLENCE**

According to Cusack et al, (1999) there are varieties of causes to domestic violence these are categorized into three broad areas;

- ❖ Verbal alteration that can escalate into violent behaviour
- ❖ Financial issues that can lead to disagreements and violence
- ❖ Offensive conduct

From the perspective of Cusack et al, (1999) the causes of domestic abuse more especially violence against women and children are Infidelity, aspect of polygamy, socio-economic reasons, disobedience, family interference, marital disharmony, outside influences and vices. School related reasons such as refusing to go to school, being late to school, refusing to perform personal chores of teachers, late payment of school fees, disobedience in school and other petty offences which include cheating in examination are other causes linked to domestic abuse. Also, argues that in spite of the free education in Ghana over a quarter of a century after independence only 44% of children is enrolled in the Primary School, and he attributes the lack of full access to education as a violation

of pupil's rights. He further cites from a newspaper article, which states that, some families were openly selling their children because of poverty and hunger.

## **2.4 EFFECTS OF DOMESTIC VIOLENCE**

Wolfe et al (1986) posit that children who are victims of domestic violence suffer traumatizing events such as injury or death. Negative feelings such as withdrawal, depression abnormal fears, regression, learning problems or a decline in school performance, truancy, bullying, difficulty in building and maintaining relationships, impulsiveness, behavioral problems, denial and emotional problems are coupled with daily activities of children who suffer or witness domestic violence. Depression has devastating effects on children eating disorders, psychosomatic illness, and difficulties at school and instances in substance abuse.

According to Wolfe et al (1986) children who were victims or witness to physical domestic violence are more likely to become perpetrators of domestic violence as adults than those who were not victimized.

## **2.5 DIFFERENTIAL EQUATION MODELING**

Modeling is one of the ways by which formulae are used or deduced to help solve a real life situation, occurrence or a problem. Mathematical models are like other types of models. The aim of every model is not to produce exact copy of 'real' object but to give a representation of some aspect of the real thing. For example, a portrait of a person, a store mannequin and a pig can all be a model of human beings; none of them is a perfect copy of human beings but each has certain aspect in common with a human. The mathematical models used in this article are based on Ordinary Differential Equations

(ODEs). Physicists first used ODEs to model the trajectories of moving objects. When applied to population or technologies, they describe continuous "trajectories" of growth or decline through time. Although populations of humans and many technological variables grow and decline in discrete numbers, continuous models are often used for simplicity when modeling large aggregates. Montroll, (1978) makes the connection between physical and population trajectories clear by proposing laws of social dynamics based on Newton's laws of mechanics.

According to Anderson, (1988), model building is a tool frequently used by planners, scientists, mathematicians etc in developing government policies and in making private decisions. A try and error approach can be disastrous. To anticipate the ultimate result without the real, we use something else that look like or act like the real thing or object to model. A photograph can be a real model in communication- one picture is worth a thousand words. Sketches are used by architect and blueprint by carpenters. If boards are cut like drawings on a manila card, a home can be built that will look like the drawing. Other changes in living systems can be modeled by mathematical equations; by varying the input on one side of the equation, one can predict the outcome symbolized by the other side. Game biologist use such models to determine how fawn production might vary if the hunting of doe is allowed in the fall. Also to find downstream oxygen levels and species diversity, a biologist needs a more complicated model using temperature, flow of water, use of oxygen by wastes and organisms, rate of replenishing oxygen and the availability of nutrient in rivers.

Model building is not only predictive but can also be descriptive; the important thing, for one to remember is that, the less complicated the system and the more information available, the greater the likelihood of a successful model. Model could also be referred

to as a formula used by planners and mathematicians etc to predict or give information about a future value or effect of any interacting object or items. Every mathematical model follows simple assumption or some basic principles. The first step is to clearly state the assumptions which are relevant to the model. It usually describes the relationship among the quantities to be studied. The next step involves completely describing the variables and parameters to be used in the model and the last step is the use of the assumption formulated in step one to derive the equation relating the quantities in step two. Quantities in every model may fall into three basic categories; that is the, dependent variable, the independent variable, and the parameters. The independent variable in this study is time. The dependent variables are quantities that are functions of time. Parameters are quantities that don't change with time (or with the independent variable) but can be adjusted (by natural causes or by scientists conducting the experiment).

Simple exponential growth is probably the most popular general model for describing positive quantities that grow in time by many orders of magnitude, as both population and world product have. It is, for example, widely used to model the last half century of economic growth (Barro & Sala-i-Martin, 1995) and the last few centuries of growth in scientific literature (de Solla Price, 1963). According to Lee, (1990), Simple variations on exponential growth, such as linear trends in growth rates, have been particularly popular in making short term population forecasts, but seem clearly inadequate for longer term forecasts. Variations on logistic growth have also long been popular in estimating various population trends (Meyer and Ausubel, 1999), but also seem clearly inadequate for modeling very long term population trends (Cohen, 1995). Many

researchers have had more empirical success explicitly modeling long term population histories with functions that describe accelerating change.

According to Von Foerster et al, (1960), the first such efforts used simple hyperbolic, and soon became infamous warnings against taking one's model too literally, since they predicted an infinite human population in the early twenty first century.

More recent related efforts Kremer, 1993; Kapitza, (1996); Johansen and Sornette, (2000) have all included corrections which could describe a recent or upcoming slowdown and other complexities, mostly to allow growth to today be approaching fundamental limits, and hence a final end to accelerating growth. Some authors have also informally summarized world history as continually accelerating change, but the more common informal summary is of human history as a sequence of specific growth modes. According to Cipolla (1967), history has, for example, been described as the slow expansion of hunter-gatherers, followed by faster growth following the domestication of plants and animals, followed by even faster growth with commerce, science, and industry. Since historical periods tend to be described as exponential growth, formal models of transitions between periods tend to be models of transitions between exponential growth asymptotes. According to Helpman (1998), upcoming transitions, such as due to computers or the internet, have also been modeled this way. Given all this, it seems natural to formally model the long term history of humanity as a sequence of exponential growth modes. Yet to my knowledge no one has done this. Some authors seem to have come close, by drawing exponential curves suggestively next to data on log-log graphs of population growth (Deevey, 1960; Kates, 1996; Livi-Bacci, 1997). According to Kates (1996), some of these authors however, do not seem to

have realized that it could be simple exponentials they were drawing, and instead of “exponential growth followed by a period of approximate stability”.

Cohen, (1995) did more explicitly consider and reject a sum of exponentials model for world population history, though without any explicit model fitting or test. Lee, (1988) considered a sum of two exponentials, one of which was decreasing. Although many populations grow exponentially for a time, no bounded system can sustain exponential growth indefinitely unless the parameters or boundaries of the system are changed. According to Kingsland (1985), the most widely used modification of the exponential growth model is the logistic. It was introduced by Verhulst (1838) but popularized in mathematical biology by Lotka, (1956).

According to Anderson, (1988); Castillo et al. (1989); Thieme and Castillo, (1993); Hyman and Li, (1997), it is important to think that the development of models of intermediate complexity works as a conceptual bridge between simple and complex model as those which implement demographic and epidemiological structure. Thinking of a mindset, of a particular idea or ideology as an epidemic is by no means a new concept. In the book The Tipping Point (Gladwell, 2000), the author goes over much of the research that has been done over the past few decades into trend research and how ideas spread between people. Aunger (2002) discusses this phenomenon as it occurs culturally, while Chilton (2005) discusses a particular case of ideas as viruses, as conveyed in the text Mein Kampf. While the field of meme tics seems largely to be philosophical and difficult to understand in nature, the basic central tenet of idea-units that are contagious much as viruses are contagious is a valuable insight. Specifically, this insight is how a particular meme or set of memes, like Domestic Violence, might be

rightly thought of as an epidemic of ideas or a social epidemic that is raging through Tamale Metropolis.

A SI model, or other compartmentalized deterministic model of an epidemic, is based on modeling the rates involved in transmission of the disease, contact between members in the population, population growth, and rates of recovery. Hethcote (2000) discusses the mathematics involved in standard and more involved SI models. Trottier and Phillippe (2001, 2002, and 2003) discuss deterministic modeling, such as SI models, and the type of sensitivity analyses and repercussions of such tactics as immunization at birth. Li et al. (2001) discuss the relevance of adding vertical transmission into deterministic models, and show the resulting changes that occur within the differential equations that make up the core of the deterministic model. Modeling an epidemic is a useful thing to do in order to gain insight into the particular factors that make up that epidemic. Some diseases lend themselves, more than others, to being targeted for elimination from the global community. According to Kretzschmar et al, (2004), one of the ways to measure an epidemic's potential for an elimination strategy is to look at the rates defined in the deterministic model. Once a meaningful model for the epidemic of Domestic Violence ideology has been developed, it is likely that these rates could be defined. According to Berryman and Kindlmann, (2008), the first step in formulating a model is to delineate the major factors governing the real-world situation that is to be modeled. Conceptualizing the problem such that all key variables are accounted for, insofar as they reflect the mechanics of the observable phenomenon, is a good method for producing a testable model. Initial sketches of a model may be done using a flow chart diagram or pseudo code, which illustrates state variables and the nature of their connections.

### 2.5.1 Malthusian Model

According to the Malthusian model (Unlimited population growth) an elementary model of population growth is based on the assumption that; the rate of growth of the population is proportional to the size of the population. This implies that the rate of change of a population depend only on the size of the population and nothing else. The quantities evolving from the assumption are as follows;

$t$ =time (independent variable)

$P$ =population (dependent variable) and

$\alpha$ =proportionality constant (parameter) between the rate of growth of the population and the size of the population

Based on the assumption the rate of growth of population  $p$  is the derivative  $\frac{dP}{dt}$ , that is

$$\frac{dP}{dt} = \alpha P \quad (2.01)$$

This is a simple differential equation of first order with a solution given as

$$P(t) = P_0 e^{\alpha t} \quad (2.02)$$

The model was used in calculating the US population census from the year 1790 to 1920. From the data of the real system the model worked for the first fifty years and started deviating there after thereby giving room for some modification to the model in order to fit the data collected. It could be seen that the assumption (first) made for the modeling process accounted for the deviation after the first fifty years. Population growth does not depend solely on the size of the population but on other factors like the

land size, the health statute of a given population and the death and birth rate of the given population. To adjust the exponential model to account for the limited environment and other limited resources;

- ❖ If the population is assumed to be small; the rate of growth of the population is proportional to its size.
- ❖ If the population is too large to be supported by its environment and other resources, the population will decrease, that the growth rate is negative. From these assumptions we have  $t$ : time,  $P$ : Population,  $\alpha$ : growth-rate coefficient for small population (parameter) and  $N$ : usually called the carrying capacity introduced by the second assumption. In this condition, it is assumed that  $P_{(t)}$  is increasing if  $P_{(t)} < N$ . However if  $P_{(t)} > N$ , we assume that  $P_{(t)}$  is decreasing.

