ROBUSTNESS OF THE QUADRATIC DISCRIMINANT FUNCTION TO CORRELATED AND SKEWED TRAINING SAMPLES

BY

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Declaration

I hereby declare that this submission is my own work towards the Master of Philosophy (M.Phil.) and that, to the best of my knowledge, it contains no material previously published by another person nor material which has been accepted for the award of any other degree of the University, except where due acknowledgement has been made in the text.

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Dedication

to my

FAMILY

with love

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I am grateful and indebted to Most High God who is able to do immeasurable more in my life. Most especially for the life he has ushered me. I also wish to express my appreciation to my lovely parents for their support in my education and to Mr. Eric Takyi Atakorah for putting me on my toes. I am especially grateful to my supervisor, Dr. (Mrs.) A. O. Adebanji for the immense supervisory support I received from her. I have learnt a lot from the positive criticisms and the pressure she mounted on me. To all the lecturers in the Department of Mathematics, KNUST, I say thank you for the knowledge you imparted on me during my postgraduate studies. Last but not least, I appreciate every effort any of my postgraduate colleagues contributed to this work, most especially Philemon Baah.

Abstract

This study investigates the asymptotic performance of Quadratic Discriminant Function and its robustness when the training samples are correlated normal or skewed. The scenarios considered were correlated normal, uncorrelated normal and skewed distributions. Three populations ($\Pi_i, i = 1, 2, 3$) with increasing group centroid separator function ($\delta = 1, 2, 3, 4, 5$) were considered. The number of predictor variables were 4, 6, and 8 with sample size ratios 1:1:1, 1:2:2 and 1:2:3. We simulated $N(\mu_i, \Sigma_i)$ of sample size 30, 60, 100, 150, 250, 300, 400, 500, 600, 700 and 2000 with MatLabR2009a for p variables in Π_1. The sizes of Π_2 and Π_3 are determined by sample ratios at 1:1:1, 1:2:2 and 1:2:3 for $n_1 : n_2 : n_3$ and these ratios also determine the prior probabilities considered. The population means were $\mu_1 = (0, 0, 0, \ldots, 0)$, $\mu_2 = (0, 0, 0, \ldots, \delta)$ and $\mu_3 = (0, 0, 0, \ldots, 2\delta)$ respectively. The covariance matrix Σ_i has $\sigma_{kl} = 0.7$ and $\sigma_k^2 = i$ for $k \neq l, \quad i = 1, 2, 3$. Reduction in error rates was more pronounced with increase in Mahalanobis distance than asymptotically. The coefficients of variation for sample size ratio 1:2:3 was more volatile under the three distributions considered. The optimal sample size ratio for the three distributions is 1:1:1. The results show the correlated normal distribution exhibits high coefficient of variation as δ increased. Further results show that the Quadratic Discriminant Function perform poorly when the training samples were skewed therefore, uncorrelated normal distribution was preferred

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Chapter 1

Introduction

1.1 Background of The Study

Discriminant analysis (DA) is a method used for finding out how a set of dependent/explanatory variables (DVs) is related to group membership, and particularly how they may be combined so as to enhance one's understanding of group differences. In more formal terms, DA aims at developing a rule for describing and subsequently predicting, if need be, group membership based on a given set of DVs. Once this rule becomes available, one may use it to make differential diagnosis, that is, group-membership prediction for particular subjects (Raykov and Marcoulides,2008).

Discriminant analysis as a topic in Multivariate Statistical Analysis has attracted much research interest over the years, with the evaluation of Discriminant Functions when the covariances matrices are unequal and the sample sizes are moderate being well explained by Wahl and Kronmal (1977). The following sections discuss some of the studies carried out on Discriminant Functions with unequal covariance matrices (Quadratic Discriminant Function).

In Scientific literatures, DA has many synonyms, such as classification, pattern recognition, character recognition, identification, prediction, and selection, depending on the type of scientific area in which it is used. The origin of DA is fairly old, and its development reflects the same broad phases as that of general statistical inferences in its applications (Giri, 2004). DA is essentially an adaptation of the regression analysis techniques for situation where the criterion variable is qualitative rather than quantitative. The assumptions for the data used in DA are that the data should be a random sample, normally distributed, homoscedastic, and uncorrelated among DVs. The terminology *discrimination* was introduced by Fisher (1936). In his early studies, Linear Function that maximizes the ratio of the between-samples variance to within-sample variances using two species (Iris Versicolor and Iris Setosa) out of the three species of the Iris data collected by Dr. E. Anderson was considered. Ever since his pioneering work, DA has been of great interest to statisticians, both theoretically and its application in other fields of study. Among them is Welch who derived the forms of Bayes rules for discriminating between two known multivariate populations with the same covariance matrix. (Rao, 1948).

Ideally, DA assumes the underlying populations to multivariate normality. In some classification problems, if there is evidence that the underlying distributions of the populations are multivariate normal, then Linear Discriminant Function (LDF) is optimum provided their covariance matrices are homogeneous. In the case of unequal covariance matrices of the normal populations, Quadratic Discriminant Function (QDF) is known to be optimum. Bahadur and Anderson (1962) considered the problem of discriminating between two unknown multivariate normal populations with different covariance matrices by looking at the discriminant rules based on linear classification functions. Departures from the assumptions of linear discriminant function analysis were explored by Krzanowski (1977), where the effects of unequal covariances on the linear discriminant method are looked at. Marks and Dunn (1974) studied the problems in the situation of linear and Quadratic Discriminant Functions where the covariance matrices differ in an orderly manner while Lachenbruch (1979) discusses the sensitivity of quadratic discriminant analysis to initial misclassification of cases. Among the relationships governing the performance of Quadratic Discriminant Analysis, those that depend on the sample size and the number of variables used were studied by Wahl and Kronmal (1977). In the study of Adebanji and Nokoe (2004), they evaluated the quadratic classifier in which they found that cross-validation error estimation procedure to be more sensitive to increased separation of variance-covariance structure than resubstitution method. The assumption of the populations being normal is met in some of the fields of application of discriminant analysis. However, there are situations in which the populations that are non-normal or near-normal often arise. The data used for discrimination are also sometimes correlated. Lachenbruch et al.(1977) studied this case by looking at the effect of non-normality on quadratic discriminant function.

1.2 Problem Statement

LDF is commonly used by the researcher because of its simplicity of form and concept. In spite of theoretical evidence supporting the use of the QDF when covariances are heterogenous, its actual employment has been sporadic because there are unanswered questions regarding its performance in the practical situation where the discriminant function must be constructed using training samples that do not satisfy the classical assumption of the model.

In this study, we investigate the performance of classification functions when the underlying distributions are non-normal (skewed), when the covariance matrices are heterogeneous and when data of interest are correlated, sample size ratios are unequal, different number of variables and varying values of group centroid separator. Hence, the specific questions that are treated in this study are

- Under what conditions will we get least error rates using QDF classification with data having the features mentioned above
- Under which sample size ratio will misclassification of observation from a group rise?

1.3 Objectives

In this study the general objective is to examine the performance of QDF under non-normality(positively skewed) and correlation within the data set by considering condition:

- Different sample size
- Varying the group centroid separator ($\delta = 1, 2, 3, 4, 5$)
- Varying the number of variables (4, 6 and 8)

1.3.1 Specific Objective

The specific objectives of this study therefore are

- to examine the performance of QDF when the training samples are skewed
- to examine the asymptotic performance of QDF when the training samples are correlated.
- to determine the sample size ratio under which the performance of QDF deteriorate
- to conduct a Monte Carlo simulation to study the performance of QDF under the above mentioned conditions.

1.4 Methodology

Considering a three population case, we examine the effect of correlation, uncorrelation and skewness considering different sample size ratios, number of variables and varying group centroid separators (δ) on classification accuracy using simulated data from these three populations. The three populations differ with respect to their mean vector and covariance matrices. This study examines the behaviour of QDF for observations from three multivariate normal populations. Sensitivity of the performance of the function to changes in

1. δ from 1 to 5 where δ is determined by the difference between the mean vectors,

2. Sample sizes are specified. Here 11 values of n_1 set at 30, 60, 100, 150, 250, 300, 400, 500, 600, 700, 2000 and the sample size of n_2 and n_3 are determined by the sample ratios at 1:1:1, 1:2:2 and 1:2:3 and these ratios also determine the prior probabilities to be considered

3. The number of variables used were $4, 6$ and 8 for each case in (2)

MatLab R2009a is used to generate normal random data for all the populations and Minitab 16 for the graphs. The leave-one-out method is used to estimate the error rates (Lanchenbruch and Mickey, 1968) in all cases.

1.5 Justification of Problem

An enormous deal of study has been made since Fisher's (1936) original work on discriminant analysis. Some estimation methods have been proposed and some sampling properties derived. However, there is little investigation done on large sample properties of these functions. A considerable number of studies had been carried out on discriminant analysis but not much is done on the effect of non-normality and correlated data in classification with respect to sample size ratios especially not on the QDF.

1.6 Organization of the Thesis

This thesis comprises of five chapters. Chapter 1 introduces the work by giving the background of the study, statement of the problem of this study, objectives of the study, methodology and justification of the problem of the study.

Framework of the study and literature are reviewed in Chapter 2. Quadratic discriminant function (QDF) as well as earlier related studies are discussed in this chapter. Chapter 3 talks about the methodology employed in our study. Chapter 4 presents and discusses the results obtained from the analysis and simulation. Finally summary of our findings, conclusion and recommendation for further research are presented in Chapter 5

Chapter 2

Literature Review

2.1 Selection of Variables in DA

In many applications of multivariate analysis, the statistician finds that he has data on very large number of variables which makes computation difficult. Murray (1977) studied the selection of variables in DA. He proposed that the procedure of selection of subset of variables were appealing from an intuitive point of view, but they produced rather disturbing results in practice. To estimate the magnitude of bias, he ran series of computer simulations in order to isolate the bias. k independent normal random variables with unit variance and the means of two populations were specified. Since he was concerned with misclassification rates and parameters were known, the optimal discriminant rule was the likelihood ratio. Three different procedures were used to search through the subsets

1. $k = 10$. Examine every subset.

2. $k = 10$. Forward selection.

3. $k = 50$. Forward selection up to ten variables.

Three different stopping procedure was also used to determine which subset to be used. They were

1. Choose the best subset of a given size.

2. Choose the best subset examined, irrespective of size.

3. Having found the best subset of size r, choose this unless the best subset of size $r+1$ gives strictly better classification

Sizes of data base used were $25, 50$ and 100. The results indicated that for the stopping rule (1) and search procedure (1) and (2), as the number of variables increased, the best error rate did not decrease monotonically. He concluded that with data base size 25, only 4 variables were needed to the true error rates of all 50 variables. Also, for the stopping rules (2) and (3) with search procedures (1) and (2), the results indicated that neither of those stopping rules detected the increased discriminatory power of large subsets. He then concluded that the apparent error rate gave misleading estimates of the true error rates.

2.2 Unequal Covariance Matrices

The pioneering work on Quadratic Discrimination was by Smith (1947). He used Fisher's Iris data in his work. He provided a full expression for the QDF and his results showed the QDF outperforming the LDF when the homogeneity of variance covariance structure is violated. .

Anderson and Bahadur (1962) also studied a slightly different work on classification into two multivariate normal populations with different mean vectors and covariance matrices.

If one attempts to use the LDF when in fact the covariance matrices are unequal, the performance on the LDF may be substantially affected. Marks and Dunn (1974) studied the performance of the LDF under this violation. Their intention was to find out whether sampling variability, arising from additional parameter estimation with increasing dimension size, reduces the quadratic's effectiveness to the point where a linear function with fewer parameters should be used instead. Their additional concern was the uncertainty about the behavior of the densities in the distribution tails which makes one reluctant to assign points far from the population means according to the quadratic. They approached these realistic problems by comparing the asymptotic and small sample performance of the QDF, best linear and Fisher's LDF for both proportional and non-proportional covariance differences under the assumption of normality and unequal covariance matrices. Two populations were used and sample sizes were from 10 to 100. The number of variates were 2 and 10. They employed the application of Monte Carlo simulation. Their results indicated that for small samples the QDF performed worse than the LDF when covariances were nearly equal and dimension was large (ie LDF was satisfactory when the covariance matrices were not too different). However, even for small samples, as covariance differences increased so did the performance of the QDF relative to Fisher's LDF. The best LDF appeared to offer little advantage in the situation where Fisher's function did better than the quadratic d but did not match the performance of the quadratic when covariance inequality was greater. It was also noted that when the means are widely separated the LDF generally do well. They concluded that for small sample sizes ($N_1, N_2 < 25$) poor performances can be expected for QDF if the dimension size is moderately large ($k > 6$).

Wahl and Kronwal in 1977 extended the study of Marks and Dunn (1974). They observed that when the dimensions size and the covariance differences are large the QDF's performance is much better than Fisher's LDF provided the sample size is sufficient. For more than 100 observations, the asymptotic results are reached fairly quickly, thus favoring the QDF. They concluded that sample size is a critical factor in choosing between the QDF and LDF. It was therefore recommended that

1. For small covariance differences and small d ($d \leq 6$) there is generally little to choose between the LDF and QDF.

2. For small samples (n_l, $n_2 < 25$) and the covariance differences and/or d large, the LDF is preferred. However, when both covariance differences and d are large, the misclassification probabilities may be too large for practical use.

3. For large covariance differences and $d > 6$, QDF is much better than LDF, provided that the samples sizes are sufficiently large.

2.3 DA Under Non-optimal Condition

In 1977, Kzarnowski studied the Fisher's Linear Discriminant Function under non-optimal conditions. He considered independent random samples, of sizes n_1 and n_2 respectively, from each of two p-variate distributions having mean vectors μ_1 and μ_2 and common covariance matrix Σ. He considered continuous and binary variables which were measured on each individual. Two populations were also looked at. They restricted their attention to when some or all the data are binary since the continuous non-normal case had already been tackled by Lachenbruch et al (1973). The specified parameters considered were the number of binary variables, means of the binary variables in the two populations, correlation between each pair of binary variables, means of continuous variables and Mahalanobis squared distances. His results indicated that in many instances the LDF may be satisfactory for classification but not for estimating risks of individuals belonging to a particular population. It was observed that the inflation of error rates for Fisher's LDF is generally greater in presence of moderate positive correlation among all the binary variables. Conversely, the largest differences in performance of Fisher's LDF between independent and correlated binary variables occur when the populations are widely separated, and it became intense as the number of binary variables were increasing. Agreement of LDF and QDF was adequate only for a moderate range constant multiple of group covariance matrix and with some amount of linear separation of populations. It becomes worse as the number of variables increase.

2.4 Evaluation of the Quadratic Classifier

Adebanji and Nokoe (2004) have considered evaluating the Quadratic classifier. They restricted their attention on two multivariate normal populations of independent variables. In addition to some theoretical result, in the case of known parameters, they conducted a Monte Carlo simulation in order to investigate the error rates. Results indicated that the total error rate computed showed that there was an increase in the error rate with re-substitution estimator for all K values. On the other hand, there was a decline across K. The Cross-validation estimator showed a steady decline for and across all values K and the recorded values showed a substantially low error rate estimates than re-substitution estimator for $K = 4$ and $K = 8$. The re-substitution estimator did not show high sensitivity in K as it was in the cross-validation estimator even with increased sample size. They also looked at relative bias where re-substitution estimator recorded its highest bias at $K = 8$ and lowest at $K = 4$ while cross-validation estimator's highest and lowest bias were at $K = 8$ and $K = 2$ respectively. The distribution of error rates was found to be closer to Gamma than to Normal distribution. They concluded that the structure of the dispersion matrix could be informative considering the use of Quadratic classifier and also in deciding on the method of estimating misclassification rate.

2.5 Effect of Initial Misclassification on QDF

According to Lachenbruch (1974), the model assumed that the observations are randomly misclassified and that each observation had the same chance of being initially misclassified is clearly unrealistic. His study attempted to present two models of non-random initial misclassification, the complete separation model which was defined as for x observation, calculate $(x-\mu_1)'(x-\mu_1) = x'x$ and $(x-\mu_2)'(x-\mu_2)$ and assign the observation to whichever population with the smaller quantity. This amounted to assigning x to Π_1 if $x_1 < \delta/2$ and to Π_2 if otherwise. It was easy to show that this led to an initial misclassification rate of $\alpha_i = \phi(\delta/2)$ in Π_1 and Π_2. The second model was the generalization of the first model. The same criterion was used, but, in addition, for an observation from Π_i to be misclassified, $(x - \Pi_i)'(x - \Pi_i)$ must be greater than a quantity, V_i. In these models, observations which were closer to the mean of the "wrong" population had a greater chance of being misclassified than others. He hoped these models were more realistic than the "equal chance" model. Monte Carlo experiments are used to evaluate the behavior of the LDF. His results indicated that (a) the actual error rates of the rules from samples with initial misclassification were only slightly affected, (b) the apparent error rates, obtained by resubstituting the observations into the calculated discriminant function, were drastically affected, and cannot be used, and (c) the Mahalanobis D^2 was greatly inflated.

Lachenbruch (1979) did a study on DA in which he considered the effects of initial misclassification on the QDF. In his simulation, a population of two with equal priori probabilities, mean of 0 and 2 and number of variables, $2, 4, 8$ and a fraction of α_i of the n_i, which are actually from the other population, were considered. To determine the effects of initial misclassification, he generated the QDFs that would result from various values of the parameters and estimated the error. He found out that although initial misclassification is not a serious problem with LDFs if initial rates are about the same, it is with QDF. The severity of the problem increases with increasing differences in covariance matrices and with increasing initial misclassification rates. He then suggested that if initial misclassification is suspected, all sample points should be carefully checked and reassigned if needed.

2.6 DA with Correlated Training Samples

If one attempts to use the LDF or QDF when in fact the training samples are correlated, the performance on the LDF or QDF may be significantly affected. Lawoko (1988) studied the performance of the LDF and QDF under the assumption of correlated training samples. In his study consideration was given to the problem of allocating an object to one of two groups on the basis of measurements on the object. His attention was on Anderson's sample LDF (W) and QDF (Z) formed from the likelihood ratio criterion. The performance of the Z and W relative to each other under correlated training observations were studied. He found that their relative performance depends on the extent of the correlation among the training observations and the size of the separation between the classes. Z was recommended over W for positively correlated training observations which followed a moving average process of order one on the basis asymptotic expansion of error rates. Under intraclass correlation model, he found that the discriminant functions formed under the model did not perform better than W and Z formed under the assumption of independent training observation. Asymptotic expected error rate for W under the model (W_m) and W were equal when the training observations follow an autoregressive process but there was a slight improvement in the overall error rate when W_m was used instead of W for numerical evaluations of the asymptotic expansions. He concluded that the efficiency of the discriminant analysis estimator is generally lowered by positively correlated training observations.

Mardia et al (1979) reported that it might be thought that a linear combination of two variables would provide a better discriminator if they were correlated than when they were uncorrelated. However, this is not necessarily so. To show this they considered two bivariate populations Π_1 and Π_2. Supposing Π_1 is $N_2(0, \Sigma)$ and Π_2 is $N_2(\mu, \Sigma)$ where $\mu = (\mu_1, \mu_2)'$ and $\Sigma = \begin{bmatrix} 1 & \rho \\ \rho & 1 \end{bmatrix}$. Now the Mahalanobis distance between the two populations is

$$\Delta^2 = \mu'\Sigma^{-1}\mu' = (\mu_1^2 + \mu_2^2 - 2\rho\mu_1\mu_2)(1 - \rho^2)$$

if the variables are uncorrelated then

$$\Delta^2 = \mu_1^2 + \mu_2^2 = \Delta_0^2$$

Correlation then improves discrimination (i.e. reduce the probability of misclassification) if and only if $\Delta^2 > \Delta_0^2$. This happens if and only if $\rho(1 + f^2)\rho - 2f > 0$ where $f = \mu_2/\mu_1$ In other words, discrimination is improved unless ρ lies between zero and $2f/(1 + f^2)$ but a small value of ρ can actually harm discrimination. Note that if $\mu_1 = \mu_2$ then any positive correlation reduces the power of discrimination.

2.7 Non-Normality

Several studies have been conducted on the effect of various types of non-normality on the QDF and LDF. Lachenbruch et al. (1973) considered the robustness of LDF. Three specific distributions and the case of independent variables were considered. These distributions were considered to be non-normal and generated from the normal distributions by using the Johnson system of transformations (log normal, inverse hyperbolic sine normal and logit normal distribution). The study indicated considerable decline in performance of the LDF (the log normal distribution used has extremely large skewness and kurtosis). They conducted Monte Carlo experiments to investigate robustness when parameters are estimated. Their results indicated that Fisher's LDF was greatly affected by non-normality in the population. Error rates for one population were greatly larger that the optimum values while the reverse was true in the other population. It was also noted that if error rates with LDF were greatly different in the two populations when a cut-off point is zero, then presence of non-normal data is indicated. They concluded that use of Fisher's LDF in non-normal situations could be badly misleading, and recommended that the data be transformed to approximate normality prior to use of the LDF and the error rates could be used as a check on the normality assumption.

Lachenbruch et al. (1977) studied the effect of non-normality on the QDF. They assumed that the data were transformable to normality, variables were independent and covariance matrices were proportional after transformation. In order to study the effect of non-normality on QDF, they generated random samples from non-normal distributions and the samples were transformed component by component by using Johnson's system of transformation. The data used represented rather mild departures from normality and the following are their observations:

1. In the computation of the overall sample standard deviation, the between sample variability of the individual error rates in the QDF on normal or non-normal distributions was quite large and for that instability of QDF is pronounced.

2. The actual error rates were considerably larger than the optimal rates in the case of zero mean difference (this is a very difficult problem in assignment)

3. The QDF for non-normal samples generally did not do substantially worse than when the QDF was applied to the normal samples which would be obtained after transformations;

4. In comparing the resubstitution method and the leave-one-out method, the resubstitution method had an unacceptably high bias. The leave-one-out method was far superior in respect of generally having a far lesser bias.

5. Attempting to obtain robust estimates of means and covariance was of little help unless the distribution was heavy-tailed or substantially skewed.

2.8 Error Rates

Evaluation of a classification procedure is by estimating the error rate. A good classification procedure should result in minimal error rates. Lachenbruch and Mickey (1968) did a study on estimation of these error rates. The discriminant function which was denoted as W by Anderson was considered. They described eight techniques to estimate error and attempted to evaluate seven of them in which the holdout method was excluded since the number of hold-out cases in each group that are misclassified is binomially distributed. It was observed that none of the methods was uniformly best for all situations, although some methods performed better than the two methods which were in use at that time.

Krzanowski and Hand (1997) considered an assessment of error rate estimators paying special attention to the leave-one-out method. The leave-one-out rule seeks to overcome the drawback of resubstitution by a process of cross-validation. The estimator was investigated in a simulation study, both in absolute terms and in comparison with a popular bootstrap estimator. The Bayes' procedure was found to give unreliable estimates of the leave-out-two which performed better than the leave-one-out. They compared the performance of the leave-one-out method with that of the 632 method, as measured by the incorrect and correct methods. They found that results leave-one-out method is even worse than had been expected. Motivated by this, extension of leave-one-out, the leave-two-out was looked at considering the varience.As expected, the leave-two-out method yields a slight variance reduction relative to the leave-one-out method, but not enough to make it a good competitor for the 632 method.

In order to study the asymptotic error rates of Linear, Quadratic and Logistic rules Kakai and Pelz (2010) conducted a Monte Calor study in $2, 3$ and 5-group discriminant analysis. The simulation study took into account the overlap of the populations ($e = 0.05$, $e = 0.1$, $e = 1.5$), their common distribution (normal, chi-square with 4, 8 and 12 df) and their heteroscedasticity degree, Γ, measured by the value of the power function, $1 - \beta$ of the homoscedasticity test related to Γ ($1 - \beta = 0.05$, $1 - \beta = 0.4$, $1 - \beta = 0.6$, $1 - \beta = 0.8$). The following observations were made

1. By considering the combination of the parameters the three rules gave similar error rates for normal homoscedastic populations.

2. For normal heteroscedastic population, the quadratic rule is theoretically Bayes rule and it presented lowest relative error irrespective of the number of groups.

3. For non-normal populations, quadratic rule still gave lowest relative error except for 2-group where logistic was the best

4. Quadratic and logistic rule were more influenced by the number of group irrespective of their lowest relative error

5. Also linear and quadratic were more influenced by non-normality

Chapter 3

Methodology

3.1 Concept of Discrimination and Classification

Discrimination and classification are multivariate techniques concerned with separating distinct sets of objects (or observations) and with allocating new objects (observations) to previously defined groups. Discriminant analysis is rather exploratory in nature. As a separative procedure, it is often employed on a one-time basis in order to investigate observed differences when causal relationships are not well understood. Classification procedures are less exploratory in the sense that they lead to well-defined rules, which can be used for assigning new objects. Classification ordinarily requires more problem structure than discrimination does. Discrimination is to describe, either graphically or algebraically, the differential features of objects (observations) from several known collections (populations). We try to find "discriminants" whose numerical values are such that the collections are separated as much as possible (separation). While classification is to sort objects (observations) into two or more labeled classes with the emphasis on deriving a rule that can be used to optimally assign new objects to the labeled classes (that is allocation). The idea of discrimination and classification frequently overlap, and the distinction between separation and allocation becomes blurred but they are always applied together in discriminant analysis.

3.2 Discrimination and Classification of Two Population

Härdle and Simar (2012) used example, the detection of "fast" and "slow" consumers of a newly introduced product. Using a consumer's characteristics like education, income, family size, amount of previous brand switching, we want to classify each consumer into the two groups just identified. Let $f_1(\mathbf{x})$ and $f_2(\mathbf{x})$ be the probability density functions associated with the $p \times 1$ vector random variable $\mathbf{X}$ for the populations Π_1 and Π_2, respectively. An object with associated measurements $\mathbf{x}$ must be assigned to either Π_1 or Π_2. Let Ω be the sample space-that is, the collection of all possible observations $\mathbf{x}$. Let R_1 be that set of $\mathbf{x}$ values for which we classify objects as Π_1 and $R_2 = \Omega - R_1$ be the remaining $\mathbf{x}$ values for which we classify objects as Π_2. Since every object must be assigned to one and only one of the two populations, the sets R_1 and R_2 are mutually exclusive and exhaustive. The main task of discriminant analysis is to find "good" regions R_j where $j = 1, 2$ such that the error of misclassification is small. In the following we describe such rules when the population distributions are known. The conditional probability, $P(2|1)$, of classifying an object as Π_2 when, in fact, it is from Π_1 is written as

$$P(2|1) = P(X \in R_2|\Pi_1) = \int_{R_2=\Omega-R_1} f_1(\mathbf{x})d\mathbf{x}$$

Similarly, the conditional probability of classifying an object as Π_1 when, in fact, it is from Π_2, $P(1|2)$ is written as

$$P(1|2) = P(X \in R_1|\Pi_2) = \int_{R_1} f_2(\mathbf{x})d\mathbf{x}$$

Where the individual integral signs represent the volume formed by the individual density functions over the region R_j.

3.3 Likelihood Ratio Discriminant Rule

It is suggested that $\mathbf{x}_0$ should be allocated to Π_1 whenever the probability of it coming from Π_1 is greater than probability of it from Π_2, to Π_2 whenever these probabilities are reversed, and by chanced to Π_1 or Π_2 whenever these probabilities are equal. By this argument, we define R_1 as the set of points for which $f_1(\mathbf{x}) \geq f_2(\mathbf{x})$, and R_2 as the set of points for which $f_1(\mathbf{x}) < f_2(\mathbf{x})$. Rewriting the above slightly, the classification rule is as follows:

$$\text{Assign } \mathbf{x}_0 \text{ to } \Pi_1 \text{ if } \frac{f_1(\mathbf{x}_0)}{f_1(\mathbf{x}_0)} \geq 1, \tag{3.1}$$

and to Π_2 when otherwise.

The allocation rule 3.1 is known as the *likelihood ratio* rule. This rule, however, fails to take into account some factors which may be important in practice. These factors are:

differential prior probabilities of observing individuals from the two populations and the differential costs of misclassification.

3.4 Prior Probability

An optimal classification rule should take into prior probabilities of occurrence into account. Let p_1 be the prior probability of Π_1 and p_2 be the prior probability of Π_2, where $p_1+p_2 = 1$. Then the overall probabilities of correctly or incorrectly classifying objects can be derived as the product of the prior and conditional classification probabilities:

$$\begin{aligned} P(\text{observation is correctly classified as} \quad \Pi_1) &= P(\text{observation comes from} \quad \Pi_1 \\ &\quad \text{and is correctly classified as} \quad \Pi_1) \\ P(X \in R_1|\Pi_1)P(\Pi_1) &= P(1|1)p_1 \end{aligned} \tag{3.2}$$

$$\begin{aligned} P(\text{observation is misclassified as} \quad \Pi_1) &= P(\text{observation comes from} \quad \Pi_2 \\ &\quad \text{and is misclassified as} \quad \Pi_1) \\ P(1|2)p_2 &= P(X \in R_1|\Pi_2)P(\Pi_2) \end{aligned} \tag{3.3}$$

$$
\begin{aligned}
P(\text{observation is correctly classified as} \quad \Pi_2) &= P(\text{observation comes from} \quad \Pi_2 \\
&\quad \text{and is correctly classified as} \quad \Pi_2) \\
P(2|2)p_2 &= P(X \in R_2|\Pi_2)P(\Pi_2) \qquad (3.4)
\end{aligned}
$$

$$
\begin{aligned}
P(\text{observation is misclassified as} \quad \Pi_2) &= P(\text{observation comes from} \quad \Pi_1 \\
&\quad \text{and is misclassified as} \quad \Pi_2) \\
P(2|1)p_1 &= P(X \in R_2|\Pi_1)P(\Pi_1) \qquad (3.5)
\end{aligned}
$$

3.5 Cost of Misclassification

According to Krzanowski (1988) another aspect of classification is cost. A rule that ignores costs may cause problems. The cost associated with each of these mistakes are $c(1|1), c(1|2), c(2|2)$ and $c(2|1)$ respectively. The costs are zero for correct classification, so the expected or average cost of misclassification is given by

$$
\begin{aligned}
ECM &= c(2|1)P(2|1)p_1 + c(1|2)P(1|2)p_2 \\
&= c(2|1)p_1 \int_{R_2} f_1(\mathbf{x})d\mathbf{x} + c(1|2)p_2 \int_{R_1} f_2(\mathbf{x})d\mathbf{x} \qquad (3.6)
\end{aligned}
$$

The best classification rule is the one that yields minimum expected cost due to misclassification, and this rule will be obtained by finding the region R_1 and R_2 minimizing ECM in 3.6 allocate $\mathbf{x}$

$$R_1 : \frac{f_1(x)}{f_2(x)} \geq \left(\frac{c(1|2)}{c(2|1)}\right)\left(\frac{p_2}{p_1}\right) \tag{3.7}$$

$$R_2 : \frac{f_1(x)}{f_2(x)} < \left(\frac{c(1|2)}{c(2|1)}\right)\left(\frac{p_2}{p_1}\right) \tag{3.8}$$

The right hand side of 3.7 and 3.8 known as the cut-off point is denoted by $\mathbf{k}$. It is clear from 3.6 that the implementation of the minimum ECM rule requires (1) the density function ratio evaluated at a new observation $\mathbf{x_0}$, (2) the cost ratio, and (3) the prior probability ratio. The appearance of ratios in the definition of the optimal classification regions is significant (Johnson and Winchern, 2007).

The likelihood ratio discriminant rule 3.1 is thus a special case of the ECM rule for equal misclassification costs and equal prior probabilities. Other special cases of Minimum Expected Cost Regions are:

(a) $p_2/p_1 = 1$ (equal prior probabilities); $k = c(1|2)/c(2|1)$

$$R_1 : \frac{f_1(\mathbf{x})}{f_2(\mathbf{x})} \geq \frac{c(1|2)}{c(2|1)} \qquad R_2 : \frac{f_1(\mathbf{x})}{f_2(\mathbf{x})} < \frac{c(1|2)}{c(2|1)}$$

(b) $c(1|2)/c(2|1) = 1$ (equal costs of misclassification); $k = p_2/p_1$

$$R_1 : \frac{f_1(\mathbf{x})}{f_2(\mathbf{x})} \geq \frac{p_2}{p_1} \qquad R_2 : \frac{f_1(\mathbf{x})}{f_2(\mathbf{x})} < \frac{p_2}{p_1}$$

When the prior probabilities are unknown, they are often considered to be equal. Similarly, the costs of misclassification are also often taken to equal when they are unknown.

3.6 Total Probability of Misclassification

Criteria other than the ECM can be used to "optimal" classification procedures. One might ignore cost of misclassification and choose R_1 and R_2 to minimize the total probability of misclassification (TMP).

$$\begin{aligned}
\text{TMP} &= P(\text{misclassifying a } \Pi_1 \text{ observation or misclassifying a } \Pi_2 \text{ observation}) \\
&= P(\text{observation comes from } \Pi_1 \text{ and is misclassified}) \\
&\quad + P(\text{observation comes from } \Pi_2 \text{ and is misclassified}) \\
&= p_1 \int_{R_2} f_1(\mathbf{x}) d\mathbf{x} + p_2 \int_{R_1} f_2(\mathbf{x}) d\mathbf{x}
\end{aligned}$$

Mathematically, this problem equivalent to minimizing the expected/average cost of misclassification when the cost of misclassification are equal.

The rule **(b)** could have been derived equivalently by minimizing TPM, where

$$TPM = P(2|1)p_1 + P(1|2)p_2 \tag{3.9}$$

is given by ECM of (2.2) but with $c(2|1)$ and $c(1|2)$ removed. It is worth mention that the rule **(b)** is equivalent to the allocation rule derived by maximizing the posterior probability of population membership.

3.7 Bayes' Classification Rule

Let

$$P(\mathbf{x} \in \Pi_i) = p_i, i = 1, 2 \tag{3.10}$$

be the *prior probabilities* that a randomly selected observation $\mathbf{x} = \mathbf{x}_0$ belongs to either Π_1 or Π_2. Suppose also that the conditional multivariate probability density of $\mathbf{x}$ for the ith class is

$$P(\mathbf{x} = \mathbf{x}_0 | \mathbf{x} \in \Pi_i) = f_i(\mathbf{x}_0), i = 1, 2. \tag{3.11}$$

From 3.10 and 3.11, Bayes' theorem yields the posterior probability,

$$P(\Pi_i|\mathbf{x}) = P(\mathbf{x} \in \Pi_i | \mathbf{x} = \mathbf{x}_0) = \frac{f_i(\mathbf{x}_0)p_i}{f_1(\mathbf{x}_0)p_1 + f_2(\mathbf{x}_0)p_2}, \tag{3.12}$$

that the observed $\mathbf{x}_0$ belongs to $\Pi_i, i = 1, 2$. For a given $\mathbf{x}_0$, a reasonable classification strategy is to assign $\mathbf{x}_0$ to the class with the higher posterior probability. This strategy is called the Bayes' classification rule. Since we are dealing with forced classification, the classification rule is

$$\text{assign } \mathbf{x}_0 \text{ to } \Pi_1 \text{ if } \frac{P(\Pi_1|\mathbf{x})}{P(\Pi_2|\mathbf{x})} \geq 1, \tag{3.13}$$

otherwise assign $\mathbf{x}_0$ to Π_2. The ratio $\frac{P(\Pi_1|\mathbf{x})}{P(\Pi_2|\mathbf{x})}$ is referred to as the "odds-ratio" that Π_1 rather than Π_2 is the correct class given the information in $\mathbf{x}_0$. Substituting 3.12 into 3.13, the Bayes' classification rule becomes

$$\text{assign } \mathbf{x}_0 \text{ to } \Pi_1 \text{ if } \frac{f_1(\mathbf{x}_0)}{f_2(\mathbf{x}_0)} \geq \frac{p_2}{p_1}, \tag{3.14}$$

and to Π_2 otherwise.(Izenman, 2008).

3.8 Distance Based Classification

We now turn our attention to classification rules for several groups based on the distance between x and the discriminating groups. We consider the case where x is multivariate normal in $\Pi_1, i = 1, 2, \ldots, g$. The Mahalanobis squared distance between x and Π_i is defined as

$$\Delta_i^2 = (\mathbf{x} - \mu_i)'\Sigma^{-1}(\mathbf{x} - \mu_i) \tag{3.15}$$

The allocation rule is allocate x to the group for which Δ_i^2 is smallest.

3.9 The Quadratic Classifier($\Sigma_1 \neq \Sigma_2$)

Suppose that the joint densities of $X' = [X_1, X_2, \ldots, X_p]$ for population Π_1 and Π_2 are given by

$$f_i(\mathbf{x}) = \frac{1}{(2\pi)^{p/2}\left|\Sigma_i\right|^{1/2}} exp\left[-\frac{1}{2}(\mathbf{x} - \mu_i)'\Sigma_i^{-1}(\mathbf{x} - \mu_i)\right] \tag{3.16}$$

The covariance matrices as well as the mean vectors are different from one another for the two populations. The regions of minimum expected cost misclassification (ECM) and minimum total probability of misclassification (TPM) depends on the ratio of the densities, $(f_1(\mathbf{x}))/(f_2(\mathbf{x})$, or equivalently, the natural logarithm of the density ratio, $\ln[(f_1(\mathbf{x})/(f_2(\mathbf{x})] = \ln[f_1(\mathbf{x})] - \ln[f_2(\mathbf{x})]$ when the multivariate normal densities have different covariance structures, the terms in the density ratio involving $\left|\Sigma_i^{1/2}\right|$ do not cancel as they do when we have equal covariance matrices and also the quadratic forms in the exponents of $f_i(\mathbf{x})$ do not combine. Therefore substituting multivariate normal densities with different covariance matrices into 3.7 and 3.8 and after taking the natural logarithms and simplifying, the likelihood of the density ratios gives the quadratic function in $\mathbf{x} \in \Pi_1$ if

$$-\frac{1}{2}\mathbf{x}'(\Sigma_1^{-1} - \Sigma_2^{-1})\mathbf{x} + (\mu_1'\Sigma_1^{-1} - \mu_2'\Sigma_2^{-1})\mathbf{x} - k \geq \ln\left[\left(\frac{c(1|2)}{c(2|1)}\right)\left(\frac{p_2}{p_1}\right)\right],$$

where

$$k = \frac{1}{2}\ln\left(\frac{|\Sigma_1|}{|\Sigma_2|}\right) + \frac{1}{2}(\mu_1'\Sigma_1^{-1}\mu_1 - \mu_2'\Sigma_2^{-1}\mu_2) \tag{3.17}$$

otherwise, $\mathbf{x} \in \Pi_1$. Considering the Mahalanobis distance, the function is sometimes written as

$$f(\mathbf{x}) = D_1^2(\mathbf{x}) - D_2^2(\mathbf{x}) + \ln\left[\frac{|\Sigma_1|}{|\Sigma_2|}\right] - 2\ln\left(\frac{p_1}{p_2}\right) \tag{3.18}$$

The quantity $D_i^2(\mathbf{x}) = (\mathbf{x} - \mu_i)'\Sigma^{-1}(\mathbf{x} - \mu_i)$ is the Mahalanobis Square Distance.

When $\Sigma_1 = \Sigma_2$ the function reduces to linear classifier rule(Adebanji and Nokoe, 2004)

In many applications, the distributions of the populations of interest may not be multivariate normal.If the data are not multivariate normal, transformation of the data to more nearly normal and a test for equality of covariance matrix can be conducted to see whether the linear rule or the quadratic rule is appropriate. This transformation is done before testing is carried out.

we can also use the linear or quadratic rule without worrying about the form of the parent population and hope that it will work reasonably well. Studies have shown that there are non-normal cases where LDF performs poorly, even though the population covariance matrices are the same. The moral is to always check the performance of any classification procedure (Johnson and Winchern, 2007).

3.10 Inferential Procedures In Discriminant Analysis

Several inferential procedures exists in discriminant function analysis. The basic ones are discussed here.

3.10.1 Test for $H_0 : \mu_1 = \mu_2$ When $\mathbf{\Sigma}_1 = \mathbf{\Sigma}_2$ Using Hotelling's T^2-Test

In the multivariate case, we wish to compare the mean vectors from two populations. We assume that two independent random samples $\mathbf{y}_{11}, \mathbf{y}_{12}, \cdots, \mathbf{y}_{1n_1}$ and $\mathbf{y}_{21}, \mathbf{y}_{22}, \cdots, \mathbf{y}_{2n_2}$ are drawn from $N_p(\mu_1, \mathbf{\Sigma}_1)$ and $N_p(\mu_2, \mathbf{\Sigma}_2)$, respectively, where $\mathbf{\Sigma}_1$ and $\mathbf{\Sigma}_2$ are unknown. In order to obtain a T^2-test, we must assume that $\mathbf{\Sigma}_1 = \mathbf{\Sigma}_2 = \mathbf{\Sigma}$, say. From the two samples, we calculate $\bar{\mathbf{y}}_1, \bar{\mathbf{y}}_2, \mathbf{W}_1 = (n_1 - 1)\mathbf{S}_1,$ and $\mathbf{W}_2 = (n_2 - 1)\mathbf{S}_2$. A pooled estimator of the covariance matrix is calculated as

$$\mathbf{S}_{pl} = \frac{\mathbf{W}_1 + \mathbf{W}_2}{n_1 + n_2 - 2},$$

for which $E(\mathbf{S}_{pl}) = \mathbf{\Sigma}$.

To test

$$H_0 : \mu_1 = \mu_2 \quad \text{versus} \quad H_1 : \mu_1 \neq \mu_2,$$

we use the test statistic

$$T^2 = \frac{n_1 n_2}{n_1 + n_2}(\bar{\mathbf{y}}_1 - \bar{\mathbf{y}}_2)'\mathbf{S}_{pl}^{-1}(\bar{\mathbf{y}}_1 - \bar{\mathbf{y}}_2),$$

which is distributed as T^2_{p,n_1+n_2-2} when H_0 is true. We reject H_0 if $T^2 \geq T^2_{\alpha,n_1+n_2-2}$.

3.10.2 Wilks's Likelihood Ratio Test

If $\mathbf{y}_{ij}, i = 1, 2, \cdots, g,\ j = 1, 2, \cdots, n$, are independently observed from $N_p(\mu_i, \mathbf{\Sigma})$, then the likelihood ratio test statistic for $H_0 : \mu_1 = \mu_2 = \cdots = \mu_g$ can be expressed as

$$\mathbf{\Lambda} = \frac{|\mathbf{E}|}{|\mathbf{E} + \mathbf{H}|}, \tag{3.19}$$

where $\mathbf{H}$ and $\mathbf{E}$ are defined as

$$\mathbf{H} = n\sum_{i=1}^{g}(\bar{\mathbf{y}}_i - \bar{\mathbf{y}})(\bar{\mathbf{y}}_i - \bar{\mathbf{y}}.)'$$

and

$$\mathbf{E} = \sum_{i=1}^{g}\sum_{j=1}^{n}(\bar{\mathbf{y}}_{ij} - \bar{\mathbf{y}}_i)(\bar{\mathbf{y}}_{ij} - \bar{\mathbf{y}}_i)'.$$

The test statistic 3.19 is distributed as the Wilks Λ-distribution. We reject $H_0 : \mu_1 = \mu_2 = \cdots = \mu_g$ if $\Lambda \leq \Lambda_{\alpha,p,\nu_H,\nu_E}$. p, ν_H and ν_E is the dimension and degrees of freedom for hypothesis and error, respectively.

3.10.3 Box's M-Test

For a one-way MANOVA with g groups ($g \geq 2$), the assumption of equality of covariance matrices can be stated as a hypothesis to be tested:

$$H_0 : \mathbf{\Sigma}_1 = \mathbf{\Sigma}_2 = \cdots = \mathbf{\Sigma}_g \tag{3.20}$$

versus H_1: at least two $\mathbf{\Sigma}_i$'s are unequal. Define $\mathbf{W}_i = \sum_{j=1}^{n_i} (\mathbf{y}_{ij} - \bar{\mathbf{y}}_i)(\mathbf{y}_{ij} - \bar{\mathbf{y}}_i)'$, and

$$M = \frac{|\mathbf{S}_1|^{\nu_1/2}|\mathbf{S}_2|^{\nu_2/2} \dots |\mathbf{S}_g|^{\nu_g/2}}{|\mathbf{S}_{pl}|^{\sum_i \nu_i/2}}, \tag{3.21}$$

where $\nu_i = n_i - 1, \mathbf{S}_i = \mathbf{W}_i/\nu_i$ is the unbiased sample covariance matrix, and $\mathbf{S}_{pl}$ is the pooled sample covariance matrix,

$$\mathbf{S}_{pl} = \frac{\sum_{i=1}^{g} \nu_i \mathbf{S}_i}{\sum_{i=1}^{g} \nu_i} = \frac{\mathbf{E}}{\nu_E}.$$

The statistic

$$u = -2(1 - c_1) \ln M \tag{3.22}$$

has an approximated χ^2-distribution with $\frac{1}{2}(k-1)p(p+1)$ degrees of freedom, where

$$c_1 = \left[\sum_{i=1}^{g} \frac{1}{\nu_i} - \frac{1}{\sum_{i=1}^{g} \nu_i} \right] \left[\frac{2p^2 + 3p - 1}{6(p+1)(k-1)} \right].$$

We reject H_0 if $u > \chi^2_\alpha$.

3.11 Classification into Several Populations

In this section, generalization of classification procedure for more than two discriminating groups (ie from 2 to $g \geq 2$) is straight forward. However, not much is known about the properties corresponding sample classification function , and in particular, their error rates have not been fully investigated. Therefore, we focus only on the Minimum ECM Classification with equal misclassification cost and Minimum TPM for multivariate normal population with unequal covariance matrices (Quadratic discriminant analysis).

3.12 Minimum ECM Classification with Equal Misclassification Cost

Allocate $\mathbf{x}_0$ to Π_k if

$$p_k f_k(\mathbf{x}) > p_i f_i(\mathbf{x}) \quad \text{for all } i \neq k \tag{3.23}$$

or, equivalently, Allocate $\mathbf{x}_0$ to Π_k if

$$\ln p_k f_k(\mathbf{x}) > \ln p_i f_i(\mathbf{x}) \quad \text{for all } i \neq k \tag{3.24}$$

Note that the classification rule in 3.23 is identical to the one that maximizes the posterior probability $P(\Pi_i|\mathbf{x})$ = P($\mathbf{x}$ comes from Π_i given that $\mathbf{x}$ is observed) where

$$P(\Pi_i|\mathbf{x}) = \frac{p_k f_k(\mathbf{x})}{\sum_{i=1}^{g} p_i f_i(\mathbf{x}} = \frac{(prior) \times (likelihood)}{\sum[(prior) \times (likelihood)]} \tag{3.25}$$

Therefore, one should keep in mind that in general minimum ECM rule must have the prior probability, misclassification cost and density function before it can be implemented.

3.13 Minimum TPM Rule for Unequal-Covariance Normal Populations

Suppose that the Π_i are multivariate normal populations, with different mean vectors μ and covariance matrices $\mathbf{\Sigma}_i \quad (i = 1, \ldots, g)$. An important special case occurs when the

$$f_i(\mathbf{x}) = \frac{1}{(2\pi)^{p/2}|\mathbf{\Sigma}_i|^{\frac{1}{2}}} \exp\{-\frac{1}{2}(\mathbf{x} - \mu_i)'\mathbf{\Sigma}_i^{-1}(\mathbf{x} - \mu_i)\}$$

with $c(i \mid i) = 0$, $c(k \mid i) = 1, k \neq i$ then

$$\begin{aligned} \ln p_k f_k(\mathbf{x}) &= \ln p_k - \left(\frac{p}{2}\right) \ln\{(2\pi)\} - \frac{1}{2}\ln|\mathbf{\Sigma}_k| - \frac{1}{2}(\mathbf{x} - \mu_k)'\mathbf{\Sigma}_k^{-1}(\mathbf{x} - \mu_k) \\ &= \max_i \ln\{p_i f_i(\mathbf{x})\} \qquad (3.26) \end{aligned}$$

The constant $(p/2)\ln(2\pi)$ can be ignored in 3.26, since it is the same for all population. Therefore, quadratic discriminant score for ith population is defined as

$$d_i^Q(\mathbf{x}) = -\frac{1}{2}\ln|\mathbf{\Sigma}_i| - \frac{1}{2}(\mathbf{x} - \mu_i)'\mathbf{\Sigma}_i^{-1}(\mathbf{x} - \mu_i) + \ln p_i \qquad (3.27)$$

The quadratic score $d_i^Q(\mathbf{x})$ is composed of contributions from the generalized variance $|\mathbf{\Sigma}_i|$, the prior probability p_i, and the square of the distance from x to the population mean μ_i. Allocate x to Π_k if the quadratic score

$$d_k^Q(\mathbf{x}) = \text{largest of } d_1^Q(\mathbf{x}), d_2^Q(\mathbf{x}), \ldots, d_g^Q(\mathbf{x}). \qquad (3.28)$$

In practice, the μ_i and Σ_i are unknown, but a training set of correctly classified observations if often available for the construction of estimates. The relevant sample quantities for population Π_i are the sample mean vector, $\bar{x}_i$, sample covariance matrix, S_i and sample size, n_i. The estimate of the quadratic discriminant score 3.28 is then

$$\hat{d}_i^Q(\mathbf{x}) = -\frac{1}{2}\ln|\mathbf{S}_i| - \frac{1}{2}(\mathbf{x} - \bar{\mathbf{x}}_i)'\mathbf{S}_i^{-1}(\mathbf{x} - \bar{\mathbf{x}}_i) + \ln p_i \quad \text{for} \quad i = 1, 2, \ldots, g \tag{3.29}$$

DA is relatively straight forward for known group-conditional densities. Generally, in practice, users of DA encounter the problem of their estimation from data. DA with other multivariate statistical techniques, has the assumptions of multivariate normality and spherical disturbance terms which provide a convenient way of specifying a parametric structure. However, in real life observations, some or all of these assumptions are sometimes violated. Therefore, it becomes necessary to use the Monte Carlo procedure with which data sets which resemble data obtained from controlled laboratory experiments and which are free of all the problems listed above are generated.

In this study, attention is focused on discrimination through the heteroscedastic normal model. Three mutually dependent normally distributed populations are considered.

3.14 Heteroscedastic Normal Model

Under a heteroscedastic normal model for the group-conditional distributions of the feature vector $\mathbf{X}$ on an entity, it is assumed that

$$\mathbf{X} \sim N(\mu_i, \Sigma_i) \quad \text{in} \quad G_i \quad \text{for} \quad i = 1, 2, \ldots, g \tag{3.30}$$

where μ_i denote group means and Σ_i denote group covariance matrix. The *ith* group-conditional density $f(\mathbf{X};\theta_i)$ is given by

$$f_i(\mathbf{X};\theta_i) = \phi(\mathbf{X};\mu_i,\Sigma_i) \tag{3.31}$$

$$= (2\pi)^{-p/2}|\mathbf{\Sigma}_i|^{-\frac{1}{2}}\exp\{-\frac{1}{2}(\mathbf{x}-\mu_i)'\mathbf{\Sigma}_i^{-1}(\mathbf{x}-\mu_i)\} \tag{3.32}$$

where θ_i consists of the elements of μ_i and the $\frac{1}{2}P(1+P)$ distinct elements of Σ_i. It is assumed that each Σ_i is non singular for the groups $\Pi_1,\ldots,\Pi_g$. If p_i denote the prior probabilities of the groups then we let:

$$\Psi_U = (p_1, p_2, \ldots, \theta_U{}')' \tag{3.33}$$

$$= (p', \theta_U{}')' \tag{3.34}$$

where θ_U consists of the elements of $\mu_1,\ldots,\mu_g$ and the distinct elements of $\Sigma_1,\ldots,\Sigma_g$. The subscript "U" emphasizes that the group variance matrices are allowed to be unequal in the specification of the model 3.30. Let the posterior probability that an entity with feature vector $\mathbf{X}$ belongs to group Π_i be denoted by $P_i((X);\Psi_U)$ for $i=1,\ldots,g$. We use the log ratios in estimating these posterior probabilities. Thus,

$$\eta_{ig}((X);\Psi_U) = \log(P_i((X);\Psi_U)/P_g((X);\Psi_U)) \tag{3.35}$$

$$= \log(P_i/P_g) + \xi_{ig}((X);\theta_U) \tag{3.36}$$

where $\xi_{ig}((X);\theta_U) = \log\{f_i(\mathbf{X};\theta_i)/f_g(\mathbf{X};\theta_g)\}$ for $(i = 1,\ldots,g-1)$ corresponds to an arbitrary choice of Π_g as the base group. Under the heteroscedastic normal model 3.30,

$$\xi_{ig} = -\frac{1}{2}\{\delta((X),\mu_i;\Sigma_i) - \delta((X),\mu_g;\Sigma_g)\} - \frac{1}{2}\{\log|\Sigma_i|/|\Sigma_g|\} \quad (i = 1'\ldots,g-1) \tag{3.37}$$

where :

$$\delta((X), \mu_i; \Sigma_i) = (\mathbf{X} - \mu_i)'\Sigma_i^{-1}(\mathbf{X} - \mu_i)$$

is the squared Mahalanobis' distance between $\mathbf{X}$ and μ_i with respect to $\Sigma_i (i = 1, \dots, g)$. The optimal or Bayes rule $r_0(\mathbf{X}; \Psi_U)$ assigns an entity with feature vector $\mathbf{X}$ to Π_g if

$$\eta_{ig}(\mathbf{X}; \Psi_U) \leq 0 \quad (i = 1, \dots, g-1)$$

is satisfied. Otherwise, the entity is assigned to Π_h if

$$\eta_{ig}(\mathbf{X}; \Psi_U) \leq \eta_{hg}(\mathbf{X}; \Psi_U) \quad (i = 1, \dots, g-1; i \neq h)$$

holds.

