SITING OF WATER BOREHOLE AT TAMALE ISLAMIC SENIOR HIGH SCHOOL USING THE ABSOLUTE 1-CENTRE PROBLEM

BY

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CHAPTER 1

INTRODUCTION

1.1BACKGROUND OF STUDY

Water is a basic necessity of life which exists as liquid under ambient conditions and can coexist in solid and gaseous states. It is a precious resource that is essential for sustaining plant and animal life. Water supply for human needs should be available to all people in good quality and adequate quantity for today and future use thereby making it a prerequisite for human health and wellbeing (UN, 2010). Also, a required quantity and quality of water must be preserved for the sustenance of crucial functions of the ecosystem.

Water covers 70.9% of the earth's surface and most of them is in the ocean. Freshwater accounts for only 2.5% of the Earth's water, and most of it is frozen in glaciers and icecaps. The remaining unfrozen fresh water is mainly found as groundwater, with only a small fraction present above ground or in the air (Green Facts, 2009).

The two main water resources are surface water and groundwater or subsurface water. Some surface waters include lakes, rivers, streams and ponds and are naturally replenished by precipitation. Surface water is highly susceptible to pollution mainly due to human activities. Groundwater is located in pore space of soil and rocks. Groundwater also comes from precipitation. Groundwater is accessed through wells and boreholes.

Freshwater is a renewable resource from the long term geologic perspective local supplies may be inadequate in the short term (Montgomery, 1997). Water is renewed through the hydrologic cycle. Water demand outpaces supply in many parts of the world due to population growth, urbanisation, increased standards of living, pollution and growing competition for water (Green Facts, 2009; Hoekstra, 2006). Therefore the abstraction of water is greater than replenishment. Water use has been growing at more than twice the rate of the population increase during this century. By 2025, as much as two thirds of the World's population could be living in countries subject to water stress a majority of them in developing countries (SIWI, 2001). The distribution of water is not also equally distributed in time and space. Therefore it is expedient to know if people (population) all over the world have access to safe drinking water and water for personal hygiene.

In the year 2000 world leaders at the United Nations (UN) Summit concluded with the adoption of global action plan to achieve the eight anti-poverty goals, known as the Millennium Development Goals (MDGs), by 2015. One of the MDG targets is to reduce by half the proportion of people without sustainable access to safe drinking water and basic sanitation.

The uses of water include agricultural, domestic (include drinking, cooking, bathing, laundry, other sanitation use), and recreational purposes. Also water is vital in the generation of electricity, fighting fire as fire hydrants, industrial processes, and religious sacraments. These are just a few to mention.

1.11 WATER SHORTAGES IN GHANA

Water is one of the most vital natural resources for all life on earth. The availability and quality of water always have played an important part in determining not only where people can live but also their quality of life. Even though there always has been plenty of fresh water on earth, water has not always been available when and where it is needed, nor is it always of suitable quality for all uses.

Water must be considered as a finite resource that has limits and boundaries to its availability and suitability for use. The balance between supply and demand for water is a delicate one. The availability of usable water has and will continue to dictate where and to what extent development will occur. Water must be in sufficient supply for an area to develop, and an area cannot continue to develop if water demand for outstrips available supply.

Ghana is well endowed with water resources. The Volta river system basin, consisting of the Oti, Daka, Pru, Sene and Afram rivers as well as the white and black Volta rivers covers 70% of the country area. Another 22% of Ghana is covered by the south western river system watershed comprising the Bia, Tano, Ankobra and Pra rivers.

The coastal river system watershed comprising the Bia, Tano, Ankobra and Pra rivers. The coastal river system watershed, comprising the Ochi-Nawuka, Ochi Amissah, Ayensu, Densu and Tordzie River, covers the remaining 8% of the country.

Furthermore, ground water is available in Mesozoic and Cenozoic sedimentary rocks and in sedimentary formation underlying the Volta basin. Despite having plentiful supplies, Ghana suffers regular water shortages due to poor distribution of rainfall and management of resources.

In recent years, the problem has been compounded by rapid and unregulated deforestation and urbanization, with resulting pressure on supplies in towns and cities.

In Ghana, statistics show that about 40% of the population is deprived of their right to portable water. In the northern part of the country, the scarcity of water results in guinea worm disease becoming an epidemic.

In the rural areas, the inability of the people to get access to portable water makes them depend on streams and rivers which they filter to make it safe for drinking. Settles in the urban areas as well go through the trauma of water scarcity, even though they do not fetch from water bodies, but depend on the Ghana Water Company Limited for their daily supply of water.

Water rationing has been introduced in many parts of the country in response to the shortage. Now large plastic yellow containers have become a feature of daily life. In many areas, people dedicate much of their time and energy to searching for water or hauling it home.

The truth is that there is water scarcity in most parts of the urban areas in Ghana. Despite considerable strides made in the delivery of safe drinking water, Ghana still grapples with frequent widespread shortage, especially in urban centers. This is as a result of obsolete equipment, low maintenance culture and limited finance for water infrastructure through national budget.

1.12 PROFILE OF TAMALE ISLAMIC SENIOR HIGH SCHOOL

The school was founded and established by the Islamic Development Bank in February 1995 with the following aims:

1. To recruit and maintain qualified Teachers to prepare students in the Sciences to pursue carrier programmers in the applied Sciences at the Tertiary Institutions.
2. To train and bring up students morally and spiritually particularly in Islam.

The school was first headed by Mr. T.A.Mahama with a student population of 76.

The school obtained a boarding status in October, 2006 and currently has a student population of about 1,188. 847 are male students while 341 are male .The total number of students in the boarding house is 960.

There are two storey building dormitories currently on campus, one each for boys and girls, along block of seven rooms and twelve round huts both occupied by male students.

There are also nine staff Bungalows,3 semi-detached and 3 detached buildings on campus occupied my senior staff members such as the Headmaster, Two Assistant Headmasters, Senior House master and Mistress and four house masters.

One of the major problems in the school is inadequate water supply.This is because the main source of water to the school is supplied from the Ghana water company. There are very few dams and boreholes in the metropolis which are inadequate to the citizenry and unfortunately, the school has none.

1.2 PROBLEM STATEMENT

Islamic Senior High school in Tamale has had its own share of problems in relation to water shortages. The problem of water supply in the school is peculiar. This is because there is no borehole in the school and most of the times water flows through the taps once or twice within a month.

This problem is compounded in the dry season. As a result of this, students in the school have been trekking long distances each day in search of water outside the school boundary to undertake their chores. There are times that the senior housemaster would even call some students during classes hours to search for water for the school kitchen.

This unpleasant development is having a toll on academic excellence of the school as punctuality to class is always affected especially in the morning.

1.3 OBJECTIVES OF THE STUDY

The objectives of the study are

1. To model the location of borehole at Tamale Islamic Senior High School (TISSEC) as an Absolute 1-centre problem.

2. Identify the optimal water borehole location at TISSEC using the Absolute 1-centre algorithm.

1.4 METHODOLOGY

This thesis focuses on locating a water borehole at TISSEC. The absolute 1-centre model is used to obtain an optimal site where the borehole can be located. The lane map out strategy is used to find the distances between all structures at TISSEC.

1.5 JUSTIFICATION OF THE STUDY

This thesis seeks to model the location of water borehole in TISSEC and to find the optimal location for the placement of a water borehole. This can be used by the school administration to alleviate the problem of acute water shortage which is been experienced by the school.

1.6 ORGANISATION OF THE STUDY

The study consists of five chapters with chapter 1 being the introduction, chapter 2 consists of literature review. In chapter 3 the method used is discussed. Chapter 4 deals with data collection, analysis of data and results whilst the chapter 5 deals with conclusions and recommendations.

CHAPTER 2

LITERATURE REVIEW

2.0 FACILITY LOCATION- AN OVERVIEW

Facility location problems have occupied an important place in operations research since the early 1960's. They investigate where to physically locate a set of facilities so as to optimize a given function subject to a set of constraints.

Facility location models are used in a wide variety of applications. Examples include locating warehouses within a supply chain to minimize the average travel time to the markets, locating hazardous material sites to minimize exposure to the public, locating railroad stations to minimize the variability of delivery schedules, locating automatic teller machines to best serve the bank's customers, and locating a coastal search and rescue station to minimize the maximum response time to maritime accidents (Hale and Molberg, 2003).

Facility location problem dates back to the 17th century when Pierre Fermat (1643), Evansta Torecilli, and Battista Cavalieri (1647) simultaneously introduced the concept, although this theory is widely contested by location analysis experts. Late in the 18th century, Pierre Varignon presented the “Varignon Frame” which was an analog solution to the planar minisum location problem. However, facility locaton started garnering more interest when Weber (1909) introduced the planar Euclidean single facility minisum problem.

Facility location has a well-developed theroretical background (Baumol and Wolfe, 1959; Brandeau and Chiu, 1989). Generally, research in this area has been focused on optimizing methodology (Brown and Gilbson, 1972; Erlenkotter, 1975; Rosenthal, White and Young, 1978; Wesolowsky, 1977). The field has seen a rapid growth in past decades, mainly due to the evolution of location theory and the advent of computer technology (Church, 1999)

Extensive effort has been devoted to solving location problems employing a wide range of objective criterion and methodology used in the decision analysis. Geoffrion (1978), for instance, includes decomposition, mixed integer linear programming, simulation and heuristics that may be used in analyzing location problems. He notes that a suitable methodology for supporting managerial decisions should be computationally efficient, lead to an optimal solution, and be capable of further testing. Other researchers stress the importance of multiple criteria that must be included in the decision analysis (Erlenkotter, 1975).

Baumol and Wolfe (1958) have solved the location problem for minimum total delivery cost with nonlinear programming. Others have incorporated stochastic functions to account for demand and/or supply (Rosenthal, White and Young, 1978; Wesolowsky, 1977). Other approaches that have been employed include dynamic programming (Geoffrion, 1978; Saaty, 1996; Tansel, Francis and Lowe, 1989), multivariate statistics using multidimensional scaling (Asami and Walters, 1989) and heusistic and search procedures (Kuehn and Hamburger, 1963).

Randhawa and West, (1995) proposed a solution approach to facility location selection problems while integrating analytical and multi-criteria decision-making models. Houshyar and White (1997) developed a mathematical model and heuristics approach that assigns N machines to N equal-sized locations on a given site such that the total adjacent flow between the machines is maximized. The proposed mode based on 0-1 integer programming formulation which may produce an optimal, but infeasible solution, followed by the heuristic which begins with the 0-1 integer solution and generates a feasible solution.

Chu (2002) presented a fuzzy TOPISI (technique for order preference by similarity to ideal solution) method-based approach for the plant location selection problems. The ratings and weight assigned by decision makers are first normalized rating of each alternative location for each criterion is then developed. A closeness coefficient is proposed to determine the ranking order of the alternatives.

Klose and Drexl (2005) reviewed in details the contributions to the current state-of-the-art related to continuous location models, network location models, mixed-integer programming models and their applications to location selection decision. Yong (2006) proposed s new fuzzy TOPSIS method which deals with the selection of plant location decision-making problems in linguistic environment, where the ratings of various alternative locations under different criteria and their weights are assessed in linguistic terms represented by fuzzy numbers.

Farahani and Asgari, (2007) presented a TOPSIS methodology to find the supportive centers with the minimum number and maximum quality of locations in military logistic system. Önüt and Soner, (2008) employed a fuzzy TOPSIS based methodology to solve the solid waste transshipment site selection problem, where the criteria weights are estimated using analytic hierarchy process (AHP). Amiri *et al.* 2009 applied TOPSIS method along with heuristics based on fuzzy goal programming to select location. The facility location selection problem is solved in three stages, i.e. (a) finding the last number distribution centers, (b) locating them in the best possible, and (c) finding the minimum cost of locating the facilities.

For a single facility problem with three existing facilities, Juel and Love (1986) proved that it is possible to determine which existing facility is the optimal location by means of simple geometrical construction.

For the multifacility location problem with no constraints on the location of the new facilities, Juel and Love (1980) derived some sufficient conditions for the coincidence of facilities that are valid in a general symmetric metric. These results were later extended by Lefebvre *et al*. (1991) to be applicable to some location problems having certain locational constraints. (Examples of other works on this subject are Francis and Cabot (1972), Calamai and Caralambus (1980), Calamai and Conn (1980) and (1982), Dax (1986 b), Overton (1983), Lefebvre *et al*. (1990), and Plastria (1992).)

Location models can be classified into three broad categories (Daskin, 1995): *p*-median models, *p*-center models and covering models. Other taxonomies of location models are discrete versus continuous demand, discrete versus continuous facility location, media versus center, location and routing problems, single versus multiple objectives, stochastic versus deterministic problems, (Drezner, 1995)

2.1 *P* – CENTER PROBLEM

The *p*-center problem (Hakimi, 1964, 1965) is the problem of minimizing the maximum distance that demand is from its closet facility given that there are pre-determined numbers of facilities. Hakimi's work established important results in location theory and sparked theoretical interest among researchers. The vertex *p*-center problem restricts the set of candidate facility sites to the nodes of the network. The absolute p -center problem permits the facilities to be anywhere along the arcs. Both versions are examined in weighted and un-weighted situations. The absolute center problem can be approximated by the vertex center problem by adding nodes to the network.

The objective of the *p*-center problem is to locate *p* new facilities, called centers, on a network G in order to minimize the maximum weighted distance between a node and its nearest facility.

The methods developed for solving this problem are quite different from those for the *p-median* problem, even though the two problems are related. The first approach developed was by Hakimi (1964) who proposed an enumerative approach for *p*=1 to specifically locate a local center on each link, and thereby to determine the overall optimal location. A more effective method was suggested by Christofides (1975), who showed that one needs to consider only a subset of the links for an optimal location. However, this approach is unable to solve general p-center problems. Erkut *et al*. (1992) presented a polynomial time, binary search algorithm to solve the distance-constrained p-center problem.

More methods have been proposed and tested to solve the p-center problem, such as the exact algorithms due to Christofides and Viola (1971), Granfinkel *et al*. (1977), and various heuristics proposed by Singer (1968).

Garfinkel et al. (1977) examined the fundamental properties of the P-centre problem in order to locate a given number of emergency facilities along a road network. He modelled the P-centre problem using integer programming and the problem was successfully solved by using a binary search technique and a combination of exact tests and heuristics.

ReVelle and Hogan (1989) formulated a *P*-centre to locate facilities so as to minimize the maximum distance within which the EMS is available with (alpha) reliability. System congestion is considered and a derived server busy probability is used to constrain the service reliability that must be satisfied for all demands.

Hochbaun and Pathria (1998) considered the emergency facility location problem that must minimize the maximum distance on the network across all time periods using the Stochastic P-centre models. The cost and distance between locations vary in each discrete time periods. The authors used k underlying networks to represent different periods and provided a polynomial-time, 3-approximation algorithm to obtain a solution for each problem.

Talwar (2002) utilized a P-centre model to locate and dispatch three emergency rescue helicopters to serve the growing EMS demands due to accidents occurring during adventure holidays such as skiing, hiking and climbing the north and south Alpine mountain ranges. One of the model's aims is to minimize the maximum (worst) response time and the author used effective heuristics to solve the problem.

Drezner (1984) presented heuristic and optimal algorithms for the p-center problem in the plane. The heuristic method yielded results for problems with up to n = 2000 and p = 10 whereas the optimal method solved problems with up to n = 30, p = 5 or n = 40, p = 4. Watson-Gandy [1984] suggested an algorithm that can optimally solve problems with up to about 50 demand points and 3 centers in reasonable time.
The p-center problem on networks has been solved by Minieka (1970) and by Toregas et al. (1971). A finite method, which is rather inefficient for large problems was suggested. An improvement based on the use of relaxations was offered by Handler and Mirchandani (1979).

Hwang et al. (1993) describe a slab-dividing approach, which is expected to efficiently solve the Euclidean p-center problem. Suzuki and Drezner (1996) propose heuristic procedures and upper bounds on the optimal solution where the demand points are distributed on a square. One of the methods they use employs the Voronoi heuristic. The same method has been recently used by Wei et al. [2006]; the authors explore the complexity of solving the continuous space p-center problem in location planning.

Agarwal and Sharir (1998) discuss efficient approximate algorithms for geometric optimization, which includes the Euclidean p-center in d dimensions. Hale and Moberg

(2004) give a broad review on location problems, which includes the Euclidean p-center problem.

Another version of the p-center problem according to Mladenovic et al (2003) deals with the minimax p center problem where the Euclidean distances are $d_{ij} = (u_i, v_j)$ and where U = $\{u_1, u_2, \dots, u_m, \}$ is a set of m users and V = $\{v_1, v_2, \dots, v_n\}$ a set of n potential locations for facilities in the plane. In these works, Tabu search and Variable Neighborhood Search methods as well as an optimal method are used, and the efficiency of these methods for small and large problems is evaluated. It should be noted that this Euclidean problem is equivalent to the p-center problem on networks where the possible location of the facilities are on the vertices and where the minimum distances between the demand and potential supply points are given. This discrete problem is also known to be NP-hard according to Kami et al (1971). Recent works on these two versions of the discrete problem include algorithms given by Caruso et al. (2003) and by Ilhan et al. (2002). The latter authors describe an efficient exact method for this p-center problem. Their algorithm finds the solution by updating, at each step, an upper or lower bound on the optimal solution. A tight lower bound to the optimal value is found in an initial phase of the algorithm, which consists of solving linear programming sub-problems.

2.2 P – MEDIAN

The *p*-median Problem (*p*-M) is to locate *p* new facilities, called medians, on the network *G* in order to minimize the sum of the weighted distances from each node to its nearest new facility (Francis *et al*., 1992).

If $p \geq 2$, then this problem can be viewed as a location-allocation problem (LAP). This is because the location of the new facilities will determine the allocation of their service in order to best satisfy the nodal demands.

Hakimi (1964) proved that in networks, a set of optimal locations will always coincide with the vertices. He also proposed an enumerative-graph theoretical approach for the problem. Revelle and Swain (1970) proposed other procedures to solve this problem after reformulating it as an integer programming (IP) problem. Jarvinen et al. (1972) also used this IP formulation and proposed a branch-and-bound algorithm for this problem. Due to the NP-hardness of the problem, several heuristic procedures have been developed, such as those of Maranzana (1964) and Teitz and Bart (1968). Beasly (1993) has also developed Lagrangian heuristics for this *p*-median location problem, based on Lagrangian relaxation and subgradient optimization concepts.

There are several variants and extensions of the *p*-median problem. One type of variant, studied by Pesamosca (1991), considers the interaction weights between the new facilities as well as the connection scheme as a tree. This case was treated as a problem Euclidean distance multifacility location problem(EMFLP) on a tree and its optimality conditions were then obtained using the optimality conditions of *p* problems of the type Euclidean single facility location problem(ESFL). Accordingly, for solving the problem Euclidean distance multifacility location problem, a fixed point algorithm was developed to iteratively solve single facility location problem(ESFL) using the Weiszfeld algorithm if differentiability is met, and otherwise, the algorithm switches over to Miehle's algorithm.

Another type of variant involves placing the capacity restrictions on the facilities to be located. When the capacity is finite, the resulting problem is called a capacitated problem; otherwise the problem is uncapacitated.

Cavalier and Sherali (1986) presented exact algorithms to solve the *p*-median problem on a chain graph and the 2-median problem on a tree graph, where the demand density functions are assumed to be piecewise uniform. For the uncapacitated *p*-median problem, Chiu (1987) addressed the 1- median problem on a general network as well as on a tree network. Dynamic location considerations on networks are addressed by Sherali (1991).

Recently, Francis et al. (1993) developed a median-row-column aggregation algorithm to solve large-scale rectilinear distance *p*-median problems. On the other hand, Sherali and Nordai (1988) gave certain localization results and algorithms for solving the capacitated *p*-median problem on a chain graph and the 2- median problem on a tree graph.

Another variant involves the treatment of a continuous demand over the network, which arises in some situations such as the location of public service facilities or in probabilistic distributions of demand. Among the contributions on this variant are Minieka (1978), Handler and Mirchandani (1979), Chiu (1987) and Derardo *et al*. (1982). Sherali and Rizzo (1991) solved an unbalanced, capacitated *p-median* problem on a chain graph with a continuum of link demands. For solving this problem, they considered two unbalanced cases, the deficit and over-capacitated cases, provided a first-order characterization of optimality for these two problems and developed an enumerative algorithm based on a partitioning of the dual space. There are still further variants that include capacity restrictions on links, probabilistic travel times on links, and maximum distance constraints.

It is worthwhile to note that the p-median model has been extended and expanded in a number of ways. These include models such as the Hierarchical Median (Narula and Ogbu,1979), the Stochastic Median (Mirchandani and Odoni,1977), the Temporal Median(Swain,1976), the Transportation Median (Neebe,1978), the Zonal Constrained Median (Church,1990) and Berman, Einav, and Handler (1991) and the Location and Scheduling Median Problems (Bloxham and Church,1991)). Further, it is easy to show that a number of location models are equivalent to one or more forms of the p median model (Hillsman, 1984), Church and ReVelle (1976), Church and Weaver (1986).

