OPTIMUM LOAN PORTFOLIO SELECTION:
A CASE STUDY OF JUABEN RURAL BANK, ASHANTI REGION

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Administration.

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DECLARATION

I hereby declare that this submission is my own work towards the Commonwealth Executive Masters in Business Administration and that, to the best of knowledge; it contains no material previously published by another person or material which has been accepted to for the award of any other degree of the university, except where due acknowledgment has been made in the text.

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(Dean of IDL)
I dedicate this work to wife and children.
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ABSTRACT

Banks receive money on current or deposit account, pay and collect cheques drawn by or paid by customers, making of advances to customers. A bank can generate revenue in a different ways including interest, transaction of fees and financial advice. The main method is through charging interest on the capital it lends out to customers. The objectives of the study were; (i) to come out with a quantitative model that will maximize the returns on loans (ii) to determine optimum loan portfolio for Juaben rural bank. Both primary and secondary data were gathered from Juaben Rural Bank. The data included; type of loans, the interest rate and the probability of bad debt associated with each type of loan. The data was then modelled as a linear programming problem. The Quantitative Manager for Windows software was used to solve the problem. It was observed that out of six million, five hundred thousand Ghana cedis (GH¢6,500,000.00) to be disbursed as loan in 2012 financial year, one million, one hundred and ten thousand, four hundred and sixteen Ghana cedis (GH¢1,110,416.00) and seven hundred and four thousand, one hundred and sixty-six Ghana cedis (GH¢704,166.00) should be given to agriculture and transport sectors respectively. The trading sector should be given one million, four hundred and eighty-nine thousand, five hundred and eighty four Ghana cedis (GH¢1,489,584.00), The cottage industry, one million, four hundred and thirty five thousand, four hundred and seventeen Ghana cedis (GH¢1,435,417.00) and salary loans one million, seven hundred and sixty thousand, four hundred and seventeen Ghana cedis (GH¢1,760,417.00). With these allocation the bank will make a maximum profit of one million, eight hundred and fifty- two thousand, four thundered eighty- five Ghana cedis eighty nine pesewas (GH¢1,852,485.89).
LIST OF TABLES

Table 3.1 General from Initial Simplex Tableau ............................. 40
Table 3.2 The Initial Tableau ..................................................... 41
Table 3.3 Second Simplex Tableau ............................................. 44
Table 3.4 Third Simplex Tableau .............................................. 45
Table 4.1 Loan Types, Interest Rate and Probability of Bad Debt ....... 48
Table 4.2 Optimum Solution for the Loan facilities ....................... 52
Table 4.3 Dural for Constraint for Loan Distribution ..................... 54
# TABLE OF CONTENTS

<table>
<thead>
<tr>
<th>Content</th>
<th>page</th>
</tr>
</thead>
<tbody>
<tr>
<td>Declaration</td>
<td>i</td>
</tr>
<tr>
<td>Dedication</td>
<td>ii</td>
</tr>
<tr>
<td>Acknowledgement</td>
<td>iii</td>
</tr>
<tr>
<td>Abstract</td>
<td>iv</td>
</tr>
<tr>
<td>List of tables</td>
<td>v</td>
</tr>
</tbody>
</table>

**Chapter One: Introduction**

1.1.0 Background to the study ............................................. 1
1.1.1 The Economic Functions of Banks .................................. 2
1.1.2 Types of Banks in Ghana ............................................. 3
1.1.3 Rural and Community Banks ......................................... 8
1.1.4 Rural Banks .......................................................... 8
1.1.5 Challenges Facing by Rural Community banks .................... 9
1.2 Statement of the Problem .............................................. 10
1.3 Objectives of the Study ............................................... 11
1.4 Research Question ..................................................... 11
1.5 Significance of the Study ............................................. 12
1.6 Scope/Limitation of the Study ........................................ 12
1.7 Justification .......................................................... 13
1.8 Organization of the Study ............................................. 13
1.9 Summary ............................................................... 14
Chapter Two: Literature Review

2.0 Introduction ................................................................. 15
2.1.0 Loans ........................................................................ 15
2.1.1 Loan Repayment ...................................................... 16
2.1.2 Loan Portfolio ........................................................ 16
2.1.3 Strategic Planning for Loan Portfolio ......................... 16
2.2 What motivate people to go for Loans ......................... 17
2.3 Factors affecting the repayment of loans ....................... 18
2.4 Portfolio Management ................................................. 18
2.5 Linear Programming .................................................... 24
2.6 Benefit of Loan Portfolio to Management and Decision Makers of Banks.. 29

Chapter Three: Research Methodology

3.1.0 Introduction ............................................................... 31
3.1.1 Organizational Profile ............................................. 31
3.1.2 Research Design .................................................... 34
3.1.3 Population and Sample ........................................... 34
3.1.4 Sources of Data ...................................................... 34
3.1.5 Data Collection Instrument .................................... 35
3.1.6 The Study Area ...................................................... 35
3.1.7 Data Analysis ........................................................ 36
3.1.8 The Linear Programming Model .......................... 36
3.1.9 Simplex Method ...................................................... 37
3.2.0 Formulation of the Problem ................................... 38
3.2.1 Algorithm for Simplex Method ............................ 38
3.2.2 Setting up Initial Simplex Tableau ......................... 39
3.2.3 Improving the Solution .......................................... 42
3.2.4 Simplex Methods with Mixed Constraints

Chapter Four: Data Presentation and Analysis

4.0 Introduction

4.1 Loan Type Interest Rate, and Probability of Bad Debt

4.2 Portfolio Selection

Chapter Five: Summary, Recommendations and Conclusion

5.0 Introduction

5.1 Summary Findings

5.2 Recommendations

5.2 Conclusion
CHAPTER ONE

INTRODUCTION

1.1.0 Background to the study

Banking means the business of receiving money on current or deposit accounts, paying and collecting cheques drawn by or paid in by consumers. Bank is a financial institution that people or businesses can keep their monies in or borrow money from.

According to Amponsah et al., (2006) banking is the transaction carried on by any individual or firm engaged in providing financial services to consumers, businesses or government enterprises. In the broadest sense banking consist of safeguarding and transfer of funds, lending or facilitating loans, guaranteeing credit-worthiness and exchange of money.

Banks provide services like accounts, which can be used like money to make payments and purchase goods and services. Savings accounts and time deposits that can be used to save money for future use. Loans can be used by consumers and to purchase goods and services.

Banking services involves supplying customers with the basic mediums-of-exchange (cash, cheque accounts, and credit cards), banks play a key role in the way goods and services are purchased. Without these familiar methods of payment, goods could only be exchanged by barter, which is extremely time-consuming and inefficient. Banks also accept money deposits from savers and then lend them to borrowers, encourage the flow of money for productive use and investments.
This work seeks to find an optimal way of allocating funds to the various loan types of Juaben Rural bank limited in order to maximize profit. A quantitative model, linear programming, (LP) shall be used to formulate the loan allocation problem.

Linear programming has become nowadays, a quantitative technique most decision makers use in solving a variety of problems related with management, from scheduling, media selection, Portfolio selection, farm planning, financial planning to capital budgeting, transportation and many others.

1.1.1. The Economic Functions of Banks

Banks provide the following economic functions among others:

(i) issue of money in the form of banknotes and current accounts subject to payment at the customer’s order.

(ii) act as both collection and paying agents for customers, participating in interbank clearing and settlement systems to collect, present, be presented with, and make payment instruments. These enables banks to economize on reserves held for settlement of payments, since inward and outward payments offset each other.

(iii) provides buffer to absorb losses without defaulting on its obligations.

(iv) lend out money to companies and individuals.

(v) store valuables and safe deposit boxes
(bog.gov.gh)
1.1.2 Types of Banks in Ghana

(a) The Central Bank of Ghana

The primary objective of the Bank of Ghana is to maintain stability in the general level of prices. Without prejudice to the Bank, it shall support the general economic policy of the government and promote economic growth and effective and efficient operation of banking and credit systems in the country, independent of instructions from the government or any other authority (bog.gov.gh).

Functions of the Central Bank of Ghana

The Bank shall perform the following functions:

(i) formulate and implement monetary policy aimed at achieving the objectives of the Bank.

(ii) provide monetary measures that stabilize the value of the currency within and outside Ghana.

(iii) regulate, supervise and direct the banking and credit system and ensure the smooth operation of the financial sector.

(iv) issue and redeem the currency notes and coins.

(v) license, regulate, promote and supervise non-banking financial institutions.

(vi) act as banker and financial adviser to the Government.

(Bank of Ghana ACT 2002, ACT 612)
(b) Commercial Banks

The Commercial banks in Ghana include, Standard Chartered Bank (Gh) Ltd., SG-SSB Limited, Barclays Bank (Gh) Ltd., Ghana Commercial Bank, Metropolitan Allied Commercial Bank, The Trust Bank, Zenith Bank, Intercontinental Bank, Standard Trust Bank, Fidelity Bank and Guaranty Trust Bank (Gh) Ltd.

Functions of Commercial Banks

The major functions of the commercial banks are to:

(i) specialize in loans to commercial and industrial businesses.

(ii) make consumer loans for automobiles and other consumer goods as well a real estate loans for both consumers and businesses.

(iii) sell bonds and other investments to customers.

