KWAME NKRUMAH UNIVERSITY OF SCIENCE AND TECHNOLOGY
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DEPARTMENT OF INDUSTRIAL MATHEMATICS

OPTIMAL LOAN PORTFOLIO
(A CASE STUDY OF ATWEABAN RURAL BANK, DUAYAW NKWANTA)

By

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A Thesis Submitted to the School of Graduate Studies, Kwame Nkrumah University of Science and Technology in partial fulfillment of the requirements for the award of a Master of Science degree in Industrial Mathematics

DECLARATION

I hereby declare that this submission is my own work towards the Master of Science degree and that, to the best of my knowledge, it contains no material previously published by another person nor material which has been accepted for award of any other degree of the university, except due acknowledgement has been made in the text.

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DEDICATION

To the Glory of God

I like to dedicate this project to my late junior sister and my mother, Gifty Asare and Miss. Margaret Amoah respectively.

ABSTRACT

Many Ghanaians both in the formal sector and the informal sector take loans for various reasons some being investment in businesses or their wards education. Others also take loans to acquire personal properties such as houses and cars. Most people rely on Banks for Loans.

Due to poor allocation of funds by most banks to prospective loan seekers the banks are not able to maximize their profits. In view of this monies that can be used for social services in the community in which they operate go into bad debt.

The main aim of this work is to develop Linear Programming model to help the Atweaban Rural Bank at Duayaw Nkwanta in the Tano North District of the Brong Ahafo Region to allocate their funds to prospective loan seekers in order for them to maximize their profits.

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CHAPTER ONE

INTRODUCTION

1.0 BACKGROUND TO THE STUDY

Ghana's financial system is based on a number of banks and non-banking financial institutions, including the Bank of Ghana, which, as the Central Bank, has the responsibility of advising the government on the implementation and control of monetary policies. Other institutions include commercial and merchant banks, discount houses, insurance companies, leasing companies, venture capital, a mortgage finance institution, and a stock exchange. Direct financing of projects in the country is provided by the commercial and other banking institutions.

In an effort to ensure systematic development of the banking system, the Central Bank (Bank of Ghana), in addition to its traditional functions (for example formulation of monetary policies), also has the responsibility to ensure that banking is responsive to the needs of the public.

In attempt to encourage the establishment of new types of financial institutions, the Bank of Ghana pursues a liberal policy with regard to entry into the banking system, and is actively involved in the promotion of development and rural banking as well as in the establishment of discount houses.

The minimum paid-up capital required for entry into the banking system is as follows:

Ghanaian Banking Business: Paid-up capital of not less than Two Thousand Ghana cedis (GH¢2000).

Foreign Banking Business: Paid-up capital of not less than Five Thousand Ghana cedis (GH¢5000), of which not less than Three Thousand cedis (GH¢3000) shall be brought into Ghana as convertible currency;

Other Banking Business: to be determined by the central bank of Ghana.

Commercial Banks: Commercial Banks are currently required to maintain a minimum of 57% of total deposits in liquid reserves. The Bank of Ghana fixes the Central Bank 'rediscount rate', which is used as the benchmark upon which commercial banks base their interest rates. There are several Commercial and Development banks in Ghana.

The National Investment Bank: Is an industrial development bank providing financial assistance to manufacturing and processing industries, including agro-industrial projects. It maintains branches in all regions of the country.

The Agricultural Development Bank: Serves principally the agricultural sector - food production, livestock breeding, poultry farming and processing of agricultural produce. It has over Thirty one (31) branches throughout Ghana.

Leasing Companies: Though 'hire purchase' activities were conducted by the banks it was not until 1992 that a leasing law was enacted in Ghana. Since then, over three leasing companies have emerged and they are offering among others equipment leasing in Ghana. These include Ghana Leasing Company Limited, General Leasing Company Limited and LeaseAfric.

Venture Capital: Venture Capital provides capital for start-ups and high risk ventures. The Ghana Venture Capital Fund Limited (GVCF) - is managed by the Venture Fund Management Company. The Commonwealth Development Corporation is the lead investor and was joined by a few local banks and other foreign financial institutions. It has focused mainly on medium-sized, indigenous growth companies with expansion projects and shied away from start-ups because of the higher risks entailed.

Mortgage Financing: The Home Finance Company (HFC) is the leading secondary mortgage financing institution in Ghana. HFC was established in 1990 as the implementing agency for a housing finance pilot scheme component for an Urban II Project provided to the Republic of Ghana by the International Development Association (World Bank).

The IDA was joined by Social Security and National Insurance Trust (SSNIT), Merchant Bank and a number of insurance companies.

Insurance Companies: There are over twenty four (24) insurance companies currently operating in Ghana.

Discount Houses: In a bid to improve financial intermediation in the country, the non-bank financial institutions comprising the insurance and trust companies have joined forces with the banking institutions to establish discount houses in order to bring into single market institutions with cash balances for their intensive and effective use. These include the Consolidated Discount House and the Securities Discount Company, Gold Coast Securities Limited, and National Trust Holding Company (NTHC).

Non-Bank Financial Institutions: Ghana's non-bank financial institutions include the Social Security and National Insurance Trust (SSNIT), the Ghana Stock Exchange, Insurance companies, discount houses and other institutions.

Traditionally, rural development credit has been provided by two types of sources: institutional and non-institutional. In rural communities, non-institutional credit is provided by moneylenders, relatives, friends, traders, commission agents, cooperatives, consumers, distributors of farm inputs, and processors of agricultural products. Research has shown that the most common providers of loans in rural areas are friends and relatives who usually charge no interest or collateral (FAO 1994).

This credit market is small, however, and the total credit from these non-institutional sources is insufficient to implement rural development programs.

For rural development to proceed at a smooth pace, larger institutional sources of credit need to be created. In Ghana, institutional sources of credit are the commercial banks, the Agricultural Development Bank, the National Investment Banks, and the Bank of Ghana Rural Banks. Until recently very few rural people, other than wealthy farmers and businessmen, had access to credit from these sources. The lack of interest in small rural credits by the National Investment Bank and the commercial banks is explained by the high cost of administering a large number of small credits spread over a wide area, coupled with the comparatively high level of default that has often accompanied small credits. The inability of rural borrowers to offer adequate security for loans, and the enormous risks associated with agricultural production, are the typical reasons given for the urban-based bias of commercial lending. The Agricultural Development Bank was created to service the rural sector in particular. It too, however, eventually began to concentrate on traditional urban-based banking activities.

To overcome many of these difficulties, the Ghanaian government, through the Bank of Ghana introduced the idea of rural banking into the country in 1976. According to the Association of Rural Banks (1992),

"The aims of Rural Banks are:

i. to stimulate banking habits among rural dwellers;

ii. to mobilize resources locked up in the rural areas into the banking systems to facilitate development; and

iii. to identify viable industries in their respective catchment [areas] for investment and development."

Due to these liberal policies of the Bank of Ghana many banks across Africa are opening branches in Ghana, this has also facilitated the opening of a lot of Rural

Banks across the country. Currently there are over One Hundred and Twenty (120) Rural Banks in the country.

Rural Banks are unit banks established to provide facilities for the rural communities in which they are located. They are owned, managed and patronized by the local people. Some of these banks also operate agencies to cater for communities that are located far from the bank's facilities.

Savings mobilized through rural banks are invested in small-scale agricultural activities, cottage industries, transportation and trading. Rural banks also provide commercial banking services such as giving loans to people within the community in which they operate.

1.1 LOANS

A loan is a type of debt. Like all debt instruments, a loan entails the redistribution of financial assets over time, between the lender and the borrower.

In a loan, the borrower initially receives or borrows an amount of money, called the principal, from the lender, and is obligated to pay back or repay an equal amount of money to the lender at a later time.

Typically, the money is paid back in regular installments, or partial repayments in an annuity, each installment is the same amount. The loan is generally provided at a cost, referred to as interest on the debt, which provides an incentive for the lender to engage in the loan. In a legal loan, each of these obligations and restrictions is enforced by contract, which can also place the borrower under additional restrictions known as loan covenants. Acting as a provider of loans is one of the principal tasks for financial institutions.

Due to poor allocation of their loan disbursement they are not able to optimize their profits when they give out these loans, hence monies that could have been used to offer social services in the community in which they operate goes into “Bad Debts”.

A model is proposed to help Rural Banks allocate their funds available for loan disbursement optimally. We used a case study of Atweaban Rural Bank at Duayaw - Nkwanta in the Tano North District of the Brong Ahafo Region.

1.2 TYPES OF LOANS

1.2.1 SECURED LOAN

A secured loan is a loan in which the borrower pledges some asset (e.g. a car or property) as collateral for the loan.

1.2.2 UNSECURED LOAN

Unsecured loans are monetary loans that are not secured against the borrower's assets. These may be available from financial institutions under many different guises or marketing packages.

1.2.3 MORTGAGE LOAN

A mortgage is a legal instrument that pledges a house or other real estate as security for repayment of a loan. By providing guarantee that the loan will be paid back, a mortgage enables a person to buy property without having the funds to pay for it outright.

1.2.4 CREDIT

A credit denotes transaction involving the transfer of money or other property on promise of repayment, usually at a fixed future date. The transferor thereby becomes a creditor, and the transferee, a debtor.

1.3 SOURCES OF LOANS

Banking is the business of providing financial services to consumers and businesses. The basic services a bank provides are checking accounts, which can be used like money to make payments and purchase goods and services; savings accounts and time deposits that can be used to save money for future use; loans that consumers and businesses can use to purchase goods and services and basic cash management services such as check cashing and foreign currency exchange.

Four types of banks specialize in offering these basic banking services; these are commercial banks, savings and loan associations, savings banks, and credit unions.

1.3.1 COMMERCIAL BANKS

A bank is any financial institution that receives, collects, transfers, pays, exchanges, lends, invests, or safeguards money for its customers. This broader definition includes many other financial institutions that are not usually thought of as banks but which nevertheless provide one or more of these broadly defined banking services.

These institutions include finance companies, investment companies, investment banks, insurance companies, pension funds, security brokers and dealers, mortgage companies.

1.3.2 CREDIT UNIONS

These are financial cooperatives and credit associations that provide loans to its members at lower rates of interest than would otherwise be available.

The capital funds of credit unions come from the purchase of shares by members, who receive yearly dividends on the basis of their investment. Credit unions are operated for the mutual benefit of their members and are usually formed by persons who share a common bond, such as membership in a church, lodge, trade union, or professional association. Many corporations have assisted their employees in establishing credit unions. The loans are usually for the acquisition of consumer goods rather than for the purchase of real estate.

1.3.3 SAVINGS AND LOANS INSTITUTIONS

Savings Institutions are banks or associations originally established to encourage personal thrift through the deposit of individual or family savings that accrued earnings in the form of interest.

1.3.4 TRUST COMPANIES

Trust Companies are corporations formed to act as trustees according to the terms of contracts known as trust agreements.

1.4 ABUSES IN LOANS

One form of abuse in the granting of loans involves granting a loan in order to put the borrower in a position that one can gain advantage over.

Another form of abuse is where the lender charges excessive interest. In different time periods and cultures the acceptable interest rate has varied, from no interest at all to unlimited interest rates. Credit card companies in some countries have been accused by consumer organizations of lending at usurious interest rates and making money out of frivolous extra charges.

Abuses can also take place in the form of the customer abusing the lender by not repaying the loan or with intent to defraud the lender.

1.5 FUNCTIONS OF BANKS

1.5.1 FUNDAMENTAL DUTIES

A Bank carries out money and credit policy in accordance with the needs of the economy and so as to maintain price stability. The Bank takes necessary measures to protect the domestic and international value of the national currency and regulates its volume and circulation. It also extends credits to banks and conducts open market operations in order to regulate money supply and liquidity in the economy.

