KWAME NKRUMAH UNIVERSITY OF SCIENCE AND TECHNOLOGY

APPLICATION OF LINEAR PROGRAMMING TO OPTIMAL CREDIT PORTFOLIO: THE CASE OF AKUAPEM RURAL BANK LTD.

BY

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(PG 3014109)

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DECLARATION

I hereby declare that this thesis is the result of my own research work towards the Master of Science certification under the supervision of Dr. Francis T. Oduro. It contains no material previously published by another person or material which has been accepted for the award of any other degree of the University, except where due acknowledgement has been made in the text.

Kwapong, Samuel Darkwa ……………………… ……………………..

PG 3014109 Signature Date

(Student Name and ID)

Certified by:

Dr. Francis T. Oduro ……………………. …………………….

(Supervisor) Signature Date

Certified by:

Mr. Kweku Darkwah ……………………… ………………………

(Head of Department) Signature Date

ABSTRACT

Efficient credit portfolio management is a key success factor of bank management. The effective management of credit risk is an essential component of a comprehensive technique to risk management and critical to the long-term success of all banking institutions.

This thesis seeks to formulate and solve a linear programming model to maximize expected credit yield subject to capacity and demand, develop risk related scenarios and interpret results in the light of credit management issues. Data was obtained from the credit and finance section of Akuapem Rural Bank Limited, Mamfe Akuapem, Ghana and was analyzed using the management scientist software to ascertain the performance of its portfolio in achieving targets been set in the contest of the LP model.

The research revealed a positive relationship between risk and the expected return on each facility of the Bank as the expected return on an asset increases as the risk on the asset also increases. The study showed that an increment in the interest rate of the bank's products yielded high values and exhibited low risk.

DEDICATION

I dedicate this study with all my love to my wife, Mrs. Evelyn Kwapong, my parents, Emmanuel A. Kwapong and Beatrice Angela Terkwor Nuertey, Master Kingsley Azu-Matey Kwapong and the Management of Akuapem Rural Bank Limited.

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CHAPTER ONE

INTRODUCTION

1.1 Background to the study

The majority of financial institutions' and banks' losses stem from outright default due to inability of customers to meet obligations in relation to lending, trading, settlement and other financial transactions. Alternatively, banks also face losses as a result of a fall in financial value of their assets due to actual or perceived deterioration in asset credit quality during recession or crisis.

In addition, bank losses can sometimes be due to unethical behavior of its staff which had happened from time to time as with Societe Generale, Barclays and Stanchart banks. The world financial market has evolved with instruments that banks and financial institutions have lost track of their real identity causing chaos in the industry. It is essential that banks manage these risks so as to reduce losses and ensure continued existence in the longer term. One major risk that needs to be effectively managed and investigated is credit risk. Bank failures, acquisitions, and consolidations have encouraged surviving banks to take a closer look at how to structure operations, build loan portfolios and improve asset quality. The forecast of a default by any credit portfolio manager is the key factor to consider by every financial institution.

Ghana's employment of the directed credit approach from the 1960's to 1980's was to intervene in the rural credits markets thus leveraging credits to farmers and rural micro enterprises. The establishment of a separate credit department tasked with the mobilization of investment funds for small scale and agriculture sectors of the

Ghanaian economy in the early 1960's by the Bank of Ghana, was part of the several attempts to implement the approach. The Ghanaian government through the central bank embarked on measures to promote effective rural development through the Directed Credit Approach. A yield of such an intervention is the establishment of the Agricultural Development bank (ABD), the first of its kind in the country as a development finance Institution. The Agricultural Development Bank (originally Agricultural Credit and Co-operative Bank) was established in 1965, by Act of Parliament to meet the banking needs of the Ghanaian agricultural sector in a profitable manner. It was given the mandate to channel resources in the form of external grants and central government funds to the small rural sector on concessionary terms (Yaron, 1994).

Despite great efforts by successive Ghanaian governments to see the success of such an initiative, amidst the imposition of a sectorial credit allocation by commercial banks, only a little above 8% of development finance institutions and commercial banks prioritized the small scale and agriculture sectors.

According to Okyere (1990), the institutional rural credit for smallholders was "non-existent" by the end of the 1960's.

The traditional commercial banks failed to be attracted to the rural sector because they believed that the rural folks were mostly of the low-income group, and were scattered over a wide and almost inaccessible areas. This coupled with the fact that such rural poor could not provide the required collateral security necessary to support effective commercial financial operations, served as a disincentive to the commercial banks. The central Bank of Ghana after a comprehensive study of the rural credit situation decided to make way for the rural banking program in the 1970's.

Rural Banks are unit banks which are owned and managed by residents in a community. They are registered under the Company's Code and are licensed by the Bank of Ghana to engage in the business of banking. Being unit banks, they are not allowed to open branches but are permitted to open agencies within their catchments areas for the purpose of mobilizing deposits.

The basic functions of Rural Banks are the mobilization of savings and the extension of credit to deserving customers in their areas of operation. It is also the belief of the Central Bank that through their financial intermediation roles, Rural Banks will act as catalysts for economic development in rural Ghana.

A lot of progress has been made since the first rural bank was established in 1976. Deposits have been mobilized, loans have been granted, the habit of savings and thrift have been inculcated in the minds of our rural dwellers, jobs have been created for our rural people as managers, accountants, project officers, cashiers, drivers, cleaners, etc. In addition, Rural Banks have developed products that facilitate fast and reliable means of moving funds from one part of the country to another through the extensive network of Rural Banks in the country.

These are all very noble achievements. But it has not been all rosy. There have been many problems. Out of the 137 Rural Banks currently operating in the country, approximately 70 have been classified as operating satisfactorily by the Central Bank of Ghana. The performances of 50 have been described as mediocre and the rest will need close monitoring and nurturing to avoid being closed down by the Central Bank.

The financial sector in Ghana is becoming increasingly competitive. In addition to the universal banks extending their services to the rural areas through their branch networks, there are institutions like Savings and Loans Companies, Finance Companies, Credit Unions, Susu Companies, Mutual Funds all of which are in the business of mobilizing deposits and granting loans. The Akuapem Rural Bank was officially commissioned on 29th August 1980 as the 12th rural bank to be established in Ghana and the 3rd in the Eastern Region. The Bank mobilizes savings from rural areas and on-lead these resources to customers in these areas for the improvement of their business and welfare. It is the mission of the bank to mobilize local resources and to use them through the credit instrument and innovative financial services to respond to the essential/developmental needs of Akuapem at the same time that the bank grows and profits are made for shareholders.

1.2 Problem Statement

The global financial crisis in 2008 has adversely increased concentrations and credit portfolio volatility (losses) in the financial sectors of the world including Ghanaian banks.

Rural banks on the other hand have had unfavorable environments and capacity constraints to mobilize scattered rural incomes at a high cost into savings. In spite of this they lend to the people with virtually no collateral to support such credits. Lending small sums of money to very poor people, who have no credit history or can offer no other security, is very expensive and risky. The consequences are that some rural banks have not only experienced liquidity problems but also customer deposits have dwindled which had led to poor financial performance and subsequent closure. Again loans are not channeled to the required sectors and cooperate social responsibilities of most rural banks have non-existent.

1.3 Objectives of the study

The study intends to analyze the credit portfolio of Akuapem Rural Bank Ltd. The objective of this research is to determine the maximum net returns which comprises of the difference between the revenue from interest and lost funds due to bad debts. The bad debt is not recoverable both as principal and interest thus reducing the total revenue. It is also to use the optimum solution to determine among the different clientele and ones that are recommended and the ones that are least attractive and make appropriate recommendations to enhance the performance of the bank's credit portfolio.

1.4 Methodology

Empirical data from the credit section of the Akuapem Rural Bank Ltd, Mamfe Akuapem will be collected and analyzed for how the portfolio fared in achieving its set targets-profits, and losses.

The Management Scientist software will be used to quantify and analyze the Bank's credit characteristics for the various customer categories amidst consideration of the risk of the portfolios.

1.5 Justification

The goals and objectives of all financial institutions if well addressed will bring about high profit margin and growth in market share, necessary for the institutions survival in the global financial world.

The study will not only bring to light critical performance indications of the bank to assist prospective investors in their investment decisions but also help in deposits growth and strengthen cooperate and social responsibilities of the Akuapem Rural Bank Ltd in its catchment area.

1.6 Organization of Thesis

The thesis is in five chapters. The first chapter is the research proposal. It covers the background, the statement of the problem, objectives of the study, methodology, justification, and the organization of the study.

The second chapter reviews the related and relevant literature.

The chapter three focuses on the methodology of the study and the profile of Akuapem Rural Bank Ltd, the case study of the research. The forth chapter is on the presentation and analysis of the data obtained from the field. The final chapter, the chapter five, is based on summary, recommendation and conclusion made by the researcher. The references and appendices will conclude the chapter.

CHAPTER TWO

LITERATURE REVIEW

2.0 Introduction

Most financial institutions find that loans are the largest and most obvious source of credit risk; however, other sources of credit risk exist throughout the activities of a bank. Financial institutions are increasingly facing credit risk in various financial instruments other than loans, including acceptances, trade financing, foreign exchange transactions, inter-bank transactions, financial futures, options, bonds, equities, swaps and in the extension of commitments and guarantees.

Since exposure to credit risk continues to be the leading source of problems in banks world-wide, banks and their regulators should be able to draw useful lessons from the past experiences. Banks should now have a keen awareness of the need to identify, measure, monitor and control credit risk as well as to determine that they hold adequate capital against these risks. It is also vital that they are adequately compensated for the risks incurred in the running of their businesses.

The Basel Committee's capital adequacy guideline aims to encourage global banking supervisors to promote sound practices for managing credit risk. The list include: (1) establishing an appropriate credit risk environment; (2) operating under a sound credit-granting process; (3) maintaining an appropriate credit administration, measurement and monitoring process; and (4) ensuring adequate controls over credit risk. A comprehensive list of procedures and recommendations by Basel II Framework can be found in Basel II: International Convergence of Capital Measurement and Capital Standards: A Revised Framework, Bank for International

Settlements. The three major pillars include minimum capital requirements, supervisory review process and market discipline. Due to the importance of credit risk management approaches, Claessens, Krahnen and Lang (2005) stressed that Basel II proposal is to encourage banks to upgrade these practices and banks with sufficiently sophisticated risk management systems have more flexibility to use their own internal systems to determine regulatory capital minimums.

Although specific credit risk management practices may differ among banks depending upon the nature and complexity of their credit activities, a comprehensive credit risk management program should address all these issues. Implementation of the credit risk management strategies should also be applied in conjunction with sound practices related to the assessment of asset quality, the adequacy of provisions and reserves, and the disclosure of credit risk.