(iv) acting as trustee and business manager for passive investor and especially as executor and administrator of estate or as guardian of a minor heir.

(v) acting as money-charger and sell monies of different nations.

(bog.gov.gh)
(c) Development Banks

The development banks include:


Development banks provide medium and long term finance through direct loans or guaranteeing loans from other sources to promote the growth and development in particular sectors of the economy such as industry, small and medium scale enterprise, trading, transport and salary loans. They also provide a wide range of technical, marketing managerial and feasibility study services among others.

Functions of Development Banks

The functions of the development banks include:

(i) the provision of capital to industries.
(ii) lends money to small scale industries.
(iii) services of mutual funds and
(iv) fund raising schemes for developing companies

(bog.gov.gh)
(d) Merchant Banks

The Merchant Banks are Ecobank Ghana Ltd., Continental Acceptance Ltd., First Atlantic Merchant Bank, CAL Bank, Merchant Bank of Ghana Ltd and HFC Bank. These banks provide capital to firms in the form of shares rather than loans. Unlike venture capital firms, they tend not to invest in new companies.

Functions of Merchant Banks

They are generally engaged in corporate banking services such as:

(i) Portfolio management; The banks decisions about investment mix and policy, matching investments to objectives, asset allocation for individuals and institutions and balancing risk against performance.

(ii) Leasing; in leasing a firm obtains the permission to use certain assets for which it must make contractual payments. The bank tends to provide funds with interest to the lessee to effect payment to the lesser.

(iii) Projects finance; it is the long term financing of infrastructure and industrial projects based upon the projected cash flows of the project. Usually, a project financing structure involves a number of equity investors, known as sponsors. The loans are most commonly secured by the project assets and paid entirely from project cash flow.

(bog.gov.gh)
(e) ARB Apex Bank

The ARB Apex Bank Ltd is a mini Central Bank in Ghana for the Rural Community Banks (RCBs) financed mainly through the Rural Financial Services Project (RFSP), which is a Government of Ghana project to holistically address the operational bottlenecks of the rural financial sector with the aim of broadening and deeping financial intermediation in the rural areas.

(f) Functions of ARB Apex Bank

The ARB Apex Bank perform the following functions:

(i) provision of cheque clearing services. This addresses the constraints of delays in cheque clearing through the big commercial banks.

(ii) development of new innovative banking products. This is to enable more rural dwellers to have access to banking products purposely designed to meet their needs.

(iii) provision of inspection services. This provision of both on-site and off-site inspection services address the problems of inadequate bookkeeping, non-observance of internal control measures.

(iv) Training of staff and directions of rural/community banks. This ensures that the staff of the management of rural/community banks possess the requisite skills to operate professionally.

(bog.gov.gh)
1.1.3 Rural and Community Banks

The Rural and Community Banks are unit banks owned by members of the rural community through purchase of shares and are licensed to provide financial intermediation in the rural areas. Rural Banks (RBs) were first initiated in 1976 to expand savings mobilization and credit services in rural areas not served by commercial and development banks.

1.1.4 Rural Banks

By the early 1970’s the Bank of Ghana realized that the normal banking institutions were not able to mobilize funds and provide services to the rural community and thereby impact adequately on the development of the country.

The Bank of Ghana set up a department at the head office called the Rural Banking Department. The department was to see to the establishment and supervision of rural banks in the country. The rural banks were to operate under the banking law of Ghana (amended in 1989, 2005), companies code and the rules and regulations of the Bank of Ghana. The first rural bank to be set up in the early 1970’s was the Nyarkrom Rural Bank at Agona Nyarkrom.

Rural banks have been recommended for their effort in making banking services available in most rural communities, which have enabled them access credit facilities to improve their business and enhance their living conditions. Some of the functions of the rural banks are:
(i) share responsibility to deepen and widen financial intermediation through
introduction of appropriate innovative instruments and products to suit the needs
of the community.

(ii) extend banking services to the remote communities and assist them to increase
productivity that would eventually translate into improved living conditions.

(iii) bring about developmental and social needs of the economy in which it operates
to stimulate job creation and create wealth.

(iv) accelerate economic transformation and growth leading to improved standard of
living and lower poverty rates in the rural communities.

In recognition of these roles, the Bank of Ghana is putting in place policies and
programmes that would strengthen rural banks and enable them perform their
developmental roles effectively.

1.1.5  Challenges facing the Rural Community Banks

The rural and community banks in Ghana face a number of challenges, which include:

(i) weak management as a result of their inability to attract qualified and competent
personnel.

(ii) low capital base, shareholders are general poor and therefore cannot make
substantial investments in the rural banks.

(iii) lack of modern technology and adequate communication facilities to promote
modern banking operations.

(iv) inadequate training due to the inability of the banks to afford the cost of training.

(v) the high risk involved in farming/agricultural operations that is, inadequate storage, marketing and processing facilities in the rural areas.

(bog.gov.gh)

1.2 Statement of the problem

The business activity of most commercial banks is lending. The loan portfolio is typically the largest asset and the predominate source of revenue. It is therefore the greatest sources of risk to a bank’s safety and soundness. This may be due to poor portfolio risk management or weakness in the economy. Loan recovery has historically been the major cause of bank losses and failures. Rural banks offer loans purposely for profits however the significant effect of lending on profitability has received little attention in the banking industry. Banks collapse because of poor loan portfolio especially banks which have huge balances of non-performing loans. Measures are therefore needed to be taken to minimize this problem.

The bank selected for research is Juaben rural bank in the Ashanti Region of Ghana. This study is undertaken to find an optimal solution of allocating funds to the various loan types of the bank using Linear Programming optimization process.
1.3 Objectives of the study

Loan provides banks the highest profit hence the administration of loan portfolio really affect the banks profit. Thus some banks failure is due to poor loan management system. The research will be in line with the use of linear programming to determine the optimal loan portfolio to reduce the bank’s operational risk during lending process.

The general objective of the research is to find out optimum loan portfolio for the bank, adherence to the regulations governing the activities of Juaben Rural Bank and their profitability levels and to make suggestions that would optimize the bank’s loan portfolio and further enhance the operation of rural banks to create more wealth for their shareholders.

Specifically the research intends to:

(i) to come out with a quantitative model that will maximize the returns on loans given out by Juaben Rural Bank.

(ii) to determine optimum loan portfolio for Juaben Rural Bank.

1.4 Research Questions

The research would seek to find answers to the following questions:

(i) is there any innovative way to maximize the returns on loans given out by Juaben Rural Bank?

(ii) how can the optimum loan portfolio of Juaben Rural Bank be determined?
1.5 Significance of the study

The study is to find out whether the loan portfolio provides adequate guidance to control the quality and quantity of credit risk. Also to determine whether the quantity of credit risk has changed or is likely to change because of portfolio changes.

Again assessing performance of the rural bank’s loan portfolio in the rural community. Suggestions would be made after the strengths and weaknesses of the different identifying loan portfolio for the rural bank. Firms in the industry particularly Rural Community Banks having similar practices would recognize the strength and challenges and review internal control of their loan portfolio.

1.6 Scope / limitation of the Study

The research will cover the loan portfolio selection policies of all branches of Juaben Rural Bank for the 2012 financial year.

This research work was conducted on just one of the numerous branches due to limited fund and time constraints. The fieldwork was scheduled to last for approximately three weeks, it was extremely costly and time-consuming to extend this research to cover all branches in the Ashanti Region. In addition, the problem of disclosure of information by the bank was encountered.
1.7 Justification

Much appreciable research work exist in areas of loans given by financial institutions and its impact on the livelihood of the people. The analysis of the loan portfolio of Juaben Rural Bank in Ashanti Region will enable the bank to optimize its annual profit, reduce the risk on loans and enact cooperate policies that would optimize its loan portfolio.

The study is intended to assist Juaben Rural bank improve the quality of its loan portfolio and make it possible for it to lend to the most productive sectors of the economy.

1.8 Organization of the Study

The thesis is organized in five chapters. Chapter one deals with the background information of the study, statement of the problem, objectives of the study, research questions and significant of the study, scope of the study and the organization of the study. Chapter two focuses on the review of the relevant literature, which covers the applications of linear programming to portfolio selection, types of loan portfolio and risk associated with loans. Chapter three describes the methodology used for the study. In chapter four we shall put forward data collection and analysis. Chapter five, which is the last chapter summaries the various findings, conclusions and recommendations.
1.9 Summary

In this chapter banking was seen as an instrument of issuing money, settlement of payments and credit intermediation. The four main types of banks were also mentioned. A brief history of rural banks and some of their social responsibilities were given. In the next chapter, we shall put forward some pertinent literature in the field of loans and linear programming.
CHAPTER TWO

LITERATURE REVIEW

2.0 Introduction

This chapter, presents relevant literature in the field of optimal loan portfolio. Linear Programming is a structured procedure that incorporates defined decision variables that are significant in determining the maximum or minimum of an objective function. The objective function is subject to quantitative equality and linear inequality constraints.

