KWAME NKRUMAH UNIVERSITY OF SCIENCE AND TECHNOLOGY

PREDICTIVE MODELING OF INSURANCE CLAIMS USING REVERSIBLE JUMP MARKOV CHAIN MONTE CARLO METHODS.

By

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Declaration

I hereby declare that this submission is our own work towards the award of the Master of Science (MSc.) and that, to the best of our knowledge, it contains no material previously published by another person nor material which had been accepted for the award of any other degree of the University, except where due acknowledgment had been made in the text.

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Dedication

I dedicate this work to my lovely wife Mrs. Lydia Adjei Suzet and dearest mum Madam Grace Afrakomah for their unflinching support throughout the entire programme.

Abstract

There has been considerable amount of attention rendered to claims reserving methods over the last few decades in actuarial science. The commonly used method of estimating claims reserves is the chain ladder technique. The underlying principle of the chain-ladder technique is that no underlying pattern to the run-off, and that each development year should be allocated a separate parameter. Applicable to a wide range of data, the chain ladder could alternatively be condemned for having too many parameters and also assumptions have to be used to estimate reserves beyond the latest development year already observed. This research seeks to explain an approach to model the development of claims run-off, using reversible jump Markov Chain Monte Carlo (RJMCMC) method. The study uses claims data from a renowed Insurance Company in Ghana; Win-
BUGS the tool used in simulating the reversible jump Markov Chain Monte Carlo (RJMCMC) method. The Bayesian methods are found to be better than the Over-dispersed Poisson model with lower predictive errors.

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Chapter 1

Introduction

1.1 Background Study

The management of loss reserved that seals up future payments from claims is a basic and central department within the industry of insurance. This very department is technically known as loss reserves accounting as it represents an involvement of complicating calculations primarily because losses are continuous and comes in at any time ranging from currently to several years down the road. This concept of loss reserving is basically an estimation on an insurer's liability from future claims; fundamentally, the allowance of an insurer to cover claims made against policies that it guarantees. Therefore when an insurer guarantees a new policy, there is immediately a recording of a premium payable articulating an asset and a claim obligation which represents a liability and it is this liability which is considered part of the unpaid losses account signifying the loss reserve. Principally, there is a fund set up for future compensation of policy holders. The amount is usually considered as provision for unsettled claims or simply claims reserves.

Modeling of claims reserves in non-life insurance is a subtle theme because the insurer requires an estimation precisely the amount of reserves to have in their stock; if the reserves are held in relatively higher amounts than actual, there is the possibility of lower profitability which has the capacity to lower the competitiveness of the company (insurer) on the business market. Meanwhile holding lower reserves has the potential of leaving the company in financial turmoil primarily due to bankruptcy. The forwarding theme in the estimation of these reserves is to compute the future amounts payable from an insurer or reinsurer's claims popularly known in actuarial science literature as Incurred But Not Reported (IBNR) claims according to Bornheutter and Ferguson (1972).

It is important in prediction of future claims to run the analysis separately from the insurer's already dissimilar existent cases or portfolios which are dependent on the type of insurance. Classically, a structure that has been established in claim reserving is the representation of the historical payments from a single line of business in a triangular form as this allows practitioners to track the time development of payments. The estimation of the future claims are based on this triangular structure known as the run-off triangle. Mathematically, this is a matrix that contains claims or loss data where each row corresponds to a year of accident and each column corresponds to the delay; the delay being the time duration between the year of accident and the year that the claim was made. A lot of literature has been devoted to claims reserving using run-off triangles by England and Verrall (2000) and Gelman et al. (2000).

Two very fundamental yet simple statistical methods highly staged for forecasting claims reserves are the Chain Ladder Method (CLM) and the BornhuetterFerguson method. The CLM makes use of data in two dimension array representing happenings and development of claims with the upper left side of the modeling matrix containing known values (previous) and these are used to forecast the remaining figures in the modeling matrix (future). The Bornhuetter-Ferguson is a Bayesian Method that incorporates an independently deprived prior estimate of ultimate expected losses as well as estimates generated by the same kind of modeling matrix. The credibility factor is used to weight the estimates with preference to more reliable projections.

These fundamental and traditional methods such as the CLM and the BornhuetterFerguson methods are deterministic in nature with no elements of probability at all as stated by England and Verrall (2002a). Over the past two and a half decades however, there has been a lot of increasing interest in stochastic reserving methods although these have only been applied by a handful of experts in the field of actuarial science England and Verrall (1999). Two things are necessary when modeling claims reserves; i.e. the provision of a ’best estimate’ of the considered reserve and secondly a provision of how precise this estimate is actually. Deterministic models are able to satisfy the first necessity but fail to provide the second. These stochastic models in addition to the first provision also present a precision of the estimate given by bringing on board the assessment of the variability of the claims reserves.

With reference to standards, the general approach is firstly specifying a model, i.e. finding an estimate of the outstanding claims under that model using the Maximum Likelihood and finally using the built model to find the precision of the estimated structure. On the other hand stochastic claims reserving first constructs the model and a method that produces the actuary’s best estimate and then uses this model to assess the uncertainty of the estimate. There have been several research into finding the best estimate using the stochastic form of the Chain Ladder Method by Verrall (2000), Mack and Venter (2000) and England and Verrall (2000).

A host of stochastic claims reserving models has been documented by Scollnik (2001) with majority of the constructed stochastic models being based on existing deterministic claims reserving models. The primary stage in the effort towards the widespread application of stochastic reserving methods was to demonstrate how the most often applied practical approaches can be formulated in statistical models which was encouraged by England and Verrall (2000) who included the use of the Chain Ladder technique.

1.2 Problem Statement

Within the application of the Chain Ladder method, there is no underlying pattern in the run-off and each development year is given a separate parameter. The merit is that the Chain Ladder technique can be applicable to a wide range of data with the demerit being that the methodology has relatively voluminous parameters. The implication here is that some other assumption has to be applied in the modeling of any potential claims beyond the latest development year already observed. Technically, this is referred to as the modeling of the tail by actuaries, or otherwise application of tail factors.

This problem of modeling the run-off tail has been investigated by many researchers using a whole range of techniques. The latest is the use of Markov Chain Monte Carlo (MCMC) technique through the application of Bayesian methods which has totally changed the trends involved in modeling run-off tails England and Verrall (2001a), Gelman et al. (2000), Lunn et al. (1998) and Lunn and Aarons (1995). In spite of this innovation there is still no room for the consideration of models which are trans-dimensional in nature. This means that it is impossible to consider models where the actual number of variables are not known Lunn et al. (2009a)

1.3 Objectives of the study

Based on the stated problem, this research will aimed at investigating the modeling the tail of a run-off triangle using a reversible jump Markov Chain Monte Carlo (RJMCMC) method. The specific objectives will be:

1. To determine the developments factors of a chain ladder technique method.
2. To estimate the reserves of a local claims run-off data using chain laddermethod.

3. To model the tail of a local claims run-off data using a reversible jumpMarkov Chain Monte Carlo (RJMCMC) method.

4. To find the prediction error of the reserve estimates given by the boththe chain ladder method and reversible jump Markov Chain Monte Carlo method.

1.4 Research Questions

The following research questions are posed in relation to addressing the problems stated in the modeling of claims run-off triangle and the objectives outlined for this research.

1. What are the developments factors of the chain ladder technique.?

2. How is the reserves of a local claims run-off data estimated using chainladder method?

3. How is the tail of local claims run-off data modeled using reversible jumpMarkov Chain Monte Carlo(RJMCMC) method?

4. What is the prediction error of the reserve estimates given by both the chainladder and the reversible jump Markov Chain Monte Carlo method?

1.5 Research Methodology

The research will investigate the modeling of tail of run-off using a reversible jump Markov Chain Monte Carlo (RJMCMC) method. The study will therefore narrow on by reviewing stochastic modeling of a claims run-off triangle that are existent in literature and consider specifically the stochastic model used in this study. Secondly an overview of Markov Chain Monte Carlo (RJMCMC) methods will be outlined. The foregoing methodologies will be employed and a numerical example will be outlined. The research will feature the use of WinBUGS as the tool for running the required simulations.

1.6 Significance of the Research

As recent as the concept of stochastic claims reserve modeling is by serving as a significant task needed within general insurance actuary, its practicality has been downplayed with majority of the practitioners sticking to very traditional methods as a means of claims reserve modeling. As discussed within the introduction however, these have the disadvantage of not providing the exact precision of the estimate provided. To some extent this tuned may be played with experience but why does try luck with experience when a few steps within modeling of the same phenomenon can help bridge the gap. This research offers an extension of that part of literature by providing an overview of the entire stochastic claims modeling process and secondly providing a justification for its application to be now widespread within the experts' environment.

1.7 Limitation of Study

The insurance sector, just as others consists of the life and non-life insurance; as the properties of life insurance, most importantly are distinct from non-life. This thesis exclusively dealt with the non-life insurance. Run off triangles usually arise particularly in non-life insurance where it may take some time after a loss until the full extent of the claims which have to be paid are known. Another limitation is that the reserves are unadjusted for inflation and undiscounted. The term reserve is used as a synonym for the sum of the Reported But Not Settled (RBNS) and the Incurred But Not Reported (IBNR).

