

NEWSPAPER DISTRIBUTION AS VEHICLE ROUTING PROBLEM

By

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DECLARATION

I hereby declare that this submission is my own work towards the Master of Science degree and that, to the best of my knowledge it contains no material previously published by another person nor material which has been accepted for the award of any degree of the University, except where due acknowledgment has been made in the text.

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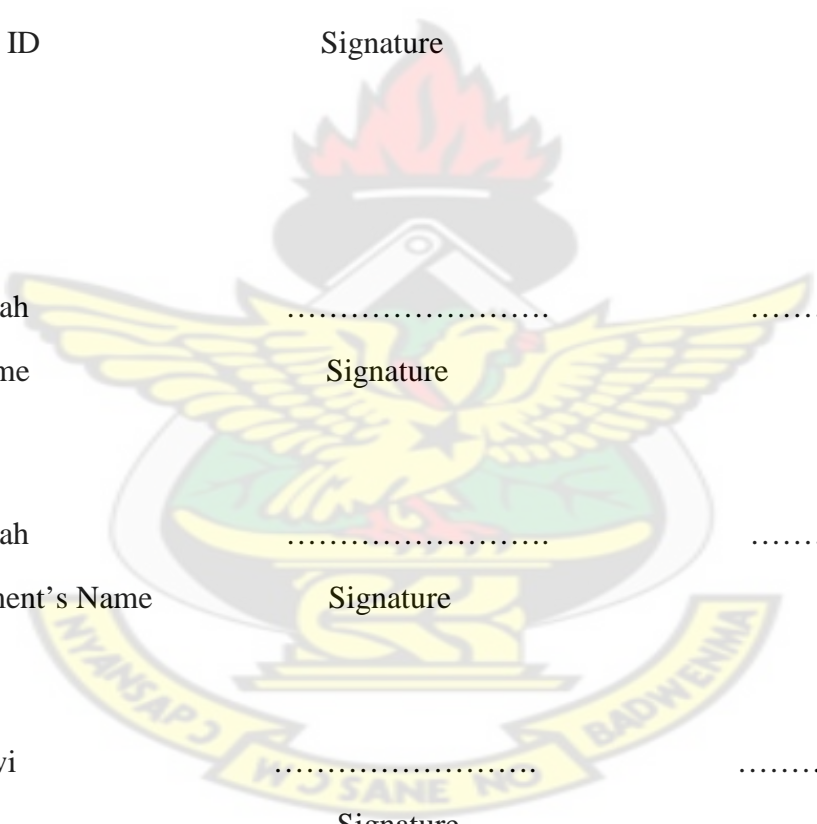
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ABSTRACT

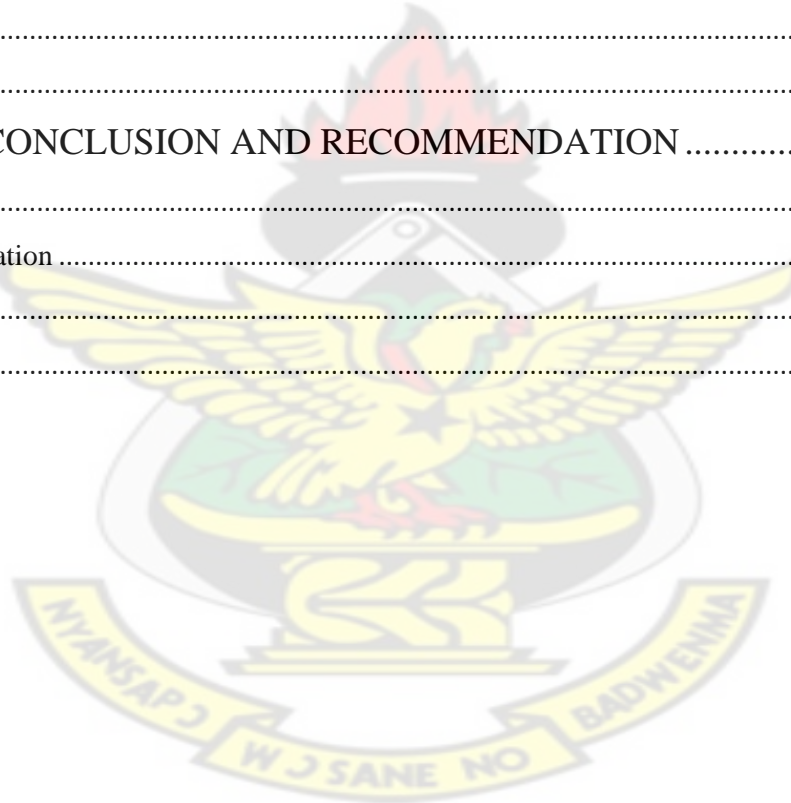
This thesis seeks to address the problem of finding distribution routes from the distribution centre in Kumasi to all the twenty-seven district capitals in Ashanti region for Graphic Communication Group Limited (GCGL) office in the region, not only to ensure timely delivery of newspapers but also cost-effective. The problem was formulated as Capacitated Vehicle Routing Problem with Time Window (CVRPTW) and the Clark and Wright's Savings algorithm was employed to solve the problem. The algorithm takes the travel time matrix as input and proceeds to find the travel time savings between all the districts. The proposed problem was solved using VRP SOLVER computer program. Comparison of results in terms of the total traveling time obtained by the Clarke and Wright savings algorithm and the actual distribution routes maintained by company showed the current total travelling time can be reduced by up to 21.9%.



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DEDICATION

I dedicate this work to my dear mother, Adwoa Serwaa Denteh.

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CHAPTER 1

INTRODUCTION

1.1 Background to the study

Newspaper publishing is a competitive market. On account of increasing distribution cost per copy, newspaper companies will want to improve the production and distribution process as well as other processes within the company in order to compete with each other and with other media such as TV, radio and other online services (Boonkleaw et al., 2009).

Newspaper is highly perishable product and requires stringent timings to be followed. Publishers have indicated that early morning distribution is very important, and even a slip of a few minutes can create a cascading effect and the value of the newspaper diminishes. At the same time, publishers print newspapers as late as possible to allow time to incorporate the most up-to-date news stories. The consequence is that, there is pressure to reduce the time taken to distribute newspapers.

In Ghana and for that matter Ashanti region, most newspapers are transferred from the national capital, Accra to the regional capitals (Distribution Centre) before it is distributed in all the districts. It is hard to believe that, regardless of the fact newspaper are delivered to the distribution centre at the earliest time; there are frequent late deliveries in all the district capitals.

The *Graphic Communication Group Ltd* (GCGL) publisher of the state-owned *Daily Graphic* and six other editions has its main printing house located in Accra and regional offices considered as Distribution Centre's and it is distributed in all 120 districts nationwide. The Ashanti region currently has the second highest circulation figure of 20,700 copies per day. The

company has six distribution vehicles and vendors in all the 27 district capitals in the region.

The distribution process is carried out six times in a week (The GCGL, Ashanti Region Office).

In trying to find answers to the question that pertain frequent late deliveries of the newspaper in the region, any operation researcher would question the degree of efficiency and effectiveness of distribution routes. My study at GCGL office in Ashanti region revealed that, distribution of newspapers depends mainly on driver's experience. Even though the problem is highly complex, they do not employ any scientific methodology for distributing newspapers in the region. This culminated to the inefficient distribution routes for delivery vehicles.

The choice of a route for Newspaper delivery vehicles is critical to planning and management phases. This is of particular importance since Newspaper distribution account for a substantial amount of total expenditure, making up approximately 23% of the total cost. To the extent possible, it is necessary to highlight the areas in which an efficient improvement is feasible in the delivery operation (Boonkleaw et al., 2009).

Efficient distribution routes are seen as an important success factor by many newspaper companies since there is encouraging evidence that on time delivery of newspapers are associated with positive improvement in sales. A good distribution system may also help to solve the problem of apparently conflicting interest between Newspaper Company and the reader. Here, indeed, time and cost is the critical factor. The efficiency of the distribution system may strongly affect the competitiveness of a newspaper (Mantel et al., 1993). Successful planning and management of Newspaper distribution is primarily focused on reducing traveling time and cost.

Guaranteed on time delivery and service quality are the main key success factors of newspaper industry, therefore it gives the company competitive advantage (Boonkleaw et al., 2010). This

makes it important to improve upon this research area in order for the newspaper company to be competitive.

1.2 Profile of study area

The Ashanti region is the third largest of ten administrative regions in Ghana. The region is centrally located in the middle belt of Ghana and lies Latitude $5^{\circ}.50''\text{N}$ and $7^{\circ}.46''\text{N}$ and Longitude $0^{\circ}.15''\text{W}$ and $2^{\circ}.25''\text{W}$ (Town and Country Planning Department-Kumasi). The region covers total area of about 24,389 square kilometers or 10.2% of total land area of Ghana and in terms of population however; it is the most populated region with a population of 4,780, 380 representing 19.4% of the total population according to 2010 population and housing census (Ghana Statistical Service).

The capital city, Kumasi is the second largest city in Ghana and its strategic geographic location makes it a brisk commercial and administrative centre and it is regarded as the commercial capital of Ghana. The region is divided into 27 districts.

1.3 Problem statement

Newspaper distribution by *Graphic Communication Group Ltd* office in Ashanti region lacks real plan for vehicle routes from the distribution centre to the district capitals in the region. This made routing very tedious and confusing sometimes among the drivers and operational inefficiency in the distribution process leading to frequent late deliveries, high transportation cost and missed delivery points.

This thesis seeks to address the problem by finding distribution routes from the distribution centre to the district capitals in Ashanti region not only to ensure timely delivery of newspapers but also cost-effective.

1.4 Objectives of the study

The objectives of this thesis are:

1. To model newspaper distribution by GCGL in the Ashanti region as Vehicle Routing Problem.
2. To determine the optimal distribution routes to all district capitals in the region.

1.5 Methodology

Distribution of newspapers by *Graphic Communication Group Ltd.* in Ashanti region will be modeled as Capacitated Vehicle Routing Problem with Time Window (CVRPTW).

The data collected from GCGL office in Ashanti region is a secondary source recorded from January 2012. The physical road distance and networks linking the all the districts capitals in the region were obtain from Google Earth and Google Map programs .The road distances are converted to time using the average speed of the distribution vehicles. The road network is represented as a graph, with district capitals as nodes and travel times linking district capitals as edges.

The Clark and Wright's Savings algorithm will be employed. The algorithm takes the travel time matrix as input and proceeds to find savings between the nodes. The proposed problem will be solved using VRP SOLVER computer program. Resources available on the internet and KNUST library will be used to obtain the needed literature for this thesis.

1.6 Justification

Optimal distribution routes from the distribution centre to all the district capitals will resolve the problem of frequent late deliveries. When this is achieved, it will impact positively by improving the competitiveness of almost all aspects of company operations including sales, logistics, production and customer satisfaction. Economically, vehicle routing can be implemented by various government agencies and businesses to cut transportation cost.

Optimal vehicle routes can avoid unnecessary fuel consumption and CO₂ emissions thereby contributing to the sustainability of the environment and health of its citizenry. Vehicle routes can provide significant benefits by reducing traffic congestion delays.

Again, the research serves as a spring board to generate interest for further research into the other aspects of vehicle routing problems in Ghana. This stems from the fact that vehicle routing problems is a multifaceted phenomenon and no one research is capable of addressing it in full. This makes this study justifiable and worthwhile.

1.7 Thesis organization

This thesis is organized into five chapters. Chapter 1 introduces the problem of Newspaper distribution. Chapter 2 reviews the relevant literature on Vehicle Routing Problems and their solutions method. In Chapter 3, the problem is described and a mathematical model of the Vehicle Routing Problem (VRP) under consideration is formulated with solution methodology. Chapter 4 consists of data collection analysis and results. Finally, Chapter 5 concludes the thesis with remarks on the results.

CHAPTER 2

LITERATURE REVIEW

2.1 Introduction to Vehicle Routing Problem (VRP)

The vehicle routing problem (VRP) is often described as the problem in which vehicles based on a central depot are required to visit a set of geographically dispersed customers in order to fulfill known customer demands. The objective is to construct a low cost, feasible set of routes – one for each vehicle. A route is a sequence of locations that a vehicle must visit along with the indication of the serve it provides. The vehicle must start and finish its tour at the depot (Marinakis and Migdalas, 2002).

Much of the motivation for studying the VRP has come from the numerous real-world applications and the considerable savings that better solutions to these problems represent. Because of this reason, VRP is an extremely active research area that has seen an exciting interplay between theory and practice. The first paper on VRP was written by Dantzig and Ramser in 1959. It describes both a practical problem concerned with delivery gasoline to gas stations by the Atlantic Refining Company and provides the first formulation of the general vehicle routing problem.

VRP falls into the category of NP-complete problem, which means that there is no known solution that can execute in polynomial time. The difficulty of solving the general VRP problem can be seen since it generalizes two well-known NP-complete problems. By relaxing the capacity constraint for each vehicle, an instance of Multiple Traveling Salesman Problem (MTSP) is

generated. On the other hand, by setting all edge cost to be zero, an instance of the Bin Packing Problem is generated.

2.2 Applications of VRP involving distribution.

Magalhães and Sousa (2006) studied the distribution of pharmaceutical goods in Northern and Central Portugal based on orders that are constantly arriving along the day to improve the quality of the service to customers. The problem was modeled as dynamic vehicle routing problem (DVRP) with objective of minimizing the delivery time. The heuristic approach adopted, cluster the pharmacies based on load and the geographical dispersion. Once the clusters have been created, the algorithm uses the least cost as the criterion for constructing the route and 2-opt exchanges procedure for the route improvement. The results showed improvements in the order of 8% was achieved, concerning the average delivery time, with no changes to the current available fleet and roughly completing the same distance.

Cho and Hsu (2008) discussed how to distribute goods of different temperatures, such as hot foods, normal temperature goods, frozen foods of logistics carrier in Taiwan. The problem was modeled as heterogeneous multi-temperature fleet vehicle routing problem (HMFVRP). The objective was to minimize total cost. The Farthest-start Nearest Neighbour (FNN) method was applied for the route construction and were improved by 2-opt and Or-opt arcs exchanges as well as inter route nodes exchanges. The computational results showed the heuristics is effective in lowering the vehicle usage cost and travel distance.

Mantel et al. (1993) presented hierarchical distribution system on Dutch regional newspaper with 140,000 papers printed per day. The problem was formulated as vehicle routing problem with time window (VRPTW). The problem was modeled by constructing the traveling time matrix

between all pairs of nodes using the Floyd's algorithm and Clarke and Wright savings technique as the vehicle routing heuristics. The results concluded that for the simplified problem, total traveling time is rather insensitive to the number of transfer nodes.

Boontho (2002) addressed the problem of lime distribution activity in Thailand of 200 nodes for three time windows of one factory and three depots. The problem was formulated as vehicle routing problem with time window (VRPTW) with the objective of minimizing the total traveling distances. The Clarke and Wright savings algorithm was implemented for the route construction procedure. Delphi software was developed based on the model to solve the problem with less computational time. The results of the software run produced up to 99.46% near the optimum solution.

Bookleaw et al. (2009) applied the vehicle routing problem to improve the newspaper distribution service in Bangkok, Thailand. The problem was formulated as vehicle routing problem with time windows (VRPTW). The objective was to minimize the total travel time of operations. The modified sweep algorithm decomposed into two natural components; clustering the vertices into feasible routes, then actual route construction, was implemented to solve the routing problem. The results of the variant VRPTW remain unsolved as modified sweep algorithms resulted the delay for 41.5% of all drop points.

Panapinum et al. (2005) studied the problem of improving the efficiency of perishable food distribution in Bangkok, Thailand. The problem was formulated as mixed integer linear program with objective of minimizing the total distance of all trucks. A software program was developed from an extension from the Clarke and Wright algorithm which allows restriction on the system.

The software run improved the results by decreasing the total distribution distance between 11% and 27% when compared the present system under practices without any additional cost.

Privé et al. (2006) studied the distribution of soft drinks and collection of recyclable containers of about 500 customers in a Quebec based company as vehicle routing problem. The problem was modeled as heterogeneous pickup and delivery vehicle routing problem (PDVRP) with the objective of minimizing the routing cost. They proposed three heuristics method for solving the problem. Nearest Neighbour heuristics for route construction and two petal based heuristics were developed for the problem. The 3-opt procedure was applied to each route in the improvement phase. The algorithms were coded in *Visual Basic* and run on a computer. Ten randomly generated instances with 150 customers each was then used to access the relative performance of the three proposed heuristics. The results obtained on the real-life case showed that a 23% distance reduction can be achieved.

Hsu and Hung (2003) presented a case study of vehicle routing problem for distribution refrigerated food in Taiwan. The problem was modeled as vehicle routing problem with time window (VRPTW) with the objective of minimizing the sum of transportation cost, inventory cost and energy cost. Their research extends VRPTW by considering randomness in refrigerated food distribution process and adopts the Time-Oriented Nearest-Neighbour Heuristic by Solomon (1983). However, the quality of the results, using the heuristics method depends on the number of customers in the problem.

Boosam et al. (2011) presented cash distribution problem of a bank in Bangkok, Thailand which has a total of 377 branches and 3 distribution centers (DC). The problem was modeled as vehicle routing problem with time window (VRPTW) and assignment problem (AP) with the objective

of minimizing the total travel time of operations. First the assignment problem clusters the branches into three groups where each group belongs to a DC and VRPTW produces routes for each DC daily. The modified sweep and Nearest Neighbour algorithms considered for solving the problem produced good results within a short processing time than the current operation of the bank.

Roy and Gelders (1981) solved a real life distribution problem of a liquid bottle product through a 3-stage logistic system; the stages of the system are plant-depot, depot-distributor and distributor-dealer. They modeled the customer allocation, depot location and transportation problem as 0-1 integer programming model with the objective function of minimization of the fleet operating costs, the depot setup cost, and delivery costs subject to supply constraints, demand constraints, truck load capacity constraints, and driver hours constraints. The problem was solved optimally by branch and bound, and Lagrangian relaxation.

Qiang and Jiuping (2008) in their paper presented a study on vehicle routing problem in the distribution of fresh agricultural products in China with variation of the number of customers' service from 150 to 300 sites every day under fuzzy environment. The problem was modeled as heterogeneous vehicle routing problem with time window (HVRPTW) with the objective of minimizing the total cost and maximizing the customer services that can be rendered to customer's acceptable delivery time simultaneously. A hybrid intelligent algorithm integrating random fuzzy simulation and genetic algorithm (GA) was designed to solve the model and was implemented using visual C++. The results indicated that waiting cost and penalty cost can significantly influence total delivery costs and the schedule of customer's service.

Ayadi and Benadada (2010) presented a paper on Liquefied Petroleum Gas (LPG) distribution for cities in Morocco consisting of scheduling the distribution of several products during a period of time, using a fleet of heterogeneous non-compartment trucks with volume meter. The problem was modeled as heterogeneous fleet with multi-trip vehicle routing problem with the objective of minimizing cost and optimizing the vehicle used. The problem was solved using genetic algorithm based on the Clarke and Wright savings method for the initialization procedure. The results showed that the genetic algorithm was able to find a good solution for LPG problem in a reasonable execution time as compared to the mathematical model with CPLEX software.

Tzeng et al. (1995) solved the problem of how to distribute and transport imported coal to each of the power plants on time in the required amounts and at the required quality under conditions of stable and supply with least delay. They formulated a LP that minimizes the cost of transportation subject to supply constraints, demand constraints, vessel constraints and handling constraints of the ports. The model was solved to yield optimum results, which is then used as input to a decision support system that help manage the coal allocation, voyage scheduling and dynamic fleet assignment.

2.3 Applications of VRP

Somprasonk and Boondiskulchok (2009) dealt with the problem of planning of operation routes for mobile medical staffs which aims for providing medical treatment and health sanitation to residences in rural and remote areas in Thailand. The problem was modeled as capacitated vehicle routing problem with time window (CVRPTW). The objective was to find feasible routes with possible lowest travelled distance as well as highest customer service level. The savings algorithm was applied for the route construction and Dijkstra's algorithm which solves single source shortest path problem was implemented for the route improvement phase. The

computational results of applying vehicle routing problem led to the reduction in transportation cost by 15% as compared to previous approach.

Sahoo et al. (2006) studied the routing of vehicles for solid-waste collection of Waste Management, Inc (WM) in North America with nearly 26,000 collection and transfer vehicles with consideration of multiple disposal trips and driver's lunch break. The problem was formulated as vehicle routing problem with time window (VRPTW). An extended insertion algorithm and a clustering-based waste algorithm were developed and implemented in a software system called WasteRoute, for routing vehicles to service commercial and residential customers. By using WasteRoute, WM significantly reduced the number of collection routes and achieved savings of \$18 million in 2003 and \$44 million in 2004.

Lu and Fu (2002) presented a case study of the school bus routing problem. It was formulated as multi-objective combinatorial optimization problem. The objectives considered included minimizing the total number of buses required, the total travel time spent by pupils at all pick-up points, which was what the school and parents are concerned with most, and the total bus travel time. It also aims at balancing the loads and travel times between buses. They proposed heuristic algorithm which was programmed and run efficiently on a PC. Numerical results were reported using test data from a kindergarten in Hong Kong. This proved to be effective as it saved 29% of total traveling times when compared to current practice.