Using the above notation, the assumption could be formulated as

$$\frac{dP}{dt} = \alpha P \text{ if } P \text{ is small} \quad (2.03)$$

$$\text{If } P \geq N, \frac{dP}{dt} < 0 \quad (2.04)$$

To make the model algebraically simple, the exponential model is modified a little to become the **Verhulst Logistic Equation**.

$$\frac{dP}{dt} = (\text{something}) P \quad (2.05)$$

That is the “something” factor is made close to 1 if  $P$  is small, but if  $P > N$ , the term “something” is made to be negative. This gives the expression.

$(\text{Something}) = 1 - \frac{P}{N}$ . This expression equals 1 if  $P = 0$  and negative if  $P > N$ . Thus the model is

$$\frac{dP}{dt} = \alpha \left(1 - \frac{P}{N}\right) P \quad (2.06)$$

The differential equation (2.06) is called the Logistic population model with  $\alpha$ : growth rate,  $N$ : carrying capacity of the system. The equation (2.06) was obtained from the Malthusian model through some added assumption and modification. This is used to determine a real life occurrence and to predict future occurrence more correctly than the exponential growth model.

### 2.5.2 The Simple Epidemic Model (SI Model)

We divide the population into two groups:

- ❖ Susceptible individuals,  $S(t)$
- ❖ Infective individuals,  $I(t)$
- ❖ Carrying capacity,  $N$

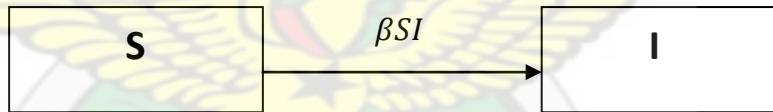


Figure (2.1)

#### Assumptions

- ❖ Population size is large and constant,  $S(t) + I(t) = N$
- ❖ No birth, death, immigration or emigration
- ❖ No recovery
- ❖ No latency
- ❖ Homogeneous mixing
- ❖ Infection rate is proportional to the number of infective, i.e.  $\beta I$

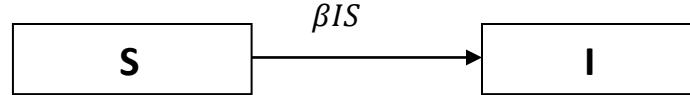


Figure (2.2)

A pair of ordinary differential equations describes this model:

$$\frac{ds}{dt} = -\beta I(t)S(t) \quad (2.07)$$

$$\frac{di}{dt} = \beta I(t)S(t) \quad (2.08)$$

But  $N = S(t) + I(t)$ , so this is equivalent to

$$S(t) = N - I(t) \quad (2.09)$$

Substituting into equation (2.08) gives:

$$\frac{di}{dt} = \beta I(t)[N - I(t)] \quad (2.10)$$

The differential equation is known as the **logistic growth equation**, proposed by Verhulst (1845) for population growth. By separation variables give

$$I(t) = \frac{NI(0)}{1+[N-I(0)]e^{-\beta Nt}} \quad (2.11)$$

## **CHAPTER 3**

### **METHODOLOGY**

#### **3.0 INTRODUCTION**

The purpose of this chapter is to develop a Differential Equation that will be used to predict the growth of Domestic Violence Against women and children. The adequacy of the model will be judge by empirically fitting it to the data collected from Domestic Violence and Victim Support Unit (DOVVSU) in Tamale.

#### **3.1 DATA**

In order to evaluate the descriptive adequacy of the model, we will obtain data sets on Domestic Violence against women and children. The data used for the model will be reported Domestic Violence cases at DOVVSU, Tamale police station. The data used will be yearly reported cases from 1999-2011.

#### **3.2 MODEL DEVELOPMENT**

##### **3.2.1 Malthusian Model**

According to the Malthusian model (Unlimited population growth) an elementary model of population growth is based on the assumption that; the rate of growth of the population is proportional to the size of the population. This implies that the rate of change of a population depend only on the size of the population and nothing else. Exponential functions come into play in situations in which the rate at which some

quantity grows or decays (i.e., increases or decreases over time) is proportional to the quantity present .The quantities evolving from the assumption are as follows;

$t \rightarrow \text{time}$  (Independent variable)

$N \rightarrow \text{Total Population size in Tamale metropolitan}$

$V \rightarrow \text{Population of Reported Domestic Violence Cases}$  (Dependent variable) and

$\alpha \rightarrow \text{Proportional constant (parameter), between the rate of growth of the population } N \text{ and the size of the population of Violence Victims } V.$   $\alpha$  tells us how fast the population of victims of Domestic Violence is changing at any given population level. It could be positive or negative. If  $\alpha$  is positive, it means the population of victims of increasing and if  $\alpha$  is negative, it means population of victims is decreasing.

Based on the assumption the rate of growth of Population of Domestic Violence Victims ( $V$ ) is the derivative  $\frac{dV}{dt}$ , implies

$$\frac{dV}{dt} = \alpha NV \quad (3.01)$$

By separation of variables,

$$\frac{dV}{V} = \alpha N dt$$

$$\int \frac{dV}{V} = \int \alpha N dt$$

$$\ln(V) = \alpha N t + c$$

$$V = e^c e^{\alpha N t}$$

$$V = Ce^{\alpha Nt}$$

At  $t = 0, V = V_0$  implies

$$V_0 = Ce^0, \quad C = V_0$$

Hence, the population growth of Domestic Violence Victims will be

$$V(t) = V e^{\alpha Nt}$$

The solution to the differential equation of first order  $\frac{dV}{dt} = \alpha NV$ , subject to  $V = V_0$ , at  $t = 0$  is given by;

$$V(t) = V_0 e^{\alpha Nt}$$

That is, if we know that the number of Domestic Violence Victims, at time  $t = 0$ , is  $V_0$  and that the rate of change is  $\frac{dV}{dt} = \alpha NV$ , then we will be able to find an expression for the number of Domestic Violence Victims present at time, (t) as

$$V(t) = V_0 e^{\alpha Nt} \quad (3.02)$$

And this model is called the exponential model. This model presents exponential growth without limit. However, in our real world, this case does not happen, because we need to consider environmental factors, food, drugs, law, education etc. I choose this model in order to start from the simplest model and based on it try to involve some factors step by step coming close to describing the actual situation and approach the goal.

### 3.2.2 The Logistic Growth Model

We will divide the population into two groups:

- ❖ Susceptible/Abusive individuals at time (t),  $S(t)$

- ❖ Domestic Violence Victim's at time (t),  $V(t)$
- ❖ Total population size,  $N$

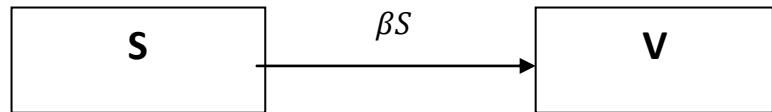


Figure (3.1)

### Assumptions of the Model

- ❖ Population size is large and constant,
- $$S(t) + V(t) = N \quad (3.03)$$
- ❖ No birth, death, immigration or emigration
  - ❖ No recovery
  - ❖ Homogeneous mixing
  - ❖ Violence spread rate is proportional to the number of Domestic Violence Victims, i.e.  $\beta V$



Figure (3.2)

A pair of ordinary differential equations will describes this model:

$$\frac{ds}{dt} = -\beta V(t)S(t) \quad (3.04)$$

$$\frac{dv}{dt} = \beta V(t)S(t) \quad (3.05)$$

But  $N = S(t) + V(t)$ , so this is equivalent  $S(t) = N - V(t)$  and substituting into equation (3.05) gives

$$\frac{dV}{dt} = \beta V(t)[N - V(t)]$$

The differential equation is known as the **Logistic Growth Model**.

We will have a nonlinear ODE,

$$\frac{dV}{dt} = \beta V(t)[N - V(t)] \quad (3.06)$$

And finally in the form

$$V(t)' = \beta V(t) \left(1 - \frac{V(t)}{N}\right) \quad (3.07)$$

First, the derivative will be zero at  $V(t) = N$ . Also; these are in fact solutions to the differential equation. These two values are called equilibrium solutions. If we start with a population of zero, there is no growth and the population stays at zero. If we start with a population in the range  $0 < V(t) < N$ , then from the differential equation we know that  $\frac{dV}{dt} > 0$  and hence  $V(t)$  is increasing. If we start at  $V(t) = N$ , the population stays at this level. Similarly, if we start with  $V > N$ , then  $\frac{dV}{dt} < 0$  and hence  $V(t)$  is decreasing.

Using the analysis we construct the following phase line diagram shown in the figure below.

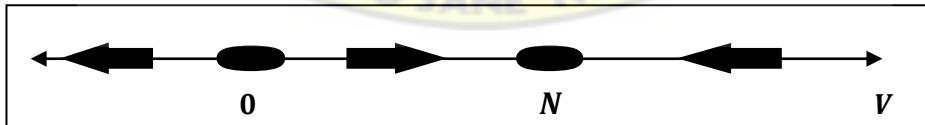


Figure (3.3)

From the above phase line diagram, we see that solutions tend toward the equilibrium at  $N$  and hence the solution  $V(t) = N$  is stable while the equilibrium at 0 is unstable. According to this model, if the population of victims of Domestic Violence is above 0, it will go to the carrying capacity  $N$  eventually.