As well illustrated by the theoretical models of O'Brien (1983) and recently modified to better reflect current practices, a set of guidelines is released to promote better understanding of credit agreements to assist the banking industry to improve their services. These guidelines include full disclosure of credit history, independent credit analysis, legal consideration, sharing credit information between agents, and prompt response to problems. Based on another study by Wu and Huang (2007) top management support is most important for risk management and mechanism to be successful.

2.0.1 Credit Criteria

Credit criteria are factors employed to determine a borrower's creditworthiness or ability to repay debt. These factors include income, amount of existing personal debt, number of accounts from other credit sources, and credit history. A lender is free to use any credit-related factor in approving or denying a credit application so long as it does not violate the equal credit protections of the Bank of Ghana prohibiting credit discrimination.

Swarens (1990) suggested that the most pervasive area of risk is an overly aggressive lending practice. It is a dangerous practice to extend lending terms beyond the useful life of the corresponding collateral. Besides that, giving out loans to borrowers who are already overloaded with debt or possess unfavorable credit history can expose banks to unnecessary default and credit risk. In order to reduce these risks, banks need to take into consideration some common applicants' particulars such as debt to income ratio, business history and performance record, credit history, and for individual loan applicants their time on the job or length of time at residence.

2.0.2 Credit Culture

A bank's credit culture is the unique combination of policies, practices, experiences, and management attitudes which defines the lending environment and determines the lending behavior acceptable to the bank. A credit culture is the glue that holds credit related issues together. More broadly, credit culture is the system of behavior, beliefs, philosophy, thought, style, and expression relating to the management of the credit function. It consists of a policy that guides credit ethic, a practice that drives lending and an audit that protects assets and credit mechanism. Any glitch in one would bring problem to the other.

Banks' hiring practices should ensure personnel are committed to strict professional ethics and comfortable in high ethical standard and behavioral environment. Mueller (1984, 1990) stressed the significance of installing a sound credit culture in order to track banks' lending strategy and objectives. This study found that interactive parts of a credit culture must match with and be built upon proven principles and high standards. On the other hand, it must also be sufficiently flexible to compensate for change (Mueller 1993). Similar to Morsman (1985), Swarens (1990) found that credit culture has emerged as an important determinant of credit quality for all types of lending. Subordinates have to be responsible and professional in order to prevent from being bias when evaluating loan applications. Management must also ensure that the reward system compensates good ethical practices and penalizes unacceptable and flawed procedures.

2.0.3 Training

Training refers to the acquisition of knowledge, skills, and competences as a result of the teaching of vocational or practical skills and knowledge that relates to specific useful skills. Training and development is the field concerned with workplace learning to improve performance. Such guidance can be generally categorized as on-the-job or off-the-job training. On-the-job describes training that is given in a normal working situation, using the actual tools, equipment, documents or materials that can be used in the actual environment. On-the-job training is usually most effective for vocational work. Off-the-job training takes place away from normal work situations which means that the employee is not regarded as a productive worker when training is taking place. An advantage of the off-the-job training is that it allows people to get away from work and totally concentrate on the training provided. This type of training is most effective for training concepts and ideas. Officers should also have good knowledge of consumer protection laws and the ability to identify alternative solutions to problems.

Swarens (1990) believes that a fully trained consumer loan officer should have superior presentation skills, good knowledge of consumer protection laws to reduce the risk of committing a discriminatory mistake. Loan officers should also have the ability to identify remedies to a problem to ensure that consumers received the best type of advice and service. Ethical standards and behavior among subordinates can also be enhanced by training and promotional opportunities.

In another study by Morsman (1985), the paper concluded that bank managers must learn from past mistakes and must be equipped with skills and knowledge to master the fundamentals of loan administration. Farrisey (1993) suggested a training program for community banks to avoid the stormy seas of imprudent lending.
This study provided a list of programs that include financial statement analysis, credit information exchange, credit reports, loan documentation and others which are essential skills required for the job.

2.0.4 Diversification

Diversification in banking involves spreading investments into a broader range of financial services or loans: business, personal, credit card, mortgage, auto and educational loan. Diversification reduces both the upside and downside potential and allows for more consistent performances under a wide range of economic conditions. Diversifications can be performed across products, industries and countries.

According to Sanford (1985), diversification can reduce risk rapidly with diversification of investment at no cost in expected profits. In other words, the business enterprise that diversified is more likely to be profitable. Total risks of loan provision fall as a variety of loan products and borrowings from different industries increase, assuming the correlation between markets is not perfect. The paper concludes that regulation provides safety and ensures soundness of the banking system.

Swarens (1990) also found that diversification by product line can be achieved by offering a wide variety of lending services. Wyman (1991) acknowledged that bank lending is basically a risky business but as long as banks have a systematic strategy to measure linkages and co-variances among industries and regions, they should be able to maximize their return at an acceptable level of risk.

2.0.5 Risk Mitigation

Risk mitigation encompasses a variety of techniques for loss prevention, loss control, and claims management. A risk mitigation program can prevent losses and reduce the cost of losses while creating a safer environment for businesses. Banks use a number of techniques to mitigate the credit risks to which they are exposed. Exposures may be collaterized by first priority claims with cash or securities. A loan exposure may be guaranteed by a third-party, or a bank may even purchase credit derivatives to offset various forms of credit risk.

A study by Blomquist (1984) reckons that loan portfolio risk can be reduced with an effective credit review of applicants and selective asset backing. When creditors have extensive rights to repossess collateral assets, there is stronger possibility that they can reduce their risks of losses on one hand and borrowers will be more responsible to pay when their assets are at stake.

2.1 Types of Credit

The principle classes of credit are as follows :mercantile or commercial credit, which merchants extend to one another to finance production and distribution of goods; investment credit, used by business firms to finance the acquisition of plant and equipment and represented by corporate bonds, long-term notes, and other proofs of indebtedness; bank credit, consisting of the deposits ,loans, and discounts of depository institutions; consumer or personal credit, which comprises advances made to individuals to enable them to meet expenses or to purchase, on a deferred-payment basis, goods or services for personal consumption; public or government credit, represented by the bond issues of national, state and municipal governments, by the nationals of foreign country, or by international banking institutions, such as the international bank for reconstruction and development.

2.2 Function of Credit

The principal function of credit is to transfer property from those who own it to those who wish to use it, as in the granting of loans by banks to individuals who plan to initiate or expand a business venture. The transfer is temporary and is made for a price, known as interest, which varies with the risk involved and also with the demand for, and supply of, credit. Credit transactions have been indispensable to the economic development of the modern world. Credit puts to use property that would otherwise lie idle, thus enabling a country to more fully employ its resources. One of the most significant differences between some nations of Africa, Asia, and South America and the advanced Western nations is the extent to which the use of credit permits the latter to keep their savings continuously at work. The presence of credit institutions rests on the readiness of people to trust one another and of courts to enforce business contracts. The lack of adequate credit facilities makes it natural and necessary for inhabitants of developing countries to hoard their savings instead of putting them to productive and profitable use. Without credit, the tremendous investments required for the development of the large-scale enterprise on which the high living standards of the West are based would have been impossible. The use of credit also makes feasible the performance of the complex operations involved in modern business without the constant handling of money. Credit operations are carried out by means of documents known as credit instruments, which include bills of exchange, money orders, checks, drafts, promissory notes, and bonds. These instruments are usually negotiable; they may legally be transferred in the same way as money. When the party issuing the instrument desires to prevent its use by anyone other than the party to whom it is issued, he or she may do so by inscribing the words “not negotiable” on the instrument.

2.3 Credit Management

Credit Management refers to the efficient blend of the four major credit policy variables to ensure prompt collection of loans granted to customers and at the same time boost their confidence in and loyalty to the bank (Van Horne, 1995). The first variable is the assessment of the quality of the customer account. This examines the ability of the customers to repay on time. The second policy variable is that of setting the credit period. In so doing, the bank ought to give enough time to allow the customers derive the full benefits of the credit. Such period must not be too long to put the bank at a disadvantage. The third variable is the discount or the enticement to credit beneficiaries to repay credit on time. Such enticement must be motivating enough before the aim can be achieved. The last variable considers the expenditure level that could be incurred in the collection exercise. This implies that the bank must not grant credit where the amount to be expended on collecting the debt will likely be greater than the debt itself. To blend these variables into an efficient workable system requires careful planning, controlling and co-ordination of all available human and material resources.

According to Asiedu-Mante (2011) credit management involves establishing formal legitimate policies and procedures that will ensure that: the proper authorities grant credit, the credit goes to the right people, the credit is granted for the productive activities or for businesses which are economically and technically viable, the appropriate size of credit is granted, the credit is recoverable and there is adequate flow of management information within the organization to monitor the credit activity.

Office of the Comptroller of the Currency defined Loan portfolio management as the process by which risks that is inherent in the credit process are managed and controlled. It involves evaluating the steps bank management takes to identify and control risk throughout the credit process. The assessment focuses on what management does to identify issues before they become problems. Office of the Comptroller of the Currency identified nine elements that should be part of a loan portfolio management process. The nine elements are:

1. Assessment of the credit culture,
2. Portfolio objectives and risk tolerance limits,
3. Management information systems,
4. Portfolio segmentation and risk diversification objectives,
5. Analysis of loans originated by other lenders,
6. Aggregate policy and underwriting exception systems,
7. Stress testing portfolios,
8. Independent and effective control functions,
9. Analysis of portfolio risk/reward trade-offs

The development of credit risk evaluation models over the years has gone through comparable analysis, statistical analysis and artificial intelligence. Below is a brief introduction of the key assumptions and values of various credit risk evaluation models.

2.3.1 Comparable Analysis in Credit Risk Management

The traditional credit risk evaluation criteria link credit risk with the default event. The key point is data mining the characteristics of both default and non-default companies to establish the identification equations and categorize the samples. The representative model of this stage is 5C analysis—character, capacity, capital, collateral and condition. People try to make a full qualitative analysis about the obligator's willingness and capability of payback from five aspects. Early models usually suffer from the subjective, empiricism and lack of objective assessments.

2.3.2 Statistical Analysis in Credit Risk Management

After Fisher's research on heuristics, there developed quickly and enormously credit risk evaluation models based on statistics, of which most represented is Edward Ahman's Z-score. Ahman observed manufacturing companies near or far from bankruptcy in 1968 and took 22 financial ratios to establish the most famous five Z-score based on the mathematical statistical screening.