1.8 Organization of the Study

The study contains five chapters altogether. The first chapter is the introduction which provides a brief background study, the problem statement, objectives based on which the study was organized, the research questions, the research methodology, significance and how the entire research has been organized. The literature review of mathematical models in both deterministic and stochastic claims reserve modeling is featured in chapter two. The third chapter deals with the general research design and precisely how the methods used in the research are explored and method of data analysis together with the organizational presentation of results. The actual presentation and analysis of the models are documented in the fourth chapter along with the necessary discussion. The final chapter is chapter five which provides the summary and findings, conclusion and recommendations for the future study.

Chapter 2

Literature Review

2.1 Introduction

There has been considerable quantum of research with respect to claims reserving techniques, initially centering deterministic models and quite recently more focused on stochastic claims reserve modeling. The concept of stochastic claims reserve modeling has been celebrated with several contributions primarily due to the solid support from management of various companies via appropriate regulations. This chapter provides a review of a number of studies related to the modeling claim reserves with respect to insurance companies. The chapter covers basic terminologies as well as concepts such as modeling insurance claims, history of insurance and its management. There is highlight of acquisition of claims processes and also features development of methodologies employed in investigating claims reserve modeling.

2.2 Basic Terminologies in Claims Reserve Modeling

High demands for the protection of risk has been the driving force behind the business and transition of insurance. The phenomenon associated with this high demand is nothing new and is pegged up because of uncertainty associated with all forms of investments. By insurance, individuals faced with considerable and unpredictable losses are provided a platform to reduce the variability of losses; this is done by the formation of a group which shares the losses incurred by the group thus reducing individual damages and losses. Insurance can be defined simply as a risk transfer mechanism.

It is simply a basic form of risk management which provides protection against anticipated future loss to life or physical assets. Traditionally, it is registered as an institution that processes a promise to reimburse the loss of damage to the insured by the insurer on payment agreed by the insured. The two main branches under insurance are life and non-life insurance Mikosch (2009). Life insurance as the name indicates is related to human life; basically, it is insurance cover that provides an amount to an insured individual or their nominated beneficiaries upon a certain event such as death of the individual who is insured. This consists of products such as whole life, endowment, medical and health insurance covers. One unique characteristic of life insurance is that it is long term with periodic (monthly, quarterly, annually, etc.) payments.

Non-life insurance on the other hand covers any form of insurance apart from the products covered by life insurance. The coverage period is one year with on-time premium payments. The form of products here include property insurance (commercial and private property against fire, water, business interruption, flooding, etc.), motor insurance, accident insurance, liability insurance, marine insurance, travel insurance, legal protection, credit insurance, aviation, epidemic insurance, etc. Reid (1978). Life and non-life insurances are modeled differently since they are entirely different contracts. Non-life insurance is the theme of this research. The term non-life insurance is popularly used in Continental Europe.
In Great Britain, the same branch of insurance is termed General Insurance. Elsewhere such as in North America, non-life insurance is referred to as property and casualty insurance Taylor (2000) .

The non-life insurance is characterized by a policy agreement between the involving parties, that is, the service provider known as the insurer and the customer known as the insured. The insured initiates the policy by making an agreed advance or fixed amount of money. These well-defined payments made by the insured to the insurer with an agreement of providing financial converge against a loss in the case of a well-specified random occurrence is known as the premium. In the event of an occurrence, the contract between the insured and the insurer provides an indication of the amount that has to be leveled out to the insured. The right of the insured party to the amount specified by the the contract is referred to as a claim or loss amount by the insured on the insurer. The claim amount which the insurer is obliged to pay in the event of an occurrence. There are usually time lags between when an event occurs and when claims finally get settled. The time it usually takes for occurrences to be reported is a factor in addition to providing clarity on the nature or degree of the occurrence. For example, a proper insurance may be settled faster compared to an accident claim in which some amount of time is required in order to establish the degree of injury or damage.

The insurer as a company upon receiving premiums sets aside a reserve that will cater for future payments or losses or claims. A claim on an insurer is the right of the insured entity to the agreed amounts in case the event said happens. Claims are demands for payment by a policyholder or an alleged third party under the terms and conditions of an insurance contract; technically, the claimant is the policyholder or third party that is asking for a payment. It is the responsibility of the insurer to ensure that there are sufficient funds, or reserves for paying out claims once they are applied for.

This essentially the reasoning behind learning about the average amount that is supposed to be paid out in a considered year (since the subject is non-life insurance). The reserved amounts should be sufficient depending on the premiums made. Also worth studying is the entire distribution of the aggregate claim for the considered year. The reserve of these claims is simply known as claims reserve; this is the outstanding claims or supply that is set up for future compensation of policy holders given claims that have already occurred. This basically constitutes amounts set aside by actuaries for claims that are not yet reported to the company (which is referred to as Incurred But Not Yet Reported (IBNYR)). There are other amounts which represent allowance for changes in the claim handlers estimate and is known as Incurred But Not Enough Reported **(IBNER)**.

Altogether, most companies combined these two amounts as Incurred But Not Reported (IBNR) according toMikosch (2009) and Mario and Wuthrich (2008). These are therefore the two types of claims in non-life insurance.

Generally, insurance companies receive their assets in the form of premiums thus rendering these receipts as deterministic in nature. Partly, the deterministic nature of these assets makes the valuation of the company involved in non-life insurance comparatively less difficult. What remains random is the number of policies sold. However the manner in which liabilities are applied to the company remain completely random in nature in comparison to the assets (premiums and volumes of policies sold); the dynamics of the liabilities therefore need to be carefully considered. The effect of not handling the balance between assets and liabilities optimally can trivially ultimately lead to the extreme effect of total dissolution of a non-life insurance company England and Verrall (2000). In settling claims based on premiums and policies sold, the difference between the assets processed and the liabilities rendered is known as surplus processing. The proper management of these surpluses process determines the success or downturn of a non-life insurance company. Another factor that influences the management of claims reserves is inflation which has been thoroughly studied by authors such as

Taylor (2000)and Mario and Wuthrich (2008).

Additional terminologies used in claims reserving include accident year, case reserves, change, etc. The accident year is defined as the year in which an accident happened and the insurer was on risk. Case reserves can be defined as estimates of amounts required to settle claims that have already been reported but not yet fully paid. Change, in the estimated or actual losses or reserves over ulterior evaluations is called development. A development factor is the quotient of the paid or incurred value for incident record evaluated at time $t + 1$ divided by the amount of that same accident record evaluated at time t. Earned premium is the part of the premium proportional to the segment of time a policy has been in force. The sum of all underwriting and operational expenses divided by the premium represents the expense ratio. The smallest divisible part of a claim is called a feature. It corresponds to a loss on coverage for one person. Incurred losses are the sum of payments and reserve changes for claims. An indication is an estimate based on analysis of the data.

Therefore a variety of mathematical methods and routines that have been researched for the estimation of total loss amounts, and this chapter is a presentation of literature of these key methodologies.

2.3 Models for Claims Reserving

Taylor (1986) investigated the classification of loss reserving models within the development of claims reserving methods for actuarial scientists. Researchers, practitioners and scholars have developed and used many methods of loss-reserving based on run-off triangles during the last few decades. These included classifications such as whether models are deterministic or stochastic, macro or micro, whether the independent variables are paid or incurred losses, etc. A very instrumental manuscript on loss reserving is by Taylor (2000). In his research he found that literature published in the previous 15 years was roughly equal in volume to prior literature. All of these methods have a common assumption, that is: all claims are settled within a fixed number of development years and that the incremental and cumulative losses from the same number of accident years are well known until the present calendar year. In that way, the losses can be represented in a triangle shape and thus is popularly referred to as run-off triangle. This means that the development of the losses of every accident year is following a development pattern which is common to all accident years.

In ’contrast’ to stochastic models being more recent, the earlier models of which the chain ladder was an example, were developed heuristically, and essentially deterministic in nature. According to Taylor and Campbell (2002), the earliest stochastic models in claims reserve modeling appear to attributed to Kramreiter and Straub (1973) and Hachemeister and Stanard (1975). There were various contributions such as from Reid (1978). After that, there were several contributions from De-Jong and Zehnwirth (1983), Pollard (1983) and Taylor and Ashe
(1983).
The concept of heuristic estimation which is typically non-stochastic was popularly used in handling the chain ladder method by Hachemeister and Stanard (1975). The research by Taylor and Ashe (1983) with the chain ladder technique was placed on an optimal basis with respect to Poisson claim counts; this however implied the technique was heuristic. Further work was conducted by England and Verrall (2002b) by placing it on an optimal basis with claim amount pegged with over-dispersed Poisson distributions.

Chronologically, the oldest of claim reserving models belonged in the 1970s including methods such as the chain ladder method, the separation method and the payment per claim finalized model Fisher and Lange (1973)and Sawkins (1979). Deterministic models were then bridged up with the addition of error terms making them stochastic models. A popular example was the model by Mack (1993) which presented a stochastic version of the traditional deterministic chain-ladder method using distribution free error terms. Earlier though, there had been research by De-Vylder (1978) featuring a least squares chain-ladder estimation where optimal parameter estimation was applied to distribution free error terms.

Other claim reserving models developed with optimal parameter estimation include the chain-ladder for triangle form of Poisson counts Hachemeister and Stanard (1975), log-normal age-to-age factor chain-ladder Hertig (1985) and the over-dispersed Poisson cells triangle chain ladder technique England and Verrall (2002b); all of which were applied if the error estimate structure added. These models from Hachemeister and Stanard (1975), Hertig (1985) and England and Verrall (2002b) were classified as static, stochastic, optimal and phenomenological; phenomenological models represent models with direct physical meaning but not directly related to aspects of the claims reserving process.