Equation (3.06) is separable so we will divide,

$$\frac{1}{V(t)[N-V(t)]} \frac{dV}{dt} = \beta$$

And integrate,

$$\int \frac{1}{V(t)[N-V(t)]} \frac{dV}{dt} dt = \int \beta dt$$

Let  $u = V(t)$ , implies  $du = dV$

$$\int \frac{1}{u[N-u]} du = \int \beta dt$$

$$\int \left[ \frac{1}{u} + \frac{1}{N-u} \right] du = \int \beta dt$$

$$\ln(u) - \ln(N-u) = \beta t + C$$

$$\ln\left(\frac{u}{N-u}\right) = \beta t + C$$

$$\frac{u}{N-u} = e^{\beta t+C}$$

$$\frac{u}{N-u} = Ce^{\beta t}$$

$$\text{But } u = V(t)$$

$$\frac{V(t)}{N-V(t)} = Ce^{\beta t}$$

$$V(t) = [N - V(t)]Ce^{\beta t}$$

$$V(t) = NCe^{\beta t} - V(t)Ce^{\beta t}$$

$$V(t) + V(t)Ce^{\beta t} = NCe^{\beta t}$$

$$V(t)[1 + Ce^{\beta t}] = NCe^{\beta t}$$

$$V(t) = \frac{NCe^{\beta t}}{1+Ce^{\beta t}} \quad (3.08)$$

At  $t=0$

$$V(0) = \frac{NCe^0}{1+Ce^0}$$

$$V(0) = \frac{NC}{1+C}$$

$$C = \frac{V(0)}{N-V(0)}$$

From equation (3.07), we have

$$V(t) = \frac{N\left(\frac{V(0)}{N-V(0)}\right)e^{\beta t}}{1+\left(\frac{V(0)}{N-V(0)}\right)e^{\beta t}}$$

$$V(t) = \frac{\frac{NV(0)e^{\beta t}}{N-V(0)}}{\frac{N-V(0)+V(0)e^{\beta t}}{N-V(0)}}$$

$$V(t) = \frac{NV(0)e^{\beta t}}{N-V(0)+V(0)e^{\beta t}}$$

$$V(t) = \frac{NV(0)e^{\beta t}}{N-V(0)+V(0)e^{\beta t}}$$

$$V(t) = \frac{NV(0)}{V(0)+[N-V(0)]e^{-\beta t}} \quad (3.09)$$

Analyzing the solution above, we see that as time increases, the size of the population of victims reaches a finite limit. Mathematically, as  $t \rightarrow +\infty$ ,  $V \rightarrow N$  and almost all women and children becomes victims of Domestic Violence.

### 3.2.3 The Violence Epidemic Model (ASV)

The ASV model just as the SIR Model in epidemiology will be used for the analysis. The victims will be mainly restricted to Tamale Metropolis. Therefore, we assume that the committing of Domestic Violence occurs mostly within a high-risk population group, such as men, women and children. Besides, Domestic Violence Victims, who are at the end-stage of the Domestic Violence infection, are strictly isolated from Abusive and susceptible population; therefore, they no longer cause any form of Violence. To simplify a real problem, we assume;

#### **Assumptions:**

- ❖ A particular population, which is reasonably restricted, is at high-risk to Domestic Violence by contact only.

- ❖ The population is uniformly mixed, so the probability of a person being a victim of Domestic Violence equally exists to every single individual within the community.
- ❖ Once victims (Women and Children) are classified into Domestic Violence Victims, they are no longer engaged in the spread of the violence.
- ❖ There are no subtractions of the population except for Violence-induced death.

### **3.2.3.1 Formulated Mathematical Model**

For the Domestic Violence victim modeling, we will consider a group of susceptible individuals; therefore a number of susceptible individuals (Women and Children) are introduced into a larger susceptible population. Besides, the infected individuals develop themselves into a group of Domestic Violence Victims, the end-stage of the violence. The population of each group will change respect to time while the violence progresses. Therefore, we will think of the change of the population as a mass balance idea,

**Rate of change in Domestic Violence population = population growth – population loss.**

Then, the terms for the population growth and population loss can be defined by the interaction of each population group in the transmission of violence. To mathematically translate, we will introduce a system of ordinary differential equations. Let's declare variables before creating ODEs.

### **3.3.3.2 Definition of Variables and Parameters**

- ❖  $N$  - Total population of a community

- ❖  $\mu$  - Recruitment rate to a population
- ❖  $\beta$  - Conversion rate (from Susceptible to Victim)
- ❖  $S(t)$  – Number of Susceptible individuals/unreported domestic violence victims at time (t)
- ❖  $V(t)$  – Number of Reported Domestic Violence Victim's at time (t)
- ❖  $A(t)$  – Number of Abusive individuals at time (t)
- ❖  $\alpha$  – Susceptible/Unreported Victims rate
- ❖  $\delta$  - Violence-induced death rate

The number of each population changes in time, so it can be expressed as a function of time, and the total population,  $N(t)$ , consists of three sub-classes,  $A(t)$ ,  $S(t)$ , and  $V(t)$ .

First, let's consider the rate of change in the Susceptible individuals,  $S(t)$ , which represents all women and children in the population who are exposed to Domestic Violence. Since there is a population growth at a rate,  $\mu$ , new incomers will immediately belong to the Abusive group,  $A(t)$ . Then, these individuals may be Susceptible by violent contact with in the Abusive group. Note that the infection occurs at a rate proportional to the number of the Susceptible and Abusive; that is  $\alpha AS$ , where a universal parameter,  $\alpha$ , represents all infective factors according to our assumptions. Thus, once individuals are infected, there will be the subtraction of population at the rate,  $\alpha AS$ , from  $A(t)$ . Along with the population growth at the rate,  $\mu$ , we finally have the equation

$$\frac{dA}{dt} = \mu - \alpha A(t)S(t).$$

Simultaneously, the infection will also invoke addition to  $S(t)$  at the same rate,  $\alpha AS$ . In the same manner, the development of Susceptible individuals,  $S(t)$ , into Domestic Violence Victim at a certain rate will result the

subtraction of population from  $S(t)$  and the addition of population to the Domestic Violence Victim,  $V(t)$ . Let  $\beta$  be the conversion rate from the Susceptible to Domestic Violence Victim, then the subtraction and the addition from  $S(t)$  to  $V(t)$  can be expressed as  $\beta S$ . Finally, Domestic Violence Victim may die at a rate  $\delta$ , which is the violence-induced death rate. As a result, a simple model can be formed as a system of differential equations,

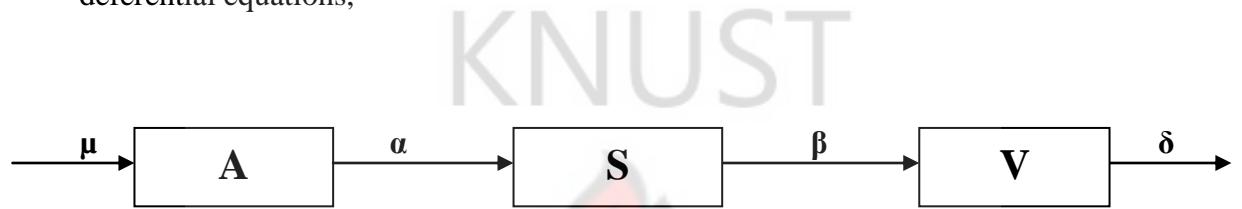


Figure (3.4)

$$\frac{dA}{dt} = \mu - \alpha A(t)S(t) \quad (3.10)$$

$$\frac{ds}{dt} = \alpha A(t)S(t) - \beta S(t) \quad (3.11)$$

$$\frac{dV}{dt} = \beta S(t) - \delta V(t) \quad (3.12)$$

We will note that the system of the differential equations is a nonlinear 3-dimensional system. Also, we will note that equation (3.10) is an inhomogeneous equation since it has a recruitment rate  $\mu$  per unit time to the Abusive class. As a result, the total population  $N(t) = A(t) + S(t) + V(t)$  will not be constant.

The  $\mu$  term is much like a “forcing term” which often arises from an external force in a physical application. The reason that we need to mention the  $\mu$  term will be explained later when we analyze the system.

## CHAPTER 4

### DATA ANALYSIS, DISCUSSION AND MODELLING

#### 4.1 ANALYSIS OF THE EPIDEMIC (ASV) MODEL

Unfortunately, we cannot explicitly solve the system of the differential equations, (3.10), (3.11) and (3.12). Instead, we can import an analytic approach to study the behavior of the infection. In the system, (3.10), (3.11) and (3.12),  $\frac{dA}{dt}$  and  $\frac{dS}{dt}$  are independent on Domestic Violence Victims,  $V$ . In other words, Domestic Violence Victims no longer affect the primary infection, so we will consider  $\frac{dA}{dt}$  and  $\frac{dS}{dt}$  only. Then, we have an equivalent 2-dimensional system for our model,

$$\frac{dA}{dt} = \mu - \alpha AS \quad (4.01)$$

$$\frac{dS}{dt} = \alpha AS - \beta S \quad (4.02)$$

From  $\frac{dA}{dt} = \mu - \alpha AS$

But  $\frac{dA}{dt} = 0$

$$0 = \mu - \alpha AS$$

$$\alpha AS = \mu$$

$$A = \frac{\mu}{\alpha S} \quad (4.03)$$

Also from,  $\frac{dS}{dt} = \alpha AS - \beta S$

$$\frac{dS}{dt} = 0$$

$$0 = \alpha AS - \beta S$$

$$\beta S = \alpha AS \Rightarrow A^* = \frac{\beta}{\alpha} \quad (4.04)$$

Therefore, from equation (4.03)

$$\alpha AS = \mu$$

$$\alpha S \left( \frac{\beta}{\alpha} \right) = \mu$$

$$\beta S = \mu \Rightarrow S^* = \frac{\mu}{\beta} \quad (4.05)$$

Primarily, we're interested in equilibrium, if any, at which both  $A$  and  $S$  do not change; that is, neither increasing nor decreasing. For that, we need to find  $A^*$  and  $S^*$  such that

both  $\frac{dA}{dt} = 0$  and  $\frac{dS}{dt} = 0$ . To do that, we will look at the intersection of  $A$  and  $S$ .