2.3.3 Artificial Analysis in Credit Risk Management

With the fast development of information technology, recent years have seen large artificial intelligence models that have been incorporated in the credit risk analysis. For instance, neural networks as a self-organizing, self-adapting and self-learning non-parameter method are very robust and accurate in predicting especially does not rigidly follow normal

2.4 Credit Administration

A typical credit administration unit of a financial Institution performs following functions:

a. **Documentation**. It is the responsibility of credit administration to ensure completeness of documentation (loan agreements, guarantees, transfer of title of collaterals etc) in accordance with approved terms and conditions. Outstanding documents should be tracked and followed up to ensure execution and receipt.

b. **Credit Disbursement**. The credit administration function should ensure that the loan application has proper approval before entering facility limits into computer systems. Disbursement should be effected only after completion of covenants, and receipt of collateral holdings. In case of exceptions necessary approval should be obtained from competent authorities.

c. **Credit monitoring**. After the loan is approved and draw down allowed, the loan should be continuously watched over. These include keeping track of borrowers' compliance with credit terms, identifying early signs of irregularity, conducting periodic valuation of collateral and monitoring timely repayments.

d. **Loan Repayment**. The obligors should be communicated ahead of time as and when the principal/markup installment becomes due. Any exceptions such as non-payment or late payment should be tagged and communicated to the management. Proper records and updates should also be made after receipt.

e. **Maintenance of Credit Files**. Institutions should devise procedural guidelines and standards for maintenance of credit files. The credit files not only include all correspondence with the borrower but should also contain sufficient information necessary to assess financial health of the borrower and its repayment performance. It need not mention that information should be filed in organized way so that external / internal auditors or SBP inspector could review it easily.

f. **Collateral and Security Documents**. Institutions should ensure that all security documents are kept in a fireproof safe under dual control. Registers for documents should be maintained to keep track of their movement.

Procedures should also be established to track and review relevant insurance coverage for certain facilities/collateral. Physical checks on security documents should be conducted on a regular basis.

2.5 Issuance of Credit

Creditors sometimes require no other assurance of repayment than the debtor's credit standing, that is, one's record of honesty in fulfilling financial obligations and one's current ability to fulfill similar obligations. Sometimes more tangible security, such as the guarantee of a third party, is required. Also, the debtor may be obliged to assign the rights to some other property, which is at least equal in value to the loan, as collateral security for payment. Bonds placed on sale by a corporation are often secured by a mortgage on the corporation's property or some part of it. Public borrowing, as by the issuance of bonds of a government, is usually unsecured, resting on the purchaser's confidence in the good faith, taxing power, and political stability of the government. When goods are sold on a deferred-payment plan, the seller may either retain legal ownership of the goods or hold a chattel mortgage until the final payment is made. The depositing of funds in a bank for safekeeping may also be regarded as a form of credit to the bank, as such funds are used for loan and investment purposes, and the bank is legally bound to repay them as an ordinary debtor.

2.6 Letter of Credit

Letter of Credit, document issued by a bank authorizing the bearer to receive money from one of its foreign branches or from another bank abroad. The order is nonnegotiable, and it specifies a maximum sum of money not to be exceeded.

Widely used by importers and exporters, the letter of credit is also made available to tourists by their home banks so that they may draw foreign currency while traveling abroad. When the instrument is directed to more than one agent, it is called a circular letter of credit.

Letters of credit are the most common means of small business financing, but they are an important financing tools for companies that engage in international trade.

2.7 Rural Banks and Credit Schemes

Andrews (2009) indicated that modern microfinance arose as a response to the fact that banks were not extending credit to the rural poor of developing countries, and as a result people were forced to endure the usurious practices of moneylenders. Though several microfinance programs came into being in different parts of the world at around the same time, the most celebrated of these —The Grameen Bank— traces its origin to 1974-1975, when a famine raged in the countryside of Bangladesh. At the time, Dr. Muhammad Yunus, the founder of Grameen Bank and co-recipient of the 2006 Nobel Peace Prize, was an economics professor at Chittagong University in rural Bangladesh. As Yunus writes in his book "Creating a World without Poverty", he found it increasingly difficult to teach elegant theories of economics and the supposedly perfect workings of the free market in the university classroom while needless death was ravaging Bangladesh. Yunus decided to go into the villages to find out why the poor were not able to bring themselves out of poverty. What he saw were "people working hard to try to help themselves—growing crops in their tiny yards, making baskets, stools and other craft items to sell, and offering their services for practically any kind of labor." Yet despite their efforts, the majority of people were unable to increase their income. Yunus' great insight came to him thanks to a village woman, who showed him that the main problem lay in the fact that “she relied on the local moneylender for the cash she needed to buy the bamboo for her stools [that she sold in the market]. But the moneylender would give her the money only if she agreed to sell him all she produces at a price he would decide. Between this unfair arrangement and the high interest on her loan, she was left with only two pennies a day as her income.”

Armed with this knowledge, Yunus and a student began identifying all the people in the village who were receiving loans from the moneylender, and found that dozens of villagers—42, in total—were borrowing an average of a measly $1.55 USD per person per week. He realized that if they could borrow the money at a reasonable interest rate, they would be able to pay back the loan and increase their income. To test the idea, he lent $26 USD to those 42 villagers, and after a successful pilot, began looking for ways of dramatically expanding the practice of micro lending. Yunus went to the local bank and suggested to the manager that he would provide loans to these poor people he had met in the village. The manager refused, arguing that the poor have no collateral, are not creditworthy and cannot read or write.
Also in other banks Yunus met the same mixture of unwillingness and disbelief. One bank was finally prepared to give loans, on the condition that Yunus provided collateral – and only after warning him several times that he would lose his money. Yunus however experienced the opposite. After a while the people who had received his loans paid back every last taka.

Because the bankers continued to refuse to grant loans to the poor without collateral from Yunus, he decided to start his own bank: a Grameen (village) Bank. He realized however that he had to develop a system that would guarantee that those who had received a loan would pay back. To ask for collateral was difficult as these poor people have hardly anything to offer as security. He devised a system of “social collateral”. Those women who wish to receive a loan need to organize themselves in groups from 5 – 10 persons. One of the group members receive a loan, and upon repayment the next one, and so on. If one member has problems repaying, the others need to assist. Thus, solidarity, cooperation and social control replaced the traditional collateral. Employees of the Grameen Bank go out to their clients – on foot, by bike or by bus – to become acquainted with them in the surroundings of their own businesses. It turns out that poor borrowers in general are very faithful and repay their debts right on time. The average repayment is higher than 95 percent, whereas banks in developing countries are accustomed to having to write off much larger percentages of their debts.

The emergence of microfinance in Ghana is not a recent phenomenon. Traditionally, people have been saving and taking loans from local lenders or group of people with the aim of starting their own businesses or for farming purposes (Asiama et al., 2007). The authors confirmed that the Canadian Catholic missionaries in 1955 first established a credit union in northern Ghana. This was the first in Africa. However Susu which is another form of microfinance scheme in Ghana is thought to have originated from Nigeria to Ghana in the early twentieth century. Asiama et al., (2007) have broadly categories micro finance institutions in Ghana into three. That is the formal suppliers such as rural and community banks, semi-formal such as credit unions and informal suppliers like susu collectors.

Rosenberg (2004) have also indicated that the exclusion of the poor from general financial services sector of the economies have resulted in the creation of micro finance institutions to address this market failure. Financial sector policies and programmes by previous governments led to the establishment of rural and community banks. These banks were required to set aside 20% of total portfolio, to promote lending to agriculture and small scale industries in the 1970s and early 1980s (Asiama J. P. et al., 2007). The bank of Ghana promoted the establishment of rural banks in Ghana, with the aim of promoting banking habits among rural households and mobilizing rural savings for agriculture, fishing, forestry and other agro-based industries.

Owusu-Frimpong (2007) asserts that rural and community banks offer both financial and non- financial services to the communities they serve. The financial services include lending, savings and other banking services such as cheque clearing for cocoa farmers. Non-financial services include supply of agricultural input to farmers. The rural banks grant loans that are payable within 4-12 months, and accept flexible payments on weekly, bi-weekly and monthly bases. In some loan applications, the banks often require the beneficiaries to be already involved in some economic activity and to deposit about 25 percent of the intended amount before the loan is approved.

The bank of Ghana promoted the establishment of rural banks in Ghana, with the aim of promoting banking habits among rural households and mobilizing rural savings for agriculture, fishing, forestry and other agro-based industries.

Addiah (2010) stated that rural banks in Ghana were created to re-enforce the government's commitment to rural credit as part of a national strategy to improve agriculture and the living conditions of rural farms. This scheme was to assist the rural dwellers especially small-scale farmers to find solutions to rural credit problems thereby increasing their output.

Asiama et al., (2007) noted that rural and community banks also play crucial role in microfinance in the country. Rural and community banks were established purposely to advance loans to small enterprises, farmers, individuals and others within their catchment areas.

Rural banks are autonomous, community based organizations regulated by the bank of Ghana. They were originally set up as community managed development banks to mobilize rural savings, furnish credit and support community initiatives. Rural banks are committed by mandate, to serve small scale economic enterprises in their service areas, although few of their clients are women and most loans are larger than the less than 300 dollars characteristics of poverty lending Programmes such as credit with education.

2.8 Default rates

Balogun and Alimi (1988) see loan default as inability of a borrower to fulfill his or her loan obligation as at when due. The costs of loan delinquencies would be felt by both the lenders and the borrowers. The lender has costs in delinquency situations, including lost interest, opportunity cost of principal, legal fees and related costs. For the borrower, the decision to default is a trade-off between the penalties in lost reputation from default versus the opportunity cost of forgoing investments due to working out the current loan.

According to Hempel and others (1994) the lending function represent the most diverse and complex activity, and the solvency and profitability of every bank largely depend on how efficient the risk of default is managed and control.

CHAPTER THREE

METHODOLOGY

3.0 INTRODUCTION

The cost of holding risk matters to every organization. Most financial decisions: whether on capital structure, dividends, investments, etc., revolve around the costs of holding risk. This issue is particularly important to banks since risk management constitutes their core business. By its very nature, banking is an attempt to manage multiple and seemingly opposing needs. Banks provide liquidity on demand to depositors through the current account and extend credit as well as liquidity to their borrowers through lines of credit. The objective function for portfolio selection problems is usually maximization of expected return or minimization of risk. The constraints usually take the form of restrictions on the type of permissible investments, state laws, company policies, maximum permissible risk. This chapter we will involve the use of linear programming to formulate and solve a portfolio selection problem.

3.1 The Model

A bank has the opportunity to finance a portfolio (continuum) of loans whose size L is derived endogenously. It funds lending out of internal funds (capital) and outside finance from investors. The supply of outside finance is perfectly elastic at a gross rate of return that is normalized to one (i.e. the risk-free net interest rate is zero).