In defining the Bayes rule, we take the costs of misclassification to be the same. However, unequal costs of misclassification can be incorporated into the formulation. In practice,θ_U is generally taken to be unknown and so must be estimated from the available training data.

3.15 Monte Carlo Studies

Monte Carlo method is a heuristic statistical technique for evaluation and simulation of intractable problems by probabilistic simulation. Its performance is similar to controlled laboratory experiments. The center in a Monte Carlo study usually is a test statistic or estimator that has unknown finite sample properties. A typical Monte Carlo study in discriminant analysis involves the specification of the number of distributions of populations, sample sizes while the variable of interest is varied to investigate its effects on the asymptotic performance of the specific function of interest.

3.16 Simulation Design

Since we wish to evaluate the performance of QDF in case of correlated training samples and non-normality of distributions, we considered QDF. A Monte Carlo study was conducted. In the simulation procedure, multivariate normally correlated random data was generated for three populations with their mean vector $\mu_1 = (0, \ldots, 0)$, $\mu_2 = (0, \ldots, \delta)$ and $\mu_3 = (0, \ldots, 2\delta)$ respectively.

The covariance matrices, Σ_i (i=1, 2, 3). Where $k \neq l$, $\sigma_{kl} = 0.7$ for all groups except the diagonal entries given as $\sigma_k^2 = i$, for $i = 1, 2, 3$. After this, the correlation in the generated data of the various populations was eliminated. Skewed data was generated from the exponent of the normally uncorrelated data (log-normal) in order to evaluate the performance of skewness on QDF. QDF was then performed in each case and the leave-one-out method was used to estimate the proportion of observations misclassified.

Factors consider in this study were

1. Mean vector separator which is set at δ from 1 to 5 where δ is determined by the difference between the mean vectors.

2. Sample sizes which are also specified. Here 14 values of n_1 set at 30, 60, 100, 150, 200, 250, 300, 400, 500, 600, 700, 800, 1000, 2000 and the sample size of n_2 and n_3 are determined by the sample ratios at 1:1:1, 1:2:2 and 1:2:3 and these ratios also determine the prior probabilities to be considered

3. The number of variables for this study is also specified. The number of variables are set at 4, 6 and 8 following Murray (1977) who considered this in selection of variables in Discriminant Analysis.

3.16.1 Subroutine for QDF

Series of subroutines were written in MatLab to perform the simulation and discrimination procedures on QDF. Below are the important ones.

3.16.1.1 Data Simulation

The subroutine below was used to generate correlated normal data.

```
%specifying the covariance matrices
for i = 1:3
    p(i).SIGMA = i*eye(nvar);
    for r = 1:nvar
        for c = 1:nvar
            if r~=c
               p(i).SIGMA(r,c) = p(i).SIGMA(r,c) + 0.7;%to ensure correlation
            end
        end
    end
end

   %normal correlated generated data
        p(i).rep(r).x = mvnrnd(p(i).mu,p(i).SIGMA,n(i));
        p(i).rep(r).x = single(p(i).rep(r).x); % converting to single precision to reduce memory usage
        p(i).rep(r).mu = mean(p(i).rep(r).x);%sample mean of group i
        p(i).rep(r).sigma = cov(p(i).rep(r).x);%sample covariance of group i
```

After QDF has been performed on the correlated normal data, the correlation in the data is then eliminated by the command `Correlation_elimination(p(i).rep(r).x’)`

```
% eliminating correlation (uncorrelated normal data)
        [p(i).rep(r).z,p(i).rep(r).zmu,p(i).rep(r).zsigma] = Correlation_elimination(p(i).rep(r).x’);
        p(i).rep(r).z = p(i).rep(r).z’;
        for k = 1:n(i)
            p(i).rep(r).x(k,:) = p(i).rep(r).z(k,:)*(i*p(i).rep(r).zsigma)^(0.5)+p(i).mu;
        end

        p(i).rep(r).mu = mean(p(i).rep(r).x); p(i).rep(r).sigma = cov(p(i).rep(r).x);
        for k = 1:n(i)
```

We focussed our attention on positively skewed data. Therefore, after QDF has been performed on the uncorrelated normal data, the data is transformed to positively skewed data by finding the exponent of the uncorrelated normal data. The subroutine below was used

```
 %generating skewed data(lognormal)

          p(i).rep(r).x = exp(p(i).rep(r).x);
```

3.16.1.2 Discrimination Procedure

```
 %quadratic discriminant score
        for i=1:3
        for k=1:n(i)
            for j=1:3
                if j==i
                    p(i).rep(r).QDF{i,k}(j)= -1/2*log(det(p(i).rep(r).sigmaH{k}))...
                        -1/2*(p(i).rep(r).holdout(k,:)-p(i).rep(r).meanH{k})*inv(p(i).rep(r).sigmaH{k})...
                        *(p(i).rep(r).holdout(k,:)-p(i).rep(r).meanH{k})’+ log(prior(j));
                else
                    p(i).rep(r).QDF{i,k}(j)= -1/2*log(det(p(j).rep(r).sigma))...
                        -1/2*(p(i).rep(r).holdout(k,:)-p(j).rep(r).mu)*inv(p(j).rep(r).sigma)...
                        *(p(i).rep(r).holdout(k,:)-p(j).rep(r).mu)’+ log(prior(j));
                end
            end
        end

%finding the maximum and allocating it
for r = 1:nrep
    rep(r).predict_label=[];
    for i= 1:3
        for k= 1:n(i)
```

```
            rep(r).qmax(i,k)= max(p(i).rep(r).QDF{i,k});
            for j= 1:3
                if rep(r).qmax(i,k)== p(i).rep(r).QDF{i,k}(j)
                    p(i).rep(r).predict_label(k)= j;
                end
            end
        end

 %developing the confusion matrix
for r = 1:nrep
    %rep(r).cm1=cfmatrix(rep(r).actual_label,rep(r).predict_label);
    rep(r).cm=cfmatrix(rep(r).actual_label,rep(r).predict_label,[1 2 3],1);
    %rep(r).pmc = 1-trace(rep(r).cm);% total proportion misclassified
end

%average confusion matrix
avgconfmat=zeros(3,3);
for j = 1:3
    for i = 1:3
        F = [];
        for r = 1:nrep
            F = horzcat(F,rep(r).cm(i,j));
        end
        avgconfmat(i,j) = mean(F);
    end
end

%average error rates for the different groups
for i = 1:3
    gp(i).err = [];
    for j = 1:3
        if j~=i
           gp(i).err =  vertcat(gp(i).err,avgconfmat(j,i));
        end
    end
    QDA.totavg_gperr(:,i) = sum(gp(i).err);
end
```

`[cor_out,norm_out,skew_out] = QuadDA(N,nratio,nvar,delta,nrep)` is main command used in the entire procedure. The hold-out observation is classified into one of the groups. The leave-one-out error rates are computed using the `cfmatrix` command, which generates a confusion matrix. This is done for each replication within the cells and later averaged over replications.

Chapter 4

Simulation Results and Discussion

4.1 Introduction

This chapter contains the results of our investigation on the effects of correlation and skewness considering sample size ratio, number of variables and mean vector (group centroid) separator on the performance of QDF. The size of population1 (n_1) is fixed throughout the study and the sizes of population 2 and population 3, n_2 and n_3 respectively are determined by the sample size ratio under consideration. We first look at QDF when the training data are correlated and then when they are uncorrelated. Skewed distribution is applied afterwards. The summary of the 11 sample sizes for each value of n_1 and sample size ratio combination is presented in the table below.

Table 4.1: Sample Size Ratios

	Sample Size (n_1)	30	60	100	150	250	300	400	500	600	700	2000
S.Size (N)	1:1:1	90	180	300	450	750	900	1200	1500	1800	2100	6000
	1:2:2	150	300	500	750	1250	1500	2000	2500	3000	3500	10000
	1:2:3	180	360	600	900	1500	1800	2400	3000	3600	4200	12000

These are the sample sizes for which we simulated the normal variate and alternate the variables of interest as required.In this empirical study, the sensitivity of normal-based QDF classification models to correlated and skewed training samples is investigated, by considering sample size increment, varying of group centroid separator and varying number of variables. Three sample size ratios of population1 to population2 to population3 (n_1 : n_2 : n_3) considered are 1:1:1, 1:2:2 and 1:2:3. Eleven sample sizes of population1 n_1 was set at 30, 60, 100, 150, 250, 300, 400, 500, 600, 700, 2000 and the total sample size is determined by the sample ratios and these ratios also determine the prior probabilities to be considered. Multivariate normal data were generated for three p-variate situations ($p = 4, \quad p = 6$ and $p = 8$) for the QDF considering the three distributions. The sample size-ratio combination was repeated for each level of δ with each scenario being replicated 100 times. The results obtained are averaged over the number of replicates.

4.2 Discussion of Simulated Results

Discussion of simulated results is carried out in section 4.2 onwards. The results of the individual population error rates, their average error rates followed by their standard deviations and then their coefficients of variation with the various total sample sizes and sample size ratios of the distributions (correlated normal, uncorrelated normal and skewed) are presented. The summary of the results which consists of the average error rates of the distributions are first presented followed by their standard deviations and then their coefficient of variation in appendix A. Some of the graphs of the average error rates and coefficients of variation of the distributions are displayed. The graphs show the total sample sizes on the horizontal axis and the average error rates or coefficients of variation on the vertical axis. Some of the results displayed in this chapter are for the various δs , number of variables and sample size ratios. The remaining results (both tabular and graphical) are shown in the appendices. In all cases, the results were recorded to four decimal places.

4.3 Effect of Sample Size on QDF

In this section, we look at the asymptotic performance of the QDF in the cases where the training data are correlated, uncorrelated, and when they are skewed. Each subsection gives the performance of each distribution.

4.3.1 Correlated Normal Distribution

Evaluating the effect of sample size on QDF with respect to the correlated normal distribution for $\delta = 1$ is present in figures 4.1 and the rest of the graphical representation of the results are shown in figure B.1 to B.4. A bird's eye view of the tables and 15 graphs reveals that generally, an increased in the total sample size decreases the error rate of a particular n_1. It was observed that the average error rate for 4, 6, and 8 variables with $\delta = 1$ were higher as compared to the other values of the δ and among the sample size ratios used, sample size ratio 1:1:1 gives the lowest average error rates as the sample size increases asymptotically. Results also show that $n_1 = 30$ gave highest average error rates and lower average error rate are for $n_1 = 2000$ for variables 4, 6 and 8. There is a rapid decrease in the average error rate from total sample size of 90 to 180 of sample size ratio (1:1:1) of 8 variables for all δ. The results of 4 variables were higher the other number of variables. $\delta = 5$ gave the lowest average error rates as the sample size increases. It was also observed that the average error rates of sample size ratio 1:1:1 and 1:2:2 were marginal for $\delta = 1$. The difference between the ratios decreased as δ increased and with maintained total sample size and the average error rates decreased as the number of variables increased. In $\delta = 5$ the performance of the three sample size ratios were marginal.

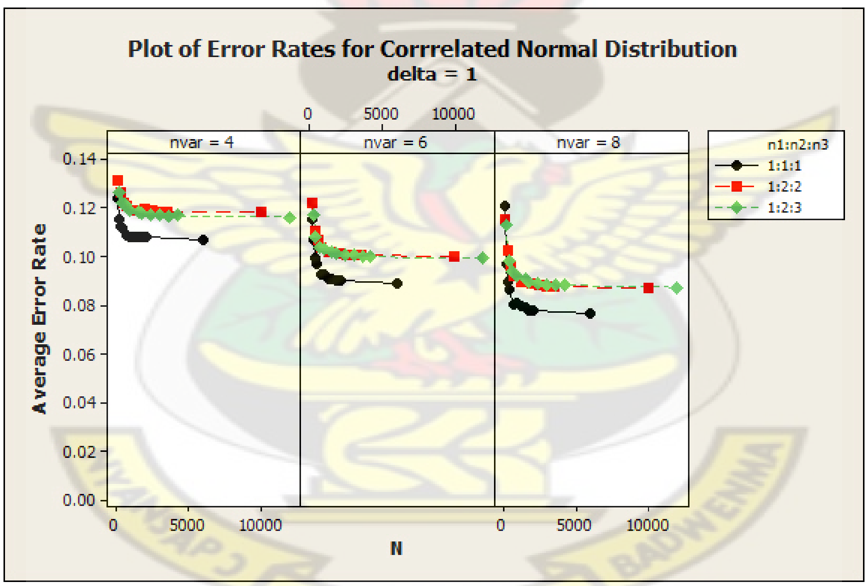

Figure 4.1: Average error rates of correlated normal distribution: $\delta = 1$

The standard deviation of the error rate from table (4.2) with the remaining tables in appendix A for the correlated normal distribution reveals that as the sample size increases, standard deviation of the error rate for sample size ratio 1:1:1 exhibit low standard deviations for $\delta = 1$ while sample size ratio 1:2:3 exhibits higher standard deviations. For a particular δ, the standard deviation decreases as the number of variables also increases. From $\delta = 2$ to $\delta = 5$, the standard deviations decreases as the sample size increases asymptotically. There is a sharp decrease of the standard deviation of sample size ratio 1:1:1 of variable 6 and 8, $\delta = 4$ and of variable 8, $\delta = 5$ as compared with their counterparts in the same case.

Table 4.2: Standard Deviations for 4 variables and a sample size ratio of (1:1:1)

Distributions		CorrNorm	UncorrNorm	Skewed
	S. Size	SD	SD	SD
$\delta = 1$	90	0.0530	0.0681	0.0875
	180	0.0536	0.0641	0.0942
	300	0.0543	0.0603	0.0983
	450	0.0547	0.0603	0.0998
	750	0.0534	0.0575	0.1031
	900	0.0526	0.0573	0.1030
	1200	0.0524	0.0568	0.1034
	1500	0.0529	0.0571	0.1051
	1800	0.0526	0.0567	0.1048
	2100	0.0531	0.0567	0.1057
	6000	0.0524	0.0559	0.1071
$\delta = 2$	90	0.0447	0.0524	0.0680
	180	0.0402	0.0469	0.0741
	300	0.0384	0.0421	0.0781
	450	0.0394	0.0424	0.0800
	750	0.0370	0.0409	0.0822
	900	0.0370	0.0415	0.0821
	1200	0.0369	0.0402	0.0847
	1500	0.0367	0.0398	0.0850
	1800	0.0372	0.0400	0.0851
	2100	0.0367	0.0399	0.0860
	6000	0.0365	0.0394	0.0883
$\delta = 3$	90	0.0300	0.0345	0.0489
	180	0.0264	0.0296	0.0493
	300	0.0245	0.0277	0.0522
	450	0.0239	0.0263	0.0535
	750	0.0232	0.0256	0.0601
	900	0.0230	0.0257	0.0593
	1200	0.0226	0.0255	0.0581
	1500	0.0225	0.0253	0.0589
	1800	0.0223	0.0248	0.0604
	2100	0.0226	0.0250	0.0606
	6000	0.0224	0.0249	0.0622
$\delta = 4$	90	0.0200	0.0236	0.0252
	180	0.0159	0.0189	0.0296
	300	0.0144	0.0173	0.0286
	450	0.0139	0.0160	0.0307
	750	0.0135	0.0158	0.0348
	900	0.0131	0.0153	0.0344
	1200	0.0124	0.0151	0.0355
	1500	0.0126	0.0151	0.0351
	1800	0.0128	0.0147	0.0378
	2100	0.0128	0.0147	0.0374
	6000	0.0125	0.0146	0.0387
$\delta = 5$	90	0.0123	0.0156	0.0158
	180	0.0094	0.0121	0.0145
	300	0.0084	0.0098	0.0128
	450	0.0078	0.0093	0.0159
	750	0.0071	0.0090	0.0152
	900	0.0069	0.0085	0.0162
	1200	0.0070	0.0085	0.0167
	1500	0.0066	0.0085	0.0164
	1800	0.0066	0.0083	0.0164
	2100	0.0065	0.0083	0.0165
	6000	0.0064	0.0081	0.0184

The coefficients of variation in correlated normal distribution in figure 4.2 and remaining graphs in figure B.5 to B.8 for $\delta = 1$, variables 4 to 8 increased exponentially and then stabilized with averagely lower variations in sample size ratio 1:2:2 and with higher variations in sample size ratio 1:2:3 as the total sample size increases asymptotically. The variations also increased as δ increased. $\delta = 3$ gives a steady coefficients of variation as the total sample size increased for variable 4 while it gave a little increase and then stabilized in variables 4 and 6. There was a decline in the coefficients of variation for $\delta = 4$ as the total sample size increased asymptotically in variable 4. The coefficients of variation increased from total sample size 150 to 500 and from 180 to 360 for sample size ratios 1:2:2 and 1:2:3 respectively for variables 6 and 8 and then decreased as the total sample size increased asymptotically. For $\delta = 5$, there was a sharp decrease in the coefficients of variation in sample size ratio 1:1:1 for all number of variables as the total sample size increased.

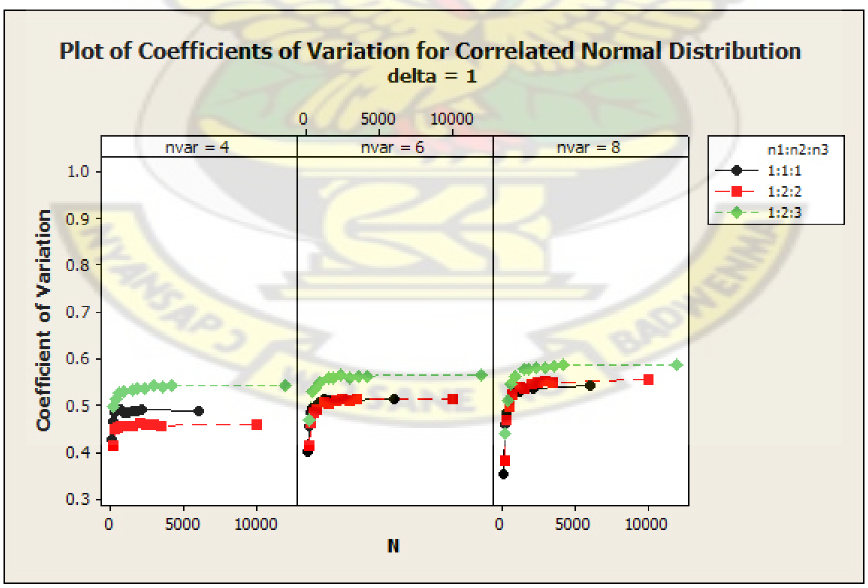

Figure 4.2: Coefficients of Variation for Correlated Normal Distribution:$\delta = 1$

4.3.2 Uncorrelated Normal Distribution

For the uncorrelated distribution the average error rate wa similar to the results obtained in the correlated normal distribution with the exception of the average error rate of sample size ratio 1:1:1 which decreased rapidly from total sample size of 90 to 180 for 8 variables in all δs. The average error rate decreased as the total sample size increased asymptotically. And it reduced when δ also increased. The graphical representation of this result for $\delta = 1$ in show in figure 4.3 with the rest in B.9 to B.12

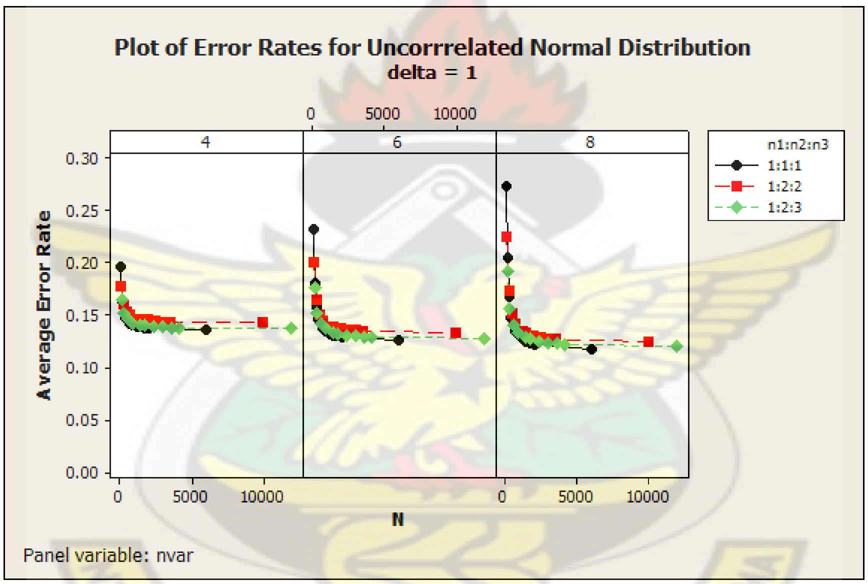

Figure 4.3: Average error rates of uncorrelated normal distribution: $\delta = 1$

The standard deviations for uncorrelated distribution decreased as the total sample size increased for $\delta = 1$ for 4, 6 and 8 variables. There was a rapid decrease (from 150 to 300) in the standard deviation of sample size ratio $1:2:2$ of 6 and 8 variables as the total sample size increases asymptotically. The sample size ratio 1:2:2 exhibited lower values of standard deviations as the total sample size increases for $\delta = 1$ and 2. For $\delta = 3, 4$ and 5, sample size ratio 1:1:1 gave lower standard deviations. The standard deviation in this distribution decreased as the sample size increased asymptotically with increasing δ. For $\delta = 5$ the standard deviations of the various sample size ratios were marginal in all the number of variables

The coefficients of variation generally increased exponentially and stabilized with increasing total sample size and number of variables in $\delta = 1$ exhibited lower variations as compared with the remaining δs as shown in figure 4.4 and figure B.13 to B.16 of appendix B.2. For $\delta =$ 4, the coefficients of variation in sample size ratio 1:1:1 decreased while the remaining ratios did not give any particular pattern for the 4 variable situation. For 4 variable situation with $\delta = 5$, the coefficients of variation decreased as the total sample size increased. The coefficient of variation of the other 6 and 8 variables situations did not show any particular pattern as the total sample size increased.

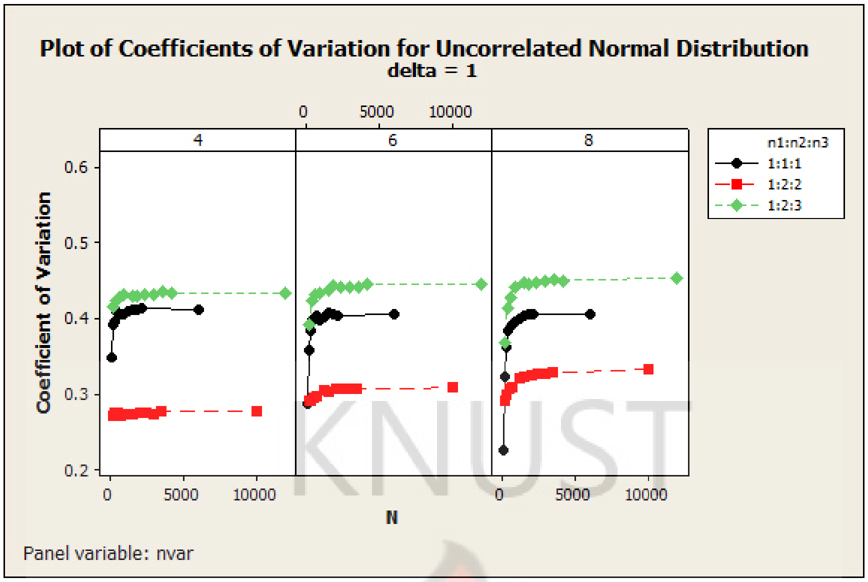

Figure 4.4: Coefficients of Variation for Uncorrelated Normal Distribution:$\delta = 1$

4.3.3 Skewed (Lognormal) Distribution

Unlike the already discussed distributions, there was an increase in the average error rates of the sample size ratios 1:2:2 and 1:2:3 as the total sample size increased asymptotically in the skewed distribution for $\delta = 1$ to 3 in figures 4.5, 4.6 and 4.7 and in figures B.17 and B.18 of appendix B.3 while the average error rates stabilized in the rest of the δs. Sample size ratio $1:1:1$ in this case gave lower average error rates. The average error rates for $\delta = 5$ were the lowest as compared to the other δs and they decreased as the total sample size increased.

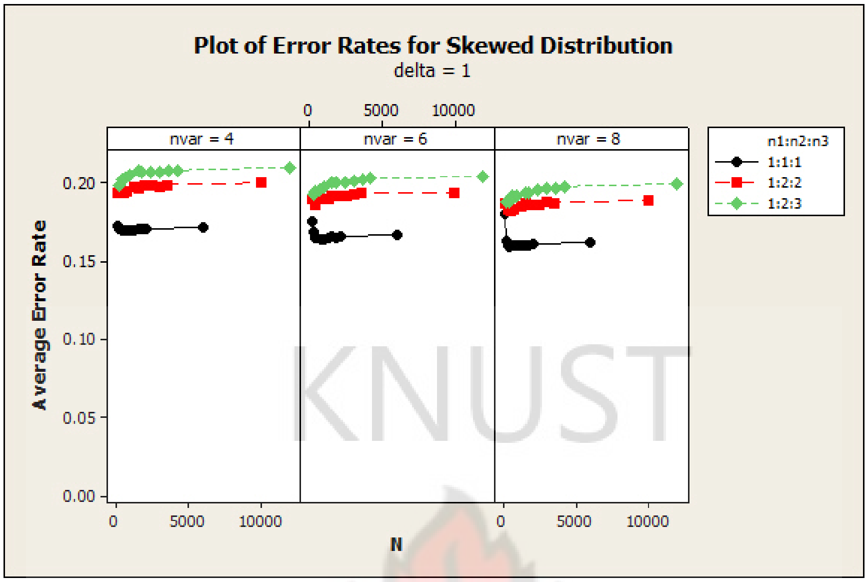

Figure 4.5: Average error rates of skewed distribution: $\delta = 1$

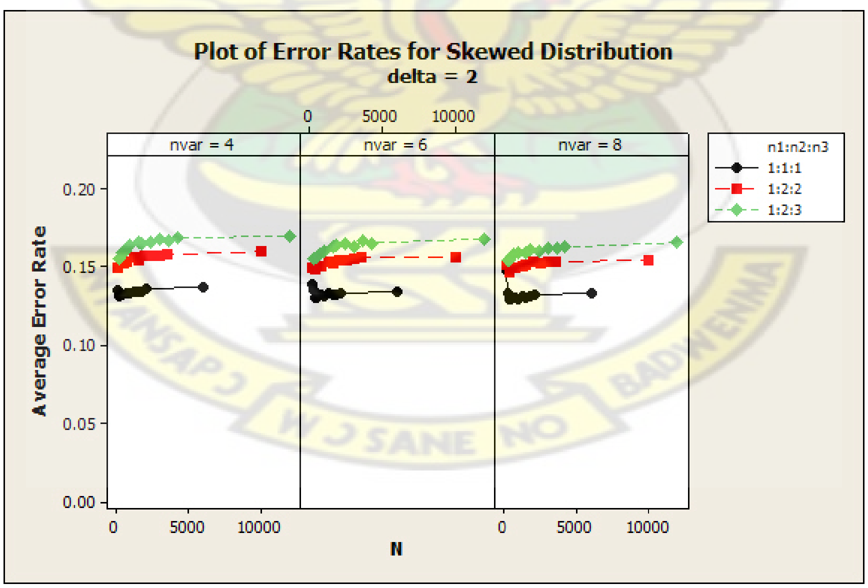

Figure 4.6: Average error rates of skewed distribution: $\delta = 2$

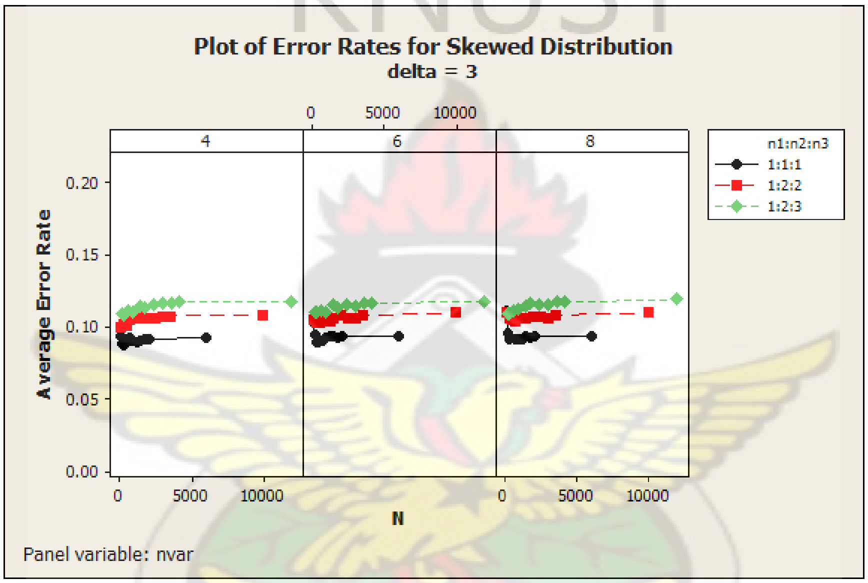

Figure 4.7: Average error rates of skewed distribution: $\delta = 3$

The standard deviations in this distribution happened to be large in $\delta = 1$ they increased and stabilized as the total sample size increased asymptotically with sample size ratio 1:1:1 giving lower deviations in the situation of 4 and 8 variables for all δs. In 6 variables case, the sample size ratio 1:1:1 and 1:2:2 reduced more than sample size ratio 1:2:3 by giving lower deviations for all giving δs as the total sample size increases.

The coefficients of variation in the skewed distribution increased exponentially with total sample size increased by exhibiting lower variabilities for all cases of variables used in $\delta = 1$ as shown in figure 4.8 and rest of the graphical representation of the results in figure B.19 to B.22. The coefficients of variation increased in $\delta = 2$ and 3 but with higher variations and did not give any particular pattern in $\delta = 4$ and 5.

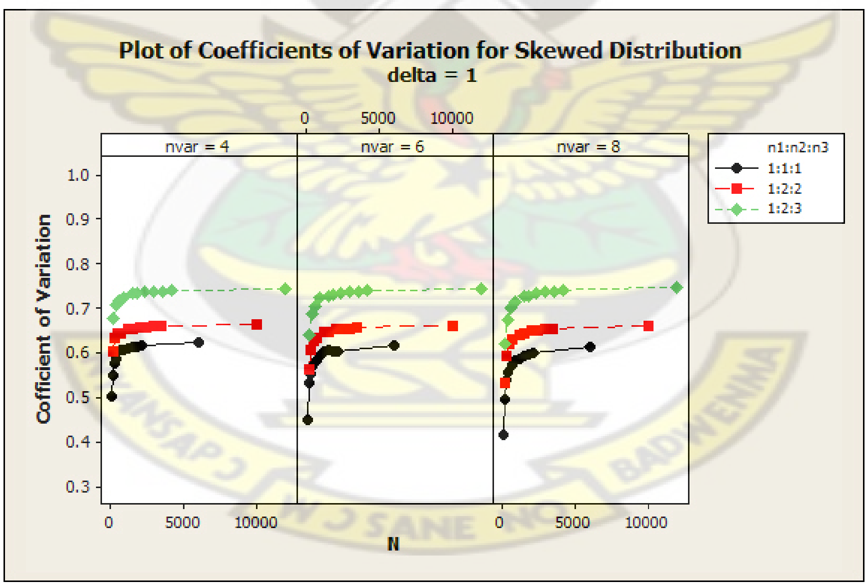

Figure 4.8: Coefficients of Variation for Skewed Distribution:$\delta = 1$

4.4 Effect of Number of Variables on QDF

The effect of number of variables on the QDF under the three mentioned distributions are discussed in the subsections. The table of results used in this section are the same for the previous section. The graphical representation of the results are shown in figures C.1 to C.10 of appendix C.

4.4.1 Correlated Normal Distribution

The graphs of the results for sample size ratio 1:1:1 of the situations of 4, 6 and 8 variables are shown in figure 4.9. It was observed that as the number of variables increased, the average error rate reduced in the correlated normal distribution. The rate at which it reduces in $\delta = 1$ for ratio 1:1:1 is better than that of the other δs. For increasing sample size ratio, as the number of variables increased, the decrease in the average error was marginal as δ increased.

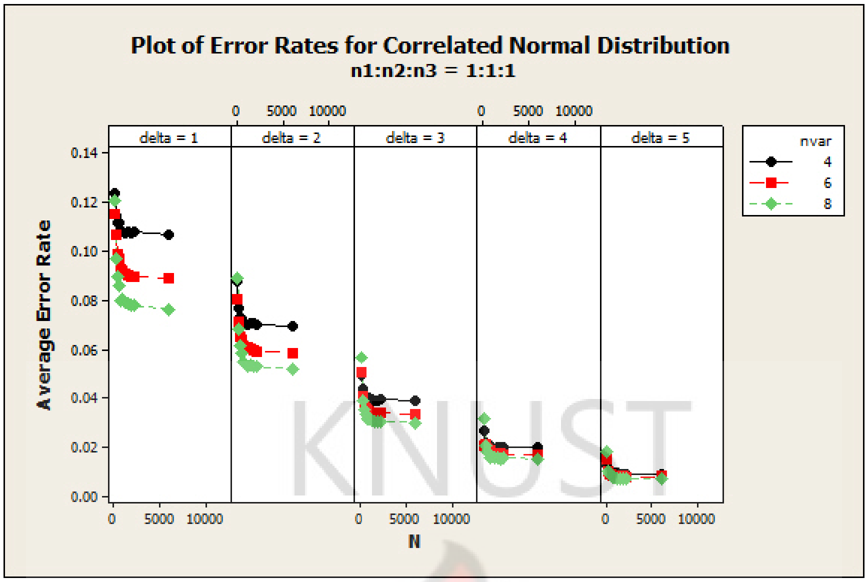

Figure 4.9: Average Error Rate for Correlated Normal Distribution:$n_1 : n_2 : n_3 = 1 : 1 : 1$

The coefficients of variation in this distribution for ratio 1:1:1 in figure 4.10 reveals that as the number of variables increased the coefficients of variation increased for variables 4, 6 and 8 from $\delta = 1$ to 3 except $\delta = 4$ and 5 in which it reduced. Yet the in the case of 8 variables the variabilities exhibited were higher than the rest in this case. For ratio 1:2:2 the coefficients of variation increased from total sample size of 150 to 2000 and stabilized for all δs as the number of variables increased except $\delta = 4$ which showed a decline in the coefficients of variation for the case of 4 and 6 variables. In $\delta = 5$, there was declination in the coefficients of variation as the number of variables increased. Sample size ratio 1:2:3 gave similar result as ratio 1:2:2

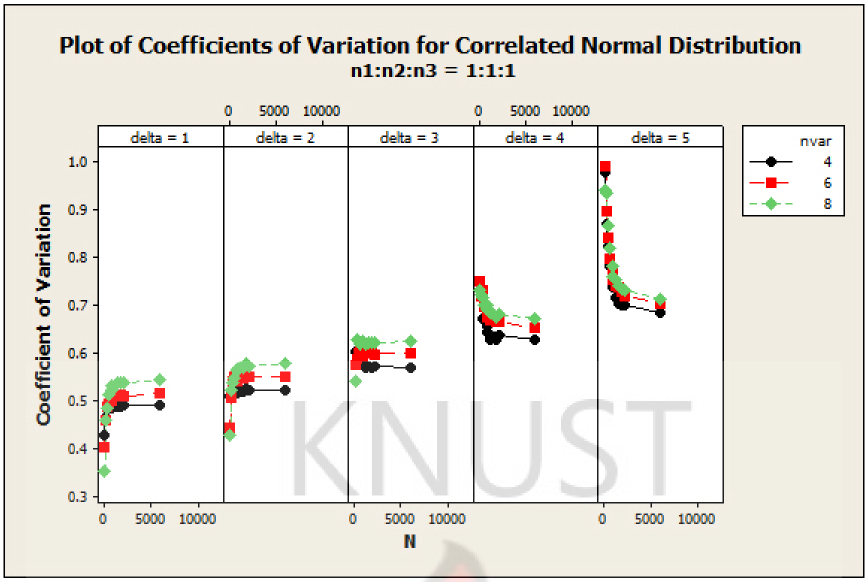

Figure 4.10: Coefficient of variation of correlated normal distribution: $n_1 : n_2 : n_3 = 1 : 1 : 1$

4.4.2 Uncorrelated Normal Distribution

The average error rate for the uncorrelated normal distribution was also looked at and it revealed that for 1:1:1, in figure 4.11, there was a sharp decline in the average error rate from total sample size 90 to 180 as the number of variables increase for all δs. It also revealed that as the number of variables increased the average error rate reduced for all sample size ratios.

The coefficients of variation in this distribution for ratio 1:1:1 in figure 4.12 showed that the variabilities increased exponentially for all δs with the exception of $\delta = 4$ and 5 for which variable 4 declined. In the case of 8 variables, about 9.65% and 11.91% increase

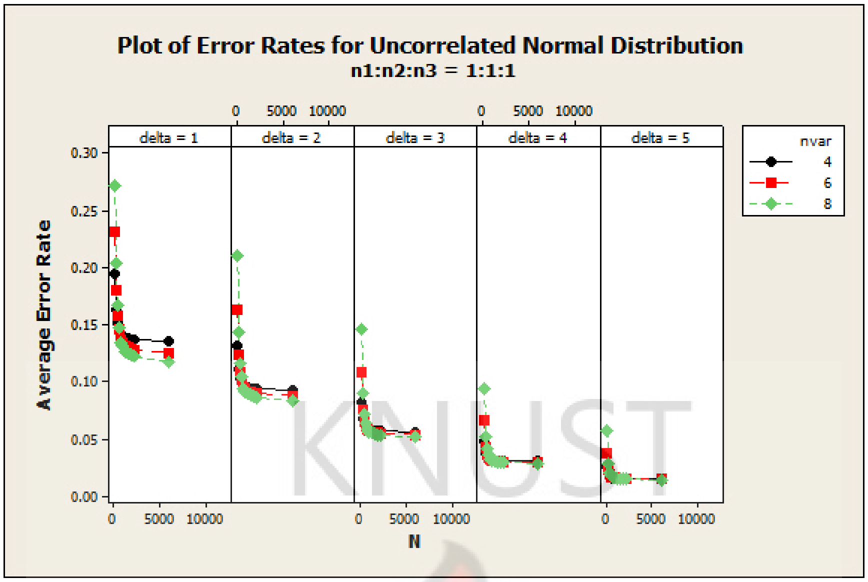

Figure 4.11: Average error rates of uncorrelated normal distribution: $n_1 : n_2 : n_3 = 1 : 1 : 1$

in variations from total sample size of 90 to 180 for all $\delta = 1$ and 2. For $\delta = 4$ and 5, the coefficients of variation for variables 4 declined from total sample size of 90 to 6000 while variables 6 and 8 increased. For $\delta = 5$, the coefficients of variations for 8 variables increased from 90 to 750 and declined to 6000. The coefficients of variation in general for this distribution increased as the number of variables increased.

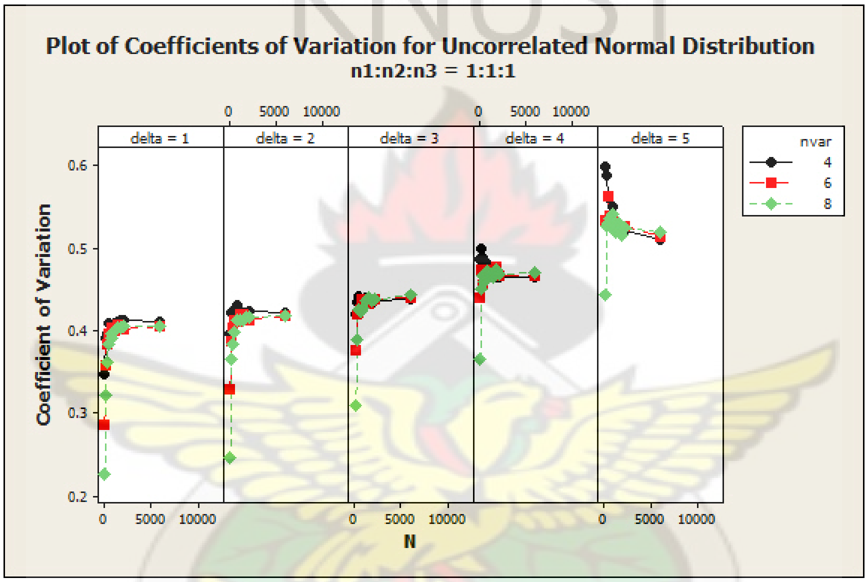

Figure 4.12: Coefficients of Variation for Uncorrelated Normal Distribution:$n_1 : n_2 : n_3 = 1 : 1 : 1$

4.4.3 Skewed (Lognormal) Distribution

The skewed distribution performs differently with increasing number of variables. For sample size ratio 1:1:1, the average error rates of the variables reduced and curved upward as the total sample size increased for all δs, as shown in figure 4.13. The average error rates of sample ratios 1:2:2 and 1:2:3 were different. Their average error rate increased as the total sample size increased and reduced with increasing number of variable for $\delta = 1$ and 2 . In $\delta = 3$ and 4 of ratios 1:2:2 and 1:2:3, as the number of variables increased the average error rate dropped from the total sample size of 150 to 300 and increased as the sample size also increased respectively while that of $\delta = 5$ decreased marginally. In general the average error rate increased as the number of variables increased with increasing δ.

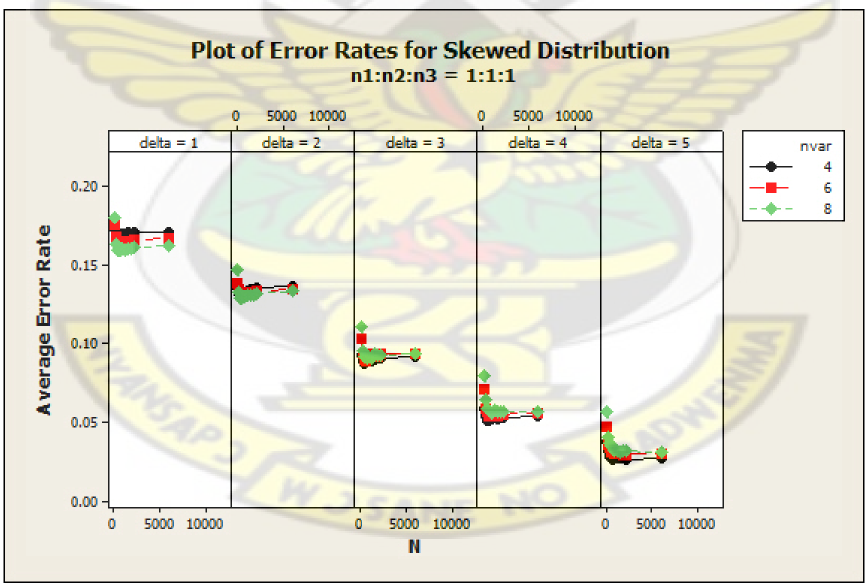

Figure 4.13: Average error rates of skewed distribution: $n_1 : n_2 : n_3 = 1 : 1 : 1$

The coefficients of variation in this distribution for ratio 1:1:1 in figure4.14reveals an increase of variabilities in the individual number of variables as δ from 1 to 4 increased. The variabilities also decreased as the number of variables increased with sharp increase in variabilities from total sample size of 90 to 180. The variabilities also increased as the sample size ratio increased.

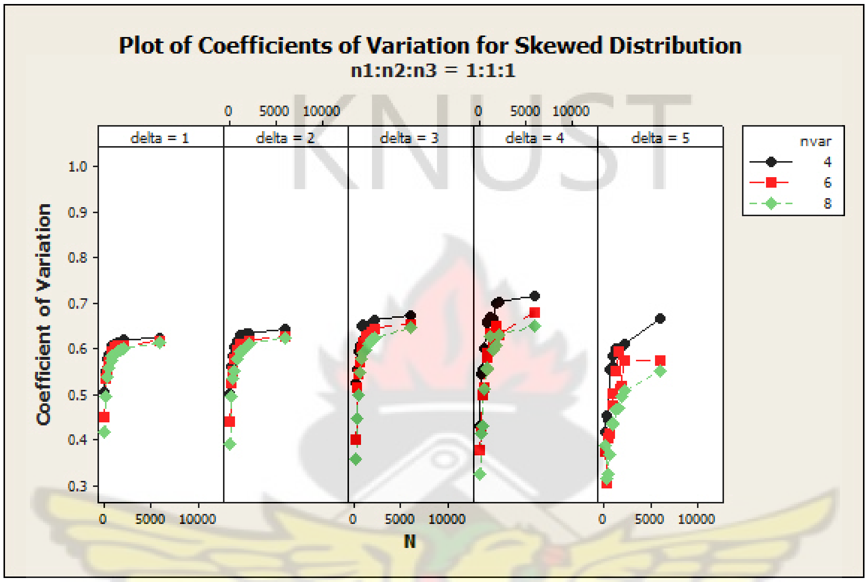

Figure 4.14: Coefficients of Variation for Skewed Distribution:$n_1 : n_2 : n_3 = 1 : 1 : 1$

4.5 Effect of Group Centroid Separator on QDF

In this section, we present results of our investigation on the effect of the Mahalanobis distance on QDF for the three distributions. The graphs for these results are shown in figure 4.15 to D.12 in appendix D.

4.5.1 Correlated Normal Distribution

Considering the correlated normal distribution in figure 4.15, it was observed that with increasing total sample size, the average error rate reduces as the δ increased and also reduced as the number of variables increased. It can be observed that there was about 2.37% drop in the average error rate from total sample size 90 to 180 for all $\delta = 1$s in the case of 8 variables. The average error rate reduced as the total sample size increased for all sample size ratios with increasing δ.

Looking at the coefficients of variation of sample size ratio 1:1:1 with increasing total sample size in figure 4.16, uniform behavior of δ was not portrayed. As coefficients of variation for $\delta = 5$ and 4 were declining, that of the rest of the δs may be increasing or reducing depending on the particular sample size ratio. In a nutshell, the correlated normal distribution and the uncorrelated normal distribution give similar result with average error rates and coefficients of variation. Therefore, with increasing δ, $\delta = 5$ gives higher coefficients of variation.

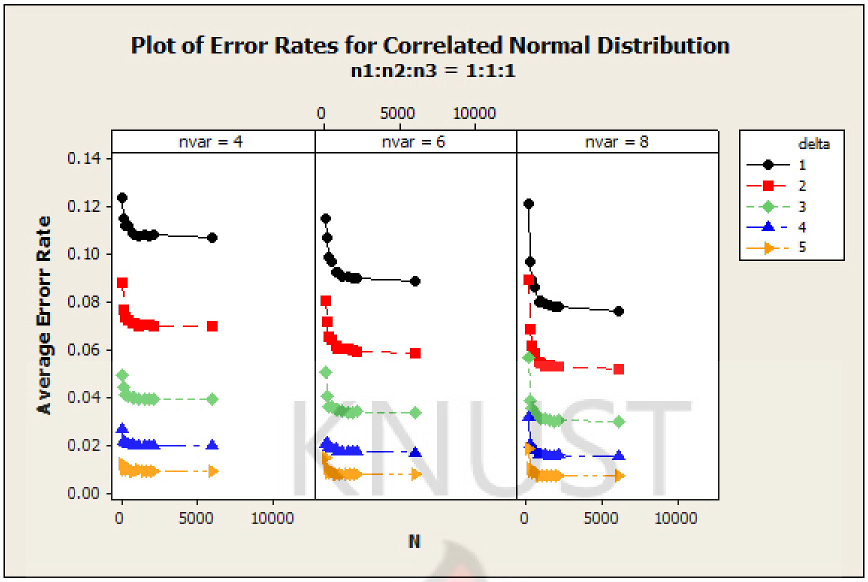

Figure 4.15: Average error rates of correlated normal distribution for δ: $n_1 : n_2 : n_3 = 1 : 1 : 1$

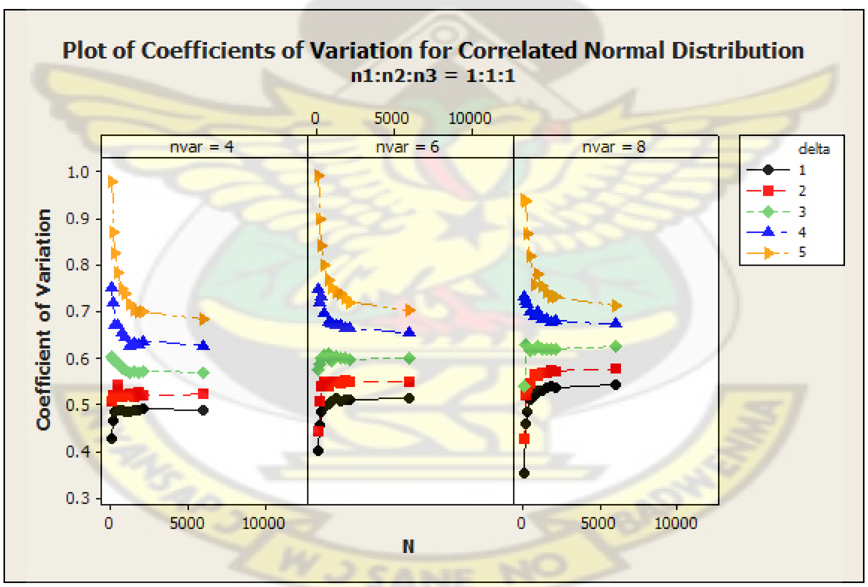

Figure 4.16: Coefficients of Variation for Correlated Normal Distribution:$n_1 : n_2 : n_3 = 1 : 1 : 1$

4.5.2 Uncorrelated Normal Distribution

We also looked the effect group centroid separator of uncorrelated on Quadratic Discriminant Function. The average error rate of the uncorrelated normal distribution for sample size ratio 1:1:1 in figure 4.17 revealed that the average error rates of the individual δs reduce as the sample size increases. There was about 3.19%, 5.09%, 6.81% drop of the average error rate for $\delta = 1$, variables 4, 6 and 8 respectively. The average error rates of $\delta = 2$ for variables 4 to 6 exhibited about 2.00% 3.99% 6.65% drop in the average error rates. In general, the average error rates decreased as δ increased irrespective of the number of variables and sample size ratios.

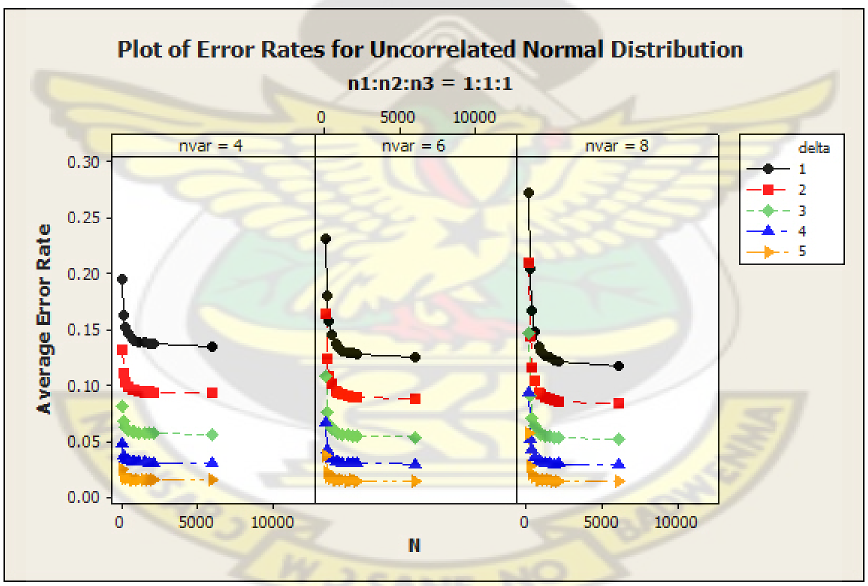

Figure 4.17: Average error rates of uncorrelated normal distribution for δ: $n_1 : n_2 : n_3 = 1 : 1 : 1$

The coefficient of variation of this distribution of sample size ratio $1 : 1 : 1$ in figure 4.18 did not show any uniform pattern in the variabilities as δ increased but in general the as δ

increased, the variabilities also increased.