Another set of stochastic models with finer structure and a micro structure with the payment per claim finalized structure by Taylor and Ashe (1983) and operational time model of claim size Reid (1978). The evolution of parameters over periods was featured via the Kalman filter Kalman (1960) which sparked a lot of existing models from static to dynamic models. Examples of such models include distribution of accident period claim payments over development periods and the popular chain ladder model by England and Verrall (2002b).

2.3.1 The Chain Ladder Methods

This model stands out as the most popular loss reserving technique with several derivations. Based on classical actuarial literature, the chain ladder model is a purely computational algorithm to estimate claim reserves. According to Verdonck and Dhaene (2007), the origination of the chain ladder method is obscured in the Antiquity of the Casualty Society. The chain ladder apart from being popular is an easy to implement technique whereby actuaries project losses from less mature stage to a more mature future. At each phase of the development, the actuary calculates the link ratio, or age-to-age factor or development factor that is the ratio of cumulative losses at a development year to that of the previous.

Immature losses move toward maturity when multiplied at each stage by the corresponding development factor, which is practically very useful. Kremer (1982) showed that the chain ladder technique is based upon a linear model. England and Verrall (2002c) in their research went further and explained how we can see in the chain ladder, the linear model Kremer (1982) was talking about. In another paper by Verrall (1989), he tried to extend and consolidate the statistical framework which enables the analysis of insurance data. In fact, that was how to enhance and improve the classical chain ladder technique. The improvements were designed to overcome two big problems that the chain ladder method faces.

The first flaw was that there was no connection between the accident years resulting in an over-parameterized model and not stable forecasts. The second point is related to the assumption that is the base of the model itself: the development pattern is assumed to be the same for all accident years. The chain ladder cannot adapt to any change with which claims are paid, or any other elements that can affect the run-off triangle. To establish a connection between

the accident years, he proposed the use of the Bayesian framework in which people can assume that the row parameters have the same prior distribution.
The over-parameterization that we talked about and that was one the main flaws of the chain ladder technique was due to the fact that in that model, the accident years were not seen as linked but rather considered as separate. He focused on a statistical analysis that allows the use of actuarial judgment. He came up with a methodology that permits other information available to be taken into account to extend the range of the analysis. This Bayes assumption can be a useful way of overcoming the very problem of over-parameterization.

England and Verrall (2002d) made an almost complete review of the existing reserving methods and models actually in use in the insurance field. The chain ladder method is based exclusively on the development factors; it often happens that the predicted result cannot be relied on with the confidence level we would like. This is particularly likely for more recent underwriting years where the development factor to predict from the actual to ultimate loss amount is relatively variable, due to the present lack of claims development. This way, actuaries thought of making use of an alternative ultimate amount, usually obtained from a supposed loss ratio.

2.3.2 The Bornhuetter-Ferguson Method

The next model for consideration in this research that is widely used for loss reserving is known as the Bornhuetter-Ferguson method. It is named after the two that developed it in 1972. England and Verrall (2002d) proposed predictors of outstanding ultimate losses and every predictor is obtained by multiplying an estimate of the expected ultimate loss by an estimator of the percentage of the outstanding loss with respect to the ultimate one. It is based on the run-off triangle like the chain-ladder but it restricts its use to the estimation of the percentage of the outstanding loss and uses the product the earned premium and an expected ultimate loss.

This method tries to stabilize the chain-ladder method and makes it less sensitive to outliers. Verrall (1989) showed that despite their different appearances, the chain ladder and the Bornhuetter-Fergusson methods have very much in common. In that paper, they first pointed out the fact that they both have a multiplicative structure when they come to the ultimate outstanding losses. They introduced a new model named the extended Bornhuetter-Ferguson.

They used the notion of development pattern to prove that the latest is not just one method among various others but a general one that comprises many other methods as special cases and leads to the Bornhuetter-Ferguson principle. After a thorough study of the Bornhuetter-Ferguson method they stated what its principle is. So, it consists of three elements: the simultaneous use of different versions of the Bornhuetter-Ferguson, the comparison of the different ultimate losses and finally, the selection of the best ones. With a numerical example, they showed that the Bornhuetter-Ferguson principle can be used to select an appropriate version of the extended Bornhuetter-Ferguson for any run-off triangle.

Up to this point, all the methods used are deterministic and give a single estimate without any information about its variability. In recent years, considerable attention has been given to discuss possible relationships between the chain ladder and some stochastic models. So, in Mack (1993), a formula for the standard error has been derived and a programmable recursive way of calculating it was also
given.

Moreover, he shows how a tail factor can be incorporated in the calculation of the standard error. Schnieper (1991) used a mixture of Bornuetter-ferguson and the chain ladder for the same purpose; but Mack's formula is specialized for the pure chain-ladder method. Later, many stochastic models were developed to give an idea of the variability of the estimates or the prediction errors.In this way, Schnieper (1991) published a paper where he described a stochastic model based on the chain ladder without any assumed specific distribution. It is called the distribution free model and it reproduces the chain ladder estimates and provides a mean of getting the standard errors. In Mark and Gary (2000), they conducted a comparative study of the distribution free and the over-dispersed Poisson models. They concluded that, both of the two models reproduce the chain ladder estimates for the claims reserve. Nevertheless, they argued that the two models are different in the sense that the true expected claims reserves, let alone estimation issues, are different.

2.4 The Markov Chain Monte Carlo Methods

Gives an almost exhaustive call of stochastic based models that have been given a lot of attention over time. As discussed previously, these stochastic methods are only significantly different from their previous deterministic counterparts by the addition of the stochastic terms England and Verrall (2002d). The first step in applying stochastic claims reserving models was a demonstration of the formulation of existing approaches in statistical models. This bridge was provided by England and Verrall (2001b), a research covering a good number of varying approaches including the discussed chain ladder technique.

The routine behind the chain ladder technique for instance offers no patternbased run-off with every development year also being placed by an allocation of a separate parameter. The implication of this on the chain ladder method is that it becomes a method that can be applied on a wider range of data with several parameters. This further implies that beyond the latest development year already observed, other assumptions have to be employed in modeling any possible claim development technically referred to as modeling the tail. The Bayesian methods, via Markov Chain Monte Carlo Methods is used to model the tail of the run-off. A review of these Bayesian methods is provided by Makov (2001). Another research devoted which provides an introductory perspective of Bayesian methods and simulation methods is by England and Verrall (2006).

The Markov Chain Monte Carlo (MCMC) was initially brought forward routinely after Monte Carlo at Los Alamos for simulating a liquid in equilibrium with its gas phase Metropolis et al. (1953). It was popularly significant because the only way to investigate thermodynamic equilibrium was centered on simulating the dynamics of the system until equilibrium was established. The MCMC simulation provided they had similar distribution did the trick of providing a platform for modeling the equilibrium of the desired variables. After several applications in a varied range of scientific communities including physics and chemistry, Gelfand and Smith (1990) facilitated an introduction of MCMC into the Bayesian community. After a while though, it was realized that majority of the Bayesian inference could be performed using MCMC, whiles on the other hand almost nothing could be done without employing MCMC Geyer (1992). The applicability of the Bayesian methods has since then been accelerated by the use of MCMC methods Gilks et al. (1996). Authors such as Congdon (2006), Ntzoufras (2009) and Scollink (2002) offers several examples based on the application of robust Bayesian models.

In the actuarial science and specifically with respect to claims reserving, Green (1995)investigated the reversible jump MCMC (RJMCMC) allowing trans-dimensional models to be analyzed; these are models with unknown number of variables or several models with fixed variable numbers thus giving allowance of consideration of interesting range of models for claims reserving. Other significant mentions in the research of reversible jump MCMC and its related issues include Ntzoufras et al. (2005), Katsis and Ntzoufras (2005), Peters et al. (2009) and Verrall and Wuthrich (2010).

2.5 Historical Background of Insurance

The first experience of man with insurance was recorded as being in the field of marine. Records, however, show that modern marine insurance was practiced in 1347. In this early form, vessel or cargo would be pledged against a loan and should the vessel not successfully complete the journey; the loan would not be repayable Iruku (1977). Another ancient maritime practice that has survived many generations virtually unchanged is that of "general average."The mode of its operation is when certain cargo is jettisoned (thrown overboard) during a journey in an attempt to save the voyage. If the journey proves successful; the owners of the cargo that was not jettisoned and was saved will contribute proportionately towards a fund out of which the unfortunate ones who lost their cargo would be paid a claim Fisher et al. (1973).

In West Africa, methods of spreading risk by the extended family system, age, groups, clans, religious groups among other social devices is called Susu or Esor which dates back to the pre colonial era . However, due to developments and modernization, this state of affairs is no longer ideal and adequate hence the need for more acceptable form of compensation.

As early as the 1920s, the British, representing agencies for insurance companies then operating in Great Britain, introduced conventional insurance to the West Africa sub region. These agencies later were transformed into insurance companies whiles for example in the case of Ghana, the government formed their own indigenous insurance company to take care of their growing insurance needs after independence. Today, Ghana has quite a bit of vibrancy in the insurance industry serving the needs of both local and foreign stakeholders, thus the need to uphold the customer in high esteem and attend to their requirements with speed and efficiency.