That is, final investors are assumed to make zero profits. The bank acts on behalf of its shareholders (insiders), whose equity holdings constitute the bank's endowment of inside capital.

E.g. the total amount paid for goods X and Y if the price for each is 20 and 5 respectively is given by 20X + 5Y.

3.1.1 A mathematical program

A mathematical program is of the form;

Find numbers x_1 and x_2 that maximize the sum $x_1 + x_2$

subject to the constraints

$x_1 \geq 0$, $x_2 \geq 0$, and

$x_1 + 2x_2 \leq 4$

$4x_1 + 2x_2 \leq 12$

$-x_1 + x_2 \leq 1$

In this problem there are two unknowns, and five constraints. All the constraints are inequalities and they are all linear in the sense that each involves an inequality in some linear function of the variables.

The first two constraints, $x_1 \geq 0$ and $x_2 \geq 0$, are special. These are called non-negativity constraints and are often found in linear programming problems. The other constraints are then called the main constraints. The function to be maximized (or minimized) is called the objective function. Here, the objective function is $x_1 + x_2$.

3.2 Linear Programming

Linear programming problems consist of a linear cost function (consisting of a certain number of variables) which is to be minimized or maximized subject to a certain number of constraints. The constraints are linear inequalities of the variables used in the cost function. The cost function is also sometimes called the objective function.

More formally, given a polytope (for example, a polygon or a polyhedron), and a real-valued affine function

$f(x_1,x_2,\ldots,x_n) = c_1x_1 + c_2x_2 + \ldots + c_nx_n + d$

defined on this polytope, a linear programming method will find a point in the polytope where this function has the smallest (or largest) value. Such points may not exist, but if they do, searching through the polytope vertices is guaranteed to find at least one of them.

Linear programs are problems that can be expressed in canonical form:

Maximize $C^T x$

Subject to $Ax \leq b$

X represents the vector of variables (to be determined), while c and b are vectors of (known) coefficients and A is a (known) matrix of coefficients. The expression to be maximized or minimized is called the objective function ($Ax \leq b$ in this case).

The equations $c^T x$ are the constraints which specify a polyhedron over which the objective function is to be optimized.

3.2.1 Standard form

Standard form is the usual and most intuitive form of describing a linear programming problem. It consists of the following three parts:

- A linear function to be maximized

  e.g. maximize $C_1x_1 + C_2x_2$

- Problem constraints of the following form

  e.g. $a_{11}x_1 + a_{12}x_2 \leq b_1$

$a_{21}x_1 + a_{22}x_2 \leq b_2$

$a_{31}x_1 + a_{32}x_2 \leq b_3$

- Non-negative variables

  e.g. $x_1 \geq 0$

  $x_2 \geq 0$.

The problem is usually expressed in matrix form, and then becomes:

maximize $c^T x$

subject to $Ax \leq b, x \geq 0$.

Other forms, such as minimization problems, problems with constraints on alternative forms, as well as problems involving negative variables can always be rewritten into an equivalent problem in standard form.

Example

Suppose that a farmer has a piece of farm land, say A square kilometers large, to be planted with either wheat or rice or some combination of the two. The farmer has a limited permissible amount F of fertilizer and P of insecticide which can be used, each of which is required in different amounts per unit area for wheat (F_1, P_1) and rice (F_2, P_2). Let S_1 be the selling price of wheat, and S_2 the price of rice. If we denote the area planted with wheat and rice by x_1 and x_2 respectively, then the optimal number of square kilometers to plant with wheat versus rice can be expressed as a linear programming problem:

Maximize $S_1x_1 + S_2x_2$ (maximize the revenue as the objective function)

Subject to $x_1 + x_2 \leq A$ (limit on total area)

$F_1x_1 + F_2x_2 \leq F$ (limit on fertilizer)

$P_1x_1 + P_2x_2 \le P$ (limit on insecticide)

$x_1 \ge 0, x_2 \ge 0$ (cannot plant a negative area)

This in matrix form becomes:

Maximize

$$\begin{bmatrix} S_1 & S_2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

subject to

$$\begin{bmatrix} 1 & 1 \\ F_1 & F_2 \\ P_1 & P_2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \le \begin{bmatrix} A \\ F \\ P \end{bmatrix}, \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \ge 0.$$

3.2.2 Augmented form (slack form)

Linear programming problems must be converted into augmented form before being solved by the simplex algorithm. This form introduces non-negative slack variables to replace inequalities with equalities in the constraints. The problem can then be written in the following block matrix form:

Maximize Z in:

$$\begin{bmatrix} 1 & -\mathbf{c}^T & 0 \\ 0 & \mathbf{A} & \mathbf{I} \end{bmatrix} \begin{bmatrix} Z \\ \mathbf{x} \\ \mathbf{x}_s \end{bmatrix} = \begin{bmatrix} 0 \\ \mathbf{b} \end{bmatrix}$$

$$X, X_s \ge 0$$

where X_S are the newly introduced slack variables, and Z is the variable to be maximized.

The example above is converted into the following augmented form:

Subject to $x_1 + x_2 + x_3 = A$ (augmented constraint)

$F_1x_1 + F_2x_2 + x_4 = F$ (augmented constraint)

$P_1x_1 + P_2x_2 + x_5 = P$ (augmented constraint)

$x_1, x_2, x_3, x_4, x_5 \geq 0$

where x_3, x_4, x_5 are (non-negative) slack variables, representing in this example the unused area, the amount of unused fertilizer, and the amount of unused insecticide. In matrix form this becomes Maximize Z in:

$$\begin{bmatrix} 1 & -S_1 & -S_2 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & F_1 & F_2 & 0 & 1 & 0 \\ 0 & P_1 & P_2 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} Z \\ x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} 0 \\ A \\ F \\ P \end{bmatrix}, \quad \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} \geq 0.$$

The standard form of a linear programming problem with n variables and m constraints can be represented as follows:

Max (min) z= $c_1x_1 + c_2x_2 + \cdots c_nx_n$

Subject to $\quad a_{11}x_1 + a_{12}x_2 + \cdots a_{1n}x_n = b_1$

$$a_{21}x_1 + a_{12}x_2 + \cdots + a_{2n}x_n = b_2$$

$$a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n = b_n$$

$x_j \geq 0$,j=1,2…n

3.2.3 Conversion into Standard Form

To develop a general solution method, the linear programming problem must first be recast in the standard form and this could arrive at in the following way. Taking the general form of the linear programming problem that is we incorporate new variable t≥0 to convert inequality into equations. In order to replace the inequality constraints by equality constraint, we introduce extra variable $t_1 \geq 0$ called a slack variable or surplus variable. A variable added to the left hand side of a "less than or equal to" constraint to convert the constraint into an equation is referred to as a slack variable and is given as follows

$$\sum_{j=1}^{n} a_{ij} x_j + t_i = b_i,\ 1 \le i \le \mathrm{p}$$

A variable $t_1 \geq 0$ subtracted from the left hand side of a greater than or equal to constraint to convert an inequality constraint into equation is referred to as a surplus Variable and is given as $\sum_{j=1}^{n} a_{ij} x_j - t_i = b_i$, p+1≤i≤k

3.2.4 Transformation of General Variables to Satisfy non-negativity Condition

Further, if there exist any variable x_i which does not satisfy the non-negativity condition $x_j \geq 0$, we put

x_i^{*}= max $(x_i, 0)$

x_i^{-}= -min $(x_i, 0)$

And replace each x_i by x_i^{+}-x_i^{-} throughout the problem. The result is a linear programming problem in standard form. Before this is solved, the right hand side of the constraint equation is made non-negative by multiplying through each constraint by (-1) if necessary.

3.3 Duality

Every linear programming problem, referred to as a primal problem, can be converted into a dual problem, which provides an upper bound to the optimal value of the primal problem. In matrix form, we can express the primal problem as:

Maximize $c^T x$

Subject to $Ax \le b, x \ge 0,$ with the corresponding symmetric *dual problem*:

Minimize $b^T y$ subject to $A^T y \ge c, y \ge 0.$

An alternative primal formulation is:

Maximize $c^T x$ subject to $Ax \le b,$ with the corresponding asymmetric dual problem; minimize $b^T y$ subject to $A^T y = c, y \ge 0$

There are two ideas fundamental to duality theory. One is the fact that (for the symmetric dual) the dual of a dual linear program is the original primal linear program. Additionally, every feasible solution for a linear program gives a bound on the optimal value of the objective function of its dual. The weak duality theorem states that the objective function value of the dual at any feasible solution is always greater than or equal to the objective function value of the primal at any feasible solution. The strong duality theorem states that if the primal has an optimal solution, x*, then the dual also has an optimal solution, y*, such that $c^T x^* = b^T y^*$.

A linear program can also be unbounded or infeasible. Duality theory tells us that if the primal is unbounded then the dual is infeasible by the weak duality theorem. Likewise, if the dual is unbounded, then the primal must be infeasible. However, it is possible for both the dual and the primal to be infeasible.

Example

Revisiting the example of the farmer who may grow wheat and rice with the set provision of some *A* land, *F* fertilizer and *P* insecticide. Assume now that unit prices for each of these means of production (inputs) are set by a planning board. The planning board's job is to minimize the total cost of procuring the set amounts of inputs while providing the farmer with a floor on the unit price of each of his crops (outputs), S_1 for wheat and S_2 for rice. This corresponds to the following linear programming problem:

minimize $Ay_A + F\ y_F + Py_P$		(minimize the total cost of the means of production as the "objective function")
Subject to	$y_A + F_1 y_F + P_1 y_P \geq S_1$	(the farmer must receive no less than S_1 for his wheat)
	$y_A + Fy_{2F} + P_2 y_P \geq S_2$	(the farmer must receive no less than S_2 for his rice)
	$y_A \geq 0, y_F \geq 0, y_P \geq 0$	(prices cannot be negative).

This in matrix form becomes:

Minimize

$$\begin{bmatrix} A & F & P \end{bmatrix} \begin{bmatrix} y_A \\ y_F \\ y_P \end{bmatrix}$$

subject to

$$\begin{bmatrix} 1 & F_1 & P_1 \\ 1 & F_2 & P_2 \end{bmatrix} \begin{bmatrix} y_A \\ y_F \\ y_P \end{bmatrix} \geq \begin{bmatrix} S_1 \\ S_2 \end{bmatrix}, \quad \begin{bmatrix} y_A \\ y_F \\ y_P \end{bmatrix} \geq 0.$$

The primal problem deals with physical quantities. With all inputs available in limited quantities, and assuming the unit prices of all outputs is known, what quantities of outputs to produce so as to maximize total revenue? The dual problem deals with economic values. With floor guarantees on all output unit prices, and assuming the available quantity of all inputs is known, what input unit pricing scheme to set so as to minimize total expenditure? To each variable in the primal space corresponds an inequality to satisfy in the dual space, both indexed by output type. To each inequality to satisfy in the primal space corresponds a variable in the dual space, both indexed by input type.