Moreover, the Bank determines the terms and types of deposits, as well as their maturity dates and validity periods, and the parity of the national currency against gold and foreign currencies. It manages gold and foreign exchange reserves and trades in foreign exchange and precious metals on the stock exchange.

The Bank, in particular, carries out the duties of financial and economic advisor, fiscal agent and treasurer to the Government.

1.5.2 FUNDAMENTAL POWERS

The Bank has the privilege of issuing banknotes and the authority to take decisions on money and credit issues and to submit proposals to the Government. The Bank determines the rediscount, discount and interest rates applicable to its own transactions.

1.5.3 ADVISORY DUTIES

The Bank presents to the Government, when required, its views with regard to measures to be taken on money and credit, and submits advisory opinions on matters related to implementation of the Banking Law or on banking and credit issues in general, upon request of the Government.

The Bank can also be consulted prior to any decision granting permission for the establishment of banks and other financial institutions, as well as for the liquidation of such institutions for which the power to liquidate rests with the Government.

1.6 PROBLEM STATEMENT

Due to poor allocation of funds some rural banks record marginal profits with some running at a lost.

The main aim of this project is to propose a linear model subject to some constraints for a newly established rural bank at Duayaw - Nkwanta named Atweaban Rural Bank to enable them disburse their funds allocated for loans optimally leading to maximization of profits.

1.7 METHODOLOGY

In order for the bank to maximize their profit, the proposed model will be based strictly on the Bank's Loan Policy and its previous history on loan disbursement. The model will be solved using the Simplex Algorithm.

The Linear Programming model has three basic components, that is the objective function which is to be optimized (Maximized or minimized), the constraints or limitation and the non negativity constraint.

In general the Linear Programming model can be formulated as follows

Let $x_1, x_2, ..., x_n$ be n decision variables with m constraints, then

The objective function:

Maximize or Minimized

$$Z = \sum_{j=1}^{n} c_j x_j$$

Subject to the m constraints

$$\sum_{i=1}^{m} \sum_{j=1}^{n} a_{ij} x_j \ (\leq = \geq) \ b_i$$

The Non negativity constraints

$$x_j \geq 0$$

$$i = 1, 2, 3...., \text{m and } j = 1, 2, 3, ..., \text{n}$$

The simplex method is an iterative procedure for solving Linear Programming Problems in a finite number of steps. This method provides an algorithm which consists of moving from one vertex of the region of feasible solution to another in such a manner that the value of the objective function at the succeeding vertex is less or more than the previous vertex. This procedure is repeated and since the number of vertices is finite, the method leads to an optimal vertex in a finite number of steps or it may indicate the existences of unbounded solution.

1.8 OBJECTIVES

The main objectives of this study are:

A. To explore ways of disbursing funds allocated for loans effectively and efficiently in order to optimize profit margin of Atweaban Rural Bank, Duayaw Nkwanta.

B. To serve as reference material in the libraries and the Internet for students who wish to undertake research into the similar field in the near future.

C. To serve as a scientific method of providing executive with an analytical and objective basis for decision making.

1.9 JUSTIFICATION

The institution of Banks is one of the fastest growing institutions in the Ghana which has a tremendous impact on the economy and the society. Among other things banks also give loans prospective loan seekers.

Outdated and ineffective loan policies can contribute to a range of problems. Introducing a loan product that is not adequately addressed in the written loan policy can create a variety of challenges for the lending staff and involve risks that management did not anticipate.

If lending authorities loan limitation are not revised when circumstances change, a Bank could be operating within guidelines that are too restrictive or too lenient. If guidelines do not comply with current laws and rules, lending decisions may not reflect best practices or regulatory requirements. A loan policy that does not anticipate risks can lead to asset quality problems and poor earnings.

The bank might run at a lost or even collapse if they are not able to retrieve all the loans they give out. Due to this, a more scientific approach must be employed by banks to ensure adequate, effective and efficient distribution of funds they have available for loans to ensure constant growth of these banks. When banks run efficiently they are able to allocate a larger amount of its funds for social services in the community in which they operate.

The proposed model is going to help banks to efficiently distribute the funds they have available for loan in order to maximize their profit. The proposed model will also help decision makers at the Bank to formulate prudent and effective loan policies. This makes this study justifiable and worthwhile.

1.10 SUMMARY

In this chapter, a brief history of Rural Banks and some of their social responsibilities were given. The objectives of the work were also presented. In the next chapter, we shall review some literature in the area of linear programming.

CHAPTER TWO

LITERATURE REVIEW

2.0 INTRODUCTION

In this section of the work, other people's works, journals of various fields of research using linear programming programs will be considered.

Stewart et al., (2008) examined the numerical implementation of a linear programming (LP) formulation of stochastic control problems involving singular stochastic processes. The decision maker has the ability to influence a diffusion process through the selection of its drift rate (a control that acts absolutely continuously in time) and may also decide to instantaneously move the process to some other level (a singular control). The first goal of the paper is to show that linear programming provides a viable approach to solving singular control problems. A second goal is the determination of the absolutely continuous control from the LP results and is intimately tied to the particular numerical implementation. The original stochastic control problem is equivalent to an infinite-dimensional linear program in which the variables are measures on appropriate bounded regions. The implementation method replaces the LP formulation involving measures by one involving the moments of the measures. This moment approach does not directly provide the optimal control in feedback form of the current state. The second goal of the paper is to show that the feedback form of the optimal control can be obtained using sensitivity analysis.

Knowledge of the presence of certain special structures can be advantageous in both the formulation and solution of linear programming problems. Thus it is desirable that linear programming software offer the option of specifying such structures explicitly. As a step in this direction, Fourer et al., (1995) described extensions to an algebraic modeling language that encompass piecewise-linear, network and related structures. Their emphasis is on the modeling considerations that motivate these extensions, and on the design issues that arise in integrating these extensions with the general-purpose features of the language. They observe that extensions sometimes make models faster to translate as well as to solve, and that they permit a "column-wise" formulation of the constraints as an alternative to the "row-wise" formulation most often associated with algebraic languages.

Consider a linear-programming problem in which the "right-hand side" is a random vector whose expected value is known and where the expected value of the objective function is to be minimized. An approximate solution is often found by replacing the "right-hand side" by its expected value and solving the resulting linear programming problem. Mandansky (1960) gave conditions for the equality of the expected value of the objective function for the optimal solution and the value of the objective function for the approximate solution; bounds on these values were also given. In addition, the relation between this problem and a related problem, where one makes an observation on the "right-hand side" and solves the (nonstochastic) linear programming problem based on this observation, was discussed.

Greenberg et al., (1986) developed a framework for model formulation and analysis to support operations and management of large-scale linear programs from the combined capabilities of camps and analyze. Both the systems were reviewed briefly and the interface which integrates the two systems was then described. The model formulation, matrix generation, and model management capability of camps and the complementary model and solution analysis capability of analyze were presented within a unified framework. Relevant generic functions were highlighted, and an example was presented in detail to illustrate the level of integration achieved in the current prototype system. Some new results on discourse models and model management support were given in a framework designed to move toward an 'intelligent' system for linear programming modeling and analysis.

Church et al., (1963) used linear programming procedures with the aid of an electronic computer to formulate fattening rations for weaned calves. Rations were formulated using digestible energy or estimated net energy, crude protein, crude fiber, calcium and phosphorus. Rations formulated on digestible energy bases had specifications for 1.24, 1.36 or 1.48 megcal. Per lb. of feed, and those with estimated net energy for 0.581, 0.638 or 0.694 megcal. Per lb. Specifications for crude protein (11.5%), calcium (0.75%), phosphorus (0.50%) and salt (0.50%) were the same for each ration. Crude fiber was restricted to a maximum of 15% and a minimum of 8%. Minimum and/or maximum specifications were used for several feedstuffs; alfalfa meal (5 and 15%), beet pulp (min. 10%) and molasses (5 and 10%).

Chemical analyses on the computer-formulated rations indicated reasonably good agreement between specifications and analyses for crude protein, crude fiber and phosphorus. Animal performance data demonstrated that estimated net energy was superior to digestible energy as a basis of ration formulation. Cattle fed the net energy-formulated rations gained more rapidly, ate more feed and energy, were more efficient, had more marbling in the rib-eye but less rib-eye area per cwt. Of carcass than did cattle fed on DE-formulated rations. Data from this trial showed that linear programming procedures can be effectively used to formulate cattle ration.

Sinha et al., (2003) proposed a modified fuzzy programming method to handle higher level multi-level decentralized programming problems (ML (D) PPs). They presented a simple and practical method to solve the same. This method overcomes the subjectivity inherent in choosing the tolerance values and the membership functions. They considered a linear ML (D) PP and applied linear programming (LP) for the optimization of the system in a supervised search procedure, supervised by the higher level decision maker (DM). The higher level DM provides the preferred values of the decision variables under his control to enable the lower level DM to search for his optimum in a narrower feasible space. The basic idea is to reduce the feasible space of a decision variable at each level until a satisfactory point is sought at the last level.

Wu et al., (2000) proposed a neural network model for linear programming that is designed to optimize radiotherapy treatment planning (RTP). This kind of neural network can be easily implemented by using a kind of `neural' electronic system in order to obtain an optimization solution in real time.

They first gave an introduction to the RTP problem and constructed a non-constraint objective function for the neural network model. They adopted a gradient algorithm to minimize the objective function and designed the structure of the neural network for RTP. Compared to traditional linear programming methods, this neural network model can reduce the time needed for convergence, the size of problems (i.e., the number of variables to be searched) and the number of extra slack and surplus variables needed. They obtained a set of optimized beam weights that resulted in a better dose distribution as compared to that obtained using the simplex algorithm under the same initial condition.

Jianq et al., (2004) proposed a novel linear programming based method to estimate arbitrary motion from two images. The proposed method always finds the global optimal solution of the linearized motion estimation energy function and thus is much more robust than traditional motion estimation schemes. As well, the method estimates the occlusion map and motion field at the same time. To further reduce the complexity of even a complexity-reduced pure linear programming method they presented a two-phase scheme for estimating the dense motion field. In the first step, they estimated a relatively sparse motion field for the edge pixels using a non-regular sampling scheme, based on the proposed linear programming method. In the second step, they set out a detail-preserving variational method to upgrade the result into a dense motion field. The proposed scheme is much faster than a purely linear programming based dense motion estimation scheme. And, since they used a global optimization method linear programming in the first estimation step, the proposed two-phase scheme was also significantly more robust than a pure variational scheme.

Vimonsatit et al., (2003) proposed a linear programming (LP) formulation for the evaluation of the plastic limit temperature of flexibly connected steel frames exposed to fire. Within a framework of discrete models and piecewise linearized yield surfaces, the formulation was derived based on the lower-bound theorem in plastic theory, which lead to a compact matrix form of an LP problem. The plastic limit temperature was determined when the equilibrium and yield conditions were satisfied. The plastic mechanism can be checked from the dual solutions in the final simplex tableau of the primal LP solutions.

Three examples were presented to investigate the effects of the partial-strength beam-to-column joints. Eigenvalue analysis of the assembled structural stiffness matrix at the predicted limit temperature was performed to check for structural instability.

The advantage of the proposed method is that it is simple, computationally efficient, and its solutions provide the necessary information at the limit temperature. The method can be used as an efficient tool to a more refined but computationally expensive step-by-step historical deformation analysis.

Jinbo et al., (2004) presented a novel linear programming approach to do protein 3-dimensional (3D) structure prediction via threading. Based on the contact map graph of the protein 3D structure template, the protein threading problem was formulated as a large scale integer programming (IP) problem. The IP formulation was then relaxed to a linear programming (LP) problem, and then solved by the canonical branch-and-bound method. The final solution is globally optimal with respect to energy functions. In particular, our energy function includes pair wise interaction preferences and allowing variable gaps which are two key factors in making the protein threading problem NP-hard. A surprising result was that, most of the time; the relaxed linear programs generate integral solutions directly. Their algorithm has been implemented as a software package RAPTOR-Rapid Protein Threading by Operation Research technique. Large scale benchmark test for fold recognition shows that RAPTOR significantly outperforms other programs at the fold similarity level. The CAFASP3 evaluation, a blind and public test by the protein structure prediction community, ranks RAPTOR as top one (1), among individual prediction servers, in terms of the recognition capability and alignment accuracy for Fold Recognition (FR) family targets. RAPTOR also performs very well in recognizing the hard Homology Modeling (HM) targets.

