KWAME NKRUMAH UNIVERSITY OF SCIENCE AND TECHNOLOGY, KUMASI

INSTITUTE OF DISTANCE LEARNING

MULTI-PERIOD CAPITAL RATIONING PROBLEM: A CASE STUDY OF THE EFFIDUASE DIOCESE OF THE METHODIST CHURCH GHANA

A THESIS SUBMITTED TO THE DEPARTMENT OF MATHEMATICS

IN PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR THE AWARD OF THE DEGREE OF MASTER OF SCIENCE (INDUSTRIAL MATHEMATICS)

By

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AUGUST, 2009

DECLARATION

I, Isaac Osei-Boadi (V. Rev.) do hereby declare that the thesis entitled 'Multi-period Capital Rationing Problem: A Case Study of the Effiduase Diocese of the Methodist Church Ghana', with the exception of quotations and references contained in published work which have all been identified and acknowledged, is entirely my own original work under the Supervision of Dr. F.T. Oduro and Dr. S.K. Amponsah.

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ABSTRACT

Due to insufficient funds available for investment, the Methodist Church Ghana is unable to initiate or implement all their viable fiscal investments. The dilemma facing the Methodist Church Ghana is that, which of the numerous fiscal investments the management should invest under the limited investment capital in order to maximize profit. To solve this fiscal investment selection problem, data were drawn from sources such as financial statements, monthly, quarterly and annual reports, and other relevant documents of the Methodist Church Ghana, Effiduase Diocese over the period 2003-2008. Financial Ratios, Continuous Probability Analysis, Linear Programming Models were used to analyze the data. Sensitivity analyses were performed on the fiscal investment parameters. The Linear Programming Model designed was suitable for solving large scale fiscal investment selection under multi-period capital rationing problems, and this produces the optimal solution quantities (i.e., the fiscal investment to be initiated), the value of the objective function (i.e., the Net Value) and the opportunity cost of the binding constraints. On the other hand, the selection of small scale fiscal investment problems was found to be solved easily by Integer Programming Models. The Mathematical Models such as Linear Programming and Integer Programming were found to be:

- Suitable for selecting fiscal investments and maximizing the returns from the batch of fiscal investments selected by the Methodist Church Ghana, Effiduase Diocese.
- Suitable for carrying out sensitivity analysis on the optimal solution to Linear Programming problems to see how sensitive the fiscal investment selection is to the changes in the parameters of the model.

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURE

LIST OF ABBREVIATIONS

ABBREVIATION	MEANING
BV	Basic Variables,
CAPM	Capital Asset Pricing
CPA	Continuous Probabilistic Analysis
EDMCG	Effiduase Diocese of the Methodist Church Ghana
IP	Integer Programming
IRR	Internal Rate of Return
LP	Linear Programming
MCR	Multi-period Capital Rationing
NPV	Net Present Value
PBP	Payback Period
PI	Profitability Index
PV	Present Value
RHS	Right Hand Side
SCR	Stochastic Capital Rationing

ACKNOWLEDGEMENT

Unto God be the glory for the great things He has done and for His unfailing love, favour, providential Grace, wisdom, strength and guidance throughout the writing of this thesis.

Without the efforts and valuable contributions of many people, this project would not have come to fruition.

I am highly indebted to my indefatigable Supervisors Dr. F. T. Oduro, Senior Lecturer, Head of Information Systems and Decision Science, KNUST School of Business & also Coordinator of the Actuarial Science Programme and Dr. S. K. Amponsah, Head of the Department of Mathematics, Faculty of Physical Sciences, KNUST, Faculty of Physical Sciences, KNUST for their time, patience, helpful critiques, comments and invaluable suggestions throughout the writing of this project.

Special thanks go to all my lecturers and mentors especially Prof. I. K. Dontwi, Dean of the Faculty of Physical Sciences, KNUST and the Ag. Director, National Institute of Mathematical Sciences, Rev. Dr. G. O. Lartey (Mathematics Dept.-KNUST), Rt. Rev. Samuel Agyeman Kwakye, Bishop of Effiduase Diocese of the Methodist Church and Very Rev. Moses Quayson, the Diocesan Bishop – Elect for their cooperation and encouragement.

I owe special thanks to the many individuals who contributed their expertise to the development of this thesis especially Mr. Godfred Owusu-Boateng, a lecturer at the Faculty of Renewable Natural Resources (FRNR), KNUST, Lewis Asare, a tutor at Juaben Senior High School for their unflinching support in diverse ways in obtaining materials and data for my work. I am most grateful.

My sincere thanks also go to Messrs Paul Cudjo and Oworae Gyamfi, both tutors of Juaben Senior High School who combined love and patience for the arduous work of typesetting and the printing of the work.

Finally, I express my profound appreciation and thanks go to my dear wife, Gladys, for her love and diverse contributions, support and encouragement to me to make my dreams come into reality.

'Thank you' is familiar words but I utter them with a deep sense of gratitude to all and sundry for your invaluable contributions.

God richly bless you.

Isaac Osei-Boadi (V. Rev.)

August, 2009

DEDICATION

I dedicate this work to my parents, wife and children.

CHAPTER ONE

GENERAL INTRODUCTION

Introduction

This chapter discusses the background to the study with the statement of the problem, it then discuses the objectives of the study giving the methodology with the justification. It continues to describe the scope and limitation of the study concluding with the organization of the thesis.

Background to the study

The church over the years has been struggling to control its expenditure in order to maximize money available for investment with less success.

A recent survey of the capital budgeting practices of large companies by Pike (1995) showed that over seventy-five percent (75%) of companies used payback as an appraisal method, often in conjunction with other techniques. The same survey showed that only seventeen percent (17%) of the companies used Net Present Value (NPV) as their primary evaluation technique in spite of generally acknowledged technical superiority of NPV over payback. This would seem to mean that much of the academic preoccupation with refining measurement techniques may be misplaced. Nevertheless, investment opportunities are of far greater importance than the particular appraisal method used, since successful investment appraisal is entirely dependent on the accuracy of the cost and revenue estimates.

The investment appraisal employed in this study is fiscal investment selection under multi-period capital rationing. Capital rationing is manifested in the situation where the firm or company in this case the church is unable to initiate all fiscal investments, which are apparently profitable because sufficient funds are not available. The situation where investment funds are expected to be limited over several periods of time is called multi-period capital rationing.

The effects of capital rationing may develop for internal purposes. For example, it may be decided that investment should be limited to the amount that can be financed solely from retained earnings or kept within a given capital budget. The external and internal factors, which impose quantitative limits, lead to two opposing view-points developing known as the "hard" and "soft" view of capital rationing.

The "hard" view is that, there is an absolute amount of money a company may borrow or Rise externally. The "soft" view on the other hand is that rationing by a quantitative limit such as an arbitrary capital expenditure budget should only be seen as a temporary administrative expedient because such a limit is not determined by the market and such a limit would not be imposed by a profit maximizing company.

Whatever the causes of the limited capital supply available for investment purposes means that, not only must each fiscal investment cover the cost of the capital but that the fiscal investment selected must maximize returns from the funds available, making some form of ranking necessary.

Ways of achieving this objective include the following rationing possibilities:

- Single period capital rationing with divisible fiscal investments.
- Single period capital rationing with indivisible fiscal investments.
- Multi-period capital rationing with divisible fiscal investments.
- Multi-period capital rationing with indivisible fiscal investments.

These investment appraisal methods would be explained in the subsequent chapters.

Statement of the problem

There is a cry in our churches that the Diocese or headquarters is taking too much money in spite of the fact that one expects to see infrastructural, or material and spiritual well being of the members. There is also the need to establish hospitals, schools, credit unions etc. The church today however cannot do without money, because it did not happen in the early church. The church leadership should therefore find time to teach the people the need to give to God. The church leadership should also develop an attitude of saving or investing part of the little funds they have, no matter how little it may be.

It is worth mentioning that the Methodist Church Ghana has properly constituted structures and policies which when followed or implemented would ensure adequate membership growth as well as its corresponding financial and material or physical infrastructural development as enshrined in the church's constitution and standing orders.

The Church has currently adopted the consolidation of all financial accounts yet the figures quoted in the consolidated accounts for the various synod or circuit financial statements do not reflect the actual situation on the ground with regards to society accounts. It has been the common practice of some societies of the Church to conceal certain revenue or property or donations they receive or under declare whatever resources they have owing to some obvious reasons.

In a situation where societies, circuits or Diocese are assessed annually, depending upon their strength, some tend to under declare all the revenue or resources they acquire perhaps with the view of evading the responsibility of being assessed heavily.

Establishment of the Methodist Church Ghana endowment fund and the Methodist Development fund Board of Trustees

The Methodist church Ghana has undergone several moves in her quest to find sustainable means of income generation or mobilization to support the growth of the church both spiritually and materially. It has been observed that in a situation where the church's leadership embarks on frequent appeals for funds, virtually during every Sunday worship, this more often than not, embarrasses most people (the congregation) who under normal circumstances, because of their social status, would be compelled to respond positively to the mention of a certain amount. These moves have imparted negatively on the church's normal Sunday Worship Service where church attendance has subsequently been dwindling especially on the part of the male adult worshippers.

It has also been observed that if the church's Worship Service are characterized by sermons on giving when in effect people consider themselves poor, the congregation feels challenged or often get embarrassed when they are not able to respond positively to the messages that are preached. This unfortunate situation is even aggravated when some monies collected from members are expended without proper documentation and so becomes very difficult to give accurate account of such monies spent when only left-overs are recorded and some procurement is made without much transparency. Church members more often than not feel disappointed when monies collected for specific purpose are diverted and spent on something else.

Owing to inappropriate accounting of monies collected at church for the purpose they are meant for, members tend to lose confidence in embracing any such move to mobilize funds for any meaningful venture that would be sustainable and ensure growth to move the Church forward.

There are a good number of church members who fail to contribute to the church's growth for their failure to fulfill their financial obligations. This calls for urgent redress of the following basic problems as inaccurate statistical returns and District / Diocese autonomy and their different modes in resource mobilization, time to start worship on Sunday morning, stewardship development and prioritizing the use of money.

The Methodist Development Fund

The Methodist Development Fund seems to attempt to create a pool of funds for general development and fiscal investment with the aim of lessening the financial constraints on her members. It however has its own limitations and constraints.

In this study we examine the data and documents on financial matters and use these to construct a model that would ensure sustainable fiscal growth in the Effiduase Diocese of the Methodist Church Ghana (EDMCG).

Objectives of Study

The purpose of this study specifically is:

(a) To formulate Linear Programming (LP) and an Integer programming (IP) models for solving appropriate EDMCG fiscal investment selection problems.

(b) To maximize the returns from the fiscal investments selected with regards to the capital limitation.

(c) To carry out sensitivity analysis on the fiscal investment parameters to see how sensitive the fiscal investment selection decision is to data.

(d) To establish a comparative analytic difference between expenditure over a three fiscal year period with its optimal solution.

Significance of Study

The study is to throw light on the inherent difficulties and problems facing EDMCG in the selection of fiscal investments with regards to the capital limitation. The models will serve as tools for solving various problems of multi-period capital rationing in Methodist Church Ghana. The study can also be a guide for policy and decision makers in other churches and organizations to maximize profit from the fiscal investment they would undertake.

Scope of the Study

The EDMCG was carved out of the Kumasi Diocese some thirteen years ago that is, in 1996. However, the study will cover the fiscal management over the period 2005-2008. It hoped that the models will be of interest to management and planners of the EDMCG in Ghana.

Data

Data for the study was purely secondary data. The data was obtained from sources such as financial statements, annual reports, monthly reports, synod reports and other relevant documents of the EDMCG. The data was critically examined and classified into tables. The fiscal investments with different lives were compared. The financial ratios and Continuous Probabilistic Analysis (CPA) were used to analyze the data. The LP and IP models for solving fiscal investment selection problems from optimization theory perspective were also presented.

Justification

The biblical passage on the parable of the talents teaches us great moral lessons to consider seriously our stewardship of money and resources. We are expected to invest our ~~resources~~ meaningfully so as to be productive. It is expected that the entire membership and or individuals be challenged to manage our resources well however small it may seem so as to give good account of our stewardship. It is hoped that the church with its defined source of income coupled with proper inventory control and systems would regenerate the needed confidence in the members and to ensure sustainable growth.

Overview

The remaining work is organized as follows:

Chapter two provides the literature review for the fiscal investment selection under multi-period capital limitation problem addressed in this thesis.

Chapter three examines the procedure and formulation of linear programming and integer programming models for solving programming problems from optimization theory perspective. In this chapter, the sources, the method of selection of data and comparison of data using financial ratios and Continuous Probabilistic Analysis (CPA) are given.