2.1.0 Loans

Loans in finance are the lending of sum of money (World Bank, 1994). In common usage it is the lending of any piece of property. A loan may be secured by the charge on the borrower’s property (as a house-purchase mortgage is) or be unsecured. There are number of conditions attached to a loan: for example, when it is to be repaid and the rate of interest to be charged on the sum of money loaned. Almost any person or any organization can make or receive a loan. The two major characteristics that vary among bank loans are the terms of the loan and the security or collate required to get the loan. For the loan term we have the long term and the short term, and of the security is secured or unsecured debt.
2.1.1 Loan Repayment

The most typical loan payment type is the fully amortizing payment in which each monthly rate has the same value overtime. The fixed monthly payment, \( P \) for a loan of, \( L \) for \( n \) months and a monthly interest rate, \( C \) is given by the relation:
appropriate. In drawing up strategic objectives, management and the board should consider establishing:

(i) goals for portfolio diversification.
(ii) how much the portfolio should contribute to the bank’s financial objectives.
(iii) loan product mix.
(iv) loan growth targets by product, market, and portfolio segment.
(v) what the bank’s geographic markets should be?
(vi) targeted industries.
(vii) targeted market share

2.2 WHAT MOTIVATE PEOPLE TO GO FOR LOANS

According to Burton (2002) engaging in loan gives a greater amount of money to fulfill ones project. Some clients find it difficult to pay for these loans but they still want to apply for it due to financial situations they find themselves. Most people apply for loans because of underlying reasons:

(i) to purchase a house or for renovation of a house.
(ii) to pay for existing loan.
(iii) to own a car.
(iv) for personal, educational purposes and others.
2.3 FACTORS AFFECTING THE REPAYMENT OF LOANS

Lending is a risky enterprise because repayment of loans can seldom be fully guaranteed. Generally inspite of the importance of loan acquisition its repayment are fraught with a number of problems (Arsyad, 2006).

Interest rates on loans are the most important factor affecting repayment of agricultural loans.

In agriculture, large rate of default in loan repayment has been a perennial problem, farming experience, and total application costs. Most of the default arise from par management procedures, loan diversion and unwillingness to repay loans.

Credit market in developing countries work ineffectively due to a number of market imperfections. These imperfections leads to loan default which include:

(i) the interest rate ceiling usually imposed by the government.
(ii) monopoly power in credit market often exercised by informal lenders.
(iii) moral hazards.
(iv) low money borrowers

2.4 PORTFOLIO MANAGEMENT

Wang et al., (2003) used a mean semi absolute deviation model for portfolio rebalancing with transaction costs and taxes. Considering the existence of a minimal purchase unit of securities, a mix integer linear programming model was proposed. Heuristic algorithm
was used efficiently to solve portfolio rebalancing problem by using real data of Shanghai Stock Exchange.

According to Kuritzes (1998) active portfolio management opens new channels to secondary market allowing internal loan originations to be directly compared with and diversified by market alternatives. The emerging of active portfolio management model has lasting impact on the structure of lending business.

The shift to active portfolio management is being driven by a number of related forces including:

(i) increasing liquification of loan market.
(ii) convergences of fixed income trading and large corporate lending.
(iii) need for improved portfolio diversification.
(iv) potential to tax and regulatory capital arbitrage (ERisk.com).

Benati (2003) indicated that one of the basic problems of applied finance is the optimal selection of stocks, with the aim of maximizing future returns and constraining risks by appropriate measure. The problem was formulated by finding the portfolio that maximizes the expected return, with the risk constraints by the worst conditional expectation. Optimal portfolio selection problem can be formulated as a linear programming instance, but with exponential number of constraints.
According to Sharpe (1967) the portfolio selection problem faced by a mutual fund manager can be formulated as a linear programming problem. This is to find those portfolio that are efficient in terms of predicted expected return and standard deviation of return, subject to legal constraints in the form of upper bounds on the proportion of the fund invested in any single security. Linear programming allows the use of an extremely simple and efficient special purpose solution algorithm.

Park (1998) suggested a new principle for choosing portfolio based on historical returns data, the optimal portfolio based on this principle is the solution to a simple linear programming problem. This principle uses minimum return rather than variance as measure of risk. In particular, the portfolio chosen minimizes the maximum loss over all past observation periods, for a given level of return. The function avoids the logical problems of a quadratic utility function implied by mean-variance portfolio selection rules. The resulting minimax portfolios are diversified; for normal return data, the portfolios are nearly equivalent to those chosen by a mean-variance rule.

According to Adams (2006) credit investing is a strange beast. The question “how much risk am I taking” is not easily answered. Traditionally with an equity portfolio the answer is usually expressed as a volatility or tracking error number. For fixed interest on credit portfolio the answer might be a duration number, an average credit rating or even a tracking error number or value of risk. Unfortunately, all these measures for credit portfolios can be significantly deficient by failing to capture the true risk profile of credit
investments. Linear programming is used to measure the credit risk of a portfolio. It seeks to highlight the benefit, flaws and assumptions of each of these approaches. Leading global credit portfolio managers are implementing risk measurement and management approaches using linear programming techniques.

According to Ogryczak (2000) portfolio selection problem is usually considered as a bi-criteria optimization problem where a reasonable trade-off between expected rate of return and risk is sought. In the classical Markowitz model the risk is measured with variance, thus generating a quadratic programming model. The Markowitz model is frequently criticized as not consistent with axiomatic models of preferences for choice under risk. Models consistent with the preference axioms are based on the relation of stochastic dominance or on expected utility theory. The former is quite easy to implement, for comparison of given portfolios, it does not offer any computational tool to analyze the portfolio selection problem. The latter, when used for the portfolio selection problem, is restrictive in modeling preferences of inventors. A linear programming model of the portfolio selection problem is developed. The model is based on the preference axioms for choice under risk.

Any investor in a credit portfolio face non-diversifiable estimation driven uncertainty about two parameters. Probability of default and asset return correlation. Bayesian inference reveals that for realistic assumptions about the portfolio’s credit and the data underlying parameter estimates, this uncertainty substantially increase the tail risk perceived by the investor. Since incorporating parameter uncertainty in a measure of tail risk is computationally demanding, linear programming derives and analyzes a closed form approximation to such a measure (Tarasher, 2009).
According to Cohen (1967) new approaches exist to measure the return-risk, trade-off in portfolio of risky debt instruments, whether bonds or loans. The use of complex, statistically based portfolio techniques to manage assets of financial institutions and fixed income portfolio is very much in its early phase and will continue to evolve, perhaps more quickly in the near future. Linear programming using the Simplex method substitutes the concept of unexpected loss for the more traditional variance of return measure used in equity securities analysis.

The manager of a bank operating in a competitive environment faces the standard goal of maximizing shareholders wealth specifically, this attempts to maximize the net worth of the bank, which in turn involves maximizing the net interest margin of the bank (among other factors, such as non-interest income). At the same time, there are significant regulatory constraints place on the banks, such as the maintenance of adequate capital, interest rate risk exposure etc.

According to Matthew et al., (2002) the genetic algorithm base technique is used to obtain an approximation to the set of pareto-optimal solutions, which increases the decision flexibility available to the bank.
Clement et al., (2004) described a methodology for measuring and optimizing the credit risk of a loan portfolio taking into account the non-normality of the credit loss distribution. Particular emphases were placed on modeling accurately joint default events for credit assets. In order to optimize portfolio credit risk, the authors minimized the conditional value at risk, a risk measure both relevant and treatable, by solving a simple linear programming problem subject to the traditional constraints of balance, portfolio expected return and trading. The outcomes, in terms of optimal portfolio compositions, assumed different default dependence structures were compared with each other. The solution of the risk minimization problem suggested how to restructure the inefficient loan portfolio in order to obtain the best risk or return profile.

According to Asemota (2009), loan management is a very complex and yet a vitally important aspect of any commercial bank operations. The balance sheet positions shows the main sources of funds as deposits and shareholders contributions. In order to operate profitably, remain solvent and consequently grow, a bank needs to properly manage its excess cash to yield returns in the form of loans. The above are achieved if the bank can honour depositors withdrawals at all times and also grant loans to credible borrowers. This is so because loans are the main portfolios of a bank that yield the highest returns.

In the world of investment, investors want to earn the highest expected return from the portfolio. The rate of expected return depends on the level of tolerance. The expected return from a portfolio of stocks is a combination of dividend and price yields. Portfolio selection and security analysis always becomes a vital area for decision making.
Markowitz (1952) divided modern portfolio theory. Recently, Chance Constrained Programming (CCP) models have been widely used for providing optimized solutions to problems that have multiply and confliction objectives and being known as multi-objective chance constrained model (Markowitz, 1952).

Investments inflows coupled with enhance economic activities has brought to the fore, the current emphasis on effective and efficient fund management. The task before the investor and fund manager is to determine the optimal asset mix in order to remain competitive. Linear programming techniques can be applied in the selection of an optimal investment portfolio as an alternative to the single asset selection method popular with fund managers (Winsten, 2003).

2.5 LINEAR PROGRAMMING

Linear programming (LP) is a highly versatile quantitative technique, which has found wide use in management and economics. It is used both as a research technique and as a planning tool, particularly at the individual firm and industry levels. In general, LP is designed to maximize or minimize a linear objective function subject to a set of linear constraints. Other related techniques are goal programming, mixed integer programming and quadratic programming. Some typical applications of linear programming include:

(i) determining the most profitable combination of enterprise or activity levels for a business firm with limited supplies of various resources.
(ii) determining the most profitable investment portfolio, given the amount of investment capital available, rates of return on various stocks, bonds and other ‘paper assets’, and limits on high-risk investments.