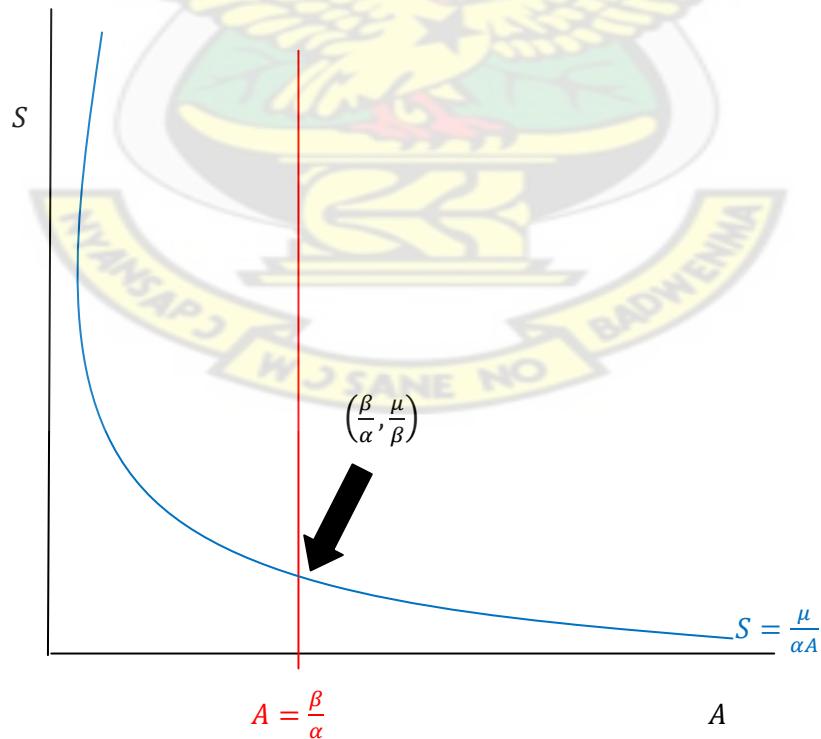


Figure (4.1)

We have one equilibrium point  $E(A^*, S^*) = \left(\frac{\beta}{\alpha}, \frac{\mu}{\beta}\right)$  in the first quadrant. Thus,

when  $A^* = \beta/\alpha$  and  $S^* = \mu/\beta$ , the infection results a condition in which there is no

change in the population of the Abusive individuals and the Susceptible and unreported victims of domestic violence. However, this is not quite enough information to predict the behavior of the violence. We need to know whether the equilibrium is stable or unstable. Since the system is nonlinear, we need to linearize the system at the equilibrium  $E(A^*, S^*)$  to determine its stability by taking the *Jacobian* matrix, which is defined as,

$$J(A^*, S^*) = \begin{pmatrix} \frac{\partial f}{\partial A} & \frac{\partial f}{\partial S} \\ \frac{\partial g}{\partial A} & \frac{\partial g}{\partial S} \end{pmatrix}_{|(A^*, S^*)}$$

Where,

$$\frac{dA}{dt} = f(A, S) \text{ and } \frac{dS}{dt} = g(A, S)$$

Then, the linearization of the system at the equilibrium  $E$  can be calculated as,

$$\frac{\partial f}{\partial A} = -\alpha S \text{ and } \frac{\partial f}{\partial S} = -\alpha A$$

$$\frac{\partial g}{\partial A} = \alpha S \text{ and } \frac{\partial g}{\partial S} = \alpha A - \beta$$

$$J(A, S) = \begin{pmatrix} -\alpha S & -\alpha A \\ \alpha S & \alpha S - \beta \end{pmatrix}$$

$$J\left(\frac{\beta}{\alpha}, \frac{\mu}{\beta}\right) = \begin{pmatrix} -\frac{\alpha\mu}{\beta} & -\beta \\ \frac{\alpha\mu}{\beta} & 0 \end{pmatrix}$$

$$|A - \lambda I| = 0$$

$$\left| \begin{pmatrix} -\alpha\mu & -\beta \\ \frac{\beta}{\alpha\mu} & 0 \\ \frac{\alpha\mu}{\beta} & 0 \end{pmatrix} - \lambda \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \right| = 0$$

$$\begin{vmatrix} -\alpha\mu - \lambda & -\beta \\ \frac{\beta}{\alpha\mu} & -\lambda \end{vmatrix} = 0$$

$$-\lambda \left( \frac{-\alpha\mu}{\beta} - \lambda \right) + \alpha\mu = 0$$

$$\lambda^2 + \left( \frac{\alpha\mu}{\beta} \right) \lambda + \alpha\mu = 0$$

From the general Quadratic formula,

$$\lambda = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\text{Where } a = 1, b = \frac{\alpha\mu}{\beta}, c = \alpha\mu$$

$$\lambda = \frac{-\frac{\alpha\mu}{\beta} \pm \sqrt{\left(\frac{\alpha\mu}{\beta}\right)^2 - 4\alpha\mu}}{2} \quad (4.06)$$

By looking at the eigenvalues of this matrix, we can determine the stability of the original nonlinear system at the equilibrium. This implies the original nonlinear system would behave like the linearized system at the equilibrium. The eigenvalues of the *Jacobian*,  $\lambda$  calculated as,

$$\lambda = \frac{-\frac{\alpha\mu}{\beta} \pm \sqrt{\left(\frac{\alpha\mu}{\beta}\right)^2 - 4\alpha\mu}}{2}$$

Since the eigenvalues,  $\lambda$  are determined by the parameters, we need to break down into two cases to determine the stability of the equilibrium.

$$\text{First, When } \frac{\alpha\mu}{\beta^2} \geq 4 \quad (4.07)$$

The eigenvalues  $\lambda$  have two negative real parts since we know,

$$\frac{\alpha\mu}{\beta} > \sqrt{\left(\frac{\alpha\mu}{\beta}\right)^2 - 4\alpha\mu} > 0.$$

The behaviour of the Domestic Violence Spread at the equilibrium is shown in the figure below.

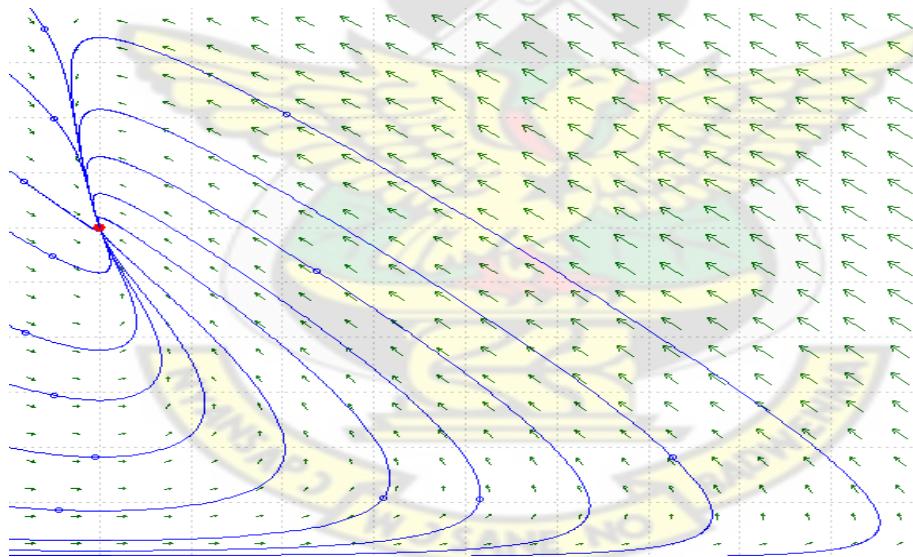


Figure (4.2)

Hence the system forms a nodal sink at the equilibrium. That implies the system have the behavior of exponential decay at the equilibrium,  $E(A^*, S^*) = \left(\beta/\alpha, \mu/\beta\right)$  which is

asymptotically stable. Thus, every solution of the system will approach to the stable equilibrium point,  $A^* = \beta/\alpha$ , and  $S^* = \mu/\beta$ , as time  $t$  goes to infinity.

Secondly, When  $\frac{\alpha\mu}{\beta^2} < 4$  (4.08)

The behaviour of the Violence at equilibrium is shown in the figure below.

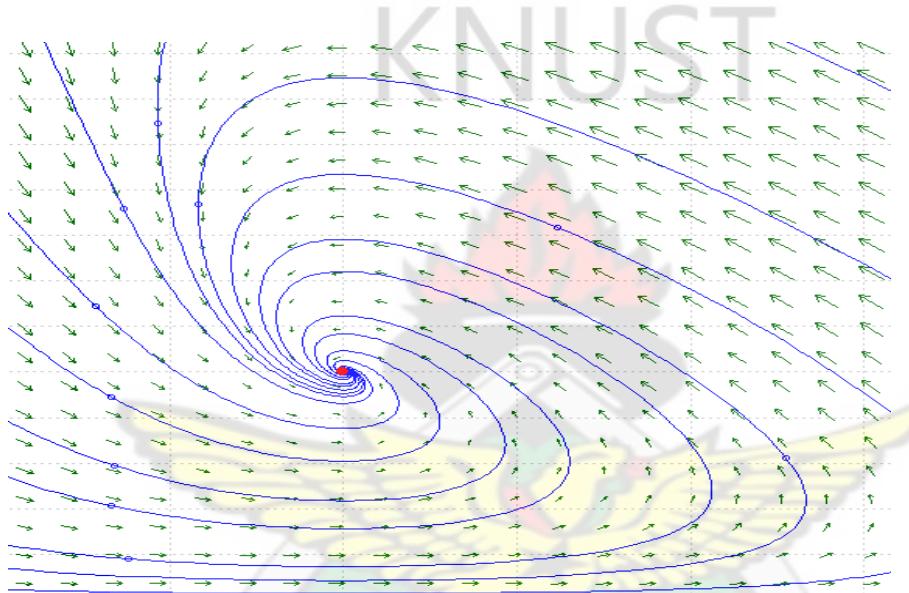


Figure (4.3)

The eigenvalues,  $\lambda$ , have complex parts along with a negative real part, hence the system forms a spiral sink at the equilibrium. That means the system have a certain form of

oscillation behavior at equilibrium,  $E(A^*, S^*) = \left(\beta/\alpha, \mu/\beta\right)$ , while its solutions still

move to stable direction. In other words, the system behaves in a damped oscillatory manner with a certain period determined by the parameters. Given the model parameters, the period of the oscillation plays a role for us to predict the further behavior of the infection.