The coefficients that bound the inequalities in the primal space are used to compute the objective in the dual space, input quantities in this example. The coefficients used to compute the objective in the primal space bound the inequalities in the dual space, output unit prices in this example.

Both the primal and the dual problems make use of the same matrix. In the primal space, this matrix expresses the consumption of physical quantities of inputs necessary to produce set quantities of outputs. In the dual space, it expresses the creation of the economic values associated with the outputs from set input unit prices.

Since each inequality can be replaced by equality and a slack variable, this means each primal variable corresponds to a dual slack variable, and each dual variable corresponds to a primal slack variable. This relation allows us to complementary slackness.

In another example, one may sometimes find it more intuitive to obtain the dual program without looking at program matrix. Consider the following linear program:

minimize $\sum_{i=1}^{m} c_i x_i + \sum_{j=1}^{n} d_j t_j \geq g_j, 1 \leq j \leq n$

subject to $\sum_{i=1}^{m} a_{ij} x_i + e_j t_j \geq g_j, 1 \leq j \leq n$

$f_i x_i + \sum b_{ij} t_j \geq h_i, 1 \leq i \leq m$

$x_i \geq 0, t_j \geq 0, 1 \leq i \leq m, 1 \leq j \leq n$

We have $m + n$ conditions and all variables are non-negative. We shall define $m + n$ dual variables: y_j and s_i. We get:

Minimize $\sum_{i=1}^{m} c_i x_i + \sum_{j=1}^{n} d_j t_j$

Subject to $\sum_{i=1}^{m} a_{ij} x_i * y_j + e_j t_j * y_j \geq g_j * y_j, 1 \leq j \leq n$

$f_i x_i * s_i + \sum_{j=1}^{n} b_{ij} t_j * s_i \geq h_i * s_i, 1 \leq i \leq m$

$x_i \geq 0, t_j \geq 0, 1 \leq i \leq m, 1 \leq j \leq n$

$y_j \geq 0, s_i \geq 0, 1 \leq j \leq i \leq m$

Since this is a minimization problem, we would like to obtain a dual program that is a lower bound of the primal. In other words, we would like the sum of all right hand side of the constraints to be the maximal under the condition that for each primal variable the sum of its coefficients does not exceed its coefficient in the linear function. For example, x_1 appears in

n+1 constraints. If we sum its constraints' coefficients we get

$a_{11} y_1 + a_{12} y_2 + ... + a_{1n} y_n + f_1 s_1$. This sum must be at most c_1. As a result we get:

Maximize $\sum_{j=1}^{n} g_j y_j + \sum_{i=1}^{m} h_i s_i$

Subject to $\sum_{j=1}^{n} a_{ij} y_j + f_i s_i \leq c_i, 1 \leq i \leq m$

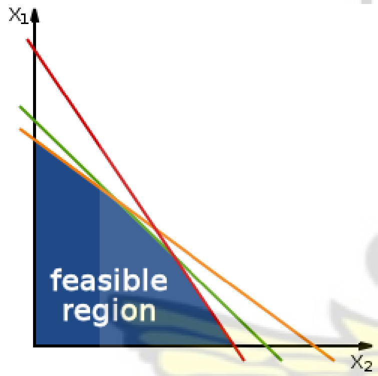
$g_j y_j + \sum b_{ij} s_i \leq d_j, 1 \leq j \leq n$

$y_j \geq 0, s_i \geq 0, 1 \leq j \leq n, 1 \leq i \leq m$

Note that we assume in our calculations steps that the program is in standard form. However, any linear program may be transformed to standard form and it is therefore not a limiting factor.

3.3.1 Feasible Region

The set of points in R^n satisfying all the constraints of the problem is called the feasible region of the problem. Any point in the feasible region is called a feasible point. If x_o denote a feasible point. If an inequality constraint $g_i(x_1 \ldots x_n) \leq b_i$, is satisfied as equality at x_0 i.e. $g_i(x_1, x_2 \ldots x_n) = b_i$, then the inequality constraint is binding or active at x_o. If the inequality constraint is satisfied as a strict inequality i.e. $g_i(x_1, x_2 \ldots x_n) \prec b_i$ then the inequality is inactive or not binding at x_o



A series of linear constraints on two variables produces a region of possible values for those variables. Solvable problems will have a feasible region in the shape of a simple polygon.

3.4 Simulated Annealing (SA)

SA consists of three parts: solution space, objective function and initial solution.

1. Solution Space: It is the group of all possible solutions and it restricts the scope of our choosing the initial solution and the new solution. In many optimization problems, besides objective functions, we also have a set of constraints. Hence; there might be some infeasible solution in the solution space. You can define the solution space exclusive of infeasible solutions or you can allow them by incorporating a penalty function to penalize the occurrence of the infeasible solution.

2. Objective function

It is the mathematical description of the optimization problem. Usually it is constructed as the sum of several optimization requirements and, when infeasible solutions are allowed, objective function needs to incorporate a penalty function.

3. Initial Solution:

It is the starting point of the algorithm. The final solution is independent of the choice of the initial solution.

3.5 The simplex Method

The simplex method refers to the idea of moving from one extreme point to another on the convex set that is formed by the constraint set and non-negativity conditions of the linear programming problem. The simplex method is algebraic in nature and is based upon the Gauss-Jordan elimination procedure.

3.5.1 Setting up the Initial Simplex Tableau

In developing a tabular approach for the simplex algorithm, we use an instructive and consistent set of notations that enhances the understanding of the process.

Terms Used

The terms used in the initial simplex tableau are as follows:

c_j= Objective function coefficients for variable j.

b_i= Right-hand-side coefficients (value) for constraint i

a_{ij}= Coefficients of variable j in constraint i

c_B= Objective function coefficients of the basic variables.

Notations which will be used extensively in the following development of the simplex method are as follows:

c row, the row of the objective function coefficients

b column, the column of the right-hand-side values of the constraints equations

[A] matrix, the matrix (with m rows and n columns) of the coefficients of the variables in the constraint equations.

3.5.2 The Simplex Algorithm

The simplex algorithm is an iterative procedure that provides a structured method for moving from one basic feasible solution to another, always maintaining the objective function until an optimal solution is obtained.

Slack variables x_1 and x_2 are restricted to be equal to be equal to or greater than zero. However, slack variables have no effect on the objective function. To solve a problem we sequentially generate a set of basic feasible solutions that corresponds to the extreme points of the feasible solution space. Naturally we first determine an initial basic feasible solution. Recall that a basic solution to a set of m equations in n variables (n>m) is obtained by setting (n-m) variables equal to zero and solving the resulting system of m equations in m variables.

The m variables are referred to as the basic variables or as the variables "in the basis". The variables are referred to as the non-basic variables or as the variables "not in the basis."

A basic feasible solution is defined as being a basic solution where the entire basic variable is non-negative (≥0).

A non-degenerate basic feasible solution is defined as being a basic solution, where all m of the basic variables are greater than zero (≥0)

Example

Maximize z = $6x_1$+$8x_2$+$0s_1$+$0s_2$

Subject to $5x_1$+$10x_2$= 60

$4x_1$+$4x_2$+$1s_1$= 40,x_1≥ 0, x_2≥ 0 ,s_1≥0, s_2≥0

In this formulation we observe that both slack variables s_1 and s_2 are restricted to be equal to or greater than zero.However,slack variables have no effect on the objective function since they are not associated with real products. To solve this problem algebraically we must be able to sequentially generate a set of basic feasible solutions that correspond to the extreme points of the feasible solution space. First, we must determine an initial basic feasible solution. A basic solution to a set of m equations in n variables (n>m) is obtained by setting (n-m) variables equal to zero and solving the resulting system of m equations in m variables. The m variables are referred to as the basic variables or as the variables "in the basis." A basic feasible solution is defined as being a basic solution where all m of the basic variables are non-negative (≥0).A non-degenerate basic feasible solution is defined as being a basic feasible, where all m of the basic variables are greater than zero (≥0).

The initial simplex tableau is a representation of the standard linear programming form, with some supplemental rows and columns. The top row of the table presents the objective function coefficients. Then, there are c_j-z_j in rows, which represent the constraint coefficients. Following is the z_j row. The row which is the net contribution per unit of the jth variable follows finally.

The leftmost column in the tableau indicates the values of the objective function coefficients associated with the basic variables, with a set of constraints, the initial column is headed "basic variable'. The next m+n columns contain the constraint coefficients; the final rightmost column displays the solution values of the basic variables.

Table 3.1 General Form-Initial Simplex Tableau

		Decision variables			Slack variables				
c_j		c_1	c_2	c_n	0	0	0	Solution	Objective function coefficients
c_B	Basic variables	x_1	x_2	x_n	1		s_m		Headings
0	s_1	a_{11}	a_{12}	a_{1n}	1	0	0		Constraints coefficient
…	s_2	a_{21}	a_{22}	a_{2n}	0	1	0		
0	…..	…..	…..	…..	…..	…..	…...		
	s_m	a_{m1}	a_{m2}	a_{mn}	0	0	1		
		z_1	z_2	z_{mn}	z_{11}		z_{1m}	Current value of objective function	
	c_j-z_j	c_1-z_1	c_2-z_2	c_{mn}-z_{mn}	c_{1_1}-z_{1_1}	c_{1_2}-z_{1_2}	c_{1_n}-z_{1_n}		Reduced cost(net contribution/unit)

3.5.3 Special Cases

Some special situations encountered in solving linear problems are:

1) Unbounded solutions where there is no limit on some entering variable, and hence there is unbounded profitability

2) No feasible solution, when constraints are so restrictive that there is no way of satisfying them.

Both of them can be detected during the simplex procedure. Both generally result from improper formulation of the problem or incorrect data. Multiple solutions to linear programming problems are possible. This is indicated when one of the outside variables has a c_j-z_j value of zero in the optimal solution.Degeneracy occurs when one or more of the solution variables equals zero.

3.5.4 Unbounded Solution

In the case of an unbounded solution the simplex method will terminate with the indication that the entering basic variable can do so only if it is allowed to assume a value of infinity. Specifically, for a maximization problem we will encounter a simplex tableau having a non-basic variable whose c_j-z_j row value is strictly greater than zero. And for this same variable all of the a_{ij} elements in its column will be zero or negative. Thus in performing the ratio test for the variable removal criterion, it will be possible only to form ratios having negative numbers or zeros as denominators.