Since its formulation the *P*-median model has been enhanced and applied to a wide range of emergency facility location problems. Carbone (1974) formulated a deterministic *P*-median model with the objective of minimizing the distance traveled by a number of users to fixed public facilities such as medical or day-care centers. Recognizing the number of users at each demand node is uncertain, the author further extended the deterministic *P*-median model to a chance constrained model. The model seeks to maximize a threshold and meanwhile ensure the probability that the total travel distance below the threshold is smaller than a specified level α.

Calvo and Marks (1973) constructed a *P*-median model to locate multi-level health care facilities including central hospitals, community hospitals and local reception centers. The model seeks to minimize distance and user costs, and maximize demand and utilization. Later, the hierarchical *P*-median model was improved by Tien et al. (1983) and Mirchandani (1987) by introducing new features and allowing various allocation schemes to overcome the deficient organization problem across hierarchies.

Paluzzi (2004) discussed and tested a *P*-median based heuristic location model for placing emergency service facilities for the city of Carbondale. The goal of this model is to determine the optimal location for placing a new fire station by minimizing the total aggregate distance from the demand sites to the fire station. The results were compared with the results from other approaches and the comparison validated the usefulness and effectiveness of the *P*-median based location model.

One major application of the *P*-median models is to dispatch EMS units such as ambulances during emergencies. Carson and Batta (1990) proposed a *P*-median model to find the dynamic ambulance positioning strategy for campus emergency service. The model uses scenarios to represent the demand conditions at different times. The ambulances are relocated in different scenarios in order to minimize the average response time to the service calls.

Berlin et al. (1976) investigated two *P*-median problems to locate hospitals and ambulances. The first problem has a major attention to patient needs and seeks to minimize the average distance from the hospitals to the demand points and the average ambulance response time from ambulance bases to demand points. In the second problem, a new objective is added in order to improve the performance of the system by minimizing the average distance from ambulance bases to hospitals.

Mandell (1998) developed a *P*-median model and used priority dispatching to optimally locate emergency units for a tiered EMS system that consists of advanced life-support (ALS) units and basic life-support (BLS) units. The model can also be used to examine other system parameters including the balance between ALS and BLS units, and different dispatch rules.

Uncertainties have also been considered in many *P*-median models. Mirchandani (1980) examined a *P*-median problem to locate fire-fighting emergency units with consideration of stochastic travel characteristics and demand patterns. The author took into account the situations that a facility may not be available to serve a demand and used a Markov process to create a system in which the states were specified according to demand distribution, service and travel time, and server availability.

Serra and Marianov (1999) implemented a *P*-median model and introduced the concept of regret and minmax objectives when locating fire station for emergency services in Barcelona. The authors explicitly addressed in their model the issue of locating facilities when there are uncertainties in demand, travel time or distance. In addition, the model uses scenarios to incorporate the variation of uncertainties and seeks to give a compromise solution by minimizing the maximum regret over the scenarios.

P-median models have also been extended to solve emergency service location problems in a queuing theory context. An example is the stochastic queue median (SQM) model due to Berman et al. (1985). The SQM model seeks to optimally dispatch mobile servers such as emergency response units to demand points and locate the facilities so as to minimize average cost of response.

2.3 COVERING PROBLEMS

Unlike the *p*-median problem which seeks to minimize the total travel distance, covering models are based on the concept of acceptable proximity. The objective of covering models is to provide "coverage" to demand points. A demand point is considered as covered only if a facility is available to service the demand point within a distance limit. Covering models can be classified according to several criteria. One of such criteria is the type of objective, which allows us to distinguish between two types of formulations.

The first type belongs to the Location Set Covering Problem (LSCP). The Location Set Covering Problem (LSCP) seeks to locate the minimum number of facilities that will cover all demands within a specified maximum distance (Toregas, et al. 1971). The problem is applied to emergency services location where a given amount of the population must be within a predefined maximum distance from a facility. The limit on maximum distance (or response time) is adopted to ensure that demands (emergency calls) are answered in timely fashion.

The second type can be classified as the Maximal Covering Location Problem (MCLP), which maximizes covered customer demand, given a limited number of facilities. The MCLP was first introduced in Church and ReVelle (1974).

Church and Meadows (1979) provided a pseudo-Hakimi property for the MCLP. This property states that for any network, there exists a finite set of points that will contain at least one of the optimal solutions to the MCLP. Daskin and Stern (1981), Hogan and ReVelle (1986), and Batta and Mannur (1990) developed the MCLP that contains a secondary "backup" coverage objective. Berman and Krass (2002) showed that the MCLP with a step coverage function is equivalent to the uncapacitated facility location problem (Cornue'jols et al., 1990). They developed two IP formulations for the problem and showed an interesting result that the LP relaxations of both formulations provide the same value of the upper bound.

In a recent paper, Berman *et al.* (2003) investigated the MCLP with a coverage decay function whose value decreases from full coverage at the lowest pre-specified radius to no coverage at the highest pre-specified radius.

Daskin (1983) provided a probabilistic formulation of the problem in which the probability of an arbitrary server being busy is specified exogenously. The objective, then, is to locate facilities so as to maximize the expected number of demand that a facility can cover. Daskin's formulation is sometimes referred to as the Maximal Expected Covering Location Problem.

Application of the set covering model includes airline crew scheduling (Desrocher et al., 1991). According to Daskin et al.(1990) it can also be applied to tool selection in flexible manufacturing systems.

Covering models are the most widespread location models for formulating the emergency facility location problems.

LSCP is an earlier statement of the emergency facility location problem by Toregas *et al.* (1971) and it aims to locate the least number of facilities that are required to cover all demand points. Since all the demand points need to be covered in LSCP, regardless of their population, remoteness, and demand quantity, the resources required for facilities could be excessive. Recognizing this problem, Church and ReVelle (1974) and White and Case (1974) developed the MCLP model that does not require full coverage to all demand points. Instead, the model seeks the maximal coverage with a given number of facilities. The MCLP, and different variants of it, have been extensively used to solve various emergency service location problems. A notable example is the work of Eaton et al. (1985) that used MCLP to plan the emergency medical service in Austin, Texas. The solution gives a reduced average emergency response time even with increased calls for service.

Schilling et al. (1979) generalized the MCLP model to locate emergency fire-fighting servers and depots in the city of Baltimore. In their model, known as FLEET (Facility Location and Equipment Emplacement Technique), two different types of servers need to be located simultaneously. A demand point is regarded as "covered" only if both servers are located within a specified distance.

The preceding models do not consider the system congestion and unavailability of the facilities. Many covering models have also been developed to address the possible congestion condition by providing redundant or back-up coverage. Daskin and Stern (1981) formulated a hierarchical objective LSCP for emergency medical service in order to find the minimum number of vehicles that are required to cover all demand areas while simultaneously maximizing the multiple coverage. Bianchi and Church (1988) proposed an EMS facility model in which they restricted the number of facilities but allowed more than one server at each facility site. Benedict (1983), Eaton et al. (1986), and Hogan and ReVelle (1986) developed MCLP models for emergency service that has a secondary "backup-coverage" objective. The models ensure that a second (backup) facility could be available to service a demand area in case that the first facility is unavailable to provide services. The backup coverage models have been popularly called as Backup Coverage Problem 1 (BACOP1). Since the models of BACOP1 require each demand point to have first coverage which is not necessary for many location problems, Hogan and ReVelle (1986) further formulated the BACOP2 model which is able to respectively maximize the population that achieve first and second coverage.

Research on emergency service covering models has also been extended to incorporate the stochastic and probabilistic characteristics of emergency situations so as to capture the complexity and uncertainty of these problems. Examples of these stochastic models can be found in recent papers by Goldberg and Paz (1991), ReVelle et al. (1996), and Beraldi and Ruszczynski (2002).

There are several approaches to model stochastic emergency service covering problems. The first approach is to use chance constrained models (Chapman and White, 1974). Daskin (1983) used an estimated parameter (q) to represent the probability that at least one server is free to serve the requests from any demand point. He formulated the Maximum Expected Covering Location Problem (MEXCLP) to place P facilities on a network with the goal to maximize the expected value of population coverage. ReVelle and Hogan (1986) later enhanced the MEXCLP and proposed the Probabilistic Location Set Covering Problem (PLSCP). In the PLSCP, an average server busy fraction (qi) and a service reliability factor (α) are defined for the demand points. Then the locations of facilities are determined such that the probability of service being available within a specified distance is maximized. The MEXCLP and PLSCP later were further modified to tackle other EMS location problems by ReVelle and Hogan (1989) as(MALP) , Bianchi and Church (1988) as(MOFLEET) , Batta *et al*. (1989) as (AMEXCLP) , Goldberg et al. (1990), and Repede and Bernardo(1994) as (TIMEXCLP).

Another approach to modeling stochastic EMS covering problems is to use scenario planning to represent possible values for parameters that may vary over the planning horizon in different emergency situations. A compromise decision is made to optimize the expected/worst-case performance or expected/worse-case regret across all scenarios. For example, Schilling (1982) extended the MCLP by incorporating scenarios to maximize the covered demands over all possible scenarios. Individual scenarios are respectively used to identify a range of good location decisions. A compromise decision is made to the final location configuration that is common to all scenarios in the horizon.

One important thrust and cornerstone in location theory is the development and application of the queuing approach in solving EMS location problems. The most well known queuing models for emergency service location problems are the hypercube and approximated hypercube by Larson (1974, 1975), which consider the congestions of the system by calculating the steady-state busy fractions of servers on a network. The hypercube model can be used to evaluate a wide variety of output performance such as vehicle utilization, average travel time, inter-district service performance, etc. Particularly important in the hypercube models is the incorporation of state-dependent interactions among facilities (mobile servers) that preclude applications of traditional location models. Larson (1979) and Brandeau and Larson (1986) later further extended and applied the hypercube models with locate-allocate heuristics for optimizing many realistic EMS systems. For example, these extended models have been successfully used to optimize the ambulance deployment problems in Boston and the EMS systems in New York. Based on the hypercube queuing model, Jarvis (1977) developed a descriptive model for operation characteristics of an EMS system with a given configuration of resources and a location model for determining the placement of ambulances to minimize average response time or other geographically based variables. Marianov and ReVelle (1996) created a realistic location model for emergency systems based on results from queuing theory. In their model, the travel times or distances along arcs of the network are considered as random variables. The goal is to place limited numbers of emergency vehicles, such as ambulances, in a way as to maximize the calls for service. Queueing models formulating other various theoretical and practical problems have also been reported by Berman and Larson (1985), Batta (1989), and Burwell et al. (1993).

CHAPTER 3

METHODOLOGY

3.1 INTRODUCTION

Facility location represents the process of identifying the best location for a service, commodity or production facility.

Facility location models can be classified into three broad categories. These are p-median, p-centre and set covering models.

3.2 SET COVERING MODEL

The set covering problem is to find a minimum cost set of facilities from among a finite set of candidate facilities so that every demand node is covered by at least one facility. This may be formulated mathematically, using the following notation

Inputs:

$$a_{ij} = \begin{cases} 1 & if\ candidate\ site\ j\ can\ cover\ demands\ at\ node\ i \\ 0 & if\ not \end{cases}$$

$$f_j \ = cost\ of\ locating\ a\ facility\ at\ candidate\ site\ j$$

Decision variables

$$X_j = \begin{cases} 1 & if\ we\ locate\ at\ candidate\ site\ j \\ 0 & if\ not \end{cases}$$

With this notation, we can formulate the set covering model as follows:

Minimize:

$$\sum_{j} f_j X_j \dots\dots\dots\dots\dots..(3.1)$$

Subject to:

$$\sum_{j} a_{ij} X_j \geq 1 \;\; \forall_i \quad \dots\dots..(3.2)$$

$$X_j = 0, 1 \quad \forall_j \dots\dots\dots\dots.(3.3)$$

The objective function (3.1) minimizes the total cost of the facilities that are selected.

Constraints (3.2) stipulate that each demand node i must be covered by at least one facility. The left hand side of (3.2) gives the number of located facilities that can cover demand node i.

These constraints may be rewritten in terms of the set N_i as follows:

$$\sum_{j \in N_i} X_j \geq 1 \; \forall_i \dots\dots\dots\dots(3.4)$$

Where N_i is the set of candidate locations j that can cover demand node i. The two forms of the constraint are equivalent. Constraints (3.3) are the integrality constraints. If all of the facility costs are identical (e.g. $f_j = 1$ for all candidate sites j), or if we simply want to minimize the number of selected facilities, the objective function may be simplified to become

Minimize:

$$\sum_{j} X_j \dots\dots\dots\dots\dots..(3.5)$$

3.3 P – MEDIAN MODEL

The p – median problem is to find the location of p facilities on a network so that the total cost is minimized. The cost of serving demands at node i is given by the product of the demand at node i and the distance between demand node i and the nearest facility to node i. This problem may be formulated using the following notation:

Let

$h_i = demand\ at\ node\ i$

$d_{ij} = distance\ between\ demand\ node\ i\ and\ candidate\ site\ j$

$p\ = number\ of\ facilities\ to\ locate.$

$$X_j = \begin{cases} 1\ if\ we\ locate\ at\ candidate\ site\ j \\ 0 \qquad\qquad\qquad\qquad if\ not \end{cases}$$

$$Y_{ij} = \begin{cases} 1\ if\ demands\ at\ node\ i\ are\ served\ by\ a\ facility\ at\ node\ j \\ 0 \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad if\ not \end{cases}$$

With this notation, the P –median problem can be formulated as follows:

Minimize

$$\sum_i \sum_j h_i d_{ij} Y_{ij} \ldots\ldots\ldots\ldots\ldots..(3.6)$$

Subject to:

$$\sum_j Y_{ij} = 1 \quad \forall_i \ldots\ldots\ldots\ldots\ldots(3.7)$$

$$\sum_j X_j = P \ldots\ldots\ldots\ldots\ldots\ldots.(3.8)$$

$$Y_{ij} - X_j \leq 0 \quad \forall_{i,j} \ldots\ldots\ldots (3.9)$$

$$X_j = 0{,}1 \quad \forall_j \ldots\ldots\ldots\ldots (3.10)$$

$$Y_{ij} = 0{,}1 \quad \forall_{i,j} \ldots\ldots\ldots (3.11)$$

The objective function (3.6) minimizes the total demand –weighted distance between each demand node and the nearest facility. Constraint (3.7) requires each demand node i to be assigned to exactly on facility j.

Constraint (3.8) states that exactly P facilities are to be located. Constraint (3.9) link the location variables (X_j) and the allocation variables (Y_{ij}). They state that demands at node i can only be assigned to a facility at location j $(Y_{ij} = 1)$ $if\ a$ facility is located at node j $(X_j = 1)$.

Constraints (3.10) and (3.11) are the standard integrality conditions.

3.4 P- CENTRE MODEL

The objective of the P –centre model is to find locations of p facilities so that all demands are covered and the maximum distance between a demand node and the nearest facility (coverage distance) is minimized. It can be said that we have relaxed the average distance.

In the P- centre model, each demand point has a weight. These weights may have different interpretations such as time per unit distance, cost per unit distance or loss per unit distance. So the problem would be seeking a centre to minimize a maximum time, cost or loss. The p-centre model is also known as a minimax problem.

3.4.1 Unweighted P-Centre Problem

Let

$d_{ij} = distance\ from\ demand\ node\ i\ to\ candidate\ facility\ j$

$h_i = demand\ at\ node\ i$

$P = number\ of\ facilities\ to\ locate$

$$X_j = \begin{cases} 1\ if\ we\ locate\ at\ candidate\ site\ j \\ 0 \qquad\qquad\qquad\qquad if\ not \end{cases}$$

$Y_{ij} = fraction\ of\ demand\ at\ node\ i\ that\ is\ served\ by\ a\ facility\ at\ node\ j$

$W = maximum\ distance\ between\ a\ demand\ node\ and\ the\ nearest\ facility$

The model is formulated as:

Minimize W ………………………………………………. (3.12)

Subject to

$$\sum_j Y_{ij} = 1 \quad \forall i \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3.13)$$

$$\sum_j X_j = P \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3.14)$$

$$Y_{ij} \leq X_j \quad \forall i, j \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3.15)$$

$$W \geq \sum_j d_{ij}\, Y_{ij} \quad \forall i \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3.16)$$

$$X_j = 0,1 \quad \forall j \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3.17)$$

$$Y_{ij} \geq 0 \quad \forall i, j \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3.18)$$

The objective function (3.12) minimizes the maximum distance between a demand node and the closest facility to the node.

Constraints (3.13) state that all of the demand at node *i* must be assigned to a facility at some node *j* for all nodes *i*.

Constraint(3.14) stipulates that P facilities are located.

Constraint(3.15) state that demands at node *i* cannot be assigned to a facility at node *j* unless a facility is located at node *j*.

Constraint(3.16) state that the maximum distance between a demand node and the nearest facility to the node (W) must be greater than the distance between any demand node *i* and the facility *j* to which it is assigned. Constraint (3.17) and (3.18) are the integrality and nonnegativity constraints, respectively.

3.4.2 P-Centre Model with Demand weighted distance

Let

$d_{ij} = distance\ from\ demand\ node\ i\ to\ candidate\ facility\ j$

$h_i = demand\ at\ node\ i$

$P = number\ of\ facilities\ to\ locate$

$$X_j = \begin{cases} 1\ if\ we\ locate\ at\ candidate\ site\ j \\ \qquad 0\ \ if\ not \end{cases}$$

$Y_{ij} = fraction\ of\ demand\ at\ node\ i\ that\ is\ served\ by\ a\ facility\ at\ node\ j$

$W = maximum\ distance\ between\ a\ demand\ node\ and\ the\ nearest\ facility$

The model is formulated as:

Minimize W ………………………………………….. (3.19)

Subject to

$$\sum_j Y_{ij} = 1 \quad \forall i \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3.20)$$

$$\sum_j X_j = P \ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots.. (3.21)$$

$$Y_{ij} \le X_j \quad \forall i,j \ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3.22)$$

$$W \geq h_i \sum_j d_{ij} Y_{ij} \quad \forall i \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots. (3.23)$$

$$X_j = 0,1 \quad \forall j \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots. (3.24)$$

$$Y_{ij} \geq 0 \quad \forall i, j \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots. (3.25)$$

3.4.3 The Absolute -1- Centre Problem

The general absolute -1-centre problem is

$$\min_{x \in G} m(x) = \max_{v \in V} d(x, V_i)$$

Subject to $x \in G$.

The above formulation implies finding the vertex and the local centres.

A point x is either a node of G or a point on an edge of G. G is a complete or weighted undirected graph with vertices having demands which indicates population.

The vertex centre is defined to be the minimum of all row maximums obtained from the matrix of all pairs shortest paths.

The local centre of an edge (p, q) is a point x_l on $the\ graph\ of\ edge\ (p, q)$ which minimizes the upper envelope.

To find the absolute centre x^*,

(a) Evaluate all vertices and find the vertex centre value

(b) Evaluate all edges to find the local centre with the minimum value

(c) Compare the two values, that is, the minimum vertex centre value and the minimum edge value. The lowest of the two is the solution x^* with cost $m(x^*)$.

Example:

Figure 3.1 below shows the network system for five cities A, B, C, D, and E.

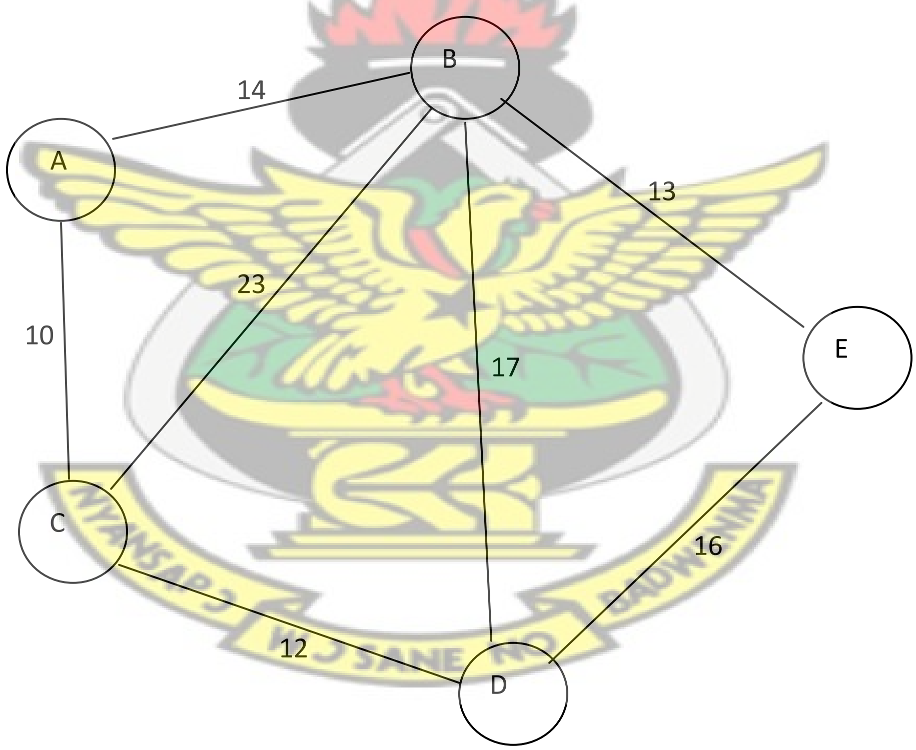


Figure3.1:Network system for five cities A,B,C,D,E

Step (a) - Finding the vertex centre

To find the vertex centre, we compute the matrix of shortest paths costs for all pairs of nodes using the Floyd's algorithm and then choose a node such that the maximum entry in its row in the matrix is the smallest among the maximum entries of all rows.