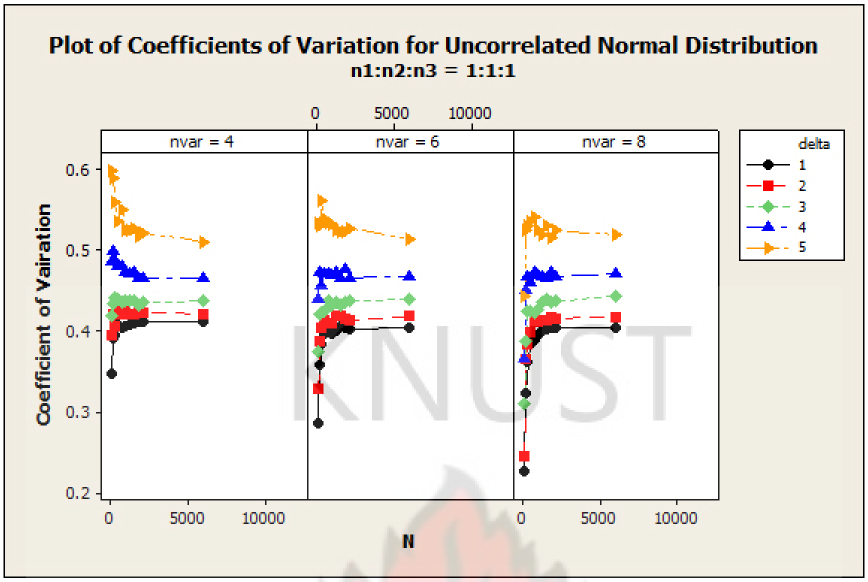

Figure 4.18: Coefficients of Variation for Uncorrelated Normal Distribution:$n_1 : n_2 : n_3 = 1 : 1 : 1$

4.5.3 Skewed (Lognormal) Distribution

The average error rate of the skewed distribution for sample size ratio 1 : 1 : 1 in figure 4.19 revealed that the average error rates of the individual δs reduced and increased a little as the sample size increased except that of $\delta =$ which decreased as the total sample size increased. As the average error rates of $\delta = 5$ reduced with increasing total sample size and sample size ratio, the rest of them increased with increasing total sample size. Whether the individual δs increased or reduced, in a nut shell, it is observed that for the average error rates of δ reduced as the sample size increased.

Figure 4.19: Average error rates of skewed distribution for δ: $n_1 : n_2 : n_3 = 1 : 1 : 1$

The coefficients of variation for sample size ratio 1 : 1 : 1 in figure **??** showed that the variabilities increased exponentially as the total sample size and δ increased. $\delta = 5$ gave lower variations in the case of 6 and 8 variables. For the rest of the sample size ratios, $\delta = 4$ and 5 gave coefficients of variation above the rest of the δs with $\delta = 1$ producing lower coefficient of variations.

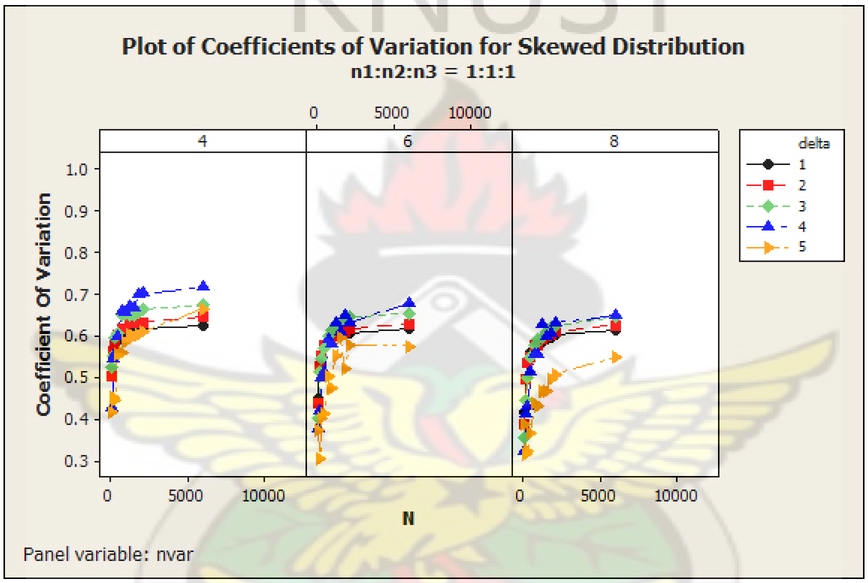

Figure 4.20: Coefficients of Variation for Skewed Distribution:$n_1 : n_2 : n_3 = 1 : 1 : 1$

4.6 Comparison of Error Rates of Correlated Normal, Uncorrelated Normal and Skewed Distribution

We present the discussion on the comparison of error rates Correlated Normal, Uncorrelated Normal and Skewed Distribution under Quadratic Discriminant Function. We present graphs of the average error rates of misclassification and coefficient of variation for the three distributions in figure E.1 to E.12 of appendix F. However, observation made on sample size ratio 1:1:1 for 4 variable case in figure 4.21 showed the average error rates of skewed distribution were the highest with correlated normal distribution exhibiting least average error rates as delta increased with increasing sample size. It can also be observed that the average error rates of the distributions decreased as δ increased and the difference between the three distributions was also decreased marginally with increasing δ. For sample size ratio 1:1:1 of variables 6 and 8, there was 5.09%, 3.99% and 3.28% drop of the average error rates from total sample size 90 to 180 for $\delta = 1, 2,$ and 3 respectively and 6.81%, 6.65% and 5.65% drop of the average error rates from total sample size 90 to 180 for $\delta = 1, 2,$ and 3 respectively in the uncorrelated normal distribution. In general, for sample size ratio 1:1:1, the average error rate decreased with increasing sample size and δ. To sum it up, as the average error rates of correlated normal and uncorrelated normal were decreasing, that of skewed distribution increased with higher average error rate for all number of variables and sample size ratios used with increasing sample size and δ followed by uncorrelated normal distribution.

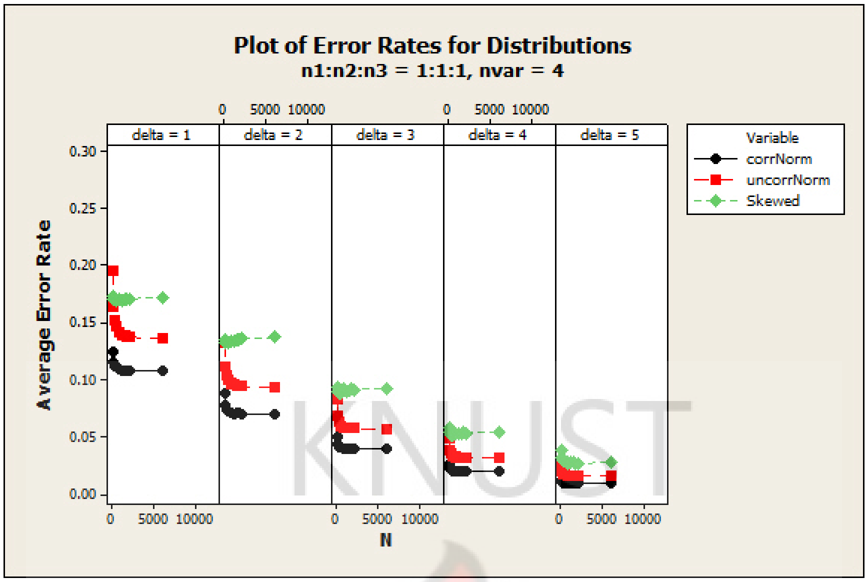

Figure 4.21: Average error rates of the three distributions for 4 variables: $n_1 : n_2 : n_3 = 1 : 1 : 1$

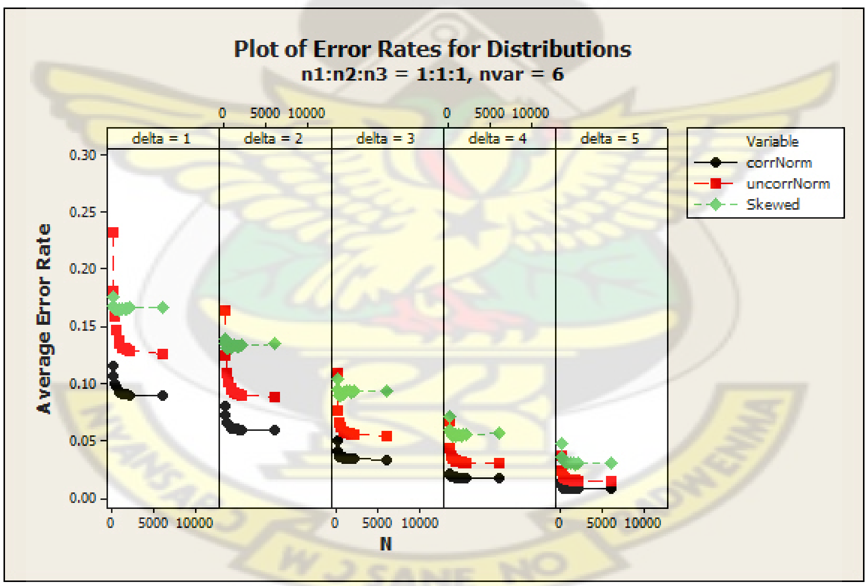

Figure 4.22: Average error rates of the three distributions for 6 variables: $n_1 : n_2 : n_3 = 1 : 1 : 1$

Looking at figures 4.24 to 4.26, the coefficients of variation for sample size ratio 1 : 1 : 1 variable 4, 6 and 8 showed that the skewed distribution gave higher variabilities for $\delta = 1$

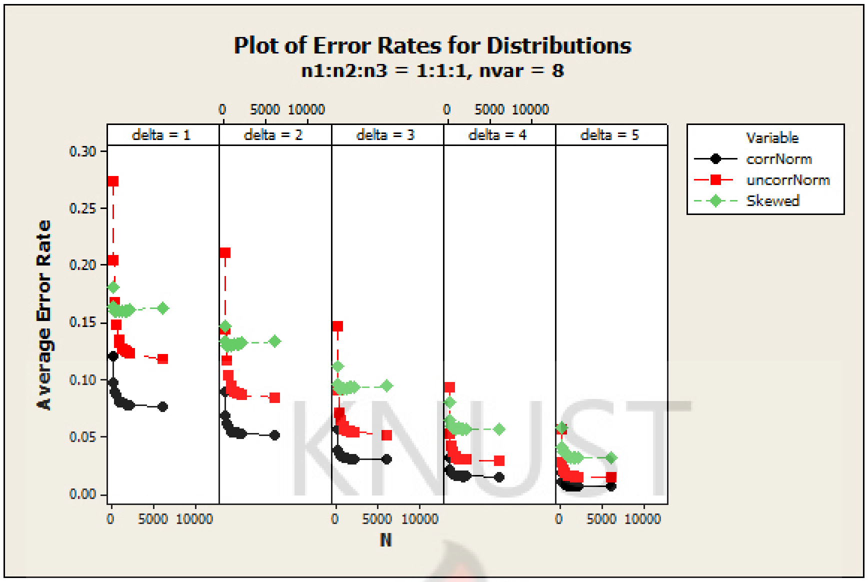

Figure 4.23: Average error rates of the three distributions for 8 variables: $n_1 : n_2 : n_3 = 1 : 1 : 1$

to 4 and it increased and stabilized with increasing sample size. The variabilities of correlated normal distribution increased as δ increased. In a nutshell, the skewed distribution exhibited the highest variabilities in all scenarios followed by the uncorrelated normal distribution.

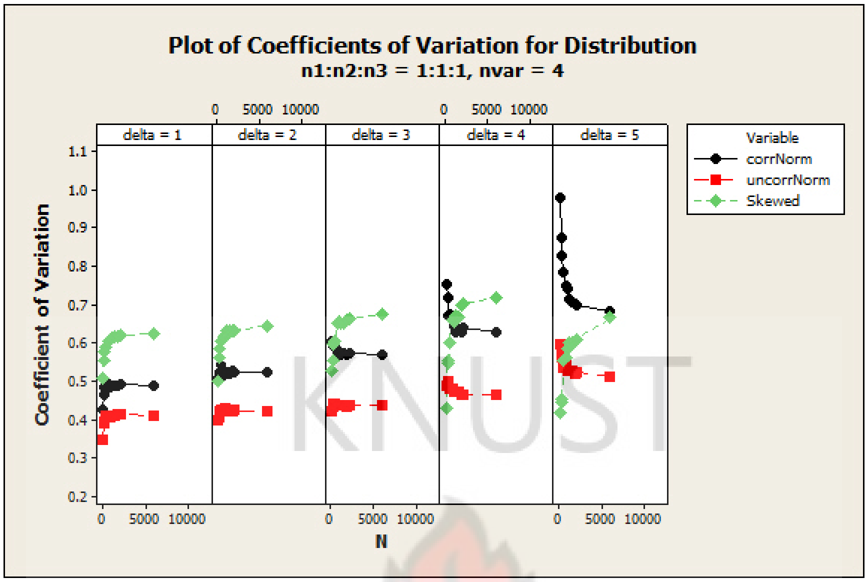

Figure 4.24: Coefficients of variation of the three distributions for 4 variables: $n_1 : n_2 : n_3 = 1 : 1 : 1$

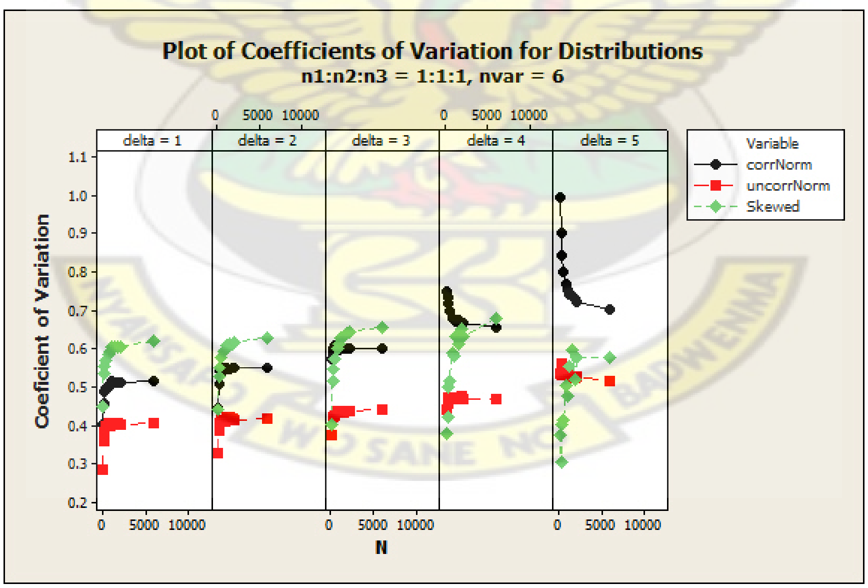

Figure 4.25: Coefficients of variation of the three distributions for 6 variables: $n_1 : n_2 : n_3 = 1 : 1 : 1$

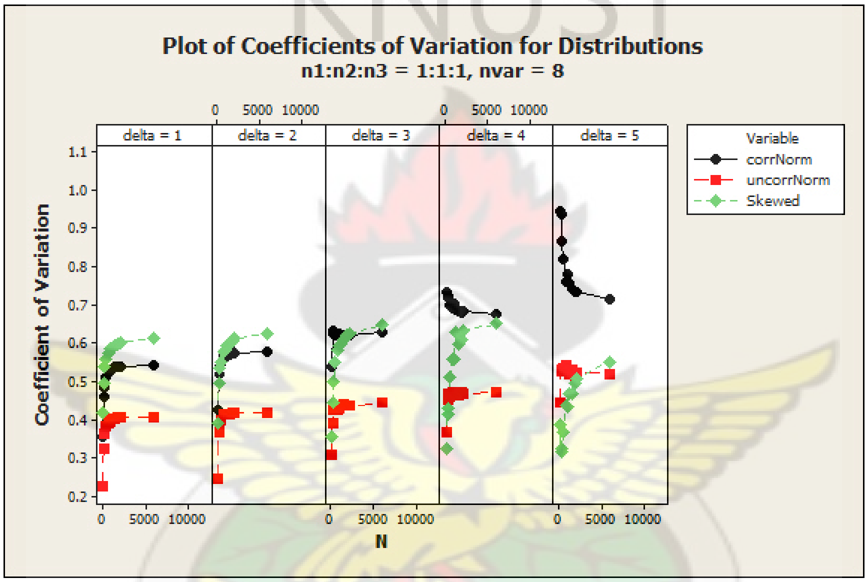

Figure 4.26: Coefficients of variation of the three distributions for 8 variables: $n_1 : n_2 : n_3 = 1 : 1 : 1$

The coefficient of variation for correlated and uncorrelated normal distributions were lower as compared with the other distribution with increasing sample size and increases with increasing δ. Yet correlated normal distribution gives higher variabilities when $\delta = 5$. For all number of variables, the coefficients of variation for all the three distributions increases with increasing δ and they give very high variabilities when the sample size ratio is $1:2:3$.

The sample size ratio at which the performance the Quadratic Discriminant Function deteriorate was looked at. Figures F.1 to F.6 in appendix F showed how the average error rates and coefficients of variation of the individual distributions performed with increasing δ as the sample size increased.

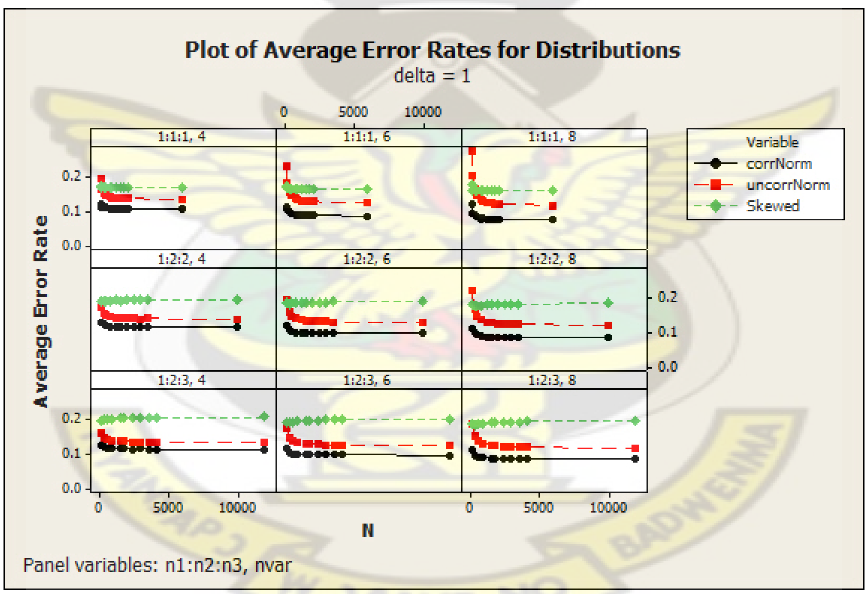

Figure 4.27: Average Error Rates for individual sample size ratios and distributions for $\delta = 1$

It can be observed that as the sample size ratio increased the average error rates also decreased with increased with increasing number of variables as shown in figures 4.27 and

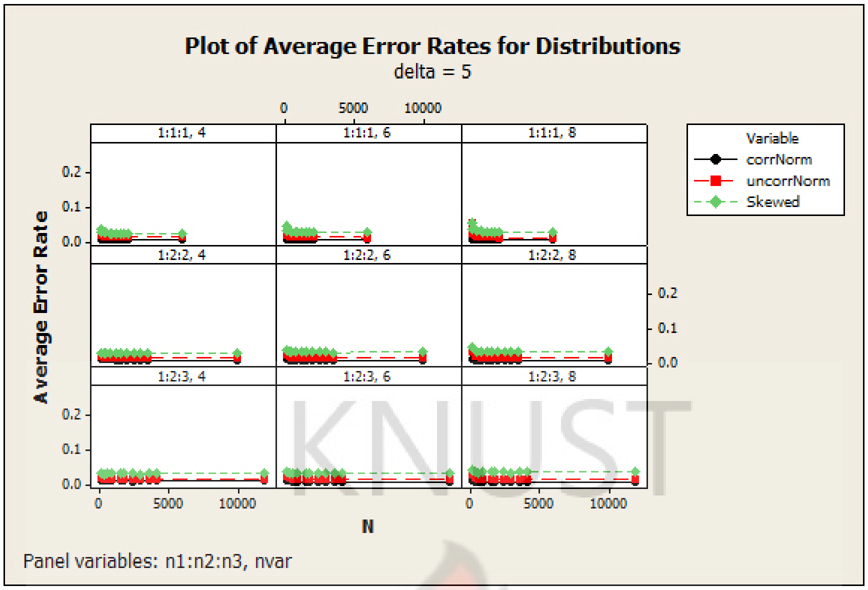

Figure 4.28: Average Error Rates for individual sample size ratios and distributions for $\delta = 5$

4.28. It was observed from figures 4.29 and 4.30 that for $\delta = 1$, the variabilities increased and the stabilized as the sample size increased for all sample size ratios. The difference between the variabilities of a particular distribution from 4 variables to 8 variables was not much. In $\delta = 5$, the variabilities for the correlated normal distributions and that of the uncorrelated normal distribution for 4 variables decreased and stabilized as the sample size increased while the rest increased and stabilized for all number of variables of all sample size ratios. In general, it is observed that the sample sample size ratio $1:1:1$ exhibited lower variabilities for all cases while sample size ratio $1:2:3$ exhibited high variabilities.

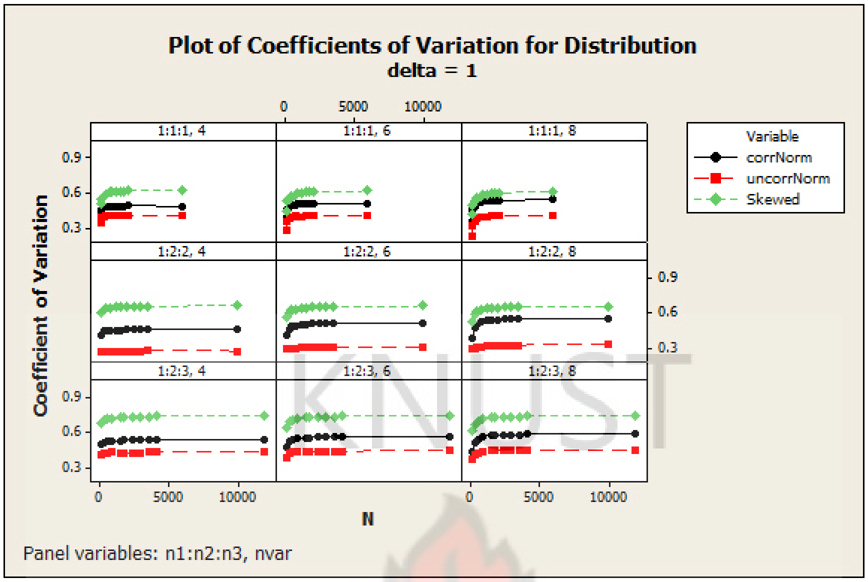

Figure 4.29: Coefficients of Variation for individual sample size ratios and distributions for $\delta = 1$

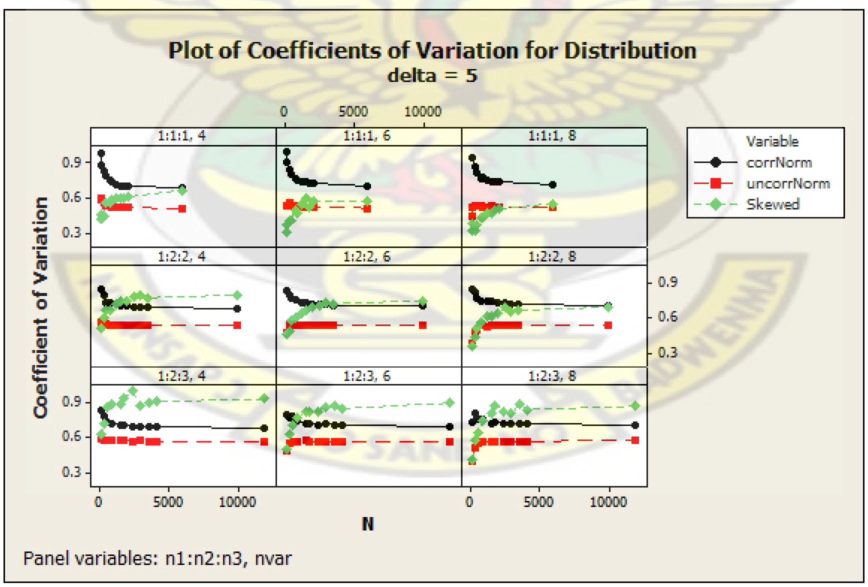

Figure 4.30: Coefficients of Variation for individual sample size ratios and distributions for $\delta = 5$

Chapter 5

Conclusion and Recommendations

5.1 Introduction

Summary of our findings from the simulation and some recommendation are made in this chapter. Below are the sections that cover them.

5.2 Findings and Conclusions

The summary of our Monte Carlo study focussed on the following:

1. The behaviour of the mean error rates and their stability as:
    - The number of variables, p increases from 4 to 6 and 8.
    - The group centroid separator δ varies from 1 to 5.
    - The sample ratios vary from 1:1:1, 1:2:2 to 1:2:3 and varying sample size.
2. A comparison of the results in (1) above for correlated normal, uncorrelated normal and skewed distributions.

The following observations were made:

- The average error rates of the skewed distribution decreased as δ increased and also increased with unequal group prior probability. The coefficients of variation of this distribution increased up to total sample size of 2500 and 2400 after which it stabilized as the number of variables increased with increasing δ (from 1 to 3) for sample size ratios 1:2:2 and 1:2:3 respectively.

- The correlated normal distribution exhibited minimum misclassification error rates and high variability as the sample size increased asymptotically.

- The performance of the Quadratic Discriminant Function deteriorated when the sample size ratio was 1:2:3 for all the three distributions as δ increased with increasing sample size.

- An overview of the results shows that reduction in misclassification error rates is more pronounced as group centroid separator increases than as sample size increases.

- As the sample size increases (i.e. n_1 from 30 to 2000) the average error rate of the three distributions become almost the same at $\delta = 5$ with little improvement in the performance by increasing sample size.

- For both correlated and uncorrelated normal distributions the average misclassification error rate decreased with increasing number of variables (from 4 to 8) and sample size ratio.

- The distributions exhibit increasing variations as δ, number of variable and sample size ratios increases. The uncorrelated normal distribution exhibited the least variability in all cases. The results obtained from this study (skewed distribution) is in conformity with Lachenbruch et al. (1977). According to them if the underlying distributions are not normal, use of QDF is not necessarily optimal and may in fact be very bad.

5.3 Recommendations

Based on the findings from our work, the following recommendations are made for Quadratic Discriminant Function when the training samples are correlated or skewed for three population case.

- To use Quadratic Discriminant Function, it is recommended that sample sizes should be fairly large and the groups well separated.
- To use correlated normal distribution, one has to check the for degree of correlation.
- Sample size ratio 1:1:1 is optimal for all distributions used in the case of coefficient of variation therefore, one should consider equal prior probabilities when using Quadratic Discriminant Function.
- Uncorrelated normal distribution should be preferred to correlated (positive) normal and skewed distributions since it has minimum coefficient of variation.

Due to minimal availability of the memory of the computer used for this study, we were restricted to some number of parameters. Therefore, the following recommendations are put across for further study with the help of much powerful computer:

- Increasing the sample size
- Increasing the group centroid separator (beyond 5)
- increasing number of variables.

The stability of the the error rates can then be well accessed.

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Appendix A

Table of Simulation Results

Table A.1: Error Rates for 4 variables and a sample ratio of (1:1:1)

Distributions		CorrNorm						UncorrNorm						Skewed					
	S. Size	π_1	π_2	π_3	G. Mean	SD	CV	π_1	π_2	π_3	G. Mean	SD	CV	π_1	π_2	π_3	G. Mean	SD	CV
$\delta = 1$	90	0.0633	0.1724	0.1361	0.1240	0.0530	0.4276	0.1160	0.2768	0.1938	0.1955	0.0681	0.3483	0.0538	0.2340	0.2311	0.1729	0.0875	0.5060
	180	0.0465	0.1654	0.1334	0.1151	0.0536	0.4656	0.0884	0.2413	0.1611	0.1636	0.0641	0.3920	0.0396	0.2305	0.2411	0.1704	0.0942	0.5528
	300	0.0405	0.1620	0.1334	0.1120	0.0543	0.4853	0.0799	0.2249	0.1517	0.1522	0.0603	0.3965	0.0328	0.2340	0.2434	0.1701	0.0983	0.5782
	450	0.0378	0.1624	0.1347	0.1116	0.0547	0.4903	0.0741	0.2196	0.1481	0.1473	0.0603	0.4092	0.0302	0.2287	0.2495	0.1695	0.0998	0.5888
	750	0.0359	0.1578	0.1330	0.1089	0.0534	0.4908	0.0715	0.2109	0.1425	0.1417	0.0575	0.4059	0.0257	0.2310	0.2532	0.1700	0.1031	0.6066
	900	0.0365	0.1572	0.1311	0.1082	0.0526	0.4857	0.0710	0.2097	0.1422	0.1409	0.0573	0.4065	0.0254	0.2307	0.2526	0.1696	0.1030	0.6073
	1200	0.0355	0.1545	0.1331	0.1077	0.0524	0.4868	0.0687	0.2069	0.1406	0.1387	0.1387	0.4096	0.0246	0.2274	0.2557	0.1692	0.1034	0.6110
	1500	0.0354	0.1567	0.1320	0.1081	0.0529	0.4893	0.0686	0.2075	0.1402	0.1388	0.0571	0.4114	0.0238	0.2294	0.2589	0.1707	0.1051	0.6155
	1800	0.0351	0.1551	0.1329	0.1077	0.0526	0.4881	0.0673	0.2053	0.1397	0.1374	0.0567	0.4123	0.0236	0.2289	0.2582	0.1702	0.1048	0.6159
	2100	0.0346	0.1558	0.1337	0.1080	0.0531	0.4913	0.0670	0.2053	0.1399	0.1374	0.0567	0.4129	0.0227	0.2295	0.2598	0.1707	0.1057	0.6194
	6000	0.0342	0.1541	0.1324	0.1069	0.0524	0.4898	0.0663	0.2028	0.1386	0.1359	0.0559	0.4116	0.0213	0.2293	0.2629	0.1712	0.1071	0.6258
$\delta = 2$	90	0.0377	0.1267	0.0997	0.0880	0.0447	0.5084	0.0754	0.1940	0.1268	0.1321	0.0524	0.3968	0.0459	0.1730	0.1877	0.1355	0.0680	0.5015
	180	0.0266	0.1107	0.0939	0.0771	0.0402	0.5215	0.0574	0.1661	0.1096	0.1111	0.0469	0.4222	0.0324	0.1681	0.1947	0.1317	0.0741	0.5626
	300	0.0234	0.1060	0.0910	0.0735	0.0384	0.5224	0.0538	0.1533	0.1039	0.1037	0.0421	0.4062	0.0275	0.1659	0.2059	0.1331	0.0781	0.5864
	450	0.0199	0.1062	0.0912	0.0724	0.0394	0.5439	0.0486	0.1502	0.1005	0.0997	0.0424	0.4251	0.0238	0.1667	0.2069	0.1324	0.0800	0.6039
	750	0.0212	0.1035	0.0899	0.0715	0.0370	0.5172	0.0466	0.1451	0.0993	0.0970	0.0409	0.4218	0.0216	0.1659	0.2125	0.1333	0.0822	0.6168
	900	0.0202	0.1023	0.0905	0.0710	0.0370	0.5217	0.0448	0.1452	0.0986	0.0962	0.0415	0.4313	0.0214	0.1656	0.2124	0.1331	0.0821	0.6171
	1200	0.0195	0.1021	0.0894	0.0703	0.0369	0.5249	0.0453	0.1426	0.0985	0.0955	0.0402	0.4209	0.0194	0.1653	0.2173	0.1340	0.0847	0.6318
	1500	0.0198	0.1010	0.0911	0.0707	0.0367	0.5191	0.0446	0.1412	0.0973	0.0944	0.0398	0.4219	0.0192	0.1662	0.2180	0.1345	0.0850	0.6319
	1800	0.0189	0.1017	0.0911	0.0706	0.0372	0.5265	0.0446	0.1418	0.0974	0.0946	0.0400	0.4230	0.0189	0.1661	0.2188	0.1346	0.0851	0.6325
	2100	0.0192	0.1008	0.0906	0.0702	0.0367	0.5222	0.0440	0.1409	0.0972	0.0940	0.0399	0.4239	0.0194	0.1660	0.2216	0.1357	0.0860	0.6335
	6000	0.0188	0.1008	0.0898	0.0698	0.0365	0.5234	0.0436	0.1396	0.0969	0.0934	0.0394	0.4223	0.0175	0.1665	0.2267	0.1369	0.0883	0.6454
$\delta = 3$	90	0.0180	0.0724	0.0586	0.0497	0.0300	0.6047	0.0454	0.1207	0.0807	0.0823	0.0345	0.4200	0.0362	0.1076	0.1356	0.0931	0.0489	0.5252
	180	0.0121	0.0640	0.0564	0.0442	0.0264	0.5983	0.0325	0.1005	0.0719	0.0683	0.0296	0.4341	0.0276	0.0998	0.1392	0.0889	0.0493	0.5543
	300	0.0095	0.0580	0.0561	0.0412	0.0245	0.5940	0.0283	0.0923	0.0677	0.0628	0.0277	0.4417	0.0226	0.0966	0.1434	0.0875	0.0522	0.5961
	450	0.0089	0.0574	0.0552	0.0405	0.0239	0.5907	0.0263	0.0880	0.0654	0.0599	0.0263	0.4395	0.0209	0.0969	0.1469	0.0882	0.0535	0.6064
	750	0.0087	0.0564	0.0554	0.0401	0.0232	0.5780	0.0257	0.0864	0.0636	0.0586	0.0256	0.4372	0.0180	0.0989	0.1598	0.0922	0.0601	0.6515
	900	0.0085	0.0563	0.0551	0.0400	0.0230	0.5763	0.0253	0.0863	0.0641	0.0586	0.0257	0.4385	0.0174	0.0957	0.1586	0.0906	0.0593	0.6545
	1200	0.0084	0.0558	0.0546	0.0396	0.0226	0.5708	0.0253	0.0864	0.0628	0.0581	0.0255	0.4393	0.0167	0.0951	0.1561	0.0893	0.0581	0.6507
	1500	0.0081	0.0552	0.0548	0.0393	0.0225	0.5727	0.0250	0.0859	0.0625	0.0578	0.0253	0.4386	0.0169	0.0961	0.1587	0.0906	0.0589	0.6508
	1800	0.0082	0.0552	0.0543	0.0393	0.0223	0.5682	0.0250	0.0844	0.0627	0.0573	0.0248	0.4330	0.0162	0.0977	0.1616	0.0918	0.0604	0.6579
	2100	0.0080	0.0561	0.0545	0.0395	0.0226	0.5728	0.0248	0.0850	0.0623	0.0574	0.0250	0.4361	0.0165	0.0950	0.1619	0.0911	0.0606	0.6646
	6000	0.0078	0.0550	0.0549	0.0393	0.0224	0.5698	0.0241	0.0841	0.0621	0.0568	0.0249	0.4380	0.0150	0.0955	0.1661	0.0922	0.0622	0.6741
$\delta = 4$	90	0.0077	0.0380	0.0341	0.0266	0.0200	0.7509	0.0230	0.0714	0.0512	0.0486	0.0236	0.4866	0.0347	0.0637	0.0774	0.0586	0.0252	0.4295
	180	0.0042	0.0327	0.0294	0.0221	0.0159	0.7195	0.0154	0.0554	0.0427	0.0379	0.0189	0.4991	0.0243	0.0535	0.0845	0.0541	0.0296	0.5470
	300	0.0036	0.0296	0.0313	0.0215	0.0144	0.6727	0.0140	0.0518	0.0403	0.0354	0.0173	0.4889	0.0200	0.0506	0.0836	0.0514	0.0286	0.5553
	450	0.0027	0.0284	0.0308	0.0207	0.0139	0.6732	0.0127	0.0486	0.0388	0.0334	0.0160	0.4802	0.0174	0.0485	0.0878	0.0512	0.0307	0.5997
	750	0.0027	0.0276	0.0312	0.0205	0.0135	0.6574	0.0119	0.0477	0.0384	0.0327	0.0158	0.4815	0.0156	0.0480	0.0943	0.0526	0.0348	0.6611
	900	0.0027	0.0279	0.0302	0.0203	0.0131	0.6455	0.0120	0.0473	0.0377	0.0323	0.0153	0.4744	0.0159	0.0462	0.0948	0.0523	0.0344	0.6567
	1200	0.0029	0.0272	0.0294	0.0198	0.0124	0.6286	0.0119	0.0469	0.0369	0.0319	0.0151	0.4725	0.0151	0.0464	0.0973	0.0529	0.0355	0.6709
	1500	0.0027	0.0266	0.0303	0.0199	0.0126	0.6353	0.0117	0.0467	0.0371	0.0318	0.0151	0.4728	0.0144	0.0468	0.0963	0.0525	0.0351	0.6679
	1800	0.0027	0.0277	0.0305	0.0203	0.0128	0.6301	0.0117	0.0459	0.0369	0.0315	0.0147	0.4660	0.0142	0.0456	0.1021	0.0540	0.0378	0.6998
	2100	0.0025	0.0271	0.0306	0.0201	0.0128	0.6382	0.0116	0.0458	0.0370	0.0315	0.0147	0.4663	0.0138	0.0445	0.1015	0.0533	0.0374	0.7025
	6000	0.0025	0.0271	0.0303	0.0200	0.0125	0.6284	0.0114	0.0456	0.0370	0.0313	0.0146	0.4656	0.0129	0.0444	0.1048	0.0540	0.0387	0.7165
$\delta = 5$	90	0.0029	0.0172	0.0177	0.0126	0.0123	0.9798	0.0114	0.0384	0.0281	0.0260	0.0156	0.5989	0.0306	0.0468	0.0361	0.0378	0.0158	0.4177
	180	0.0011	0.0149	0.0163	0.0108	0.0094	0.8717	0.0071	0.0301	0.0244	0.0205	0.0121	0.5892	0.0217	0.0326	0.0420	0.0321	0.0145	0.4533
	300	0.0008	0.0149	0.0150	0.0102	0.0084	0.8252	0.0060	0.0255	0.0208	0.0174	0.0098	0.5613	0.0180	0.0268	0.0418	0.0289	0.0128	0.4448
	450	0.0007	0.0133	0.0160	0.0100	0.0078	0.7838	0.0055	0.0246	0.0218	0.0173	0.0093	0.5367	0.0160	0.0251	0.0449	0.0287	0.0159	0.5562
	750	0.0006	0.0126	0.0153	0.0095	0.0071	0.7489	0.0044	0.0234	0.0210	0.0163	0.0090	0.5507	0.0137	0.0223	0.0454	0.0271	0.0152	0.5619
	900	0.0006	0.0123	0.0150	0.0093	0.0069	0.7399	0.0049	0.0230	0.0205	0.0161	0.0085	0.5262	0.0135	0.0222	0.0472	0.0277	0.0162	0.5850
	1200	0.0007	0.0128	0.0159	0.0098	0.0070	0.7158	0.0048	0.0231	0.0206	0.0162	0.0085	0.5260	0.0130	0.0216	0.0487	0.0278	0.0167	0.6027
	1500	0.0007	0.0125	0.0147	0.0093	0.0066	0.7058	0.0046	0.0233	0.0203	0.0161	0.0085	0.5272	0.0127	0.0210	0.0486	0.0274	0.0164	0.5991
	1800	0.0007	0.0123	0.0150	0.0093	0.0066	0.7016	0.0048	0.0232	0.0202	0.0160	0.0083	0.5182	0.0123	0.0211	0.0481	0.0272	0.0164	0.6034
	2100	0.0006	0.0122	0.0152	0.0093	0.0065	0.7002	0.0045	0.0228	0.0203	0.0159	0.0083	0.5221	0.0122	0.0207	0.0481	0.0270	0.0165	0.6103
	6000	0.0006	0.0123	0.0152	0.0094	0.0064	0.6843	0.0046	0.0226	0.0202	0.0158	0.0081	0.5109	0.0115	0.0194	0.0521	0.0277	0.0184	0.6653

Table A.2: Error Rates for 4 variables and a sample ratio of (1:2:2)

Distributions		CorrNorm						UncorrNorm						Skewed					
	S. Size	π_1	π_2	π_3	G. Mean	SD	CV	π_1	π_2	π_3	G. Mean	SD	CV	π_1	π_2	π_3	G. Mean	SD	CV
	150	0.0607	0.1721	0.1617	0.1315	0.0545	0.4148	0.1175	0.2247	0.1921	0.1781	0.0485	0.2723	0.0320	0.2646	0.2843	0.1936	0.1171	0.6046
	300	0.0497	0.1658	0.1644	0.1266	0.0569	0.4495	0.1024	0.2000	0.1789	0.1604	0.0442	0.2753	0.0233	0.2635	0.2964	0.1944	0.1233	0.6340
	500	0.0460	0.1608	0.1596	0.1222	0.0556	0.4549	0.0964	0.1913	0.1728	0.1535	0.0423	0.2756	0.0204	0.2584	0.3009	0.1932	0.1245	0.6445
	750	0.0444	0.1587	0.1595	0.1209	0.0552	0.4564	0.0942	0.1851	0.1710	0.1501	0.0408	0.2717	0.0193	0.2614	0.3025	0.1944	0.1257	0.6467
	1250	0.0432	0.1571	0.1571	0.1191	0.0545	0.4572	0.0914	0.1808	0.1679	0.1467	0.0401	0.2731	0.0169	0.2646	0.3093	0.1969	0.1292	0.6560
$\delta = 1$	1500	0.0429	0.1558	0.1585	0.1191	0.0544	0.4571	0.0907	0.1792	0.1675	0.1458	0.0397	0.2725	0.0172	0.2639	0.3088	0.1966	0.1287	0.6546
	2000	0.0422	0.1581	0.1593	0.1199	0.0554	0.4619	0.0898	0.1794	0.1675	0.1456	0.0401	0.2755	0.0160	0.2659	0.3120	0.1980	0.1306	0.6595
	2500	0.0423	0.1565	0.1586	0.1192	0.0547	0.4592	0.0891	0.1777	0.1663	0.1444	0.0396	0.2747	0.0159	0.2657	0.3130	0.1982	0.1308	0.6600
	3000	0.0417	0.1551	0.1588	0.1185	0.0547	0.4612	0.0890	0.1759	0.1662	0.1437	0.0392	0.2727	0.0156	0.2646	0.3128	0.1976	0.1307	0.6612
	3500	0.0421	0.1548	0.1580	0.1183	0.0542	0.4579	0.0881	0.1779	0.1659	0.1439	0.0401	0.2783	0.0159	0.2663	0.3141	0.1987	0.1312	0.6602
	10000	0.0414	0.1553	0.1584	0.1184	0.0546	0.4611	0.0875	0.1758	0.1657	0.1430	0.0396	0.2768	0.0143	0.2666	0.3190	0.2000	0.1333	0.6667
	150	0.0333	0.1173	0.1105	0.0870	0.0432	0.4958	0.0713	0.1640	0.1317	0.1223	0.0412	0.3369	0.0283	0.1873	0.2331	0.1495	0.0910	0.6084
	300	0.0282	0.1168	0.1088	0.0846	0.0428	0.5055	0.0590	0.1526	0.1209	0.1108	0.0406	0.3660	0.0197	0.1892	0.2477	0.1522	0.0984	0.6467
	500	0.0241	0.1094	0.1093	0.0809	0.0415	0.5132	0.0541	0.1420	0.1206	0.1056	0.0383	0.3628	0.0180	0.1871	0.2512	0.1521	0.0995	0.6543
	750	0.0234	0.1091	0.1077	0.0801	0.0409	0.5105	0.0536	0.1394	0.1187	0.1039	0.0372	0.3578	0.0161	0.1886	0.2555	0.1534	0.1019	0.6644
	1250	0.0225	0.1076	0.1087	0.0796	0.0409	0.5143	0.0513	0.1373	0.1170	0.1019	0.0372	0.3650	0.0143	0.1918	0.2631	0.1564	0.1054	0.6741
$\delta = 2$	1500	0.0221	0.1073	0.1072	0.0788	0.0406	0.5154	0.0512	0.1370	0.1169	0.1017	0.0370	0.3642	0.0137	0.1888	0.2605	0.1543	0.1045	0.6769
	2000	0.0217	0.1071	0.1080	0.0789	0.0408	0.5173	0.0504	0.1370	0.1161	0.1012	0.0372	0.3672	0.0131	0.1937	0.2649	0.1572	0.1068	0.6792
	2500	0.0219	0.1069	0.1075	0.0788	0.0405	0.5149	0.0502	0.1358	0.1165	0.1008	0.0369	0.3660	0.0127	0.1926	0.2649	0.1567	0.1067	0.6810
	3000	0.0216	0.1072	0.1076	0.0788	0.0407	0.5170	0.0497	0.1362	0.1164	0.1008	0.0372	0.3694	0.0127	0.1913	0.2661	0.1567	0.1069	0.6822
	3500	0.0216	0.1078	0.1073	0.0789	0.0408	0.5169	0.0498	0.1360	0.1153	0.1004	0.0369	0.3678	0.0128	0.1919	0.2688	0.1578	0.1078	0.6831
	10000	0.0212	0.1065	0.1078	0.0785	0.0407	0.5179	0.0491	0.1346	0.1158	0.0999	0.0368	0.3685	0.0119	0.1925	0.2748	0.1597	0.1102	0.6901
	150	0.0147	0.0698	0.0686	0.0511	0.0293	0.5743	0.0393	0.1059	0.0833	0.0762	0.0302	0.3971	0.0225	0.1137	0.1625	0.0996	0.0611	0.6142
	300	0.0111	0.0648	0.0658	0.0472	0.0276	0.5851	0.0310	0.0963	0.0771	0.0681	0.0290	0.4257	0.0172	0.1087	0.1796	0.1019	0.0693	0.6800
	500	0.0104	0.0630	0.0651	0.0462	0.0265	0.5748	0.0291	0.0914	0.0769	0.0658	0.0276	0.4188	0.0150	0.1079	0.1805	0.1011	0.0697	0.6896
	750	0.0101	0.0633	0.0653	0.0462	0.0266	0.5749	0.0276	0.0895	0.0758	0.0643	0.0272	0.4230	0.0133	0.1115	0.1906	0.1051	0.0742	0.7061
	1250	0.0091	0.0611	0.0662	0.0455	0.0264	0.5815	0.0256	0.0875	0.0752	0.0628	0.0271	0.4314	0.0119	0.1124	0.1953	0.1066	0.0766	0.7186
$\delta = 3$	1500	0.0088	0.0620	0.0660	0.0456	0.0265	0.5813	0.0263	0.0877	0.0750	0.0630	0.0267	0.4247	0.0120	0.1113	0.1955	0.1062	0.0762	0.7177
	2000	0.0087	0.0614	0.0660	0.0454	0.0263	0.5798	0.0257	0.0865	0.0744	0.0622	0.0265	0.4262	0.0115	0.1106	0.1956	0.1059	0.0763	0.7209
	2500	0.0087	0.0615	0.0656	0.0453	0.0262	0.5797	0.0259	0.0864	0.0748	0.0624	0.0264	0.4233	0.0109	0.1095	0.1964	0.1056	0.0765	0.7242
	3000	0.0084	0.0609	0.0655	0.0449	0.0262	0.5826	0.0253	0.0864	0.0749	0.0622	0.0267	0.4291	0.0107	0.1117	0.1988	0.1071	0.0779	0.7271
	3500	0.0084	0.0615	0.0660	0.0453	0.0264	0.5823	0.0254	0.0856	0.0745	0.0618	0.0263	0.4255	0.0105	0.1093	0.2028	0.1075	0.0795	0.7395
	10000	0.0085	0.0608	0.0657	0.0450	0.0260	0.5780	0.0251	0.0854	0.0743	0.0616	0.0263	0.4272	0.0100	0.1104	0.2036	0.1080	0.0795	0.7364
	150	0.0064	0.0363	0.0373	0.0267	0.0186	0.6951	0.0212	0.0584	0.0503	0.0433	0.0185	0.4279	0.0214	0.0596	0.1046	0.0619	0.0409	0.6618
	300	0.0038	0.0339	0.0359	0.0245	0.0165	0.6733	0.0137	0.0539	0.0463	0.0380	0.0188	0.4955	0.0150	0.0544	0.1079	0.0591	0.0408	0.6909
	500	0.0031	0.0307	0.0374	0.0237	0.0160	0.6745	0.0132	0.0514	0.0469	0.0372	0.0178	0.4786	0.0129	0.0552	0.1097	0.0593	0.0424	0.7153
	750	0.0027	0.0320	0.0367	0.0238	0.0157	0.6588	0.0124	0.0501	0.0453	0.0359	0.0173	0.4817	0.0114	0.0525	0.1139	0.0593	0.0441	0.7444
	1250	0.0029	0.0311	0.0366	0.0236	0.0153	0.6492	0.0123	0.0491	0.0444	0.0352	0.0167	0.4726	0.0103	0.0534	0.1195	0.0611	0.0464	0.7590
$\delta = 4$	1500	0.0027	0.0309	0.0360	0.0232	0.0150	0.6471	0.0117	0.0490	0.0449	0.0352	0.0170	0.4823	0.0099	0.0518	0.1201	0.0606	0.0468	0.7719
	2000	0.0029	0.0317	0.0367	0.0238	0.0152	0.6391	0.0116	0.0492	0.0446	0.0351	0.0169	0.4827	0.0099	0.0516	0.1206	0.0607	0.0469	0.7724
	2500	0.0027	0.0272	0.0301	0.0200	0.0125	0.6271	0.0117	0.0456	0.0372	0.0315	0.0146	0.4632	0.0140	0.0439	0.1022	0.0534	0.0383	0.7182
	3000	0.0027	0.0309	0.0363	0.0233	0.0149	0.6414	0.0115	0.0485	0.0443	0.0348	0.0167	0.4796	0.0092	0.0507	0.1283	0.0628	0.0510	0.8124
	3500	0.0027	0.0305	0.0364	0.0232	0.0149	0.6419	0.0115	0.0483	0.0442	0.0346	0.0166	0.4794	0.0092	0.0507	0.1247	0.0615	0.0487	0.7911
	10000	0.0025	0.0308	0.0363	0.0232	0.0149	0.6402	0.0113	0.0483	0.0442	0.0346	0.0166	0.4798	0.0085	0.0501	0.1279	0.0622	0.0502	0.8069
	150	0.0023	0.0179	0.0209	0.0137	0.0117	0.8511	0.0092	0.0321	0.0283	0.0232	0.0133	0.5715	0.0184	0.0376	0.0437	0.0332	0.0171	0.5139
	300	0.0012	0.0163	0.0190	0.0122	0.0097	0.7962	0.0058	0.0279	0.0265	0.0201	0.0115	0.5730	0.0134	0.0291	0.0520	0.0315	0.0190	0.6039
	500	0.0011	0.0160	0.0185	0.0119	0.0088	0.7383	0.0053	0.0269	0.0246	0.0189	0.0104	0.5509	0.0113	0.0287	0.0550	0.0317	0.0214	0.6740
	750	0.0008	0.0154	0.0186	0.0116	0.0085	0.7298	0.0052	0.0266	0.0244	0.0187	0.0102	0.5443	0.0100	0.0267	0.0560	0.0309	0.0208	0.6745
	1250	0.0007	0.0147	0.0183	0.0112	0.0081	0.7214	0.0047	0.0252	0.0246	0.0182	0.0182	0.5467	0.0092	0.0247	0.0599	0.0313	0.0226	0.7244
$\delta = 5$	1500	0.0008	0.0148	0.0181	0.0112	0.0079	0.7030	0.0046	0.0251	0.0241	0.0179	0.0097	0.5424	0.0087	0.0242	0.0609	0.0313	0.0234	0.7478
	2000	0.0006	0.0143	0.0179	0.0109	0.0078	0.7102	0.0045	0.0248	0.0246	0.0179	0.0098	0.5433	0.0084	0.0236	0.0606	0.0309	0.0232	0.7531
	2500	0.0007	0.0145	0.0185	0.0112	0.0078	0.6960	0.0046	0.0247	0.0246	0.0179	0.0096	0.5362	0.0082	0.0236	0.0632	0.0317	0.0251	0.7916
	3000	0.0006	0.0142	0.0181	0.0110	0.0077	0.6988	0.0045	0.0246	0.0239	0.0177	0.0095	0.5357	0.0080	0.0230	0.0641	0.0317	0.0254	0.7996
	3500	0.0007	0.0145	0.0184	0.0112	0.0078	0.6951	0.0045	0.0245	0.0244	0.0178	0.0095	0.5365	0.0080	0.0230	0.0639	0.0316	0.0246	0.7774
	10000	0.0006	0.0143	0.0180	0.0110	0.0076	0.6862	0.0043	0.0247	0.0242	0.0177	0.0095	0.5382	0.0074	0.0217	0.0654	0.0315	0.0252	0.7993