If we have unbounded solution, none of the current basic variables can be driven from solution by the introduction of a new basic variable, even if that new basic variable assumes an infinitely large value. Generally arriving at an unbounded solution indicates that the problem was originally wrongly formulated within the constraints set and needs reformulation.

3.5.5 Multiple Solutions

There may be more than one optimal solution to a linear programming problem. In this case two or more basic solutions will have the same optimum profit or cost .In the optimal tableau of the example the solution has no c_j -z_j value negative, indicating an optimum has been reached. Imagine, however for example variable x_2,is not a solution variable and

c_j -z_j =0.this indicates that variable x_2 can be brought into the solution without increasing or decreasing the cost. The optimal solution lies within the corner points of the feasible area and any convex combination of these corner points is an optimum solution.

3.5.6 No feasible solution

The lack of a feasible solution can be detected in the simplex table. At some point in the procedure a solution occurs that would appear to be optimal. All the coefficients in the C_J -Z_J rows are non positive if maximizing or nonnegative if minimizing. However one of the solution variables is an artificial variable. Computer programs would stop at this point with a message that no feasible solution exist.

In a business situation the lack of a feasible solution generally indicate an error in formulating the problem or in entering the data.

3.5.7 Degeneracy

Theoretically, if there is a tie between two or more variables to be removed from the basis at some iteration prior to reaching the optimal solutions, a situation known as cycling can occur. Cycling can occur if the arbitrary choice between the tied variables for a variable to be removed from the basis will always generate a degenerate feasible solution in which all of the tied variables reach zero simultaneously s the entering basic variable is increased.

3.6 The simplex procedure

The simplex procedure involves the following processes:

3.6.1 Modify the constraints

Add a slack variable for each less than or equal to constraint. Add an artificial variable for each equality constraint. Add both an artificial and a surplus variable for each greater than or less than constraint. For each artificial variable, assign a very large cost (negative profit) in the objective function.

3.6.2 Initial Solution

Identification of the initial solution which composes of the slack and artificial variables.

3.6.3 Check for Optimality

The current solution is optimal if: for maximization all coefficients in the c_j - z_j, are zero or negative .for minimum all coefficient in c_j - z_j row are zero or positive. If optimum has been reached, stop the simplex procedure.

3.6.4 Entering Variable

This is the non-basic variable associated with the largest positive (maximum) or negative (minimum) coefficient in the c_j - z_j row.

3.6.5 Leaving variable

For each row, calculate the ratio of the values in the "solution values column" divided by the coefficient in the entering variable column. Ignore any ratios that are negative .the leaving variable is the row having the smallest ratio.

3.6.6 Resolve the equation

a) Identify the pivot element as the coefficient in the entering variable row.

b) Divide the entire coefficient in the leaving row by the pivot element.

c) Modify the other rows, possibly including the objective function row as

New row = old row – coefficient of entering variable column x row obtained in step 3.7.6(b).

Calculation of z_j, for the jth column: for each row, multiply the substitution coefficient by the c_j value for that row and sum. The total is z_j.repeat for all columns.

Calculation of c_j - z_j , Subtract the z_j from the original objective function coefficient at the top of the table.

3.6.7 Return to step 7

3.6.8 Economic interpretation of the c_j - z_j values

First consider the economic interpretation of the c_j - z_j values, the bottom row of the simplex table. Recall that z_j is the opportunity cost of introducing one unit of variable j into the solution .the cost of replacing or subtracting for other solution variables. Since c_j is the unit profit, the c_j - z_j value is the net profit resulting from introducing one unit of j into the solution.

3.7 Convex Region

A region of space is said to be convex if the portion of the line segment between any two points in the region also lies in the region.

3.7.1 Convex function

A function f(x) is said to be convex if the set of points (x, v) where $v \geq f(x)$ from a convex region.

3.7.2 Convex Mathematical Program

A mathematical programming model is said to be convex if it involves the minimization or maximization of a convex function over a convex function feasible region. A function which is non-differentiable but is a convex function can be minimize over a convex feasible region.

3.8 Sensitivity Analysis

Sensitivity Analysis is designed to study the effect of changes in the parameters of the linear programming model on the optimal solution. The ultimate objective of the analysis is to obtain information about possible new optimum solutions with minimal additional computations. An example is the worth of a revenue unit? This problem deals with the study of the sensitivity of the optimum solution to changes in the right hand side of the constraints. If the constraints represent a limited resource, the problem reduces to studying the effect of changing the availability of the resource. The specific goal of this sensitivity problem is to determine the effect of changes in the right hand side of constraints on the optimum objective value. In essence the results are given as predetermined ranges of the right hand side within which the objective optimum value will change at a given constant rate. It enables us to answer such questions as would you like to buy any more of the resource? If so what price should one pay? How many units should a consumer buy at that price? Similar questions can be asked about selling resources, even though a resource may be currently used in making products, at some price it is worthwhile to forgo products and sell it. These considerations are of interest because they lead to decisions that can increase profit or reduce cost.

Another part of the sensitivity analysis deals with impart of changes in available resources or other conditions which are expressed in the right side of constrains.

Sensitivity analysis involves,

1) First finding the shadow price, the marginal values for a unit change in the Right-Hand-Side value of the constraint

2) Second finding the range of values over which the shadow price holds (R.H.S Ranges)

3) Third which objective function coefficient ranges. These ranges indicate the changes in the objective function coefficient within the optimal solution remaining the same. In general, determining the sensitivity of the optimal solution to changes is easiest analytically when only one parameter changes.

3.9 Concept of Markowitz programming- Modern portfolio theory (MPT)

The fundamental concept behind MPT is that the assets in an investment portfolio should not be selected individually, each on their own merits. Rather, it is important to consider how each asset changes in price relative to how every other asset in the portfolio changes in price.

Investing is a tradeoff between risk and expected return. In general, assets with higher expected returns are riskier. For a given amount of risk, MPT describes how to select a portfolio with the highest possible expected return. Or, for a given expected return, MPT explains how to select a portfolio with the lowest possible risk (the targeted expected return cannot be more than the highest-returning available security, of course, unless negative holdings of assets are possible).

MPT therefore is a form of diversification and for specific quantitative definitions of risk or return; it explains how to find the best possible diversification strategy.

CHAPTER FOUR

DATA ANALYSIS AND MODELLING

4.0 INTRODUCTION

This chapter deals with the presentation and analysis of data which were collected from the credit department of the Akuapem Rural Bank Limited. The data was analyzed using the Management Scientist Software based on the objectives of the study. A brief explanation of the Bank's profile is followed by a formulation and solution of the bank's credit portfolio problem with five decision variables and four constraints. The solution of the problem continues involving five decision variables and a number of constraints.

In discussing the computer solution for each of these problems, we focus on the interpretation of the computer output including both the optimal solution and sensitivity analysis information.

4.1 Profile of Bank

Akuapem Rural Bank Limited was established in the year 1980. This was in response to the need and the concern to make institutional credit and other formal financial and banking services easily available to the people of Akuapem and its environs. It has its head office at Mamfe in the Akuapem North District of the Eastern Region. The main purpose of its inception was to help inculcate the habit of saving into the rural folk within its catchment areas and alleviate rural poverty by granting credit to the people. Today, the bank has four (4) branches (Mamfe – Akuapem, Nsawam, Koforidua and Adukrom) and three (3) agencies (Adawso-Akuapem, Larteh- Akuapem and Okorase-Akuapem) in the Eastern region of Ghana. The operations of the bank are mainly focused on income generating activities by advancing loans to its customers, especially, women to help them start businesses on their own. The bank has a total of 126 employees (53 males and 69 females) currently tasked to running affairs in the various branches and agencies.

Akuapem Rural Bank limited has a loan policy involving a total of Fifteen million Ghana Cedis (GH¢ 15,000,000.00). Being a full-service facility, the bank has been tasked to grant loans to different clientele. The following table 4.1 provides the types of loans, the interest rate charged by the bank, and the probability of bad debt as estimated from past experiences.

Table 4.1 Types of Loan and corresponding Interest Rate and Risk

Types of Loan	Interest Rate	Probability of bad debt
Cottage Industry Loan	0.28	0.04
Transport Loan	0.32	0.05
Agricultural Loan	0.30	0.02
Salary Loan	0.30	0.01
Micro Finance Loan	0.28	0.02

Bad debts are assumed unrecoverable and hence produce no interest revenue.

Competition with other financial institutions in the Eastern region has compelled the Akuapem Rural Bank

1. To resolve that at least 60% of funds be allocated to transport, Agricultural, and cottage industry loans in other to assist farmers in the region.

2. To allocate at least 40% of available funds to Salary and Micro Finance loans.

3. To state a policy specifying that the overall ration for bad debts on all loans may not exceed 0.03.

4.2 MODELLING

4.2.1 Mathematical Model

The decision variables of the model used are defined as follows:

x_1= Cottage Industry loan amount

x_2= Transport loan amount

x_3= Agricultural loan amount

x_4= Salary loan amount

x_5= Micro Finance loan amount

The Akuapem Rural Bank's objective is to maximize its returns comprising of the difference between the revenue from interest and lost funds due to bad debts .Since bad debts are not recoverable both as principal and interest the objective function may be written as

Let

x_j= units of returns for the jth group.

P_j=probability of bad debt for the jth group.

$I_j(1- P_j)x_j$= units of revenue for the jth group.

Net revenue = $\sum_{j=1}^{n} I_j(1-P_j)x_j$

Maximize: $\sum_{j=1}^{n} I_j(1-P_j)x_j$ =objective function

Incorporating data from Table 4.1 the objective function can be written as:

Maximize

$$Z= 0.28(1\text{-}0.04)x_1 + 0.32(1\text{-}0.05)x_2 + 0.30(1\text{-}0.02)x_3 + 0.30(1\text{-}0.01)x_4 + 0.28(1\text{-}0.02)x_5$$

This function simplifies to

$Z = 0.27x_1+0.30x_2+0.29x_3+0.30x_4+0.27x_5$

The problem has four constraints:

In accordance with current policy of the bank the four constraints may be constructed as follows.