Interestingly, in either case, we can observe a stable equilibrium point,  $A^* = \beta/\alpha$ , and  $S^* = \mu/\beta$ , where the violence would have a steady state. Now, an interesting question comes out. The parametric conditions with inequality, (4.07) and (4.08), come purely from the mathematical analysis and we have no clear insight about how they are related to the behaviour of the violence spread. Keeping this in mind, we will reconsider the system of the differential equations, (4.01) and (4.02), which originally gives us a general picture of the violence interaction. If we nondimensionalized the system,

$$\frac{dA}{dt} = \mu - \alpha AS$$

$$\frac{dS}{dt} = \alpha AS - \beta S$$

With the characteristic scales  $c_A = \frac{\beta}{\alpha}$  for the Abusive class A,  $c_S = \frac{\beta}{\alpha}$  for the Susceptible and Domestic Violence Victim class S, and  $c_T = \frac{1}{\beta}$  for Time t, we end up with a nondimensionalized system of the differential equations as the following.

$$\frac{d\tilde{A}}{d\tilde{t}} = \frac{\alpha\mu}{\beta^2} - \tilde{A}\tilde{S} \quad (4.09)$$

$$\frac{d\tilde{S}}{d\tilde{t}} = \tilde{S}(\tilde{A} - 1) \quad (4.10)$$

Where  $\frac{\alpha\mu}{\beta^2}$  is a free parameter,  $\rho$ .

Now, we can easily notice that the free parameter,  $\rho$ , in the nondimensionalized system is same as the parametric conditions, (4.07) and (4.08). That means the overall behaviour of the violence can be observed by changing the free parameter,  $\rho = \frac{\alpha\mu}{\beta^2}$  with the nondimensionalized equations. If we simply look at the nondimensionalized violence epidemic model, the free parameter,  $\rho$ , can be interpreted as recruitment to the

population of a Violence-high-risk Tamale metropolitan,  $N$ ; of course, the recruitment then immediately belongs to the Abusive class,  $A$ , as the violence spread is in progress. This phenomenon is similar to the fact that the original differential equations, (4.01) and (4.02), has the  $\mu$  term, which represents new entrance to the Abusive class per unit time. Note that, in the dimensionless system of differential equations, the dimension of  $\mu$ ,

$$[\mu] = \frac{P}{T}, \text{ where } [A] = [S] = P \text{ and } [t] = T.$$

Recall the “*forcing term*” that we mentioned earlier. It becomes now clearer that the  $\mu$  term acts like a “*forcing term*” which makes the behavior of the violence change. In fact, it is more accurate to say the free parameter,  $\rho = \frac{\alpha\mu}{\beta^2}$ , determines the behavior of the overall violence epidemic. The below figures visually present the different behavior of the violence spread at the equilibrium.

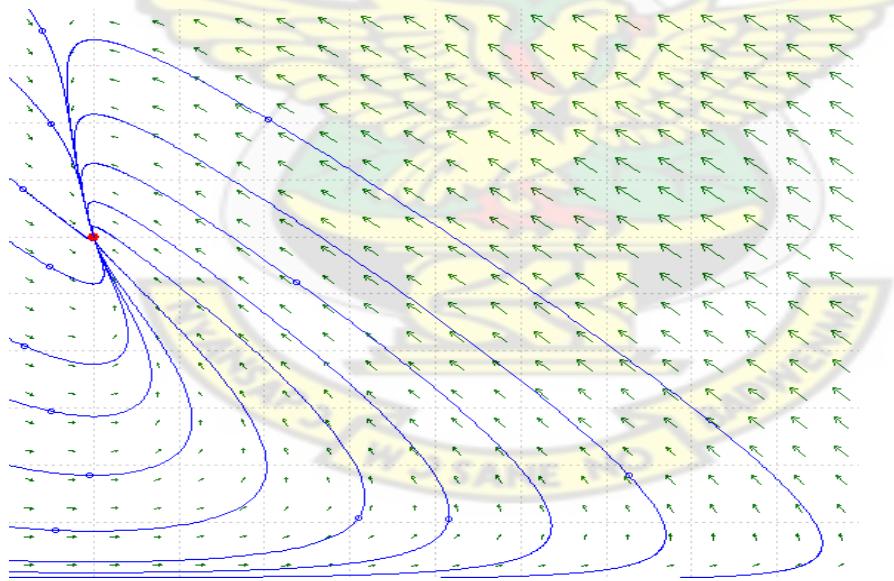


Figure (4.2) when  $\rho \geq 4$ , the solutions in the phase-plane move to the nodal sink equilibrium,  $(A^*, S^*) = \left(\frac{\beta}{\alpha}, \frac{\mu}{\beta}\right)$ .

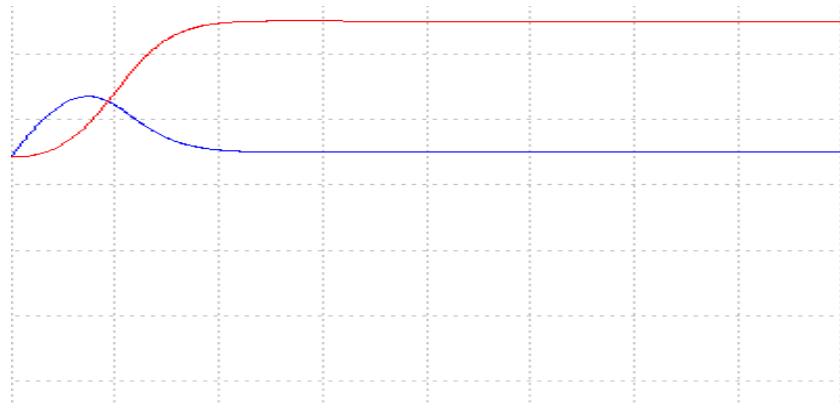


Figure (4.4) when  $\rho \geq 4$ , the curve below ( $A$ ) and the curve above ( $S$ ) approach to a steady state of the violence as  $t \rightarrow \infty$ .

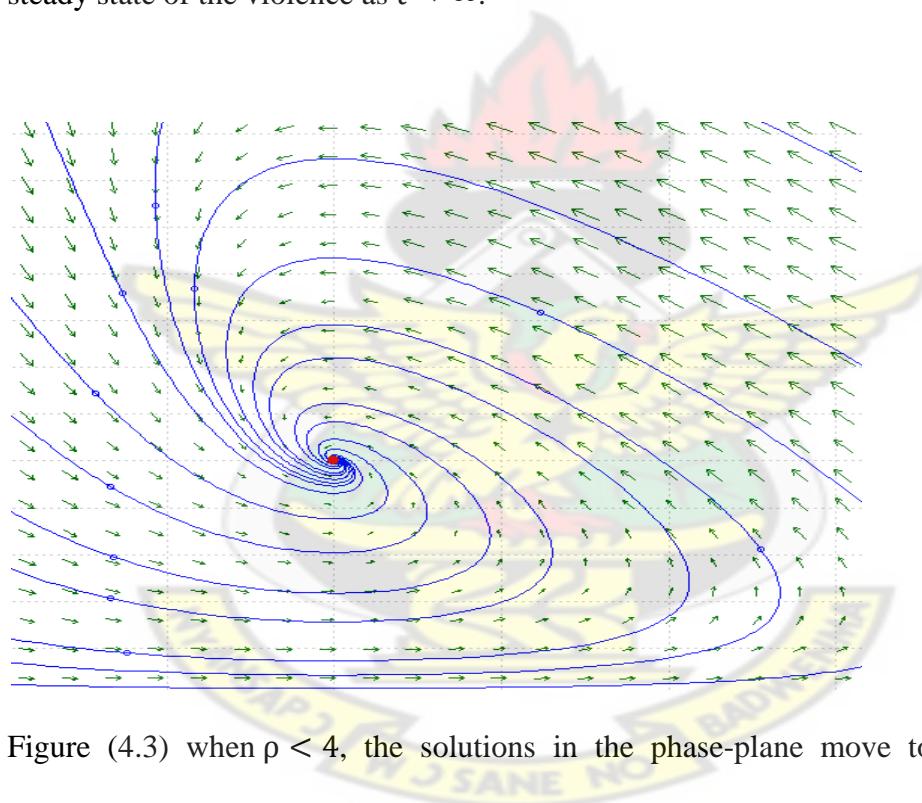


Figure (4.3) when  $\rho < 4$ , the solutions in the phase-plane move to the spiral sink equilibrium,  $E(A^*, S^*) = \left(\frac{\beta}{\alpha}, \frac{\mu}{\beta}\right)$ .

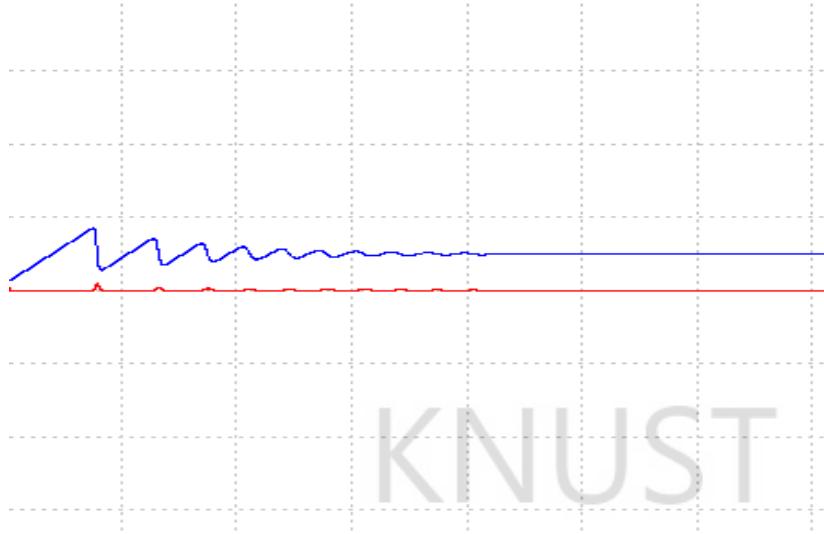


Figure (4.5) when  $\rho < 4$ , the curve above ( $A$ ) and the curve below ( $S$ ) approach to a steady state, in oscillation manner, as  $t \rightarrow \infty$

The general behavior of the violence epidemic as shown in the figures is predicted under the initial condition of the violence epidemic, starting close enough to the equilibrium point, and as time goes to infinity. However, what happens the very beginning of the violence epidemic? The question might be a more realistic concern especially for the short history of Domestic Violence infection reflected on the real data in table (4.1).