Table A.3: Error Rates for 4 variables and a sample ratio of (1:2:3)

Distributions		CorrNorm						UncorrNorm						Skewed					
	S. Size	π_1	π_2	π_3	G. Mean	SD	CV	π_1	π_2	π_3	G. Mean	SD	CV	π_1	π_2	π_3	G. Mean	SD	CV
	180	0.0493	0.1944	0.1360	0.1266	0.0629	0.4970	0.0958	0.2561	0.1443	0.1654	0.0688	0.4157	0.0266	0.2192	0.3497	0.1985	0.1347	0.6787
	360	0.0423	0.1913	0.1334	0.1223	0.0631	0.5156	0.0849	0.2364	0.1329	0.1514	0.0642	0.4238	0.0195	0.2216	0.3650	0.2020	0.1430	0.7078
	600	0.0392	0.1924	0.1328	0.1215	0.0640	0.5268	0.0799	0.2279	0.1293	0.1457	0.0622	0.4267	0.0179	0.2195	0.3719	0.2031	0.1459	0.7184
	900	0.0382	0.1899	0.1293	0.1191	0.0631	0.5301	0.0774	0.2237	0.1256	0.1422	0.0614	0.4318	0.0157	0.2221	0.3779	0.2052	0.1491	0.7266
	1500	0.0367	0.1888	0.1294	0.1183	0.0631	0.5332	0.0759	0.2208	0.1255	0.1407	0.0605	0.4295	0.0143	0.2234	0.3853	0.2077	0.1524	0.7339
$\delta = 1$	1800	0.0361	0.1888	0.1291	0.1180	0.0633	0.5362	0.0754	0.2193	0.1247	0.1398	0.0600	0.4293	0.0143	0.2216	0.3844	0.2068	0.1521	0.7354
	2400	0.0356	0.1881	0.1280	0.1172	0.0631	0.5381	0.0745	0.2179	0.1237	0.1387	0.0597	0.4308	0.0135	0.2214	0.3863	0.2071	0.1530	0.7390
	3000	0.0352	0.1895	0.1277	0.1175	0.0637	0.5422	0.0747	0.2177	0.1231	0.1385	0.0596	0.4304	0.0136	0.2222	0.3863	0.2074	0.1529	0.7375
	3600	0.0351	0.1881	0.1276	0.1169	0.0632	0.5406	0.0736	0.2175	0.1229	0.1380	0.0599	0.4343	0.0131	0.2226	0.3882	0.2080	0.1539	0.7402
	4200	0.0350	0.1887	0.1272	0.1170	0.0634	0.5424	0.0739	0.2173	0.1230	0.1381	0.0597	0.4324	0.0128	0.2233	0.3882	0.2081	0.1541	0.7406
	12000	0.0346	0.1876	0.1267	0.1163	0.0630	0.5420	0.0728	0.2158	0.1223	0.1370	0.0594	0.4337	0.0121	0.2236	0.3949	0.2102	0.1569	0.7463
	180	0.0282	0.1299	0.1042	0.0874	0.0472	0.5395	0.0604	0.1707	0.1132	0.1148	0.0471	0.4106	0.0240	0.1561	0.2862	0.1554	0.1100	0.7077
	360	0.0224	0.1235	0.1003	0.0821	0.0452	0.5509	0.0495	0.1579	0.1071	0.1048	0.0455	0.4338	0.0174	0.1560	0.3024	0.1586	0.1183	0.7457
	600	0.0208	0.1224	0.1002	0.0811	0.0450	0.5545	0.0461	0.1543	0.1056	0.1020	0.0448	0.4396	0.0146	0.1586	0.3079	0.1604	0.1211	0.7550
	900	0.0191	0.1207	0.0984	0.0794	0.0444	0.5594	0.0444	0.1525	0.1037	0.1002	0.0448	0.4473	0.0135	0.1600	0.3165	0.1633	0.1249	0.7649
	1500	0.0191	0.1178	0.0997	0.0789	0.0435	0.5514	0.0433	0.1481	0.1034	0.0983	0.0433	0.4402	0.0119	0.1607	0.3234	0.1654	0.1281	0.7747
$\delta = 2$	1800	0.0187	0.1188	0.0991	0.0789	0.0437	0.5534	0.0425	0.1492	0.1032	0.0983	0.0440	0.4480	0.0117	0.1581	0.3239	0.1646	0.1283	0.7799
	2400	0.0187	0.1190	0.0988	0.0788	0.0437	0.5541	0.0420	0.1490	0.1028	0.0979	0.0441	0.4501	0.0115	0.1593	0.3275	0.1661	0.1298	0.7817
	3000	0.0182	0.1185	0.0987	0.0785	0.0437	0.5568	0.0415	0.1480	0.1029	0.0975	0.0438	0.4497	0.0109	0.1612	0.3317	0.1679	0.1318	0.7847
	3600	0.0179	0.1181	0.0989	0.0783	0.0437	0.5579	0.0414	0.1477	0.1024	0.0972	0.0437	0.4502	0.0107	0.1597	0.3288	0.1664	0.1305	0.7842
	4200	0.0179	0.1183	0.0992	0.0784	0.0437	0.5577	0.0414	0.1473	0.1023	0.0970	0.0436	0.4492	0.0107	0.1609	0.3328	0.1681	0.1321	0.7859
	12000	0.0178	0.1192	0.0988	0.0786	0.0439	0.5587	0.0411	0.1470	0.1022	0.0968	0.0435	0.4495	0.0098	0.1617	0.3382	0.1699	0.1346	0.7920
	180	0.0135	0.0716	0.0650	0.0500	0.0300	0.6002	0.0328	0.1072	0.0761	0.0720	0.0328	0.4553	0.0202	0.0942	0.2136	0.1093	0.0836	0.7645
	360	0.0093	0.0709	0.0634	0.0479	0.0291	0.6082	0.0261	0.0984	0.0730	0.0658	0.0311	0.4720	0.0142	0.0910	0.2178	0.1077	0.0866	0.8047
	600	0.0082	0.0680	0.0631	0.0464	0.0282	0.6070	0.0232	0.0946	0.0712	0.0630	0.0304	0.4818	0.0125	0.0935	0.2281	0.1114	0.0913	0.8197
	900	0.0077	0.0669	0.0636	0.0461	0.0280	0.6076	0.0227	0.0941	0.0714	0.0627	0.0302	0.4812	0.0113	0.0906	0.2289	0.1103	0.0915	0.8300
	1500	0.0076	0.0664	0.0643	0.0461	0.0276	0.5992	0.0216	0.0913	0.0705	0.0611	0.0295	0.4825	0.0099	0.0920	0.2404	0.1141	0.0966	0.8470
$\delta = 3$	1800	0.0074	0.0664	0.0643	0.0460	0.0277	0.6016	0.0217	0.0912	0.0711	0.0613	0.0294	0.4796	0.0095	0.0925	0.2392	0.1137	0.0963	0.8464
	2400	0.0072	0.0673	0.0634	0.0460	0.0277	0.6034	0.0217	0.0909	0.0697	0.0608	0.0292	0.4799	0.0093	0.0920	0.2439	0.1150	0.0982	0.8534
	3000	0.0074	0.0657	0.0625	0.0452	0.0270	0.5974	0.0215	0.0901	0.0700	0.0605	0.0290	0.4782	0.0092	0.0913	0.2480	0.1162	0.1001	0.8618
	3600	0.0071	0.0665	0.0635	0.0457	0.0275	0.6023	0.0211	0.0903	0.0700	0.0605	0.0292	0.4824	0.0090	0.0919	0.2477	0.1162	0.0999	0.8593
	4200	0.0070	0.0663	0.0637	0.0456	0.0275	0.6029	0.0211	0.0909	0.0703	0.0608	0.0294	0.4840	0.0090	0.0930	0.2494	0.1171	0.1007	0.8594
	12000	0.0069	0.0661	0.0637	0.0456	0.0275	0.6030	0.0209	0.0900	0.0701	0.0604	0.0291	0.4826	0.0083	0.0919	0.2534	0.1179	0.1023	0.8678
	180	0.0043	0.0362	0.0360	0.0255	0.0180	0.7063	0.0162	0.0601	0.0467	0.0410	0.0204	0.4973	0.0177	0.0529	0.1253	0.0653	0.0498	0.7620
	360	0.0032	0.0362	0.0350	0.0248	0.0168	0.6763	0.0123	0.0561	0.0449	0.0378	0.0195	0.5177	0.0133	0.0497	0.1237	0.0622	0.0484	0.7770
	600	0.0026	0.0347	0.0363	0.0245	0.0165	0.6745	0.0109	0.0535	0.0441	0.0362	0.0188	0.5200	0.0108	0.0460	0.1383	0.0650	0.0565	0.8696
	900	0.0029	0.0343	0.0358	0.0243	0.0159	0.6538	0.0105	0.0519	0.0435	0.0353	0.0183	0.5177	0.0098	0.0449	0.1415	0.0654	0.0577	0.8828
	1500	0.0025	0.0344	0.0359	0.0243	0.0158	0.6506	0.0102	0.0509	0.0431	0.0347	0.0179	0.5159	0.0085	0.0447	0.1502	0.0678	0.0615	0.9078
$\delta = 4$	1800	0.0023	0.0338	0.0364	0.0241	0.0158	0.6546	0.0099	0.0503	0.0434	0.0345	0.0178	0.5164	0.0084	0.0424	0.1519	0.0676	0.0635	0.9395
	2400	0.0022	0.0329	0.0360	0.0237	0.0155	0.6530	0.0097	0.0504	0.0431	0.0344	0.0179	0.5201	0.0082	0.0430	0.1474	0.0662	0.0601	0.9080
	3000	0.0023	0.0337	0.0359	0.0240	0.0156	0.6488	0.0098	0.0506	0.0429	0.0345	0.0178	0.5179	0.0078	0.0431	0.1534	0.0681	0.0634	0.9304
	3600	0.0022	0.0335	0.0364	0.0240	0.0156	0.6515	0.0094	0.0502	0.0435	0.0344	0.0180	0.5230	0.0078	0.0434	0.1505	0.0672	0.0620	0.9225
	4200	0.0021	0.0332	0.0360	0.0238	0.0155	0.6525	0.0093	0.0499	0.0432	0.0341	0.0179	0.5237	0.0074	0.0429	0.1571	0.0692	0.0652	0.9433
	12000	0.0022	0.0332	0.0360	0.0238	0.0154	0.6470	0.0093	0.0500	0.0429	0.0341	0.0178	0.5226	0.0071	0.0427	0.1611	0.0703	0.0665	0.9464
	180	0.0014	0.0186	0.0192	0.0131	0.0110	0.8433	0.0067	0.0325	0.0268	0.0220	0.0132	0.5985	0.0161	0.0326	0.0588	0.0358	0.0228	0.6360
	360	0.0008	0.0171	0.0190	0.0123	0.0096	0.7791	0.0053	0.0285	0.0258	0.0199	0.0115	0.5775	0.0114	0.0260	0.0610	0.0328	0.0234	0.7149
	600	0.0008	0.0157	0.0184	0.0116	0.0086	0.7392	0.0045	0.0266	0.0251	0.0187	0.0108	0.5761	0.0093	0.0245	0.0713	0.0350	0.0301	0.8596
	900	0.0006	0.0165	0.0186	0.0119	0.0086	0.7212	0.0041	0.0267	0.0247	0.0185	0.0107	0.5799	0.0081	0.0220	0.0716	0.0339	0.0302	0.8917
	1500	0.0007	0.0154	0.0188	0.0116	0.0082	0.7031	0.0039	0.0258	0.0243	0.0180	0.0103	0.5752	0.0075	0.0210	0.0729	0.0338	0.0299	0.8841
$\delta = 5$	1800	0.0006	0.0159	0.0183	0.0116	0.0082	0.7045	0.0037	0.0266	0.0247	0.0183	0.0106	0.5793	0.0071	0.0209	0.0758	0.0346	0.0324	0.9356
	2400	0.0005	0.0153	0.0179	0.0112	0.0079	0.7012	0.0038	0.0258	0.0241	0.0179	0.0102	0.5683	0.0070	0.0204	0.0784	0.0353	0.0353	0.9998
	3000	0.0006	0.0158	0.0184	0.0116	0.0081	0.6959	0.0036	0.0260	0.0241	0.0179	0.0103	0.5736	0.0067	0.0203	0.0719	0.0330	0.0289	0.8768
	3600	0.0006	0.0156	0.0183	0.0115	0.0080	0.6940	0.0038	0.0257	0.0241	0.0179	0.0101	0.5650	0.0068	0.0202	0.0758	0.0342	0.0310	0.9051
	4200	0.0005	0.0156	0.0185	0.0115	0.0080	0.6965	0.0037	0.0258	0.0238	0.0178	0.0101	0.5672	0.0066	0.0191	0.0753	0.0337	0.0307	0.9118
	12000	0.0005	0.0155	0.0184	0.0115	0.0079	0.6867	0.0037	0.0254	0.0242	0.0178	0.0100	0.5644	0.0062	0.0190	0.0787	0.0346	0.0323	0.9334

Table A.4: Error Rates for 6 variables and a sample ratio of (1:1:1)

Distributions		CorrNorm						UncorrNorm						Skewed					
	S. Size	π_1	π_2	π_3	G. Mean	SD	CV	π_1	π_2	π_3	G. Mean	SD	CV	π_1	π_2	π_3	G. Mean	SD	CV
	90	0.0682	0.1578	0.1197	0.1152	0.0463	0.4022	0.1467	0.3009	0.2484	0.2320	0.0665	0.2867	0.0694	0.2343	0.2220	0.1753	0.0790	0.4508
	180	0.0436	0.1484	0.1284	0.1068	0.0490	0.4585	0.1048	0.2599	0.1787	0.1811	0.0650	0.3586	0.0437	0.2284	0.2324	0.1682	0.0900	0.5350
	300	0.0345	0.1401	0.1227	0.0991	0.0482	0.4868	0.0863	0.2322	0.1557	0.1581	0.0607	0.3841	0.0378	0.2203	0.2350	0.1644	0.0910	0.5535
	450	0.0319	0.1359	0.1234	0.0971	0.0480	0.4944	0.0770	0.2169	0.1452	0.1464	0.0581	0.3970	0.0334	0.2223	0.2381	0.1646	0.0940	0.5712
	750	0.0285	0.1309	0.1186	0.0926	0.0465	0.5016	0.0701	0.2049	0.1374	0.1374	0.0556	0.4046	0.0299	0.2211	0.2419	0.1643	0.0961	0.5848
$\delta = 1$	900	0.0273	0.1297	0.1204	0.0925	0.0469	0.5073	0.0693	0.1997	0.1364	0.1351	0.0538	0.3984	0.0279	0.2201	0.2436	0.1639	0.0973	0.5937
	1200	0.0258	0.1276	0.1190	0.0908	0.0467	0.5139	0.0669	0.1953	0.1328	0.1317	0.0528	0.4012	0.0264	0.2207	0.2480	0.1650	0.0993	0.6053
	1500	0.0262	0.1276	0.1179	0.0905	0.0462	0.5099	0.0653	0.1948	0.1316	0.1306	0.0533	0.4078	0.0250	0.2232	0.2484	0.1655	0.1004	0.6066
	1800	0.0254	0.1257	0.1189	0.0900	0.0462	0.5128	0.0650	0.1934	0.1322	0.1302	0.0527	0.4050	0.0254	0.2207	0.2494	0.1652	0.1001	0.6057
	2100	0.0257	0.1255	0.1185	0.0899	0.0459	0.5105	0.0644	0.1906	0.1311	0.1287	0.0519	0.4031	0.0253	0.2203	0.2517	0.1658	0.1005	0.6065
	6000	0.0244	0.1246	0.1182	0.0891	0.0459	0.5158	0.0622	0.1871	0.1294	0.1262	0.0512	0.4055	0.0227	0.2208	0.2566	0.1667	0.1032	0.6190
	90	0.0449	0.1110	0.0858	0.0806	0.0358	0.4450	0.1063	0.2296	0.1566	0.1642	0.0541	0.3296	0.0604	0.1774	0.1776	0.1385	0.0609	0.4397
	180	0.0256	0.1015	0.0881	0.0717	0.0364	0.5074	0.0696	0.1831	0.1202	0.1243	0.0482	0.3876	0.0389	0.1713	0.1938	0.1347	0.0710	0.5273
	300	0.0186	0.0931	0.0851	0.0656	0.0355	0.5416	0.0567	0.1606	0.1075	0.1083	0.0438	0.4047	0.0329	0.1632	0.1957	0.1306	0.0721	0.5520
	450	0.0168	0.0909	0.0848	0.0642	0.0352	0.5492	0.0511	0.1518	0.1018	0.1016	0.0420	0.4138	0.0281	0.1636	0.2005	0.1308	0.0757	0.5786
	750	0.0156	0.0863	0.0830	0.0616	0.0333	0.5408	0.0467	0.1407	0.0982	0.0952	0.0391	0.4109	0.0251	0.1664	0.2047	0.1321	0.0783	0.5929
$\delta = 2$	900	0.0147	0.0854	0.0820	0.0607	0.0335	0.5513	0.0457	0.1388	0.0973	0.0939	0.0386	0.4110	0.0250	0.1618	0.2070	0.1313	0.0786	0.5985
	1200	0.0146	0.0863	0.0819	0.0609	0.0334	0.5488	0.0444	0.1378	0.0942	0.0921	0.0387	0.4199	0.0233	0.1631	0.2133	0.1332	0.0812	0.6099
	1500	0.0145	0.0846	0.0821	0.0604	0.0330	0.5469	0.0430	0.1356	0.0949	0.0911	0.0383	0.4202	0.0228	0.1622	0.2126	0.1325	0.0809	0.6108
	1800	0.0137	0.0826	0.0832	0.0598	0.0330	0.5520	0.0430	0.1343	0.0941	0.0904	0.0377	0.4170	0.0225	0.1621	0.2123	0.1323	0.0809	0.6114
	2100	0.0136	0.0821	0.0820	0.0593	0.0326	0.5505	0.0430	0.1332	0.0935	0.0899	0.0372	0.4136	0.0216	0.1634	0.2159	0.1336	0.0826	0.6185
	6000	0.0133	0.0818	0.0817	0.0589	0.0324	0.5502	0.0410	0.1308	0.0926	0.0881	0.0369	0.4191	0.0201	0.1629	0.2202	0.1344	0.0845	0.6287
	90	0.0207	0.0738	0.0568	0.0504	0.0290	0.5753	0.0661	0.1567	0.1042	0.1090	0.0410	0.3762	0.0548	0.1208	0.1347	0.1034	0.0415	0.4017
	180	0.0118	0.0574	0.0530	0.0407	0.0239	0.5878	0.0406	0.1130	0.0749	0.0762	0.0321	0.4212	0.0343	0.1089	0.1400	0.0944	0.0487	0.5156
	300	0.0082	0.0511	0.0499	0.0364	0.0219	0.6019	0.0327	0.0976	0.0673	0.0659	0.0279	0.4230	0.0284	0.1008	0.1403	0.0898	0.0491	0.5463
	450	0.0069	0.0514	0.0500	0.0361	0.0220	0.6095	0.0288	0.0907	0.0647	0.0614	0.0262	0.4259	0.0256	0.0980	0.1450	0.0896	0.0513	0.5727
	750	0.0060	0.0485	0.0511	0.0352	0.0216	0.6124	0.0257	0.0866	0.0631	0.0585	0.0256	0.4383	0.0217	0.0995	0.1508	0.0907	0.0545	0.6009
$\delta = 3$	900	0.0065	0.0482	0.0500	0.0349	0.0208	0.5957	0.0257	0.0853	0.0631	0.0580	0.0251	0.4325	0.0214	0.0995	0.1564	0.0924	0.0567	0.6133
	1200	0.0059	0.0470	0.0504	0.0344	0.0208	0.6034	0.0246	0.0839	0.0623	0.0569	0.0249	0.4372	0.0204	0.1000	0.1606	0.0936	0.0587	0.6269
	1500	0.0058	0.0464	0.0501	0.0341	0.0205	0.6027	0.0246	0.0825	0.0612	0.0561	0.0243	0.4330	0.0195	0.1002	0.1604	0.0934	0.0589	0.6308
	1800	0.0057	0.0470	0.0492	0.0340	0.0204	0.6017	0.0240	0.0821	0.0610	0.0557	0.0243	0.4367	0.0192	0.0974	0.1608	0.0925	0.0591	0.6388
	2100	0.0057	0.0465	0.0501	0.0341	0.0204	0.5991	0.0236	0.0815	0.0608	0.0553	0.0242	0.4379	0.0190	0.0980	0.1644	0.0938	0.0605	0.6448
	6000	0.0053	0.0460	0.0497	0.0336	0.0202	0.6017	0.0227	0.0802	0.0600	0.0543	0.0239	0.4404	0.0175	0.0974	0.1665	0.0938	0.0614	0.6550
	90	0.0086	0.0431	0.0311	0.0207	0.0276	0.7506	0.0371	0.0988	0.0644	0.0668	0.0294	0.4404	0.0480	0.0834	0.0821	0.0712	0.0269	0.3777
	180	0.0040	0.0309	0.0292	0.0214	0.0154	0.7193	0.0196	0.0646	0.0453	0.0431	0.0205	0.4743	0.0314	0.0647	0.0800	0.0590	0.0249	0.4212
	300	0.0024	0.0275	0.0277	0.0192	0.0141	0.7321	0.0165	0.0542	0.0405	0.0371	0.0170	0.4571	0.0258	0.0577	0.0866	0.0567	0.0284	0.5000
	450	0.0020	0.0252	0.0294	0.0189	0.0132	0.6980	0.0136	0.0499	0.0397	0.0344	0.0162	0.4711	0.0222	0.0553	0.0848	0.0541	0.0278	0.5150
	750	0.0021	0.0255	0.0287	0.0187	0.0127	0.6789	0.0124	0.0480	0.0378	0.0327	0.0154	0.4722	0.0194	0.0532	0.0939	0.0555	0.0328	0.5910
$\delta = 4$	900	0.0019	0.0237	0.0270	0.0175	0.0118	0.6766	0.0120	0.0464	0.0373	0.0319	0.0150	0.4696	0.0190	0.0508	0.0931	0.0543	0.0316	0.5827
	1200	0.0018	0.0237	0.0276	0.0177	0.0119	0.6708	0.0115	0.0458	0.0372	0.0315	0.0149	0.4739	0.0175	0.0505	0.0986	0.0556	0.0352	0.6335
	1500	0.0016	0.0232	0.0275	0.0175	0.0118	0.6736	0.0117	0.0455	0.0373	0.0315	0.0147	0.4667	0.0176	0.0497	0.0957	0.0543	0.0333	0.6123
	1800	0.0017	0.0232	0.0280	0.0176	0.0118	0.6664	0.0109	0.0450	0.0364	0.0308	0.0147	0.4775	0.0171	0.0494	0.1010	0.0558	0.0363	0.6509
	2100	0.0016	0.0232	0.0273	0.0173	0.0115	0.6659	0.0112	0.0443	0.0363	0.0306	0.0143	0.4667	0.0169	0.0492	0.0990	0.0550	0.0348	0.6322
	6000	0.0017	0.0227	0.0273	0.0172	0.0113	0.6546	0.0108	0.0436	0.0360	0.0301	0.0141	0.4675	0.0154	0.0475	0.1066	0.0565	0.0384	0.6797
	90	0.0036	0.0238	0.0180	0.0151	0.0150	0.9928	0.0178	0.0554	0.0391	0.0374	0.0200	0.5345	0.0443	0.0600	0.0373	0.0472	0.0177	0.3757
	180	0.0015	0.0161	0.0143	0.0106	0.0096	0.8996	0.0095	0.0344	0.0267	0.0235	0.0125	0.5305	0.0297	0.0423	0.0372	0.0364	0.0111	0.3050
	300	0.0008	0.0133	0.0128	0.0090	0.0076	0.8421	0.0065	0.0284	0.0222	0.0191	0.0107	0.5632	0.0232	0.0351	0.0443	0.0342	0.0138	0.4039
	450	0.0005	0.0121	0.0144	0.0090	0.0072	0.7999	0.0058	0.0248	0.0221	0.0176	0.0095	0.5404	0.0197	0.0311	0.0435	0.0314	0.0130	0.4131
	750	0.0004	0.0115	0.0134	0.0084	0.0065	0.7699	0.0050	0.0239	0.0207	0.0165	0.0088	0.5347	0.0172	0.0277	0.0485	0.0311	0.0157	0.5030
$\delta = 5$	900	0.0003	0.0109	0.0136	0.0083	0.0063	0.7539	0.0049	0.0239	0.0207	0.0165	0.0088	0.5330	0.0168	0.0279	0.0465	0.0304	0.0144	0.4757
	1200	0.0003	0.0110	0.0130	0.0081	0.0060	0.7423	0.0048	0.0233	0.0203	0.0161	0.0085	0.5253	0.0159	0.0259	0.0504	0.0307	0.0170	0.5536
	1500	0.0004	0.0109	0.0137	0.0083	0.0061	0.7380	0.0046	0.0222	0.0198	0.0155	0.0081	0.5226	0.0156	0.0248	0.0526	0.0310	0.0185	0.5959
	1800	0.0003	0.0109	0.0139	0.0084	0.0061	0.7295	0.0045	0.0224	0.0205	0.0158	0.0083	0.5226	0.0152	0.0252	0.0492	0.0299	0.0155	0.5193
	2100	0.0004	0.0106	0.0130	0.0080	0.0058	0.7208	0.0043	0.0224	0.0199	0.0155	0.0082	0.5275	0.0147	0.0244	0.0508	0.0300	0.0173	0.5766
	6000	0.0004	0.0107	0.0137	0.0082	0.0058	0.7038	0.0044	0.0218	0.0198	0.0153	0.0079	0.5145	0.0141	0.0227	0.0533	0.0300	0.0173	0.5766

Table A.5: Error Rates for 6 variables and a sample ratio of (1:2:2)

Distributions		CorrNorm						UncorrNorm						Skewed					
	S. Size	π_1	π_2	π_3	G. Mean	SD	CV	π_1	π_2	π_3	G. Mean	SD	CV	π_1	π_2	π_3	G. Mean	SD	CV
	150	0.0573	0.1625	0.1468	0.1222	0.0509	0.4166	0.1249	0.2581	0.2179	0.2003	0.0584	0.2915	0.0423	0.2531	0.2744	0.1899	0.1073	0.5651
	300	0.0415	0.1457	0.1450	0.1107	0.0513	0.4635	0.1023	0.2117	0.1790	0.1645	0.0477	0.2902	0.0292	0.2496	0.2801	0.1863	0.1131	0.6072
	500	0.0352	0.1408	0.1450	0.1070	0.0521	0.4872	0.0915	0.1936	0.1680	0.1510	0.0445	0.2945	0.0239	0.2540	0.2921	0.1900	0.1195	0.6287
	750	0.0329	0.1362	0.1427	0.1039	0.0512	0.4925	0.0870	0.1861	0.1630	0.1454	0.0432	0.2973	0.0215	0.2555	0.2918	0.1896	0.1205	0.6356
	1250	0.0297	0.1343	0.1418	0.1019	0.0516	0.5067	0.0812	0.1783	0.1589	0.1395	0.0424	0.3042	0.0190	0.2531	0.2978	0.1900	0.1229	0.6470
$\delta = 1$	1500	0.0301	0.1343	0.1419	0.1021	0.0515	0.5043	0.0807	0.1772	0.1583	0.1388	0.0422	0.3039	0.0186	0.2567	0.3007	0.1920	0.1245	0.6483
	2000	0.0286	0.1334	0.1422	0.1014	0.0520	0.5124	0.0790	0.1754	0.1572	0.1372	0.0422	0.3073	0.0174	0.2545	0.3026	0.1915	0.1252	0.6534
	2500	0.0283	0.1322	0.1421	0.1009	0.0518	0.5136	0.0781	0.1733	0.1568	0.1361	0.0418	0.3075	0.0168	0.2558	0.3033	0.1920	0.1258	0.6553
	3000	0.0286	0.1317	0.1412	0.1005	0.0513	0.5103	0.0779	0.1728	0.1563	0.1357	0.0416	0.3068	0.0170	0.2553	0.3047	0.1924	0.1261	0.6554
	3500	0.0282	0.1319	0.1420	0.1007	0.0517	0.5130	0.0775	0.1715	0.1559	0.1350	0.0414	0.3066	0.0160	0.2572	0.3067	0.1933	0.1274	0.6591
	10000	0.0276	0.1304	0.1413	0.0998	0.0514	0.5149	0.0759	0.1691	0.1546	0.1332	0.0411	0.3085	0.0151	0.2565	0.3102	0.1939	0.1286	0.6633
	150	0.0367	0.1132	0.1080	0.0860	0.0389	0.4526	0.0885	0.1875	0.1443	0.1401	0.0436	0.3109	0.0361	0.1888	0.2229	0.1493	0.0848	0.5679
	300	0.0234	0.1019	0.0999	0.0751	0.0390	0.5194	0.0661	0.1582	0.1257	0.1167	0.0398	0.3411	0.0255	0.1827	0.2370	0.1484	0.0919	0.6190
	500	0.0200	0.0925	0.0982	0.0702	0.0370	0.5270	0.0567	0.1421	0.1175	0.1054	0.0368	0.3492	0.0201	0.1863	0.2441	0.1502	0.0964	0.6418
	750	0.0180	0.0932	0.0987	0.0700	0.0377	0.5383	0.0525	0.1383	0.1159	0.1022	0.0371	0.3628	0.0183	0.1856	0.2484	0.1508	0.0982	0.6515
	1250	0.0161	0.0913	0.0997	0.0690	0.0381	0.5524	0.0494	0.1344	0.1142	0.0993	0.0367	0.3699	0.0159	0.1872	0.2554	0.1528	0.1014	0.6638
$\delta = 2$	1500	0.0158	0.0917	0.0981	0.0685	0.0378	0.5520	0.0481	0.1334	0.1129	0.0981	0.0367	0.3743	0.0157	0.1884	0.2538	0.1526	0.1012	0.6632
	2000	0.0157	0.0910	0.0987	0.0685	0.0378	0.5518	0.0473	0.1315	0.1114	0.0967	0.0362	0.3744	0.0149	0.1906	0.2586	0.1547	0.1034	0.6686
	2500	0.0151	0.0903	0.0986	0.0680	0.0379	0.5568	0.0469	0.1302	0.1120	0.0964	0.0360	0.3740	0.0143	0.1881	0.2601	0.1542	0.1038	0.6734
	3000	0.0150	0.0910	0.0995	0.0685	0.0383	0.5587	0.0463	0.1301	0.1117	0.0960	0.0362	0.3767	0.0142	0.1887	0.2630	0.1553	0.1049	0.6751
	3500	0.0148	0.0900	0.0980	0.0676	0.0377	0.5581	0.0458	0.1297	0.1112	0.0956	0.0362	0.3785	0.0141	0.1906	0.2634	0.1560	0.1052	0.6742
	10000	0.0144	0.0895	0.0987	0.0675	0.0379	0.5612	0.0452	0.1283	0.1109	0.0948	0.0359	0.3785	0.0131	0.1886	0.2672	0.1563	0.1065	0.6815
	150	0.0181	0.0697	0.0654	0.0511	0.0281	0.5503	0.0541	0.1212	0.0924	0.0892	0.0306	0.3423	0.0311	0.1183	0.1661	0.1052	0.0605	0.5753
	300	0.0111	0.0581	0.0595	0.0429	0.0248	0.5773	0.0367	0.0985	0.0812	0.0721	0.0277	0.3833	0.0213	0.1131	0.1740	0.1028	0.0657	0.6387
	500	0.0078	0.0562	0.0598	0.0413	0.0249	0.6039	0.0305	0.0918	0.0760	0.0661	0.0268	0.4057	0.0176	0.1112	0.1797	0.1029	0.0684	0.0684
	750	0.0072	0.0533	0.0598	0.0401	0.0243	0.6069	0.0281	0.0873	0.0757	0.0637	0.0262	0.4116	0.0156	0.1138	0.1886	0.1060	0.0729	0.6876
	1250	0.0062	0.0528	0.0612	0.0401	0.0247	0.6161	0.0258	0.0847	0.0727	0.0611	0.0258	0.4224	0.0140	0.1102	0.1873	0.1039	0.0723	0.6958
$\delta = 3$	1500	0.0063	0.0524	0.0597	0.0395	0.0240	0.6089	0.0250	0.0847	0.0733	0.0610	0.0262	0.4292	0.0135	0.1125	0.1909	0.1056	0.0735	0.6957
	2000	0.0057	0.0523	0.0602	0.0394	0.0244	0.6194	0.0248	0.0837	0.0735	0.0607	0.0260	0.4280	0.0129	0.1125	0.1983	0.1079	0.0770	0.7139
	2500	0.0059	0.0529	0.0598	0.0395	0.0242	0.6129	0.0245	0.0835	0.0722	0.0601	0.0258	0.4296	0.0125	0.1121	0.1942	0.1062	0.0751	0.7067
	3000	0.0060	0.0523	0.0602	0.0395	0.0241	0.6110	0.0239	0.0836	0.0725	0.0600	0.0261	0.4352	0.0123	0.1118	0.1953	0.1065	0.0758	0.7115
	3500	0.0057	0.0527	0.0606	0.0397	0.0245	0.6166	0.0239	0.0830	0.0722	0.0597	0.0259	0.4330	0.0122	0.1135	0.1979	0.1079	0.0769	0.7132
	10000	0.0055	0.0512	0.0598	0.0388	0.0239	0.6168	0.0233	0.0815	0.0720	0.0589	0.0256	0.4343	0.0114	0.1117	0.2060	0.1097	0.0800	0.7293
	150	0.0092	0.0361	0.0371	0.0275	0.0173	0.6315	0.0283	0.0704	0.0562	0.0516	0.0206	0.3987	0.028	0.0714	0.0994	0.0663	0.0353	0.5333
	300	0.0035	0.0305	0.0345	0.0228	0.0157	0.6890	0.0182	0.0582	0.0489	0.0417	0.0185	0.4423	0.0200	0.0653	0.1027	0.0627	0.0372	0.5937
	500	0.0022	0.0291	0.0328	0.0214	0.0149	0.6989	0.0142	0.0531	0.0453	0.0375	0.0177	0.4708	0.0161	0.0601	0.1072	0.0611	0.0401	0.6560
	750	0.0022	0.0279	0.0328	0.0210	0.0140	0.6673	0.0129	0.0508	0.0456	0.0364	0.0174	0.4765	0.0138	0.0581	0.1164	0.0627	0.0438	0.6984
	1250	0.0019	0.0269	0.0325	0.0204	0.0138	0.6768	0.0123	0.0477	0.0439	0.0346	0.0163	0.4697	0.0122	0.0562	0.1198	0.0627	0.0460	0.7330
$\delta = 4$	1500	0.0019	0.0266	0.0332	0.0206	0.0139	0.6735	0.0117	0.0479	0.0439	0.0345	0.0165	0.4778	0.0120	0.0573	0.1211	0.0635	0.0464	0.7305
	2000	0.0018	0.0264	0.0323	0.0202	0.0135	0.6701	0.0113	0.0470	0.0436	0.0339	0.0163	0.4798	0.0116	0.0560	0.1219	0.0632	0.0468	0.7402
	2500	0.0017	0.0267	0.0325	0.0203	0.0136	0.6703	0.0108	0.0472	0.0431	0.0337	0.0164	0.4875	0.0108	0.0559	0.1193	0.0620	0.0454	0.7325
	3000	0.0015	0.0260	0.0328	0.0201	0.0136	0.6773	0.0111	0.0468	0.0433	0.0337	0.0162	0.4814	0.0111	0.0545	0.1274	0.0643	0.0496	0.7717
	3500	0.0016	0.0267	0.0331	0.0205	0.0138	0.6743	0.0107	0.0465	0.0433	0.0335	0.0163	0.4871	0.0107	0.0553	0.1245	0.0635	0.0477	0.7511
	10000	0.0016	0.0264	0.0326	0.0202	0.0135	0.6659	0.0106	0.0465	0.0428	0.0333	0.0162	0.4859	0.0100	0.0542	0.1300	0.0648	0.0502	0.7748
	150	0.0027	0.0175	0.0181	0.0127	0.0107	0.8427	0.0147	0.0391	0.0317	0.0285	0.0136	0.4761	0.0268	0.0494	0.0489	0.0417	0.0193	0.4638
	300	0.0011	0.0144	0.0168	0.0108	0.0086	0.7990	0.0071	0.0300	0.0274	0.0215	0.0117	0.5452	0.0181	0.0380	0.0523	0.0362	0.0177	0.4899
	500	0.0007	0.0136	0.0162	0.0101	0.0078	0.7684	0.0059	0.0273	0.0246	0.0193	0.0103	0.5361	0.0142	0.0349	0.0547	0.0346	0.0200	0.5770
	750	0.0004	0.0133	0.0167	0.0101	0.0077	0.7614	0.0052	0.0258	0.0258	0.0189	0.0103	0.5457	0.0124	0.0315	0.0572	0.0337	0.0205	0.6091
	1250	0.0004	0.0126	0.0165	0.0099	0.0073	0.7355	0.0047	0.0250	0.0246	0.0181	0.0098	0.5421	0.0112	0.0296	0.0599	0.0336	0.0215	0.6401
$\delta = 5$	1500	0.0003	0.0123	0.0161	0.0096	0.0070	0.7326	0.0047	0.0244	0.0238	0.0176	0.0095	0.5359	0.0108	0.0292	0.0615	0.0338	0.0228	0.6735
	2000	0.0004	0.0125	0.0159	0.0096	0.0069	0.7218	0.0045	0.0244	0.0240	0.0177	0.0095	0.5390	0.0103	0.0280	0.0626	0.0336	0.0235	0.6997
	2500	0.0004	0.0123	0.0161	0.0096	0.0069	0.7238	0.0043	0.0239	0.0241	0.0174	0.0095	0.5435	0.0099	0.0274	0.0633	0.0336	0.0238	0.7085
	3000	0.0004	0.0124	0.0160	0.0096	0.0069	0.7157	0.0042	0.0242	0.0238	0.0174	0.0095	0.5451	0.0096	0.0278	0.0651	0.0342	0.0249	0.7304
	3500	0.0004	0.0124	0.0161	0.0096	0.0068	0.7114	0.0041	0.0240	0.0235	0.0172	0.0094	0.5451	0.0095	0.0270	0.0633	0.0333	0.0241	0.7246
	10000	0.0004	0.0124	0.0162	0.0097	0.0068	0.7060	0.0043	0.0239	0.0238	0.0173	0.0093	0.5368	0.0088	0.0253	0.0673	0.0338	0.0254	0.7527

Table A.6: Error Rates for 6 variables and a sample ratio of (1:2:3)

Distributions		CorrNorm						UncorrNorm						Skewed					
	S. Size	π_1	π_2	π_3	G. Mean	SD	CV	π_1	π_2	π_3	G. Mean	SD	CV	π_1	π_2	π_3	G. Mean	SD	CV
$\delta = 1$	180	0.0477	0.1737	0.1312	0.1175	0.0553	0.4705	0.1047	0.2657	0.1561	0.1755	0.0686	0.3912	0.0364	0.2107	0.3315	0.1929	0.1235	0.6405
	360	0.0332	0.1651	0.1249	0.1077	0.0572	0.5307	0.0850	0.2371	0.1344	0.1522	0.0644	0.4234	0.0245	0.2103	0.3478	0.1942	0.1339	0.6894
	600	0.0293	0.1592	0.1233	0.1039	0.0558	0.5367	0.0760	0.2222	0.1275	0.1419	0.0612	0.4317	0.0194	0.2119	0.3536	0.1949	0.1378	0.7068
	900	0.0270	0.1606	0.1222	0.1033	0.0569	0.5506	0.0725	0.2159	0.1250	0.1378	0.0597	0.4335	0.0171	0.2101	0.3660	0.1977	0.1435	0.7257
	1500	0.0248	0.1590	0.1215	0.1018	0.0570	0.5598	0.0681	0.2096	0.1233	0.1336	0.0585	0.4381	0.0156	0.2149	0.3696	0.2000	0.1455	0.7276
	1800	0.0248	0.1581	0.1207	0.1012	0.0565	0.5586	0.0669	0.2084	0.1213	0.1322	0.0586	0.4433	0.0154	0.2133	0.3726	0.2004	0.1466	0.7316
	2400	0.0236	0.1576	0.1197	0.1003	0.0567	0.5651	0.0662	0.2063	0.1211	0.1312	0.0579	0.4413	0.0146	0.2139	0.3733	0.2006	0.1472	0.7339
	3000	0.0241	0.1569	0.1203	0.1004	0.0563	0.5607	0.0654	0.2049	0.1208	0.1304	0.0576	0.4415	0.0145	0.2129	0.3769	0.2014	0.1487	0.7381
	3600	0.0238	0.1573	0.1187	0.0999	0.0564	0.5639	0.0651	0.2036	0.1199	0.1295	0.0572	0.4413	0.0136	0.2145	0.3779	0.2020	0.1494	0.7396
	4200	0.0232	0.1559	0.1200	0.0997	0.0562	0.5641	0.0643	0.2035	0.1200	0.1293	0.0574	0.4441	0.0134	0.2151	0.3796	0.2027	0.1502	0.7410
	12000	0.0229	0.1562	0.1191	0.0994	0.0563	0.5668	0.0631	0.2015	0.1190	0.1279	0.0570	0.4453	0.0126	0.2151	0.3846	0.2041	0.1524	0.7468
$\delta = 2$	180	0.0315	0.1226	0.0935	0.0825	0.0412	0.4992	0.0752	0.1923	0.1195	0.1290	0.0499	0.3866	0.0327	0.1566	0.2756	0.1550	0.1024	0.6608
	360	0.0200	0.1099	0.0920	0.0740	0.0405	0.5480	0.0546	0.1650	0.1074	0.1090	0.0460	0.4218	0.0206	0.1561	0.2906	0.1558	0.1121	0.7195
	600	0.0155	0.1047	0.0935	0.0712	0.0408	0.5725	0.0471	0.1543	0.1044	0.1019	0.0444	0.4358	0.0171	0.1555	0.3022	0.1583	0.1177	0.7438
	900	0.0146	0.1052	0.0924	0.0707	0.0407	0.5751	0.0436	0.1484	0.1024	0.0981	0.0434	0.4425	0.0146	0.1554	0.3096	0.1599	0.1215	0.7598
	1500	0.0136	0.1030	0.0918	0.0695	0.0403	0.5798	0.0415	0.1446	0.1001	0.0954	0.0426	0.4462	0.0133	0.1562	0.3186	0.1627	0.1257	0.7726
	1800	0.0131	0.1015	0.0916	0.0687	0.0399	0.5806	0.0403	0.1433	0.0996	0.0944	0.0425	0.4500	0.0131	0.1575	0.3196	0.1634	0.1261	0.7715
	2400	0.0128	0.1014	0.0915	0.0686	0.0399	0.5823	0.0393	0.1425	0.0997	0.0938	0.0425	0.4534	0.0123	0.1586	0.3231	0.1647	0.1277	0.7756
	3000	0.0125	0.1015	0.0915	0.0685	0.0401	0.5848	0.0389	0.1418	0.0992	0.0933	0.0424	0.4547	0.0123	0.1568	0.3208	0.1633	0.1266	0.7756
	3600	0.0123	0.1011	0.0917	0.0684	0.0401	0.5860	0.0388	0.1410	0.0994	0.0930	0.0422	0.4531	0.0117	0.1607	0.3288	0.1671	0.1302	0.7793
	4200	0.0124	0.1009	0.0911	0.0681	0.0398	0.5838	0.0385	0.1403	0.0985	0.0924	0.0419	0.4540	0.0118	0.1573	0.3263	0.1651	0.1291	0.7819
	12000	0.0118	0.1007	0.0917	0.0681	0.0401	0.5885	0.0375	0.1396	0.0986	0.0919	0.0420	0.4574	0.0109	0.1587	0.3342	0.1679	0.1325	0.7890
$\delta = 3$	180	0.0144	0.0714	0.0603	0.0487	0.0286	0.5874	0.0444	0.1228	0.0820	0.0831	0.0343	0.4129	0.0270	0.0992	0.2030	0.1097	0.0767	0.6989
	360	0.0092	0.0610	0.0595	0.0432	0.0257	0.5947	0.0317	0.1014	0.0736	0.0689	0.0299	0.4333	0.0183	0.0956	0.2158	0.1099	0.0841	0.7656
	600	0.0068	0.0599	0.0584	0.0417	0.0257	0.6154	0.0259	0.0947	0.0718	0.0642	0.0292	0.4558	0.0150	0.0936	0.2237	0.1108	0.0878	0.7930
	900	0.0055	0.0605	0.0583	0.0414	0.0261	0.6303	0.0230	0.0910	0.0699	0.0613	0.0289	0.4717	0.0136	0.0915	0.2245	0.1099	0.0881	0.8020
	1500	0.0055	0.0575	0.0583	0.0404	0.0251	0.6205	0.0214	0.0895	0.0688	0.0599	0.0288	0.4811	0.0117	0.0958	0.2390	0.1155	0.0954	0.8258
	1800	0.0052	0.0585	0.0586	0.0408	0.0255	0.6264	0.0212	0.0889	0.0687	0.0596	0.0287	0.4808	0.0113	0.0935	0.2364	0.1138	0.0941	0.8275
	2400	0.0049	0.0571	0.0589	0.0403	0.0253	0.6277	0.0207	0.0879	0.0689	0.0591	0.0285	0.4818	0.0111	0.0946	0.2403	0.1154	0.0959	0.8312
	3000	0.0050	0.0570	0.0585	0.0401	0.0251	0.6246	0.0203	0.0875	0.0685	0.0588	0.0285	0.4843	0.0103	0.0950	0.2383	0.1145	0.0947	0.8272
	3600	0.0046	0.0570	0.0588	0.0401	0.0253	0.6310	0.0202	0.0872	0.0684	0.0586	0.0284	0.4839	0.0104	0.0935	0.2456	0.1165	0.0984	0.8444
	4200	0.0048	0.0570	0.0580	0.0399	0.0250	0.6260	0.0199	0.0869	0.0680	0.0583	0.0283	0.4863	0.0099	0.0952	0.2437	0.1163	0.0975	0.8384
	12000	0.0046	0.0564	0.0578	0.0396	0.0248	0.6270	0.0195	0.0864	0.0679	0.0579	0.0283	0.4885	0.0094	0.0933	0.2500	0.1176	0.1002	0.8520
$\delta = 4$	180	0.0063	0.0384	0.0342	0.0263	0.0178	0.6770	0.0248	0.0690	0.0499	0.0479	0.0204	0.4265	0.0251	0.0605	0.1188	0.0681	0.0432	0.6335
	360	0.0030	0.0337	0.0330	0.0233	0.0160	0.6893	0.0151	0.0602	0.0468	0.0407	0.0200	0.4919	0.0164	0.0559	0.1262	0.0662	0.0479	0.7235
	600	0.0025	0.0322	0.0334	0.0227	0.0153	0.6761	0.0119	0.0542	0.0448	0.0369	0.0188	0.5091	0.0133	0.0498	0.1354	0.0662	0.0536	0.8096
	900	0.0018	0.0306	0.0334	0.0219	0.0149	0.6791	0.0109	0.0522	0.0433	0.0355	0.0182	0.5128	0.0121	0.0493	0.1419	0.0678	0.0571	0.8428
	1500	0.0016	0.0291	0.0324	0.0210	0.0142	0.6737	0.0098	0.0504	0.0426	0.0343	0.0179	0.5211	0.0102	0.0486	0.1469	0.0686	0.0592	0.8635
	1800	0.0017	0.0294	0.0330	0.0213	0.0143	0.6713	0.0095	0.0498	0.0428	0.0340	0.0178	0.5227	0.0103	0.0477	0.1458	0.0679	0.0586	0.8625
	2400	0.0016	0.0290	0.0323	0.0210	0.0141	0.6703	0.0095	0.0494	0.0421	0.0336	0.0176	0.5216	0.0095	0.0463	0.1524	0.0694	0.0625	0.9011
	3000	0.0014	0.0290	0.0323	0.0209	0.0140	0.6714	0.0092	0.0492	0.0416	0.0333	0.0175	0.5258	0.0092	0.0461	0.1551	0.0701	0.0634	0.9044
	3600	0.0014	0.0288	0.0322	0.0208	0.0140	0.6702	0.0093	0.0491	0.0418	0.0334	0.0174	0.5219	0.0090	0.0463	0.1516	0.0690	0.0612	0.8877
	4200	0.0014	0.0287	0.0325	0.0209	0.0140	0.6694	0.0091	0.0489	0.0420	0.0333	0.0175	0.5242	0.0088	0.0458	0.1546	0.0697	0.0631	0.9047
	12000	0.0013	0.0285	0.0325	0.0208	0.0139	0.6694	0.0088	0.0483	0.0418	0.0330	0.0174	0.5260	0.0082	0.0460	0.1595	0.0712	0.0651	0.9147
$\delta = 5$	180	0.0026	0.0209	0.0181	0.0139	0.0111	0.7967	0.0128	0.0376	0.0286	0.0263	0.0129	0.4887	0.0226	0.0392	0.0572	0.0396	0.0200	0.5050
	360	0.0010	0.0164	0.0174	0.0116	0.0090	0.7786	0.0063	0.0306	0.0255	0.0208	0.0116	0.5569	0.0152	0.0339	0.0627	0.0373	0.0234	0.6294
	600	0.0005	0.0141	0.0160	0.0102	0.0080	0.7825	0.0049	0.0278	0.0248	0.0192	0.0109	0.5684	0.0121	0.0291	0.0655	0.0355	0.0253	0.7127
	900	0.0004	0.0135	0.0164	0.0101	0.0075	0.7436	0.0043	0.0263	0.0241	0.0182	0.0104	0.5697	0.0108	0.0279	0.0700	0.0362	0.0281	0.7753
	1500	0.0003	0.0142	0.0163	0.0103	0.0074	0.7225	0.0038	0.0268	0.0239	0.0182	0.0105	0.5768	0.0092	0.0246	0.0750	0.0363	0.0298	0.8227
	1800	0.0003	0.0138	0.0162	0.0101	0.0073	0.7241	0.0039	0.0257	0.0233	0.0176	0.0100	0.5680	0.0089	0.0242	0.0733	0.0355	0.0290	0.8178
	2400	0.0003	0.0135	0.0164	0.0101	0.0072	0.7140	0.0039	0.0253	0.0236	0.0176	0.0099	0.5621	0.0088	0.0239	0.0756	0.0361	0.0299	0.8290
	3000	0.0003	0.0135	0.0161	0.0100	0.0071	0.7150	0.0037	0.0252	0.0233	0.0174	0.0099	0.5671	0.0082	0.0238	0.0785	0.0368	0.0320	0.8686
	3600	0.0003	0.0132	0.0162	0.0099	0.0070	0.7113	0.0036	0.0250	0.0236	0.0174	0.0099	0.5680	0.0079	0.0235	0.0804	0.0373	0.0327	0.8759
	4200	0.0003	0.0132	0.0162	0.0099	0.0070	0.7107	0.0034	0.0248	0.0233	0.0172	0.0098	0.5719	0.0079	0.0231	0.0777	0.0362	0.0308	0.8519
	12000	0.0003	0.0134	0.0162	0.0100	0.0070	0.7014	0.0034	0.0248	0.0233	0.0171	0.0098	0.5721	0.0074	0.0217	0.0835	0.0375	0.0337	0.8991