Groër et al. (2008) dealt with the problem of balancing routes for billing meter readers. The problem was modeled as balanced billing cycle vehicle routing problem (BBCVRP) with the objective of reducing meter reading cost and balancing workload. They developed a three-phase heuristic technique for meter reading routes. Initial routes are generated using modified version

of the VRP record-to-record travel time algorithm. Billing day is assigned to each route using assignment problem and customers are transitioned to their billing days using mixed integer program (MIP). The algorithm was coded in C/C++ and used the GMPL in conjunction with CPLEX. The test performance when run on an existing set of routes containing between 3,000 and 4,000 customers produced efficient and balanced target routes along with a set of intermediate routes.

Ping and Keja (2007) studied scheduling of postal vehicles, including mail collection from district post offices and delivery to general post office in Hong Kong. The objectives pursued for this study were maximization of resource utilization and minimization of operation cost. Integer linear programming model of the vehicle routing problem (VRP) was developed in an effort to obtain optimal solutions. The results of the proposed model after it was implemented in CPLEX produced optimal vehicle routes and schedules.

Oppen and Løkketangen (2008) presented the Livestock Collection Problem (LCP) from the Norwegian meat industry which consists of constructing a route plan for transporting animals from farms to a slaughterhouse. The problem was seen as a multi-tour vehicle routing problem. To find a solution to the LCP, both Tabu search heuristic and exact method based on column generation were developed. The computational testing of the exact solution procedure shows that column generation algorithm was capable of optimally solving instances with up to approximately 25 orders whilst the solutions generated by the Tabu Search heuristic are between 12% and 21% better than manual ones when measured by total distance driven by all vehicles.

Abounacer et al. (2009) proposed a two-population meta-heuristics to the professional staff transportation problem (PSTP). PSTP has to do with building vehicle routes for transporting staff

of one or several companies in order to minimize total cost of transport, taking into account the level of service offered to its users. The meta-heuristic developed in their paper included ACO and Genetic algorithm (GA). Experimental results proved both techniques to be effective.

Madsen et al. (1995) presented an algorithm for a real-life multi vehicle dynamic dial-a-ride problem consisting of up to 300 daily requests for the transportation of elderly and handicapped people in Copenhagen. The problem which was modeled as dynamic vehicle routing problem (DVRP) had many constraints such as time windows, multi dimensional capacity restrictions customer priorities and a heterogeneous vehicle fleet. Many objectives taking into account user satisfaction and service cost were considered through the use of weight parameters. When new request arrives, it is inserted in a current route using an efficient insertion algorithm. Computational results on a real-life instance with instances with up to 300 requests and 24 vehicles have shown that the algorithm was fast enough to be used in a dynamic context and that it is capable of producing good quality solutions.

Tiejun and Yijun (2010) dealt with the problem of routing emergency rescue vehicles such as ambulances, fire engines, police cars, emergency logistic cars etc. in an important way. Their problem was modeled as dynamic vehicle routing problem (DVRP) with the objective of minimizing service time. The study applied the technologies of GIS, GPS, RS, and GSM/GPRS etc. into designing of vehicle routing system, using geographic information as the carrier of other types of information. The Genetic algorithm (GA) was simulated for solving the problem. Simulation results indicated the efficiency of vehicle routing and the quality of emergency rescue services can be significantly improved with the proposed new method.

Horn (2002) developed software for demand-responsive passenger services such as taxis and variable route buses. His problem which is dynamic vehicle routing problem (DVRP) includes time window restrictions for the dynamic request, capacity constraints and book cancellations. The insertion of new requests is done through a minimum cost insertion scheme which is supplemented by a local search procedure executed periodically. The author developed heuristic for strategically relocating the idle vehicles, taking into account future request patterns. Computational experiments were carried out based on data from a taxi company in Gold Coast, Australia. Results from the tested instances, which had around 4200 request and 220 taxis in 24 hour period, have shown that the proposed heuristic performs well on dynamic instances of large size.

Le Blanc et al. (2006) presented a paper dealing with collection of containers from vehicle dismantlers in the Netherlands. In their problem, they considered vehicles can carry two containers at a time. Their heuristic is a two-step procedure, first generating candidates' routes, then selecting from these routes using a set partitioning approach. They reported potential cost savings of over 18% compared with the current system.

Leung et al. (1990) developed an optimization-based approach for a point-to-point route planning that arises in many large-scale delivery systems, such as communication, rail, mail, and package delivery. In these settings, a firm, which must ship goods between many origin and destination pairs on a network, needs to specify a route for each origin-destination pair so as to minimize transportation cost. They developed a mixed multi-commodity flow formulation of the route planning problem, which contains sixteen million 0-1 variables, which is beyond the capacity of general IP code. The problem was decomposed into two smaller sub-problems, each

amendable to solution by a combination of optimization and heuristic techniques. They adopted solution methods based on Lagrangian relaxation for each sub-problem.

Coene et al. (2008) proposed several heuristics algorithms for a routing problem of a Belgian company collecting waste at slaughterhouses, butchers and supermarket. The company was responsible for collecting high-risk and low-risk waste categories of animal waste. Both wastes need to be collected separately. Comparison results in terms of the total traveling time between the proposed solutions and the current routes were presented. For the low-risk waste, the results indicated that the current total travelling time can be reduced by up to 15.5%, whereas the travelling time for the collection of high-risk waste can be reduced by up to 9%.

2.4 Models and Formulation of VRP

Schittekat et al. (2006) formulated the school bus routing problem using a single objective integer programming model VRP by introducing several other interesting additional features. They considered a set of potential stops as well as a set of students who can walk to one or more of these potential stops. The goal of their routing problem was to select a subset of stops that will actually be visited by the buses; determine which stops each student should walk to; and develop a set of tours that minimize the total distance travelled by all buses. The problem was solved using commercial integer programming solver and results on small instances were discussed.

Xu et al (2003) formulated a practical pickup and delivery problem (PPDP) using multiple vehicle types that are available to fill a set of pickup and delivery orders having multiple pickup and delivery time windows. In addition to Hours-of-Service (HOS) rules, vehicle capacity, vehicle compatibility to orders and compatibility between orders, as well as nested precedence relationships in truck loading and unloading (LIFO) were taken into consideration. The cost of a

trip was determined by several factors, such as fixed charge, total mileage, total waiting time and total layover of a driver. The problem was solved using the column generation procedure.

Bent and Hentenryck (2003) modeled the dynamic vehicle routing with stochastic service times and customer locations using Multiple Plan Approach (MPA) and Multiple Scenario Approach (MSA). Multiple plan approach's main idea was to generate and maintain all possible sets of routing plans along with the main routing plan. The random customer requests were ranked according to a priority. The requests were checked against the available plans. If they match one of the available plans, it was accepted, otherwise it was rejected. In case of the multiple scenario approach, the requests were sampled from a general log normal distribution and the routing plans were generated according to sample requests. Hence future customer requests were accommodated by switching from the main routing plan to the available plan. The approach yielded good results.

Achuthan et al. (1994) formulated an Integer Programming model to solve a vehicle routing problem (VRP) with the objective of distance minimization for the delivery of a single commodity from a centralized depot to a number of specified customers locations with known demands using a fleet of vehicles that have common capacity and maximum distance restrictions. They introduced a new sub-tour elimination constraint and solved the problem optimally using the branch and bound method and used the CPLEX software to solve the relaxed sub-problems.

Malandraki and Daskin (1992) proposed a mixed integer linear programming formulation of Time Dependent Vehicle Routing Problem (TDVRP) that considers travel time as step function, time windows for serving the customer and the maximum allowable duration for each route. The

distance is a time dependent function as the speed of the vehicle does not remain constant due to the variable traffic density. The objective was to minimize the total route time of all vehicles (includes travel time, service time and waiting time at all nodes). The nearest neighbourhood technique, sequential route construction heuristic, simultaneous route construction TDVRP heuristic, cutting plane heuristics were used to solve the problem. The results obtained from each of the method were compared using various criteria.

Chang et al. (1997) proposed a revised multi objective mixed-integer programming model (MIP) for analyzing the optimal path in a waste collection network within a geographic information system (GIS) environment. They demonstrated the integration of the MIP and the GIS for the management of solid waste in Kaohsiung, Taiwan. The computational results show a reduction of around 36.46% in distance travelled and 6.03% in collection time compared to the current system.

Azi et al. (2007) described an exact algorithm for solving a problem where the same vehicle performs several routes to serve a set of customers with time windows. A two phases method was proposed based on an elementary shortest path algorithm with resource constraints. In the first phase, all non-dominated feasible routes were generated, while in the second phase, some routes were selected and sequenced to form the vehicle workday.

Miller (1995) presented branch-and-bound algorithm for solving VRP. The lower bounds were derived by relaxing the subtour elimination and vehicle capacity constraints to yield a perfect b-matching problem. The subtour elimination and vehicle capacity constraints were expressed by a single family of inequalities called generalized subtour elimination constraints. Bounds were

strengthen by using Lagrange multipliers to enforce subtour elimination and capacity constraints. This method increased the size of the instance solvable by branch and bound effectively.

Baldacci et al. (2007) presented an exact algorithm for the capacitated vehicle routing problem (CVRP) based on the set partitioning formulation with additional cuts that correspond to capacity and clique inequalities. The exact algorithm used a bounding procedure that found a near optimal dual solution of the LP-relaxation of the resulting mathematical formulation by combining three dual ascent heuristics. The first dual heuristic was based on the



problem and pricing sub-problem. The LP relaxation of the master problem was solved using CPLEX 9.0 and a limited-search-wit-bound algorithm was developed to efficiently solve the pricing sub-problem. The proposed model was tested on 11 problem instances and compared with the cutting plane approach. The results obtain gaps which were consistently smaller than those generated by the cutting plane approach.

Mourão and Almeida (2000) solved a capacitated arc routing problem (CARP) with side constraints for a refuse collection VRP using two lower-bounding methods to incorporate the side constraints and a three-phase heuristics to generate a near optimal solution from the solution obtained with first lower-bounding method. Then, the feasible solution from the heuristic represents an upper bound to the problem. The model formulation they developed is a route-first, cluster-second method.

2.5 Application of algorithms used in solving vehicle routing problem (VRP)

Czech and Czarnas (2001) used parallel simulated annealing algorithm to solve vehicle routing problem with time window (VRPTW) without any backhauling. The waiting time, service time, earliest time and latest time were all taken into consideration while solving the problem. The objective had to minimize the number of vehicles used as well as to reduce total distance traveled. The paper also discusses the criterion to select the initial temperature and also termination criteria of when the search should stopped. Excellent conclusions on what should be done to get good optimization results were summarized.

Hirsch et al. (2007) used four different tabu search approaches and a few post optimization heuristics to solve the timber transport vehicle routing problem (TTVRP). Though every approach's objective was to minimize the empty truck movements. Even though all approaches

produce good results, it was found that the solution got stuck to local optima most of the time. Hence a post-optimization improvement heuristics, namely 2-opt, was done to produce a much better solution. It was observed that the heuristics produced a result closer to the lower bound produced by other CPLEX solvers.

Ioachim et al. (1995) proposed a mini-cluster first, route second approach using column generation to solve a multi-vehicles, door-to-door, handicapped transportation system with time windows. A mini-cluster is a set of geographically and temporally cohesive transportation requests that can feasibly be served by the same vehicle. Specifically, they designed vehicles to simultaneously accommodate three different types of handicapped persons: the ambulatories that use regular seats, those in folding wheel-chairs, and those in non-folding wheel-chairs. Their objective was to minimize total mini-cluster cost, that is, the sum between the total internal travel time and the estimated external travel time. They tested a large scale problem with 2545 requests generated by an operation day in Toronto, Canada.

Bell and McMullen (2004) applied a metaheuristics method of ant colony optimization (ACO) to establish a set of vehicle routing problems. They modified the ACO algorithm to solve the traveling salesman problem (TSP) in order to allow the search of multiple routes of the vehicle routing problem (VRP). Experimental results exhibited the success of the algorithm in finding within 1% of known optimal solution. The usage of multi ant colonies provided a comparatively competitive solution technique especially for large problem. Also the size of the candidates list used within the algorithm became a significant factor in finding improved solution. The computational times for the algorithm compared favourably with other solution methods.

Nazif and Lee (2010) proposed an “Optimal Crossover Genetic Algorithm for Vehicle Routing Problem with Time Windows”. In their work, they considered a set of vehicle with limits on capacity and travel times available to a service set of customers with demands and earliest and latest time for serving. The objective was to find routes for the vehicles to service all customers at minimal cost without violating capacity and travel time constraints and time window set by customers. Their proposed algorithm was tested with benchmark instances and also compared with other heuristics in the literature. The results proved the competitiveness of the proposed algorithm in terms of quality of the solution found.

Bräysy et al. (2004) presented a heuristic search method that hybridized ideas of evolutionary computation with some other search techniques, such as Tabu Search (TS) or Simulated Annealing (SA) which have also been used for solving VRPs. Most of the hybrid methods presented used local search mutation instead of the random mutation operators. In the first phase, an initial solution was created by either the cheapest insertion heuristic or the sectoring based genetic algorithm GIDEON. The second phase applied one of the following search procedures that use the λ -interchange mechanism: a local search descent procedure, a SA algorithm or a hybrid SA and TS, where TS is combined with the SA-based acceptance criterion to decide which moves to accept from the candidate list. The main feature of the local search procedures was that infeasible solutions with penalties were been allowed if considered attractive (Braysy et al., 2004).

Osman (1993) investigated three different algorithms namely descent, hybrid simulated annealing with tabu search and tabu search algorithms for their performance in solving vehicle routing problem (VRP) with capacity and distance constraints. The well known Clarke and Wright savings procedure was used to find an initial feasible solution. The λ - Interchange

generation mechanism was used to search the neighbourhood of the current solution. This helped in reducing the computational time by 50%. The simulated annealing algorithm produced results better than descent algorithm but the variance of the quality of the solution and that of the computational time was found to vary significantly. Tabu Search was the best of all algorithms used but it required a lot of storage space for selection of the alternative solution.

Thangiah et al. (1995) examine the vehicle routing problems with time deadlines (VRPTD), i.e., without earliest time widow. They developed two heuristics based on principles of time oriented sweep and cheapest insertion procedures for solving the VRPTD, followed by λ -interchanges of Osman (1993). They concluded the two proposed heuristics performed well for problems in which the customers were tightly clustered or long deadlines.

Xu and Kelly (1996) applied and more sophisticated neighbourhood structures to tabu search to solve the VRP. Computational experience showed that the tabu search was the best heuristic for the capacitated vehicle routing problem (CVRP) to date. Its success was due to some key implementation ideas. These included the allowance of infeasible solutions during the search procedure, the use of self-adjusting parameters, diversification and intensification and so on. These ideas combined to both the success of improving the quality of solution and the saving of running time.

Bullnheimer et al. (1998) provided two applications of the Ant System (AS) to the vehicle routing problem. In these applications, they developed two hybrid ant systems in which each vehicle route produced in a given iteration was improved by the 2-opt heuristic before the trail update. In their procedure, they used a number of 'elitist ants' to account for the best solutions. The result of their method was encouraging.

Kontoravidis and Bard (1995) proposed greedy randomized adaptive search procedure (GRASP) to VRP with time window. The objective was to address the problem of finding the minimum number of vehicles required to visit a set of nodes subject to time window constraints. They also considered a secondary object centered on minimizing the total distance travelled. Feasible solution was obtained from GRASP for standard 100 nodes data set as well as for a number of real-world problems with up to 417 customers. Experimental results revealed that their proposed procedure out performs techniques existing at the time and required only a small fraction of time taken by exact method. They gauged the quality of solutions by applying three different lower bounding heuristics. The first considers the “bin parking” aspect of the problem with respect to vehicle capacity, the second was based on the maximum clique associated with customers’ incompatibility, and the third exploits the time windows constraints.

Tillard et al. (1997) described a tabu search heuristics for vehicle routing problem with soft time windows. Their problem allows lateness at customer location although a penalty is incurred and added to the objective value. In the tabu search, a neighbourhood of current solution was created through an exchange procedure that swaps sequences of consecutive customers (segments) between routes. The tabu search also exploits and adaptive memory that contains routes of the best previously visited solutions. New starting points for the tabu search were produced through combination of routes taken from different solution found in the memory.

Brando and Mercer (1996) used the tabu search heuristic to solve the multi-trip vehicle routing and scheduling in a real distribution problem, taking into account not only the constraints that are common to the basic routing problem, but also the following; during each day a vehicle can make more than one trip, customers delivery time windows, multi capacity vehicles, access to

some customers was restricted to some vehicles, and drivers have maximum driving time with breaks.

Gronalt et al. (2003) applied the Clarke and Wright savings algorithm to solve pickup and delivery of truckloads under time window constraints. A logistics service provider studied, accepts orders from customers requiring shipments between two locations, and serves the others from a number of distribution centers. Thus, shipments occur between the pickup location of an order and the closest distribution center, between distribution centers and the delivery location of an order. The problem was solved to optimality.

Karadimas et al. (2007) presented an ant colony system (ACS) for determining waste collection routes for the Municipality of Athens (MoA). The collection involved 72 loading spots. Comparison results of the empirical method used by MoA were presented. The route length of the empirical model was 9850, whilst the ACS route was 7328. Thus, the improvement was approximately 25.6%.

2.6 Development of theory of algorithms for solving VRP

Clarke and Wright (1964) developed an algorithm for the vehicle routing problem based on the computation of the *savings* for combining two customers into the same route. The savings

that the savings formula becomes

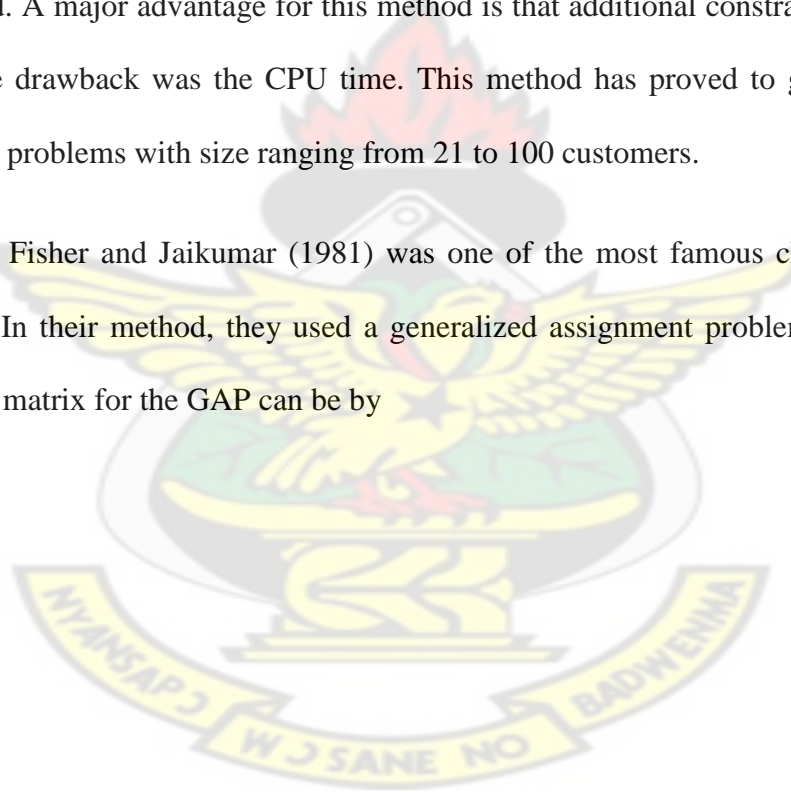
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the maximal route length is not exceeded. In other words, feasible clusters are formed by rotating a ray centered at the depot. Then improve each vehicle route by solving the TSP.

Foster and Ryan (1976) in an attempt to employ that many optimal solutions show a petal or an almost petal structure, proposed a similar approach to the sweep algorithm by assigning polar coordinate angles and record all customers accordingly. The authors listed all feasible routes with a petal structure and solved a linear program to optimality in which they selected a set of feasible petal routes (a spanning petal) such that each customer is visited and the total traveled time is minimized. A major advantage for this method is that additional constraints can be easily be added, but the drawback was the CPU time. This method has proved to give near optimal results for several problems with size ranging from 21 to 100 customers.

The algorithm of Fisher and Jaikumar (1981) was one of the most famous cluster-first, route-second methods. In their method, they used a generalized assignment problem (GAP) to form clusters. The cost matrix for the GAP can be by



the emerging route is the vertex yielding

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phase, a set of feasible solutions (termed population in Genetic Algorithm) are constructed by using one of the existing classical tour constructive heuristics. During selection phase, a portion of solutions are selected from the population based on a probability function. The solution whose total cost (termed fitness in Genetic Algorithm) is small is more likely to be selected. At the recombination phase, the solutions that are selected are merged into a single one. Solution feasibility is maintained throughout the course of merge. The last phase, mutation, is aimed at optimizing the solution that is generated at recombination.