Consider the differential equation (4.02) of the system,

$$\frac{dS}{dt} = \alpha AS - \beta S$$

At the very beginning of the violence epidemic, the populations of the Susceptible individuals( $S$ ) and the Domestic Violence Victims( $V$ ) are almost negligible compared to the Abusive individuals( $A$ ). Therefore,  $N = A + S + V$ , becomes  $N \approx A$ . Besides, the relatively long incubation time of Domestic Violence to the insignificance of the term  $-\beta S$  which represent the conversion from the Susceptible to Domestic Violence Victim. Ignorance of Domestic Violence with most cases not been reported could also cause

$-\beta S \approx 0$  during that particular time span. Now, according to these assumptions, we have an equivalent differential equation,

$$\frac{ds}{dt} = \alpha NS \quad (4.11)$$

By inspection, we can easily obtain the solution of the equation (4.09) is  $S = S_0 e^{\alpha N t}$ .

Clearly,  $\alpha > 0$  as long as the epidemic is in effect and  $N > 0$  since it's a population. Therefore, no matter how small the parameters are, the domestic Violence spread will exponentially grow in the beginning of the epidemic. This is what's actually happening in the real world at least in last 13 years according to the data from DOVVSU.

## 4.2 ANALYSIS OF THE LOGISTIC GROWTH MODEL

### 4.2.1 Graph the Data

Considering the population sizes for domestic Violence victims in Tamale for the years between 1999 and 2011, we will derive a mathematical model for the Victims. Using the data below, we can plot the graph below.

<b>YEARS</b>	<b>CASES</b>
1999	15
2000	27
2001	53
2002	94
2003	128
2004	142
2005	158
2006	161
2007	172
2008	174
2009	181
2010	187
2011	191

Table (4.1)

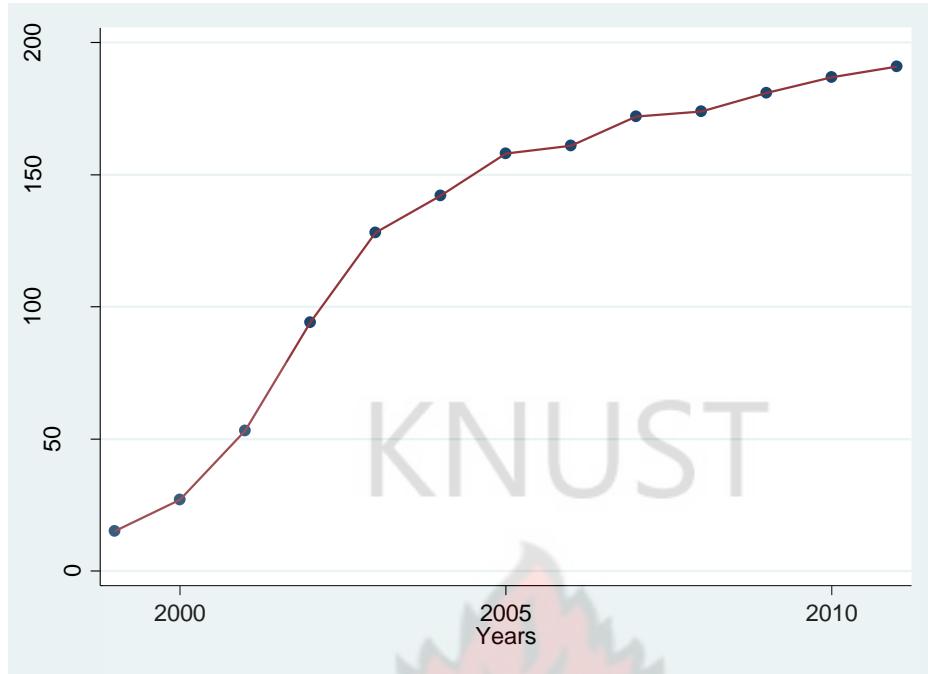


Figure (4.6)

#### 4.2.2 Logistic Model for the Data

Let's consider that the logistic growth model with form:

$$V(t)' = \beta V(t) \left(1 - \frac{V(t)}{N}\right) \quad (4.12)$$

In order to show that model (4.01) is logistic, we need to focus on the following questions:

- 1) How to tell whether a given set of data is reasonably logistic?
- 2) What parameter  $\beta$  and  $N$  will be good fit?

Since we have discrete data, then we describe the model using a difference equation. We use previous values from the system to calculate the new ones. The equation (4.12) can be expressed by the difference equation version as the following equation:

$$V(t+1) - V(t) = \beta V \left(1 - \frac{V}{N}\right) \quad (4.13)$$

It can be rewritten as:

$$\frac{\Delta V}{V} = \beta \left(1 - \frac{V}{N}\right) \quad (4.14)$$

The equation (4.14) says that the ratio of  $\Delta V$  and  $V$  is a linear function of  $V$ .

Now we have testing of logistic behaviour for the model:

First of all, let's consider the left hand side (LHS) of equation (4.14). We calculate the difference of the populations for two consecutive years, and then use those differences against the corresponding function values. Next, we plot the ratios and the corresponding function values. At last, if we can show that the plots are approximately linear, then the model equation (4.14) is reasonable. That is to say, the model has the form (4.12) and it is Logistic.

Calculating the ratios on the left hand side of (4.12) yields:

$$1. \quad a_1 = \frac{V(2000)-V(1999)}{V(1999)} = \frac{27-15}{15} = 0.8000;$$

$$2. \quad a_2 = \frac{V(2001)-V(2000)}{V(2000)} = \frac{53-27}{27} = 0.9629;$$

$$3. \quad a_3 = \frac{V(2002)-V(2001)}{V(2001)} = \frac{94-53}{53} = 0.7736;$$

$$4. \quad a_4 = \frac{V(2003)-V(2002)}{V(2002)} = \frac{128-94}{94} = 0.3617;$$

$$5. \quad a_5 = \frac{V(2004)-V(2003)}{V(2003)} = \frac{142-128}{128} = 0.1094;$$

$$6. \quad a_6 = \frac{V(2005)-V(2004)}{V(2004)} = \frac{158-142}{142} = 0.1127;$$

$$7. \quad a_7 = \frac{V(2006)-V(2005)}{V(2005)} = \frac{161-158}{158} = 0.0189;$$

$$8. \quad a_8 = \frac{V(2007)-V(2006)}{V(2006)} = \frac{172-161}{161} = 0.0683;$$

$$9. \quad a_9 = \frac{V(2008)-V(2007)}{V(2007)} = \frac{174-172}{172} = 0.0116;$$

$$10. a_{10} = \frac{V(2009)-V(2008)}{V(2008)} = \frac{181-174}{174} = 0.0402;$$

$$11. a_{11} = \frac{V(2010)-V(2009)}{V(2009)} = \frac{187-181}{181} = 0.0331;$$

$$12. a_{12} = \frac{V(2011)-V(2010)}{V(2010)} = \frac{191-187}{187} = 0.0214;$$

Thus, we have the following list of data:

$a$	$V(t)$
0.8000	15
0.9629	27
0.7736	53
0.3617	94
0.1094	128
0.1127	142
0.0189	158
0.0683	161
0.0116	172
0.0402	174
0.0331	181
0.0214	187

Table (4.2)

Plotting the Least Square approximation graph by using the data from Table (4. 2) obtain following graph.

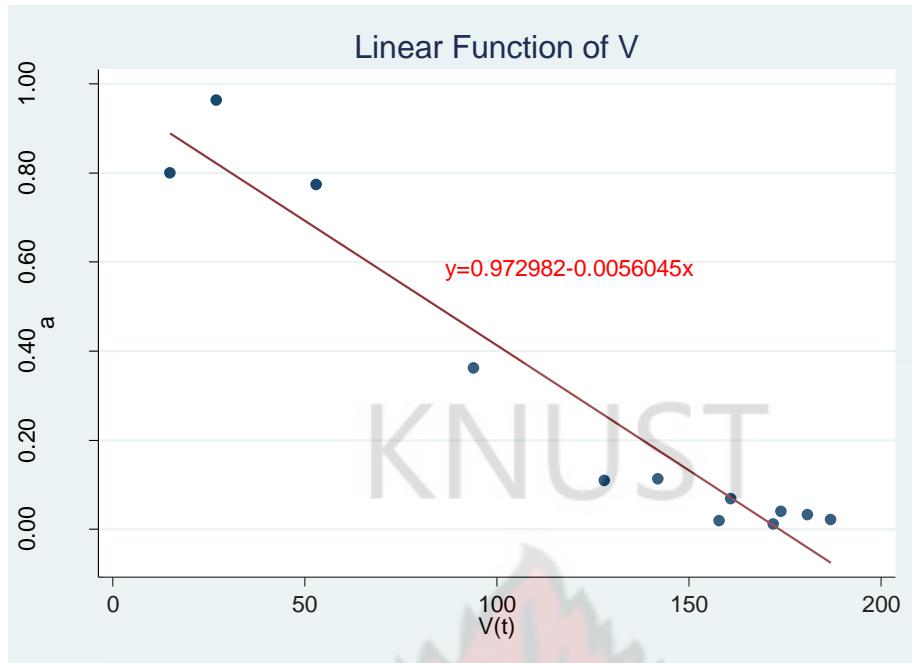


Figure (4.7)

As we can see in Figure (4.7), at various cases of Domestic Violence Victim plotted levels  $V(t)$  at time  $t$ , we can calculate corresponding ratios  $a$ . Based on these points we plot Least Square Approximation graph.