An Excel Solver program for solving the LP and IP problems is formulated and provided. In addition, the secondary data extracted from the published financial statements, monthly/quarterly reports, synod reports and yearly reports of EDMCG from 2003 – 2008 and other supporting documentary evidence are used to formulate suitable LP and IP objective function and decision variables that will assist the EDMCG to select viable fiscal investment within its capital limitation. This is followed by sensitivity analysis of the results and the findings. Chapter four is for the discussions, conclusions and recommendations of the study.

Limitations

There are no doubts that in the right circumstances, the LP and the IP can be useful methods of dealing with multi-period capital rationing problems. There are, however, few assumptions and limitations, which are worth mentioning. These include the assumption that;

- All functions which are linear may not be realistic.
- The fiscal investments and the constraints all being dependant of one another.
- The cash flows results and the constraints which are known with certainty may also not be realistic.

The researchers are also aware that there are other techniques like Payback Period (PBP) and Internal Rate of Return (IRR) for selecting optimal fiscal investments. However, due to the number of serious limitations they present, only NPV was used in the formulation of the objective functions of the models.

CHAPTER TWO

LITERATURE REVIEW

Introduction

This chapter reviews the literature for the fiscal investment selection under multi-period capital limitation problems addressed in this thesis. It is upon this background information that the programming models are formulated.

Definition of Terms

Multi-period capital rationing

The multi-period capital rationing is embarked upon where investment funds are expected to be limited over several periods. In such circumstances, it becomes difficult to choose the fiscal investments, that is, some starting immediately, first or second or others a couple of periods later which yield the maximum returns and yet which remain within the capital limits.

The problem becomes one of the optimizing a factor, (e.g. NPV) where resources are limited and the funds available over the period are considered. This will be recognized as a situation where linear programming and integer programming could be used. Both IP and LP have been used successfully in solving multi-period capital rationing problems. Specifically, the LP method is usually used to solve divisible fiscal investments, that is, where a fractional part of a fiscal investment is desired to be undertaken. Where the fiscal investments are not divisible, the only feasible solution method is IP method.

Fiscal investment Selection

The appropriate choice of investment fiscal investments depends primarily on the nature of cash flows generated by the fiscal investments, the risk level associated with the cash flows and the budgetary limitations of the corporation over time. For the past four decades, researchers have attempted to present the working investment choice model that considers the various aspects of the budgetary process.

In 1963, the first mathematical programming formulation of the multi-period capital rationing problem was provided. In his formulation, the net discount cash inflows for fiscal investments are maximized while cash outflows and availability of resources are maintained in each period. This formulation withstood many criticisms over the past three decades. The majority of these criticisms are based on three main features of the model. These being the appropriate selection of an objective functions, the determination of suitable discount rate to account for fiscal investment returns and the inability of the model to deal with uncertain budgetary constraints.

In the seminal work by Weingartner (1963), the author provided a framework using deterministic linear programming approach. His model uses Net Present Value (NPV) as its objective function. The value associated with the timing of a particular cash flow is adjusted by an appropriate discount rate (KIRA, 2000).

It is evident from the above survey that the LP model for fiscal investment evaluation under capital rationing made use of fiscal investment cash flows NPV, IRR and other investment evaluation techniques. It is therefore imperative that these methods are examined to see how they could be incorporated into the formulation of LP and IP in the proposed study.

Fiscal investment and Cash Flows

Every decision the company makes is a capital budgeting decision whenever it changes the company's cash flows and considers launching a new fiscal investment. This involves a phase where the new product is advertised and distributed. Hence the company will have the cash outflows for paying advertising agencies, distributors, transportation services, etc. Then, for the period of time the company may have cash inflows from the sale of the product in the future. Thus, two types of cash flow are identified; cash inflow and cash outflow from the fiscal investment. Cash flow items include:

Cash inflows

- The fiscal investment revenues
- Payment of Assessment
- Camp meeting and Synod proceeds
- Annual Harvests
- Any other cash inflows caused by accepting a fiscal investment.

Cash Outflows

- Initial investment in acquiring the assets.
- Fiscal investment costs, labour, materials, etc.
- Connexional Assessment
- Any other cash outflow caused by accepting a fiscal investment.

The difficulty with making these decisions are that, typically many cash flows are affected and they usually extend over a long period of time. Investment appraisal criteria could be employed in analyzing capital budgeting decisions by aggregating the multiple of the cash flows into one number. Thus, all cash flows have to be included in the analysis whenever they are affected by the decision (Brav *et. al*, 1999).

Determination of Cash Flows in the NPV analysis

The incremental net cash flow of an investment proposal is defined as the difference between the company's cash flows if the investment fiscal investment is undertaken and the company's cash flows if the investment fiscal investment is not undertaken. That is, the net cash flow generated by certain assets is given by the equation:

Net Cash Flow = Cash Inflow – Cash Outflow

= Revenue – Expenses – Capital Expenditure – Taxes...(1)

The income tax paid is determined by:

Taxes = t (Revenue – Expenses – Depreciation)...........................(2)

Where t is the corporate tax rate. The depreciation is not a cash expenses and it only affects cash flows through its effects on taxes.

Substituting equation (1) into equation (2) yields an expression for the company's cash flow:

Cash Flow = (1-t)(Revenue–Expenses) + t(Depreciation) – Capital Expenditure...(3)

The term t (Depreciation) is sometimes known as the depreciation tax shield

Investment Appraisal Techniques

The investment evaluation techniques under consideration in this study include:

- Net Present Value (NPV)
- Discount Rate (DR)
- P Multi-period capital rationing with divisible fiscal investments.
- Payback Period (PBP)
- Internal Rate of Return (IRR)
- Profitability Index (PI)

Net Present Value (NPV)

The investment appraisal measure which the researcher used is the original Net Present Value (NPV). The NPV of a fiscal investment is defined as the present value of all future cash flows by an investment, less the cost of initial cost of investment.

Let each cash inflow/outflow be discounted back to its PV. They are summed.

Therefore

$$NPV = \sum_{t=1}^{n} \frac{C_t}{(1+r)^t} - C_o$$

Where,

t – the time of the cash flow,

n – the total time of the fiscal investment,

r – the discount rate,

c_t – the net cash flow (the amount cash) at time t, and

c_o – capital outlay at the beginning of the investment time ($t = 0$)

NPV is an indicator of how much value an investment or fiscal investment adds to the value of the firm. With a particular fiscal investment, if c_t is a positive value, the fiscal investment is in the status of discounted cash inflow in the time of t. If c_t is a negative value, the fiscal investment is in the status of discounted cash outflow in the time of t; appropriately risked fiscal investments a positive NPV may be accepted. This does not necessarily mean that they should be undertaken since NPV at the cost of capital may not account for opportunity, i.e. comparism with other available investments. In financial theory, if there is a choice between two mutually exclusive alternatives, the one yielding the higher NPV should be selected. The following sums the NPVs in various situations.

If	It means	Then
NPV >0	the investment would add value to the firm	the fiscal investment may be accepted
NPV <0	the investment would subtract value from the firm	the fiscal investment should be rejected
NPV =0	neither gain nor loss value for the firm	it should be indifferent in the decision whether to accept or the investment would reject the fiscal investment. This fiscal investment adds no monetary value. Decision should be based on other criteria, e.g. strategic positioning or other factors not explicitly included in the calculations.

Source: Baker, 2007

However, NPV = 0 does not mean that a fiscal investment is only expected to break, even in the sense of undiscounted profit or loss (earnings). It will show net total positive cash flow and earnings over its life. In sum, it is optimal to make a decision that generates positive NPV of their incremental cash values. If there are more than two alternatives, it is optimal to choose the alternative that generates the highest NPV.

Illustration

X Corporation must decide whether to introduce a new product line. The new product will have startup costs, operational costs, and incoming cash flows over six years. This fiscal investment will have immediate (t =0) cash outflow of GH¢ 100, 000.00 (which might include machinery, and employee training cost). Other cash outflows for years 1-6 are expected to be GH¢5,000.00 per year. Cash inflows are expected to be GH¢30,000.00 per year for years 1-6. All cash flows are after tax, and there are no cash flows expected after year 6. The required rate of return is 10%. The Present Value (PV) can be calculated for each year:

T=0-GH¢100,000.00/1.10^0 =-GH¢100,000.00PV.

T=1 (GH¢30,000.00-GH¢5,000.00)/GH¢1.10^1=GH¢22,727PV.

T=2 (GH¢30,000.00-GH¢5,000.00)/1.10^2 =GH¢20,661PV.

T=3 (GH¢30,000.00-GH¢5,000.00)/1.10^3 =GH¢18,783PV.

T=4 (GH¢30,000.00-GH¢5,000.00)/1.10^4 =GH¢17,075PV.

T=5 (GH¢30,000.00-GH¢5,000.00)/1.10^5 =GH¢15,523PV.

T=6 (GH¢30,000.00-GH¢5,000.00)/1.10^6 =GH¢14,112PV.

The sum of these present values is the net present value, which equals GH¢8,881. Since the NPV is greater than zero, the corporation should invest in the fiscal investment.

Discount Rate

The rate used to discount future cash flows to their present values is a key input of this process. Most firms have a well defined policy regarding their capital structure, so the weighted average cost of capital (after tax) is used with all fiscal investments. Some people believe that it is appropriate to use higher discount rates to adjust for riskier fiscal investments. Another method is to use a variable discount rate with higher rates applied to cash flows occurring further along the time span, (reflecting the yield curve premium for long-term debt).

Another approach for the discount rate is to decide the rate, which the capital needed for the fiscal investment could return if invested in an alternative venture. If, for example, the capital require for fiscal investment A can earn 5% elsewhere, use this discount rate in the NPV calculation to allow a direct comparism to be made between fiscal investment A and other alternative. Obviously, NPV value obtained using variable discount rates with the years of investment duration better reflects the real situation than that calculated from a constant discount rate for the entire investment duration (Harrell and Harpaz, 2005).

For some professional investors, their investment funds are committed to target a specified rate of return. In such cases, the rate of return should be selected as the discount rate for the NPV calculation. In this way, a direct comparism can be made between the profitability of the fiscal investment and the desire rate of the return. To some extent, the selection of the discount rate is dependant on the use to which it will be put. If the intent is simply to determine whether a fiscal investment will add value to a company, using the firm's weighted average cost of capital may be appropriate. If trying to decide between alternative investments in order to maximize the value of the firm, the corporate investment rate would probably be a better choice. Using variable rate over time or discounting "guaranteed" cash flows which is different from "at risk" cash flow may be a superior methodology, but it is seldom used in practice. Using the discount rate to adjust for risk is often difficult to do in practice (especially internationally), and is really difficult to do well. An alternative to using discount factor to adjust for risk is to explicitly correct the cash flows for the risk elements and then discount at the firm's rate.

Payback Period (PBP)

Numerous surveys have shown that payback is a popular technique for appraising fiscal investments either on its own or in conjunction with other methods. Payback can be defined as the period, usually expressed in years, which it takes for fiscal investment net cash inflows to recoup the original investment. The usual decision rule is to accept the fiscal investment with the shortest payback period. The payback has several advantages and disadvantages. Among these are:

Advantages

(i) Uses fiscal investment cash flows rather than accounting profits and hence is more objectively based.

(ii) Favors quick return fiscal investments which may produce faster growth for companies and hence liquidity.

Disadvantages:

(i) Payback does not measure overall fiscal investment worth because it does not consider cash flows after payback period.

(ii) It provides only crude measure of timing of fiscal investment cash flows.

In spite of any theoretical disadvantages, payback is undoubtedly the most popular appraisal criterion in practice (Pike, 1995).

Internal Rate Return (IRR)

The IRR of a fiscal investment is the rate which equates the NPV of the fiscal investment's cash flow to zero; or equivalently the rate of return which equates the PV of inflows to the PV of outflows.

Internal Rate of Return Rule

IRR is return that equates initial investment with PV of each cash flow.

$$O = -C_0 + \sum_{t=1}^{t} C_t \left[\frac{1}{(1+IRR)^t}\right]$$

Where;

t- the time of the cash flow,

0- zero,

C_t- the net cash flow (the amount of cash) at time t, and

C_o- the capital outlay at the beginning of the investment time ($t = 0$).

The decision rules include:

Accept fiscal investments with IRR > r

Reject fiscal investments with IRR< r

The problems with IRR are:

(a) Ignores Value Creation (Scale).

(b) Assumes cash flows being reinvested at IRR.

(c) Multiply IRRs if later cash flows are negative.

Profitability Index (PI)

Another investment appraisal technique, the PI, is used when the companies or firms have only a limited supply of capital with which to invest is positive NPV fiscal investments. This type of problem is referred to as capital rationing. Given that the objective is to maximize shareholders' wealth, the objective in the capital rationing problem is to identify that subset of fiscal investments that collectively have the highest aggregate NPV. To assist in that evaluation, this method requires that each fiscal investment's PI is computed using:

Profitability Index (PI) = $\frac{NPV}{I}$, where I = Initial investment.