Table A.7: Error Rates for 8 variables and a sample ratio of (1:1:1)

Distributions		CorrNorm						UncorrNorm						Skewed					
	S. Size	π_1	π_2	π_3	G. Mean	SD	CV	π_1	π_2	π_3	G. Mean	SD	CV	π_1	π_2	π_3	G. Mean	SD	CV
	90	0.0779	0.1623	0.1228	0.1210	0.0428	0.3540	0.1887	0.3240	0.3051	0.2726	0.0619	0.2269	0.0810	0.2360	0.2229	0.1800	0.0750	0.4169
	180	0.0415	0.1368	0.1130	0.0971	0.0446	0.4593	0.1242	0.2827	0.2066	0.2045	0.0661	0.3234	0.0521	0.2218	0.2154	0.1631	0.0810	0.4967
	300	0.0320	0.1266	0.1101	0.0896	0.0434	0.4851	0.0962	0.2416	0.1640	0.1673	0.0606	0.3624	0.0398	0.2152	0.2244	0.1598	0.0864	0.5403
	450	0.0262	0.1229	0.1095	0.0862	0.0441	0.5122	0.0811	0.2188	0.1449	0.1483	0.0570	0.3845	0.0351	0.2146	0.2287	0.1595	0.0892	0.5590
	750	0.0223	0.1114	0.1071	0.0803	0.0419	0.5220	0.0716	0.1995	0.1346	0.1352	0.0529	0.3911	0.0316	0.2114	0.2359	0.1596	0.0918	0.5749
$\delta = 1$	900	0.0214	0.1119	0.1094	0.0809	0.0429	0.5308	0.0686	0.1951	0.1315	0.1318	0.0522	0.3963	0.0296	0.2150	0.2365	0.1603	0.0936	0.5835
	1200	0.0206	0.1109	0.1070	0.0795	0.0423	0.5315	0.0651	0.1893	0.1286	0.1276	0.0511	0.4006	0.0289	0.2107	0.2395	0.1597	0.0938	0.5871
	1500	0.0195	0.1097	0.1078	0.0790	0.0425	0.5382	0.0631	0.1859	0.1270	0.1253	0.0506	0.4034	0.0274	0.2109	0.2415	0.1599	0.0950	0.5940
	1800	0.0190	0.1080	0.1070	0.0780	0.0421	0.5397	0.0622	0.1843	0.1252	0.1239	0.0501	0.4047	0.0267	0.2122	0.2421	0.1603	0.0957	0.5969
	2100	0.0192	0.1078	0.1070	0.0780	0.0419	0.5372	0.0611	0.1820	0.1247	0.1226	0.0497	0.4051	0.0259	0.2128	0.2451	0.1613	0.0971	0.6020
	6000	0.0179	0.1048	0.1068	0.0765	0.0416	0.5441	0.0577	0.1744	0.1224	0.1181	0.0479	0.4054	0.0236	0.2129	0.2502	0.1622	0.0995	0.6133
	90	0.0571	0.1252	0.0858	0.0894	0.0382	0.4273	0.1501	0.2679	0.2139	0.2106	0.0520	0.2467	0.0766	0.1887	0.1763	0.1472	0.0574	0.3898
	180	0.0251	0.0996	0.0814	0.0687	0.0358	0.5210	0.0869	0.2091	0.1362	0.1441	0.0527	0.3658	0.0447	0.1739	0.1803	0.1330	0.0661	0.4972
	300	0.0185	0.0879	0.0790	0.0618	0.0332	0.5378	0.0652	0.1714	0.1126	0.1164	0.0447	0.3843	0.0355	0.1619	0.1903	0.1292	0.0691	0.5350
	450	0.0156	0.0826	0.0779	0.0587	0.0322	0.5478	0.0549	0.1540	0.1035	0.1041	0.0416	0.3992	0.0324	0.1612	0.1942	0.1293	0.0713	0.5515
	750	0.0121	0.0780	0.0756	0.0552	0.0313	0.5668	0.0465	0.1398	0.0962	0.0942	0.0389	0.4128	0.0280	0.1601	0.2018	0.1300	0.0751	0.5778
$\delta = 2$	900	0.0123	0.0756	0.0756	0.0545	0.0306	0.5622	0.0452	0.1368	0.0942	0.0921	0.0380	0.4127	0.0270	0.1618	0.2004	0.1298	0.0752	0.5798
	1200	0.0112	0.0735	0.0755	0.0534	0.0304	0.5698	0.0432	0.1329	0.0934	0.0898	0.0372	0.4138	0.0253	0.1622	0.2065	0.1313	0.0780	0.5937
	1500	0.0113	0.0740	0.0768	0.0540	0.0307	0.5683	0.0423	0.1308	0.0925	0.0885	0.0367	0.4140	0.0240	0.1613	0.2062	0.1305	0.0782	0.5990
	1800	0.0103	0.0728	0.0761	0.0531	0.0306	0.5774	0.0413	0.1298	0.0918	0.0876	0.0366	0.4175	0.0240	0.1598	0.2103	0.1314	0.0794	0.6040
	2100	0.0105	0.0725	0.0761	0.0530	0.0304	0.5737	0.0406	0.1276	0.0905	0.0862	0.0359	0.4167	0.0226	0.1609	0.2124	0.1319	0.0808	0.6122
	6000	0.0099	0.0705	0.0756	0.0520	0.0300	0.5777	0.0388	0.1241	0.0891	0.0840	0.0352	0.4186	0.0208	0.1609	0.2183	0.1333	0.0833	0.6247
	90	0.0299	0.0849	0.0551	0.0566	0.0306	0.5399	0.1009	0.2013	0.1394	0.1472	0.0458	0.3109	0.0687	0.1382	0.1258	0.1109	0.0396	0.3570
	180	0.0105	0.0587	0.0472	0.0388	0.0245	0.6303	0.0523	0.1329	0.0868	0.0907	0.0353	0.3893	0.0429	0.1117	0.1321	0.0955	0.0427	0.4465
	300	0.0069	0.0525	0.0467	0.0353	0.0221	0.6259	0.0358	0.1068	0.0718	0.0714	0.0304	0.4260	0.0331	0.1057	0.1370	0.0919	0.0459	0.4993
	450	0.0064	0.0467	0.0476	0.0335	0.0208	0.6191	0.0306	0.0947	0.0669	0.0641	0.0272	0.4248	0.0286	0.1037	0.1453	0.0925	0.0507	0.5485
	750	0.0051	0.0436	0.0467	0.0318	0.0197	0.6218	0.0274	0.0872	0.0634	0.0594	0.0252	0.4245	0.0244	0.0998	0.1498	0.0913	0.0530	0.5798
$\delta = 2$	900	0.0048	0.0429	0.0468	0.0315	0.0197	0.6261	0.0255	0.0830	0.0624	0.0570	0.0244	0.4279	0.0238	0.0994	0.1521	0.0918	0.0918	0.5926
	1200	0.0046	0.0430	0.0461	0.0312	0.0194	0.6211	0.0241	0.0816	0.0615	0.0557	0.0243	0.4360	0.0226	0.0986	0.1532	0.0915	0.0547	0.5977
	1500	0.0045	0.0413	0.0462	0.0307	0.0191	0.6215	0.0234	0.0811	0.0604	0.0550	0.0242	0.4401	0.0218	0.1011	0.1584	0.0937	0.0572	0.6106
	1800	0.0043	0.0412	0.0455	0.0303	0.0189	0.6223	0.0233	0.0794	0.0594	0.0540	0.0235	0.4357	0.0211	0.0983	0.1592	0.0929	0.0576	0.6205
	2100	0.0042	0.0407	0.0465	0.0305	0.0189	0.6216	0.0228	0.0790	0.0597	0.0539	0.0236	0.4375	0.0207	0.0986	0.1601	0.0931	0.0581	0.6239
	6000	0.0038	0.0404	0.0463	0.0302	0.0189	0.6271	0.0212	0.0764	0.0584	0.0520	0.0231	0.4443	0.0187	0.0969	0.1660	0.0939	0.0607	0.6469
	90	0.0123	0.0494	0.0339	0.0319	0.0234	0.7332	0.0613	0.1327	0.0871	0.0937	0.0343	0.3658	0.0618	0.0964	0.0811	0.0798	0.0259	0.3246
	180	0.0041	0.0309	0.0273	0.0208	0.0150	0.7233	0.0263	0.0779	0.0524	0.0522	0.0236	0.4515	0.0371	0.0729	0.0831	0.0644	0.0266	0.4135
	300	0.0025	0.0268	0.0265	0.0186	0.0133	0.7180	0.0179	0.0607	0.0404	0.0424	0.0189	0.4675	0.0305	0.0634	0.0825	0.0588	0.0253	0.4309
	450	0.0022	0.0253	0.0262	0.0179	0.0125	0.6998	0.0152	0.0533	0.0410	0.0365	0.0168	0.4608	0.0258	0.0579	0.0905	0.0581	0.0298	0.5132
	750	0.0015	0.0213	0.0253	0.0160	0.0111	0.6914	0.0125	0.0477	0.0388	0.0330	0.0156	0.4730	0.0220	0.0548	0.0942	0.0570	0.0318	0.5569
$\delta = 4$	900	0.0012	0.0228	0.0255	0.0165	0.0115	0.7013	0.0123	0.0467	0.0367	0.0319	0.0150	0.4695	0.0215	0.0532	0.0935	0.0561	0.0312	0.5571
	1200	0.0014	0.0211	0.0258	0.0161	0.0110	0.6850	0.0117	0.0454	0.0364	0.0312	0.0146	0.4688	0.0198	0.0529	0.1022	0.0583	0.0366	0.6280
	1500	0.0012	0.0210	0.0248	0.0157	0.0107	0.6840	0.0114	0.0439	0.0360	0.0304	0.0142	0.4659	0.0195	0.0521	0.0984	0.0567	0.0339	0.5987
	1800	0.0012	0.0208	0.0244	0.0155	0.0105	0.6777	0.0109	0.0436	0.0358	0.0301	0.0143	0.4737	0.0191	0.0515	0.0999	0.0568	0.0346	0.6080
	2100	0.0012	0.0213	0.0256	0.0160	0.0109	0.6811	0.0111	0.0439	0.0360	0.0303	0.0142	0.4684	0.0186	0.0506	0.1027	0.0573	0.0362	0.6311
	6000	0.0011	0.0202	0.0248	0.0154	0.0104	0.6743	0.0103	0.0420	0.0349	0.0291	0.0137	0.4715	0.0169	0.0492	0.1052	0.0571	0.0371	0.6501
	90	0.0056	0.0316	0.0184	0.0185	0.0174	0.9418	0.0327	0.0828	0.0559	0.0571	0.0254	0.4441	0.0592	0.0757	0.0376	0.0575	0.0223	0.3884
	180	0.0011	0.0166	0.0137	0.0105	0.0098	0.9369	0.0119	0.0430	0.0303	0.0284	0.0149	0.5262	0.0343	0.0501	0.0389	0.0411	0.0130	0.3153
	300	0.0006	0.0138	0.0133	0.0092	0.0080	0.8672	0.0078	0.0313	0.0254	0.0215	0.0114	0.5304	0.0276	0.0406	0.0432	0.0371	0.0120	0.3246
	450	0.0004	0.0126	0.0131	0.0087	0.0071	0.8201	0.0062	0.0277	0.0228	0.0189	0.0101	0.5356	0.0233	0.0369	0.0458	0.0353	0.0130	0.3670
	750	0.0003	0.0106	0.0123	0.0077	0.0059	0.7595	0.0049	0.0244	0.0212	0.0168	0.0091	0.5413	0.0207	0.0318	0.0493	0.0339	0.0149	0.4385
$\delta = 5$	900	0.0003	0.0104	0.0124	0.0077	0.0060	0.7813	0.0053	0.0242	0.0208	0.0168	0.0088	0.5253	0.0194	0.0303	0.0480	0.0325	0.0141	0.4329
	1200	0.0003	0.0096	0.0121	0.0073	0.0055	0.7549	0.0048	0.0229	0.0202	0.0160	0.0083	0.5199	0.0186	0.0288	0.0495	0.0323	0.0151	0.4672
	1500	0.0002	0.0100	0.0124	0.0075	0.0056	0.7423	0.0044	0.0226	0.0202	0.0157	0.0084	0.5309	0.0179	0.0286	0.0489	0.0318	0.0149	0.4685
	1800	0.0003	0.0096	0.0122	0.0074	0.0054	0.7344	0.0046	0.0220	0.0198	0.0155	0.0080	0.5163	0.0171	0.0278	0.0509	0.0319	0.0158	0.4952
	2100	0.0003	0.0098	0.0124	0.0075	0.0055	0.7332	0.0043	0.0219	0.0197	0.0153	0.0080	0.5248	0.0168	0.0270	0.0523	0.0320	0.0163	0.5082
	6000	0.0002	0.0095	0.0123	0.0073	0.0052	0.7139	0.0041	0.0210	0.0194	0.0148	0.0077	0.5201	0.0154	0.0250	0.0544	0.0316	0.0174	0.5511

Table A.8: Error Rates for 8 variables and a sample ratio of (1:2:2)

Distributions		CorrNorm						UncorrNorm						Skewed					
	S. Size	π_1	π_2	π_3	G. Mean	SD	CV	π_1	π_2	π_3	G. Mean	SD	CV	π_1	π_2	π_3	G. Mean	SD	CV
$\delta = 1$	150	0.0627	0.1525	0.1319	0.1157	0.0443	0.3832	0.1375	0.2876	0.2506	0.2252	0.0658	0.2921	0.0503	0.2459	0.2644	0.1869	0.0996	0.5331
	300	0.0375	0.1360	0.1337	0.1024	0.0482	0.4710	0.1067	0.2274	0.1846	0.1729	0.0516	0.2984	0.0319	0.2453	0.2721	0.1831	0.1089	0.5948
	500	0.0300	0.1270	0.1307	0.0959	0.0478	0.4986	0.0908	0.2003	0.1659	0.1524	0.0469	0.3075	0.0250	0.2422	0.2784	0.1819	0.1128	0.6201
	750	0.0254	0.1211	0.1299	0.0921	0.0482	0.5230	0.0832	0.1854	0.1577	0.1421	0.0440	0.3094	0.0224	0.2432	0.2845	0.1834	0.1158	0.6316
	1250	0.0223	0.1178	0.1286	0.0896	0.0483	0.5392	0.0756	0.1758	0.1513	0.1343	0.0431	0.3211	0.0200	0.2451	0.2883	0.1845	0.1182	0.6408
	1500	0.0223	0.1177	0.1282	0.0894	0.0482	0.5386	0.0746	0.1736	0.1513	0.1332	0.0429	0.3221	0.0187	0.2483	0.2922	0.1864	0.1205	0.6466
	2000	0.0211	0.1166	0.1282	0.0886	0.0484	0.5460	0.0722	0.1698	0.1486	0.1302	0.0422	0.3244	0.0178	0.2462	0.2935	0.1859	0.1208	0.6501
	2500	0.0203	0.1150	0.1293	0.0882	0.0486	0.5513	0.0709	0.1683	0.1478	0.1290	0.0422	0.3272	0.0174	0.2456	0.2950	0.1860	0.1213	0.6523
	3000	0.0202	0.1145	0.1284	0.0877	0.0484	0.5516	0.0700	0.1662	0.1472	0.1278	0.0419	0.3274	0.0169	0.2474	0.2978	0.1874	0.1226	0.6545
	3500	0.0204	0.1142	0.1284	0.0877	0.0482	0.5494	0.0695	0.1651	0.1467	0.1271	0.0417	0.3278	0.0166	0.2474	0.2975	0.1872	0.1228	0.6560
	10000	0.0191	0.1131	0.1279	0.0867	0.0483	0.5571	0.0671	0.1621	0.1456	0.1250	0.0416	0.3327	0.0153	0.2474	0.3041	0.1890	0.1253	0.6628
$\delta = 2$	150	0.0417	0.1117	0.0994	0.0843	0.0358	0.4246	0.1093	0.2211	0.1602	0.1635	0.0488	0.2982	0.0443	0.1875	0.2217	0.1512	0.0809	0.5349
	300	0.0226	0.0951	0.0918	0.0698	0.0356	0.5094	0.0744	0.1686	0.1301	0.1244	0.0404	0.3249	0.0286	0.1835	0.2280	0.1467	0.0875	0.5965
	500	0.0170	0.0892	0.0925	0.0662	0.0359	0.5429	0.0604	0.1471	0.1203	0.1093	0.0373	0.3410	0.0225	0.1880	0.2371	0.1492	0.0934	0.6258
	750	0.0146	0.0845	0.0909	0.0633	0.0355	0.5609	0.0533	0.1385	0.1137	0.1018	0.0365	0.3582	0.0203	0.1835	0.2459	0.1499	0.0961	0.6414
	1250	0.0127	0.0821	0.0908	0.0619	0.0355	0.5738	0.0477	0.1321	0.1109	0.0969	0.0363	0.3742	0.0173	0.1839	0.2495	0.1502	0.0986	0.6563
	1500	0.0119	0.0814	0.0913	0.0616	0.0357	0.5804	0.0466	0.1297	0.1097	0.0953	0.0358	0.3754	0.0174	0.1849	0.2510	0.1511	0.0991	0.6560
	2000	0.0115	0.0799	0.0907	0.0607	0.0354	0.5832	0.0447	0.1263	0.1089	0.0933	0.0354	0.3794	0.0156	0.1866	0.2565	0.1529	0.1019	0.6665
	2500	0.0108	0.0792	0.0910	0.0603	0.0356	0.5903	0.0440	0.1257	0.1084	0.0927	0.0354	0.3819	0.0154	0.1849	0.2552	0.1519	0.1013	0.6669
	3000	0.0108	0.0793	0.0912	0.0604	0.0357	0.5904	0.0437	0.1248	0.1077	0.0920	0.0351	0.3819	0.0147	0.1865	0.2599	0.1537	0.1033	0.6721
	3500	0.0107	0.0789	0.0907	0.0601	0.0355	0.5901	0.0431	0.1241	0.1077	0.0916	0.0352	0.3841	0.0148	0.1863	0.2576	0.1529	0.1024	0.6696
	10000	0.0103	0.0782	0.0908	0.0598	0.0354	0.5930	0.0415	0.1221	0.1068	0.0901	0.0351	0.3891	0.0136	0.1868	0.2635	0.1546	0.1048	0.6780
$\delta = 3$	150	0.0236	0.0697	0.0581	0.0505	0.0253	0.5018	0.0769	0.1471	0.1045	0.1095	0.0323	0.2945	0.0394	0.1269	0.1635	0.1099	0.0570	0.5186
	300	0.0104	0.0571	0.0578	0.0418	0.0241	0.5756	0.0426	0.1067	0.0827	0.0774	0.0282	0.3641	0.0254	0.1173	0.1747	0.1058	0.0646	0.6106
	500	0.0076	0.0532	0.0562	0.0390	0.0232	0.5944	0.0329	0.0958	0.0780	0.0689	0.0689	0.3982	0.0197	0.1168	0.1780	0.1048	0.0673	0.6421
	750	0.0058	0.0490	0.0555	0.0368	0.0229	0.6224	0.0284	0.0890	0.0742	0.0639	0.0265	0.4157	0.0180	0.1128	0.1805	0.1038	0.0681	0.6563
	1250	0.0051	0.0465	0.0552	0.0356	0.0223	0.6273	0.0258	0.0841	0.0720	0.0606	0.0256	0.4216	0.0154	0.1131	0.1887	0.1057	0.0723	0.6841
	1500	0.0048	0.0475	0.0553	0.0359	0.0227	0.6333	0.0249	0.0828	0.0714	0.0597	0.0254	0.4258	0.0149	0.1139	0.1885	0.1058	0.0723	0.6833
	2000	0.0044	0.0469	0.0555	0.0356	0.0227	0.6375	0.0236	0.0810	0.0714	0.0587	0.0254	0.4321	0.0141	0.1121	0.1950	0.1071	0.0747	0.6977
	2500	0.0044	0.0463	0.0557	0.0355	0.0225	0.6356	0.0233	0.0811	0.0708	0.0584	0.0254	0.4345	0.0135	0.1146	0.1924	0.1068	0.0743	0.6951
	3000	0.0042	0.0463	0.0555	0.0353	0.0225	0.6380	0.0230	0.0797	0.0705	0.0577	0.0250	0.4333	0.0135	0.1120	0.1940	0.1065	0.0746	0.7005
	3500	0.0042	0.0456	0.0551	0.0350	0.0223	0.6368	0.0226	0.0798	0.0702	0.0575	0.0252	0.4377	0.0131	0.1142	0.1973	0.1082	0.0760	0.7023
	10000	0.0039	0.0454	0.0553	0.0348	0.0224	0.6418	0.0218	0.0785	0.0697	0.0567	0.0250	0.4408	0.0122	0.1132	0.2039	0.1097	0.0789	0.7189
$\delta = 4$	150	0.0115	0.0377	0.0328	0.0273	0.0165	0.6056	0.0464	0.0883	0.0632	0.0660	0.0213	0.3231	0.0357	0.0825	0.0987	0.0723	0.0335	0.4632
	300	0.0037	0.0291	0.0301	0.0210	0.0143	0.6826	0.0219	0.0607	0.0508	0.0445	0.0181	0.4079	0.0232	0.0679	0.1091	0.0667	0.0391	0.5864
	500	0.0026	0.0287	0.0316	0.0210	0.0141	0.6728	0.0166	0.0546	0.0463	0.0392	0.0172	0.4398	0.0184	0.0645	0.1087	0.0639	0.0395	0.6181
	750	0.0017	0.0259	0.0301	0.0192	0.0133	0.6919	0.0138	0.0503	0.0449	0.0364	0.0168	0.4611	0.0159	0.0594	0.1144	0.0632	0.0422	0.6678
	1250	0.0015	0.0257	0.0313	0.0195	0.0134	0.6881	0.0120	0.0483	0.0440	0.0347	0.0165	0.4753	0.0139	0.0596	0.1189	0.0641	0.0446	0.6953
	1500	0.0013	0.0248	0.0306	0.0189	0.0130	0.6892	0.0115	0.0477	0.0430	0.0341	0.0164	0.4822	0.0136	0.0585	0.1169	0.0630	0.0433	0.6870
	2000	0.0013	0.0246	0.0301	0.0187	0.0128	0.6864	0.0107	0.0470	0.0430	0.0336	0.0165	0.4904	0.0129	0.0594	0.1187	0.0637	0.0444	0.6972
	2500	0.0012	0.0239	0.0303	0.0185	0.0127	0.6880	0.0109	0.0464	0.0424	0.0332	0.0161	0.4830	0.0125	0.0579	0.1235	0.0646	0.0468	0.7238
	3000	0.0012	0.0241	0.0300	0.0184	0.0126	0.6847	0.0106	0.0459	0.0422	0.0329	0.0160	0.4870	0.0120	0.0576	0.1212	0.0636	0.0458	0.7206
	3500	0.0012	0.0237	0.0301	0.0183	0.0125	0.6841	0.0106	0.0450	0.0422	0.0326	0.0157	0.4819	0.0119	0.0572	0.1261	0.0651	0.0484	0.7434
	10000	0.0011	0.0233	0.0299	0.0181	0.0124	0.6858	0.0099	0.0447	0.0418	0.0321	0.0158	0.4919	0.0109	0.0563	0.1296	0.0656	0.0494	0.7526
$\delta = 5$	150	0.0038	0.0205	0.0170	0.0138	0.0117	0.8519	0.0241	0.0504	0.0360	0.0369	0.0144	0.3894	0.0351	0.0599	0.0454	0.0468	0.0169	0.3612
	300	0.0010	0.0147	0.0155	0.0104	0.0085	0.8207	0.0102	0.0332	0.0280	0.0238	0.0116	0.4876	0.0214	0.0445	0.0512	0.0390	0.0173	0.4434
	500	0.0008	0.0137	0.0153	0.0100	0.0077	0.7685	0.0078	0.0291	0.0262	0.0210	0.0105	0.4977	0.0165	0.0393	0.0565	0.0374	0.0195	0.5212
	750	0.0006	0.0123	0.0149	0.0093	0.0069	0.7444	0.0053	0.0265	0.0246	0.0188	0.0102	0.5405	0.0142	0.0369	0.0599	0.0370	0.0208	0.5634
	1250	0.0004	0.0120	0.0149	0.0091	0.0068	0.7427	0.0051	0.0251	0.0239	0.0180	0.0096	0.5305	0.0128	0.0337	0.0608	0.0358	0.0220	0.6153
	1500	0.0003	0.0116	0.0149	0.0090	0.0067	0.7430	0.0047	0.0247	0.0237	0.0177	0.0095	0.5373	0.0120	0.0323	0.0609	0.0351	0.0216	0.6160
	2000	0.0003	0.0111	0.0148	0.0088	0.0064	0.7328	0.0043	0.0242	0.0234	0.0173	0.0095	0.5468	0.0117	0.0314	0.0622	0.0351	0.0225	0.6405
	2500	0.0002	0.0115	0.0150	0.0089	0.0065	0.7335	0.0043	0.0242	0.0236	0.0173	0.0095	0.5457	0.0112	0.0302	0.0665	0.0359	0.0248	0.68962
	3000	0.0002	0.0112	0.0144	0.0086	0.0062	0.7237	0.0043	0.0240	0.0231	0.0171	0.0093	0.5401	0.0111	0.0307	0.0641	0.0353	0.0230	0.6518
	3500	0.0003	0.0114	0.0147	0.0088	0.0064	0.7234	0.0041	0.0238	0.0231	0.0170	0.0093	0.5452	0.0106	0.0296	0.0641	0.0347	0.0232	0.6686
	10000	0.0003	0.0109	0.0145	0.0085	0.0061	0.7142	0.0039	0.0231	0.0231	0.0167	0.0091	0.5463	0.0098	0.0284	0.0676	0.0353	0.0245	0.6948

Table A.9: Error Rates for 8 variables and a sample ratio of (1:2:3)

Distributions		CorrNorm						UncorrNorm						Skewed					
	S. Size	π_1	π_2	π_3	G. Mean	SD	CV	π_1	π_2	π_3	G. Mean	SD	CV	π_1	π_2	π_3	G. Mean	SD	CV
$\delta = 1$	180	0.0516	0.1646	0.1233	0.1132	0.0500	0.4414	0.1156	0.2827	0.1766	0.1916	0.0704	0.3675	0.0404	0.2028	0.3212	0.1882	0.1172	0.6227
	360	0.0318	0.1464	0.1155	0.0979	0.0502	0.5126	0.0890	0.2408	0.1374	0.1557	0.0644	0.4133	0.0274	0.2021	0.3336	0.1877	0.1267	0.6752
	600	0.0247	0.1422	0.1151	0.0940	0.0513	0.5461	0.0755	0.2187	0.1270	0.1404	0.0600	0.4275	0.0212	0.2058	0.3469	0.1913	0.1342	0.7017
	900	0.0214	0.1407	0.1141	0.0921	0.0517	0.5620	0.0688	0.2105	0.1218	0.1337	0.0590	0.4410	0.0189	0.2040	0.3531	0.1920	0.1374	0.7155
	1500	0.0187	0.1396	0.1135	0.0906	0.0524	0.5784	0.0632	0.2027	0.1200	0.1286	0.0576	0.4479	0.0166	0.2034	0.3594	0.1931	0.1407	0.7286
	1800	0.0184	0.1373	0.1131	0.0896	0.0517	0.5771	0.0624	0.2000	0.1198	0.1274	0.0568	0.4456	0.0161	0.2053	0.3589	0.1934	0.1407	0.7276
	2400	0.0177	0.1367	0.1119	0.0887	0.0516	0.5813	0.0599	0.1949	0.1177	0.1241	0.0556	0.4477	0.0151	0.2065	0.3654	0.1956	0.1437	0.7346
	3000	0.0171	0.1356	0.1120	0.0882	0.0515	0.5832	0.0592	0.1939	0.1170	0.1234	0.0554	0.4490	0.0144	0.2066	0.3684	0.1965	0.1451	0.7387
	3600	0.0168	0.1355	0.1122	0.0882	0.0516	0.5849	0.0583	0.1931	0.1167	0.1227	0.0554	0.4513	0.0143	0.2057	0.3686	0.1962	0.1452	0.7402
	4200	0.0165	0.1354	0.1125	0.0881	0.0517	0.5868	0.0580	0.1919	0.1168	0.1222	0.0550	0.4499	0.0139	0.2067	0.3724	0.1976	0.1469	0.7433
	120000	0.0159	0.1343	0.1114	0.0872	0.0514	0.5893	0.0559	0.1890	0.1158	0.1202	0.0546	0.4538	0.0129	0.2071	0.3777	0.1992	0.1494	0.7498
$\delta = 2$	180	0.0366	0.1201	0.0891	0.0819	0.0386	0.4708	0.0912	0.2126	0.1301	0.1446	0.0527	0.3640	0.0366	0.1576	0.2684	0.1542	0.0973	0.6311
	360	0.0194	0.1025	0.0861	0.0693	0.0376	0.5421	0.0611	0.1702	0.1097	0.1137	0.0457	0.4023	0.0233	0.1539	0.2851	0.1541	0.1086	0.7048
	600	0.0154	0.0966	0.0858	0.0659	0.0373	0.5654	0.0496	0.1558	0.1040	0.1031	0.0441	0.4275	0.0188	0.1558	0.2996	0.1580	0.1159	0.7333
	900	0.0124	0.0951	0.0867	0.0647	0.0379	0.5862	0.0443	0.1488	0.1006	0.0979	0.0432	0.4411	0.0167	0.1530	0.3065	0.1588	0.1193	0.7517
	1500	0.0104	0.0926	0.0846	0.0625	0.0375	0.5996	0.0401	0.1412	0.0975	0.0929	0.0418	0.4497	0.0144	0.1543	0.3085	0.1591	0.1211	0.7611
	1800	0.0100	0.0905	0.0853	0.0619	0.0372	0.6004	0.0391	0.1402	0.0980	0.0924	0.0418	0.4518	0.0139	0.1544	0.3153	0.1612	0.1240	0.7689
	2400	0.0096	0.0898	0.0847	0.0614	0.0370	0.6030	0.0379	0.1376	0.0965	0.0907	0.0412	0.4539	0.0133	0.1531	0.3126	0.1597	0.1228	0.7692
	3000	0.0094	0.0899	0.0851	0.0614	0.0371	0.6040	0.0365	0.1366	0.0969	0.0900	0.0413	0.4594	0.0128	0.1544	0.3184	0.1619	0.1255	0.7754
	3600	0.0089	0.0892	0.0849	0.0610	0.0371	0.6089	0.0360	0.1358	0.0961	0.0893	0.0412	0.4614	0.0126	0.1545	0.3185	0.1619	0.1256	0.7762
	4200	0.0089	0.0890	0.0849	0.0609	0.0371	0.6082	0.0359	0.1347	0.0960	0.0889	0.0408	0.4590	0.0122	0.1554	0.3199	0.1625	0.1262	0.7769
	12000	0.0086	0.0880	0.0846	0.0604	0.0368	0.6088	0.0343	0.1324	0.0948	0.0872	0.0405	0.46466	0.0114	0.1551	0.3299	0.1655	0.1307	0.7897
$\delta = 3$	180	0.0199	0.0753	0.0569	0.0507	0.0271	0.5352	0.0617	0.1408	0.0889	0.0971	0.0352	0.3626	0.0329	0.1047	0.1896	0.1091	0.0690	0.6322
	360	0.0097	0.0611	0.0527	0.0411	0.0244	0.5927	0.0363	0.1087	0.0764	0.0738	0.0308	0.4168	0.0220	0.0983	0.2086	0.1096	0.0787	0.7179
	600	0.0062	0.0578	0.0535	0.0392	0.0244	0.6221	0.0281	0.0972	0.0702	0.0652	0.0292	0.4476	0.0170	0.0963	0.2205	0.1113	0.0853	0.7666
	900	0.0046	0.0548	0.0542	0.0378	0.0243	0.6408	0.0238	0.0916	0.0693	0.0616	0.0288	0.4678	0.0145	0.0957	0.2260	0.1121	0.0890	0.7940
	1500	0.0040	0.0525	0.0545	0.0370	0.0238	0.6426	0.0212	0.0887	0.0682	0.0594	0.0286	0.4820	0.0128	0.0953	0.2362	0.1148	0.0938	0.8175
	1800	0.0041	0.0516	0.0546	0.0368	0.0235	0.6386	0.0206	0.0859	0.0680	0.0581	0.0278	0.4786	0.0125	0.0975	0.2390	0.1163	0.0951	0.8175
	2400	0.0037	0.0516	0.0542	0.0365	0.0236	0.6451	0.0197	0.0847	0.0666	0.0570	0.0276	0.4840	0.0117	0.0951	0.2388	0.1152	0.0948	0.8228
	3000	0.0037	0.0514	0.0544	0.0365	0.0235	0.6428	0.0197	0.0851	0.0667	0.0572	0.0277	0.4847	0.0115	0.0940	0.2409	0.1155	0.0958	0.8293
	3600	0.0036	0.0500	0.0540	0.0358	0.0230	0.6429	0.0194	0.0841	0.0661	0.0565	0.0274	0.4848	0.0111	0.0951	0.2452	0.1171	0.0976	0.8336
	4200	0.0034	0.0504	0.0542	0.0360	0.0233	0.6472	0.0187	0.0837	0.0667	0.0564	0.0277	0.4909	0.0111	0.0953	0.2445	0.1170	0.0974	0.8330
	12000	0.0032	0.0501	0.0541	0.0358	0.0232	0.6473	0.0182	0.0824	0.0660	0.0555	0.0273	0.4915	0.0101	0.0943	0.2529	0.1191	0.1013	0.8507
$\delta = 4$	180	0.0106	0.0416	0.0332	0.0285	0.0178	0.6271	0.0381	0.0846	0.0561	0.0596	0.0218	0.3662	0.0329	0.0684	0.1152	0.0722	0.0401	0.5558
	360	0.0029	0.0319	0.0306	0.0218	0.0149	0.6811	0.0196	0.0631	0.0466	0.0431	0.0191	0.4443	0.0197	0.0578	0.1245	0.0674	0.0463	0.6878
	600	0.0021	0.0298	0.0310	0.0210	0.0144	0.6872	0.0131	0.0551	0.0444	0.0375	0.0185	0.4921	0.0156	0.0527	0.1379	0.0687	0.0541	0.7875
	900	0.0015	0.0279	0.0303	0.0199	0.0137	0.6860	0.0111	0.0532	0.0434	0.0359	0.0183	0.5108	0.0137	0.0520	0.1381	0.0679	0.0539	0.7933
	1500	0.0012	0.0279	0.0309	0.0200	0.0138	0.6897	0.0099	0.0503	0.0418	0.0340	0.0177	0.5208	0.0113	0.0513	0.1467	0.0698	0.0584	0.8373
	1800	0.0013	0.0263	0.0301	0.0192	0.0131	0.6802	0.0097	0.0491	0.0419	0.0336	0.0174	0.5172	0.0114	0.0498	0.1479	0.0697	0.0596	0.8542
	2400	0.0011	0.0265	0.0307	0.0194	0.0133	0.6855	0.0089	0.0482	0.0421	0.0330	0.0175	0.5289	0.0105	0.0489	0.1519	0.0704	0.0616	0.8753
	3000	0.0011	0.0262	0.0300	0.0191	0.0130	0.6837	0.0089	0.0478	0.0411	0.0326	0.0171	0.5250	0.0102	0.0491	0.1543	0.0712	0.0626	0.8785
	3600	0.0010	0.0263	0.0301	0.0191	0.0131	0.6831	0.0088	0.0477	0.0411	0.0325	0.0171	0.5262	0.0099	0.0495	0.1531	0.0708	0.0613	0.8650
	4200	0.0010	0.0259	0.0297	0.0189	0.0129	0.6826	0.0088	0.0474	0.0409	0.0324	0.0170	0.5248	0.0099	0.0490	0.1528	0.0706	0.0614	0.8703
	12000	0.0010	0.0257	0.0299	0.0188	0.0128	0.6813	0.0084	0.0467	0.0406	0.0319	0.0169	0.5289	0.0089	0.0473	0.1620	0.0727	0.0658	0.9048
$\delta = 5$	180	0.0046	0.0204	0.0164	0.0138	0.0101	0.7291	0.0212	0.0463	0.0318	0.0331	0.0133	0.4012	0.0281	0.0483	0.0541	0.0435	0.0177	0.4069
	360	0.0009	0.0159	0.0163	0.0110	0.0089	0.8117	0.0084	0.0339	0.0265	0.0229	0.0119	0.5192	0.0178	0.0376	0.0658	0.0404	0.0234	0.5803
	600	0.0005	0.0143	0.0148	0.0098	0.0075	0.7625	0.0054	0.0293	0.0246	0.0197	0.0110	0.5559	0.0140	0.0323	0.0667	0.0377	0.0244	0.6463
	900	0.0003	0.0131	0.0142	0.0092	0.0070	0.7612	0.0044	0.0270	0.0243	0.0186	0.0105	0.5640	0.0118	0.0307	0.0743	0.0390	0.0292	0.7486
	1500	0.0003	0.0129	0.0154	0.0095	0.0069	0.7224	0.0040	0.0257	0.0241	0.0179	0.0102	0.5693	0.0106	0.0281	0.0791	0.0393	0.0317	0.8058
	1800	0.0002	0.0126	0.0151	0.0093	0.0068	0.7332	0.0040	0.0253	0.0236	0.0176	0.0099	0.5634	0.0100	0.0274	0.0777	0.0383	0.0336	0.8772
	2400	0.0002	0.0122	0.0144	0.0089	0.0065	0.7264	0.0036	0.0251	0.0230	0.0172	0.0098	0.5705	0.0094	0.0263	0.0782	0.0380	0.0311	0.8184
	3000	0.0002	0.0127	0.0149	0.0093	0.0066	0.7163	0.0035	0.0248	0.0231	0.0171	0.0098	0.5727	0.0090	0.0261	0.0771	0.0374	0.0302	0.8066
	3600	0.0002	0.0123	0.0148	0.0091	0.0065	0.7178	0.0035	0.0245	0.0230	0.0170	0.0097	0.5713	0.0090	0.0258	0.0810	0.0386	0.0343	0.8877
	4200	0.0002	0.0120	0.0146	0.0089	0.0064	0.7170	0.0034	0.0243	0.0230	0.0169	0.0097	0.5714	0.0092	0.0256	0.0805	0.0384	0.0324	0.8430
	12000	0.0002	0.0121	0.0147	0.0090	0.0064	0.7082	0.0032	0.0243	0.0228	0.0168	0.0097	0.5760	0.0081	0.0234	0.0846	0.0387	0.0337	0.8709

Table A.10: Summary of Results for Error Rates of 4 variables and a sample ratio of (1:1:1)

Distributions		CorrNorm			UncorrNorm			Skewed		
	S. Size	G. Mean	SD	CV	G. Mean	SD	CV	G. Mean	SD	CV
$\delta = 1$	90	0.1240	0.0530	0.4276	0.1955	0.0681	0.3483	0.1729	0.0875	0.5060
	180	0.1151	0.0536	0.4656	0.1636	0.0641	0.3920	0.1704	0.0942	0.5528
	300	0.1120	0.0543	0.4853	0.1522	0.0603	0.3965	0.1701	0.0983	0.5782
	450	0.1116	0.0547	0.4903	0.1473	0.0603	0.4092	0.1695	0.0998	0.5888
	750	0.1089	0.0534	0.4908	0.1417	0.0575	0.4059	0.1700	0.1031	0.6066
	900	0.1082	0.0526	0.4857	0.1409	0.0573	0.4065	0.1696	0.1030	0.6073
	1200	0.1077	0.0524	0.4868	0.1387	0.0568	0.4096	0.1692	0.1034	0.6110
	1500	0.1081	0.0529	0.4893	0.1388	0.0571	0.4114	0.1707	0.1051	0.6155
	1800	0.1077	0.0526	0.4881	0.1374	0.0567	0.4123	0.1702	0.1048	0.6159
	2100	0.1080	0.0531	0.4913	0.1374	0.0567	0.4129	0.1707	0.1057	0.6194
	6000	0.1069	0.0524	0.4898	0.1359	0.0559	0.4116	0.1712	0.1071	0.6258
$\delta = 2$	90	0.0880	0.0447	0.5084	0.1321	0.0524	0.3968	0.1355	0.0680	0.5015
	180	0.0771	0.0402	0.5215	0.1111	0.0469	0.4222	0.1317	0.0741	0.5626
	300	0.0735	0.0384	0.5224	0.1037	0.0421	0.4062	0.1331	0.0781	0.5864
	450	0.0724	0.0394	0.5439	0.0997	0.0424	0.4251	0.1324	0.0800	0.6039
	750	0.0715	0.0370	0.5172	0.0970	0.0409	0.4218	0.1333	0.0822	0.6168
	900	0.0710	0.0370	0.5217	0.0962	0.0415	0.4313	0.1331	0.0821	0.6171
	1200	0.0703	0.0369	0.5249	0.0955	0.0402	0.4209	0.1340	0.0847	0.6318
	1500	0.0707	0.0367	0.5191	0.0944	0.0398	0.4219	0.1345	0.0850	0.6319
	1800	0.0706	0.0372	0.5265	0.0946	0.0400	0.4230	0.1346	0.0851	0.6325
	2100	0.0702	0.0367	0.5222	0.0940	0.0399	0.4239	0.1357	0.0860	0.6335
	6000	0.0698	0.0365	0.5234	0.0934	0.0394	0.4223	0.1369	0.0883	0.6454
$\delta = 3$	90	0.0497	0.0300	0.6047	0.0823	0.0345	0.4200	0.0931	0.0489	0.5252
	180	0.0442	0.0264	0.5983	0.0683	0.0296	0.4341	0.0889	0.0493	0.5543
	300	0.0412	0.0245	0.5940	0.0628	0.0277	0.4417	0.0875	0.0522	0.5961
	450	0.0405	0.0239	0.5907	0.0599	0.0263	0.4395	0.0882	0.0535	0.6064
	750	0.0401	0.0232	0.5780	0.0586	0.0256	0.4372	0.0922	0.0601	0.6515
	900	0.0400	0.0230	0.5763	0.0586	0.0257	0.4385	0.0906	0.0593	0.6545
	1200	0.0396	0.0226	0.5708	0.0581	0.0255	0.4393	0.0893	0.0581	0.6507
	1500	0.0393	0.0225	0.5727	0.0578	0.0253	0.4386	0.0906	0.0589	0.6508
	1800	0.0393	0.0223	0.5682	0.0573	0.0248	0.4330	0.0918	0.0604	0.6579
	2100	0.0395	0.0226	0.5728	0.0574	0.0250	0.4361	0.0911	0.0606	0.6646
	6000	0.0393	0.0224	0.5698	0.0568	0.0249	0.4380	0.0922	0.0622	0.6741
$\delta = 4$	90	0.0266	0.0200	0.7509	0.0486	0.0236	0.4866	0.0586	0.0252	0.4295
	180	0.0221	0.0159	0.7195	0.0379	0.0189	0.4991	0.0541	0.0296	0.5470
	300	0.0215	0.0144	0.6727	0.0354	0.0173	0.4889	0.0514	0.0286	0.5553
	450	0.0207	0.0139	0.6732	0.0334	0.0160	0.4802	0.0512	0.0307	0.5997
	750	0.0205	0.0135	0.6574	0.0327	0.0158	0.4815	0.0526	0.0348	0.6611
	900	0.0203	0.0131	0.6455	0.0323	0.0153	0.4744	0.0523	0.0344	0.6567
	1200	0.0198	0.0124	0.6286	0.0319	0.0151	0.4725	0.0529	0.0355	0.6709
	1500	0.0199	0.0126	0.6353	0.0318	0.0151	0.4728	0.05247	0.0351	0.6679
	1800	0.0203	0.0128	0.6301	0.0315	0.0147	0.4660	0.0540	0.0378	0.6998
	2100	0.0201	0.0128	0.6382	0.0315	0.0147	0.4663	0.0533	0.0374	0.7025
	6000	0.0200	0.0125	0.6284	0.0313	0.0146	0.4656	0.0540	0.0387	0.7165
$\delta = 5$	90	0.0126	0.0123	0.9798	0.0260	0.0156	0.5989	0.0378	0.0158	0.4177
	180	0.0108	0.0094	0.8717	0.0205	0.0121	0.5892	0.0321	0.0145	0.4533
	300	0.0102	0.0084	0.8252	0.0174	0.0098	0.5613	0.0289	0.0128	0.4448
	450	0.0100	0.0078	0.7838	0.0173	0.0093	0.5367	0.0287	0.0159	0.5562
	750	0.0095	0.0071	0.7489	0.0163	0.0090	0.5507	0.0271	0.0152	0.5619
	900	0.0093	0.0069	0.7399	0.0161	0.0085	0.5262	0.0277	0.0162	0.5850
	1200	0.0098	0.0070	0.7158	0.0162	0.0085	0.5260	0.0278	0.0167	0.6027
	1500	0.0093	0.0066	0.7058	0.0161	0.0085	0.5272	0.0274	0.0164	0.5991
	1800	0.0093	0.0066	0.7016	0.0160	0.0083	0.5182	0.0272	0.0164	0.6034
	2100	0.0093	0.0065	0.7002	0.0159	0.0083	0.5221	0.0270	0.0165	0.6103
	6000	0.0094	0.0064	0.6843	0.0158	0.0081	0.5109	0.0277	0.0184	0.6653

Table A.11: Summary of Results for Error Rates of 4 variables and a sample ratio of (1:2:2)

Distributions		CorrNorm			UncorrNorm			Skewed		
	S. Size	G. Mean	SD	CV	G. Mean	SD	CV	G. Mean	SD	CV
$\delta = 1$	150	0.1315	0.0545	0.4148	0.1781	0.0485	0.2723	0.1936	0.1171	0.6046
	300	0.1266	0.0569	0.4495	0.1604	0.0442	0.2753	0.1944	0.1233	0.6340
	500	0.1222	0.0556	0.4549	0.1535	0.0423	0.2756	0.1932	0.1245	0.6445
	750	0.1209	0.0552	0.4564	0.1501	0.0408	0.2717	0.1944	0.1257	0.6467
	1250	0.1191	0.0545	0.4572	0.1467	0.0401	0.2731	0.1969	0.1292	0.6560
	1500	0.1191	0.0544	0.4571	0.1458	0.0397	0.2725	0.1966	0.1287	0.6546
	2000	0.1199	0.0554	0.4619	0.1456	0.0401	0.2755	0.1980	0.1306	0.6595
	2500	0.1192	0.0547	0.4592	0.1444	0.0396	0.2747	0.1982	0.1308	0.6600
	3000	0.1185	0.0547	0.4612	0.1437	0.0392	0.2727	0.1976	0.1307	0.6612
	3500	0.1183	0.0542	0.4579	0.1439	0.0401	0.2783	0.1987	0.1312	0.6602
	10000	0.1184	0.0546	0.4611	0.1430	0.0396	0.2768	0.2000	0.1333	0.6667
$\delta = 2$	150	0.0870	0.0432	0.4958	0.1223	0.0412	0.3369	0.1495	0.0910	0.6084
	300	0.0846	0.0428	0.5055	0.1108	0.0406	0.3660	0.1522	0.0984	0.6467
	500	0.0809	0.0415	0.5132	0.1056	0.0383	0.3628	0.1521	0.0995	0.6543
	750	0.0801	0.0409	0.5105	0.1039	0.0372	0.3578	0.1534	0.1019	0.6644
	1250	0.0796	0.0409	0.5143	0.1019	0.0372	0.3650	0.1564	0.1054	0.6741
	1500	0.0788	0.0406	0.5154	0.1017	0.0370	0.3642	0.1543	0.1045	0.6769
	2000	0.0789	0.0408	0.5173	0.1012	0.0372	0.3672	0.1572	0.1068	0.6792
	2500	0.0788	0.0405	0.5149	0.1008	0.0369	0.3660	0.1567	0.1067	0.6810
	3000	0.0788	0.0407	0.5170	0.1008	0.0372	0.3694	0.1567	0.1069	0.6822
	3500	0.0789	0.0408	0.5169	0.1004	0.0369	0.3678	0.1578	0.1078	0.6831
	10000	0.0785	0.0407	0.5179	0.0999	0.0368	0.3685	0.1597	0.1102	0.6901
$\delta = 3$	150	0.0511	0.0293	0.5743	0.0762	0.0302	0.3971	0.0996	0.0611	0.6142
	300	0.0472	0.0276	0.5851	0.0681	0.0290	0.4257	0.1019	0.0693	0.6800
	500	0.0462	0.0265	0.5748	0.0658	0.0276	0.4188	0.1011	0.0697	0.6896
	750	0.0462	0.0266	0.5749	0.0643	0.0272	0.4230	0.1051	0.0742	0.7061
	1250	0.0455	0.0264	0.5815	0.0628	0.0271	0.4314	0.1066	0.0766	0.7186
	1500	0.0456	0.0265	0.5813	0.0630	0.0267	0.4247	0.1062	0.0762	0.7177
	2000	0.0454	0.0263	0.5798	0.0622	0.0265	0.4262	0.1059	0.0763	0.7209
	2500	0.0453	0.0262	0.5797	0.0624	0.0264	0.4233	0.1056	0.0765	0.7242
	3000	0.0449	0.0262	0.5826	0.0622	0.0267	0.4291	0.1071	0.0779	0.7271
	3500	0.0453	0.0264	0.5823	0.0618	0.0263	0.4255	0.1075	0.0795	0.7395
	10000	0.0450	0.0260	0.5780	0.0616	0.0263	0.4272	0.1080	0.0795	0.7364
$\delta = 4$	150	0.0267	0.0185	0.6951	0.0433	0.0185	0.4279	0.0619	0.0409	0.6618
	300	0.0245	0.0165	0.6733	0.0380	0.0188	0.4955	0.0591	0.0408	0.6909
	500	0.0237	0.0160	0.6745	0.0372	0.0178	0.4786	0.0593	0.0424	0.7153
	750	0.0238	0.0157	0.6588	0.0359	0.0173	0.4817	0.0593	0.0441	0.7444
	1250	0.0236	0.0153	0.6492	0.0352	0.0167	0.4726	0.0611	0.0464	0.7590
	1500	0.0232	0.0150	0.6471	0.0352	0.0170	0.4823	0.0606	0.0468	0.7719
	2000	0.0238	0.0152	0.6391	0.0351	0.0169	0.4827	0.0607	0.0469	0.7724
	2500	0.0200	0.0125	0.6271	0.0315	0.0146	0.4632	0.0534	0.0383	0.7182
	3000	0.0233	0.0149	0.6414	0.0348	0.0167	0.4796	0.0628	0.0510	0.8124
	3500	0.0232	0.0149	0.6419	0.0346	0.0166	0.4794	0.0615	0.0487	0.7911
	10000	0.0232	0.0149	0.6402	0.0346	0.0166	0.4798	0.0622	0.0502	0.8069
$\delta = 5$	150	0.0137	0.0117	0.8511	0.0232	0.0133	0.5715	0.0332	0.0171	0.5139
	300	0.0122	0.0097	0.7962	0.0201	0.0115	0.5730	0.0315	0.0190	0.6039
	500	0.0119	0.0088	0.7383	0.0189	0.0104	0.5509	0.0317	0.0214	0.6740
	750	0.0116	0.0085	0.7298	0.0187	0.0102	0.5443	0.0309	0.0208	0.6745
	1250	0.0112	0.0081	0.7214	0.0182	0.0095	0.5467	0.0313	0.0226	0.7244
	1500	0.0112	0.0079	0.7030	0.0179	0.0097	0.5424	0.0313	0.0234	0.7478
	2000	0.0109	0.0078	0.7102	0.0179	0.0098	0.5433	0.0309	0.0232	0.7531
	2500	0.0112	0.0078	0.6960	0.0179	0.0096	0.5362	0.0317	0.0251	0.7916
	3000	0.0110	0.0077	0.6988	0.0177	0.0095	0.5357	0.0317	0.0254	0.7996
	3500	0.0112	0.0078	0.6951	0.0178	0.0095	0.5365	0.0316	0.0246	0.7774
	10000	0.0110	0.0076	0.6862	0.0177	0.0095	0.5382	0.0315	0.0252	0.7993

Table A.12: Summary of Results for Error Rates of 4 variables and a sample ratio of (1:2:3)