Looking at the graph, we can see that most of the data points are close to this line. The overall resulting plot is approximately linear. Therefore, our assumption for the equation (4.12) is reasonable. That is the present model (4.12) shows that the given data is logistic.

#### 4.2.3 Determining the Values of $\beta$ and $N$

In the Least Square Approximation graph, figure (4.7), we know the equation for the line, which is,

$$y = 0.972982 - 0.0056045x \quad (4.15)$$

Substituting the point  $V(1999)$  into this equation, we obtain,

$$y_1 = 0.972982 - 0.0056045(15)$$

$$y_1 = 0.972982 - 0.084068$$

$$y_1 = 0.888915$$

Similarly, substituting  $V(2000)$  into it, we obtain

$$y_2 = 0.972982 - 0.0056045(27)$$

$$y_2 = 0.972982 - 0.151322$$

$$y_2 = 0.821661$$

That is to say, we can get values of the ratio,  $\alpha$ , where  $y = \alpha = \frac{\Delta V}{V}$ .

Then we have,

$$y_1 = 0.0888915 \text{ and } y_2 = 0.821661$$

Substituting the data of 1999 and 2000 to the equation (4.14) gives,

$$0.888915 = \beta \left(1 - \frac{15}{N}\right) \quad (4.16)$$

$$0.821661 = \beta \left(1 - \frac{27}{N}\right) \quad (4.17)$$

Suppose that  $\beta, N \neq 0$ , and divide equation (4.16) by equation (4.17), we can get that:

$$\frac{0.888915}{0.821661} = \frac{\beta \left(1 - \frac{15}{N}\right)}{\beta \left(1 - \frac{27}{N}\right)}$$

$$1.08185 = \frac{1 - \frac{15}{N}}{1 - \frac{27}{N}}$$

$$1.08185 = \frac{\frac{N-15}{N}}{\frac{N-27}{N}}$$

$$1.08185 = \frac{N-15}{N-27}$$

$$1.08185(N-27) = N-15$$

$$1.08185N - 29.21 = N - 15$$

$$0.08185N = 14.21$$

$$N = \frac{14.21}{0.08185}$$

$$N = 173.6103$$

$$N \approx 174$$

From equation (4.16), we obtain the value of  $\beta$ ,

$$0.888915 = \beta \left(1 - \frac{15}{174}\right)$$

$$0.888915 = \beta(1 - 0.086207)$$

$$0.888915 = 0.913793\beta$$

$$\beta = \frac{0.888915}{0.913793}$$

$$\beta = 0.972775$$

Therefore, the model is

$$V(t)' = 0.972775V \left(1 - \frac{V}{174}\right) \quad (4.18)$$

As we know, the size for the number of cases tends to the carrying capacity  $N$ . In this case, the size is bounded by 174 victims of Domestic Violence. In another words, the limiting number for this population model is 174 Victims.

#### 4.2.4 The Solution for the Logistic Model

Rewriting equation (4.18), we have

$$\frac{dV}{dt} = 0.972775V \left(1 - \frac{V}{174}\right)$$

$$\frac{dV}{dt} = 0.972775V - 0.005591V^2 \quad (4.19)$$

By separation of variables, equation (4.19) gives

$$\int \frac{dV}{V(0.972775 - 0.005591V)} = t + c \quad (4.20)$$

Also because

$$\frac{1}{V(0.972775 - 0.005591V)} = \frac{1}{0.972775} \left( \frac{1}{V} + \frac{0.005591}{0.972775 - 0.005591V} \right)$$

The equation (4.20) can be written as

$$\frac{1}{0.972775} \int \left( \frac{1}{V} + \frac{0.005591}{0.972775 - 0.005591V} \right) dV = t + c$$

Let  $t = 0$  corresponds to the size of cases in 1999, 15. Then we have

$$V_0 = 15$$

Using the condition  $V_0 = 15$  at  $t = 0$ , we get

$$c = \frac{1}{0.972775} [\ln(15) - \ln(0.972775 - 0.005591(15))]$$

$$c = \frac{1}{0.972775} [2.70805 - \ln(0.88891)]$$

$$c = \frac{1}{0.972775} [2.70805 + 0.117759]$$

$$c = \frac{1}{0.972775} [2.82581]$$

$$c = 2.90489$$

Thus equation (4.20) becomes

$$\frac{1}{0.972775} (\ln V - \ln(0.972775 - 0.005591V)) = t + 2.90489$$

$$\ln V - \ln(0.972775 - 0.005591V) = 0.972775t + 2.8258$$

$$\ln \left( \frac{V}{0.972775 - 0.005591V} \right) = 0.972775t + 2.8258$$

$$\frac{V}{0.972775 - 0.005591V} = e^{0.972775t + 2.8258}$$

$$\frac{V}{0.972775 - 0.005591V} = 16.9201e^{0.972775t}$$

$$V = 16.9201e^{0.972775t}(0.972775 - 0.005591V)$$

$$V = 16.4595e^{0.972775t} - 0.0946Ve^{0.972775t}$$

$$V(1 + 0.0946e^{0.972775t}) = 16.4595e^{0.972775t}$$

$$V = \frac{16.4595e^{0.972775t}}{1+0.0946e^{0.972775t}}$$

Dividing through by  $0.0946e^{0.972775t}$  gives

$$V = \frac{173.9905}{1+10.5708e^{-0.972775t}} \quad (4.21)$$

If we take the limit of solution (4.21) as  $t \rightarrow \infty$ , we can see that,  $V(t) \rightarrow 174$ , this shows that there is a limit to the growth of  $V$ . The limiting number is 174 Victims. From equation (4.21), we can get the predicted cases for each year. The following table shows time( $t$ ), actual cases( $V$ ) and predicted cases( $V(t)$ ) for Domestic Violence against women and children.



Time (t)	Actual Cases V	Predicted Cases V(t)
0	15	15.0379
1	27	34.8271
2	53	69.3045
3	94	110.5040
4	128	143.1060
5	142	160.8710
6	158	168.7920
7	161	171.9940
8	172	173.2360
9	174	173.7100
10	181	173.8900
11	187	173.9590
12	191	173.9840
13		173.9940
14		173.9980
15		173.9990
16		174
17		174
18		174

Table (4.3), as  $t \rightarrow \infty$ , we can see that,  $V(t) \rightarrow 174$ .

Assuming now, the carrying capacity,  $N = 1000$ . Then from equation (4.16) we obtain the value of  $\beta$ ;

$$0.888915 = \beta \left(1 - \frac{15}{1000}\right)$$

$$0.888915 = \beta(1 - 0.015)$$

$$0.888915 = 0.985\beta$$

$$\beta = \frac{0.888915}{0.985}$$

$$\beta = 0.902452$$

Then, the model becomes

$$V(t)' = 0.902452V \left(1 - \frac{v}{1000}\right) \quad (4.22)$$

When solve gives

$$V(t) = \frac{1000.4900}{1+65.6987e^{-0.902452t}} \quad (4.23)$$

From equation (4.25), we obtain the predicted cases of Domestic Violence in the table below;

Time (t)	Predicted Cases $V(t)$
0	14.992
1	36.172
2	84.697
3	185.772
4	360.021
5	581.073
6	773.754
7	893.983
8	954.110
9	980.866
10	992.151
11	996.802
12	998.700
13	999.472
14	999.786
15	999.913
16	999.965
17	999.986
18	999.994
19	999.998
20	999.999

21	1000.000
22	1000.000

Table (4.5), as  $t \rightarrow \infty$ , we can see that,  $V(t) \rightarrow 1000$ .

Finally, assuming the carrying capacity is equal to the population of females in the Northern region according to the provisional results of the Population and Housing Census of Ghana. That is  $N = 1257855$

Then from equation (4.16) we obtain the value of  $\beta$ ;

$$0.888915 = \beta \left(1 - \frac{15}{1257855}\right)$$

$$0.888915 = \beta(1 - 0.000012)$$

$$0.888915 = 0.999988\beta$$

$$\beta = \frac{0.888915}{0.999988}$$

$$\beta = 0.888926$$

Then, the model becomes

$$V(t)' = 0.888926V \left(1 - \frac{v}{1257855}\right) \quad (4.24)$$

When solve gives

$$V(t) = \frac{1250008.3333}{1+83333.3333e^{-0.888926t}} \quad (4.25)$$

From equation (4.27), we obtain the predicted cases of Domestic Violence in the table below;

Time (t)	Predicted Cases $V(t)$
0	14.9999
1	36.4860
2	88.7512
3	215.8669
4	524.9698
5	1276.2295
6	3099.9146
7	7513.8979
8	18121.6303
9	43184.3213
10	100092.9939
11	218423.1079
12	424947.3826
13	695155.9456
14	941182.6164
15	1101434.4142
16	1184333.0296
17	1222147.3674
18	1238402.4356
19	1245210.9520
20	1248031.6770

21	1249194.9782
22	1249673.8372
23	1249870.8013
24	1249951.7906
25	1249995.0881
26	1249998.7771
27	1250004.4047
28	1250006.7183
29	1250007.6693
30	1250008.0603
31	1250008.2211
32	1250008.2871
33	1250008.3143
34	1250008.3255
35	1250008.3301
36	1250008.3319
37	1250008.3327
38	1250008.3330
39	1250008.3332
40	1250008.333

Table (4.6), as  $t \rightarrow \infty$ , we can see that,  $V(t) \rightarrow 1250008$ .

In conclusion, equation (4.21), (4.23) and (4.25) shows that there is a limit to the growth Domestic Violence Cases in Tamale Metropolitan.

## CHAPTER 5

### SUMMARY, CONCLUSION AND RECOMMENDATION

#### 5.1 SUMMARY OF RESULTS

##### 5.1.1 The Violence Epidemic Model

The prevalent equilibrium  $E = (A^*, S^*) = \left(\frac{\beta}{\alpha}, \frac{\mu}{\beta}\right)$  is asymptotically stable. When  $\rho \geq 4$ , a relatively larger recruitment rate to the Abusive/Susceptible class, the violence epidemic slows down after some time. In this case, if a group of Susceptible/Unreported Victim is introduced to the Abusive/Susceptible class, the population of the Abusive/Susceptible class will approach to  $A^* = \frac{\beta}{\alpha}$  while the populations of the Susceptible/Unreported Victim class approach to  $S^* = \frac{\mu}{\beta}$ , as time increases. On the other hand, when  $\rho < 4$ , a relatively smaller recruitment rate to the Abusive/Susceptible class, the violence epidemic slow down with a certain period due to oscillation in terms of the model parameters while the population of  $A$  and  $S$  still approach to the same equilibrium point. However, in the very beginning of the violence epidemic, we could observe the violence spread obeying the Exponential Growth,  $S = S_0 e^{\alpha N t}$ .