Kas et al., (1996) studied linear inverse problems where a vector with positive components was chosen from a feasible set defined by linear constraints. The problem requires the minimization of a certain function which is a measure of distance from a priori guess. An explicit and perfect dual of the resulting programming problem was shown, the corresponding duality theorem and optimality criteria were proven, and an algorithm solution was proposed.

Nace et al., (2006) introduced the lexicographically minimum load linear programming problem, and they provided a polynomial approach followed by the proof of correctness. This problem has applications in numerous areas where it is desirable to achieve an equitable distribution or sharing of resources.

They considered the application of their technique to the problem of lexicographically minimum load in capacitated multi-commodity networks and discussed a special non-linear case, the so-called Klein rock load function. They defined the lexicographically maximum load linear programming problem and deduced a similar approach.

An application in the lexicographically maximum concurrent flow problem was depicted followed by a discussion on the minimum balance problem as a special case of the lexicographically maximum load problem.

Konickova (2006) said a linear programming problem whose coefficients are prescribed by intervals is called strongly unbounded if each linear programming problem obtained by fixing coefficients in these intervals is unbounded. In the main result of the paper a necessary and sufficient condition for strong unboundedness of an interval linear programming problem was described. In order to have a full picture they also showed conditions for strong feasibility and strong solvability of this problem. The necessary and sufficient conditions for strong feasibility, strong solvability and strong unboundedness can be verified by checking the appropriate properties by the finite algorithms. Checking strong feasibility and checking strong solvability are NP-hard. This shows that checking strong unboundedness is NP-hard as well.

Optimal solutions of Linear Programming problems may become severely infeasible if the nominal data is slightly perturbed. Aharon et al., (2000) demonstrated this phenomenon by studying ninety (90) LPs from the well-known NETLIB collection. They then applied the Robust Optimization methodology to produce “robust” solutions of the above LPs which are in a sense immured against uncertainty. Surprisingly, for the NETLIB problems these robust solutions nearly lose nothing in optimality.

A linear programming problem in an inequality form having a bounded solution is solved error-free using an algorithm that sorts the inequalities, removes the redundant ones, and uses the p-adic arithmetic. Lakshmikantham et al., (1997).

Yoshito (2004) considered the problem of finite dimensional approximation of the dual problem in abstract linear programming approach to control system design. A constraint qualification that guarantees the existence of a sequence of finite dimensional dual problems that computes the true optimal value. The result is based on the averaging integration by a probability measures.

A matrix is sought that solves a given dual pair of systems of linear algebraic equations. Necessary and sufficient conditions for the existence of solutions to this problem were obtained, and the form of the solutions was found. The form of the solution with the minimal Euclidean norm was indicated. Conditions for this solution to be a rank one matrix were examined. On the basis of these results, an analysis was performed for the following two problems: modifying the coefficient matrix for a dual pair of linear programs (which can be improper) to ensure the existence of given solutions for these programs, and modifying the coefficient matrix for a dual pair of improper linear programs to minimize its Euclidean norm. Necessary and sufficient conditions for the solvability of the first problem were given, and the form of its solutions was described. For the second problem, a method for the reduction to a nonlinear constrained minimization problem was indicated, necessary conditions for the existence of solutions were found, and the form of solutions was described. Numerical results were presented. Erokhin (2007).

Biswal et al., (1998) developed an approach to solve probabilistic linear programming problems with exponential random variables. The first step involves obtaining the probability density function (p.d.f.) of the linear combination of n independent exponential random variables. Probabilistic constraints are then transformed to the deterministic constraints using the p.d.f. The resulting non-linear deterministic model is then solved using a non-linear programming solution method.

Frangioni et al., (2009) discussed a general framework for outer approximation type algorithms for the canonical DC optimization problem. The algorithms rely on a polar reformulation of the problem and exploit an approximated oracle in order to check global optimality. Consequently, approximate optimality conditions were introduced and bounds on the quality of the approximate global optimal solution were obtained. A thorough analysis of properties which guarantee convergence was carried out; two families of conditions were introduced which lead to design six implementable algorithms, whose convergence can be proved within a unified framework.

Cherubini et al., (2009) described an optimization model which aims at minimizing the maximum link utilization of IP telecommunication networks under the joint use of the traditional IGP protocols and the more sophisticated MPLS-TE technology. The survivability of the network was taken into account in the optimization process implementing the path restoration scheme.

This scheme benefits of the Fast Re-Route (FRR) capability allowing service providers to offer high availability and high revenue SLAs (Service Level Agreements). The hybrid IGP/MPLS approach relies on the formulation of an innovative Linear Programming mathematical model that, while optimizing the network utilization, provides optimal user performance, efficient use of network resources, and 100% survivability in case of single link failure. The possibility of performing an optimal exploitation of the network resources throughout the joint use of the IGP and MPLS protocols provides a flexible tool for the ISP (Internet Service Provider) networks traffic engineers. The efficiency of the proposed approach was validated by a wide experimentation performed on synthetic and real networks. The obtained results showed that a small number of LSP tunnels have to be set up in order to significantly reduce the congestion level of the network while at the same time guaranteeing the survivability of the network.

They applied this approach to a quadratic-cost single-commodity network design problem, comparing the newly developed algorithm with those based on both the standard continuous relaxation and the two usual variants of PR relaxation.

Harlan et al., (1983) reported on the solution to optimality of ten large-scale zero-one linear programming problems. All problem data come from real-world industrial applications and are characterized by sparse constraint matrices with rational data. About half of the sample problems have no apparent special structure; the remainder show structural characteristics that their computational procedures do not exploit directly.

By today's standards, their methodology produced impressive computational results, particularly on sparse problems having no apparent special structure.

The computational results on problems with up to two thousand seven hundred and fifty (2750) variables strongly confirm their hypothesis that a combination of problem preprocessing, cutting planes, and clever branch-and-bound techniques permit the optimization of sparse large-scale zero-one linear programming problems, even those with no apparent special structure, in reasonable computation times. Their results indicate that cutting-planes related to the facets of the underlying polytope are an indispensable tool for the exact solution of this class of problem. To arrive at these conclusions, they designed an experimental computer system PIPX that uses the IBM linear programming system MPSX/370 and the IBM integer programming system MIP/370 as building blocks. The entire system is automatic and requires no manual intervention.

In contrast, it is common practice for today's mixed integer programming solvers to just discard infeasible sub problems and the information they reveal.

In the maximum feasible subsystem problem, given an infeasible linear system $Ax \geq b$, one wishes to find a feasible subsystem containing a maximum number of inequalities. This NP-hard problem has interesting applications in a variety of fields. In some challenging applications in telecommunications and computational biology one faces very large maximum feasible subsystem instances with up to millions of inequalities in thousands of variables. Belotti et al., (2005) proposed to tackle large-scale instances of

Maximum feasible subsystem using randomized and thermal variants of the classical relaxation method for solving systems of linear inequalities.

They established lower bounds on the probability that these methods identify an optimal solution within a given number of iterations.

These bounds, which are expressed as a function of a condition number of the input data, imply that with probability one these randomized methods identify an optimal solution after finitely many iterations. Computational results obtained for medium- to large-scale instances arising in the design of linear classifiers, in the planning of digital video broadcasts and in the modeling of the energy functions driving protein folding, indicate that an efficient implementation of such a method perform very well in practice.

Industrial switching involves moving materials on rail cars within or between industrial complexes and connecting with other rail carriers. Planning tasks include the making up of trains with a minimum shunting effort, the feasible and timely routing through an in-plant rail network on short paths, and assigning and scheduling of locomotives under safety and network capacity aspects. A human planner must often resort to routine and simple heuristics, not least for the reason of unavailability of computer aided suggestions. Marco et al., (2005) proposed mixed integer programming models to capture the whole process at once in order to obtain optimal or provably good solutions. Column generation allows them to work with linear programming relaxations with a huge number of variables.

This popular technique has almost attained an industry standard level and usually enables one to set up appropriate models quickly. The challenge hides in the actual implementation which still needs tailoring to the particular application. Their work is based on practical data from a German in-plant railroad.

Gay (1997) told how to make solvers work with AMPL's solve command. It describes an interface library, amplsolver.a, whose source is available from netlib as individual files, as gzip-compressed files, or in a single tar file. Examples include programs for listing LPs, automatic conversion to the LP dual (shell-script as solver), solvers for various nonlinear problems (with first and sometimes second derivatives computed by automatic differentiation), and getting C or Fortran 77 for non-linear constraints, objectives and their first derivatives. Drivers for various well known linear, mixed-integer, and nonlinear solvers provide more examples.

Practical large-scale mathematical programming involves more than just the application of an algorithm to minimize or maximize an objective function. Before any optimizing routine can be invoked, considerable effort must be expended to formulate the underlying model and to generate the requisite computational data structures. AMPL is a new language designed to make these steps easier and less error-prone.

AMPL closely resembles the symbolic algebraic notation that many modelers use to describe mathematical programs, yet it is regular and formal enough to be processed by a computer system; it is particularly notable for the generality of its syntax and for the variety of its indexing operations.

Fourer et al., (1990) implemented a translator that takes as input a linear AMPL model and associated data, and produces output suitable for standard linear programming optimizers.

Both the language and the translator admit straightforward extensions to more general mathematical programs that incorporate nonlinear expressions or discrete variables.

Diverse problems in optimization, engineering, and economics have natural formulations in terms of complementarity conditions, which state (in their simplest form) that either a certain nonnegative variable must be zero or a corresponding inequality must hold with equality, or both. A variety of algorithms have been devised for solving problems expressed in terms of complementarity conditions.

It is thus attractive to consider extending algebraic modeling languages, which are widely, used for sending ordinary equations and inequality constraints to solvers, so that they can express complementarity problems directly. Ferris et al., (1999) described an extension to the AMPL modeling language that can express the most common complementarity conditions in a concise and flexible way, through the introduction of a single new “complements” operator.

They presented details of an efficient implementation that incorporates an augmented pre-solve phase to simplify complementarity problems, and that converts complementarity conditions to a canonical form convenient for solvers.

Column generation algorithms are instrumental in many areas of applied optimization, where linear programs with an enormous number of columns need to be solved.

Although successfully employed in many applications, these approaches suffer from well-known instability issues that somewhat limit their efficiency. Building on the theory developed for non-differentiable optimization algorithms, a large class of stabilized column generation algorithms can be defined which avoid the instability issues by using an explicit stabilizing term in the dual; this amounts at considering a (generalized) augmented Lagrangian of the primal master problem. Since the theory allows for a great degree of flexibility in the choice and in the management of the stabilizing term, one can use piecewise-linear or quadratic functions that can be efficiently dealt with off-the-shelf solvers.

The effectiveness in practice of this approach is demonstrated by extensive computational experiments on large-scale Vehicle and Crew Scheduling problems. Also, the results of a detailed computational study on the impact of the different choices in the stabilization term (shape of the function, parameters), and their relationships with the quality of the initial dual estimates, on the overall effectiveness of the approach are reported, providing practical guidelines for selecting the most appropriate variant in different situations. Amor et al., (2009).

A simplified non-linear dynamic model of greenhouse crop growth with constraints on the state and the control signal is presented. The weather is assumed to be known. The optimization criterion is to minimize the heating cost.

The resulting optimal control problem is analyzed from the point of view of the Pontryagin maximum Principle. It is shown that this particular problem can be solved numerically by linear programming, and that it can also be formulated as a network flow problem.