Thangiah et al. (1991) developed a method called GIDEON that assigns customers to vehicles by partitioning the customers into sectors by genetic algorithm and customers within each formed sector are routed using the cheapest insertion method. In the next step the routes are improved using the



phase, two parent solutions are merged into single one, so as to guarantee the feasibility of the new solution. Two types of crossover operators are used to modify a randomly selected route or to insert a route into the other parent solution. A special repair operator is then applied to the offspring to generate a new feasible solution. Mutation operators are aimed at reducing the number of routes. Finally, in order to locally optimize the solution, mutation operator based on Or-opt exchanges is included.

Berger et al. (1998) propose a method based on the hybridization of a genetic algorithm with well-known construction heuristics. The authors omit the coding issues and represent a solution by a set of feasible routes. The initial population is created with nearest neighbor heuristic. The fitness values of the individuals are based on the number of routes and the total distance of the corresponding solution and for the selection purpose the authors use the so called roulette-wheel scheme. In this scheme, the probability of selecting an individual is proportional to its fitness. The proposed crossover operator combines iteratively various routes r_1 of parent solution P1 with as subset of customers, formed by r_2 nearest-neighbour routes from parent solution P2. A removal procedure is first carried out to remove some key customer nodes from r_1 . Then an insertion heuristic coupled to a random customer acceptance is locally applied to build a feasible route, considering the partial route r_1 as an initial solution. The mutation operators are aimed at reducing the number of routes of solutions having only a few customers and locally reordering routes.

Colomi et al. (1991) developed ant colony algorithm based on observed behavior of real ant colonies in search of food. Namely, ants communicate the information about food sources by using pheromones to mark the paths which lead to food. The main underlying idea is to associate pheromone concentration

link

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Kirkpatrick et al. (1983) introduced Simulated Annealing (SA) based on thermodynamic process of annealing in physics. It works by searching the set of all possible solution, thus reducing the chance of getting stuck in a poor local optimum by performing *uphill* moves to inferior solutions under the control of a randomized scheme. This is done by probabilistically allowing a solution that is worse than the current solution to replace the current solution. Specifically, if a move from one solution

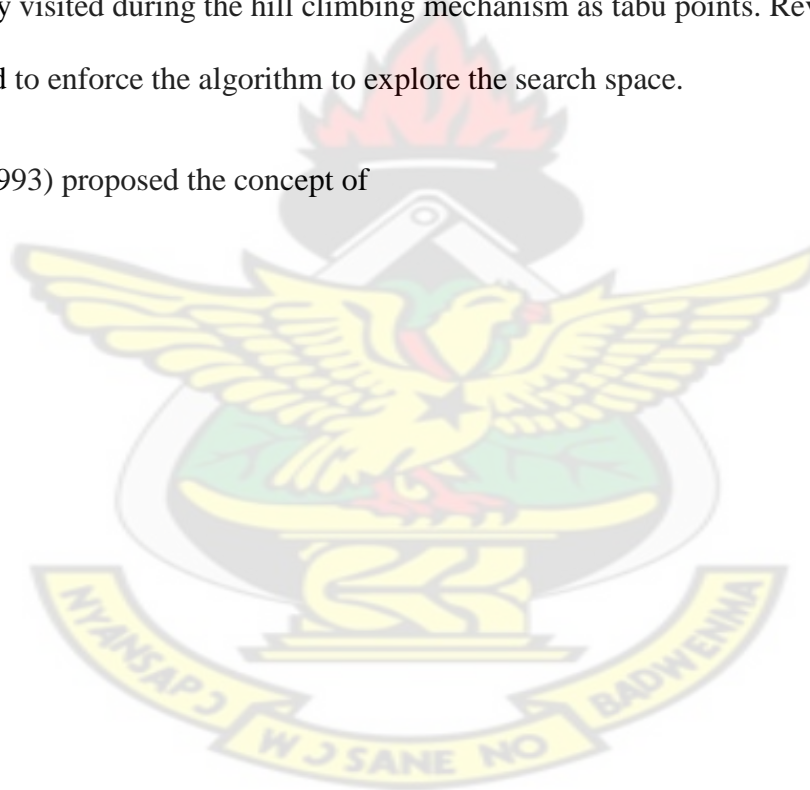
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strings of customers between every two routes. String relocation is used to insert a customer or a string of customers from one route into another route. A string mix is a mix of string cross, exchange or relocate customers or strings of customers, depending on which yields the greatest savings.

Glover (1986) proposed a deterministic search mechanism (with limited memory) called Tabu Search. The search is based on a hill climbing mechanism where memory is used to escape from the local optima. Hill climbing may get stuck at local optima. Tabu search keeps in memory the points previously visited during the hill climbing mechanism as tabu points. Revisiting the tabu points is avoided to enforce the algorithm to explore the search space.

Osman (1991, 1993) proposed the concept of



This may result in creating a new route or deleting one. To limit the neighbourhood size, only randomly selected subsets of vertices are considered for reinsertion in other routes. The concept of a penalized objective

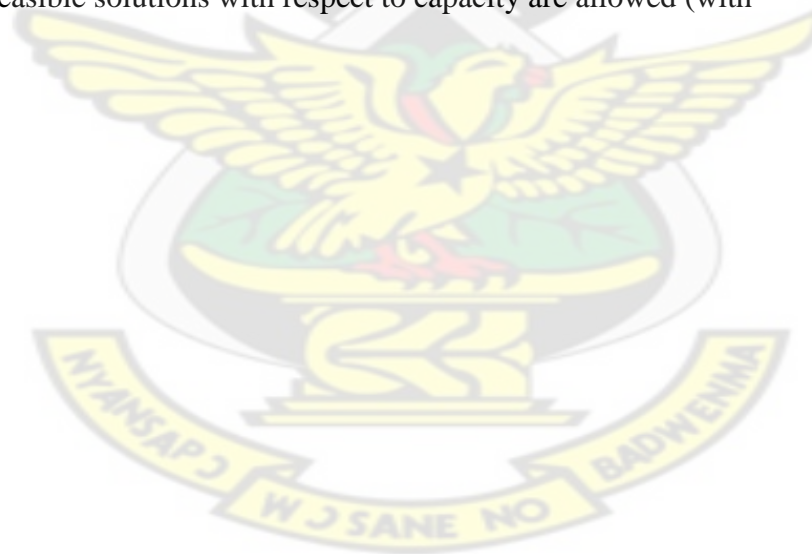
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subregion boundaries are updated periodically to provide a diversification effect. In non-planar problems, regions are defined through the computation of shortest spanning arborescence rooted at the depot. Taillard's algorithm remains to this day one of the most effective Tabu Search heuristics for the VRP. On the fourteen CMT instances, it produced twelve of the best known solutions.

Xu and Kelly (1996) in their Tabu Search algorithm defined neighbours by oscillating between ejection chains and vertex swaps between two routes. The ejection chains are determined by minimizing a flow on a network. Since several customers can be removed from the same route or reinserted into same route, all arc cost are approximations only. As in some previous algorithms, individual routes are periodically reoptimized by means of 3-opt and 2-opt operations.

Intermediate infeasible solutions with respect to capacity are allowed (with



CHAPTER 3

METHODOLOGY

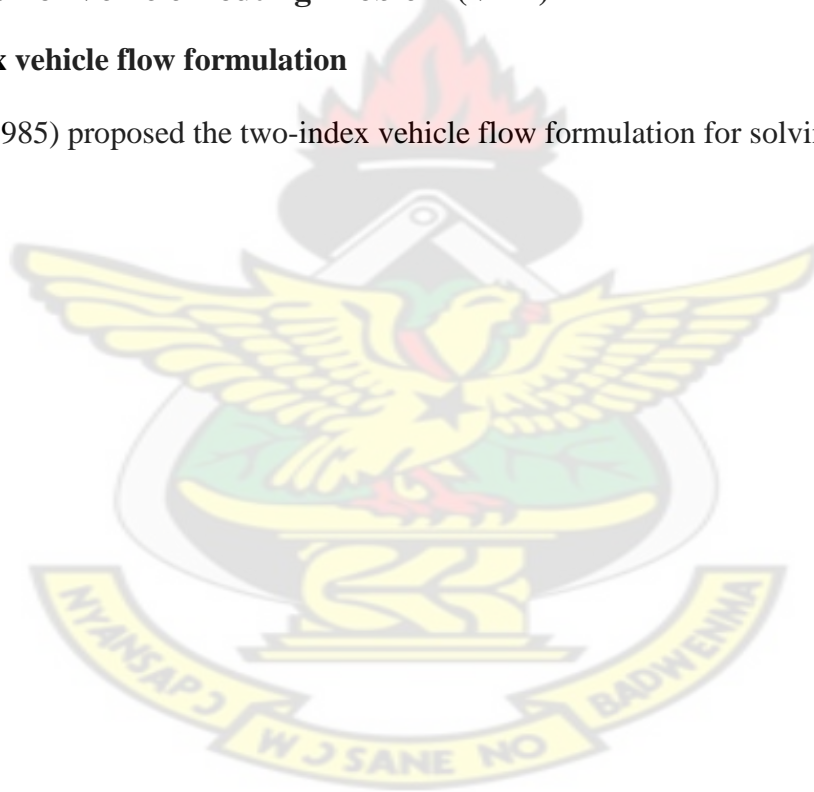
This chapter presents relevant fundamentals that will help us to come out with an appropriate Vehicle Routing Problem Model for Newspaper distribution of Graphic Communication Group Limited (GCGL) and the best way it will be solved.

3.1 Formulation of Vehicle Routing Problem (VRP)

3.1.1 Two-index vehicle flow formulation

Laporte et al., (1985) proposed the two-index vehicle flow formulation for solving VRPs.

Let



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3.1.3. Description for the Newspaper distribution Problem.

A mathematical description of VRP for newspaper distribution problem in this case may be defined as follows. Let $G = (V, A)$ be a network where

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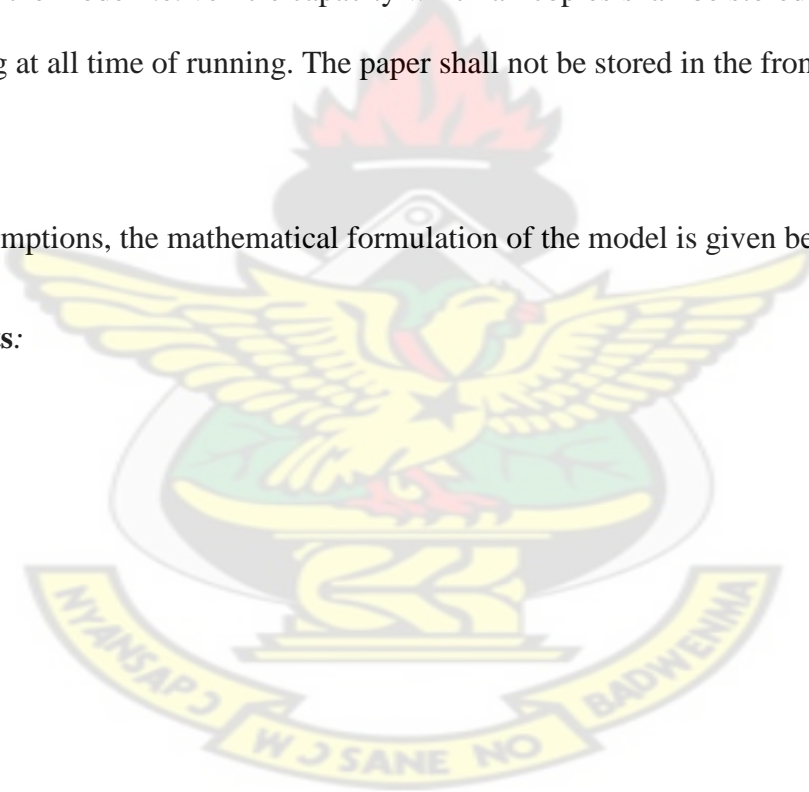
Constraints in this problem are

- Total of n vehicles are available.
- Hour of operations: there are time window of $T=180$ minutes for delivering newspapers to the last stops/customer and $T= 60$ minutes for returning to the depot.

However, there are other constraints that did not define in this research due to intangible factor cannot be part of the model i.e. vehicle capacity which all copies shall be stored behind the truck with door closing at all time of running. The paper shall not be stored in the front seat or the roof of the truck.

Under these assumptions, the mathematical formulation of the model is given below:

Definition of sets:



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3.2 Solution Method

3.2.1 Route construction heuristic for VRP

Route construction heuristics are used to build feasible routes for VRP. Some of them select one unrouted node after the other, based on some cost (or distance/time) minimization criterion, until all nodes are routed and a feasible solution has been created. These algorithms often consider restrictions of vehicle capacity and time window constraints which must not be violated. Sequential methods construct one route at a time and parallel methods allow building more routes at the same time.

3.2.1.1 The Clarke and Wright Savings Heuristic

One of the most widely used heuristics to get an initial solution to the VRP is the Clarke-Wright (CW) algorithm (1964). The algorithm is simple and produces good results. We consider a case where there is one depot, which is the starting point for vehicles. We begin the procedure by sending one vehicle to every customer, which travels back to the depot with a total distance of

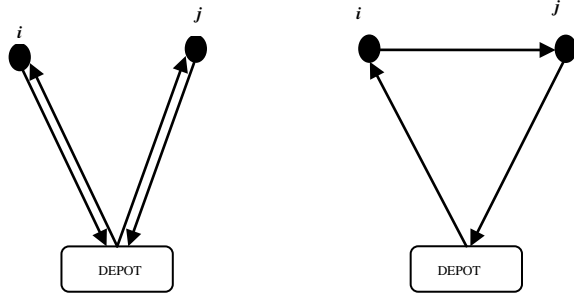
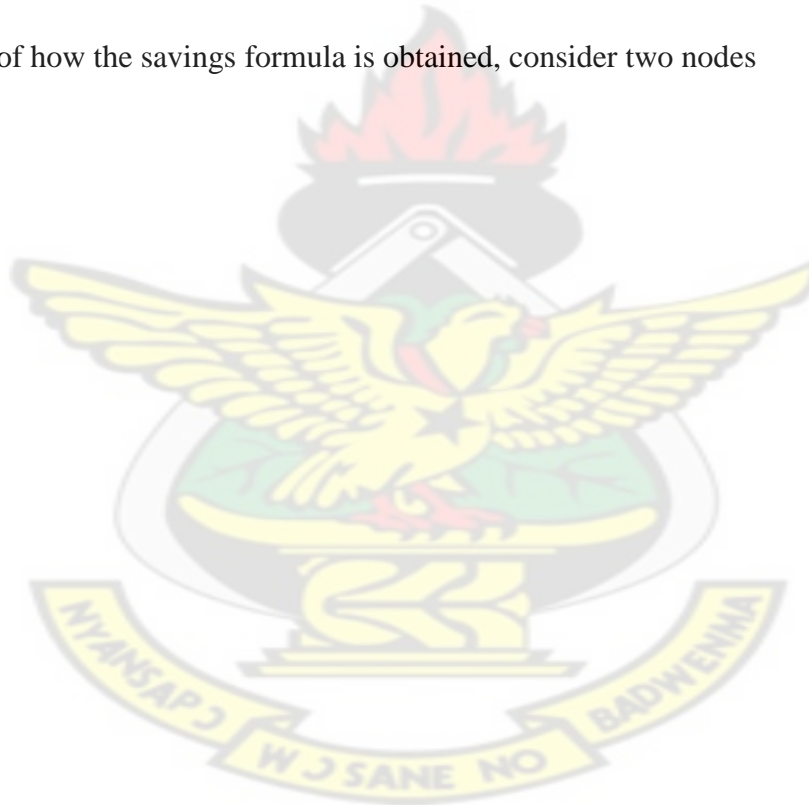


Figure 3.1 Illustration of the savings concept

In the left, customers i and j are served by separate routes and in the right the routes are combined by inserting customer j after i .

For an example of how the savings formula is obtained, consider two nodes



Algorithm:

STEP 1: Calculate the savings

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Illustration of the Clarke and Wright Savings Heuristics

Example 3.1 We consider a problem with 5 customers. The transportation cost between all pairs of points are shown in Table 3.1, where 0 represents the depot (the cost are symmetric).

Table 3.1 Cost Matrix

| | 0 | 1 | 2 | 3 | 4 | 5 |
|---|----|----|----|----|----|----|
| 0 | - | 28 | 31 | 20 | 25 | 34 |
| 1 | 28 | - | 21 | 29 | 26 | 20 |
| 2 | 31 | 21 | - | 38 | 20 | 32 |
| 3 | 20 | 29 | 38 | - | 30 | 27 |
| 4 | 25 | 26 | 20 | 30 | - | 25 |
| 5 | 34 | 20 | 32 | 27 | 30 | - |

The customers' demands that must be delivered from the depot are given in the following table.

The vehicle capacity

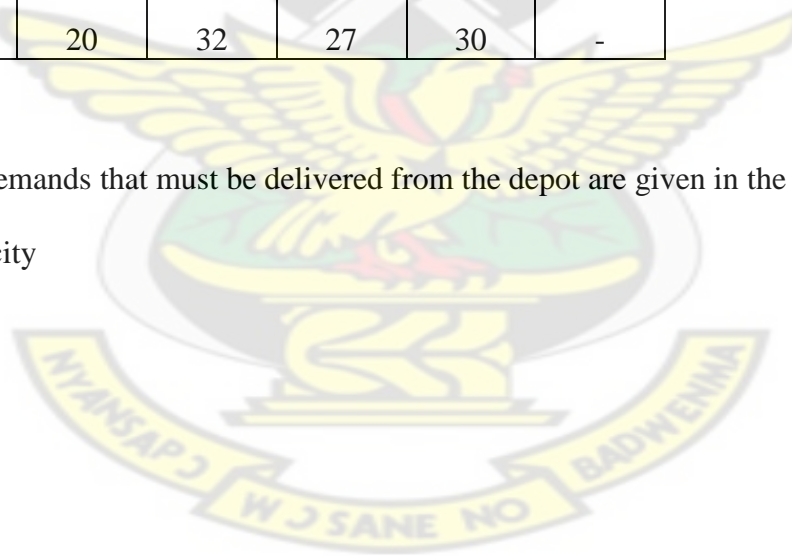
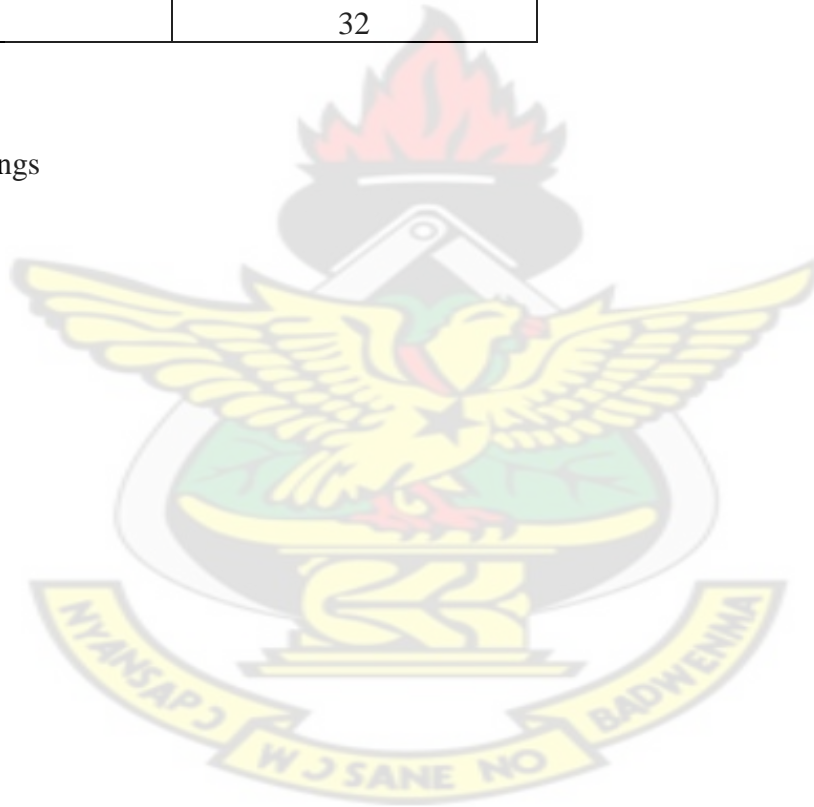


Table 3.2 Customer Demand

| Customer | Quantity |
|----------|----------|
| 1 | 37 |
| 2 | 35 |
| 3 | 30 |
| 4 | 25 |
| 5 | 32 |

Step 1: The savings



The table showing all the calculated savings in presented below.

Table 3.3 Savings Matrix

| | | | | | |
|---|----|----|----|----|----|
| | 1 | 2 | 3 | 4 | 5 |
| 1 | - | 38 | 19 | 27 | 42 |
| 2 | 38 | - | 13 | 36 | 33 |
| 3 | 19 | 13 | - | 15 | 27 |
| 4 | 27 | 36 | 15 | - | 34 |
| 5 | 42 | 33 | 27 | 34 | - |

Step 2: Now the point pairs are sorted in descending order of the savings. This gives the sorted list of point pairs:

Table 3.4 Savings list

| <i>Link</i> | <i>Savings</i> |
|-------------|----------------|
| 1-5 | 42 |
| 1-2 | 38 |
| 2-4 | 36 |
| 4-5 | 34 |
| 2-5 | 33 |
| 1-4 | 27 |
| 3-5 | 27 |
| 1-3 | 19 |
| 3-4 | 15 |
| 2-3 | 13 |

Step 3a: The sequential savings of Clarke and Wright algorithm

Edge [1, 5]: Join route 0-1-0 to 0-5-0: results **0-1-5-0**, Quantity = 69 < K.