By using the Floyd Warshall algorithm, we obtain a matrix of the shortest paths of the network of figure 1. The algorithm then computes the distance, d (p, q), for all node pairs p and q as shown in table 3.1.

	A	B	C	D	E	ROW MAX
A		14	10	22	27	27
B	14		23	17	13	23
C	10	23		12	28	28
D	22	17	12		16	22
E	27	13	28	16		28

Table 3.1: Matrix of all pairs shortest paths of figure 1

From table 1, the minimum maximum entry on all the rows occurs at D with m(D)=22.D is therefore the vertex centre.

Step (b) – Finding the local centre

The local centre for each edge can be found as shown below. Consider an edge (p ,q) with a point x on it as shown in figure 2 below

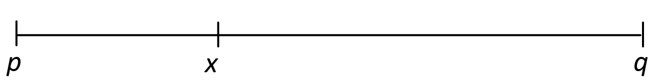


Figure 3.2 An edge (p,q) with a point x on it

An edge is a distance between two vertices p and q and denoted by c(p,q).

A shortest path is the total distance between two vertices which is not direct but passing through other vertices.This is denoted by d(p,q) described as the minimum path cost. That is

$$d(p,q) = min \sum_{i=0}^{n} c\left(V_{i,}V_{i+1}\right) \dots\dots\dots\dots\dots..(3.26)$$

Assuming we want to move from x to V_i(V_i being any node or vertex on G), we find the minimum path cost by moving to V_i through p or q as shown in figure 3.3 below

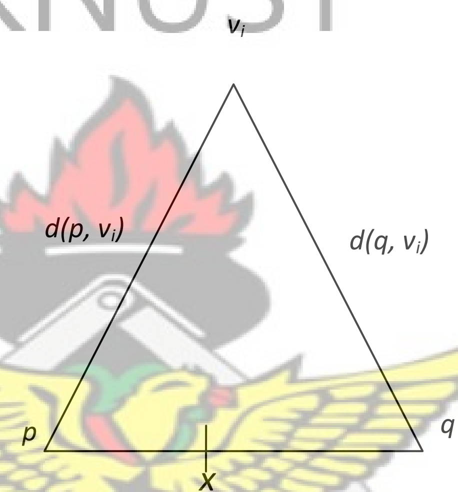


Figure 3.3: Movement from x to V_i through either p or q

From the diagram above,

d (p, x)= x

d (x, q) = c(p, q) – x

d (p, x) + d (x, q) = c(p, q)

Also movement from x to V_i can be done in two directions that is, through p or q giving rise to the equations below.

$$y_1 = x + d(p, V_i) \dots\dots\dots\dots\dots\dots(3.27)$$

$$y_2 = c(p,q) - x + d(q,V_i) \dots\dots (3.28)$$

y_1 is the distance from x to V_i through p and y_2 is the distance from x to V_i through q.

As x moves along the edge (p ,q), there will be a point when the two distances or costs will be equal. At this point, $y_1 = y_2$ and the kink point can be found.

Solving for the point of equal costs, we have

$$x + d(p,V_i) = c(p,q) - x + d(q,V_i)$$

$$x + x = c(p,q) + d(q,V_i) - d(p,V_i)$$

$$2\,x = c(p,q) + d(q,V_i) - d(p,V_i)$$

$$x = \frac{c(p,q) + d(q,V_i) - d(p,V_i)}{2},$$

where x can be denoted by X_m being the point of maximum cost. As V_i assumes all the nodes on the network, a number of equations will be generated under equations (3.27) and (3.28).These equations will be sketched on the same axes in the range $0 \le x \le c(p,q)$ obtained from solving for the kink point for each pair of equations.

An upper envelope is then obtained by tracing all paths of lines beyond which there are no higher points for the x value in the given range on the graph. The local centre $X_{pq} = X_l$ is the point that minimizes the upper envelope.

Using figure 3.1 above, we evaluate all edges on the given network as follows to demonstrate how the absolute centre can be found on a given network.

(a) Location on edge (C, D)

Consider $m(x) = \begin{cases} y_1 = x + d(p,V_i) \\ y_2 = c(p,q) - x + d(q,V_i) \end{cases}$

Let $p = C$ and $q = D$

When $i = C$,

$y_1 = x + d(C, C)$ and $y_2 = c(C, D) - x + d(D, C)$

$$y_1 = x \text{ and } y_2 = 24 - x$$

For kink point, $x = 24 - x$

$x = 12$

When $i = D$,

$$y_1 = x + d(C, D) \quad , y_2 = c(C, D) - x + d(D, D)$$

$$y_1 = x + 12 \quad , \quad y_2 = 12 - x$$

For kink point, $x + 12 = 12 - x$

$$x = 0$$

The range therefore is $0 \leq x \leq 12$

The equations to be sketched are

$$y_1 = x \qquad 0 \leq x \leq 12 \ldots\ldots\ldots (i)$$

$$y_2 = 12 - x \qquad 0 \leq x \leq 12 \ldots\ldots \qquad (ii)$$

The other two equations are rejected because they fall outside the range of x.

When $i = A$,

$$y_1 = x + d(C, A) \quad , y_2 = c(C, D) - x + d(D, A)$$

$$y_1 = x + 10 \quad , \quad y_2 = 34 - x$$

For kink point, $x + 10 = 34 - x$

$$x = 12$$

Equations to be sketched are

$$y_1 = x + 10 \qquad 0 \leq x \leq 12 \ldots\ldots (iii)$$

$$y_2 = 34 - x \qquad x = 12 \ldots\ldots\ldots (iv)$$

When $i = B$,

$$y_1 = x + d(C,B) \qquad , y_2 = c(C,D) - x + d(D,B)$$

$$y_1 = x + 23 \qquad , \quad y_2 = 29 - x$$

For kink point,

$$x + 23 = 29 - x$$

$$x = 3$$

Equations to be sketched are

$$y_1 = x + 23 \qquad 0 \leq x \leq 3 \ldots\ldots (v)$$

$$y_2 = 29 - x \qquad 3 \leq x \leq 12 \ldots\ldots (vi)$$

When $i = E$,

$$y_1 = x + d(C,E) \qquad , y_2 = c(C,D) - x + d(D,E)$$

$$y_1 = x + 28 \ \ and \ \ y_2 = 28 - x$$

For kink point,

$$x + 28 = 28 - x$$

$$x = 0$$

Equations to be sketched are

$$y_1 = x + 28 \qquad x = 0 \ldots\ldots..(vii)$$

$$y_2 = 28 - x \qquad 0 \leq x \leq 12 \ldots\ldots (viii)$$

The eight equations above are then sketched on the same axes as shown below. From this graph, the minimum cost or distance can be found using the upper envelop.

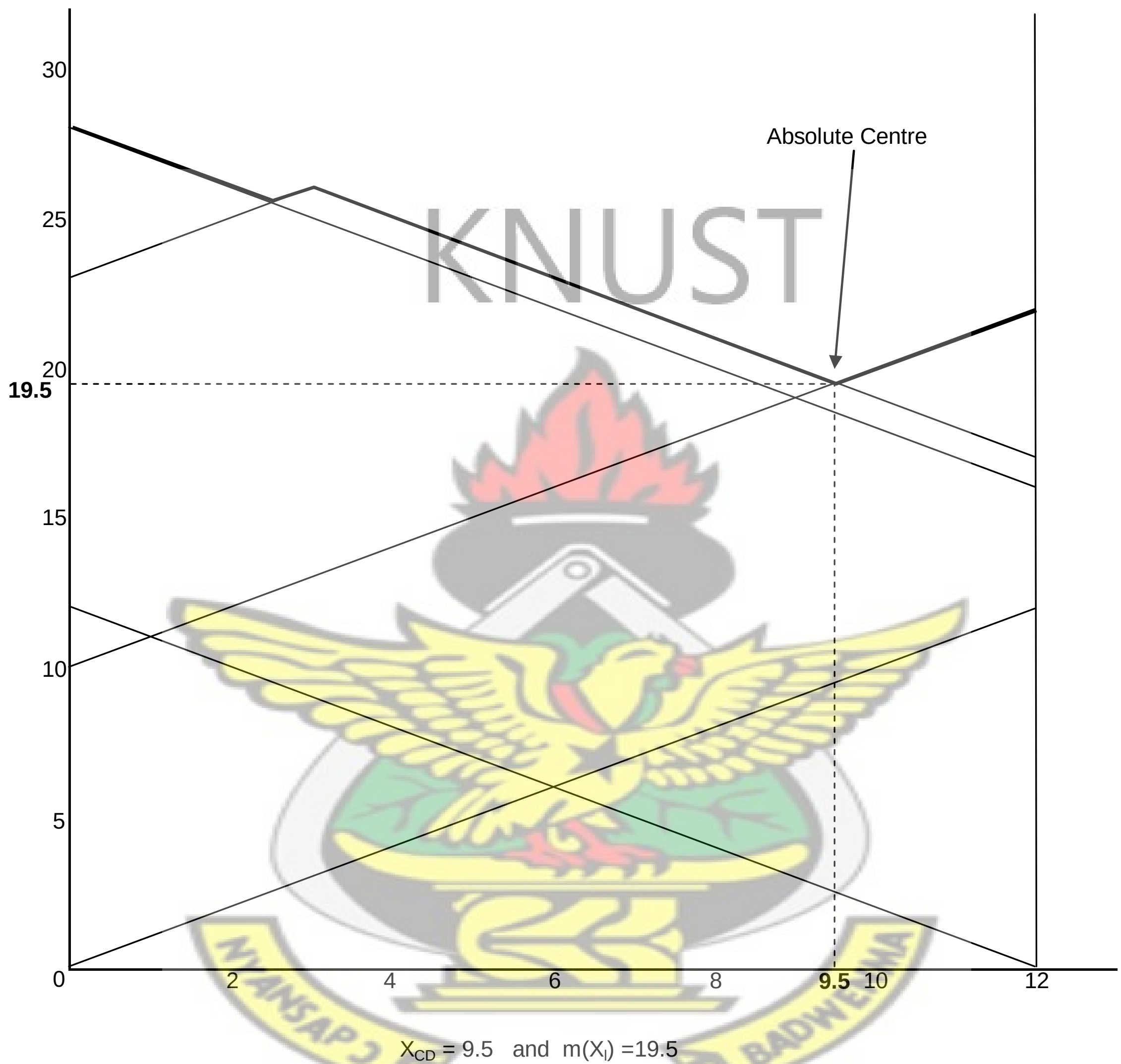


Table 3.2 below shows all the eight equations generated using the two equations

$$y_1 = x + d(p, V_i)$$
$$y_2 = c(p,q) - x + d(q, V_i)$$

on all the edges in figure 3.1. The kink point obtained for solving for the path of equal distance as well as the local centre obtained after the construction of the upper envelope is shown.

Edge	C(p,q)	Lines	x_{kink}	x_l	$m(x_l)$
(C,A)	12	$y_1 = x$ $\quad y_2 = 12 - x$	$x = 0, x = 12$	$9.5m$	19.5m
		$y_1 = x + 10$ $\quad y_2 = 34 - x$	$x = 12$		
		$y_1 = x + 23$ $\quad y_2 = 29 - x$	$x = 3$		
		$y_1 = x + 28$ $\quad y_2 = 28 - x$	$x = 0$		
(B,C)	23	$y_1 = x$ $\quad y_2 = 12 - x$	$x = 0, x = 12$	3m	20m
		$y_1 = x + 10$ $\quad y_2 = 34 - x$	$x = 12$		
		$y_1 = x + 23$ $\quad y_2 = 29 - x$	$x = 3$		
		$y_1 = x + 28$ $\quad y_2 = 28 - x$	$x = 0$		
(A,C)	10	$y_1 = x$ $\quad y_2 = 10 - x$	$x = 0, x = 10$	0m	27m
		$y_1 = x + 14$ $\quad y_2 = 33 - x$	$x = 9.5$		
		$y_1 = x + 17$ $\quad y_2 = 35 - x$	$x = 9$		
		$y_1 = x + 13$ $\quad y_2 = 51 - x$	$x = 19$		
(A,B)	14	$y_1 = x$ $\quad y_2 = 12 - x$	$x = 0, x = 12$	10.5m	20.5m
		$y_1 = x + 10$ $\quad y_2 = 37 - x$	$x = 13.5$		
		$y_1 = x + 22$ $\quad y_2 = 31 - x$	$x = 14.5$		
		$y_1 = x + 27$ $\quad y_2 = 27 - x$	$x = 0$		
(B,D)	17	$y_1 = x$ $\quad y_2 = 17 - x$	$x = 0, x = 17$	7.5m	21.5m
		$y_1 = x + 14$ $\quad y_2 = 39 - x$	$x = 12.5$		
		$y_1 = x + 23$ $\quad y_2 = 29 - x$	$x = 3$		
		$y_1 = x + 13$ $\quad y_2 = 33 - x$	$x = 10$		
(B,E)	13	$y_1 = x$ $\quad y_2 = 13 - x$	$x = 0, x = 13$	0m	23m
		$y_1 = x + 14$ $\quad y_2 = 40 - x$	$x = 13$		
		$y_1 = x + 23$ $\quad y_2 = 41 - x$	$x = 9$		
		$y_1 = x + 17$ $\quad y_2 = 28 - x$	$x = 6$		

Table 3.2: Equations, kink points and local centres of all edges in figure 3.1

Invariance of local center with respect to the choice of any vertex as the origin

Upon thorough research it has been established and verified that any of the end nodes of an edge can be chosen as the origin since the graph obtained is a reflection of the graph when the other node is set as the origin. The axis of reflection may pass through any of the nodes. The edge (C, D) of figure 1 shall therefore be used to illustrate this fact.

(a) Location on edge (D, C)

Consider $m(x) = \begin{cases} y_1 = x + d(p, V_i) \\ y_2 = c(p,q) - x + d(q, V_i) \end{cases}$

Let $p = D$ and $q = C$

When $i = D$,

$y_1 = x + d(D,D)$ and $y_2 = c(D,C) - x + d(C,D)$

$$y_1 = x \text{ and } y_2 = 24 - x$$

For kink point, $x = 24 - x$

$x = 12$

When $i = C$,

$$y_1 = x + d(D,C) \quad , y_2 = c(D,C) - x + d(D,D)$$

$$y_1 = x + 12 \quad , \quad y_2 = 12 - x$$

For kink point, $x + 12 = 12 - x$

$$x = 0$$

The range therefore is $0 \le x \le 12$

The equations to be sketched are

$$y_1 = x \qquad 0 \le x \le 12 \ldots\ldots\ldots(i)$$

$$y_2 = 12 - x \qquad 0 \le x \le 12 \ldots\ldots \qquad (ii)$$

The other two equations are rejected because they fall outside the range of x.