According to Gormley (2008), the business of insurance could be the possible solution to the catastrophic food crises affecting third world nations like Ghana. In an article titled "Industry can help avert price disaster "published in the "Insurance Day", Gormley (2008) stated that a study by the French Agricultural Research Centre for International Development, said, "insurance industry could play a major part in solving the underlying problems causing rising food prices. Ghana is mainly an agriculture country with majority of its workers engaged in farming cash crops consisting primarily of cocoa products, which typically provide about two-thirds of export revenues, timber products, coconuts and shea nuts, which produce an edible fat, and coffee. Ghana also has established a successful program for non-traditional agricultural products for export, including pineapples, cashews, pepper, cassava, yams, plantains, maize, rice, peanuts, millet, and sorghum. Fish, poultry, and meat also are important dietary staples. Ghana has an estimated population of twenty four million people.

The New Insurance Act 2006 forms the basis for insurance regulation in Ghana, which is enforced by the National Insurance Commission ("NIC"). Besides establishing a minimum paid up capital level of US1m (including reserves), insurers are also required to maintain an adequate total assets to total liabilities ratio ,which is currently set at 150%. Further guidelines are stipulated with regards to the quality of assets, with investments required to equate to a minimum 55% of total assets by December 2010, whilst investments inequities and properties are limited to 30% and 20% of total investments respectively. The non-life insurance market remains relatively small, with industry Gross Written Premium (GWP) totalling GHS 226.8m (or US 156m) in 2009. Given that 23 registered insurers compete in this market (with further entrants expected in the medium term), competition is intense, with market share predominantly contested via premium reductions.

The insurance industry does not produce a tangible, physical product but it rather renders services. Insurance is among the most complicated and least understood services in today Ghana's economy. The major factor, which contributes to this misunderstanding, is the highly complicated nature of the insurance policy itself. Individual policyholders remain confused by the small prints and its legality hence poor response in lowly educated areas like Ghana.

Chapter 3

Methodology

3.1 Introduction

Stochastic claims reserve methods built from deterministic claims reserve methods have received considerable attention in the recent actuarial literature with increasing application from experts lately. This chapter is a presentation of the Methodology used for analyzing the claims reserve in this research. The chapter presents both deterministic and Stochastic claims reserve modeling concepts and tools with emphasis on the Reversible Jump Markov Chain Monte Carlo (RJMCMC) method. Theoretical models are presented together with WinBUGS which is a statistical software for Bayesian analysis that is popularly used for simulating Markov Chain Monte Carlo (MCMC) methods.

3.2 Basic Concepts and Data Representation

The basic assumption for majority of the existing methods for claims reserve estimation is that the data is presented in the form of a *run-off triangle*. This presentation places the data into a *period of origin* and *development period*. The former i.e. the period of origin relates to the year in which the claims were reported or particularly when the policy relating the claim was underwritten. The period of development on the other hand indicates the entire period from the period or origin within which the claims were paid, incurred or reported. Incidentally, this means that the development year of the year of origin is the development year zero.

The general form of the run-off is represented in the Table 3.1.

Table 3.1: Claims Run-off Triangle Structure

Year of Origin	Year of development						
	0	1	2	$\cdots$ j	$\cdots$	$n-1$	n
0	$\chi_{0,0}$	$\chi_{0,1}$	$\chi_{0,2}$	$\cdots$	$\cdots$	$\chi_{0,n-1}$	$\chi_{0,n}$
1	$\chi_{1,0}$	$\chi_{1,1}$	$\chi_{1,2}$	$\cdots$	$\cdots$	$\chi_{1,n-1}$	
...	...	...	$\ddots$	$\cdots$	$\cdots$		
i	$\chi_{i,0}$	$\chi_{i,1}$	$\chi_{i,2}$	$\cdots$	$\chi_{i,n-i}$		
...	...						
n	$\chi_{n,0}$						

In Table 3.1, a claim cohort is based on the defining terms of the claims for a particular origin period and development period. If for example the very entry in the run-off triangle in Table 3.1 is the value of the claim paid in the development year j, then the claim having occurred is in year of origin i.

We also consider key assumptions aside the run-off triangle in the build-up of the methodology of claim for the avoidance of ambiguity and for the notation to be universal are outlined in this section. We assume at all claims are settled in n periods of time and there are a total of n periods of past claims data. The unit of time here is year, but can be effected to reflect quarterly or monthly captures. The claims development triangle has indices $i \in \{0,1,...,I\}$ and $j \in \{0,1,...,J\}$ given that $I = J$. The index i represents the accident and j represents the development year as discussed previously.

Let $X_{i,j}$ represent the incremental claim amounts for accident year i and development year j; this is also known as the incremental payments of the change of reported claims. Let R represent the total outstanding claims liability. The claims that are incurred or the sum of all reported claims is termed as the **cumulative claims amounts** and denoted by $C_{i,j}$. Mathematically, the cumulative claim amounts for accident year i up to the development year j are given by

$$C_{i,j} = X_{i,1} + X_{i,2} + X_{i,3} + \cdots + X_{i,j} = \sum_{k=1}^{j} X_{i,k} \tag{3.1}$$

and

$$R_i = X_{i,n+2-i} + X_{i,m+3-i} + X_{i,n+4-i} + \cdots + X_{i,m} = C_{i,n} - C_{i,I-i} \tag{3.2}$$

and the total outstanding loss liabilities for all accident years given by

$$R = R_2 + R_3 + R_4 + \cdots + R_n \tag{3.3}$$

There is a general assumption that the last development year is given by I ($\Rightarrow X_{i,j} \equiv 0, \forall j > I$ given that the last observed accident year is I. The ultimate claim amount of a particular accident year i is $C_{i,I}$. Very often, the observations

$\mathbf{D}_I$ at I which is the calendar year is defined by

$$\mathbf{D}_I = \{X_{i,j}, \quad i + j \leq I\} \tag{3.4}$$

and the random variables are predicted in its complement

$$\mathbf{D}_I^c = \{X_{i,j}, \quad i + j > I\} \tag{3.5}$$

We also use Bayesian modeling which is primarily based on Bayes Theorem assuming that all parameters are unknown random variables.

3.2.1 The Basic Chain Ladder Models

This is labeled in actuarial literature as one of the oldest actuarial techniques in application for the estimation of loss reserves. It is relatively intuitively natural and was initially built as a non-Stochastic model without the inclusion of a random component. It assumes all external factors such as claim cost inflation, change in ,ox pf business, settlement claim changes are ignored.

The basic chain ladder model therefore takes the form based on Ntzoufras (2009).

$$C_{i,j} = x_i y_j + \epsilon_{ij} \tag{3.6}$$

where $C_{i,j}$ $j = 1,...,n$ represent the claim amount in accident year i and development year j; x_i represents the ultimate total claim cost in a particular accident year i and y_j represents the proportion of total payments made by the end of development j.

Given that the amount of claim written in development year j with respect to accident year i is represented by $S_{i,j}$, then

$$C_{i,1} = S_{i,1} \tag{3.7}$$

$$C_{i,j+1} = C_{i,j} + S_{i,j+1} \qquad \forall i,j; j \leq n - i + 1 \tag{3.8}$$

The factors y_j are assumed to be constant by the basic chain-ladder technique for all years of the accident. Given that D_j represents the ratio of the cumulative payments made by the end of the year j to the expected value of the cumulative payments made by the end of the year $j - 1$, then the estimation of D_j is given by

$$D_j = \frac{\sum_{i=1}^{n-j+1} C_{i,j}}{\sum_{i=1}^{n-j+1} C_{i,j-1}} \qquad j = 2,...,n \tag{3.9}$$

The factor D_j are therefore calculated by summing each column in the run-off triangle given in Table 3.1; this takes into consideration the ratio to previous column total excluding the final entry. We define another parameter λ_j by the product of $(n - j)D_j$'s. Mathematically we will define λ_j by

$$\lambda_j = \prod_{k=j+1}^{n} D_k \qquad j = 1,...,n - 1 \tag{3.10}$$

The parameter λ_j allows us to compute the claim amount still outstanding at the end of year $(i + j)$ with respect to the accident year i by $C_{i,j}(\lambda_j - 1)$.

let additionally define D_{n+} as the ratio of outstanding liability at the end of the development year n for year of accident 1 to cumulative amount of claims $C_{1,n}$. This implies that D_{n+} represents the estimate of the outstanding liability given at the end of the development year n (for year for origin 1). This essentially implies that Equation (3.10) can be expressed as

$$\lambda_j = \prod_{l=k+1}^{n} D_k \qquad j = 1, \dots, n - 1 \tag{3.11}$$

$$\lambda_n = D_{n+} \tag{3.12}$$

This methodology described involving the basic chain-ladder technique assuming that claim cost inflation, change in mix of business and all forms of external factors can be ignored provides estimation which can be applied to complete the run-off of the latter years of origin up to the point for which past experience is available. After from this basic chain-ladder method which has ignored factors, there are other chain-ladder techniques such as the inflation adjusted chain ladder technique and the separation technique.