1. Total funds: the total amount allocated for all loans is 15 million cedis.

$x_1 + x_2 + x_3+x_4+x_5 \le 15$

2. Salary and Micro Finance Loans (in millions)

$x_4+x_5 \ge 6$

3. Agricultural loans, Transport loans and Cottage Industry (in millions)

$x_1+x_2+x_3 \ge 9$

4. Limit on Bad debts

$$\frac{0.04x_1+0.05x_2+0.02x_3+0.01x_4+0.02x_5}{x_1+x_2+x_3+x_4+x_5} \le 0.03$$

Simplifying to

$0.01x_1+0.02x_2 - 0.01x_3-0.02x_4-0.01x_5 \le 0$

Non-negativity: $x_1, x_2, x_3, x_4, x_5 \ge 0$

4.2.2 Scenario 1: All four constraints

Maximize $z = 0.27x_1+0.30x_2+0.29x_3+0.30x_4+0.27x_5$

Subject to

$x_1 + x_2 + x_3+x_4+x_5 \le 15$

$x_4+x_5 \ge 6$

$x_1+x_2+x_3 \ge 9$

$0.01x_1+0.02x_2 - 0.01x_3-0.02x_4-0.01x_5 \le 0$

The solution for scenario 1 using the Management Scientist Software is given in Table 4.2.

Table 4.2 Solution for Four Constraints

VARIABLE	VALUE
x_1	0.000
x_2	7.000
x_3	2.000
x_4	6.000
x_5	0.000

The objective Function value = 4.480

Table 4.3 gives the Dual for the Decision Variables.

Table 4.3 Dual Prices

CONSTRAINT	SLACK/SURPLUS	DUAL PRICES
1	0.000	0.307
2	0.000	0.000
3	0.000	-0.013
4	0.000	0.333

Table 4.4 gives the objective coefficient ranges corresponding to the maintenance of the solution given in Table 4.3 after undergoing a Sensitivity Analysis.

Table 4.4 Objective Coefficient Ranges

VARIABLE	LOWER LIMIT	CURRENT VALUE	UPPER LIMIT
x_1	No Lower Limit	0.270	0.297
x_2	0.290	0.300	No Upper Limit
x_3	0.210	0.290	0.300
x_4	0.287	0.300	No Upper Limit
x_5	No Lower Limit	0.270	0.303

Sensitivity analysis for the Right Hand Side ranges of the constraints is shown in Table 4.5

Table 4.5 Right Hand Side Ranges

CONSTRAINT	LOWER LIMIT	CURRENT VALUE	UPPER LIMIT
1	15.000	15.000	18.000
2	No Lower Limit	6.000	6.000
3	7.500	9.000	9.000
4	-0.210	0.000	0.060

The objective function value indicates that the optimal solution to the problem will provide a maximum profit of 4.480 million cedis. Values of the variables for cottage Industry loans is 0.000, transport loans is 7.000, agricultural loans is 2.000, salary loans is 6.000 and microfinance is also 0.000.This shows that Transport, Agricultural and Salary loans are recommended and the rest i.e. Cottage Industry and Micro Finance loans are less attractive. The dual price associated with constraint 3 (i.e. -0.013) is negative which indicates that an increase in the amount allocated to the corresponding decision variable will have an unfavorable effect on the net return.

4.2.3 Scenario2: Five Constraints

Management's assessment of the optimal solution under scenario 1 and the onward deliberation on the objective function value calls for a critical look at the unallocated loans.

The Bank's management will not consider a solution without allocations for both the cottage industry and micro finance loans; hence the requests for a scenario with an increase in the interest rate of the micro finance loan facility. Management nonetheless is interested in increasing profits rather than a reduction. An increase of 3% of the interest rate of micro finance loan which happens to be a risky product and a requirement that the amount of money allocated to the facility must be at least 15% of the total loan portfolio will result in a new problem and solution as shown below: We then have$x_5 \geq$ 0.15 x15 or $x_5 \geq$ 2.25 and

Maximize z = $0.27x_1+0.30x_2+0.29x_3+0.30x_4+0.30x_5$

Subject to

$x_1 + x_2 + x_3+x_4+x_5 \leq 15$

$x_4+x_5 \geq 6$

$x_1+x_2+x_3 \geq 9$

$0.01x_1+0.02x_2 - 0.01x_3-0.02\, x_4-0.01x_5 \leq 0$

$x_5 \geq 2.25$

The solution for the problem in scenario 2 is given in Table 4.6.

Table 4.6 Solution for five constraints

VARIABLE	VALUE
x_1	0.000
x_2	6.250
x_3	2.750
x_4	3.750
x_5	2.250

The objective Function value = 4.473

Table 4.7 below gives the Dual for the Decision Variables.

Table 4.7 Dual Prices

CONSTRAINT	SLACK/SURPLUS	DUAL PRICES
1	0.000	0.307
2	0.000	0.000
3	0.000	-0.013
4	0.000	0.333
5	0.000	-0.003

Table 4.8 gives the objective coefficient ranges corresponding to the maintenance of the solution given in Table 4.7 after undergoing a Sensitivity Analysis.

Table 4.8 Objective Coefficient Ranges

VARIABLE	LOWER LIMIT	CURRENT VALUE	UPPER LIMIT
x_1	No Lower Limit	0.270	0.297
x_2	0.290	0.300	No Upper Limit
x_3	0.210	0.290	0.300
x_4	0.297	0.300	No Upper Limit
x_5	No Lower Limit	0.300	0.303

Sensitivity analysis for the Right Hand Side ranges of the constraints is shown in Table 4.9

Table 4.9 Right Hand Side Ranges

CONSTRAINT	LOWER LIMIT	CURRENT VALUE	UPPER LIMIT
1	15.000	15.000	19.125
2	No Lower Limit	6.000	6.000
3	6.938	9.000	9.000
4	-0.188	0.000	0.083
5	0.000	2.250	6.000

The objective function value indicates that the optimal solution to the problem will provide a maximum profit of 4.473 million cedis which according to the Bank's management is commendable. The optimal values of the decision variables are 0.000, 6.250, 2.750, 3.750, 2.250 for x_1 ,x_2,x_3,x_4 and x_5 respectively. Thus the bank should allocate 6.250 million cedis to transport loans, 2.750 million cedis to agricultural loans, 3.750 million cedis to salary loans and 2.250 million cedis to micro finance loans. Cottage industry loans should be discarded though the sensitivity analysis on both the objective coefficient and right hand side ranges indicated 0.297 and 19.125 respectively for its upper limit.

4.2.4 Scenario3: Six Constraints

The management of the bank wishes to include the cottage industry loan which had no allocations, it is therefore prudent to add another constraint so that all loans will be allocated. An assessment of the optimal solution from scenario 2 has resulted in

Management's addition of the requirement that the amount of loans allocated to the cottage industry should be at most 12% of the total loan portfolio rather than an increment in the interest rate.

We then have $x_1 \leq 1.8$

Adding this new constraint to the problem restructures it to be as follows:

Maximize z = $0.27x_1+0.30x_2+0.29x_3+0.30x_4+0.27x_5$

Subject to

$x_1 + x_2 + x_3+x_4+x_5 \leq 15$

$x_4+x_5 \geq 6$

$x_1+x_2+x_3 \geq 9$

$0.01x_1+0.02x_2 - 0.01x_3-0.02x_4-0.01x_5 \leq 0$

$x_5 \geq 2.7$

$x_1 \leq 1.8$

The solution for the problem using the Management Scientist Software is given in Table 4.10.

Table 4.10 Solution for Six Constraints

VARIABLE	VALUE
x_1	0.000
x_2	6.100
x_3	2.900
x_4	3.300
x_5	2.700

The objective Function value = 4.390

Table 4.11 below gives the Dual for the Decision Variables.

Table 4.11Dual Prices

CONSTRAINT	SLACK/SURPLUS	DUAL PRICES
1	0.000	0.307
2	0.000	0.000
3	0.000	-0.013
4	0.000	0.333
5	0.000	-0.033
6	1.800	0.000

Table 4.12 gives the objective coefficient ranges corresponding to the maintenance of the solution given in Table 4.11 after undergoing a Sensitivity Analysis.

Table 4.12 Objective Coefficient Ranges

VARIABLE	LOWER LIMIT	CURRENT VALUE	UPPER LIMIT
x_1	No Lower Limit	0.270	0.297
x_2	0.290	0.300	No Upper Limit
x_3	0.210	0.290	0.300
x_4	0.287	0.300	No Upper Limit
x_5	No Lower Limit	0.270	0.303

A Sensitivity analysis for the Right Hand Side ranges is shown in Table 4.13

Table 4.13 Right Hand Side Ranges

CONSTRAINT	LOWER LIMIT	CURRENT VALUE	UPPER LIMIT
1	15.000	15.000	19.350
2	No Lower Limit	6.000	6.000
3	6.825	9.000	9.000
4	-0.183	0.000	0.087
5	0.000	2.700	6.000
6	0.000	1.800	No Upper Limit

The objective function value section shows that the optimal solution to the problem will provide a maximum profit of 4.390 million cedis. The optimal values of the decision variables are x_1=0.000, x_2=6.100, x_3=2.900, x_4=3.300 and x_5 =2.700. Thus the bank should allocate 6.100 million cedis to transport loans, 2.900 million cedis to agricultural loans, 3.300 million cedis to salary loans and 2.700 million cedis to micro finance loans. Cottage industry loans should be discarded once more as was the case in scenario 2.

4.2.5 Scenario4: Increased interest rates for the most risky product.

Management still requests for the inclusion of cottage industry hence the need to increase its interest rates. A critical look at the sensitivity analysis of the loan facility under the previous scenarios calls for an increase of 4% of the rate on cottage industry which happens to be the riskiest product. This will result in a new problem and solution as shown below:

Maximize z = $0.31x_1+0.30x_2+0.29x_3+0.30x_4+0.27x_5$

Subject to, $x_1 + x_2 + x_3+x_4+x_5 \leq 15$

$x_4+x_5 \geq 6$, $x_1+x_2+x_3 \geq 9$, $0.01x_1+0.02x_2 - 0.01x_3-0.02\, x_4-0.01x_5 \leq 0$

$x_5 \geq 2.7$, $x_1 \leq 1.8$

Table 4.14 Increased interest rates for most risky product.

VARIABLE	VALUE
x_1	1.800
x_2	4.900
x_3	2.300
x_4	3.300
x_5	2.700

The objective Function value = 4.414

Table 4.15 below gives the Dual for the Decision Variables.

Table 4.15 Dual Prices

CONSTRAINT	SLACK/SURPLUS	DUAL PRICES
1	0.000	0.307
2	0.000	0.000
3	0.000	-0.013
4	0.000	0.333
5	0.000	-0.033
6	0.000	0.013

Table 4.16 gives the objective coefficient ranges corresponding to the maintenance of the solution given in Table 4.15 after undergoing a Sensitivity Analysis.