Edge [1, 2]: Join route 0-1-5-0 and 0-2-0: results **0-2-1-5-0**, Quantity = 104 > K.

Edge [2, 4]: not connected to Edge [1, 5].

Edge [4, 5]: Join route 0-4-0 to 0-1-5-0 results **0-1-5-4-0**, Quantity = 94 < K.

Edge [2, 5]: Condition in Step 3b doesn't hold. Node 5 is interior node.

Edge [1, 4]: nodes belong to the same route.

Edge [3, 5]: Condition in Step 3b doesn't hold. Node 5 is interior node.

Edge [1, 3]: Join route 0-3-0 to 0-1-5-4-0 results **0-3-1-5-4-0**, Quantity = 124 > K.

Edge [3, 4]: Join route 0-3-0 to 0-1-5-4-0 results **0--1-5-4-3-0**, Quantity = 124 > K.

Edge [2, 3]: Join route 0-2-0 to 0-3-0: results **0-2-3-0**, Quantity = 65 < K.

| Route. | Quantity. | Cost. |
|---------------|------------------|--------------|
| 0-1-5-4-0 | 94 | 98 |
| 0-2-3-0 | 65 | 89 |

Total Cost = 187.

Step 3b: The parallel savings of Clarke and Wright algorithm

Edge [1, 5]: Join route 0-1-0 to 0-5-0: results **0-1-5-0**, Quantity = 69 < K.

Edge [1, 2]: Join route 0-1-5-0 and 0-2-0: results **0-2-1-5-0**, Quantity = 104 > K.

Edge [2, 4]: Join route 0-2-0 to 0-4-0: results **0-2-4-0**, Quantity = 60 < K.

Edge [4, 5]: Already included in a route.

Edge [2, 5]: Already included in a route.

Edge [1, 4]: Already included in a route.

Edge [3, 5]: Join route 0-3-0 to 0-1-5-0 results **0-1-5-3-0**, Quantity = 99 < K.

Edge [1, 3]: nodes belong to the same route.

Edge [3, 4]: Already included in a route.

Edge [2, 3]: Already included in a route.

| Route. | Quantity. | Cost. |
|---------------|------------------|--------------|
| 0-1-5-3-0 | 99 | 95 |
| 0-2-4-0 | 60 | 76 |

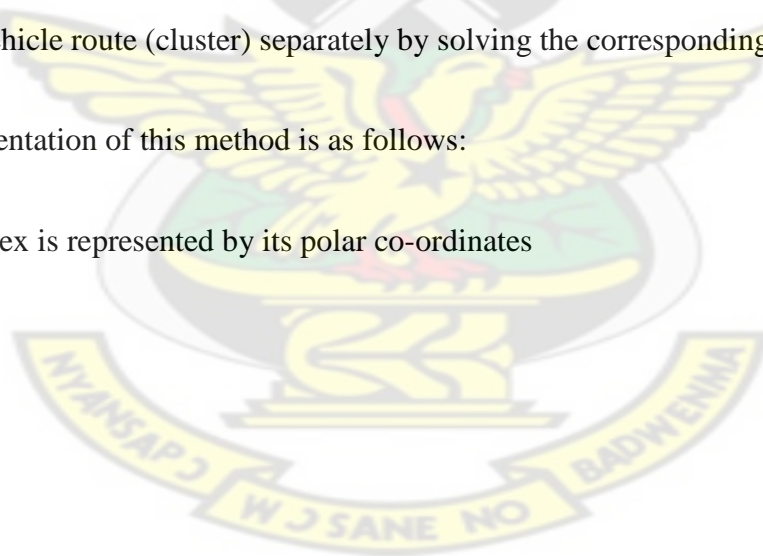
Total Cost =171.

3.2.1.2 The Sweep algorithm

One of the most known two-phase constructive methods for the VRP is the sweep algorithm. The algorithm belongs to the cluster-first, route-second methods. In the first phase the algorithm decomposes the VRP problem by clustering customers in m -TSP problems. The customer cluster is conducted by two criteria. The positions of all customers are transformed into polar coordinates with the depot in the origin of the coordinate system. The first criterion for grouping customers is the minimal angle. The second criterion matches the capacity of the vehicle which is assigned to the cluster, so that the total demands of all the selected customers has to be less than or equal to the capacity of the vehicle. The first and the second criteria are combined so that the assignment of customers to groups is performed by increasing the angular coordinate from 0 to the value where capacity of the assigned vehicle for that cluster is exhausted. The last step optimizes each vehicle route (cluster) separately by solving the corresponding TSP.

A simple implementation of this method is as follows:

Assume each vertex is represented by its polar co-ordinates



Step 3: Starting from the unrouted node having the smallest angle, sweep all nodes by increasing the polar angle as long as the vehicle capacity is not exceeded to form a cluster.

Step 4: Create new cluster by resuming the sweep where the last one left off.

Step 5: Optimize each route (cluster) by solving a corresponding TSP.

Illustration of the Sweep Algorithm

Example 3.2 Consider 10 customers where customers' coordinates and demands and its cost matrix are shown in Table 3.5 and Table 3.6 respectively. With vehicle capacity as

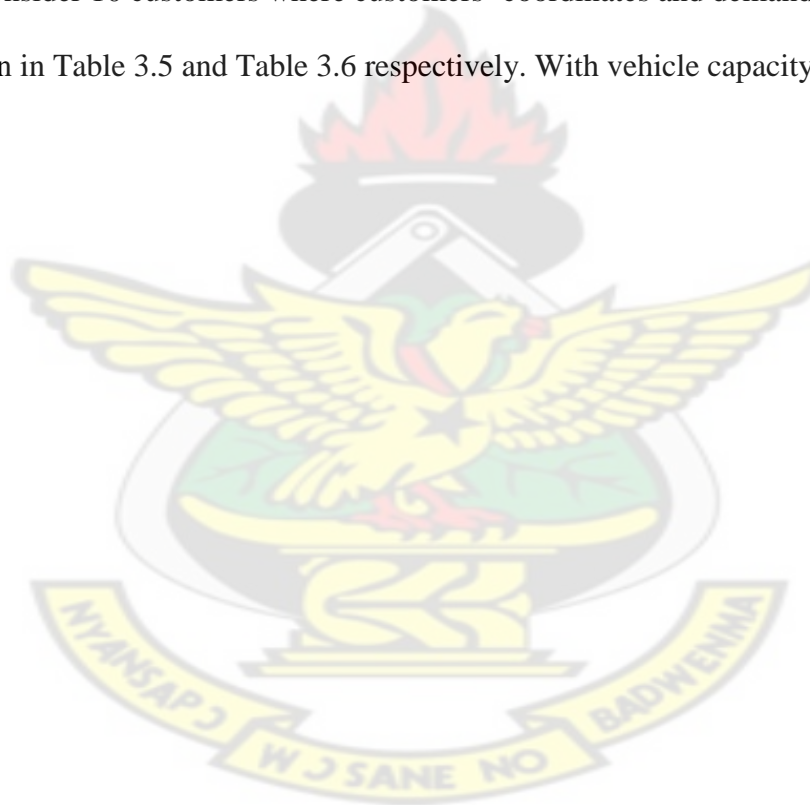


Table 3.6 Cost Matrix

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STEP 1: We compute the polar coordinates



STEP 2: Rank the nodes in ascending order of their polar angle (

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3.2.1.3 The Sequential Insertion algorithm

A route is first initialized with a “seed” customer and the remaining unrouted customers are added into the route until it is full with respect to capacity constraint. The seed customers are selected by finding either the geographically farthest unrouted customer in relation to the depot or unrouted customer with the biggest demand. For each unrouted customer the feasible insertion place in the emerging route with its minimal insertion cost as a weighted average of a additional distance is computed first. The customer for whom the cost difference between insertion in the new route and in the emerging route is the largest is selected. The selected customer is then inserted in the route and the new calculation and selection is repeated until capacity resources are exhausted. If unrouted customers remain, the initializations and insertion procedures are then repeated until all customers are serviced.

The method uses two subsequently defined criteria



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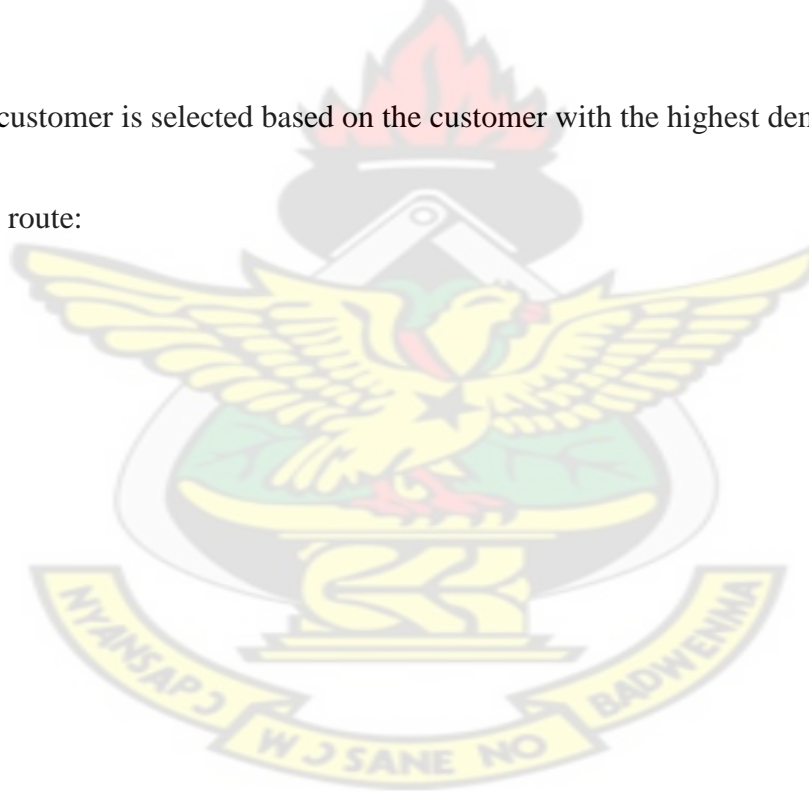


Table 3.10 Distance matrix

| Node | 0 | 1 | 2 | 3 | 4 | 5 |
|------|----|----|----|----|----|----|
| 0 | 0 | 42 | 32 | 29 | 53 | 23 |
| 1 | 42 | 0 | 73 | 23 | 95 | 48 |
| 2 | 32 | 73 | 0 | 60 | 23 | 33 |
| 3 | 29 | 23 | 60 | 0 | 81 | 46 |
| 4 | 53 | 95 | 23 | 81 | 0 | 55 |
| 5 | 23 | 48 | 33 | 46 | 55 | 0 |

STEP 1: 'Seed' customer is selected based on the customer with the highest demand

The partial route:



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3.2.2 Route Improvement Heuristic for VRP

3.2.2.1 Local Search Heuristic

Given a solution, generated by construction heuristics, we can apply some modifications on the solution to improve its quality. Two types of improvement algorithms based on local search heuristics can be applied to VRP solutions. *Intra-route heuristics* optimize each route separately by means of a TSP improvement heuristic. *Inter-route heuristics* consist of moving vertices to different routes.

a. 2-Opt Heuristics

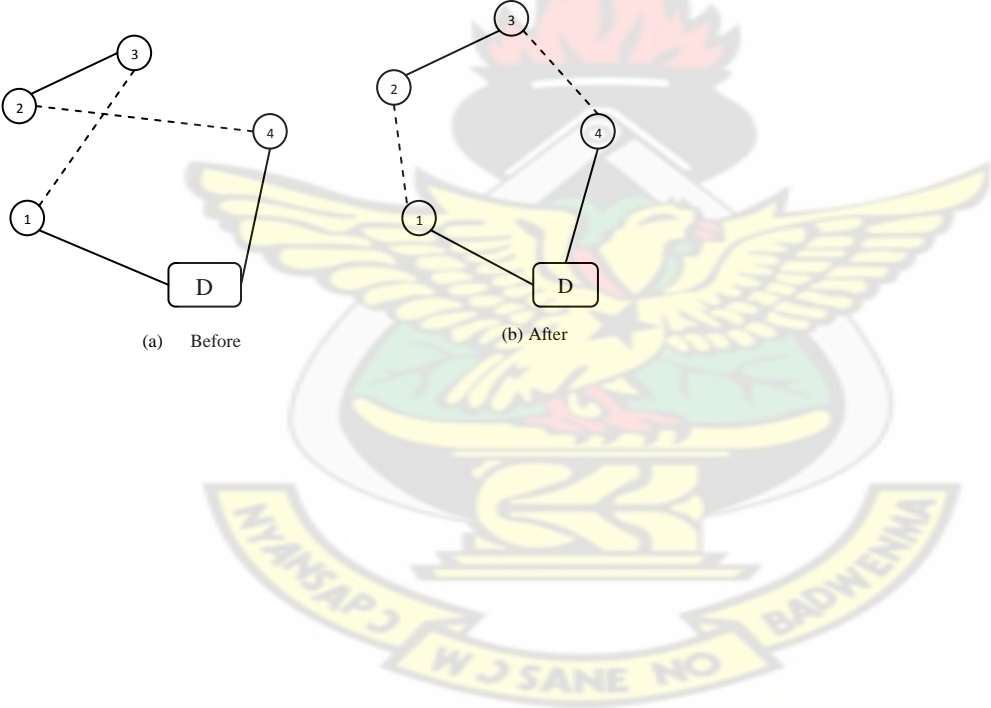
The 2-Opt operation is a way of improving an existing solution. It involves swapping a pair of edges between any four nodes to reduce the distance in the VRP and the objective function. The algorithm involves looping through all pairs of edges. The first switch that reduces the objective function is accepted and the loop is ended. The swaps could be between routes (inter routes) or within a route (intra route). If the swap is within a route then the total drop off amount in the route remains the same, however, the order in which the vehicles visit every customer is changed. If the swap is between routes then the total drop off amount in each route can change. The swaps with maximum gain are exchanged.

Algorithm

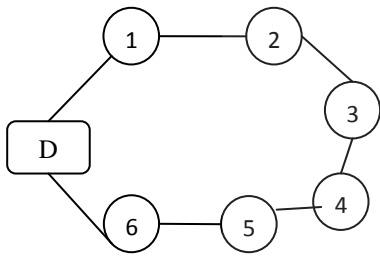
STEP 1: Let

STEP 2: For every node

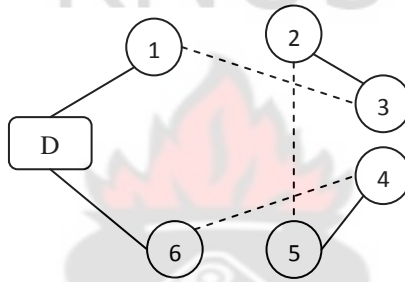
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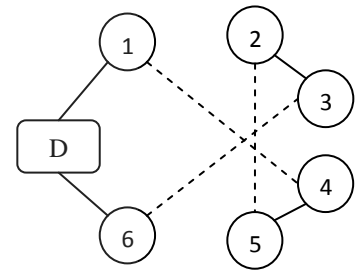
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(a)



(b)

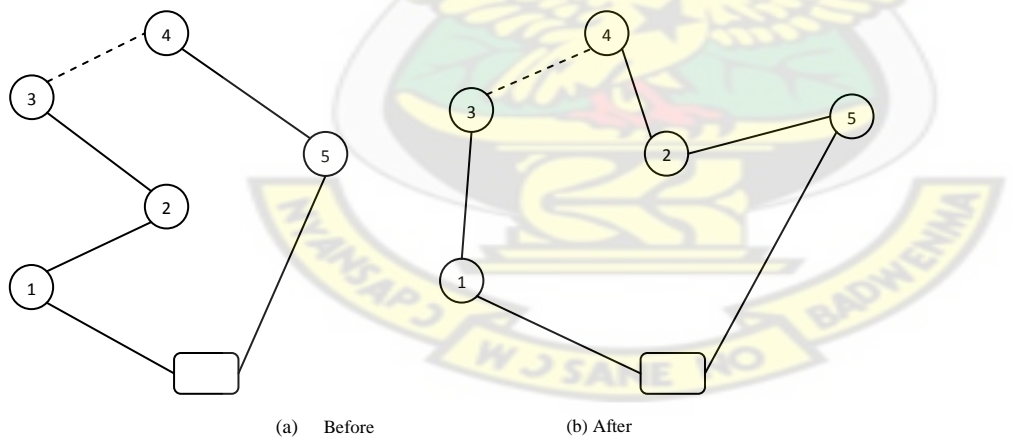


(c)



Or-opt algorithm can be described as follows:

STEP 1: Consider an initial route and set



3.2.2.2. Iterative Search Heuristics

a. Tabu Search

Tabu search has traditionally been used on combinatorial optimization problems and frequently has been applied to many integer programming problems, such as routing and scheduling, traveling salesman and others. The basic concept of Tabu search was presented by Glover (1990) who described it as “meta-heuristic superimposed on another heuristic”. The overall approach is to avoid cycles by forbidding or penalizing moves which take the solution, in the next iteration, to points in the solution space previously visited (hence “tabu”).

According to Glover (1990), Tabu search is composed of three primary features: (1) the use of flexible attribute-based memory structures designed to permit evaluation criteria; (2) a control mechanism for employing the memory structures based on the interplay between conditions that restrict and free the search process; and (3) the incorporation of different time horizons, from short term to long term to implement strategies for intensifying and diversifying the search.

Tabu search begins by moving to a local minimum. To avoid revisiting the steps used the method records recent moves in one or more tabu lists. The original aim was not to prevent a previous move from being repeated, but rather to ensure it was not revisited. Tabu lists are historical in nature and form the tabu search memory. The role of the memory can change as the algorithm proceeds. For initialization, in each iteration, the objective is to make a rough examination of the solution space, known as “diversification”, but as locations of the candidate solutions are identified, the search is more focused to produce local optimal solutions in a process of “intensification”. In other words, *diversification strategies* drive the search into new regions,

while *intensification strategies* reinforce move combination and solution features historically found good.

In many cases, various implementation models of the Tabu search method can be achieved by changing the size, variability of the tabu memory to a particular domain.

The main limitation of a local search method (i.e., the hill climbing procedure) is that it might stop at local optima that might be far from the global optimum. As one of the heuristic approaches to overcome this shortcoming, Tabu search (TS) algorithm imitate an intelligent attitude by using an adaptive memory and can therefore avoid being entrapped at the local optima with the aid of a memory function.

In each iteration, the tabu search explores the solution space by moving from a solution to the solution with the best objective function value in its neighbourhood, even in the case that this might cause the deterioration of the objective. In order to avoid cycling, solutions that were recently examined are declared forbidden or “tabu” for a certain number of iterations (i.e., called *tabu tenure* or *tabu duration*) and associated attributes with the tabu solutions are also stored. The tabu status of a solution might be overridden if it corresponds to a new best solution, which condition is called “Aspiration criterion”. There are groups of Tabu search methods that use either short term memory or intermediate and long term memory strategies. The recency-based memory functions require specifying the tabu tenure

Illustration of the Tabu Search Algorithm

Example 3.4 Consider 5 customers where customers' coordinates and demands and its distance matrix are shown in Table 3.11 and 3.12 respectively. Set route to be as follows

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Iteration 3

| Delete Link | Add Link | Route | VRP cost |
|-------------|----------|----------------------|------------|
| 24 & 31 | 23 & 41 | 0 2 3 4 1 5 0 | 356 |
| 24 & 15 | 21 & 45 | 0 2 1 3 4 5 0 | 308 |
| 24 & 50 | 25 & 40 | 0 2 5 1 3 4 0 | 271 |
| 43* & 15 | 41 & 35 | 0 2 4 1 3 5 0 | 248 |
| 43 & 50 | 45 & 30 | 0 2 4 5 1 3 0 | 258 |
| 31 & 50 | 35 & 10 | 0 2 4 3 5 1 0 | 289 |

Aspiration = 356

Current solution: 0 2 3 4 1 5 0

Tabu list: 10 (1) 50 (1)

41 (1) 15 (2)

43 (2) 24 (3)

35 (2) 23 (3)

31 (3) 41 (3)

Iteration 4

| Delete Link | Add Link | Route | VRP cost | |
|-------------|-----------|----------------------|------------|----------|
| 23* & 41* | 24* & 31* | 0 2 4 3 1 5 0 | 247 | 12 |
| 23* & 15* | 21 & 35* | 0 2 1 4 3 5 0 | 356 | 7 |
| 23* & 50* | 25 & 30 | 0 2 5 1 4 3 0 | 318 | 4 |
| 34* & 15* | 31* & 45* | 0 2 3 1 4 5 0 | 309 | 6 |
| 34* & 50* | 35* & 40 | 0 2 3 5 1 4 0 | 334 | 3 |
| 41* & 50* | 45* & 10* | 0 2 3 4 5 1 0 | 335 | 6 |

Aspiration = 248

Current solution: 0 2 3 5 1 4 0

Tabu list: 40 (3)

41 (1) 35 (1) 31 (2)

24 (2) 15 (1) 41 (2)

23 (3) 34 (3) 43 (1)

The process is repeated until iteration = 10

Iteration 5: 0 2 3 1 5 4 0 [292]

Iteration 6: 0 2 4 1 5 3 0 [279]

Iteration 7: 0 2 4 5 1 3 0 [258]

Iteration 8: 0 2 3 5 1 4 0 [334]

Iteration 9: 0 2 3 1 5 4 0 [271]

Iteration 10

| Delete Link | Add Link | Route | VRP cost | |
|-------------|-----------|----------------------|------------|----------|
| 23* & 15 | 24 & 35* | 0 2 1 3 5 4 0 | 292 | 5 |
| 23* & 54 | 25 & 34 | 0 2 5 1 3 4 0 | 271 | 2 |
| 23* & 40* | 24* & 30 | 0 2 4 1 5 3 0 | 279 | 6 |
| 31* & 54* | 35* & 14* | 0 2 3 5 1 4 0 | 334 | 9 |
| 31* & 40* | 34 & 10 | 0 2 3 4 5 1 0 | 335 | 5 |
| 15* & 40* | 14* & 50 | 0 2 3 1 4 5 0 | 309 | 5 |

Aspiration = 248

Current solution: 0 2 5 1 3 4 0

The Tabu Search starts from aspiration cost of 269 yields by initial solution. After that all neighbourhood that can be reached from initial solution is examined in iteration 1. Among the neighbourhood, route 0 2 4 1 3 5 0 gives the least cost (248) and it is less than the aspiration cost, thus we accept it as current solution and override the aspiration cost. From the current solution, we continue to iteration 2 and examine again the neighbourhood and find that route 0 2 4 3 1 5 0 produces least cost (247). Although this move is tabu, we accept this as new current solution and override the aspiration cost since this cost is less than the aspiration cost, the aspiration cost becomes 247 and freeing the tabu status of this move.