The fiscal investment's Pi is ranked from highest to lowest and then selected from the top of the list until the capital budget is exhausted. The idea behind the PI method is that it provides the subset of fiscal investments that can maximize the aggregate NPV. In general the PI is of limited usefulness and the use of NPV is considered safer.

Fiscal investment Valuation

Several authors employed the single period capital asset pricing model (CAPM) (Lintner and Sharpe, 1965) to value long-lived capital assets. The study by Bogue and Rol, (1960) was an early attempt in this direction. They demonstrated that the single period CAPM has limitations in valuing long-term capital fiscal investments. Their approach was later explored and extended by Brennah, (1998) which used the continuous time version of the CAPM and were able to drive the fiscal investment's value assuming simple expectation formulation of the future cash flows.

In the study reported by Harrell and Harpaz (2005), they investigated the valuation of a capital investment fiscal investment stipulating that a time-varying normal stochastic process with unknown means generates the fiscal investment's cash flows. Investors are assumed to be Byesian decision makers under uncertainty in the sense that they combine their prior information with evidence from the observed fiscal investment's cash flows to sequentially about unknown mean cash flows. It is assumed that CAPM of Lintner and Sharpe (1965) valid now, and in the future periods, as it provides the basis of valuation framework in the capital budgeting problem. In this context they derived the valuation formulae and betas of capital investment fiscal investments under different scenarios regarding the behavior of the fiscal investments cash flows. Also, they investigated the inter-temporary behavior of the fiscal investment's betas for various parameter values, and examined the variety of the traditional textbook valuation formulae. The application of the convention NPV technique in capital budgeting under uncertainty is criticized.

The stochastic capital rationing (SCR) model developed by KIRA, (2000) does not directly consider the issue of uncertain fiscal investment cash flows in its analysis. Rather, they developed a procedure for the capital budgeting problem wherein both uncertainty in budgetary constraints and returns can be addressed simultaneously. This is realized by utilizing the SCR mode and by considering varying standard deviations of fiscal investment returns in generating the optimal composition of fiscal investments. Many authors and researchers consider the possibility of information upgrading in fiscal investment valuation, but do not present a specific model delineating how learning can be formally embedded into the multi-period capital rationing problems. This study is a step in bridging this gap.

The purpose of this study is to develop or formulate LP and IP models for solving multi-period capital rationing problems. Specifically, the LP model will be designated to solve multi-period capital rationing (MCR) with divisible fiscal investment problems while IP will be used to solve MCR with indivisible fiscal investment problems. The models seek to produce optimal solution quantities (i.e. the fiscal investments to be initiated), the value of the objective function (i.e. the total NPV) and the shadow costs (i.e. opportunity costs of the binding constraints).

Linear Programming - Mathematical Model

Linear programming (LP) is a widely used mathematical modeling technique designed to help managers in planning and decision making relative to resource allocation. *LP* is a technique that helps in resource allocation decisions. *Programming* refers to modeling and solving a problem mathematically.

Examples of Successful LP Applications

1. Development of a production schedule that will satisfy future demands for a firm's production while *minimizing* total production and inventory costs

2. Selection of product mix in a factory to make best use of machine-hours and labor-hours available while *maximizing* the firm's products

3. Determination of grades of petroleum products to yield the *maximum* profit

4. Selection of different blends of raw materials to feed mills to produce finished feed combinations at *minimum* cost

5. Determination of a distribution system that will *minimize* total shipping cost from several warehouses to various market locations

Requirements of a Linear Programming Problem

All LP problems have 4 properties in common:

- All problems seek to maximize or minimize some quantity (the objective function).
- The presence of restrictions or constraints limits the degree to which we can pursue our objective.
- There must be alternative courses of action to choose from.
- The objective and constraints in linear programming problems must be expressed in terms of <u>*linear*</u> equations or inequalities.

5 Basic Assumptions of Linear Programming

1. Certainty:
   - numbers in the objective and constraints are known with certainty and do not change during the period being studied
2. Proportionality:
   - exists in the objective and constraints
   - constancy between production increases and resource utilization
3. Additivity:
   - the total of all activities equals the sum of the individual activities

4. Divisibility:

- solutions need not be in whole numbers (integers)
- solutions are divisible, and may take any fractional value

5. Non-negativity:

- all answers or variables are greater than or equal to ($\geq$) zero
- negative values of physical quantities are impossible

Formulating Linear Programming Problems

Formulating a linear program involves developing a mathematical model to represent the managerial problem. Once the managerial problem is understood, one can begin to develop the mathematical statement of the problem.

Steps in LP Formulations

In formulating LP one needs to:

1. Completely understand the managerial problem being faced.

2. Identify the objective and the constraints.

3. Define the decision variables.

4. Use the decision variables to write mathematical expressions for the objective function and the constraints.

Linear Programming (LP)

A Linear Programming (LP) is one of the most widely used optimization techniques and perhaps the most effective (Amponsah, 2006). The term Linear Programming was coined by George Dantzig in 1947 to refer the problems in which both the objective function and the constraints are linear.

A Linear Programming is the problem of optimizing linear objective in the decision variables $x_1, x_2, \ldots\ldots x_n$ subject to linear equality or inequality constraints on the decision variables.

Standard Form of the Linear Programming Problem

In the standard form, the Linear Programming problem is expressed as;

$$\text{Maximize } F = \sum_{j=1}^{n} c_j x_j \quad \ldots\ldots(1)$$

$$\text{Subject to } \sum_{j=1}^{n} a(i,j)x_j = b_i, \quad i = 1, 2 \ldots\ldots, m \quad \ldots\ldots(2)$$

$$l_j \le x_j \le u_j, \ j = 1, 2 \ldots\ldots n, \quad \ldots\ldots(3)$$

where c_j are the objective function coefficients, $a(i,\ j)$ and b_j are parameters in m linear inequality constraints and l_j and u_j are lower and upper bounds with $l_j \le u_j$. Both l_j and u_j, may be positive or negative.

A linear programming problem can be expressed more conveniently using matrices;

$$\text{Minimize } F = C^t X \quad \ldots\ldots(4)$$

Subject to: AX = b..(5)

$l \leq x \leq u$..(6)

A is *m x n* matrix whose *(i,j)* element is the constraint coefficient *a(i,j)* and *c, b, l, u,* are factors whose complements are c_j, b_j, u_j respectively. If any of the equations (1-5) were redundant, that is, linear combinations of the orders, they could be deducted without changing any solutions of the system. If there is no solution or if there is one solution for equation (5), there can be optimization. Thus the case of greatest interest is where the system of equation (5) has more than unknown equations and has at least two and potentially and infinite number of solutions. This occurs if and only if

$n > m$, and

Rank (A) = m

These conditions are assumed to be true in what follows. The problem of linear programming is to first detect whether solution exist, and if so, to find one yielding the minimum F.

Some Basic Definitions

1. A feasible solution to LP problem is a vector $X = (X_1, X_2, \ldots\ldots\ldots, X_n)$ that satisfies the equation AX = b and the bounds $l \leq x \leq u$.

2. A linear programming (LP) is feasible if there exists a feasible solution otherwise it is said to be infeasible.

3. A basic feasible solution is a basic solution in which variables satisfy their bounds $l_j \le x_j \le u_j$

4. A non degenerate basic feasible solution in which all basic variables X_j are strictly between their bounds, that is, $l_j \le x_j \le u_j$.

5. An optimal solution $X = (X_1, X_2, \ldots\ldots X_n)$ is a feasible solution subject to C^TX, $AX = b$ and $X \le 0$. (Lasdon and Powell, 1998)

Equivalent Forms of LP

A linear programming can take on several forms. It might be maximizing instead of minimizing. The LP can have a combination of equality and inequality constraints. Some variables may be restricted to be non positive instead of non negative, or be unrestricted in sign. Two forms are said to be equivalent if they have the same set of optimal solutions or are either infeasible or unbounded.

1. A maximization problem can be expressed as a minimization problem;

i.e. maximize $C^TX \equiv$ minimize $- C^TX$

2. An equality can be represented as a pair of inequalities

$$a_i^T x = b_i \equiv \begin{cases} a_i^T x \le b_i \\ a_i^T x \ge b_i \end{cases} \text{ OR } \begin{cases} a_i^T x \ge b_i \\ a_i^T x \le -b_i^T \end{cases}$$

3. By adding a slack variable, an inequality can be represented as a combination of equality and non-negative constraints. For example $\sum_{j=1}^{n} a(i,j)x_j \le b_j$

Then slack variable is defined as $s_i \ge o$ such that;

$\sum_{j=1}^{n} a(i,j)x_j + s_i = b_i$, and the inequality becomes an equality.

Similarly, if the inequality is $\sum_{j=1}^{n} a(i,j)_j j \geq b_i$, it is written as

$$\sum_{j=1}^{n} a(i,j)x_j - s_i = b_i$$

4. A non-positive constraint can be expressed as non-negative constraint if we replace X_j everywhere with $-y_j$ and impose the $y_j \geq 0$ condition.

5. X may be unrestricted in sign. In such a case, X_j is replaced everywhere by $\overset{+}{X}_j - \overset{-}{X}_j$, adding the constraints $\overset{+}{X}_{k,} \overset{-}{X}_j \geq 0$.

In general, an inequality can be represented using a combination of equality and non-negative constraints, and vice versa. Using these rules;

Minimize $\{C^T X, \text{st } AX = b\}$ can be transformed into minimize

$\{A\overset{+}{X} - A\overset{-}{X} - s = b, \overset{+}{X}, \overset{-}{X}, s \geq 0\}$. The former LP is said to be in canonical form, the later in standard form.

Duality

For any given linear programming problem called Primal, there is an associated linear programming called Dual Problem. Duality is an important concept in linear programming fiscal investment since it is algorithmic and allows a proof of optimality.

Rules for Taking Dual Problems

If PRIMAL problem (P) is minimal problem, then the DUAL (D) problem is maximum problem and vice versa. In general, using the rule of transforming a linear programming in standard canonical form, the dual (D) of primal (P) is;

Minimize $Z = C^T X$

Subject to $AX \leq b,$

$X_j \geq 0,$

where $X = (X_1, X_2, \ldots\ldots\ldots, X_n)^T$ is any n –factor

$C = (C_1, C_2, \ldots\ldots\ldots, C_n)^T$ is any n factor

$A = a(i,j)$ *is* $m \times n$ matrix and

$b = (b_1, b_2, \ldots\ldots, b_n)T$ is any m-vector in DUAL (D).

Maximize $W = b^T y$

Subject to $A^T y \geq C, y \geq 0.$

The variables in P are called Primal variables and the variables in the Dual problem are called Dual variables. The general rules for converting primal problem of any form into dual problem can be summarized as shown in Table 3.1:

Table 3.1

PRIMAL PROBLEM	DUAL PROBLEM
Maximization	***Manimization***
• Coefficients of objective function	• Right hand sides of constraints
• Coefficients of i^{th} constraints	• Coefficient of i^{th} variable
• i^{th} constraint is an inequality of the form $\leq$	• i^{th} variable satisfies ≥ 0
• i^{th} constraint is an inequality (=)	• i^{th} variable is unrestricted
• i^{th} variable is unrestricted	• i^{th} constraint is an inequality (=)
• i^{th} variable satisfy ≥ 0	• i^{th} constraint is an inequality of the type $\geq$
• Number of variables	• Number of constraints
• Number of constraints	• Number of variables
• If inequality of type $\geq$ occurs in the maximization problem compared to the type $\leq$ by multiplying through by -1	• If inequality of type $\leq$ occurs in maximization problem convert to type $\geq$ by multiplying through by -1

Solution Techniques for Linear Programming

There several are approaches for solving the linear programming problems. Among these techniques are:

1. Graphical Approach
2. Simplex Algorithms
3. LINDO Software
4. QSB Package
5. MATLAB
6. YE's Interior Point Algorithms
7. Microsoft Excel 2003

The most convenient and effective technique in use now is the Simplex Algorithm. Essentially, the Simplex Algorithm starts at one vertex of the feasible region and moves (at each iteration) to another (adjacent) vertex, improving (or leaving unchanged), the object function as it does so, until it reaches the vertex corresponding to the optimal linear programming solution.

The Simplex Algorithm for solving LP's was developed by Dantzig, (1940). A number of different versions of algorithms have now been developed. One of these later versions, called the Reversed Simplex Algorithms (Appendix B) form the basis of the most modern computer packages for solving LP's (Arsham, 1999).