The solution is presented and it is found that, in cold weather, it is worthwhile deviating from the "blueprint" and instead heating during the night when the thermal screens are in place and the heat loss small, while maintaining the minimal allowed temperature during the daytime. Assuming a perfect weather forecast, heating costs in the analyzed case are lowered by 22%, relative to a "blueprint" operation of keeping the temperature constant. Gutman et al., (2006).

The main objective of this work was to evaluate, through Integer Programming, the consequences of using Linear Programming with post rounding out of the responses, with emphasis on even-aged forest regulation.

Thus, a simplified forest regulation problem was proposed out and solved by model by means of Linear Programming, Linear Programming with post rounding out, and Integer Programming. It was concluded that the rounding out of responses obtained by the model solved by Linear Programming led to an unviable solution for the proposed regulation problem. The same did not occur with the Integer Programming model, which presented a viable, optimal regulation plan, showing that, from a mathematical viewpoint, responses with rounding out of solution using Linear Programming models should not be adopted. Fernandes (2003).

2.1 SUMMARY

In this chapter, other research works done by some scholars' in connection with Linear Programming Problems were reviewed. In the next chapter, we shall put forward guidelines used to model and solve Linear Programming Problems.

CHAPTER THREE

METHODOLOGY

3.0 INTRODUCTION

This part of the work reviews relevant fundamentals that will help us to come out with an appropriate linear model and the best way it will be solved.

3.1 LINEAR PROGRAMMING

Linear programming is a mathematical technique that deals with the optimization (maximizing or minimizing) of a linear function known as objective function subject to a set of linear equations or inequalities known as constraints. It is a mathematical technique which involves the allocation of scarce resources in an optimum manner, on the basis of a given criterion of optimality. The technique used here is linear because the decision variables in any given situation generate straight line when graphed. It is also programming because it involves the movement from one feasible solution to another until the best possible solution is attained.

A variable or decision variables usually represent things that can be adjusted or controlled. An objective function can be defined as a mathematical expression that combines the variables to express your goal and the constraints are expressions that combine variables to express limits on the possible solutions.

Generally we have constrained problems and unconstrained optimization.

3.1.1 UNCONSTRAINED OPTIMIZATION

Unconstrained optimization finds the highest point (or lowest point) on an objective function. For optimization to be required there must be more than one solution available, any point on the function is a solution, and because the single variable is real-valued function, there are an infinite number of solutions. Some kind of optimization process is then required in order to choose the very best solution from among those available. Best solution can mean the solution that provides the most profit or consumes the least of some limited resource.

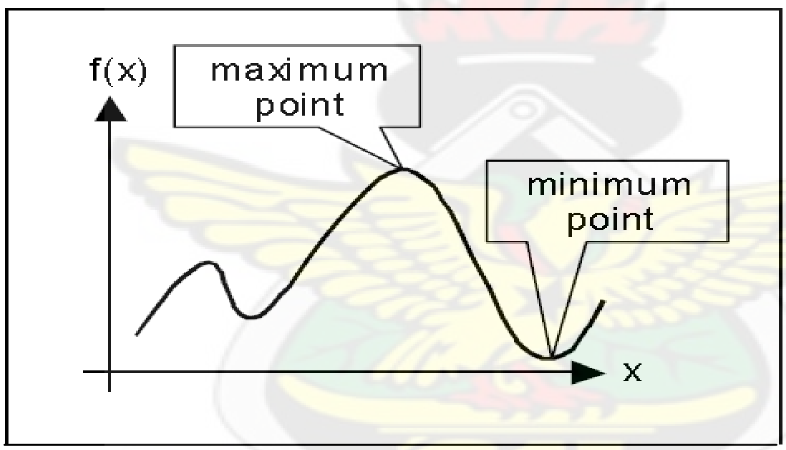

Figure 3.1 : Simple unconstrained Optimization.

3.1.2 CONSTRAINED OPTIMIZATION

Constrained optimization is much harder than unconstrained optimization. In contrained optimization you still have to find the best point of the function, but have to respect various constrains while doing so. Unlike unconstrined problems the best solution may not occur at the top of the peak or at the bottom of the valley, the best solution might occur halfway up a peak when a contraint prohibits movement further up.

3.2 METHODS OF SOLVING LINEAR PROGRAMMING

Basically, there are two methods of solving a linear programming problem. These are

i. The graphical (Geometrical) Method
ii. The simplex (Algebraic) Method

3.2.1 THE GRAPHICAL METHOD

This method of solving Linear Programming Problem is applicable to problems involving only two decision variables. The following steps can be followed in solving Linear Programming Problem using the graphical approach;

STEP 1

Locate and identify or define the decisions variables in accordance with problem given.

STEP 2

Formulate the problem in a standard Linear Programming model. The standard Linear Programming model consists of the objective function which is either to maximize or minimize the constraints which are either inequality or an equation.

Generally, if the problem is of maximized type, the inequality used is the less than or equal to $(\leq)$, unless otherwise specified. On the other hand, minimization problem goes with greater than or equal to $(\geq)$ unless otherwise stated. The non negativity constraint must also be stated.

STEP 3

Consider each of the inequality as an equation and plot each equation on the graph as each will geometrically represents a straight line.

STEP 4

Mark the appropriate region.

If the inequality constraint corresponding to that line is less than or equal to, then the region below the line lying in the first quadrant (due to the non negativity of the decision variables) is shaded. For the inequality constraint corresponding with greater than or equal to, the region above the line in the first quadrant is shaded.

STEP 5

The points lying in common region will satisfy all the constraints simultaneously. The common region thus obtained is called the feasible region. Feasible Region also referred to as Feasibility Polygon is the region common to all constraints in any given problem. It contains all the feasible or possible solutions to the problem. Points in the feasible region do not contravene any of the constraints.

There may be a situation where a constraint may not touch the feasible region; such constraint is known as redundant constraint.

The edges or vertex of the feasible region is called extreme points or corner points and these are the points used to obtain the optimal solution. The optimal solution is the solution that maximizes or minimizes the objective function as the case may be.

STEP 6

To obtain the optimum solution theoretically, a line of equal profits or line of equal cost is drawn to represent the objective function after assigning a value say zero for the objective function so as to for a straight line passing through the origin. Stretch the objective function line till the extreme points of the feasible region.

In the maximization case this line will stop farthest from the origin or the last extreme point the line touches before it completely leaves the feasible region gives the optimal solution.

In the case of minimization, this line will stop nearest to the origin and passing through at least one corner of the feasible region or the first extreme point it touches before it enters the feasible region is the optimum solution.

In practice however, we determine the coordinates of the feasibility polygon and then substitute these coordinates into the objective function. If the problem is a maximum case, the one that gives the maximum value is the optimum solution; otherwise the one that gives the minimum value will give the optimal solution.

STEP 7

Draw the necessary conclusion.

3.2.2 TYPES OF GRAPHICAL SOLUTION

As the Linear Programming Model is based on the use of linear inequalities, there is the likelihood that, in solving the LP problem, there may be an instance when one may come across different forms of solutions.

3.2.2.1 A UNIQUE OPTIMAL SOLUTION

This is where the solution to the problem occurs at one and only one extreme point of the feasible region. That is, the combination that gives the highest contribution or profit or the minimum cost or time depending on the problem at hand.

3.2.2.2 INFINITELY MANY SOLUTIONS

This is where the optimal solution to the problem is obtained at more than one extreme point. This implies that there is no unique solution to the problem. When this happens, the assumption made is that the graph of the objective function is parallel to at least one of the constraints binding the feasible region. Thus two or more different points may give the same value. Thus all points on this line will give an optimal solution. Figure 3.2 shows graphical representation of infinitely many solutions.

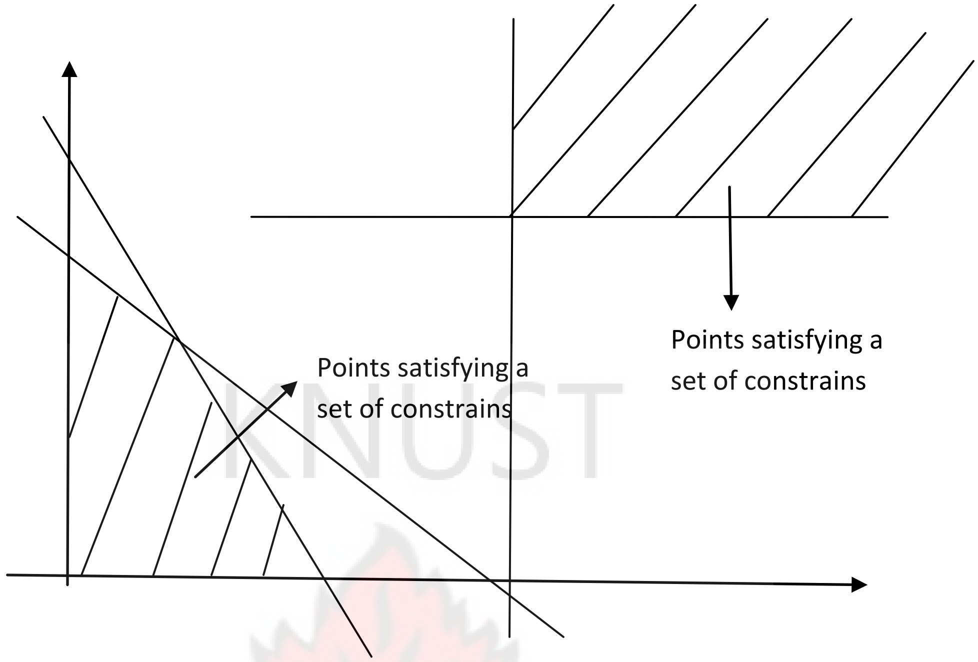

Figure 3.2: graphical representation of many solutions.

3.2.2.3 UNBOUNDED SOLUTION

This is a situation where the feasible region is not enclosed by constraints. In such situation, there may or may not be an optimal solution.

However, in all cases if the feasible region is unbounded, then there exists no maximum solution but rather a minimum solution.

To illustrate unbounded solution, let us consider a numerical example.

Maximize $\quad Z = 20x_1 + 10x_2$

Subject to

$x_1 \geq 2$

$$x_2 \le 5$$

$$x_1 \ge 0, x_2 \ge 0$$

Graphing the feasible region as shown in Figure 3.3, it is part of the feasible region that is shown since the feasible region extends indefinitely in the direction of the x_1 - axis.

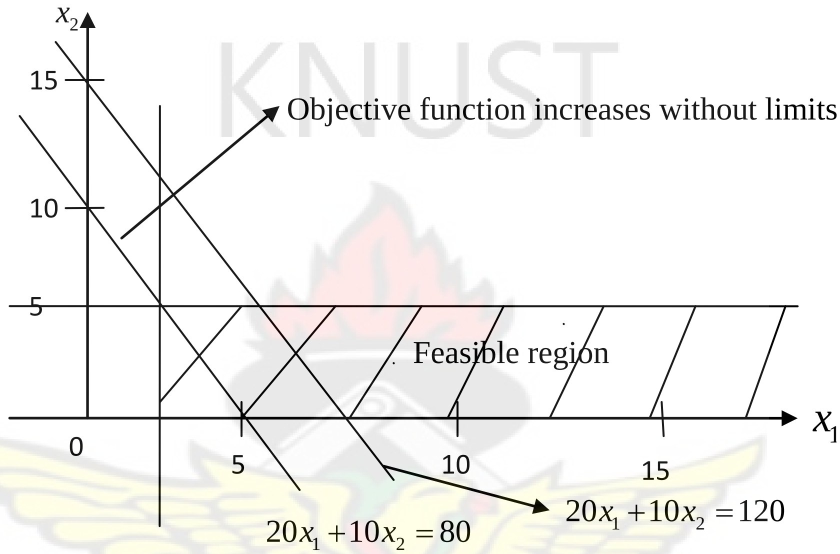

Figure 3.3: the feasible region of unbounded solution

3.2.2.4 NO SOLUTION

There may also be a situation where there is no solution to the problem at hand. In such case, there will be no feasible region hence; the bounded area will be empty.