At iteration 3, the least cost (248) is gained by route 0 2 4 1 3 5 0. Since this cost is not lower than aspiration cost and this move is tabu, thus we go to the 2nd best. This process is repeated until non tabu move is found. Finally the process stop at route 0 2 3 4 1 5 0 produced from drop 24 and 31 and add 23 and 41 which are non tabu moves, so we accept it as current solution but we do not need to update the aspiration cost. When it comes to iteration 4, the entire move is tabu. So one of the neighbourhood had to be free with aspiration by default (freeing the least tabu tenure) to be the current solution for next iteration. The least tabu tenure is 2, provided by 0 2 5

1 3 4 0. These search process is repeated until stopping criteria (maximum number of iterations is equal to 10) is met. Note that at iteration 8, the current solution is cycling back to previous current solution at iteration 4, and the next iteration (iteration 9) still produces the same route as obtained in iteration 5.

b. Genetic Algorithm

The genetic algorithm (GA) is another type of well-known modern heuristics. It can also be viewed as a form of neighborhood search, although its original inspiration comes from population genetics. Unlike SA and TS, GA makes of a population of solutions, from which, using selective breeding and recombination strategies, better and better solutions can be produced. Simple genetic 'operators' such as crossover and mutations are used to construct new solutions from pieces of old ones, in such a way that for many problems, the population steadily improves.

Solutions to genetic algorithm are represented by data structures, often a fixed length vector, called chromosomes. Chromosomes are composed of genes which have value called alleles, and the position of the gene in the chromosomes is called locus (Goldberg, 1989). Chromosomes can be represented by vectors, strings, arrays and tables. Alleles can be represented by a simple binary system of zeros and ones, integers, real number, and letter of alphabet, or other symbols (Gen and Cheng 1997). Each chromosome has a fitness value which is indicative of the suitability of the chromosome as a solution to the problem (Goldberg, 1989).

A simple genetic algorithm can be summarized as follows:

1. Representation: Encode the characteristics of each individual in the initial population as a chromosome (typically, a chromosome is a bit string). Set the current population to this initial population.
2. Reproduction/ Selection: Select two parent chromosomes from the current population and a chromosome with high fitness is more likely to be selected.
3. Recombination: Generate two offspring from the two parents by exchanging sub strings between parent chromosomes (crossover).
4. Mutation: this is a random change of a bit position in offspring chromosome.
5. Repeat steps (2), (3), (4), until the number of chromosomes in the new population is the same as in the old population.
6. Set the current population to the new population of chromosomes.

This procedure is repeated for a fixed number of generations, or until convergence to a population of similar individuals is obtained. Then, the best chromosome generated during the search is decoded into the corresponding individual (Potvin and Bengio, 1996).

Selection Methods

In selection process, chromosomes are selected from the population to be parents for crossover. According to Darwin's theory of evolution the best ones survive to create new offspring. There are many methods in selecting the best chromosomes. Some of them will be described in following (Obitko, 1998).

- a. Roulette Wheel Selection

Parents are selected according to their fitness. The better the chromosomes are, the more chances to be selected they have. Imagine a roulette wheel where all the chromosomes in the

population are placed. The size of the section in the roulette wheel is proportional to the value of the fitness function of every chromosome: the bigger the value, the larger the selection. Example is shown in Figure 3.5 below.

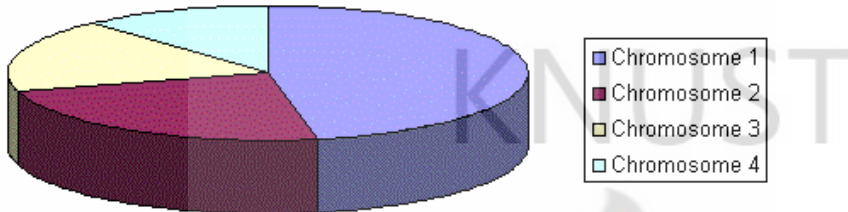


Figure 3.5 Illustration of Roulette Wheel Selection

A marble is thrown in the roulette wheel and the chromosome where it stops is selected. Clearly, the chromosomes with bigger fitness value will be selected more times. This process can be described by the following algorithm.

1. [Sum] Calculate the sum of all chromosomes fitnesses in population –sum

selection ranks the population first and then every chromosome receives fitness value determined by this ranking. The worst will have the fitness 1, the second worst 2 etc. and the best will have fitness N (number of chromosomes in population). Figure 3.6 and Figure 3.7 describes how the situation changes after fitness to the numbers determined by ranking.

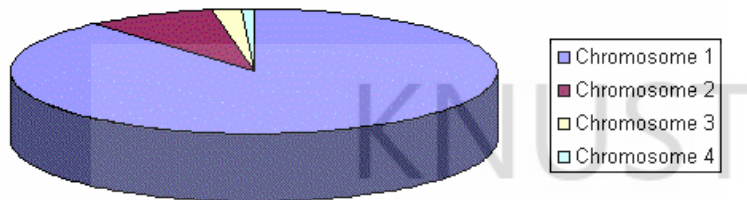


Figure 3.6 Situation before ranking (graph of fitnesses)

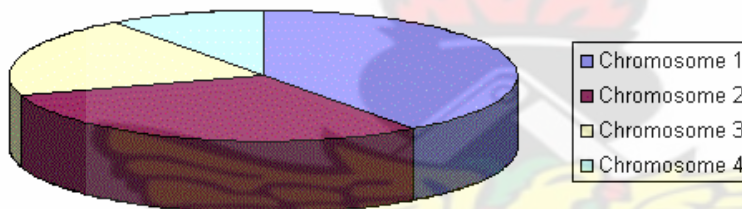


Figure 3.7 Situation after ranking (graph of order of numbers)

Now all the chromosomes have a chance to be selected. However this method can lead to slower convergence, because the best chromosomes do not differ so much from other ones.

c. Steady-State Selection.

This is not a particular method of selecting parents. The main idea of this type of selecting to the new population is that a big part of chromosomes can survive to next generation. The steady-state selection GA works in the following way. In every generation a few good (with higher fitness) chromosomes are selected for creating new offspring. Then some bad (with lower fitness) chromosomes are removed and the new offspring is placed in their place. The rest of population survives to new generation.

d. Elitism

When creating a new population by crossover and mutation, we have a big chance, that we will lose the best chromosome. Elitism is the name of the method that first copies the best chromosome (or few best chromosomes) to the new population. The rest of the population is constructed in ways described above. Elitism can rapidly increase the performance of GA, because it prevents a loss of the best found solution (Obitko, 1998).

Recombination

The simplest type of recombination is one point crossover. As illustrated in Loft and Snow (2006) where chromosomes are strings of symbols (here, 0's and 1's), there are two chromosomes; one from Parent A and one from Parent B, both of the same length. A crossover point is selected randomly. The two chromosomes are cut at this point, and a new chromosome is formed by using the chromosome from Parent A before the crossover point and from Parent B after the crossover point. This is depicted in Figure 3.8.

| | | | | | | | | |
|-----------|---|---|---|---|---|---|---|---|
| Parent A: | 0 | 1 | 1 | 0 | 1 | 0 | 1 | 1 |
| Parent B: | 1 | 0 | 1 | 1 | 1 | 0 | 0 | 0 |
| Offspring | 0 | 1 | 1 | 1 | 1 | 0 | 0 | 0 |

Figure 3.8 One point crossover

A first generalization of one point crossover is two point crossover. In two point crossover, two crossover points are randomly chosen. Genetic material is copied from A before the first crossover point. Between the crossover points, material is taken from Parent B. After the second crossover point, material is again taken from A. this is depicted in Figure 3.9

| | | | | | | | | | | |
|-----------|---|---|--|---|---|---|--|---|---|---|
| Parent A: | 0 | 1 | | 1 | 0 | 1 | | 0 | 1 | 1 |
| Parent B: | 1 | 0 | | 1 | 1 | 1 | | 0 | 0 | 0 |
| Offspring | 0 | 1 | | 1 | 1 | 1 | | 0 | 1 | 1 |

Figure 3.9 Two point crossover

From two point crossover, one can imagine three point crossover, four point, five point, and so on. The logical conclusion of this is uniform crossover. In uniform crossover, each symbol in the offspring's chromosome is chosen randomly. To be equal to the corresponding symbol of the chromosome of either Parent A or Parent B as shown in Figure 3.10 (The shaded symbols are the ones chosen for the offspring).

| | | | | | | | | |
|-----------|---|---|---|---|---|---|---|---|
| Parent A: | 0 | 1 | 1 | 0 | 1 | 0 | 1 | 1 |
| Parent B: | 1 | 0 | 1 | 1 | 1 | 0 | 0 | 0 |
| Offspring | 0 | 0 | 1 | 0 | 1 | 0 | 0 | 1 |

Figure 3.10 Uniform crossover

Loft and Snow (2006) observed that if a pattern appears in the chromosomes of both parents, then recombination will preserve that pattern. For example, both parents above have 1's as the third and fifth symbol in their chromosomes and 0 as the sixth then in one point, two point, and uniform crossover the offspring has the same pattern. Other crossovers are Partially Match Crossover (PMX), Cycle Crossover (CX), Order Crossover (OX), Matrix Crossover (MX) and Modified Order Crossover (MOX) (Bryant, 2000).

Mutation

Mutation is a genetic operator that alters one or more gene values in a chromosome from its initial state. This can result in entirely new gene values being added to the gene pool. With these new gene values, the Genetic Algorithm may be able to arrive at a better solution than what was previously possible. Mutation is an important part of the genetic search as it helps to prevent the population from stagnating at any local optima. Mutation occurs during evolution according to a user-definable mutation probability rate. This probability rate should usually be set low. If it is set too high, the search will turn into a primitive random search (Mitchell, 1996).

The primary purpose of mutation is to increase variation into a population. Mutation is the most important in populations where the initial population may be a small subset of all possible solutions. It is possible, for example, that every instance of an essential bit might be zero in the initial population. In such a case, crossover could never set that bit to one; mutation, however, can set that bit to one. Mutation takes place after crossover has been performed. Mutation changes randomly the new offspring. For binary encoding, it is done by switching a few randomly chosen bits from 1 to 0 or from 0 to 1 (Mitchell, 1996).

A variety of mutation methods are used in GAs, e.g. inversion, insertion, displacement and reciprocal exchange mutation. Inversion mutation selects two positions within a chromosome at random and then inverts the substring between these two positions. Insertion is where an individual is randomly selected and inserted in a random position. Displacement is where a sub-tour is selected at random and inserted in a random position. Whereas reciprocal exchange where we select two positions at random and swap them (Gen and Cheng, 1997).

Illustration of Genetic Algorithm

Example 3.5 In this example, the GA parameters were set at the following values.

- Population size $N= 10$
- Probability of crossover = 0.75
- Probability of mutation = 0.05
- Number of generation= 10

The initial population generated by Nearest Neighbour (NN) and Farthest Insertion (FI) heuristics and randomly generated candidate solutions were listed in Table 3.13. Chromosome that was identical to the other chromosomes was replaced by generating candidate solution randomly until there are no clones.

Table 3.13 Population in the initial Generation (Generation = 0)

| Chromosome No | Generated by | Route | Route without clones |
|---------------|--------------|---------------|----------------------|
| 1 | FI | 0 2 4 5 3 1 0 | 0 2 4 5 3 1 0 |
| 2 | NN | 0 2 4 5 3 1 0 | 0 2 1 5 3 4 0 |
| 3 | Random | 0 2 1 3 5 4 0 | 0 2 3 1 5 4 0 |
| 4 | Random | 0 2 3 1 5 4 0 | 0 2 3 1 5 4 0 |
| 5 | Random | 0 2 4 1 3 5 0 | 0 2 4 1 3 5 0 |
| 6 | Random | 0 2 4 5 3 1 0 | 0 2 1 4 5 3 0 |
| 7 | Random | 0 2 3 5 1 4 0 | 0 2 3 5 1 4 0 |
| 8 | Random | 0 2 3 5 4 1 0 | 0 2 3 5 4 1 0 |
| 9 | Random | 0 2 5 1 4 3 0 | 0 2 5 1 4 3 0 |
| 10 | Random | 0 2 5 1 3 4 0 | 0 2 5 1 3 4 0 |

After the population was built, the evaluation and selection took place. The VRP objective function is an evaluation measure for discriminating good and bad solutions. The smaller the VRP objective function, the better the solution. In roulette wheel selection, each individual in the population is assigned a roulette wheel slot sized in proportion to its fitness. That is, good

solutions have a larger slot size than the less fit solution. Set VRP objective function to be

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mutate by generating random number between 0 and 1. If the random number is less than the mutation rate, the chromosome is picked for mutation. The offspring then replaced the worst parents until the population size is equal to 10. If none of the offspring is better than the parents, then the population just remains the same. These procedures were repeated until the number of generation is equal to 10.

Table 3.15 Crossover and Swap Mutation Process in the 1st generation

| Mating Pool | Random No. of crossover | Parent and Offspring | Random No. for mutation | After crossover & mutation | Cost | Cost Rank | New Population |
|---------------|-------------------------|----------------------|-------------------------|----------------------------|------|-----------|----------------|
| 0 2 1 4 5 3 0 | 0.853103 | 0 2 <u>1</u> 4 5 3 0 | 0.04655 | 0 2 4 1 5 3 0 | 279 | 3 | 0 2 4 1 5 3 0 |
| 0 2 4 1 3 5 0 | 0.125565 | 0 2 4 1 3 5 0 | 0.60446 | 0 2 4 1 3 5 0 | 248 | 1 | 0 2 4 1 3 5 0 |
| 0 2 1 4 5 3 0 | 0.391876 | 0 2 1 4 5 3 0 | 0.92949 | 0 2 1 4 5 3 0 | 336 | | 0 2 3 5 4 1 0 |
| 0 2 3 5 4 1 0 | 0.600514 | 0 2 3 5 1 4 0 | 0.31041 | 0 2 3 5 4 1 0 | 330 | 9 | 0 2 1 3 5 4 0 |
| 0 2 3 5 1 4 0 | 0.340618 | 0 2 1 3 5 4 0 | 0.88872 | 0 2 3 5 1 4 0 | 334 | | 0 2 3 5 4 1 0 |
| 0 2 1 3 5 4 0 | 0.524349 | 0 2 3 5 4 1 0 | 0.71238 | 0 2 1 3 5 4 0 | 303 | 7 | 0 2 1 3 5 4 0 |
| 0 2 3 5 4 1 0 | 0.979810 | 0 2 1 4 5 3 0 | 0.93372 | 0 2 3 5 4 1 0 | 330 | 10 | 0 2 3 1 5 4 0 |
| 0 2 1 5 4 3 0 | 0.948004 | 0 2 1 3 5 4 0 | 0.20853 | 0 2 1 4 5 3 0 | 336 | | 0 2 5 1 3 4 0 |
| 0 2 1 3 5 4 0 | 0.878514 | 0 2 1 3 5 4 0 | 0.24742 | 0 2 1 3 5 4 0 | 303 | 6 | 0 2 1 5 4 3 0 |
| 0 2 3 1 5 4 0 | 0.230942 | 0 2 3 1 5 4 0 | 0.19517 | 0 2 3 1 5 4 0 | 292 | 4 | 0 2 1 3 5 4 0 |
| | | 0 2 5 1 3 4 0 | 0.60567 | 0 2 5 1 3 4 0 | 271 | 2 | |
| | | 0 2 1 5 4 3 0 | 0.16681 | 0 2 1 5 4 3 0 | 319 | 8 | |
| | | 0 2 1 3 5 4 0 | 0.29362 | 0 2 1 3 5 4 0 | 303 | 5 | |
| AVERAGE | | | | | | | 297.8 |
| BEST COST | | | | | | | 248 |

From generation to generation, we may get better and better chromosomes in the population as shown in Table 3.14. The parent chromosomes have already been selected according to their fitness, so the next population (which included the parents which did not undergo crossover) is among the fittest in the population and the population was gradually, on average, increase its fitness (which mean the VRP objective function is decreasing). The problem is the condition when the two parents have the same allele at a given gene the OX crossover will not change that. In other words, the offspring remains the same like their parents and the average of the fitness

does not change. Mutation is designed to overcome this problem in order to add diversity to the population and ensure that it is possible to explore the entire search space. But in fact, using swap mutation with the probability of 0.05 did not make any substantial change that could create a new better chromosome. So, better mutation operator is needed to explore more the search region.

c. Ant Colony Optimization (ACO)

Ant Colony Optimization (Dorigo et al., 1992) has been inspired by the foraging behaviour of real ants. Ants randomly explore the surrounding of the anthill; when they find food, they return to the nest depositing a *pheromone trail*, a trace of a chemical substance that can be smelled by other ants. Ants can follow various paths to the food source and back, but it has been observed by (Deneubourg et al., 1990) that, thanks to the reinforcement of the pheromone trail by successive passages, only the shortest path remains in use, since ants prefer to follow stronger pheromone concentrations.

Pheromone reinforcement is autocatalytic, since the shortest path, the least time will be taken to travel back and forth, and therefore, while ants on longer paths are still in transit, the ants on the shortest path can restart the route again, reinforcing the pheromone trail on the shortest path. Over time, the majority of the ants will travel on that path, while a minority will still choose alternative paths. The behavior of this minority is important, since it allows exploring the environment to find even better solutions, which initially were not considered. The choice of the path is therefore probabilistic and, while it is strongly influenced by the pheromone intensity, it still allows for random deviations from the current best solution.

The ACO algorithm replicates this behaviour, adding some features to make it more efficient in the computer implementation. Ants are implemented as a set of concurrent and asynchronous agents. They construct a solution visiting a series of nodes on a graph. They select the move along an edge to the next node to visit according to two parameters: trails and attractiveness. As real ants, also artificial ants will prefer in most cases a deterministic choice of the path, based on the selection of the path with the strongest pheromone and on the highest attractiveness. Yet, in a fraction of cases, the choice will be made probabilistically, through guided by attractiveness and trails.

The attractiveness



evaporation reduces pheromone over all trails iteration by iteration, usually by exponential decay.

There are several ACO metaheuristic implementations that differ in the way the artificial pheromone is used and updated. The first prototype is Ant System: when an artificial ant is at node

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performance, measured by algorithm complexity and by computational results. ACS introduced the concepts of *local* and *global update*, and of the *pseudo-random-proportional* transition rule, which are at the basis of any successful ACO implementation.

In ACS, at the end of each iteration, global update uses only the best solution, computed so far, to update the pheromone trail. The only edges that are modified are those edges

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chosen node is the one with the best

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Table 3.17 Parameter settings for numerical example

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STEP 1: Initial Pheromone Declaration



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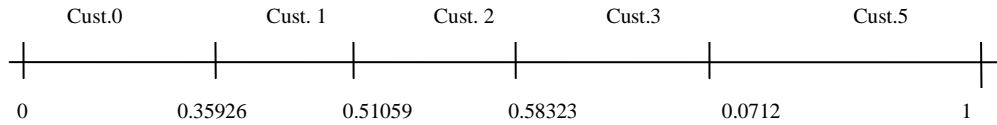


Figure 3.12 Roulette wheel for selecting next customer to move to

A random number generated,

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Ant 1: 1- 0 - 4 -5 - 2- 3 -1

Ant 2: 0- 1 - 2 -5 - 4- 3 -1

Before the total expected costs for all tours are evaluated, the customers' point sequence are rearranged such that they start and end at node 0 as follow for total expected cost calculation purpose.