The inflation adjusted chain ladder technique for example adopts a generalized model and introduces an assumption of an index of claims cost in the form

$$C_{i,j} = S_i R_j \cdot X_{i+j} + e_{ij} \tag{3.13}$$

with the parameters given as

$$C_{i,j} = s_i r_j \cdot \varphi_{i+j} + e_{ij} \tag{3.14}$$

where $C_{i,j}$ represents the claim payments in year of origin i and development year j; $_i$ represents the ultimate cost in real terms and claims incurred within period of origin i, r_j represents proportion of total payments in real terms made in development year j and φ_{i+j} are the assumed indices of claim costs. Another popular technique is the separation method which has a generalized model of the form

$$C_{i,j} = S_i R_j \cdot X_{i+j} + e_{ij} \tag{3.15}$$

with parameters defined by

$$C_{i,j} = n_i r_j \cdot \varphi_{i+j} + e_{ij} \tag{3.16}$$

In Equation (3.16), the number of claims incurred in year of origin i is denoted by n_i and φ_{i+j} is derived from the data than the typical assumption from external sources.

In the next section we consider the design of methodology for Stochastic reserve models.

3.3 Stochastic Claims Reserving Models

There has been increased amount of interest in Stochastic claims reserving methods although they are being used by a notable number of experts in the actuarial science industry. A number of reasons for this include inadequate understanding of these methods, rigid nature of the Stochastic method and lack of simulators to ensure easy manipulation of these methods amongst others. The discussion of Stochastic methods are constantly highlighted against existing deterministic methods across all disciplines; the actuarial science discipline is no exception with Stochastic claims reserving methods also discussed and analyzed alongside deterministic claims reserve techniques. There is no debate however that Stochastic models hold the potential in exhibiting better estimates compared to traditional methods primarily because they incorporate some variability of claims reserves and are able to ideally estimate full distribution of possible outcomes

3.3.1 Stochastic Chain Ladder Models

The basic chain-ladder as discussed previously makes use of cumulative data coupled with the derivation of development factors and ratios. In the Stochastic sense, it does not make any difference whether the data use dare incremental or the cumulative data. The assumption of the triangle form of the data is still maintained in the chain-ladder Stochastic model. This is primarily because of the simplicity of this nature of data; this therefore has data from early origin years considered as fully run-off or other parts of the triangle missing.

The incremental data is therefore defined as

$$C_{i,j} \qquad i = 1,\ldots,n \qquad j = 1,\ldots,n - i + 1$$

where same as the notation in the deterministic sense, i represents the accident year or year of writing in the row of the triangle and the column j describes the delay measured in years. Based on the notation, the cumulative claims are given by

$$D_{i,j} = \sum_{k=1}^{j} C_{i,k} \tag{3.17}$$

with development factors denoted by $\{\lambda_j; j = 2,\ldots,n\}$.

Based on the chain-ladder technique the estimation of the development factors are given as

$$\tilde{\lambda}_j = \frac{\sum_{i=1}^{n-j+1} D_{i,j}}{\sum_{i=1}^{n-j+1} D_{i,j-1}} \tag{3.18}$$

After arriving at the estimates, the $\hat{\lambda}_j$'s are applied to the latest cumulative claims to produce forecasts as follows

$$\hat{D}_{i,n-i+2} = D_{i,n-i+1}\hat{\lambda}_{j n-i+2} \quad (3.19)$$

$$\hat{D}_{i,k} = \hat{D}_{i,k-1}\hat{\lambda}_{k} \quad (3.20)$$

In the most simplest of forms, the chain-ladder technique consists of the forecasts of ultimate claims, which represents the latest delay year so far observed exclusive of any tail factors.

The concept of the prediction error defined as the standard deviation of the distribution expressed as a percentage of possible reserve outcomes. The prediction error relates to the forecast variations, considering the uncertainty associated with parameter estimation and the variability inherent in the forecasting of data.

A statistical model formulated with assumptions about the data is the right step in addressing the issue of obtaining the prediction error. Since the main aim is to obtain a stochastic model similar to the chain ladder model, the predicted estimates should be the same as those of the chain-ladder method. Two ways to do this is either to specify distributions for the data or specify the first two moments. The next subsection discusses an example of a stochastic model that gives the same estimates as that of the chain ladder technique.

3.3.2 Over-Dispersed Poisson Model

There are a number of stochastic models that gives the same estimates of the chain ladder technique. The over-dispersed Poisson (ODP) is one of the particular well known and assumes that incremental claims have an over-dispersed Poisson distribution. An ODP looks like a Poisson distribution in that the variance function is equal to the mean, but it also includes the dispersion parameter φ

Thus,

$$C_{ij} \sim IID\ ODP_0(m_{ij}) \quad (3.21)$$

where IID denotes independent, identically distributed with $E(C_{ij}) = m_{ij}$ and

$Var(C_{ij}) = \varphi m_{ij}$

This model is primarily used due to its ability to give the same outstanding claim estimates as the chain ladder technique. For this condition to be satisfied i.e. for the Over-dispersed Poisson to provide the same reserves estimates as the chain ladder technique, the mean is modeled using

$$\log(m_{ij}) = c + \alpha_i + \beta_j \tag{3.22}$$

This is recognized as a generalized linear model in which the responses C_{ij} are estimated with a logarithmic link function and linear predictor, m_{ij}. The predictor structure is suitable for fitting the chain ladder technique in that there is a parameter column for each row i and a parameter for each column j. The corner constraints are applied due to the overparametrization of the model as follows:

$$\begin{cases} \log(m_{ij}) = c + \alpha_i + \beta_j \\ \alpha_1 = 0 \quad \beta_1 = 0, \end{cases} \tag{3.23}$$

β_j, the column parameter determines the run off structure of the data. There is no particular shape of the run off patterns due to the one parameter for each column and is consistent with the chain ladder model.

3.3.3 The Re-parametrization of the Run-off Shape

Because of the trans-dimensional approach adopted, the run-off shape undergoes re-parametrization. Although the expected shape of the run-off is exponentialdecaying tail in the presence of Bayesian model, this primarily departs from the expectation. The implication is that taking the logarithm of the run-off will produce a straight line and the second differences of the parameters $\{\beta_j : j = 2,3,\ldots,n\}$ is almost zero. Note that the requirement for the application of this model is that the parameters should be tested as to whether they are assumed to be zero. This means that if we have a parameter implementation in which the zero, it would be accepted. The exact form of parametrization used is as follows:

Consider

$$\nabla\beta_j = \beta_j - \beta_{j-1} \qquad \text{and} \qquad \nabla^2\beta_j - 2\beta_{j-1} + \beta_{j-2} \qquad j = 2,3,\ldots,n$$

where $\nabla\beta_j$-gradient of the log development pattern and $\nabla^2\beta_j$ - change in the gradient. This means that if $\nabla^2\beta_j$ is zero, the log development follows a straight line with non-zero values of $\nabla^2\beta_j$; meaning departures from the straight line. A matrix representation which will be used in the trans-dimensional Bayesian model according to Lunn et al. (2009b) is given as;

$$\begin{pmatrix}\nabla\beta_3\\ \nabla\beta_4\\ \nabla\beta_5\\ \vdots\\ \nabla\beta_n\end{pmatrix} = \begin{pmatrix}1 & 0 & \cdots & \cdots & 0\\ 1 & 1 & 0 & \cdots & 0\\ 1 & 1 & 1 & \cdots & 0\\ \vdots & \vdots & \vdots & \ddots & \vdots\\ 1 & 1 & 1 & \cdots & 1\end{pmatrix}\begin{pmatrix}\nabla^2\beta_3\\ \nabla^2\beta_4\\ \nabla^2\beta_5\\ \vdots\\ \nabla^2\beta_n\end{pmatrix} \qquad (3.24)$$

$\nabla\beta_n$ ⯑⯑

1

For the sake of recasting, the extrapolation is done beyond the latest development year to obtain tail factors by assuming $\nabla^2 = 0$; this implies that $\beta_j = 2\beta_j - \beta_{j-2}$. This consideration of the latter set of parameters allows the application of the trans-dimensional Bayesian model to $\{\nabla^2\beta_j\}_{j=3}^{n}$.

3.4 Basic Terminology of Markov Chain

3.4.1 Markov Chain

A significant concept in this research is that of Markov Chains. This basically represents a set of sequence $X_1, X_2, \ldots$ of random elements such that the conditional distribution of X_{n+1} given $X_1, X_2, \ldots, X_n$ depends only on X_n. In this instance we refer to the set as a *Markov Chain* and the set in which X_i takes values as the *State Space* of the Markov Chain. A Markov Chain is said to have a stationary transition probabilities given that the conditional distribution of X_{n+1} given X_n does not depend on n. A transition probability distribution is reverisble with respect to an intial ditribution if, for the Markov chain $X_1, X_2, \ldots$ they specify, the distribution of pairs (X_i, X_{i+1}) is exchangeable; this is the type of

Markov chain of interest in this research referred to as the *Markov Chain Monte Carlo (MCMC)*

3.5 Bayesian Modeling, RJMCMC and WinBUGS

Software

The parameter specification used in the research are as follows; using the assumption that I represents the observed data and its distribution is $f(I|\theta,\mathbf{M})$ where $\mathbf{M}$ is the model and θ represents the number of parameters and both are assumed to be unknown. The prior distributions $f(\mathbf{M})$ and $f(\theta|\mathbf{M})$ are assigned to the model and the posterior distribution is given by

$$f(\mathbf{M},\theta|I) \propto f(I|\theta,\mathbf{M})f(\theta|\mathbf{M})f(\mathbf{M}) \tag{3.25}$$

and the parameter vector is

$$\theta = (c,\alpha_1,...,\alpha_n,\beta_2,\nabla\beta_3,\nabla^2\beta_j, \qquad j \in \mathbf{M} \tag{3.26}$$

In contrast to most applications in statistics where the objective is identification of optimal model and its application to statistical inference, the main objective of claims reserves modeling is the predictive distribution of future claims Verrall and Wuthrich (2010). The predictive distribution for the incremental future claims $C_{i,j}$ is

$$f(C_{i,k}|\mathbf{M},I) = \int f(C_{i,j}|\mathbf{M},\theta)f(\theta|M,I)d\theta \quad i+k>n+1 \tag{3.27}$$

There are basically two different approaches in the context of trans-dimensional model in accounting for model uncertainty. The first is to make a choice of the most likely model from the Bayesian analysis in producing predictive distribution known as a Maximum A Posterior (MAP) estimator given by

$$f(C_{i,k}|I) \approx f(C_{i,k}|\mathbf{M}_{\max},I) \tag{3.28}$$

The second approach is to estimate the predictive distribution by finding the average of the models using their weights as the posterior probabilities. This is second approach known as Bayesian Model Averaging (BMA) which has forecasts given by

$$f(C_{i,j}|I) = \sum_{\mathbf{M}} f(C_{i,j}|\mathbf{M},I)P(\mathbf{M},|I) \tag{3.29}$$

Equation (3.29) is the approach adopted in this research.