3.2.3 EXAMPLE OF A GRAPHICAL METHOD SOLUTION

A bicycle company produces two kinds of bicycles by hand. These were mountain bikes and street racers. The company wishes to determine the rates at which each type of bicycle should be produced in order to maximize profits on the sales of the bicycles on the assumption that all the bicycles produced will be sold.

Two mountain bikes and three racers are produced per day respectively and producing each type requires the same amount of time on the metal finishing machine, this machine can process at most a total of four bicycles a day of either type. The profit generated on the mountain bikes and the racers are GH¢15 and GH¢12 respectively. The above problem is formulate as follow

x_1 =Number of mountain bikes produce per day

x_2 =Number of racers produced per day

Maximize $z = 15x_1 + 10x_2$ (in GH¢ per day)

$x_1 \leq 2$ (constraint for mountain bikes per day)

$x_2 \leq 3$ (constraint for racers per day)

$x_1 + x_2 \leq 4$ (production limit for metal finishing machine per day)

$x1 \geq 0$ and $x2 \geq 0$

A graph of the constraints is plotted as follows

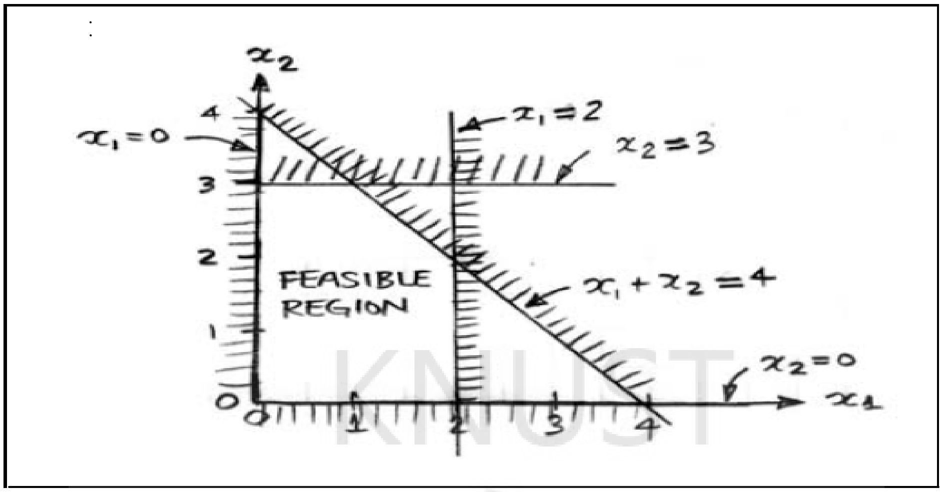

Figure 3.4: the feasible region of the bicycle company.

The limiting value of each of the constraint is shown as a line. Each constraint eliminates part of the plane. For example the vertical line labeled "$x_1 = 2$" is the limiting value of the inequality $x_1 \le 2$. All points to the right of the line violate the constraint (i.e. the infeasible region). The areas eliminated by the constraints are shaded. The unshaded area represents points that are not eliminated by any constraint, and is called feasible region.

To find a point in the feasible region gives the largest valued of the objective function. One way to do this is to randomly choose feasible points and to calculate the value of the objective function at those points, keeping the point that gives the best value of the objective.

Because there are an infinite number of points in the feasible region, this is not very effective because there is no guarantee that the best point will be found, or even that an objective function value that is close to the best possible value will be found.

An efficient search technique based on a couple of simple observations is developed. A line of equal profits is drawn to represent the objective function after assigning a value say zero for the objective function so as to get a straight line passing through the origin. The objective function line is stretched till the extreme points of the feasible region.

Figure 3.5 shows the constant profit lines drawn and indicates the optimal solution.

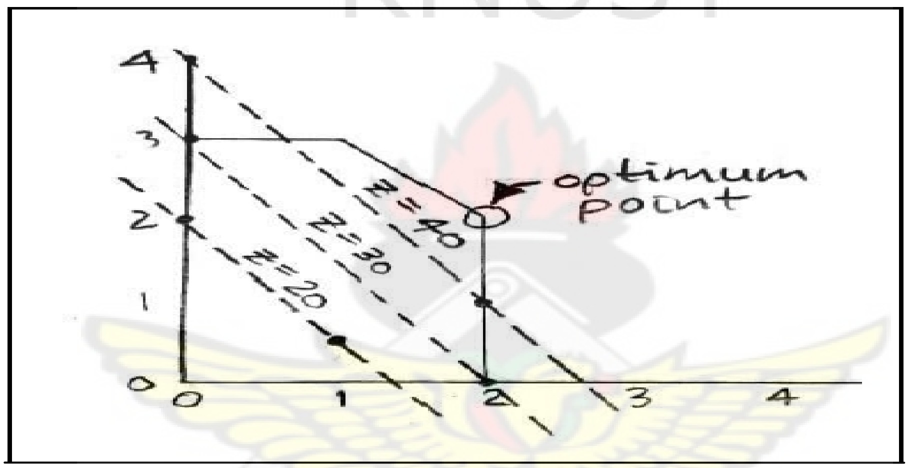

Figure 3.5: Constant Profit Lines for Bicycle Company.

As shown in Figure 3.5 the points are having the same value of Z (value of the objective function) form a line.

This is easy to understand if Z is replaced by specific value that can be plotted like $Z = 15x_1 + 10x_2$ becomes the line $15x_1 + 10x_2 = 20$ plotted in Figure 3.5.

Figure 3.5 also shows that all of the constant – profit lines are parallel. This is because all of the constant – profit line equations differing only by the selected value of Z. If the slope of any constant – profit line is to find, the Z constant will disappear, the slope of the entire constant – profit lines are the same.

Another observation is that the value of Z is higher for the constant – profit lines towards the upper right in Figure 3.5 and the last point is $(2,2)$ with $Z = 50$. This is the solution to the linear programming, the feasible point that has the best value of the objective function.

In some cases, the objective function has exactly the same slope as a face of the feasible region and the first contact is between the objective function and this face, as in Figure 3.6.

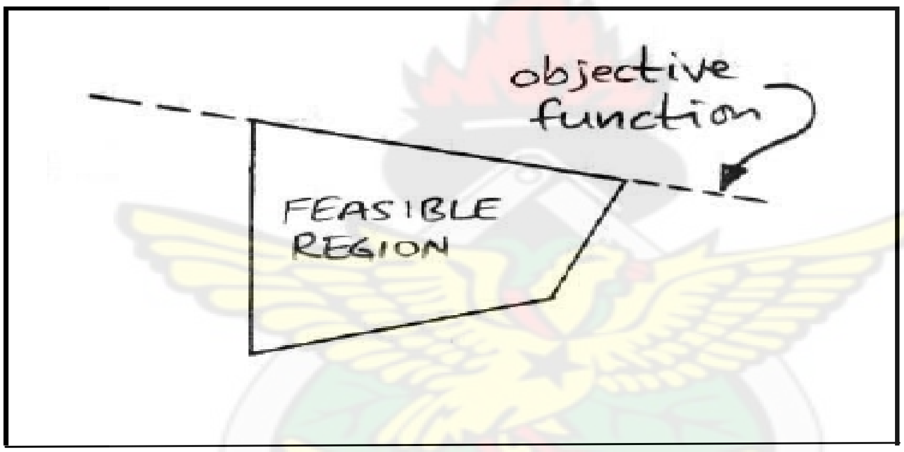

Figure 3.6: the slope of the objective function exactly matches the slope of the face of the feasible region.

This means that all of the points on that face have the same value of the objective function, and all are optimum, that is there are multiple optima. Though, if a face has first contact, then the corner points of the face also have first contact.

The important idea is that first contact between the objective function and the feasible region always involves at least one corner point. Hence, an optimum solution to the linear programming is always at a corner point or extreme point.

3.2.4 THE STANDARD FORM LINEAR PROGRAMMING

Linear programs can have objective functions that are to be maximized or minimized, constraints that are of three types $(\leq, \geq, =)$, and variables that have upper and lower bounds. An important subset of the possible LPs is the standard form LP. A standard form LP has these characteristics:

- The objective function must be maximized,
- All constraints are $\leq$ type,
- All constraints right hand side are nonnegative,
- All variables are restricted to non-negativity.

A standard form LP is the simplest form of linear program and most significant property of a standard form LP is that the origin (all variables set to zero) is always a feasible Corner point. This is because all standard form LPs have the kind of shape illustrated in Figure 3.7.

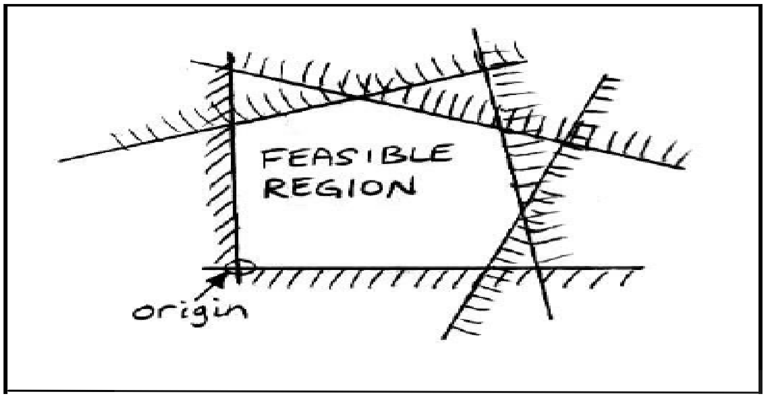

Figure 3.7: the origin is always a feasible extreme point or corner point in a standard form LP.

3.3 SLACK AND SURPLUS VARIABLES

A slack variable is associated with the $(\leq)$ constraint and represents the amount by which the right-hand side of the constraint exceeds its left-hand side. For constraints of the type $(\leq)$, the right-hand side normally represents the limited resource, whereas its left-hand side represents the usage of this limited resource by the different activities (variables) of the model. In this regard, the slack variable represents the unused amount of the resource.

A surplus variable is identified with a $(\geq)$ constraint and represents the excess of the left-hand side over the right-hand side. Constraints of the type $(\geq)$ normally set minimum specification requirements, in which case the surplus variable would represent excess amount by which the minimum specification is satisfied.

To illustrate the slack and surplus variables, let us consider the following problem.

Minimize $z = 3x_1 + 2x_2$

Subject to

$$x_1 + x_2 \leq 4$$

$$2x_1 + x_2 \geq 2$$

$$x_1, x_2 \geq 0.$$

The optimal solution to the problem above is $(1, 0)$. Then substituting the values $x_1 = 1$ and $x_2 = 0$ into the above constraints, we have

$1 + 0 = 1 \leq 4$ ……………………(1)

$2(1) + 0 = 2 \geq 2$ ………………..(2)

From the constraints equations (1) and (2), the slack variable with respect to equation (1) is $4 - 1 = 3$ and the surplus with respect to equation (2) is $2 - 2 = 0$.

3.4 SIMPLEX METHOD

The simplex method is the name given to the solution algorithm for solving linear programming problems developed by George Dantzig in 1947. A simplex is an n-dimensional convex figure that has exactly $n+1$ extreme points. For example, a simplex in two dimensions is a triangle, and in three dimensions is a tetrahedron. The simplex method refers to the idea of moving from one extreme point to another on the convex set that is formed by the constraint set and non-negativity conditions of the linear programming problem.

The solution algorithm is an iterative procedure having fixed computational rules that leads to a solution to the problem in a finite number of steps (i.e., converges to an answer). The simplex method is algebraic in nature and is based upon the Gauss-Jordan elimination procedure.