Ant 1: 0 - 4 -5 - 2- 3 -1- 0, Total expected cost = 59.8349 units

Ant 2: 0- 1 - 2 -5 - 4- 3 - 0, Total expected cost = 56.5243 units

STEP 5: Local Search (Descent Method)

After all ants have completed their tour, all ants will be brought to a local minimum by a local search called Descent Method. For each of the iteration, two customers' points are randomly chosen for swapping their positions in the given sequence in accepted tour and its total expected cost is computed. The iterations will be stopped after 10 consecutive non-improving solutions and the final best solution is recorded.

Local minima for both ants are as follow:

Ant 1: 0 - 4 -5 - 3- 2 -1- 0, Total expected cost = 54.6327 units

Ant 2: 0- 2 - 1 -4 - 5- 3 - 0, Total expected cost = 55.0749 units

STEP 6: Global pheromone updating

The global best tour is the globally best tour from the beginning of the trial, the one and only ant that produces the global best tour is allowed to deposit the pheromone on the edges that belong to the global best tour using the equation as follow:



d. Simulated Annealing

Simulated annealing is stochastic approach for solving combinatorial optimization problems. It uses iterative improvement technique that allowed accepting non-improving neighboring solutions to avoid being trapped at a poor local optimum with a certain probability.

It starts with a feasible initial solution (which is defined as current solution

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The probability of accepting a non-improving solution is controlled by two factors which are the different of the objective functions and the temperature. The higher the value of

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STEP 4: If

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Second Iteration

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Updating the temperature

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CHAPTER 4

DATA COLLECTION, ANALYSIS AND RESULTS

4.1 Data Collection

The data was secondary data and collected from Graphic Communication Group Limited (GCGL) office in Ashanti region. The data is the average number of Newspaper copies circulated in each of the twenty seven (27) districts in Ashanti region in January 2012 per day. Through interviews of the officer-in-charge of circulation and drivers of the distribution vehicles data on routes used for distribution was collected. The districts are represented as nodes.

The map of Ashanti region showing all the districts is resented in Figure 4.1.

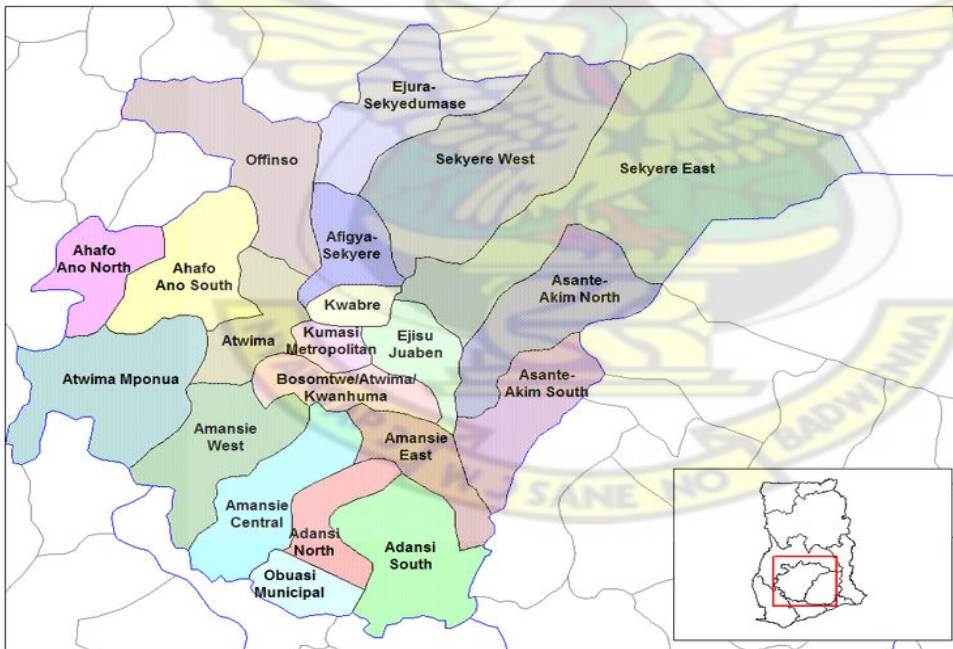


Figure 4.1 Map the districts in Ashanti region.

Table 4.1 below shows the 27 districts in Ashanti region and the number of copies circulated.

The districts are represented as nodes.

Table 4.1 Districts and Copies circulated.

| Node | District | Capital | Average Copies |
|-----------|----------------------|---------------|----------------|
| 1 (Depot) | Kumasi Metropolis | Kumasi | 6,220 |
| 2 | Adansi North | Fomena | 550 |
| 3 | Adansi South | New Edubiase | 540 |
| 4 | Afigya-Kwabre | Kodie | 550 |
| 5 | Ahafo Ano North | Tepa | 480 |
| 6 | Ahafo Ano South | Mankranso | 500 |
| 7 | Amansie Central | Jacobi | 650 |
| 8 | Amansie West | Manso-Nkwanta | 630 |
| 9 | Asante Akim North | Konongo | 700 |
| 10 | Asante Akim South | Juaso | 460 |
| 11 | Atwima Kwanwoma | Foase Kokoben | 620 |
| 12 | Atwima Mponua | Nyinahin | 350 |
| 13 | Atwima Nwabiagya | Nkawie | 450 |
| 14 | Bekwai Municipal | Bekwai | 620 |
| 15 | Bosome Freho | Asiwa | 350 |
| 16 | Bosomtwe | Kuntenase | 500 |
| 17 | Ejisu-Juaben | Ejisu | 650 |
| 18 | Ejura-Sekyeredumase | Ejura | 400 |
| 19 | Kwabre East | Mamponteng | 450 |
| 20 | Mampong Municipal | Mampong | 670 |
| 21 | Obuasi Municipal | Obuasi | 750 |
| 22 | Offinso Municipal | Offinso | 750 |
| 23 | Offinso North | Akomandan | 650 |
| 24 | Sekyere Afram Plains | Kumawu | 480 |
| 25 | Sekyere Central | Nsuta | 500 |
| 26 | Sekyere East | Effiduase | 650 |
| 27 | Sekyere South | Agona | 600 |

Acquiring some parameters, such as road distance and transportation time, is not easy. It will consume much time and money. Therefore applying assisting tools like Google Earth and Google map programs was taken into consideration. Some real data, both shortest distance and transportation time, were collected to compare with the data obtained from Google Earth and Google Map programs. By the method of statistical hypothesis testing, it was found that the distances did not differ from the real data with 0.05 significant levels as so the travelling time. Therefore, in this study, road distance data were taken from Google Earth and Google Map programs.

Table 4.2 below displays the matrix of the edge distance (in Km) of the direct road link between the twenty seven (27) district capitals in Ashanti region. The edge

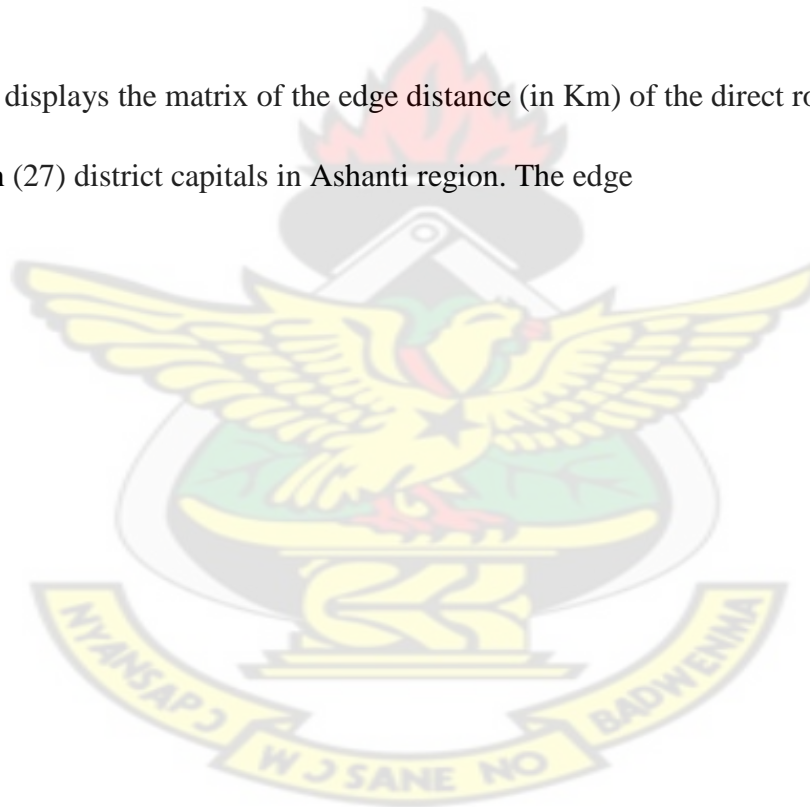


Table 4.2 Edge Distances Matrix (kilometers).

| Node | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 | 25 | 26 | 27 | | |
|------|------|------|------|------|------|------|------|------|------|------|------|----|------|------|------|------|------|------|------|------|------|------|------|----|------|------|------|------|------|
| 1 | - | ∞ | ∞ | 13.6 | ∞ | 34.7 | ∞ | ∞ | ∞ | ∞ | 29.4 | ∞ | 25.1 | ∞ | 57.8 | 26.5 | 13.5 | ∞ | 15.1 | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | | |
| 2 | ∞ | - | 30.7 | ∞ | ∞ | ∞ | 26.6 | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | 22.5 | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | 23.5 | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | | |
| 3 | ∞ | 30.7 | - | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | 53.2 | 58.3 | ∞ | ∞ | ∞ | ∞ | ∞ | 45.0 | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | | |
| 4 | 13.6 | ∞ | ∞ | - | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | 17.8 | ∞ | ∞ | ∞ | ∞ | ∞ | | |
| 5 | ∞ | ∞ | ∞ | ∞ | - | 44.0 | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | | |
| 6 | 34.7 | ∞ | ∞ | ∞ | 44.0 | - | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | | |
| 7 | ∞ | 26.6 | ∞ | ∞ | ∞ | ∞ | - | ∞ | ∞ | ∞ | 14.0 | ∞ | ∞ | 16.9 | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | 29.1 | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | | |
| 8 | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | - | ∞ | ∞ | 44.7 | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | | |
| 9 | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | - | 14.2 | ∞ | ∞ | ∞ | ∞ | 33.8 | 40.7 | 37.5 | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | 57.9 | ∞ | 49.1 | | |
| 10 | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | 14.2 | - | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | | |
| 11 | 29.4 | ∞ | ∞ | ∞ | ∞ | ∞ | 14.0 | 44.7 | ∞ | ∞ | - | ∞ | ∞ | 9.0 | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | 36.4 | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | | |
| 12 | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | - | 35.8 | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | | |
| 13 | 25.1 | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | 35.8 | - | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | | |
| 14 | ∞ | 22.5 | 53.2 | ∞ | ∞ | ∞ | 16.9 | ∞ | ∞ | ∞ | 9.0 | ∞ | ∞ | - | 35.6 | 18.7 | ∞ | ∞ | ∞ | ∞ | 39.3 | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | | |
| 15 | 57.8 | ∞ | 58.2 | ∞ | ∞ | ∞ | ∞ | ∞ | 33.8 | ∞ | ∞ | ∞ | ∞ | ∞ | 35.6 | - | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | | |
| 16 | 26.5 | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | 40.7 | ∞ | ∞ | ∞ | ∞ | ∞ | 18.7 | ∞ | - | 28.4 | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | | |
| 17 | 13.5 | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | 37.5 | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | 28.4 | - | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | 27.5 | 34.3 | | |
| 18 | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | - | 15.5 | 40.0 | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | | |
| 19 | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | 15.1 | - | ∞ | ∞ | 24.6 | ∞ | ∞ | ∞ | 24.1 | 20.7 | |
| 20 | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | 40.0 | ∞ | - | ∞ | ∞ | ∞ | ∞ | 8.2 | ∞ | 20.3 | |
| 21 | ∞ | 23.5 | 45.0 | ∞ | ∞ | ∞ | 29.1 | ∞ | ∞ | ∞ | 36.4 | ∞ | ∞ | 39.3 | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | - | ∞ | ∞ | ∞ | ∞ | ∞ | | |
| 22 | ∞ | ∞ | ∞ | 17.8 | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | - | 65.0 | ∞ | ∞ | ∞ | 24.2 | | |
| 23 | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | 65.0 | - | ∞ | ∞ | ∞ | | |
| 24 | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | 57.9 | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | - | ∞ | 17.1 | | |
| 25 | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | 8.2 | ∞ | ∞ | ∞ | ∞ | - | 22.1 | 19.5 | |
| 26 | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | 49.1 | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | 17.1 | 22.1 | - | 15.4 |
| 27 | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | 34.3 | ∞ | 20.7 | 20.3 | ∞ | 24.2 | ∞ | ∞ | 19.5 | 15.4 | - | |

4.2 Data Analysis

The Floyd-Warshall algorithm was used to compute all pair shortest path of the data in Table 4.2 to obtain a complete all pairs shortest path distance matrix which will be used for our analysis.

The shortest path distance matrix is a complete symmetric undirected graph with edges



Table 4.3 Distance Matrix (kilometers).

| Node | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 | 25 | 26 | 27 |
|------|------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| 1 | - | 60.9 | 91.6 | 13.6 | 78.7 | 34.7 | 43.4 | 74.1 | 51.0 | 65.2 | 29.4 | 60.9 | 25.1 | 38.4 | 57.8 | 26.5 | 13.5 | 71.1 | 56.0 | 68.1 | 65.8 | 31.4 | 96.4 | 58.1 | 63.1 | 41 | 47.8 |
| 2 | 60.9 | - | 30.7 | 74.5 | 139.6 | 95.6 | 26.6 | 76.2 | 81.9 | 96.1 | 31.5 | 121.8 | 86.0 | 22.5 | 58.1 | 41.2 | 69.6 | 132.0 | 116.9 | 124.2 | 23.5 | 92.3 | 157.3 | 114.2 | 119.2 | 97.1 | 103.9 |
| 3 | 91.6 | 30.7 | - | 105.2 | 170.3 | 126.3 | 57.3 | 106.9 | 92.0 | 106.2 | 62.2 | 152.5 | 116.7 | 53.2 | 58.2 | 71.9 | 100.3 | 162.7 | 147.6 | 154.9 | 45.0 | 123.0 | 188.0 | 144.9 | 149.9 | 127.8 | 134.6 |
| 4 | 13.6 | 74.5 | 105.2 | - | 92.3 | 48.3 | 57.0 | 87.7 | 64.6 | 78.8 | 43.0 | 74.5 | 38.7 | 52.0 | 71.4 | 40.1 | 27.1 | 57.5 | 42.4 | 62.3 | 79.4 | 17.8 | 82.8 | 71.7 | 61.5 | 54.6 | 42.0 |
| 5 | 78.7 | 139.6 | 170.3 | 92.3 | - | 44.0 | 122.1 | 152.8 | 129.7 | 143.9 | 108.1 | 139.6 | 103.8 | 117.1 | 136.5 | 105.2 | 92.2 | 149.8 | 134.7 | 146.8 | 144.5 | 110.1 | 175.1 | 136.8 | 141.8 | 119.7 | 126.5 |
| 6 | 34.7 | 95.6 | 126.3 | 48.3 | 44.0 | - | 78.1 | 108.8 | 85.7 | 99.9 | 64.1 | 95.60 | 59.8 | 73.1 | 92.5 | 61.2 | 48.2 | 105.8 | 90.7 | 102.8 | 100.5 | 66.1 | 131.1 | 92.8 | 97.8 | 75.7 | 82.5 |
| 7 | 43.4 | 26.6 | 57.3 | 57.0 | 122.1 | 78.1 | - | 58.7 | 76.3 | 90.5 | 14.0 | 104.3 | 68.5 | 16.9 | 52.5 | 35.6 | 56.9 | 114.5 | 99.4 | 111.5 | 29.1 | 74.8 | 139.8 | 101.5 | 106.5 | 84.4 | 91.2 |
| 8 | 51.0 | 76.2 | 106.9 | 87.7 | 152.8 | 108.8 | 58.7 | - | 113.1 | 127.3 | 44.7 | 135.0 | 99.2 | 53.7 | 89.3 | 72.4 | 87.6 | 145.2 | 130.1 | 142.2 | 81.1 | 105.5 | 170.5 | 132.2 | 137.2 | 115.1 | 121.9 |
| 9 | 65.2 | 81.9 | 92.0 | 64.6 | 129.7 | 85.7 | 76.3 | 113.1 | - | 14.2 | 68.4 | 111.9 | 76.1 | 59.4 | 33.8 | 40.7 | 37.5 | 88.3 | 73.2 | 79.4 | 98.7 | 82.4 | 147.4 | 57.9 | 71.2 | 49.1 | 64.5 |
| 10 | 29.4 | 96.1 | 106.2 | 78.8 | 143.9 | 99.9 | 90.5 | 127.3 | 14.2 | - | 82.6 | 126.1 | 90.3 | 73.6 | 48.0 | 54.9 | 51.7 | 102.5 | 87.4 | 93.6 | 112.9 | 96.4 | 161.6 | 72.1 | 85.4 | 63.3 | 78.7 |
| 11 | 29.4 | 31.5 | 62.2 | 43.0 | 108.1 | 64.1 | 14.0 | 44.7 | 68.4 | 82.6 | - | 90.3 | 54.5 | 9.0 | 44.6 | 27.7 | 42.9 | 100.5 | 85.4 | 97.5 | 36.4 | 60.8 | 125.8 | 87.5 | 92.5 | 70.4 | 77.2 |
| 12 | 60.9 | 121.8 | 152.5 | 74.5 | 139.6 | 95.6 | 104.3 | 135.0 | 111.9 | 126.1 | 90.3 | - | 35.8 | 99.3 | 118.7 | 87.4 | 74.4 | 132.0 | 116.9 | 129.0 | 126.7 | 92.3 | 157.3 | 119.0 | 124.0 | 101.9 | 108.7 |
| 13 | 25.1 | 86.0 | 116.7 | 38.7 | 103.8 | 59.8 | 68.5 | 99.2 | 76.1 | 90.3 | 54.5 | 35.8 | - | 63.5 | 82.9 | 51.6 | 38.6 | 96.2 | 81.4 | 93.2 | 90.9 | 56.5 | 121.5 | 83.2 | 88.2 | 66.1 | 72.9 |
| 14 | 38.4 | 22.5 | 53.2 | 52.0 | 117.1 | 73.1 | 16.9 | 53.7 | 59.4 | 73.6 | 9.0 | 99.3 | 63.5 | - | 35.6 | 18.7 | 47.1 | 109.5 | 94.4 | 101.7 | 39.3 | 69.8 | 134.8 | 91.7 | 96.7 | 74.6 | 81.4 |
| 15 | 57.8 | 58.1 | 58.2 | 71.4 | 136.5 | 92.5 | 52.5 | 89.3 | 33.8 | 48.0 | 44.6 | 118.7 | 82.9 | 35.6 | - | 54.3 | 71.3 | 122.1 | 107.0 | 113.2 | 74.9 | 89.2 | 154.2 | 91.7 | 105.0 | 82.9 | 98.3 |
| 16 | 26.5 | 41.2 | 71.9 | 40.1 | 105.2 | 61.2 | 35.6 | 72.4 | 40.7 | 54.9 | 27.7 | 87.4 | 51.6 | 18.7 | 54.3 | - | 28.4 | 95.1 | 80.0 | 83.0 | 58.0 | 57.9 | 122.9 | 73.0 | 78.0 | 55.9 | 62.7 |
| 17 | 13.5 | 69.6 | 100.3 | 27.1 | 92.2 | 48.2 | 56.9 | 87.6 | 37.5 | 51.7 | 42.9 | 74.4 | 38.6 | 47.1 | 71.3 | 28.4 | - | 66.7 | 51.6 | 54.6 | 79.3 | 44.9 | 109.9 | 44.6 | 49.6 | 27.5 | 34.3 |
| 18 | 71.1 | 132.0 | 162.7 | 57.5 | 149.8 | 105.8 | 114.5 | 145.2 | 88.3 | 102.5 | 100.5 | 132.0 | 96.2 | 109.5 | 122.1 | 95.1 | 66.7 | - | 15.1 | 40.0 | 136.9 | 39.7 | 104.7 | 56.3 | 48.2 | 39.2 | 35.8 |
| 19 | 56.0 | 116.9 | 147.6 | 42.4 | 134.7 | 90.7 | 99.4 | 130.1 | 73.2 | 87.4 | 85.4 | 116.9 | 81.1 | 94.4 | 107.0 | 80.0 | 51.6 | 15.1 | - | 41.0 | 121.8 | 24.6 | 89.6 | 41.2 | 40.2 | 24.1 | 20.7 |
| 20 | 68.1 | 124.2 | 154.9 | 62.3 | 146.8 | 102.8 | 111.5 | 142.2 | 79.4 | 93.6 | 97.5 | 129.0 | 93.2 | 101.7 | 133.2 | 83.0 | 54.6 | 40.0 | 41.0 | - | 133.9 | 44.5 | 109.5 | 47.4 | 8.2 | 30.3 | 20.3 |
| 21 | 65.8 | 23.5 | 45.0 | 79.4 | 144.5 | 100.5 | 29.1 | 81.1 | 98.7 | 112.9 | 36.4 | 126.7 | 90.9 | 39.3 | 74.9 | 58.0 | 79.3 | 136.9 | 121.8 | 133.9 | - | 97.2 | 162.2 | 123.9 | 128.9 | 106.8 | 113.6 |
| 22 | 31.4 | 92.3 | 123.0 | 17.8 | 110.1 | 66.1 | 74.8 | 105.5 | 82.4 | 96.6 | 60.8 | 92.3 | 56.5 | 69.8 | 89.2 | 57.9 | 44.9 | 39.7 | 24.6 | 44.5 | 97.2 | - | 65.0 | 56.7 | 43.7 | 39.6 | 24.2 |
| 23 | 96.4 | 157.3 | 188.0 | 82.8 | 175.1 | 131.1 | 139.8 | 170.5 | 147.4 | 161.6 | 125.8 | 157.3 | 121.5 | 134.8 | 154.2 | 122.9 | 109.9 | 104.7 | 89.6 | 109.5 | 162.2 | 65.0 | - | 121.7 | 108.7 | 104.6 | 89.2 |
| 24 | 96.4 | 114.2 | 144.9 | 71.7 | 136.8 | 92.8 | 101.5 | 132.5 | 57.9 | 72.1 | 87.5 | 119.0 | 83.2 | 91.7 | 91.7 | 73.0 | 44.6 | 56.3 | 41.2 | 47.4 | 123.9 | 56.7 | 121.7 | - | 39.2 | 17.1 | 32.5 |
| 25 | 63.1 | 119.2 | 149.9 | 61.5 | 141.8 | 97.8 | 106.5 | 137.2 | 71.2 | 85.4 | 92.5 | 124.0 | 88.2 | 96.7 | 105.0 | 78.0 | 49.6 | 48.2 | 40.2 | 8.2 | 128.9 | 43.7 | 108.7 | 39.2 | - | 22.1 | 19.5 |
| 26 | 41 | 97.1 | 127.8 | 54.6 | 119.7 | 75.7 | 84.4 | 115.1 | 49.1 | 63.3 | 70.4 | 101.9 | 66.1 | 74.6 | 82.9 | 55.9 | 27.5 | 39.2 | 24.1 | 30.3 | 106.8 | 39.6 | 104.6 | 17.1 | 22.1 | - | 15.4 |
| 27 | 47.8 | 103.9 | 134.6 | 42.0 | 126.5 | 82.5 | 91.2 | 121.9 | 64.5 | 78.7 | 77.2 | 108.7 | 72.9 | 81.4 | 98.3 | 62.7 | 34.3 | 35.8 | 20.7 | 20.3 | 113.6 | 24.2 | 89.2 | 32.5 | 19.5 | 15.4 | - |