##### 5.1.2 The Logistic Growth Model

A model for the population of victims of Domestic Violence against Women and Children in Tamale Metropolitan from 1999 to 2011 was developed. That is, the differential equation that approximately models this population is

$$V' = 0.972775V \left(1 - \frac{V}{174}\right) \quad (5.10)$$

Also the present model is showed to be Logistic. From the model we can predict the population limitation of victims of Domestic Violence in Tamale. The number is 174, and from the data collected from DOVVSU in Tamale showed the situation in Tamale has reached its limit. The graph of the solution (4.21) has the general appearance shown in the following graph.

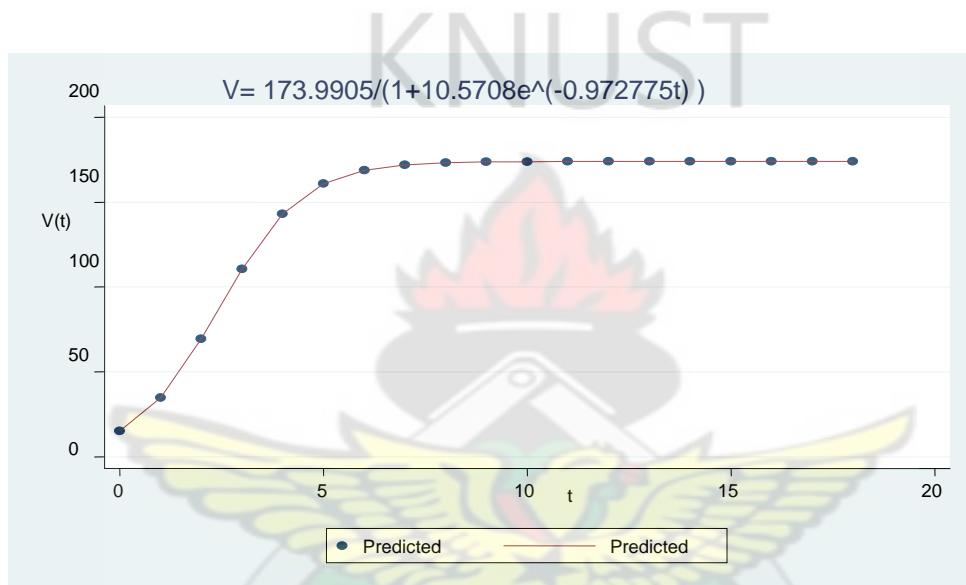


Figure (5.1)

Moreover, we find the solution for differential equation (5.10). Basically, we compared actual values with the predicted values and match the given data as shown in the graph below. It shows that the mathematical model (5.10) we obtain does have potential as a possible Logistic Growth.

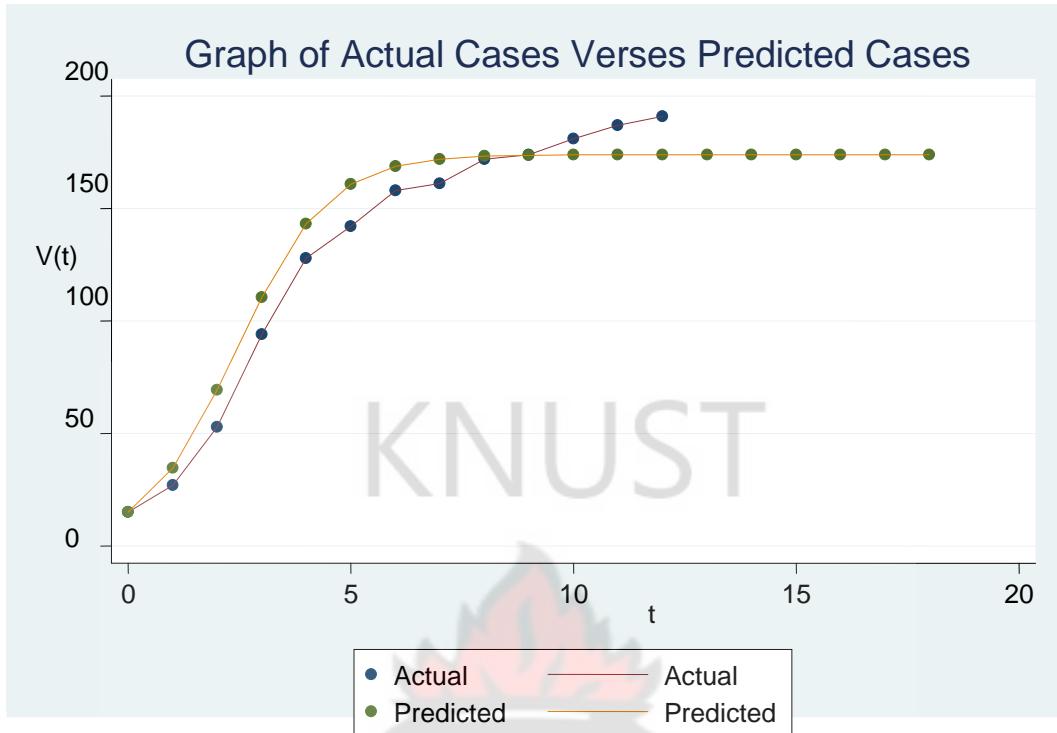


Figure (5.2)

## 5.2 CONCLUSION

A simple continuous model can mirror a primary interaction of Domestic Violence spread, using a system of Ordinary Differential Equations called the Violence Epidemic Model (ASV). The modeling is accomplished by formulating the rate of change of each population with their interactions. According to the results, by changing an environmental-control parameter,  $\rho$ , the long-term behavior of Domestic Violence against Women and children spread changes until the spread reaches a steady state. When  $\rho \geq 4$ , the solutions in the phase-plane move to the nodal sink equilibrium state

$$E(A^*, S^*) = \left( \frac{\beta}{\alpha}, \frac{\mu}{\beta} \right).$$

When  $\rho < 4$ , the solutions in the phase-plane move to the spiral sink equilibrium state  $E(A^*, S^*) = \left(\frac{\beta}{\alpha}, \frac{\mu}{\beta}\right)$ . The result also shows that the Violence spreads in the very beginning obeying the Exponential growth Model  $S = S_0 e^{\alpha N t}$ .

Knowing the simplified model might be somewhat inappropriate for the dynamics of Domestic Violence spread, though, fundamental mathematical modeling techniques are accomplished in this research to conceptualize a physical phenomenon. Analytic approaches are conveniently used to understand the factors governing the behavior of the model.

Also, the values collected from Domestic Violence and Victim Support Unit of the Ghana Police, Tamale was tested using the Logistic Growth Model;

$V(t)' = 0.972775V \left(1 - \frac{V}{174}\right)$  And the solution is  $V = \frac{173.9905}{1+10.5708e^{-0.972775t}}$ . The solution shows that there is a limit to the growth of Domestic Violence Victims  $V$  as time  $t \rightarrow \infty$ . The limiting number is 174 Victims. Assumptions of different carrying capacity of Tamale Metropolitan were considered and its Logistic Growth Model was deduced. Their result shows that there is a limit to the growth of Domestic Violence Victims over time period. The Logistic Growth Model was tested using data values and it proved to predict accurate figures of Domestic Violence Victims.

### 5.3 RECOMMENDATION

- ❖ From the Violence Epidemic Model, we assume that all Susceptible/ Unreported victims have an equal probability to be abuse from the abusive individuals, which is not necessary true in a real problem. Thus, an improvement can be made by accounting for the violence spread probability which is embedded in the

Susceptible/Unreported victim's rate,  $\alpha$  for the model in this research. In addition, the model neglects natural death, non-Domestic Violence-related subtraction from each population group. Due to the lack precise numerical data for Domestic Violence Victims and its relatively short history at DOVVSU Tamale, it is identified that estimating the model parameters is considerably difficult. Indeed, in our model, there is an unknown constant of entry rate to the Abusive/Susceptible class. In general, an in-depth study of the dynamics of the spread of Domestic Violence along with modeling technique is required for the improvement of the model.

- ❖ The Logistic Growth Model has proved obviously not suitable for the change in population (carrying capacity) of Domestic Violence Victims in Tamale. Students and Researchers consider looking into this area to develop a model that will give the best mathematical expression (formulae) needed for systems with such characteristics.
- ❖ The Government and other Civil Society Groups should consider adopting other new models such as this one which is simpler. The focus is that, resources should be directed to Abusive/Susceptible class and Susceptible/Unreported Victim class.

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## APPENDICES

### APPENDIX A:

The data collected from Domestic Violence and Victims Support Unit, Tamale Police station from 1999-2011.

YEARS	CASES
1999	15
2000	27
2001	53
2002	94
2003	128
2004	142
2005	158
2006	161
2007	172
2008	174
2009	181
2010	187
2011	191

## APPENDIX B:

Differential Equations Models and their Solutions,

For a carrying capacity of **171**;

$$\text{Model: } V(t)' = 0.972775V \left(1 - \frac{V}{174}\right)$$

$$\text{Solution: } V = \frac{173.9905}{1 + 10.5708e^{-0.972775t}}$$

KNUST

For a carrying capacity of **1000**;

$$\text{Model: } V(t)' = 0.902452V \left(1 - \frac{V}{1000}\right)$$

$$\text{Solution: } V(t) = \frac{1000.4900}{1 + 65.6987e^{-0.902452t}}$$

For a carrying capacity of **1257855**;

$$\text{Model: } V(t)' = 0.888926V \left(1 - \frac{V}{1257855}\right)$$

$$\text{Solution: } V(t) = \frac{1250008.3333}{1 + 83333.3333e^{-0.888926t}}$$