(iii) formulating mixtures to combine ingredients such that a required overall composition of the mix is satisfied at least cost. Important applications are fuel and fertilizer blending and determination of livestock rations or supplementary feeds.

(iv) scheduling the various tasks in a construction project so as to complete the overall project in minimal time or at minimal cost and

(v) determining the location and size of storage facilities and processing plants together with the distribution pattern, so as to minimize the total of transport, storage and processing costs.

Lyn (2002) discussed how one can use linear programming to estimate the interest rates for the prices of bonds. In the personal sector finance, where lending is far greater than the higher profile cooperate sector, linear programming can be used to develop credit scorecards.

According to Al-Faraj et al., (1990) decision makers are always faced with the problem of determine optimal allocation of limited resources. The problem is to determine the best combination of activities levels, which does not use more resources than are actually available and at the same time maximize output, revenue, service level or minimize cost.
Jao (2000) indicated that linear programming techniques have long been used in many areas of economics analysis and business administration. Their application to bank management appears to be a relatively new development.

According to Pearson (2007), Linear Programming (LP) is useful in managerial decisions, product mix, make-buy, media selection, marketing research, loan Portfolio selection, shipping and transportation and multi-period scheduling.

Among all the optimization techniques, linear programming is perhaps the most used and best understood by the business and industrial community, Wu (1989). Linear programming deals with optimization problems that can be modeled with a linear objective function subject to a set of linear constraints. The objective of these problems is either to minimize resources for a fixed level of performance, or to maximize performance at a fixed level of resources.

According to Falkie et al., (1972) linear programming models have been used to solve many production planning problems for a multi-plant operations serving several customers. The main factors considered are availability of resources, inventory restrictions and demand requirements. The objective function to be optimized is the total annual net returns from the operations.

Amponsah et al., (2006) modeled a banking policy for Atwima Kwanwoma Rural Bank. The banks policy of granting loans were modeled as a linear programming problem with respect to profit and budget constraints on the loan portfolios. Their research showed
greater profit and expansion of service if recommendation were to be implemented. The bank found their policy proposal suitable for implementation.

A company may invest in short-term trade credit (TC) corporate bonds (CB), gold stocks (GS) and construction loans (CL). To encourage a diversified portfolio, limits are placed on the amount that can be committed to any one type of investment. The company may have certain amount of money available for immediate investment and wishes to do two things.

(i) maximize the interest earned on the investments made over a certain period.
(ii) satisfy the diversification requirement as set out by the board of directors. These specifics can be achieved through linear programming model of the investment.

According to Gendzio and Grothey (2005) recognized linear programming as a powerful tool to help decision making under uncertainty in financial planning. It shows how portfolio optimization problems with sizes measured in millions of constraints and decision variables featuring constraints on semi-variance, skewness or nonlinear activity functions in the objective can be solved.

Most business resource allocation problems require the decision maker to take into account various types of constraints, such as capital, labour, legal and behavior restrictions. Linear programming techniques can be used to provide relatively simple and realistic situations to problems involving constrained resources allocation decisions. A
wide production, finance, marketing and distribution problems have been formulated linear framework.

Rasmussen et al., (2000) considered the dynamics of the Danish mortgage loan system several models are prepared to reject the choices of a mortgage,). The models were formulated as multi stage stochastic integer programs, which are difficult to solve for more than ten (10) stages. Linear Programming was used to obtain near optimal solutions for large problem instances.

End of life (EOL) products are purchased from a number of suppliers in order to be disassembled into individual components to satisfy the demand for specified number of components. However, there are a lot of uncertainties that complicate the process. Multi-criteria disassembly to order (DTO) model was developed to take into consideration multiple system uncertainties and solves it using linear physical programming (LPP). This helps in finding the best combination of take-back EOL products to be purchase from every supplier that would satisfy the demand and achieve the aspiration levels of multiple goals.

Cash-flow matching is an important and practical tool for managing interest rate risk. Interest rate fluctuations are major risk for the insurance and pension industry. If assets are invested shorter than the corresponding liabilities reinvestment risk arises because interest rate can fall. On the other hand if assets are longer than the liabilities, then liquidation risk or market risk exist. An insurer or pension fund faces the problem of
constructing from the current available universe of non callable and default free fixed income securities on investment portfolio that will meet the future liability payments. With a finite amount of resources, the decision maker seeks an initial investment portfolio with minimum cost such that it cash flow will at least meet the projected liability payment for each and every period in the planning horizon. Duality theory of linear programming provides insight for generalizing and solving cash-flow matching problem. (Transactions of society of Actuaries, 1990)

Amponsah et al., (2011) presented an optimal loan allocation mix policy from the steady state distribution of loan disbursement process. Using monthly data on actual loan disbursement of four loan types for a period of twenty-four months, by using a transition matrix. From the estimated probability transition matrix, the study state distribution indicated that in a long run, trade loan should constituted 77.3% of the total loan, 10.3% for service loan, 2.0% for production loan and 10.4% for susu loan.

2.6 BENEFITS OF LOAN PORTFOLIO TO MANAGEMENT AND DECISION MAKERS OF BANKS

Light et al., (2005) indicated that loan portfolio management is one of the responsibilities critical to the success of an institution. It is the dynamic process of managing an institution’s primary earning assets to achieve the primary objectives of the boards strategic business and capital plans.
Loan portfolio encompasses all systems and processes used by management to adequately plan, direct, control and monitor the institutions lending operations. Loan portfolio ensures that all material aspect of lending operations are adequately controlled relative to the institutions risk bearing capacity.

Loan portfolio helps management and decision makers in the analysis of how business results are achieved, whether such results will continue and how the institution can optimize its opportunity and provide great benefit to its members.

Loan portfolio also helps decision makers to measure the portfolio risk both for short term returns and hold long term strategy. Finally, it helps managers to minimize the funding of cost while lending against the market risk.
CHAPTER THREE

RESEARCH METHODOLOGY

3.1.0 Introduction

This chapter discusses the profile of Juaben Rural Bank in Asahnti Region of Ghana, the research methodology and linear programming.

PROFILE OF JUABEN RURAL BANK

3.1.1 Organizational profile

Juaben rural bank was established as one of the numerous rural banks in Ghana. It was registered on the 24th October, 1984 with Certificate of Incorporation No. 25435. However, Bank of Ghana licensed it as a Rural Bank on 12th March 1985 under licensed No.105.

Juaben Rural bank limited was established in a town called Juaben in the Ejisu-Juaben district of the Ashanti region of Ghana. The bank has its head office at Juaben, and can boast of seven other branches at Ejisu, Kwaso, Bonwire, Aboaso, Roman Hill, Magazine New Road and Sepe Timpom.

VISION

The vision of Juaben rural bank is to be the most preferred Rural Bank in Ashanti Region by 2012.
MISSION

Its mission is to ensure that its products and services meet the expectations of its varied and segmented customers. The management of the bank has stated that it will accomplish this mission by utilizing highly qualified, competent and well-motivated staff to provide quality and innovative products/services that responds to the dynamic market.

The hallmark of Juaben Rural Bank is to maintain its integrity in all its operations at all times.

CORE VALUES

The bank believes that with humility they will serve their customers and motivate their staff and retain them to keep the Bank as a going concern. The need to understand customer and staff needs and to respond to such needs at all times.

The hallmark of Juaben Rural Bank is to maintain its integrity in all its operations at all times. There is the need to succeed, this calls for total commitment from staff on continual basis on the grounds of fairness and integrity.

CORE PRODUCTS

The core products of the bank are lending and savings products.
Lending

The bank lends to the following Sectors:

Agriculture (cash crops, food crops, livestock, and fishing), Cottage Industry (agro-based and non-agro based), transport loans, trading and others

Lending Products

The lending products for Juaben Rural Bank are overdraft and loans

The various types of loan provided by the bank are:

(i.) salary loans

(ii) normal loans – Individuals, Enterprises, Corporate entities etc.

(iii) institutional loans -churches

(iv) susu Loans

(v) funeral Loans

(vi) micro Finance Loans

Savings Products

The savings products of Juaben Rural Bank includes:

(i) savings deposit accounts,

(ii) susu deposit accounts,
(iii) current/demand deposit accounts and

(iii) term/fixed deposit accounts.

3.1.2 Research Design

The main sampling technique was purposive sampling procedure. Purposive sampling is when the people selected are the key individuals who can give the information require for study. Questionnaire was designed for the bank loan officers to obtain information on the loan type, interest rate and probability of bad debt. The main branch was served with questionnaire. The questionnaire focused on how the bank adjust the loan condition to reflect the risk of lending and how to select a loan facility that would maximize profit with minimum resources and minimize risk on the loan portfolio in other to obtain an optimal portfolio.

3.1.3 Population and Sample

A case study research design was use to conduct this study, the population consist of two credit officers and a manager of Juaben rural bank.

3.1.4 Sources of Data

The data employed in this study are both primary and secondary. The primary data was collected through questionnaire administration and secondary data was obtained from
journals, publications and financial statement of the bank. This was used to determine the profitability of the bank and to find out its lending management practices. The analysis of the financial statement revealed the overall profit of the bank from loan and other financial activities.