When $i = A$,

$$y_1 = x + d(D,A) \quad , y_2 = c(D,C) - x + d(C,A)$$

$$y_1 = x + 22 \quad , \quad y_2 = 22 - x$$

For kink point, $2x = 0$

$$x = 0$$

Equations to be sketched are

$$y_1 = x + 22 \quad x = 0 \dots \dots (iii)$$

$$y_2 = 22 - x \quad 0 \leq x \leq 12 \dots \dots \dots (iv)$$

When $i = B$,

$$y_1 = x + d(D,B) \quad , y_2 = c(D,C) - x + d(C,B)$$

$$y_1 = x + 17 \quad , \quad y_2 = 35 - x$$

For kink point,

$$x + 17 = 35 - x$$

$$x = 9$$

Equations to be sketched are

$$y_1 = x + 17 \quad 0 \leq x \leq 9 \dots \dots (v)$$

$$y_2 = 35 - x \quad 9 \leq x \leq 12 \dots \dots (vi)$$

When $i = E$,

$$y_1 = x + d(D,E) \quad , y_2 = c(D,C) - x + d(C,E)$$

$$y_1 = x + 16 \ \ and \ \ y_2 = 40 - x$$

For kink point,

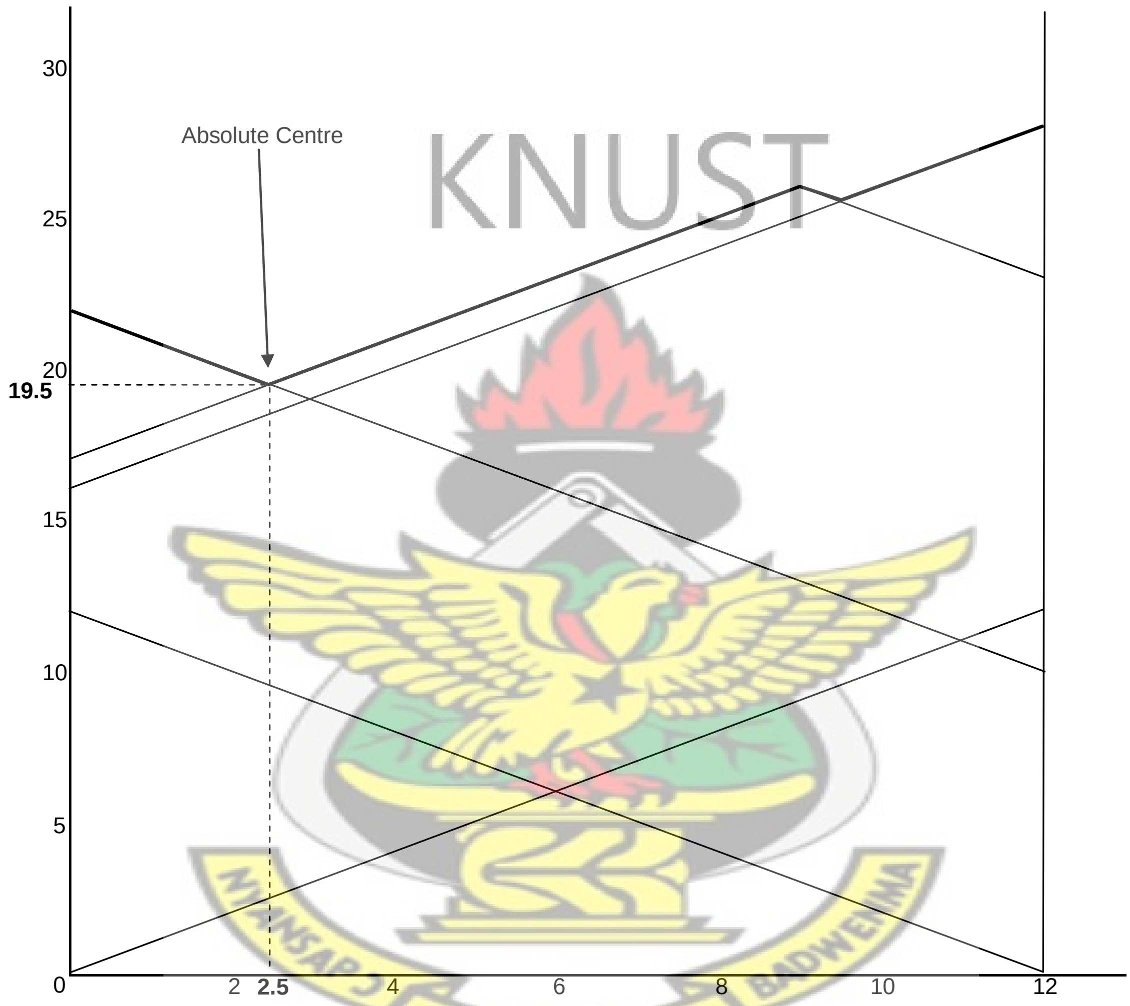
$$2x = 24$$

$$x = 12$$

Equations to be sketched are

$$y_1 = x + 16 \qquad\qquad 0 \le x \le 12 \dots\dots..(vii)$$

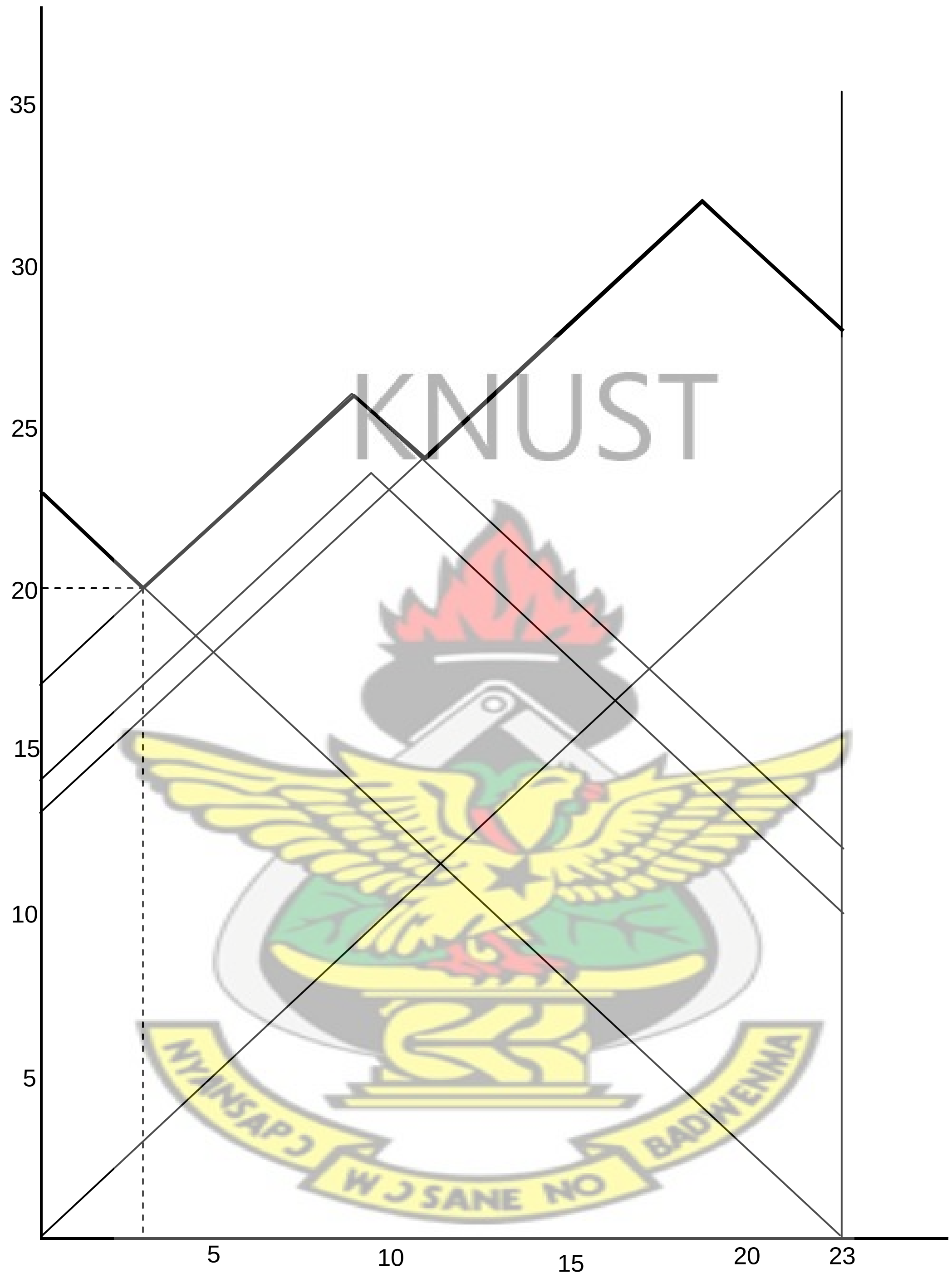
$$y_2 = 40 - x \qquad x = 12 \dots\dots(viii)$$



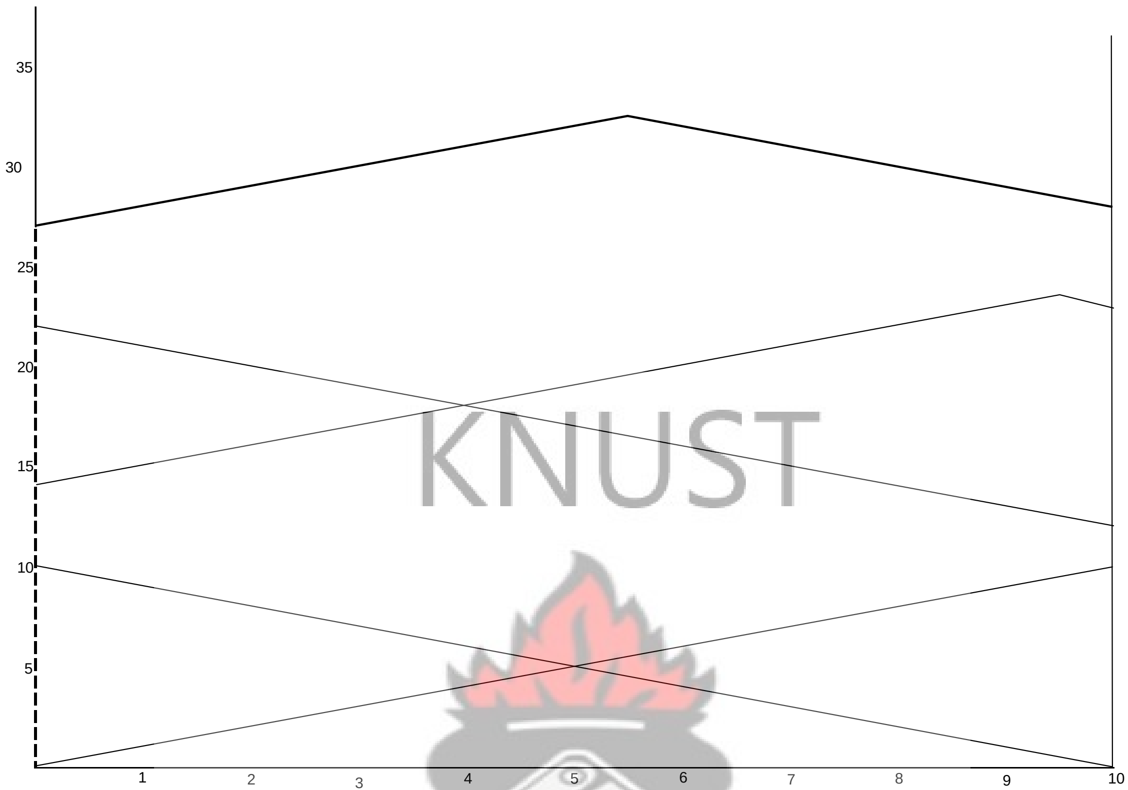
$X_{DC} = 2.5$ and $m(X_j) = 19.5$

Construction of the Upper Envelope

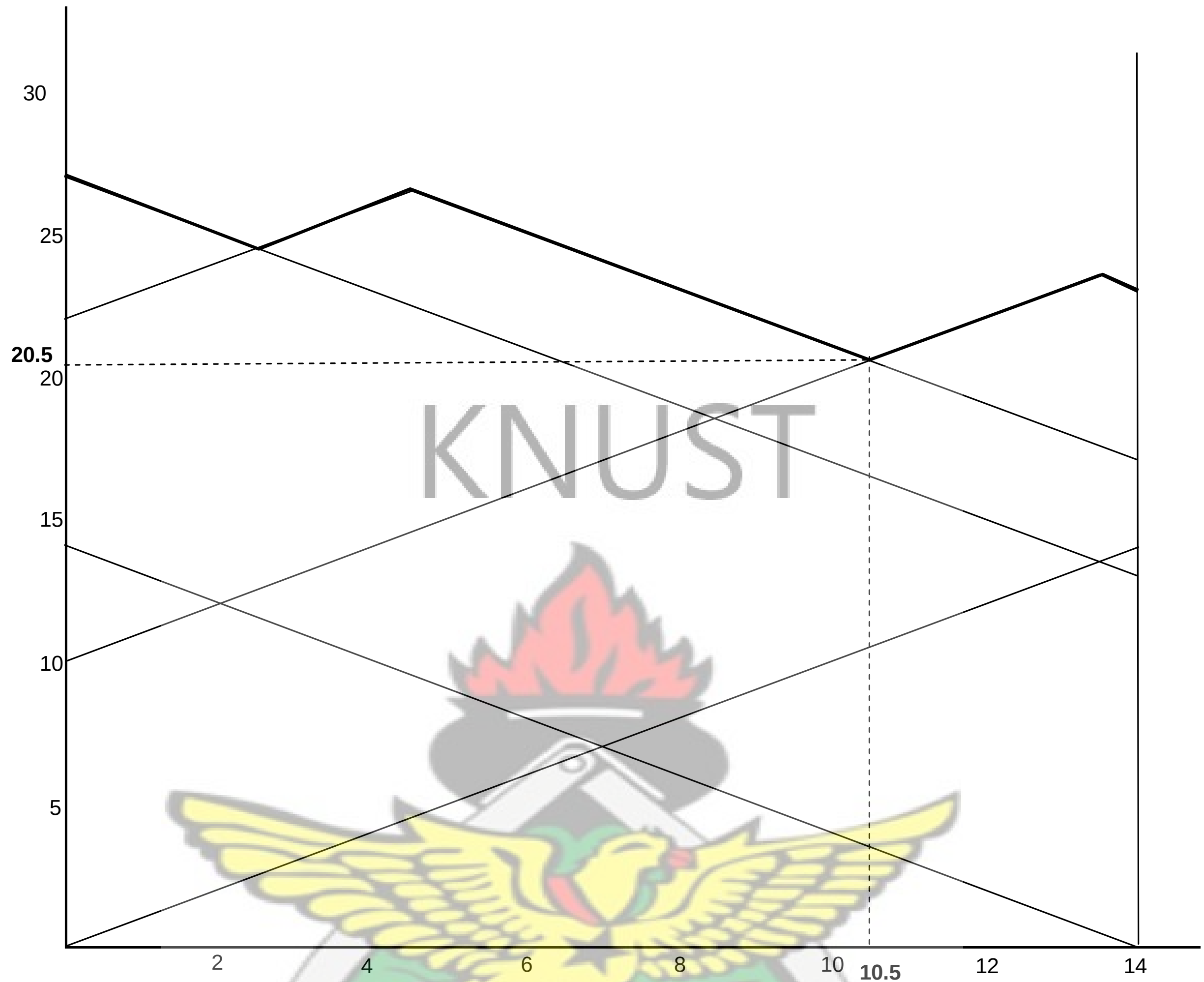
After sketching all the equations resulting from the location on an edge on the same axes as shown in the diagram above, there is the need to construct an 'upper envelope' which gives the minimum cost/distance of a shortest path from x to a farthest node on a given edge. To construct the upper envelope, we trace all paths of lines beyond which there are no higher points for the same x value in the given range. These paths are indicated by thick lines as shown in the diagram above. The diagrams below show the graphs of the remaining edges in the example above with their corresponding minimum costs.



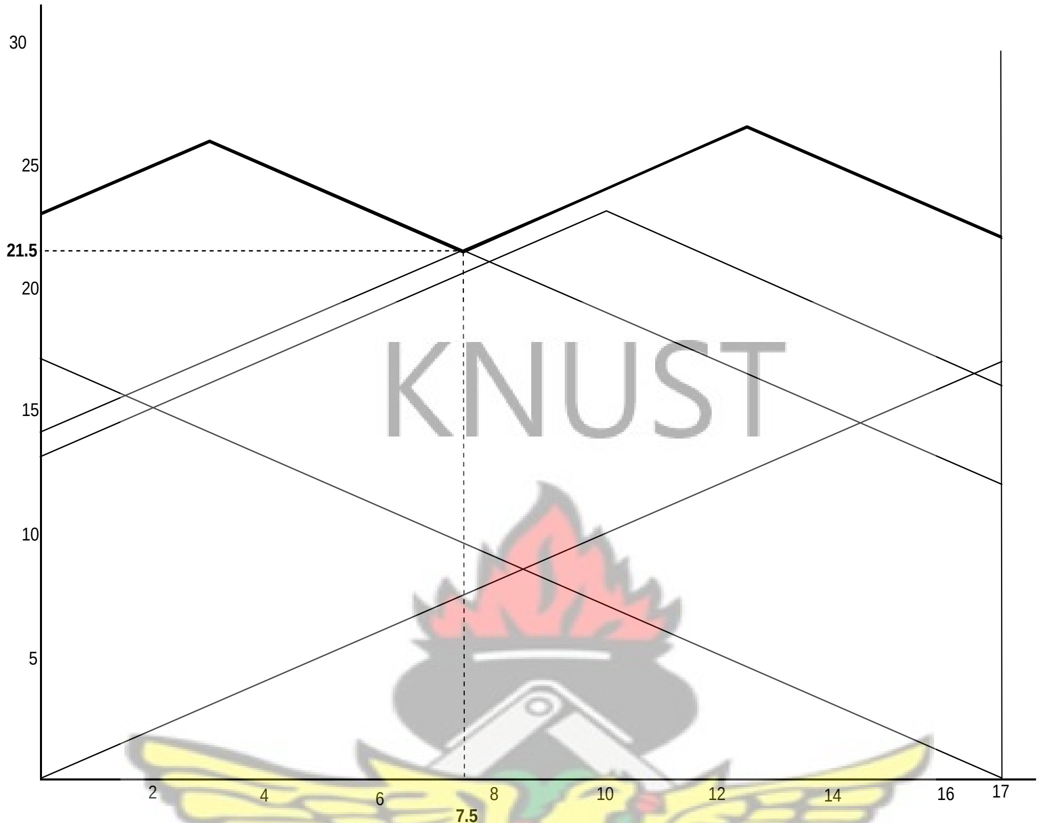
$X_{BC} = 3$ and $m(X_I) = 20$



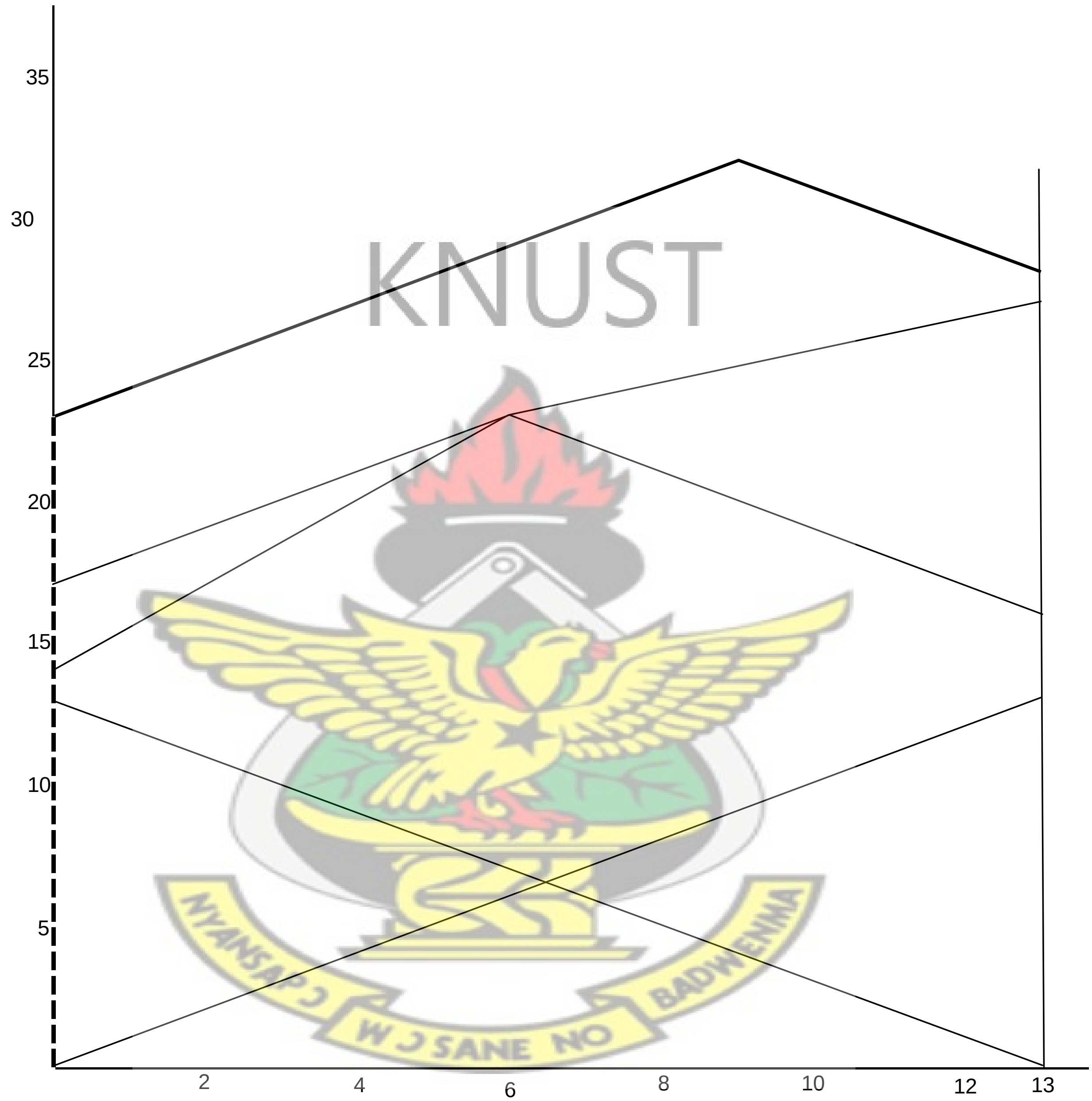
$X_{AC} = 0$ and $m(X_i) = 27$



$X_{AB} = 10.5$ and $m(X_I) = 20.5$



X_{BD} = 7.5 and $m(X_i)$ =21.5



$X_{BE} = 0$ and $m(X_l) = 23$

CHAPTER 4

DATA ANALYSIS

4.1 Data

In this chapter, a sector map of TISSEC showing the exact locations of the structures in the school and the distances between them is provided. The researcher measured the distances with a surveyors' tape with the aid of teachers and students of Tamale Islamic Senior high School.

4.2Locations considered

School administration (A)

Boy's dormitory (B)

Fourth year classrooms(C)

P.T.A dormitory (D)

Round houses (E)

Girl's dormitory (F)

Seniors dormitory (G)

Headmasters house (H)

Assistant headmaster's (Academic) house (I)

Assistant headmaster's (Administration) house (J)

Table 4.1 below shows the direct distances in metres between the locations considered. The first column shows the number, column 2 shows the edge and the third column is the edge distance in metres. As stated in chapter 3, an edge is the distance between two vertices p and q. A, B, C,etc refer to the various nodes.

NUMBER	EDGES CONSIDERED	DISTANCE(METRES)
1	(A,B)	353
2	(A,C)	58
3	(A,D)	75
4	(E,A)	110
5	(F,A)	184
6	(G,A)	247
7	(B,D)	313
8	(B,F)	435
9	(CD)	120
10	(G,E)	185
11	(E,F)	74
12	(E,J)	264
13	(F,J)	338
14	(H,G)	20
15	(H,I)	14
16	(I,J)	45

Table 4.1: Edge and Distance Table

The data above was then developed into a network as shown below.