3.5.1 Reversible Jump MCMC

The predictive distribution of the outstanding incremental claims as a combination of Equation (3.27) and (3.29) is

$$f(C_{i,k}|I) = \int f(C_{i,k}|\mathbf{M},\theta) f(\mathbf{M},\theta|I)\, d(\mathbf{M},\theta) \qquad (3.30)$$

In the situation where an exact form of this distribution is not obtainable in the closed, numerical methods via simulation have proved to be very effective. The simulation methods have their foundation in Markov Chains which are generated such that it has the same equilibrium distribution as the posterior distribution Metropolis et al. (1953). Using this analogy, the predictive distribution in (3.30) is given by the Monte Carlo Average

$$f(C_{i,j}|I) = \frac{1}{N} \sum_{a=1}^{N} f(C_{i,k}|M^{(B+ta)}, \theta^{(B+ta)}) \qquad (3.31)$$

where B is the burn-in time, that is the time before the Markov chain has converged to its equilibrium distribution, t - thinning parameter that is often set to t=1 and may be altered to t¿1 if the serial correlation of the output Markov Chain is high. The methodology of MCMC provides a framework of generating the Markov Chain. The next state give the current state $(\mathbf{M}^{(b)},\theta^{(b)}$ is

$$(\mathbf{M}^{(b+1)}, \theta^{(b+1)}) = \begin{cases} (\mathbf{M},\theta) & \text{if } (\mathbf{M},\theta) \text{ is accepted} \\ (\mathbf{M}^{(b)},\theta^{(b)}) & \text{if } (\mathbf{M},\theta) \text{ is rejected} \end{cases} \qquad (3.32)$$

Using Gibbs sampling, the model $\mathbf{M} = \mathbf{M}^{(b)}$ is kept fixed, whereas a set of parameters in θ is updated based on its conditional posterior distribution. Because the proposal distribution is the best one, all proposals are accepted. A difficult sample selection based on the conditional distribution evokes the use of the more standard Metropolis-Hastings (MH) algorithm defined by

$$P(\text{accept}(\mathbf{M},\theta)) = \min\left(1, \frac{f(\theta|\mathbf{M}^{(b)}, I)\pi(\theta^{(b)}|\theta)}{f(\theta^{(b)}|\mathbf{M}^{(b)}, I)\pi(\theta|\theta^{(b)})}\right) \qquad (3.33)$$

A generalization of the MH algorithm is the Reversible Jump MCMC allowing for jumps between different models. In contrast to the criteria in Equation (3.33), the acceptance of the probability is given by

$$P(\text{accept}(\mathbf{M},\theta)) = \min\left(1, \frac{f(\mathbf{M},\theta|I)\pi(\mathbf{M}^{(b)},\theta^{(b)}|M,\theta)}{f(\mathbf{M}^{(b)},\theta^{(b)}|I)\pi(\mathbf{M},\theta|\mathbf{M}^{(b)},\theta^{(b)})}\left|\frac{\partial(\theta,v)}{\partial(\theta^{(b)},u)}\right|\right) \qquad (3.34)$$

which has a partial derivative representative of the Jacobian which explains the different forms of parametrization of the two models.

3.5.2 Trans-dimensional Modeling with WinBUGS

As discussed previously, this research employs the use of a statistical software for simulating the claims reserves known as WinBUGS. WinBUGS is a freeware with the aim of making practical MCMC methods available to all statisticians. Specifically, WinBUGS is a standalone package or can be called from other software such as the R statistical package and is a flexible platform for Bayesian analysis of complex statistical modeling via Markov Chain Monte Carlo (MCMC) methods.

According to Lunn et al. (2009b), there are two streams of modes suitable within WinBUGS.

With the trans-dimensional model by Lunn et al. (2009b) defined in terms of an unknown number of entries associated with the run-off, we employ the reparametrization by using $\{\nabla^2\beta_j : j = 4,5,...,n\}$ as the entities. We denoted the number of parameters which is unknown at prior by k and this is initially specified with a binomial distribution with parameters $n-3$ and $\frac{1}{2}$. The parametrization of the model is given by

… … (3.35)

$$\begin{pmatrix}\nabla\psi_3\\ \nabla\psi_4\\ \nabla\psi_5\\ \vdots\\ \nabla\psi_n\end{pmatrix} = \begin{pmatrix}1 & 0 & \cdots & & 0\\ 1 & 1 & 0 & \cdots & 0\\ 1 & 1 & 1 & \cdots & 0\\ \vdots & & & \ddots & \vdots\\ 1 & \cdots & \cdots & \cdots & 1\end{pmatrix}\begin{pmatrix}\nabla^2\beta_3\\ \nabla^2\beta_4\\ \nabla^2\beta_5\\ \vdots\\ \nabla^2\beta_n\end{pmatrix}$$

with ψ begin chosen to coincide with Lunn et al. (2009b) which directly enables a comparison to be made. $\psi_j = \nabla\beta_{j+2}$ for $j = 1,2,...,n-2$ and the parameters $\beta_3,\beta_4,...,\beta_n$ estimated using $\beta_j = \beta_{j-1} + \psi_{j-2}$; $j = 3,4,...,n$.

The form of parametrization described ensures the possibility of constructing a Bayesian model via a trans-dimensional model within WinBUGS given that the second difference of the run-off paramters is $\{\nabla^2\beta_j : j = 4,5,...,n\}$ and is treated as an optional parameter. Three different alternatives are possible

Dimension Move :	Propose new $\|\mathbf{M}\|$,$\mathbf{M}$ and θ in this order
Configuration Move :	Propose new $\mathbf{M}$ and θ in this order, with $\|\mathbf{M}\|$ fixed
Coefficients Move :	Propose new θ with $\mathbf{M}$

The completion of the model lies in the specifying of the prior distributions. This is provided as follows: The first necessary replacement is to condition the model, M using prior distribution begin uniform as $P(\mathbf{M}) = 2^{-(n-3)}$ for all members. In

WinBUGS, this usually is set by default for trans-dimensional models. Particularly, key is the conditional via the prior distributions of the optimal parameters $\{\nabla^2\beta_j : j = 4,5,...,n\}$ being set by default such that they are independently normally distributed with

$$E[\nabla^2\beta_j] = 0; \qquad Var[\nabla^2\beta_j] = \tau \tag{3.36}$$

The prior distribution of the hyperparameter τ which is an inverse gamma distribution with variance say 1000, is large

$$\tau^{-1} \sim \mathbf{N}(0.001, 0.001) \tag{3.37}$$

There are other non-informative prior distributions for the remaining parameters all normally distributed with mean zero and variance 10,000.

$$c \sim \mathbf{N}(0, 10{,}000)$$

$$\{\alpha_i : i = 1,2,3,...,n\} \sim \text{independent } \mathbf{N}(0, 10{,}000)$$

$$\beta_2 \sim \mathbf{N}(0, 10{,}000)$$

$$\nabla\beta_3 \sim \mathbf{N}(0, 10{,}000)$$

The variance of the prior normal distributions are chosen to be relatively large and this could be made even as large as 100,000 although this would be insignificant.

A significant reduction on the other hand will alter the results.

3.5.3 Convergence Diagnostics

For the MCMC algorithm to effectively work, it is important to have the Markov Chain $\{(\mathbf{M}^{(b)}, \theta^{(b)})\}$ reach the equilibrium posterior distribution and run for a significantly long time. This is possible by setting appropriately the burn-in time B, thinning parameter t and the chain length N. There are several methods of convergence diagnostics Cowles and Carlin (1996), Brooks and Gelman (1998) and Geweke (1991) which considers Gibbs sampling and data augmentation with a suggestion of a test of differences between posterior means of the early and late parts of the Markov Chain via spectral methods with time series.