3.1.5 Data Collection Instrument

Two sets of data are employed for the analysis, these are primary and secondary data. The primary data was collected through questionnaire administration. Secondary data was collected from published statement of the bank and it was used to assess the banks financial standing. The financial statement was to assess the portfolio, outstanding loan at the end of each period, the percentage of loans to total inflows and provision on bad debts.

3.1.6 The Study Area

The study area was the main branch (Juaben) of the bank which is located in the Ejisu Juaben District in the Ashanti Region of the country.
3.1.7 Data Analysis

An analytical tool such as Quantitative Manager (QM) for Windows, which is Management Scientist software in linear programming was used to determine the optimum loan portfolio between the different loan types.

The objective of this project is to offer a technique that could be applied in the selection of an optimal loan portfolio using a linear programming model. Linear programming approach is selected because it solves efficiently resource allocation problems and loan.

3.1.8 The Linear Programming Model

Linear programming is a mathematical technique that deals with the optimization (maximizing or minimizing) of a linear function known as objective function subject to a set of linear equations or inequalities known as constraints. It is a mathematical technique, which involves the allocation of scarce resources in an optimum manner, on the basis of a given criterion of optimality. The technique used here is linear because the decision variables in any given situation generate straight line when graphed. It is also programming because it involves the movement from one feasible solution to another until the best possible solution is attained.

A variable or decision variables usually represent things that can be adjusted or controlled. An objective function can be defined as a mathematical expression that combines the variables to express your goal and the constraints are expressions that combine variables to express limits on the possible solutions.
### 3.1.9 Simplex Method

The simplex method is the name given to the solution algorithm for solving linear programming problems developed by George Dantzig in 1947. A simplex is an n-dimensional convex figure that has exactly \((n+1)\) extreme points. For example, a simplex in two dimensions is a triangle, and in three dimensions is a tetrahedron. The simplex method refers to the idea of moving from one extreme point to another on the convex set that is formed by the constraint set and non-negativity conditions of the linear programming problem.

The solution algorithm is an iterative procedure having fixed computational rules that leads to a solution to the problem in a finite number of steps (i.e., converges to an answer). The simplex method is algebraic in nature and is based upon the Gauss-Jordan elimination procedure.

The principle underlying the simplex method involves the use of the algorithm which is made up of two phase, where each phase involves a special sequence of number of elementary row operations known as pivoting. A pivot operation consist of finite number of \(m\) elementary row operations which replace a given system of linear equations by an equivalent system in which replace a given system of linear equations by an equivalent system in which a specified decision variables appears in only one of the system and has a unit coefficient.

The algorithm has two phases, the phase of the algorithm, is finding an initial basic feasible solution (BFS) to the original problem and the second phase, consists of finding an optimal solution to the problem which begins from the initial basic feasible solution.
3.2.0 Formulation of the Problem

The objective function to be Maximized or Minimized is given by

\[ Z = C_1X_1 + C_2X_2 + \ldots + C_nX_n \]

Subject to the \( m \) constraints given by

\[ a_{11}x_1 + a_{12}x_2 + \ldots + a_{1n}x_n \leq b_1 \]
\[ a_{21}x_1 + a_{22}x_2 + \ldots + a_{2n}x_n \leq b_2 \]
\[ a_{m1}x_1 + a_{m2}x_2 + \ldots + a_{mn}x_n \leq b_m \]

The Non negativity constraints

\[ x_1 \geq 0, \ x_2 \geq 0 \ldots x_n \geq 0 \]

Where \( c_j, a_{ij} \) and \( b_i \) are all known constraints and greater than zero and \( i=1,2,3\ldots,m \) and \( j=1,2,3,\ldots,n \).

3.2.1 Algorithm for Simplex Method

A basic feasible solution to the system of m linear constraint equations and n variables is required as a starting point for the simplex method. From this starting point, the simplex successively generates better basic feasible solutions to the system of linear equation. We proceed to develop a tabular approach for the simplex algorithm. The purpose of the tableau form is to provide an initial basic feasible solution that is required to get simplex
method started. It must be noted that basic variable appear once and have coefficient of positive one.

### 3.2.2 Setting up Initial Simplex Tableau

In developing a tabular approach we adopt these notations as used in the initial simplex tableau.

- $c_j =$ objective function coefficients for variable $j$
- $b_j =$ right – hand side coefficients (value) for constraints $i$
- $a_j =$ coefficients variable $j$ in constraints $i$
- $c_B =$ objective function coefficients of the basic variables
- $C_j – Z_j =$ the net evaluation per unit of $j$ – th variable
- $[A]$ matrix = the matrix (with m rows and n columns) of the coefficients of the variable in the constraint equations.
Table 3.1: General from – Initial Simplex Tableau

<table>
<thead>
<tr>
<th></th>
<th>Decision variables</th>
<th>Slack Variables</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>c_i</td>
<td>c_2</td>
<td>..</td>
</tr>
<tr>
<td>c_B</td>
<td>Basic variables</td>
<td>x_1</td>
<td>x_2</td>
</tr>
<tr>
<td></td>
<td>s_m</td>
<td>a_m1</td>
<td>a_m2</td>
</tr>
<tr>
<td></td>
<td>Z_j</td>
<td>Z_1</td>
<td>Z_2</td>
</tr>
<tr>
<td></td>
<td>c_i-Z_j</td>
<td>c_1-Z_1</td>
<td>c_2-Z_2</td>
</tr>
</tbody>
</table>

Illustrative example

Maximize \( Z = 6x_1 + 8x_2 \)

Subject to

\[
5x_1 + 10x_2 \leq 60
\]

\[
4x_1 + 4x_2 \leq 40
\]

\[
x_1, x_2 \geq 0
\]

The above example can be restated in the standard form as follows:

Maximize \( Z = 6x_1 + 8x_2 + 0s_1 + 0s_2 \)
Subject to

\[ 5x_1 + 10x_2 + s_1 = 60 \]

\[ 4x_1 + 4x_2 + s_2 = 40 \]

\[ x_1, x_2, s_1, s_2 \geq 0 \]

Transferring to the initial simplex tableau, we have Table 3.1.1

<table>
<thead>
<tr>
<th>Pivot Row</th>
<th>Basic variable</th>
<th>x₁</th>
<th>x₂</th>
<th>s₁</th>
<th>s₂</th>
<th>solution</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>s₁</td>
<td>5</td>
<td></td>
<td>1</td>
<td>0</td>
<td>60</td>
</tr>
<tr>
<td>0</td>
<td>s₂</td>
<td>4</td>
<td>4</td>
<td>0</td>
<td>1</td>
<td>40</td>
</tr>
<tr>
<td>0</td>
<td>Zjit</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

The current basic variables always form an identity matrix within the simplex tableau. Note that the basic variables form a basis matrix that is an identity matrix (I). From the initial tableau, the solution values can be read directly in the rightmost column. The values of \( z_j \) row are calculated by multiplying the elements in the \( c_n \) column by the corresponding elements in the columns of the \([A]\) matrix and summing them. Each value in the \( C_j - Z_j \) row represents the net profit or net contribution that is added by producing
one unit of product (if $C_j - Z_j$ is positive) or the net profit or net contribution that is subtracted by producing one unit of product $j$ (if $C_j - Z_j$ is negative).

Since all the $z_j$ values ($j=1,\ldots,4$) are equal to zero in the simplex tableau, we proceed to generate a new basic feasible solution (extreme point) that yields a better value for the objective function. This is accomplished by selecting one of current non-basic variables to be made basic and one of the current basic variables to be made non-basic in such a fashion that the new basic feasible solution yields an improved value for the objective function. This process is called changing that basic or iterating.

### 3.2.3 Improving the Solution

The criteria for which a variable should enter or leave basis is summarized as follows:

**Variable Entry Criteria:** The variable entry criterion is based upon the value in the $C_j - Z_j$ row of the simple tableau. For a maximization problem, the variable selected for entry is the one having the largest (most positive) value of $C_j - Z_j$. When all values of $C_j - Z_j$ are zero or negative, the optimal solution has been obtained.

**Variable Removing Criterion:** The variable removal criterion is based upon the ratios formed as the values ($b_i$) in the “right-hand-side” column are divided by the corresponding values ($a_{ij}$ coefficients) in the column for the variable selected to enter the basis. Ignore any $a_{ij}$ values in the column that are zero or negative (ie., do not compute the ratio). The variable chosen to be removed from the basis is the one having the
smallest ratio. In the case of ties for the smallest ratio between two or more variables, break the tie arbitrarily (i.e., simply choose one of the variables for removal). This variable removal criterion remains the same for both maximization and minimization problems.

Applying the variable entry and removal criteria to our present maximization problem, \( x_2 \) is chosen as the variable to enter basis and \( s_1 \) leaves the basis. Thus, the current basic variable \( s_1 \) is replaced by non-basic variable \( (x_2) \).

Now that we have determined the new elements in basis and those not in basis, we proceed to determine the new solution through pivoting \( x_2 \) into basis and pivoting \( s_1 \) out of basis. The pivoting process involves performing elementary row operations on the rows of the simplex tableau to solve the system of constraint equations in terms of the new set of basic variables. We initiate pivoting processing by identifying the variable, \( x_2 \), to be entered into the basis by denoting its corresponding column as the pivot column in Table 3.2. Similarly, we identify the variable, \( s_1 \), to be removed from the basis by specifying the pivot row which is the row it corresponds to as in Table 3.2. The element at the intersection of the pivot column and pivot is referred to as pivot element. The two-step pivoting process proceeds as follows:

**Step I:** Convert the pivot element to one by dividing all values in the pivot row by pivot element (10). This new row is entered in the next tableau, Table 3.3.