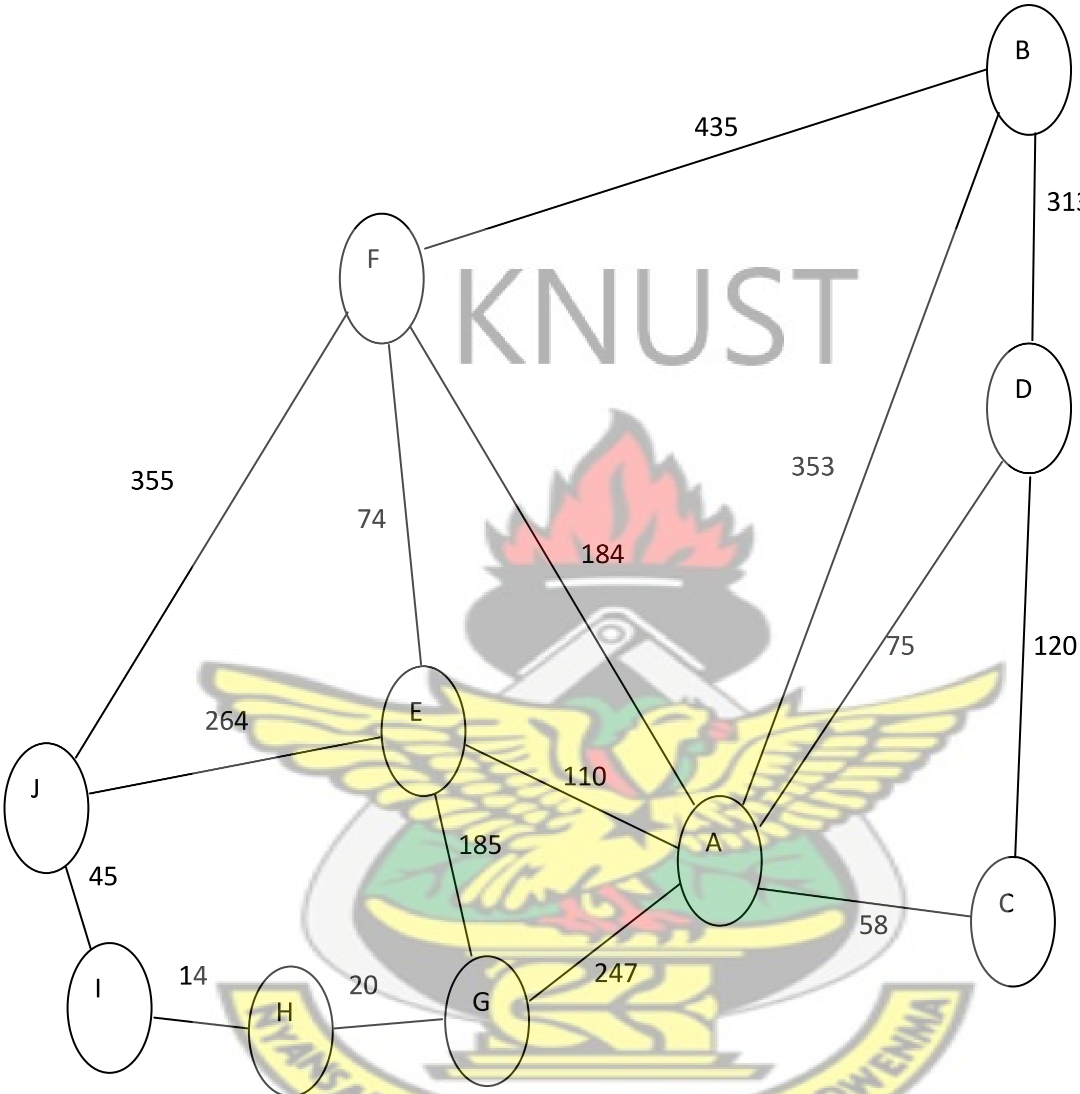


Figure 4.1 Network for structures in Tamale Islamic School Senior High School

The letters A, B, C etc in figure 4.1 above represent the various structures on Tamale Islamic Senior High School. The network shows how the structures are linked up.

The matrix of the shortest paths using the Floyd- Warshall algorithm for the network is obtained as shown below.

	A	B	C	D	E	F	G	H	I	J	ROW MAX
A		353	58	75	110	184	247	267	281	326	**353**
B	353		411	313	463	435	600	620	634	679	679
C	58	411		120	168	242	305	325	339	384	411
D	75	313	120		185	259	322	342	356	401	401
E	110	463	168	185		74	185	205	219	264	463
F	184	435	242	259	74		259	279	293	338	435
G	247	600	305	322	185	259		20	34	79	600
H	267	620	325	342	205	279	20		14	59	620
I	281	634	339	356	219	293	34	14		45	634
J	326	679	384	401	264	338	79	59	45		679

Table 4.2: Matrix of all pairs shortest paths for the network in figure7

4.3 Locating the Vertex Centre

The node or vertex center is then chosen as a node such that the maximum entry in its row in the matrix is the smallest among the maximum entries of all rows. From Table 4.2 above the rows with the minimum maximum entry occur at node A with a maximum distance of 353 metres thus the vertex centre for the network above is node A with $m(A) = 353$.

Since step 1 of the two step algorithm for finding the absolute 1-centre is a time consuming one, the researcher demonstrated how the local centres were calculated by finding the local centre on edge (C,A) only. The remaining are put in table form without their calculations in appendix A.

Location on edge (C, A)

Consider $m(x) = \begin{cases} y_1 = x + d(p, V_i) \\ y_2 = c(p,q) - x + d(q, V_i) \end{cases} \quad for\ i = A, B, C, D, E$

When $i = C$,

$y_1 = x + d(C, C)$ and $y_2 = c(C, A) - x + d(A, C)$

$$y_1 = x\ and\ y_2 = 116 - x$$

For kink point, $x = 116 - x$

$x = 58$

When $i = A$,

$$y_1 = x + d(C, A) \quad , y_2 = c(C, A) - x + d(A, A)$$

$$y_1 = x + 58 \quad , \quad y_2 = 58 - x$$

For kink point, $x + 58 = 58 - x$

$$x = 0$$

The range therefore is $0 \leq x \leq 58$

The equations to be sketched are

$$y_1 = x \qquad 0 \leq x \leq 58 \qquad \dots\dots\dots (i)$$

$$y_2 = 58 - x \qquad 0 \leq x \leq 58 \quad \dots\dots \qquad (ii)$$

The other two equations are rejected because they fall outside the range.

When $i = B$,

$$y_1 = x + d(C, B) \quad , y_2 = c(C, A) - x + d(A, B)$$

$$y_1 = x + 411 \quad , \quad y_2 = 411 - x$$

For kink point, $x + 411 = 411 - x$

$$x = 0$$

Equations to be sketched are

$$y_1 = x + 411 \qquad\qquad x = 0 \ldots\ldots (iii)$$

$$y_2 = 411 - x \qquad 0 \le x \le 58 \ldots\ldots\ldots (iv)$$

When $i = D$,

$$y_1 = x + d(C,D) \qquad , y_2 = c(C,A) - x + d(A,D)$$

$$y_1 = x + 120 \qquad , \quad y_2 = 133 - x$$

For kink point,

$$2x = 13$$

$$x = 6.5$$

Equations to be sketched are

$$y_1 = x + 120 \qquad 0 \le x \le 6.5 \ldots\ldots (v)$$

$$y_2 = 133 - x \qquad 6.5 \le x \le 58 \ldots\ldots (vi)$$

When $i = E$,

$$y_1 = x + d(C,E) \qquad , y_2 = c(C,A) - x + d(A,E)$$

$$y_1 = x + 168 \;\; and \;\; y_2 = 168 - x$$

For kink point,

$$x + 168 = 168 - x$$

$$x = 0$$

Equations to be sketched are

$$y_1 = x + 168 \qquad\qquad x = 0 \ldots\ldots (vii)$$

$$y_2 = 168 - x \qquad\qquad 0 \le x \le 58 \ldots\ldots (viii)$$

When $i = F$,

$$y_1 = x + d(C,F) \qquad , y_2 = c(C,A) - x + d(A,F)$$

$$y_1 = x + 242 \qquad , \qquad y_2 = 242 - x$$

For kink point,

$$x + 242 = 242 - x$$

$$2x = 0$$

$$x = 0$$

Equations to be sketched are

$$y_1 = x + 242 \qquad x = 0 \ldots \ldots (ix)$$

$$y_2 = 242 - x \qquad 0 \le x \le 58 \ldots \ldots \ldots (x)$$

When $i = G$,

$$y_1 = x + d(C, G) \qquad , y_2 = c(C, A) - x + d(A, G)$$

$$y_1 = x + 305 \qquad , \quad y_2 = 305 - x$$

For kink point,

$$2x = 0$$

$$x = 0$$

Equations to be sketched are

$$y_1 = x + 305 \qquad x = 58 \ldots \ldots (xi)$$

$$y_2 = 305 - x \qquad 0 \le x \le 58 \ldots \ldots (xii)$$

When $i = H$,

$$y_1 = x + d(C, H) \qquad , y_2 = c(C, A) - x + d(A, H)$$

$$y_1 = x + 325 \qquad and \;\; y_2 = 325 - x$$

For kink point,

$$2x = 0$$

$$x = 0$$

Equations to be sketched are

$$y_1 = x + 325 \qquad x = 0 \ldots \ldots (xiii)$$

$$y_2 = 325 - x \qquad 0 \le x \le 58 \ldots \ldots (xiv)$$

When $i = I$,

$$y_1 = x + d(C, I) \qquad , y_2 = c(C, A) - x + d(A, I)$$

$$y_1 = x + 339 \qquad , \quad y_2 = 339 - x$$

For kink point,

$$2x = 0$$

$$x = 0$$

Equations to be sketched are

$$y_1 = x + 339 \qquad x = 58 \ldots\ldots (xiv)$$

$$y_2 = 339 - x \qquad 0 \leq x \leq 58 \ldots\ldots (xv)$$

When $i = J$,

$$y_1 = x + d(C, J) \qquad , y_2 = c(C, A) - x + d(A, J)$$

$$y_1 = x + 384 \qquad and \;\; y_2 = 384 - x$$

For kink point,

$$2x = 0$$

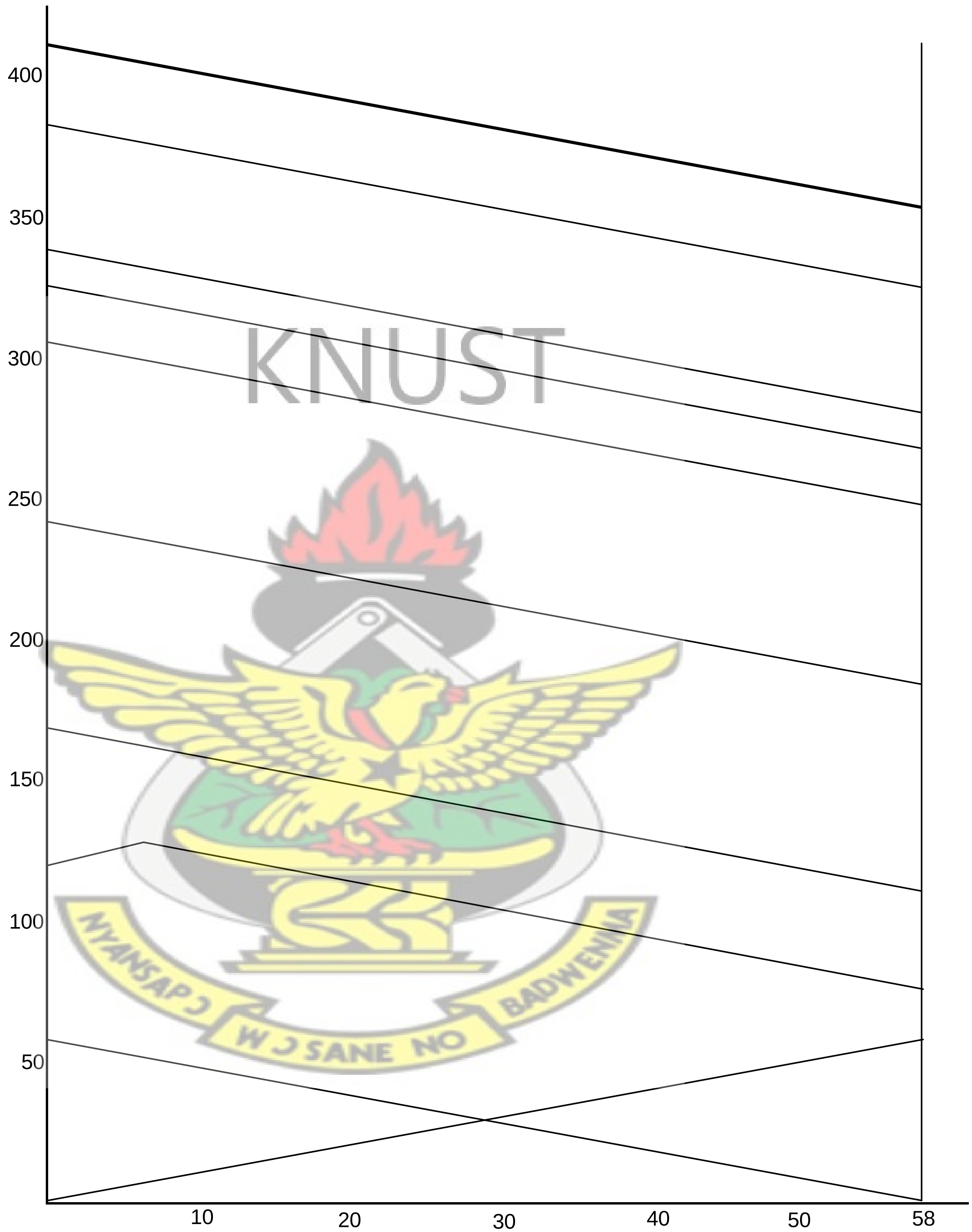
$$x = 0$$

Equations to be sketched are

$$y_1 = x + 384 \qquad x = 58 \ldots\ldots..(xvi)$$

$$y_2 = 384 - x \qquad 0 \leq \; x \leq 58 \ldots\ldots (xvii)$$

The graph of all the equations obtained is shown below



$X_{CA} = 58$ and $m(X_I) = 353$

The local centres for the remaining edges are shown in the table below.

NO.	EDGES	x_l	**Maximum distance**$[\boldsymbol{m}(\boldsymbol{X_l})]$
1	(A,B)	25m from node A	340m
2	(A,C)	At node A	353m
3	(A,D)	32m from node A	357.5m
4	(E,A)	At node A	353m
5	(F,A)	At node A	353m
6	(G,A)	At node A	353m
7	(B,D)	At node D	401m
8	(B,F)	386m from node B	384m
9	(CD)	24m from node C	404.5
10	(G,E)	At node E	463m
11	(E,F)	At node F	435m
12	(E,J)	At node E	463m
13	(F,J)	At node F	435m
14	(H,G)	At node G	600m
15	(H,I)	At node H	620m
16	(I,J)	At node I	634m

Table 4.3: Local centres obtained for all edges in figure 4.1

Conclusion

From the table above, the smallest $m(x_l)$ corresponds to the point x^*,on the edge (A,B) which is 25m from A and 228m from node B .Since this value is minimum compared to the vertex centre, it is taken as the absolute centre. Thus, the absolute centre is $X_{(AB)} = 340m$.

This means that the maximum distance from the point $x^* = X_{(AB)}$ (absolute center) to the farthest node on the network is 340 metres.Also, the water borehole facility should be located in the neighbourhood of 25m from node A(school administration).

CHAPTER 5

Conclusions and Recommendations

5.0 Conclusions

Islamic Senior High School in Tamale has had its own share of problems in relation to water shortages. This is because the school has no borehole and water flows through the taps once or twice within a month.

An optimal location on Tamale Islamic Senior High School is found using the absolute- 1-centre method for the establishment of a water borehole facility. The distances in metres between the structures on the school compound were measured and the absolute 1 – centre algorithm used to find the strategic position for its placement.

The solution to the absolute 1-centre problem was obtained by locating the vertex centre to be node A(school administration) with distance 353metres. The local centre was also found to be 25m from A and 228m from node B with a distance of 340metres.Since the local centre had a least value compared to the vertex centre, it is taken as the absolute centre of the network. This means that the facility must be located in the neighbourhood of 25m from node A(school administration).

In finding the local centre for any edge, one of the ends of that edge must be taken as the origin. This project has proved experimentally that any of the end nodes of an edge can be chosen as the origin since the graph obtained is a reflection of the other graph when the other node is set as the origin. The axis of reflection may pass through any of the nodes.

5.1 Recommendations

I recommend that Tamale Islamic Senior High School should use the findings of this research to establish a water borehole facility to reduce the problem of water shortages and the stress that both students and teachers face during the dry season.

I also recommend that expert advise on the site so selected be considered as to whether the site has enough water for future use by the citizenry.

I also want to recommend further research on the p-centre model by students.

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Appendix A

The tables below shows the various lines for all the edges under consideration in chapter 4,their corresponding kink points, and their local centres

Lines		x_{kink}	x_l	$m(x_l)$
$y_1 = x$	$y_2 = 58 - x$	$x = 0, x = 58$	58m	353m
$y_1 = x + 411$	$y_2 = 411 - x$	$x = 0$		
$y_1 = x + 120$	$y_2 = 133 - x$	$x = 65$		
$y_1 = x + 242$	$y_2 = 242 - x$	$x = 0$		
$y_1 = x + 305$	$y_2 = 305 - x$	$x = 0$		
$y_1 = x + 325$	$y_2 = 325 - x$	$x = 0$		
$y_1 = x + 3\,39$	$y_2 = 339 - x$	$x = 0$		
$y_1 = x + 384$	$y_2 = 384 - x$	$x = 0$		
$y_1 = x + 168$	$y_2 = 168 - x$	$x = 0$		

Table 4.3: Lines, kink points and local centres obtained on edge (C,A)

Lines		x_{kink}	x_l	$m(x_l)$
$y_1 = x$	$y_2 = 353 - x$	$x = 0, x = 353$	25m	340m
$y_1 = x + 58$	$y_2 = 764 - x$	$x = 353$		
$y_1 = x + 75$	$y_2 = 666 - x$	$x = 295.5$		
$y_1 = x + 110$	$y_2 = 816 - x$	$x = 353$		
$y_1 = x + 184$	$y_2 = 788 - x$	$x = 302$		
$y_1 = x + 247$	$y_2 = 953 - x$	$x = 353$		
$y_1 = x + 267$	$y_2 = 973 - x$	$x = 353$		
$y_1 = x + 281$	$y_2 = 987 - x$	$x = 353$		
$y_1 = x + 326$	$y_2 = 1032 - x$	$x = 353$		

Table 4.4: Lines , kink points and local centres obtained on edge (A,B)

Lines		x_{kink}	x_l	$m(x_l)$
$y_1 = x$	$y_2 = 75 - x$	$x = 0, x = 75$	32m	357.5m
$y_1 = x + 353$	$y_2 = 388 - x$	$x = 17.5$		
$y_1 = x + 58$	$y_2 = 195 - x$	$x = 68.5$		
$y_1 = x + 110$	$y_2 = 260 - x$	$x = 75$		
$y_1 = x + 184$	$y_2 = 334 - x$	$x = 75$		
$y_1 = x + 247$	$y_2 = 397 - x$	$x = 75$		
$y_1 = x + 267$	$y_2 = 417 - x$	$x = 75$		
$y_1 = x + 281$	$y_2 = 431 - x$	$x = 75$		
$y_1 = x + 326$	$y_2 = 476 - x$	$x = 75$		

Table 4.5: Lines , kink points and local centres obtained on edge (A,D)

Lines		x_{kink}	x_l	$m(x_l)$
$y_1 = x$	$y_2 = 110 - x$	$x = 0, x = 110$	110m	353m
$y_1 = x + 463$	$y_2 = 463 - x$	$x = 0$		
$y_1 = x + 168$	$y_2 = 168 - x$	$x = 0$		
$y_1 = x + 185$	$y_2 = 185 - x$	$x = 0$		
$y_1 = x + 74$	$y_2 = 294 - x$	$x = 110$		
$y_1 = x + 185$	$y_2 = 357 - x$	$x = 86$		
$y_1 = x + 205$	$y_2 = 377 - x$	$x = 86$		
$y_1 = x + 219$	$y_2 = 391 - x$	$x = 86$		
$y_1 = x + 264$	$y_2 = 436 - x$	$x = 86$		

Table 4.6: Lines , kink points and local centres obtained on edge (E,A)

Lines		x_{kink}	x_l	$m(x_l)$
$y_1 = x$	$y_2 = 110 - x$	$x = 0, x = 184$	184m	353m
$y_1 = x + 435$	$y_2 = 537 - x$	$x = 51$		
$y_1 = x + 242$	$y_2 = 242 - x$	$x = 0$		
$y_1 = x + 259$	$y_2 = 259 - x$	$x = 0$		
$y_1 = x + 74$	$y_2 = 294 - x$	$x = 110$		
$y_1 = x + 259$	$y_2 = 431 - x$	$x = 86$		
$y_1 = x + 279$	$y_2 = 451 - x$	$x = 86$		
$y_1 = x + 293$	$y_2 = 466 - x$	$x = 86$		
$y_1 = x + 338$	$y_2 = 510 - x$	$x = 86$		

Table 4.7: Lines , kink points and local centres obtained on edge (F,A)

Lines		x_{kink}	x_l	$m(x_l)$
$y_1 = x$	$y_2 = 494 - x$	$x = 0, x = 247$	247m	353m
$y_1 = x + 600$	$y_2 = 600 - x$	$x = 0$		
$y_1 = x + 305$	$y_2 = 305 - x$	$x = 0$		
$y_1 = x + 322$	$y_2 = 322 - x$	$x = 0$		
$y_1 = x + 185$	$y_2 = 357 - x$	$x = 86$		
$y_1 = x + 259$	$y_2 = 431 - x$	$x = 86$		
$y_1 = x + 20$	$y_2 = 514 - x$	$x = 247$		
$y_1 = x + 34$	$y_2 = 528 - x$	$x = 247$		
$y_1 = x + 79$	$y_2 = 573 - x$	$x = 247$		

Table 4.8: Lines , kink points and local centres obtained on edge (G,A)

Lines		x_{kink}	x_l	$m(x_l)$
$y_1 = x$	$y_2 = 313 - x$	$x = 0, x = 313$	313m	401m
$y_1 = x + 353$	$y_2 = 388 - x$	$x = 17.5$		
$y_1 = x + 411$	$y_2 = 433 - x$	$x = 11$		
$y_1 = x + 463$	$y_2 = 498 - x$	$x = 17.5$		
$y_1 = x + 435$	$y_2 = 572 - x$	$x = 68.5$		
$y_1 = x + 600$	$y_2 = 635 - x$	$x = 17.5$		
$y_1 = x + 620$	$y_2 = 655 - x$	$x = 17.5$		
$y_1 = x + 634$	$y_2 = 669 - x$	$x = 17.5$		
$y_1 = x + 679$	$y_2 = 714 - x$	$x = 17.5$		