Table 4.16 Objective Coefficient Ranges

VARIABLE	LOWER LIMIT	CURRENT VALUE	UPPER LIMIT
x_1	0.297	0.310	No Upper Limit
x_2	0.290	0.300	0.320
x_3	No Lower Limit	0.290	0.300
x_4	0.287	0.300	No Upper Limit
x_5	No Lower Limit	0.270	0.303

A Sensitivity analysis for the Right Hand Side ranges is given in Table 4.17.

Table 4.17 Right Hand Side Ranges

CONSTRAINT	LOWER LIMIT	CURRENT VALUE	UPPER LIMIT
1	15.000	15.000	18.450
2	No Lower Limit	6.000	6.000
3	7.275	9.000	9.000
4	-0.147	0.000	0.069
5	0.000	2.700	6.000
6	0.000	1.800	8.700

The objective function value section shows that the optimal solution to the problem will provide a maximum profit of 4.414 million cedis. The optimal values of the decision variables are x_1=1.800, x_2=4.900, x_3=2.300, x_4=3.300 and x_5 =2.700. Thus the bank should allocate 1.800 million cedis to cottage industry, 4.900 million cedis to transport loans, 2.300 million cedis to agricultural loans, 3.300 million cedis to salary loans and 2.700 million cedis to micro finance loans.

4.2.6 Scenario5: Reduced mandates –Two Constraints.

Maintaining the interest rates of the products and reducing the mandates to two constraints will give the problem as follows:

Maximize z = $0.27x_1+0.30x_2+0.29x_3+0.30x_4+0.27x_5$

Subject to

$x_1 + x_2 + x_3+x_4+x_5 \leq 15$, $x_4+x_5\geq 6$

The optimal solution for the problem in scenario 5 is shown in Table 4.18 below.

Table 4.18 Reduced mandates- Two Constraints.

VARIABLE	VALUE
x_1	0.000
x_2	9.000
x_3	0.000
x_4	6.000
x_5	0.000

The objective Function value = 4.500

Table 4.19 gives the objective coefficient ranges corresponding to the maintenance of the solution given in Table 4.18 after undergoing a Sensitivity Analysis.

Table 4.19 Objective Coefficient Ranges

VARIABLE	LOWER LIMIT	CURRENT VALUE	UPPER LIMIT
x_1	No Lower Limit	0.270	0.300
x_2	0.300	0.300	No Upper Limit
x_3	No Lower Limit	0.290	0.300
x_4	0.270	0.300	0.300
x_5	No Lower Limit	0.270	0.300

The objective function value section shows that the optimal solution to the problem will provide a maximum profit of 4.500 million cedis. The optimal values of the decision variables are x_1=0.000, x_2=9.000, x_3=0.000, x_4=6.000 and x_5=0.000.The bank should allocate 9.000 million cedis to transport loans and 6.000 million cedis to salary loans. Cottage industry, Agricultural and Micro finance loans should be discarded.

4.2.7 Scenario6: Reduced mandate –One Constraint.

The problem becomes

Maximize z = 0.27x_1+0.30x_2+0.29x_3+0.30x_4+0.27x_5

Subject to

x_1 + x_2 + x_3+x_4+$x_5 \leq 15$

The solution is shown in Table 4.20.

Table 4.20 Reduced mandates- One Constraint.

VARIABLE	VALUE
x_1	0.000
x_2	15.000
x_3	0.000
x_4	0.000
x_5	0.000

The objective Function value = 4.500

Table 4.21 gives the objective coefficient ranges corresponding to the maintenance of the solution given in Table 4.20 after undergoing a Sensitivity Analysis.

Table 4.21 Objective Coefficient Ranges

VARIABLE	LOWER LIMIT	CURRENT VALUE	UPPER LIMIT
x_1	No Lower Limit	0.270	0.300
x_2	0.300	0.300	No Upper Limit
x_3	No Lower Limit	0.290	0.300
x_4	No Lower Limit	0.300	0.300
x_5	No Lower Limit	0.270	0.300

The objective function value section shows that the optimal solution to the problem will provide a maximum profit of 4.500 million cedis. The optimal values of the decision variables are x_1=0.000, x_2=15.000, x_3=0.000, x_4= 0.000 and x_5=0.000.The bank should allocate all funds to transport loans and discard the rest.

4.2.8 Scenario7: Reduced mandates –Increased Interest rates and One Constraint.

A critical look at the sensitivity analysis coupled with Management's insistence on profit increases recommends an increment in the interest rates of all the products by 3% and a reduction on the mandates to only one constraint gives the problem as below:

Maximize z = $0.30x_1+0.33x_2+0.22x_3+0.33x_4+0.30x_5$

Subject to

$x_1 + x_2 + x_3+x_4+x_5 \leq 15$

The optimal solution using the Management Scientist Software is shown in Table 4.22.

Table 4.22 Increased Rates and One constraint Reduced mandate.

VARIABLE	VALUE
x_1	0.000
x_2	15.000
x_3	0.000
x_4	0.000
x_5	0.000

The objective Function value = 4.950

Table 4.23 gives the objective coefficient ranges corresponding to the maintenance of the solution given in Table 4.22 after undergoing a Sensitivity Analysis.

Table 4.23 Objective Coefficient Ranges

VARIABLE	LOWER LIMIT	CURRENT VALUE	UPPER LIMIT
x_1	No Lower Limit	0.300	0.330
x_2	0.330	0.330	No Upper Limit
x_3	No Lower Limit	0.320	0.330
x_4	No Lower Limit	0.330	0.330
x_5	No Lower Limit	0.300	0.330

The objective function value indicates that the optimal solution to the problem will provide an annual return of 4.950 million cedis. The optimal values of the decision variables are x_1=0.000, x_2=15.000, x_3=0.000, x_4= 0.000 and x_5=0.000. Thus all funds should be allocated to transport loans and the rest discarded as per the solution of the scenario.

Table 4.24 Summary of All the Scenarios

SCENARIO	INTEREST RATE	OBJECTIVE FUNCTION VALUE	No.OF ALLOCATION
One	Remains the same	4.480	3
Two	Increased for Micro finance loan	4.473	4
Three	Remains the same	4.390	4
Four	Increased for most risky product	4.414	5

Table 4.25 Summary of the Scenarios with reduced mandates

SCENARIO	INTEREST RATE	OBJECTIVE FUNCTION VALUE	No. OF ALLOCATION
Five	Remains the same	4.500	2
Six	Remains the same	4.500	1
Seven	Three percent increase on all products	4.950	1

A critical assessment of the data in both tables 4.24 and 4.25 indicates that the highest profit of 4.950 million cedis was obtained under scenario seven which had a constraint and an increased interest rate. This result will not be conducive for the banks clients since there is only one allocation from this scenario and management should look at more allocations rather than profit only.

Scenarios one to four produced not only good profits but also a higher number of allocations. Scenario one produced a profit of 4.480 million cedis with three allocations, thus cottage industry and micro finance had no allocations since their interest rates were low. An addition of a constraint resulted in a profit of 4.473 million cedis and an increment of the allocations to four. Another constraint added with the aim of allocating all the loans also yielded a profit of 4.390 million cedis but still four allocations were made. The cottage industry loan facility was not allocated due to its high risk.

An increase in the interest rate of cottage industry to 4% as indicated in scenario four resulted in a profit of 4.414 million cedis and an allocation of all five facilities.

A reduction of the mandates to two and one constraint(s) in scenarios five and six respectively yielded the same profit amount of 4.500 million cedis even though the interest rates remained the same, thus either of the above solutions can be employed by the Bank’s management in other to maximize profits.

CHAPTER FIVE

CONCLUSIONS AND RECOMMENDATIONS

5.1 CONCLUSIONS

This study set out to assess the credit portfolio of Akuapem Rural Bank Limited in the Eastern Region. The main challenge of the study was that there had not been enough studies in the bank regarding their credit portfolio; hence there were little documentation in the area of optimal credit portfolio. This situation creates an uncertainty as to whether the bank will adhere to the results from this study to positively impact on managing its various credit facilities.

The Akuapem Rural Bank offers Salary loans, Micro Finance loans, Transport loans, Cottage industry loans and Agricultural loans to its customers and there is a positive relationship between risk and the expected return on each facility. The expected return on an asset increases as the risk on the asset also increases, this is evident when an investor takes some risk and is expected to be compensated with a higher return.

An increment in the interest rate of most of the products offered by the Bank yielded high values and exhibited low risk. The various solutions from the optimization model analysis has focused on maximizing profits for each scenario as explained in the summary and Management's adherence will help their credit portfolio. This model can be further improved if optimization is also done inside each asset class taking into account all the credit class of the assets.

5.2 RECOMMENDATIONS

According to the analysis from the present study and the results, the following items are suggested for an optimal credit portfolio for the Akuapem Rural Bank Limited.

1. Banks and other financial institutions (especially the Akuapem Rural Bank Limited) should employ scientific and mathematical methods in the conduct of their businesses, apart from loan disbursements.
2. The Akuapem Rural Bank's Management should consider increasing the interest rate of cottage industry loans in other to make it profitable as is the case with the other four facilities.

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APPENDIX A

DEFINITION OF TERMS

Constraint An equation or inequality that rules out certain combinations of decision variables as feasible solutions.

Constraint function The left-hand side of a constraint (i.e. the portion of the constraint containing the variables).

Objective function All linear programs have a linear objective function that is to be either maximized or minimized. It is used to measure the profit or cost of a particular solution.

Solution Any set of values for the variables.

Optimal solution A feasible solution that maximizes or minimizes the value of the objective function.

Non-negativity constraints A set of constraints that requires all variables to be nonnegative.

Mathematical model A representation of a problem where the objective and all the constraint conditions are described by mathematical expressions.

Linear program A mathematical model with a linear objective function, a set of linear constraints and nonnegative variables.

Linear function A mathematical expression in which the variables appear in separate terms and are raised to the first power.

Feasible solution A solution that satisfies all the constraints

Feasible region The set of all feasible solutions 81

Slack variable A variable added to the left-hand side of a less-than or equal to constraint to convert the constraint into an equality.

Unboundedness A maximization linear programming problem is said to be unbounded if the value of the solution may be made infinitely large without violating any of the constraints.

Sensitivity analysis The evaluation of how changes in the coefficients of a linear programming problem affect the optimal solution to the problem.

Range of optimality The range of values over which an objective function coefficient may vary without causing any change in the values of the decision variables in the optimal solution.

Dual price The improvement in the value of the optimal solution per unit increase in a constraint right-hand-side value.

Degeneracy When one or more of the basic variables have a value of zero.

Simplex tableau A table used to keep track of the calculations made when the simplex method is employed.