**Step II:** The objective of the second step is to obtain zeros in all the elements of the pivot column, except, of course for the pivot element itself. This is done by elementary row operations involving adding or subtracting the appropriate multiple of the new pivot row.
or from the other rows. Performing these calculations, the results are as presented in Table 3.3

<table>
<thead>
<tr>
<th>Column</th>
<th>Pivot column</th>
</tr>
</thead>
<tbody>
<tr>
<td>$c_j$</td>
<td>6</td>
</tr>
<tr>
<td>$c_B$ Basic variable</td>
<td>$x_1$ 8 0 0</td>
</tr>
<tr>
<td>$s_1$</td>
<td>8</td>
</tr>
<tr>
<td>$s_2$ solution</td>
<td>0</td>
</tr>
</tbody>
</table>

The second simplex tableau can be construed as shown in Table 3.3 Note that the columns that correspond to the current basic variables $x_2$ (real variable) and $s_2$ (slack variable) from a basis $[B]$ which is identity matrix. The values in the $z_j$ row and $C_j - Z_j$ row are computed in the same way as in the initial simplex tableau. Observe that $C_j - Z_j = 2 (0)$ and so the optimal solution has not been obtained and continue the iteration since we are maximizing.
We continue the process by determining the variables leaving the basis and which is entering the basis using the variable entry and removing criteria stated earlier. The outcome is summarized in Table 3.4

<table>
<thead>
<tr>
<th>( c_j )</th>
<th>6</th>
<th>8</th>
<th>0</th>
<th>0</th>
<th>solution</th>
</tr>
</thead>
<tbody>
<tr>
<td>( c_B )</td>
<td>Basic Variables</td>
<td>( x_1 )</td>
<td>( x_2 )</td>
<td>( s_1 )</td>
<td>( s_2 )</td>
</tr>
<tr>
<td>8</td>
<td>( x_2 )</td>
<td>0</td>
<td>1</td>
<td></td>
<td>2</td>
</tr>
<tr>
<td>6</td>
<td>( x_1 )</td>
<td>1</td>
<td>0</td>
<td></td>
<td>8</td>
</tr>
<tr>
<td>( Z_j )</td>
<td>6</td>
<td>8</td>
<td></td>
<td>1</td>
<td>64</td>
</tr>
<tr>
<td>( c_j - Z_j )</td>
<td>0</td>
<td>0</td>
<td></td>
<td>-1</td>
<td></td>
</tr>
</tbody>
</table>

Observe that in this third simplex tableau all \( c_j - Z_j \) values are either zero or negative. We have thus obtained the optimal solution with \( x_1 = 8 \), \( x_2 = 2 \), \( s_1 = 0 \), \( s_2 = 0 \) and the optimal value of \( z = 64 \).

The optimal solution suggests that the profit will be maximized when eight products of \( x_1 \) and two products of \( x_2 \) are produced.

### 3.2.4 Simplex Method with Mixed Constraints

Some Linear Programming problem may consists of a mixture of \( \leq \), \( = \) and \( \geq \) sign in the constraints and wish to maximized or minimized the objective function. Such mixture of signs in the constraints is referred to as mixed constraints.

The following procedure is followed when dealing with problem with mixed constraints.
**STEP 1:** Ensuring that the objective function is to be maximized. If it is to be minimized then we convert it into a problem of maximization by \( \text{Max } W = -\text{Min } (-Z) \)

**STEP 2:** For each constraints involving ‘greater or equal to’ we convert to ‘less than or equal to’ that is, constraints of the form

\[
a_{21}x_1 + a_{22}x_2 + \ldots + a_{2n}x_n \geq b_2
\]

Is multiplied by negative one to obtain

\[
-a_{21}x_1 - a_{22}x_2 - \ldots - a_{2n}x_n \leq -b_2
\]

**STEP 3:** Replace constraints

\[
a_{21}x_1 + a_{22}x_2 + \ldots + a_{2n}x_n = b_2
\]

by

\[
a_{21}x_1 + a_{22}x_2 + \ldots + a_{2n}x_n \leq b_2
\]

and

\[
a_{21}x_1 + a_{22}x_2 + \ldots + a_{2n}x_n \geq b_2
\]

Where the latter is written as

\[
-a_{21}x_1 - a_{22}x_2 - \ldots - a_{2n}x_n \leq -b_2
\]

**STEP 4:** Form the initial simplex tableau

**STEP 5:** If there exist no negative appearing on the RIGHT HAND SIDE column of the initial tableau, proceed to obtain the optimum basic feasible solution.
**STEP 6:** If there exist a negative entry on the Right Hand Side column of the initial tableau,

(i) Identify the most negative at the Right Hand Side, this row is the pivot row.

(ii) Select the most negative entry in the pivoting row to the left of the Right Hand Side. This entry is the pivot element.

(iii) Reduce the pivot element to 1 and the other entries on the pivot column to 0 using elementary row operation.

**STEP 7:** Repeat step 6 as long as there is a negative entry on the Right Hand Side column. When no negative entry exist on the Right Hand Side column, except in the last row, we proceed to find the optimal solution.
CHAPTER FOUR

DATA PRESENTATION AND ANALYSIS

4.0 Introduction

This chapter deals with data presentation and analysis of the study. A linear programming model was proposed to solve the problem. For this study, data was collected from Juaben rural bank limited with focus on the loan records for the 2010 financial year. The data included the sum of all the types of loans offered by the bank. The bank has five loan portfolios, analysis would be based on the amount allocated to each portfolio, associated bad debt and the interest rate on each type of loan.

4.1 Loan Type, Interest Rate and Probability of Bad Debt

Table 4.1 Loan types, Interest rates, Probability of bad debt.

<table>
<thead>
<tr>
<th>Loan types</th>
<th>Interest rate (%)</th>
<th>Probability of bad debt</th>
</tr>
</thead>
<tbody>
<tr>
<td>Agriculture</td>
<td>29.5</td>
<td>3.2</td>
</tr>
<tr>
<td>Cottage industry</td>
<td>30.2</td>
<td>5.4</td>
</tr>
<tr>
<td>Transport</td>
<td>25.7</td>
<td>3.3</td>
</tr>
<tr>
<td>Trading</td>
<td>29.9</td>
<td>3.6</td>
</tr>
<tr>
<td>Salary</td>
<td>30.7</td>
<td>3.0</td>
</tr>
</tbody>
</table>

Source – Juaben rural bank
Juaben rural banks has categories their loans into five (5) types namely agriculture, transport, cottage industry, salary and trading loans. Agricultural loan comprises of loans to cash crop farming, animal farming, poultry, oil plantation and agro chemical sellers. Bulk of the amount allocate to agric goes to the oil palm plantation. The bank believe that most of the clients under the agricultural sector depends on the climatic change that is rainfall season. When it rains normally there is bumper harvest but the reverse is the case when rain delay. As a result of this the bank being profit oriented knows that the probability of high default rate on the part of the farmers will account to the probability of bad debt of 3.2%. With the poultry there is not enough revenue because of the inflows of foreign poultry products. The competition is so keen and when loan is granted to such sector the returns should be high hence the 29.9%.

Cottage industry comprises of kente and basket weaving individuals, local distillers (apeteshie), soap production and foot wear designers. The bank grants at a higher rate of 30.2% looking at the members in that sector, granting facility to them comes with higher risk. Thus the rate of default is very high since competition in such market is very competitive and sometimes distillers runs into all sort of problems. If a bank wants to enter into such market then the return should be higher, looking at the probability of bad debts of 5.4% explain the high nature of the industry. From the data gathered cottage industry constitute the highest of the unpaid loan.
Transportation sector is made up of spare parts dealers, car importers, and transport owners. The bank grants loan facility to the above group, and the default rate under this sector is also higher due to the rampant road accidents in Ghana. Import duties also make prices higher, and based on this, the probability of bad debts is around 3.3%. This accounts for the fact that even though the risk is higher, the returns on loans are not encouraging. The bank therefore offers a little assistance to that sector (i.e., 25.7%).

Trading comprises those who buy and sell within the country and those who import and sell within the country, for instance, goods from Togo, China, etc. The bank offers assistance to these traders at the rate of 29.9% with the idea that the traders having market in Ghana will pay back every facility on time, but that has not been the case. Due to inappropriate record keeping, inability to separate the business from the owners, lack of succession planning, the rate of default is very high, hence 3.6% as the probability of bad debt.

Salary loans are granted to individuals whose salary passes through the banks. At the end of every month, loan repayment is deducted at source, and also since individuals access the facility with the previous payment voucher and guarantors, the probability of bad debt is 3% which is the least at the bank. The bank therefore finds it convenient to grant loan to salary workers at an interest rate of 30.7%.
Revenue contribution generated indicated that salary loan contributes 0.2956 out of every one Ghana cedi (GH¢1.00) of the total profit. Salary sector therefore contributes the highest amount to the total profit and this confirms its highest interest rate of 30.7% and lowers probability of bad debt of 3.0%.