Sensitivity Analysis

Sensitivity Analysis (or Post-optimal Analysis) allows the researcher to observe the effect of changes in parameters of linear programming problems on the optimal solution. Given the LP package, it is easier to change the data to see how the solution changes (if at all) as certain key data items change.

As a by-product using the Simplex Algorithm, researchers get sensitivity information,

(a) For the variables, the Reduced Cost (also known as opportunity cost) column which gives each variable an estimate of how the object functions will change if the variable is made non-zero. This is called "Reduced Cost" for the variables. The Reduced Cost can also be interpreted as the amount by which objective function coefficient for a variable need change becomes non-zero.

(b) For each constraint column headed shadow price indicates by how much the objective function will change if the right hand side of the corresponding ~~constraint is~~ changed. This is known as the "Marginal Value" for the constraint.

LP Formulation and Applications

The conditions for a mathematical model to be a linear programming are;

(a) All variables must be continuous (i.e. can take fractional value).

(b) A single objective (minimum or maximum) must be indicated.

(c) The objective and the constraints are linear (i.e. any term is either a constant or constant multiples of an unknown).

The LP is important in everyday life because, many practical problems can be formulated as LP and also there an algorithm (called the Simplex Algorithm) which enables the researcher to solve LP numerically relatively easily (Padberg & Hoffman, 1995).

Integer Linear Programming (IP)

In many of the problems, certain variables should have been regarded as Integer values. But for the sake of convenience, the variables are allowed to take functional values reasoning that the variables are likely to be so large that any fractional part could be neglected. Whilst that is accepted in some situations, in many cases, a numeric solution must be found such that the variables take only integer values. The problems in which this is the case are called Integer Programs (IP) and the subject of solving such programs is called Integer Programming. Thus Integer Programming is the subset of

Linear Programming in which all the variables are required to be non-negative integers.

General Forms of Integer Programming (IP)

1. An IP in which all the variables are required to be integers is called Pure IP problem, i.e. maximize $Z = \sum_{j=1}^{n} a_j X_j$ (objective function)

Subject to $\sum_{j=1}^{n} a_{(i,j)} X_j \le b_j$ (constraints)

$X_j \le 0, X_j$ Integer

$j = 1, 2.....n$

2. An IP in which only some of the variables are required to be integers is called Mixed Integer Programming,
i.e. Maximize $Z = \sum Q_j X_j$

Subject to $\sum_{j=1}^{n} a_{(i,j)} X_j \le P_j$

$\sum_{j=1}^{n} X_j + \sum y_j \ge W$

$X_j \ge 0$, y = 0, or 1

$j = 1, 2....n$

3. An IP problem in which ~~all the~~ variables must be equal to 0 or 1 is called a Zero-~~one I~~nteger Programming,

i.e. Maximize $Z = \sum Q_j X_j$

Subject to $\sum_{j=1}^{n} d_j X_j$ less than D_j

$$X_j = 0 \text{ or } 1$$

4. The LP obtained by omitting all integers or 0-1 constraints as variables is called the LP Relaxation of LP.

For example, the LP relaxation of (1) is

Maximize $Z = 3X_1 + 2X_2$

Subject to $X_1 + X_2 \leq 6$

$X_1, X_2 \geq 0$

And the LP relaxation of (2) is

Maximize $Z = X_1 + X_2$

Subject to $X_1 + 2X_2 \leq 2$

$2X_1 +, X_2 \leq 0$

$X_1, X_2 \geq 0$

Any IP may be viewed as LP relaxation plus some additional constraints, the constraints that state which variables must be integers or be 0 or 1. Hence the LP relaxation is less constrained or a more relaxed version of the LP. This means that the feasible region for an IP must be contained in the feasible region for the corresponding LP relaxation. LP relaxation for any IP is a maximize problem. This implies that;

Optimal value for LP relaxation $\geq$ Optimal Z value for LP.

This result plays a key role in the discussion of the solution of IP.

Solution Techniques for IP

There is general purpose (independent of LP being solved) and computationally effective able to solve large LP) algorithms (Simplex or interior point) for solving LP problems. However, there is no similar general purpose and computationally effective algorithm exists for solving IP problems. This means that IP problems are harder to solve than LP problems. There are, at least three different approaches for solving IP problems, although they are frequently combined into "hybrid" solution procedures in computational practice. They are;

- Enumerative techniques
- Relaxation and decomposition techniques
- Cutting planes approaches based on polyhedral combinatory.

The most effective and simplest approach to solving a pure IP problem is to numerate all finitely many possibilities. However, due to "combinatorial explosion" resulting from the parameter "size" only the smallest instances could be solved by such an approach. Sometimes one can implicitly eliminate many possibilities by domination or feasibility arguments. Besides straight-forward or implicit enumerations, the most commonly used enumerative approach is called BRANCH and BOUND, where the "branching" refers to the enumeration part of the solution technique and "bounding" refers to the fathoming of possible solutions by comparison to a known upper or lower bound on the solution value (Doig and Land, 1960).

To obtain an upper bound on the problem (i.e. in maximizing problem), the problem is relaxed in a way which makes the solution to the relaxed problem, relatively easy to solve. All commercial branch-and-bound codes relax the problem by dropping the integrality conditions and solve the resultant continuous LP problem over the set P. If the solution to the relaxed linear programming satisfies the integrality restrictions, the solution obtained is optimal. If the LP is infeasible, then so is the integer program. Otherwise, at least one of the integer variables is fractional in the LP solution. One chooses one or such fractional variables and "branches" to create two or more sub problems which exclude the prior solution but do not eliminate any feasible integer solutions. These new problems constitute "nodes" on branching tree, and an LP problem is solved for each node created. Nodes can be fathomed if the solution to the sub problem is feasible, satisfies all the integrality restrictions, or has an objective function value than a known integer solution (Powell and Lasdon, 1998).

Illustration

Consider the LP problem, (parameters measured in \$m);

$$\text{Maximize } Z = 0.2X_1+0.3X_2+0.5X_3+0.1X_4$$

$$\text{Subject to} \quad 0.5X_1+1.0X_2+0.1.5X_3+0.1X_4 \leq 3.1$$

$$0.3X_1+0.8X_2+1.5X_3+0.4X_4 \leq 2.5$$

$$0.2X_1+0.2X_2+0.3X_3+0.1X_4 \leq 0.4$$

$$X_j = 0 \text{ or } 1, j = 1, 2, 3, 4.$$

What makes this problem difficult is the fact that the variables are restricted to integers (zero or one). If the variables are allowed to be fractional (takes all values between zero and one for example) then LP would be obtained which can easily be solved.

To solve this LP relaxation of the problem, the $x_j = 0$ or 1, $j = 1, 2, 3, 4$ is replaced by $0 < x_j < 1$, $j = 1, 2, 3, 4$. Then using MATLAB packages gives the solution $x_2 = 0.5$, $x_3 = 1$, $x_1 = x_4 = 0$ of value 0.65 (i.e. the objective function value of the optimal linear programming solution is 0.65).

Now, the optimal integer solution is ≤ 0.65, i.e. this value of 0.65 is an upper bound on the optimal integer solution. This is because when the integrality constraint is relaxed, the solution value ends up with at least that of the optimal integer solution (and may be better).

Consider this LP relaxation solution. The variable x_2 is fractional needs to be an integer. To remove this troublesome fractional value, two new problems can be generated:

- original LP relaxation plus $x_2 = 0$
- original LP relaxation plus $x_2 = 1$

Then the optimal integer solution to the original problem is contained in one of these two new problems. This process of taking a fractional variable (a variable which takes a fractional value in the LP relaxation) and explicitly constraining it to each of its integer values is known as branching. It can be represented diagrammatically as below (in a tree diagram, which is how the name tree search arises).

The solution to these two new LP relaxations problems are given below:

- ~~P1~~ – original LP relaxation plus $x_2 = 0$, solution $x_1 = 0.5$, $x_3 = 1$, $x_2 = x_4 = 0$ of value 0.6

- P2 – original LP relaxation plus $x_2 = 1$, solution $x_2 = 1$, $x_3 = 0.67$, $x_1 = x_4 = 0$ of value 0.63

This can be represented diagrammatically as below:

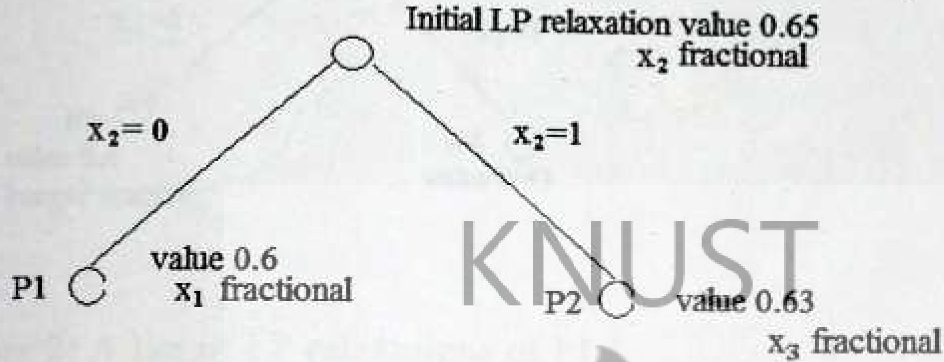

Figure 1: A solution to two new LP relaxations problems

To find the optimal integer solution, the process is repeated, i.e. choosing one of these two problems, choosing one fractional variable and generating two new problems to be solved.

Choosing problem P1 means branch x_1 to get a list of LP relaxations as:

- P3 – original LP relaxation plus $x_2 = 0$(P1) plus $x_1 = 0$, solution $x_3 = x_4=1$, $x_1 = x_2 = 0$ of value 0.6
- P4 – original LP relaxation plus $x_2 = 0$(P1) plus $x_1 = 1$, solution $x_1 = 1$, $x_3=0.67$, $x_2 = x_4 = 0$ of value 0.53
- P2 – original LP relaxation plus $x_2 = 1$, solution $x_2 = 1$, $x_3=0.67$,

$x_1 = x_4 = 0$ of value 0.63. This can again be represented diagrammatically as below.

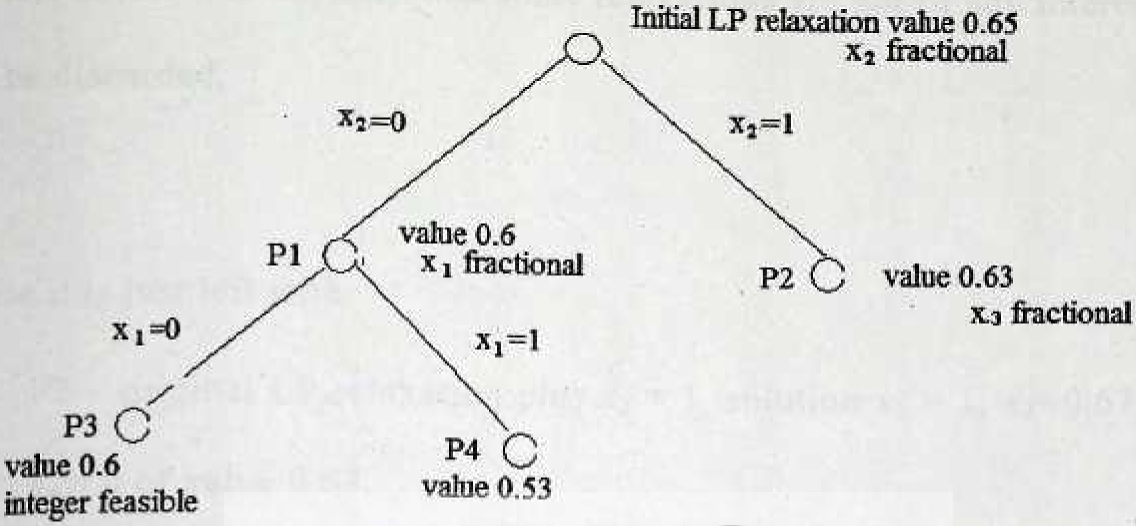

Figure 2: A list of LP relaxations of P1

At this stage, an integer feasible solution of value 0.6 at P3 is identified. There are fractional variables so no branching is necessary and P3 can be dropped from the list of LP relaxation.

Hence, the new information about the optimal (best) integer solution is that, it lies between 0.6 and 0.65 (inclusive).

Considering P4, it has value 0.53 and has a fractional variable (x_3). However if the branching were to be on x_3, any objective function solution values that would be obtained after branching can never be better (higher) than 0.53. As already the integer feasible solution value is 0.6, P4 can be dropped from the list of LP relaxations since branching from it could never find an improved feasible solution. This is known as ***bounding***- using a known feasible solution to identify that some relaxations are not of any interest and can be discarded.