Distributions		CorrNorm			UncorrNorm			Skewed		
	S. Size	G. Mean	SD	CV	G. Mean	SD	CV	G. Mean	SD	CV
	180	0.1266	0.0629	0.4970	0.1654	0.0688	0.4157	0.1985	0.1347	0.6787
	360	0.1223	0.0631	0.5156	0.1514	0.0642	0.4238	0.2020	0.1430	0.7078
	600	0.1215	0.0640	0.5268	0.1457	0.0622	0.4267	0.2031	0.1459	0.7184
	900	0.1191	0.0631	0.5301	0.1422	0.0614	0.4318	0.2052	0.1491	0.7266
	1500	0.1183	0.0631	0.5332	0.1407	0.0605	0.4295	0.2077	0.1524	0.7339
$\delta = 1$	1800	0.1180	0.0633	0.5362	0.1398	0.0600	0.4293	0.2068	0.1521	0.7354
	2400	0.1172	0.0631	0.5381	0.1387	0.0597	0.4308	0.2071	0.1530	0.7390
	3000	0.1175	0.0637	0.5422	0.1385	0.0596	0.4304	0.2074	0.1529	0.7375
	3600	0.1169	0.0632	0.5406	0.1380	0.0599	0.4343	0.2080	0.1539	0.7402
	4200	0.1170	0.0634	0.5424	0.1381	0.0597	0.4324	0.2081	0.1541	0.7406
	12000	0.1163	0.0630	0.5420	0.1370	0.0594	0.4337	0.2102	0.1569	0.7463
	180	0.0874	0.0472	0.5395	0.1148	0.0471	0.4106	0.1554	0.1100	0.7077
	360	0.0821	0.0452	0.5509	0.1048	0.0455	0.4338	0.1586	0.1183	0.7457
	600	0.0811	0.0450	0.5545	0.1020	0.0448	0.4396	0.1604	0.1211	0.7550
	900	0.0794	0.0444	0.5594	0.1002	0.0448	0.4473	0.1633	0.1249	0.7649
	1500	0.0789	0.0435	0.5514	0.0983	0.0433	0.4402	0.1654	0.1281	0.7747
$\delta = 2$	1800	0.0789	0.0437	0.5534	0.0983	0.0440	0.4480	0.1646	0.1283	0.7799
	2400	0.0788	0.0437	0.5541	0.0979	0.0441	0.4501	0.1661	0.1298	0.7817
	3000	0.0785	0.0437	0.5568	0.0975	0.0438	0.4497	0.1679	0.1318	0.7847
	3600	0.0783	0.0437	0.5579	0.0972	0.0437	0.4502	0.1664	0.1305	0.7842
	4200	0.0784	0.0437	0.5577	0.0970	0.0436	0.4492	0.1681	0.1321	0.7859
	12000	0.0786	0.0439	0.5587	0.0968	0.0435	0.4495	0.1699	0.1346	0.7920
	180	0.0500	0.0300	0.6002	0.0720	0.0328	0.4553	0.1093	0.0836	0.7645
	360	0.0479	0.0291	0.6082	0.0658	0.0311	0.4720	0.1077	0.0866	0.8047
	600	0.0464	0.0282	0.6070	0.0630	0.0304	0.4818	0.1114	0.0913	0.8197
	900	0.0461	0.0280	0.6076	0.0627	0.0302	0.4812	0.1103	0.0915	0.8300
	1500	0.0461	0.0276	0.5992	0.0611	0.0295	0.4825	0.1141	0.0966	0.8470
$\delta = 3$	1800	0.0460	0.0277	0.6016	0.0613	0.0294	0.4796	0.1137	0.0963	0.8464
	2400	0.0460	0.0277	0.6034	0.0608	0.0292	0.4799	0.1150	0.0982	0.8534
	3000	0.0452	0.0270	0.5974	0.0605	0.0290	0.4782	0.1162	0.1001	0.8618
	3600	0.0457	0.0275	0.6023	0.0605	0.0292	0.4824	0.1162	0.0999	0.8593
	4200	0.0456	0.0275	0.6029	0.0608	0.0294	0.4840	0.1171	0.1007	0.8594
	12000	0.0456	0.0275	0.6030	0.0604	0.0291	0.4826	0.1179	0.1023	0.8678
	180	0.0255	0.0180	0.7063	0.0410	0.0204	0.4973	0.0653	0.0498	0.7620
	360	0.0248	0.0168	0.6763	0.0378	0.0195	0.5177	0.0622	0.0484	0.7770
	600	0.0245	0.0165	0.6745	0.0362	0.0188	0.5200	0.0650	0.0565	0.8696
	900	0.0243	0.0159	0.6538	0.0353	0.0183	0.5177	0.0654	0.0577	0.8828
	1500	0.0243	0.0158	0.6506	0.0347	0.0179	0.5159	0.0678	0.0615	0.9078
$\delta = 4$	1800	0.0241	0.0158	0.6546	0.0345	0.0178	0.5164	0.0676	0.0635	0.9395
	2400	0.0237	0.0155	0.6530	0.0344	0.0179	0.5201	0.0662	0.0601	0.9080
	3000	0.0240	0.0156	0.6488	0.0345	0.0178	0.5179	0.0681	0.0634	0.9304
	3600	0.0240	0.0156	0.6515	0.0344	0.0180	0.5230	0.0672	0.0620	0.9225
	4200	0.0238	0.0155	0.6525	0.0341	0.0179	0.5237	0.0692	0.0652	0.9433
	12000	0.0238	0.0154	0.6470	0.0341	0.0178	0.5226	0.0703	0.0665	0.9464
	180	0.0131	0.0110	0.8433	0.0220	0.0132	0.5985	0.0358	0.0228	0.6360
	360	0.0123	0.0096	0.7791	0.0199	0.0115	0.5775	0.0328	0.0234	0.7149
	600	0.0116	0.0086	0.7392	0.0187	0.0108	0.5761	0.0350	0.0301	0.8596
	900	0.0119	0.0086	0.7212	0.0185	0.0107	0.5799	0.0339	0.0302	0.8917
	1500	0.0116	0.0082	0.7031	0.0180	0.0103	0.5752	0.0338	0.0299	0.8841
$\delta = 5$	1800	0.0116	0.0082	0.7045	0.0183	0.0106	0.5793	0.0346	0.0324	0.9356
	2400	0.0112	0.0079	0.7012	0.0179	0.0102	0.5683	0.0353	0.0353	0.9998
	3000	0.0116	0.0081	0.6959	0.0179	0.0103	0.5736	0.0330	0.0289	0.8768
	3600	0.0115	0.0080	0.6940	0.0179	0.0101	0.5650	0.0342	0.0310	0.9051
	4200	0.0115	0.0080	0.6965	0.0178	0.0101	0.5672	0.0337	0.0307	0.9118
	12000	0.0115	0.0079	0.6867	0.0178	0.0100	0.5644	0.0346	0.0323	0.9334

Table A.13: Summary of Results for Error Rates of 6 variables and a sample ratio of (1:1:1)

Distributions		CorrNorm			UncorrNorm			Skewed		
	S. Size	G. Mean	SD	CV	G. Mean	SD	CV	G. Mean	SD	CV
$\delta = 1$	90	0.1152	0.0463	0.4022	0.2320	0.0665	0.2867	0.1753	0.0790	0.4508
	180	0.1068	0.0490	0.4585	0.1811	0.0650	0.3586	0.1682	0.0900	0.5350
	300	0.0991	0.0482	0.4868	0.1581	0.0607	0.3841	0.1644	0.0910	0.5535
	450	0.0971	0.0480	0.4944	0.1464	0.0581	0.3970	0.1646	0.0940	0.5712
	750	0.0926	0.0465	0.5016	0.1374	0.0556	0.4046	0.1643	0.0961	0.5848
	900	0.0925	0.0469	0.5073	0.1351	0.0538	0.3984	0.1639	0.0973	0.5937
	1200	0.0908	0.0467	0.5139	0.1317	0.0528	0.4012	0.1650	0.0993	0.6053
	1500	0.0905	0.0462	0.5099	0.1306	0.0533	0.4078	0.1655	0.1004	0.6066
	1800	0.0900	0.0462	0.5128	0.1302	0.0527	0.4050	0.1652	0.1001	0.6057
	2100	0.0899	0.0459	0.5105	0.1287	0.0519	0.4031	0.1658	0.1005	0.6065
	6000	0.0891	0.0459	0.5158	0.1262	0.0512	0.4055	0.1667	0.1032	0.6190
$\delta = 2$	90	0.0806	0.0358	0.4450	0.1642	0.0541	0.3296	0.1385	0.0609	0.4397
	180	0.0717	0.0364	0.5074	0.1243	0.0482	0.3876	0.1347	0.0710	0.5273
	300	0.0656	0.0355	0.5416	0.1083	0.0438	0.4047	0.1306	0.0721	0.5520
	450	0.0642	0.0352	0.5492	0.1016	0.0420	0.4138	0.1308	0.0757	0.5786
	750	0.0616	0.0333	0.5408	0.0952	0.0391	0.4109	0.1321	0.0783	0.5929
	900	0.0607	0.0335	0.5513	0.0939	0.0386	0.4110	0.1313	0.0786	0.5985
	1200	0.0609	0.0334	0.5488	0.0921	0.0387	0.4199	0.1332	0.0812	0.6099
	1500	0.0604	0.0330	0.5469	0.0911	0.0383	0.4202	0.1325	0.0809	0.6108
	1800	0.0598	0.0330	0.5520	0.0904	0.0377	0.4170	0.1323	0.0809	0.6114
	2100	0.0593	0.0326	0.5505	0.0899	0.0372	0.4136	0.1336	0.0826	0.6185
	6000	0.0589	0.0324	0.5502	0.0881	0.0369	0.4191	0.1344	0.0845	0.6287
$\delta = 3$	90	0.0504	0.0290	0.5753	0.1090	0.0410	0.3762	0.1034	0.0415	0.4017
	180	0.0407	0.0239	0.5878	0.0762	0.0321	0.4212	0.0944	0.0487	0.5156
	300	0.0364	0.0219	0.6019	0.0659	0.0279	0.4230	0.0898	0.0491	0.5463
	450	0.0361	0.0220	0.6095	0.0614	0.0262	0.4259	0.0896	0.0513	0.5727
	750	0.0352	0.0216	0.6124	0.0585	0.0256	0.4383	0.0907	0.0545	0.6009
	900	0.0349	0.0208	0.5957	0.0580	0.0251	0.4325	0.0924	0.0567	0.6133
	1200	0.0344	0.0208	0.6034	0.0569	0.0249	0.4372	0.0936	0.0587	0.6269
	1500	0.0341	0.0205	0.6027	0.0561	0.0243	0.4330	0.0934	0.0589	0.6308
	1800	0.0340	0.0204	0.6017	0.0557	0.0243	0.4367	0.0925	0.0591	0.6388
	2100	0.0341	0.0204	0.5991	0.0553	0.0242	0.4379	0.0938	0.0605	0.6448
	6000	0.0336	0.0202	0.6017	0.0543	0.0239	0.4404	0.0938	0.0614	0.6550
$\delta = 4$	90	0.0207	0.0276	0.7506	0.0668	0.0294	0.4404	0.0712	0.0269	0.3777
	180	0.0214	0.0154	0.7193	0.0431	0.0205	0.4743	0.0590	0.0249	0.4212
	300	0.0192	0.0141	0.7321	0.0371	0.0170	0.4571	0.0567	0.0284	0.5000
	450	0.0189	0.0132	0.6980	0.0344	0.0162	0.4711	0.0541	0.0278	0.5150
	750	0.0187	0.0127	0.6789	0.0327	0.0154	0.4722	0.0555	0.0328	0.5910
	900	0.0175	0.0118	0.6766	0.0319	0.0150	0.4696	0.0543	0.0316	0.5827
	1200	0.0177	0.0119	0.6708	0.0315	0.0149	0.4739	0.0556	0.0352	0.6335
	1500	0.0175	0.0118	0.6736	0.0315	0.0147	0.4667	0.0543	0.0333	0.6123
	1800	0.0176	0.0118	0.6664	0.0308	0.0147	0.4775	0.0558	0.0363	0.6509
	2100	0.0173	0.0115	0.6659	0.0306	0.0143	0.4667	0.0550	0.0348	0.6322
	6000	0.0172	0.0113	0.6546	0.0301	0.0141	0.4675	0.0565	0.0384	0.6797
$\delta = 5$	90	0.0151	0.0150	0.9928	0.0374	0.0200	0.5345	0.0472	0.0177	0.3757
	180	0.0106	0.0096	0.8996	0.0235	0.0125	0.5305	0.0364	0.0111	0.3050
	300	0.0090	0.0076	0.8421	0.0191	0.0107	0.5632	0.0342	0.0138	0.4039
	450	0.0090	0.0072	0.7999	0.0176	0.0095	0.5404	0.0314	0.0130	0.4131
	750	0.0084	0.0065	0.7699	0.0165	0.0088	0.5347	0.0311	0.0157	0.5030
	900	0.0083	0.0063	0.7539	0.0165	0.0088	0.5330	0.0304	0.0144	0.4757
	1200	0.0081	0.0060	0.7423	0.0161	0.0085	0.5253	0.0307	0.0170	0.5536
	1500	0.0083	0.0061	0.7380	0.0155	0.0081	0.5226	0.0310	0.0185	0.5959
	1800	0.0084	0.0061	0.7295	0.0158	0.0083	0.5226	0.0299	0.0155	0.5193
	2100	0.0080	0.0058	0.7208	0.0155	0.0082	0.5275	0.0300	0.0173	0.5766
	6000	0.0082	0.0058	0.7038	0.0153	0.0079	0.5145	0.0300	0.0173	0.5766

Table A.14: Summary of Results for Error Rates of 6 variables and a sample ratio of (1:2:2)

Distributions		CorrNorm			UncorrNorm			Skewed		
	S. Size	G. Mean	SD	CV	G. Mean	SD	CV	G. Mean	SD	CV
$\delta = 1$	150	0.1222	0.0509	0.4166	0.2003	0.0584	0.2915	0.1899	0.1073	0.5651
	300	0.1107	0.0513	0.4635	0.1645	0.0477	0.2902	0.1863	0.1131	0.6072
	500	0.1070	0.0521	0.4872	0.1510	0.0445	0.2945	0.1900	0.1195	0.6287
	750	0.1039	0.0512	0.4925	0.1454	0.0432	0.2973	0.1896	0.1205	0.6356
	1250	0.1019	0.0516	0.5067	0.1395	0.0424	0.3042	0.1900	0.1229	0.6470
	1500	0.1021	0.0515	0.5043	0.1388	0.0422	0.3039	0.1920	0.1245	0.6483
	2000	0.1014	0.0520	0.5124	0.1372	0.0422	0.3073	0.1915	0.1252	0.6534
	2500	0.1009	0.0518	0.5136	0.1361	0.0418	0.3075	0.1920	0.1258	0.6553
	3000	0.1005	0.0513	0.5103	0.1357	0.0416	0.3068	0.1924	0.1261	0.6554
	3500	0.1007	0.0517	0.5130	0.1350	0.0414	0.3066	0.1933	0.1274	0.6591
	10000	0.0998	0.0514	0.5149	0.1332	0.0411	0.3085	0.1939	0.1286	0.6633
$\delta = 2$	150	0.0860	0.0389	0.4526	0.1401	0.0436	0.3109	0.1493	0.0848	0.5679
	300	0.0751	0.0390	0.5194	0.1167	0.0398	0.3411	0.1484	0.0919	0.6190
	500	0.0702	0.0370	0.5270	0.1054	0.0368	0.3492	0.1502	0.0964	0.6418
	750	0.0700	0.0377	0.5383	0.1022	0.0371	0.3628	0.1508	0.0982	0.6515
	1250	0.0690	0.0381	0.5524	0.0993	0.0367	0.3699	0.1528	0.1014	0.6638
	1500	0.0685	0.0378	0.5520	0.0981	0.0367	0.3743	0.1526	0.1012	0.6632
	2000	0.0685	0.0378	0.5518	0.0967	0.0362	0.3744	0.1547	0.1034	0.6686
	2500	0.0680	0.0379	0.5568	0.0964	0.0360	0.3740	0.1542	0.1038	0.6734
	3000	0.0685	0.0383	0.5587	0.0960	0.0362	0.3767	0.1553	0.1049	0.6751
	3500	0.0676	0.0377	0.5581	0.0956	0.0362	0.3785	0.1560	0.1052	0.6742
	10000	0.0675	0.0379	0.5612	0.0948	0.0359	0.3785	0.1563	0.1065	0.6815
$\delta = 3$	150	0.0511	0.0281	0.5503	0.0892	0.0306	0.3423	0.1052	0.0605	0.5753
	300	0.0429	0.0248	0.5773	0.0721	0.0277	0.3833	0.1028	0.0657	0.6387
	500	0.0413	0.0249	0.6039	0.0661	0.0268	0.4057	0.1029	0.0684	0.6655
	750	0.0401	0.0243	0.6069	0.0637	0.0262	0.4116	0.1060	0.0729	0.6876
	1250	0.0401	0.0247	0.6161	0.0611	0.0258	0.4224	0.1039	0.0723	0.6958
	1500	0.0395	0.0240	0.6089	0.0610	0.0262	0.4292	0.1056	0.0735	0.6957
	2000	0.0394	0.0244	0.6194	0.0607	0.0260	0.4280	0.1079	0.0770	0.7139
	2500	0.0395	0.0242	0.6129	0.0601	0.0258	0.4296	0.1062	0.0751	0.7067
	3000	0.0395	0.0241	0.6110	0.0600	0.0261	0.4352	0.1065	0.0758	0.7115
	3500	0.0397	0.0245	0.6166	0.0597	0.0259	0.4330	0.1079	0.0769	0.7132
	10000	0.0388	0.0239	0.6168	0.0589	0.0256	0.4343	0.1097	0.0800	0.7293
$\delta = 4$	150	0.0275	0.0173	0.6315	0.0516	0.0206	0.3987	0.0663	0.0353	0.5333
	300	0.0228	0.0157	0.6890	0.0417	0.0185	0.4423	0.0627	0.0372	0.5937
	500	0.0214	0.0149	0.6989	0.0375	0.0177	0.4708	0.0611	0.0401	0.6560
	750	0.0210	0.0140	0.6673	0.0364	0.0174	0.4765	0.0627	0.0438	0.6984
	1250	0.0204	0.0138	0.6768	0.0346	0.0163	0.4697	0.0627	0.0460	0.7330
	1500	0.0206	0.0139	0.6735	0.0345	0.0165	0.4778	0.0635	0.0464	0.7305
	2000	0.0202	0.0135	0.6701	0.0339	0.0163	0.4798	0.0632	0.0468	0.7402
	2500	0.0203	0.0136	0.6703	0.0337	0.0164	0.4875	0.0620	0.0454	0.7325
	3000	0.0201	0.0136	0.6773	0.0337	0.0162	0.4814	0.0643	0.0496	0.7717
	3500	0.0205	0.0138	0.6743	0.0335	0.0163	0.4871	0.0635	0.0477	0.7511
	10000	0.0202	0.0135	0.6659	0.0333	0.0162	0.4859	0.0648	0.0502	0.7748
$\delta = 5$	150	0.0127	0.0107	0.8427	0.0285	0.0136	0.4761	0.0417	0.0193	0.4638
	300	0.0108	0.0086	0.7990	0.0215	0.0117	0.5452	0.0362	0.0177	0.4899
	500	0.0101	0.0078	0.7684	0.0193	0.0103	0.5361	0.0346	0.0200	0.5770
	750	0.0101	0.0077	0.7614	0.0189	0.0103	0.5457	0.0337	0.0205	0.6091
	1250	0.0099	0.0073	0.7355	0.0181	0.0098	0.5421	0.0336	0.0215	0.6401
	1500	0.0096	0.0070	0.7326	0.0176	0.0095	0.5359	0.0338	0.0228	0.6735
	2000	0.0096	0.0069	0.7218	0.0177	0.0095	0.5390	0.0336	0.0235	0.6997
	2500	0.0096	0.0069	0.7238	0.0174	0.0095	0.5435	0.0336	0.0238	0.7085
	3000	0.0096	0.0069	0.7157	0.0174	0.0095	0.5451	0.0342	0.0249	0.7304
	3500	0.0096	0.0068	0.7114	0.0172	0.0094	0.5451	0.0333	0.0241	0.7246
	10000	0.0097	0.0068	0.7060	0.0173	0.0093	0.5368	0.0338	0.0254	0.7527

Table A.15: Summary of Results for Error Rates of 6 variables and a sample ratio of (1:2:3)

Distributions		CorrNorm			UncorrNorm			Skewed		
	S. Size	G. Mean	SD	CV	G. Mean	SD	CV	G. Mean	SD	CV
$\delta = 1$	180	0.1175	0.0553	0.4705	0.1755	0.0686	0.3912	0.1929	0.1235	0.6405
	360	0.1077	0.0572	0.5307	0.1522	0.0644	0.4234	0.1942	0.1339	0.6894
	600	0.1039	0.0558	0.5367	0.1419	0.0612	0.4317	0.1949	0.1378	0.7068
	900	0.1033	0.0569	0.5506	0.1378	0.0597	0.4335	0.1977	0.1435	0.7257
	1500	0.1018	0.0570	0.5598	0.1336	0.0585	0.4381	0.2000	0.1455	0.7276
	1800	0.1012	0.0565	0.5586	0.1322	0.0586	0.4433	0.2004	0.1466	0.7316
	2400	0.1003	0.0567	0.5651	0.1312	0.0579	0.4413	0.2006	0.1472	0.7339
	3000	0.1004	0.0563	0.5607	0.1304	0.0576	0.4415	0.2014	0.1487	0.7381
	3600	0.0999	0.0564	0.5639	0.1295	0.0572	0.4413	0.2020	0.1494	0.7396
	4200	0.0997	0.0562	0.5641	0.1293	0.0574	0.4441	0.2027	0.1502	0.7410
	12000	0.0994	0.0563	0.5668	0.1279	0.0570	0.4453	0.2041	0.1524	0.7468
$\delta = 2$	180	0.0825	0.0412	0.4992	0.1290	0.0499	0.3866	0.1550	0.1024	0.6608
	360	0.0740	0.0405	0.5480	0.1090	0.0460	0.4218	0.1558	0.1121	0.7195
	600	0.0712	0.0408	0.5725	0.1019	0.0444	0.4358	0.1583	0.1177	0.7438
	900	0.0707	0.0407	0.5751	0.0981	0.0434	0.4425	0.1599	0.1215	0.7598
	1500	0.0695	0.0403	0.5798	0.0954	0.0426	0.4462	0.1627	0.1257	0.7726
	1800	0.0687	0.0399	0.5806	0.0944	0.0425	0.4500	0.1634	0.1261	0.7715
	2400	0.0686	0.0399	0.5823	0.0938	0.0425	0.4534	0.1647	0.1277	0.7756
	3000	0.0685	0.0401	0.5848	0.0933	0.0424	0.4547	0.1633	0.1266	0.7756
	3600	0.0684	0.0401	0.5860	0.0930	0.0422	0.4531	0.1671	0.1302	0.7793
	4200	0.0681	0.0398	0.5838	0.0924	0.0419	0.4540	0.1651	0.1291	0.7819
	12000	0.0681	0.0401	0.5885	0.0919	0.0420	0.4574	0.1679	0.1325	0.7890
$\delta = 3$	180	0.0487	0.0286	0.5874	0.0831	0.0343	0.4129	0.1097	0.0767	0.6989
	360	0.0432	0.0257	0.5947	0.0689	0.0299	0.4333	0.1099	0.0841	0.7656
	600	0.0417	0.0257	0.6154	0.0642	0.0292	0.4558	0.1108	0.0878	0.7930
	900	0.0414	0.0261	0.6303	0.0613	0.0289	0.4717	0.1099	0.0881	0.8020
	1500	0.0404	0.0251	0.6205	0.0599	0.0288	0.4811	0.1155	0.0954	0.8258
	1800	0.0408	0.0255	0.6264	0.0596	0.0287	0.4808	0.1138	0.0941	0.8275
	2400	0.0403	0.0253	0.6277	0.0591	0.0285	0.4818	0.1154	0.0959	0.8312
	3000	0.0401	0.0251	0.6246	0.0588	0.0285	0.4843	0.1145	0.0947	0.8272
	3600	0.0401	0.0253	0.6310	0.0586	0.0284	0.4839	0.1165	0.0984	0.8444
	4200	0.0399	0.0250	0.6260	0.0583	0.0283	0.4863	0.1163	0.0975	0.8384
	12000	0.0396	0.0248	0.6270	0.0579	0.0283	0.4885	0.1176	0.1002	0.8520
$\delta = 4$	180	0.0263	0.0178	0.6770	0.0479	0.0204	0.4265	0.0681	0.0432	0.6335
	360	0.0233	0.0160	0.6893	0.0407	0.0200	0.4919	0.0662	0.0479	0.7235
	600	0.0227	0.0153	0.6761	0.0369	0.0188	0.5091	0.0662	0.0536	0.8096
	900	0.0219	0.0149	0.6791	0.0355	0.0182	0.5128	0.0678	0.0571	0.8428
	1500	0.0210	0.0142	0.6737	0.0343	0.0179	0.5211	0.0686	0.0592	0.8635
	1800	0.0213	0.0143	0.6713	0.0340	0.0178	0.5227	0.0679	0.0586	0.8625
	2400	0.0210	0.0141	0.6703	0.0336	0.0176	0.5216	0.0694	0.0625	0.9011
	3000	0.0209	0.0140	0.6714	0.0333	0.0175	0.5258	0.0701	0.0634	0.9044
	3600	0.0208	0.0140	0.6702	0.0334	0.0174	0.5219	0.0690	0.0612	0.8877
	4200	0.0209	0.0140	0.6694	0.0333	0.0175	0.5242	0.0697	0.0631	0.9047
	12000	0.0208	0.0139	0.6694	0.0330	0.0174	0.5260	0.0712	0.0651	0.9147
$\delta = 5$	180	0.0139	0.0111	0.7967	0.0263	0.0129	0.4887	0.0396	0.0200	0.5050
	360	0.0116	0.0090	0.7786	0.0208	0.0116	0.5569	0.0373	0.0234	0.6294
	600	0.0102	0.0080	0.7825	0.0192	0.0109	0.5684	0.0355	0.0253	0.7127
	900	0.0101	0.0075	0.7436	0.0182	0.0104	0.5697	0.0362	0.0281	0.7753
	1500	0.0103	0.0074	0.7225	0.0182	0.0105	0.5768	0.0363	0.0298	0.8227
	1800	0.0101	0.0073	0.7241	0.0176	0.0100	0.5680	0.0355	0.0290	0.8178
	2400	0.0101	0.0072	0.7140	0.0176	0.0099	0.5621	0.0361	0.0299	0.8290
	3000	0.0100	0.0071	0.7150	0.0174	0.0099	0.5671	0.0368	0.0320	0.8686
	3600	0.0099	0.0070	0.7113	0.0174	0.0099	0.5680	0.0373	0.0327	0.8759
	4200	0.0099	0.0070	0.7107	0.0172	0.0098	0.5719	0.0362	0.0308	0.8519
	12000	0.0100	0.0070	0.7014	0.0171	0.0098	0.5721	0.0375	0.0337	0.8991

Table A.16: Summary of Results for Error Rates of 8 variables and a sample ratio of (1:1:1)

Distributions		CorrNorm			UncorrNorm			Skewed		
	S. Size	G. Mean	SD	CV	G. Mean	SD	CV	G. Mean	SD	CV
	90	0.1210	0.0428	0.3540	0.2726	0.0619	0.2269	0.1800	0.0750	0.4169
	180	0.0971	0.0446	0.4593	0.2045	0.0661	0.3234	0.1631	0.0810	0.4967
	300	0.0896	0.0434	0.4851	0.1673	0.0606	0.3624	0.1598	0.0864	0.5403
	450	0.0862	0.0441	0.5122	0.1483	0.0570	0.3845	0.1595	0.0892	0.5590
	750	0.0803	0.0419	0.5220	0.1352	0.0529	0.3911	0.1596	0.0918	0.5749
$\delta = 1$	900	0.0809	0.0429	0.5308	0.1318	0.0522	0.3963	0.1603	0.0936	0.5835
	1200	0.0795	0.0423	0.5315	0.1276	0.0511	0.4006	0.1597	0.0938	0.5871
	1500	0.0790	0.0425	0.5382	0.1253	0.0506	0.4034	0.1599	0.0950	0.5940
	1800	0.0780	0.0421	0.5397	0.1239	0.0501	0.4047	0.1603	0.0957	0.5969
	2100	0.0780	0.0419	0.5372	0.1226	0.0497	0.4051	0.1613	0.0971	0.6020
	6000	0.0765	0.0416	0.5441	0.1181	0.0479	0.4054	0.1622	0.0995	0.6133
	90	0.0894	0.0382	0.4273	0.2106	0.0520	0.2467	0.1472	0.0574	0.3898
	180	0.0687	0.0358	0.5210	0.1441	0.0527	0.3658	0.1330	0.0661	0.4972
	300	0.0618	0.0332	0.5378	0.1164	0.0447	0.3843	0.1292	0.0691	0.5350
	450	0.0587	0.0322	0.5478	0.1041	0.0416	0.3992	0.1293	0.0713	0.5515
	750	0.0552	0.0313	0.5668	0.0942	0.0389	0.4128	0.1300	0.0751	0.5778
$\delta = 2$	900	0.0545	0.0306	0.5622	0.0921	0.0380	0.4127	0.1298	0.0752	0.5798
	1200	0.0534	0.0304	0.5698	0.0898	0.0372	0.4138	0.1313	0.0780	0.5937
	1500	0.0540	0.0307	0.5683	0.0885	0.0367	0.4140	0.1305	0.0782	0.5990
	1800	0.0531	0.0306	0.5774	0.0876	0.0366	0.4175	0.1314	0.0794	0.6040
	2100	0.0530	0.0304	0.5737	0.0862	0.0359	0.4167	0.1319	0.0808	0.6122
	6000	0.0520	0.0300	0.5777	0.0840	0.0352	0.4186	0.1333	0.0833	0.6247
	90	0.0566	0.0306	0.5399	0.1472	0.0458	0.3109	0.1109	0.0396	0.3570
	180	0.0388	0.0245	0.6303	0.0907	0.0353	0.3893	0.0955	0.0427	0.4465
	300	0.0353	0.0221	0.6259	0.0714	0.0304	0.4260	0.0919	0.0459	0.4993
	450	0.0335	0.0208	0.6191	0.0641	0.0272	0.4248	0.0925	0.0507	0.5485
	750	0.0318	0.0197	0.6218	0.0594	0.0252	0.4245	0.0913	0.0530	0.5798
	900	0.0315	0.0197	0.6261	0.0570	0.0244	0.4279	0.0918	0.0544	0.5926
$\delta = 3$	1200	0.0312	0.0194	0.6211	0.0557	0.0243	0.4360	0.0915	0.0547	0.5977
	1500	0.0307	0.0191	0.6215	0.0550	0.0242	0.4401	0.0937	0.0572	0.6106
	1800	0.0303	0.0189	0.6223	0.0540	0.0235	0.4357	0.0929	0.0576	0.6205
	2100	0.0305	0.0189	0.6216	0.0539	0.0236	0.4375	0.0931	0.0581	0.6239
	6000	0.0302	0.0189	0.6271	0.0520	0.0231	0.4443	0.0939	0.0607	0.6469
	90	0.0319	0.0234	0.7332	0.0937	0.0343	0.3658	0.0798	0.0259	0.3246
	180	0.0208	0.0150	0.7233	0.0522	0.0236	0.4515	0.0644	0.0266	0.4135
	300	0.0186	0.0133	0.7180	0.0424	0.0189	0.4675	0.0588	0.0253	0.4309
	450	0.0179	0.0125	0.6998	0.0365	0.0168	0.4608	0.0581	0.0298	0.5132
	750	0.0160	0.0111	0.6914	0.0330	0.0156	0.4730	0.0570	0.0318	0.5569
	900	0.0165	0.0115	0.7013	0.0319	0.0150	0.4695	0.0561	0.0312	0.5571
$\delta = 4$	1200	0.0161	0.0110	0.6850	0.0312	0.0146	0.4688	0.0583	0.0366	0.6280
	1500	0.0157	0.0107	0.6840	0.0304	0.0142	0.4659	0.0567	0.0339	0.5987
	1800	0.0155	0.0105	0.6777	0.0301	0.0143	0.4737	0.0568	0.0346	0.6080
	2100	0.0160	0.0109	0.6811	0.0303	0.0142	0.4684	0.0573	0.0362	0.6311
	6000	0.0154	0.0104	0.6743	0.0291	0.0137	0.4715	0.0571	0.0371	0.6501
	90	0.0185	0.0174	0.9418	0.0571	0.0254	0.4441	0.0575	0.0223	0.3884
	180	0.0105	0.0098	0.9369	0.0284	0.0149	0.5262	0.0411	0.0130	0.3153
	300	0.0092	0.0080	0.8672	0.0215	0.0114	0.5304	0.0371	0.0120	0.3246
	450	0.0087	0.0071	0.8201	0.0189	0.0101	0.5356	0.0353	0.0130	0.3670
	750	0.0077	0.0059	0.7595	0.0168	0.0091	0.5413	0.0339	0.0149	0.4385
	900	0.0077	0.0060	0.7813	0.0168	0.0088	0.5253	0.0325	0.0141	0.4329
$\delta = 5$	1200	0.0073	0.0055	0.7549	0.0160	0.0083	0.5199	0.0323	0.0151	0.4672
	1500	0.0075	0.0056	0.7423	0.0157	0.0084	0.5309	0.0318	0.0149	0.4685
	1800	0.0074	0.0054	0.7344	0.0155	0.0080	0.5163	0.0319	0.0158	0.4952
	2100	0.0075	0.0055	0.7332	0.0153	0.0080	0.5248	0.0320	0.0163	0.5082
	6000	0.0073	0.0052	0.7139	0.0148	0.0077	0.5201	0.0316	0.0174	0.5511

Table A.17: Summary of Results for Error Rates of 8 variables and a sample ratio of (1:2:2)

Distributions		CorrNorm			UncorrNorm			Skewed		
	S. Size	G. Mean	SD	CV	G. Mean	SD	CV	G. Mean	SD	CV
$\delta = 1$	150	0.1157	0.0443	0.3832	0.2252	0.0658	0.2921	0.1869	0.0996	0.5331
	300	0.1024	0.0482	0.4710	0.1729	0.0516	0.2984	0.1831	0.1089	0.5948
	500	0.0959	0.0478	0.4986	0.1524	0.0469	0.3075	0.1819	0.1128	0.6201
	750	0.0921	0.0482	0.5230	0.1421	0.0440	0.3094	0.1834	0.1158	0.6316
	1250	0.0896	0.0483	0.5392	0.1343	0.0431	0.3211	0.1845	0.1182	0.6408
	1500	0.0894	0.0482	0.5386	0.1332	0.0429	0.3221	0.1864	0.1205	0.6466
	2000	0.0886	0.0484	0.5460	0.1302	0.0422	0.3244	0.1859	0.1208	0.6501
	2500	0.0882	0.0486	0.5513	0.1290	0.0422	0.3272	0.1860	0.1213	0.6523
	3000	0.0877	0.0484	0.5516	0.1278	0.0419	0.3274	0.1874	0.1226	0.6545
	3500	0.0877	0.0482	0.5494	0.1271	0.0417	0.3278	0.1872	0.1228	0.6560
	10000	0.0867	0.0483	0.5571	0.1250	0.0416	0.3327	0.1890	0.1253	0.6628
$\delta = 2$	150	0.0843	0.0358	0.4246	0.1635	0.0488	0.2982	0.1512	0.0809	0.5349
	300	0.0698	0.0356	0.5094	0.1244	0.0404	0.3249	0.1467	0.0875	0.5965
	500	0.0662	0.0359	0.5429	0.1093	0.0373	0.3410	0.1492	0.0934	0.6258
	750	0.0633	0.0355	0.5609	0.1018	0.0365	0.3582	0.1499	0.0961	0.6414
	1250	0.0619	0.0355	0.5738	0.0969	0.0363	0.3742	0.1502	0.0986	0.6563
	1500	0.0616	0.0357	0.5804	0.0953	0.0358	0.3754	0.1511	0.0991	0.6560
	2000	0.0607	0.0354	0.5832	0.0933	0.0354	0.3794	0.1529	0.1019	0.6665
	2500	0.0603	0.0356	0.5903	0.0927	0.0354	0.3819	0.1519	0.1013	0.6669
	3000	0.0604	0.0357	0.5904	0.0920	0.0351	0.3819	0.1537	0.1033	0.6721
	3500	0.0601	0.0355	0.5901	0.0916	0.0352	0.3841	0.1529	0.1024	0.6696
	10000	0.0598	0.0354	0.5930	0.0901	0.0351	0.3891	0.1546	0.1048	0.6780
$\delta = 3$	150	0.0505	0.0253	0.5018	0.1095	0.0323	0.2945	0.1099	0.0570	0.5186
	300	0.0418	0.0241	0.5756	0.0774	0.0282	0.3641	0.1058	0.0646	0.6106
	500	0.0390	0.0232	0.5944	0.0689	0.0274	0.3982	0.1048	0.0673	0.6421
	750	0.0368	0.0229	0.6224	0.0639	0.0265	0.4157	0.1038	0.0681	0.6563
	1250	0.0356	0.0223	0.6273	0.0606	0.0256	0.4216	0.1057	0.0723	0.6841
	1500	0.0359	0.0227	0.6333	0.0597	0.0254	0.4258	0.1058	0.0723	0.6833
	2000	0.0356	0.0227	0.6375	0.0587	0.0254	0.4321	0.1071	0.0747	0.6977
	2500	0.0355	0.0225	0.6356	0.0584	0.0254	0.4345	0.1068	0.0743	0.6951
	3000	0.0353	0.0225	0.6380	0.0577	0.0250	0.4333	0.1065	0.0746	0.7005
	3500	0.0350	0.0223	0.6368	0.0575	0.0252	0.4377	0.1082	0.0760	0.7023
	10000	0.0348	0.0224	0.6418	0.0567	0.0250	0.4408	0.1097	0.0789	0.7189
$\delta = 4$	150	0.0273	0.0165	0.6056	0.0660	0.0213	0.3231	0.0723	0.0335	0.4632
	300	0.0210	0.0143	0.6826	0.0445	0.0181	0.4079	0.0667	0.0391	0.5864
	500	0.0210	0.0141	0.6728	0.0392	0.0172	0.4398	0.0639	0.0395	0.6181
	750	0.0192	0.0133	0.6919	0.0364	0.0168	0.4611	0.0632	0.0422	0.6678
	1250	0.0195	0.0134	0.6881	0.0347	0.0165	0.4753	0.0641	0.0446	0.6953
	1500	0.0189	0.0130	0.6892	0.0341	0.0164	0.4822	0.0630	0.0433	0.6870
	2000	0.0187	0.0128	0.6864	0.0336	0.0165	0.4904	0.0637	0.0444	0.6972
	2500	0.0185	0.0127	0.6880	0.0332	0.0161	0.4830	0.0646	0.0468	0.7238
	3000	0.0184	0.0126	0.6847	0.0329	0.0160	0.4870	0.0636	0.0458	0.7206
	3500	0.0183	0.0125	0.6841	0.0326	0.0157	0.4819	0.0651	0.0484	0.7434
	10000	0.0181	0.0124	0.6858	0.0321	0.0158	0.4919	0.0656	0.0494	0.7526
$\delta = 5$	150	0.0138	0.0117	0.8519	0.0369	0.0144	0.3894	0.0468	0.0169	0.3612
	300	0.0104	0.0085	0.8207	0.0238	0.0116	0.4876	0.0390	0.0173	0.4434
	500	0.0100	0.0077	0.7685	0.0210	0.0105	0.4977	0.0374	0.0195	0.5212
	750	0.0093	0.0069	0.7444	0.0188	0.0102	0.5405	0.0370	0.0208	0.5634
	1250	0.0091	0.0068	0.7427	0.0180	0.0096	0.5305	0.0358	0.0220	0.6153
	1500	0.0090	0.0067	0.7430	0.0177	0.0095	0.5373	0.0351	0.0216	0.6160
	2000	0.0088	0.0064	0.7328	0.0173	0.0095	0.5468	0.0351	0.0225	0.6405
	2500	0.0089	0.0065	0.7335	0.0173	0.0095	0.5457	0.0359	0.0248	0.6896
	3000	0.0086	0.0062	0.7237	0.0171	0.0093	0.5401	0.0353	0.0230	0.6518
	3500	0.0088	0.0064	0.7234	0.0170	0.0093	0.5452	0.0347	0.0232	0.6686
	10000	0.0085	0.0061	0.7142	0.0167	0.0091	0.5463	0.0353	0.0245	0.6948

Table A.18: Summary of Results for Error Rates of 8 variables and a sample ratio of (1:2:3)

Distributions		CorrNorm			UncorrNorm			Skewed		
	S. Size	G. Mean	SD	CV	G. Mean	SD	CV	G. Mean	SD	CV
$\delta = 1$	180	0.1132	0.0500	0.4414	0.1916	0.0704	0.3675	0.1882	0.1172	0.6227
	360	0.0979	0.0502	0.5126	0.1557	0.0644	0.4133	0.1877	0.1267	0.6752
	600	0.0940	0.0513	0.5461	0.1404	0.0600	0.4275	0.1913	0.1342	0.7017
	900	0.0921	0.0517	0.5620	0.1337	0.0590	0.4410	0.1920	0.1374	0.7155
	1500	0.0906	0.0524	0.5784	0.1286	0.0576	0.4479	0.1931	0.1407	0.7286
	1800	0.0896	0.0517	0.5771	0.1274	0.0568	0.4456	0.1934	0.1407	0.7276
	2400	0.0887	0.0516	0.5813	0.1241	0.0556	0.4477	0.1956	0.1437	0.7346
	3000	0.0882	0.0515	0.5832	0.1234	0.0554	0.4490	0.1965	0.1451	0.7387
	3600	0.0882	0.0516	0.5849	0.1227	0.0554	0.4513	0.1962	0.1452	0.7402
	4200	0.0881	0.0517	0.5868	0.1222	0.0550	0.4499	0.1976	0.1469	0.7433
	12000	0.0872	0.0514	0.5893	0.1202	0.0546	0.4538	0.1992	0.1494	0.7498
$\delta = 2$	180	0.0819	0.0386	0.4708	0.1446	0.0527	0.3640	0.1542	0.0973	0.6311
	360	0.0693	0.0376	0.5421	0.1137	0.0457	0.4023	0.1541	0.1086	0.7048
	600	0.0659	0.0373	0.5654	0.1031	0.0441	0.4275	0.1580	0.1159	0.7333
	900	0.0647	0.0379	0.5862	0.0979	0.0432	0.4411	0.1588	0.1193	0.7517
	1500	0.0625	0.0375	0.5996	0.0929	0.0418	0.4497	0.1591	0.1211	0.7611
	1800	0.0619	0.0372	0.6004	0.0924	0.0418	0.4518	0.1612	0.1240	0.7689
	2400	0.0614	0.0370	0.6030	0.0907	0.0412	0.4539	0.1597	0.1228	0.7692
	3000	0.0614	0.0371	0.6040	0.0900	0.0413	0.4594	0.1619	0.1255	0.7754
	3600	0.0610	0.0371	0.6089	0.0893	0.0412	0.4614	0.1619	0.1256	0.7762
	4200	0.0609	0.0371	0.6082	0.0889	0.0408	0.4590	0.1625	0.1262	0.7769
	12000	0.0604	0.0368	0.6088	0.0872	0.0405	0.4646	0.1655	0.1307	0.7897
$\delta = 3$	180	0.0507	0.0271	0.5352	0.0971	0.0352	0.3626	0.1091	0.0690	0.6322
	360	0.0411	0.0244	0.5927	0.0738	0.0308	0.4168	0.1096	0.0787	0.7179
	600	0.0392	0.0244	0.6221	0.0652	0.0292	0.4476	0.1113	0.0853	0.7666
	900	0.0378	0.0243	0.6408	0.0616	0.0288	0.4678	0.1121	0.0890	0.7940
	1500	0.0370	0.0238	0.6426	0.0594	0.0286	0.4820	0.1148	0.0938	0.8175
	1800	0.0368	0.0235	0.6386	0.0581	0.0278	0.4786	0.1163	0.0951	0.8175
	2400	0.0365	0.0236	0.6451	0.0570	0.0276	0.4840	0.1152	0.0948	0.8228
	3000	0.0365	0.0235	0.6428	0.0572	0.0277	0.4847	0.1155	0.0958	0.8293
	3600	0.0358	0.0230	0.6429	0.0565	0.0274	0.4848	0.1171	0.0976	0.8336
	4200	0.0360	0.0233	0.6472	0.0564	0.0277	0.4909	0.1170	0.0974	0.8330
	12000	0.0358	0.0232	0.6473	0.0555	0.0273	0.4915	0.1191	0.1013	0.8507
$\delta = 4$	180	0.0285	0.0178	0.6271	0.0596	0.0218	0.3662	0.0722	0.0401	0.5558
	360	0.0218	0.0149	0.6811	0.0431	0.0191	0.4443	0.0674	0.0463	0.6878
	600	0.0210	0.0144	0.6872	0.0375	0.0185	0.4921	0.0687	0.0541	0.7875
	900	0.0199	0.0137	0.6860	0.0359	0.0183	0.5108	0.0679	0.0539	0.7933
	1500	0.0200	0.0138	0.6897	0.0340	0.0177	0.5208	0.0698	0.0584	0.8373
	1800	0.0192	0.0131	0.6802	0.0336	0.0174	0.5172	0.0697	0.0596	0.8542
	2400	0.0194	0.0133	0.6855	0.0330	0.0175	0.5289	0.0704	0.0616	0.8753
	3000	0.0191	0.0130	0.6837	0.0326	0.0171	0.5250	0.0712	0.0626	0.8785
	3600	0.0191	0.0131	0.6831	0.0325	0.0171	0.5262	0.0708	0.0613	0.8650
	4200	0.0189	0.0129	0.6826	0.0324	0.0170	0.5248	0.0706	0.0614	0.8703
	12000	0.0188	0.0128	0.6813	0.0319	0.0169	0.5289	0.0727	0.0658	0.9048
$\delta = 5$	180	0.0138	0.0101	0.7291	0.0331	0.0133	0.4012	0.0435	0.0177	0.4069
	360	0.0110	0.0089	0.8117	0.0229	0.0119	0.5192	0.0404	0.0234	0.5803
	600	0.0098	0.0075	0.7625	0.0197	0.0110	0.5559	0.0377	0.0244	0.6463
	900	0.0092	0.0070	0.7612	0.0186	0.0105	0.5640	0.0390	0.0292	0.7486
	1500	0.0095	0.0069	0.7224	0.0179	0.0102	0.5693	0.0393	0.0317	0.8058
	1800	0.0093	0.0068	0.7332	0.0176	0.0099	0.5634	0.0383	0.0336	0.8772
	2400	0.0089	0.0065	0.7264	0.0172	0.0098	0.5705	0.0380	0.0311	0.8184
	3000	0.0093	0.0066	0.7163	0.0171	0.0098	0.5727	0.0374	0.0302	0.8066
	3600	0.0091	0.0065	0.7178	0.0170	0.0097	0.5713	0.0386	0.0343	0.8877
	4200	0.0089	0.0064	0.7170	0.0169	0.0097	0.5714	0.0384	0.0324	0.8430
	12000	0.0090	0.0064	0.7082	0.0168	0.0097	0.5760	0.0387	0.0337	0.8709

Appendix B

Graphs for Effect of Sample Size on Quadratic Discriminant Function

B.1 Graphs of Effect of Sample Size on Correlated Normal Distribution

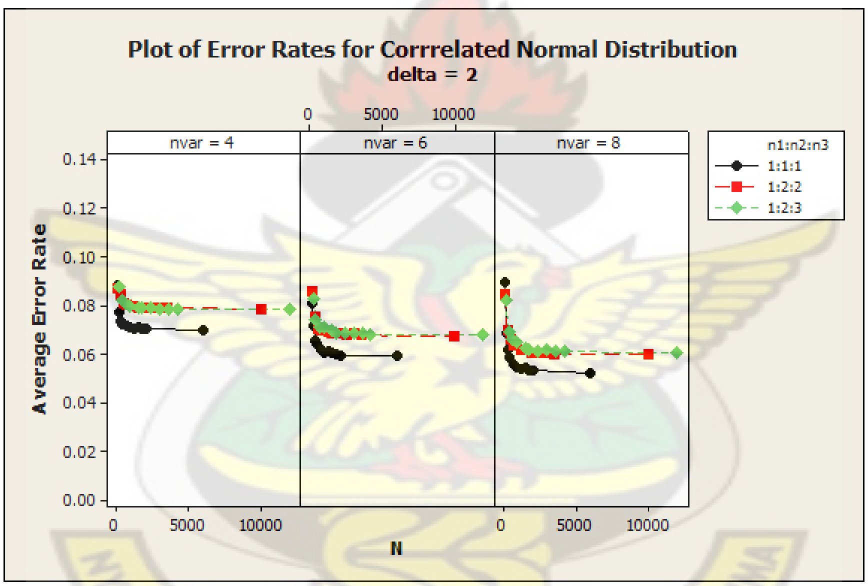

Figure B.1: Average Error Rate for Correlated Normal Distribution:$\delta = 2$

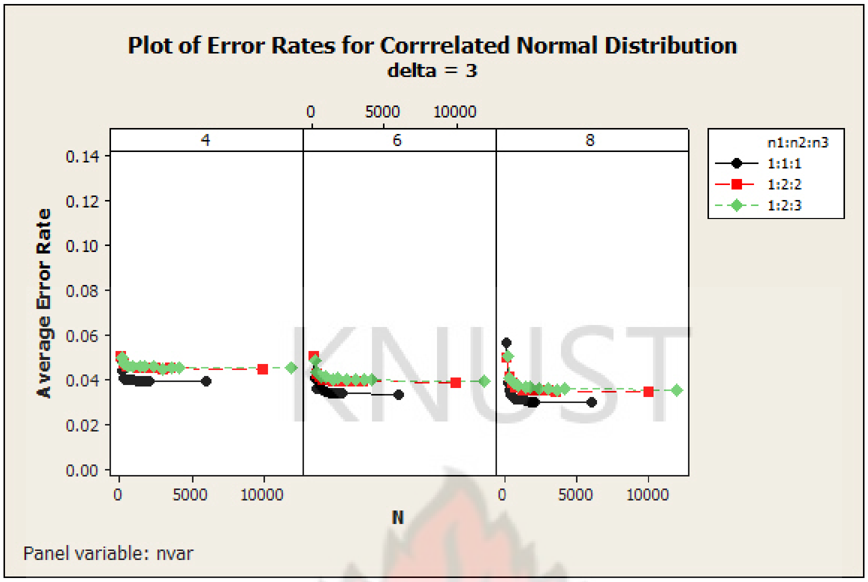

Figure B.2: Average Error Rate for Correlated Normal Distribution:$\delta = 3$

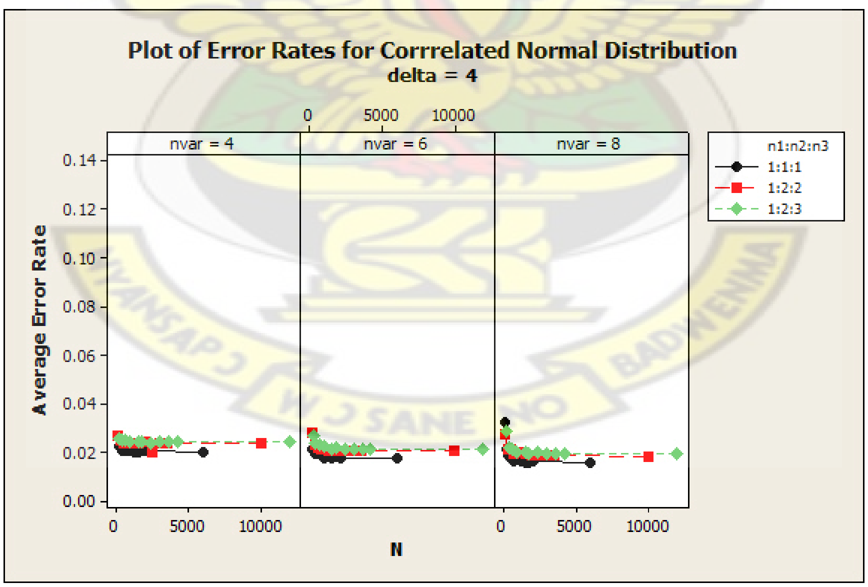

Figure B.3: Average Error Rate for Correlated Normal Distribution:$\delta = 4$

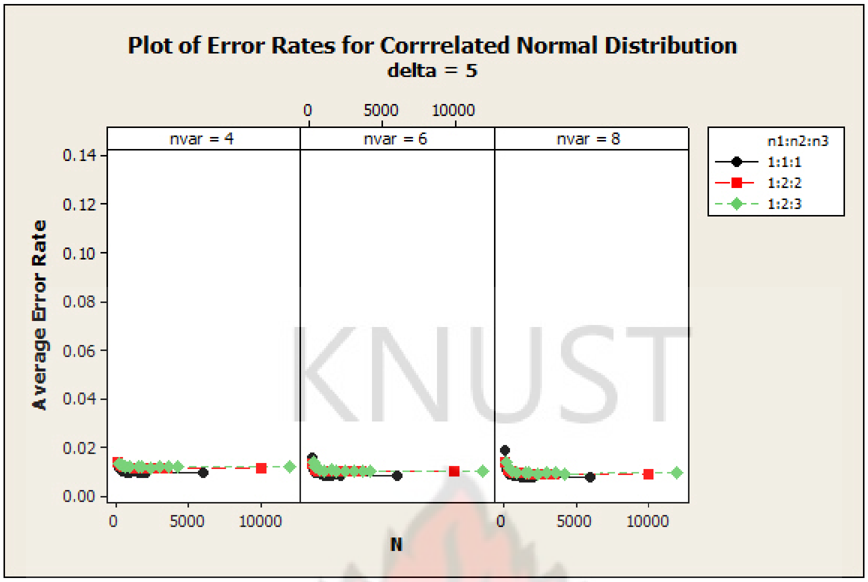

Figure B.4: Average Error Rate for Correlated Normal Distribution:$\delta = 5$

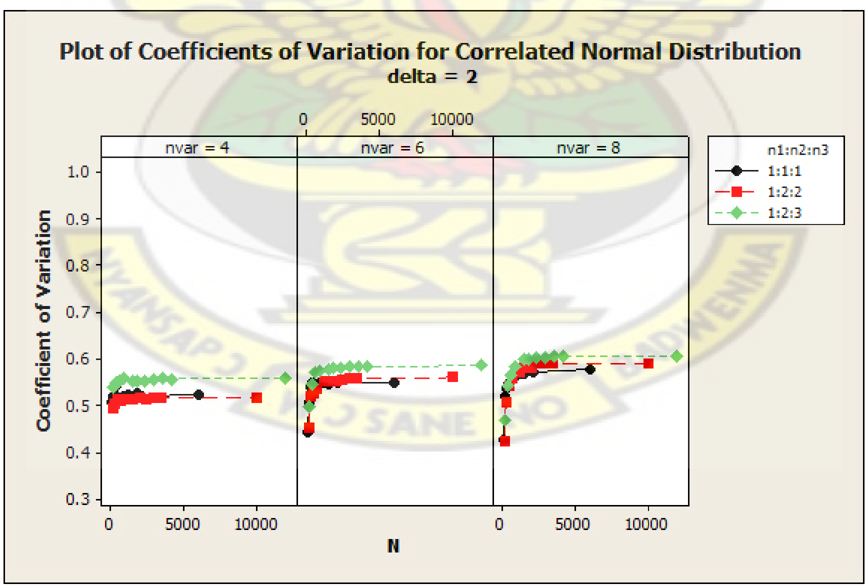

Figure B.5: Coefficients of Variation for Correlated Normal Distribution:$\delta = 2$