Table 4.9: Lines , kink points and local centres obtained on edge (B,D)

Lines		x_{kink}	x_l	$m(x_l)$
$y_1 = x$	$y_2 = 435 - x$	$x = 0, x = 435$	386m	384m
$y_1 = x + 353$	$y_2 = 619 - x$	$x = 133$		
$y_1 = x + 411$	$y_2 = 677 - x$	$x = 133$		
$y_1 = x + 313$	$y_2 = 694 - x$	$x = 190.5$		
$y_1 = x + 463$	$y_2 = 509 - x$	$x = 23$		
$y_1 = x + 600$	$y_2 = 694 - x$	$x = 47$		
$y_1 = x + 620$	$y_2 = 714 - x$	$x = 47$		
$y_1 = x + 634$	$y_2 = 728 - x$	$x = 47$		
$y_1 = x + 679$	$y_2 = 773 - x$	$x = 47$		

Table 4.10: Lines , kink points and local centres obtained on edge (B,F)

Lines	x_{kink}	x_l	$m(x_l)$
$y_1 = x$ $y_2 = 120 - x$	$x = 0, x = 120$	24m	404.5m
$y_1 = x + 58$ $y_2 = 195 - x$	$x = 68.5$		
$y_1 = x + 411$ $y_2 = 433 - x$	$x = 11$		
$y_1 = x + 168$ $y_2 = 305 - x$	$x = 68.5$		
$y_1 = x + 24\ 2$ $y_2 = 379 - x$	$x = 68.5$		
$y_1 = x + 305$ $y_2 = 442 - x$	$x = 68.5$		
$y_1 = x + 325$ $y_2 = 462 - x$	$x = 68.5$		
$y_1 = x + 339$ $y_2 = 476 - x$	$x = 68.5$		
$y_1 = x + 384$ $y_2 = 521 - x$	$x = 68.5$		

Table 4.11: Lines, kink points and local centres obtained on edge (C,D)

Lines	x_{kink}	x_l	$m(x_l)$
$y_1 = x$ $y_2 = 185 - x$	$x = 0, x = 185$	185m	463m
$y_1 = x + 247$ $y_2 = 295 - x$	$x = 24$		
$y_1 = x + 305$ $y_2 = 353 - x$	$x = 24$		
$y_1 = x + 322$ $y_2 = 370 - x$	$x = 24$		
$y_1 = x + 259$ $y_2 = 259 - x$	$x = 0$		
$y_1 = x + 20$ $y_2 = 390 - x$	$x = 185$		
$y_1 = x + 34$ $y_2 = 404 - x$	$x = 185$		
$y_1 = x + 34$ $y_2 = 404 - x$	$x = 185$		
$y_1 = x + 79$ $y_2 = 449 - x$	$x = 185$		

Table 4.12: Lines, kink points and local centres obtained on edge (G,E)

Lines	x_{kink}	x_l	$m(x_l)$
$y_1 = x$ $y_2 = 74 - x$	$x = 0, x = 74$	74m	435m
$y_1 = x + 110$ $y_2 = 258 - x$	$x = 74$		
$y_1 = x + 463$ $y_2 = 509 - x$	$x = 23$		
$y_1 = x + 168$ $y_2 = 316 - x$	$x = 74$		
$y_1 = x + 185$ $y_2 = 333 - x$	$x = 74$		
$y_1 = x + 185$ $y_2 = 333 - x$	$x = 74$		
$y_1 = x + 205$ $y_2 = 353 - x$	$x = 74$		
$y_1 = x + 219$ $y_2 = 367 - x$	$x = 74$		
$y_1 = x + 264$ $y_2 = 412 - x$	$x = 74$		

Table 4.13: Lines, kink points and local centres obtained on edge (E,F)

Lines	x_{kink}	x_l	$m(x_l)$
$y_1 = x \qquad y_2 = 264 - x$	$x = 0, x = 264$	264m	463m
$y_1 = x + 326 \qquad y_2 = 374 - x$	$x = 24$		
$y_1 = x + 679 \qquad y_2 = 727 - x$	$x = 24$		
$y_1 = x + 384 \qquad y_2 = 433 - x$	$x = 24$		
$y_1 = x + 401 \qquad y_2 = 449 - x$	$x = 24$		
$y_1 = x + 338 \qquad y_2 = 338 - x$	$x = 0$		
$y_1 = x + 79 \qquad y_2 = 449 - x$	$x = 185$		
$y_1 = x + 59 \qquad y_2 = 469 - x$	$x = 205$		
$y_1 = x + 45 \qquad y_2 = 484 - x$	$x = 219$		

Table 4.14: Lines , kink points and local centres obtained on edge (J,E)

Lines	x_{kink}	x_l	$m(x_l)$
$y_1 = x \qquad y_2 = 338 - x$	$x = 0, x = 338$	0m	435m
$y_1 = x + 184 \qquad y_2 = 664 - x$	$x = 240$		
$y_1 = x + 435 \qquad y_2 = 1017 - x$	$x = 291$		
$y_1 = x + 242 \qquad y_2 = 722 - x$	$x = 240$		
$y_1 = x + 74 \qquad y_2 = 602 - x$	$x = 264$		
$y_1 = x + 259 \qquad y_2 = 417 - x$	$x = 79$		
$y_1 = x + 259 \qquad y_2 = 739 - x$	$x = 240$		
$y_1 = x + 279 \qquad y_2 = 397 - x$	$x = 59$		
$y_1 = x + 293 \qquad y_2 = 383 - x$	$x = 45$		

Table 4.15: Lines, kink points and local centres obtained on edge (F,J)

Lines	x_{kink}	x_l	$m(x_l)$
$y_1 = x \qquad y_2 = 20 - x$	$x = 0, x = 20$	0m	600m
$y_1 = x + 267 \qquad y_2 = 627 - x$	$x = 0$		
$y_1 = x + 620 \qquad y_2 = 620 - x$	$x = 0$		
$y_1 = x + 325 \qquad y_2 = 325 - x$	$x = 0$		
$y_1 = x + 342 \qquad y_2 = 342 - x$	$x = 0$		
$y_1 = x + 205 \qquad y_2 = 205 - x$	$x = 0$		
$y_1 = x + 279 \qquad y_2 = 279 - x$	$x = 0$		
$y_1 = x + 14 \qquad y_2 = 54 - x$	$x = 20$		
$y_1 = x + 59 \qquad y_2 = 99 - x$	$x = 20$		

Table 4.16: Lines, kink points and local centres obtained on edge (G,H)

Lines	x_{kink}	x_l	$m(x_l)$
$y_1 = x$ $\quad y_2 = 14 - x$	$x = 0, x = 14$	0m	620m
$y_1 = x + 267$ $\quad y_2 = 295 - x$	$x = 14$		
$y_1 = x + 620$ $\quad y_2 = 648 - x$	$x = 14$		
$y_1 = x + 325$ $\quad y_2 = 353 - x$	$x = 14$		
$y_1 = x + 342$ $\quad y_2 = 370 - x$	$x = 14$		
$y_1 = x + 205$ $\quad y_2 = 233 - x$	$x = 14$		
$y_1 = x + 279$ $\quad y_2 = 307 - x$	$x = 14$		
$y_1 = x + 20$ $\quad y_2 = 48 - x$	$x = 14$		
$y_1 = x + 59$ $\quad y_2 = 59 - x$	$x = 0$		

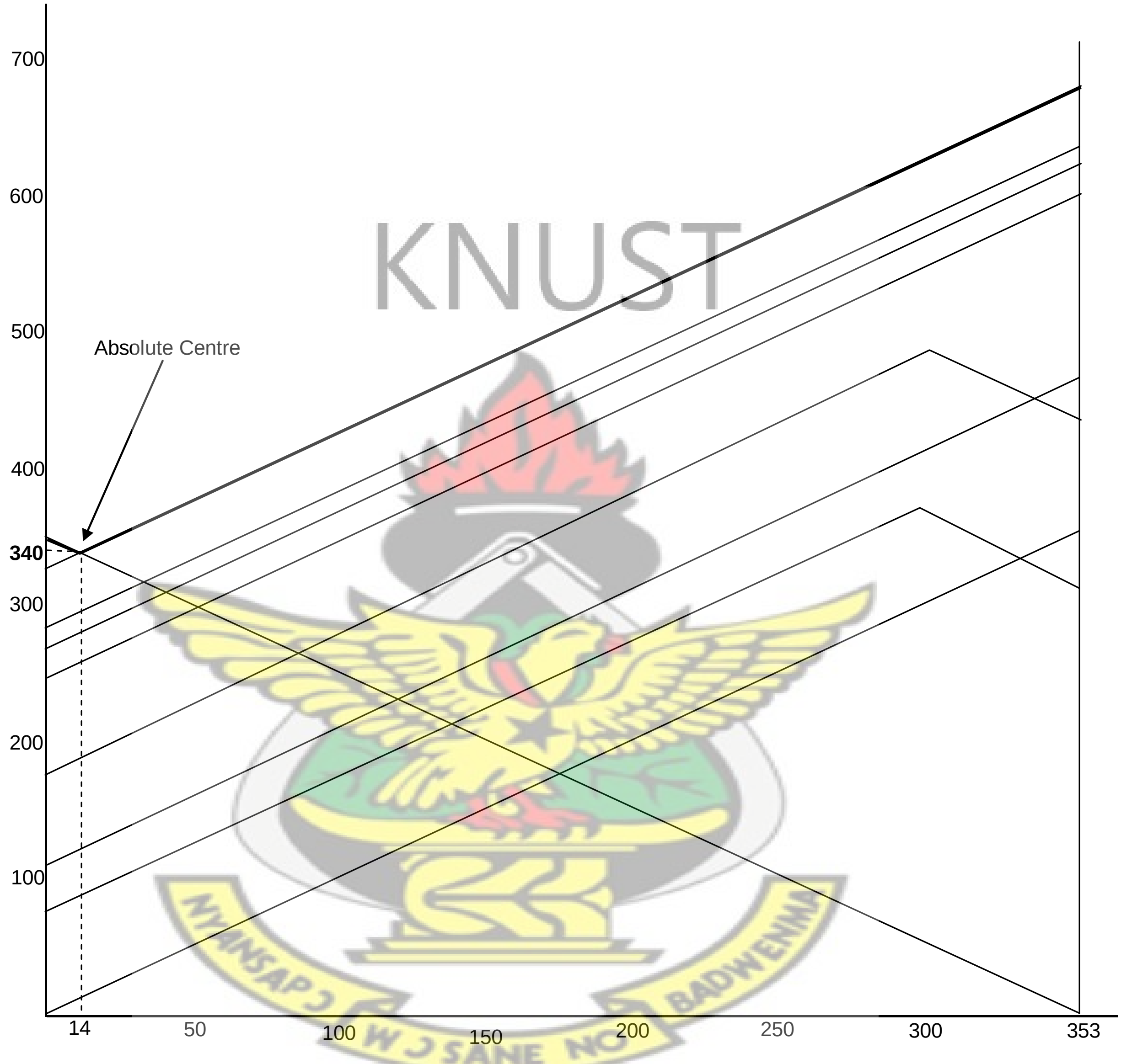
Table 4.17: Lines, kink points and local centres obtained on edge (H,I)

Lines	x_{kink}	x_l	$m(x_l)$
$y_1 = x$ $\quad y_2 = 45 - x$	$x = 0, x = 45$	0m	634m
$y_1 = x + 281$ $\quad y_2 = 371 - x$	$x = 45$		
$y_1 = x + 634$ $\quad y_2 = 724 - x$	$x = 45$		
$y_1 = x + 339$ $\quad y_2 = 429 - x$	$x = 45$		
$y_1 = x + 356$ $\quad y_2 = 446 - x$	$x = 45$		
$y_1 = x + 219$ $\quad y_2 = 309 - x$	$x = 45$		
$y_1 = x + 293$ $\quad y_2 = 383 - x$	$x = 45$		
$y_1 = x + 34$ $\quad y_2 = 124 - x$	$x = 45$		
$y_1 = x + 14$ $\quad y_2 = 104 - x$	$x = 45$		

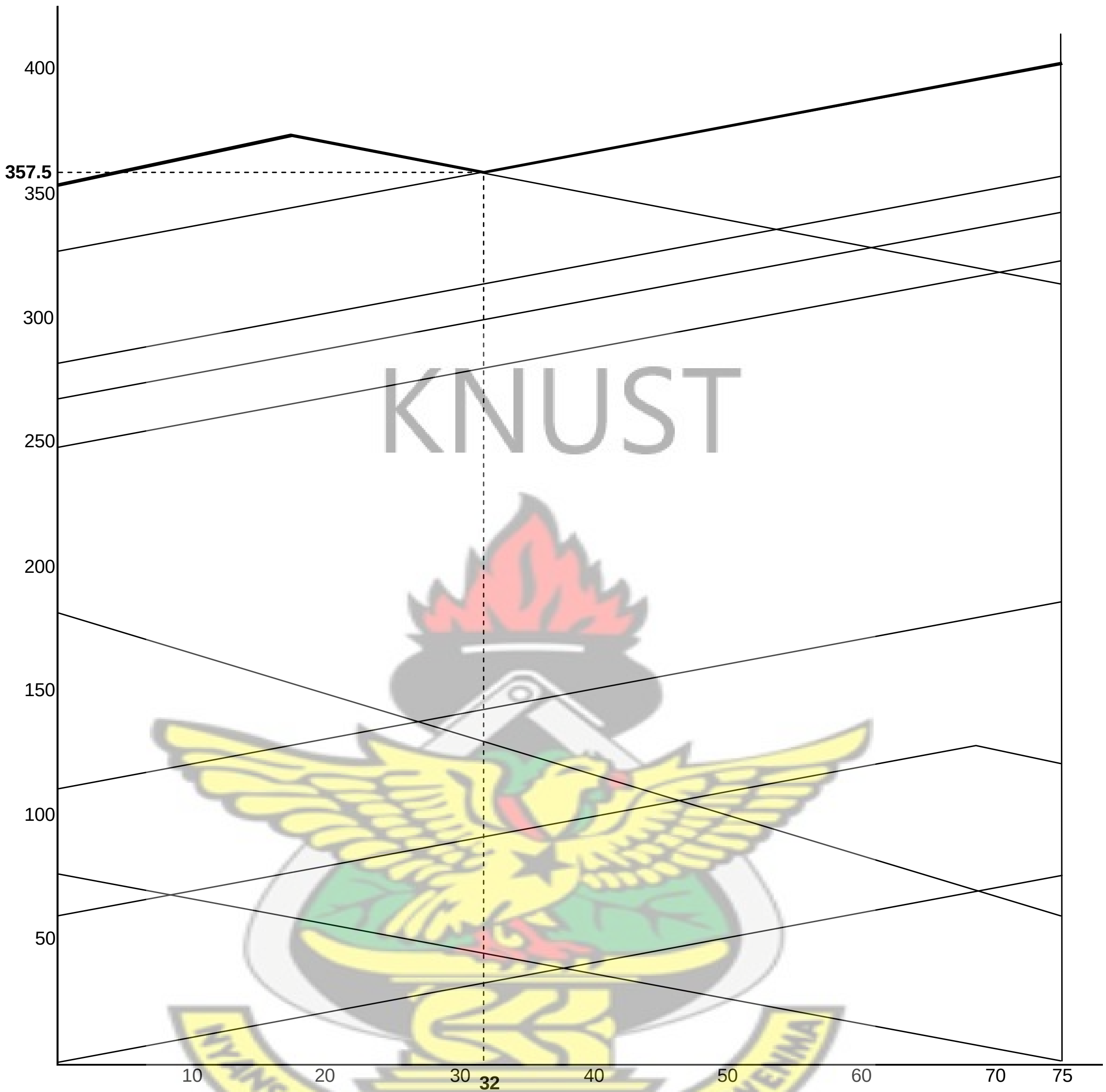
Table 4.18: Lines, kink points and local centres obtained on edge (I,J)