The *R* package boa Smith (2007)is a software for monitoring MCMC convergence. It firstly selects a variable being a scalar function of the parameter vector θ which may be α_i,β_j or the outstanding reserve for the accident year i. The second step is to run parallel chains $m \geq 2$ and monitor the chosen variable of interest. The third step is to compare the sample variance within W and sample variance between B/N using the Potential Scale Reduction Factor (PSRF) given by

$$PSRF = \sqrt{\frac{N-1}{N} + \frac{m+1}{mN}\frac{B}{W}} \tag{3.38}$$

The PSRF is compared to 1 as a value of almost 1 indicates convergence. Additionally, a connected scale reduction factor

$$CSRF = \sqrt{\frac{df+3}{df+1}} \tag{3.39}$$

is calculated. This basically accounts for sampling variability in the estimate of the true variance of the variable of interest where df is the estimate degree of freedom associated with the method of moments. The software gives the 0.5 and 0.975 quantile greater than 120 interpreted as evidence of non-convergence for a variable Smith (2007).

The next chapter is presentation of simulation results of claims reserves based on the methodology presented in this chapter. The WinBUGS is employed in supporting the theoretical concepts built in this chapter.

Chapter 4

Results and Discussion

4.1 Introduction

This chapter is a presentation of and statistical analysis of claims data collected from a renowned Insuarnce Company in Ghana, which operates as a dominant insurance company especially in the non-life insurance industry. The Insurance Company runs non-life insurance products such as fire, home, motor, etc. and other personal and corporate polices. The claims data in GH S from the Insurance Company spanned from 10 years, 2005 to 2014 and is shown Table (4.1).

Table 4.1: Incremental Claims Data

	Development Year									
i,j	1	2	3	4	5	6	7	8	9	10
1	135,295	89,258	73,381	49,261	229533	31,741	16,592	12,507	7,613	3,350
2	132,487	85,952	72,789	50,113	37,890	28,560	19,645	13,679	8,768	
3	127,975	100,356	70,247	47,037	30,474	25,041	17,057	11,952		
4	128,783	93,432	67,917	51,145	31,315	34,370	15,612			
5	130,367	80,819	68,546	53,692	35,418	29,137				
6	118,947	96,779	74,484	55,741	28,187					
7	125,808	81,610	69,340	46,113						
8	123,462	90,290	71,859							
9	119,306	79,523								
10	133,621									

The methodology outlined in the previous chapter is used to analyze the incremental claims reserve data in Table (4.1) using Windows Bayesian Inference Using Gibbs Sampling (**WinBUGS**) software, version 1.4.3, a stand-alone package. WinBUGS is designed for Bayesian analysis with the use of Markov Chain Monte Carlo (MCMC) methods. The reserve statistics are based on 500000 simulations run with initial Burn in cost of 5000. The WinBUGS code used for the simulation in this research has been documented in the Appendix of this research.

4.2 Analysis of Results

The modeling of the claims reserves starts with the cumulative of the claims reserve data as shown in Table (4.2) using the claims reserves in Table (4.1).

Table 4.2: Cumulative Claims Data

	1	2	3	4	5	6	7	8	9	10
1	135,295	224,553	287,934	347,195	376,728	408,469	425,061	437,568	445,181	448,531
2	132,487	218,439	291,228	341,341	379,231	407,791	441,115	449,883		
3	127,975	228,331	298,578	345,615	376,089	401,130	418,187	430,139		
4	128,783	211,186	290,132	341,277	372,592	406,962	422,574			
5	130,367	211,186	279,732	333,424	368,842	397,979				
6	118,947	215,726	290,210	345,951	374,138					
7	125,808	207,418	276,758	322,871						
8	123,462	213,752	285,611							
9	119,306	198,829								
10	133,621									

The next is to fit the straightforward Over-dispersed Poisson (ODP) model with a constant scale parameter which gives the same results as the chain ladder method as noted in the methodology.

Table 4.3: Chain-Ladder Development Factors

j	f_{ij}
2	1.6985
3	1.3265
4	1.1744
5	1.0938
6	1.0795
7	1.0424
8	1.0300
9	1.0186
10	1.0075

The development factors are shown in Table (4.3) and corresponding chain ladder estimates are given Table (4.4).

Table 4.4: Chain-Ladder Reserve Estimates (in GHS)

j	
2	3,385
3	11,316
4	24,134
5	40,578
6	70,901
7	97,225
8	150,818
9	204,176
10	326,401
Overall	928,934

4.2.1 Parameter Estimation: ODP and Bayesian Models

The results of the maximum likelihood parameter estimates as a consequence of the fit of the chain ladder ODP model is shown in Table (4.5).

Table 4.5: Comparison of Estimates of the Parameters of the ODP Model and the Bayesian Model using 95% prediction intervals for the Bayesian Model

Parameter	ODP	Posterior Mean	Posterior PI
Constant $\tilde{c}$	11.7775	11.7519	(11.4028,11.7546)
$\tilde{\alpha}_2$	0.0105	0.0136	(-0.0572,0.0843)
$\tilde{\alpha}_3$	-0.0159	-0.0139	(-0.0855,0.0583)
$\tilde{\alpha}_4$	-0.0041	-0.0032	(-0.0757,0.0697)
$\tilde{\alpha}_5$	-0.0225	-0.0193	(-0.0931,0.0543)
$\tilde{\alpha}_6$	-0.0078	-0.0074	(-0.0850,0.0660)
$\tilde{\alpha}_7$	-0.0655	-0.0616	(-0.1407,0.1776)
$\tilde{\alpha}_8$	-0.0274	-0.0255	(-0.1080,0.0566)
$\tilde{\alpha}_9$	-0.1070	-0.1063	(-0.1998,-0.0136)
$\tilde{\alpha}_{10}$	0.0253	0.0260	(-0.0826,0.1338)
$\tilde{\beta}_2$	-0.3588	-0.3569	(-0.4059,-0.3085)
$\tilde{\beta}_3$	-0.5897	-0.5936	(-0.6516,-0.5386)
$\tilde{\beta}_4$	-0.9341	-0.9489	(-1.0070,-0.8210)
$\tilde{\beta}_5$	-1.3932	-1.3554	(-1.4190,-1.2490)
$\tilde{\beta}_6$	-1.4699	-1.5340	(-1.6530,-1.4370)
$\tilde{\beta}_7$	-2.0209	-1.9632	(-2.0630,-1.8720)
$\tilde{\beta}_8$	-2.3254	-2.3721	(-2.4980,-2.2390)
$\tilde{\beta}_9$	-2.7721	-2.8410	(-3.0320,-26150)
$\tilde{\beta}_{10}$	-3.6608	-3.3530	(-3.8180,-2.9470)
Dispersion parameter	292.7238		

The parameter estimates i.e. c, α_i and β_j are all estimated based on the Bayesian approach such that $\tilde{c} = E(c|\mathbf{I}$, $\tilde{\alpha} = E(\alpha_i|\mathbf{I})$ and $\tilde{\beta} = E(\beta_j|\mathbf{I})$ from the output of the MCMC method. Table (4.6) shows the output of the parameter estimates for the claims reserves.

4.2.2 Estimates of Outstanding Claims

A plot of the comparison between the estimates (logarithm) of the ODP model and the two Bayesian model (the first without a tail and the second with tail) is shown in Figure (4.1). Based on the patterns displayed in Figure (4.1), all models

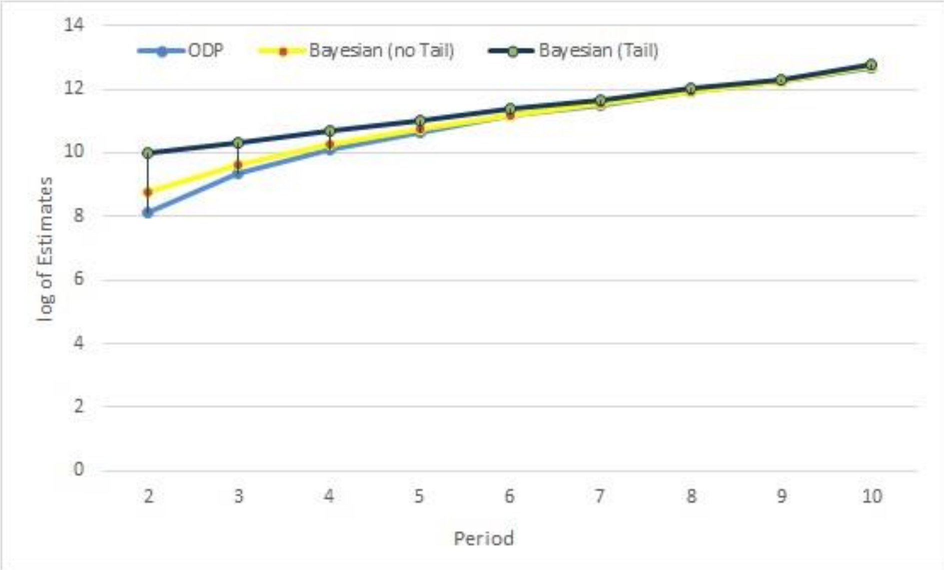

Figure 4.1: Logarithm plot of the estimates for the ODP and Two Bayesian Models

indicate some level of convergence as the period increases. The differences in the estimates provided are only glare in the initial periods. The estimates from the respective models are similar as time increases.

The errors associated with each of the models together with the actual period estimates is shown Table (4.6). Based on this result, the Bayesian model without tail returns the lowest prediction error of 5%. This result from the simulation is parallel and consistent with the findings of Verrall et al. (2010) who run the the RJMCMC using data from Taylor and Ashe (1983). The prediction errors from the Bayesian methods are generally lower and this occurrence is attributed to the effect of the reflection of the smoothing of the run-off shape posed within the Bayesian Methods.