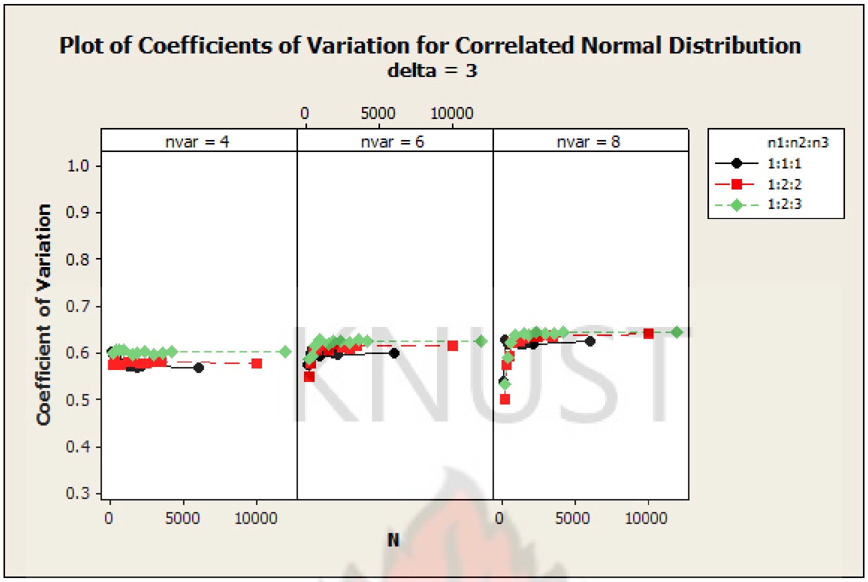

Figure B.6: Coefficients of Variation for Correlated Normal Distribution:$\delta = 3$

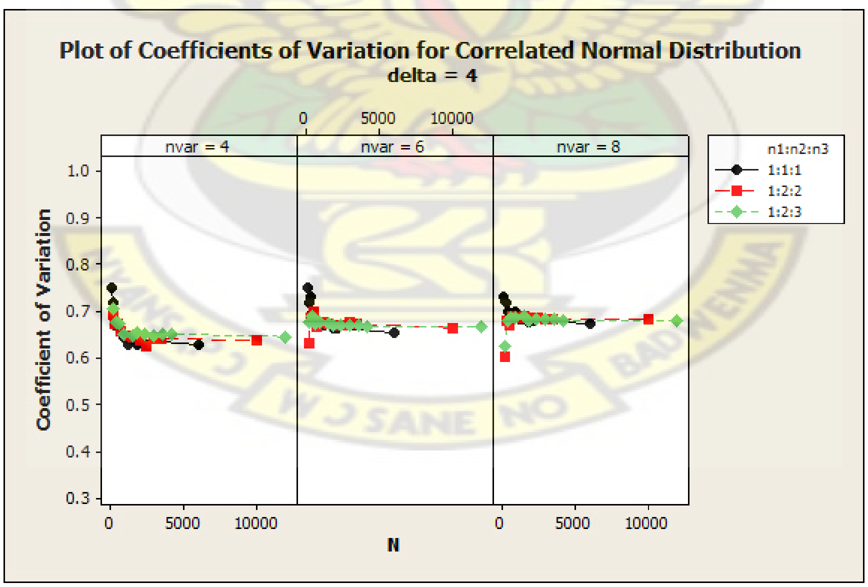

Figure B.7: Coefficients of Variation for Correlated Normal Distribution:$\delta = 4$

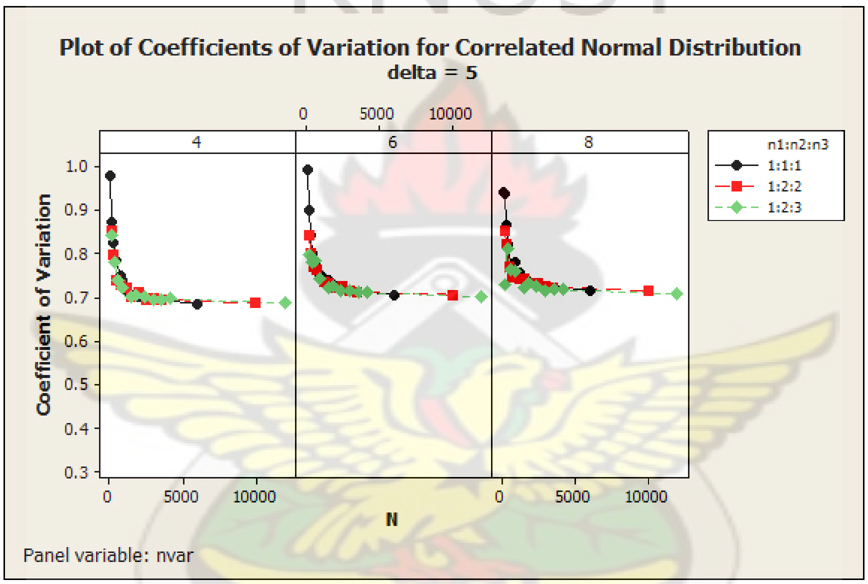

Figure B.8: Coefficients of Variation for Correlated Normal Distribution:$\delta = 5$

B.2 Graphs of Effect of Sample Size on Uncorrelated Normal Distribution

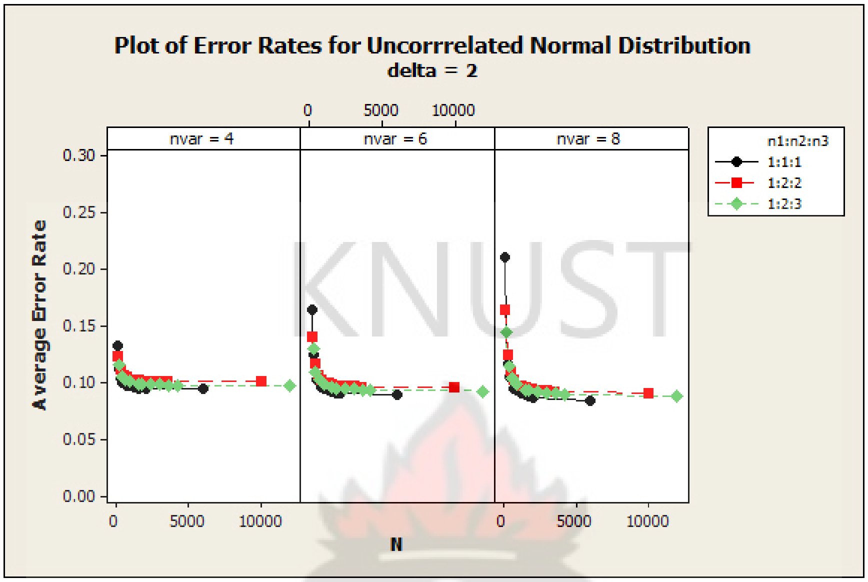

Figure B.9: Average Error Rate for Uncorrelated Normal Distribution:$\delta = 2$

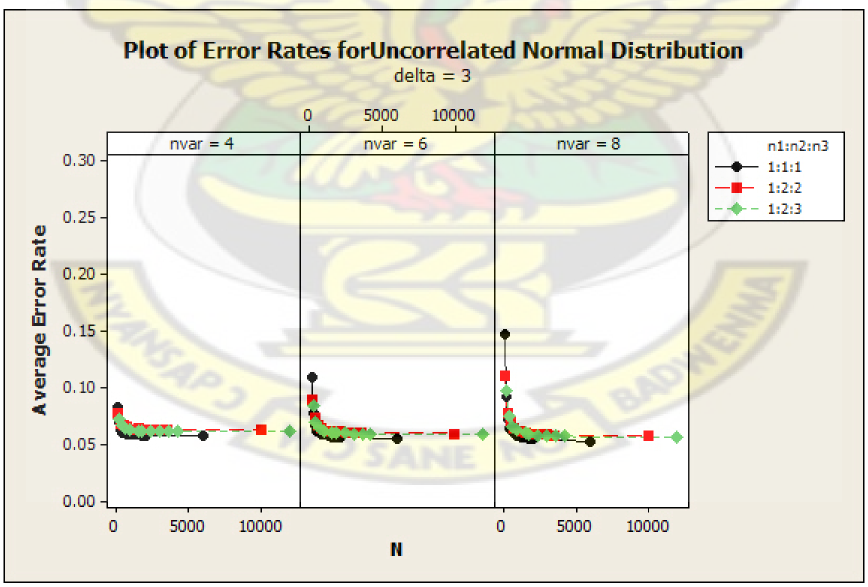

Figure B.10: Average Error Rate for Uncorrelated Normal Distribution:$\delta = 3$

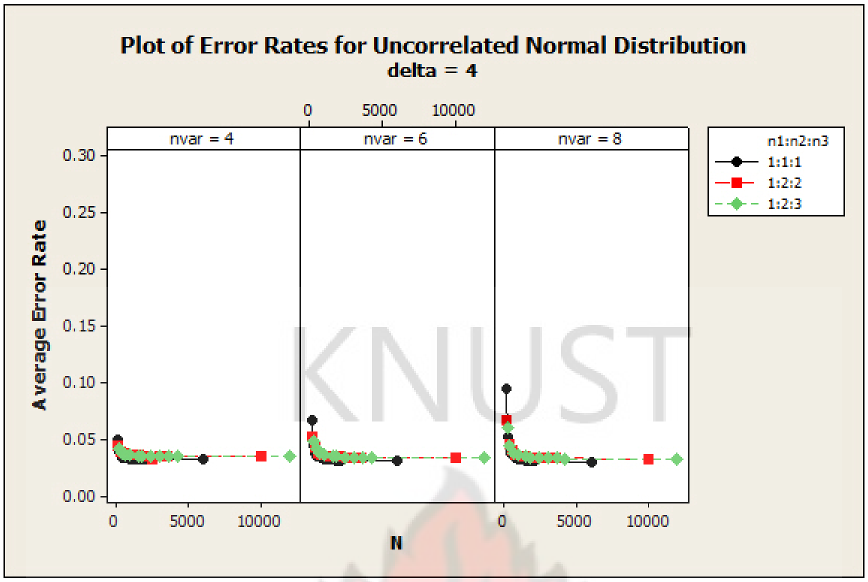

Figure B.11: Average Error Rate for Uncorrelated Normal Distribution:$\delta = 4$

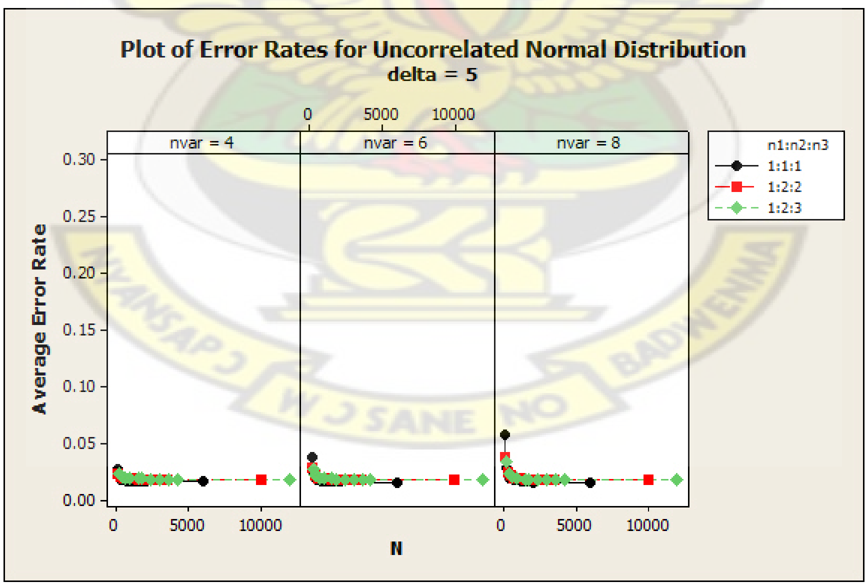

Figure B.12: Average Error Rate for Uncorrelated Normal Distribution:$\delta = 5$

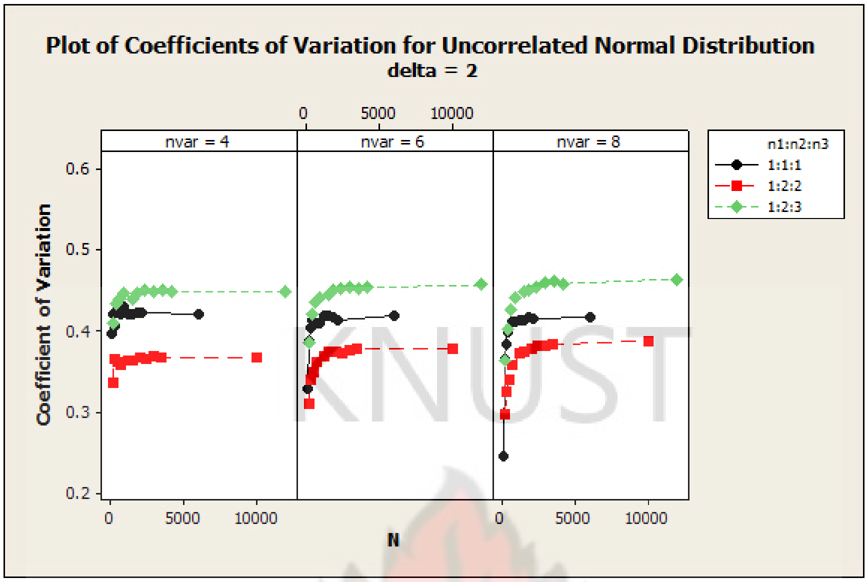

Figure B.13: Coefficients of Variation for Uncorrelated Normal Distribution:$\delta = 2$

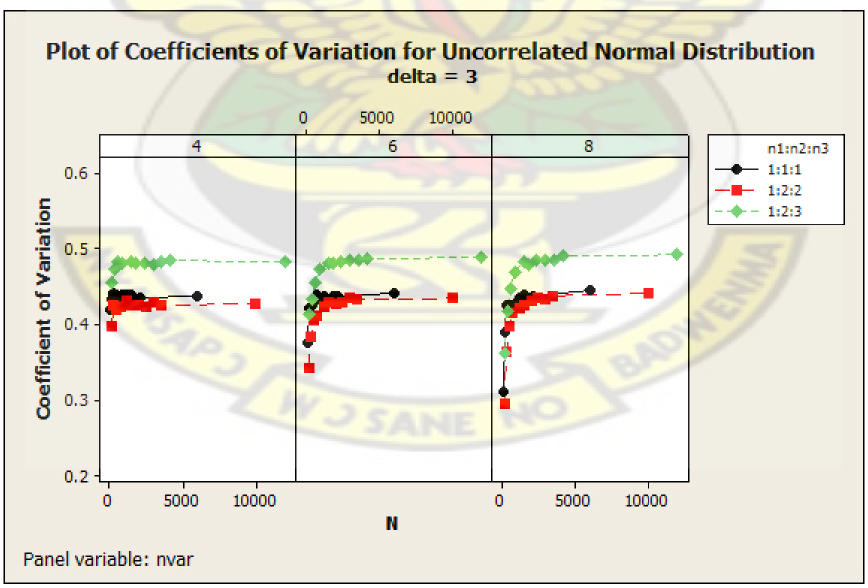

Figure B.14: Coefficients of Variation for Uncorrelated Normal Distribution:$\delta = 3$

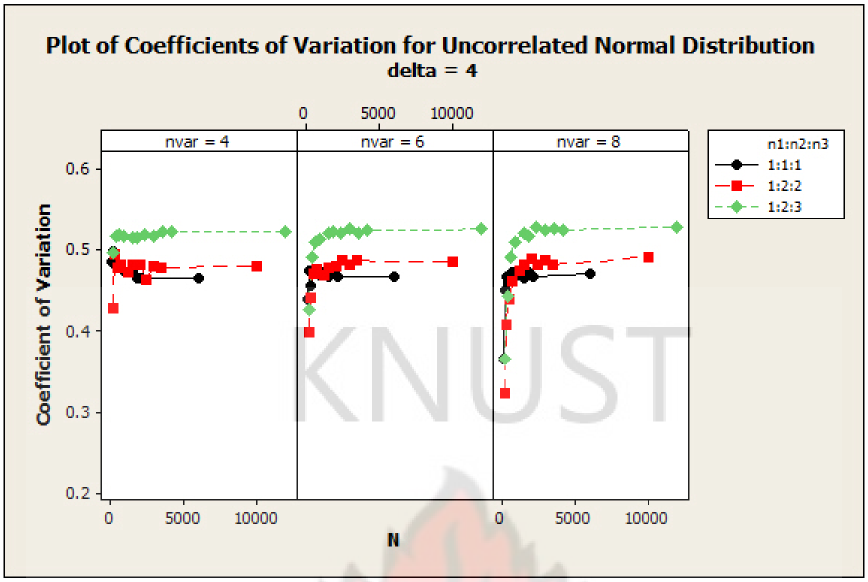

Figure B.15: Coefficients of Variation for Uncorrelated Normal Distribution:$\delta = 4$

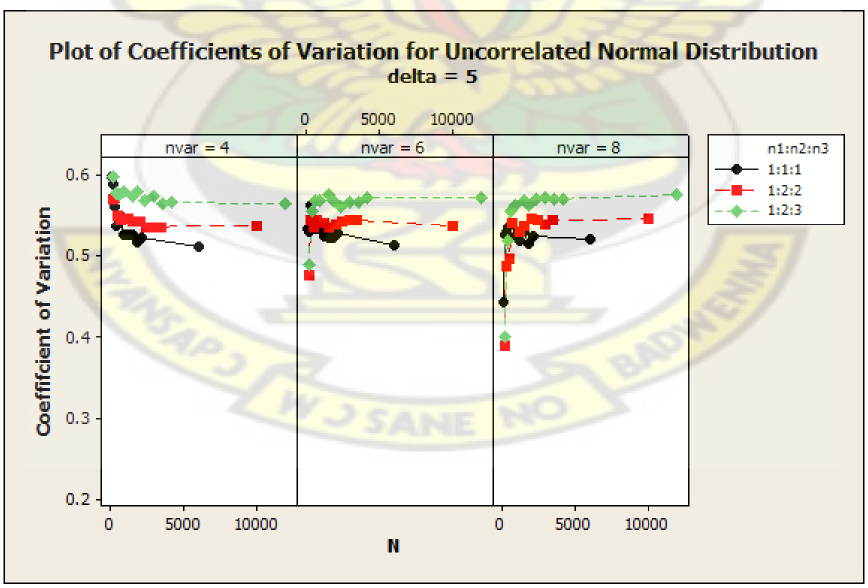

Figure B.16: Coefficients of Variation for Uncorrelated Normal Distribution:$\delta = 5$

B.3 Graphs of Effect of Sample Size on Skewed Distribution

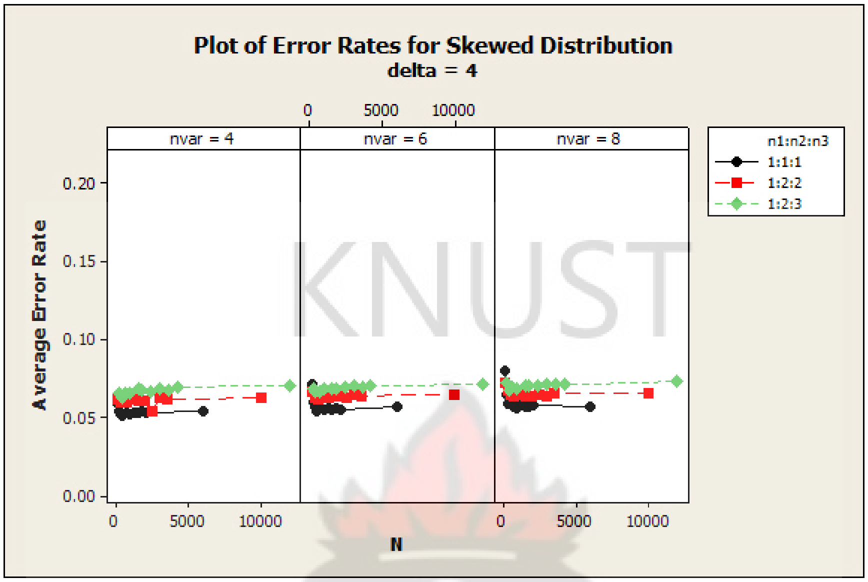

Figure B.17: Average Error Rate for Skewed Distribution:$\delta = 4$

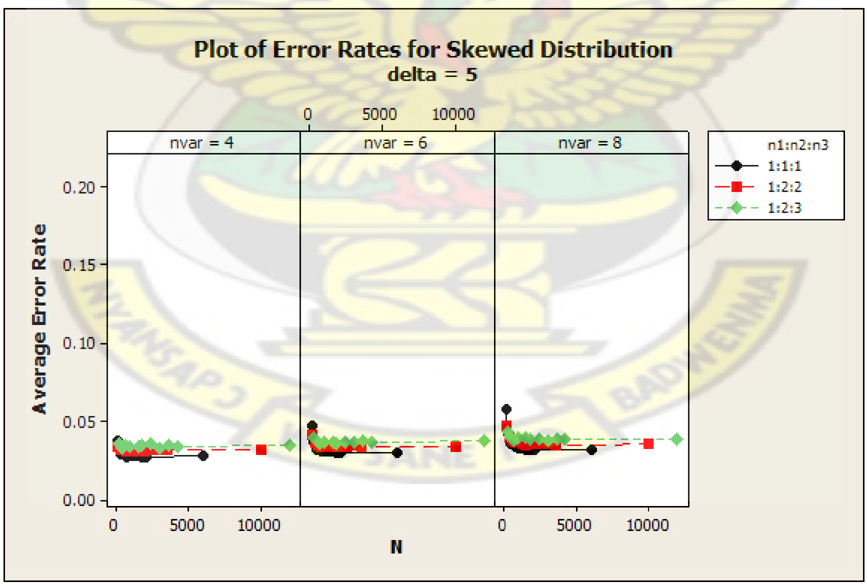

Figure B.18: Average Error Rate for Skewed Distribution:$\delta = 5$

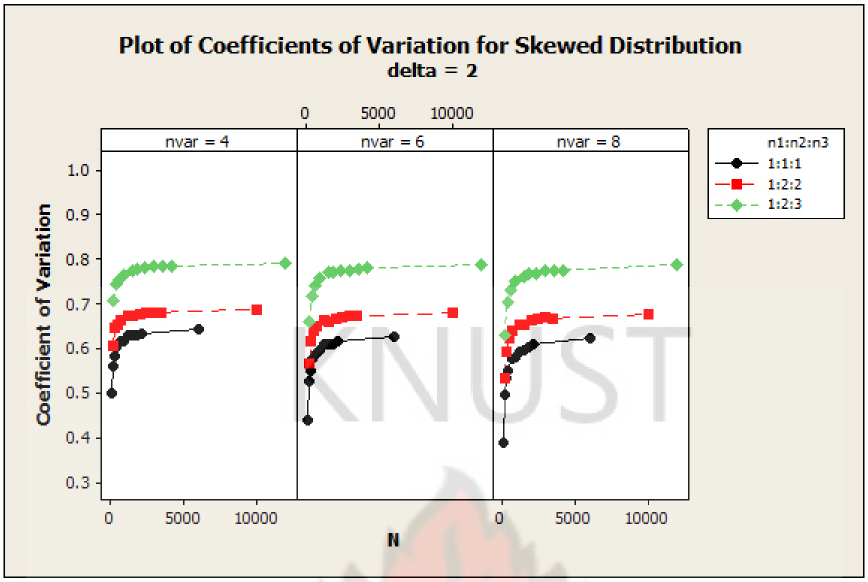

Figure B.19: Coefficients of Variation for Skewed Distribution:$\delta = 2$

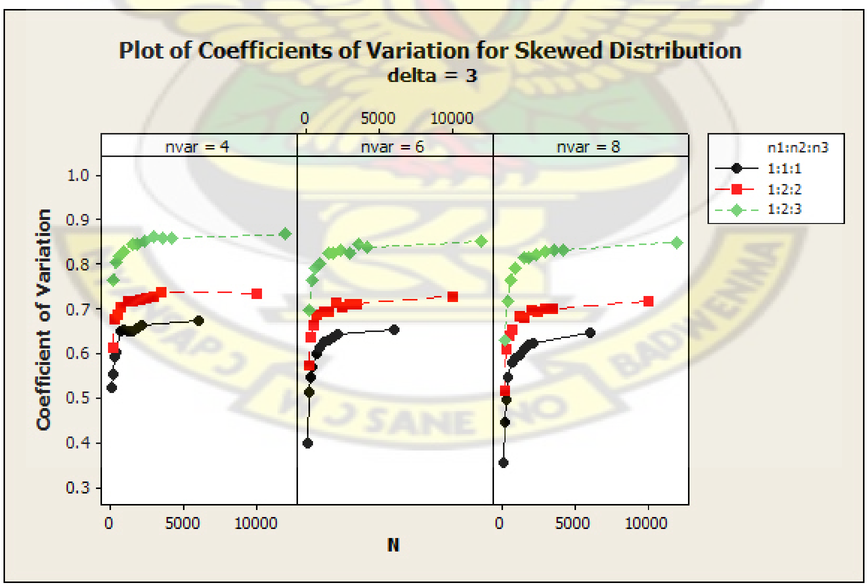

Figure B.20: Coefficients of Variation for Skewed Distribution:$\delta = 3$

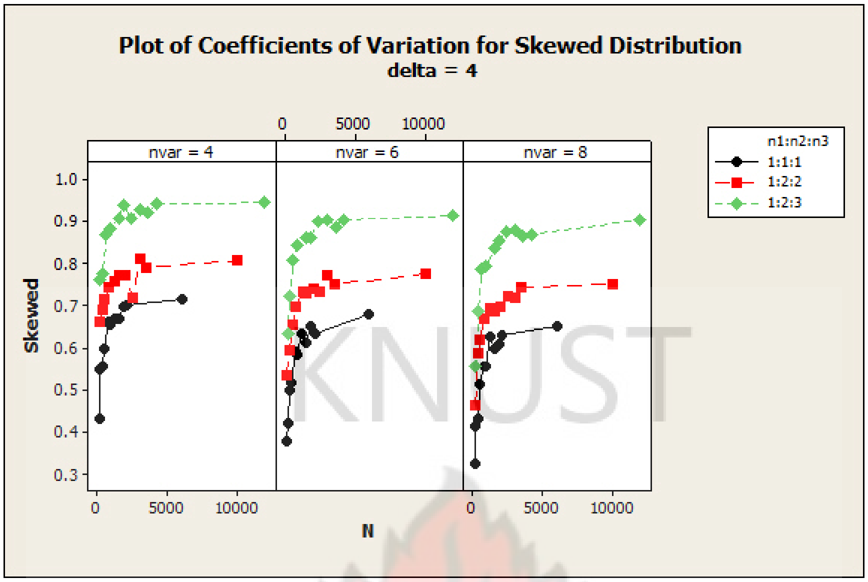

Figure B.21: Coefficients of Variation for Skew Distribution:$\delta = 4$

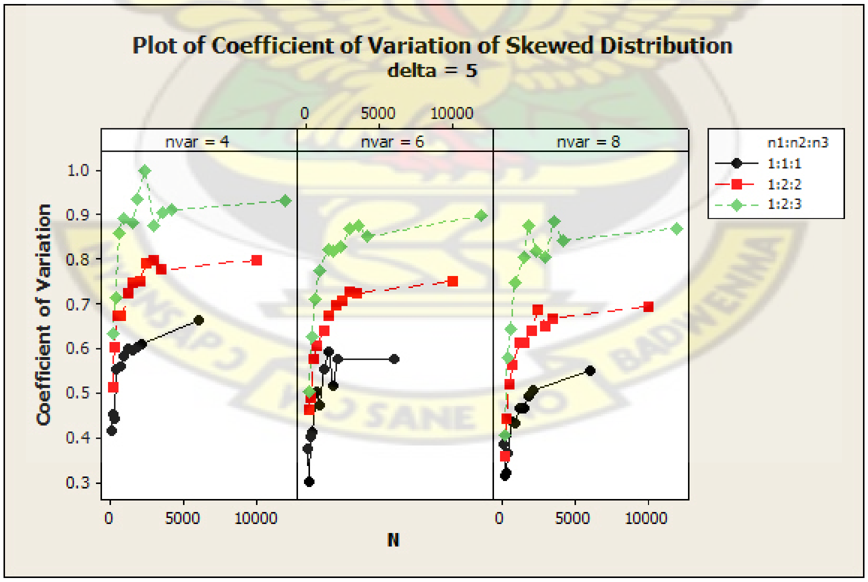

Figure B.22: Coefficients of Variation for Skewed Distribution:$\delta = 5$

Appendix C

Graphs for Effect of Number of Variables on Quadratic Discriminant Function

C.1 Graphs of Effect of Number of Variable on Correlated Normal Distribution

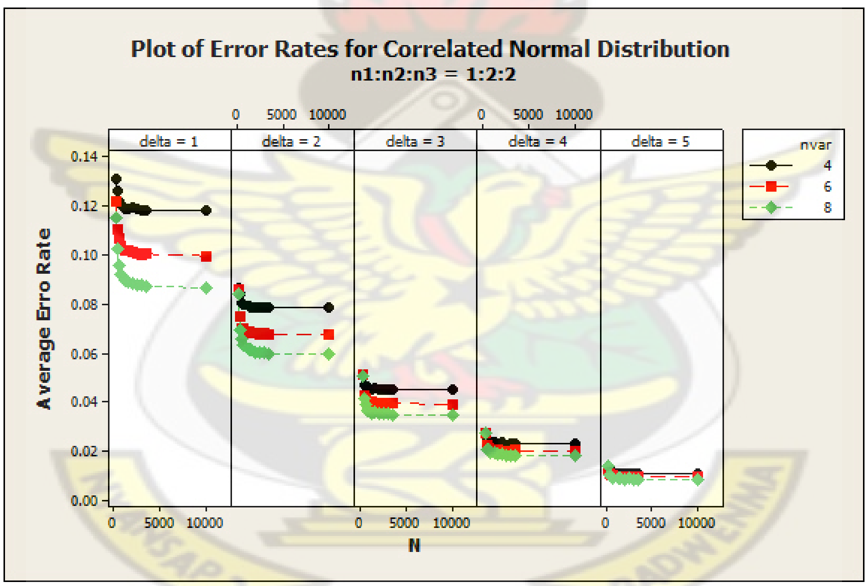

Figure C.1: Average Error Rate for Correlated Normal Distribution:$n_1 : n_2 : n_3 = 1 : 2 : 2$

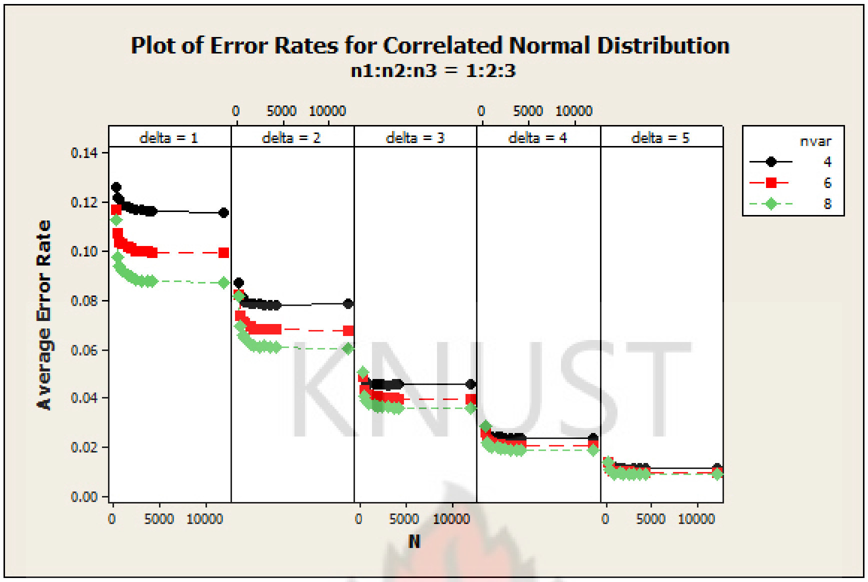

Figure C.2: Average Error Rate for Correlated Normal Distribution:$n_1 : n_2 : n_3 = 1 : 2 : 3$

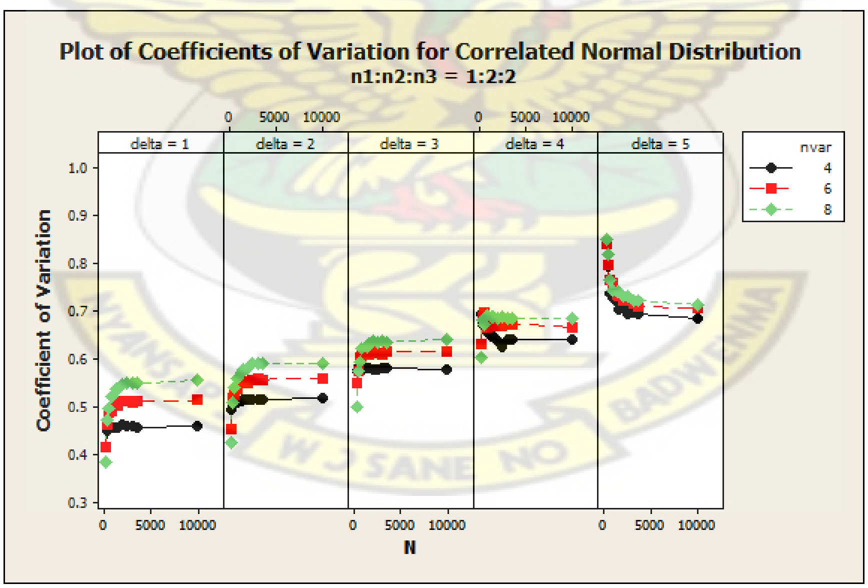

Figure C.3: Coefficients of Variation for Correlated Normal Distribution:$n_1 : n_2 : n_3 = 1 : 2 : 2$

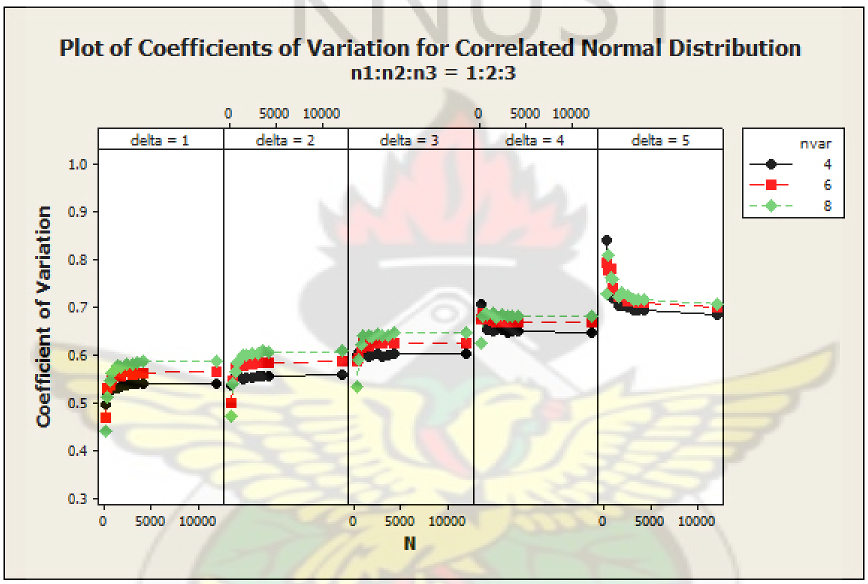

Figure C.4: Coefficients of Variation for Correlated Normal Distribution:$n_1 : n_2 : n_3 = 1 : 2 : 3$

C.2 Graphs of Effect of Number of Variables on Uncorrelated Normal Distribution

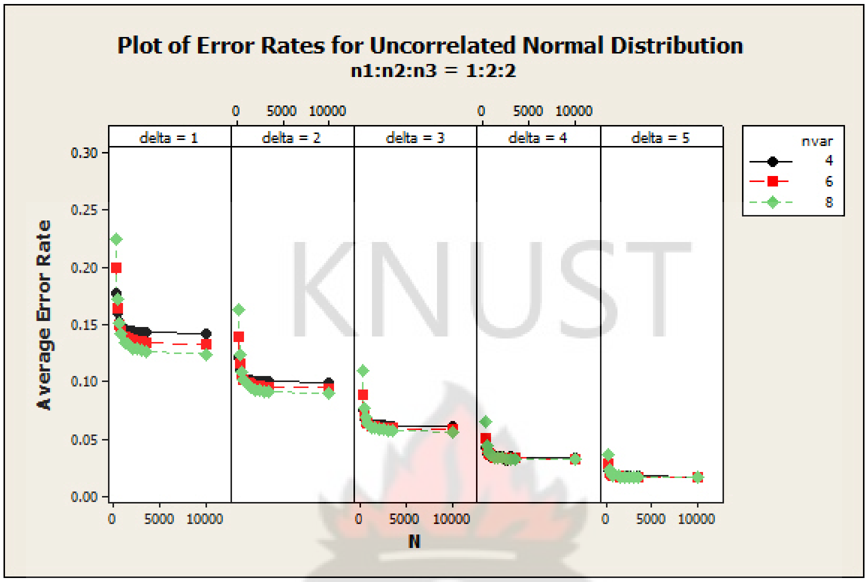

Figure C.5: Average Error Rate for Uncorrelated Normal Distribution:$n_1 : n_2 : n_3 = 1 : 2 : 2$

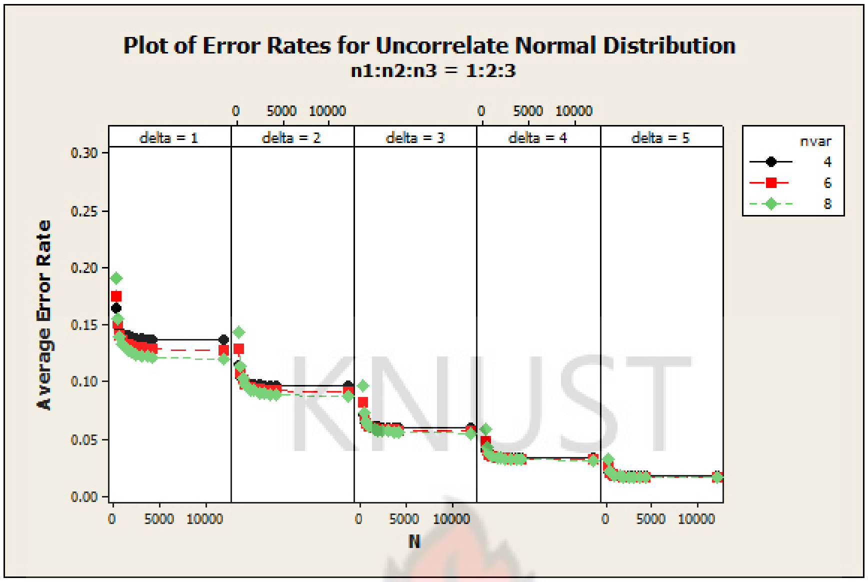

Figure C.6: Average Error Rate for Uncorrelated Normal Distribution:$n_1 : n_2 : n_3 = 1 : 2 : 3$

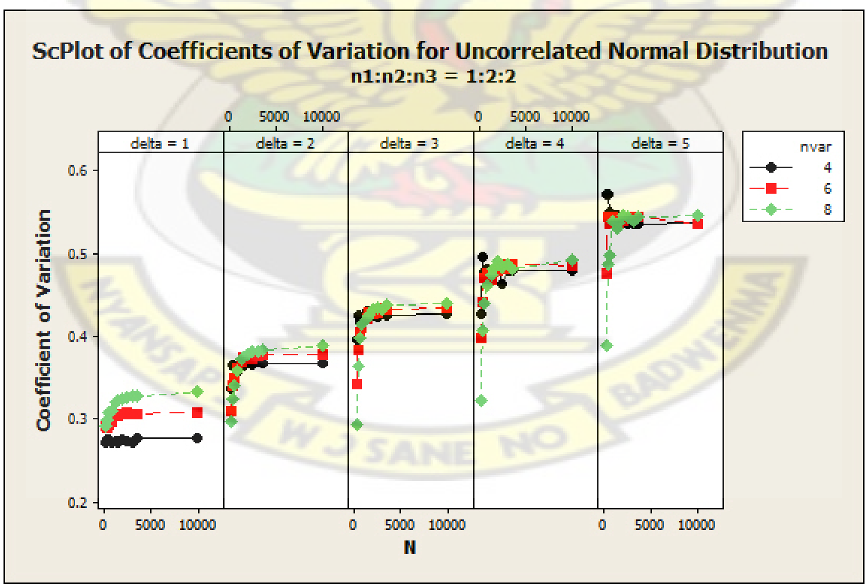

Figure C.7: Coefficients of Variation for Uncorrelated Normal Distribution:$n_1 : n_2 : n_3 = 1 : 2 : 2$

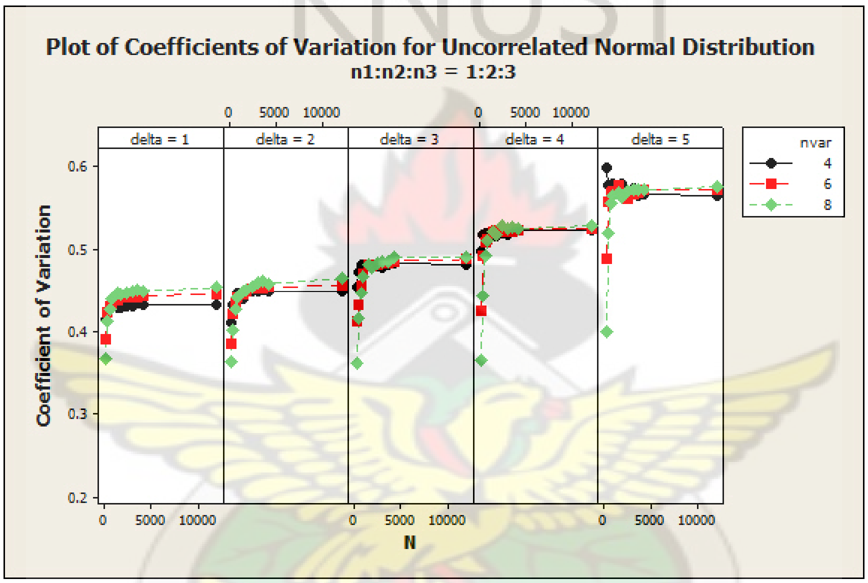

Figure C.8: Coefficients of Variation for Uncorrelated Normal Distribution:$n_1 : n_2 : n_3 = 1 : 2 : 3$

C.3 Graphs of Effect of Number of Variables on Skewed Distribution

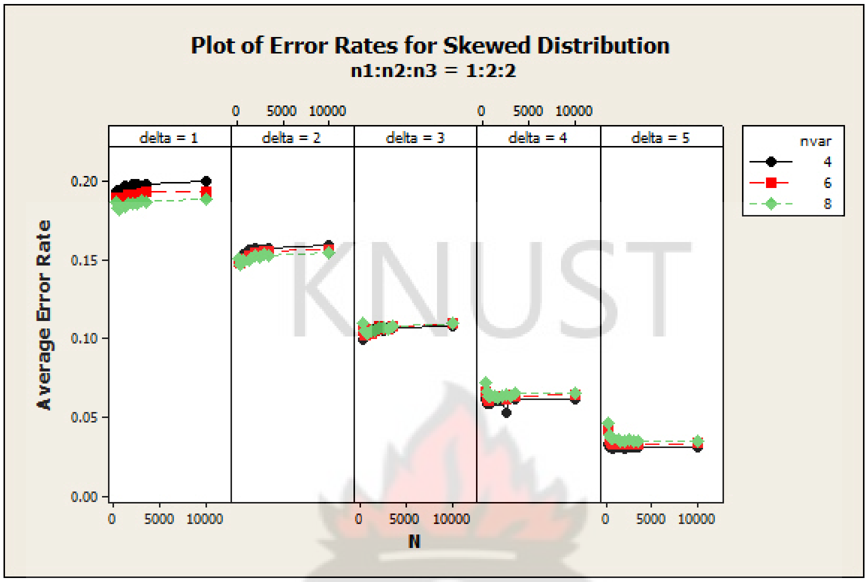

Figure C.9: Average Error Rate for Skewed Distribution:$n_1 : n_2 : n_3 = 1 : 2 : 2$

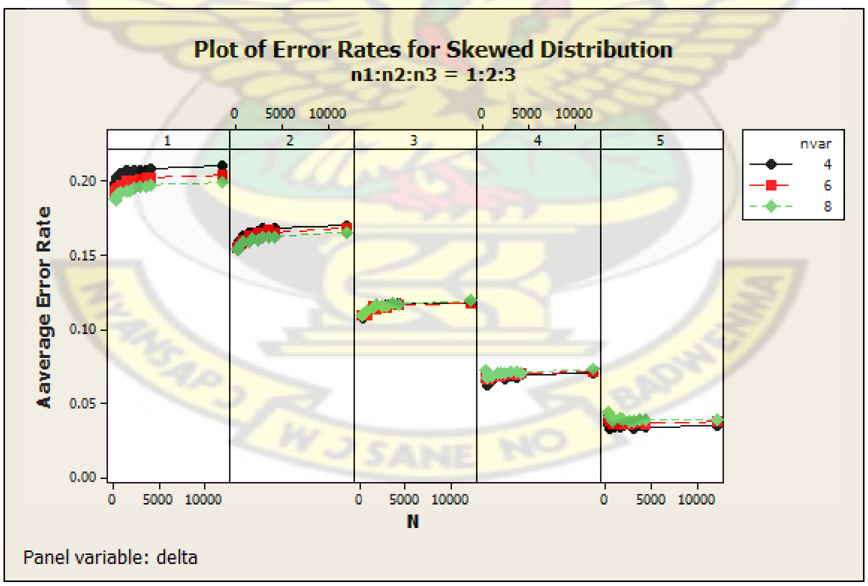

Figure C.10: Average Error Rate for Skewed Distribution:$n_1 : n_2 : n_3 = 1 : 2 : 3$

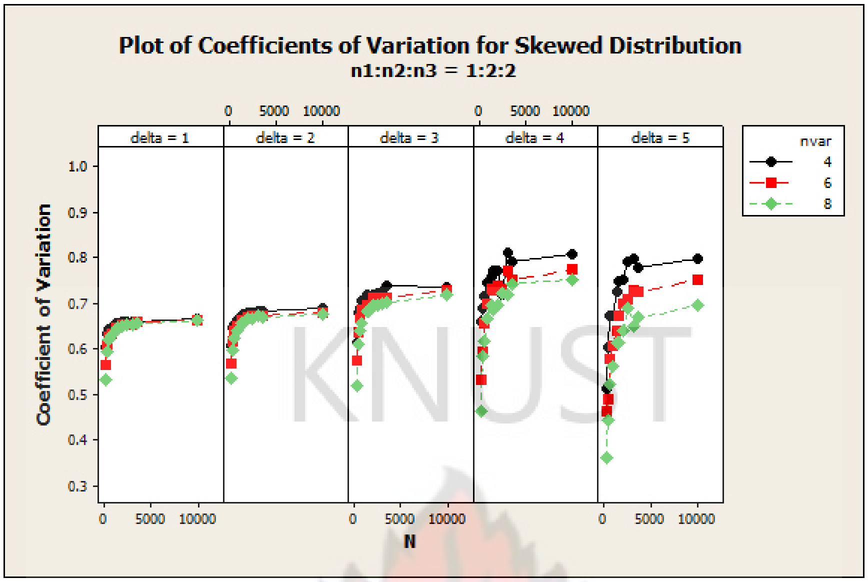

Figure C.11: Coefficients of Variation for Skewed Distribution:$n_1 : n_2 : n_3 = 1 : 2 : 2$

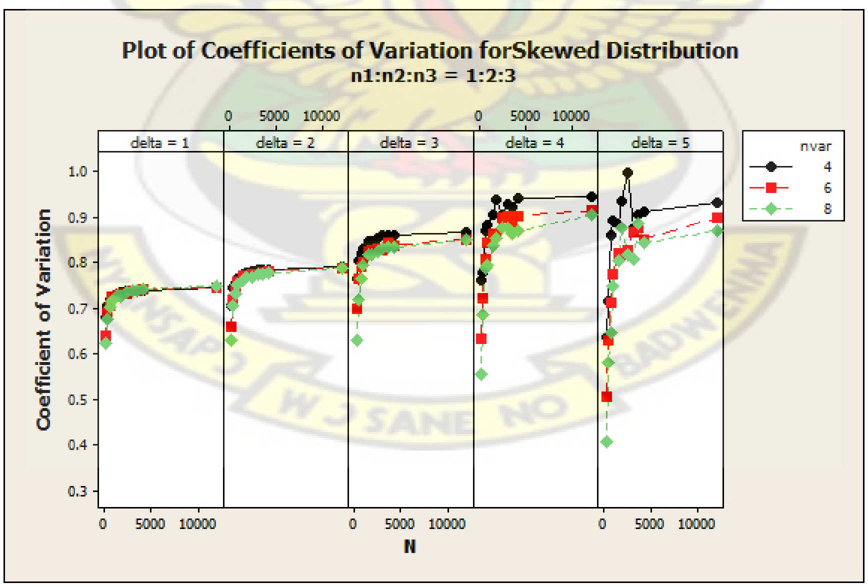

Figure C.12: Coefficients of Variation for Skewed Distribution:$n_1 : n_2 : n_3 = 1 : 2 : 3$