Trading, agricultural and cottage industry also contribute tremendously to the total profit. Thus out of every one Ghana cedi (GH¢1.00) of the total profit trading, agriculture, and cottage industry contribute 0.2882, 0.2856 and 0.2857 respectively.

Transport contribute the lowest amount to the total profit as it contributes only 0.2485 out of every one Ghana cedis (GH¢1.00) of the total profit. This therefore explain why it has the highest probability of bad debt of 5.4%.

4.2 Portfolio Selection

In this section we examine the amount of loans to be granted in 2012 financial year. This projection is base on the bank’s attempt to maximize interest earned on loans and minimize the lost on loans.
Table 4.2 Optimum Solution for the loan facilities

<table>
<thead>
<tr>
<th>Variable</th>
<th>Value</th>
<th>Reduced Cost</th>
<th>Original Val</th>
<th>Lower Bound</th>
<th>Upper Bound</th>
</tr>
</thead>
<tbody>
<tr>
<td>X1</td>
<td>1,110,416</td>
<td>0.0</td>
<td>0.29</td>
<td>0.28</td>
<td>0.58</td>
</tr>
<tr>
<td>X2</td>
<td>1,760,417</td>
<td>0.0</td>
<td>0.29</td>
<td>0.24</td>
<td>0.29</td>
</tr>
<tr>
<td>X3</td>
<td>704,166</td>
<td>0.0</td>
<td>0.25</td>
<td>0.24</td>
<td>0.29</td>
</tr>
<tr>
<td>X4</td>
<td>1,489,584</td>
<td>0.0</td>
<td>0.29</td>
<td>0.2</td>
<td>0.29</td>
</tr>
<tr>
<td>X5</td>
<td>1,435,417</td>
<td>0.0</td>
<td>0.30</td>
<td>0.30</td>
<td>Infinity</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Constraint</th>
<th>Dual Value</th>
<th>Slack/Surplus</th>
<th>Original Val</th>
<th>Lower Bound</th>
<th>Upper Bound</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constraint 1</td>
<td>0.3115</td>
<td>0.0</td>
<td>6,500,000.</td>
<td>4,473,530.</td>
<td>8,098,649.</td>
</tr>
<tr>
<td>Constraint 2</td>
<td>0.0</td>
<td>2,596,751.</td>
<td>3,250.</td>
<td>-infinity</td>
<td>2,600,000.</td>
</tr>
<tr>
<td>Constraint 3</td>
<td>0.0</td>
<td>2,789,583.</td>
<td>0.0</td>
<td>-infinity</td>
<td>2,789,584.</td>
</tr>
<tr>
<td>Constraint 4</td>
<td>-0.0356</td>
<td>0.0</td>
<td>3,575,000.</td>
<td>2,620,968.</td>
<td>6,035,714.</td>
</tr>
<tr>
<td>Constraint 5</td>
<td>-0.0172</td>
<td>0.0</td>
<td>2,600,000.</td>
<td>628,333.4</td>
<td>6,045,000.</td>
</tr>
<tr>
<td>Constraint 6</td>
<td>-0.0006</td>
<td>0.0</td>
<td>0.0</td>
<td>-infinity</td>
<td>4,921,429.</td>
</tr>
<tr>
<td>Constraint 7</td>
<td>0.1792</td>
<td>0.0</td>
<td>0.0</td>
<td>-233,187.5</td>
<td>147,875.</td>
</tr>
</tbody>
</table>

Source: Field data

From Table 4.2, it has been indicated that, the model for the loan has optimal values for the loan facilities, it indicates that an amount of one million one hundred and ten thousand four hundred and sixteen (GH¢ 1,110,416) Ghana cedis should be allocated for the agricultural loan. An amount of one million seven hundred and sixty thousand, four
hundred and seventeen (GH¢ 1,760,417) should also be allocated for Cottage industry loan.

Moreover, seven hundred and four thousand one hundred and sixty six cedis (GH¢ 704,166) should also be allocated for the Transport loan 704,166. One million, four hundred and eighty nine thousand, five hundred eighty four (1,489,584) should also be made available for the Trading loan facility. Besides, an amount of one million, four hundred and thirty five thousand, four hundred and seventeen (GH¢ 1,435,417) Ghana cedis should be made for the Salary loan.

The model suggests that, loan facilities can still be varied and yet still provides an optimum profit for the bank. As shown in Table 4.2, the lower bound suggest the minimum percentage of the entire loan facility and upper bound for a maximum percentage that can be allocated to a particular loan facility and yet makes the profits remain an optimum
Table 4.3. Dual for Constraints for loan distribution

<table>
<thead>
<tr>
<th>Constraints</th>
<th>X1</th>
<th>X2</th>
<th>X3</th>
<th>X4</th>
<th>X5</th>
<th>RHS</th>
<th>Dual</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maximize</td>
<td>0.2856</td>
<td>0.2857</td>
<td>0.2485</td>
<td>0.2882</td>
<td>0.2978</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Constraint1</td>
<td>1.</td>
<td>1.</td>
<td>0.</td>
<td>1.</td>
<td>1.</td>
<td>≤ 6,500,000.</td>
<td>0.3115</td>
</tr>
<tr>
<td>Constraint2</td>
<td>1.</td>
<td>0.</td>
<td>1.</td>
<td>1.</td>
<td>0.</td>
<td>≥ 3,250.</td>
<td>0.</td>
</tr>
<tr>
<td>Constraint3</td>
<td>-1.</td>
<td>1.</td>
<td>1.</td>
<td>0.</td>
<td>1.</td>
<td>≥ 0.</td>
<td>0.</td>
</tr>
<tr>
<td>Constraint4</td>
<td>1.</td>
<td>1.</td>
<td>1.</td>
<td>0.</td>
<td>0.</td>
<td>≥ 3,575,000.</td>
<td>-0.0356</td>
</tr>
<tr>
<td>Constraint5</td>
<td>1.</td>
<td>-0.4</td>
<td>1.</td>
<td>1.</td>
<td>1.</td>
<td>≥ 2,600,000.</td>
<td>-0.0172</td>
</tr>
<tr>
<td>Constraint6</td>
<td>-1.</td>
<td>1.</td>
<td>-1.</td>
<td>1.</td>
<td>-1.</td>
<td>≥ 0.</td>
<td>-0.0006</td>
</tr>
<tr>
<td>Constraint7</td>
<td>0.15</td>
<td>0.02</td>
<td>-0.06</td>
<td>-0.03</td>
<td>-0.08</td>
<td>≤ 0.</td>
<td>0.1792</td>
</tr>
<tr>
<td>Solution-&gt;</td>
<td>1,110,417</td>
<td>1,760,417</td>
<td>704,166.7</td>
<td>1,489,584.</td>
<td>1,435,417</td>
<td>1,852,485.89</td>
<td></td>
</tr>
</tbody>
</table>

Source: Field data

Table 4.3 shows the dual values for the constraints. The dual value is the change in optimal value or profit per unit change in the resource or right hand side of the various constraints. The Table 4.3 shows that the dual value corresponding to the first constraint is 0.3115. This indicates that the optimal profits will increase by 0.3115 for each increase in the total amount to be disbursed. Thus it is advisable to increase the amount available for allocation if extra funds are available. Similarly, constraint seven will also cause for the profit to be increase by 0.1792 for each increase in the seventh constraint. Which indicates that, if the bad debt is reduced, it will increase the bad debt ratio with the total loan facility thereby decreasing the risk involved and increasing the profit?
However, a values of -0.0356, -0.0172, -0.0006 show that there is a decrease of -0.0356 for constraint 4, -0.0172 for constraint 5 and -0.0006 for constraint 6 in the value of the objective function per unit increase in the amount allocated to satisfy these constraints.

Moreover, the studies have shown in Table 4.3 that, the model will yield an optimum amount of one million eight hundred and fifty two thousand, four hundred and eighty five Ghana cedis eighty-nine pesewas (GHS 1,852,485.89) as a profit from the loan disbursement.
CHAPTER FIVE

SUMMARY, RECOMMENDATIONS AND CONCLUSION

5.0 INTRODUCTION

This chapter presents the summary of the results, conclusions and recommendations of the study to help Juaben Rural Bank in order to optimize profit margin.

5.1 SUMMARY OF FINDINGS

The thesis was conducted in order to analyse the possibility of improving the profit on loans given out by the Juaben Rural bank. Data on loans types, corresponding interest rates and probability of bad debt on each loan type were gathered. The problem was modelled as a linear programming problem. The Quantitative Manager for Windows (QMW) software was used to solve the problem.

From the findings the bank’s loan portfolio are grouped into five (5). These are cottage industry, trading, salary, agricultural and transport loans. The cottage industry refers to those in the distilleries, kente and basket weaving, soap production and other similar industries. The transport includes car importers, transport owners and spare parts dealers. Salary loans are loans given to those who earn salary. The other types are agriculture and trading, agricultural is made up of the poultry, animal farming, oil palm plantation, cash crop production and agro chemical sellers. Trading refers to buyers and sellers within the country and those importers and sellers within the country.
From the findings agriculture loan should not be encouraged due to its high risk of repayment by the farmers. Other reasons attributed to the non-payment of the agriculture loan are: bad farming practices which lead to crop failure, unpredictable and poor weather conditions, infestation by pest and the abundance of produce during the harvest season which gives poor returns in terms of income. From the above observations it is not prudent for the bank to offer farmers with agriculture loans.