Hence it is just left with:

- P2 – original LP relaxation plus $x_2 = 1$, solution $x_2 = 1$, x_3=0.67, $x_1 = x_4 = 0$ of value 0.63.

Branching on x_3 gives;

- P5 – original LP relaxation plus $x_2 = 1$(P2) plus $x_3 = 0$, solution $x_1 = 1$, $x_3 = x_4 = 0$ of value 0.5
- P6 – original LP relaxation plus $x_2 = 1$(P2) plus $x_3 = 1$, problem infeasible.

Neither of P5 or P6 can lead to further branching so the process is completed and the optimal integer solution value is 0.6 which corresponds to $x_3 = x_4 = 1$, $x_1 = x_2 = 0$

The entire process leading to this optimal solution (and to prove that it is optimal) is shown graphically below:

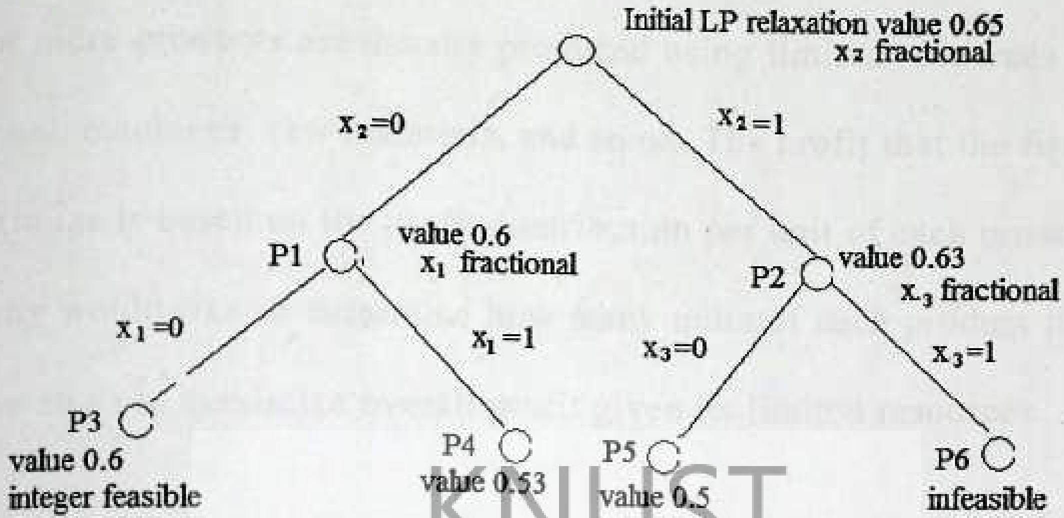

Figure 3: The optimal integer solution for the problem

Thus, the optimal integer solution for this problem is $0.6m

Applications of IP

The major application areas which IP can be applied are;

(a) Manpower scheduling problems concerned with security personnel

(b) Church location problems

(c) Capital budgeting problems

(d) Traveling salespersons problems

(e) The cutting stock problems

(f) Vehicle routing problems

The *Product Mix Problem*

Two or more products are usually produced using limited resources such as personnel, machines, raw materials, and so on. The profit that the firm seeks to maximize is based on the profit contribution per unit of each product. The company would like to determine how many units of each product it should produce so as to maximize overall profit given its limited resources.

CHAPTER THREE

DATA ANALYSIS AND MODELLING

Introduction

This chapter examines the sources and methods of collection of data. It is also proposed to provide the statistical analysis, basic methods and formulation of IP and LP models for solving the Effiduase Diocese of the Methodist Church Ghana's (EDMCG) fiscal investment selection under multi-period capital rationing problems which sought to maximize investment while minimizing expenditure.

It also entails the extraction of secondary data from published financial statements, monthly and quarterly reports, synod and connexional reports of EDMCG from 2003 – 2008 and other supporting documentary evidence formulated in this chapter. The data is used to formulate suitable LP and IP objective function and decision variables that will assist the EDMCG to select viable fiscal investment within its capital limitation

Sources and Data Collection

A capital budgeting is the process of considering alternative capital investment and selecting those alternatives that provide the most profitable return on available funds, within the framework of the company's goals and objectives. A capital investment is any available alternative, to purchase, build, lease or renovate buildings, equipment, or other long range major items of property or fiscal investments. The alternative selected usually involves large sums of money and results in a large increase on fixed assets for several years. Once a company builds a plant or undertakes some other capital expenditure, the company becomes less flexible regarding future plans (Hermanson and Maher, 1992).

Based on this fact, most of the necessary information about cash inflows and cash outflows of the EDMCG fiscal investments were extracted from the EDMCG financial statements, monthly and synod reports for the period 2005-2008. The annual net cash flow which is the difference between the cash inflows and cash outflows during each period for the under listed investments were then estimated and recorded (Table 1). The concepts of capital budgeting can be applied to other churches, not-for-profit organizations, such as universities, school districts, cities and not-for-profit hospitals. Since these organizations are not subject to as many taxes as profit-making organizations, the cash flows related to taxes are usually zero or near zero (Ibid p.125). Due to this, the tax factor in the estimation of the annual ~~net~~ cash flows for the EDMCG investments was ignored (equation 3, chapter 2).

The EDMCG investments are classified into short term and long term investments. The discount factors are also estimated at cost of capital of ten percent (10%) for each cash inflow for each investment and the corresponding NPV at ten percent (10%) (Table 1). The distribution of cash outlay for EDMCG short term investments are shown in Table 2. Table 2 also shows the capital requirement for each investment, available capital at each period and the corresponding capital returns.

Table 1:

Annual Net Cash Flow for EDMCG Large Scale Fiscal investment for 2003-2008 in Ghana GH¢ x 10^2

Year	2003	2004	2005	2006	2007	2008	NPV at 10%	PI
F. invest./Period	0	1	2	3	4	5		
Treasury Bill	100	-100	-200	-400	600	500	264	0.82
Fixed Deposit	400	500	-1000	1200	-1400	1200	719	0.80
Savings	250	200	-360	500	-400	0	237	0.92
School Project	30	50	60	60	150	-90	217	0.33
Agriculture	10	20	-10	0	30	50	72	0.72
Discount Factors	1.00	0.909	0.826	0.751	0.683	0.621		
Capital Limitation Q_j	550	500						

Sources: EDMCG

Table 2:

Distribution of Capital Requirement for EDMCG Small Scale Fiscal investment for 2003-2008 in GH¢ x 10^2

Year	2001	2002	2003	2004	2005	2006	Capital
F. invest./Period	0	1	2	3	4	5	Returns
Stores (X_1)	50	30	20	20	60	40	50
Moringa Project (X_2)	10	80	20	20	30	60	30
Soap Making (X_3)	15	15	30	40	60	0	50
B. Tie & Dye (X_4)	10	40	10	10	0	10	10
Bee-Hive Prod. (X_5)	10	0	10	20	50	10	20
Available Capital R_j	80	145	90	100	165	80	

Source: EDMCG

Statistical Analysis

The continuous probabilistic analysis (CPA) was used for the analysis of the cash flows and NPV of the various fiscal investments. This shows the variability of the fiscal investments at outcomes, which results from the variability of the individual fiscal investment cash flows. This enabled the researcher to make probability assignment of the likelihood of the various fiscal investment cash flows and variability (risk) of using the fiscal investment's NPV in the proposed model.

The most useful measure for statistical purpose is the standard deviation. Initially, the mean (NPV) is computed, followed by the dispersion and variance of the period's cash flows. The fiscal investments' standard deviation is then obtained by combining the discounted standard deviations of the individual cash flows, using what is known as the statistical sum. Having calculated the means and standard deviation of the various fiscal investments, the relative variability of the distribution of the fiscal investment cash flows were then computed, using the formula;

Coefficient of variation = $\delta_i/_\chi$ x 100%, where δ_i is the standard deviation and χ is mean of the cash flows. Results were then compared and recorded (Tables 3 & 4).

Table 3:

Distribution of Large Scale Fiscal investment Cash Flows (2003-2008)

Fiscal investment	Mean ($\bar{x}$)	Standard Deviation (δ_i)	Coefficient of Variation /%
Treasury Bill	266.67	205.48	77
Fixed Deposit	250.00	373.05	39
Saving	285.00	160.59	56
School Project	73.33	38.59	53
Agriculture	20.00	16.32	82

Table 4:

Distribution of Small Scale Fiscal investment Cash Flow (2003-2008)

Fiscal investment	Mean ($\bar{x}$)	Standard Deviation (δ_i)	Coefficient of Variation (%)
Stores for Hiring (X_1)	36.67	14.91	41
Moringa Project (X_2)	36.67	24.94	68
Soap Making (X_3)	26.67	19.51	73
Batik Tie and Dye (X_4)	13.33	12.47	93
Bee hive Production (X_5)	16.6	15.99	96

The results in Table 3 and 4 enable the researchers to make probability statement about the fiscal investment outcomes which reflects the variability's expected in each period's cash flows and the distribution of the competing fiscal investment to be compared favourably.

LP Model for Fiscal investment Selection under Multi-capital Rationing Problem

Based on the above information about LP, a formal formulation of LP Model for solving EDMCG fiscal investment selection problem is presented below.

Conditions

- The capital value for investment is denoted by Q
- The problem facing the EDMCG is that, which fiscal investment or problems of a fiscal investment it should initiate with Q

Steps

(i) The fiscal investment NPV's are determined using

$$B_j\,(\text{NPV}) = \sum_{t=1}^{n}\left[\frac{C_t}{(1+rj)^t}\right],$$

where t = 0, 1; j = 1, 2,….5 and C is the cash flows.

Assuming the results, NPV of the fiscal investments to be; Treasury Bill(A) = β_1, Fixed Deposit (B) β_2, Savings (C) = β_3, School Project (D) = β_4 and Agriculture (E) = β_5.

(ii) Formulating the problem as LP means defining the objective function, decision variables and constraint. The objective function of EDMCG is to maximize NPV. That is;

Maximize $Z = \beta_1 X_A + \beta_2 X_B + \beta_3 X_C + \beta_4 X_D + \beta_5 X_E$

where the decision variables (X_j);

X_A is proportion of fiscal investment A to be initiated (j =1),

X_B is proportion of fiscal investment B to be initiated (j =2),

X_C is proportion of fiscal investment C to be initiated (j =3),

X_D is proportion of fiscal investment D to be initiated (j =4),

X_E is proportion of fiscal investment E to be initiated (j =5),

(iii) Subject to constraint in the EDMCG problem are the budgetary limitations in periods 0 and 1 (Table 1). Taking the periods separately, gives:

Capital at time (t) = 0,

$$a_{(1,1)}X_A+a_{(1,2)}X_B+a_{(1,3)}X_C+a_{(1,4)}X_D+a_{(1,5)}X_E \leq Q_1$$

Capital at time (t) = 1,

$$a_{(2,1)}X_A+a_{(2,2)}X_B+a_{(2,3)}X_C+a_{(2,4)}X_D+a_{(2,5)}X_E \leq Q_2$$

To ensure that a fiscal investment is not accepted more than once or negative fiscal investments are accepted, the constraints regarding the proportions of the fiscal investments are specified as shown below;

$$X_A, X_B, X_C, X_D, X_E \leq 1$$

$$X_A, X_B, X_C, X_D, X_E \geq 0$$

where $a_{(i,j)}$ = cash flow for each period for each fiscal investment.

The whole formulation in a compact form is thus;

$$\text{Maximize } Z = \beta_1X_1+\beta_2X_2+\beta_3X_3+\beta_4X_4+\beta_5X_5$$

Subject to

$$a_{(1,1)}X_1+a_{(1,2)}X_2+a_{(1,3)}X_3+a_{(1,4)}X_4+a_{(1,5)}X_5 \leq Q_1$$

$$a_{(2,1)}X_1+a_{(2,2)}X_2+a_{(2,3)}X_3+a_{(2,4)}X_4+a_{(2,5)}X_5 \leq Q_2$$

$$
\begin{array}{llllll}
X_1 & & & & & \le 1 \\
& X_2 & & & & \le 1 \\
& & X_3 & & & \le 1 \\
& & & X_4 & & \le 1 \\
& & & & X_5 & \le 1 \\
X_1 & & & & & \ge 0 \\
& X_2 & & & & \ge 0 \\
& & X_3 & & & \ge 0 \\
& & & X_4 & & \ge 0 \\
& & & & X_5 & \ge 0
\end{array}
$$

A more general form is as follows:

$$\text{Maximize } Z = \sum_{j-1}^{N} \beta_j X_j \qquad \text{(objective function)}$$

$$\text{Subject to} \qquad \sum_{j=1}^{N} a_{(i,j)} X_j \le Q_j \qquad \text{(constraints)}$$

$$i = 1, 2 \ldots\ldots m$$

$$0 \le x_j \le 1$$

$$J = 1, 2 \ldots\ldots N,$$

where $\{a_{(i,j)}, Q_j \text{ and } \beta_j\}$ are given and N is the number of fiscal investments to be invested.