Since the objective is to plan distribution routes that will reduce the frequent late deliveries, the road distance are converted to time. The conversion of travelled distance into time for checking whether travelled time is within the limitation of available time window of the distribution vehicles can be presented as in (1)

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Table 4.4 Time Matrix (minutes).

| Node | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 | 25 | 26 | 27 |
|------|----|-----|-----|----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|----|-----|----|-----|-----|-----|-----|-----|-----|-----|-----|
| 1 | - | 50 | 73 | 14 | 77 | 32 | 40 | 66 | 43 | 54 | 26 | 55 | 27 | 33 | 55 | 30 | 13 | 76 | 18 | 54 | 57 | 27 | 41 | 60 | 56 | 41 | 35 |
| 2 | 50 | - | 23 | 64 | 127 | 82 | 28 | 64 | 75 | 86 | 24 | 105 | 77 | 17 | 44 | 38 | 63 | 86 | 68 | 104 | 20 | 77 | 91 | 110 | 106 | 91 | 85 |
| 3 | 73 | 23 | - | 87 | 150 | 105 | 51 | 87 | 85 | 96 | 47 | 128 | 100 | 40 | 54 | 61 | 86 | 109 | 91 | 127 | 36 | 100 | 114 | 133 | 129 | 114 | 108 |
| 4 | 14 | 64 | 87 | - | 91 | 46 | 54 | 80 | 57 | 68 | 40 | 69 | 41 | 47 | 69 | 44 | 27 | 50 | 32 | 58 | 71 | 13 | 55 | 74 | 60 | 55 | 39 |
| 5 | 77 | 127 | 150 | 91 | - | 45 | 117 | 143 | 120 | 131 | 103 | 35 | 63 | 110 | 132 | 107 | 90 | 113 | 95 | 131 | 134 | 104 | 118 | 137 | 133 | 118 | 112 |
| 6 | 32 | 82 | 105 | 46 | 45 | - | 72 | 98 | 75 | 86 | 58 | 80 | 59 | 65 | 87 | 62 | 45 | 68 | 50 | 86 | 89 | 59 | 73 | 92 | 88 | 73 | 67 |
| 7 | 40 | 28 | 51 | 54 | 117 | 72 | - | 54 | 73 | 84 | 14 | 95 | 67 | 15 | 42 | 36 | 53 | 76 | 58 | 94 | 27 | 67 | 81 | 100 | 96 | 81 | 75 |
| 8 | 66 | 64 | 87 | 80 | 143 | 98 | 54 | - | 105 | 116 | 40 | 121 | 93 | 47 | 74 | 68 | 79 | 102 | 84 | 120 | 71 | 93 | 107 | 126 | 122 | 107 | 101 |
| 9 | 43 | 75 | 85 | 57 | 120 | 75 | 73 | 105 | - | 11 | 65 | 98 | 70 | 58 | 31 | 40 | 30 | 79 | 61 | 80 | 91 | 70 | 84 | 58 | 68 | 45 | 61 |
| 10 | 54 | 86 | 96 | 68 | 131 | 86 | 84 | 116 | 11 | - | 76 | 109 | 81 | 69 | 42 | 51 | 41 | 90 | 72 | 91 | 102 | 81 | 95 | 69 | 79 | 56 | 72 |
| 11 | 26 | 24 | 47 | 40 | 103 | 58 | 14 | 40 | 65 | 76 | - | 81 | 53 | 7 | 34 | 28 | 39 | 62 | 44 | 80 | 31 | 53 | 67 | 86 | 82 | 67 | 61 |
| 12 | 55 | 105 | 128 | 69 | 35 | 80 | 95 | 121 | 98 | 109 | 81 | - | 28 | 88 | 110 | 85 | 68 | 91 | 73 | 109 | 112 | 82 | 96 | 115 | 111 | 96 | 90 |
| 13 | 27 | 77 | 100 | 41 | 63 | 59 | 67 | 93 | 70 | 81 | 53 | 28 | - | 60 | 82 | 57 | 40 | 63 | 45 | 81 | 84 | 54 | 68 | 87 | 83 | 68 | 62 |
| 14 | 33 | 17 | 40 | 47 | 110 | 65 | 15 | 47 | 58 | 69 | 7 | 88 | 60 | - | 27 | 21 | 46 | 69 | 51 | 87 | 33 | 60 | 74 | 93 | 89 | 74 | 68 |
| 15 | 55 | 44 | 54 | 69 | 132 | 87 | 42 | 74 | 31 | 42 | 34 | 110 | 82 | 27 | - | 48 | 61 | 91 | 73 | 109 | 60 | 82 | 96 | 89 | 99 | 76 | 90 |
| 16 | 30 | 38 | 61 | 44 | 107 | 62 | 36 | 68 | 40 | 51 | 28 | 85 | 57 | 21 | 48 | - | 31 | 66 | 48 | 84 | 54 | 57 | 71 | 78 | 82 | 59 | 65 |
| 17 | 13 | 63 | 86 | 27 | 90 | 45 | 53 | 79 | 30 | 41 | 39 | 68 | 40 | 46 | 61 | 31 | - | 49 | 31 | 59 | 70 | 40 | 54 | 47 | 51 | 28 | 40 |
| 18 | 76 | 86 | 109 | 50 | 113 | 68 | 76 | 102 | 79 | 90 | 62 | 91 | 63 | 69 | 91 | 66 | 49 | - | 18 | 33 | 93 | 61 | 41 | 61 | 45 | 42 | 35 |
| 19 | 18 | 68 | 91 | 32 | 95 | 50 | 58 | 84 | 61 | 72 | 44 | 73 | 45 | 51 | 73 | 48 | 31 | 18 | - | 36 | 75 | 43 | 23 | 43 | 38 | 24 | 17 |
| 20 | 54 | 104 | 127 | 58 | 131 | 86 | 94 | 120 | 80 | 91 | 80 | 109 | 81 | 87 | 109 | 84 | 59 | 33 | 36 | - | 111 | 45 | 59 | 54 | 12 | 35 | 19 |
| 21 | 57 | 20 | 36 | 71 | 134 | 89 | 27 | 71 | 91 | 102 | 31 | 112 | 84 | 33 | 60 | 54 | 70 | 93 | 75 | 111 | - | 84 | 98 | 117 | 113 | 98 | 92 |
| 22 | 27 | 77 | 100 | 13 | 104 | 59 | 67 | 93 | 70 | 81 | 53 | 82 | 54 | 60 | 82 | 57 | 40 | 61 | 43 | 45 | 84 | - | 49 | 61 | 47 | 42 | 26 |
| 23 | 41 | 91 | 114 | 55 | 118 | 73 | 81 | 107 | 84 | 95 | 67 | 96 | 68 | 74 | 96 | 71 | 54 | 41 | 23 | 59 | 98 | 49 | - | 66 | 61 | 47 | 40 |
| 24 | 60 | 110 | 133 | 74 | 137 | 92 | 100 | 126 | 58 | 69 | 86 | 115 | 87 | 93 | 89 | 78 | 47 | 61 | 43 | 54 | 117 | 61 | 66 | - | 42 | 19 | 35 |
| 25 | 56 | 106 | 129 | 60 | 133 | 88 | 96 | 122 | 68 | 79 | 82 | 111 | 83 | 89 | 99 | 82 | 51 | 45 | 38 | 12 | 113 | 47 | 61 | 42 | - | 23 | 21 |
| 26 | 41 | 91 | 114 | 55 | 118 | 73 | 81 | 107 | 45 | 56 | 67 | 96 | 68 | 74 | 76 | 59 | 28 | 42 | 24 | 35 | 98 | 42 | 47 | 19 | 23 | - | 16 |
| 27 | 35 | 85 | 108 | 39 | 112 | 67 | 75 | 101 | 61 | 72 | 61 | 90 | 62 | 68 | 90 | 65 | 40 | 35 | 17 | 19 | 92 | 26 | 40 | 35 | 21 | 16 | - |

4.3 Problem description of the proposed model for GCGL.

A mathematical description of VRP for newspaper distribution problem in this case may be defined as follows. Let $G = (V, A)$ be a network where

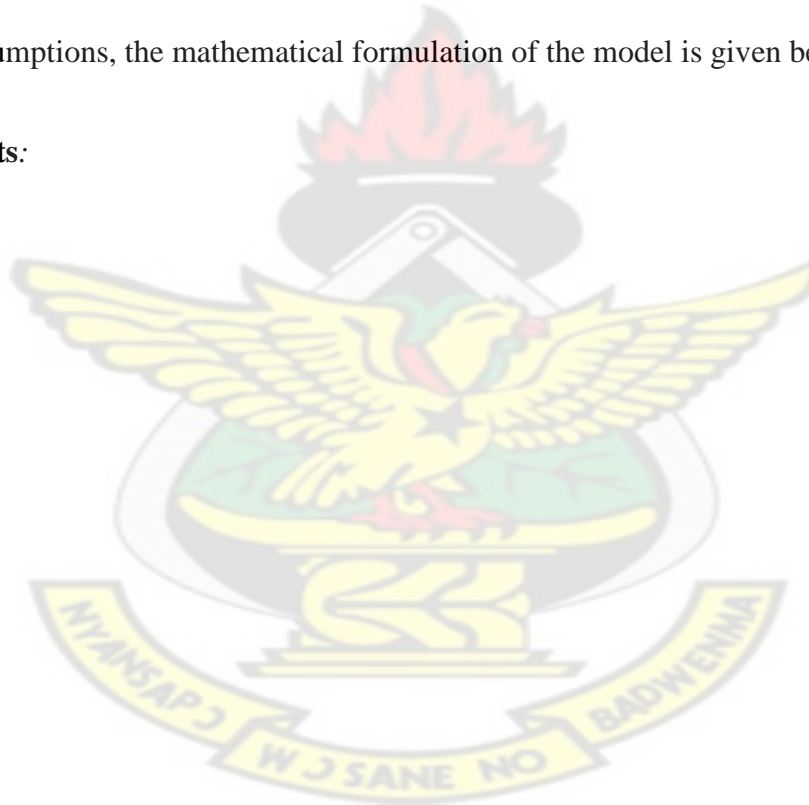


- Total of 6 vehicles are available.
- Hour of operations: there are time window of $T=180$ minutes for delivering newspapers to the last stops/customer and $T= 60$ minutes for returning to the depot.

However, there are other constraints that did not define in this research due to intangible factor cannot be part of the model i.e. vehicle capacity which all copies shall be stored behind the truck with door closing at all time of running. The paper shall not be stored in the front seat or the roof of the truck.

Under these assumptions, the mathematical formulation of the model is given below:

Definition of sets:



Decision Variables:

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Explanation of the model

Objective function (1) minimizes the total tour time, which includes of traveling time and unloading time to a complete tour. Constraints (2) ensure that each district capital is visited exactly once by a vehicle. Constraints (3) show that all vehicles must start its route from the depot. Constraints (2) and (3) together ensures that each vehicle return to the depot. Constraints (4) are balance constraint which implies that a vehicle should enter and leave the district. Constraints (5) state that the total demand on a route should not exceed the maximum capacity of the vehicle. Constraints (6) ensures that the traveling time of the vehicle should not exceed the time window, where $T=180$ minutes. Constraint (7) is sub-tour elimination constraint. It ensures that the route cannot form a loop without including the depot. Constraints (8) and (9) are binary constraints.

4.4 The Clarke and Wright Savings Algorithm

The algorithm used to solve the problem in this thesis is based on the concept of Clark and Wright (1964) which is a constructive method. The original Clarke and Wright method did not deal with the time, it only deals with the distance, customer demand and the vehicle capacity. In this thesis, the time factors, which are very important for the real business world, are inserted in the algorithm. The algorithm proceeds as follows:

STEP 1: Calculate the travel time savings:

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4.5 Computational Procedures

A heuristic program developed by Snyder (2003) called VRP SOLVER which implements an adaptation of the Clarke-Wright savings algorithm for vehicle routing problems described in section 4.5 and the mathematical model described in section 4.4 of this chapter was implemented to solve the problem.

The program takes input from a text file listing each customer's location and demand. It builds vehicle routes that visit every city exactly once and that obey user-specified vehicle volume and distance limits.

Results are displayed in graphical (map) form and in a text form. After an initial solution is built, various improvement heuristics are performed. These include the well-known 2-opt and Or-opt operations (the Or-opt uses group sizes of 1, 2, and 3), as well as a swap operation in which two customers on different routes may be removed from their routes and inserted into the opposite route.

The main screen of the program in appendix A. allows you to load and view the data, set options, and run the model. If the data are successfully read, they will appear in chart form in the box in the middle of the window. At the bottom of the main screen are two boxes in which you can set the parameters of the problem. The "Truck Capacity" box specifies the maximum allowable volume a vehicle may carry (in units of demand). The "Truck Distance Limit" box specifies the maximum allowable distance a truck may travel in its round trip. The "Distances" button displays a window showing the distance matrix among pairs of cities in the data and allowing you to choose how distances are computed in appendix B. The "Options" button displays a dialog box in which you can define the number of iterations for the algorithm.

After performing 13 runs of the program with each run consisting of 15 iterations the best results of the problem was obtained with 0.85 seconds of computational time. The program for obtaining our solutions was run on an Intel® CORE with 2.13 GHz processor speed and 4GB RAM running Windows 7 Ultimate.

4.6 Results

Table 4.5 is the summary of results of the data analyzed using the VRP SOLVER. The first column represents the vehicle numbers whilst in the second column the sequence of the vehicle routes starting and ending at the depot is displayed. The third and fourth columns represent the total travel time for each vehicle and the number of copies distributed on the route respectively.

Table 4.5 Summary of Clarke and Wright heuristics results.

| Vehicle Number | Route | Total Travel Time (minutes) | Number of Copies |
|----------------|--------------------|-----------------------------|------------------|
| 1 | 1-7-21-3-2-1 | 176 | 2490 |
| 2 | 1-4-22-23-18-19-1 | 178 | 2800 |
| 3 | 1-13-12-5-6-1 | 167 | 1780 |
| 4 | 1-16-14-11-8-1 | 164 | 2370 |
| 5 | 1-15-10-9-17-1 | 151 | 2160 |
| 6 | 1-26-24-25-20-27-1 | 168 | 2900 |

It can be observed from the results in Table 4.5, that six vehicle routes are constructed with the traveling time for each route not exceeding the time window constraint of 180 minutes and the load not exceeding the vehicle capacity of 3000 copies per vehicle. It can also be observed from the table that all the routes start and end at the depot.

Figure 4.2 below shows the vehicle routes constructed by the Clarke and Wright algorithm using the VRP SOLVER. Each of the routes is represented by different colour with each consisting of node numbers as district capitals provided in Table 4.1.

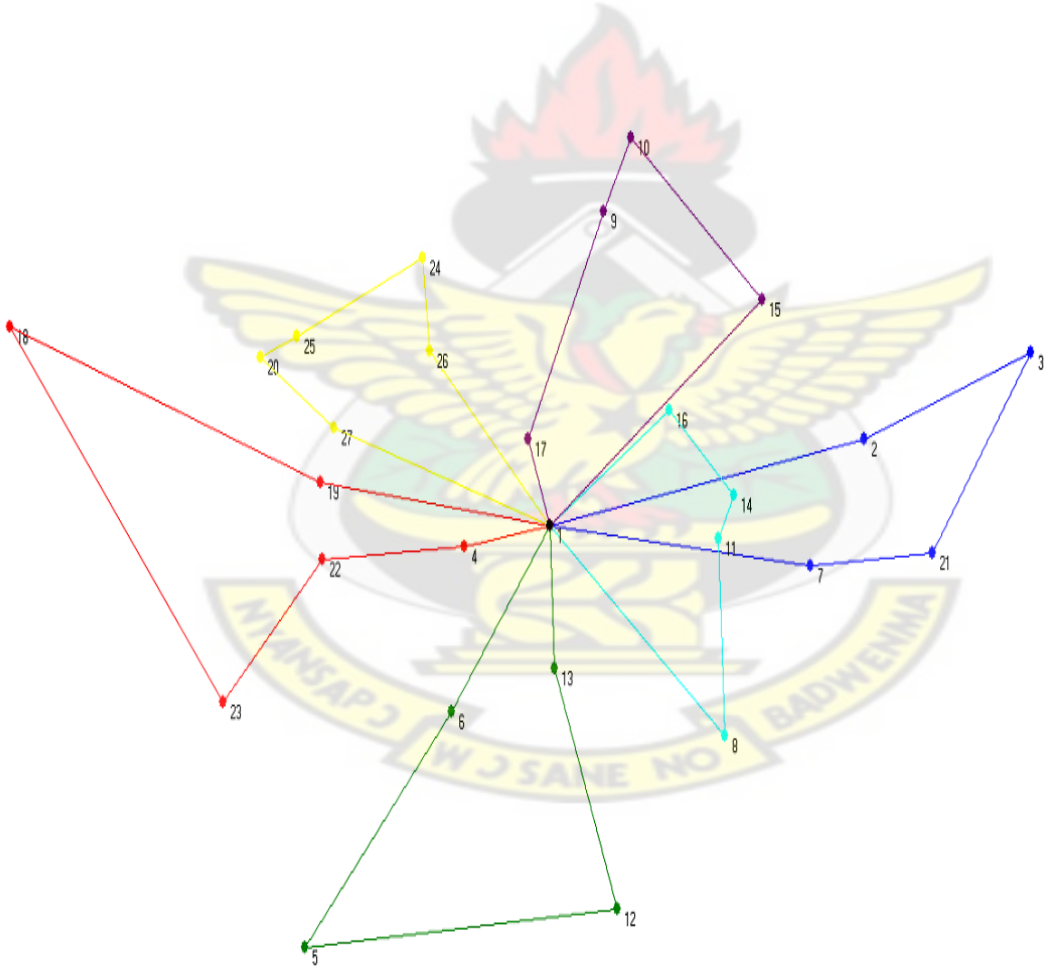


Figure 4.2 Clarke and Wright algorithm Distribution Routes

Table 4.6 presents comparison of the results obtained by the Clarke and Wright algorithm and distribution routes used by GCGL vehicles with respect to the total time for each route. In column 1 the vehicle numbers are presented and the GCGL routes are presented in column 2, whilst the route obtained from the Clarke and Wright heuristics results displayed in column 4. Column 3 and 5 is the total time (in minutes) to complete a tour. The last column shows the percentage improvement of the total travelled time of the routes operated by GCGL vehicles.