The principle underlying the simplex method involves the use of the algorithm which is made up of two phase, where each phase involves a special sequence of number of elementary row operations known as pivoting. A pivot operation consist of finite number of m elementary row operations which replace a given system of linear equations by an equivalent system in which a specified decision variables appears in only one of the system and has a unit coefficient.

The algorithm has two phases, the first phase of the algorithm, is finding an initial basic feasible solution (BFS) to the original problem and the second phase, consists of finding an optimal solution to the problem which begins from the initial basic feasible solution.

3.4.1 FORMULATION OF THE PROBLEM

The objective function to be Maximized or Minimized is given by

$$Z = c_1 x_1 + c_2 x_2 + \ldots + c_n x_n$$

Subject to the m constraints given by

$$a_{11} x_1 + a_{12} x_2 + \ldots + a_{1n} x_n \leq \mathrm{b}_1$$

$$a_{21}x_1 + a_{22}x_2 + ... + a_{2n}x_n \leq \mathrm{b}_2$$

.

.

$$a_{m1}x_1 + a_{m2}x_2 + ... + a_{mn}x_n \leq \mathrm{b}_\mathrm{m}$$

The Non negativity constraints

$$x_1 \geq 0, \mathrm{x}_2 \geq 0 \ldots \mathrm{x}_\mathrm{n} \geq 0$$

Where c_j, a_{ij} and b_j are all known constants and greater than zero and $i = 1, 2, 3...., \mathrm{m}$ and $\mathrm{j} = 1, 2, 3, ..., \mathrm{n}$.

3.4.2 ALGORITHM FOR SIMPLEX METHOD

A basic feasible solution to the system of m linear constraint equations and n variables is required as a starting point for the simplex method. From this starting point, the simplex successively generates better basic feasible solutions to the system of linear equations. We proceed to develop a tabular approach for the simplex algorithm. The purpose of the tableau form is to provide an initial basic feasible solution that is required to get simlex method started. It must be noted that basic variables appear once and have coefficient of positive one.

3.4.2.1 SETTING UP INITIAL SIMPLEX TABLEAU

In developing a tabular approach we adopt these notations as used in the initial simplex tableau.

c_j = objective function coefficients for variable j

b_i = right – hand side coefficients (value) for constraint i

a_{ij} = coefficients variable j in constraint i

c_B = objective function coefficients of the basic variables

$C_j - Z_j$ = the net evaluation per unit of $j-th$ variable

$[A]$ matrix = the matrix (with m rows and n columns) of the coefficients of the variables in the constraint equations.

Table 3.1: General form – Initial Simplex Tableau.

		Decision variables				Slack Variables						
c_j		c_1	c_2	..	c_n		0	0	..	0	Solution	(objective function coefficients)
c_B	Basic Variables	x_1	x_2	..	x_n		s_1	s_2	..	s_m		(Headings)
0	s_1	a_{11}	a_{12}	..	a_{1n}		1	0	..	0		(Constraints coefficients)
…	s_2	a_{21}	a_{22}	..	a_{2n}		0	1	..	0		
0	…	…	…	..	…		…	…	..	…		
	s_m	a_{m1}	a_{m2}	..	a_{mn}		0	0	..	1		
	Z_j	Z_1	Z_2	..	Z_{mn}	..	Z_{11}	Z_{12}	..	Z_{1m}	Current value of objective function	
	$c_j - Z_j$	$c_1 - Z_1$	$c_2 - Z_2$	..	$c_{mn} - Z_{mn}$	..	$c_{1_1} - Z_{1_1}$	$c_{1_2} - Z_{1_2}$	..	$c_{1_n} - Z_{1_n}$		Reduced cost (Net contribution/unit)

Example 3.1

Maximize $Z = 6x_1 + 8x_2$

Subject to

$$5x_1 + 10x_2 \leq 60$$

$$4x_1 + 4x_2 \leq 40$$

$$x_1, x_2 \geq 0.$$

The above example can be restated in the standard form as follows:

Maximize $Z = 6x_1 + 8x_2 + 0s_1 + 0s_2$

Subject to

$$5x_1 + 10x_2 + s_1 = 60$$

$$4x_1 + 4x_2 \qquad + s_2 = 40$$

$$x_1, x_2, s_1, s_2 \geq 0 .$$

Transferring to the initial simplex tableau, we have table 3.1.1

Table 3.1.1: The Initial Tableau (Example 3.1)

				Pivot column			
		c_j	6	8	0	0	
	c_B	Basic variable	x_1	x_2	s_1	s_2	Solution
Pivot row	0	s_1	5	10	1	0	60
	0	s_2	4	4	0	1	40
		Z_j	0	0	0	0	0
		$c_j - Z_j$	6	8	0	0	

Pivot element

The current basic variables always form an identity matrix within the simplex tableau. Note that the basic variables form a basis matrix that is an identity matrix (I).

From the initial tableau, the solution values can be read directly in the rightmost column. The values of z_j row are calculated by multiplying the elements in the c_B column by the corresponding elements in the columns of the $[A]$ matrix and summing them. Each value in the $(C_j - Z_j)$ row represents the net profit or net contribution that is added by producing one unit of product (if $C_j - Z_j$ is positive) or the net profit or net contribution that is subtracted by producing one unit of product (if $C_j - Z_j$ is negative).

Since all the z_j values (j = 1,..,4) are equal to zero in the simplex tableau, we proceed to generate a new basic feasible solution (extreme point) that yields a better value for the objective function. This is accomplished by selecting one of current non – basic variables to be made basic and one of the current basic variables to be made non – basic in such a fashion that the new basic feasible solution yields an improved value for the objective function. This process is called changing the basis or iterating.

3.4.2.2 IMPROVING THE SOLUTION

The criteria for which a variable should enter or leave basis is summarized as follows:

Variable Entry Criteria: The variable entry criterion is based upon the values in the $(C_j - Z_j)$ row of the simplex tableau. For a maximization problem, the variable selected for entry is the one having the largest (most positive) value of $(C_j - Z_j)$. When all values of $(C_j - Z_j)$ are zero or negative, the optimal solution has been obtained.

Variable Removing Criterion: The variable removal criterion is based upon the ratios formed as the values (b_i) in the "right-hand-side" column are divided by the corresponding values (a_{ij} coefficients) in the column for the variable selected to enter the basis. Ignore any a_{ij} values in the column that are zero or negative (i.e., do not compute the ratio). The variable chosen to be removed from the basis is the one having the smallest ratio. In the case of ties for the smallest ratio between two or more variables, break the tie arbitrarily (i.e., simply choose one of the variables for removal). This variable removal criterion remains the same for both maximization and minimization problems.

Applying the variable entry and removal criteria to our present maximization problem x_2 is chosen as the variable to enter basis and s_1 leaves the basis. Thus the current basic variable s_1 is replaced by non – basic variable (x_2).

Now that we have determined the new elements in basis and that not in basis we proceed to determine the new solution through pivoting x_2 into basis and pivoting s_1 out of basis. The pivoting process involves performing elementary row operations on the rows of the simplex tableau to solve the system of constrain equations in terms of the new set of basic variables. We initiate the pivoting processing by identifying the variable, x_2, to be entered into the basis by denoting its corresponding column as the pivot column in Table 3.1.1. Similarly, we identify the variable, s_1, to be removed from the basis by specifying the pivot row which is the row it corresponds as in Table 3.1.1. The element at the intersection of the pivot column and pivot is referred to as pivot element. The two – step pivoting process proceeds as follows:

Step I: Convert the pivot element to one by dividing all values in the pivot row by pivot element (10). This new row is entered in the next tableau, Table 3.1.2.

Step II: The objective of the second step is to obtain zeros in all the elements of the pivot column, except, of course for the pivot element itself. This is done by elementary row operations involving adding or subtracting the appropriate multiple of the new pivot row or from the other rows. Performing these calculations, the results are as presented in Table 3.1.2.

Table 3.1.2: Second Simplex Tableau (Example 3.1)							
			Pivot column				
		c_j	6	8	0	0	
	c_B	Basic variable	x_1	x_2	s_1	s_2	Solution
	8	x_2	$\frac{1}{2}$	1	$\frac{1}{10}$	0	6
Pivot row	0	s_2	2	0	$-\frac{2}{5}$	1	16
		Z_j	4	8	$\frac{4}{3}$	0	48
		$c_j - Z_j$	2	0	$-\frac{4}{3}$	0	
Pivot element							

The second simplex tableau can be constructed as shown in Table 3.1.2. Notice that the columns that correspond to the current basic variables x_2 (real variable) and s_2(slack variable) form a basis $[B]$which is identity matrix. The values in the z_j row and $(C_j - Z_j)$ row are computed in the same way as in the initial simplex tableau. Observe that $C_j - Z_j$ = 2 (> 0) and so the optimal solution has not been obtained and continue the iteration since we are maximizing.

We continue the process by determining the variables leaving the basis and which is entering the basis using the variable entry and removing criteria stated earlier. The outcome is summarized in Table 3.1.3.

Table 3.1.3: Third Simplex Tableau (Optimal Solution)

	c_j	6	8	0	0	
c_B	Basic Variables	x_1	x_2	s_1	s_2	Solution
8	x_2	0	1	$\frac{1}{3}$	$-\frac{1}{4}$	2
6	x_1	1	0	$-\frac{1}{5}$	$\frac{1}{2}$	8
	Z_j	6	8	$\frac{2}{5}$	1	64
	$c_j - Z_j$	0	0	$-\frac{2}{5}$	-1	

Observe that in this third simplex tableau all $c_j - Z_j$ values are either zero or negative. We have thus obtained the optimal solution with $x_1 = 8$, $x_2 = 2$, $s_1 = 0$, $s_2 = 0$ and the optimal value of $z = 64$.

The optimal solution suggests that the profit will be maximized when eight products of x_1 and two products of x_2 are produced.

3.4.3 SIMPLEX METHOD WITH MIXED CONSTRAINTS

Some Linear Programming problem may consists of a mixture of ≤, =, and ≥ sign in the constraints and wish to maximized or minimized the objective function. Such mixture of signs in the constraints is referred to as mixed constraints.

The following procedure is followed when dealing with problem with mixed constraints.

STEP1: Ensuring that the objective function is to be maximized. If it is to be minimized then we convert it into a problem of maximization by

Max W = -Min (-Z)

STEP2: For each constraints involving 'greater or equal to' we convert to 'less than or equal to' that is, constraints of the form

$$a_{21}x_1 + a_{22}x_2 + ... + a_{2n}x_n \geq \mathrm{b}_2$$

Is multiplied by negative one to obtain

$$-a_{21}x_1 - a_{22}x_2 - ... - a_{2n}x_n \leq -\mathrm{b}_2$$

STEP 3: Replace constraints

$$a_{21}x_1 + a_{22}x_2 + ... + a_{2n}x_n = \mathrm{b}_2$$

by

$$a_{21}x_1 + a_{22}x_2 + ... + a_{2n}x_n \leq \mathrm{b}_2$$

and

$$a_{21}x_1 + a_{22}x_2 + ... + a_{2n}x_n \geq \mathrm{b}_2$$

Where the latter is written as

$$-a_{21}x_1 - a_{22}x_2 - ... - a_{2n}x_n \leq -\mathrm{b}_2$$

STEP 4: Form the initial simplex tableau

STEP 5: If there exist no negative entry appearing on the RIGHT HAND SIDE column of the initial tableau, proceed to obtain the optimum basic feasible solution

STEP 6: If there exist a negative entry on the Right Hand Side column of the initial tableau,

i. identify the most negative at the Right Hand Side , this row is the pivot row

ii. Select the most negative entry in the pivoting row to the left of the Right Hand Side. This entry is the pivot element

iii. Reduce the pivot element to 1 and the other entries on the pivot column to 0 using elementary row operation

STEP 7: Repeat step 6 as long as there is a negative entry on the Right Hand Side column. When no negative entry exists on the Right Hand Side column, except in the last row, we proceed to find the optimal solution.