Assumptions

The assumptions made in formulating the LP model include:

(i) All the fiscal investments constraints are independent on one another.

(ii) Equal investment opportunities are assumed for the fiscal investments for each period.

(iii) The cash flows, resources and constraints are known with certainty.

Alternative Solution to the Problem Using IP

The branch and the bound method is the method used by the QSB Software which was employed for the analysis. The data input for the problem representing initial problem presented above is given below:

Variable	X_1	X_2	X_3	X_4	Direction	R.H.S.
Maximize	0.2	0.3	0.5	0.1		
C_1	0.5	1.0	1.5	0.1	$\le$	3.1
C_2	0.3	0.8	1.5	0.4	$\le$	2.5
C_3	0.2	0.2	0.3	0.1	$\le$	0.4
Lower Bound	0	0	0	0		
Upper Bound	1	1	1	1		
Variable Type	Binary	Binary	Binary	Binary		

The solution is shown below:

Date	Decision Variable	Solution Value	Unit cost or Profit	Total Contribution	Reduced Cost	Basis Status
1	X1	0	0.2000	0	0.2000	At bound
2	X2	0	0.3000	0	0.3000	At bound
3	X3	1.0000	0.5000	0.5000	0	basic
4	X4	1.0000	0.1000	0.1000	0	basic
	Objective	Function	(maximum) = 0.6000			

Here, the optimal decision is to choose to do fiscal investments 3 and 4 (Baker and Harrell, 2007).

IP Model for Fiscal investment Selection under Multi-period Capital Rationing Problem

Based on the background information about IP, a formulation of an IP model for solving EDMCG fiscal investment selection problems is presented below:

Conditions

1. The problem facing EDMCG is that, it cannot invest in all n-fiscal investments suitable for investments which run n-years.
2. The fiscal investment characteristics show that $\sum d_{(i,j)}$ is greater than R_j where $d_{(i,j)}$ is the least capital requirement for j fiscal investments and R_j is the capital for investment.

The decision problem is that, which fiscal investments EDMCG should be selected in order to maximize the total returns. To formulate IP for the EDMCG problem, the following steps were used:

Step 1: Decision variables

To define decision variables,

$$\text{let } X_j = \begin{cases} 1, & \text{if EDMCG invest in fiscalinvesment } j \\ 0, & \text{if EDMCG does not invest in fiscalinvesment } j \end{cases}$$

j = 1, 2,.....N

That is, the X_j are integer variables which must take one of the two possible values (zero or one). This represents binary decision (e.g. do fiscal investment = 1 or not to do fiscal investment = 0)

Step 2: Constraints

To define the constraints,

let $d_{(i,j)}$ = capital requirements for j fiscal investments,

R_j = available capital for j fiscal investments for each year.

Then the constraints relating to availability of capital funds each year are;

$$\sum_{j=1}^{N} d_{(i,j)} X_j \le R_j, \qquad i = 1, 2, \ldots m$$

$$X_j = 0 \text{ or } 1 \qquad j = 1, 2, \ldots N$$

Step 3: Objective Function

Let the total profit be $\sum P_j X_j$

Then the total Return is maximized as:

$$\text{Maximize } Z = \sum_{j=1}^{N} P_j X_j$$

This gives a complete IP model for EDMCG problem as;

$$\text{Maximize } Z = \sum_{j=1}^{N} P_j X_j \qquad \text{(objective function)}$$

$$i = 1, 2, \ldots m$$

$$X_j = 0 \text{ or } 1 \qquad \textit{(non-negative constraints)}$$

$$j = 1, 2, \ldots N$$

A more compact form is;

Maximize $Z = P_1X_1+P_2X_2+\ldots\ldots+P_NX_N$

Subject to

$$d_{(1,1)}X_1+d_{(1,2)}X_2+\ldots\ldots+d_{(1,N)}X_N \leq R_1$$

$$d_{(2,1)}X_1+d_{(2,2)}X_2+\ldots\ldots+d_{(2,N)}X_N \leq R_2$$

- -

- -

- -

$$d_{(m,1)}X_1+d_{(m,2)}X_2+\ldots\ldots+d_{(m,N)}X_N \leq R_N$$

$X_j = 0$ or 1

$j = 1, 2, \ldots N$

This completes the IP model.

Comments

- In writing down the complete IP, the inclusion of the information $x_j = 0$ or 1 ($j = 1, \ldots\ldots, N$) serves as a remainder that the variables are integers.
- The zero-one nature of the decision variable means that always a single term or item will be captured, i.e. a fiscal investment is either accepted or rejected.
- The objective and constraints are linear (i.e. any term in the constraints/objective is either a constant or a constant multiplied by an unknown). Here, only a linear integer programming (IP with a linear objective and linear constraints) is under consideration.

Non-linear integer programming, however, are outside the scope of this thesis.

Implementation of LP Model

Applying LP model;

Maximize $Z = \beta_j X_j$

Subject to $\sum_{j=1}^{N} a_{(i,j)} X_j \le b_j$

$i = 1, 2, \ldots\ldots, m$

$0 \le X_j \le 1,$

$j = 1, 2, \ldots\ldots, n,$

to the EDMCG capital rationing data (Table 1), the church's fiscal investment selection problem can be formulated as LP has shown below:

Maximize $Z = 264X_1 + 719X_2 + 237X_3 + 217X_4 + 72X_5$

Subject to $100X_1 + 400X_2 + 250X_3 + 30X_4 + 10X_5 \le 550$

$100X_1 + 500X_2 + 200X_3 + 50X_4 + 20X_5 \le 500$

$0 < X_j < 1, \quad j = 1, 2, \ldots, 5$

In this LP problem the assumption is that, fractional variables will be acceptable. Since the amount required for each period is reasonably large, this will not cause too many problems.

Solution to the LP, Using MATLAB Package

There are more than two variables in the LP problem, hence it is considered as large scale LP problem that can be solved easily by software package like MATLAB. The LP modular in the MATLAB package is called "Linprog".

To solve an LP problem,

$$\min_{x} f^T X \text{ such that}$$

$$AX \leq b$$

$$A_{eg}X = b_{eg}$$

$$lb \leq x \leq ub$$

where *f, x, b, beg, lb* and *ub* are vectors and *A* and *Aeg* are matrices, using linprog, the syntax is;

x = linprog (f, A, b, Aeq, beq)

x = linprog (f, A, b, Aeq, beq, lb, ub)

x = linprog (f, A, b, Aeq, beq, lb, ub,x0)

x = linprog (f, A, b, Aeq, beq, lb, ub,x0, options)

[X fval] = linprog (...)

[X, fval, exitflag] = linprog (...)

[X, fval, exitflag, output] = linprog (...)

[X, fval, exitflag, output, lambda] = linprog (...)

Description

Linprog solves linear programming problems.

x = linprog (f, A, b, Aeq, beq) solves min *f(x)* such that $Ax \leq b$.

x = linprog (f, A, b, Aeq, beq) solves the problem above while additionally satisfying the equality constraints *Aeq.x = beq.Set A = []* *and b = []* if no inequalities exist.

x = linprog (f, A, b, Aeq[X,fval, exitflag, output, lambda])= linprog ([X,fval, exitflag, output, lambda]) = linprog (Aeq, beq, lb, ub) defines a set of lower and upper bounds on the design variables,

x = linprog (f, A, b, Aeq, beq, lb, ub, x0) sets the starting point to *x0*. This option is only available with the medium-scale algorithm (the Large Scale option is said to 'off' using optimset). The default large-scale algorithm and the simplest algorithm ignore any starting point.

x = linprog (f, A, b, Aeq, beq, lb, ub,x0, options) minimizes with the optimization options specified in the structure options. Use optimset to set these options.

[X fval] = linprog (...) returns the value of the objective function fun at the solution *x:fval = f(x)*.

[X, lambda, exitflag] = linprog (...) returns a value exitflag that describes the exit condition

[X, lambda, exitflag, output] = linprog (...) returns a structure output that contains information about the optimization.

[X,fval, exitflag, output, lambda] = linprog (...) returns a structure lambda whose fields ~~contain the~~ Lagrange multipliers at the solution *x*.

Input the parameters of the LP into the "linprog" as

f = [-264,-719,-237,-217,-72]

$$A = \begin{bmatrix} 100 & 400 & 250 & 30 & 10 \\ 100 & 500 & 200 & 50 & 20 \end{bmatrix}$$

b = [550,500]

lb = zeros (5,1)

ub = ones (5,1)

b_{eg} = []

A_{eg} = []

[X,fval, exitflag, output, lambda] = linprog (f, A, b, Aeq, beq, lb, ub, [], []);

Enter

X,fval, lambda.ineqlin, lambda.lower

Enter

The solution to the LP problem is shown below;

Decision Variable	Solution Variables	Unit cost or Profit	Total Contribution	Shadow Price	Reduction Cost
Treasury Bill (X_1)	1.00	264.00	264.00	0	0
Fixed Deposit (X_2)	0.66	719.00	474.54	1.438	0
Savings (X_3)	0	237.00	0	0	50.60
School Project (X_4)	1.00	217.00	217.00	0	0
Agriculture (X_5)	1.00	72.00	72.00	0	0

Optimal solution (Max. objective function) = GH¢1027.5x10^2

Interpretation of Solution

The solution indicates the investment in Treasury bill (X_1 = 1), providing agricultural input (X_3 =1), and School project (X_4=1) can be done by the EDMCG within its capital limitation. At the same time, 0.66 portion of investment in fixed deposit (X_2) can be initiated while (X_3=0) indicates that the church cannot invest in savings within the time frame with the limited capital. This investment plan uses all the funds available in year zero and year one.

The shadow price indicates that the amount by which the NPV of the optimal plan (i.e.GH¢1027.5x10^2) could be increased if the budgetary constraint increases.

For every GH¢ 1x10^2 relaxation of the constraint in period two, GH¢1.438x10^2 extra NPV could be obtained. The shadow price also indicates that extra funds in period zero are not required. That is, the marginal value or dual value is not needed in the first constraint. The reduced cost of 50.6 in X_3 indicate the amount by which the objective coefficient for a variable X_3 needs to change before becoming non-zero is GH¢50.6x10^2

The slack or surplus column gives, for a particular constraint, the different between the left hand side of the constraint when evaluated at the LP optimal (i.e. evaluated at X_j) and the right hand side of the constraint. However, in the LP solution, all the constraint are tight or binding (i.e. have zero surplus or slack). None of the constraints is loose (i.e. have non-zero surplus). All these facts may give EDMCG management some guidance in their considerations of the various alternative sources of capital.

Sensitivity Analysis

As explained early on, sensitivity analysis or post optimal analysis permits the EDMCG to observe the effect of change in the parameters of the LP problem on the optimal solution.

At this point, we shall study the impact of changing;

(a) The objective function coefficient (cost coefficient of our LP for multi-capital rationing problem)

(b) The right hand side (RHS) coefficient of the constraint of our LP model.

The MATLAB LP package provides the sensitive information (the reduced cost) and shadow price. Hence, the data items concerns are varied and the LP resolved to see how the solution changes as certain parameter change.

Changing the RHS Coefficient of Constraints

The shadow price of j[th] constraint gives the amount by which the optimal Z-value is increased in the maximized problem if the RHS coefficient of constraint is increased by one. Hence, the b_2 is increased by one (i.e. 500+1 =501) and the LP resolve to get;

Decision Variable	Solution Variables	Unit cost or Profit	Total Contribution	Shadow Price	Reduction Cost
Treasury Bill (X_1)	1.00	264.00	264.00	0	0
School (X_2)	0.66	719.00	474.54	1.438	0
Health (X_3)	0	237.00	0	0	50.60
Electrification (X_4)	1.00	217.00	217.00	0	0
Agriculture (X_5)	1.00	72.00	72.00	0	0

Optimal solution (maximum objective function) = GH¢1029

Interpretation of Solution

At the optimal solution, a unit change in the constraint X_3 does do not affect the solution values and reduced cost or opportunity cost. Also, the unit increased in the capital funds for the constraints (savings(X_3)) has created a marginal value of 1.5 in the optimal objective function value (NPV) for a fiscal investment. That is, the optimal objective function increased from 1027.6 to 1029, with a difference of 1.5 which is approximately equal to the shadow price of 1.438 for the constraint X_3.

From this observation, it can be deduced that in the maximization problem, if RHS of the j^{th} constraint is increased by an amount Δb_j, then assuming the current optimal solution, the new optimal Z-value can be found from;

New optimal Z-value = Old optimal Z-value+Δb_j (constraint j shadow prices).