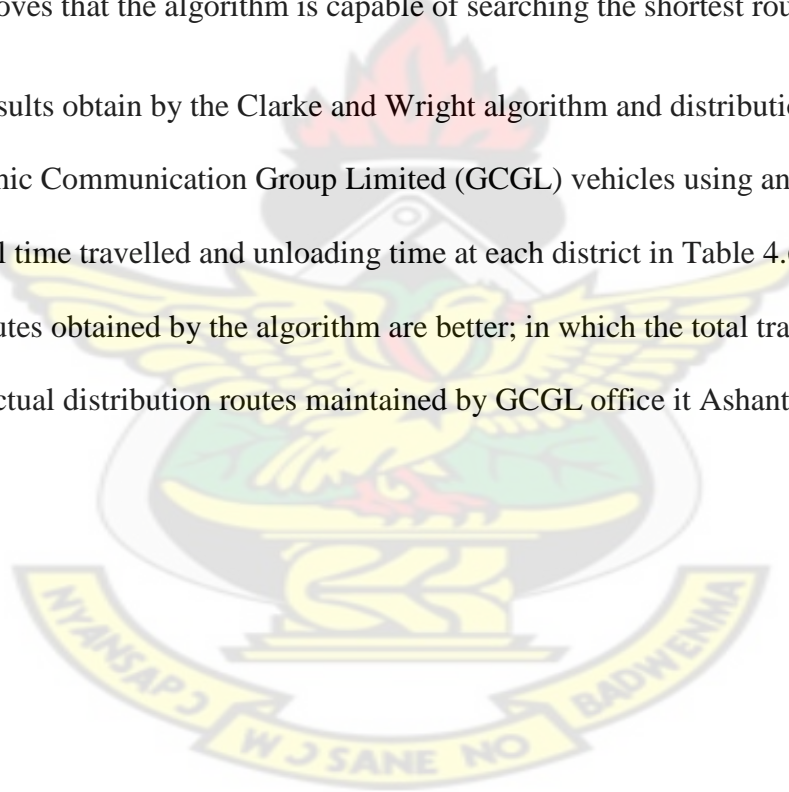
Table 4.6 Comparison GCGL routes and Clarke and Wright algorithm

| Vehicle No. | GCGL | | Clarke and Wright algorithm | | Improvement |
|-------------|--------------------|----------------------|-----------------------------|----------------------|-------------|
| | Route | Total Time (minutes) | Route | Total Time (minutes) | |
| 1 | 1-4-22-23-5-1 | 271 | 1-7-21-3-2-1 | 176 | 35.1 |
| 2 | 1-6-13-12-8-1 | 306 | 1-4-22-23-18-19-1 | 178 | 41.8 |
| 3 | 1-11-7-21-3-1 | 176 | 1-13-12-5-6-1 | 167 | 5.1 |
| 4 | 1-16-15-14-2-1 | 172 | 1-16-14-11-8-1 | 164 | 4.7 |
| 5 | 1-19-27-25-20-18-1 | 177 | 1-15-10-9-17-1 | 151 | 14.7 |
| 6 | 1-17-26-24-9-10-1 | 183 | 1-26-24-25-20-27-1 | 168 | 8.2 |
| TOTAL | 1285 | | 1004 | | 21.9 |

4.7 Discussion

The results presented in the previous section are obtained by assuming that the demands at nodes are known and all the vehicles travel at constant speed. It is shown that the Clarke and Wright can be implemented to vehicle routing for Newspaper distribution. It is a fact that the Clarke and Wright algorithm has contributed a significant improvement in generating distribution routes and reducing the travel time of the current routes maintained by the company. The result also proves that the algorithm is capable of searching the shortest route.

Comparison of results obtain by the Clarke and Wright algorithm and distribution routes operated by Graphic Communication Group Limited (GCGL) vehicles using an objective which combines the total time travelled and unloading time at each district in Table 4.6, shows that all the six vehicle routes obtained by the algorithm are better; in which the total travel time is 21.9% shorter than the actual distribution routes maintained by GCGL office it Ashanti region.



CHAPTER 5

CONCLUSION AND RECOMMENDATION

5.1 Conclusion

The Newspaper distribution problem of Graphic Communication Group Limited (GCGL) office in Ashanti region was modeled as capacitated vehicle routing problem with time window (CVRPTW) to reflect the characteristics of the problem.

Construction of distribution routes in order to minimize delivery time of newspapers to all the district capitals in the region was based on Clarke and Wright savings algorithm. The algorithm is integrated in computer software called VRP SOLVER which is used to analyze the data collected from the GCGL office in the region with time constraints to obtain optimal solution for this problem. Table 5.1 shows the routes for the distribution vehicles to all the district capitals in Ashanti region and the total to complete the tour.

Table 5.1 Vehicle Routes to the district capitals.

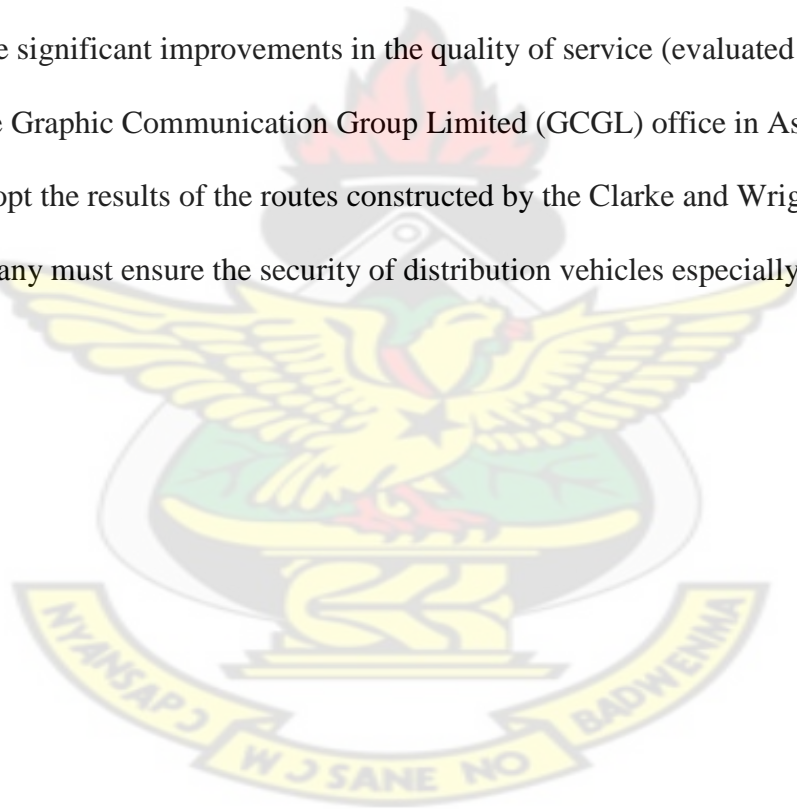
| Vehicle | Route | Total Time (mins.) |
|---------|---|--------------------|
| 1 | Kumasi – Jacobu – Obuasi – New Edubiase – Fomena – Kumasi. | 176 |
| 2 | Kumasi – Kodie – Offinso – Akomandan – Ejura–Mamponteng–Kumasi. | 178 |
| 3 | Kumasi – Nkawie – Nyinahin – Tapa – Mankranso – Kumasi. | 167 |
| 4 | Kumasi – Kuntense – Bekwai – Foase –Manso Nkwanta– Kumasi. | 164 |
| 5 | Kumasi – Asiswa – Juaso – Konongo – Ejisu – Kumasi. | 151 |
| 6 | Kumasi – Effiduase – Kumawu – Nsuta – Mampong – Agona– Kumasi. | 168 |

The results obtain from the algorithm showed improvements of 22% of overall total time traveled compared to the routes operated by distribution vehicles of GCGL hence satisfying the objective of the study.

5.2 Recommendation

Upon a study of application of Clarke and Wright savings algorithm to newspaper distribution of GCGL office in Ashanti region as Capacitated Vehicle Routing Problem Time Window (CVRPTW), the following recommendation could be considered:

1. To achieve significant improvements in the quality of service (evaluated by delivery times), the Graphic Communication Group Limited (GCGL) office in Ashanti region should adopt the results of the routes constructed by the Clarke and Wright algorithm.
2. The company must ensure the security of distribution vehicles especially the commercial ones.



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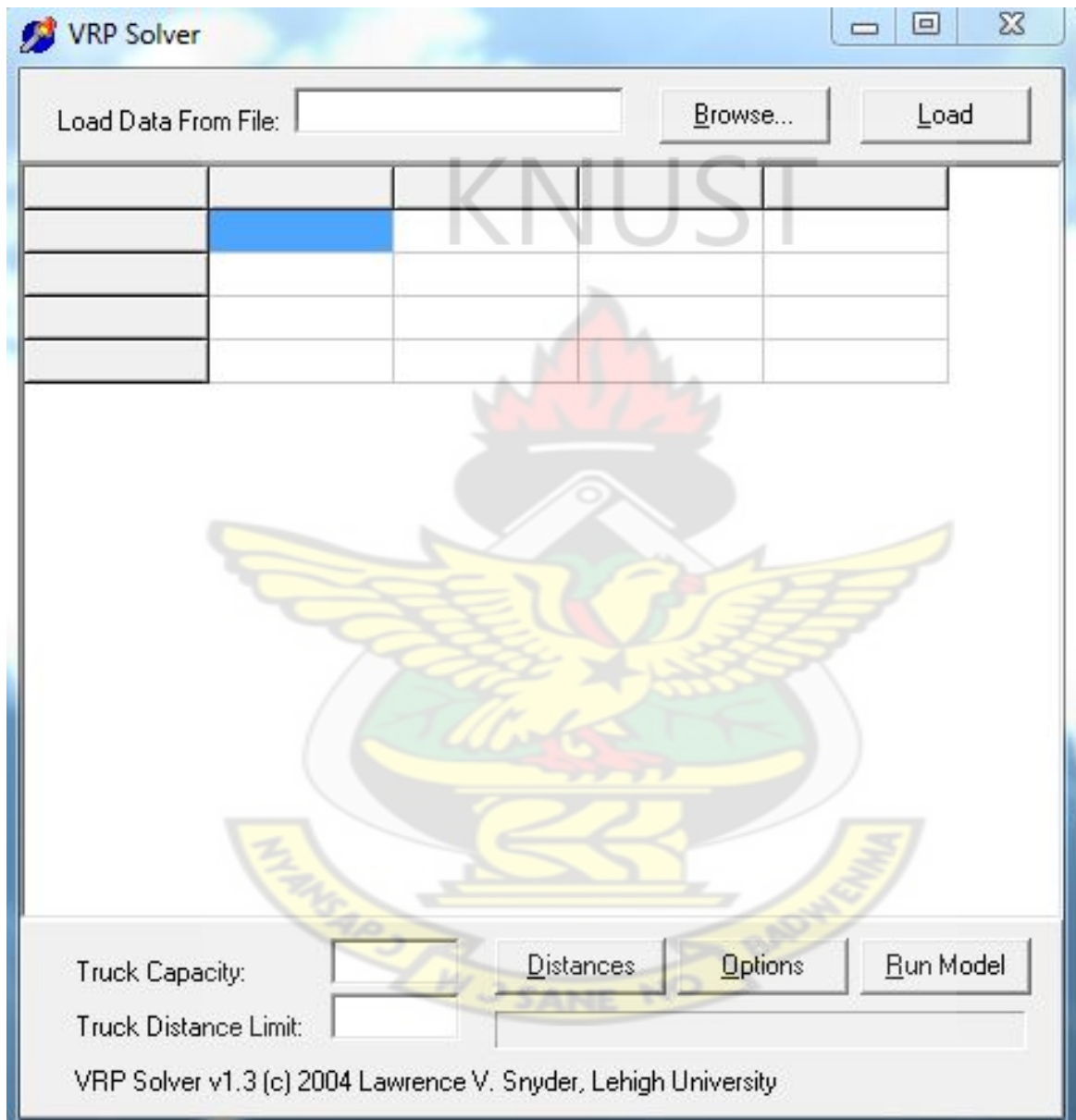
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APPENDICES

Appendix A. Main screen of VRP SOLVER.



Appendix B. Distance Matrix Window.

Distances

Distance Preferences
 Euclidean Great Circle Round Distances to Nearest Integer
 Load From File: C:\Users\Wallace\Documents\symr Browse...
 Generate Distances
 Save Distances

Distance matrix reflects current settings.

| | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 |
|----|--------|--------|-------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|-------|--------|-------|--------|--------|
| 1 | 50.00 | 73.00 | 14.00 | 77.00 | 32.00 | 40.00 | 66.00 | 43.00 | 54.00 | 26.00 | 95.00 | 27.00 | 33.00 | 55.00 | 30.00 | 13.00 | 36.00 | 18.00 | 54.00 | 57.00 |
| 2 | 0.00 | 23.00 | 64.00 | 127.00 | 82.00 | 28.00 | 64.00 | 75.00 | 86.00 | 24.00 | 105.00 | 77.00 | 17.00 | 44.00 | 38.00 | 63.00 | 86.00 | 68.00 | 104.00 | 20.00 |
| 3 | 23.00 | 0.00 | 87.00 | 150.00 | 105.00 | 51.00 | 87.00 | 85.00 | 96.00 | 47.00 | 128.00 | 100.00 | 40.00 | 54.00 | 61.00 | 86.00 | 109.00 | 91.00 | 127.00 | 36.00 |
| 4 | 64.00 | 87.00 | 0.00 | 91.00 | 46.00 | 54.00 | 80.00 | 57.00 | 68.00 | 40.00 | 69.00 | 41.00 | 47.00 | 69.00 | 44.00 | 27.00 | 50.00 | 32.00 | 58.00 | 71.00 |
| 5 | 127.00 | 150.00 | 91.00 | 0.00 | 45.00 | 117.00 | 143.00 | 120.00 | 131.00 | 103.00 | 35.00 | 63.00 | 110.00 | 132.00 | 107.00 | 90.00 | 113.00 | 95.00 | 131.00 | 134.00 |
| 6 | 82.00 | 105.00 | 46.00 | 45.00 | 0.00 | 72.00 | 98.00 | 75.00 | 86.00 | 58.00 | 80.00 | 59.00 | 65.00 | 87.00 | 62.00 | 45.00 | 68.00 | 50.00 | 86.00 | 89.00 |
| 7 | 28.00 | 51.00 | 54.00 | 117.00 | 72.00 | 0.00 | 54.00 | 73.00 | 84.00 | 14.00 | 95.00 | 67.00 | 15.00 | 42.00 | 36.00 | 53.00 | 76.00 | 58.00 | 94.00 | 27.00 |
| 8 | 64.00 | 87.00 | 80.00 | 143.00 | 98.00 | 54.00 | 0.00 | 105.00 | 116.00 | 40.00 | 121.00 | 93.00 | 47.00 | 74.00 | 68.00 | 79.00 | 102.00 | 84.00 | 120.00 | 71.00 |
| 9 | 75.00 | 85.00 | 57.00 | 120.00 | 75.00 | 73.00 | 105.00 | 0.00 | 11.00 | 65.00 | 98.00 | 70.00 | 58.00 | 31.00 | 40.00 | 30.00 | 79.00 | 61.00 | 80.00 | 91.00 |
| 10 | 86.00 | 96.00 | 68.00 | 131.00 | 86.00 | 84.00 | 116.00 | 11.00 | 0.00 | 76.00 | 109.00 | 81.00 | 69.00 | 42.00 | 51.00 | 41.00 | 90.00 | 72.00 | 91.00 | 102.00 |
| 11 | 24.00 | 47.00 | 40.00 | 103.00 | 58.00 | 14.00 | 40.00 | 65.00 | 76.00 | 0.00 | 81.00 | 53.00 | 7.00 | 34.00 | 28.00 | 39.00 | 62.00 | 44.00 | 80.00 | 31.00 |
| 12 | 105.00 | 128.00 | 69.00 | 35.00 | 80.00 | 95.00 | 121.00 | 98.00 | 109.00 | 81.00 | 0.00 | 28.00 | 88.00 | 110.00 | 85.00 | 68.00 | 91.00 | 73.00 | 109.00 | 112.00 |
| 13 | 77.00 | 100.00 | 41.00 | 63.00 | 59.00 | 67.00 | 93.00 | 70.00 | 81.00 | 53.00 | 28.00 | 0.00 | 60.00 | 82.00 | 57.00 | 40.00 | 63.00 | 45.00 | 81.00 | 84.00 |
| 14 | 17.00 | 40.00 | 47.00 | 110.00 | 65.00 | 15.00 | 47.00 | 58.00 | 69.00 | 7.00 | 88.00 | 60.00 | 0.00 | 27.00 | 21.00 | 46.00 | 69.00 | 51.00 | 87.00 | 33.00 |
| 15 | 44.00 | 54.00 | 69.00 | 132.00 | 87.00 | 42.00 | 74.00 | 31.00 | 42.00 | 34.00 | 110.00 | 82.00 | 27.00 | 0.00 | 48.00 | 61.00 | 91.00 | 73.00 | 109.00 | 60.00 |
| 16 | 38.00 | 61.00 | 44.00 | 107.00 | 62.00 | 36.00 | 68.00 | 40.00 | 51.00 | 28.00 | 85.00 | 57.00 | 21.00 | 48.00 | 0.00 | 31.00 | 66.00 | 48.00 | 84.00 | 54.00 |
| 17 | 63.00 | 86.00 | 27.00 | 90.00 | 45.00 | 53.00 | 79.00 | 30.00 | 41.00 | 39.00 | 68.00 | 40.00 | 46.00 | 61.00 | 31.00 | 0.00 | 49.00 | 31.00 | 59.00 | 70.00 |
| 18 | 86.00 | 109.00 | 50.00 | 113.00 | 68.00 | 76.00 | 102.00 | 79.00 | 90.00 | 62.00 | 91.00 | 63.00 | 69.00 | 91.00 | 66.00 | 49.00 | 0.00 | 18.00 | 33.00 | 93.00 |
| 19 | 68.00 | 91.00 | 32.00 | 95.00 | 50.00 | 58.00 | 84.00 | 61.00 | 72.00 | 44.00 | 73.00 | 45.00 | 51.00 | 73.00 | 48.00 | 31.00 | 18.00 | 0.00 | 36.00 | 75.00 |
| 20 | 104.00 | 127.00 | 58.00 | 131.00 | 86.00 | 94.00 | 120.00 | 80.00 | 91.00 | 80.00 | 109.00 | 81.00 | 87.00 | 109.00 | 84.00 | 59.00 | 33.00 | 36.00 | 0.00 | 111.00 |
| 21 | 20.00 | 36.00 | 71.00 | 134.00 | 89.00 | 27.00 | 71.00 | 91.00 | 102.00 | 31.00 | 112.00 | 84.00 | 33.00 | 60.00 | 54.00 | 70.00 | 93.00 | 75.00 | 111.00 | 0.00 |
| 22 | 77.00 | 100.00 | 13.00 | 104.00 | 59.00 | 67.00 | 93.00 | 70.00 | 81.00 | 53.00 | 82.00 | 54.00 | 60.00 | 82.00 | 57.00 | 40.00 | 61.00 | 43.00 | 45.00 | 84.00 |
| 23 | 91.00 | 114.00 | 55.00 | 118.00 | 73.00 | 81.00 | 107.00 | 84.00 | 95.00 | 67.00 | 96.00 | 68.00 | 74.00 | 96.00 | 71.00 | 54.00 | 41.00 | 23.00 | 59.00 | 98.00 |
| 24 | 110.00 | 133.00 | 74.00 | 137.00 | 92.00 | 100.00 | 126.00 | 58.00 | 69.00 | 86.00 | 115.00 | 87.00 | 93.00 | 89.00 | 78.00 | 47.00 | 61.00 | 43.00 | 54.00 | 117.00 |
| 25 | 106.00 | 129.00 | 60.00 | 133.00 | 88.00 | 96.00 | 122.00 | 68.00 | 79.00 | 82.00 | 111.00 | 83.00 | 89.00 | 99.00 | 82.00 | 51.00 | 45.00 | 38.00 | 12.00 | 113.00 |
| 26 | 91.00 | 114.00 | 55.00 | 118.00 | 73.00 | 81.00 | 107.00 | 45.00 | 56.00 | 67.00 | 96.00 | 68.00 | 74.00 | 76.00 | 59.00 | 28.00 | 42.00 | 24.00 | 35.00 | 98.00 |
| 27 | 85.00 | 108.00 | 39.00 | 112.00 | 67.00 | 75.00 | 101.00 | 61.00 | 72.00 | 61.00 | 90.00 | 62.00 | 68.00 | 90.00 | 65.00 | 40.00 | 35.00 | 17.00 | 19.00 | 92.00 |