3.5 DUALITY

Corresponding to any given linear programming problem called the primal problem, is another linear programming problem called the Dual Problem. Since a given linear programming problem can be stated in several forms (standard form, canonical form, etc), it follows that the forms of the dual problem will depend on the form of the primal problem.

A fundamental of the primal dual-relationship is that the optimal solution to either the primal or the dual problem also provides optimal solution to the other.

A maximization problems with all the less-than or equal to constraint and the non-negative requirement for the decision variables is said to be in canonical form as in example 3.3 used below. If the dual problem has optimal solution, then the primal also has an optimal solution and vice versa. The values of the optimal solution to the dual and primal are equal

These are rules for converting the primal problem in any form into its dual

Table 3.2: Converting of primal problem to dual form

PRIMAL PROBLEM	DUAL PROBLEM
Maximization	Minimization
Coefficient of objective function	Right hand sides of constraint
Coefficient of i^{th} constraint	Coefficient of i^{th} variable
i^{th} constraint is an inequality of the form $\leq$	i^{th} variable satisfies ≥ 0
i^{th} constraint is an equality	i^{th} variable is unrestricted
i^{th} variable is unrestricted	i^{th} constraint is an equality
i^{th} variable satisfies ≥ 0	i^{th} constraint is an inequality of the type $\geq$
Number of variables	Number of Constraints
Number of Constraint	Number of variables

Note: Tableau can be read both ways

Example 3.2

(a) Find the dual of the LP problem.

Maximize $z = 3x_1 + x_2 + 4x_3$

Subject to

$$3x_1 + 3x_2 + x_3 \le 18$$

$$2x_1 + 2x_2 + 4x_3 = 12$$

$$x_1, x_3 \ge 0, x_2 \text{ unrestricted}$$

The dual is given by

Minimize $u = 18w_1 + 12w_2$

Subject to

$$3w_1 + 2w_2 \ge 3$$

$$3w_1 + 2w_2 = 1$$

$$w_1 + 4w_2 \ge 4$$

$$w_1 \ge 0, w_2 \text{ unrestricted.}$$

3.6 UNCONSTRAINED VARIABLES

In many practical situations, we may want to allow one or more of the decision variables, the x_j to be unconstrained in sign, that is either positive or negative.

We have already noted that the use of the simplex method requires that all the decision variables must be non negative at each iteration. However, by some simple algebraic manipulations, we can convert a linear programming problem involving variables that are unconstrained in sign into an equivalent problem having only non negative variables. This is accomplished by expressing each of the unconstrained variables as the difference of two non negative variables.

Assume the variable x_1 to be unconstrained in sign.

Define two new variables $x_j' \geq 0$ and $x_j'' \geq 0$

Let $x = x_1' - x_1''$.

Thus, when $x_1' \geq 0$ and $x_1'' \geq 0$

$x_1' \leq x_1''$ then $x_1 \leq 0$, and the desired result has been achieved. The unconstrained variable must be replaced by the two new variables wherever it appears in the linear programming model that is in both the objective function and the constraint set.

3.7 DEGENERACY

A linear program is said to be degenerate if one or more basic variables have a value zero. This occurs whenever there is a tie in the minimum ratio prior to reaching the optimal solution. This may result in cycling, that is the procedure could possibly alternate between the same set of non optimal basic feasible solutions and never reach the optimal solution.

In order to overcome this problem, the following steps may be used to break the tie between the key row tie

1. Select the rows where the ties are found for determining the key row.

2. Find the coefficient of the slack variable and divide each coefficient by the coefficients in the key column in order to break the tie. If the ratios at this stage do not break the tie, find the similar ratios for the coefficient of the decision variables.

3. Compare the resulting ratio column by column

4. Select the row which has the smallest ratio and this now becomes the key row.

3.8 TYPES OF SIMPLEX METHOD SOLUTIONS

The simplex method will always terminate in a finite number of steps with an indication that a unique optimal solution has been obtained or that one of three special cases has occurred. These special cases are:

1. Alternative optimal solutions

2. Unbounded solutions

3. Infeasible solutions

3.8.1 ALTERNATIVE OPTIMAL SOLUTIONS

The simplex method provides a clear indication of the presence of alternative or multiple, optimal solutions upon its termination. These alternative optimal solutions can be recognized by considering the $(c_j - Z_j)$ row. Assume that we are maximizing and remember that when all $(c_j - Z_j)$ values are all negative, we know that an optimal solution has been obtained. Now, the presence of an alternative optimal solution will be indicated by the fact that for some variable not in the basis, the corresponding $(c_j - Z_j)$ value will equal zero.

Thus, this variable can be entered into the basis, the appropriate variable can be removed from the basis, and the value of the objective function will not change. In this manner, the various alternative optimal solutions can be determined.

3.8.2 UNBOUNDED SOLUTIONS

In the case of an unbounded solution, the simplex method will terminate with the indication that the entering basic variable can do so only if it is allowed to assume a value of infinity ($+\infty$). Specifically, for a maximization problem we will encounter a simplex tableau having a non basic variable whose $(c_j - Z_j)$ row value is strictly greater than zero.

And for this same variable all of the a_{ij} elements in its column will be zero or negative value (i.e. every coefficient in the pivot column will be either negative or zero). Thus, in performing the ratio test for the variable removal criterion, it will be possible only to form ratios having negative numbers or zeros as denominators. Negative numbers in the denominators cannot be considered since this will result in the introduction of a basic variable at a negative level (i.e. an infeasible solution would result). Zeros in the denominator will produce a ratio having an undefined value and would indicate that the entering basic variable should be increased indefinitely (i.e. infinitely) without any of the current basic variables being driven from the basis.

Therefore, if we have an unbounded solution, none of the current basic variables can be driven from solution by the introduction of a new basic variable, even if that new basic variable assumes an infinitely large value.

Generally, arriving at an unbounded solution indicates that the problem was originally misformulated within the constraint set and needs reformulation.

3.8.3 INFEASIBLE SOLUTION

An indication that no feasible solution is possible will be given by the fact that at least one of the artificial variables, which should be driven to zero by the simplex method will be present as a positive basic variable in the solution that appears to be optimal. For example, assume we are solving a maximization problem in which artificial variables are required. Then, at some iteration we achieve a solution in which all the

$(c_j - Z_j)$ values are zero or negative, but which has one or more artificial variables as positive basic variables.

When an infeasible solution is indicated the management science analyst should carefully reconsider the construction of the model, because the model is either improperly formulated or two or more of the constraints are incompatible. Reformulation of the model is mandatory for cases in which the no feasible solution condition is indicated.

3.9 SENSITIVITY ANALYSIS

Suppose that you have just completed a linear programming solution which has a major impact. How much will the result change if your basic data is slightly wrong? Will that have a minor impact on your result? Will it give a completely different outcome, or change the outcome only slightly?

These are the kind of questions addressed by sensitivity analysis. It allows us to observe the effect of changes in the parameters in the LP problem on the optimal solution. It is also useful when the values of the problem parameters are not known. Formally, the question is this; is my optimum solution sensitive to a small change in one of the original problem coefficient. This sort of examination of impact of the input data on output results is very crucial. The procedure and algorithm of mathematical programming are important, but the problems that really appear in practice are usually associated with data: getting it all, and getting accurate data. What is required in sensitivity analysis is which data has significant impact on your results.

There are several ways to approach sensitivity analysis. If your model is small enough to solve quite quickly, you can simply change the initial data and solve the model again to see what results you get. At the extreme, if your model is very large and takes a long time to solve, you can apply formal methods of classical sensitivity analysis. The classical methods rely on the relationships between the initial tableau and any later tableau to quickly update the optimum solution when changes are made to the coefficient of the original tableau. Finally on the state of sensitivity analysis: you are typically limited to analyzing the impact of changing only one coefficient at a time. There are few accepted techniques for changing several coefficients at once.

3.9.1 CHANGE OBJECTIVE FUNCTION COEFFICIENT

A change of the coefficients of the objective function does not affect the values of the variables directly. So as we change the values of the objective function coefficients we should ensure that the optimality conditions are not violated. The range of values over which an objective function coefficient may vary without any change in the optimal solution is known as the range by those coefficient values that maintain $(c_j - z_j \leq 0)$.

The computation for the range of optimality can be categorized into two; that for the basic variables and also for the non-basic variable.

Example 3.3

Maximize $z = 50x_1 + 40x_2$

Subject to

$$3x_1 + 5x_2 \le 150$$

$$x_2 \le 20$$

$$8x_1 + 5x_2 \le 300$$

$$x_1, x_2 \ge 0.$$

Figure 3.3: Final Simplex Tableau (Example 3.3)

		x_1	x_2	s_1	s_2	s_3	
Basic Variables	C_B	50	40	0	0	0	Solution
x_2	40	0	1	$\frac{8}{25}$	0	$-\frac{3}{25}$	12
s_2	0	0	0	$-\frac{8}{25}$	1	$\frac{3}{25}$	8
x_1	50	1	0	$-\frac{5}{25}$	0	$\frac{5}{25}$	30
	z_j	50	40	$\frac{14}{5}$	0	$\frac{26}{5}$	1980
	$c_j - z_j$	0	0	$-\frac{14}{5}$	0	$-\frac{26}{5}$	

For any non-basic variable, the range of optimality will be $(-\infty < c_j \leq Z_j)$ in the maximization problem.

For the basic variable x_1 and x_2 the lower and upper limits of the coefficient within which the different solutions remains optimal can be computed by finding the ratio of $(c_j - z_j)$ to x_j values in the final simplex tableau. The smallest positive value for the ratio gives the extent to which it can be increased and the negative value with the smallest absolute value gives the extent to which it can be decreased. This is illustrated below.

x_1	1	0	$-\frac{5}{25}$	0	$\frac{5}{25}$
$c_j - z_j$	0	0	$-\frac{14}{5}$	0	$-\frac{26}{5}$
$(c_j - z_j)/x_1$			14		-26

The range of optimality for c_1 is $24 \leq c_1 \leq 64$

3.9.2 CHANGING A RIGHT HAND SIDE CONSTRAINT.

Right hand side constraints normally represent a limitation on the resources, and are likely to change in practice as business conditions change. An overall procedure for examining proposed changes to the right hand side of constraints is to check whether the proposed changes is within the allowable range of changes for the right hand side of the constraint. So an optimal tableau will continue satisfying the optimal conditions regardless of the altered values of the right hand side coefficients. The change in value of the objective function per unit increase in the constraints right hand side value is known as shadow price. When Simplex methods is used to solve LP problem, the values of the shadow price are found in the Z_j of the final Simplex tableau.

Let us again consider the example 3.3

Maximize $z = 50x_1 + 40x_2$

Subject to

$$3x_1 + 5x_2 \leq 150$$

$$x_2 \leq 20$$

$$8x_1 + 5x_2 \leq 300$$

$$x_1, x_2 \geq 0$$

Basic	C_B	50	40	0	0	0	Solution
x_2	40	0	1	$\frac{8}{25}$	0	$-\frac{3}{25}$	12
s_2	0	0	0	$-\frac{8}{25}$	1	$\frac{3}{25}$	8
x_1	50	1	0	$-\frac{5}{25}$	0	$\frac{5}{25}$	30
	z_j	50	40	$\frac{14}{5}$	0	$\frac{26}{5}$	1980
	$c_j - z_j$	0	0	$-\frac{14}{5}$	0	$-\frac{26}{5}$	

The shadow prices with respect to each of the constraints are the z_j values of the variables s_1, s_2 and s_3 respectively: since they represent the unused resources. s_2 has an optimal value of 8 which means that the second constraint has an excess and so additional resources are unnecessary hence shadow price is 0.

The constraint with $s_1 = 0$ the user is to pay the right hand side up to $\frac{14}{5}$. This means that in the problem above it would not allow any slack to occur in the first constraint unless it is worth more than $\frac{14}{5}$.