This analysis is very useful in planning because it enables the management to identify the most sensitive parameters or elements in the fiscal investments. Once the elements of the fiscal investments are identified, further analysis and study can take place on these elements trying to establish the likelihood of the variability and the range of values that might be expected to make a more reasoned decision whether or not to proceed with the fiscal investment.

Changing the Coefficient of the Objective Function

The reduced cost, 50.6 in X_3, row shows the amount by which the objective function coefficient for the ~~variable~~ X_3 should change to make it non-zero. Hence the coefficient of X_3 in the objective function is altered by +50.6 and the LP problem resolved.

Solving by the MATLAB package gives:

Decision Variable	Solution Variables	Unit cost or Profit	Total Contribution	Shadow Price	Reduction Cost
Treasury Bill (X_1)	1.0000	264.00	264.00	0	0
Fixed deposit (X_2)	0.38042	719.00	273.52	1.438	0
Savings (X_3)	0.69895	287.00	201.20	0	0
School project (X_4)	1.0000	217.00	217.00	0	0
Agriculture (X_5)	1.0000	72.00	72.00	0	0

Optimal solution (maximum objective function) = GH¢1027.5×10^2 .

Explanation

Addition of the reduced cost of the 50.6 on the role of variable (X_2) to its corresponding coefficient in the objective function effects no changes in the shadow prices with solution values for variables X_1, X_4 ,X_5 and the optimal objective function. However, there were sharp variations in some of the optimal values. The coefficient of variable (X_2) decreased from 0.662 to 0.38042 while variable X_3 increased from 0 to 0.69895. Thus, increasing the NPV per unit on the variable X_3 impacts a sharp change on the optimal solution. Given the sensitivity analysis of one or more of the key factors of fiscal investments like this, the management's task is to decide whether the fiscal investment is worthwhile.

Implementation of IP Model

IP works reasonably well where there is a hierarchy of decision to be made. For instance, building a new factory enables various consequential activities to take place. Although the solution depends on the values of all the decision variables, setting the values of the most important ones restricts the values of the decision variables representing the consequential activities. In such a case, the IP code will usually be worked out for itself which are the most important decisions and determine those first or it can be assisted by specifying the hierarchy of decisions explicitly. Based on these IP principles, the IP model for solving multi-period capital rationing problems;

Maximize $\quad Z = \sum_{j=i}^{N} P_j X_j$

Subject to $\quad \sum_{j=i}^{N} d_{(i,j)} X_j \leq R_j ,$

$\quad X_j = 0$ or $1, \qquad j = 1, 2, \ldots N$

is applied to the EDMCG small scale fiscal investment selection problem data (Table 2) as follows:

Purpose

The EDMCG fiscal investment manager wishes to select from N-potential fiscal investments for investments so that by the end of n-years, the fiscal investment selected will maximize returns from these fiscal investments with regards to his capital limitation. The problem can be formulated, (using the data Table 2) as zero-one IP problem (do a fiscal investment = 1, not do fiscal investment= 0). Putting the data into the IP model gives:

$$\text{Maximize } Z = 50X_1+30X_2+50X_3+10X_4+20X_5$$

$$\begin{aligned} \text{Subject to}\quad & 50X_1+10X_2+15X_3+10X_4+10X_5 \le 80 \\ & 0.3X_1+0.8X_2+1.5X_3+0.4X_4 \le 2.5 \\ & 30X_1+80X_2+15X_3+40X_4+0X_5 \le 145 \\ & 20X_1+20X_2+30X_3+10X_4+10X_5 \le 90 \\ & 20X_1+20X_2+40X_3+10X_4+20X_5 \le 100 \\ & 60X_1+30X_2+60X_3+0X_4+50X_5 \le 165 \\ & 40X_1+60X_2+0X_3+10X_4+10X_5 \le 80 \\ & X_j = 0 \text{ or } 1,\ j = 1, 2, 3, 4, 5. \end{aligned}$$

where stores = X_1, moringa project = X_2 , soap production = X_3 , Batik Tie and Dye = X_4, and Bee-Hive production = X_5. This is a Binary decision problem and can be solved easily by MATLAB software as shown below:

Solution to IP, using MATLAB

The IP model in the MATLAB package is called "bintprog". To solve an IP problem of the form;

$$\min_{x} f^T X \text{ such that}$$

$$AX \leq b$$

$$A_{eq} = b_{eq}$$

All variables are integer;

where *f, b,* and *beq* are vectors, *A* and *Aeq* are matrices, and the solution *x* is required to be a binary integer vector –that is, its entries can only take on the values 0 or 1. Using "bintprog", the syntax is

x = bintprog *(f)*

x = bintprog *(f, A, b)*

x = bintprog *(f, A, b, Aeq, beq)*

x = bintprog *(f, A, b, Aeq, beq, x0)*

x = bintprog *(f, A, b, Aeq, beq, x0, options)*

[X, fval, exitflag] = bintprog (...)

[X, fval, exitflag, output] = bintprog (...)

Description

x = *b*intprog *(f)* solves the binary integer programming problem

$$\min_{x} imize\ fX$$

x = bintprog *(f, A, b)* solves the binary integer programming problem

$$\min_{x} imize\ fX \text{ such that } AX \leq b$$

x = bintprog (f, A, b, Aeq, beq) solves the preceding problem with the additional equality constraint. *Aeq = beq*

x = bintprog (f, A, b, Aeq, beq, x0) sets the starting point for the algorithm to *x0* is not in the feasible region, bintprog uses the default initial point.

x = linprog (f, A, b, Aeq, beq, x0, options) minimizes with the default optimization options replaced by values in the structure options, which you can create using the function optimset.

[x, fval] = bintprog (…) returns *fval,* the value of the objective function at *x*.

[X,fval, exitflag] = bintprog (…) returns exitflag that describes the exit condition of bintprog.

[X,fval, exitflag, output] = bintprog (…) returns a structure output that contains information about the optimixation.

Input the parameters of IP into the "bintprog" as f = [-50,-30,-50,-10,-20]

$$A = \begin{bmatrix} 50 & 10 & 15 & 10 & 10 \\ 30 & 80 & 18 & 15 & 0 \\ 20 & 20 & 30 & 10 & 10 \\ 20 & 20 & 40 & 10 & 20 \\ 60 & 30 & 60 & 0 & 50 \\ 40 & 60 & 0 & 10 & 10 \end{bmatrix}$$

B = [80, 145, 90, 100,165, 80]

[X, fval, exitflag, output] = bintprog (*f, A, b*)

The solution to the IP problem (output from bintprog) is shown below;

Decision Variable	Solution Variables	Unit cost or Profit	Total Contribution	Reduction Cost
Stores (X_1)	1	50	50	0
Moringa project (X_2)	0	30	0	30
Soap Making (X_3)	1	50	50	0
Batik T&D (X_4)	1	10	10	0
Bee-Hive prod (X_5)	0	20	0	20
Optimal objective function value				= GH¢110×10^2

Interpretation of Solution

The optimal decision is to choose to do fiscal investments; Stores, Soap and Batik Tie and Dye whilst the EDMCG cannot undertake the moringa project and the Bee-Hive production within its capital limitation for the next five years unless the capital investment is reviewed.

The optimal decision achieves maximum returns of GH¢110×10^2 instead of GH¢160×10^2. That is, the EDMCG has been able to recover about sixty-nine percent (69%) of the targeted returns from the batch of the fiscal investment selected. It is evident that the model has assisted the fiscal investment manager to select a large number of the viable fiscal investments that can maximize profit. This is better than relying on an ad-hoc judgmental approach to the fiscal investment. The model can be used for sensitivity analysis, for instance, to examine how sensitive the fiscal investment selection decision is to changes of the model.

Extension to the LP and IP Models

The extensions to the models include;

- Fiscal investment of different lengths
- Fiscal investments with different start/end dates
- Adding capital inflows from completed fiscal investments
- Fiscal investments with staged returns
- Carrying unused capital forward from year to year
- Mutually exclusive fiscal investments (can have one or the other but not both)
- Fiscal investments with a time window for the start time.

For the amendment of LP and IP models to deal with these extensions (Appendix A)

Findings

(i) The LP and IP models revealed the following facts about fiscal investment parameters. The models;

- Enabled the EDMCG to maximize profit rather than depending on an ad-hoc judgmental approach to fiscal investment.
- Enabled the EDMCG to deal with a larger number of viable investment opportunities.
- Assisted the EDMCG fiscal investment manager to see that a unit change in the capital funds constraint x_j could create marginal value in the optimal objective function (NPV).
- A change in the coefficient objective function by the reduced cost could cause a sharp variation in some of the coefficient of constraints x_j . A unit increase in NPV would impact some changes in the optimal solution.

(ii) The study has shown that in the fiscal investment selection problems where proportions or fractional parts of a fiscal investment and a whole fiscal investment(s) is/are desired to be initiated within a period, the best decision tool is the LP model. Whenever only a whole fiscal investment is desired to be invested, the IP model is the suitable decision making tool.

(iii) It has also been found that the following computer software packages;

(a)MATLAB

(b)QSB package

(c)Microsoft Excel 2003

could be used to analyze and solve the problem at stake.

CHAPTER FOUR

SUMMARY, CONCLUSIONS AND RECOMMENDATIONS

Introduction

The results from the previous chapters are summarized in this chapter. Conclusions, discussions and recommendations are made about the findings as well.

Summary

The primary objective in this thesis was to design linear programming and integer programming models for solving Effiduase Diocese of the Methodist Church Ghana's fiscal investment selection under multi-period capital rationing problems. A second objective was to maximize the return from the fiscal investments selected with regard to the capital limitation, and also use the models to carry out sensitivity analysis on the fiscal investment parameters in order to assist fiscal investment managements decide effectively which fiscal investments are worth undertaking.

Data used for thesis is purely secondary data extracted from financial statements, annual reports, synod reports, monthly reports and other relevant documents from the Diocese. The financial ratios such as NPV, Profitability Index, and also LP and IP models were used in the analysis of the data. The discount factors at cost of capital at ten percent (10%) for each cash flow for each fiscal investment, NPV's at percent (10%) and relative profitability index (PI) computed and the results tabulated (Table 1).

LP model is designed to solve EDMCG large scale fiscal investment selection problems and this produces the optimal solution quantities (i.e. the fiscal investments to be initiated), the value of the objective function (i.e. the total NPV) and opportunity cost of the building constraints. On the other hand, selection of the small scale fiscal investment problems is solved effectively by IP model or quantitative modeling techniques.

Factors which come into play in choosing which IP solution method is appropriate are:

(i) The size of the IP (variables and constraints)
(ii) Time available to build the model (formulation plus solution algorithm)
(iii) Time available for computer solution once the model has been built
(iv) Experience

The solutions to both LP and IP models can be given by software such as:

(i) MATLAB
(ii) QSB Package
(iii) Microsoft Excel 2003

The model can be used to carry out sensitivity analysis, for instance, to examine how sensitive the fiscal investment selection decision is to change in the parameters of the model. This invariably helps the fiscal investment management to decide effectively on the fiscal investments which are worth undertaking.

Discussions and Conclusions

As explained earlier on, the idea behind the profitability index (PI) is that the PI provides the subset of fiscal investments that maximizes the aggregate NPV. However, this is not always the case. The fiscal investments were ranked with the most highly PI first i.e. Savings, ninety-two percent (92%) viable, followed by Treasury Bill , eighty-two percent (82%), Fixed Deposit, eighty percent (80%) and Agriculture, seventy-seven percent (77%). This provided the highest aggregate NPV value of GH¢1392. This value is much higher than the LP model optimal objective function of GH¢1027.5. The total capital available for investment is GH¢1050. The PI thus leads to wrong conclusion, a decision that could have reduced the EDMCG wealth.

The larger coefficient of variation of Agriculture, eighty-two percent (82%) and Treasury Bill of seventy-seven percent (77%) means that the probability of selecting these fiscal investments are more certain than others.

The LP model assisted the fiscal investment manager to accept the hundred percent (100%) investment in Treasury Bill, hundred percent (100%) agriculture inputs, hundred percent (100%) school projects and sixty-six percent (66%) investment in fixed deposit but failed to accept the investment in savings at the total cost of GH¢1027.5×10^2. This yielded an acceptable optimal decision.

Much of the information obtainable as by-product of the solution of the LP model can be useful to management in estimating the changes without going to the expense of resolving the LP. Similarly, using the IP model, the optimal decision for maximizing the returns is to do the fiscal investment: stores, soap production and Batik tie and dye at the cost of GH¢110. One helpful observation is that, the mathematical models supports fiscal investment management in the following ways. The model will enable the fiscal investment managers in:

(i) Problem identification, that is, diagnosis of the problem from its symptoms if not obvious, delineation of the sub problem to studies establishment of objectives, limitation and requirements.