Transport industry registered 10.83% of the share of the loan disbursed. This indicates that it is not very economical to give loan to such facility if other sectors are improving tremendously. This goes to confirm the lowest interest rate of 25.7% being charged by the bank as the loan is not attractive.

Dual value is the change in the optimal value or profit per unit change. Positive dual value indicates that the optimal profit will increase by the margin for each of the total amount to be disbursed. The negative dual value reduces profit. From the findings the two positive dual values 0.3115 and 0.1792 indicates that the optimum profit will increase by the same margins for each increase in the total amount to be disbursed.

The findings also revealed that higher amount should be given to cottage industry, salary workers and trading in order to optimize profit. Although cottage industry has the highest
probability of bad debt (ie. 5.4%) greater amount of the profit comes from that sector as they form about 50% of bank’s customers.

5.2 RECOMMENDATIONS

One of the most successful and important application of quantitative analysis to solving business problems has been in the areas of given out loans. From the conclusions it was realized that using quantitative methods to give out loans help banks to increase their profits.

It is therefore recommended that the banks should concentrate on cottage industry, trading and salary loans which forms 27.08%, 22.92% and 21.93% respectively of the loans disbursed in order to optimize profit in both the short and long terms.

It is recommended that Juaben Rural Bank should adapt this model in the allocation of funds reserved for loans. Again banks be educated to employ quantitative method to find an appropriate quantitative model to help them disburse funds of the banks more efficiently.
Lastly, I recommend that apart from loan disbursement, banks and other financial institution should employ scientific and mathematical methods in most of the business they conduct.

A further study might also research into bad loan problem in different sectors of the bank’s lending activities.

5.3 CONCLUSION

It was observed that out of the total amount of six million five hundred Ghana cedis (GH¢6,500,000) to be disbursed as loan in 2012 financial year, one million one hundred and ten thousand four hundred and sixteen Ghana cedis (GH¢1,110,416) should be given to agriculture \((x_1)\) but not two million two hundred seventy eight Ghana cedis (GH¢2,278,000).

The transport sector \((x_3)\) should receive an amount of seven hundred and four thousand one hundred and sixty six (GH¢704,166.00). Also trading \((x_4)\) should be given one million four hundred and eighty nine thousand five hundred eighty four (GH¢ 1,489,584), one million four hundred and thirty five thousand four hundred and seventeen Ghana cedis (GH¢ 1,435,417.00), should be allocated for salary loans and cottage industry \((x_2)\) one million seven hundred and sixty thousand four hundred and seventeen (GH¢ 1,760,417.00).
With these allocations, the bank can make an optimum profit of one million eight hundred and fifty two thousand four hundred and eighty five ceids eighty nine pesewas (GH¢1,852,485.89) per annum.
REFERENCES


Light, M., Rosser, B. and Hayward, S. (2005). *Realizing the Benefits of Project and Portfolio Management*


APPENDIX 1

KWAME NKRUMAH UNIVERSITY OF SCIENCE AND TECHNOLOGY
INSTITUTE OF DISTANCE LEARNING
RESEARCH QUESTIONNAIRE

This questionnaire seeks to collect data for the study ‘Optimum loan portfolio selection’
Case study of Juabeng Rural Bank Limited Ashanti Region.
The data is solely needed for academic purpose and would be conducted in a highly confidential manner. Thank you.

Please tick the appropriate one.

1. Which of the following areas is the bank investing in 2012?
   A. Agricultural [ ]    B. Transport [ ]    C. Trading [ ]
   D. Cottage industry [ ] E. Salary [ ]

2. How much is being proposed for investment in loans for 2012?
   ………………………………………………………………………………………………………

3. What are the respective interest rate?
   A. Agricultural …………  B. Transport …………C. Trading……
   D. Cottage industry …………  E. Salary……

4. Do you provide for bad debt?
   Yes [ ]    No [ ]

5. What is the probability of bad dept on each portfolio?
   A. Agricultural …………  B. Transport …………C. Trading……
   D. Cottage industry …………  E. Salary……
APPENDIX 2

**Decision:** How much to invest in each of the five (5) investment options?

**Objective:** Maximize interest earned

**The linear programming model (LP)**

The main objective of the model is to maximize profit on each loan portfolio. The variables of the model can be define as:

- $x_1 =$ Agriculture loan
- $x_2 =$ Cottage industry loan
- $x_3 =$ Transport loan
- $x_4 =$ Trading loan
- $x_5 =$ Salary loan

**Model Formation**

The objective function to be maximized:

Maximize $Z = \sum [ \text{interest rate} \times (1-\text{probability of bad debt}) \times \text{loan type}] - [\text{(probability of bad debt)} \times \text{(loan type)}]$

Maximize $z = 0.295 (0.968) x_1 + 0.302 (0.946) x_2 + 0.257 (0.967) x_3 + 0.299$
(0.964) x₄ + 0.307 (0.97) x₅

Maximize Z = 0.28615x₁ + 0.285692x₂ + 0.248519x₃ + 0.288236x₄ + 0.29779x₅

Revenue Contribution

<table>
<thead>
<tr>
<th>Type of loan</th>
<th>Interest rate</th>
<th>Probability of bad debt</th>
<th>Revenue</th>
</tr>
</thead>
<tbody>
<tr>
<td>x₁</td>
<td>29.5</td>
<td>3.2</td>
<td>0.2856</td>
</tr>
<tr>
<td>x₂</td>
<td>30.2</td>
<td>5.4</td>
<td>0.2857</td>
</tr>
<tr>
<td>x₃</td>
<td>25.7</td>
<td>3.3</td>
<td>0.2485</td>
</tr>
<tr>
<td>x₄</td>
<td>29.9</td>
<td>3.6</td>
<td>0.2882</td>
</tr>
<tr>
<td>x₅</td>
<td>30.7</td>
<td>3.0</td>
<td>0.2978</td>
</tr>
</tbody>
</table>

Source Author own calculation

Maximize R = Total Revenue

Maximize = 0.2856x₁+0.2857x₂+0.2485x₃+0.2882x₄+0.2978x₅

The problem has eight (8) constraints:

1. The total loan to be disbursed is six million five hundred thousand Ghana cedis.
   (GH¢ 6,500,000.00)
   \[ x₁ + x₂ + x₃ + x₄ + x₅ \leq 6,500,000 \]

2. Allocate at least 50% of the total fund to agricultural and trading loans.
   \[ x₁ + x₄ \geq 0.5 \times 6,500,000 \]
APPENDIX 4

\[ x_1 + x_4 \geq 3,250 \]

(3) The sum of salary loan, cottage industry loan and transport loan exceeds agricultural loan.

\[ x_2 + x_3 + x_5 \geq x_1 \]

\[ -x_1 + x_2 + x_3 + x_5 \geq 0 \]

(4) The sum of agricultural, cottage industry and transport loans should be equal or exceed 55% of the total loan.

\[ x_1 + x_2 + x_3 \geq .55 (6,500,000) \]

\[ x_1 + x_2 + x_3 \geq 3,575,000 \]

(5) The sum of agricultural loan, transport loan and trading should be equal or exceed the sum of 40% of total loan and 40% of cottage industry loan

\[ x_3 + x_4 \geq 0.4 x_2 + 0.4 (6,500,000) \]

\[ x_1 - 0.4 x_2 + x_3 + x_4 \geq 2,600,000 \]

(6) Non-negativity condition

\[ x_1 \geq 0, \ x_2 \geq 0, \ x_3 \geq 0, \ x_4 \geq 0, \ x_5 \geq 0 \]

(7) The sum of trading loan and cottage industry loan should be greater or equal to the sum of agricultural loan and salary loan

\[ x_4 + x_2 \geq x_1 + x_5 \]

(8) The total ratio for bad debt on all loans should not exceed 20% of the total loan.

\[
\frac{0.295 x_1 + 0.302 x_2 + 0.257 x_3 + 0.299 x_4 + 0.307 x_5}{x_1 + x_2 + x_3 + x_4 + x_5} \leq 0.2
\]

\[
0.295 x_1 + 0.302 x_2 + 0.257 x_3 + 0.299 x_4 + 0.307 x_5 \leq 0.2 (x_1 + x_2 + x_3 + x_4 + x_5)
\]
The linear programming model for the problem is

Maximize $Z = -0.28615x_1 + 0.285692x_2 + 0.248519x_3 + 0.288236x_4 + 0.29779x_5$

Subject to:

1. $x_1 + x_2 + x_3 + x_4 + x_5 \leq 6,500,000$
2. $x_1 + x_4 \geq 3,250$
3. $-x_1 + x_2 + x_3 + x_5 \geq 0$
4. $x_1 + x_2 + x_3 \geq 3,575,000$
5. $x_1 - 0.4x_2 + x_3 + x_4 \geq 2,600,000$
6. $x_1 \geq 0, x_2 \geq 0, x_3 \geq 0, x_4 \geq 0, x_5 \geq 0$
7. $x_4 + x_2 \geq x_1 + x_5$
8. $0.095x_1 + 0.102x_2 - 0.057x_3 - 0.099x_4 - 0.107x_5 \leq 0$

The model was then solved by using the Quantitative Manager (QM) for windows (software).