Appendix D

Graphs for Effect of Group Centroid Separator on Quadratic Discriminant Function

D.1 Graphs of Effect of Group Centroid Separator on Correlated Normal Distribution

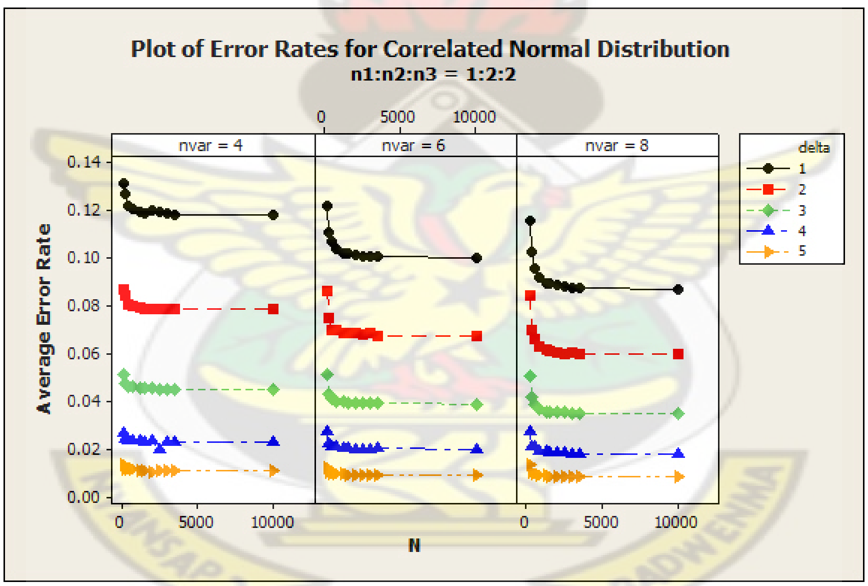

Figure D.1: Average Error Rate for Correlated Normal Distribution:$n_1 : n_2 : n_3 = 1 : 2 : 2$

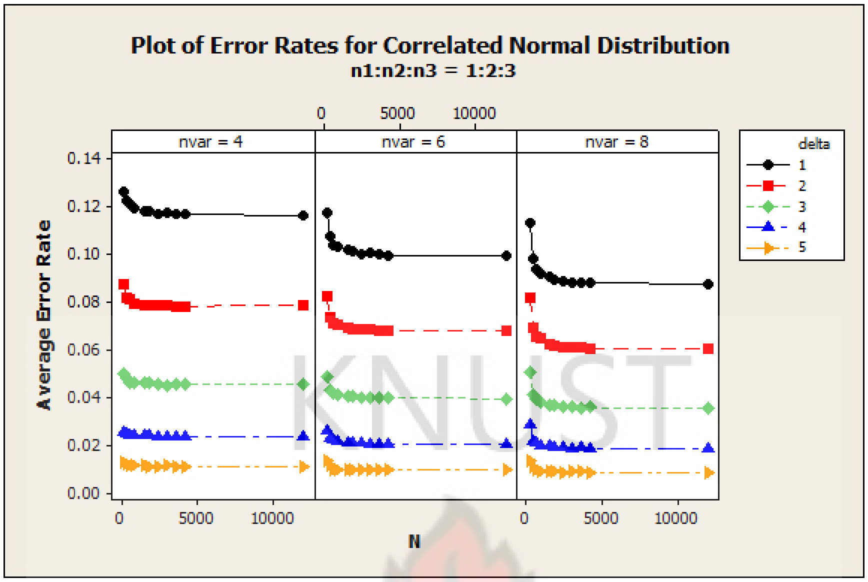

Figure D.2: Average Error Rate for Correlated Normal Distribution:$n_1 : n_2 : n_3 == 1 : 2 : 3$

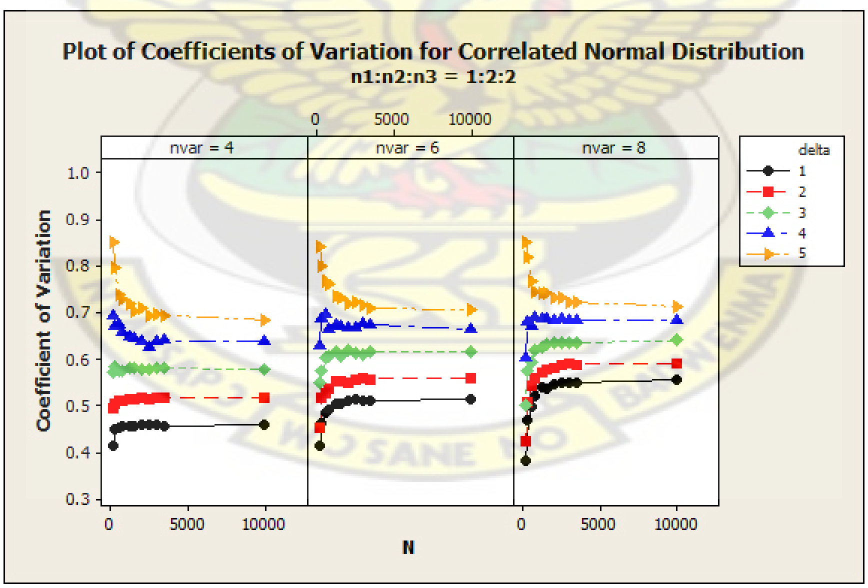

Figure D.3: Coefficients of Variation for Correlated Normal Distribution:$n_1 : n_2 : n_3 = 1 : 2 : 2$

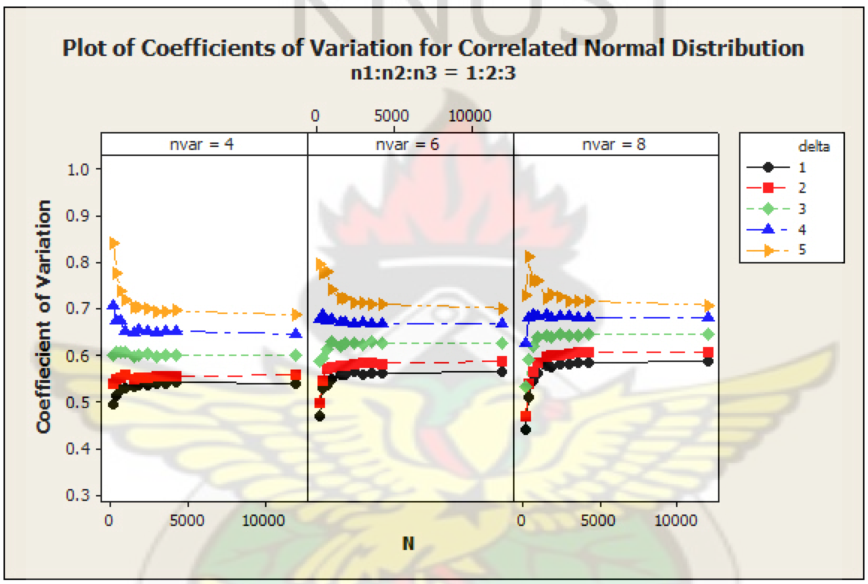

Figure D.4: Coefficients of Variation for Correlated Normal Distribution:$n_1 : n_2 : n_3 = 1 : 2 : 3$

D.2 Graphs of Effect of Group Centroid Separator on Uncorrelated Normal Distribution

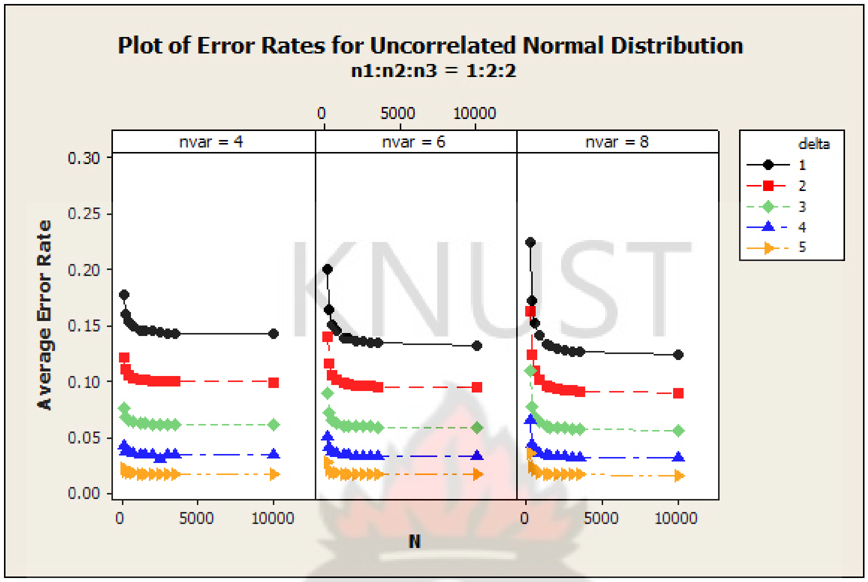

Figure D.5: Average Error Rate for Uncorrelated Normal Distribution:$n_1 : n_2 : n_3 = 1 : 2 : 2$

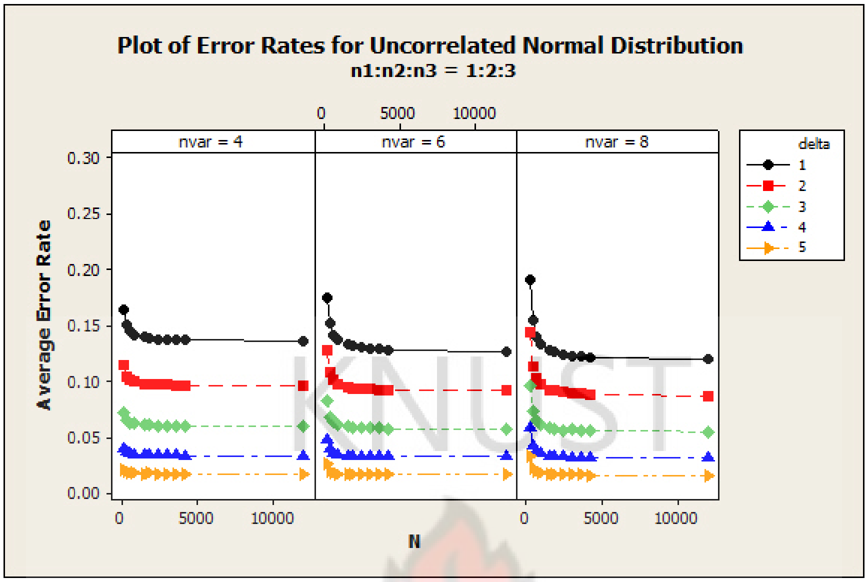

Figure D.6: Average Error Rate for Uncorrelated Normal Distribution:$n_1 : n_2 : n_3 = 1 : 2 : 3$

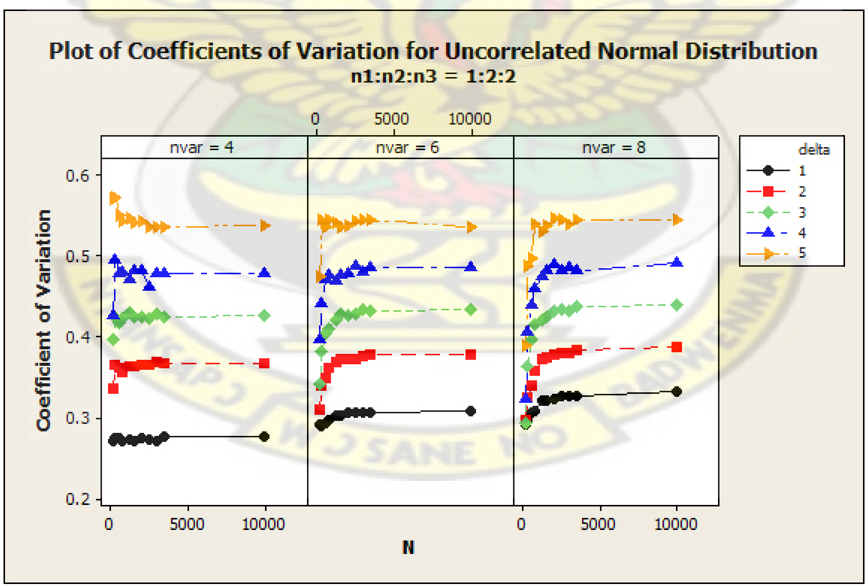

Figure D.7: Coefficients of Variation for Uncorrelated Normal Distribution:$n_1 : n_2 : n_3 = 1 : 2 : 2$

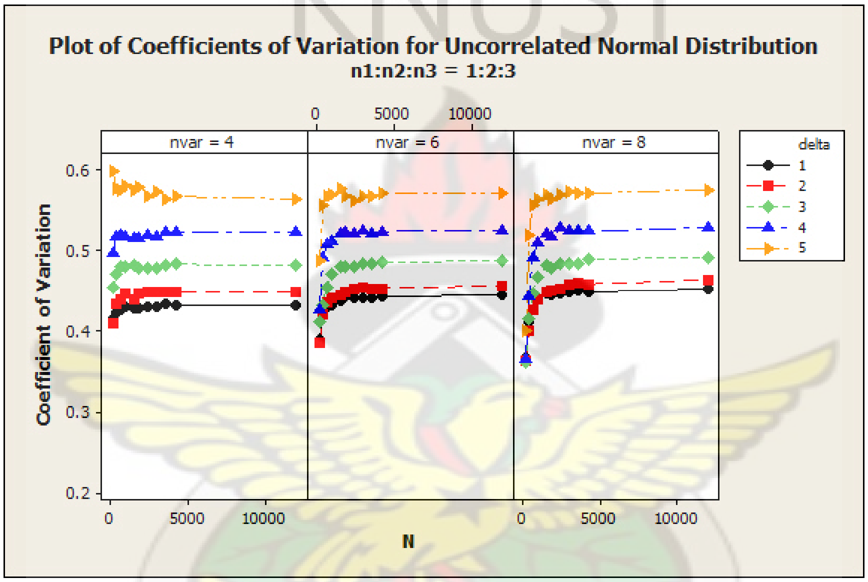

Figure D.8: Coefficients of Variation for Uncorrelated Normal Distribution:$n_1 : n_2 : n_3 = 1 : 2 : 3$

D.3 Graphs of Effect of Group Centroid Separator on Skewed Distribution

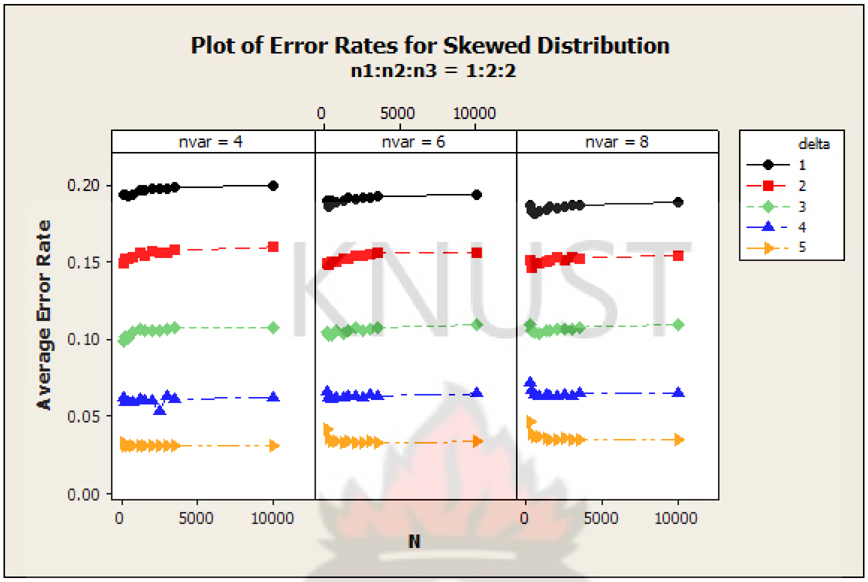

Figure D.9: Average Error Rate for Skewed Distribution:$n_1 : n_2 : n_3 = 1 : 2 : 2$

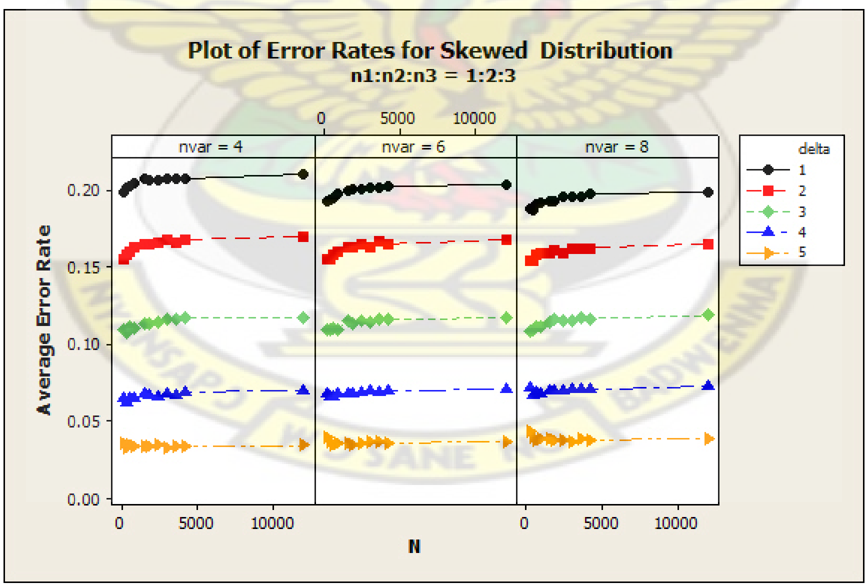

Figure D.10: Average Error Rate for Skewed Distribution:$n_1 : n_2 : n_3 = 1 : 2 : 3$

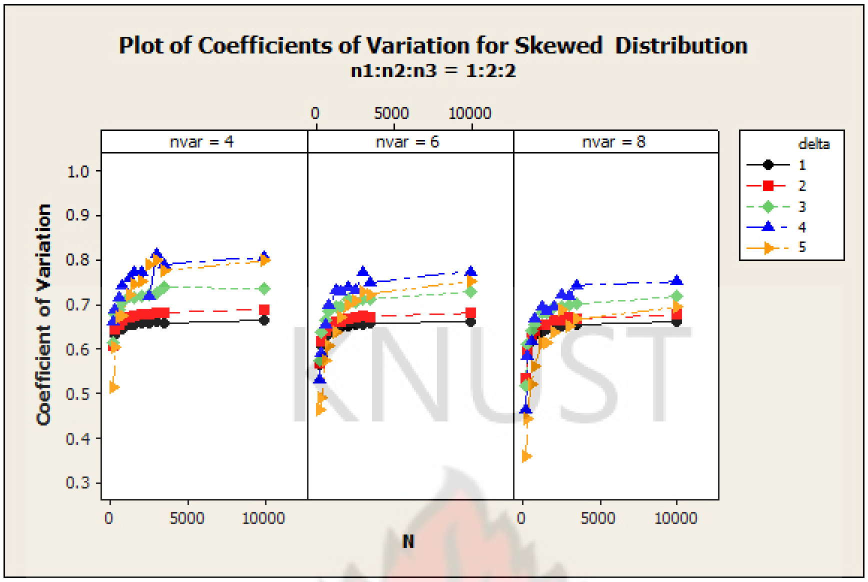

Figure D.11: Coefficients of Variation for Skewed Distribution:$n_1 : n_2 : n_3 = 1 : 2 : 2$

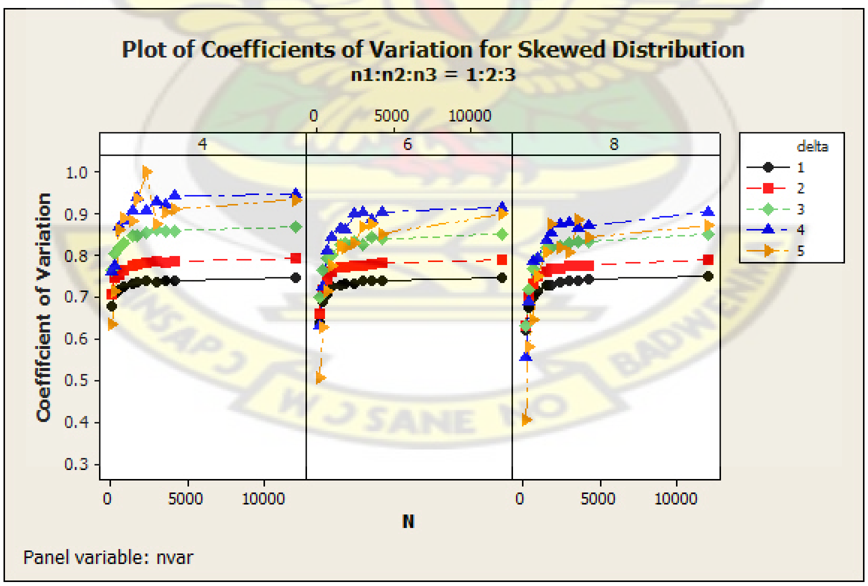

Figure D.12: Coefficients of Variation for Skewed Distribution:$n_1 : n_2 : n_3 = 1 : 2 : 3$

Appendix E

Graphs of Comparison of Error Rates of the Three Distributions

E.1 Graphs of Comparison of Error Rates of the Three Distributions for Sample Size Ratio $n_1 : n_2 : n_3 = 1 : 2 : 2$

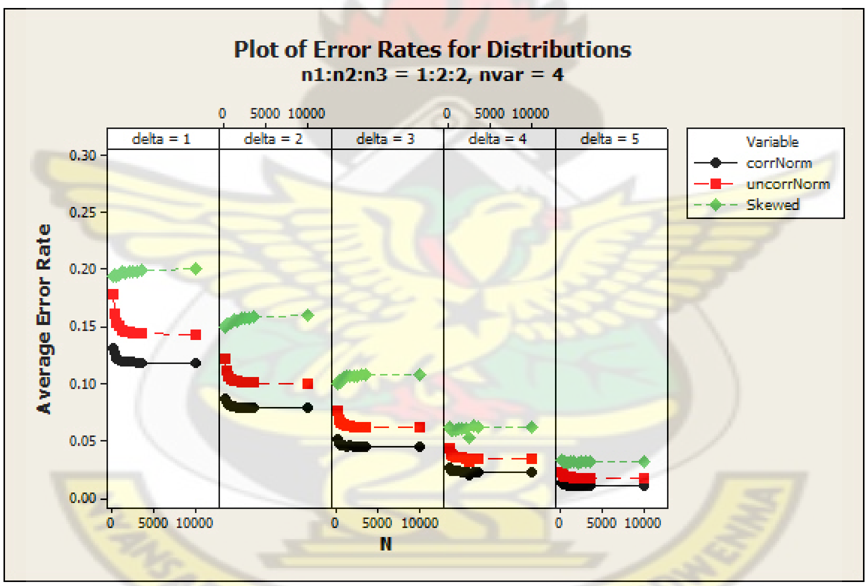

Figure E.1: Average error rates of the three distributions for 4 variables: $n_1 : n_2 : n_3 = 1 : 2 : 2$

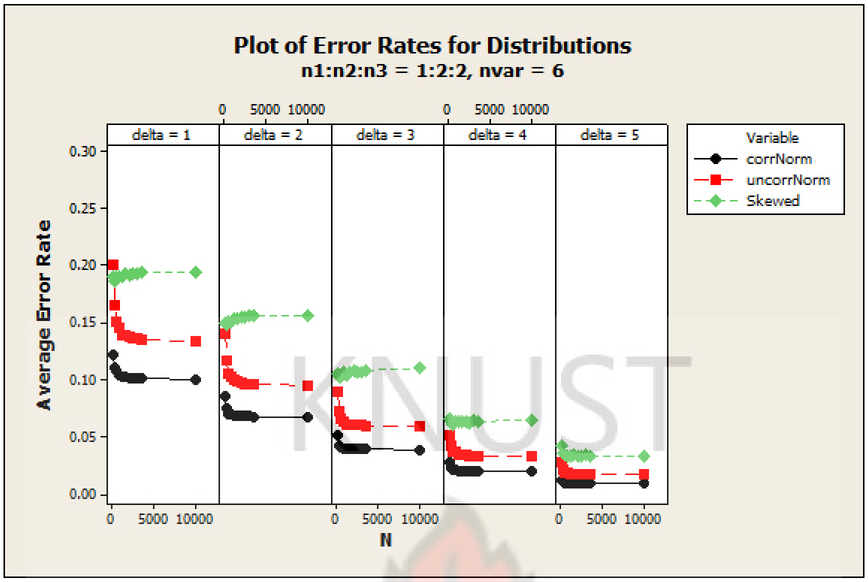

Figure E.2: Average error rates of the three distributions for 6 variables: $n_1 : n_2 : n_3 = 1 : 2 : 2$

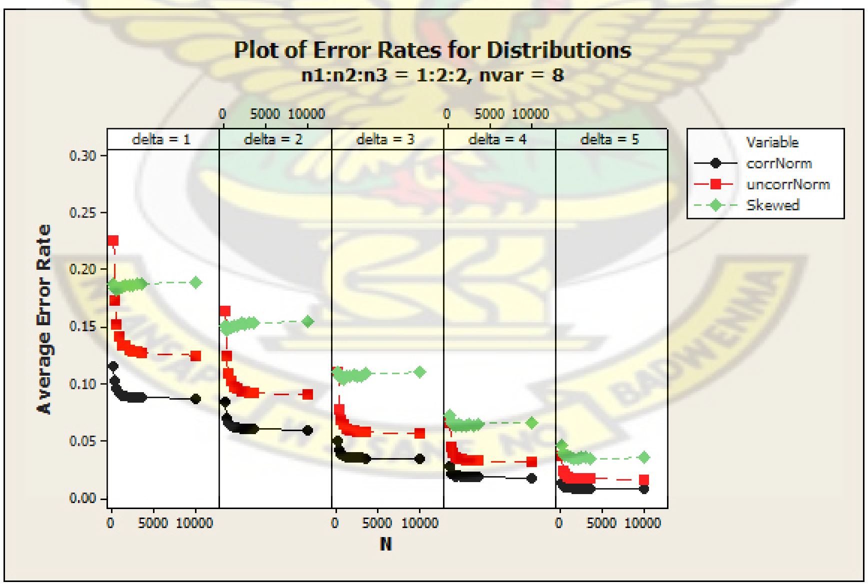

Figure E.3: Average error rates of the three distributions for 8 variables: $n_1 : n_2 : n_3 = 1 : 2 : 2$

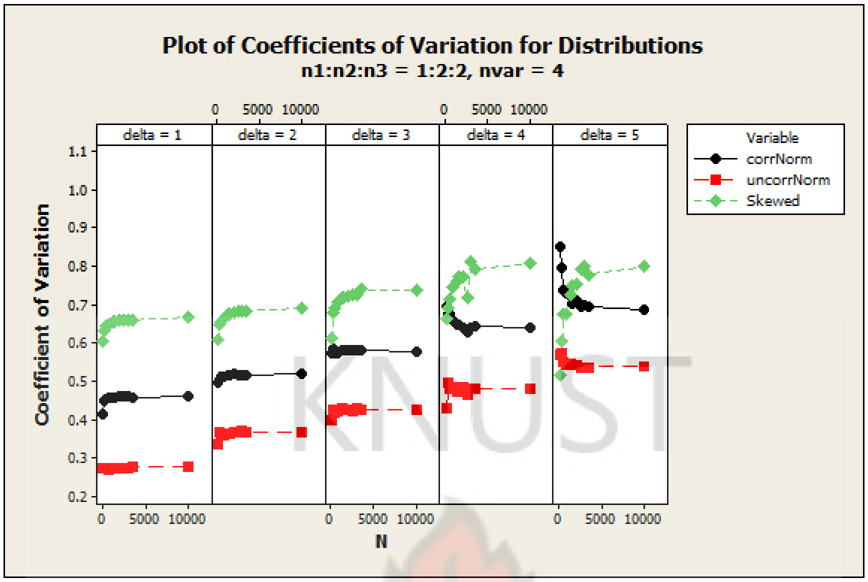

Figure E.4: Coefficients of Variation of the three distributions for 4 variables: $n_1 : n_2 : n_3 = 1 : 2 : 2$

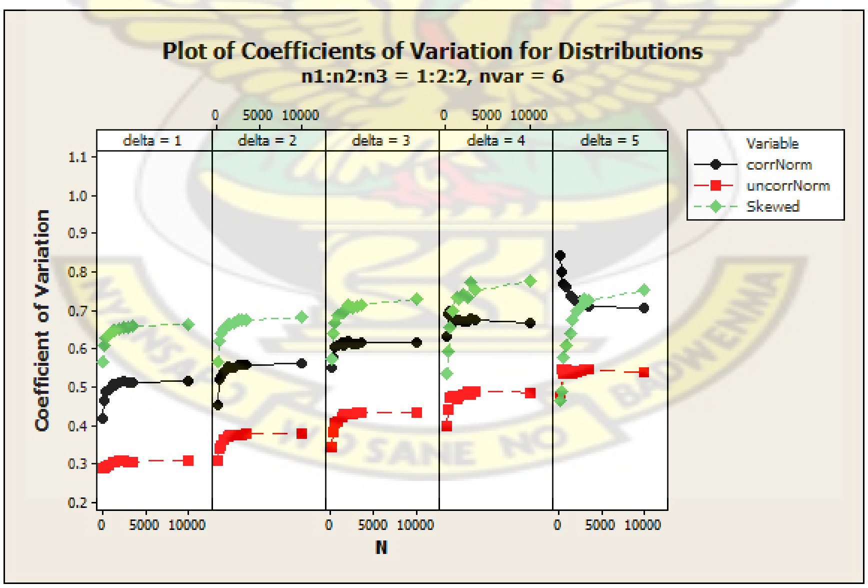

Figure E.5: Coefficients of Variation of the three distributions for 6 variables: $n_1 : n_2 : n_3 = 1 : 2 : 2$

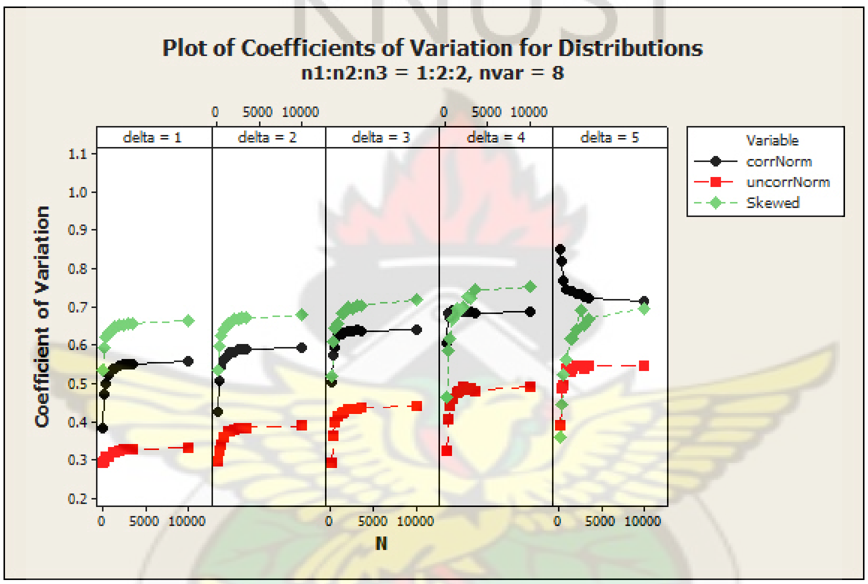

Figure E.6: Coefficients of Variation of the three distributions for 8 variables: $n_1 : n_2 : n_3 = 1 : 2 : 2$

E.2 Graphs of Comparison of Error Rates of the Three Distributions for Sample Size Ratio $n_1 : n_2 : n_3 = 1 : 2 : 3$

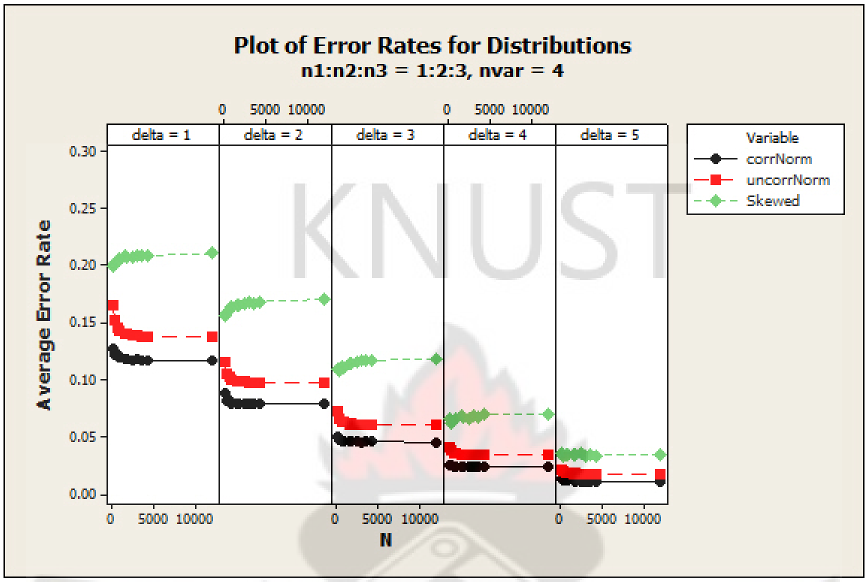

Figure E.7: Average error rates of the three distributions for 4 variables: $n_1 : n_2 : n_3 = 1 : 2 : 3$

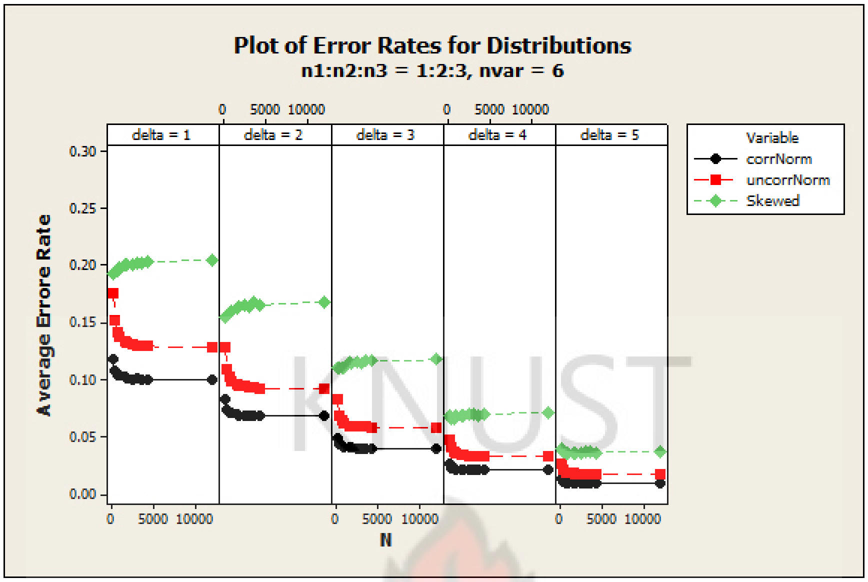

Figure E.8: Average error rates of the three distributions for 6 variables: $n_1 : n_2 : n_3 = 1 : 2 : 3$

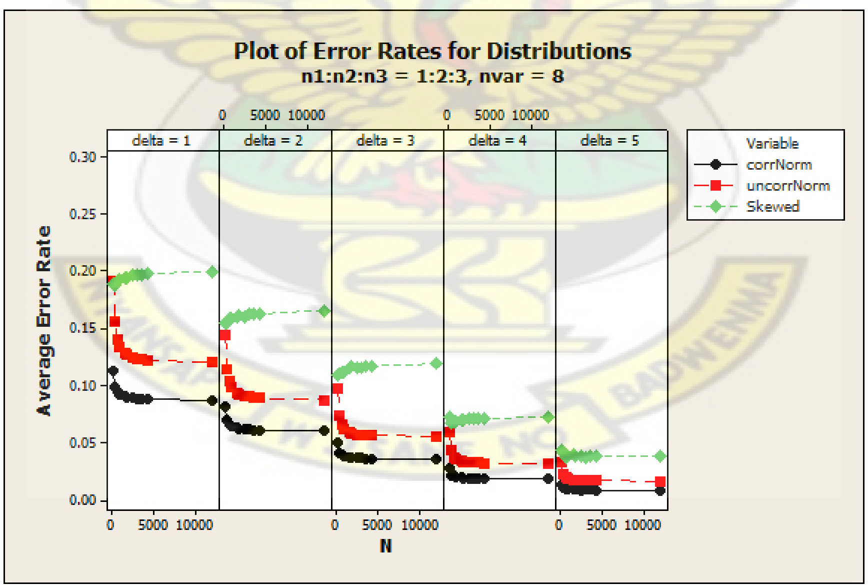

Figure E.9: Average error rates of the three distributions for 8 variables: $n_1 : n_2 : n_3 = 1 : 2 : 3$

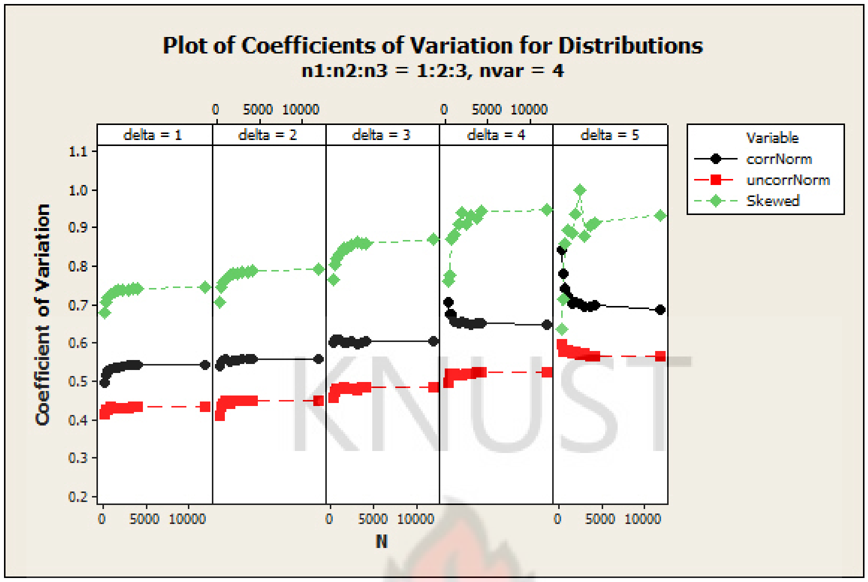

Figure E.10: Coefficients of Variation of the three distributions for 4 variables: $n_1 : n_2 : n_3 = 1 : 2 : 3$

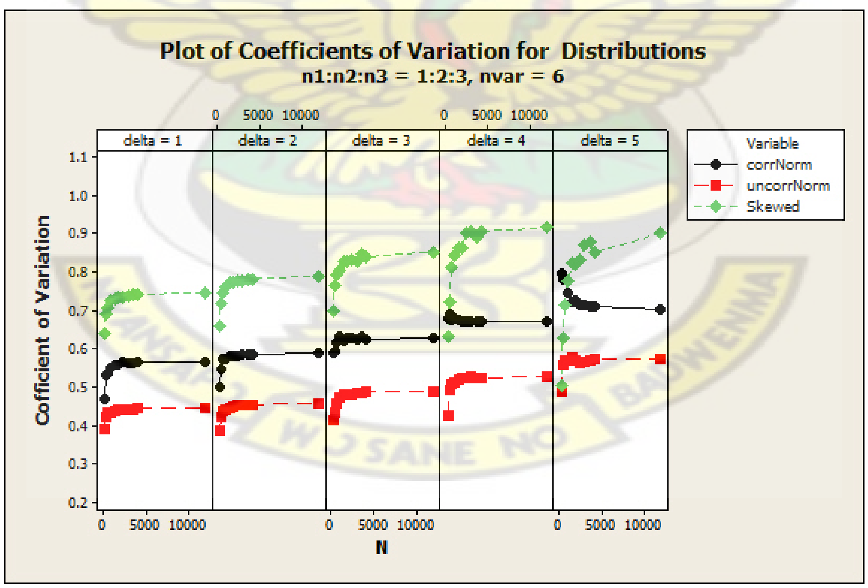

Figure E.11: Coefficients of Variation of the three distributions for 6 variables: $n_1 : n_2 : n_3 = 1 : 2 : 3$

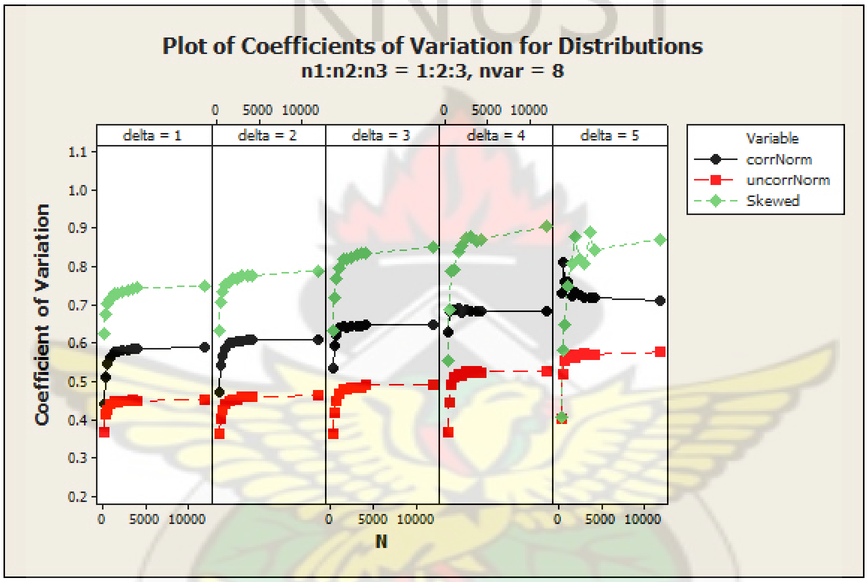

Figure E.12: Coefficients of Variation of the three distributions for 8 variables: $n_1 : n_2 : n_3 = 1 : 2 : 3$

Appendix F

Graphs of Comparison of Error Rates and Coefficients of Variation of the Three Distributions (Sample Size Ratio)

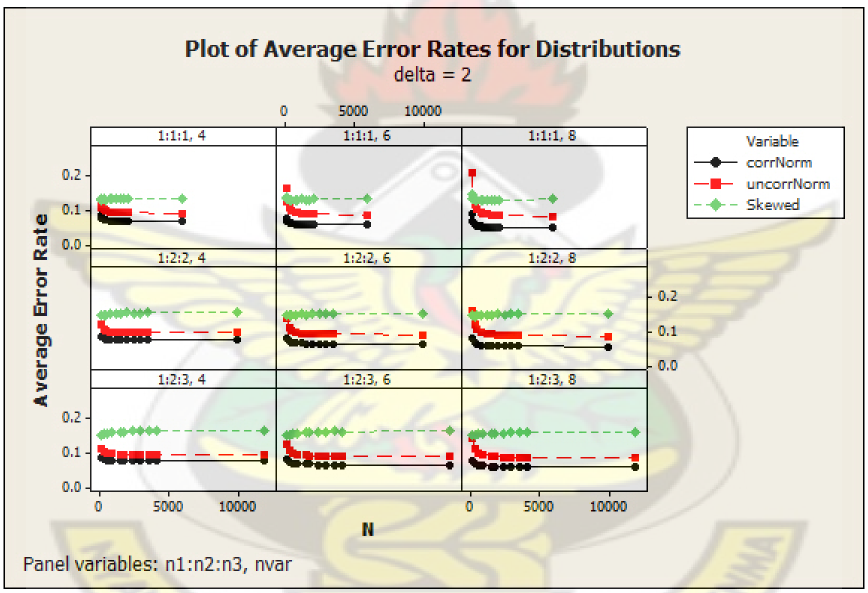

Figure F.1: Average Error Rates for individual sample size ratios and distributions for $\delta = 2$

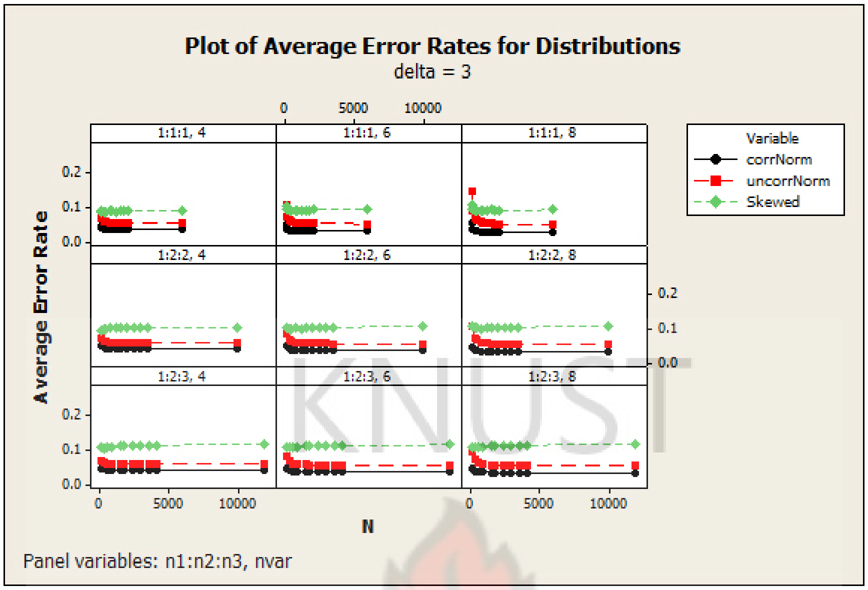

Figure F.2: Average Error Rates for individual sample size ratios and distributions for $\delta = 3$

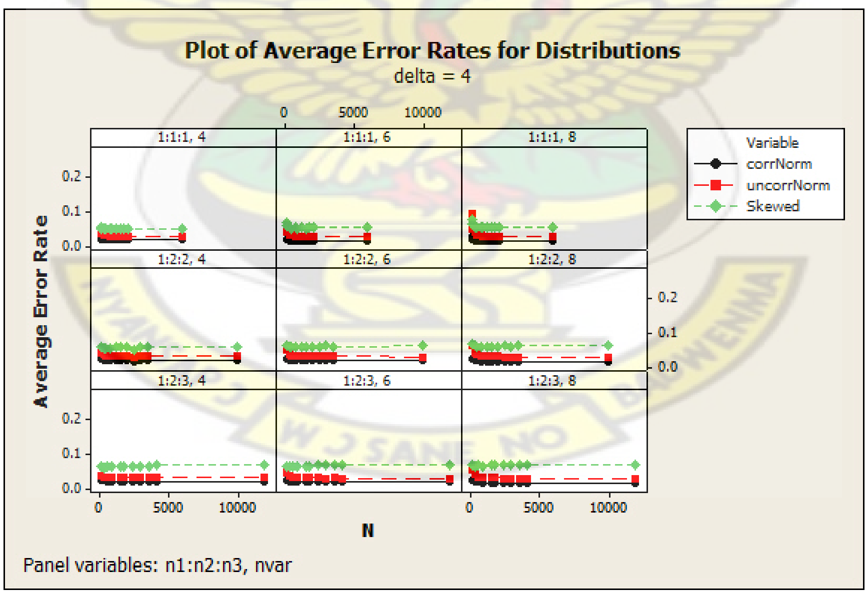

Figure F.3: Average Error Rates for individual sample size ratios and distributions for $\delta = 4$

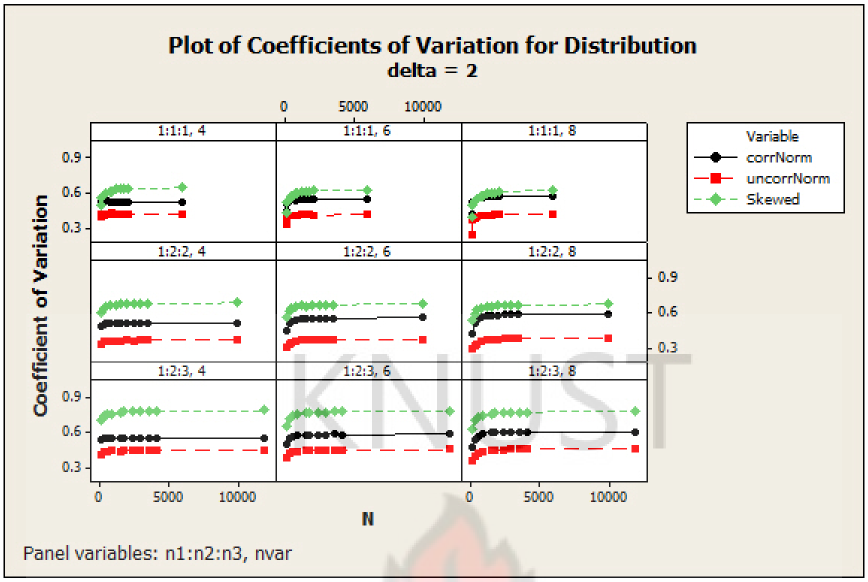

Figure F.4: Coefficients of Variation for individual sample size ratios and distributions for $\delta = 2$

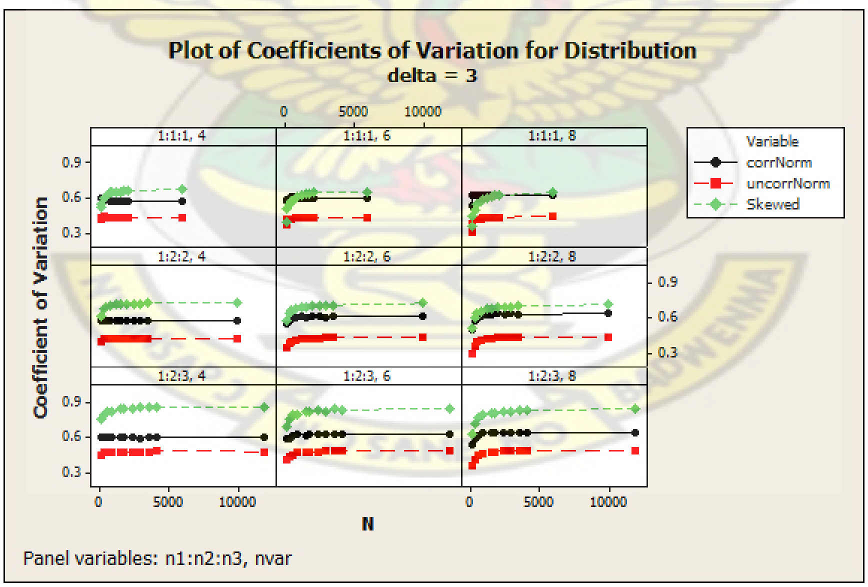

Figure F.5: Coefficients of Variation for individual sample size ratios and distributions for $\delta = 3$

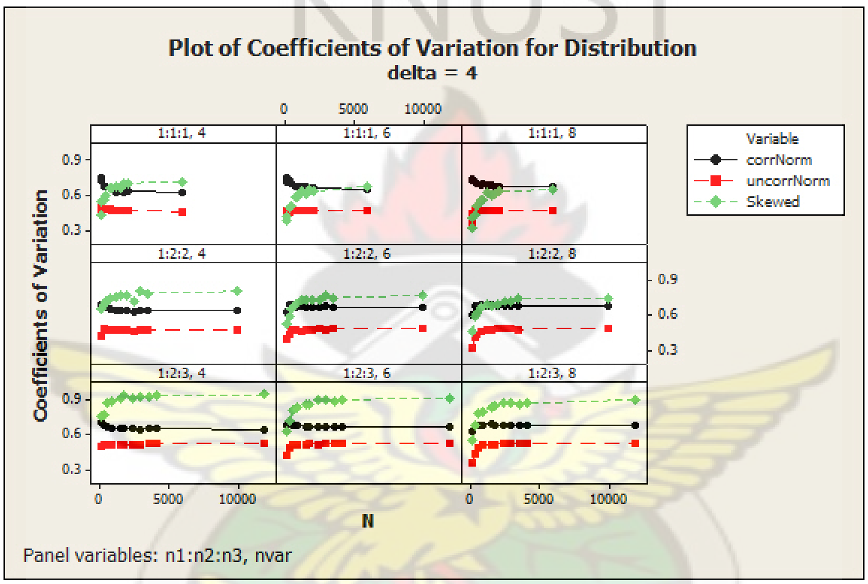

Figure F.6: Coefficients of Variation for individual sample size ratios and distributions for $\delta = 4$