(ii) Formulation of fiscal investment selection problem as LP and IP.

(iii) Model validation. This involves running the algorithm for the model on the computer in order to ensure that the input data is free from errors.

(iv) Solution of the model, that is, standard computer package or especially developed algorithms can be used to solve the model. The solutions are many under varying assumptions to establish sensitivity.

(v)Implementation, that is, the implementation of the results of the study or algorithm for solving the models serves as an operational tool.

The advantage of using a software package to solve LP and IP models, rather than a judgmental approach to fiscal investment selection problem are:

(i) Actually to maximize profit, rather than believing that the judgmental solutions maximize profit. This may end up having bad judgment.

(ii) Making the fiscal investment decision one that can be solved in a routine operational manner on a computer rather than having to exercise judgment each and every time solution to a problem is desired.

(iii) Those that can be appropriately formulated as LP are almost always better solved by computer than by people.

(iv) Carry out sensitivity analysis very easily using a computer.

Recommendations

To encourage the use of LP and IP models to solve fiscal investment selection problems;

- The decision rule where capital rationing exist is to maximize the return from the project(s) selected rather than simply accepting or rejecting decisions of the fiscal investments in isolation.
- Studies have shown that in the fiscal investment selection problems where precautions or fractional part of a fiscal investments and a whole fiscal investment(s) are desired to be initiated within a period, the best decision tool is the LP model. Whenever only a whole fiscal investment is desired to be invested, the IP model and or quantitative modeling technique is the suitable decision making tool.
- Due to the large amount of data involved and the complexity of the mathematical technique involved, fiscal investment managers are highly implored to use computer software packages such as MATLAB, QSB and Microsoft Excel to solve capital rationing problems, particularly for risk evaluation (NPV) and sensitivity analysis.
- The models can be modified to deal with the following extensions:
    - (i) Projects of a different length
    - (ii) Projects with different starts/end dates

(iii) Adding capital inflows from computed projects

(iv) Projects with stage returns

(v) Carrying unused capital forward from year to year

(vi) Mutually exclusive projects (can have one or the other but not the both)

(vii) Projects with a time window for the start time.

This in no small way is going to lessen the demand of the church on its members towards their financial commitments to the church and to ensure sustainable fiscal growth in the church.

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APPENDIX A

EXTENSIONS TO THE MATHEMATICAL MODELS OF FISCAL INVESTMENT SELECTION

Consider the IP model for EDMCG fiscal investment selection problems:

$$\text{Maximize } Z = \sum_{j=1}^{N} P_j X_j \qquad \text{(objective function)}$$

$$\text{Subject to} \quad \sum_{j=1}^{N} d_{(i,j)} X_j \le R_j \qquad \text{(constraints)}$$

$$i = 1, 2, \ldots m$$

$$X_j = 0 \text{ or } 1 \qquad \text{(non-negative constraints)}$$

$$j = 1, 2, \ldots N$$

This basic model can be modified to deal with extension or fiscal investments with the following characteristics:

Fiscal investments of different lengths

Fiscal investments of different lengths are easily dealt with, just set their capital requirement in any year in which the fiscal investment does not exist (i.e. has not started or has already ended) to zero. For example if fiscal investment X_1 only runs for two years (instead of three years) and hence finishes in year two then the capital requirement constraint for year three becomes (let j = 1, 2, 3, 4):

$0X_1+d_{(3,2)}X_2+d_{(3,3)}X_3+d_{(3,4)}X_4 \leq R_3$

Fiscal investments with different starts/end dates

Fiscal investments with different starts/end dates are dealt with in a similar manner as fiscal investments of different lengths, just set their capital requirements in any year in which they do not exist (i.e. they have not started or already ended) to zero. For example if fiscal investment X_1 starts and ends in year two; fiscal investment X_2 starts in year two and fiscal investment X_4 ends in year two then the capital requirement constraint become:

$0X_1+0X_2+d_{(1,3)}X_3+d_{(1,4)}X_4 \leq R_3$ (Year 1)

$d_{(2,1)}X_1+d_{(2,2)}X_2+ d_{(2,3)}X_3+d_{(2,4)}X_4 \leq R_2$ (Year 2)

$0X_1+d_{(3,2)}X_2+d_{(3,3)}X_3+0X_4 \leq R_3$ (Year 3)

Adding capital inflows from completed fiscal investments

If fiscal investment X_1 finishes in year two and all of the return from fiscal investment X_1 is available as capital in year three then this can be formulated by changing the capital requirement constraint for year three to:

$0X_1+d_{(3,2)}X_2+d_{(3,3)}X_3+d_{(3,4)}X_4 \leq R_3+d_{(3,2)}X_1$

It can observed that, the $0X_1$ above as fiscal investment X_1 finishes in year two and hence have no capital requirement in year three.

A question arises here in that the return from a fiscal investment is put into the capital for future years then should the counting the return comes from the fiscal investment in the objective function as well? One way to address this for fiscal investment X_1 here is to say that the return is split into two-one part y_1 (say) that is counted as return taken in the objective and one part y_2 (say) that is taken as return used for future capital in year three. Then amending the formulation gives $y_1 + y_2 = d_{(3,1)} X_1$ a balancing equality equation to correctly account for the return (since the choice may be not to do fiscal investment X_1)

$$0X_1+d_{(3,2)}X_2+d_{(3,3)}X_3+d_{(3,4)}X_4 \le R_3+y_2$$

to account for the capital added in year three

$$\text{Maximize } Z = y_1+P_2X_2+P_3X_3+P_4X_4$$

to account for the return declared as profit.

It is observed that, y_1 and y_2 (both ≥ 0) are continuous (fractional) variables so adding them in this way yields are mixed-integer program (MIP).

Solve this MIP ~~numerically, would~~ yield optimal split between the return from ~~fiscal~~ investment X_1 as return in the objective (y_1) and reinvesting it as available capital in year three (y_2).

Fiscal investments with staged returns

In this extension, a fiscal investment gives a return at various stages over its lifetime, and this return can (perhaps) be used as capital to fund ongoing (or new) fiscal investments. To illustrate how this can be formulated considered fiscal investment X_1 which gives a total return of P_1. Supposed now that this fiscal investment gives a return of q (such that $q+S = P_1$) at the end of year two, and the remaining return of S in year three. Suppose further that all of this "early" return can be used as available capital in year three. Then the capital requirement constraint in year three becomes:

$$d_{(3,1}+X_1+d_{(3,2)}X_2+d_{(3,3)}X_3+d_{(3,4)}X_4 \leq R$$

Carrying unused capital forward from year to year

In the example as currently formulated there is capital available in each year (R_1 in year one, R_2 in year two and R_3 in year three). In any particular year all of this capital may not be consumed by the fiscal investment that will be chosen to do. Suppose that it is allowed to carry forward (from to year) r% of any capital that is unused. To formulate this, introduce linear (fractional) variables C_1 and C_2 (≥ 0) with C_1 being the unused capital in year one and C_2 being the unused capital in year two. Then the constraints of the problem become:

$d_{(1,1)}X_1+d_{(1,2)}X_2+ d_{(1,3)}X_3+d_{(1,4)}X_4 +C_1=R_1$ (Year 1)

$d_{(2,1)}X_1+d_{(2,2)}X_2+ d_{(2,3)}X_3+d_{(2,4)}X_4 +C_2=R_2+0.01\ C_{1r}$ (Year 2)

$d_{(3,1)}X_1+d_{(3,2)}X_2+ d_{(3,3)}X_3+d_{(3,4)}X_4 =R_3+0.01\ C_{2r}$ (Year 3)

Note that year one and two have been employed in the equality relationship:

Capital used + unused capital = capital available

The introduction of additional variables (C_1 and C_2) makes the task of formulating the problem easier.

Mutually exclusive fiscal investments (can have one or the other but not both)

Suppose that fiscal investments X_3 and X_4 are mutually exclusive, i.e. the MA can choose to do one, or other, of these fiscal investments but not both. Then this can be formulated by adding to the problem the constraint:

$X_3 + X_4 \leq 1$

This allows the EDMCG to do neither of the fiscal investments ($X_3 = X_4 = 0$). If the managements wish to insist that they should do exactly one of these fiscal investments then $X_3 + X_4 =1$

Fiscal investments with a time window for the start time

Suppose that X_1 can start either in year one, when it has the characteristics given above, or in year two, when it has a different characteristic – still the same return of P1 but a capital requirement of K_1 in year two and K_2 in year three, where K_1, K_2 are different capital requirement. Then these can be formulated by introducing a new zero=one variable y with $y = 1$ representing choosing to do fiscal investment X_1 starting in year two, $y = 0$ representing not choosing fiscal investment X_1 starting in year two. Then the capital requirement constraint for year two and three become:

$$d_{(2,1)}X_1+d_{(2,2)}X_2+ d_{(2,3)}X_3+d_{(2,4)}X_4 +k_{1y}\leq R_2 \text{ (Year 2)}$$

$$d_{(3,1)}X_1+d_{(3,2)}X_2+ d_{(3,3)}X_3+d_{(3,4)}X_4 +k_{2y}\leq R_{3} \text{ (Year 3)}$$

The EDMCG also needs to add a constraint to prevent fiscal investment X_1 being started at more than one start time (i.e. fiscal investment X_1 starting in year one and fiscal investment X_1 in year two are mutually exclusive)

$X_1 + y \leq 1$

and the objective becomes

$$\text{Maximize } Z = P_1X_1+ P_2X_2+ P_3X_3+ P_4X_4+ P_{1y}.$$

APPENDIX B

THE REVISED SIMPLEX ALGORITHM

Consider the LP,

$$\text{Maximize } z = \sum_{j=1}^{N} C_j X_j$$

$$\text{Subject to } \sum_{j=1}^{N} a_{(i,j)} X_j \le b_j\,,\; i = 1, 2, \ldots, m$$

$$X_j > 0,\; j = 1, 2, \ldots, N$$

To show how to create a tableau for any set of basic variables, BV, we can first describe the following notation (assumed the LP has m constraints).

BV = any of basic variables (the first element of BV is the basic variable in the second constraint, and so on. Thus BV_j is the basic variable for constraint j in the desired tableau).

b = right hand side vector of the original tableau's constraints.

aj = column for X_j in the constraint of the original problem.

$B = m \times m$ matrix where the j^{th} constraint is the column for BV_j in the original constraint.

C_j = coefficient of X_j in the objective function.

$C_{BV} = 1 \times m$ row vector whose j^{th} element is the objective function coefficient for BV_j.

$u_i = m \times 1$ row vector with i^{th} element one and all other elements equal to zero.

Now it can be deduced that:

$B^{-1}a_i$ = column for X_j in BV tableau...(1)

$C_{BV} B^{-1} a_i - C_j$ = coefficient of X_j in row zero...(2)

$B^{-1}b$ = right hand side of constraint in BV tableau......................................(3)

$C_{BV}B^{-1}u_i$ = coefficient of slack variable X_j in BV in row zero......................(4)

$C_{BV}B^{-1}(-u_i)$ = coefficient of excess variable e_i in BV row..............................(5)

$M + C_{BV}B^{-1}u_i$ = coefficient of artificial variable a_i in BV row zero (in maximum problem)..(6)

$C_{BV}B^{-1} b$ = right hand side of BV row..(7)

If the BV, B^{-1} and the original tableau are known formulae (1)-(7) will enable the reader to compute any part of the simplex tableau for any set of basic variables BV. This means that if a computer is programmed to perform the simplex algorithm, all the computer needs to store on any pivot is the current set of basic variables, B^{-1}, and the initial tableau. Then (1)-(7) can be used to generate any portion of the simplex tableau. This idea is the basis of the revised simplex algorithm.

APPENDIX C

SUMMARY OF IMPORTANT FORMULAS

Payback period (PBP)

This is the cost of investment divided by the cash flow period.

i.e. $PBP = \dfrac{\text{cost of investment}}{\text{cash flow period}}$ ....(1)

Net Present Value (NPV)

The Net Present Value (including the time of money) of initial and future flows is given by the equation

NPV =

$$\sum_{t=1}^{N}\left(\frac{C_t}{(1+rp)^t}\right)$$ ....(2)

Internal Rate of Return (IRR)

This is the interest or discount rate of which the future net cash flows equals the initial cash outlay.

O =

$$\sum_{t=1}^{N}\left[\frac{C_t}{(1+IRR)^T}\right]-C$$ ....(3)

Profitability Index (PI)

This is the NPV per unit initial investment.

$$\text{i.e. Profitability Index} = \frac{\text{NPV}}{\text{Initial Investment}} \quad \text{...(4)}$$

Net Cash Flow = Cash Inflow – Cash Outflow...........