For maximization problem with a greater than or equal to constraint, the value of the shadow price will be less or equal to zero because one unit increase on the right hand side cannot help. It makes it more difficult to satisfy the constraint. As a result, the optimal value of the objective function will decrease when the right hand value of the greater than constraint is increased. The shadow price gives a negative number because of the decrease. The shadow price is given by the negative of z_j values of the corresponding artificial variable in the final Simplex tableau.

The range of feasibility for a less than or equal to constraint is given by

$$\begin{pmatrix} b_1 \\ b_2 \\ \vdots \\ b_m \end{pmatrix} + \begin{pmatrix} a_{1j} \\ a_{2j} \\ \vdots \\ a_{mj} \end{pmatrix} \geq \begin{pmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{pmatrix}$$

b = current optimal solution

a_{ij} = the column corresponding to the slack variable associated with the constraint.

The range can be established by the maximum of the lower limits and the minimum of the upper limits.

From the above final simplex tableau suppose that the range of constraints 1 is to be determined then we have

$$\begin{pmatrix} 12 \\ 8 \\ 38 \end{pmatrix} + \Delta b_1 \begin{pmatrix} 8/25 \\ -8/25 \\ -5/25 \end{pmatrix} = \begin{pmatrix} 12+8/25\Delta b_1 \\ 8-8/25\Delta b_1 \\ 30-5/25\Delta b_1 \end{pmatrix} \geq \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\Rightarrow 12+8/25\Delta b_1 \geq 0 \rightarrow \Delta b_1 = -37.5$$

$$8-8/25\Delta b_1 \geq 0 \rightarrow \Delta b_1 = 25$$

$$30-5/25\Delta b_1 \geq 0 \rightarrow \Delta b_1 = 150$$

Applying the conditions we will have -37.5 $\leq \Delta b_1 \leq$ 25 the initial amount on the right hand side is 150 it therefore follows that

$$-37.5+150 \leq \Delta b_1 \leq 25+150$$

$\Leftrightarrow 112.5 \leq b_1 \leq 175$ which is the range of optimality for b_i

3.9.3 SIMULTANEOUS CHANGES

If two or more objective functions coefficient or resources (right-hand-side) are to be changed then for each coefficient or constraint, compute the percentage or allowable increase or decrease represented by the change. If the sum of the percentage for all changes does not exceed 100%, then it satisfies 100% rule and the changes are allowable.

3.10 SUMMARY

In this chapter, Linear Programming and Simplex method were discussed. The analysis on the Linear Programming and Simplex method also form part of the discussion in this section of the work. In the next chapter, the data collected from the bank will be used to formulate the linear model and solve using the Simplex method.

CHAPTER FOUR

DATA COLLECTION AND MODELING

4.0 INTRODUCTION

In this chapter we analyze the data taken from Atweaban Rural Bank, A model is proposed and solved to help Atweaban Rural Bank maximize its net profit.

A banking institution, Atweaban Rural Bank, is in the process of formulating a loan policy involving a total of GH¢120,000. Being a full-service facility, the bank is obligated to grant loans to different clientele. The table 4.1 below provides the type of loans, the interest rate charged by the bank, and the probability of bad debt as estimated from past years.

Table 4.1: Loans available to the Atweaban Rural Bank.

Type of Loan	Interest rate	Probability of bad debt
Salary	0.28	0.02
Commercial	0.3	0.12
Farm	0.3	0.2
Funeral	0.4	0.01
Susu	0.3	0.03

Bad debts are assumed unrecoverable and hence produce no interest revenue. For policy reasons, there are limits on how the bank allocates the funds. The competition with other banking institutions in the area requires that the bank:

- Allocate at least 50% of the total funds to salary loan and commercial loan.
- To optimize profit salary loan must be at least greater than 50% of the farm loan, funeral loan and susu loan.
- The sum of Salary loan and susu loan must be at least greater than 50% of commercial loan, farm loan and funeral loan.
- The sum of farm loan and funeral loan must be at least 25% of the total funds.
- The sum of commercial and farm loans must be at least 29% of the total funds.
- Allocate at least 5% of the total funds to farm loan.
- The bank also stated that the total ratio for the bad debt on all loans may not exceed 0.05.

4.1 PROPOSED MODEL

The variables of the model can be defined as follows:

$x_1 =$ Salary loan (in thousands of cedis)

$x_2 =$ Commercial loan

$x_3 =$ Farm loan

$x_4 =$ Funeral loan

$x_5 =$ Susu loan

The objective of the Atweaban Rural Bank is to maximize its net return comprised of the difference between the revenue from interest and lost funds due to dad debts.

Objective function:

$$\begin{aligned} \textit{Maximize } Z = {} & 0.28(0.98x_1) + 0.3(0.88x_2) + 0.3(0.8x_3) + 0.4(0.99x_4) + 0.3(0.97x_5) \\ & - 0.02x_1 - 0.12x_2 - 0.2x_3 - 0.01x_4 - 0.03x_5 \end{aligned}$$

$$\textit{Max. } Z = 0.2544x_1 + 0.144x_2 + 0.04x_3 + 0.386x_4 + 0.261x_5$$

The problem has nine constrains:

1. Limit on total funds available

   $$x_1 + x_2 + x_3 + x_4 + x_5 \le 120000$$

2. Limit on Salary and Commercial loans

   $$x_1 + x_2 \ge 0.5 \times 120000$$

   $$x_1 + x_2 \ge 60000$$

3. Limit on salary loan compare to farm, funeral and susu loans

   $$x_1 \ge 0.5(x_3 + x_4 + x_5)$$

   $$x_1 - 0.5x_3 - 0.5x_4 - 0.5x_5 \ge 0$$

4. Limit on Salary and Susu loans compare to commercial, farm and funeral loans

$$x_1 + x_5 \geq 0.5(x_2 + x_3 + x_4)$$

$$x_1 - 0.5x_2 - 0.5x_3 - 0.5x_4 + x_5 \geq 0$$

5. Limit on farm and funeral loans

$$x_3 + x_4 \geq 0.25 \times 120000$$

$$x_3 + x_4 \geq 30000$$

6. Limit on commercial and farm loans

$$x_2 + x_3 \geq 0.29 \times 120000$$

$$x_2 + x_3 \geq 34800$$

7. Limit on farm loan

$$x_3 \geq 0.05 \times 120000$$

$$x_3 \geq 6000$$

8. Limit on bad debts

$$\frac{0.02x_1 + 0.12x_2 + 0.2x_3 + 0.01x_4 + 0.03x_5}{x_1 + x_2 + x_3 + x_4 + x_5} \leq 0.05$$

$$-0.03x_1 + 0.07x_2 + 0.15x_3 - 0.04x_4 - 0.02x_5 \leq 0$$

9. Non-negativity

$x_1 \geq 0,\ x_2 \geq 0,\ x_3 \geq 0,\ x_4 \geq 0,\ x_5 \geq 0.$

The following is the output returned by the management Scientist solver for the above model.

Maximize $Z = 0.2544x_1 + 0.144x_2 + 0.04x_3 + 0.386x_4 + 0.261x_5$

Subject to:

1) $x_1 + x_2 + x_3 + x_4 + x_5 \leq 120000$

2) $x_1 + x_2 \geq 60000$

3) $x_1 - 0.5x_3 - 0.5x_4 - 0.5x_5 \geq 0$

4) $x_1 - 0.5x_2 - 0.5x_3 - 0.5x_4 + x_5 \geq 0$

5) $x_3 + x_4 \geq 30000$

6) $x_2 + x_3 \geq 34800$

7) $x_3 \geq 6000$

8) $-0.03x_1 + 0.07x_2 + 0.15x_3 - 0.04x_4 - 0.02x_5 \leq 0$

4.2 OPTIMAL SOLUTION

Objective Function Value = 32068.4800

Variable	Value	Reduced Costs
x_1	31200.0000	0.0000
x_2	28800.0000	0.0000
x_3	6000.0000	0.0000
x_4	45200.0000	0.0000
x_5	8800.0000	0.0000

Constraint	Slack/Surplus	Dual Prices
1	0.0000	0.3443
2	1200.0000	0.0000
3	0.0000	-0.0833
4	21200.0000	0.0000
5	0.0000	-0.2354

6	0.0000	-0.1106
7	0.0000	-0.0066
8	4.0000	0.0000

OBJECTIVE COEFFICIENT RANGES

Variable	Lower Limit	Current Value	Upper Limit
------------	---------------	---------------	---------------
x_1	0.0190	0.2544	0.2610
x_2	0.0334	0.1440	0.3794
x_3	No Lower Limit	0.0400	0.1506
x_4	0.2610	0.3860	No Upper Limit
x_5	0.2544	0.2610	0.3860

RIGHT HAND SIDE RANGES

Constraint	Lower Limit	Current Value	Upper Limit
------------	---------------	---------------	---------------
1	119880.0000	120000.0000	122400.0000
2	No Lower Limit	0.0000	1200.0000

3	-13200.0000	0.0000	300.0000
4	No Lower Limit	30000.0000	51200.0000
5	26000.0000	34800.0000	34833.3333
6	4800.0000	6000.0000	6057.1429
7	59600.0000	60000.0000	68800.0000
8	-4.0000	0.0000	No Upper Limit

4.3 DISCUSSION OF RESULTS

There are several things to observe about this output data. The reduced costs for x_1, x_2, x_3, x_4 *and* x_5 are zero. This is because the reduced costs are the objective function coefficients of the original variables, and since x_1, x_2, x_3, x_4 *and* x_5 are basic at the optimum, their objective function coefficients must be zero when the tableau is put into proper form. This is always true, either the variable is zero (non-basic), or the reduced cost or dual price is zero. It is also seen that the pattern holds for the slack and surplus variables too. The dual prices for constraints (1), (3), (5), (6) and (7) are nonzero at the optimum because they correspond to the five active constraints at the optimum, hence their slack variables are non-basic (value is zero), so the dual prices can be nonzero. When both the variable and the associated reduced cost or dual prices are zero, then we have either degeneracy if the variable is basic or multiple optima if the variable is non-basic.

It must be noted that the optimal solution with $x_1 =$ GH¢31200, $x_2 =$ GH¢28800, $x_3 =$GH¢6000, $x_4 =$GH¢45200 and $x_5 =$ GH¢8800 show that the bank should allocate funds to all the types of loans since none of them with value of zero.

4.4 SUMMARY

In this chapter, data collected from the bank were used to formulate the proposed model and the output results were also discussed. The next chapter, which supposes to be the final chapter of the work, presents the conclusions and recommendations of the study.

CHAPTER FIVE

CONCLUSIONS AND RECOMMENDATIONS

5.0 INTRODUCTION

This chapter presents the conclusions drawn from the study and makes some recommendations to help the Atweaban Rural Bank in order to optimize the profit margin.

5.1 CONCLUSIONS

Reading through this work, we realize most banks in the country do not have any scientific method for given out loans. Due to this, most banks are not able to optimize their profits, which intern affects their socio economic contributions in the areas in which they operate. A model has also been proposed to help Atweaban Rural Banks disburse their funds available for loans. Our model shows that if Atweaban Rural Bank adapts to the model they can be able to make an annual profit of GH¢32068.4800 on loans alone. Hence we conclude that the scientific method we used to develop our proposed model can have a dramatic increase in the profit margin of the Bank should they adapt to it.

5.2 RECOMMENDATIONS

From the conclusion we realized that using scientific methods to give out loans helps banks to increase their profits. Hence we recommend Atweaban Rural Bank should adapt this model in their allocation of funds reserved for loans.

Secondly, it is recommended that Banks be educated to employ mathematicians to use scientific methods to find an appropriate mathematical model to help them disburse funds of the banks more efficiently.

Lastly, it is recommended that apart from loan disbursement, banks and other financial institutions should employ scientific methods and mathematical methods in most of the businesses they conduct.

5.3 SUMMARY

In this chapter of the work which is also the final chapter, discussed conclusions and some recommendations of the entire work.

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