Table 4.6: Outstanding claims estimates for Bayesian and ODP Models (in GHS)

	ODP		Bayesian Model Without Tail		Bayesian Model With Tail	
Row	Estimate	PE	Estimate	PE	Estimate	PE
1					7,206	48%
2	3,385	42%	4,727	22%	12,040	37%
3	11,316	21%	12,130	15%	19,250	26%
4	24,134	14%	24,380	10%	31,570	17%

5	40,578	10%	41,930	6%	49,010	11%
6	70,901	9%	70,190	5%	77,340	9%
7	97,225	7%	98,840	4%	105,600	6%
8	150,818	6%	151,600	4%	158,600	5%
9	204,176	6%	204,500	4%	211,000	5%
10	326,401	6%	327,100	5%	334,000	5%
Total	928,934	6%	935,397	5%	1,006,116	7%

From Table (4.6), the over-dispersed Poisson Model returns a prediction error of 6%, slightly higher than the Bayesian model without tail. The Bayesian model with tail however returns the largest prediction error of 5% which is lower but consistent with the results from Verrall et al. (2010).

Chapter 5

Conclusion and Recommendations

5.1 Introduction

The main objective of this research was to model the claims run-off triangle by the use of the reversible jump Markov Chain Monte Carlo (RJMCMC) method. This by way of advantage is primarily to model claims run-off using an objective methodology. The RJMCMC method is said to have great potential in contrast to existing methods for modeling claims, Verrall et al. (2010). The research uses WinBUGS to run the simulation of claims data from an Insurance Company in Ghana. This chapter is a presentation of the conclusions and recommendations.

5.2 Major Findings

The chain ladder ODP model gives a lower result for the claims reserves GHS 928,934 as compared to the Bayesian GHS 935,397 and GHS 1,006,116 without and with tail respectively. The results clearly showed the chain ladder ODP model under estimates the outstanding claims liabilities mainly due to the low development factors. Clearly be observed in most recent years that, the prediction errors expressed as a percentage of the reserve estimate are large primarily due to the estimation error. The claims reserves are increased by ≈GHS 71,000 between the Bayesian with or without tail giving roughly GHS 7,100 per accident year $i \in \{1,.....10\}$ that is for the ten year period.

5.3 Conclusion

The research concludes that based on the findings from the data analysis, the claims reserve is best estimated using the Bayesian methods. The findings show that the prediction errors of the Bayesian models are relatively lower. This is because of the smoothing effect within the Bayesian methods. Conclusion can be drawn that, the tail factor is an important quantity in estimating oustanding claim laibilities in terms of both the claims reserves but also for the uncertainty in these claims reserves. The results may be general for claims reserves data since the findings here are consistent with the findings of Verrall et al. (2010).

5.4 Recommendation

The concept of problem solving is constantly evolving with more refined solutions to pertaining problems across all disciplines. The study recommends that further research work be done in examination of claims reserve data from several other insurance companies in Ghana to ascertain the generality of the Bayesian Methods producing lower predictive errors in comparison to ad hoc methods. There should be innovative ways of getting Actuarial scientists and experts in claims reserve modeling to use these new trending methodologies with obvious advantage over the ad hoc methods which have been demonstrated to be simple yet less reliable. More studies should be devoted to software development to enhance usability of developed models for claims reserves.

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Appendix

WinBUGS Code for the Simulation

```
model {
# The likelihood is constructed using the zeros trick. The data are first divi
# by 1000 for computational efficiency.
for( i in 1 : 55 )
  {
Z[i] <- Y[i]/1000 log(mu[i]) <- cons+alpha[row[i]] + beta[col[i]]; zeros[i]<- 0 zeros[i] ~ dpois(phi[i])
phi[i] <- ((mu[i]-Z[i])-Z[i]*log(mu[i]/Z[i]))/scale        # MINUS log likelihood }

# psi is not available directly, and so we create an artificial variable, b1,
#which is essentially equal to psi.
for (i in 1:8) {
b1[i]~dnorm(psi[i],100000) }
```

```
# This section sets up the trans-dimensional model for the run-off parameters beta[1]<-0 beta[2]<-
beta2 beta2~dnorm(0,0.0001) for (i in 1:8) { beta[i+2]<-beta[i+1]+b1[i]} for (i in 1:5) { beta[10+i]<-
beta[9+i]+b1[8]} psi[1:8]<-jump.lin.pred.int(X[1:8,1:7],k1,tau,0,0.0001) tau~dgamma(0.001,0.001)
id<-jump.model.id(psi[1:8]) k1~dbin(0.5,7)

# As suggested by England and Verrall (2006), we use a gamma
#distribution with the same mean and variance as the ODP for forecasting.
for( i in 56 : 100 ) {
log(mu[i]) <- cons+alpha[row[i]] + beta[col[i]]; fa[i] <-max(0.01,1000*mu[i]/sc fb[i] <- 1/scale
Z[i] ~ dgamma(fa[i], fb[i])
}

for( i in 1 : 100 ) {
fit[i] <- Z[i]
}

for(i in 1:50) { log(muT[i])<-
cons+alpha[rowT[i]]+beta[colT[i]] faT[i]<-
max(0.01,1000*muT[i]/scale) fbT[i]<-1/scale
ZT[i]~dgamma(faT[i],fbT[i])
}

for (i in 1:10) {
Tail[i]<-sum(ZT[5*(i-1)+1:5*i])
}

scale <- 0.2927 cons~dnorm(0.0,0.0001)
alpha[1]<- 0 for (k in 2:10) {alpha[k]~
dnorm(0.0,0.0001)}
```

```
R[1] <-0
R[2] <- fit[56]
R[3] <- sum(fit[57:58])
R[4] <- sum(fit[59:61])
R[5] <- sum(fit[62:65])
R[6] <- sum(fit[66:70])
R[7] <- sum(fit[71:76])
R[8] <- sum(fit[77:83])
R[9] <- sum(fit[84:91])
R[10] <- sum(fit[92:100]) Total <-
sum(R[2:10])

for (i in 1:10) {
RT[i]<-R[i]+Tail[i]
} TotalT<-sum(RT[1:10])
}
#INITIAL VALUES list(alpha = c(NA,0,0,0,0,0,0,0,0,0), b1 = c(0,0,0,0,0,0,0,0,0),
cons=0, tau=1, beta2=1)

 # DATA list(row=c(1,1,1,1,1,1,1,1,1,1,2,2,2,2,2,2,2,2,2,3,3,3,3,3,3,
 3,3,4,4,4,4,4,4,4,5,5,5,5,5,5,6,6,6,6,6,7,7,7,7,8,8,8,9,9,10
 ,2,3,3,4,4,4,5,5,5,5,6,6,6,6,6,7,7,7,7,7,7,8,8,8,8,8,8,8,
 9,9,9,9,9,9,9,9,10,10,10,10,10,10,10,10,10),
 col=c(1,2,3,4,5,6,7,8,9,10,1,2,3,4,5,6,7,8,9,1,2,3,4,5,6,7,8,
 1,2,3,4,5,6,7,1,2,3,4,5,6,1,2,3,4,5,1,2,3,4,1,2,3,1,2,1,
 10,9,10,8,9,10,7,8,9,10,6,7,8,9,10,5,6,7,8,9,10,
 4,5,6,7,8,9,10,3,4,5,6,7,8,9,10,2,3,4,5,6,7,8,9,10),
 Y=c(135295,89258,73381,49261,29533,31741,16592,12507,
7613,3350,132487,85952,72789,50113,37890,28560,19645,
```

```
13679,8768,127975,100356,70247,47037,30474,25041,
17057,11952,128783,93432,67917,51145,31315,34370,
15612,130367,80819,68546,53692,35418,29137,
118947,96779,74484,55741,28187,
125808,81610,69340,46113,
123462,90290,71859,
119306,79523,
133621,
NA,NA,NA,NA,NA,NA,NA,NA,NA,NA,NA,NA,NA,NA,NA,NA,NA,NA,
NA,NA,NA,NA,NA,NA,NA,NA,NA,NA,NA,NA,NA,NA,NA,NA,NA,NA,
NA,NA,NA,NA,NA,NA,NA,NA,NA),
ZT=c(NA,NA,NA,NA,NA,NA,NA,NA,NA,NA,NA,NA,NA,NA,NA,NA,
NA,NA,NA,NA,NA,NA,NA,NA,NA,NA,NA,NA,NA,NA,NA,NA,NA,NA,
NA,NA,NA,NA,NA,NA,NA,NA,NA,NA,NA,NA,NA,NA,NA,NA), rowT=c(
1,1,1,1,1,
2,2,2,2,2,
3,3,3,3,3,
4,4,4,4,4,
5,5,5,5,5,
6,6,6,6,6,
7,7,7,7,7,
8,8,8,8,8,
9,9,9,9,9, 10,10,10,10,10),
colT=c( 11,12,13,14,15,
11,12,13,14,15,
11,12,13,14,15,
11,12,13,14,15,
11,12,13,14,15,
11,12,13,14,15,
```

```
11,12,13,14,15,
11,12,13,14,15,
11,12,13,14,15,
11,12,13,14,15),
X=structure(
.Data = c(
0,0,0,0,0,0,0,
1,0,0,0,0,0,0,
1,1,0,0,0,0,0,
1,1,1,0,0,0,0,
1,1,1,1,0,0,0,
1,1,1,1,1,0,0,
1,1,1,1,1,1,0,
1,1,1,1,1,1,1),
.Dim = c(8,7)))
```