Appendix B

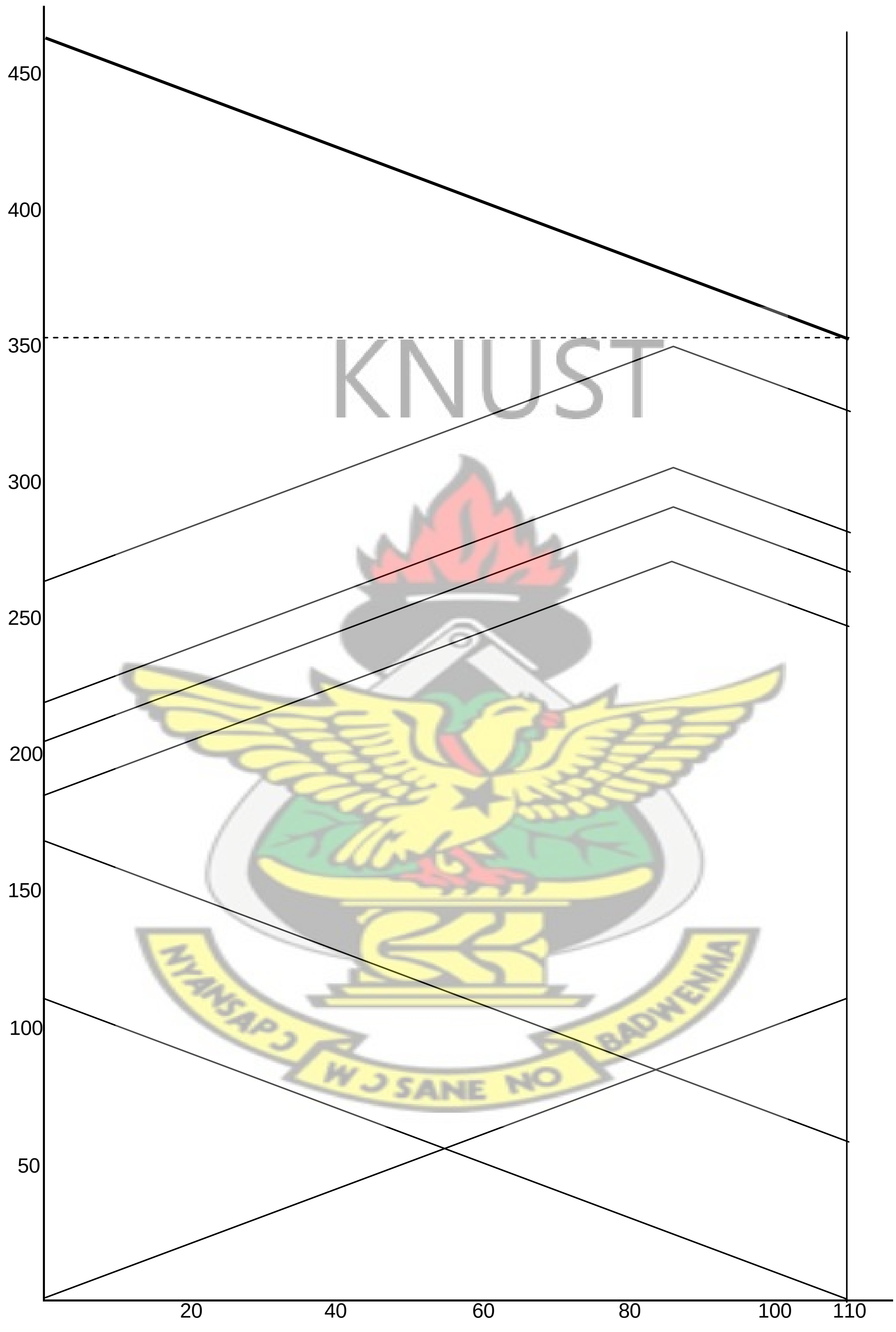
Graphs of remaining edges in chapter 4



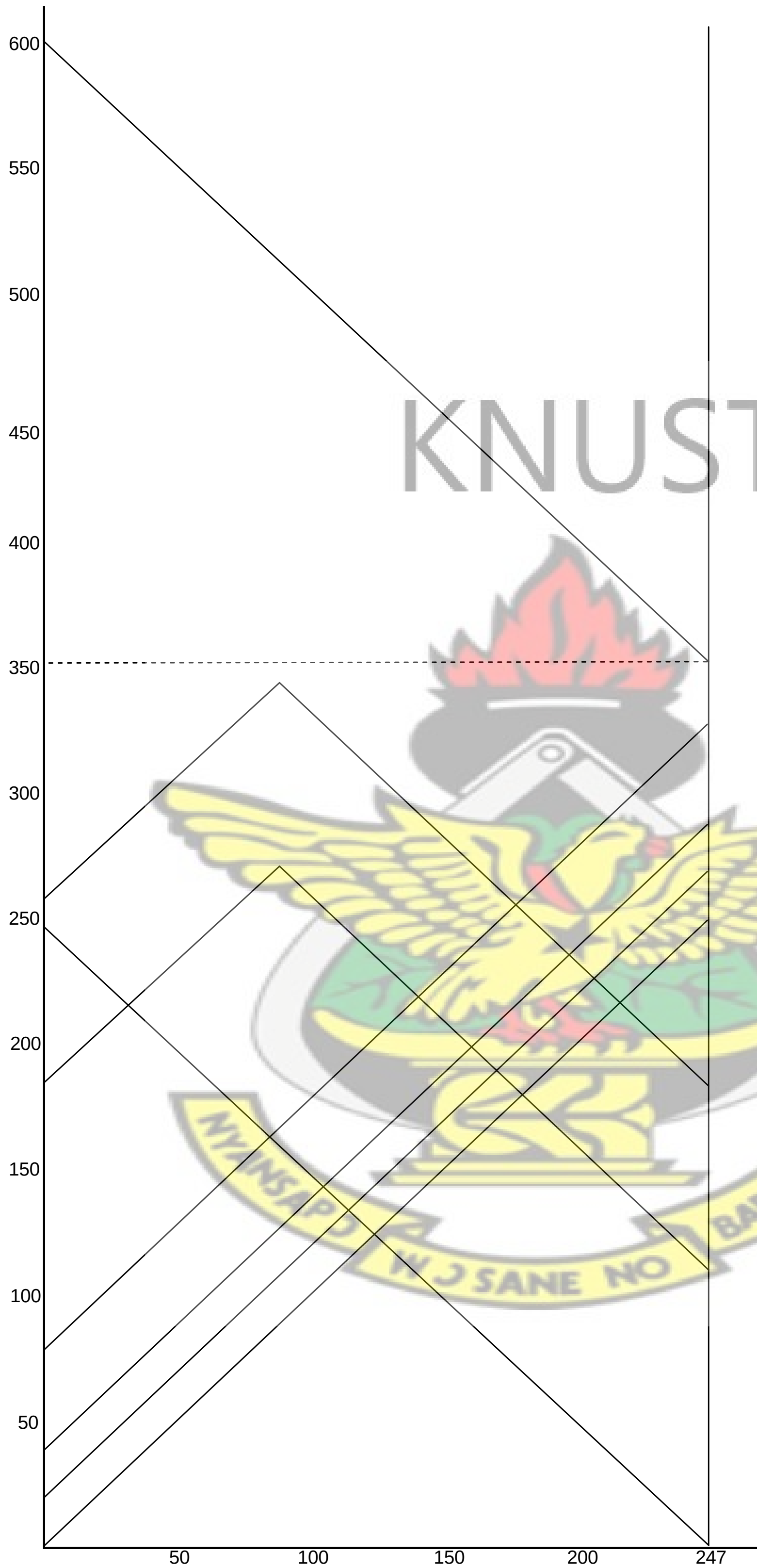
$X_{AB} = 14$ and $m(X_I) = 340$



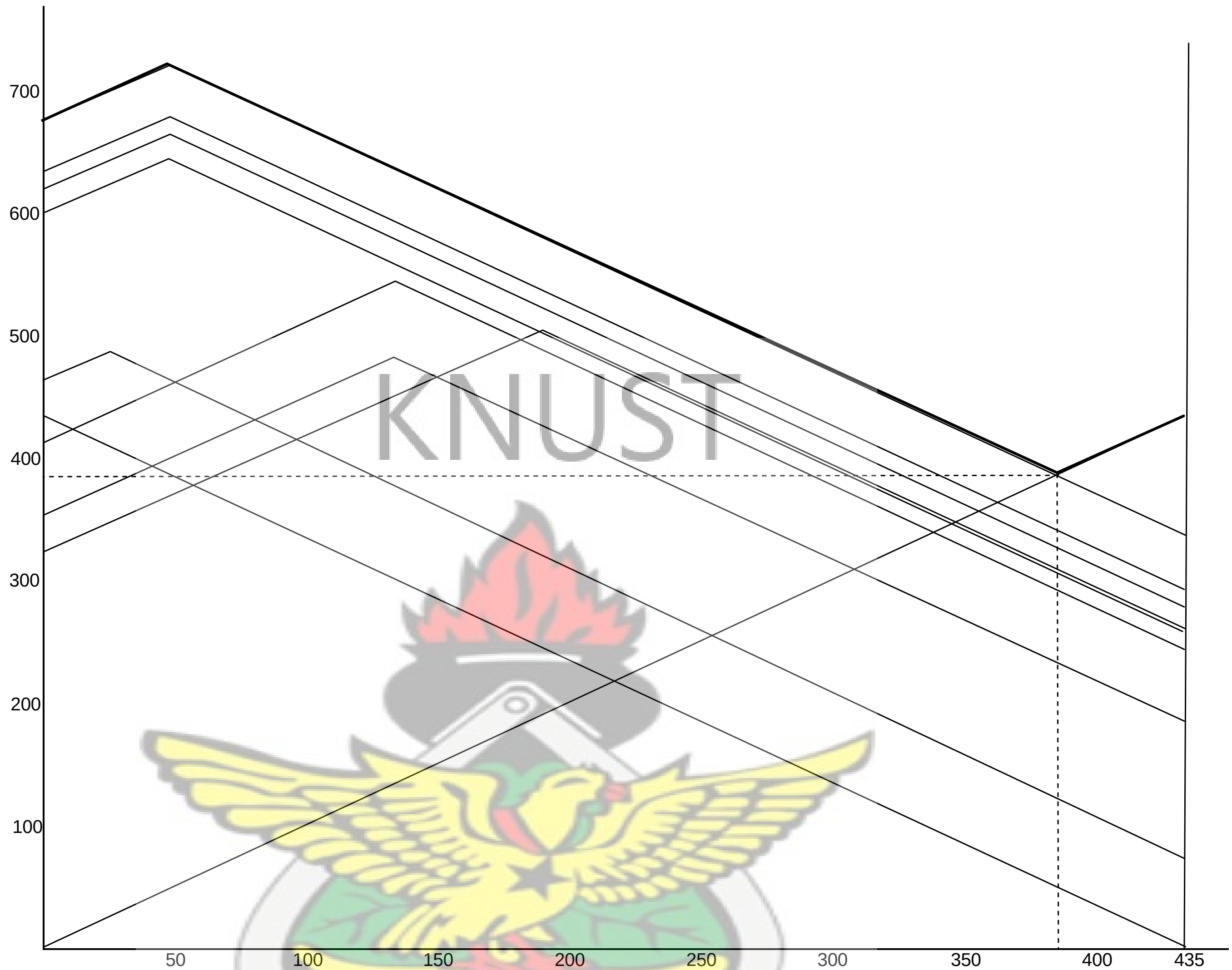
$X_{AD} = 32$ and $m(X_I) = 357.5$



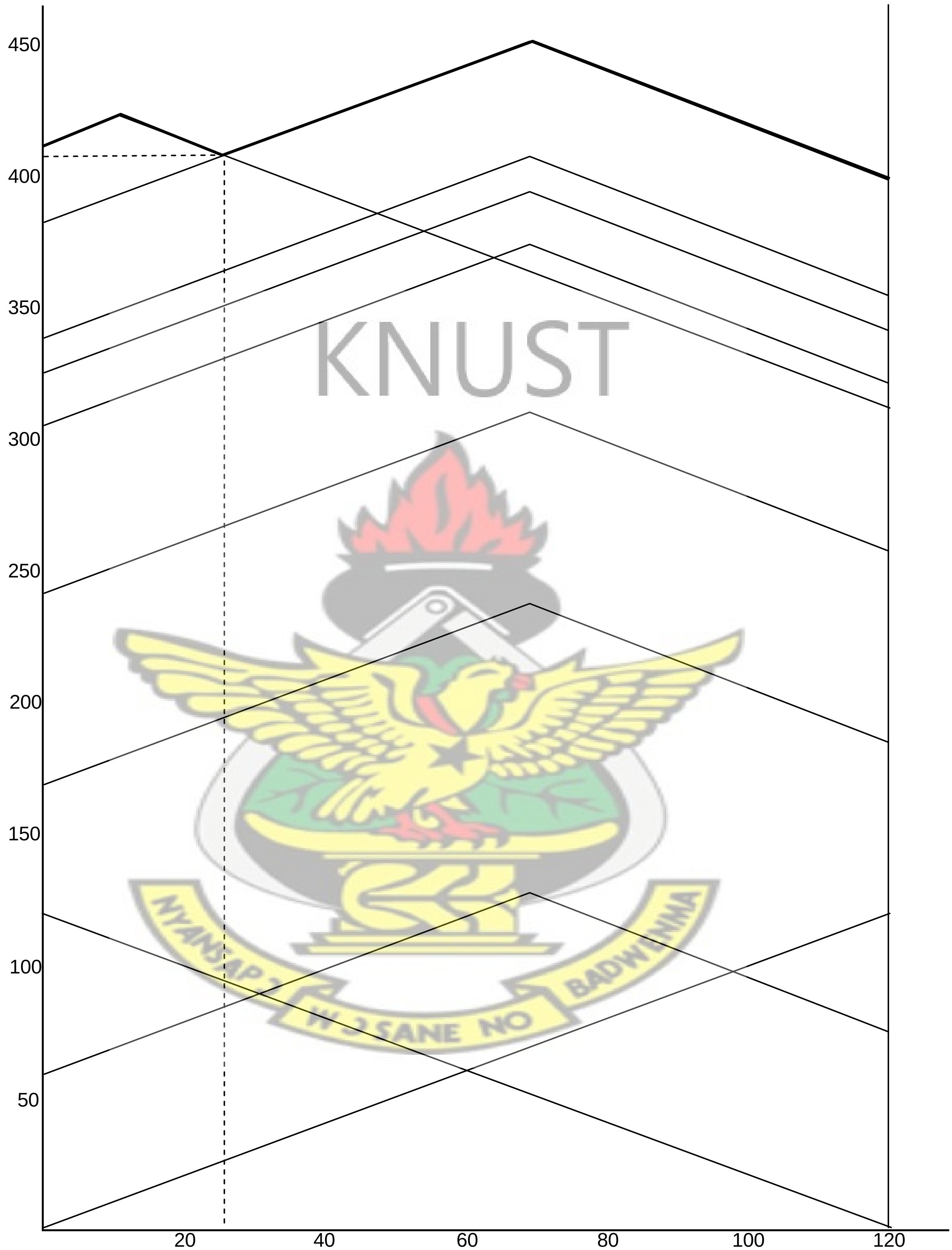
$X_{EA} = 110$ and $m(X_I) = 353$



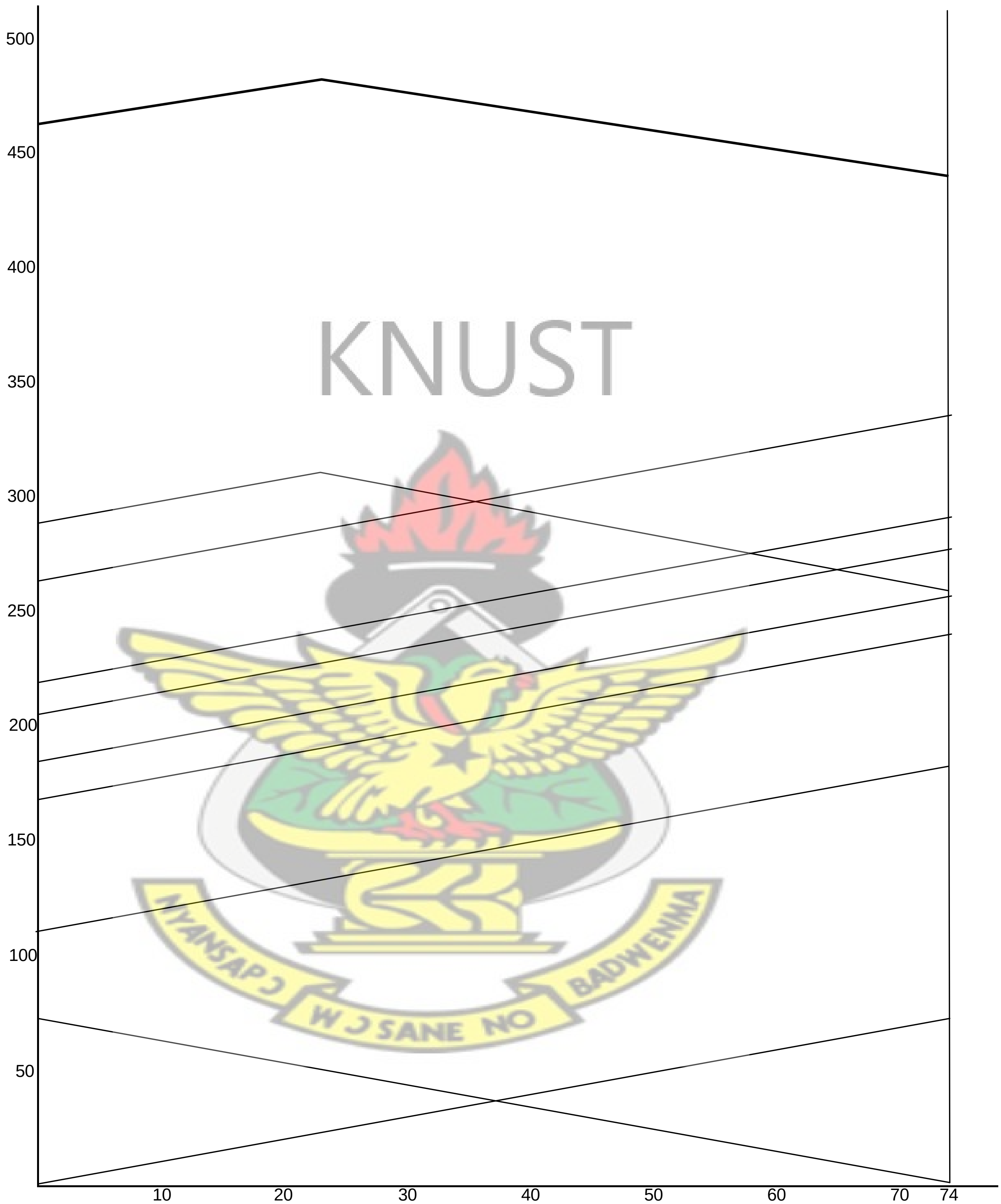
$X_{GA} = 247$ and $m(X_I) = 353$



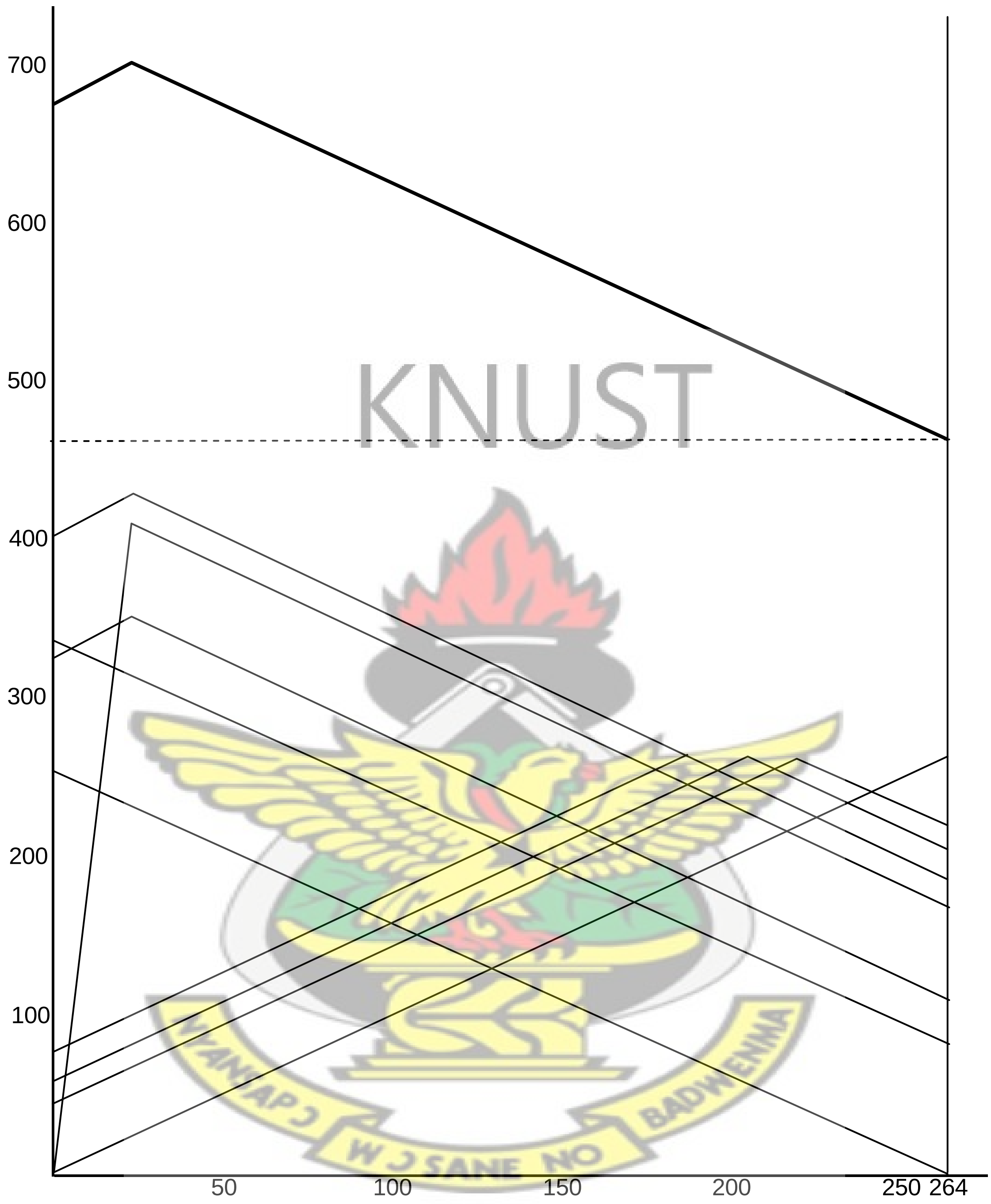
$X_{BF} = 391$ and $m(X_l) = 384$



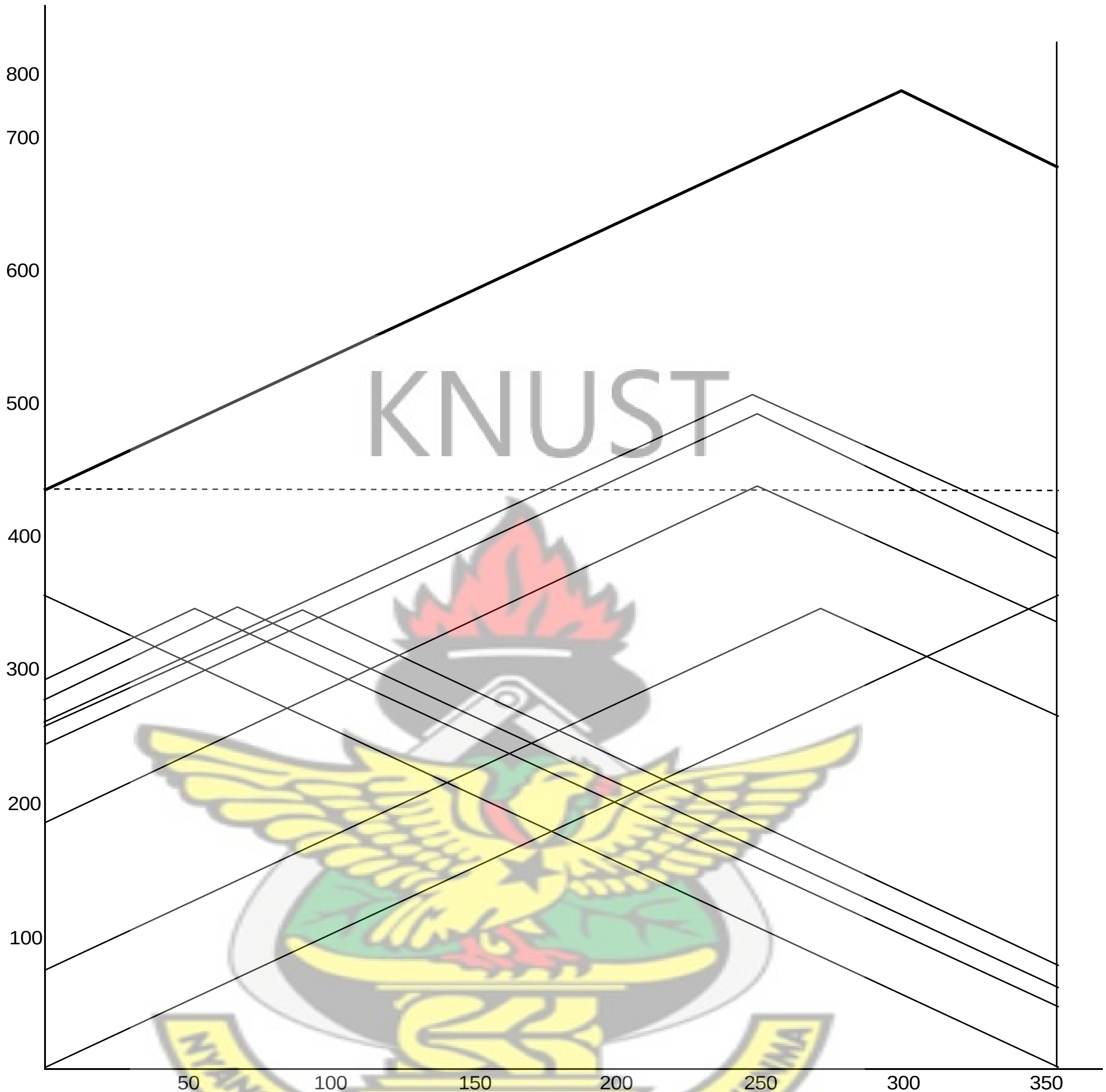
$X_{CD} = 24$ and $m(X_I) = 404.5$



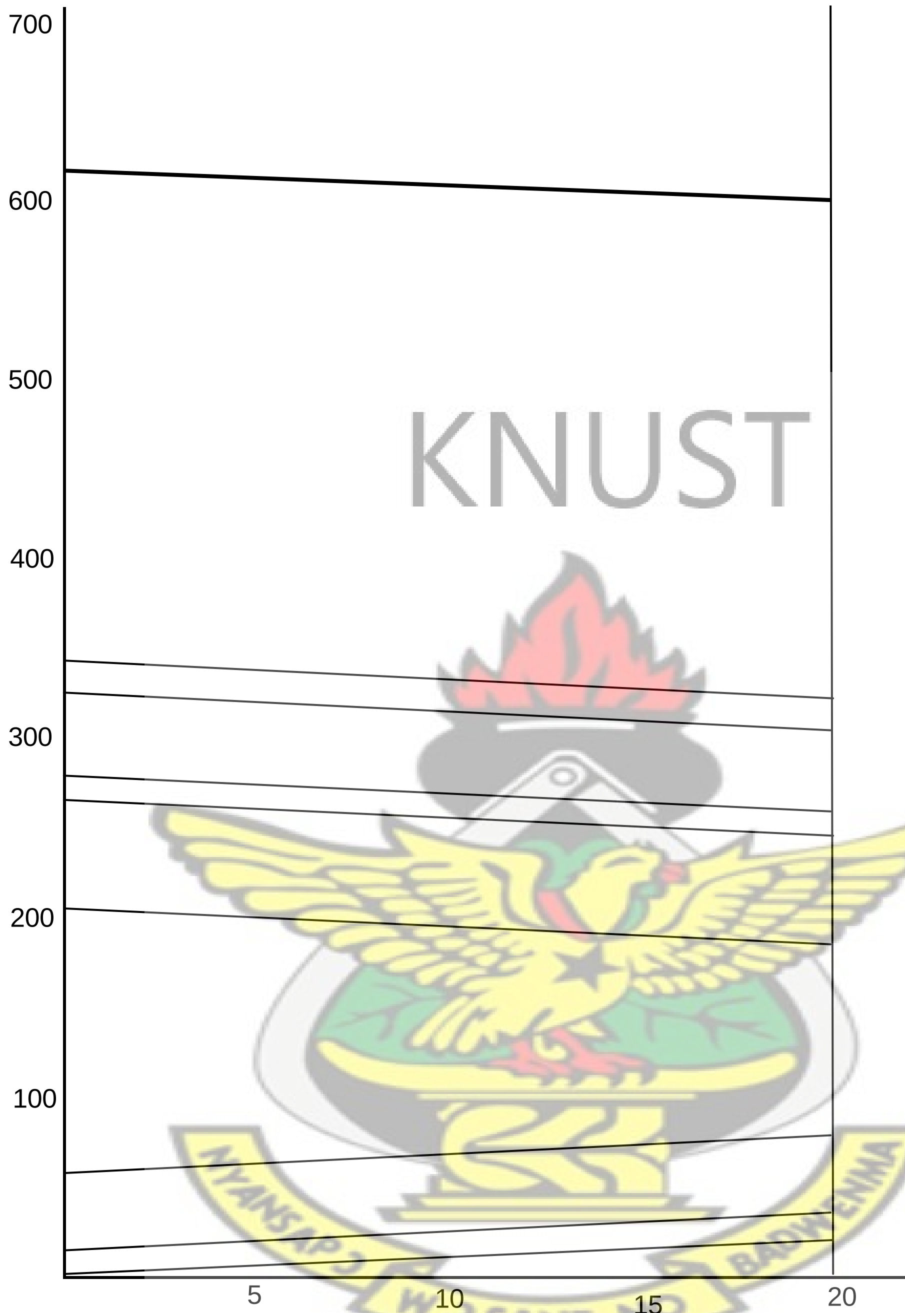
$X_{EF} = 74$ and $m(X_i) = 435$



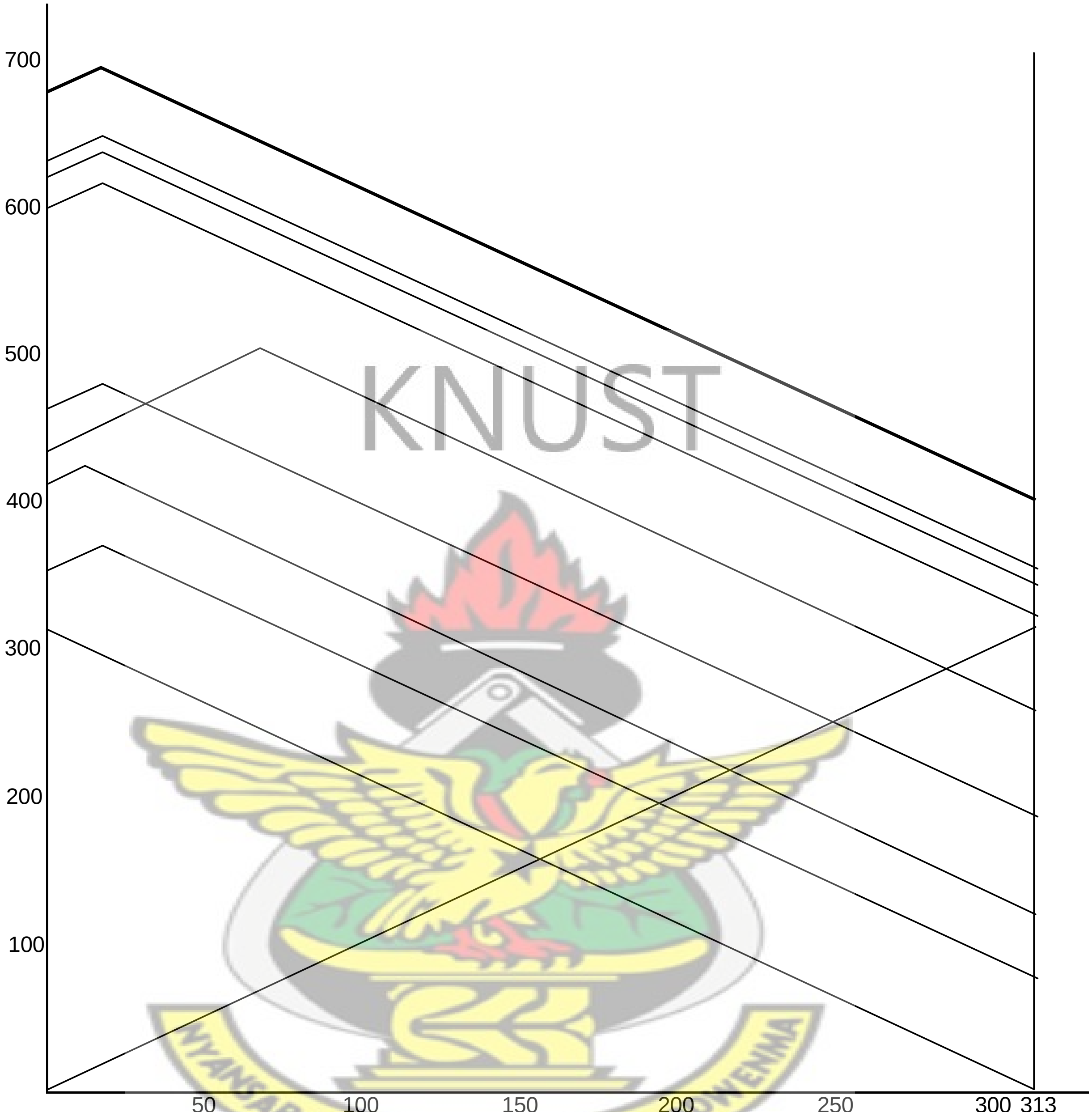
X_{JE} = 264 and $m(X_l)$ = 463



X_{FJ} = 355 and $m(X_j)$ = 435

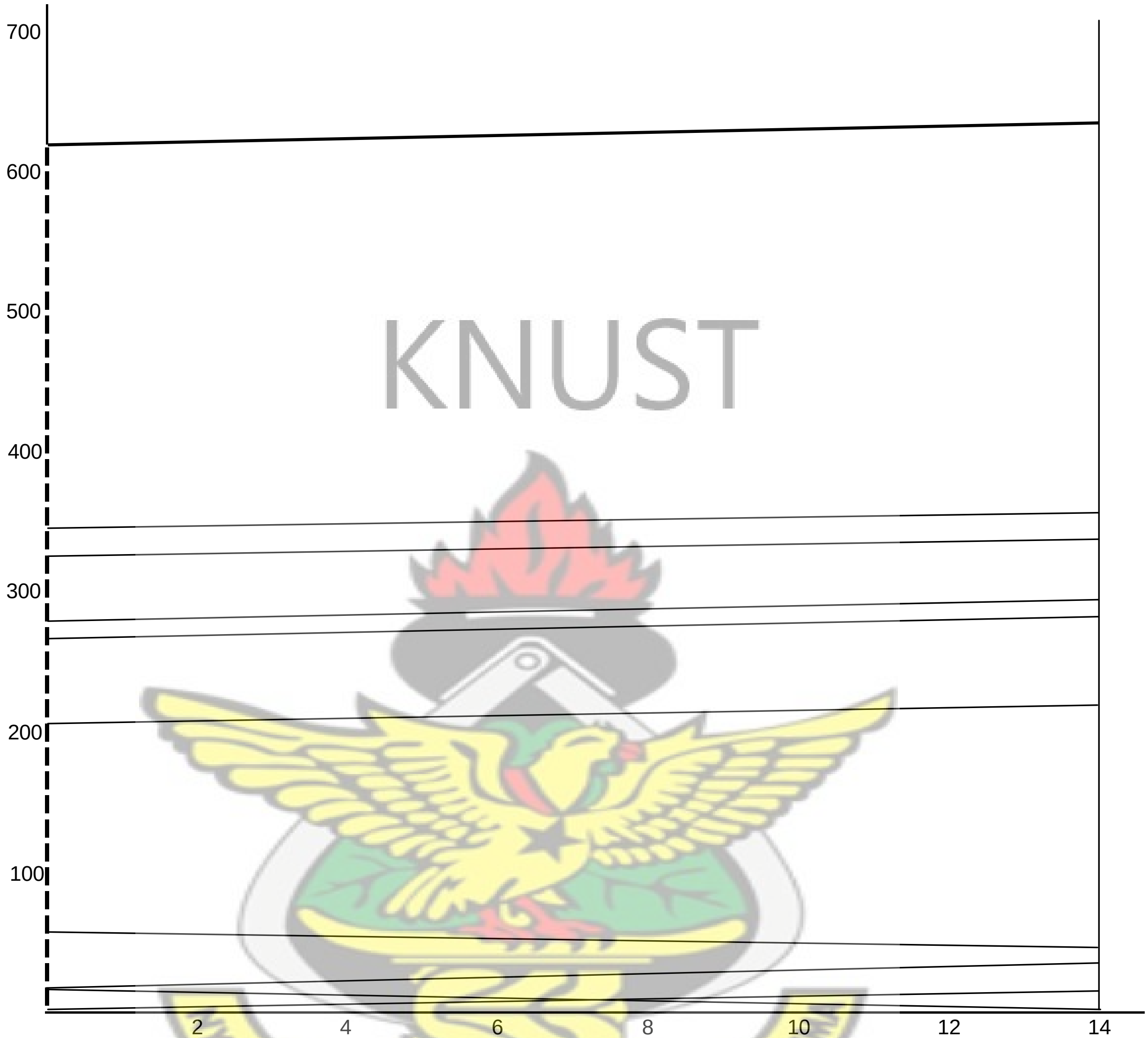


$X_{GH} = 0$ and $m(X_I) = 600$

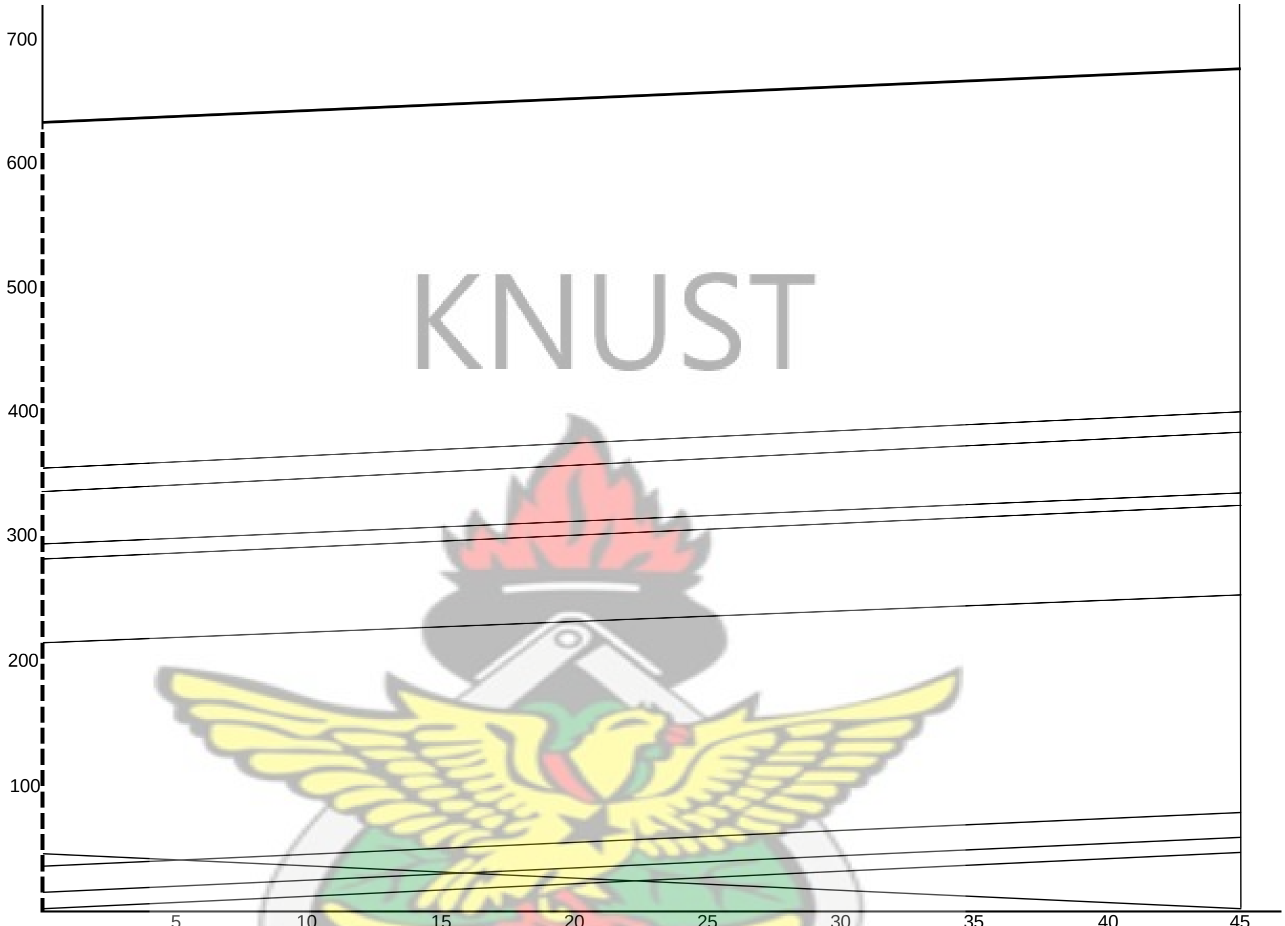


X_{BD} = 313 and $m(X_I)$ = 401

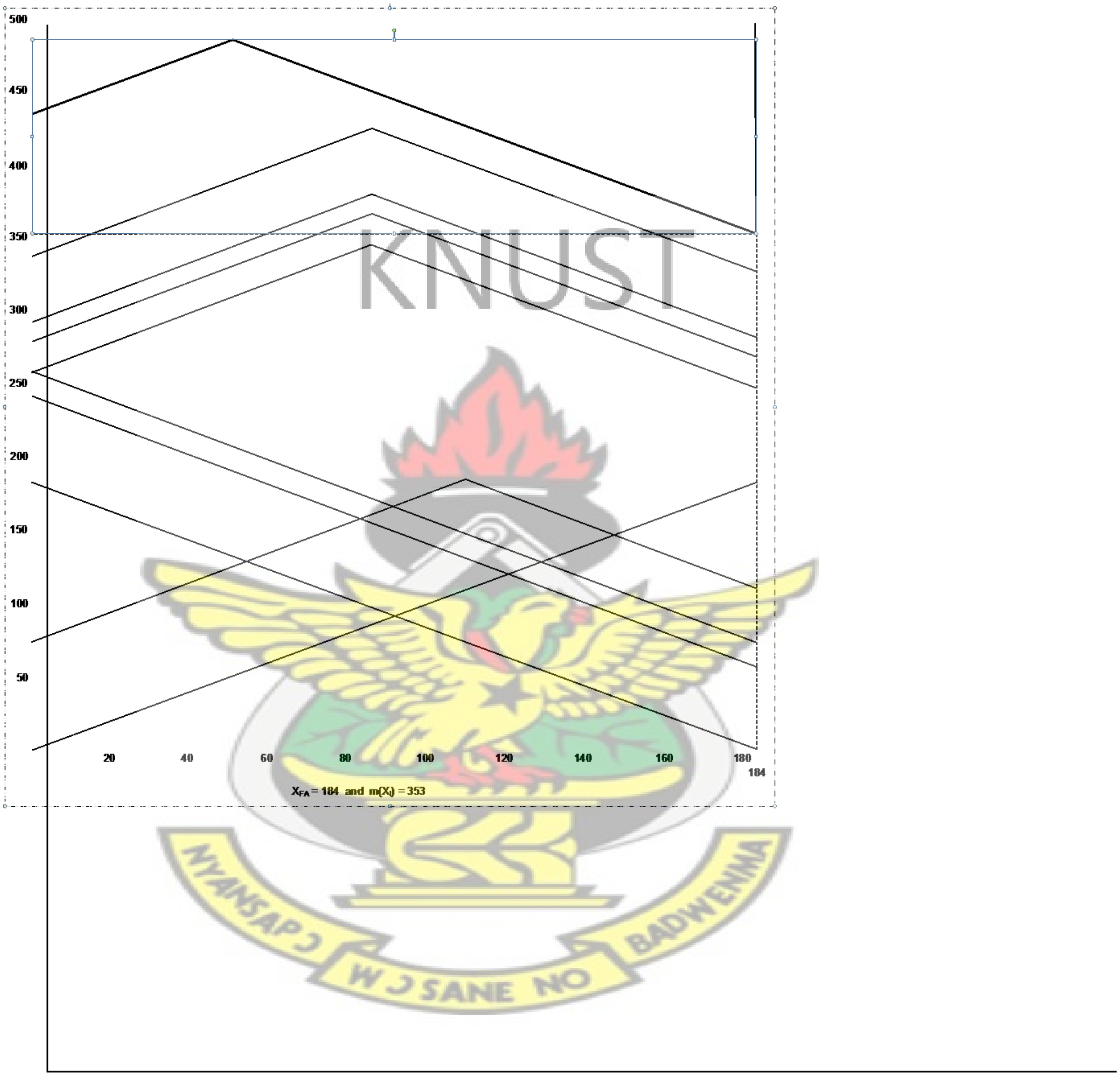
X_{GE} = 185 and m(X_l) = 463



$X_{HI} = 0$ and $m(X_I) = 620$



$X_{IJ} = 0$ and $m(X_j) = 634$


500
450
400
350
300
250
200
150
100
50
20
40
60
80
100
120
140
160
180
184
X_{FA} = 184 and $m(X_i)$ = 353